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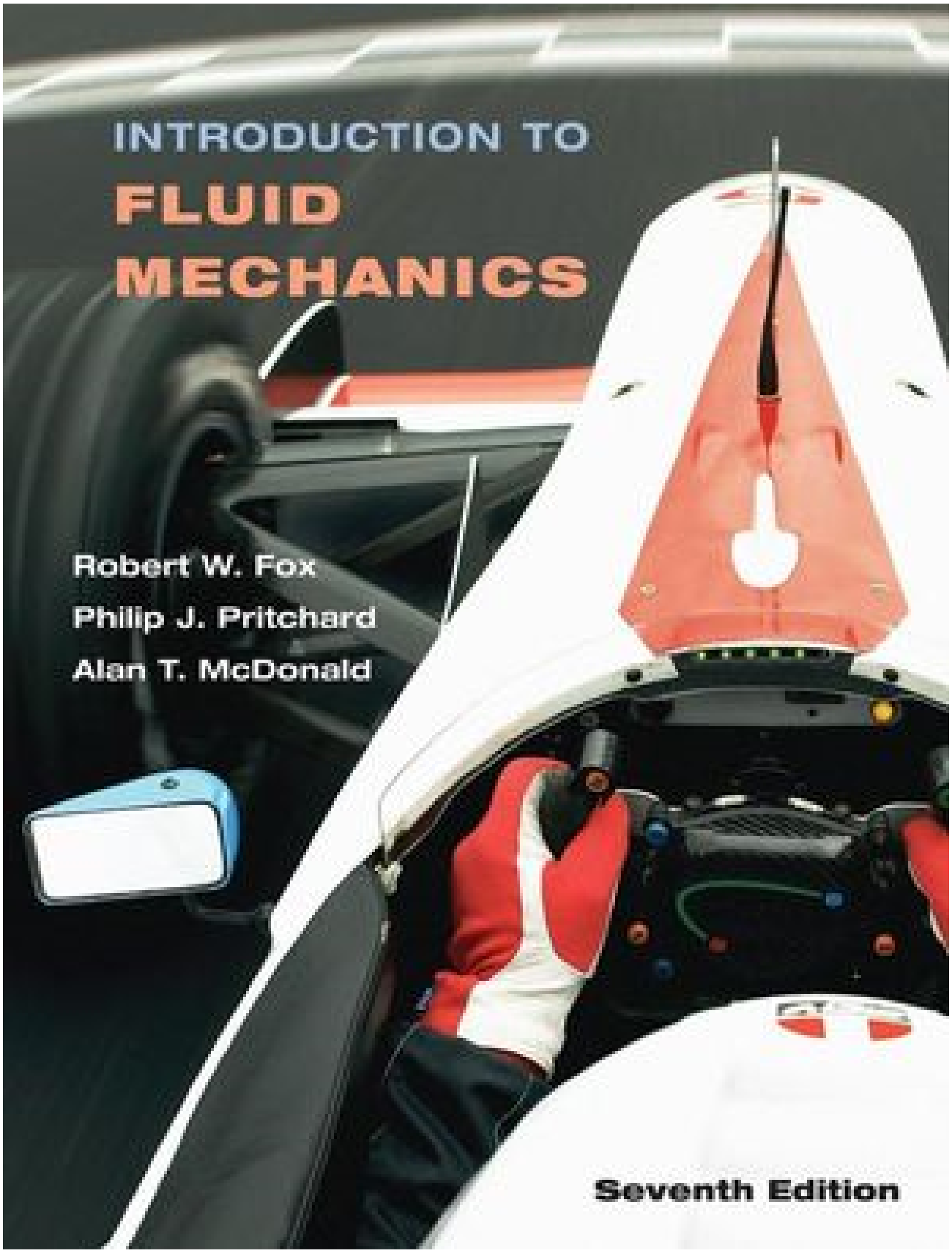
VISITANOS PARA DESARGALOS GRATIS.

SOLUTION MANUAL FOR



INTRODUCTION TO

FLUID MECHANICS

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Seventh Edition

Problem 1.1

[3]

1.1 A number of common substances are

Tar
‘‘Silly Putty’’
Modeling clay
Wax
Sand
Jello
Toothpaste
Shaving cream

Some of these materials exhibit characteristics of both solid and fluid behavior under different conditions. Explain and give examples.

Given: Common Substances

Tar
“Silly Putty”
Modeling clay
Wax
Sand
Jello
Toothpaste
Shaving cream

Some of these substances exhibit characteristics of solids and fluids under different conditions.

Find: Explain and give examples.

Solution: Tar, Wax, and Jello behave as solids at room temperature or below at ordinary pressures. At high pressures or over long periods, they exhibit fluid characteristics. At higher temperatures, all three liquefy and become viscous fluids.

Modeling clay and silly putty show fluid behavior when sheared slowly. However, they fracture under suddenly applied stress, which is a characteristic of solids.

Toothpaste behaves as a solid when at rest in the tube. When the tube is squeezed hard, toothpaste “flows” out the spout, showing fluid behavior. Shaving cream behaves similarly.

Sand acts solid when in repose (a sand “pile”). However, it “flows” from a spout or down a steep incline.

Problem 1.2

1.2 Give a word statement of each of the five basic conservation laws stated in Section 1-4, as they apply to a system.

Given: Five basic conservation laws stated in Section 1-4.

Write: A word statement of each, as they apply to a system.

Solution: Assume that laws are to be written for a *system*.

a. Conservation of mass — The mass of a system is constant by definition.

b. Newton's second law of motion — The net force acting on a system is directly proportional to the product of the system mass times its acceleration.

c. First law of thermodynamics — The change in stored energy of a system equals the net energy added to the system as heat and work.

d. Second law of thermodynamics — The entropy of any isolated system cannot decrease during any process between equilibrium states.

e. Principle of angular momentum — The net torque acting on a system is equal to the rate of change of angular momentum of the system.

Problem 1.3

1.3 Discuss the physics of skipping a stone across the water surface of a lake. Compare these mechanisms with a stone as it bounces after being thrown along a roadway.

Open-Ended Problem Statement: Consider the physics of "skipping" a stone across the water surface of a lake. Compare these mechanisms with a stone as it bounces after being thrown along a roadway.

Discussion: Observation and experience suggest two behaviors when a stone is thrown along a water surface:

1. If the angle between the path of the stone and the water surface is steep the stone may penetrate the water surface. Some momentum of the stone will be converted to momentum of the water in the resulting splash. After penetrating the water surface, the high drag* of the water will slow the stone quickly. Then, because the stone is heavier than water it will sink.
2. If the angle between the path of the stone and the water surface is shallow the stone may not penetrate the water surface. The splash will be smaller than if the stone penetrated the water surface. This will transfer less momentum to the water, causing less reduction in speed of the stone. The only drag force on the stone will be from friction on the water surface. The drag will be momentary, causing the stone to lose only a portion of its kinetic energy. Instead of sinking, the stone may skip off the surface and become airborne again.

When the stone is thrown with speed and angle just right, it may skip several times across the water surface. With each skip the stone loses some forward speed. After several skips the stone loses enough forward speed to penetrate the surface and sink into the water.

Observation suggests that the shape of the stone significantly affects skipping. Essentially spherical stones may be made to skip with considerable effort and skill from the thrower. Flatter, more disc-shaped stones are more likely to skip, provided they are thrown with the flat surface(s) essentially parallel to the water surface; spin may be used to stabilize the stone in flight.

By contrast, no stone can ever penetrate the pavement of a roadway. Each collision between stone and roadway will be inelastic; friction between the road surface and stone will affect the motion of the stone only slightly. Regardless of the initial angle between the path of the stone and the surface of the roadway, the stone may bounce several times, then finally it will roll to a stop.

The shape of the stone is unlikely to affect trajectory of bouncing from a roadway significantly.

Problem 1.4

1.4 The barrel of a bicycle tire pump becomes quite warm during use. Explain the mechanisms responsible for the temperature increase.

Open-Ended Problem Statement: The barrel of a bicycle tire pump becomes quite warm during use. Explain the mechanisms responsible for the temperature increase.

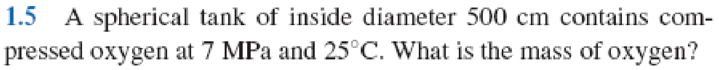
Discussion: Two phenomena are responsible for the temperature increase: (1) friction between the pump piston and barrel and (2) temperature rise of the air as it is compressed in the pump barrel.

Friction between the pump piston and barrel converts mechanical energy (force on the piston moving through a distance) into thermal energy as a result of friction. Lubricating the piston helps to provide a good seal with the pump barrel and reduces friction (and therefore force) between the piston and barrel.

Temperature of the trapped air rises as it is compressed. The compression is not adiabatic because it occurs during a finite time interval. Heat is transferred from the warm compressed air in the pump barrel to the cooler surroundings. This raises the temperature of the barrel, making its outside surface warm (or even hot!) to the touch.

Problem 1.5

[1]



1.5 A spherical tank of inside diameter 500 cm contains compressed oxygen at 7 MPa and 25°C. What is the mass of oxygen?

Given: Data on oxygen tank.

Find: Mass of oxygen.

Solution: Compute tank volume, and then use oxygen density (Table A.6) to find the mass.

The given or available data is: $D = 500 \cdot \text{cm}$ $\quad p = 7 \cdot \text{MPa}$ $\quad T = (25 + 273) \cdot \text{K}$ $\quad T = 298\,\text{K}$

$$R_{O2} = 259.8 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}} \qquad \text{(Table A.6)}$$

The governing equation is the ideal gas equation

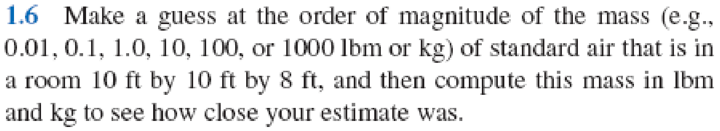
$$p = \rho \cdot R_{O2} \cdot T \qquad \text{and} \quad \rho = \frac{M}{V}$$

where *V* is the tank volume $\quad V = \frac{\pi \cdot D^3}{6} \qquad V = \frac{\pi}{6} \times (5 \cdot \text{m})^3 \qquad V = 65.4 \cdot \text{m}^3$

Hence $\quad M = V \cdot \rho = \frac{p \cdot V}{R_{O2} \cdot T} \qquad M = 7 \times 10^6 \cdot \frac{\text{N}}{\text{m}^2} \times 65.4 \cdot \text{m}^3 \times \frac{1}{259.8} \cdot \frac{\text{kg} \cdot \text{K}}{\text{N} \cdot \text{m}} \times \frac{1}{298} \cdot \frac{1}{\text{K}} \qquad M = 5913\,\text{kg}$

Problem 1.6

[1]



1.6 Make a guess at the order of magnitude of the mass (e.g., 0.01, 0.1, 1.0, 10, 100, or 1000 lbm or kg) of standard air that is in a room 10 ft by 10 ft by 8 ft, and then compute this mass in lbm and kg to see how close your estimate was.

Given: Dimensions of a room

Find: Mass of air

Solution:

Basic equation: $\rho = \frac{p}{R_{air} \cdot T}$

Given or available data

$p = 14.7\text{psi}$ $\quad T = (59 + 460)\text{R}$ $\quad R_{air} = 53.33 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot \text{R}}$

$V = 10 \cdot \text{ft} \times 10 \cdot \text{ft} \times 8 \cdot \text{ft}$ $\quad V = 800\,\text{ft}^3$

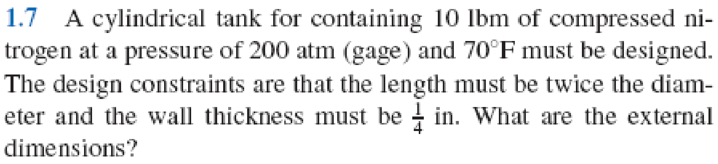
Then

$\rho = \frac{p}{R_{air} \cdot T}$ $\quad \rho = 0.076 \frac{\text{lbm}}{\text{ft}^3}$ $\quad \rho = 0.00238 \frac{\text{slug}}{\text{ft}^3}$ $\quad \rho = 1.23 \frac{\text{kg}}{\text{m}^3}$

$M = \rho \cdot V$ $\quad M = 61.2\,\text{lbm}$ $\quad M = 1.90\,\text{slug}$ $\quad M = 27.8\,\text{kg}$

Problem 1.7

[2]


1.7 A cylindrical tank for containing 10 lbm of compressed nitrogen at a pressure of 200 atm (gage) and 70°F must be designed. The design constraints are that the length must be twice the diameter and the wall thickness must be $\frac{1}{4}$ in. What are the external dimensions?

Given: Mass of nitrogen, and design constraints on tank dimensions.

Find: External dimensions.

Solution: Use given geometric data and nitrogen mass, with data from Table A.6.

The given or available data is: $M = 10 \cdot \text{lbm}$ $\quad p = (200+1)\cdot\text{atm}$ $\quad p = 2.95\times 10^{3}\cdot\text{psi}$

$T = (70+460)\cdot\text{K}$ $\quad T = 954\cdot\text{R}$ $\quad R_{N2} = 55.16\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}}$ (Table A.6)

The governing equation is the ideal gas equation $\quad p = \rho\cdot R_{N2}\cdot T$ and $\rho = \frac{M}{V}$

where *V* is the tank volume $\quad V = \frac{\pi\cdot D^2}{4}\cdot L$ $\quad$ where $\quad L = 2\cdot D$

Combining these equations:

Hence

$$M = V\cdot\rho = \frac{p\cdot V}{R_{N2}\cdot T} = \frac{p}{R_{N2}\cdot T}\cdot\frac{\pi\cdot D^2}{4}\cdot L = \frac{p}{R_{N2}\cdot T}\cdot\frac{\pi\cdot D^2}{4}\cdot 2\cdot D = \frac{p\cdot\pi\cdot D^3}{2\cdot R_{N2}\cdot T}$$

Solving for D

$$D = \left(\frac{2\cdot R_{N2}\cdot T\cdot M}{p\cdot\pi}\right)^{\frac{1}{3}} \qquad D = \left[\frac{2}{\pi}\times 55.16\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}}\times 954\cdot\text{K}\times 10\cdot\text{lbm}\times\frac{1}{2950}\cdot\frac{\text{in}^2}{\text{lbf}}\times\left(\frac{\text{ft}}{12\cdot\text{in}}\right)^2\right]^{\frac{1}{3}}$$

$D = 1.12\cdot\text{ft}$ $\quad D = 13.5\cdot\text{in}$ $\quad L = 2\cdot D$ $\quad L = 27\cdot\text{in}$

These are internal dimensions; the external ones are 1/4 in. larger: $L = 27.25\cdot\text{in}$ $\quad D = 13.75\cdot\text{in}$

Problem 1.8

[3]

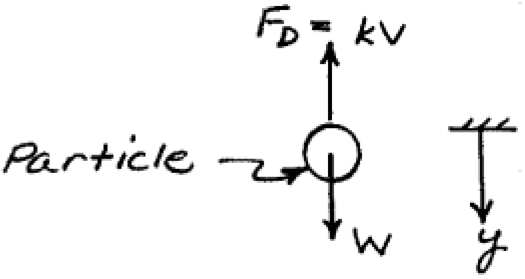
1.8 Very small particles moving in fluids are known to experience a drag force proportional to speed. Consider a particle of net weight *W* dropped in a fluid. The particle experiences a drag force, $F_D = kV$, where V is the particle speed. Determine the time required for the particle to accelerate from rest to 95 percent of its terminal speed, V_t, in terms of k, W, and g.

Given: Small particle accelerating from rest in a fluid. Net weight is W, resisting force $F_D = kV$, where V is speed.

Find: Time required to reach 95 percent of terminal speed, V_t.

Solution: Consider the particle to be a system. Apply Newton's second law.

Basic equation: $\sum F_y = ma_y$



Assumptions:

1. W is net weight
2. Resisting force acts opposite to V

Then $$\sum F_y = W - kV = ma_y = m\frac{dV}{dt} = \frac{W}{g}\frac{dV}{dt}$$

or $$\frac{dV}{dt} = g(1 - \frac{k}{W}V)$$

Separating variables, $$\frac{dV}{1 - \frac{k}{W}V} = g\,dt$$

Integrating, noting that velocity is zero initially, $$\int_0^V \frac{dV}{1 - \frac{k}{W}V} = -\frac{W}{k}\ln(1 - \frac{k}{W}V)\Bigg]_0^V = \int_0^t g dt = gt$$

or $$1-\frac{k}{W}V = e^{-\frac{kgt}{W}};\quad V = \frac{W}{k}\left[1-e^{-\frac{kgt}{W}}\right]$$

But $V \to V_t$ as $t \to \infty$, so $V_t = \frac{W}{k}$. Therefore $$\frac{V}{V_t} = 1-e^{-\frac{kgt}{W}}$$

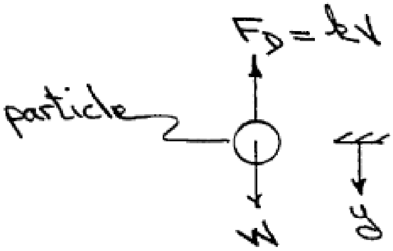
When $\frac{V}{V_t} = 0.95$, then $e^{-\frac{kgt}{W}} = 0.05$ and $\frac{kgt}{W} = 3$. Thus t = 3 W/gk

Problem 1.9

[2]

1.9 Consider again the small particle of Problem 1.8. Express the distance required to reach 95 percent of its terminal speed in terms of *g*, *k*, and *W*.

Given: Small particle accelerating from rest in a fluid. Net weight is W, resisting force is F_D = kV, where V is speed.



Find: Distance required to reach 95 percent of terminal speed, V_t.

Solution: Consider the particle to be a system. Apply Newton's second law.

Basic equation: $\sum F_y = ma_y$

Assumptions:

1. W is net weight.
2. Resisting force acts opposite to V.

Then, $\sum F_y = W - kV = ma_y = m\frac{dV}{dt} = \frac{W}{g}V\frac{dV}{dy}$ or $1 - \frac{k}{W}V = \frac{V}{g}\frac{dV}{dy}$

At terminal speed, $a_y = 0$ and $V = V_t = \frac{W}{k}$. Then $1 - \frac{V}{V_g} = \frac{1}{g}V\frac{dV}{dy}$

Separating variables $\frac{V\,dV}{1 - \frac{1}{V_t}V} = g\,dy$

Integrating, noting that velocity is zero initially

$$gy = \int_0^{0.95V_t} \frac{V\,dV}{1 - \frac{1}{V_t}V} = \left[-VV_t - V_t^2 \ln\left(1 - \frac{V}{V_t}\right)\right]_0^{0.95V_t}$$

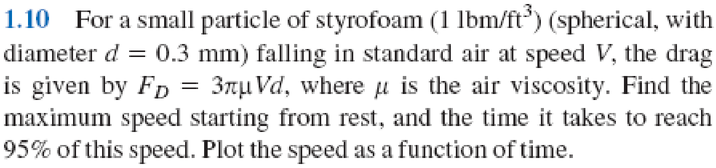
$$gy = -0.95V_t^2 - V_t^2 \ln(1 - 0.95) - V_t^2 \ln(1)$$

$$gy = -V_t^2\left[0.95 + \ln 0.05\right] = 2.05\,V_t^2$$

$$\therefore y = \frac{2.05}{g}V_t^2 = 2.05\frac{W^2}{gt^2}$$

Problem 1.10

[3]



1.10 For a small particle of styrofoam (1 lbm/ft^3) (spherical, with diameter $d = 0.3$ mm) falling in standard air at speed V, the drag is given by $F_D = 3\pi\mu V d$, where μ is the air viscosity. Find the maximum speed starting from rest, and the time it takes to reach 95% of this speed. Plot the speed as a function of time.

Given: Data on sphere and formula for drag.

Find: Maximum speed, time to reach 95% of this speed, and plot speed as a function of time.

Solution: Use given data and data in Appendices, and integrate equation of motion by separating variables.

The data provided, or available in the Appendices, are:

$$\rho_{air} = 1.17\cdot\frac{kg}{m^3} \qquad \mu = 1.8\times 10^{-5}\cdot\frac{N\cdot s}{m^2} \qquad \rho_w = 999\cdot\frac{kg}{m^3} \qquad SG_{Sty} = 0.016 \qquad d = 0.3\cdot mm$$

Then the density of the sphere is $\rho_{Sty} = SG_{Sty}\cdot\rho_w$ $\qquad \rho_{Sty} = 16\,\frac{kg}{m^3}$

The sphere mass is $M = \rho_{Sty}\cdot\frac{\pi\cdot d^3}{6} = 16\cdot\frac{kg}{m^3}\times\pi\times\frac{(0.0003\cdot m)^3}{6}$ $\qquad M = 2.26\times 10^{-10}\,kg$

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $\qquad M\cdot g = 3\cdot\pi\cdot V\cdot d$

so

$$V_{max} = \frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d} = \frac{1}{3\cdot\pi}\times 2.26\times 10^{-10}\cdot kg\times 9.81\cdot\frac{m}{s^2}\times\frac{m^2}{1.8\times 10^{-5}\cdot N\cdot s}\times\frac{1}{0.0003\cdot m} \qquad V_{max} = 0.0435\,\frac{m}{s}$$

Newton's 2nd law for the general motion is (ignoring buoyancy effects) $\qquad M\cdot\frac{dV}{dt} = M\cdot g - 3\cdot\pi\cdot\mu\cdot V\cdot d$

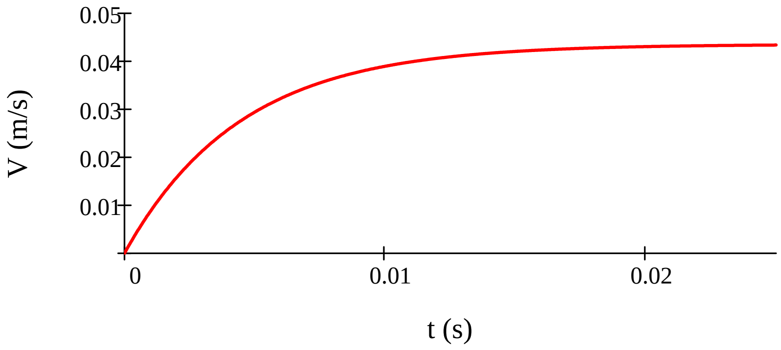
so

$$\frac{dV}{g - \frac{3\cdot\pi\cdot\mu\cdot d}{M}\cdot V} = dt$$

Integrating and using limits

$$V(t) = \frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d}\cdot\left(1 - e^{\frac{-3\cdot\pi\cdot\mu\cdot d}{M}\cdot t}\right)$$

Using the given data



The time to reach 95% of maximum speed is obtained from $$\frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d}\cdot\left(1-e^{\frac{-3\cdot\pi\cdot\mu\cdot d}{M}\cdot t}\right)=0.95\cdot V_{max}$$

so $$t=-\frac{M}{3\cdot\pi\cdot\mu\cdot d}\cdot\ln\left(1-\frac{0.95\cdot V_{max}\cdot 3\cdot\pi\cdot\mu\cdot d}{M\cdot g}\right)$$

Substituting values $t = 0.0133\,s$

The plot can also be done in *Excel.*

Problem 1.11

[4]

1.11 In a combustion process, gasoline particles are to be dropped in air. The particles must drop at least 25 cm in 1 s. Find the diameter d of droplets required for this. (The drag on these particles is given by $F_D = 3\pi\mu V d$, where V is the particle speed and μ is the air viscosity. To solve this problem use *Excel*'s *Goal Seek*.)

Given: Data on sphere and formula for drag.

Find: Diameter of gasoline droplets that take 1 second to fall 25 cm.

Solution: Use given data and data in Appendices; integrate equation of motion by separating variables.

The data provided, or available in the Appendices, are:

$$\mu = 1.8\times 10^{-5}\cdot\frac{\mathrm{N\cdot s}}{\mathrm{m}^2} \qquad \rho_w = 999\cdot\frac{\mathrm{kg}}{\mathrm{m}^3} \qquad SG_{gas} = 0.72 \qquad \rho_{gas} = SG_{gas}\cdot\rho_w \qquad \rho_{gas} = 719\frac{\mathrm{kg}}{\mathrm{m}^3}$$

Newton's 2nd law for the sphere (mass M) is (ignoring buoyancy effects) $M\cdot\frac{dV}{dt} = M\cdot g - 3\cdot\pi\cdot\mu\cdot V\cdot d$

so
$$\frac{dV}{g - \frac{3\cdot\pi\cdot\mu\cdot d}{M}\cdot V} = dt$$

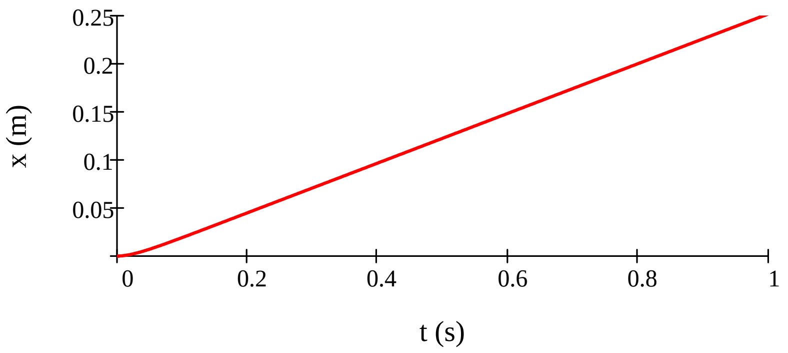
Integrating and using limits
$$V(t) = \frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d}\cdot\left(1 - e^{\frac{-3\cdot\pi\cdot\mu\cdot d}{M}\cdot t}\right)$$

Integrating again
$$x(t) = \frac{M\cdot g}{3\cdot\pi\cdot\mu\cdot d}\cdot\left[t + \frac{M}{3\cdot\pi\cdot\mu\cdot d}\cdot\left(e^{\frac{-3\cdot\pi\cdot\mu\cdot d}{M}\cdot t} - 1\right)\right]$$

Replacing M with an expression involving diameter d $\quad M = \rho_{gas}\cdot\frac{\pi\cdot d^3}{6} \qquad x(t) = \frac{\rho_{gas}\cdot d^2\cdot g}{18\cdot\mu}\cdot\left[t + \frac{\rho_{gas}\cdot d^2}{18\cdot\mu}\cdot\left(e^{\frac{-18\cdot\mu}{\rho_{gas}\cdot d^2}\cdot t} - 1\right)\right]$

This equation must be solved for d so that $x(1\cdot s) = 1\cdot m$. The answer can be obtained from manual iteration, or by using *Excel's Goal Seek*. (See this in the corresponding Excel workbook.)

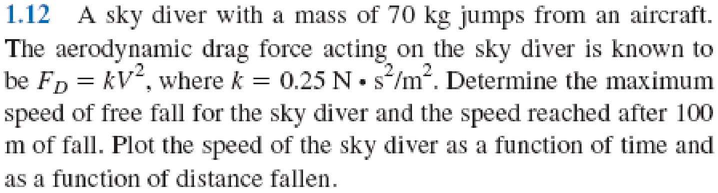
$$d = 0.109\cdot mm$$



Note That the particle quickly reaches terminal speed, so that a simpler approximate solution would be to solve $Mg = 3\pi\mu Vd$ for d, with $V = 0.25$ m/s (allowing for the fact that M is a function of d)!

Problem 1.12

[4]



1.12 A sky diver with a mass of 70 kg jumps from an aircraft. The aerodynamic drag force acting on the sky diver is known to be $F_D = kV^2$, where $k = 0.25\ \mathrm{N \cdot s^2/m^2}$. Determine the maximum speed of free fall for the sky diver and the speed reached after 100 m of fall. Plot the speed of the sky diver as a function of time and as a function of distance fallen.

Given: Data on sky diver: $M = 70 \cdot \mathrm{kg}$ $\quad k = 0.25 \cdot \dfrac{\mathrm{N \cdot s^2}}{\mathrm{m^2}}$

Find: Maximum speed; speed after 100 m; plot speed as function of time and distance.

Solution: Use given data; integrate equation of motion by separating variables.

Treat the sky diver as a system; apply Newton's 2nd law:

Newton's 2nd law for the sky diver (mass M) is (ignoring buoyancy effects): $M \cdot \dfrac{dV}{dt} = M \cdot g - k \cdot V^2$ (1)

(a) For terminal speed V_t, acceleration is zero, so $M \cdot g - k \cdot V^2 = 0$ so $V_t = \sqrt{\dfrac{M \cdot g}{k}}$

$$V_t = \left(75 \cdot \mathrm{kg} \times 9.81 \cdot \frac{\mathrm{m}}{\mathrm{s}^2} \times \frac{\mathrm{m}^2}{0.25 \cdot \mathrm{N} \cdot \mathrm{s}^2} \cdot \frac{\mathrm{N \cdot s^2}}{\mathrm{kg \times m}}\right)^{\frac{1}{2}} \qquad V_t = 54.2 \frac{\mathrm{m}}{\mathrm{s}}$$

(b) For V at y = 100 m we need to find $V(y)$. From (1) $M \cdot \dfrac{dV}{dt} = M \cdot \dfrac{dV}{dy} \cdot \dfrac{dy}{dt} = M \cdot V \cdot \dfrac{dV}{dt} = M \cdot g - k \cdot V^2$

Separating variables and integrating:

$$\int_0^V \frac{V}{1 - \dfrac{k \cdot V^2}{M \cdot g}}\, dV = \int_0^y g\, dy$$

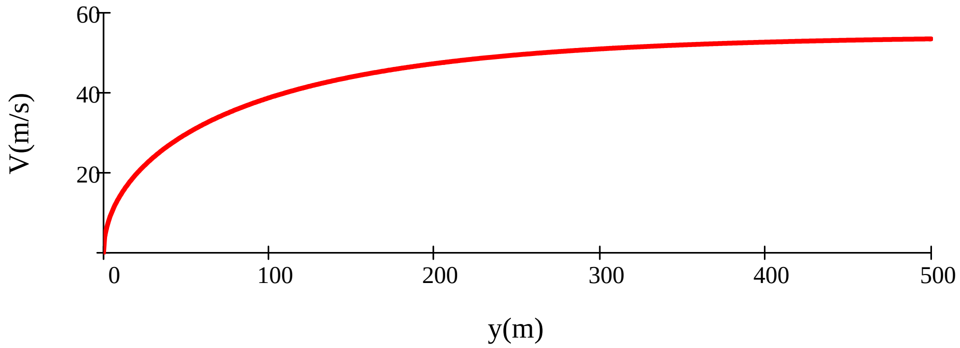
so $\ln\left(1 - \dfrac{k \cdot V^2}{M \cdot g}\right) = -\dfrac{2 \cdot k}{M} y$ $\qquad$ or $\qquad V^2 = \dfrac{M \cdot g}{k} \cdot \left(1 - e^{-\frac{2 \cdot k \cdot y}{M}}\right)$

Hence

$$V(y) = V_t \cdot \left(1 - e^{-\frac{2 \cdot k \cdot y}{M}}\right)^{\frac{1}{2}}$$

For y = 100 m:

$$V(100 \cdot \mathrm{m}) = 54.2 \cdot \frac{\mathrm{m}}{\mathrm{s}} \cdot \left(1 - e^{-2 \times 0.25 \cdot \frac{\mathrm{N \cdot s^2}}{\mathrm{m^2}} \times 100 \cdot \mathrm{m} \times \frac{1}{70 \cdot \mathrm{kg}} \times \frac{\mathrm{kg \cdot m}}{\mathrm{s^2 \cdot N}}}\right)^{\frac{1}{2}} \qquad V(100 \cdot \mathrm{m}) = 38.8 \cdot \frac{\mathrm{m}}{\mathrm{s}}$$

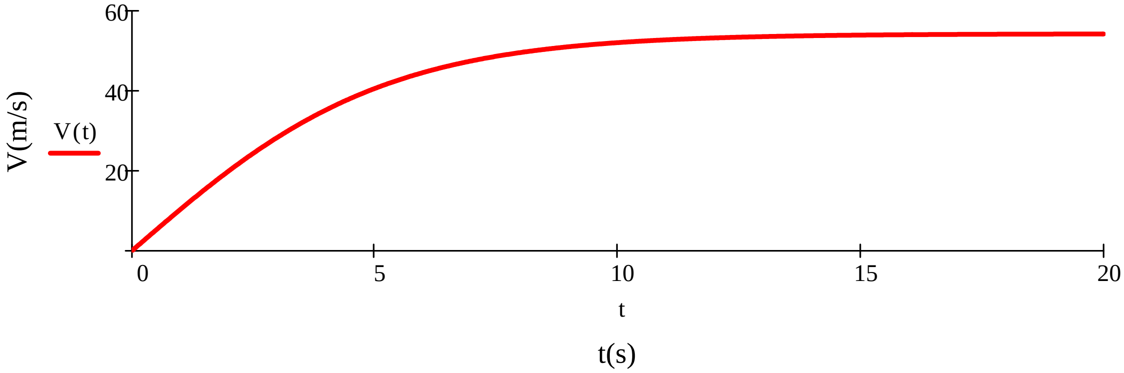


(c) For *V*(*t*) we need to integrate (1) with respect to *t*: $M\cdot\frac{dV}{dt} = M\cdot g - k\cdot V^2$

Separating variables and integrating: $$\int_0^V \frac{V}{\frac{M\cdot g}{k} - V^2}\,dV = \int_0^t 1\,dt$$

so $$t = \frac{1}{2}\cdot\sqrt{\frac{M}{k\cdot g}}\cdot\ln\left(\left|\frac{\sqrt{\frac{M\cdot g}{k}} + V}{\sqrt{\frac{M\cdot g}{k}} - V}\right|\right) = \frac{1}{2}\cdot\sqrt{\frac{M}{k\cdot g}}\cdot\ln\left(\frac{\left|V_t + V\right|}{\left|V_t - V\right|}\right)$$

Rearranging $$V(t) = V_t\cdot\frac{\left(e^{2\cdot\sqrt{\frac{k\cdot g}{M}}\cdot t} - 1\right)}{\left(e^{2\cdot\sqrt{\frac{k\cdot g}{M}}\cdot t} + 1\right)}$$ or $$V(t) = V_t\cdot\tanh\left(V_t\cdot\frac{k}{M}\cdot t\right)$$



The two graphs can also be plotted in Excel.

Problem 1.13 [5]

1.13 For Problem 1.12, the initial horizontal speed of the skydiver is 70 m/s. As she falls, the k value for the vertical drag remains as before, but the value for horizontal motion is $k = 0.05$ $N \cdot s/m^2$. Compute and plot the 2D trajectory of the skydiver.

Given: Data on sky diver: $M = 70 \cdot kg$ $\quad k_{vert} = 0.25 \cdot \frac{N \cdot s^2}{m^2}$ $\quad k_{horiz} = 0.05 \cdot \frac{N \cdot s^2}{m^2}$ $\quad U_0 = 70 \cdot \frac{m}{s}$

Find: Plot of trajectory.

Solution: Use given data; integrate equation of motion by separating variables.

Treat the sky diver as a system; apply Newton's 2nd law in horizontal and vertical directions:

Vertical: Newton's 2nd law for the sky diver (mass M) is (ignoring buoyancy effects): $M \cdot \frac{dV}{dt} = M \cdot g - k_{vert} \cdot V^2$ (1)

For *V*(*t*) we need to integrate (1) with respect to *t*:

Separating variables and integrating:

$$\int_0^V \frac{V}{\frac{M \cdot g}{k_{vert}} - V^2}\, dV = \int_0^t 1\, dt$$

so

$$t = \frac{1}{2} \cdot \sqrt{\frac{M}{k_{vert} \cdot g}} \cdot \ln\left(\left| \frac{\sqrt{\frac{M \cdot g}{k_{vert}}} + V}{\sqrt{\frac{M \cdot g}{k_{vert}}} - V} \right|\right)$$

Rearranging

$$V(t) = \sqrt{\frac{M \cdot g}{k_{vert}}} \cdot \frac{\left(e^{2 \cdot \sqrt{\frac{k_{vert} \cdot g}{M}} \cdot t} - 1\right)}{\left(e^{2 \cdot \sqrt{\frac{k_{vert} \cdot g}{M}} \cdot t} + 1\right)}$$

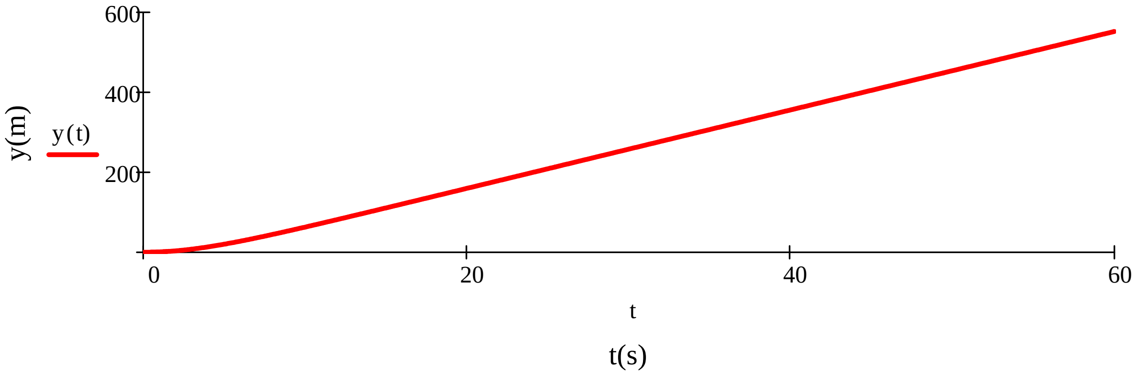
so

$$V(t) = \sqrt{\frac{M \cdot g}{k_{vert}}} \cdot \tanh\left(\sqrt{\frac{k_{vert} \cdot g}{M}} \cdot t\right)$$

For *y*(*t*) we need to integrate again: $\frac{dy}{dt} = V$ or $y = \int V\, dt$

$$y(t) = \int_0^t V(t)\, dt = \int_0^t \sqrt{\frac{M \cdot g}{k_{vert}}} \cdot \tanh\left(\sqrt{\frac{k_{vert} \cdot g}{M}} \cdot t\right) dt = \sqrt{\frac{M \cdot g}{k_{vert}}} \cdot \ln\left(\cosh\left(\sqrt{\frac{k_{vert} \cdot g}{M}} \cdot t\right)\right)$$

$$y(t) = \sqrt{\frac{M \cdot g}{k_{vert}}} \cdot \ln\left(\cosh\left(\sqrt{\frac{k_{vert} \cdot g}{M}} \cdot t\right)\right)$$



Horizontal: Newton's 2nd law for the sky diver (mass M) is: $$M\cdot\frac{dU}{dt} = -k_{horiz}\cdot U^2 \qquad (2)$$

For *U*(*t*) we need to integrate (2) with respect to *t*:

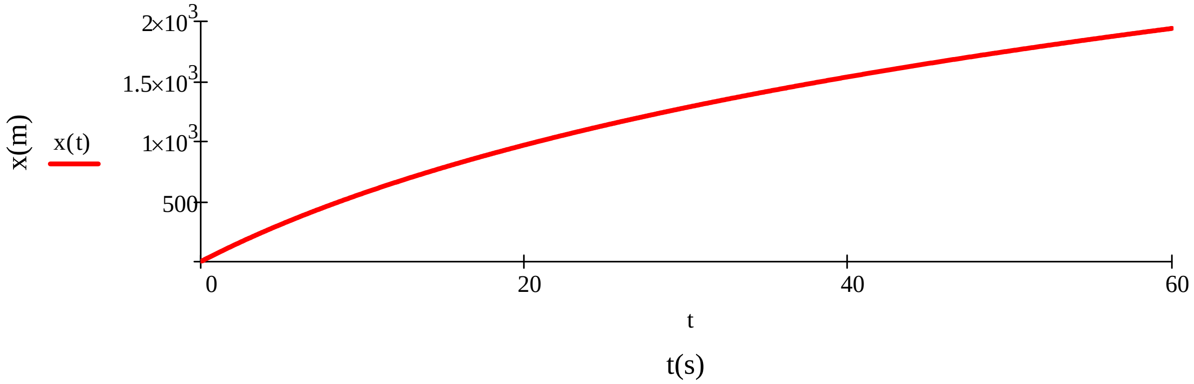
Separating variables and integrating: $$\int_{U_0}^{U} \frac{1}{U^2}\,dU = \int_0^t -\frac{k_{horiz}}{M}\,dt \qquad \text{so} \qquad -\frac{k_{horiz}}{M}\cdot t = -\frac{1}{U} + \frac{1}{U_0}$$

Rearranging or $$U(t) = \frac{U_0}{1 + \frac{k_{horiz}\cdot U_0}{M}\cdot t}$$

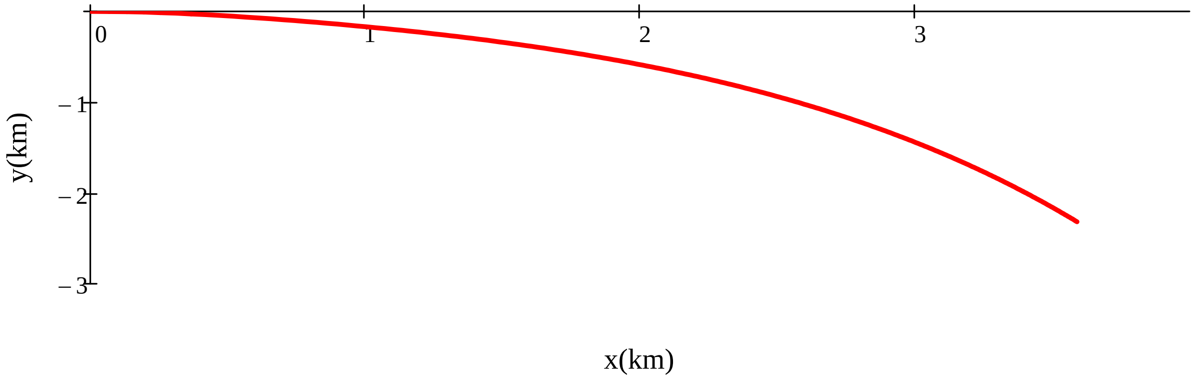
For *x*(*t*) we need to integrate again: $$\frac{dx}{dt} = U \qquad \text{or} \qquad x = \int U\,dt$$

$$x(t) = \int_0^t U(t)\,dt = \int_0^t \frac{U_0}{1 + \frac{k_{horiz}\cdot U_0}{M}\cdot t}\,dt = \frac{M}{k_{horiz}}\cdot\ln\left(\frac{k_{horiz}\cdot U_0}{M}\cdot t + 1\right)$$

$$x(t) = \frac{M}{k_{horiz}}\cdot\ln\left(\frac{k_{horiz}\cdot U_0}{M}\cdot t + 1\right)$$



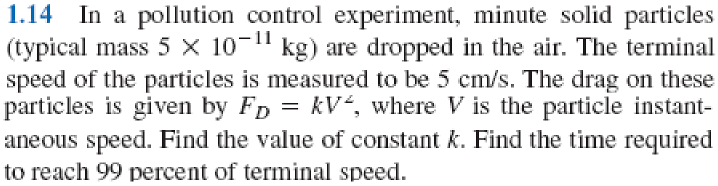
Plotting the trajectory:



These plots can also be done in Excel.

Problem 1.14

[3]



1.14 In a pollution control experiment, minute solid particles (typical mass 5×10^{-11} kg) are dropped in the air. The terminal speed of the particles is measured to be 5 cm/s. The drag on these particles is given by $F_D = kV^2$, where V is the particle instantaneous speed. Find the value of constant k. Find the time required to reach 99 percent of terminal speed.

Given: Data on sphere and terminal speed.

Find: Drag constant *k*, and time to reach 99% of terminal speed.

Solution: Use given data; integrate equation of motion by separating variables.

The data provided are: $M = 5 \cdot 10^{-11} \cdot \text{kg}$ $\quad V_t = 5 \cdot \frac{\text{cm}}{\text{s}}$

Newton's 2nd law for the general motion is (ignoring buoyancy effects) $\quad M \cdot \frac{dV}{dt} = M \cdot g - k \cdot V \quad (1)$

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $\quad M \cdot g = k \cdot V_t \quad$ so $\quad k = \frac{M \cdot g}{V_t}$

$$k = \frac{M \cdot g}{V_t} = 5 \times 10^{-11} \cdot \text{kg} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times \frac{\text{s}}{0.05 \cdot \text{m}} \qquad k = 9.81 \times 10^{-9} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}}$$

To find the time to reach 99% of V_t, we need $V(t)$. From 1, separating variables $\quad \frac{dV}{g - \frac{k}{M} \cdot V} = dt$

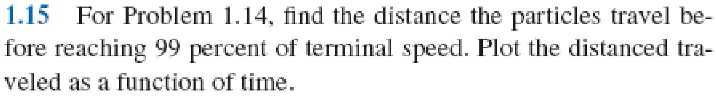
Integrating and using limits $\quad t = -\frac{M}{k} \cdot \ln\left(1 - \frac{k}{M \cdot g} \cdot V\right)$

We must evaluate this when $\quad V = 0.99 \cdot V_t \qquad V = 4.95 \cdot \frac{\text{cm}}{\text{s}}$

$$t = 5 \times 10^{-11} \cdot \text{kg} \times \frac{\text{m}}{9.81 \times 10^{-9} \cdot \text{N} \cdot \text{s}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \cdot \ln\left(1 - 9.81 \cdot 10^{-9} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}} \times \frac{1}{5 \times 10^{-11} \cdot \text{kg}} \times \frac{\text{s}^2}{9.81 \cdot \text{m}} \times \frac{0.0495 \cdot \text{m}}{\text{s}} \times \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2}\right)$$

$$t = 0.0235 \, \text{s}$$

Problem 1.15 [5]



1.15 For Problem 1.14, find the distance the particles travel before reaching 99 percent of terminal speed. Plot the distanced traveled as a function of time.

Given: Data on sphere and terminal speed from Problem 1.14.

Find: Distance traveled to reach 99% of terminal speed; plot of distance versus time.

Solution: Use given data; integrate equation of motion by separating variables.

The data provided are: $M = 5\cdot10^{-11}\cdot kg$ $\quad V_t = 5\cdot\frac{cm}{s}$

Newton's 2nd law for the general motion is (ignoring buoyancy effects) $\quad M\cdot\frac{dV}{dt} = M\cdot g - k\cdot V \qquad (1)$

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $\quad M\cdot g = k\cdot V_t \quad$ so $\quad k = \frac{M\cdot g}{V_t}$

$$k = \frac{M\cdot g}{V_t} = 5\times10^{-11}\cdot kg\times9.81\cdot\frac{m}{s^2}\times\frac{s}{0.05\cdot m} \qquad k = 9.81\times10^{-9}\cdot\frac{N\cdot s}{m}$$

To find the distance to reach 99% of V_t, we need $V(y)$. From 1: $\quad M\cdot\frac{dV}{dt} = M\cdot\frac{dy}{dt}\cdot\frac{dV}{dy} = M\cdot V\cdot\frac{dV}{dy} = M\cdot g - k\cdot V$

Separating variables $\quad \dfrac{V\cdot dV}{g-\frac{k}{M}\cdot V} = dy$

Integrating and using limits $\quad y = -\frac{M^2\cdot g}{k^2}\cdot\ln\left(1-\frac{k}{M\cdot g}\cdot V\right) - \frac{M}{k}\cdot V$

We must evaluate this when $\quad V = 0.99\cdot V_t \qquad V = 4.95\cdot\frac{cm}{s}$

$$y = \left(5\times10^{-11}\cdot kg\right)^2\times\frac{9.81\cdot m}{s^2}\times\left(\frac{m}{9.81\times10^{-9}\cdot N\cdot s}\right)^2\times\left(\frac{N\cdot s^2}{kg\cdot m}\right)^2\cdot\ln\left(1-9.81\cdot10^{-9}\cdot\frac{N\cdot s}{m}\times\frac{1}{5\times10^{-11}\cdot kg}\times\frac{s^2}{9.81\cdot m}\times\frac{0.0495\cdot m}{s}\times\frac{kg\cdot m}{N\cdot s^2}\right)\ldots$$
$$+-5\times10^{-11}\cdot kg\times\frac{m}{9.81\times10^{-9}\cdot N\cdot s}\times\frac{0.0495\cdot m}{s}\times\frac{N\cdot s^2}{kg\cdot m}$$

$$y = 0.922\cdot mm$$

Alternatively we could use the approach of Problem 1.14 and first find the time to reach terminal speed, and use this time in y(t) to find the above value of y:

From 1, separating variables $\quad \dfrac{dV}{g-\frac{k}{M}\cdot V} = dt$

Integrating and using limits $\quad t = -\frac{M}{k}\cdot\ln\left(1-\frac{k}{M\cdot g}\cdot V\right) \qquad (2)$

We must evaluate this when $V = 0.99 \cdot V_t$ $\quad V = 4.95 \cdot \dfrac{cm}{s}$

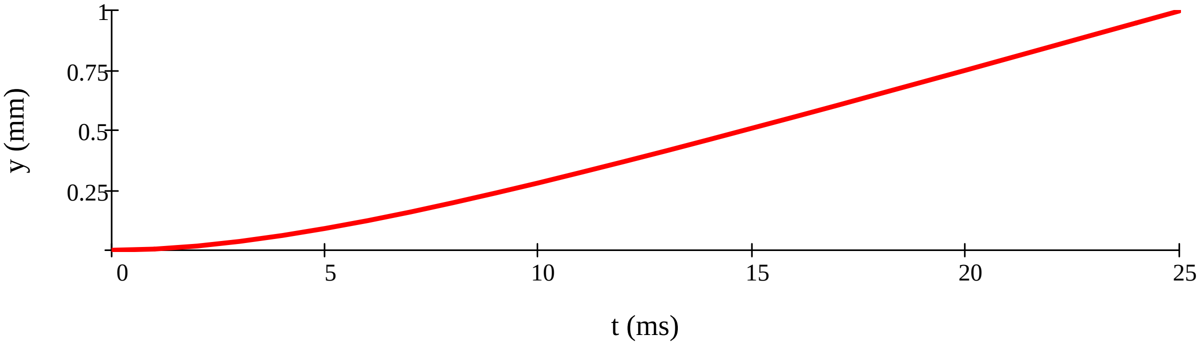
$$t = 5 \times 10^{-11} \cdot kg \times \frac{m}{9.81 \times 10^{-9} \cdot N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} \cdot \ln\left(1 - 9.81 \cdot 10^{-9} \cdot \frac{N \cdot s}{m} \times \frac{1}{5 \times 10^{-11} \cdot kg} \times \frac{s^2}{9.81 \cdot m} \times \frac{0.0495 \cdot m}{s} \times \frac{kg \cdot m}{N \cdot s^2}\right) \qquad t = 0.0235\,s$$

From 2, after rearranging $\quad V = \dfrac{dy}{dt} = \dfrac{M \cdot g}{k} \cdot \left(1 - e^{-\frac{k}{M} \cdot t}\right)$

Integrating and using limits $\quad y = \dfrac{M \cdot g}{k} \cdot \left[t + \dfrac{M}{k} \cdot \left(e^{-\frac{k}{M} \cdot t} - 1\right)\right]$

$$y = 5 \times 10^{-11} \cdot kg \times \frac{9.81 \cdot m}{s^2} \times \frac{m}{9.81 \times 10^{-9} \cdot N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} \cdot \left[0.0235 \cdot s \;\ldots \atop + 5 \times 10^{-11} \cdot kg \times \dfrac{m}{9.81 \times 10^{-9} \cdot N \cdot s} \times \dfrac{N \cdot s^2}{kg \cdot m} \cdot \left(e^{-\frac{9.81 \cdot 10^{-9}}{5 \cdot 10^{-11}} \cdot .0235} - 1\right)\right]$$

$y = 0.922 \cdot mm$



This plot can also be presented in Excel.

Problem 1.16

[3]

1.16 The English perfected the longbow as a weapon after the Medieval period. In the hands of a skilled archer, the longbow was reputed to be accurate at ranges to 100 meters or more. If the maximum altitude of an arrow is less than $h = 10$ m while traveling to a target 100 m away from the archer, and neglecting air resistance, estimate the speed and angle at which the arrow must leave the bow. Plot the required release speed and angle as a function of height *h*.

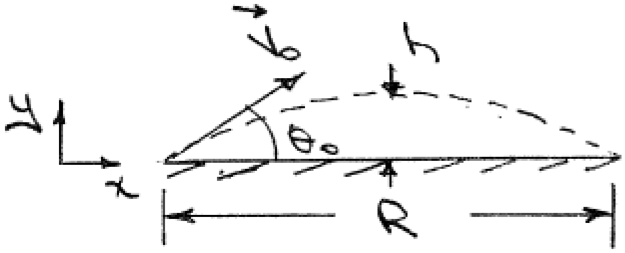
Given: Long bow at range, R = 100 m. Maximum height of arrow is h = 10 m. Neglect air resistance.

Find: Estimate of (a) speed, and (b) angle, of arrow leaving the bow.

Plot: (a) release speed, and (b) angle, as a function of h

Solution: Let $\overrightarrow{V_0} = u_0\hat{i} + v_0\hat{j} = V_0(\cos\theta_0\hat{i} + \sin\theta_0\hat{j})$

$\Sigma F_y = m\frac{dv}{dt} = -mg$, so $v = v_0 - gt$, and $t_f = 2t_{v=0} = 2v_0/g$



Also, $$mv\frac{dv}{dy} = -mg,\ v\,dv = -g\,dy,\ 0 - \frac{v_0^2}{2} = -gh$$

Thus $$h = v_0^2\big/2g \tag{1}$$

$$\Sigma F_x = m\frac{du}{dt} = 0,\ \text{so } u = u_0 = \text{const, and } R = u_0 t_f = \frac{2u_0v_0}{g} \tag{2}$$

From

1. $v_0^2 = 2gh$ (3)

2. $u_0 = \dfrac{gR}{2v_0} = \dfrac{gR}{2\sqrt{2gh}} \qquad \therefore\ u_0^2 = \dfrac{gR^2}{8h}$

Then $$V_0^2 = u_0^2 + v_0^2 = \frac{gR^2}{8h} + 2gh \quad \text{and} \quad V_0 = \left[2gh + \frac{gR^2}{8h}\right]^{\frac{1}{2}} \tag{4}$$

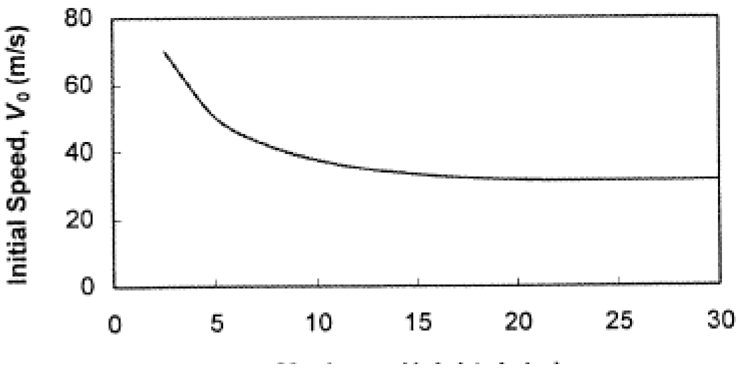
$$V_0 = \left[2 \times 9.81\ \frac{m}{s^2} \times 10\ m + \frac{9.81}{8}\ \frac{m}{s^2} \times (100)^2\ m^2 \times \frac{1}{10\ m}\right]^{\frac{1}{2}} = 37.7\ m/s$$

From Eq. 3 $$v_0 = \sqrt{2gh} = V_0 \sin\theta, \theta = \sin^{-1}\frac{\sqrt{2gh}}{V_0} \tag{5}$$

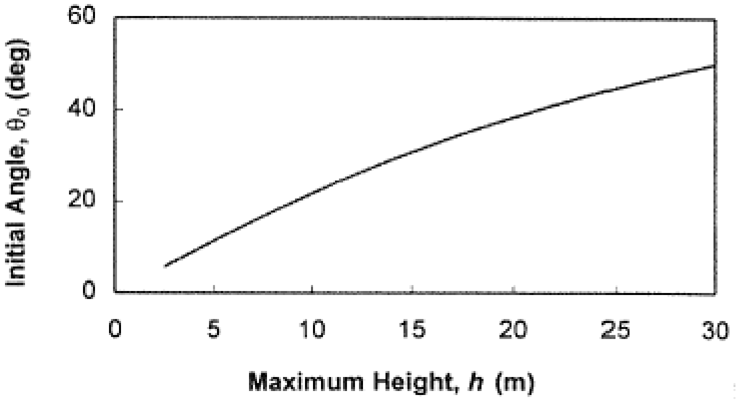
$$\theta = \sin^{-1}\left[\left(2 \times 9.81\frac{m}{s} \times 10\ m\right)^{\frac{1}{2}} \frac{s}{37.7\ m}\right] = 21.8°$$

Plots of $V_0 = V_0(h)$ {Eq. 4} and $\theta_0 = \theta_0(h)$ {Eq. 5} are presented below

Eq. 4: Initial Speed vs. Max. Height

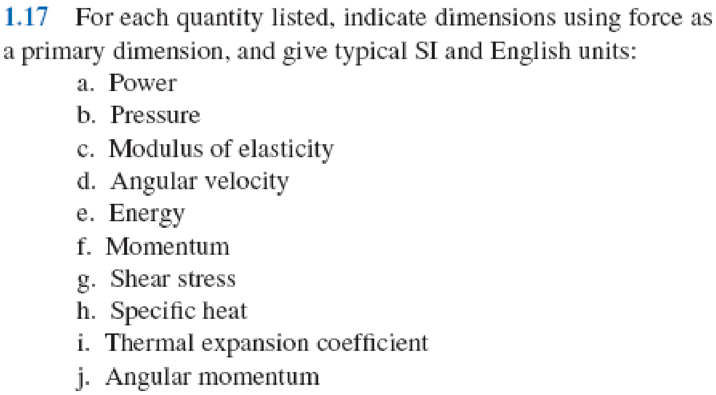


Eq. 5: Initial Angle vs. Max. Height



Problem 1.17

[2]



1.17 For each quantity listed, indicate dimensions using force as a primary dimension, and give typical SI and English units:

a. Power
b. Pressure
c. Modulus of elasticity
d. Angular velocity
e. Energy
f. Momentum
g. Shear stress
h. Specific heat
i. Thermal expansion coefficient
j. Angular momentum

Given: Basic dimensions F, L, t and T.

Find: Dimensional representation of quantities below, and typical units in SI and English systems.

Solution:

(a) Power $\quad \text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}} = \frac{F \cdot L}{t} \qquad \frac{N \cdot m}{s} \qquad \frac{lbf \cdot ft}{s}$

(b) Pressure $\quad \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} \qquad \frac{N}{m^2} \qquad \frac{lbf}{ft^2}$

(c) Modulus of elasticity $\quad \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} \qquad \frac{N}{m^2} \qquad \frac{lbf}{ft^2}$

(d) Angular velocity $\quad \text{AngularVelocity} = \frac{\text{Radians}}{\text{Time}} = \frac{1}{t} \qquad \frac{1}{s} \qquad \frac{1}{s}$

(e) Energy $\quad \text{Energy} = \text{Force} \times \text{Distance} = F \cdot L \qquad N \cdot m \qquad lbf \cdot ft$

(f) Momentum $\quad \text{Momentum} = \text{Mass} \times \text{Velocity} = M \cdot \frac{L}{t}$

From Newton's 2nd law $\quad \text{Force} = \text{Mass} \times \text{Acceleration}$ so $\quad F = M \cdot \frac{L}{t^2}$ or $\quad M = \frac{F \cdot t^2}{L}$

Hence $\quad \text{Momentum} = M \cdot \frac{L}{t} = \frac{F \cdot t^2 \cdot L}{L \cdot t} = F \cdot t \qquad N \cdot s \qquad lbf \cdot s$

(g) Shear stress $\quad \text{ShearStress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} \qquad \frac{N}{m^2} \qquad \frac{lbf}{ft^2}$

(h) Specific heat $\quad \text{SpecificHeat} = \frac{\text{Energy}}{\text{Mass} \times \text{Temperature}} = \frac{F \cdot L}{M \cdot T} = \frac{F \cdot L}{\left(\frac{F \cdot t^2}{L}\right) \cdot T} = \frac{L^2}{t^2 \cdot T} \qquad \frac{m^2}{s^2 \cdot K} \qquad \frac{ft^2}{s^2 \cdot R}$

(i) Thermal expansion coefficient $\quad \text{ThermalExpansionCoefficient} = \frac{\frac{\text{LengthChange}}{\text{Length}}}{\text{Temperature}} = \frac{1}{T} \qquad \frac{1}{K} \qquad \frac{1}{R}$

(j) Angular momentum $\quad \text{AngularMomentum} = \text{Momentum} \times \text{Distance} = F \cdot t \cdot L \qquad N \cdot m \cdot s \qquad lbf \cdot ft \cdot s$

Problem 1.18

[2]

1.18 For each quantity listed, indicate dimensions using mass as a primary dimension, and give typical SI and English units:

a. Power
b. Pressure
c. Modulus of elasticity
d. Angular velocity
e. Energy
f. Moment of a force
g. Momentum
h. Shear stress
i. Strain
j. Angular momentum

Given: Basic dimensions M, L, t and T.

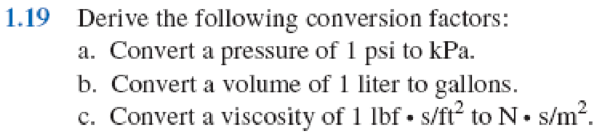
Find: Dimensional representation of quantities below, and typical units in SI and English systems.

Solution:

(a) Power	$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}} = \frac{F \cdot L}{t}$		
	From Newton's 2nd law $\text{Force} = \text{Mass} \times \text{Acceleration}$ so $F = \frac{M \cdot L}{t^2}$		
	Hence $\text{Power} = \frac{F \cdot L}{t} = \frac{M \cdot L \cdot L}{t^2 \cdot t} = \frac{M \cdot L^2}{t^3}$	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$	$\frac{\text{slug}\,\text{ft}^2}{\text{s}^3}$
(b) Pressure	$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} = \frac{M \cdot L}{t^2 \cdot L^2} = \frac{M}{L \cdot t^2}$	$\frac{\text{kg}}{\text{m} \cdot \text{s}^2}$	$\frac{\text{slug}}{\text{ft} \cdot \text{s}^2}$
(c) Modulus of elasticity	$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} = \frac{M \cdot L}{t^2 \cdot L^2} = \frac{M}{L \cdot t^2}$	$\frac{\text{kg}}{\text{m} \cdot \text{s}^2}$	$\frac{\text{slug}}{\text{ft} \cdot \text{s}^2}$
(d) Angular velocity	$\text{AngularVelocity} = \frac{\text{Radians}}{\text{Time}} = \frac{1}{t}$	$\frac{1}{\text{s}}$	$\frac{1}{\text{s}}$
(e) Energy	$\text{Energy} = \text{Force} \times \text{Distance} = F \cdot L = \frac{M \cdot L \cdot L}{t^2} = \frac{M \cdot L^2}{t^2}$	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$	$\frac{\text{slug} \cdot \text{ft}^2}{\text{s}^2}$
(f) Moment of a force	$\text{MomentOfForce} = \text{Force} \times \text{Length} = F \cdot L = \frac{M \cdot L \cdot L}{t^2} = \frac{M \cdot L^2}{t^2}$	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$	$\frac{\text{slug} \cdot \text{ft}^2}{\text{s}^2}$
(g) Momentum	$\text{Momentum} = \text{Mass} \times \text{Velocity} = M \cdot \frac{L}{t} = \frac{M \cdot L}{t}$	$\frac{\text{kg} \cdot \text{m}}{\text{s}}$	$\frac{\text{slug} \cdot \text{ft}}{\text{s}}$
(h) Shear stress	$\text{ShearStress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} = \frac{M \cdot L}{t^2 \cdot L^2} = \frac{M}{L \cdot t^2}$	$\frac{\text{kg}}{\text{m} \cdot \text{s}^2}$	$\frac{\text{slug}}{\text{ft} \cdot \text{s}^2}$
(i) Strain	$\text{Strain} = \frac{\text{LengthChange}}{\text{Length}} = \frac{L}{L}$	Dimensionless	
(j) Angular momentum	$\text{AngularMomentum} = \text{Momentum} \times \text{Distance} = \frac{M \cdot L}{t} \cdot L = \frac{M \cdot L^2}{t}$	$\frac{\text{kg} \cdot \text{m}^2}{\text{s}}$	$\frac{\text{slugs} \cdot \text{ft}^2}{\text{s}}$

Problem 1.19

[1]



1.19 Derive the following conversion factors:

a. Convert a pressure of 1 psi to kPa.

b. Convert a volume of 1 liter to gallons.

c. Convert a viscosity of 1 lbf • s/ft^2 to N • s/m^2.

Given: Pressure, volume and density data in certain units

Find: Convert to different units

Solution:

Using data from tables (e.g. Table G.2)

(a) $$1 \cdot \text{psi} = 1 \cdot \text{psi} \times \frac{6895 \cdot \text{Pa}}{1 \cdot \text{psi}} \times \frac{1 \cdot \text{kPa}}{1000 \cdot \text{Pa}} = 6.89 \cdot \text{kPa}$$

(b) $$1 \cdot \text{liter} = 1 \cdot \text{liter} \times \frac{1 \cdot \text{quart}}{0.946 \cdot \text{liter}} \times \frac{1 \cdot \text{gal}}{4 \cdot \text{quart}} = 0.264 \cdot \text{gal}$$

(c) $$1 \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2} = 1 \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2} \times \frac{4.448 \cdot \text{N}}{1 \cdot \text{lbf}} \times \left(\frac{\frac{1}{12} \cdot \text{ft}}{0.0254 \cdot \text{m}} \right)^2 = 47.9 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

Problem 1.20

[1]

1.20 Derive the following conversion factors:

a. Convert a viscosity of 1 m^2/s to ft^2/s.

b. Convert a power of 100 W to horsepower.

c. Convert a specific energy of 1 kJ/kg to Btu/lbm.

Given: Viscosity, power, and specific energy data in certain units

Find: Convert to different units

Solution:

Using data from tables (e.g. Table G.2)

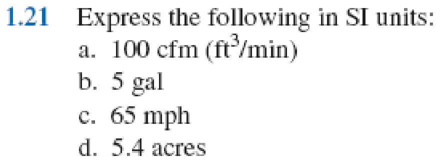
(a) $$1\cdot\frac{m^2}{s} = 1\cdot\frac{m^2}{s}\times\left(\frac{\frac{1}{12}\cdot ft}{0.0254\cdot m}\right)^2 = 10.76\cdot\frac{ft^2}{s}$$

(b) $$100\cdot W = 100\cdot W\times\frac{1\cdot hp}{746\cdot W} = 0.134\cdot hp$$

(c) $$1\cdot\frac{kJ}{kg} = 1\cdot\frac{kJ}{kg}\times\frac{1000\cdot J}{1\cdot kJ}\times\frac{1\cdot Btu}{1055\cdot J}\times\frac{0.454\cdot kg}{1\cdot lbm} = 0.43\cdot\frac{Btu}{lbm}$$

Problem 1.21

[1]



1.21 Express the following in SI units:

a. 100 cfm (ft^3/min)
b. 5 gal
c. 65 mph
d. 5.4 acres

Given: Quantities in English Engineering (or customary) units.

Find: Quantities in SI units.

Solution: Use Table G.2 and other sources (e.g., Google)

(a) $$100\cdot\frac{ft^3}{m} = 100\cdot\frac{ft^3}{min}\times\left(\frac{0.0254\cdot m}{1\cdot in}\times\frac{12\cdot in}{1\cdot ft}\right)^3\times\frac{1\cdot min}{60\cdot s} = 0.0472\cdot\frac{m^3}{s}$$

(b) $$5\cdot gal = 5\cdot gal\times\frac{231\cdot in^3}{1\cdot gal}\times\left(\frac{0.0254\cdot m}{1\cdot in}\right)^3 = 0.0189\cdot m^3$$

(c) $$65\cdot mph = 65\cdot\frac{mile}{hr}\times\frac{1852\cdot m}{1\cdot mile}\times\frac{1\cdot hr}{3600\cdot s} = 29.1\cdot\frac{m}{s}$$

(d) $$5.4\cdot acres = 5.4\cdot acre\times\frac{4047\cdot m^3}{1\cdot acre} = 2.19\times 10^4\cdot m^2$$

Problem 1.22

[1]

1.22 Express the following in BG units:

a. 50 m^2
b. 250 cc
c. 100 kW
d. 5 lbf • s/ft^2

Given: Quantities in SI (or other) units.

Find: Quantities in BG units.

Solution: Use Table G.2.

(a) $$50\cdot m^2 = 50\cdot m^2 \times \left(\frac{1\cdot in}{0.0254\cdot m} \times \frac{1\cdot ft}{12\cdot in}\right)^2 = 538\cdot ft^2$$

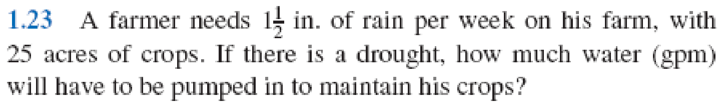
(b) $$250\cdot cc = 250\cdot cm^3 \times \left(\frac{1\cdot m}{100\cdot cm} \times \frac{1\cdot in}{0.0254\cdot m} \times \frac{1\cdot ft}{12\cdot in}\right)^3 = 8.83\times 10^{-3}\cdot ft^3$$

(c) $$100\cdot kW = 100\cdot kW \times \frac{1000\cdot W}{1\cdot kW} \times \frac{1\cdot hp}{746\cdot W} = 134\cdot hp$$

(d) $5\cdot\frac{lbf\cdot s}{ft^2}$ is already in BG units

Problem 1.23

[1]



1.23 A farmer needs $1\frac{1}{2}$ in. of rain per week on his farm, with 25 acres of crops. If there is a drought, how much water (gpm) will have to be pumped in to maintain his crops?

Given: Acreage of land, and water needs.

Find: Water flow rate (gpm) to water crops.

Solution: Use Table G.2 and other sources (e.g., Google) as needed.

The volume flow rate needed is

$$Q = \frac{1.5 \cdot \text{in}}{\text{week}} \times 25 \cdot \text{acres}$$

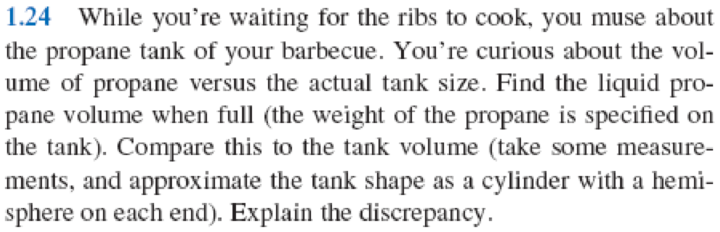
Performing unit conversions

$$Q = \frac{1.5 \cdot \text{in} \times 25 \cdot \text{acre}}{\text{week}} = \frac{1.5 \cdot \text{in} \times 25 \cdot \text{acre}}{\text{week}} \times \frac{4.36 \times 10^{4} \cdot \text{ft}^{2}}{1 \cdot \text{acre}} \times \left(\frac{12 \cdot \text{in}}{1 \cdot \text{ft}}\right)^{2} \times \frac{1 \cdot \text{week}}{7 \cdot \text{day}} \times \frac{1 \cdot \text{day}}{24 \cdot \text{hr}} \times \frac{1 \cdot \text{hr}}{60 \cdot \text{min}}$$

$$Q = 101 \cdot \text{gpm}$$

Problem 1.24

[2]



1.24 While you're waiting for the ribs to cook, you muse about the propane tank of your barbecue. You're curious about the volume of propane versus the actual tank size. Find the liquid propane volume when full (the weight of the propane is specified on the tank). Compare this to the tank volume (take some measurements, and approximate the tank shape as a cylinder with a hemisphere on each end). Explain the discrepancy.

Given: Geometry of tank, and weight of propane.

Find: Volume of propane, and tank volume; explain the discrepancy.

Solution: Use Table G.2 and other sources (e.g., Google) as needed.

The author's tank is approximately 12 in in diameter, and the cylindrical part is about 8 in. The weight of propane specified is 17 lb.

The tank diameter is $D = 12 \cdot \text{in}$

The tank cylindrical height is $L = 8 \cdot \text{in}$

The mass of propane is $m_{prop} = 17 \cdot \text{lbm}$

The specific gravity of propane is $SG_{prop} = 0.495$

The density of water is $\rho = 998 \cdot \frac{\text{kg}}{\text{m}^3}$

The volume of propane is given by $V_{prop} = \frac{m_{prop}}{\rho_{prop}} = \frac{m_{prop}}{SG_{prop} \cdot \rho}$

$$V_{prop} = 17 \cdot \text{lbm} \times \frac{1}{0.495} \times \frac{\text{m}^3}{998 \cdot \text{kg}} \times \frac{0.454 \cdot \text{kg}}{1 \cdot \text{lbm}} \times \left(\frac{1 \cdot \text{in}}{0.0254 \cdot \text{m}}\right)^3$$

$$V_{prop} = 953 \cdot \text{in}^3$$

The volume of the tank is given by a cylinder diameter D length L, $\pi D^2 L/4$ and a sphere (two halves) given by $\pi D^3/6$

$$V_{tank} = \frac{\pi \cdot D^2}{4} \cdot L + \frac{\pi \cdot D^3}{6}$$

$$V_{tank} = \frac{\pi \cdot (12 \cdot \text{in})^2}{4} \cdot 8 \cdot \text{in} + \pi \cdot \frac{(12 \cdot \text{in})^3}{6}$$

$$V_{tank} = 1810 \cdot \text{in}^3$$

The ratio of propane to tank volumes is $\frac{V_{prop}}{V_{tank}} = 53 \cdot \%$

This seems low, and can be explained by a) tanks are not filled completely, b) the geometry of the tank gave an overestimate of the volume (th ends are not really hemispheres, and we have not allowed for tank wall thickness).

Problem 1.25

[1]

1.25 The density of mercury is given as 26.3 slug/ft^3. Calculate the specific gravity and the specific volume in m^3/kg of the mercury. Calculate the specific weight in lbf/ft^3 on Earth and on the moon. Acceleration of gravity on the moon is 5.47 ft/s^2.

Given: Density of mercury is $\rho = 26.3$ slug/ft^3.

Acceleration of gravity on moon is $g_m = 5.47$ ft/s^2.

Find:

a. Specific gravity of mercury.

b. Specific volume of mercury, in m^3/kg.

c. Specific weight on Earth.

d. Specific weight on moon.

Solution: Apply definitions: $\gamma \equiv \rho g,\ v \equiv 1/\rho,\ \mathrm{SG} \equiv \rho/\rho_{\mathrm{H_2O}}$

Thus

$$\mathrm{SG} = 26.3\frac{\text{slug}}{\text{ft}^3} \times \frac{\text{ft}^3}{1.94\ \text{slug}} = 13.6$$

$$v = \frac{\text{ft}^3}{26.3\ \text{slug}} \times (0.3048)^3\frac{\text{m}^3}{\text{ft}^3} \times \frac{\text{slug}}{32.2\ \text{lbm}} \times \frac{\text{lbm}}{0.4536\ \text{kg}} = 7.37 \times 10^{-5}\ \text{m}^3/\text{kg}$$

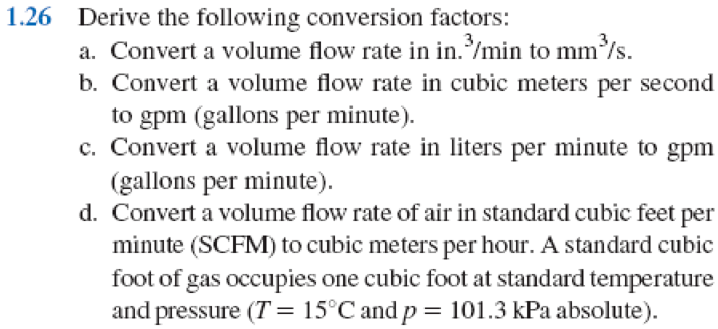
On Earth,
$$\gamma_E = 26.3\frac{\text{slug}}{\text{ft}^3} \times 32.2\frac{\text{ft}}{\text{s}^2} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = 847\ \text{lbf}/\text{ft}^3$$

On the moon,
$$\gamma_m = 26.3\frac{\text{slug}}{\text{ft}^3} \times 5.47\frac{\text{ft}}{\text{s}^2} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = 144\ \text{lbf}/\text{ft}^3$$

{Note that the mass based quantities (SG and ν) are independent of gravity.}

Problem 1.26

[1]



1.26 Derive the following conversion factors:

a. Convert a volume flow rate in in.^{3}/min to mm^3/s.

b. Convert a volume flow rate in cubic meters per second to gpm (gallons per minute).

c. Convert a volume flow rate in liters per minute to gpm (gallons per minute).

d. Convert a volume flow rate of air in standard cubic feet per minute (SCFM) to cubic meters per hour. A standard cubic foot of gas occupies one cubic foot at standard temperature and pressure ($T = 15°\text{C}$ and $p = 101.3$ kPa absolute).

Given: Data in given units

Find: Convert to different units

Solution:

(a)
$$1\cdot\frac{\text{in}^3}{\text{min}} = 1\cdot\frac{\text{in}^3}{\text{min}}\times\left(\frac{0.0254\cdot\text{m}}{1\cdot\text{in}}\times\frac{1000\cdot\text{mm}}{1\cdot\text{m}}\right)^3\times\frac{1\cdot\text{min}}{60\cdot\text{s}} = 273\cdot\frac{\text{mm}^3}{\text{s}}$$

(b)
$$1\cdot\frac{\text{m}^3}{\text{s}} = 1\cdot\frac{\text{m}^3}{\text{s}}\times\frac{1\cdot\text{gal}}{4\times 0.000946\cdot\text{m}^3}\times\frac{60\cdot\text{s}}{1\cdot\text{min}} = 15850\cdot\text{gpm}$$

(c)
$$1\cdot\frac{\text{liter}}{\text{min}} = 1\cdot\frac{\text{liter}}{\text{min}}\times\frac{1\cdot\text{gal}}{4\times 0.946\cdot\text{liter}}\times\frac{60\cdot\text{s}}{1\cdot\text{min}} = 0.264\cdot\text{gpm}$$

(d)
$$1\cdot\text{SCFM} = 1\cdot\frac{\text{ft}^3}{\text{min}}\times\left(\frac{0.0254\cdot\text{m}}{\frac{1}{12}\cdot\text{ft}}\right)^3\times\frac{60\cdot\text{min}}{1\cdot\text{hr}} = 1.70\cdot\frac{\text{m}^3}{\text{hr}}$$

Problem 1.27

[1]

1.27 The kilogram force is commonly used in Europe as a unit of force. (As in the U.S. customary system, where 1 lbf is the force exerted by a mass of 1 lbm in standard gravity, 1 kgf is the force exerted by a mass of 1 kg in standard gravity.) Moderate pressures, such as those for auto or truck tires, are conveniently expressed in units of kgf/cm^2. Convert 32 psig to these units.

Given: In European usage, 1 kgf is the force exerted on 1 kg mass in standard gravity.

Find: Convert 32 psi to units of kgf/cm^2.

Solution: Apply Newton's second law.

Basic equation: F = ma

The force exerted on 1 kg in standard gravity is $$F = 1\,\text{kg} \times 9.81\frac{\text{m}}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 9.81\,\text{N} = 1\,\text{kgf}$$

Setting up a conversion from psi to kgf/cm^2, $$1\frac{\text{lbf}}{\text{in.}^2} = 1\frac{\text{lbf}}{\text{in.}^2} \times 4.448\frac{\text{N}}{\text{lbf}} \times \frac{\text{in.}^2}{(2.54)^2\ \text{cm}^2} \times \frac{\text{kgf}}{9.81\,\text{N}} = 0.0703\frac{\text{kgf}}{\text{cm}^2}$$

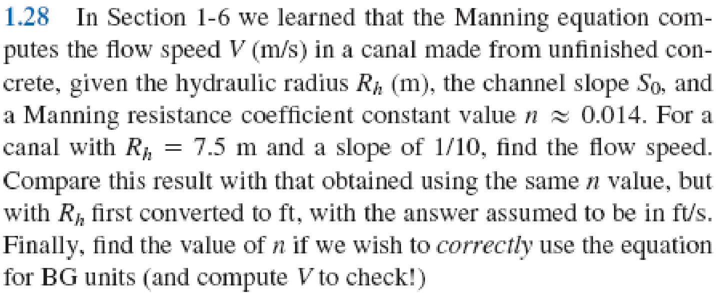
or $$1 \equiv \frac{0.0703\,\text{kgf}/\text{cm}^2}{\text{psi}}$$

Thus $$32\text{ psi} = 32\text{ psi} \times \frac{0.0703\,\text{kgf}/\text{cm}^2}{\text{psi}}$$

$$32\text{ psi} = 2.25\,\text{kgf}/\text{cm}^2$$

Problem 1.28

[3]



1.28 In Section 1-6 we learned that the Manning equation computes the flow speed V (m/s) in a canal made from unfinished concrete, given the hydraulic radius R_h (m), the channel slope S_0, and a Manning resistance coefficient constant value $n \approx 0.014$. For a canal with $R_h = 7.5$ m and a slope of 1/10, find the flow speed. Compare this result with that obtained using the same n value, but with R_h first converted to ft, with the answer assumed to be in ft/s. Finally, find the value of n if we wish to *correctly* use the equation for BG units (and compute V to check!)

Given: Information on canal geometry.

Find: Flow speed using the Manning equation, correctly and incorrectly!

Solution: Use Table G.2 and other sources (e.g., Google) as needed.

The Manning equation is $$V = \frac{R_h^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}}{n}$$ which assumes R_h in meters and V in m/s.

The given data is $R_h = 7.5\cdot m$ $\quad S_0 = \frac{1}{10}$ $\quad n = 0.014$

Hence $$V = \frac{7.5^{\frac{2}{3}}\cdot\left(\frac{1}{10}\right)^{\frac{1}{2}}}{0.014} \qquad V = 86.5\cdot\frac{m}{s}$$

(Note that we don't cancel units; we just write m/s next to the answer! Note also this is a very high speed due to the extreme slope S_0.)

Using the equation incorrectly: $$R_h = 7.5\cdot m\times\frac{1\cdot in}{0.0254\cdot m}\times\frac{1\cdot ft}{12\cdot in} \qquad R_h = 24.6\cdot ft$$

Hence $$V = \frac{24.6^{\frac{2}{3}}\cdot\left(\frac{1}{10}\right)^{\frac{1}{2}}}{0.014} \qquad V = 191\cdot\frac{ft}{s}$$

(Note that we again don't cancel units; we just write ft/s next to the answer!)

This incorrect use does not provide the correct answer $$V = 191\cdot\frac{ft}{s}\times\frac{12\cdot in}{1\cdot ft}\times\frac{0.0254\cdot m}{1\cdot in} \qquad V = 58.2\,\frac{m}{s}$$ which is wrong!

This demonstrates that for this "engineering" equation we must be careful in its use!

To generate a Manning equation valid for R_h in ft and V in ft/s, we need to do the following:

$$V\left(\frac{ft}{s}\right) = V\left(\frac{m}{s}\right)\times\frac{1\cdot in}{0.0254\cdot m}\times\frac{1\cdot ft}{12\cdot in} = \frac{R_h(m)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}}{n}\times\left(\frac{1\cdot in}{0.0254\cdot m}\times\frac{1\cdot ft}{12\cdot in}\right)$$

$$V\left(\frac{ft}{s}\right) = \frac{R_h(ft)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}}{n}\times\left(\frac{1\cdot in}{0.0254\cdot m}\times\frac{1\cdot ft}{12\cdot in}\right)^{-\frac{2}{3}}\times\left(\frac{1\cdot in}{0.0254\cdot m}\times\frac{1\cdot ft}{12\cdot in}\right) = \frac{R_h(ft)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}}{n}\times\left(\frac{1\cdot in}{0.0254\cdot m}\times\frac{1\cdot ft}{12\cdot in}\right)^{\frac{1}{3}}$$

In using this equation, we ignore the units and just evaluate the conversion factor $\left(\frac{1}{.0254}\cdot\frac{1}{12}\right)^{\frac{1}{3}} = 1.49$

Hence $$V\left(\frac{ft}{s}\right) = \frac{1.49\cdot R_h(ft)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}}{n}$$

Handbooks sometimes provide this form of the Manning equation for direct use with BG units. In our case we are asked to instead define a new value for n:

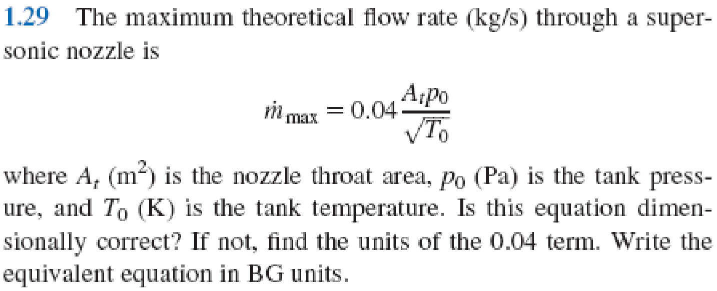
$$n_{BG} = \frac{n}{1.49} \qquad n_{BG} = 0.0094 \qquad \text{where} \qquad V\left(\frac{ft}{s}\right) = \frac{R_h(ft)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}}{n_{BG}}$$

Using this equation with Rh = 24.6 ft: $V = \frac{24.6^{\frac{2}{3}}\cdot\left(\frac{1}{10}\right)^{\frac{1}{2}}}{0.0094}$ $\qquad V = 284\frac{ft}{s}$

Converting to m/s $\qquad V = 284\cdot\frac{ft}{s}\times\frac{12\cdot in}{1\cdot ft}\times\frac{0.0254\cdot m}{1\cdot in}$ $\qquad V = 86.6\frac{m}{s}$ which is the correct answer!

Problem 1.29

[2]



1.29 The maximum theoretical flow rate (kg/s) through a supersonic nozzle is

$$\dot{m}_{max} = 0.04\frac{A_t p_0}{\sqrt{T_0}}$$

where A_t (m^2) is the nozzle throat area, p_0 (Pa) is the tank pressure, and T_0 (K) is the tank temperature. Is this equation dimensionally correct? If not, find the units of the 0.04 term. Write the equivalent equation in BG units.

Given: Equation for maximum flow rate.

Find: Whether it is dimensionally correct. If not, find units of 0.04 term. Write a BG version of the equation

Solution: Rearrange equation to check units of 0.04 term. Then use conversions from Table G.2 or other sources (e.g., Google)

"Solving" the equation for the constant 0.04: $0.04 = \frac{m_{max}\cdot\sqrt{T_0}}{A_t\cdot p_0}$

Substituting the units of the terms on the right, the units of the constant are

$$\frac{kg}{s}\times K^{\frac{1}{2}}\times\frac{1}{m^2}\times\frac{1}{Pa} = \frac{kg}{s}\times K^{\frac{1}{2}}\times\frac{1}{m^2}\times\frac{m^2}{N}\times\frac{N\cdot s^2}{kg\cdot m} = \frac{K^{\frac{1}{2}}\cdot s}{m}$$

Hence the constant is actually $c = 0.04\cdot\frac{K^{\frac{1}{2}}\cdot s}{m}$

For BG units we could start with the equation and convert each term (e.g., A_t), and combine the result into a new constant, or simply convert c directly:

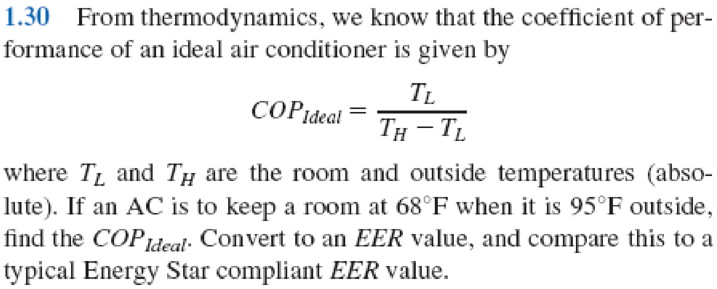
$$c = 0.04\cdot\frac{K^{\frac{1}{2}}\cdot s}{m} = 0.04\times\left(\frac{1.8\cdot R}{K}\right)^{\frac{1}{2}}\times\frac{0.0254\cdot m}{1\cdot in}\times\frac{12\cdot in}{1\cdot ft}$$

$c = 0.0164\cdot\frac{R^{\frac{1}{2}}\cdot s}{ft}$ so $m_{max} = 0.0164\cdot\frac{A_t\cdot p_0}{\sqrt{T_0}}$ with A_t in ft^2, p_0 in lbf/ft^2, and T_0 in R.

This value of c assumes p is in lbf/ft^2. For p in psi we need an additional conversion:

$c = 0.0164\cdot\frac{R^{\frac{1}{2}}\cdot s}{ft}\times\left(\frac{12\cdot in}{1\cdot ft}\right)^2$ $c = 2.36\cdot\frac{R^{\frac{1}{2}}\cdot in^2\cdot s}{ft^3}$ so $m_{max} = 2.36\cdot\frac{A_t\cdot p_0}{\sqrt{T_0}}$ with A_t in ft^2, p_0 in psi, and T_0 in R.

Problem 1.30



1.30 From thermodynamics, we know that the coefficient of performance of an ideal air conditioner is given by

$$COP_{Ideal} = \frac{T_L}{T_H - T_L}$$

where T_L and T_H are the room and outside temperatures (absolute). If an AC is to keep a room at 68°F when it is 95°F outside, find the COP_{Ideal}. Convert to an *EER* value, and compare this to a typical Energy Star compliant *EER* value.

Given: Equation for COP and temperature data.

Find: $\mathrm{COP_{Ideal}}$, EER, and compare to a typical Energy Star compliant EER value.

Solution: Use the COP equation. Then use conversions from Table G.2 or other sources (e.g., Google) to find the EER.

The given data is $T_L = (68+460)\cdot R$ $\quad T_L = 528\cdot R$ $\quad T_H = (95+460)\cdot R$ $\quad T_H = 555\cdot R$

The $\mathrm{COP_{Ideal}}$ is
$$\mathrm{COP_{Ideal}} = \frac{T_L}{T_H - T_L} = \frac{525}{555-528} = 19.4$$

The EER is a similar measure to COP except the cooling rate (numerator) is in BTU/hr and the electrical input (denominator) is in W:

$$\mathrm{EER_{Ideal}} = \mathrm{COP_{Ideal}} \times \frac{\frac{BTU}{hr}}{W} = 19.4 \times \frac{2545\cdot\frac{BTU}{hr}}{746\cdot W} = 66.2\cdot\frac{\frac{BTU}{hr}}{W}$$

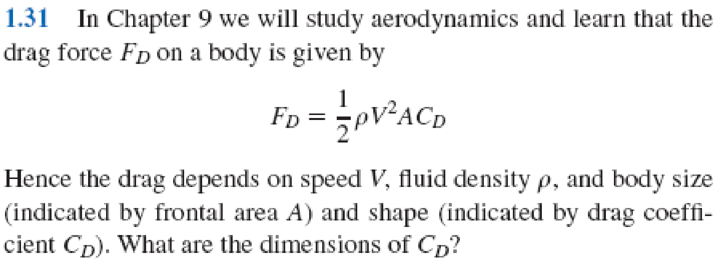
This compares to Energy Star compliant values of about 15 BTU/hr/W! We have some way to go! We can define the isentropic efficiency as

$$\eta_{isen} = \frac{\mathrm{EER_{Actual}}}{\mathrm{EER_{Ideal}}}$$

Hence the isentropic efficiency of a very good AC is about 22.5%.

Problem 1.31

[1]



1.31 In Chapter 9 we will study aerodynamics and learn that the drag force F_D on a body is given by

$$F_D = \frac{1}{2}\rho V^2 A C_D$$

Hence the drag depends on speed V, fluid density ρ, and body size (indicated by frontal area A) and shape (indicated by drag coefficient C_D). What are the dimensions of C_D?

Given: Equation for drag on a body.

Find: Dimensions of C_D.

Solution: Use the drag equation. Then "solve" for CD and use dimensions.

The drag equation is $F_D = \frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D$

"Solving" for C_D, and using dimensions $C_D = \dfrac{2\cdot F_D}{\rho\cdot V^2\cdot A}$

$$C_D = \frac{F}{\frac{M}{L^3}\times\left(\frac{L}{t}\right)^2\times L^2}$$

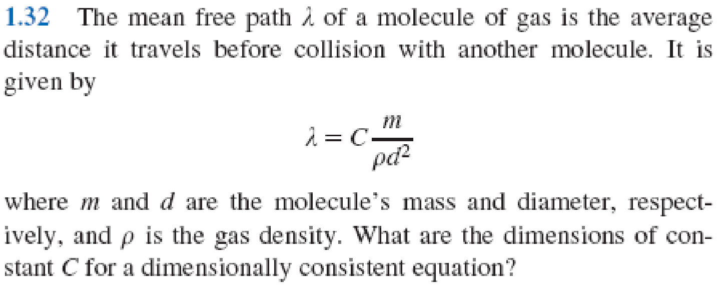
But, From Newton's 2nd law $\text{Force} = \text{Mass}\cdot\text{Acceleration}$ or $F = M\cdot\frac{L}{t^2}$

Hence $C_D = \dfrac{F}{\frac{M}{L^3}\times\left(\frac{L}{t}\right)^2\times L^2} = \dfrac{M\cdot L}{t^2}\times\dfrac{L^3}{M}\times\dfrac{t^2}{L^2}\times\dfrac{1}{L^2} = 0$

The drag coefficient is dimensionless.

Problem 1.32

[1]



1.32 The mean free path λ of a molecule of gas is the average distance it travels before collision with another molecule. It is given by

$$\lambda = C\frac{m}{\rho d^2}$$

where m and d are the molecule's mass and diameter, respectively, and ρ is the gas density. What are the dimensions of constant C for a dimensionally consistent equation?

Given: Equation for mean free path of a molecule.

Find: Dimensions of C for a diemsionally consistent equation.

Solution: Use the mean free path equation. Then "solve" for C and use dimensions.

The mean free path equation is $\lambda = C\cdot\frac{m}{\rho\cdot d^2}$

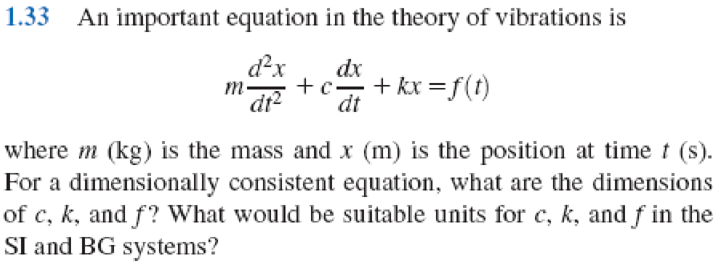
"Solving" for C, and using dimensions $C = \frac{\lambda\cdot\rho\cdot d^2}{m}$

$$C = \frac{L\times\frac{M}{L^3}\times L^2}{M} = 0$$

The drag constant C is dimensionless.

Problem 1.33

[1]


1.33 An important equation in the theory of vibrations is

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

where m (kg) is the mass and x (m) is the position at time t (s). For a dimensionally consistent equation, what are the dimensions of c, k, and f? What would be suitable units for c, k, and f in the SI and BG systems?

Given: Equation for vibrations.

Find: Dimensions of c, k and f for a dimensionally consistent equation. Also, suitable units in SI and BG systems.

Solution: Use the vibration equation to find the diemsions of each quantity

The first term of the equation is $m\cdot\frac{d^2x}{dt^2}$

The dimensions of this are $M\times\frac{L}{t^2}$

Each of the other terms must also have these dimensions.

Hence $c\cdot\frac{dx}{dt} = \frac{M\cdot L}{t^2}$ so $c\times\frac{L}{t} = \frac{M\cdot L}{t^2}$ and $c = \frac{M}{t}$

$k\cdot x = \frac{M\cdot L}{t^2}$ so $k\times L = \frac{M\cdot L}{t^2}$ and $k = \frac{M}{t^2}$

$f = \frac{M\cdot L}{t^2}$

Suitable units for c, k, and f are c: $\frac{kg}{s}$ $\frac{slug}{s}$ k: $\frac{kg}{s^2}$ $\frac{slug}{s^2}$ f: $\frac{kg\cdot m}{s^2}$ $\frac{slug\cdot ft}{s^2}$

Note that c is a damping (viscous) friction term, k is a spring constant, and f is a forcing function. These are more typically expressed using F rather than M (mass). From Newton's 2nd law:

$$F = M\cdot\frac{L}{t^2} \qquad \text{or} \qquad M = \frac{F\cdot t^2}{L}$$

Using this in the dimensions and units for c, k, and f we fin $c = \frac{F\cdot t^2}{L\cdot t} = \frac{F\cdot t}{L}$ $k = \frac{F\cdot t^2}{L\cdot t^2} = \frac{F}{L}$ $f = F$

c: $\frac{N\cdot s}{m}$ $\frac{lbf\cdot s}{ft}$ k: $\frac{N}{m}$ $\frac{lbf}{ft}$ f: N lbf

Problem 1.34

1.34 A parameter that is often used in describing pump performance is the specific speed, $N_{S_{cu}}$, given by

$$N_{S_{cu}} = \frac{N(\text{rpm})\,[Q(\text{gpm})]^{1/2}}{[H(\text{ft})]^{3/4}}$$

What are the units of specific speed? A particular pump has a specific speed of 2000. What will be the specific speed in SI units (angular velocity in rad/s)?

Given: Specific speed in customary units

Find: Units; Specific speed in SI units

Solution:

The units are $\dfrac{\text{rpm}\cdot\text{gpm}^{\frac{1}{2}}}{\text{ft}^{\frac{3}{4}}}$ or $\dfrac{\text{ft}^{\frac{3}{4}}}{\text{s}^{\frac{3}{2}}}$

Using data from tables (e.g. Table G.2)

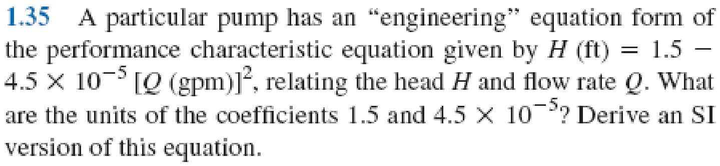
$$N_{Scu} = 2000\cdot\frac{\text{rpm}\cdot\text{gpm}^{\frac{1}{2}}}{\text{ft}^{\frac{3}{4}}}$$

$$N_{Scu} = 2000\times\frac{\text{rpm}\cdot\text{gpm}^{\frac{1}{2}}}{\text{ft}^{\frac{3}{4}}}\times\frac{2\cdot\pi\cdot\text{rad}}{1\cdot\text{rev}}\times\frac{1\cdot\text{min}}{60\cdot\text{s}}\times\left(\frac{4\times 0.000946\cdot\text{m}^3}{1\cdot\text{gal}}\cdot\frac{1\cdot\text{min}}{60\cdot\text{s}}\right)^{\frac{1}{2}}\times\left(\frac{\frac{1}{12}\cdot\text{ft}}{0.0254\cdot\text{m}}\right)^{\frac{3}{4}}$$

$$N_{Scu} = 4.06\cdot\frac{\frac{\text{rad}}{\text{s}}\cdot\left(\frac{\text{m}^3}{\text{s}}\right)^{\frac{1}{2}}}{\text{m}^{\frac{3}{4}}}$$

Problem 1.35

[1]



1.35 A particular pump has an "engineering" equation form of the performance characteristic equation given by H (ft) = 1.5 − 4.5 × 10^{-5} [Q (gpm)]^2, relating the head H and flow rate Q. What are the units of the coefficients 1.5 and 4.5 × 10^{-5}? Derive an SI version of this equation.

Given: "Engineering" equation for a pump

Find: SI version

Solution:

The dimensions of "1.5" are ft.

The dimensions of "4.5 x 10^{-5}" are ft/gpm^2.

Using data from tables (e.g. Table G.2), the SI versions of these coefficients can be obtained

$$1.5\cdot\text{ft} = 1.5\cdot\text{ft}\times\frac{0.0254\cdot\text{m}}{\frac{1}{12}\cdot\text{ft}} = 0.457\cdot\text{m}$$

$$4.5\times10^{-5}\cdot\frac{\text{ft}}{\text{gpm}^2} = 4.5\cdot10^{-5}\cdot\frac{\text{ft}}{\text{gpm}^2}\times\frac{0.0254\cdot\text{m}}{\frac{1}{12}\cdot\text{ft}}\times\left(\frac{1\cdot\text{gal}}{4\cdot\text{quart}}\cdot\frac{1\text{quart}}{0.000946\cdot\text{m}^3}\cdot\frac{60\cdot\text{s}}{1\text{min}}\right)^2$$

$$4.5\cdot10^{-5}\cdot\frac{\text{ft}}{\text{gpm}^2} = 3450\cdot\frac{\text{m}}{\left(\frac{\text{m}^3}{\text{s}}\right)^2}$$

The equation is

$$\text{H}(\text{m}) = 0.457 - 3450\cdot\left(\text{Q}\left(\frac{\text{m}^3}{\text{s}}\right)\right)^2$$

Problem 1.36

[2]

1.36 A container weighs 3.5 lbf when empty. When filled with water at 90°F, the mass of the container and its contents is 2.5 slug. Find the weight of water in the container, and its volume in cubic feet, using data from Appendix A.

Given: Empty container weighing 3.5 lbf when empty, has a mass of 2.5 slug when filled with water at 90°F.

Find:

a. Weight of water in the container

b. Container volume in ft^3

Solution: Basic equation: $F = ma$

Weight is the force of gravity on a body, $W = mg$

Then

$$W_t = W_{H_2O} + W_c$$

$$W_{H_2O} = W_t - W_c = mg - W_c$$

$$W_{H_2O} = 2.5\,\text{slug} \times 32.2\frac{\text{ft}}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} - 3.5\,\text{lbf} = 77.0\,\text{lbf}$$

The volume is given by

$$\forall = \frac{M_{H_2O}}{\rho} = \frac{M_{H_2O}g}{\rho g} = \frac{W_{H_2O}}{\rho g}$$

From Table A.7, $\rho = 1.93\,\text{slug/ft}^3$ at $T = 90°\text{F}$

$$\therefore\ \forall = 77.0\,\text{lbf} \times \frac{\text{ft}^3}{1.93\,\text{slug}} \times \frac{\text{s}^2}{32.2\,\text{ft}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 1.24\,\text{ft}^3$$

Problem 1.37

[2]

1.37 Calculate the density of standard air in a laboratory from the ideal gas equation of state. Estimate the experimental uncertainty in the air density calculated for standard conditions (29.9 in. of mercury and 59°F) if the uncertainty in measuring the barometer height is ±0.1 in. of mercury and the uncertainty in measuring temperature is ±0.5°F. (Note that 29.9 in. of mercury corresponds to 14.7 psia.)

Given: Air at standard conditions – p = 29.9 in Hg, T = 59°F

Uncertainty: in p is ± 0.1 in Hg, in T is ± 0.5°F

Note that 29.9 in Hg corresponds to 14.7 psia

Find:

a. air density using ideal gas equation of state.

b. estimate of uncertainty in calculated value.

Solution:

$$\rho = \frac{p}{RT} = 14.7\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lb}\cdot{}^\circ\text{R}}{53.3\ \text{ft}\cdot\text{lbf}} \times \frac{1}{519^\circ\,\text{R}} \times 144\frac{\text{in}^2}{\text{ft}^2}$$

$$\rho = 0.0765\ \text{lbm}/\text{ft}^3$$

The uncertainty in density is given by

$$u_\rho = \left[\left(\frac{p}{\rho}\frac{\partial\rho}{\partial p}u_p\right)^2 + \left(\frac{T}{\rho}\frac{\partial\rho}{\partial T}u_T\right)^2\right]^{1/2}$$

$$\frac{p}{\rho}\frac{\partial\rho}{\partial p} = RT\frac{1}{RT} = \frac{RT}{RT} = 1; \qquad u_p = \frac{\pm 0.1}{29.9} = \pm 0.334\%$$

$$\frac{T}{\rho}\frac{\partial\rho}{\partial T} = \frac{T}{\rho}\left(-\frac{p}{RT^2}\right) = -\frac{p}{\rho RT} = -1; \qquad u_T = \frac{\pm 0.5}{460+59} = \pm 0.0963\%$$

Then

$$u_\rho = \left[\left(u_p\right)^2 + \left(-u_T\right)^2\right]^{1/2} = \pm\left[(0.334)^2 + (-0.0963)^2\right]$$

$$u_\rho = \pm 0.348\%\ \left(\pm 2.66\times 10^{-4}\ \text{lbm}/\text{ft}^3\right)$$

Problem 1.38

[2]

1.38 Repeat the calculation of uncertainty described in Problem 1.37 for air in a freezer. Assume the measured barometer height is 759 ± 1 mm of mercury and the temperature is –20 ± 0.5 C. [Note that 759 mm of mercury corresponds to 101 kPa (abs).]

Given: Air at pressure, p = 759 ± 1 mm Hg and temperature, T = –20 ± 0.5°C.

Note that 759 mm Hg corresponds to 101 kPa.

Find:

a. Air density using ideal gas equation of state

b. Estimate of uncertainty in calculated value

Solution: $\rho = \frac{p}{RT} = 101 \times 10^3 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\ \text{N}\cdot\text{m}} \times \frac{1}{253\ \text{K}} = 1.39\ \text{kg}/\text{m}^3$

$$u_\rho = \left[\left(\frac{p}{\rho}\frac{\partial\rho}{\partial p}u_p\right)^2 + \left(\frac{T}{\rho}\frac{\partial\rho}{\partial T}u_T\right)^2\right]^{1/2}$$

The uncertainty in density is given by

$$\frac{p}{\rho}\frac{\partial\rho}{\partial p} = RT\frac{1}{RT} = 1; \qquad u_p = \frac{\pm 1}{759} = \pm 0.132\%$$

$$\frac{T}{\rho}\frac{\partial\rho}{\partial T} = \frac{T}{\rho}\left(-\frac{p}{RT^2}\right) = -\frac{p}{\rho RT} = -1; \qquad u_T = \frac{\pm 0.5}{273 - 20} = \pm 0.198\%$$

Then

$$u_\rho = \left[(u_p)^2 + (-u_T)^2\right]^{1/2} = \pm\left[(0.132)^2 + (-0.198)^2\right]^{1/2}$$

$$u_\rho = \pm 0.238\% \quad \left(\pm 3.31 \times 10^{-3}\ \text{kg}/\text{m}^3\right)$$

Problem 1.39 [2]

1.39 The mass of the standard American golf ball is 1.62 ± 0.01 oz and its mean diameter is 1.68 ± 0.01 in. Determine the density and specific gravity of the American golf ball. Estimate the uncertainties in the calculated values.

Given:

Standard American golf ball:
$$\begin{aligned} &m = 1.62 \pm 0.01\,\text{oz} \quad (20\text{ to }1)\\ &D = 1.68 \pm 0.01\,\text{in.} \quad (20\text{ to }1)\end{aligned}$$

Find:

a. Density and specific gravity.

b. Estimate uncertainties in calculated values.

Solution:

Density is mass per unit volume, so

$$\rho = \frac{m}{\forall} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{m}{(D/2)^3} = \frac{6}{\pi}\frac{m}{D^3}$$

$$\rho = \frac{6}{\pi} \times 1.62\text{ oz} \times \frac{1}{(1.68)^3\text{ in.}^3} \times \frac{0.4536\text{ kg}}{16\text{ oz}} \times \frac{\text{in.}^3}{(0.0254)^3\text{ m}^3} = 1130\text{ kg/m}^3$$

and

$$SG = \frac{\rho}{\rho H_2O} = 1130\,\frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{1000\text{ kg}} = 1.13$$

The uncertainty in density is given by

$$u_\rho = \pm\left[\left(\frac{m}{\rho}\frac{\partial\rho}{\partial m}u_m\right)^2 + \left(\frac{D}{\rho}\frac{\partial\rho}{\partial D}u_D\right)^2\right]^{1/2}$$

$$\frac{m}{\rho}\frac{\partial\rho}{\partial m} = \frac{m}{\rho}\frac{1}{\forall} = \frac{\forall}{\forall} = 1;\;\; u_m = \pm\frac{0.01}{1.62} = \pm 0.617\text{ percent}$$

$$\frac{D}{\rho}\frac{\partial\rho}{\partial D} = \frac{D}{\rho}\left(-3\,\frac{6}{\pi}\frac{m}{D^4}\right) = \frac{\pi D^4}{6\text{ m}}\left(-3\frac{6}{\pi}\frac{m}{D^4}\right) = -3;\;\; u_D = \pm 0.595\text{ percent}$$

Thus

$$\begin{aligned} u_\rho &= \pm\left[(u_m)^2 + (-3u_D)^2\right]^{1/2}\\ &= \pm\left\{(0.617)^2 + \left[-3(0.595)^2\right]\right\}^{\frac{1}{2}}\\ u_\rho &= \pm 1.89\text{ percent }\left(\pm 21.4\text{ kg/m}^3\right)\\ u_{SG} &= u_\rho = \pm 1.89\text{ percent }(\pm 0.0214)\end{aligned}$$

Finally,

$$\begin{aligned} \rho &= 1130 \pm 21.4\text{ kg/m}^3\text{ (20 to 1)}\\ SG &= 1.13 \pm 0.0214\text{ (20 to 1)}\end{aligned}$$

Problem 1.40

[2]

1.40 The mass flow rate in a water flow system determined by collecting the discharge over a timed interval is 0.2 kg/s. The scales used can be read to the nearest 0.05 kg and the stopwatch is accurate to 0.2 s. Estimate the precision with which the flow rate can be calculated for time intervals of (a) 10 s and (b) 1 min.

Given: Mass flow rate of water determined by collecting discharge over a timed interval is 0.2 kg/s.

Scales can be read to nearest 0.05 kg.

Stopwatch can be read to nearest 0.2 s.

Find: Estimate precision of flow rate calculation for time intervals of (a) 10 s, and (b) 1 min.

Solution: Apply methodology of uncertainty analysis, Appendix F:

Computing equations:

$$\dot{m} = \frac{\Delta m}{\Delta t}$$

$$u_{\dot{m}} = \pm\left[\left(\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m}u_{\Delta m}\right)^2 + \left(\frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t}u_{\Delta t}\right)^2\right]^{\frac{1}{2}}$$

Thus

$$\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m} = \Delta t\left(\frac{1}{\Delta t}\right) = 1 \quad \text{and} \quad \frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t} = \frac{\Delta t^2}{\Delta m}\left[(-1)\frac{\Delta m}{\Delta t^2}\right] = -1$$

The uncertainties are expected to be ± half the least counts of the measuring instruments.

Tabulating results:

Time Interval, Δt(s)	**Error in Δt(s)**	**Uncertainty in Δt (percent)**	**Water Collected, Δm(kg)**	**Error in Δm(kg)**	**Uncertainty in Δm (percent)**	**Uncertainty in $\dot{m}$ (percent)**
10	± 0.10	± 1.0	2.0	± 0.025	± 1.25	± 1.60
60	± 0.10	± 0.167	12.0	± 0.025	± 0.208	± 0.267

A time interval of about 15 seconds should be chosen to reduce the uncertainty in results to ± 1 percent.

Problem 1.41

[2]

1.41 A can of pet food has the following internal dimensions: 102 mm height and 73 mm diameter (each ±1 mm at odds of 20 to 1). The label lists the mass of the contents as 397 g. Evaluate the magnitude and estimated uncertainty of the density of the pet food if the mass value is accurate to ±1 g at the same odds.

Given: Pet food can

$$H = 102 \pm 1 \text{ mm (20 to 1)}$$
$$D = 73 \pm 1 \text{ mm (20 to 1)}$$
$$m = 397 \pm 1 \text{ g (20 to 1)}$$

Find: Magnitude and estimated uncertainty of pet food density.

Solution: Density is

$$\rho = \frac{m}{\forall} = \frac{m}{\pi R^2 H} = \frac{4}{\pi}\frac{m}{D^2 H} \quad \text{or} \quad \rho = \rho\,(m, D, H)$$

From uncertainty analysis

$$u_\rho = \pm\left[\left(\frac{m}{\rho}\frac{\partial\rho}{\partial m}u_m\right)^2 + \left(\frac{D}{\rho}\frac{\partial\rho}{\partial D}u_D\right)^2 + \left(\frac{H}{\rho}\frac{\partial\rho}{\partial H}u_H\right)^2\right]^{\frac{1}{2}}$$

Evaluating,

$$\frac{m}{\rho}\frac{\partial\rho}{\partial m} = \frac{m}{\rho}\frac{4}{\pi}\frac{1}{D^2 H} = \frac{1}{\rho}\frac{4\,m}{\pi D^2 H} = 1; \qquad u_m = \frac{\pm 1}{397} = \pm 0.252\%$$

$$\frac{D}{\rho}\frac{\partial\rho}{\partial D} = \frac{D}{\rho}(-2)\frac{4m}{\pi D^3 H} = (-2)\frac{1}{\rho}\frac{4m}{\pi D^2 H} = -2; \qquad u_D = \frac{\pm 1}{73} = \pm 1.37\%$$

$$\frac{H}{\rho}\frac{\partial\rho}{\partial H} = \frac{H}{\rho}(-1)\frac{4m}{\pi D^2 H^2} = (-1)\frac{1}{\rho}\frac{4m}{\pi D^2 H} = -1; \qquad u_H = \frac{\pm 1}{102} = \pm 0.980\%$$

Substituting

$$u_\rho = \pm\left\{[(1)(0.252)]^2 + [(-2)(1.37)]^2 + [(-1)(0.980)]^2\right\}^{\frac{1}{2}}$$
$$u_\rho = \pm 2.92 \text{ percent}$$

$$\forall = \frac{\pi}{4}D^2 H = \frac{\pi}{4} \times (73)^2\ \text{mm}^2 \times 102\ \text{mm} \times \frac{\text{m}^3}{10^9\ \text{mm}^3} = 4.27 \times 10^{-4}\ \text{m}^3$$

$$\rho = \frac{m}{\forall} = \frac{397\ \text{g}}{4.27 \times 10^{-4}\ \text{m}^3} \times \frac{\text{kg}}{1000\ \text{g}} = 930\ \text{kg/m}^3$$

Thus $\rho = 930 \pm 27.2\ \text{kg/m}^3$ (20 to 1)

Problem 1.42 [2]

1.42 The mass of the standard British golf ball is 45.9 ± 0.3 g and its mean diameter is 41.1 ± 0.3 mm. Determine the density and specific gravity of the British golf ball. Estimate the uncertainties in the calculated values.

Given: Standard British golf ball:

$$m = 45.9 \pm 0.3\ \text{g} \quad (20\ \text{to}\ 1)$$
$$D = 41.1 \pm 0.3\ \text{mm} \quad (20\ \text{to}\ 1)$$

Find:

a. Density and specific gravity

b. Estimate of uncertainties in calculated values.

Solution: Density is mass per unit volume, so

$$\rho = \frac{m}{\forall} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{m}{(D/2)^3} = \frac{6}{\pi}\frac{m}{D^3}$$

$$\rho = \frac{6}{\pi} \times 0.0459\ \text{kg} \times \frac{1}{(0.0411)^3}\ \text{m}^3 = 1260\ \text{kg}/\text{m}^3$$

and

$$SG = \frac{\rho}{\rho_{H_2O}} = 1260\ \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{1000\ \text{kg}} = 1.26$$

The uncertainty in density is given by

$$u_\rho = \pm\left[\left(\frac{m}{\rho}\frac{\partial\rho}{\partial m}u_m\right)^2 + \left(\frac{D}{\rho}\frac{\partial\rho}{\partial D}u_D\right)^2\right]^{1/2}$$

$$\frac{m}{\rho}\frac{\partial\rho}{\partial m} = \frac{m}{\rho}\frac{1}{\forall} = \frac{\forall}{\forall} = 1; \quad u_m = \pm\frac{0.3}{45.9} = \pm 0.654\%$$

$$\frac{D}{\rho}\frac{\partial\rho}{\partial D} = \frac{D}{\rho}\left(-3\frac{6}{\pi}\frac{m}{D^4}\right) = -3\left(\frac{6m}{\pi D^3 \rho}\right) = -3$$

$$u_D = \pm\frac{0.3}{41.1} = 0.730\%$$

Thus

$$u_\rho = \pm[(u_m)^2 + (-3u_D)^2]^{1/2} = \pm\left\{(0.654)^2 + [-3(0.730)]^2\right\}^{1/2}$$

$$u_\rho = \pm 2.29\%\ (\pm 28.9\ \text{kg}/\text{m}^3)$$

$$u_{SG} = u_\rho = \pm 2.29\%\ (\pm 0.0289)$$

Summarizing

$$\rho = 1260 \pm 28.9\ \text{kg}/\text{m}^3\ (20\ \text{to}\ 1)$$

$$SG = 1.26 \pm 0.0289\ (20\ \text{to}\ 1)$$

Problem 1.43

[3]

1.43 The mass flow rate of water in a tube is measured using a beaker to catch water during a timed interval. The nominal mass flow rate is 100 g/s. Assume that mass is measured using a balance with a least count of 1 g and a maximum capacity of 1 kg, and that the timer has a least count of 0.1 s. Estimate the time intervals and uncertainties in measured mass flow rate that would result from using 100, 500, and 1000 mL beakers. Would there be any advantage in using the largest beaker? Assume the tare mass of the empty 1000 mL beaker is 500 g.

Given: Nominal mass flow rate of water determined by collecting discharge (in a beaker) over a timed interval is $\dot{m} = 100 \text{ g/s}$

- Scales have capacity of 1 kg, with least count of 1 g.
- Timer has least count of 0.1 s.
- Beakers with volume of 100, 500, 1000 mL are available – tare mass of 1000 mL beaker is 500 g.

Find: Estimate (a) time intervals, and (b) uncertainties, in measuring mass flow rate from using each of the three beakers.

Solution: To estimate time intervals assume beaker is filled to maximum volume in case of 100 and 500 mL beakers and to maximum allowable mass of water (500 g) in case of 1000 mL beaker.

Then $\dot{m} = \frac{\Delta m}{\Delta t}$ and $\Delta t = \frac{\Delta m}{\dot{m}} = \frac{\rho \Delta \forall}{\dot{m}}$

Tabulating results

$$\begin{array}{llll} \Delta\forall = & 100 \text{ mL} & 500 \text{ mL} & 1000 \text{ mL} \\ \Delta t = & 1 \text{ s} & 5 \text{ s} & 5 \text{ s} \end{array}$$

Apply the methodology of uncertainty analysis, Appendix E Computing equation:

$$u_{\dot{m}} = \pm\left[\left(\frac{\Delta m}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta m}u_{\Delta m}\right)^2 + \left(\frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t}u_{\Delta t}\right)^2\right]^{1/2}$$

The uncertainties are expected to be ± half the least counts of the measuring instruments

$$\delta\Delta m = \pm 0.5 \text{ g} \qquad \delta\Delta t = 0.05 \text{ s}$$

$$\frac{\Delta m}{\dot{m}} = \frac{\partial \dot{m}}{\partial \Delta m} = \Delta t\left(\frac{1}{\Delta t}\right) = 1 \qquad \text{and} \qquad \frac{\Delta t}{\dot{m}}\frac{\partial \dot{m}}{\partial \Delta t} = \frac{(\Delta t)^2}{\Delta m}\left[-\frac{\Delta m}{(\Delta t)^2}\right] = -1$$

$$\therefore u_{\dot{m}} = \pm\left[\left(u_{\Delta m}\right)^2 + \left(-u_{\Delta t}\right)^2\right]^{1/2}$$

Tabulating results:

Beaker Volume $\Delta\forall$ (mL)	Water Collected Δm(g)	Error in Δm(g)	Uncertainty in Δm (percent)	Time Interval Δt(s)	Error in Δt(s)	Uncertainty in Δt (percent)	in $\dot{m}$ (percent)
100	100	± 0.50	± 0.50	1.0	± 0.05	± 5.0	± 5.03
500	500	± 0.50	± 0.10	5.0	± 0.05	± 1.0	± 1.0
1000	500	± 0.50	± 0.10	5.0	± 0.05	± 1.0	± 1.0

Since the scales have a capacity of 1 kg and the tare mass of the 1000 mL beaker is 500 g, there is no advantage in using the larger beaker. The uncertainty in $\dot{m}$ could be reduced to ± 0.50 percent by using the large beaker if a scale with greater capacity the same least count were available

Problem 1.44

1.44 The estimated dimensions of a soda can are *D* = 66.0 ± 0.5 mm and *H* = 110 ± 0.5 mm. Measure the mass of a full can and an empty can using a kitchen scale or postal scale. Estimate the volume of soda contained in the can. From your measurements estimate the depth to which the can is filled and the uncertainty in the estimate. Assume the value of SG = 1.055, as supplied by the bottler.

Given: Soda can with estimated dimensions D = 66.0 ± 0.5 mm, H = 110 ± 0.5 mm. Soda has SG = 1.055

Find:

a. volume of soda in the can (based on measured mass of full and empty can).

b. estimate average depth to which the can is filled and the uncertainty in the estimate.

Solution: Measurements on a can of coke give

$$m_f = 386.5 \pm 0.50 \text{ g}, \quad m_e = 17.5 \pm 0.50 \text{ g} \therefore m = m_f - m_e = 369 \pm u_m \text{ g}$$

$$u_m = \pm\left[\left(\frac{m_f}{m}\frac{\partial m}{\partial m_f}u_{m_f}\right)^2 + \left(\frac{m_e}{m}\frac{\partial m}{\partial m_e}u_{m_e}\right)^2\right]^{1/2}$$

$$u_{m_f} = \pm\frac{0.5\text{ g}}{386.5\text{ g}} = \pm 0.00129, \quad u_{m_e} = \pm\frac{0.50}{17.5} = 0.0286$$

$$\therefore u_m = \pm\left\{\left[\frac{386.5}{369}(1)(0.00129)\right]^2 + \left[\frac{17.5}{369}(-1)(0.0286)\right]^2\right\}^{1/2} = 0.0019$$

Density is mass per unit volume and SG = ρ/ρH_2O so

$$\forall = \frac{m}{\rho} = \frac{m}{\rho H_2O \text{ SG}} = 369 \text{ g} \times \frac{\text{m}^3}{1000 \text{ kg}} \times \frac{1}{1.055} \times \frac{\text{kg}}{1000 \text{ g}} = 350 \times 10^{-6} \text{ m}^3$$

The reference value ρH_2O is assumed to be precise. Since SG is specified to three places beyond the decimal point, assume u_{SG} = ± 0.001. Then

$$u_v = \pm\left[\left(\frac{m}{v}\frac{\partial v}{\partial m}u_m\right)^2 + \left(\frac{m}{SG}\frac{\partial v}{\partial SG}\right)^2\right]^{1/2} = \pm\left\{[(1)\ u_m]^2 + [(-1)\ u_{SG}]^2\right\}^{1/2}$$

$$u_v = \pm\left\{[(1)(0.0019)]^2 + [(-1)(0.001)]^2\right\}^{1/2} = 0.0021 \text{ or } 0.21\%$$

$$\forall = \frac{\pi D^2}{4}L \text{ or } L = \frac{4\forall}{\pi D^2} = \frac{4}{\pi} \times \frac{350 \times 10^{-6}\ \text{m}^3}{(0.066)^2\ \text{m}^2} \times \frac{10^3\ \text{mm}}{\text{m}} = 102\ \text{mm}$$

$$u_L = \pm\left[\left(\frac{\forall}{L}\frac{\partial L}{\partial \forall}u_\forall\right)^2\right] + \left[\left(\frac{D}{L}\frac{\partial L}{\partial D}u_D\right)^2\right]^{1/2}$$

$$\frac{\forall}{L}\frac{\partial L}{\partial \forall} = \frac{\pi D^2}{4} \times \frac{4}{\pi D^2} = 1\ u_D = \pm\frac{0.5\ \text{mm}}{66\ \text{mm}} = 0.0076$$

$$\frac{D}{L}\frac{\partial L}{\partial D} = D\frac{\pi D^2}{4\forall} \times \frac{4\forall}{\pi}\left(-\frac{2}{D^3}\right) = -2$$

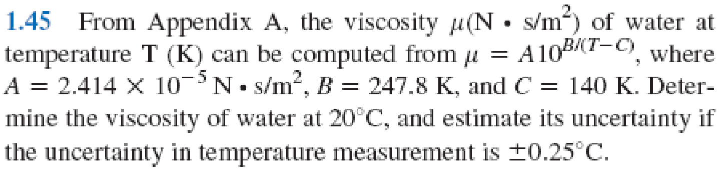
$$u_L = \pm\left\{[(1)(0.0021)]^2 + [(-2)\ (0.0076)]^2\right\}^{1/2} = 0.0153 \text{ or } 1.53\%$$

Note:

1. Printing on the can states the content as 355 ml. This suggests that the implied accuracy of the SG value may be over stated.
2. Results suggest that over seven percent of the can height is void of soda.

Problem 1.45

[3]



1.45 From Appendix A, the viscosity $\mu(\mathrm{N \cdot s/m^2})$ of water at temperature **T** (K) can be computed from $\mu = A10^{B/(T-C)}$, where $A = 2.414 \times 10^{-5}\ \mathrm{N \cdot s/m^2}$, $B = 247.8$ K, and $C = 140$ K. Determine the viscosity of water at 20°C, and estimate its uncertainty if the uncertainty in temperature measurement is ±0.25°C.

Given: Data on water

Find: Viscosity; Uncertainty in viscosity

Solution:

The data is: $A = 2.414 \times 10^{-5} \cdot \frac{\mathrm{N \cdot s}}{\mathrm{m^2}}$ $\quad B = 247.8 \cdot \mathrm{K}$ $\quad C = 140 \cdot \mathrm{K}$ $\quad T = 293 \cdot \mathrm{K}$

The uncertainty in temperature is $u_T = \frac{0.25 \cdot \mathrm{K}}{293 \cdot \mathrm{K}}$ $\quad u_T = 0.085 \cdot \%$

Also $\mu(T) = A \cdot 10^{\frac{B}{(T-C)}}$ Evaluating $\mu(T) = 1.01 \times 10^{-3} \cdot \frac{\mathrm{N \cdot s}}{\mathrm{m^2}}$

For the uncertainty

$$\frac{d}{dT}\mu(T) = -\frac{A \cdot B \cdot \ln(10)}{10^{\frac{B}{C-T}} \cdot (C-T)^2}$$

Hence

$$u_\mu(T) = \left| \frac{T}{\mu(T)} \cdot \frac{d}{dT}\mu(T) \cdot u_T \right| = \frac{\ln(10) \cdot \left| B \cdot T \cdot u_T \right|}{\left(\left| C - T \right| \right)^2}$$

Evaluating $u_\mu(T) = 0.609 \cdot \%$

Problem 1.46

1.46 An enthusiast magazine publishes data from its road tests on the lateral acceleration capability of cars. The measurements are made using a 150-ft-diameter skid pad. Assume the vehicle path deviates from the circle by ±2 ft and that the vehicle speed is read from a fifth-wheel speed-measuring system to ±0.5 mph. Estimate the experimental uncertainty in a reported lateral acceleration of 0.7 *g*. How would you improve the experimental procedure to reduce the uncertainty?

Given: Lateral acceleration, a = 0.70 g, measured on 150-ft diameter skid pad.

$$\left.\begin{array}{l}\text{Path deviation: } \pm 2\text{ ft}\\ \text{Vehicle speed: } \pm 0.5\text{ mph}\end{array}\right\}\text{measurement uncertainty}$$

Find:

a. Estimate uncertainty in lateral acceleration.

b. How could experimental procedure be improved?

Solution: Lateral acceleration is given by $a = V^2/R$.

From Appendix F, $u_a = \pm[(2u_v)^2 + (u_R)^2]^{1/2}$

From the given data, $$V^2 = aR; \quad V = \sqrt{aR} = \left[0.70 \times \frac{32.2\text{ ft}}{s^2} \times 75\text{ ft}\right]^{1/2} = 41.1\text{ ft / s}$$

Then $$u_v = \pm\frac{\delta V}{V} = \pm 0.5\frac{\text{mi}}{\text{hr}} \times \frac{\text{s}}{41.1\text{ ft}} \times 5280\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\text{ s}} = \pm 0.0178$$

and $$u_R = \pm\frac{\delta R}{R} = \pm 2\text{ ft} \times \frac{1}{75\text{ ft}} = \pm 0.0267$$

so $$u_a = \pm\left[(2 \times 0.0178)^2 + (0.0267)^2\right]^{1/2} = \pm 0.0445$$
$$u_a = \pm 4.45\text{ percent}$$

Experimental procedure could be improved by using a larger circle, assuming the absolute errors in measurement are constant.

For

$$D = 400 \text{ ft}, \quad R = 200 \text{ ft}$$

$$V = \sqrt{aR} = \left[0.70 \times \frac{32.2 \text{ ft}}{s^2} \times 200 \text{ ft}\right]^{1/2} = 67.1 \text{ ft / s} = 45.8 \text{ mph}$$

$$u_v = \pm\frac{0.5 \text{ mph}}{45.8 \text{ mph}} = \pm 0.0109; \; u_R = \pm\frac{2 \text{ ft}}{200 \text{ ft}} = \pm 0.0100$$

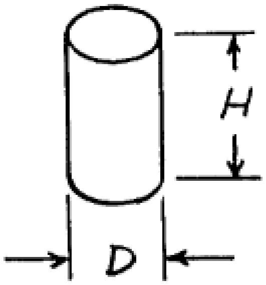
$$u_a = \pm\left[(2 \times 0.0109)^2 + (0.0100)^2\right]^{1/2} = \pm 0.0240 \text{ or } \pm 2.4 \text{ percent}$$

Problem 1.47

1.47 Using the nominal dimensions of the soda can given in Problem 1.44, determine the precision with which the diameter and height must be measured to estimate the volume of the can within an uncertainty of ±0.5 percent.

Given: Dimensions of soda can:

$$D = 66\ \text{mm}$$
$$H = 110\ \text{mm}$$



Find: Measurement precision needed to allow volume to be estimated with an uncertainty of ± 0.5 percent or less.

Solution: Use the methods of Appendix F:

Computing equations:

$$\forall = \frac{\pi D^2 H}{4}$$

$$u_\forall = \pm\left[\left(\frac{H}{\forall}\frac{\partial\forall}{\partial H}u_H\right)^2 + \left(\frac{D}{\forall}\frac{\partial\forall}{\partial D}u_D\right)^2\right]^{\frac{1}{2}}$$

Since $\forall = \frac{\pi D^2 H}{4}$, then $\frac{\partial\forall}{\partial H} = \frac{\pi D^2}{4}$ and $\frac{\partial\forall}{\partial D} = \frac{\pi DH}{2}$

Let $u_D = \pm\frac{\delta x}{D}$ and $u_H = \pm\frac{\delta x}{H}$, substituting,

$$u_\forall = \pm\left[\left(\frac{4H}{\pi D^2 H}\frac{\pi D^2}{4}\frac{\delta x}{H}\right)^2 + \left(\frac{4D}{\pi D^2 H}\frac{\pi DH}{2}\frac{\delta x}{D}\right)^2\right]^{\frac{1}{2}} = \pm\left[\left(\frac{\delta x}{H}\right)^2 + \left(\frac{2\delta x}{D}\right)^2\right]^{\frac{1}{2}}$$

Solving,

$$u_\forall^{\ 2} = \left(\frac{\delta x}{H}\right)^2 + \left(\frac{2\delta x}{D}\right)^2 = (\delta x)^2\left[\left(\frac{1}{H}\right)^2 + \left(\frac{2}{D}\right)^2\right]$$

$$\delta x = \pm \frac{u_{\forall}}{\left[\left(\frac{1}{H}\right)^2 + \left(\frac{2}{D}\right)^2\right]^{\frac{1}{2}}} = \pm \frac{0.005}{\left[\left(\frac{1}{110\text{ mm}}\right)^2 + \left(\frac{2}{66\text{ mm}}\right)^2\right]^{\frac{1}{2}}} = \pm 0.158\text{ mm}$$

Check:

$$u_H = \pm \frac{\delta x}{H} = \pm \frac{0.158\text{ mm}}{110\text{ mm}} = \pm 1.44 \times 10^{-3}$$

$$u_D = \pm \frac{\delta x}{D} = \pm \frac{0.158\text{ mm}}{66\text{ mm}} = \pm 2.39 \times 10^{-3}$$

$$u_{\forall} = \pm[(u_H)^2 + (2u_D)^2]^{\frac{1}{2}} = \pm[(0.00144)^2 + (0.00478)^2]^{\frac{1}{2}} = \pm 0.00499$$

If δx represents half the least count, a minimum resolution of about $2\,\delta x \approx 0.32$ mm is needed.

Problem 1.48

[4]

Given: American golf ball, $m = 1.62 \pm 0.01$ oz, $D = 1.68$ in.

Find: Precision to which D must be measured to estimate density within uncertainty of ± 1 percent.

Solution: Apply uncertainty concepts

Definition: Density, $\rho \equiv \frac{m}{\forall}$ $\qquad \forall = \frac{4}{3}\pi R^3 = \frac{\pi D^3}{6}$

Computing equation: $u_R = \pm\left[\left(\frac{x_1}{R}\frac{\partial R}{\partial x_1}u_{x_1}\right)^2 + \cdots\right]^{1/2}$

From the definition, $\rho = \frac{m}{\pi D^3/6} = \frac{6m}{\pi D^3} = \rho(m, D)$

Thus $\frac{m}{\rho}\frac{\partial \rho}{\partial m} = 1$ and $\frac{D}{\rho}\frac{\partial \rho}{\partial D} = 3$, so

$$u_\rho = \pm\left[(1\,u_m)^2 + (3\,u_D)^2\right]^{1/2}$$

$$u_\rho^2 = u_m^2 + 9\,u_D^2$$

Solving, $u_D = \pm\frac{1}{3}\left[u_\rho^2 - u_m^2\right]^{\frac{1}{2}}$

From the data given, $u_\rho = \pm 0.0100$

$$u_m = \frac{\pm 0.01 \text{ oz}}{1.62 \text{ oz}} = \pm 0.00617$$

$$u_D = \pm\frac{1}{3}\left[(0.0100)^2 - (0.00617)^2\right]^{\frac{1}{2}} = \pm 0.00262 \text{ or } \pm 0.262\,\%$$

Since $u_D = \pm\frac{\delta D}{D}$, then

$$\delta D = \pm D\,u_D = \pm 1.68 \text{ in.} \times 0.00262 = \pm 0.00441 \text{ in.}$$

δ

The ball diameter must be measured to a precision of ±0.00441 in. (±0.112 mm) or better to estimate density within ±1 percent. A micrometer or caliper could be used.

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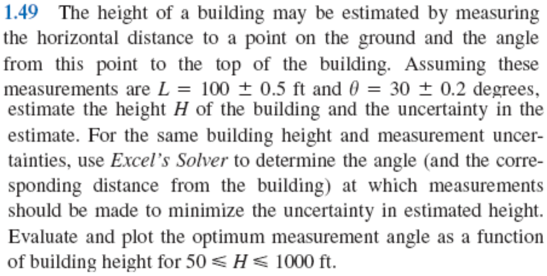
Problem 1.49

[4]



1.49 The height of a building may be estimated by measuring the horizontal distance to a point on the ground and the angle from this point to the top of the building. Assuming these measurements are $L = 100 \pm 0.5$ ft and $\theta = 30 \pm 0.2$ degrees, estimate the height *H* of the building and the uncertainty in the estimate. For the same building height and measurement uncertainties, use *Excel's Solver* to determine the angle (and the corresponding distance from the building) at which measurements should be made to minimize the uncertainty in estimated height. Evaluate and plot the optimum measurement angle as a function of building height for $50 \leqslant H \leqslant 1000$ ft.

Given: Data on length and angle measurements

Find: Height; Angle for minimum uncertainty in height; Plot

Solution:

The data is: $L = 100 \cdot ft$ $\quad \delta L = 0.5 \cdot ft$ $\quad \theta = 30 \cdot deg$ $\quad \delta\theta = 0.2 \cdot deg$

Uncertainties: $u_L = \frac{\delta L}{L}$ $\quad u_L = 0.5\%$ $\quad u_\theta = \frac{\delta\theta}{\theta}$ $\quad u_\theta = 0.667\%$

The height is: $H = L \cdot \tan(\theta)$ $\quad H = 57.7 ft$ $\quad$ with uncertainty $\quad u_H = \sqrt{\left(\frac{L}{H} \cdot \frac{\partial}{\partial L} H \cdot u_L\right)^2 + \left(\frac{\theta}{H} \cdot \frac{\partial}{\partial \theta} H \cdot u_\theta\right)^2}$

Hence with $\frac{\partial}{\partial L} H = \tan(\theta)$ $\quad \frac{\partial}{\partial \theta} H = L \cdot \left(1 + \tan(\theta)^2\right)$ $\quad u_H = \sqrt{\left(\frac{L}{H} \cdot \tan(\theta) \cdot u_L\right)^2 + \left[\frac{L \cdot \theta}{H} \cdot \left(1 + \tan(\theta)^2\right) \cdot u_\theta\right]^2}$

Evaluating $u_H = 0.949\%$ $\quad$ and $\quad \delta H = u_H \cdot H$ $\quad \delta H = 0.548 ft$

The height is then $H = 57.7 ft$ +/- $\delta H = 0.548 ft$

To plot u_H versus θ for a given H we need to replace L, u_L and u_θ with functions of θ. Doing this and simplifying

$$u_H(\theta) = \sqrt{\left(\tan(\theta) \cdot \frac{\delta L}{H}\right)^2 + \left[\frac{\delta\theta}{\tan(\theta)} \cdot \left(1 + \tan(\theta)^2\right)\right]^2}$$

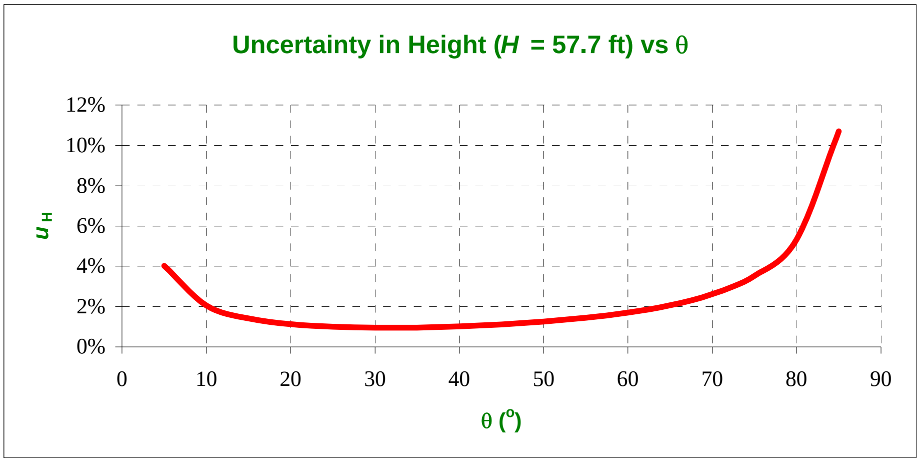
Given data:

H =	57.7	ft
δ*L* =	0.5	ft
δθ =	0.2	deg

For this building height, we are to vary θ (and therefore *L*) to minimize the uncertainty u_H.

Plotting u_H vs θ

θ (deg)	u_H
5	4.02%
10	2.05%
15	1.42%
20	1.13%
25	1.00%
30	0.95%
35	0.96%
40	1.02%
45	1.11%
50	1.25%
55	1.44%
60	1.70%
65	2.07%
70	2.62%
75	3.52%
80	5.32%
85	10.69%

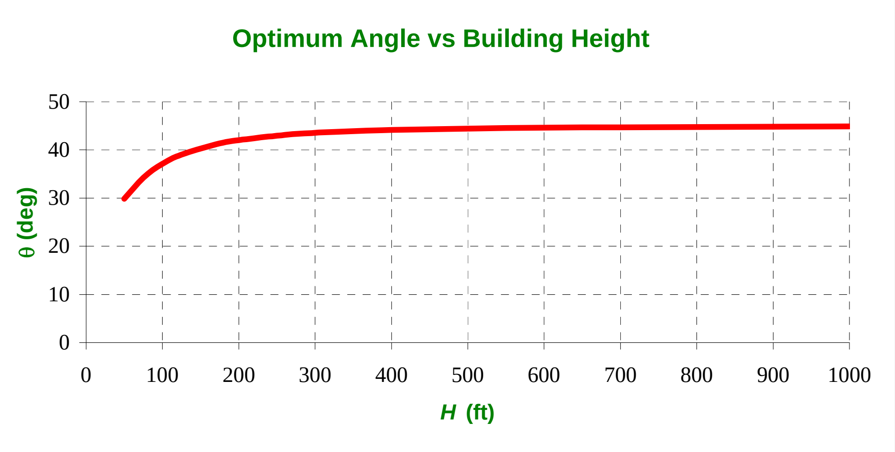


Optimizing using *Solver*

θ (deg)	u_H
31.4	0.947%

To find the optimum θ as a function of building height *H* we need a more complex *Solver*

H (ft)	θ (deg)	u_H
50	29.9	0.992%
75	34.3	0.877%
100	37.1	0.818%
125	39.0	0.784%
175	41.3	0.747%
200	42.0	0.737%
250	43.0	0.724%
300	43.5	0.717%
400	44.1	0.709%
500	44.4	0.705%
600	44.6	0.703%
700	44.7	0.702%
800	44.8	0.701%
900	44.8	0.700%
1000	44.9	0.700%



Use *Solver* to vary ALL θ's to minimize the total u_H!

Total u_H's: 11.3%

Problem 1.50

[5]

1.50 In the design of a medical instrument it is desired to dispense 1 cubic millimeter of liquid using a piston-cylinder syringe made from molded plastic. The molding operation produces plastic parts with estimated dimensional uncertainties of ±0.002 in. Estimate the uncertainty in dispensed volume that results from the uncertainties in the dimensions of the device. Plot on the same graph the uncertainty in length, diameter, and volume dispensed as a function of cylinder diameter *D* from *D* = 0.5 to 2 mm. Determine the ratio of stroke length to bore diameter that gives a design with minimum uncertainty in volume dispensed. Is the result influenced by the magnitude of the dimensional uncertainty?

Given:

Piston-cylinder device to have $\forall = 1\,\text{mm}^3$.

Molded plastic parts with dimensional uncertainties, $\delta = \pm\,0.002$ in.

Find:

a. Estimate of uncertainty in dispensed volume that results from the dimensional uncertainties.

b. Determine the ratio of stroke length to bore diameter that minimizes $u_\forall$; plot of the results.

c. Is this result influenced by the magnitude of δ?

Solution:

Apply uncertainty concepts from Appendix F:

Computing equation: $$\forall = \frac{\pi D^2 L}{4};\quad u_\forall = \pm\left[\left(\frac{L}{\forall}\frac{\partial\forall}{\partial L}u_L\right)^2 + \left(\frac{D}{\forall}\frac{\partial\forall}{\partial D}u_D\right)^2\right]^{\frac{1}{2}}$$

From $\forall$, $\frac{L}{\forall}\frac{\partial\forall}{\partial L} = 1$, and $\frac{D}{\forall}\frac{\partial\forall}{\partial D} = 2$, so $u_\forall = \pm[u_L^2 + (2u_D)^2]^{\frac{1}{2}}$

The dimensional uncertainty is $\delta = \pm 0.002\text{ in.} \times 25.4\,\frac{\text{mm}}{\text{in.}} = \pm 0.0508\text{ mm}$

Assume D = 1 mm. Then $L = \frac{4\forall}{\pi D^2} = \frac{4}{\pi} \times 1\,\text{mm}^3 \times \frac{1}{(1)^2\,\text{mm}^2} = 1.27\text{ mm}$

$$\left.\begin{aligned} u_D &= \pm\frac{\delta}{D} = \pm\frac{0.0508}{1} = \pm 5.08\text{ percent} \\ u_L &= \pm\frac{\delta}{L} = \pm\frac{0.0508}{1.27} = \pm 4.00\text{ percent} \end{aligned}\right\} u_\forall = \pm[(4.00)^2 + (2(5.08))^2]^{\frac{1}{2}}$$

$$u_\forall = \pm 10.9\text{ percent}$$

To minimize $u_\forall$, substitute in terms of D:

$$u_\forall = \pm[(u_L)^2 + (2u_D)^2] = \pm\left[\left(\frac{\delta}{L}\right)^2 + \left(2\frac{\delta}{D}\right)^2\right]^{\frac{1}{2}} = \pm\left[\left(\frac{\pi D^2}{4\forall}\delta\right)^2 + \left(2\frac{\delta}{D}\right)^2\right]^{\frac{1}{2}}$$

This will be minimum when D is such that $\partial[]/\partial D = 0$, or

$$\frac{\partial[]}{\partial D} = \left(\frac{\pi\delta}{4\forall}\right)^2 4D^3 + (2\delta)^2\left(-2\frac{1}{D^3}\right) = 0;\;\; D^6 = 2\left(\frac{4\forall}{\pi}\right)^2;\;\; D = 2^{\frac{1}{6}}\left(\frac{4\forall}{\pi}\right)^{\frac{1}{3}}$$

Thus $$D_{opt} = 2^{\frac{1}{6}}\left(\frac{4}{\pi}\times 1\,\text{mm}^3\right)^{\frac{1}{3}} = 1.22\text{ mm}$$

The corresponding L is $$L_{opt} = \frac{4\forall}{\pi D^2} = \frac{4}{\pi}\times 1\,\text{mm}^3 \times \frac{1}{(1.22)^2\,\text{mm}^2} = 0.855\text{ mm}$$

The optimum stroke-to-bore ratio is $L/D)_{opt} = \frac{0.855\text{ mm}}{1.22\text{ mm}} = 0.701$ (see table and plot on next page)

Note that δ drops out of the optimization equation. This optimum L/D is independent of the magnitude of δ

However, the magnitude of the optimum $u_\forall$ increases as δ increases.

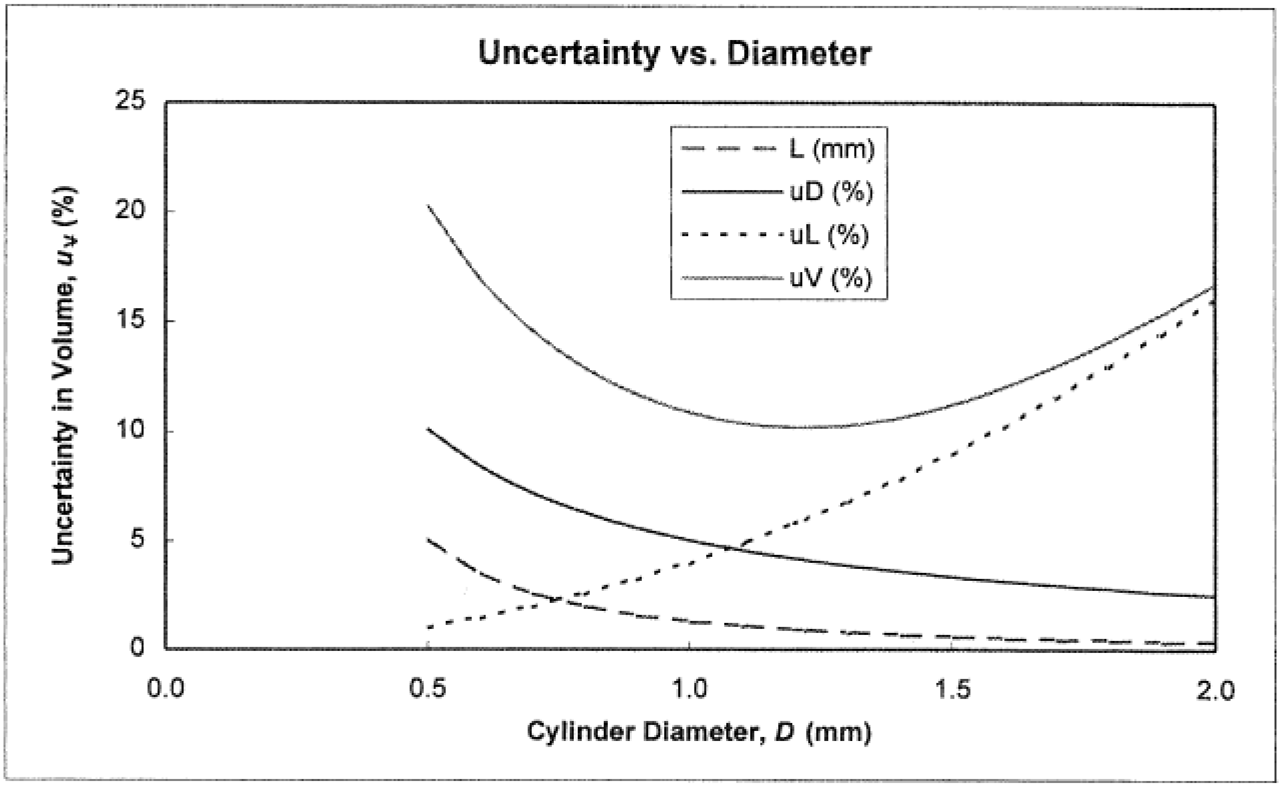
Uncertainty in volume of cylinder:

$\delta = 0.002$ in. 0.0508 mm

$\forall = 1\ \text{mm}^3$

***D* (mm)**	***L* (mm)**	***L/D* (---)**	**u_D(%)**	**u_L(%)**	**$u_\forall$ (%)**
0.5	5.09	10.2	10.2	1.00	20.3
0.6	3.54	5.89	8.47	1.44	17.0
0.7	2.60	3.71	7.26	1.96	14.6
0.8	1.99	2.49	6.35	2.55	13.0
0.9	1.57	1.75	5.64	3.23	11.7
1.0	1.27	1.27	5.08	3.99	10.9
1.1	1.05	0.957	4.62	4.83	10.4
1.2	0.884	0.737	4.23	5.75	10.2
1.22	0.855	0.701	4.16	5.94	10.2
1.3	0.753	0.580	3.91	6.74	10.3

1.4	0.650	0.464	3.63	7.82	10.7
1.5	0.566	0.377	3.39	8.98	11.2
1.6	0.497	0.311	3.18	10.2	12.0
1.7	0.441	0.259	2.99	11.5	13.0
1.8	0.393	0.218	2.82	12.9	14.1
1.9	0.353	0.186	2.67	14.4	15.4
2.0	0.318	0.159	2.54	16.0	16.7
2.1	0.289	0.137	2.42	17.6	18.2
2.2	0.263	0.120	2.31	19.3	19.9
2.3	0.241	0.105	2.21	21.1	21.6
2.4	0.221	0.092	2.12	23.0	23.4
2.5	0.204	0.081	2.03	24.9	25.3



Problem 2.1

[1]

2.1 For the velocity fields given below, determine:

a. whether the flow field is one-, two-, or three-dimensional, and why.

b. whether the flow is steady or unsteady, and why.

c. (The quantities a and b are constants.)

(1) $\vec{V} = [ay^2 e^{-bt}]\hat{i}$ (2) $\vec{V} = ax^2\hat{i} + bx\hat{j} + c\hat{k}$

(3) $\vec{V} = axy\hat{i} - byt\hat{j}$ (4) $\vec{V} = ax\hat{i} - by\hat{j} + ct\hat{k}$

(5) $\vec{V} = [ae^{-bx}]\hat{i} + bt^2\hat{j}$ (6) $\vec{V} = a(x^2 + y^2)^{1/2}(1/z^3)\hat{k}$

(7) $\vec{V} = (ax + t)\hat{i} - by^2\hat{j}$ (8) $\vec{V} = ax^2\hat{i} + bxz\hat{j} + cy\hat{k}$

Given: Velocity fields

Find: Whether flows are 1, 2 or 3D, steady or unsteady.

Solution:

(1)	$\vec{V} = \vec{V}(y)$	1D	$\vec{V} = \vec{V}(t)$	Unsteady
(2)	$\vec{V} = \vec{V}(x)$	1D	$\vec{V} \neq \vec{V}(t)$	Steady
(3)	$\vec{V} = \vec{V}(x,y)$	2D	$\vec{V} = \vec{V}(t)$	Unsteady
(4)	$\vec{V} = \vec{V}(x,y)$	2D	$\vec{V} = \vec{V}(t)$	Unsteady
(5)	$\vec{V} = \vec{V}(x)$	1D	$\vec{V} = \vec{V}(t)$	Unsteady
(6)	$\vec{V} = \vec{V}(x,y,z)$	3D	$\vec{V} \neq \vec{V}(t)$	Steady
(7)	$\vec{V} = \vec{V}(x,y)$	2D	$\vec{V} = \vec{V}(t)$	Unsteady
(8)	$\vec{V} = \vec{V}(x,y,z)$	3D	$\vec{V} \neq \vec{V}(t)$	Steady

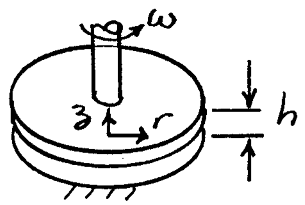
Problem 2.2

[2]

Given: Viscous liquid sheared between parallel disks.

Upper disk rotates, lower fixed.

Velocity field is $\vec{V} = \hat{e}_\theta r\omega z/h$.



Find: (a) Dimensions of velocity field.
(b) Satisfy physical boundary conditions.

Solution: To find dimensions, compare to $\vec{V} = \vec{V}(x, y, z)$ form.

The given field is $\vec{V} = \vec{V}(r, z)$. Two space coordinates are included, so field is 2-D.

2-D

Flow must satisfy the no-slip condition:

(1) At lower disk, $\vec{V} = 0$, since stationary.

$z = 0$, so $\vec{V} = \hat{e}_\theta r\omega(0)/h = 0$ $\therefore$ satisfied

$z =$

(2) At upper disk, $\vec{V} = \hat{e}_\theta r\omega$, since it rotates as a solid body.

$z = h$, so $\vec{V} = \hat{e}_\theta r\omega(h)/h = \hat{e}_\theta r\omega$ $\therefore$ satisfied

$z =$

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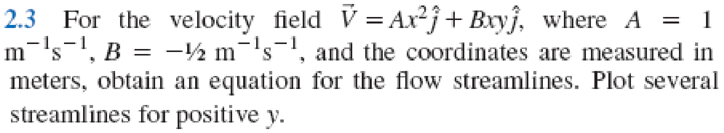
Problem 2.3

[1]



2.3 For the velocity field $\vec{V} = Ax^2\hat{j} + Bxy\hat{j}$, where $A = 1$ $\text{m}^{-1}\text{s}^{-1}$, $B = -½\ \text{m}^{-1}\text{s}^{-1}$, and the coordinates are measured in meters, obtain an equation for the flow streamlines. Plot several streamlines for positive y.

Given: Velocity field

Find: Equation for streamlines

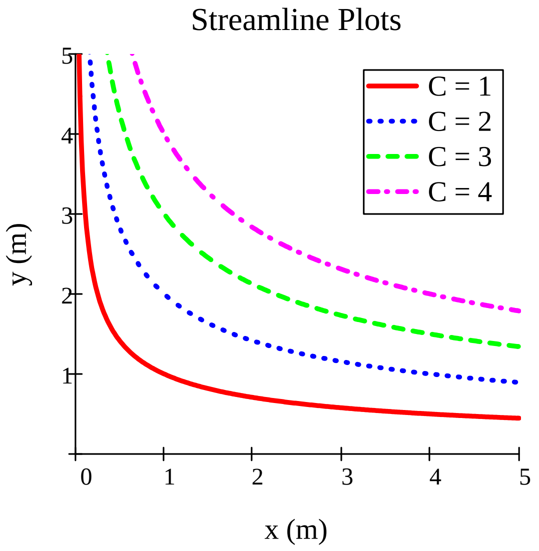
Solution:

For streamlines $\frac{v}{u} = \frac{dy}{dx} = \frac{B \cdot x \cdot y}{A \cdot x^2} = \frac{B \cdot y}{A \cdot x}$

So, separating variables $\frac{dy}{y} = \frac{B}{A} \cdot \frac{dx}{x}$

Integrating $\ln(y) = \frac{B}{A} \cdot \ln(x) + c = -\frac{1}{2} \cdot \ln(x) + c$

The solution is $y = \frac{C}{\sqrt{x}}$



The plot can be easily done in *Excel*.

Problem 2.4

[2]

Given: Velocity field, $\vec{V} = ax\hat{i} - by\hat{j}$ $(a = b = 1\,\text{sec}^{-1})$

Find: Equation for the flow streamlines, and

Plot: Representative streamlines for $x \geq 0$ and $y \geq 0$

Solution:

The slope of the streamlines in the x-y plane is given by

$$\frac{dy}{dx} = \frac{v}{u}$$

For $\vec{V} = ax\hat{i} - by\hat{j}$, then $u = ax$, $v = -by$. Hence

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{b}{a}\frac{y}{x}$$

To solve the differential equation, separate variables and integrate

$$\int \frac{dy}{y} = -\int \frac{b}{a}\frac{dx}{x}$$

$$\ln y = -\frac{b}{a}\ln x + \text{constant}$$

$$\ln y = \ln x^{-\frac{b}{a}} + \ln c \qquad \text{where constant} = \ln c$$

Then

$$y = c\,x^{-\frac{b}{a}} \longleftarrow y(x)$$

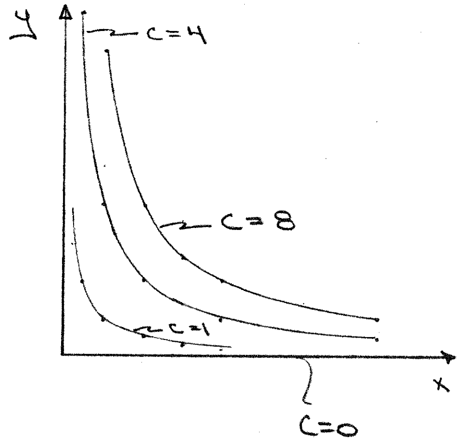
or alternately $x = \left(\frac{y}{c}\right)^{-\frac{a}{b}} = \left(\frac{c}{y}\right)^{\frac{a}{b}}$

For a given velocity field, the constants a and b are fixed. Different streamlines are obtained by assigning different values to the constant of integration, c.

Since $a = b = 1\,\text{sec}^{-1}$, then $a/b = 1$, and the streamlines are given by the equation

$$y = cx^{-1} = \frac{c}{x} \qquad \text{or} \qquad x = \frac{c}{y}$$

For $c = 0$ $y = 0$ for all x and $x = 0$ for all y.



The equation $y = \frac{c}{x}$ is the equation of a hyperbola.

Curves are shown for different values of c

Problem 2.5

[2]

2.5 A velocity field is given by $\vec{V} = ax\hat{i} - bty\hat{j}$, where $a = 1\ \text{s}^{-1}$ and $b = 1\ \text{s}^{-2}$. Find the equation of the streamlines at any time t. Plot several streamlines in the first quadrant at $t = 0$ s, $t = 1$ s, and $t = 20$ s.

Given: Velocity field

Find: Equation for streamlines; Plot streamlines

Solution:

For streamlines $$\frac{v}{u} = \frac{dy}{dx} = \frac{-b\cdot t\cdot y}{a\cdot x}$$

So, separating variables $$\frac{dy}{y} = \frac{-b\cdot t}{a}\cdot\frac{dx}{x}$$

Integrating $$\ln(y) = \frac{-b\cdot t}{a}\cdot\ln(x)$$

The solution is $$y = c\cdot x^{\frac{-b}{a}\cdot t}$$

For $t = 0$ s $\quad y = c \qquad$ For $t = 1$ s $\quad y = \frac{c}{x} \qquad$ For $t = 20$ s $\quad y = c\cdot x^{-20}$

t = 0

	c = 1	c = 2	c = 3
x	**y**	**y**	**y**
0.05	1.00	2.00	3.00
0.10	1.00	2.00	3.00
0.20	1.00	2.00	3.00
0.30	1.00	2.00	3.00
0.40	1.00	2.00	3.00
0.50	1.00	2.00	3.00
0.60	1.00	2.00	3.00
0.70	1.00	2.00	3.00
0.80	1.00	2.00	3.00
0.90	1.00	2.00	3.00
1.00	1.00	2.00	3.00
1.10	1.00	2.00	3.00
1.20	1.00	2.00	3.00
1.30	1.00	2.00	3.00
1.40	1.00	2.00	3.00
1.50	1.00	2.00	3.00
1.60	1.00	2.00	3.00
1.70	1.00	2.00	3.00
1.80	1.00	2.00	3.00
1.90	1.00	2.00	3.00
2.00	1.00	2.00	3.00

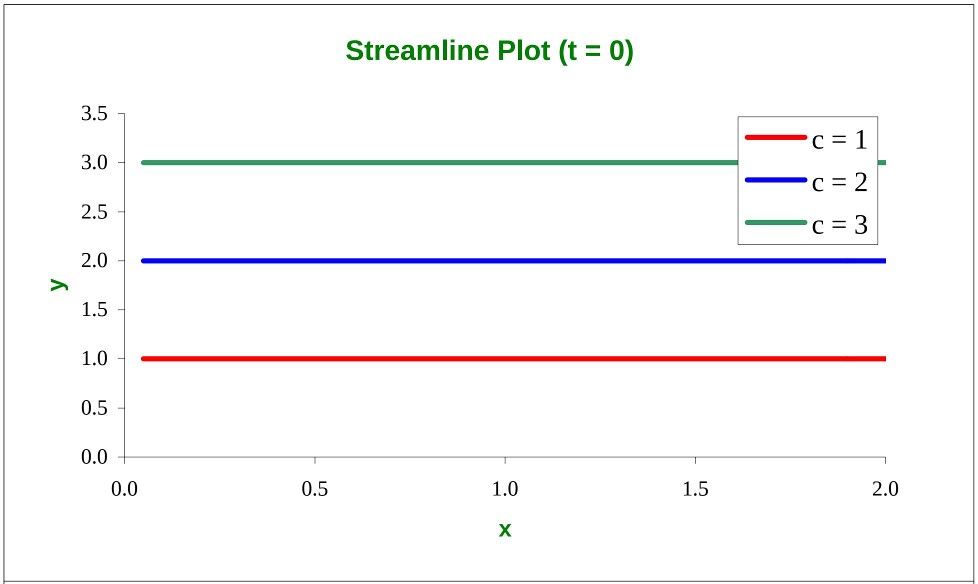
t =1 s

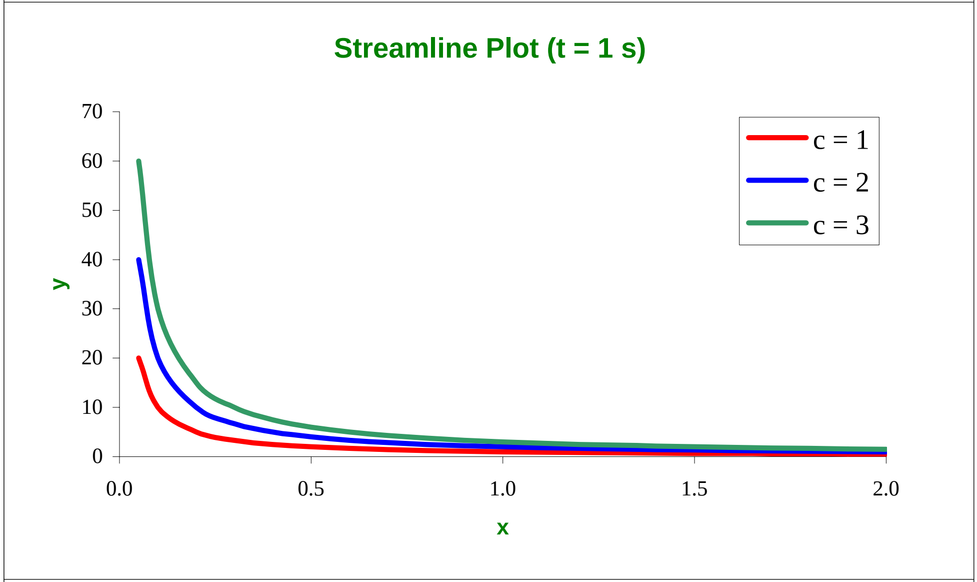
(### means too large to view)

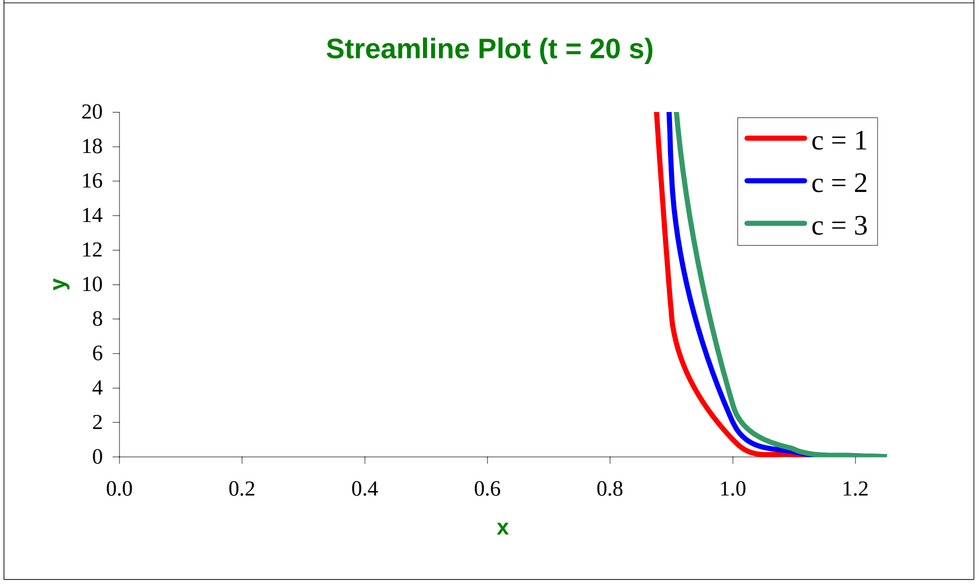
	c = 1	c = 2	c = 3
x	**y**	**y**	**y**
0.05	20.00	40.00	60.00
0.10	10.00	20.00	30.00
0.20	5.00	10.00	15.00
0.30	3.33	6.67	10.00
0.40	2.50	5.00	7.50
0.50	2.00	4.00	6.00
0.60	1.67	3.33	5.00
0.70	1.43	2.86	4.29
0.80	1.25	2.50	3.75
0.90	1.11	2.22	3.33
1.00	1.00	2.00	3.00
1.10	0.91	1.82	2.73
1.20	0.83	1.67	2.50
1.30	0.77	1.54	2.31
1.40	0.71	1.43	2.14
1.50	0.67	1.33	2.00
1.60	0.63	1.25	1.88
1.70	0.59	1.18	1.76
1.80	0.56	1.11	1.67
1.90	0.53	1.05	1.58
2.00	0.50	1.00	1.50

t = 20 s

	c = 1	c = 2	c = 3
x	**y**	**y**	**y**
0.05	######	######	######
0.10	######	######	######
0.20	######	######	######
0.30	######	######	######
0.40	######	######	######
0.50	######	######	######
0.60	######	######	######
0.70	######	######	######
0.80	86.74	173.47	260.21
0.90	8.23	16.45	24.68
1.00	1.00	2.00	3.00
1.10	0.15	0.30	0.45
1.20	0.03	0.05	0.08
1.30	0.01	0.01	0.02
1.40	0.00	0.00	0.00
1.50	0.00	0.00	0.00
1.60	0.00	0.00	0.00
1.70	0.00	0.00	0.00
1.80	0.00	0.00	0.00
1.90	0.00	0.00	0.00
2.00	0.00	0.00	0.00

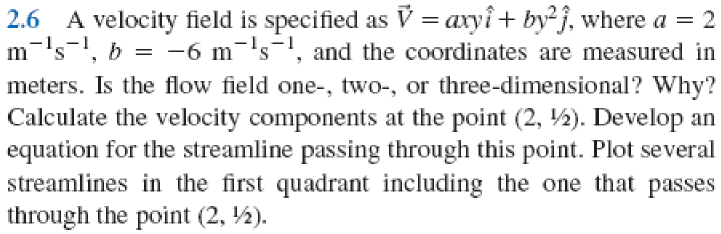

Streamline Plot (t = 0)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0.0
0.5
1.0
1.5
2.0
x
c = 1
c = 2
c = 3


Streamline Plot (t = 1 s)
70
60
50
40
30
20
10
0
y
0.0
0.5
1.0
1.5
2.0
x
c = 1
c = 2
c = 3


Streamline Plot (t = 20 s)
20
18
16
14
12
10
8
6
4
2
0
y
0.0
0.2
0.4
0.6
0.8
1.0
1.2
x
c = 1
c = 2
c = 3

Problem 2.6

[1]



2.6 A velocity field is specified as $\vec{V} = axy\hat{i} + by^2\hat{j}$, where $a = 2\ \mathrm{m^{-1}s^{-1}}$, $b = -6\ \mathrm{m^{-1}s^{-1}}$, and the coordinates are measured in meters. Is the flow field one-, two-, or three-dimensional? Why? Calculate the velocity components at the point (2, ½). Develop an equation for the streamline passing through this point. Plot several streamlines in the first quadrant including the one that passes through the point (2, ½).

Given: Velocity field

Find: Whether field is 1D, 2D or 3D; Velocity components at (2,1/2); Equation for streamlines; Plot

Solution:

The velocity field is a function of *x* and *y*. It is therefore 2D.

At point (2,1/2), the velocity components are $u = a\cdot x\cdot y = 2\cdot\frac{1}{\mathrm{m\cdot s}}\times 2\cdot\mathrm{m}\times\frac{1}{2}\cdot\mathrm{m}$ $\quad u = 2\cdot\frac{\mathrm{m}}{\mathrm{s}}$

$$v = b\cdot y^2 = -6\cdot\frac{1}{\mathrm{m\cdot s}}\times\left(\frac{1}{2}\cdot\mathrm{m}\right)^2 \qquad v = -\frac{3}{2}\cdot\frac{\mathrm{m}}{\mathrm{s}}$$

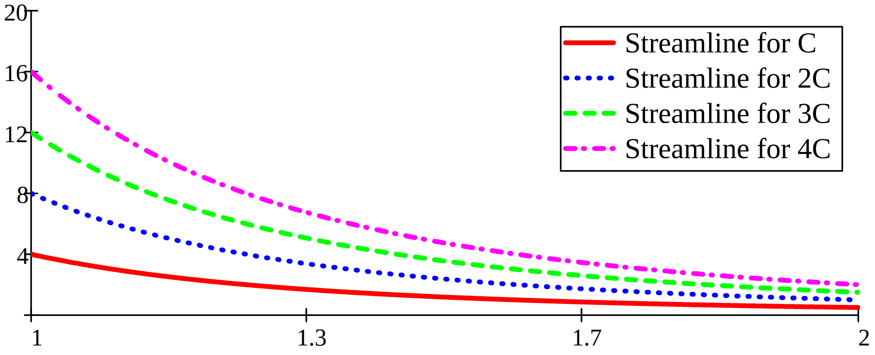
For streamlines $\quad \frac{v}{u} = \frac{dy}{dx} = \frac{b\cdot y^2}{a\cdot x\cdot y} = \frac{b\cdot y}{a\cdot x}$

So, separating variables $\quad \frac{dy}{y} = \frac{b}{a}\cdot\frac{dx}{x}$

Integrating $\quad \ln(y) = \frac{b}{a}\cdot\ln(x) + c \qquad y = C\cdot x^{\frac{b}{a}}$

The solution is $\quad y = C\cdot x^{-3}$

The streamline passing through point (2,1/2) is given by $\quad \frac{1}{2} = C\cdot 2^{-3} \qquad C = \frac{1}{2}\cdot 2^3 \qquad C = 4 \qquad y = \frac{4}{x^3}$



This can be plotted in *Excel.*

Problem 2.7

[2]

2.7 A velocity field is given by $\vec{V} = ax^3\hat{\imath} + bxy^3\hat{\jmath}$, where $a = 1\ \mathrm{m^{-2}\ s^{-1}}$ and $b = 1\ \mathrm{m^{-3}\ s^{-1}}$. Find the equation of the streamlines. Plot several streamlines in the first quadrant.

Given: Velocity field

Find: Equation for streamlines; Plot streamlines

Solution:

Streamlines are given by
$$\frac{v}{u} = \frac{dy}{dx} = \frac{b\cdot x\cdot y^3}{a\cdot x^3}$$

So, separating variables
$$\frac{dy}{y^3} = \frac{b\cdot dx}{a\cdot x^2}$$

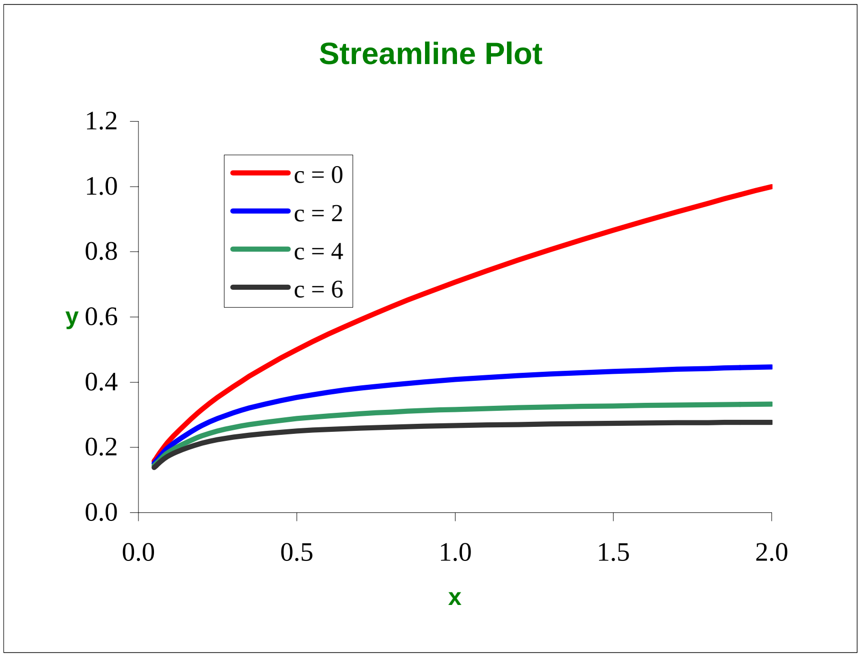
Integrating
$$-\frac{1}{2\cdot y^2} = \frac{b}{a}\cdot\left(-\frac{1}{x}\right) + C$$

The solution is
$$y = \frac{1}{\sqrt{2\cdot\left(\frac{b}{a\cdot x} + C\right)}}$$

Note: For convenience the sign of C is changed.

a = **1**
b = **1**

C =	**0**	**2**	**4**	**6**
x	**y**	**y**	**y**	**y**
0.05	0.16	0.15	0.14	0.14
0.10	0.22	0.20	0.19	0.18
0.20	0.32	0.27	0.24	0.21
0.30	0.39	0.31	0.26	0.23
0.40	0.45	0.33	0.28	0.24
0.50	0.50	0.35	0.29	0.25
0.60	0.55	0.37	0.30	0.26
0.70	0.59	0.38	0.30	0.26
0.80	0.63	0.39	0.31	0.26
0.90	0.67	0.40	0.31	0.27
1.00	0.71	0.41	0.32	0.27
1.10	0.74	0.41	0.32	0.27
1.20	0.77	0.42	0.32	0.27
1.30	0.81	0.42	0.32	0.27
1.40	0.84	0.43	0.33	0.27
1.50	0.87	0.43	0.33	0.27
1.60	0.89	0.44	0.33	0.27
1.70	0.92	0.44	0.33	0.28
1.80	0.95	0.44	0.33	0.28
1.90	0.97	0.44	0.33	0.28
2.00	1.00	0.45	0.33	0.28



Problem 2.8

[2]

2.8 A flow is described by the velocity field $\vec{V} = (Ax + B)\hat{i} + (-Ay)\hat{j}$, where A = 10 ft/s/ft and B = 20 ft/ s. Plot a few streamlines in the xy plane, including the one that passes through the point $(x, y) = (1, 2)$.

Given: Velocity field

Find: Plot streamlines

Solution:

Streamlines are given by $\frac{v}{u} = \frac{dy}{dx} = \frac{-A \cdot y}{A \cdot x + B}$

So, separating variables $\frac{dy}{-A \cdot y} = \frac{dx}{A \cdot x + B}$

Integrating $-\frac{1}{A}\ln(y) = \frac{1}{A}\cdot\ln\left(x + \frac{B}{A}\right)$

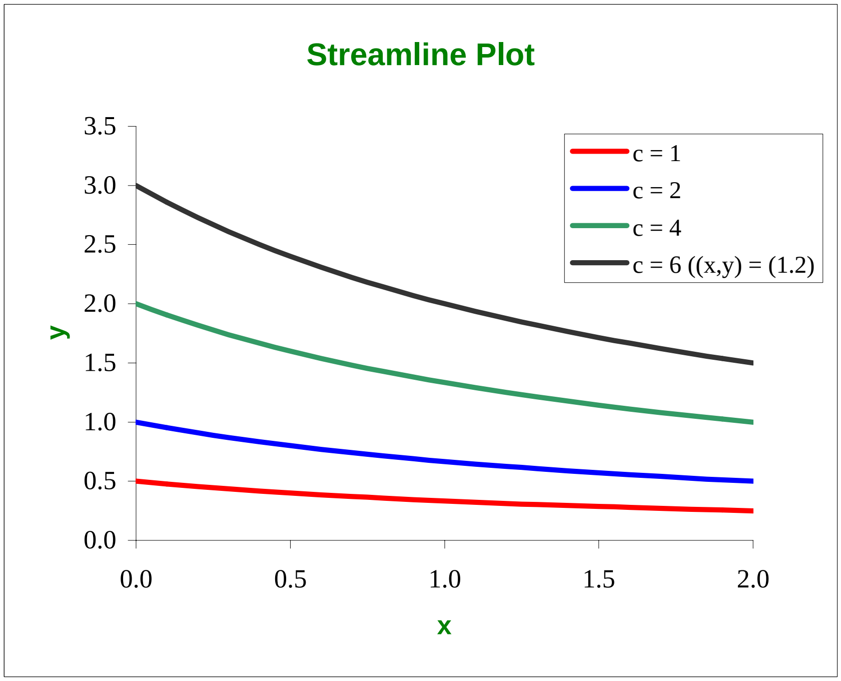
The solution is $y = \frac{C}{x + \frac{B}{A}}$

For the streamline that passes through point $(x,y) = (1,2)$ $C = y\cdot\left(x + \frac{B}{A}\right) = 2\cdot\left(1 + \frac{20}{10}\right) = 6$ $y = \frac{6}{x + \frac{20}{10}}$

$$y = \frac{6}{x + 2}$$

A = **10**
B = **20**
C =

	1	**2**	**4**	**6**
x	**y**	**y**	**y**	**y**
0.00	0.50	1.00	2.00	3.00
0.10	0.48	0.95	1.90	2.86
0.20	0.45	0.91	1.82	2.73
0.30	0.43	0.87	1.74	2.61
0.40	0.42	0.83	1.67	2.50
0.50	0.40	0.80	1.60	2.40
0.60	0.38	0.77	1.54	2.31
0.70	0.37	0.74	1.48	2.22
0.80	0.36	0.71	1.43	2.14
0.90	0.34	0.69	1.38	2.07
1.00	0.33	0.67	1.33	2.00
1.10	0.32	0.65	1.29	1.94
1.20	0.31	0.63	1.25	1.88
1.30	0.30	0.61	1.21	1.82
1.40	0.29	0.59	1.18	1.76
1.50	0.29	0.57	1.14	1.71
1.60	0.28	0.56	1.11	1.67
1.70	0.27	0.54	1.08	1.62
1.80	0.26	0.53	1.05	1.58
1.90	0.26	0.51	1.03	1.54
2.00	0.25	0.50	1.00	1.50



Problem 2.9

2.9 The velocity for a steady, incompressible flow in the xy plane is given by $\vec{V} = \hat{i}A/x + \hat{j}Ay/x^2$, where $A = 2\ \mathrm{m^2/s}$, and the coordinates are measured in meters. Obtain an equation for the streamline that passes through the point $(x, y) = (1, 3)$. Calculate the time required for a fluid particle to move from $x = 1$ m to $x = 2$ m in this flow field.

Given: Velocity field

Find: Equation for streamline through (1,3)

Solution:

For streamlines
$$\frac{v}{u} = \frac{dy}{dx} = \frac{A\cdot\frac{y}{x^2}}{\frac{A}{x}} = \frac{y}{x}$$

So, separating variables
$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating
$$\ln(y) = \ln(x) + c$$

The solution is
$$y = C\cdot x$$
which is the equation of a straight line.

For the streamline through point (1,3)
$$3 = C\cdot 1 \qquad C = 3 \quad \text{and} \quad y = 3\cdot x$$

For a particle
$$u_p = \frac{dx}{dt} = \frac{A}{x} \quad \text{or} \quad x\cdot dx = A\cdot dt \qquad x = \sqrt{2\cdot A\cdot t + c} \qquad t = \frac{x^2}{2\cdot A} - \frac{c}{2\cdot A}$$

Hence the time for a particle to go from x = 1 to x = 2 m is

$$\Delta t = t(x = 2) - t(x = 1)$$

$$\Delta t = \frac{(2\cdot m)^2 - c}{2\cdot A} - \frac{(1\cdot m)^2 - c}{2\cdot A} = \frac{4\cdot m^2 - 1\cdot m^2}{2\times 2\cdot\frac{m^2}{s}}$$

$$\Delta t = 0.75\cdot s$$

Problem 2.10

2.10 The flow field for an atmospheric flow is given by

$$\vec{V} = -\frac{Ky}{2\pi(x^2+y^2)}\hat{i} + \frac{Kx}{2\pi(x^2+y^2)}\hat{j}$$

where $K = 5 \times 10^4$ m^2/s and the x and y coordinates are parallel to the local latitude and longitude. Plot the velocity magnitude along the x axis, along the y axis, and along the line $y = x$. For each plot use the range -10 km $\le x$ or $y \le 10$ km, excluding $|x|$ or $|y| \le 100$ m. Find the equation for the streamlines and sketch several of them. What does this flow field model?

Given: Flow field

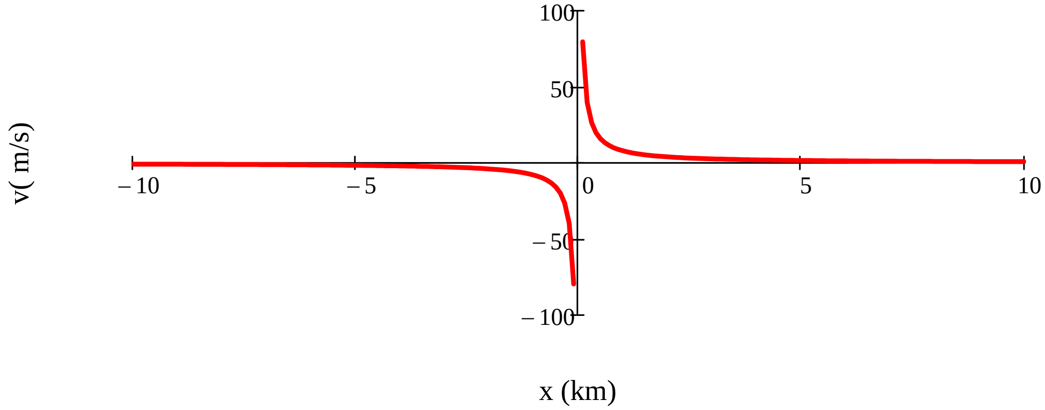
Find: Plot of velocity magnitude along axes, and y = x; Equation of streamlines

Solution:

On the x axis, y = 0, so

$$u = -\frac{K\cdot y}{2\cdot\pi\cdot\left(x^2+y^2\right)} = 0 \qquad v = \frac{K\cdot x}{2\cdot\pi\cdot\left(x^2+y^2\right)} = \frac{K}{2\cdot\pi\cdot x}$$

Plotting



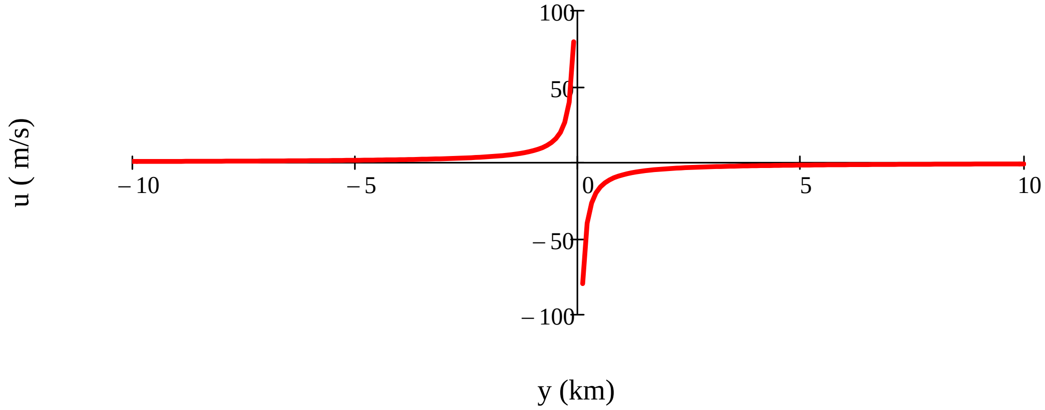
The velocity is perpendicular to the axis, is very high close to the origin, and falls off to zero.

This can also be plotted in Excel.

On the y axis, x = 0, so

$$u = -\frac{K\cdot y}{2\cdot\pi\cdot\left(x^2+y^2\right)} = -\frac{K}{2\cdot\pi\cdot y} \qquad v = \frac{K\cdot x}{2\cdot\pi\cdot\left(x^2+y^2\right)} = 0$$

Plotting



The velocity is perpendicular to the axis, is very high close to the origin, and falls off to zero.

This can also be plotted in Excel.

On the y = x axis $u = -\frac{K \cdot x}{2 \cdot \pi \cdot \left(x^2 + x^2\right)} = -\frac{K}{4 \cdot \pi \cdot x}$ $v = \frac{K \cdot x}{2 \cdot \pi \cdot \left(x^2 + x^2\right)} = \frac{K}{4 \cdot \pi \cdot x}$

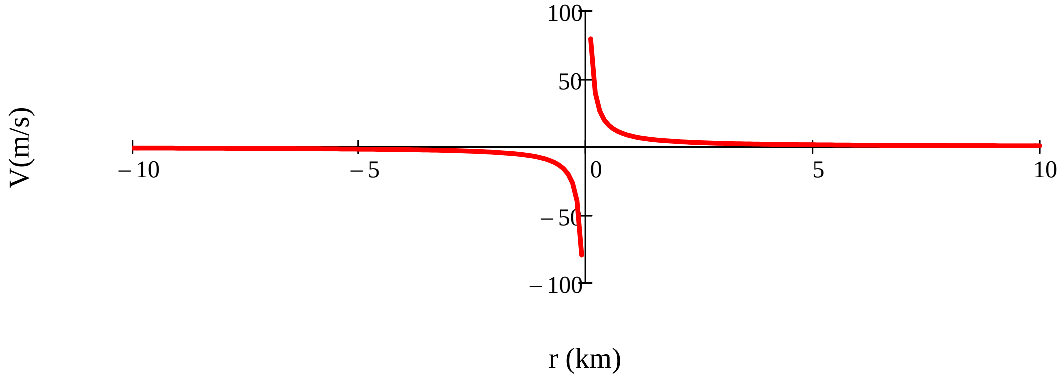
The flow is perpendicular to line y = x: Slope of line y = x: 1

Slope of trajectory of motion: $\frac{u}{v} = -1$

If we define the radial position: $r = \sqrt{x^2 + y^2}$ then along y = x $r = \sqrt{x^2 + x^2} = \sqrt{2} \cdot x$

Then the magnitude of the velocity along y = x is $V = \sqrt{u^2 + v^2} = \frac{K}{4 \cdot \pi} \cdot \sqrt{\frac{1}{x^2} + \frac{1}{x^2}} = \frac{K}{2 \cdot \pi \cdot \sqrt{2} \cdot x} = \frac{K}{2 \cdot \pi \cdot r}$

Plotting



This can also be plotted in Excel.

For streamlines $\frac{v}{u} = \frac{dy}{dx} = \frac{\frac{K \cdot x}{2 \cdot \pi \cdot \left(x^2 + y^2\right)}}{-\frac{K \cdot y}{2 \cdot \pi \cdot \left(x^2 + y^2\right)}} = -\frac{x}{y}$

So, separating variables $y \cdot dy = -x \cdot dx$

Integrating $\frac{y^2}{2} = -\frac{x^2}{2} + c$

The solution is $x^2 + y^2 = C$ which is the equation of a circle.

Streamlines form a set of concentric circles.

This flow models a vortex flow. See Example 5.6 for streamline plots. Streamlines are circular, and the velocity approaches infinity as we approach the center. In Problem 2.11, we see that the streamlines are also circular. In a real tornado, at large distances from the center, the velocities behave as in this problem; close to the center, they behave as in Problem 2.11.

Problem 2.11

2.11 The flow field for an atmospheric flow is given by

$$\vec{V} = -\frac{My}{2\pi}\hat{i} + \frac{Mx}{2\pi}\hat{j}$$

where $M = 0.5\ \mathrm{s}^{-1}$ and the x and y coordinates are parallel to the local latitude and longitude. Plot the velocity magnitude along the x axis, along the y axis, and along the line $y = x$. For each plot use the range $-10\ \mathrm{km} \le x$ or $y \le 10$ km, excluding $|x|$ or $|y| \le 100$ m. Find the equation for the streamlines and sketch several of them. What does this flow field model?

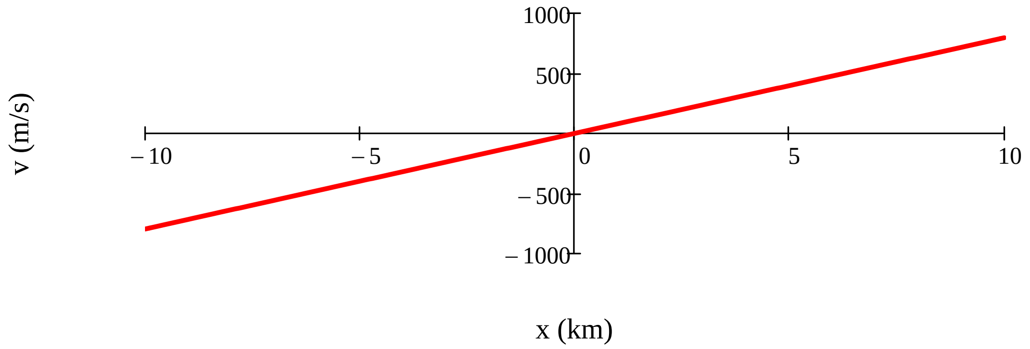
Given: Flow field

Find: Plot of velocity magnitude along axes, and y = x; Equation for streamlines

Solution:

On the x axis, y = 0, so $u = -\frac{M\cdot y}{2\cdot\pi} = 0$ $v = \frac{M\cdot x}{2\cdot\pi}$

Plotting

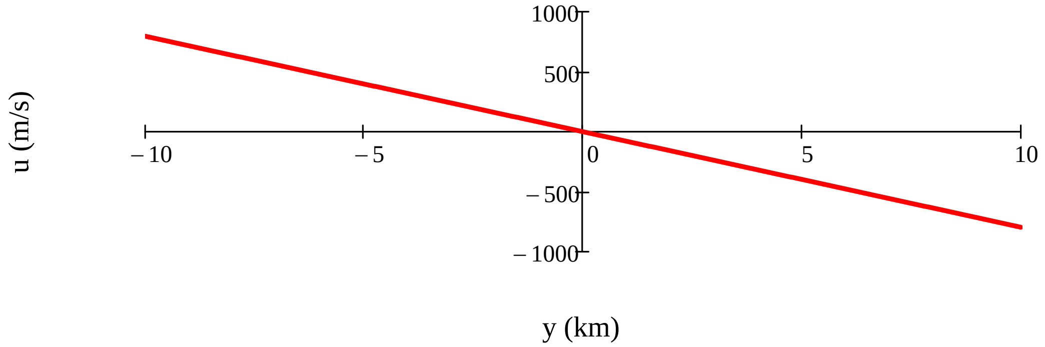


The velocity is perpendicular to the axis and increases linearly with distance x.

This can also be plotted in Excel.

On the y axis, x = 0, so $u = -\frac{M\cdot y}{2\cdot\pi}$ $v = \frac{M\cdot x}{2\cdot\pi} = 0$

Plotting



The velocity is perpendicular to the axis and increases linearly with distance y.

This can also be plotted in Excel.

On the y = x axis $u = -\frac{M \cdot y}{2 \cdot \pi} = -\frac{M \cdot x}{2 \cdot \pi}$ $v = \frac{M \cdot x}{2 \cdot \pi}$

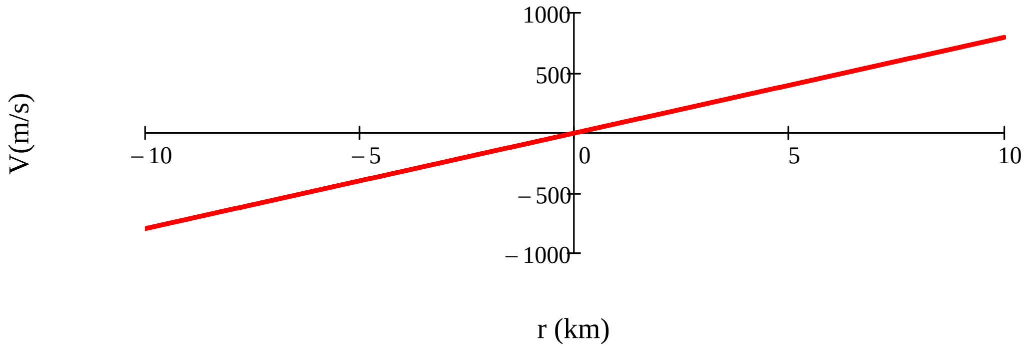
The flow is perpendicular to line y = x: Slope of line y = x: 1

Slope of trajectory of motion: $\frac{u}{v} = -1$

If we define the radial position: $r = \sqrt{x^2 + y^2}$ then along y = x $r = \sqrt{x^2 + x^2} = \sqrt{2} \cdot x$

Then the magnitude of the velocity along y = x is $V = \sqrt{u^2 + v^2} = \frac{M}{2 \cdot \pi} \cdot \sqrt{x^2 + x^2} = \frac{M \cdot \sqrt{2} \cdot x}{2 \cdot \pi} = \frac{M \cdot r}{2 \cdot \pi}$

Plotting



This can also be plotted in Excel.

For streamlines $\frac{v}{u} = \frac{dy}{dx} = \frac{\frac{M \cdot x}{2 \cdot \pi}}{-\frac{M \cdot y}{2 \cdot \pi}} = -\frac{x}{y}$

So, separating variables $y \cdot dy = -x \cdot dx$

Integrating $\frac{y^2}{2} = -\frac{x^2}{2} + c$

The solution is $x^2 + y^2 = C$ which is the equation of a circle.

The streamlines form a set of concentric circles.

This flow models a rigid body vortex flow. See Example 5.6 for streamline plots. Streamlines are circular, and the velocity approaches zer as we approach the center. In Problem 2.10, we see that the streamlines are also circular. In a real tornado, at large distances from the center, the velocities behave as in Problem 2.10; close to the center, they behave as in this problem.

Problem 2.12

[3]

2.12 A flow field flow is given by

$$\vec{V} = -\frac{qx}{2\pi(x^2+y^2)}\hat{i} - \frac{qy}{2\pi(x^2+y^2)}\hat{j}$$

where $q = 2 \times 10^4$ m^2/s. Plot the velocity magnitude along the x axis, along the y axis, and along the line $y = x$. For each plot use the range -10 km $\le x$ or $y \le 10$ km, excluding $|x|$ or $|y| \le 100$ m. Find the equation for the streamlines and sketch several of them. What does this flow field model?

Given: Flow field

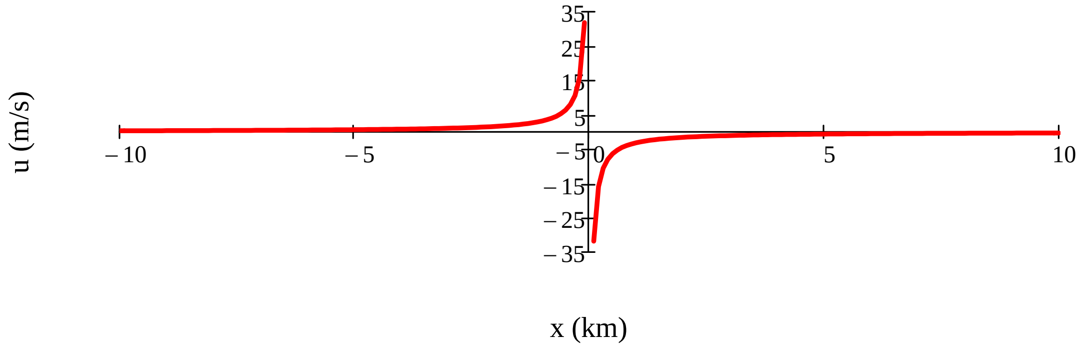
Find: Plot of velocity magnitude along axes, and y = x; Equations of streamlines

Solution:

On the x axis, y = 0, so

$$u = -\frac{q \cdot x}{2\cdot\pi\cdot\left(x^2+y^2\right)} = -\frac{q}{2\cdot\pi\cdot x} \qquad v = -\frac{q \cdot y}{2\cdot\pi\cdot\left(x^2+y^2\right)} = 0$$

Plotting

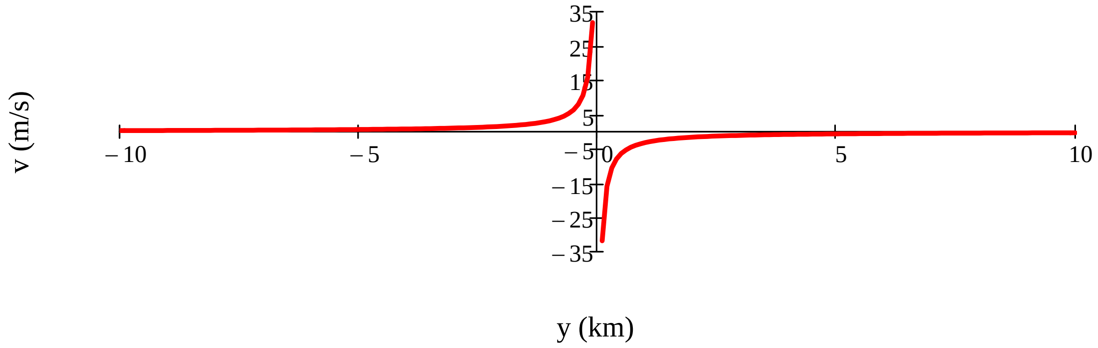


The velocity is very high close to the origin, and falls off to zero. It is also along the axis. This can be plotted in *Excel.*

On the y axis, x = 0, so

$$u = -\frac{q \cdot x}{2\cdot\pi\cdot\left(x^2+y^2\right)} = 0 \qquad v = -\frac{q \cdot y}{2\cdot\pi\cdot\left(x^2+y^2\right)} = -\frac{q}{2\cdot\pi\cdot y}$$

Plotting



The velocity is again very high close to the origin, and falls off to zero. It is also along the axis.

This can also be plotted in *Excel.*

On the y = x axis $u = -\frac{q \cdot x}{2 \cdot \pi \cdot \left(x^2 + x^2\right)} = -\frac{q}{4 \cdot \pi \cdot x}$ $v = -\frac{q \cdot x}{2 \cdot \pi \cdot \left(x^2 + x^2\right)} = -\frac{q}{4 \cdot \pi \cdot x}$

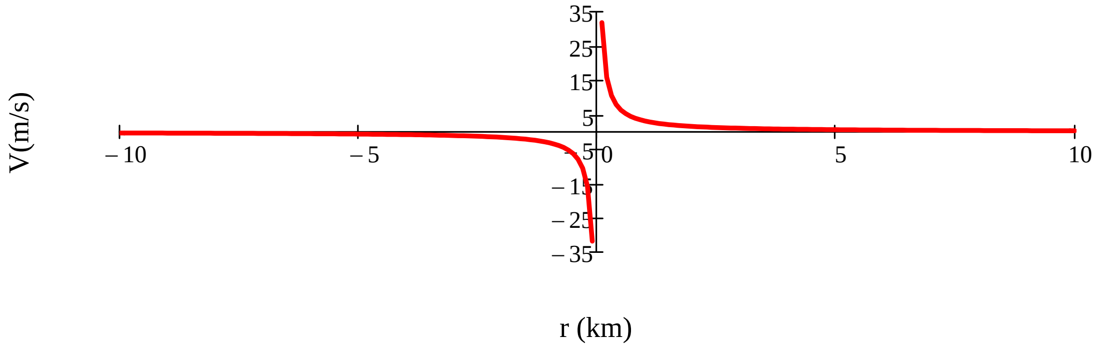
The flow is parallel to line y = x: Slope of line y = x: 1

Slope of trajectory of motion: $\frac{v}{u} = 1$

If we define the radial position: $r = \sqrt{x^2 + y^2}$ then along y = x $r = \sqrt{x^2 + x^2} = \sqrt{2} \cdot x$

Then the magnitude of the velocity along y = x is $V = \sqrt{u^2 + v^2} = \frac{q}{4 \cdot \pi} \cdot \sqrt{\frac{1}{x^2} + \frac{1}{x^2}} = \frac{q}{2 \cdot \pi \cdot \sqrt{2} \cdot x} = \frac{q}{2 \cdot \pi \cdot r}$

Plotting



This can also be plotted in Excel.

For streamlines $$\frac{v}{u} = \frac{dy}{dx} = \frac{-\frac{q \cdot y}{2 \cdot \pi \cdot \left(x^2 + y^2\right)}}{-\frac{q \cdot x}{2 \cdot \pi \cdot \left(x^2 + y^2\right)}} = \frac{y}{x}$$

So, separating variables $\frac{dy}{y} = \frac{dx}{x}$

Integrating $\ln(y) = \ln(x) + c$

The solution is $y = C \cdot x$ which is the equation of a straight line.

This flow field corresponds to a sink (discussed in Chapter 6).

Problem 2.13

[2]

Given: Velocity field $\vec{V} = ax\hat{i} - by\hat{j}$, where $a = b = 1\ s^{-1}$.

Find: (a) Show that particle motion is described by the parametric equations $x_p = c_1 e^{at}$ and $y_p = c_2 e^{-bt}$

(b) Obtain equation of pathline for particle located at (1,2) at $t=0$

(c) Compare pathline with streamline through same point

Solution

(a) A particle moving in the velocity field $\vec{V} = ax\hat{i} - by\hat{j}$ will have velocity components $u = ax$, $v = -by$

Thus $u_p = \frac{dx}{dt} = ax$ or $\frac{dx}{x} = a\,dt$ and $\int \frac{dx}{x} = \int a\,dt$ ...(1)

$v_p = \frac{dy}{dt} = -by$ or $\frac{dy}{y} = -b\,dt$ and $\int \frac{dy}{y} = -\int b\,dt$ ...(2)

Integrating Eqs. (1) and (2) we obtain

$$\left.\begin{array}{lll} \ln x = at + \ln c_1 & \text{or } \frac{x}{c_1} = e^{at} & \text{and } x = c_1 e^{at} \\ \ln y = -bt + \ln c_2 & \text{or } \frac{y}{c_2} = e^{-bt} & \text{and } y = c_2 e^{-bt} \end{array}\right\} \leftarrow \text{Q.E.D.}$$

(b) To obtain the equation of the pathline we eliminate t from the parametric equations.

$x = c_1 e^{at}$ $\therefore \ln \frac{x}{c_1} = at$ or $t = \frac{1}{a} \ln \frac{x}{c_1}$

$y = c_2 e^{-bt}$ $\therefore \ln \frac{y}{c_2} = -bt$ or $t = -\frac{1}{b} \ln \frac{y}{c_2}$

Equating expressions for t, we obtain

$\frac{1}{a} \ln \frac{x}{c_1} = -\frac{1}{b} \ln \frac{y}{c_2}$ or $-\frac{b}{a} \ln \frac{x}{c_1} = \ln \frac{y}{c_2}$

Thus $\left(\frac{x}{c_1}\right)^{-b/a} = \frac{y}{c_2}$ or $y\left(\frac{x}{c_1}\right)^{b/a} = c_2$

At $t=0$ $x = 1 = c_1$, $y = 2 = c_2$. Since $a = b$, then the pathline of the particle is $xy = 2$. ← Pathline

(c) The streamline in the x-y plane has slope $\frac{dy}{dx} = \frac{v}{u} = -\frac{b}{a}\frac{y}{x}$

Thus $\frac{dy}{y} + \frac{b}{a}\frac{dx}{x} = 0$. This can be integrated to obtain

$\ln y + \frac{b}{a} \ln x = \text{constant} = \ln c$

Simplifying we obtain $y x^{b/a} = c$. With $b = a$, the equation of the streamline through point (1,2) is then

$xy = 2$ ← Streamline

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Problem 2.14

[2]

2.14 A velocity field is given by $\vec{V} = ayt\hat{i} - bx\hat{j}$, where $a = 1$ s^{-2} and $b = 4$ s^{-1}. Find the equation of the streamlines at any time t. Plot several streamlines at $t = 0$ s, $t = 1$ s, and $t = 20$ s.

Given: Velocity field

Find: Equation of streamlines; Plot streamlines

Solution:

Streamlines are given by $\frac{v}{u} = \frac{dy}{dx} = \frac{-b \cdot x}{a \cdot y \cdot t}$

So, separating variables $a \cdot t \cdot y \cdot dy = -b \cdot x \cdot dx$

Integrating $\frac{1}{2} \cdot a \cdot t \cdot y^2 = -\frac{1}{2} \cdot b \cdot x^2 + C$

The solution is $y = \sqrt{C - \frac{b \cdot x^2}{a \cdot t}}$

For $t = 0$ s $\quad x = c \qquad$ For $t = 1$ s $\quad y = \sqrt{C - 4 \cdot x^2} \qquad$ For $t = 20$ s $\quad y = \sqrt{C - \frac{x^2}{5}}$

t = 0

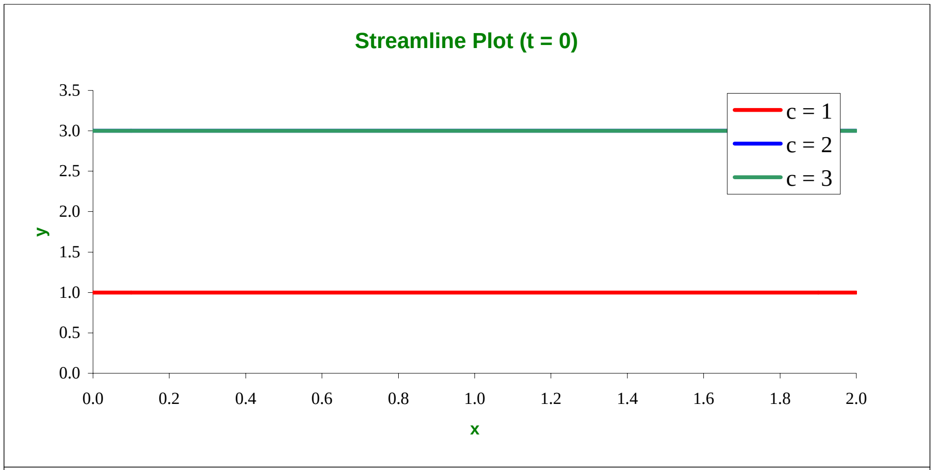
	C = 1	C = 2	C = 3
x	y	y	y
0.00	1.00	2.00	3.00
0.10	1.00	2.00	3.00
0.20	1.00	2.00	3.00
0.30	1.00	2.00	3.00
0.40	1.00	2.00	3.00
0.50	1.00	2.00	3.00
0.60	1.00	2.00	3.00
0.70	1.00	2.00	3.00
0.80	1.00	2.00	3.00
0.90	1.00	2.00	3.00
1.00	1.00	2.00	3.00
1.10	1.00	2.00	3.00
1.20	1.00	2.00	3.00
1.30	1.00	2.00	3.00
1.40	1.00	2.00	3.00
1.50	1.00	2.00	3.00
1.60	1.00	2.00	3.00
1.70	1.00	2.00	3.00
1.80	1.00	2.00	3.00
1.90	1.00	2.00	3.00
2.00	1.00	2.00	3.00

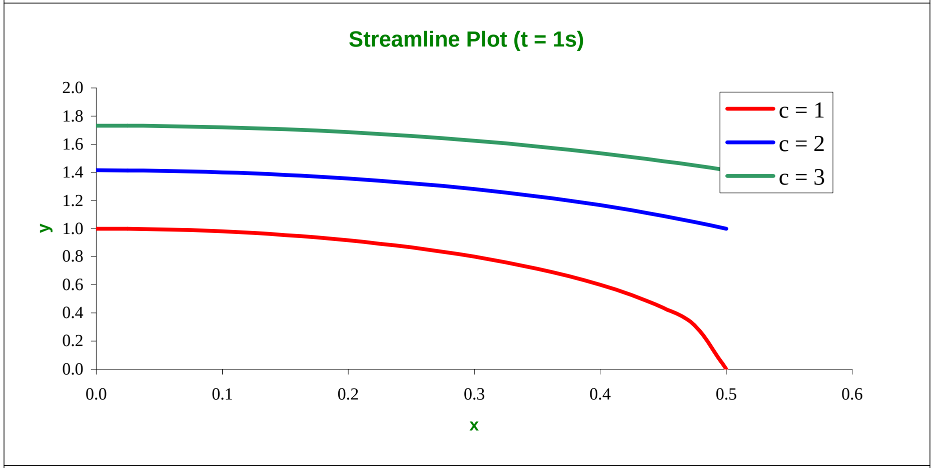
t =1 s

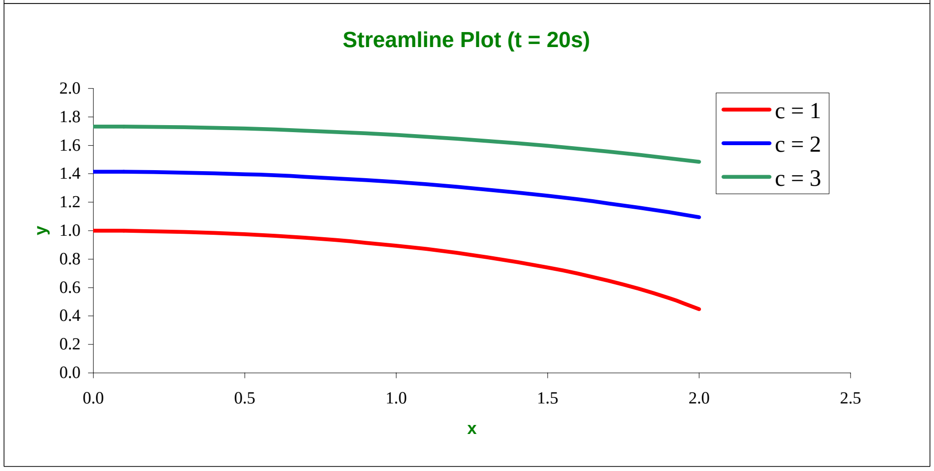
	C = 1	C = 2	C = 3
x	y	y	y
0.000	1.00	1.41	1.73
0.025	1.00	1.41	1.73
0.050	0.99	1.41	1.73
0.075	0.99	1.41	1.73
0.100	0.98	1.40	1.72
0.125	0.97	1.39	1.71
0.150	0.95	1.38	1.71
0.175	0.94	1.37	1.70
0.200	0.92	1.36	1.69
0.225	0.89	1.34	1.67
0.250	0.87	1.32	1.66
0.275	0.84	1.30	1.64
0.300	0.80	1.28	1.62
0.325	0.76	1.26	1.61
0.350	0.71	1.23	1.58
0.375	0.66	1.20	1.56
0.400	0.60	1.17	1.54
0.425	0.53	1.13	1.51
0.450	0.44	1.09	1.48
0.475	0.31	1.05	1.45
0.500	0.00	1.00	1.41

t = 20 s

	C = 1	C = 2	C = 3
x	y	y	y
0.00	1.00	1.41	1.73
0.10	1.00	1.41	1.73
0.20	1.00	1.41	1.73
0.30	0.99	1.41	1.73
0.40	0.98	1.40	1.72
0.50	0.97	1.40	1.72
0.60	0.96	1.39	1.71
0.70	0.95	1.38	1.70
0.80	0.93	1.37	1.69
0.90	0.92	1.36	1.68
1.00	0.89	1.34	1.67
1.10	0.87	1.33	1.66
1.20	0.84	1.31	1.65
1.30	0.81	1.29	1.63
1.40	0.78	1.27	1.61
1.50	0.74	1.24	1.60
1.60	0.70	1.22	1.58
1.70	0.65	1.19	1.56
1.80	0.59	1.16	1.53
1.90	0.53	1.13	1.51
2.00	0.45	1.10	1.48

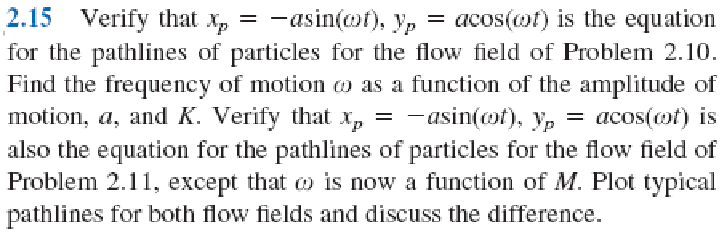

Streamline Plot (t = 0)
y
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
x
c = 1
c = 2
c = 3


Streamline Plot (t = 1s)
y
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x
c = 1
c = 2
c = 3


Streamline Plot (t = 20s)
y
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
x
c = 1
c = 2
c = 3

Problem 2.15

[4]



2.15 Verify that $x_p = -a\sin(\omega t)$, $y_p = a\cos(\omega t)$ is the equation for the pathlines of particles for the flow field of Problem 2.10. Find the frequency of motion ω as a function of the amplitude of motion, a, and K. Verify that $x_p = -a\sin(\omega t)$, $y_p = a\cos(\omega t)$ is also the equation for the pathlines of particles for the flow field of Problem 2.11, except that ω is now a function of M. Plot typical pathlines for both flow fields and discuss the difference.

Given: Pathlines of particles

Find: Conditions that make them satisfy Problem 2.10 flow field; Also Problem 2.11 flow field; Plot pathlines

Solution:

The given pathlines are $x_p = -a\cdot\sin(\omega\cdot t)$ $\quad$ $y_p = a\cdot\cos(\omega\cdot t)$

The velocity field of Problem 2.10 is $u = -\dfrac{K\cdot y}{2\cdot\pi\cdot\left(x^2+y^2\right)}$ $\quad$ $v = \dfrac{K\cdot x}{2\cdot\pi\cdot\left(x^2+y^2\right)}$

If the pathlines are correct we should be able to substitute x_p and y_p into the velocity field to find the velocity as a function of time:

$$u = -\frac{K\cdot y}{2\cdot\pi\cdot\left(x^2+y^2\right)} = -\frac{K\cdot a\cdot\cos(\omega\cdot t)}{2\cdot\pi\cdot\left(a^2\cdot\sin(\omega\cdot t)^2+a^2\cdot\cos(\omega\cdot t)^2\right)} = -\frac{K\cdot\cos(\omega\cdot t)}{2\cdot\pi\cdot a} \qquad (1)$$

$$v = \frac{K\cdot x}{2\cdot\pi\cdot\left(x^2+y^2\right)} = -\frac{K\cdot(-a\cdot\sin(\omega\cdot t))}{2\cdot\pi\cdot\left(a^2\cdot\sin(\omega\cdot t)^2+a^2\cdot\cos(\omega\cdot t)^2\right)} = -\frac{K\cdot\sin(\omega\cdot t)}{2\cdot\pi\cdot a} \qquad (2)$$

We should also be able to find the velocity field as a function of time from the pathline equations (Eq. 2.9):

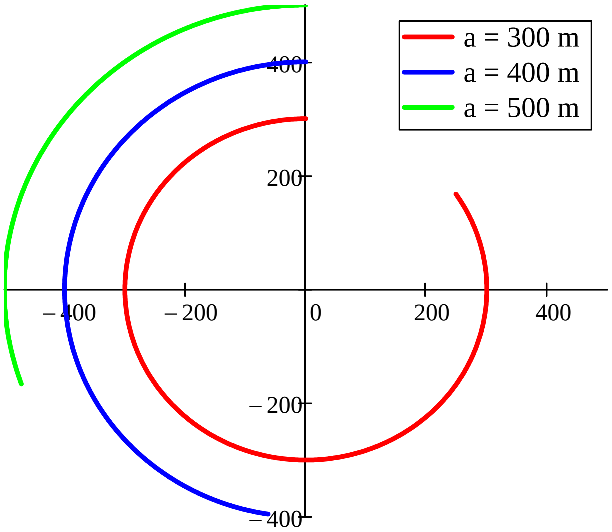
$$\frac{dx_p}{dt} = u \qquad \frac{dx_p}{dt} = v \qquad (2.9)$$

$$u = \frac{dx_p}{dt} = -a\cdot\omega\cdot\cos(\omega\cdot t) \qquad v = \frac{dy_p}{dt} = -a\cdot\omega\cdot\sin(\omega\cdot t) \qquad (3)$$

Comparing Eqs. 1, 2 and 3 $\quad$ $u = -a\cdot\omega\cdot\cos(\omega\cdot t) = -\dfrac{K\cdot\cos(\omega\cdot t)}{2\cdot\pi\cdot a}$ $\quad$ $v = -a\cdot\omega\cdot\sin(\omega\cdot t) = -\dfrac{K\cdot\sin(\omega\cdot t)}{2\cdot\pi\cdot a}$

Hence we see that $\quad$ $a\cdot\omega = \dfrac{K}{2\cdot\pi\cdot a}$ $\quad$ or $\quad$ $\omega = \dfrac{K}{2\cdot\pi\cdot a^2}$ $\quad$ for the pathlines to be correct.

The pathlines are



To plot this in Excel, compute x_p and y_p for t ranging from 0 to 60 s, with ω given by the above formula. Plot y_p versus x_p. Note that outer particles travel much slower!

This is the free vortex flow discussed in Example 5.6

The velocity field of Problem 2.11 is $u = -\frac{M \cdot y}{2 \cdot \pi}$ $v = \frac{M \cdot x}{2 \cdot \pi}$

If the pathlines are correct we should be able to substitute x_p and y_p into the velocity field to find the velocity as a function of time:

$$u = -\frac{M \cdot y}{2 \cdot \pi} = -\frac{M \cdot (a \cdot \cos(\omega \cdot t))}{2 \cdot \pi} = -\frac{M \cdot a \cdot \cos(\omega \cdot t)}{2 \cdot \pi} \qquad (4)$$

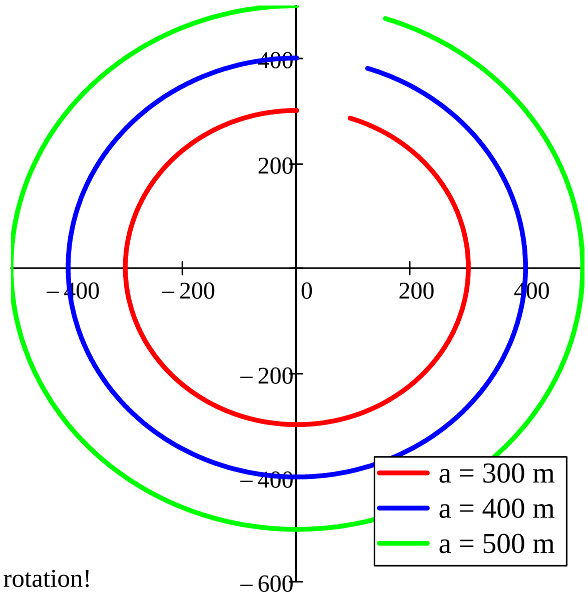
$$v = \frac{M \cdot x}{2 \cdot \pi} = \frac{M \cdot (-a \cdot \sin(\omega \cdot t))}{2 \cdot \pi} = -\frac{M \cdot a \cdot \sin(\omega \cdot t)}{2 \cdot \pi} \qquad (5)$$

Recall that $u = \frac{dx_p}{dt} = -a \cdot \omega \cdot \cos(\omega \cdot t)$ $v = \frac{dy_p}{dt} = -a \cdot \omega \cdot \sin(\omega \cdot t)$ (3)

Comparing Eqs. 1, 4 and 5 $u = -a \cdot \omega \cdot \cos(\omega \cdot t) = -\frac{M \cdot a \cdot \cos(\omega \cdot t)}{2 \cdot \pi}$ $v = -a \cdot \omega \cdot \sin(\omega \cdot t) = -\frac{M \cdot a \cdot \sin(\omega \cdot t)}{2 \cdot \pi}$

Hence we see that $\omega = \frac{M}{2 \cdot \pi}$ for the pathlines to be correct.

The pathlines



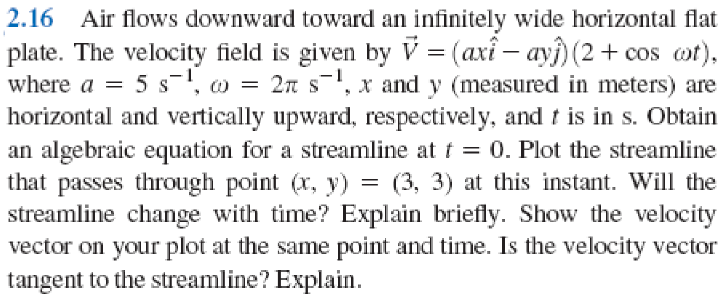
To plot this in Excel, compute x_p and y_p for t ranging from 0 to 75 s, with ω given by the above formula. Plot y_p versus x_p. Note that outer particles travel faster!

This is the forced vortex flow discussed in Example 5.6

Note that this is rigid body rotation!

Problem 2.16

[2]



2.16 Air flows downward toward an infinitely wide horizontal flat plate. The velocity field is given by $\vec{V} = (ax\hat{i} - ay\hat{j})(2 + \cos\ \omega t)$, where $a = 5\ \text{s}^{-1}$, $\omega = 2\pi\ \text{s}^{-1}$, x and y (measured in meters) are horizontal and vertically upward, respectively, and t is in s. Obtain an algebraic equation for a streamline at $t = 0$. Plot the streamline that passes through point $(x, y) = (3, 3)$ at this instant. Will the streamline change with time? Explain briefly. Show the velocity vector on your plot at the same point and time. Is the velocity vector tangent to the streamline? Explain.

Given: Time-varying velocity field

Find: Streamlines at t = 0 s; Streamline through (3,3); velocity vector; will streamlines change with time

Solution:

For streamlines $\frac{v}{u} = \frac{dy}{dx} = -\frac{a \cdot y \cdot (2 + \cos(\omega \cdot t))}{a \cdot x \cdot (2 + \cos(\omega \cdot t))} = -\frac{y}{x}$

At t = 0 (actually all times!) $\frac{dy}{dx} = -\frac{y}{x}$

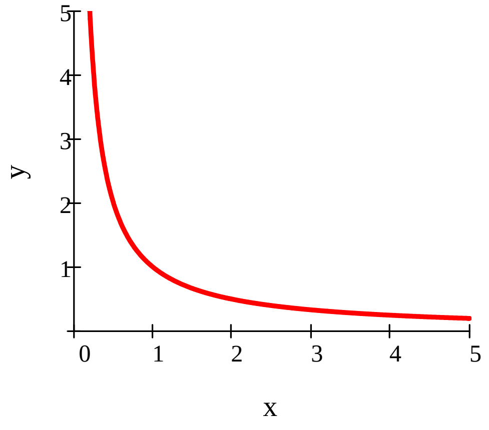
So, separating variables $\frac{dy}{y} = -\frac{dx}{x}$

Integrating $\ln(y) = -\ln(x) + c$

The solution is $y = \frac{C}{x}$ which is the equation of a hyperbola.

For the streamline through point (3,3) $C = \frac{3}{3}$ $C = 1$ and $y = \frac{1}{x}$

The streamlines will not change with time since dy/dx does not change with time.



At t = 0 $u = a \cdot x \cdot (2 + \cos(\omega \cdot t)) = 5 \cdot \frac{1}{s} \times 3 \cdot m \times 3$

$u = 45 \cdot \frac{m}{s}$

$v = -a \cdot y \cdot (2 + \cos(\omega \cdot t)) = 5 \cdot \frac{1}{s} \times 3 \cdot m \times 3$

$v = -45 \cdot \frac{m}{s}$

The velocity vector is tangent to the curve;

Tangent of curve at (3,3) is $\frac{dy}{dx} = -\frac{y}{x} = -1$

Direction of velocity at (3,3) is $\frac{v}{u} = -1$

This curve can be plotted in Excel.

Problem 2.17

[3]

Given: Velocity field $\vec{V} = Bx(1+At)\hat{i} + Cy\hat{j}$, with $A = 0.5\,s^{-1}$, $B = C = 1\,s^{-1}$; coordinates measured in meters.

Plot: the pathline of the particle that passed through the point (1,1,0) at time t=0.
Compare with the streamlines through the same point at the instants t = 0, 1, and 2s

Solution:

For a particle, $u = \frac{dx}{dt}$ and $v = dy/dt$

then

$$u = Bx(1+At) = \frac{dx}{dt}, \quad \int_{x_0}^{x}\frac{dx}{x} = \int_0^t B(1+At)\,dt$$

$$\ln\frac{x}{x_0} = B\left[t+\frac{1}{2}At^2\right]_0^t = B\left[t+\frac{1}{2}At^2\right] \quad \therefore\ x = x_0 e^{B(t+\frac{1}{2}At^2)}$$

$$v = Cy = dy/dt, \quad \int_0^t C\,dt = \int_{y_0}^{y}\frac{dy}{y} \quad \therefore\ y = y_0 e^{Ct}$$

$\Big\}\ x, y$

The pathline may be plotted by varying t as shown below

The streamline is found (at given t) from $\left.\frac{dy}{dx}\right)_{\text{streamline}} = \frac{v}{u}$

Then

$$\frac{dy}{dx} = \frac{Cy}{Bx(1+At)} \quad \text{and} \quad (1+At)\frac{dy}{y} = \frac{C}{B}\frac{dx}{x}$$

and

$$(1+At)\ln y = \frac{C}{B}\ln x + \ln c_1, \quad c_1 x^{C/B} = y^{(1+At)}$$

Streamline through point (1,1,0) gives $c_1 = 1$. Then on substituting for A, B, and C we obtain

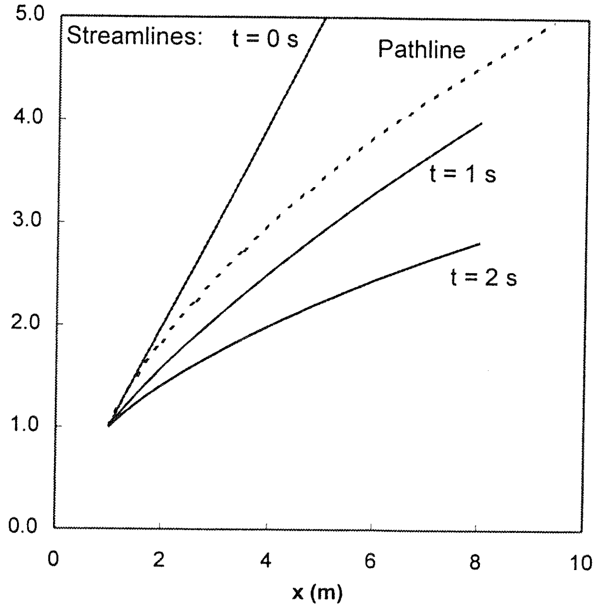
$$x = y^{(1+0.5t)}$$

At $t=0$, $x = y$

$t = 1s$, $x = y^{1.5}$

$t = 2s$, $x = y^2$

Streamline



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Problem 2.18

[3]

Given: Velocity field $\vec{V} = A\hat{i} + Bt\hat{j}$, where $A = 2$ m/s, $B = 0.6$ m/s^2, and coordinates are in meters.

Find: (a) position functions for particle located at $(x_0, y_0) = 1,1$ at time $t = 0$

(b) algebraic expression for pathline of particle of part (a)

Plot: the pathline and compare with streamline through the same point at $t = 0, 1, 2$ s.

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Solution:

For a particle $u = \frac{dx}{dt}$ and $v = \frac{dy}{dt}$

Then,

$$u = A = dx/dt, \quad \int_{x_0}^{x} dx = \int_0^t A\,dt \quad \text{and} \quad x = x_0 + At \qquad (1a)$$

$$v = Bt = dy/dt, \quad \int_{y_0}^{y} dy = \int_0^t Bt\,dt \quad \text{and} \quad y = y_0 + \tfrac{1}{2}Bt^2 \qquad (1b)$$

Substituting values for A, B, x_0, and y_0, then

$$x = 1 + 2t \quad \text{and} \quad y = 1 + 0.30\,t^2 \longleftarrow \underline{\quad x, y}$$

(b) To determine the pathline for the particle, we eliminate t from the paremetric equations of part (a).

From Eq. 1a, $t = (x - x_0)/A$. Substituting into Eq. (1b), then

$$y - y_0 = \frac{B(x - x_0)^2}{2A^2} \qquad (2)$$

Substituting numerical values,

$$y = 1 + 0.075\,(x - 1)^2 \longleftarrow \underline{\quad \text{pathline}}$$

(c) The steamline is found (at given t) from $\left.\frac{dy}{dx}\right)_s = \frac{v}{u}$

$$\left.\frac{dy}{dx}\right)_{\text{streamline}} = \frac{v}{u} = \frac{Bt}{A}$$

$$\therefore y = \frac{Bt}{A}x + c$$

Through point (1,1)

$$c = 1 - \frac{0.6}{2}t = 1 - 0.3t$$

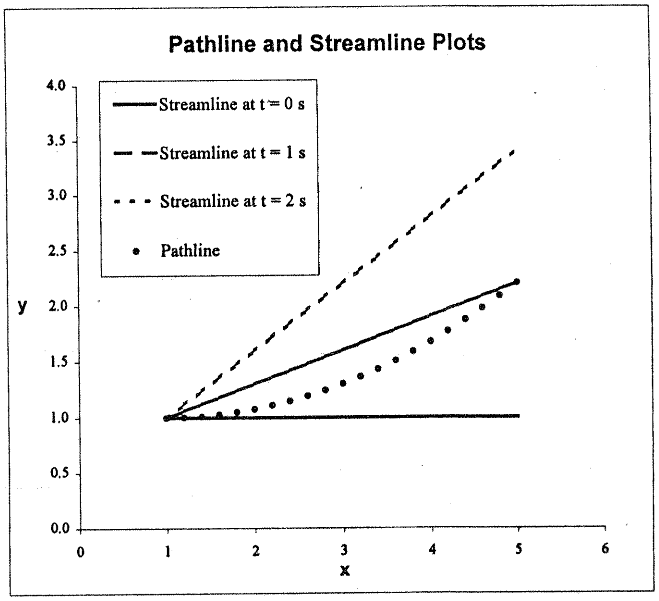
$$y = 1 + 0.3\,t(x - 1) \longleftarrow$$

Streamline through (1,1)

@ $t = 0$, $y = 1$

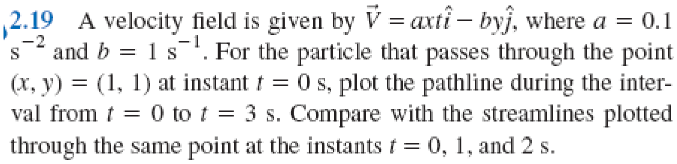
$t = 1$ s, $y = 1 + 0.3\,(x - 1)$

$t = 2$ s, $y = 1 + 0.6\,(x - 1)$



Problem 2.19

[3]



2.19 A velocity field is given by $\vec{V} = axt\hat{i} - by\hat{j}$, where $a = 0.1$ s^{-2} and $b = 1$ s^{-1}. For the particle that passes through the point $(x, y) = (1, 1)$ at instant $t = 0$ s, plot the pathline during the interval from $t = 0$ to $t = 3$ s. Compare with the streamlines plotted through the same point at the instants $t = 0$, 1, and 2 s.

Given: Velocity field

Find: Plot pathlines and streamlines

Solution:

Pathlines are given by $\quad \frac{dx}{dt} = u = a \cdot x \cdot t \qquad \frac{dy}{dt} = v = -b \cdot y$

So, separating variables $\quad \frac{dx}{x} = a \cdot t \cdot dt \qquad \frac{dy}{y} = -b \cdot dt$

Integrating $\quad \ln(x) = \frac{1}{2} \cdot a \cdot t^2 + c_1 \qquad \ln(y) = -b \cdot t + c_2$

For initial position (x_0, y_0) $\quad x = x_0 \cdot e^{\frac{a}{2} \cdot t^2} \qquad y = y_0 \cdot e^{-b \cdot t}$

Using the given data, and IC $(x_0, y_0) = (1,1)$ at $t = 0$

$$x = e^{0.05 \cdot t^2} \qquad y = e^{-t}$$

Streamlines are given by $\quad \frac{v}{u} = \frac{dy}{dx} = \frac{-b \cdot y}{a \cdot x \cdot t}$

So, separating variables $\quad \frac{dy}{y} = -\frac{b}{a \cdot t} \cdot \frac{dx}{x}$

Integrating $\quad \ln(y) = -\frac{b}{a \cdot t} \cdot \ln(x) + C$

The solution is $\quad y = C \cdot x^{-\frac{b}{a \cdot t}}$

For streamline at (1,1) at $t = 0$ s $\quad x = c$

For streamline at (1,1) at $t = 1$ s $\quad y = x^{-10}$

For streamline at (1,1) at $t = 2$ s $\quad y = x^{-5}$

Pathline

t	x	y
0.00	1.00	1.00
0.25	1.00	0.78
0.50	1.01	0.61
0.75	1.03	0.47
1.00	1.05	0.37
1.25	1.08	0.29
1.50	1.12	0.22
1.75	1.17	0.17
2.00	1.22	0.14
2.25	1.29	0.11
2.50	1.37	0.08
2.75	1.46	0.06
3.00	1.57	0.05
3.25	1.70	0.04
3.50	1.85	0.03
3.75	2.02	0.02
4.00	2.23	0.02
4.25	2.47	0.01
4.50	2.75	0.01
4.75	3.09	0.01
5.00	3.49	0.01

Streamlines

t = 0

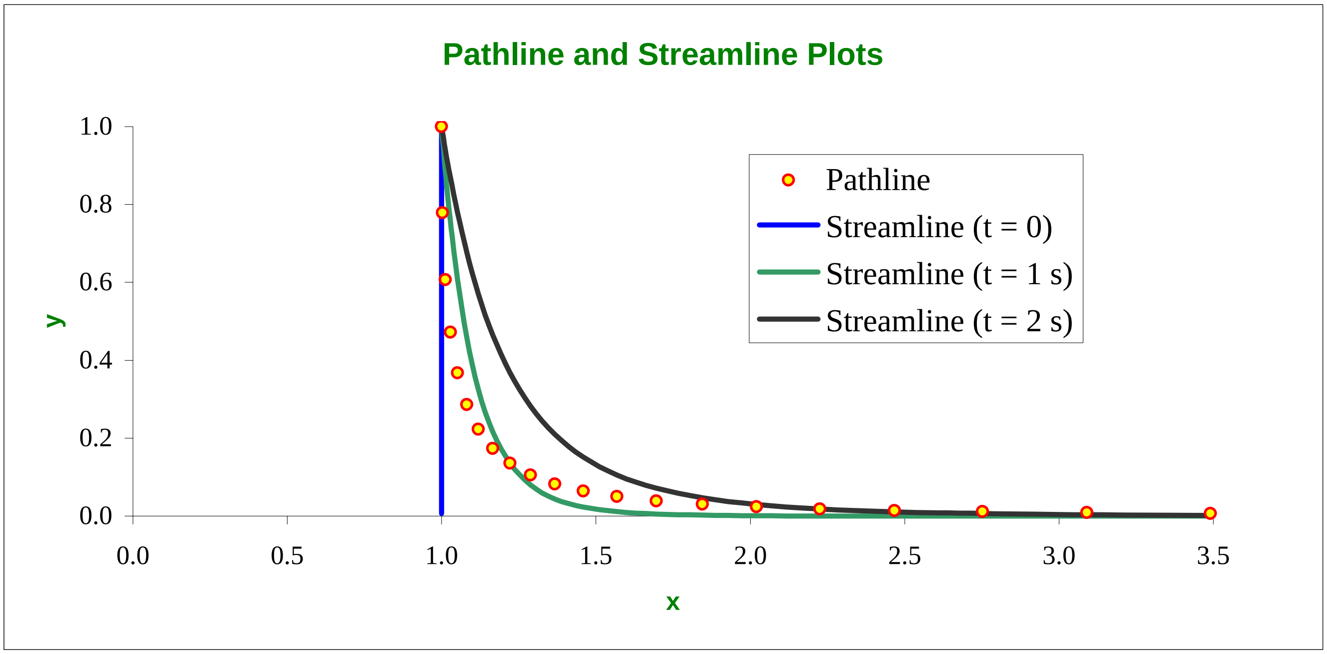
x	y
1.00	1.00
1.00	0.78
1.00	0.61
1.00	0.47
1.00	0.37
1.00	0.29
1.00	0.22
1.00	0.17
1.00	0.14
1.00	0.11
1.00	0.08
1.00	0.06
1.00	0.05
1.00	0.04
1.00	0.03
1.00	0.02
1.00	0.02
1.00	0.01
1.00	0.01
1.00	0.01
1.00	0.01

t = 1 s

x	y
1.00	1.00
1.00	0.97
1.01	0.88
1.03	0.75
1.05	0.61
1.08	0.46
1.12	0.32
1.17	0.22
1.22	0.14
1.29	0.08
1.37	0.04
1.46	0.02
1.57	0.01
1.70	0.01
1.85	0.00
2.02	0.00
2.23	0.00
2.47	0.00
2.75	0.00
3.09	0.00
3.49	0.00

t = 2 s

x	y
1.00	1.00
1.00	0.98
1.01	0.94
1.03	0.87
1.05	0.78
1.08	0.68
1.12	0.57
1.17	0.47
1.22	0.37
1.29	0.28
1.37	0.21
1.46	0.15
1.57	0.11
1.70	0.07
1.85	0.05
2.02	0.03
2.23	0.02
2.47	0.01
2.75	0.01
3.09	0.00
3.49	0.00



Problem 2.20

[3]

Given: Velocity field $\vec{V} = ax\hat{i} + by(1+ct)\hat{j}$, where $a=b=2\,s^{-1}$, $c = 0.4\,s^{-1}$, and coordinates are measured in meters

Plot: the pathline (during the interval $0 \le t \le 1.5\,s$) of the particle that passed through the point $(x_0, y_0) = (1,1)$ at time $t=0$.
Compare with the streamline plotted through the same point at $t = 0, 1,$ and $1.5\,s$

Solution:

For a particle, $u = dx/dt$ and $v = dy/dt$

then $u = dx/dt = ax$, $\int_{x_0}^{x} \frac{dx}{x} = \int_0^t a\,dt$, $\ln\frac{x}{x_0} = at$, $x = x_0 e^{at}$

Also $v = dy/dt = by(1+ct)$, $\int_{y_0}^{y} \frac{dy}{y} = \int_0^t b(1+ct)\,dt$

$\ln y/y_0 = b(t + \frac{1}{2}ct^2)$ $y = y_0 e^{b(t+\frac{1}{2}ct^2)}$

Substituting for $a, b, c, x_0,$ and y_0

$x = e^{2t}$, $y = e^{(2t+0.4t^2)}$ ← (x,y) pathline

The streamline is found (at given t) from $dy/dx)_s = v/u$

then $\frac{dy}{dx} = \frac{by(1+ct)}{ax}$, $\int_{y_0}^{y} \frac{dy}{y} = \int_{x_0}^{x} \frac{b(1+ct)}{a} \frac{dx}{x}$, $\ln\frac{y}{y_0} = \frac{b(1+ct)}{a} \ln\frac{x}{x_0}$

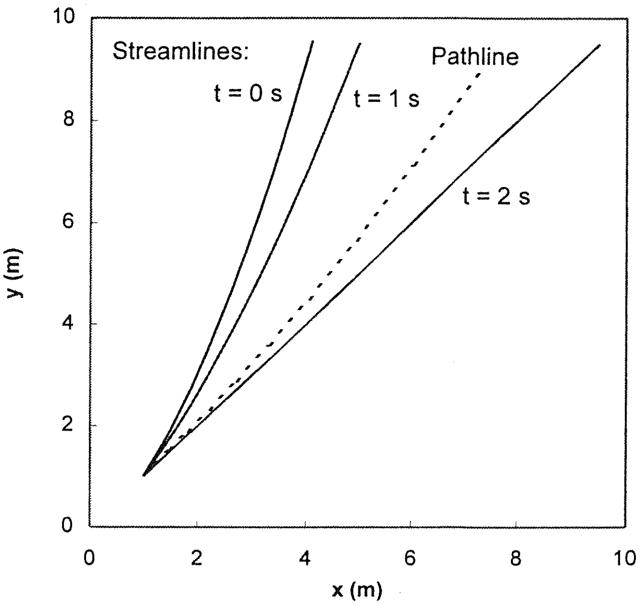
$y = y_0 \left(\frac{x}{x_0}\right)^{\frac{b(1+ct)}{a}}$. Substituting for $a, b, c, x_0,$ and y_0

$y = x^{(1+0.4t)}$ ← streamline

At $t=0$, $y = x$

$t = 1\,s$, $y = x^{1.4}$

$t = 1.5\,s$, $y = x^{1.6}$



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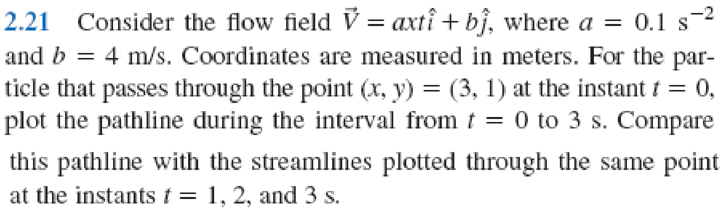
Problem 2.21



2.21 Consider the flow field $\vec{V} = axt\hat{i} + b\hat{j}$, where $a = 0.1\ \text{s}^{-2}$ and $b = 4$ m/s. Coordinates are measured in meters. For the particle that passes through the point $(x, y) = (3, 1)$ at the instant $t = 0$, plot the pathline during the interval from $t = 0$ to 3 s. Compare this pathline with the streamlines plotted through the same point at the instants $t = 1$, 2, and 3 s.

Given: Flow field

Find: Pathline for particle starting at (3,1); Streamlines through same point at t = 1, 2, and 3 s

Solution:

For particle paths $\quad \dfrac{dx}{dt} = u = a\cdot x\cdot t \quad$ and $\quad \dfrac{dy}{dt} = v = b$

Separating variables and integrating $\quad \dfrac{dx}{x} = a\cdot t\cdot dt \quad$ or $\quad \ln(x) = \dfrac{1}{2}\cdot a\cdot t^2 + c_1$

$dy = b\cdot dt \quad$ or $\quad y = b\cdot t + c_2$

Using initial condition (x,y) = (3,1) and the given values for a and b

$c_1 = \ln(3\cdot m) \quad$ and $\quad c_2 = 1\cdot m$

The pathline is then $\quad x = 3\cdot e^{0.05\cdot t^2} \quad$ and $\quad y = 4\cdot t + 1$

For streamlines (at any time t) $\quad \dfrac{v}{u} = \dfrac{dy}{dx} = \dfrac{b}{a\cdot x\cdot t}$

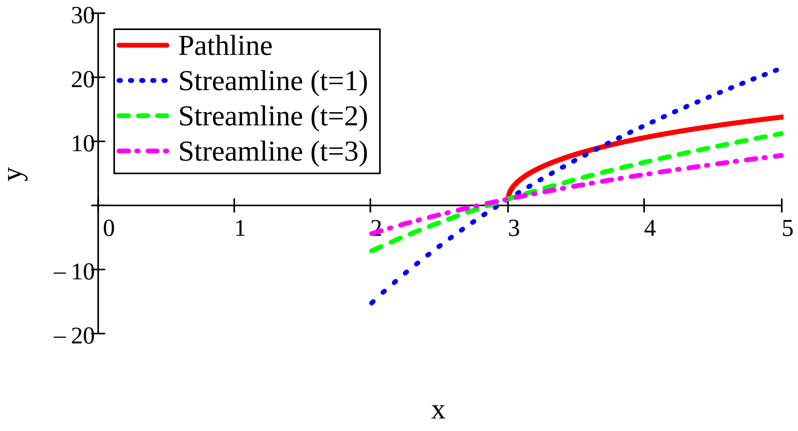
So, separating variables $\quad dy = \dfrac{b}{a\cdot t}\cdot\dfrac{dx}{x}$

Integrating $\quad y = \dfrac{b}{a\cdot t}\cdot\ln(x) + c$

We are interested in instantaneous streamlines at various times that always pass through point (3,1). Using a and b values:

$$c = y - \frac{b}{a\cdot t}\cdot\ln(x) = 1 - \frac{4}{0.1\cdot t}\cdot\ln(3)$$

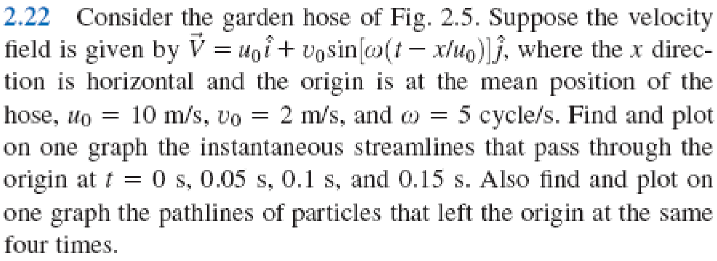
The streamline equation is $\quad y = 1 + \dfrac{40}{t}\cdot\ln\left(\dfrac{x}{3}\right)$



These curves can be plotted in Excel.

Problem 2.22

[4]



2.22 Consider the garden hose of Fig. 2.5. Suppose the velocity field is given by $\vec{V} = u_0\hat{i} + v_0 \sin[\omega(t - x/u_0)]\hat{j}$, where the x direction is horizontal and the origin is at the mean position of the hose, $u_0 = 10$ m/s, $v_0 = 2$ m/s, and $\omega = 5$ cycle/s. Find and plot on one graph the instantaneous streamlines that pass through the origin at $t = 0$ s, 0.05 s, 0.1 s, and 0.15 s. Also find and plot on one graph the pathlines of particles that left the origin at the same four times.

Given: Velocity field

Find: Plot streamlines that are at origin at various times and pathlines that left origin at these times

Solution:

For streamlines $$\frac{v}{u} = \frac{dy}{dx} = \frac{v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]}{u_0}$$

So, separating variables (t=const) $$dy = \frac{v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]}{u_0} \cdot dx$$

Integrating $$y = \frac{v_0 \cdot \cos\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]}{\omega} + c$$

Using condition y = 0 when x = 0 $$y = \frac{v_0 \cdot \left[\cos\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right] - \cos(\omega \cdot t)\right]}{\omega}$$ This gives streamlines y(x) at each time t

For particle paths, first find x(t) $$\frac{dx}{dt} = u = u_0$$

Separating variables and integrating $dx = u_0 \cdot dt$ or $x = u_0 \cdot t + c_1$

Using initial condition x = 0 at t = τ $c_1 = -u_0 \cdot \tau$ $x = u_0 \cdot (t - \tau)$

For y(t) we have $$\frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left(t - \frac{x}{u_0}\right)\right]$$ so $$\frac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left[t - \frac{u_0 \cdot (t - \tau)}{u_0}\right]\right]$$

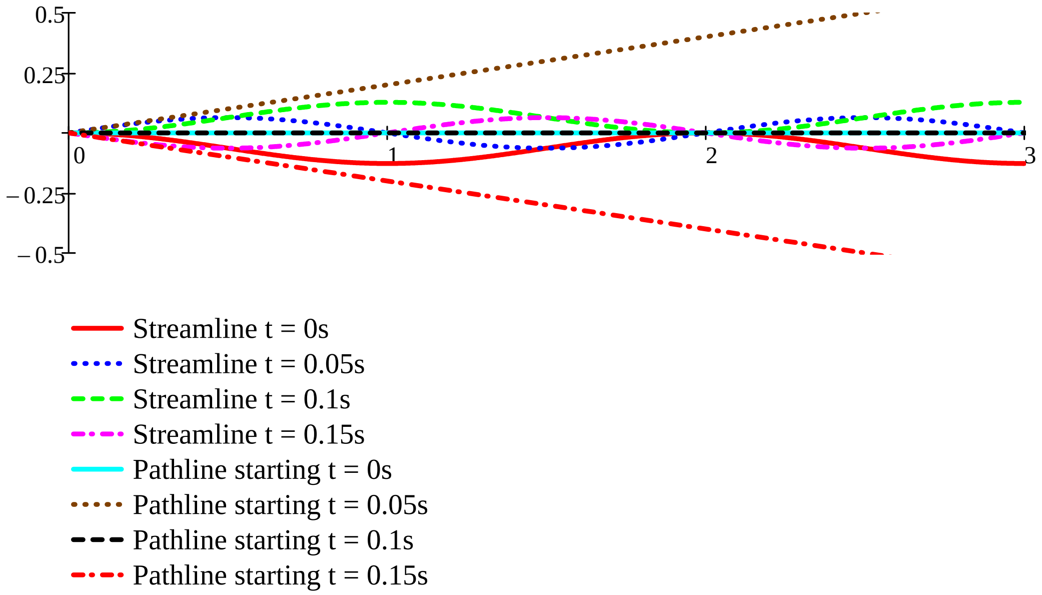
and $$\frac{dy}{dt} = v = v_0 \cdot \sin(\omega \cdot \tau)$$

Separating variables and integrating $dy = v_0 \cdot \sin(\omega \cdot \tau) \cdot dt$ $y = v_0 \cdot \sin(\omega \cdot \tau) \cdot t + c_2$

Using initial condition y = 0 at t = τ $c_2 = -v_0 \cdot \sin(\omega \cdot \tau) \cdot \tau$ $y = v_0 \cdot \sin(\omega \cdot \tau) \cdot (t - \tau)$

The pathline is then

$x(t, \tau) = u_0 \cdot (t - \tau)$ $y(t, \tau) = v_0 \cdot \sin(\omega \cdot \tau) \cdot (t - \tau)$ These terms give the path of a particle (x(t),y(t)) that started at t = τ.

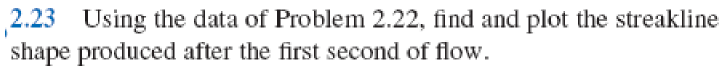


The streamlines are sinusoids; the pathlines are straight (once a water particle is fired it travels in a straight line).

These curves can be plotted in *Excel*.

Problem 2.23

[5]



2.23 Using the data of Problem 2.22, find and plot the streakline shape produced after the first second of flow.

Given: Velocity field

Find: Plot streakline for first second of flow

Solution:

Following the discussion leading up to Eq. 2.10, we first find equations for the pathlines in form

$$x_p(t) = x\left(t, x_0, y_0, t_0\right) \qquad \text{and} \qquad y_p(t) = y\left(t, x_0, y_0, t_0\right)$$

where x_0, y_0 is the position of the particle at $t = t_0$, and re-interprete the results as streaklines

$$x_{st}\left(t_0\right) = x\left(t, x_0, y_0, t_0\right) \qquad \text{and} \qquad y_{st}\left(t_0\right) = y\left(t, x_0, y_0, t_0\right)$$

which gives the streakline at t, where x_0, y_0 is the point at which dye is released (t_0 is varied from 0 to t)

For particle paths, first find x(t) $\quad \dfrac{dx}{dt} = u = u_0$

Separating variables and integrating $\quad dx = u_0 \cdot dt \qquad$ or $\qquad x = x_0 + u_0 \cdot \left(t - t_0\right)$

For y(t) we have $\quad \dfrac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left(t - \dfrac{x}{u_0}\right)\right] \qquad$ so $\qquad \dfrac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left[t - \dfrac{x_0 + u_0 \cdot \left(t - t_0\right)}{u_0}\right]\right]$

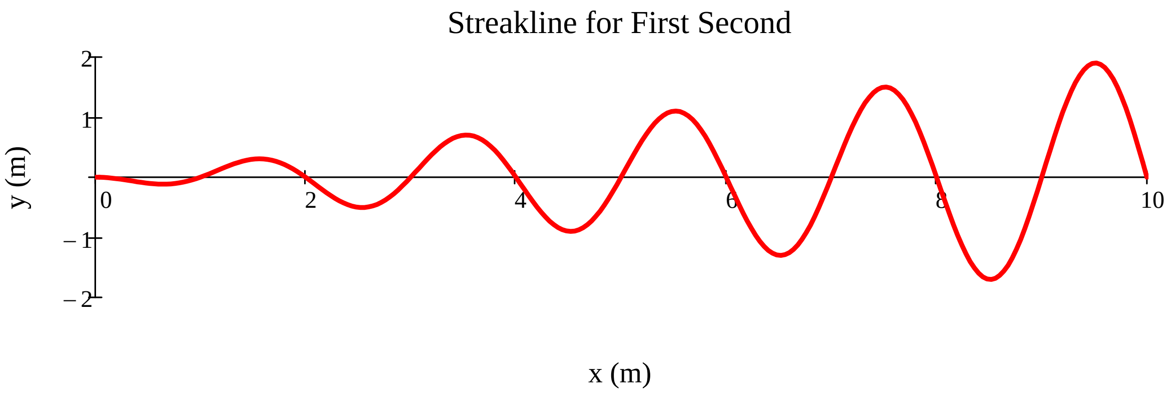
and $\quad \dfrac{dy}{dt} = v = v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \dfrac{x_0}{u_0}\right)\right]$

Separating variables and integrating $\quad dy = v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \dfrac{x_0}{u_0}\right)\right] \cdot dt \qquad y = y_0 + v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \dfrac{x_0}{u_0}\right)\right] \cdot \left(t - t_0\right)$

The streakline is then $\quad x_{st}\left(t_0\right) = x_0 + u_0\left(t - t_0\right) \qquad y_{st}\left(t_0\right) = y_0 + v_0 \cdot \sin\left[\omega \cdot \left(t_0 - \dfrac{x_0}{u_0}\right)\right] \cdot \left(t - t_0\right)$

With $\quad x_0 = y_0 = 0$

$$x_{st}\left(t_0\right) = u_0 \cdot \left(t - t_0\right) \qquad y_{st}\left(t_0\right) = v_0 \cdot \sin\left[\omega \cdot \left(t_0\right)\right] \cdot \left(t - t_0\right)$$



This curve can be plotted in *Excel*. For t = 1, t_0 ranges from 0 to t.

Problem 2.24

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Given: Velocity field $\vec{V} = Bx(1+At)\hat{i} + Cy\hat{j}$, with $A = 0.5\,s^{-1}$, $B = C = 1\,s^{-1}$; coordinates measured in meters.

Plot: the streakline formed by particles that passed through point $(x_0, y_0, z_0) = (1,1,0)$ during interval from $t = 0$ to $t = 3\,s$.
Compare with streamlines through point at $t = 0$, 1, and 2 s

Solution

Streakline at $t = 3\,s$ connects particles that passed through point $(1,1,0)$ at earlier times $t_0 = 0, 1$, and 2 s

For a particle, $u = \frac{dx}{dt}$ and $v = dy/dt$

Then

$$u = Bx(1+At) = \frac{dx}{dt}, \quad \int_{x_0}^{x} \frac{dx}{x} = \int_{t_0}^{t} B(1+At)\,dt$$

$$\therefore \ln\frac{x}{x_0} = B\left[t + \tfrac{1}{2}At^2\right]_{t_0}^{t} = B\left[(t-t_0) + \tfrac{1}{2}AB(t^2-t_0^2)\right]$$

$$x = x_0 e^{B[(t-t_0) + \frac{1}{2}AB(t^2-t_0^2)]} \qquad (1a)$$

Also $v = Cy = dy/dt$, $\int_{t_0}^{t} C\,dt = \int_{y_0}^{y}\frac{dy}{y}$, $\therefore y = y_0 e^{C(t-t_0)}$ (1b)

The velocity vector is tangent to the streamline

$$\left.\frac{dy}{dx}\right|_{\text{streamline}} = \frac{v}{u} = \frac{Cy}{Bx(1+At)} \quad \text{and} \quad (1+At)\frac{dy}{y} = \frac{C}{B}\frac{dx}{x}$$

Then

$$(1+At)\ln y = \frac{C}{B}\ln x + \ln c_1 \quad \text{and} \quad c_1 x^{C/B} = y^{(1+At)}$$

Streamline through point $(1,1,0)$ gives $c_1 = 1$. Then on substituting for A, B, and C we obtain

$$x = y^{(1+0.5t)} \qquad \text{Streamline}$$

At $t = 0$ $x = y$
$t = 1\,s$ $x = y^{1.5}$
$t = 2\,s$ $x = y^2$
} These streamlines through $(1,1,0)$ are shown on the plot

Points on the streakline have coordinates given by Eqs 1a & 1b

$$x = x_0 e^{B[(t-t_0) + \frac{1}{2}AB(t^2-t_0^2)]} \qquad y = y_0 e^{C(t-t_0)}$$

Substituting for A, B, and C

$$x = x_0 e^{[(t-t_0) + 0.25(t^2-t_0^2)]} \qquad y = y_0 e^{(t-t_0)}$$

The streakline through $(x_0, y_0) = (1,1)$ at time $t = 3\,s$ is obtained by substituting $x_0 = 1$, $y_0 = 1$, $t = 3\,s$ and varying t_0 in these equations.

Thus, $x = e^{[(3-t_0)+0.25(9-t_0^2)}$

$y = e^{(3-t_0)}$

give points (obtained by varying t_0) on the streakline through (1,1,0) at t = 3s.

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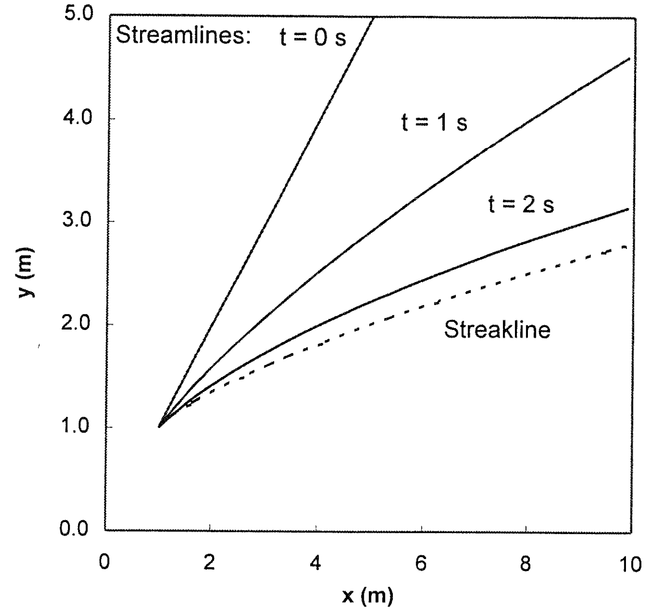


Problem 2.25

Given: Velocity field $\vec{V} = ax(1+bt)\hat{i} + cy\hat{j}$, where $a=c=1\,s^{-1}$, $b = 0.2\,s^{-1}$, and coordinates are measured in meters.

Plot: the streakline that passes through the point $(x_0, y_0) = (1,1)$ during the interval $0 \le t \le 3$s.
Compare with the streamlines plotted through the same point at $t = 0$, 1, and 2s

Solution:

Streakline at $t = 3$s connects particles that passed through point (x_0, y_0) at earlier times $\tau = 0, 1, 2$, and 3s.

For a particle, $u = dx/dt$ and $v = dy/dt$

Then

$$u = ax(1+bt) = \frac{dx}{dt} \quad \text{and} \quad \int_{x_0}^{x} \frac{dx}{x} = \int_{\tau}^{t} a(1+bt)\,dt$$

$$\ln\frac{x}{x_0} = a\left(t + \frac{b}{2}t^2\right)\Big]_{\tau}^{t} = a\left[(t-\tau) + \frac{b}{2}(t^2 - \tau^2)\right]$$

$$x = x_0\, e^{a[(t-\tau) + \frac{b}{2}(t^2-\tau^2)]}$$

Also $v = \frac{dy}{dt} = cy$, $\int_{y_0}^{y} \frac{dy}{y} = \int_{\tau}^{t} c\,dt$, $\ln\frac{y}{y_0} = c(t-\tau)$, $y = y_0\, e^{c(t-\tau)}$

Substituting for a, b, c, x_0, and y_0, gives:

$$x = e^{[t-\tau] + 0.1(t^2-\tau^2)}, \quad y = e^{(t-\tau)} \quad \longleftarrow \underline{(x,y)\text{ streakline}}$$

The streakline may be plotted by substituting values for τ in the range $0 \le \tau \le 3$s as shown below.

The streamline is found (at given t) from $dy/dx)_s = \frac{v}{u}$

Thus

$$dy/dx = \frac{cy}{ax(1+bt)} \quad \text{and} \quad \int_{y_0}^{y} \frac{dy}{y} = \int_{x_0}^{x} \frac{c}{a(1+bt)} \frac{dx}{x}$$

$$\ln\frac{y}{y_0} = \frac{c}{a(1+bt)} \ln\frac{x}{x_0} \quad \text{or} \quad y = y_0 \left[\frac{x}{x_0}\right]^{c/a(1+bt)}$$

Substituting values for x_0, y_0, a, b, c, then

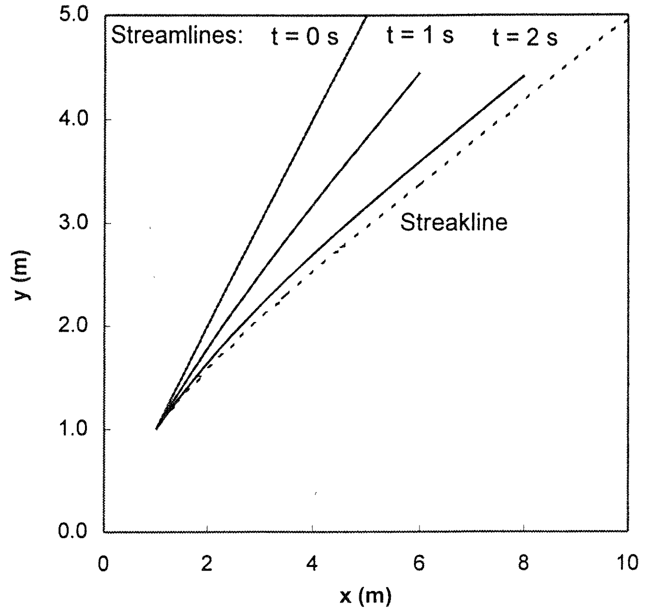
$$y = x^{1/(1+0.2t)} \quad \text{or} \quad x = y^{(1+0.2t)} \quad \longleftarrow \underline{\text{streamline}}$$

At $t = 0$, $x = y$

$t = 1$s, $x = y^{1.2}$

$t = 2$s, $x = y^{1.4}$

Problem 2.25


Streamlines:
t = 0 s
t = 1 s
t = 2 s
Streakline
y (m)
x (m)
0.0
1.0
2.0
3.0
4.0
5.0
0
2
4
6
8
10

Problem 2.26

[4] Part 1/2

Given: Velocity field $\vec{V} = axt\hat{i} + b\hat{j}$, where $a = 0.2\ s^{-1}$, $b = 1\ m/s$, and coordinates are in meters.

Plot: the pathline (during the interval $0 \le t \le 3s$) of the particle that passed through the point $(x_0, y_0) = (1, 2)$ at time $t = 0$

Compare with the streakline through the same point at the instant $t = 3s$.

Solution:

The pathline and streakline are based on parametric equations for a particle.

For a particle $u = dx/dt$ and $v = dy/dt$

Then $u = \frac{dx}{dt} = axt$, $\int_{x_0}^{x} \frac{dx}{x} = \int_{t_0}^{t} at\,dt$, $\ln\frac{x}{x_0} = \frac{1}{2}a(t^2 - t_0^2)$

$$x = x_0 e^{\frac{1}{2}a(t^2 - t_0^2)}$$

Also $v = dy/dt = b$, $\int_{y_0}^{y} dy = \int_{t_0}^{t} b\,dt$, $y = y_0 + b(t - t_0)$

In the above equations, x_0, y_0 are coordinates of particle at t_0

(a) The pathline is obtained by following the particle that passed through the point $(x_0, y_0) = (1, 2)$ at time $t_0 = 0$

$$\text{Thus}\quad \left.\begin{aligned} x &= x_0 e^{\frac{1}{2}at^2} = e^{0.1t^2} \\ y &= y_0 + bt = 2 + t \end{aligned}\right\} \longleftarrow \underline{(x, y)\text{ pathline}}$$

The pathline may be plotted by varying t $(0 \le t \le 3s)$ as shown below

(b) The streakline is obtained by locating (and connecting) at time $t = 3s$, all the particles that passed through the point $(x_0, y_0) = (1, 2)$ at some earlier time t_0.

$$\text{Thus}\quad \left.\begin{aligned} x &= x_0 e^{\frac{1}{2}a(9 - t_0^2)} = e^{0.1(9 - t_0^2)} \\ y &= y_0 + b(t - t_0) = 2 + (3 - t_0) = 5 - t_0 \end{aligned}\right\} \longleftarrow \underline{(x, y)\text{ streakline}}$$

The streakline may be plotted by varying t_0 $(0 \le t_0 \le 3s)$ as shown below.

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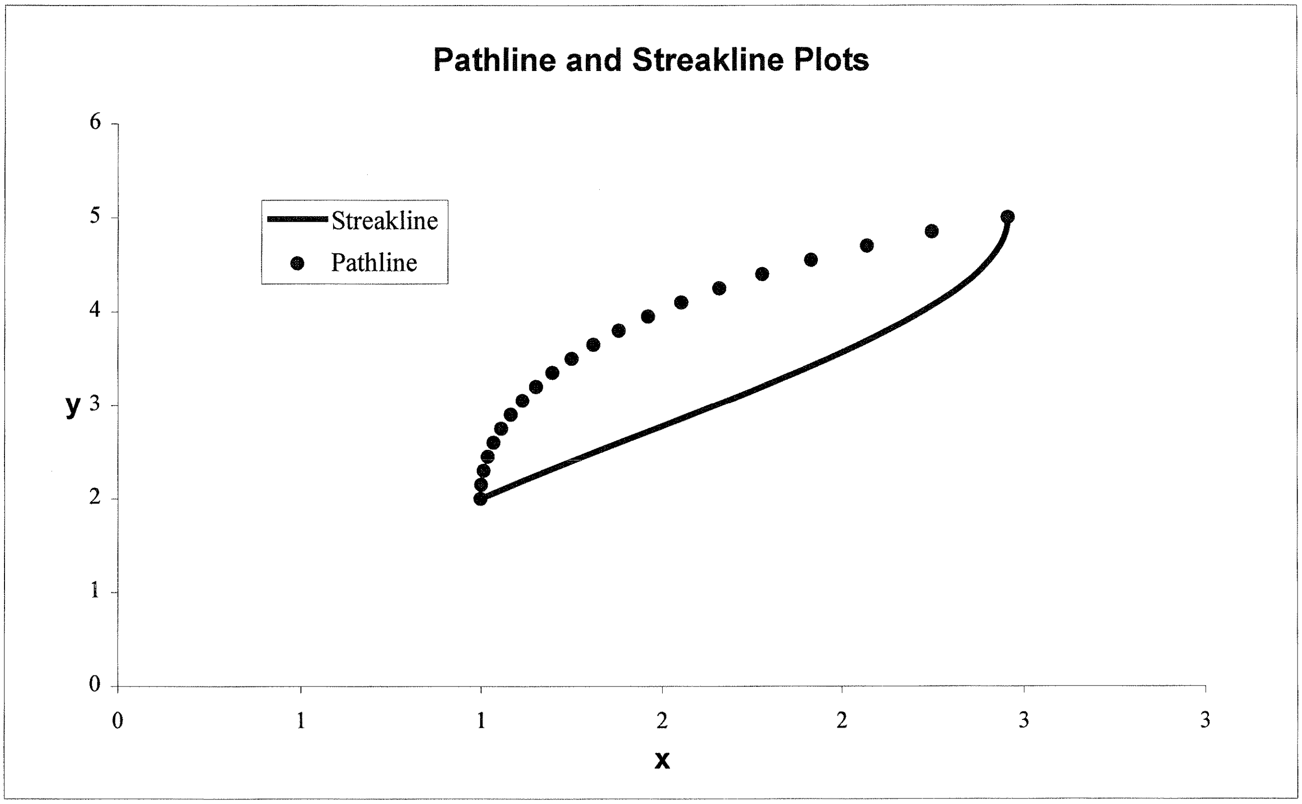
Problem 2.26


Pathline and Streakline Plots
Streakline
Pathline
y
x
0
1
2
3
4
5
6
0
1
1
2
2
3
3

Problem 2.27

[3]

2.27 Tiny hydrogen bubbles are being used as tracers to visualize a flow. All the bubbles are generated at the origin ($x = 0$, $y = 0$). The velocity field is unsteady and obeys the equations:

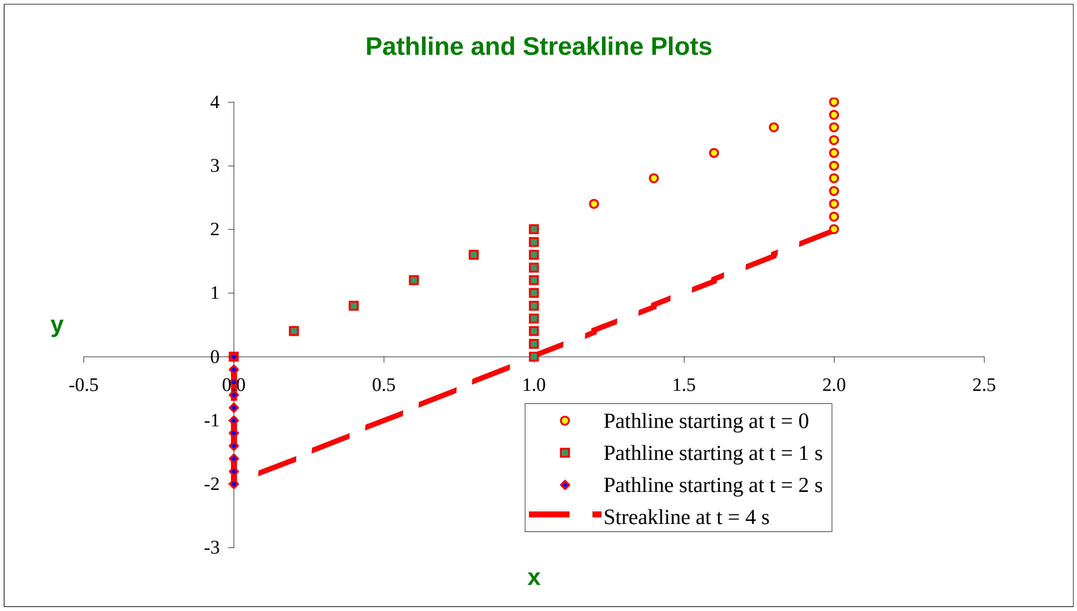
$$
\begin{array}{lll}
u = 1\ \text{m/s} & v = 2\ \text{m/s} & 0 \le t < 2\ \text{s} \\
u = 0 & v = -1\ \text{m/s} & 0 \le t \le 4\ \text{s}
\end{array}
$$

Plot the pathlines of bubbles that leave the origin at $t = 0$, 1, 2, 3, and 4 s. Mark the locations of these five bubbles at $t = 4$ s. Use a dashed line to indicate the position of a streakline at $t = 4$ s.

Solution

The particle starting at t = 3 s follows the particle starting at t = 2 s;
The particle starting at t = 4 s doesn't move!

Pathlines:	**Starting at t = 0**		**Starting at t = 1 s**		**Starting at t = 2 s**		**Streakline at t = 4 s**	
t	**x**	**y**	**x**	**y**	**x**	**y**	**x**	**y**
0.00	0.00	0.00					2.00	2.00
0.20	0.20	0.40					1.80	1.60
0.40	0.40	0.80					1.60	1.20
0.60	0.60	1.20					1.40	0.80
0.80	0.80	1.60					1.20	0.40
1.00	1.00	2.00	0.00	0.00			1.00	0.00
1.20	1.20	2.40	0.20	0.40			0.80	-0.40
1.40	1.40	2.80	0.40	0.80			0.60	-0.80
1.60	1.60	3.20	0.60	1.20			0.40	-1.20
1.80	1.80	3.60	0.80	1.60			0.20	-1.60
2.00	2.00	4.00	1.00	2.00	0.00	0.00	0.00	-2.00
2.20	2.00	3.80	1.00	1.80	0.00	-0.20	0.00	-1.80
2.40	2.00	3.60	1.00	1.60	0.00	-0.40	0.00	-1.60
2.60	2.00	3.40	1.00	1.40	0.00	-0.60	0.00	-1.40
2.80	2.00	3.20	1.00	1.20	0.00	-0.80	0.00	-1.20
3.00	2.00	3.00	1.00	1.00	0.00	-1.00	0.00	-1.00
3.20	2.00	2.80	1.00	0.80	0.00	-1.20	0.00	-0.80
3.40	2.00	2.60	1.00	0.60	0.00	-1.40	0.00	-0.60
3.60	2.00	2.40	1.00	0.40	0.00	-1.60	0.00	-0.40
3.80	2.00	2.20	1.00	0.20	0.00	-1.80	0.00	-0.20
4.00	2.00	2.00	1.00	0.00	0.00	-2.00	0.00	0.00



Problem 2.28

[4]

2.28 A flow is described by velocity field, $\vec{V} = ay^2\hat{i} + b\hat{j}$, where $a = 1\ \text{m}^{-1}\,\text{s}^{-1}$ and $b = 2$ m/s. Coordinates are measured in meters. Obtain the equation for the streamline passing through point (6, 6). At $t = 1$ s, what are the coordinates of the particle that passed through point (1, 4) at $t = 0$? At $t = 3$ s, what are the coordinates of the particle that passed through point $(-3, 0)$ 2 s earlier? Show that pathlines, streamlines, and streaklines for this flow coincide.

Given: 2D velocity field

Find: Streamlines passing through (6,6); Coordinates of particle starting at (1,4); that pathlines, streamlines and streaklines coincide

Solution:

For streamlines $\quad \dfrac{v}{u} = \dfrac{dy}{dx} = \dfrac{b}{a \cdot y^2} \quad$ or $\quad \displaystyle\int a \cdot y^2\,dy = \int b\,dx$

Integrating $\quad \dfrac{a \cdot y^3}{3} = b \cdot x + C$

For the streamline through point (6,6) $\quad C = 60 \quad$ and $\quad y^3 = 6 \cdot x + 180$

For particle that passed through (1,4) at t = 0 $\quad u = \dfrac{dx}{dt} = a \cdot y^2 \qquad \displaystyle\int 1\,dx = x - x_0 = \int a \cdot y^2\,dt \quad$ but we need y(t)

$$v = \frac{dy}{dt} = b \qquad \int 1\,dy = \int b\,dt \qquad y = y_0 + b \cdot t = y_0 + 2 \cdot t$$

Then $\quad \displaystyle x - x_0 = \int_0^t a \cdot \left(y_0 + b \cdot t\right)^2 dt \qquad x = x_0 + a \cdot \left(y_0^2 \cdot t + b \cdot y_0 \cdot t^2 + \frac{b^2 \cdot t^3}{3}\right)$

Hence, with $\quad x_0 = 1 \quad y_0 = 4 \qquad x = 1 + 16 \cdot t + 8 \cdot t^2 + \dfrac{4}{3} \cdot t^3 \quad$ At t = 1 s $\quad x = 26.3 \cdot \text{m}$

$$y = 4 + 2 \cdot t \qquad y = 6 \cdot \text{m}$$

For particle that passed through (-3,0) at t = 1 $\quad \displaystyle\int 1\,dy = \int b\,dt \qquad y = y_0 + b \cdot \left(t - t_0\right)$

$$x - x_0 = \int_{t_0}^{t} a \cdot \left(y_0 + b \cdot t\right)^2 dt \qquad x = x_0 + a \cdot \left[y_0^2 \cdot \left(t - t_0\right) + b \cdot y_0 \cdot \left(t^2 - t_0^2\right) + \frac{b^2}{3} \cdot \left(t^3 - t_0^3\right)\right]$$

Hence, with $x_0 = -3$, $y_0 = 0$ at $t_0 = 1$ $\quad x = -3 + \dfrac{4}{3} \cdot \left(t^3 - 1\right) = \dfrac{1}{3} \cdot \left(4 \cdot t^3 - 13\right) \qquad y = 2 \cdot (t - 1)$

Evaluating at t = 3 $\quad x = 31.7 \cdot \text{m} \qquad y = 4 \cdot \text{m}$

This is a steady flow, so pathlines, streamlines and streaklines always coincide

Problem 2.29

[4] Part 1/2

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Given: Velocity field in xy plane, $\vec{V} = a\hat{i} + bx\hat{j}$, where $a = 2\ \text{m/s}$ and $b = 1\ \text{s}^{-1}$.

Find: (a) Equation for streamline through $(x,y) = (2,5)$.

(b) At $t = 2\ \text{s}$, coordinates of particle $(0,4)$ at $t = 0$.

(c) At $t = 3\ \text{s}$, coordinates of particle $(1, 4.25)$ at $t = 1\ \text{s}$.

(d) compare pathline, streamline, streakline.

Solution: For a streamline $\dfrac{dx}{u} = \dfrac{dy}{v}$

For $\vec{V} = a\hat{i} + bx\hat{j}$, $u = a$ and $v = bx$, so $\dfrac{dx}{a} = \dfrac{dy}{bx}$ or

$$x\,dx = \frac{a}{b}\,dy$$

Integrating

$$\frac{x^2}{2} = \frac{a}{b}y + c' \quad \text{or} \quad y = \frac{b}{2a}x^2 + c$$

Evaluating c at $(x,y) = (2,5)$,

$$c = y - \frac{b}{2a}x^2 = 5\,\text{m} - \frac{1}{2} \times \frac{1}{\text{s}} \times \frac{\text{s}}{2\,\text{m}}\,(2\,\text{m})^2 = 4\,\text{m}$$

Streamline through $(x,y) = (2,5)$ is $\quad y = \dfrac{x^2}{4} + 4$ (a)

To locate particles, derive parametric equations

$$u_p = \frac{dx}{dt} = a, \quad dx = a\,dt, \quad \text{and} \quad x - x_0 = a(t - t_0)$$

$$v_p = \frac{dy}{dt} = bx, \quad dy = bx\,dt = b(x_0 + at - at_0)$$

$$y - y_0 = bx_0(t - t_0) + \frac{a}{2}(t^2 - t_0^2) - at_0(t - t_0)$$

For the particle at $(x_0, y_0) = (0,4)$ at $t = 0$,

$x = 0 + at \qquad$ so at $t = 2\,\text{s}$, $\ x = 2\,\dfrac{\text{m}}{\text{s}} \times 2\,\text{s} = 4\,\text{m}$

$y = 4 + \dfrac{at^2}{2} \qquad$ so at $t = 2\,\text{s}$, $\ y = 4 + \dfrac{1}{2} \times 2\,\dfrac{\text{m}}{\text{s}} \times (2)^2\text{s}^2$

$y = 8\,\text{m}$ (b)

Problem 2.29

[4] Part 2/2

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For the particle at $(x,y) = (1, 4.25)$ at $t = 1\,s$,

$$x = x_0 + a(t - t_0) = 1 + a(t-1)$$

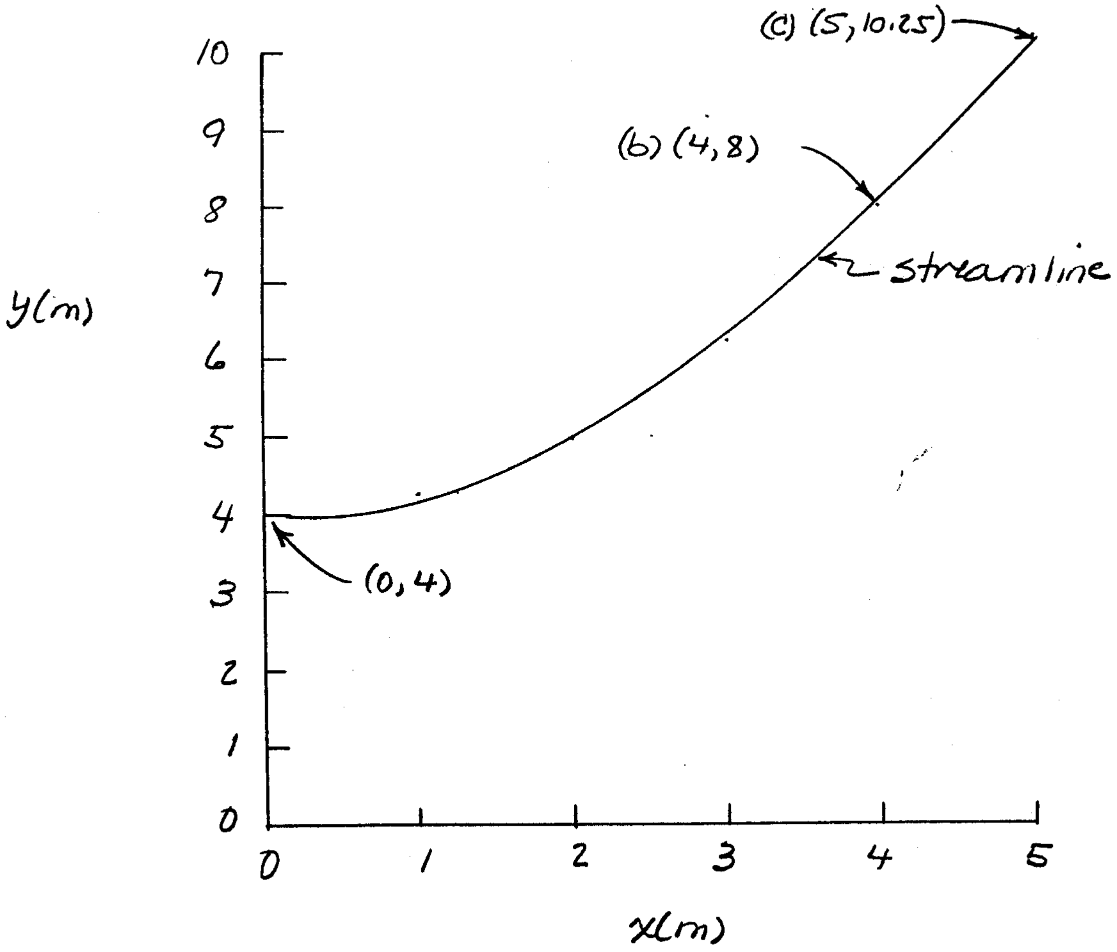
so at $t = 3\,s$, $x = 1 + 2\frac{m}{s}(3-1)s = 5m$

$$y = y_0 + bx_0(t - t_0) + \frac{a}{2}(t^2 - t_0^2) - at_0(t - t_0)$$

$$= 4.25 + \frac{1}{s} \times 1m \times (t-1) + \frac{1}{2} \times 2\frac{m}{s}(t^2 - 1) - 2\frac{m}{s} \times 1s\,(t-1)$$

so at $t = 3\,s$, $y = 4.25 + 2 + 8 - 4 = 10.25\ m$ (c)

All these points lie on the same streamline, as shown below:



For this steady flow, streamlines, pathlines, and streaklines coincide, as expected.

Problem 2.30

[4] Part 1/2

Given: Velocity field $\vec{V} = ay\hat{i} + bt\hat{j}$, where $a = 1\ s^{-1}$; $b = 0.5\ m/s^2$; t in s.

Find: (a) At t = 2 s, particle that passed (1, 2) at t = 0 s

(b) At t = 3 s, particle that passed (1, 2) at t = 2 s

(c) Plot pathline and streakline through (1, 2); compare with streamlines at t = 0, 1, 2 s.

Solution: Pathline and streakline are based on parametric equations for a particle. Thus

$$v = \frac{dy}{dt} = bt, \text{ so } dy = bt\,dt, \text{ and } y - y_0 = \frac{b}{2}(t^2 - t_0^2)$$

and $$u = \frac{dx}{dt} = ay = a\left[y_0 + \frac{b}{2}(t^2 - t_0^2)\right]$$

so $$x\Big]_{x_0}^{x} = a\left[y_0 t + \frac{b}{2}\left(\frac{t^3}{3} - t_0^2 t\right)\right]_{t_0}^{t} \; ; \; x = x_0 + ay_0(t - t_0) + \frac{ab}{2}\left(\frac{t^3 - t_0^3}{3} + t_0^2(t_0 - t)\right)$$

where x_0, y_0 are coordinates of particle at t_0.

For (a), $t_0 = 0$, and $(x_0, y_0) = (1, 2)$. Thus at t = 2 s, $y = y_0 + \frac{bt^2}{2}$

$$y = 2\,m + \frac{1}{2} \times 0.5\,\frac{m}{s^2} \times (2)^2 s^2 = 3.00\,m$$

$$x = 1\,m + \frac{1}{s} \times 2\,m\,(2-0)\,s + \frac{1}{2} \times \frac{1}{s} \times 0.5\,\frac{m}{s^2}\left(\frac{(2)^3 - 0}{3} + 0\right)s^3 = 5.67\,m$$

$t_0 = 0$ (a)
At t = 2 s, (x, y) = (5.67, 3.00) m

For (b), $t_0 = 2$ s, and $(x_0, y_0) = (1, 2)$. Thus at t = 3 s, the particle is at

$$y(3) = 2\,m + \frac{1}{2} \times 0.5\,\frac{m}{s^2}\left[(3)^2 - (2)^2\right] s^2 = 3.25\,m$$

$$x(3) = 1\,m + \frac{1}{s} \times 2\,m\,(3-2)\,s + \frac{1}{2} \times \frac{1}{s} \times 0.5\,\frac{m}{s^2}\left(\frac{(3)^3 - (2)^3}{3} + (2)^2(2-3)\right)s^3 = 3.58\,m$$

At t = 3 s, t_0 = 2 s (b)
(x, y) = (3.58, 3.25) m

For (c), the streakline may be plotted at any t by varying t_0, as shown on the next page.

The streamline is found (at given t) from $\frac{dx}{u} = \frac{dy}{v}$

Substituting u = ay and v = bt, $dx = \frac{ay}{bt}\,dy$ or $y^2 = \frac{2bt}{a}x + c$

Thus $c = y_0^2 - \frac{2bt}{a}x_0$

For t = 0, $y^2 = c$; at $(x_0, y_0) = (1, 2)$, then c = 4

t = 1, $y^2 = \frac{2b}{a}x + c$; at $(x_0, y_0) = (1, 2)$, then c = 3

t = 2, $y^2 = \frac{4b}{a}x + c$; at (x, y) = (1, 2), c = 2; for t = 3 s, c = 1

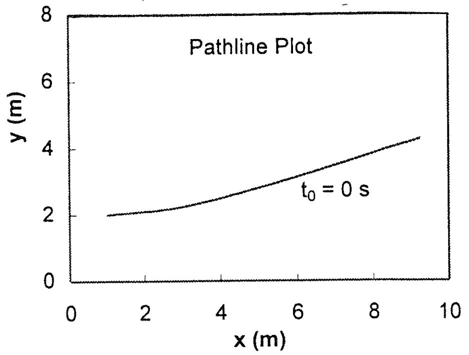
(c)

Problem 2.30

Recall $\vec{V} = ay\hat{i} + bt\hat{j}$, where $a = 1\ s^{-1}$; $b = 0.5\ m/s^2$; $(x_0, y_0) = (1, 2)$ m.

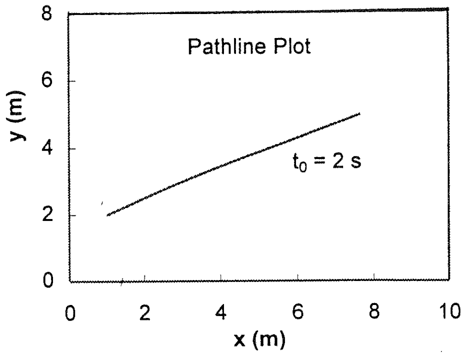
Part (a): Pathline of particle located at (x_0, y_0) at $t_0 = 0$ s:

t_0 (s)	t (s)	x (m)	y (m)
0	0	1.00	2.00
0	1	3.08	2.25
0	2	5.67	3.00
0	3	9.25	4.25



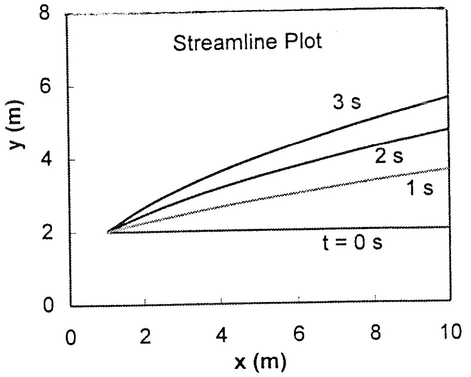
Part (b): Pathline of particle located at (x_0, y_0) at $t_0 = 2$ s:

t_0 (s)	t (s)	x (m)	y (m)
2	2	1.00	2.00
2	3	3.58	3.25
2	4	7.67	5.00



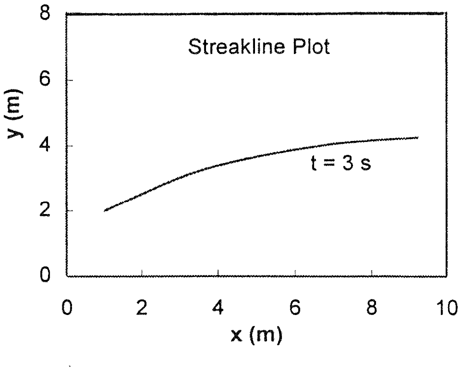
Part (c): Streamlines through point (x_0, y_0) at $t = 0, 1, 2,$ and 3 s:

	t (s)	0	1	2	3
	c =	4.0	3.0	2.0	1.0
t_0 (s)	x (m)	y (m)	y (m)	y (m)	y (m)
0	1	2.00	2.00	2.00	2.00
0	2	2.00	2.24	2.45	2.65
0	3	2.00	2.45	2.83	3.16
0	4	2.00	2.65	3.16	3.61
0	5	2.00	2.83	3.46	4.00
0	6	2.00	3.00	3.74	4.36
0	7	2.00	3.16	4.00	4.69
0	8	2.00	3.32	4.24	5.00
0	9	2.00	3.46	4.47	5.29
0	10	2.00	3.61	4.69	5.57



Streakline at t = 3 s of particles that passed thru point (x_0, y_0):

t_0 (s)	t (s)	x (m)	y (m)
0	3	9.25	4.25
1	3	6.67	4.00
2	3	3.58	3.25
3	3	1.00	2.00



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Problem 2.31

Given: Velocity field $\vec{V} = at\hat{i} + b\hat{j}$, where $a = 0.4\ m/s^2$, $b = 2\ m/s$, and coordinates are measured in meters

Find: (a) At t = 2s, coordinates of particle that passed through $(x_0, y_0) = (2,1)$ at $t=0$
b) At t = 3s, coordinates of the particle that passed through (x_0, y_0) at $t = 2s$

Plot: the pathline and streakline through point (2,1); compare with the streamlines through the same point at t=0, 1, 2s

Solution:

The pathline and streakline are based on parametric equations for a particle.

For a particle $u = dx/dt$ and $v = dy/dt$

Thus $u = \frac{dx}{dt} = at$, $\int_{x_0}^{x} dx = \int_{t_0}^{t} at\,dt$, $x = x_0 + \frac{1}{2}a(t^2 - t_0^2)$ (1a)

$v = \frac{dy}{dt} = b$, $\int_{y_0}^{y} dy = \int_{t_0}^{t} b\,dt$, $y = y_0 + b(t - t_0)$. (1b)

In the above equations, x_0, y_0 are coordinates of the particle at time t_0

(a) The pathline is obtained by following the particle that passed through the point $(x_0, y_0) = (2,1)$ at time $t_0 = 0$

Thus $\left.\begin{aligned} x &= x_0 + \tfrac{1}{2}at^2 = 2 + 0.2t^2 \\ y &= y_0 + bt = 1 + 2t \end{aligned}\right\} \longleftarrow$ (x,y) pathline

At t = 2s, particle is at (x,y) = (2.8, 5) m $\longleftarrow$ (a)

The pathline may be plotted by varying t ($0 \le t \le 3s$) as shown below

b) The streakline is obtained by locating (and connecting) at time t = 3s, all the particles that passed through the point $(x_0, y_0) = (2,1)$ at some earlier time t_0

Thus $\left.\begin{aligned} x &= x_0 + \tfrac{1}{2}a(9 - t_0^2) = 2 + 0.2(9 - t_0^2) \\ y &= y_0 + b(t - t_0) = 1 + 2(3 - t_0) \end{aligned}\right\} \longleftarrow$ (x,y) streakline

At t = 2s, particle is at (x,y) = (3,3) $\longleftarrow$ b)

The streakline may be plotted by varying t_0 ($0 \le t_0 \le 3s$) as shown below.

The streamline is found (at given t) from $\left.\frac{dy}{dx}\right)_s = \frac{v}{u}$

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Problem 2.31

Then, $dy/dx = \frac{b}{at}$, $\int_{y_0}^{y} dy = \int_{x_0}^{x} \frac{b}{at}\,dx$, $y - y_0 = \frac{b}{at}(x - x_0)$

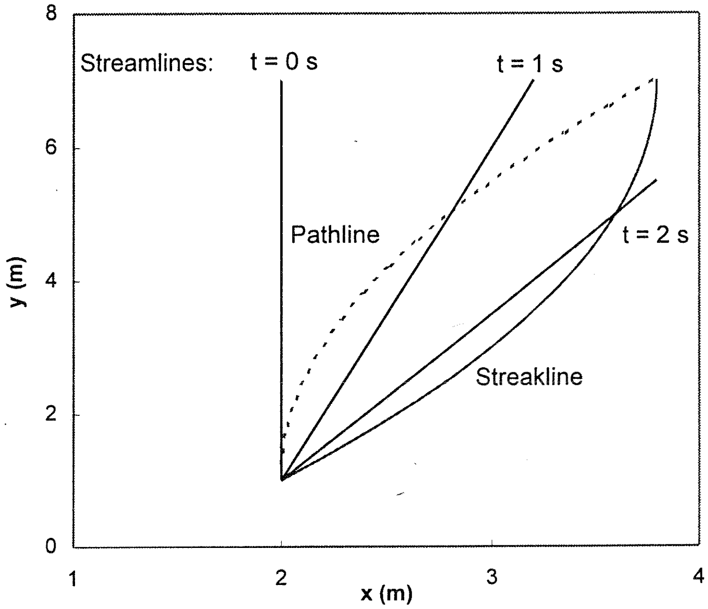
Streamline through point (2,1) gives $y - 1 = \frac{b}{at}(x-2)$

$y = 1 + \frac{5(x-2)}{t}$ or $x = 2 + \frac{t}{5}(y-1)$ ← Streamline

At $t=0$, $x=2$

$t=1$, $y = 5x-9$

$t=2$, $y = 2.5x-4$



Problem 2.32

[2]

Given: Variation of air viscosity with temperature (absolute) is

$$\mu = \frac{bT^{1/2}}{1+S/T}$$

where $b = 1.458\times10^{-6}\ \mathrm{kg/m.s.K^{1/2}}$, $S = 110.4\,\mathrm{K}$

Find: Equation for calculating air viscosity in British Gravitational units as a function of absolute temperature in degrees Rankine. Check result using data from Appendix A

Solution:

Convert constants

$$b = 1.458\times10^{-6}\frac{\mathrm{kg}}{\mathrm{m.s.K^{1/2}}}\times\frac{\mathrm{lbm}}{0.4536\,\mathrm{kg}}\times\frac{\mathrm{slug}}{32.17\,\mathrm{lbm}}\times\frac{\mathrm{lbf.s^2}}{\mathrm{slug.ft}}\times\frac{0.3048\,\mathrm{m}}{\mathrm{ft}}\times\left(\frac{5\,\mathrm{K}}{9\,{}^\circ\mathrm{R}}\right)^{1/2}$$

$$b = 2.27\times10^{-8}\ \mathrm{lbf.s/ft^2.{}^\circ R^{1/2}}$$

$$S = 110.4\,\mathrm{K}\times\frac{9\,{}^\circ\mathrm{R}}{5\,\mathrm{K}} = 198.7\,{}^\circ\mathrm{R}$$

Then in British Gravitational Units

$$\mu = \frac{2.27\times10^{-8}\,T^{1/2}}{1+198.7/T} \longleftarrow \mu$$

where units of T are °R; μ is in $\mathrm{lbf.s/ft^2}$

Evaluate at $T = 80\,{}^\circ\mathrm{F}$ ($539.7\,{}^\circ\mathrm{R}$)

$$\mu = \frac{2.27\times10^{-8}\times(539.7)^{1/2}}{1+198.7/539.7} = 3.855\times10^{-7}\ \mathrm{lbf.s/ft^2}$$

From Table A.9 (Appendix A) at $T = 80\,{}^\circ\mathrm{F}$

$\mu = 3.86\times10^{-7}\ \mathrm{lbf.s/ft^2}$ ✓ check.

13-782 500 SHEETS, FILLER 5 SQUARE
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Problem 2.33

Given: Variation of air viscosity with temperature (absolute) is

$$\mu = \frac{bT^{1/2}}{1+S/T}$$

where $b = 1.458 \times 10^{-6} \ \frac{kg}{m \cdot s \cdot K^{1/2}}$

$S = 110.4\ K$

Find: Equation for kinematic viscosity of air (in SI units) as a function of temperature at atmospheric pressure. Assume ideal gas behavior. Check result using data from Appendix A.

Solution:

For an ideal gas, $p = \rho RT$. From Table A.6, $R = 286.9\ N \cdot m/kg \cdot K$

The kinematic viscosity, $\nu \equiv \mu/\rho$

$$\therefore \nu = \frac{\mu}{\rho} = \frac{\mu RT}{p} = \frac{RT}{p}\frac{bT^{1/2}}{1+S/T} = \frac{Rb}{p}\frac{T^{3/2}}{1+S/T} = \frac{b'T^{3/2}}{1+S/T}$$

where $b' = \frac{Rb}{p} = 286.9 \frac{\cancel{N} \cdot \cancel{m}}{\cancel{kg} \cdot K} \times 1.458 \times 10^{-6} \frac{\cancel{kg}}{\cancel{m} \cdot s \cdot K^{1/2}} \times \frac{m^2}{101.3 \times 10^3\ N}$

$b' = 4.129 \times 10^{-9}\ m^2/s \cdot K^{3/2}$

$$\therefore \nu = \frac{b'T^{3/2}}{1+S/T}$$ ν

where $b' = 4.129 \times 10^{-9}\ m^2/s \cdot K^{3/2}$, $S = 110.4\ K$

units of T are (K); ν is in m^2/s

Evaluate at $T = 20°C = 293.2\ K$

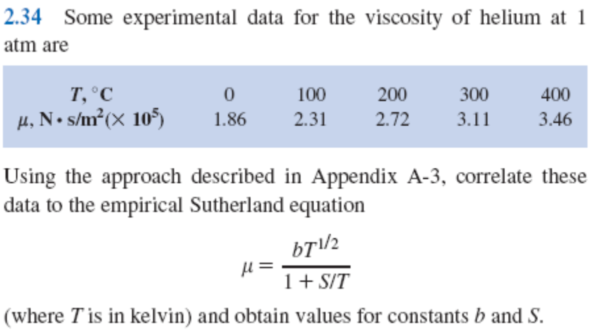
$$\nu = \frac{4.129 \times 10^{-9}\ (293.2)^{3/2}}{1 + 110.4/293.2} = 1.506 \times 10^{-5}\ m^2/s$$

From Table A.10 (Appendix A) at $T = 20°C$

$\nu = 1.51 \times 10^{-5}\ m^2/s$ ✓ check

Problem 2.34

[3]



2.34 Some experimental data for the viscosity of helium at 1 atm are

T, °C	0	100	200	300	400
μ, N · s/m^2($\times 10^5$)	1.86	2.31	2.72	3.11	3.46

Using the approach described in Appendix A-3, correlate these data to the empirical Sutherland equation

$$\mu = \frac{bT^{1/2}}{1 + S/T}$$

(where T is in kelvin) and obtain values for constants b and S.

Given: Viscosity data

Find: Obtain values for coefficients in Sutherland equation

Solution:

Data:

T (°C)	T (K)	μ(x10^5)
0	273	1.86E-05
100	373	2.31E-05
200	473	2.72E-05
300	573	3.11E-05
400	673	3.46E-05

Using procedure of Appendix A.3:

T (K)	$T^{3/2}/\mu$
273	2.43E+08
373	3.12E+08
473	3.78E+08
573	4.41E+08
673	5.05E+08

The equation to solve for coefficients S and b is

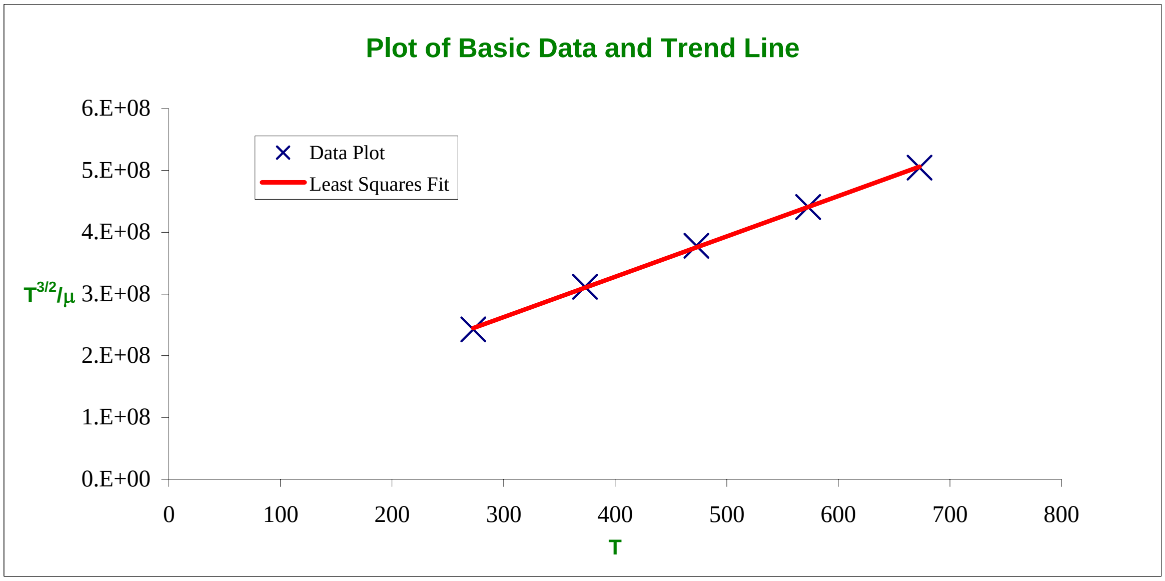
$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{b}\right)T + \frac{S}{b}$$

From the built-in *Excel Linear Regression* functions:

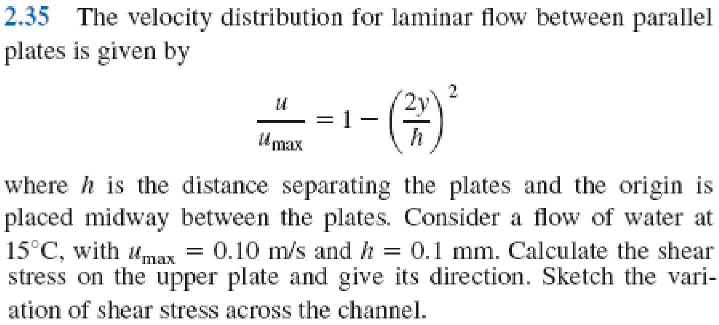
Slope = 6.534E+05
Intercept = 6.660E+07
R^2 = 0.9996

Hence:

b = 1.531E-06 kg/m·s·K$^{1/2}$
S = 101.9 K



Problem 2.35



2.35 The velocity distribution for laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider a flow of water at 15°C, with $u_{\max} = 0.10$ m/s and $h = 0.1$ mm. Calculate the shear stress on the upper plate and give its direction. Sketch the variation of shear stress across the channel.

Given: Velocity distribution between flat plates

Find: Shear stress on upper plate; Sketch stress distribution

Solution:

Basic equation $\quad \tau_{yx} = \mu\cdot\frac{du}{dy} \qquad \frac{du}{dy} = \frac{d}{dy}u_{\max}\cdot\left[1 - \left(\frac{2\cdot y}{h}\right)^2\right] = u_{\max}\cdot\left(-\frac{4}{h^2}\right)\cdot 2\cdot y = -\frac{8\cdot u_{\max}\cdot y}{h^2}$

$$\tau_{yx} = -\frac{8\cdot\mu\cdot u_{\max}\cdot y}{h^2}$$

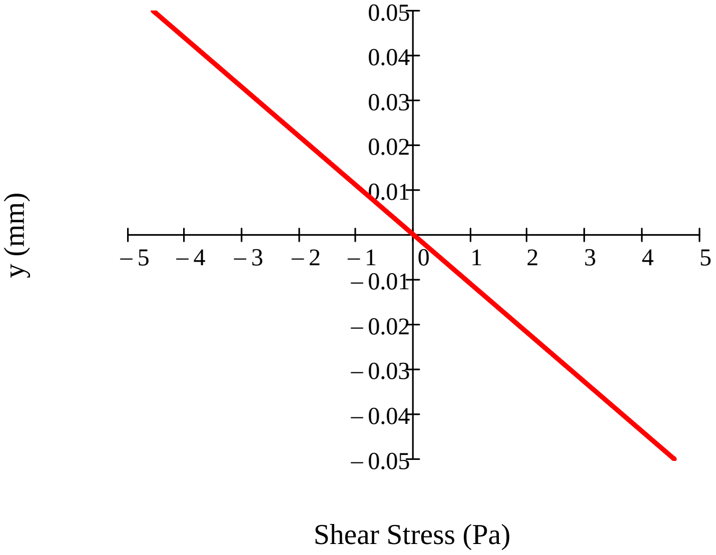
At the upper surface $\quad y = \frac{h}{2} \quad$ and $\quad h = 0.1\cdot\text{mm} \qquad u_{\max} = 0.1\cdot\frac{\text{m}}{\text{s}} \qquad \mu = 1.14\times 10^{-3}\cdot\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ (Table A.8)

Hence $\quad \tau_{yx} = -8\times 1.14\times 10^{-3}\cdot\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times 0.1\cdot\frac{\text{m}}{\text{s}}\times\frac{0.1}{2}\cdot\text{mm}\times\frac{1\cdot\text{m}}{1000\cdot\text{mm}}\times\left(\frac{1}{0.1\cdot\text{mm}}\times\frac{1000\cdot\text{mm}}{1\cdot\text{m}}\right)^2$

$$\tau_{yx} = -4.56\cdot\frac{\text{N}}{\text{m}^2}$$

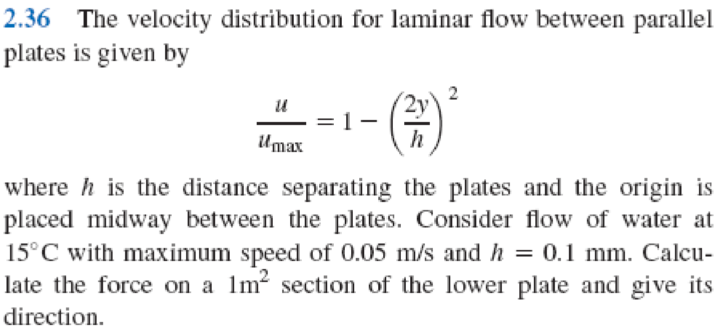
The upper plate is a minus y surface. Since $\tau_{yx} < 0$, the shear stress on the upper plate must act in the plus x direction.

The shear stress varies linearly with y $\qquad \tau_{yx}(y) = -\left(\frac{8\cdot\mu\cdot u_{\max}}{h^2}\right)\cdot y$



Problem 2.36

[2]



2.36 The velocity distribution for laminar flow between parallel plates is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{2y}{h}\right)^2$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider flow of water at 15°C with maximum speed of 0.05 m/s and $h = 0.1$ mm. Calculate the force on a 1m^2 section of the lower plate and give its direction.

Given: Velocity distribution between parallel plates

Find: Force on lower plate

Solution:

Basic equations $\quad F = \tau_{yx} \cdot A \qquad \tau_{yx} = \mu \cdot \frac{du}{dy}$

$$\frac{du}{dy} = \frac{d}{dy} u_{max} \cdot \left[1 - \left(\frac{2 \cdot y}{h}\right)^2\right] = u_{max} \cdot \left(-\frac{4}{h^2}\right) \cdot 2 \cdot y = -\frac{8 \cdot u_{max} \cdot y}{h^2}$$

so $\quad \tau_{yx} = -\frac{8 \cdot \mu \cdot u_{max} \cdot y}{h^2} \quad$ and $\quad F = -\frac{8 \cdot A \cdot \mu \cdot u_{max} \cdot y}{h^2}$

At the lower surface $\quad y = -\frac{h}{2} \quad$ and $\quad h = 0.1 \cdot mm \qquad A = 1 \cdot m^2$

$$u_{max} = 0.05 \cdot \frac{m}{s} \qquad \mu = 1.14 \times 10^{-3} \cdot \frac{N \cdot s}{m^2} \qquad \text{(Table A.8)}$$

Hence

$$F = -8 \times 1 \cdot m^2 \times 1.14 \times 10^{-3} \cdot \frac{N \cdot s}{m^2} \times 0.05 \cdot \frac{m}{s} \times \frac{-0.1}{2} \cdot mm \times \frac{1 \cdot m}{1000 \cdot mm} \times \left(\frac{1}{0.1} \cdot \frac{1}{mm} \times \frac{1000 \cdot mm}{1 \cdot m}\right)^2$$

$F = 2.28 \cdot N \qquad$ (to the right)

Problem 2.37

[2]

Explain how an ice skate interacts with the ice surface. What mechanism acts to reduce sliding friction between skate and ice?

Open-Ended Problem Statement: Explain how an ice skate interacts with the ice surface. What mechanism acts to reduce sliding friction between skate and ice?

Discussion: The normal freezing and melting temperature of ice is 0°C (32°F) at atmospheric pressure. The melting temperature of ice decreases as pressure is increased. Therefore ice can be caused to melt at a temperature below the normal melting temperature when the ice is subjected to increased pressure.

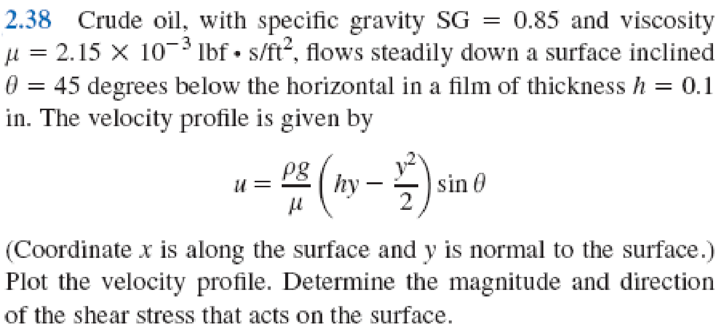
A skater is supported by relatively narrow blades with a short contact against the ice. The blade of a typical skate is less than 3 mm wide. The length of blade in contact with the ice may be just ten or so millimeters. With a 3 mm by 10 mm contact patch, a 75 kg skater is supported by a pressure between skate blade and ice on the order of tens of megaPascals (hundreds of atmospheres). Such a pressure is enough to cause ice to melt rapidly.

When pressure is applied to the ice surface by the skater, a thin surface layer of ice melts to become liquid water and the skate glides on this thin liquid film. Viscous friction is quite small, so the effective friction coefficient is much smaller than for sliding friction.

The magnitude of the viscous drag force acting on each skate blade depends on the speed of the skater, the area of contact, and the thickness of the water layer on top of the ice.

The phenomenon of static friction giving way to viscous friction is similar to the hydroplaning of a pneumatic tire caused by a layer of water on the road surface.

Problem 2.38



2.38 Crude oil, with specific gravity SG = 0.85 and viscosity $\mu = 2.15 \times 10^{-3}$ lbf • s/ft^2, flows steadily down a surface inclined $\theta = 45$ degrees below the horizontal in a film of thickness $h = 0.1$ in. The velocity profile is given by

$$u = \frac{\rho g}{\mu}\left(hy - \frac{y^2}{2}\right)\sin\theta$$

(Coordinate x is along the surface and y is normal to the surface.) Plot the velocity profile. Determine the magnitude and direction of the shear stress that acts on the surface.

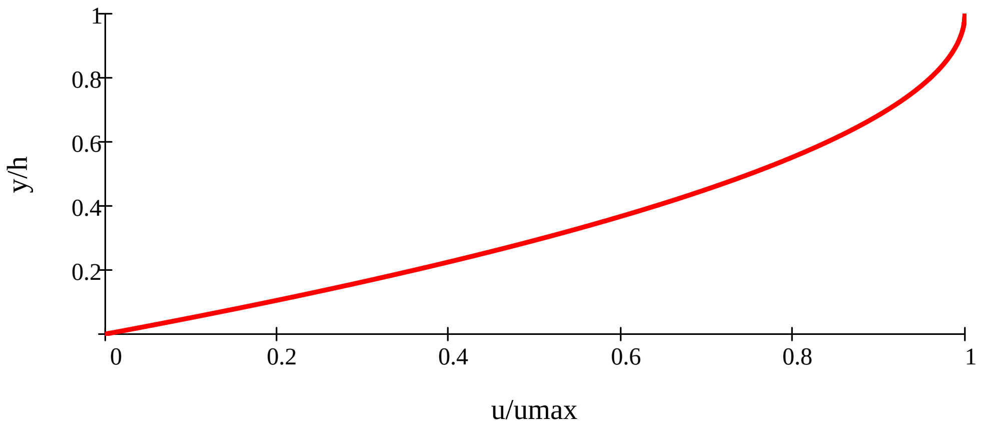
Given:
Velocity profile

Find:
Plot of velocity profile; shear stress on surface

Solution:

The velocity profile is $u = \frac{\rho \cdot g}{\mu}\cdot\left(h\cdot y - \frac{y^2}{2}\right)\cdot\sin(\theta)$ so the maximum velocity is at y = h $u_{max} = \frac{\rho \cdot g}{\mu}\cdot\frac{h^2}{2}\cdot\sin(\theta)$

Hence we can plot $\frac{u}{u_{max}} = 2\cdot\left[\frac{y}{h} - \frac{1}{2}\cdot\left(\frac{y}{h}\right)^2\right]$



This graph can be plotted in *Excel*

The given data is $h = 0.1\cdot in$ $\mu = 2.15 \times 10^{-3}\cdot\frac{lbf\cdot s}{ft^2}$ $\theta = 45\cdot deg$

Basic equation $\tau_{yx} = \mu\cdot\frac{du}{dy}$ $\tau_{yx} = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{d}{dy}\frac{\rho\cdot g}{\mu}\cdot\left(h\cdot y - \frac{y^2}{2}\right)\cdot\sin(\theta) = \rho\cdot g\cdot(h - y)\cdot\sin(\theta)$

At the surface y = 0 $\tau_{yx} = \rho\cdot g\cdot h\cdot\sin(\theta)$

Hence $\tau_{yx} = 0.85 \times 1.94\cdot\frac{slug}{ft^3} \times 32.2\cdot\frac{ft}{s^2} \times 0.1\cdot in \times \frac{1\cdot ft}{12\cdot in} \times \sin(45\cdot deg) \times \frac{lbf\cdot s^2}{slug\cdot ft}$ $\tau_{yx} = 0.313\cdot\frac{lbf}{ft^2}$

The surface is a positive y surface. Since $\tau_{yx} > 0$, the shear stress on the surface must act in the plus x direction.

Problem 2.39

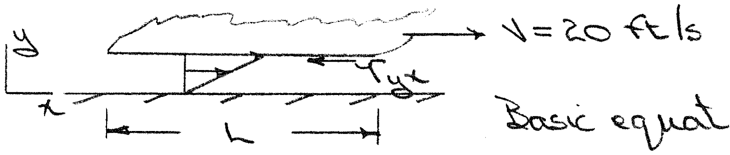
[2]

Given: Skater, of weight $W = 100$ lbf, glides on one skate at speed $V = 20$ ft/s. Skate blade, of length $L = 11.5$ in and width $w = 0.125$ in. glides on thin film of water of height $h = 5.75 \times 10^{-5}$ in.

Find: the deceleration of the skater due to viscous shear.

Solution:

Model flow as one-dimensional shear flow



Basic equation: $\tau_{yx} = \mu \dfrac{du}{dy}$

Assumptions:
1. Newtonian fluid
2. Linear velocity profile
3. Neglect end effects.

From Table A.7, Appendix A, at 32°F

$\mu = 3.66 \times 10^{-5}$ lbf.s/ft^2

$$\tau_{yx} = \mu \frac{du}{dy} = \mu \frac{V}{h} = 3.66 \times 10^{-5} \frac{\text{lbf.s}}{\text{ft}^2} \times 20 \frac{\text{ft}}{\text{s}} \times \frac{1}{5.75 \times 10^{-5}\,\text{in}} \times \frac{12\,\text{in}}{\text{ft}}$$

$\tau_{yx} = 153$ lbf/ft^2

$\Sigma F_x = m a_x \qquad \therefore \; \tau_{yx} A = -\dfrac{W}{g} a_x$

$$a_x = -\frac{\tau_{yx} A g}{W} = -\frac{\tau_{yx} L w g}{W}$$

$$= -153 \frac{\cancel{\text{lbf}}}{\cancel{\text{ft}^2}} \times 11.5\,\text{in} \times 0.125\,\cancel{\text{in}} \times 32.2 \frac{\text{ft}}{\text{s}^2} \times \frac{1}{100\,\cancel{\text{lbf}}} \times \frac{\cancel{\text{ft}^2}}{144\,\cancel{\text{in}^2}}$$

$a_x = -0.491$ ft/s^2 ←——— a_x

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42 389 200 SHEETS 5 SQUARE
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Problem 2.40

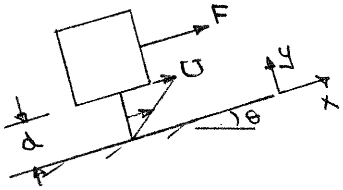
[2]

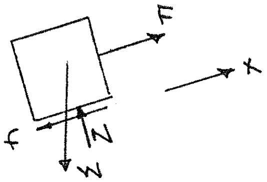
Given: Block of weight 10 lbf, 10 in. on each edge, is pulled up a plane, inclined at 25° to the horizontal, over a film of SAE 10W oil at 100°F. The speed of the block is constant at 2 ft/s and the oil film thickness is 0.001 in. Velocity profile in film is linear.

Find: Force required.

Solution:

Since the block is moving at constant velocity, U, then $\Sigma \vec{F}_{ext} = 0$. Consider the forces along the direction of motion and look at a free body diagram of the block.





Since $\Sigma F_x = 0$, then $F - f - W \sin\theta = 0$

Now the friction force, $f = \tau A$

where $\tau = \mu \frac{du}{dy}$

For small gap (linear velocity profile) $\tau = \mu \frac{U}{d}$

Hence

$$f = \tau A = \mu \frac{U}{d} A$$

and

$$F - \mu \frac{U}{d} A - W \sin\theta = 0$$

Thus

$$F = \mu \frac{U}{d} A + W \sin\theta$$

From Fig A.2, Appendix A, for SAE 10W oil @ 100°F (38°C), $\mu = 3.7 \times 10^{-2}\ \text{N}\cdot\text{s/m}^2$

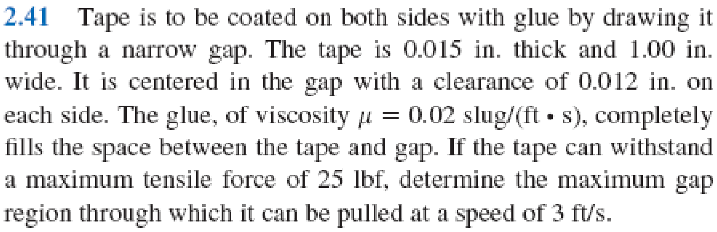
$$F = \mu \frac{U}{d} A + W \sin\theta$$

$$= 3.7 \times 10^{-2} \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2.09 \times 10^{-2} \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2} \cdot \frac{\text{m}^2}{\text{N}\cdot\text{s}} \times 2 \frac{\text{ft}}{\text{s}} \times (10)^2\,\text{in}^2 \times \frac{1}{0.001\,\text{in}} \times \frac{\text{ft}}{12\,\text{in}} + 10\,\text{lbf} \sin 25°$$

$F = 17.1$ lbf $\longleftarrow$ F

Problem 2.41

[2]



2.41 Tape is to be coated on both sides with glue by drawing it through a narrow gap. The tape is 0.015 in. thick and 1.00 in. wide. It is centered in the gap with a clearance of 0.012 in. on each side. The glue, of viscosity $\mu = 0.02$ slug/(ft • s), completely fills the space between the tape and gap. If the tape can withstand a maximum tensile force of 25 lbf, determine the maximum gap region through which it can be pulled at a speed of 3 ft/s.

Given: Data on tape mechanism

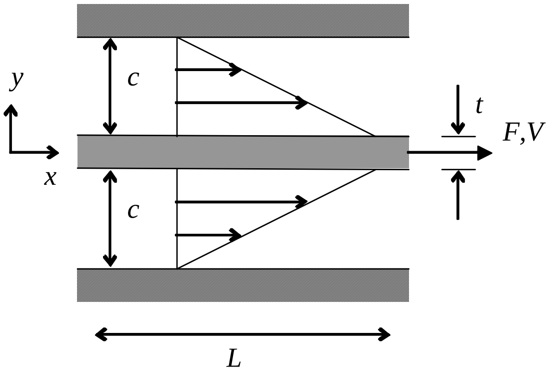
Find: Maximum gap region that can be pulled without breaking tape

Solution:

Basic equation $\tau_{yx} = \mu \cdot \frac{du}{dy}$ and $F = \tau_{yx} \cdot A$

Here F is the force on each side of the tape; the total force is then $F_T = 2 \cdot F = 2 \cdot \tau_{yx} \cdot A$

The velocity gradient is linear as shown $\frac{du}{dy} = \frac{V - 0}{c} = \frac{V}{c}$



The area of contact is $A = w \cdot L$

Combining these results

$$F_T = 2 \cdot \mu \cdot \frac{V}{c} \cdot w \cdot L$$

Solving for L $L = \frac{F_T \cdot c}{2 \cdot \mu \cdot V \cdot w}$

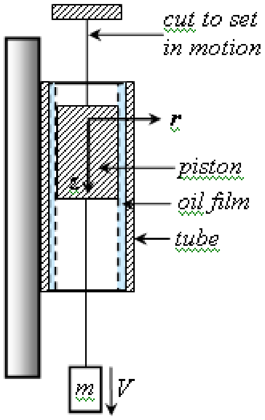
The given data is $F_T = 25 \cdot \text{lbf}$ $c = 0.012 \cdot \text{in}$ $\mu = 0.02 \cdot \frac{\text{slug}}{\text{ft} \cdot \text{s}}$ $V = 3 \cdot \frac{\text{ft}}{\text{s}}$ $w = 1 \cdot \text{in}$

Hence $L = 25 \cdot \text{lbf} \times 0.012 \cdot \text{in} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \times \frac{1}{2} \times \frac{1}{0.02} \cdot \frac{\text{ft} \cdot \text{s}}{\text{slug}} \times \frac{1}{3} \cdot \frac{\text{s}}{\text{ft}} \times \frac{1}{1} \cdot \frac{1}{\text{in}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \times \frac{\text{slug} \cdot \text{ft}}{\text{s}^2 \cdot \text{lbf}}$ $L = 2.5\,\text{ft}$

Problem 2.42

[2]

2.42 A 73-mm-diameter aluminum (SG = 2.64) piston of 100-mm length resides in a stationary 75-mm-inner-diameter steel tube lined with SAE 10W-30 oil at 25°C. A mass $m = 2$ kg is suspended from the free end of the piston. The piston is set into motion by cutting a support cord. What is the terminal velocity of mass m? Assume a linear velocity profile within the oil.



Given: Flow data on apparatus

Find: The terminal velocity of mass *m*

Solution:

Given data: $D_{piston} = 73 \cdot mm$ $\quad D_{tube} = 75 \cdot mm$ $\quad Mass = 2 \cdot kg$ $\quad L = 100 \cdot mm$ $\quad SG_{Al} = 2.64$

Reference data: $\rho_{water} = 1000 \cdot \frac{kg}{m^3}$ (maximum density of water)

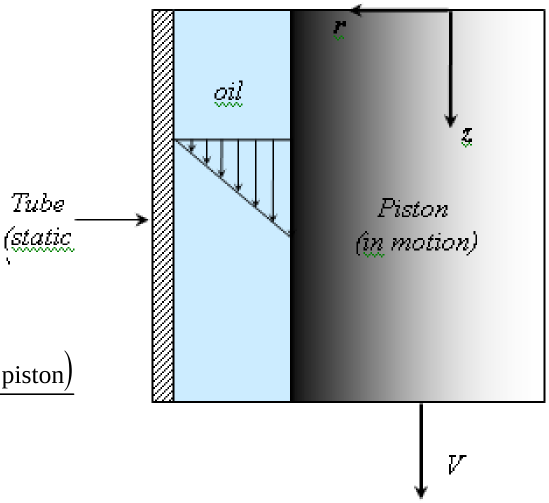
From Fig. A.2:, the dynamic viscosity of SAE 10W-30 oil at 25^{o}C is: $\mu = 0.13 \cdot \frac{N \cdot s}{m^2}$

The terminal velocity of the mass *m* is equivalent to the terminal velocity of the piston. At that terminal speed, the acceleration of the piston is zero. Therefore, all forces acting on the piston must be balanced. This means that the force driving the motion (i.e. the weight of mass *m* and the piston) balances the viscous forces acting on the surface of the piston. Thus, at r = R_{piston}:

$$\left[Mass + SG_{Al} \cdot \rho_{water} \cdot \left(\frac{\pi \cdot D_{piston}^{\;2} \cdot L}{4}\right)\right] \cdot g = \tau_{rz} \cdot A = \left(\mu \cdot \frac{d}{dr} V_z\right) \cdot \left(\pi \cdot D_{piston} \cdot L\right)$$

The velocity profile within the oil film is linear ...

Therefore $$\frac{d}{dr} V_z = \frac{V}{\left(\frac{D_{tube} - D_{piston}}{2}\right)}$$



Thus, the terminal velocity of the piston, *V*, is:

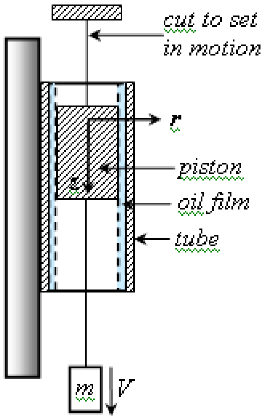
$$V = \frac{g \cdot \left(SG_{Al} \cdot \rho_{water} \cdot \pi \cdot D_{piston}^{\;2} \cdot L + 4 \cdot Mass\right) \cdot \left(D_{tube} - D_{piston}\right)}{8 \cdot \mu \cdot \pi \cdot D_{piston} \cdot L}$$

or $V = 10.2 \frac{m}{s}$

Problem 2.43

[3]

2.43 The piston in Problem 2.42 is traveling at terminal speed. The mass *m* now disconnects from the piston. Plot the piston speed vs. time. How long does it take the piston to come within 1 percent of its new terminal speed?



Given: Flow data on apparatus

Find: Sketch of piston speed vs time; the time needed for the piston to reach 99% of its new terminal speed.

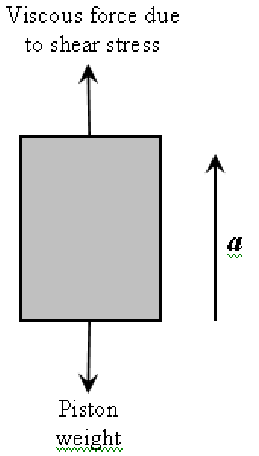
Solution:

Given data: $D_{piston} = 73 \cdot mm$ $\quad D_{tube} = 75 \cdot mm$ $\quad L = 100 \cdot mm$ $\quad SG_{Al} = 2.64$ $\quad V_0 = 10.2 \cdot \frac{m}{s}$

Reference data: $\rho_{water} = 1000 \cdot \frac{kg}{m^3}$ (maximum density of water) (From Problem 2.42)

From Fig. A.2, the dynamic viscosity of SAE 10W-30 oil at 25°C is: $\mu = 0.13 \cdot \frac{N \cdot s}{m^2}$

The free body diagram of the piston after the cord is cut is:



Piston weight: $W_{piston} = SG_{Al} \cdot \rho_{water} \cdot g \cdot \left(\frac{\pi \cdot D_{piston}^2}{4} \right) \cdot L$

Viscous force: $F_{viscous}(V) = \tau_{rz} \cdot A$ or $F_{viscous}(V) = \mu \cdot \left[\frac{V}{\frac{1}{2} \cdot \left(D_{tube} - D_{piston} \right)} \right] \cdot \left(\pi \cdot D_{piston} \cdot L \right)$

Applying Newton's second law: $m_{piston} \cdot \frac{dV}{dt} = W_{piston} - F_{viscous}(V)$

Therefore $\frac{dV}{dt} = g - a \cdot V$ where $a = \frac{8 \cdot \mu}{SG_{Al} \cdot \rho_{water} \cdot D_{piston} \cdot \left(D_{tube} - D_{piston} \right)}$

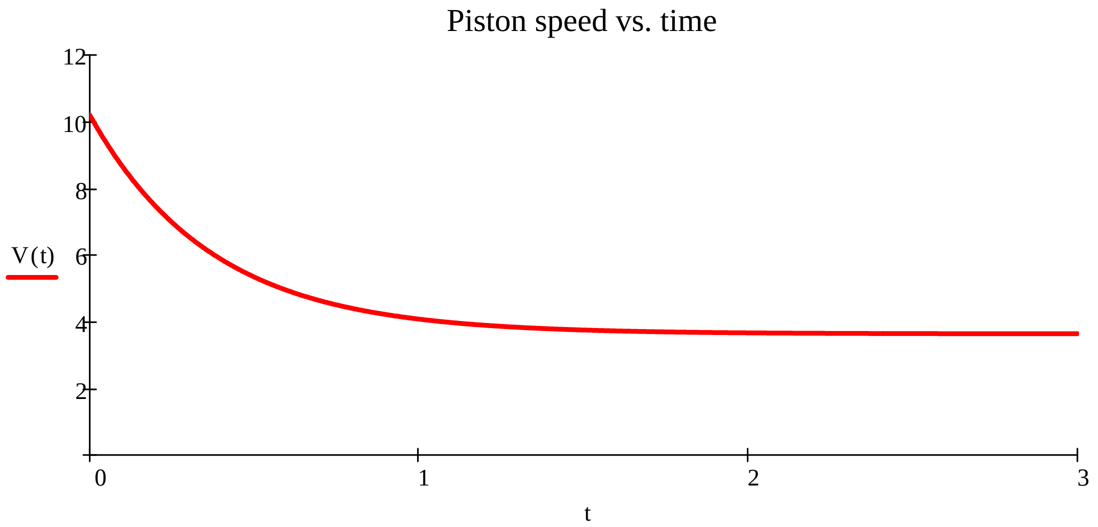
If $V = g - a \cdot V$ then $\frac{dX}{dt} = -a \cdot \frac{dV}{dt}$

The differential equation becomes $\frac{dX}{dt} = -a \cdot X$ where $X(0) = g - a \cdot V_0$

The solution to this differential equation is: $X(t) = X_0 \cdot e^{-a \cdot t}$ or $g - a \cdot V(t) = \left(g - a \cdot V_0\right) \cdot e^{-a \cdot t}$

Therefore $V(t) = \left(V_0 - \frac{g}{a}\right) \cdot e^{(-a \cdot t)} + \frac{g}{a}$

Plotting piston speed vs. time (which can be done in Excel)

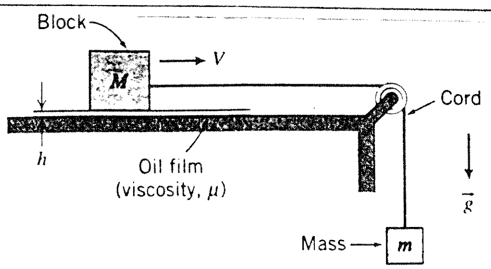


The terminal speed of the piston, V_t, is evaluated as *t* approaches infinity

$$V_t = \frac{g}{a} \qquad \text{or} \qquad V_t = 3.63 \frac{m}{s}$$

The time needed for the piston to slow down to within 1% of its terminal velocity is:

$$t = \frac{1}{a} \cdot \ln\left(\frac{V_0 - \frac{g}{a}}{1.01 \cdot V_t - \frac{g}{a}}\right) \qquad \text{or} \qquad t = 1.93\,s$$



Given: Block of mass M slides on thin film of oil of thickness h. Contact area of block is A. At time t=0, mass m is released from rest.

$M = 5\,kg$, $m = 1\,kg$, $A = 25\,cm^2$, $h = 0.5\,mm$

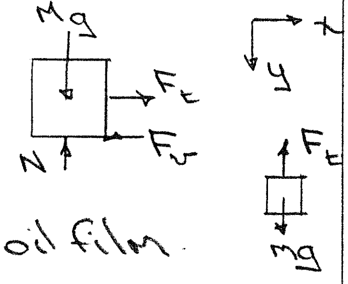
Find: (a) Expression for viscous force on block when moving at speed V

(b) Differential equation governing block speed as a function of time

(c) Expression for block speed $V = V(t)$; plot

(d) If $V = 1\,m/s$ at $t = 1\,s$, find μ

Solution:



Basic equations: $\tau_{yx} = \mu \dfrac{du}{dy}$ $\qquad \Sigma \vec{F} = m\vec{a}$

Assumptions: (1) Newtonian fluid

(2) Linear velocity profile in oil film

Then, $F_v = \tau A = \mu \dfrac{du}{dy} A = \mu \dfrac{\Delta u}{\Delta y} A = \mu \dfrac{V}{h} A \longleftarrow$ F_v

For the block, $\Sigma F_x = F_t - F_v = M \dfrac{dV_b}{dt}$ (1)

For the falling mass $\Sigma F_y = mg - F_t = m \dfrac{dV_m}{dt}$, or

$F_t = mg - m \dfrac{dV_m}{dt}$ (2)

Since $V_b = V_m = V$, then substituting from Eq. (2) into (1) gives

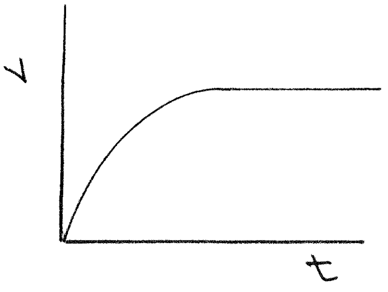
$$mg - m\frac{dV}{dt} - F_v = M \frac{dV}{dt} = mg - m\frac{dV}{dt} - \mu \frac{V}{h} A$$

Finally, $mg - \mu \dfrac{V}{h} A = (M+m) \dfrac{dV}{dt} \longleftarrow$ Diff. Eq.

To solve we separate variables and integrate

$$t = \int_0^t dt = \int_0^V \frac{(M+m)}{mg - \mu \frac{V}{h} A} dV = -(M+m)\frac{h}{\mu A} \ln\left(mg - \frac{\mu V A}{h}\right)\Big]_0^V$$

$$t = -\frac{(M+m)h}{\mu A} \ln\left(1 - \frac{\mu V A}{mgh}\right)$$



Taking antilogarithms,

$$1 - \frac{\mu V A}{mgh} = e^{-\frac{\mu A t}{(M+m)h}}$$

Solving for V,

$$V = \frac{mgh}{\mu A}\left(1 - e^{-\frac{\mu A t}{(M+m)h}}\right) \longleftarrow V$$

The velocity increases exponentially to $V_{max} = \dfrac{mgh}{\mu A}$

Problem 2.44

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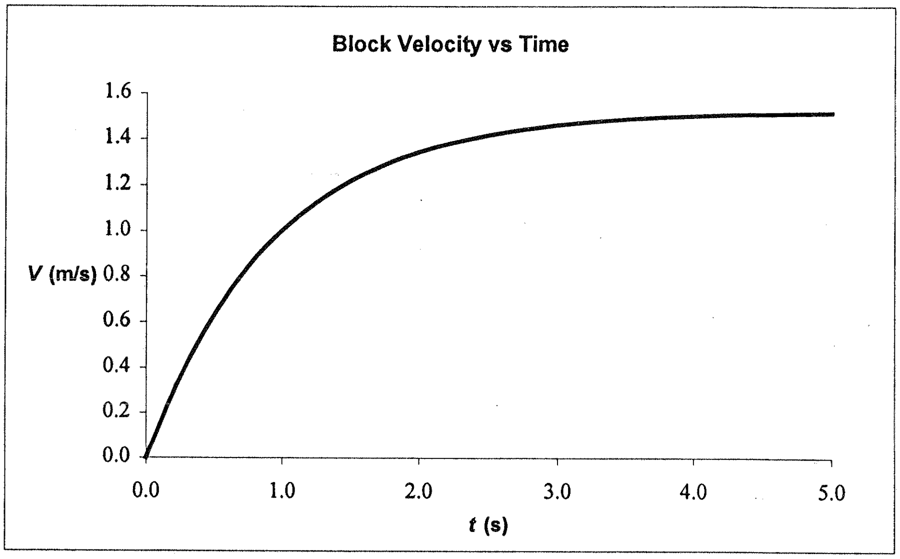
Using Excel's solver, with $V = 1$ m/s at $t = 1$s, the oil viscosity is found to be $\mu = 1.29$ N.s/m² ← μ

The $V(t)$ plot, with $M = 5$ kg, $m = 1$ kg, $A = 25$ cm², $h = 0.5$ mm and $\mu = 1.29$ N.s/m², is generated from

$$V = \frac{Mgh}{\mu A}\left[1 - e^{-\frac{\mu A t}{(M+m)h}}\right]$$

t (s)	*V* (m/s)
0.00	0.00
0.25	0.36
0.50	0.63
0.75	0.84
1.00	1.00
1.25	1.12
1.50	1.22
1.75	1.29
2.00	1.34
2.25	1.39
2.50	1.42
2.75	1.44
3.00	1.46
3.25	1.47
3.50	1.49
3.75	1.49
4.00	1.50
4.25	1.51
4.50	1.51
4.75	1.51
5.00	1.51

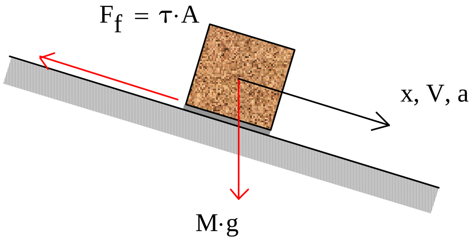
$M =$ 5 kg
$m =$ 1 kg
$A =$ 25 cm²
$h =$ 0.5 mm
$\mu = \boxed{1.29}$ N.s/m² (From *Solver* or *Goal Seek*)



Problem 2.45

[4]

2.45 A block 0.1 m square, with 5 kg mass, slides down a smooth incline, 30° below the horizontal, on a film of SAE 30 oil at 20°C that is 0.20 mm thick. If the block is released from rest at $t = 0$, what is its initial acceleration? Derive an expression for the speed of the block as a function of time. Plot the curve for $V(t)$. Find the speed after 0.1 s. If we want the mass to instead reach a speed of 0.3 m/s at this time, find the viscosity μ of the oil we would have to use.



Given: Data on the block and incline

Find: Initial acceleration; formula for speed of block; plot; find speed after 0.1 s. Find oil viscosity if speed is 0.3 m/s after 0.1 s

Solution:

Given data $M = 5\cdot kg$ $A = (0.1\cdot m)^2$ $d = 0.2\cdot mm$ $\theta = 30\cdot deg$

From Fig. A.2 $\mu = 0.4\cdot\frac{N\cdot s}{m^2}$

Applying Newton's 2nd law to initial instant (no frictic $M\cdot a = M\cdot g\cdot\sin(\theta) - F_f = M\cdot g\cdot\sin(\theta)$

so $a_{init} = g\cdot\sin(\theta) = 9.81\cdot\frac{m}{s^2}\times\sin(30\cdot deg)$ $a_{init} = 4.9\frac{m}{s^2}$

Applying Newton's 2nd law at any instant $M\cdot a = M\cdot g\cdot\sin(\theta) - F_f$ and $F_f = \tau\cdot A = \mu\cdot\frac{du}{dy}\cdot A = \mu\cdot\frac{V}{d}\cdot A$

so $M\cdot a = M\cdot\frac{dV}{dt} = M\cdot g\cdot\sin(\theta) - \frac{\mu\cdot A}{d}\cdot V$

Separating variables

$$\frac{dV}{g\cdot\sin(\theta) - \frac{\mu\cdot A}{M\cdot d}\cdot V} = dt$$

Integrating and using limits

$$-\frac{M\cdot d}{\mu\cdot A}\cdot\ln\left(1 - \frac{\mu\cdot A}{M\cdot g\cdot d\cdot\sin(\theta)}\cdot V\right) = t$$

or

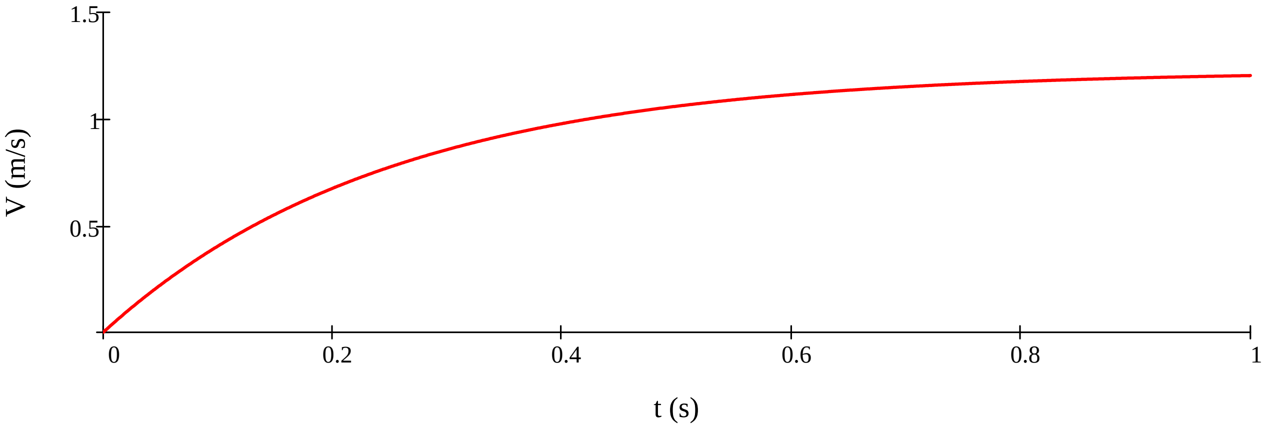
$$V(t) = \frac{M\cdot g\cdot d\cdot\sin(\theta)}{\mu\cdot A}\cdot\left(1 - e^{\frac{-\mu\cdot A}{M\cdot d}\cdot t}\right)$$

At t = 0.1 s

$$V = 5\cdot kg\times 9.81\cdot\frac{m}{s^2}\times 0.0002\cdot m\cdot\sin(30\cdot deg)\times\frac{m^2}{0.4\cdot N\cdot s\cdot(0.1\cdot m)^2}\times\frac{N\cdot s^2}{kg\cdot m}\times\left[1 - e^{-\left(\frac{0.4\cdot 0.01}{5\cdot 0.0002}\cdot 0.1\right)}\right]$$

$$V(0.1\cdot s) = 0.404\cdot\frac{m}{s}$$

The plot looks like



To find the viscosity for which V(0.1 s) = 0.3 m/s, we must solve

$$V(t = 0.1\cdot s) = \frac{M\cdot g\cdot d\cdot \sin(\theta)}{\mu\cdot A}\cdot\left[1 - e^{\frac{-\mu\cdot A}{M\cdot d}\cdot(t=0.1\cdot s)}\right]$$

The viscosity μ is implicit in this equation, so solution must be found by manual iteration, or by any of a number of classic root-finding numerical methods, or by using *Excel*'s *Goal Seek*

Using *Excel*: $\mu = 1.08\cdot\frac{N\cdot s}{m^2}$

Problem 2.46

[3]

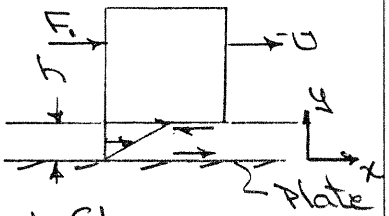
Given: Block of mass M moves at steady speed U under influence of constant force F_1 on a thin film of oil of thickness h and viscosity μ; block is square, a mm on a side.

Find: (a) Magnitude and direction of shear stress acting on bottom of block and supporting plate.

(b) Expression for time required to lose 95 % of its initial speed when force is suddenly removed.

(c) Expect shape of speed vs time curve.

Solution:



Basic equations: $\tau_{yx} = \mu \frac{du}{dy}$ $\quad \Sigma \vec{F} = m\vec{a}$

Assumptions: (1) Newtonian fluid

(2) Linear velocity profile in oil film

$$\tau_{yx} = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y} = \mu \frac{U}{h} \longleftarrow \tau_{yx}$$

Bottom of block is $-y$ surface, so τ_{yx} acts to left

Plate surface is $+y$ surface, so τ_{yx} acts to right

Viscous shear force on block is $F_v = \tau A = \tau a^2 = \frac{\mu U a^2}{h}$

When F_1 is removed, block slows under action of F_v

$$\Sigma F_x = m\frac{dU}{dt} = -F_v = -\frac{\mu U a^2}{h}$$

Separating variables and integrating we have

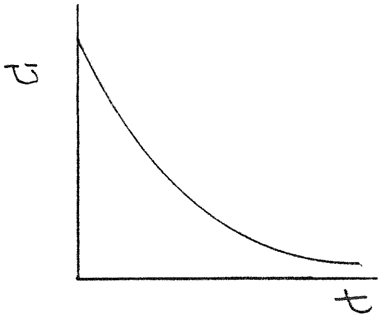
$$\int_{U_i}^{U} \frac{dU}{U} = -\int_0^t \frac{\mu a^2}{mh}\, dt$$

Then

$$\ln \frac{U}{U_i} = -\frac{\mu a^2}{mh} t \quad \text{----(1)}$$

and

$$t = -\frac{mh}{\mu a^2} \ln \frac{U}{U_i}$$



For $U/U_i = 0.05$

$$t = 3.0 \frac{mh}{\mu a^2} \longleftarrow t$$

From Eq.(1) we can write

$$U = U_i e^{-\frac{\mu a^2}{h} t}$$

The speed thus decreases exponentially with time.

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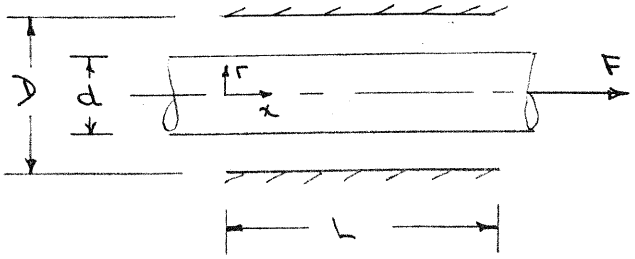
Problem 2.47

Given: Wire, of diameter d, is to be coated with varnish by drawing it through a circular die of diameter, D, and length, L.



$d = 0.9\ \text{mm}$, $D = 1.0\ \text{mm}$, $L = 50\ \text{mm}$

Varnish, $\mu = 20$ centipoise fills the space between wire and die

Wire is drawn through at speed, $V = 50\ \text{m/s}$

Find: Force required to pull the wire

Solution

$$\Sigma F_x = ma_x$$

Since V_{wire} = constant, applied force must be sufficient to balance friction force, F_f

$$F_f = \tau A \quad \text{where } \tau = \mu \frac{du}{dr} \quad \text{and } A = \pi d L$$

Assuming a linear velocity distribution in varnish

$$\tau_s = \mu \left.\frac{du}{dr}\right)_s = \mu \frac{V_{D/2} - V_{d/2}}{D/2 - d/2} = -\mu \frac{V}{(D-d)/2}$$

(negative stress on positive r surface must act in negative x direction)

$$F - F_f = 0$$

$$F = \tau A = \mu \frac{2V}{(D-d)} \times \pi d L$$

$$F = 20\ \text{cp} \times \frac{\text{gm}}{100\ \text{cm.s.cp}} \times 2\pi \times 50 \frac{\text{m}}{\text{s}} \times 0.9\ \text{mm} \times 50\ \text{mm} \times \frac{1}{0.1\ \text{mm}} \times \frac{\text{cm}}{10\ \text{mm}} \times \frac{\text{kg}}{1000\ \text{gm}} \times \frac{\text{N.s}^2}{\text{kg.m}}$$

$$F = 2.83\ \text{N} \longleftarrow F$$

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Problem 2.48

2.48 A double-pipe heat exchanger consists of two concentric fluid-carrying pipes used to transfer heat between nonmixing fluids. The figure shown below is a full-section view of a 0.85-m length of the double-pipe apparatus.

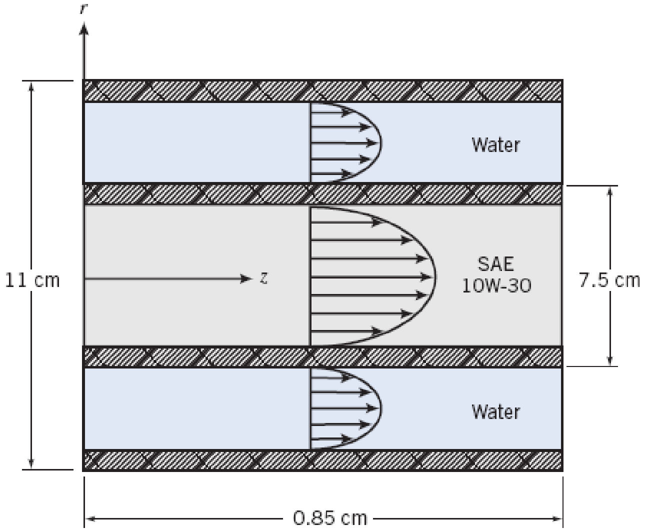
SAE10W-30 oil at 100°C flows through the 7.5-cm-outer-diameter inside pipe. Water at 10°C flows through the annulus between the inside pipe and the 11-cm-outer-diameter outside pipe. The wall thickness of each pipe is 3 mm. The theoretical velocity profiles for laminar flow through a pipe and annulus are:

Inner pipe: $$u_z(r) = u_{\max}\left[1 - \left(\frac{r}{R_{i,\text{ inside}}}\right)^2\right]$$

where: $$u_{\max} = \frac{R_{i,\text{ inside}}^2 \Delta P}{4\mu L}$$

Annulus: $$u_z(r) = \frac{1}{4\mu}\left(\frac{\Delta P}{L}\right) \times \left[R_{i,\text{ outside}}^2 - r^2 - \frac{R_{o,\text{ inside}}^2 - R_{i,\text{ outside}}^2}{\ln\left(\frac{R_{i,\text{ outside}}}{R_{o,\text{ inside}}}\right)} \cdot \ln\left(\frac{r}{R_{i,\text{ outside}}}\right)\right]$$

Show that the no-slip condition is satisfied by these expressions. The pressure drop across the given length is 2.5 Pa and 8 Pa for the water and oil flows, respectively. If both flows are in the same direction (along the $+z$ axis), what is the net viscous force acting on the inner pipe?



NOTE: Figure is wrong - length is 0.85 m

Given:
Data on double pipe heat exchanger

Find:
Whether no-slip is satisfied; net viscous force on inner pipe

Solution:

For the oil, the velocity profile is $$u_z(r) = u_{max}\cdot\left[1 - \left(\frac{r}{R_{ii}}\right)^2\right] \quad \text{where} \quad u_{max} = \frac{R_{ii}^2\cdot\Delta p}{4\cdot\mu\cdot L}$$

Check the no-slip condition. When $r = R_{ii}$ $$u_z(R_{ii}) = u_{max}\cdot\left[1 - \left(\frac{R_{ii}}{R_{ii}}\right)^2\right] = 0$$

For the water, the velocity profile is $$u_z(r) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left(R_{io}^2 - r^2 - \frac{R_{oi}^2 - R_{io}^2}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{r}{Rio}\right)\right)$$

Check the no-slip condition. When $r = R_{oi}$ $$u_z(R_{oi}) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left(R_{io}^2 - R_{oi}^2 - \frac{R_{oi}^2 - R_{io}^2}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{R_{oi}}{Rio}\right)\right)$$

$$u_z(R_{oi}) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left[R_{io}^2 - R_{oi}^2 + \left(R_{oi}^2 - R_{io}^2\right)\right] = 0$$

When $r = R_{io}$

$$u_z\left(R_{io}\right) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left(R_{io}^{2} - R_{io}^{2} - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{R_{io}}{R_{io}}\right)\right) = 0$$

The no-slip condition holds on all three surfaces.

The given data is $R_{ii} = \frac{7.5\cdot cm}{2} - 3\cdot mm$ $\quad R_{ii} = 3.45\cdot cm$ $\quad R_{io} = \frac{7.5\cdot cm}{2}$ $\quad R_{io} = 3.75\cdot cm$ $\quad R_{oi} = \frac{11\cdot cm}{2} - 3\cdot mm$ $\quad R_{oi} = 5.2\cdot cm$

$\Delta p_w = 2.5\cdot Pa$ $\qquad \Delta p_{oil} = 8\cdot Pa$ $\qquad L = 0.85\cdot m$

The viscosity of water at 10ºC is (Fig. A.2) $\quad \mu_w = 1.25\times 10^{-3}\cdot\frac{N\cdot s}{m^2}$

The viscosity of SAE 10-30 oil at 100ºC is (Fig. A.2) $\quad \mu_{oil} = 1\times 10^{-2}\cdot\frac{N\cdot s}{m^2}$

For each, shear stress is given by $\quad \tau_{rx} = \mu\cdot\frac{du}{dr}$

For water

$$\tau_{rx} = \mu\cdot\frac{du_z(r)}{dr} = \mu_w\cdot\frac{d}{dr}\left[\frac{1}{4\cdot\mu_w}\cdot\frac{\Delta p_w}{L}\cdot\left(R_{io}^{2} - r^{2} - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{r}{Rio}\right)\right)\right]$$

$$\tau_{rx} = \frac{1}{4}\cdot\frac{\Delta p_w}{L}\cdot\left(-2\cdot r - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)\cdot r}\right)$$

so on the pipe surface

$$F_w = \tau_{rx}\cdot A = \frac{1}{4}\cdot\frac{\Delta p_w}{L}\cdot\left(-2\cdot R_{io} - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)\cdot R_{io}}\right)\cdot 2\cdot\pi\cdot R_{io}\cdot L$$

$$F_w = \Delta p_w\cdot\pi\cdot\left(-R_{io}^{2} - \frac{R_{oi}^{2} - R_{io}^{2}}{2\cdot\ln\left(\frac{R_{io}}{R_{oi}}\right)}\right)$$

Hence

$$F_w = 2.5\cdot\frac{N}{m^2}\times\pi\times\left[-\left(3.75\cdot cm\times\frac{1\cdot m}{100\cdot cm}\right)^{2} - \frac{\left[(5.2\cdot cm)^{2} - (3.75\cdot cm)^{2}\right]\times\left(\frac{1\cdot m}{100\cdot cm}\right)^{2}}{2\cdot\ln\left(\frac{3.75}{5.2}\right)}\right]$$

$$F_w = 0.00454\,N$$

This is the force on the r-negative surface of the fluid; on the outer pipe itself we also have $F_w = 0.00454\,N$

For oil

$$\tau_{rx} = \mu\cdot\frac{du_z(r)}{dr} = \mu_{oil}\cdot\frac{d}{dr}\,u_{max}\cdot\left[1 - \left(\frac{r}{R_{ii}}\right)^{2}\right] = -\frac{2\cdot\mu_{oil}\cdot u_{max}\cdot r}{R_{ii}^{2}} = -\frac{\Delta p_{oil}\cdot r}{2\cdot L}$$

so on the pipe surface

$$F_{oil} = \tau_{rx}\cdot A = -\frac{\Delta p_{oil}\cdot Rii}{2\cdot L}\cdot 2\cdot\pi\cdot R_{ii}\cdot L = -\Delta p_{oil}\cdot\pi\cdot R_{ii}^{2}$$

This should not be a surprise: the pressure drop just balances the friction!

Hence $F_{oil} = -8 \cdot \frac{N}{m^2} \times \pi \times \left(3.45 \cdot cm \times \frac{1 \cdot m}{100 \cdot cm}\right)^2$ $F_{oil} = -0.0299\,N$

This is the force on the r-positive surface of the fluid; on the pipe it is equal and opposite $F_{oil} = 0.0299\,N$

The total force is $F = F_w + F_{oil}$ $F = 0.0345\,N$

Note we didn't need the viscosities because all quantities depend on the Δp's!

Problem 2.49

[3]

2.49 Repeat Problem 2.48 assuming a counterflow arrangement, where the oil flows in the $+z$ direction and the water flows in the $-z$ direction.

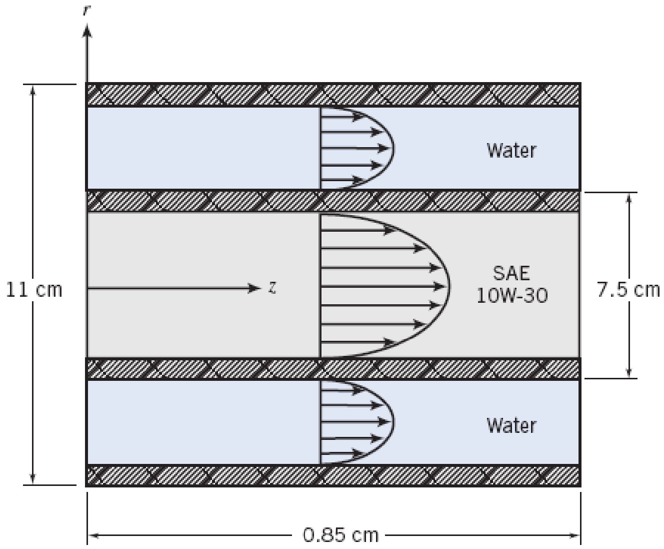
SAE10W-30 oil at 100°C flows through the 7.5-cm-outer-diameter inside pipe. Water at 10°C flows through the annulus between the inside pipe and the 11-cm-outer-diameter outside pipe. The wall thickness of each pipe is 3 mm. The theoretical velocity profiles for laminar flow through a pipe and annulus are:

Inner pipe: $$u_z(r) = u_{\max}\left[1 - \left(\frac{r}{R_{i,\text{ inside}}}\right)^2\right]$$

$$\text{where: } u_{\max} = \frac{R^2_{i,\text{ inside}}\Delta P}{4\mu L}$$

Annulus: $$u_z(r) = \frac{1}{4\mu}\left(\frac{\Delta P}{L}\right) \times \left[R^2_{i,\text{ outside}} - r^2 - \frac{R^2_{o,\text{ inside}} - R^2_{i,\text{ outside}}}{\ln\left(\frac{R_{i,\text{ outside}}}{R_{o,\text{ inside}}}\right)} \cdot \ln\left(\frac{r}{R_{i,\text{ outside}}}\right)\right]$$

Show that the no-slip condition is satisfied by these expressions. The pressure drop across the given length is 2.5 Pa and 8 Pa for the water and oil flows, respectively. If both flows are in the same direction (along the $+z$ axis), what is the net viscous force acting on the inner pipe?



NOTE: Figure is wrong - length is 0.85 m

Given: Data on counterflow heat exchanger

Find: Whether no-slip is satisfied; net viscous force on inner pipe

Solution:

The analysis for Problem 2.48 is repeated, except the oil flows in reverse, so the pressure drop is -2.5 Pa not 2.5 Pa.

For the oil, the velocity profile is $u_z(r) = u_{\max}\cdot\left[1 - \left(\frac{r}{R_{ii}}\right)^2\right]$ where $u_{\max} = \frac{R_{ii}^{\ 2}\cdot\Delta p}{4\cdot\mu\cdot L}$

Check the no-slip condition. When $r = R_{ii}$ $\quad u_z\left(R_{ii}\right) = u_{\max}\cdot\left[1 - \left(\frac{R_{ii}}{R_{ii}}\right)^2\right] = 0$

For the water, the velocity profile is $$u_z(r) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left(R_{io}^{\ 2} - r^2 - \frac{R_{oi}^{\ 2} - R_{io}^{\ 2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{r}{R_{io}}\right)\right)$$

Check the no-slip condition. When $r = R_{oi}$ $$u_z\left(R_{oi}\right) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left(R_{io}^{\ 2} - R_{oi}^{\ 2} - \frac{R_{oi}^{\ 2} - R_{io}^{\ 2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{R_{oi}}{R_{io}}\right)\right)$$

$$u_z\left(R_{oi}\right) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left[R_{io}^{\ 2} - R_{oi}^{\ 2} + \left(R_{oi}^{\ 2} - R_{io}^{\ 2}\right)\right] = 0$$

When $r = R_{io}$
$$u_z\left(R_{io}\right) = \frac{1}{4\cdot\mu}\cdot\frac{\Delta p}{L}\cdot\left(R_{io}^{2} - R_{io}^{2} - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{R_{io}}{R_{io}}\right)\right) = 0$$

The no-slip condition holds on all three surfaces.

The given data is $R_{ii} = \frac{7.5\cdot cm}{2} - 3\cdot mm$ $R_{ii} = 3.45\cdot cm$ $R_{io} = \frac{7.5\cdot cm}{2}$ $R_{io} = 3.75\cdot cm$ $R_{oi} = \frac{11\cdot cm}{2} - 3\cdot mm$ $R_{oi} = 5.2\cdot cm$

$\Delta p_w = -2.5\cdot Pa$ $\Delta p_{oil} = 8\cdot Pa$ $L = 0.85\cdot m$

The viscosity of water at 10ºC is (Fig. A.2) $\mu_w = 1.25\times 10^{-3}\cdot\frac{N\cdot s}{m^{2}}$

The viscosity of SAE 10-30 oil at 100ºC is (Fig. A.2) $\mu_{oil} = 1\times 10^{-2}\cdot\frac{N\cdot s}{m^{2}}$

For each, shear stress is given by $\tau_{rx} = \mu\cdot\frac{du}{dr}$

For water
$$\tau_{rx} = \mu\cdot\frac{du_z(r)}{dr} = \mu_w\cdot\frac{d}{dr}\left[\frac{1}{4\cdot\mu_w}\cdot\frac{\Delta p_w}{L}\cdot\left(R_{io}^{2} - r^{2} - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)}\cdot\ln\left(\frac{r}{R_{io}}\right)\right)\right]$$

$$\tau_{rx} = \frac{1}{4}\cdot\frac{\Delta p_w}{L}\cdot\left(-2\cdot r - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)\cdot r}\right)$$

so on the pipe surface
$$F_w = \tau_{rx}\cdot A = \frac{1}{4}\cdot\frac{\Delta p_w}{L}\cdot\left(-2\cdot R_{io} - \frac{R_{oi}^{2} - R_{io}^{2}}{\ln\left(\frac{R_{io}}{R_{oi}}\right)\cdot R_{io}}\right)\cdot 2\cdot\pi\cdot R_{io}\cdot L$$

$$F_w = \Delta p_w\cdot\pi\cdot\left(-R_{io}^{2} - \frac{R_{oi}^{2} - R_{io}^{2}}{2\cdot\ln\left(\frac{R_{io}}{R_{oi}}\right)}\right)$$

Hence
$$F_w = -2.5\cdot\frac{N}{m^{2}}\times\pi\times\left[-\left[(3.75\cdot cm)\times\frac{1\cdot m}{100\cdot cm}\right]^{2} - \frac{\left[(5.2\cdot cm)^{2} - (3.75\cdot cm)^{2}\right]\times\left(\frac{1\cdot m}{100\cdot cm}\right)^{2}}{2\cdot\ln\left(\frac{3.75}{5.2}\right)}\right]$$

$$F_w = -0.00454\,N$$

This is the force on the r-negative surface of the fluid; on the outer pipe itself we also have $F_w = -0.00454\,N$

For oil
$$\tau_{rx} = \mu\cdot\frac{du_z(r)}{dr} = \mu_{oil}\cdot\frac{d}{dr}u_{max}\cdot\left[1 - \left(\frac{r}{R_{ii}}\right)^{2}\right] = -\frac{2\cdot\mu_{oil}\cdot u_{max}\cdot r}{R_{ii}^{2}} = -\frac{\Delta p_{oil}\cdot r}{2\cdot L}$$

so on the pipe surface
$$F_{oil} = \tau_{rx}\cdot A = -\frac{\Delta p_{oil}\cdot Rii}{2\cdot L}\cdot 2\cdot\pi\cdot R_{ii}\cdot L = -\Delta p_{oil}\cdot\pi\cdot R_{ii}^{2}$$

This should not be a surprise: the pressure drop just balances the friction!

Hence $F_{oil} = -8 \cdot \frac{N}{m^2} \times \pi \times \left(3.45 \cdot cm \times \frac{1 \cdot m}{100 \cdot cm}\right)^2$ $F_{oil} = -0.0299\,N$

This is the force on the r-positive surface of the fluid; on the pipe it is equal and opposite $F_{oil} = 0.0299\,N$

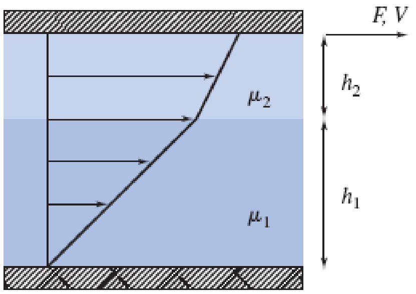
The total force is $F = F_w + F_{oil}$ $F = 0.0254\,N$

Note we didn't need the viscosities because all quantities depend on the Δp's!

Problem 2.50

[2]

2.50 Fluids of viscosities $\mu_1 = 0.1\ \mathrm{N{\bullet}s/m^2}$ and $\mu_2 = 0.15\ \mathrm{N \bullet s/m^2}$ are contained between two plates (each plate is 1 $\mathrm{m^2}$ in area). The thicknesses are $h_1 = 0.5$ mm and $h_2 = 0.3$ mm, respectively. Find the force F to make the upper plate move at a speed of 1 m/s. What is the fluid velocity at the interface between the two fluids?



Given: Flow between two plates

Find: Force to move upper plate; Interface velocity

Solution:

The shear stress is the same throughout (the velocity gradients are linear, and the stresses in the fluid at the interface must be equal and opposite).

Hence $\tau = \mu_1 \cdot \frac{du_1}{dy} = \mu_2 \cdot \frac{du_2}{dy}$ or $\mu_1 \cdot \frac{V_i}{h_1} = \mu_2 \cdot \frac{(V - V_i)}{h_2}$ where V_i is the interface velocity

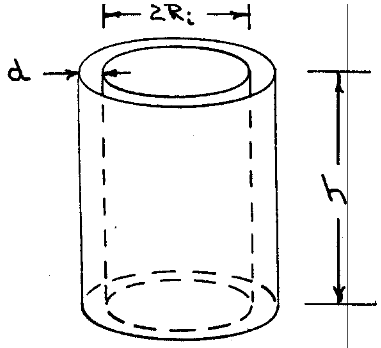
Solving for the interface velocity V_i

$$V_i = \frac{V}{1 + \frac{\mu_1}{\mu_2} \cdot \frac{h_2}{h_1}} = \frac{1 \cdot \frac{m}{s}}{1 + \frac{0.1}{0.15} \cdot \frac{0.3}{0.5}} \qquad V_i = 0.714 \frac{m}{s}$$

Then the force required is

$$F = \tau \cdot A = \mu_1 \cdot \frac{V_i}{h_1} \cdot A = 0.1 \cdot \frac{N \cdot s}{m^2} \times 0.714 \cdot \frac{m}{s} \times \frac{1}{0.5 \cdot mm} \times \frac{1000 \cdot mm}{1 \cdot m} \times 1 \cdot m^2 \qquad F = 143\,N$$

Problem 2.51



Given: Concentric cylinder viscometer

$R_i = 37.5\,mm$, $d = 0.02\,mm$, $h = 150\,mm$

Inner cylinder rotates at $\omega = 100\,rpm$, under torque, $T = 0.021\,N.m$

Find: Viscosity of liquid in clearance gap.

Solution

The imposed torque must balance the resisting torque of the shear force.

The shear force is given by $F = \tau A$ where $A = 2\pi R_i h$

For a Newtonian fluid $\tau = \mu \frac{du}{dy}$

Since the velocity profile is assumed to be linear, $\tau = \mu \frac{V}{d}$

where V is the tangential velocity of the inner cylinder, $V = R_i \omega$

Thus,

$$F = \tau A = \mu \frac{V}{d} 2\pi R_i h = \frac{2\pi \mu R_i^2 \omega h}{d}$$

and the torque $T = R_i F = \frac{2\pi \mu R_i^3 \omega h}{d}$

Solving for μ,

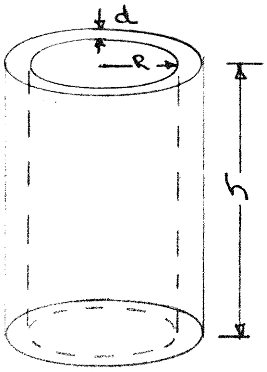
$$\mu = \frac{Td}{2\pi R_i^3 \omega h} = 0.021\,N.m \times 0.02\,mm \times \frac{1}{2\pi} \times \frac{1}{(37.5)^3\,mm^3} \times \frac{min}{100\,rev} \times \frac{1}{150\,mm}$$

$$\times \frac{rev}{2\pi\,rad} \times \frac{60\,s}{min} \times (1000)^3 \frac{mm^3}{m^3}$$

$\mu = 8.07 \times 10^{-4}\ N.s/m^2$ ← μ

Problem 2.52

[2]



Given: Concentric cylinder viscometer.

$R = 2.0$ in $d = 0.001$ in $h = 8$ in.

Inner cylinder rotates at 400 rpm

Gap filled with castor oil at 90°F.

Determine: Torque required to rotate the inner cylinder

Solution:

The required torque must balance the resisting torque of the shear force.

The shear force is given by $F = \tau A$ where $A = 2\pi R h$

For a Newtonian fluid $\tau = \mu \frac{du}{dy}$

For small gap (linear profile) $\tau = \mu \frac{V}{d}$.

where V = tangential velocity of inner cylinder $= R\omega$

Hence

$$F = \tau A = \mu \frac{R\omega}{d} 2\pi R h = \frac{2\pi \mu R^2 \omega h}{d}$$

and the torque $T = RF = \dfrac{2\pi \mu R^3 \omega h}{d}$

From Fig A.2, for castor oil at 90°F (32°C), $\mu = 3.80 \times 10^{-1}$ N.s/m²

Substituting numerical values.

$$T = \frac{2\pi \mu R^3 \omega h}{d} = 2\pi \times 3.80 \times 10^{-1} \frac{N.s}{m^2} \times 2.09 \times 10^{-2} \frac{lbf.s.m^2}{ft^2.N.s} \times (2.0)^3 in^3 \times 400 \frac{rev}{min} \times 8 in \times \frac{1}{10^{-3} in}$$

$$\times 2\pi \frac{rad}{rev} \times \frac{min}{60 s} \times \frac{ft^3}{1728 in^3}$$

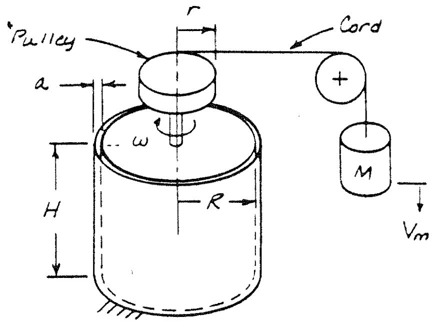
$T = 77.4$ ft·lbf ← Torqu

Problem 2.53

[2]

Given: Concentric-cylinder viscometer, driven by falling mass.

$M = 0.10$ kg $\quad r = 25$ mm
$R = 50$ mm $\quad a = 0.20$ mm
$H = 80$ mm $\quad V_m = 30$ mm/s



After starting transient, $V_m = \text{const.}$

Find: (a) An algebraic expression for viscosity of the liquid, in terms of M, g, V_m, r, R, a, and H.

(b) Evaluate using the data given.

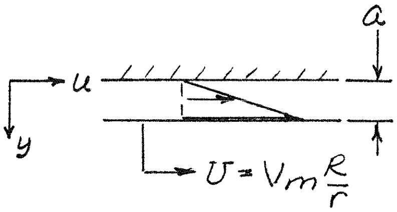
Solution: Apply Newton's law of viscosity.

Basic equations: $\tau = \mu \dfrac{du}{dy} \qquad \Sigma M = 0 \qquad T = \tau A R$

Assumptions: (1) Newtonian liquid
(2) Narrow gap, so linear velocity profile
(3) Steady angular speed

Summing torques on the rotor

$$\Sigma M = Mgr - \tau A R = I\overset{=0(3)}{\cancel{\alpha}} = 0 \; ; \; A = 2\pi R H$$



Because $a \ll R$, treat the gap as plane. Then

$$\tau = \mu \frac{du}{dy} \approx \mu \frac{\Delta u}{\Delta y} = \mu \frac{U - 0}{a - 0} = \mu \frac{U}{a} = \frac{\mu V_m R}{a r}$$

Substituting,

$$Mgr - \frac{\mu V_m R}{a r} 2\pi R H \, R = Mgr - \frac{2\pi \mu V_m R^3 H}{a r} = 0$$

so

$$\mu = \frac{M g r^2 a}{2\pi V_m R^3 H}$$ μ

Evaluating for the given data

$$\mu = \frac{1}{2\pi} \times 0.10\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times (0.025)^2\,\text{m}^2 \times 0.0002\,\text{m} \times \frac{\text{s}}{0.030\,\text{m}} \times \frac{1}{(0.050)^3\,\text{m}^3} \times \frac{1}{0.080\,\text{m}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$\mu = 0.0651\ \text{N}\cdot\text{s}/\text{m}^2 \quad (65.1\ \text{mPa}\cdot\text{s})$$ μ

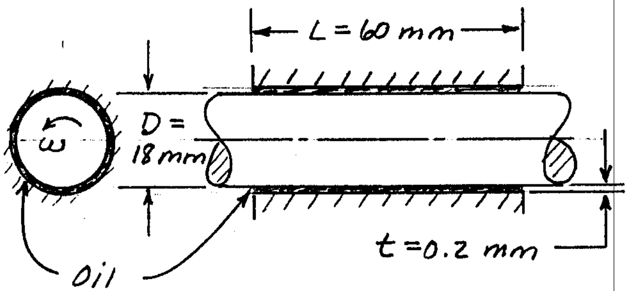
Problem 2.54

[2]

Given: Shaft turning inside stationary journal as shown, $N = 20$ rps.

Torque, $T = 0.0036\ \text{N}\cdot\text{m}$

Find: Estimate viscosity of oil.

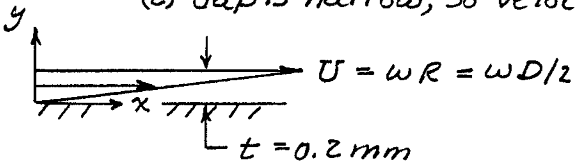


Solution: Basic equation $\tau_{yx} = \mu \frac{du}{dy}$

Assumptions: (1) Newtonian fluid

(2) Gap is narrow, so velocity profile is linear, $\frac{du}{dy} \approx \frac{\Delta u}{\Delta y}$

Then



Shear stress is

$$\tau_{yx} \approx \mu \frac{\Delta u}{\Delta y} = \mu \frac{U}{t} = \frac{\mu \omega D}{2t}$$

Neglecting end effects, torque is

$$T = FR = \tau_{yx} A R = \tau_{yx} (\pi D L) \frac{D}{2} = \frac{\mu \pi \omega D^3 L}{4t}$$

Solving for viscosity

$$\mu = \frac{4tT}{\pi \omega D^3 L}$$

$$= \frac{4}{\pi} \times 0.2\ \text{mm} \times 0.0036\ \text{N}\cdot\text{m} \times \frac{\text{s}}{20\ \text{rev}} \times \frac{1}{(18)^3\ \text{mm}^3} \times \frac{1}{60\ \text{mm}} \times \frac{\text{rev}}{2\pi\ \text{rad}} \times (1000)^3 \frac{\text{mm}^3}{\text{m}^3}$$

$$\mu = 0.0208\ \text{N}\cdot\text{s}/\text{m}^2 \longleftarrow \mu$$

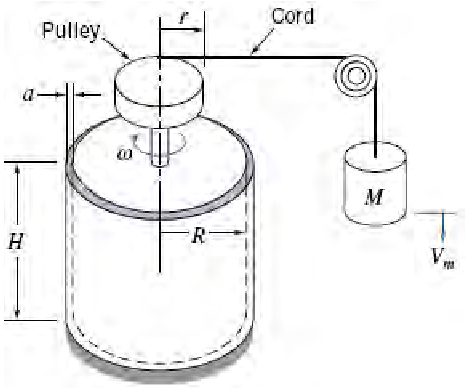
{ From Fig. A.2, this oil appears somewhat less viscous than SAE 10W, assuming the oil is at room temperature. }

NATIONAL 42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE

Problem 2.55

[4]

2.55 The viscometer of Problem 2.53 is being used to verify that the viscosity of a particular fluid is $\mu = 0.1\ \mathrm{N \cdot s/m^2}$. Unfortunately the cord snaps during the experiment. How long will it take the cylinder to lose 99% of its speed? The moment of inertia of the cylinder/pulley system is $0.0273\ \mathrm{kg \cdot m^2}$.



Given: Data on the viscometer

Find: Time for viscometer to lose 99% of speed

Solution:

The given data is $R = 50 \cdot mm$ $\quad H = 80 \cdot mm$ $\quad a = 0.20 \cdot mm$ $\quad I = 0.0273 \cdot kg \cdot m^2$ $\quad \mu = 0.1 \cdot \frac{N \cdot s}{m^2}$

The equation of motion for the slowing viscometer is $\quad I \cdot \alpha = \text{Torque} = -\tau \cdot A \cdot R$

where α is the angular acceleration and τ is the viscous stress, and A is the surface area of the viscometer

The stress is given by $$\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{V-0}{a} = \frac{\mu \cdot V}{a} = \frac{\mu \cdot R \cdot \omega}{a}$$

where *V* and ω are the instantaneous linear and angular velocities.

Hence $$I \cdot \alpha = I \cdot \frac{d\omega}{dt} = -\frac{\mu \cdot R \cdot \omega}{a} \cdot A \cdot R = \frac{\mu \cdot R^2 \cdot A}{a} \cdot \omega$$

Separating variables $$\frac{d\omega}{\omega} = -\frac{\mu \cdot R^2 \cdot A}{a \cdot I} \cdot dt$$

Integrating and using IC $\omega = \omega_0$ $$\omega(t) = \omega_0 \cdot e^{-\frac{\mu \cdot R^2 \cdot A}{a \cdot I} \cdot t}$$

The time to slow down by 99% is obtained from solving $\quad 0.01 \cdot \omega_0 = \omega_0 \cdot e^{-\frac{\mu \cdot R^2 \cdot A}{a \cdot I} \cdot t}$ $\quad$ so $\quad t = -\frac{a \cdot I}{\mu \cdot R^2 \cdot A} \cdot \ln(0.01)$

Note that $\quad A = 2 \cdot \pi \cdot R \cdot H$ $\quad$ so $\quad t = -\frac{a \cdot I}{2 \cdot \pi \cdot \mu \cdot R^3 \cdot H} \cdot \ln(0.01)$

$$t = -\frac{0.0002 \cdot m \cdot 0.0273 \cdot kg \cdot m^2}{2 \cdot \pi} \cdot \frac{m^2}{0.1 \cdot N \cdot s} \cdot \frac{1}{(0.05 \cdot m)^3} \cdot \frac{1}{0.08 \cdot m} \cdot \frac{N \cdot s^2}{kg \cdot m} \cdot \ln(0.01) \qquad t = 4.00\,s$$

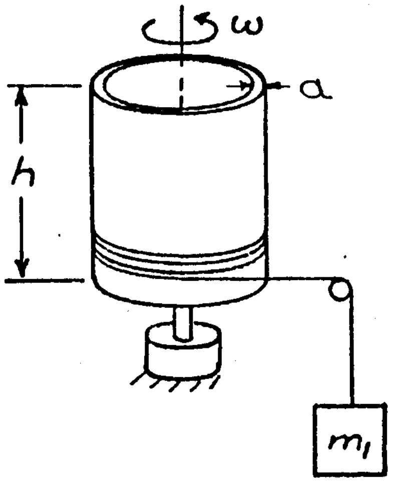
Problem 2.56

[4]___

Given: Thin outer cylinder (mass, M_2, and radius R) of a concentric-cylinder viscometer is driven by the falling mass, m_1. Clearance between outer cylinder and stationary inner cylinder is a. Bearing friction, air resistance and mass of liquid in the viscometer may be neglected

Find: (a) algebraic expression for the torque due to viscous shear acting on cylinder at angular speed ω.
(b) differential equation and solution for $\omega(t)$
(c) expression for ω_{max}

Solution:

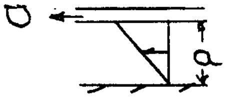


Basic equations: $\tau = \mu \frac{du}{dy}$

$\Sigma F = ma \quad , \Sigma M = I\alpha$

Assume: (1) Newtonian fluid
(2) linear velocity profile

In the gap,



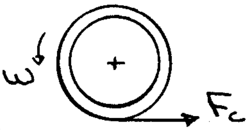
$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{a} = \frac{\mu R \omega}{a}$$

$$T = \tau A R = \frac{\mu R \omega}{a}(2\pi R h) R$$

$$T = \frac{2\pi R^3 \mu h}{a}\,\omega$$

T

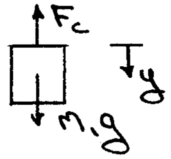
During acceleration, let the tension in the cord be F_c



For the cylinder $\Sigma M = F_c R - T = I\alpha = M_2 R^2 \frac{d\omega}{dt}$ ...(1)

For the mass $\Sigma F_y = m_1 g - F_c = ma = m_1 \frac{dV}{dt} = m_1 R \frac{d\omega}{dt}$ ...(2)

$\therefore F_c = m_1 g - m_1 R \frac{d\omega}{dt}$



Substituting into eq. (1)

$$m_1 g R - \frac{2\pi R^3 \mu h}{a}\omega = (m_1 + M_2) R^2 \frac{d\omega}{dt}$$

Let $m_1 g R = b$, $-2\pi R^3 \mu h / a = c$, $(m_1 + M_2) R^2 = f$

Then, $b + c\omega = f \frac{d\omega}{dt}$ or $\int_0^t \frac{1}{f}\,dt = \int_0^\omega \frac{d\omega}{(b + c\omega)}$

Integrating, $\frac{1}{f} t = \frac{1}{c} \ln(b + c\omega)\Big]_0^\omega = \frac{1}{c} \ln \frac{(b + c\omega)}{b} = \frac{1}{c}\ln\left(1 + \frac{c}{b}\omega\right)$.

$$\frac{c}{f} t = \ln\left(1 + \frac{c}{b}\omega\right) \Rightarrow e^{\frac{c}{f}t} = \left(1 + \frac{c}{b}\omega\right) \Rightarrow \omega = \frac{b}{c}\left(e^{\frac{c}{f}t} - 1\right)$$

Substituting for b, c, and f

$$\omega = \frac{m_1 g \cancel{R} a}{2\pi R^{\cancel{3}} \mu h}\left(1 - e^{\frac{-2\pi R^3 \mu h}{a(m_1 + M_2) R^2} t}\right) = \frac{mga}{2\pi R^2 \mu h}\left[1 - e^{-\frac{2\pi R \mu h}{a(m_1 + M_2)} t}\right]$$

ω(t)

Maximum ω occurs at $t \to \infty$

$$\omega_{max} = \frac{mga}{2\pi R^2 \mu h}$$

ω_{max}

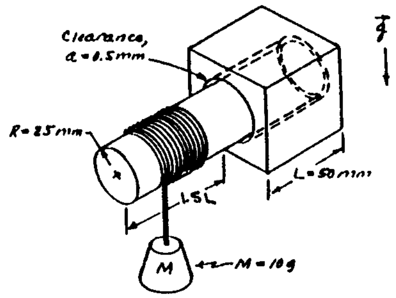
Problem 2.57

Given: Circular aluminum shaft in journal.

Symmetric clearance gap

filled with SAE 10W-30 at 30°C.

Shaft turned by mass and cord.



Find: (a) Develop and solve a differential equation for angular speed as a function of time.
(b) Calculate maximum angular speed.
(c) Estimate time needed to reach 95 percent of maximum speed.

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
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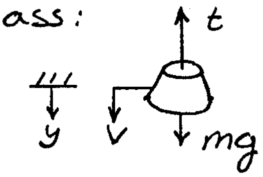
Solution: Apply summation of torques and Newton's second law.

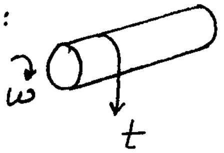
Basic equations: $\Sigma T = I\frac{d\omega}{dt}$ $\Sigma F = m\frac{dV}{dt}$ $V = R\omega$

For the mass:



$$\Sigma F_y = mg - t = m\frac{dV}{dt} = mR\frac{d\omega}{dt} \qquad (1)$$

For the shaft:



$$\Sigma T = tR - T_{viscous} = I\frac{d\omega}{dt} \qquad (2)$$

$$T_{viscous} = \tau R A = \mu\frac{V}{a}R\,2\pi RL = \frac{2\pi\mu\omega R^3 L}{a}$$

Assume: (1) Newtonian liquid, (2) Small gap, (3) Linear profile

Then Eq. 2 becomes $tR - \frac{2\pi\mu R^3 L}{a}\omega = I\frac{d\omega}{dt}$; $I = \frac{1}{2}MR^2$ (3)

Multiplying Eq. 1 by R and combining with Eq. 3 gives

$$mgR - mR^2\frac{d\omega}{dt} - \frac{2\pi\mu R^3 L}{a}\omega = I\frac{d\omega}{dt} \quad \text{or} \quad mgR - \frac{2\pi\mu R^3 L}{a}\omega = (I + mR^2)\frac{d\omega}{dt} \qquad (4)$$

This may be written $A - B\omega = C\frac{d\omega}{dt}$ where $A = mgR$, $B = \frac{2\pi\mu R^3 L}{a}$, $C = I + mR^2$

Separating variables $\frac{d\omega}{A - B\omega} = \frac{dt}{C}$

Integrating $\int_0^\omega \frac{d\omega}{A - B\omega} = -\frac{1}{B}\ln(A - B\omega)\Big]_0^\omega = -\frac{1}{B}\ln\left(1 - \frac{B\omega}{A}\right) = \int_0^t \frac{dt}{C} = \frac{t}{C}$

Simplifying $1 - \frac{B\omega}{A} = e^{-Bt/C}$ or $\omega = \frac{A}{B}\left[1 - e^{-Bt/C}\right]$ (5) $\omega(t)$

The maximum angular speed ($t \to \infty$) is $\omega = A/B$.

$$A = mgR = 0.010\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 0.025\,\text{m} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 2.45 \times 10^{-3}\,\text{N}\cdot\text{m}$$

$$B = \frac{2\pi\mu R^3 L}{a} = 2\pi \times 0.095\,\frac{\text{kg}}{\text{m}\cdot\text{s}} \times (0.025)^3\,\text{m}^3 \times 0.050\,\text{m} \times \frac{1}{0.0005\,\text{m}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 9.33 \times 10^{-4}\,\text{N}\cdot\text{m}\cdot\text{s}$$

Problem 2.57

Evaluating, $\omega_{max} = \frac{A}{B} = 2.45\times10^{-5}\,\mathrm{N\cdot m} \times \frac{1}{9.33\times10^{-4}\,\mathrm{N\cdot m\cdot sec}} = 2.63\ \mathrm{rad/s}$.

Thus

$$\omega_{max} = 2.63\,\frac{\mathrm{rad}}{\mathrm{s}} \times \frac{\mathrm{rev}}{2\pi\,\mathrm{rad}} \times 60\,\frac{\mathrm{s}}{\mathrm{min}} = 25.1\ \mathrm{rpm}$$

ω_{ma}

From Eq. 5, $\omega = 0.95\,\omega_{max}$ when $e^{-Bt/C} = 0.05$, or $Bt/C \simeq 3$; $t \simeq \frac{3C}{B}$

$$C = I + mR^2 = \frac{1}{2}MR^2 + mR^2 = \left(\frac{1}{2}M + m\right)R^2$$

$$M = \pi R^2(1.5L + L)\rho = 2.5\pi R^2 L\, SG\, \rho_w$$

$$M = 2.5\pi \times (0.025)^2\,\mathrm{m}^2 \times 0.050\,\mathrm{m} \times (2.64)1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} = 0.648\ \mathrm{kg}$$

$$C = \left(\frac{1}{2}\times 0.648\ \mathrm{kg} + 0.010\ \mathrm{kg}\right)(0.025)^2\,\mathrm{m}^2 = 2.09\times10^{-4}\ \mathrm{kg\cdot m^2}$$

Thus

$$t_{95} = 3 \times 2.09\times10^{-4}\,\mathrm{kg\cdot m^2} \times \frac{1}{9.33\times10^{-4}\,\mathrm{N\cdot m\cdot s}} \times \frac{\mathrm{N\cdot s^2}}{\mathrm{kg\cdot m}} = 0.671\ \mathrm{s}$$

t_{95}

{The terminal speed could have been computed from Eq. 4 by setting $d\omega/dt \to 0$, without solving the differential equation.}

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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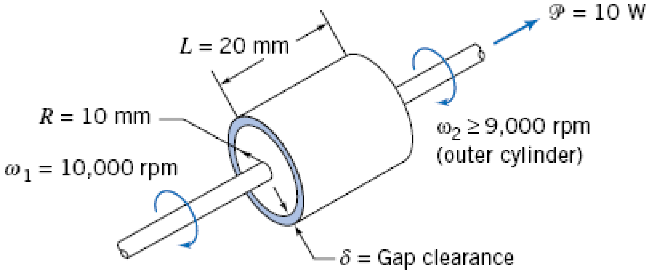
Problem 2.58

[3]

2.58 A shock-free coupling for a low-power mechanical drive is to be made from a pair of concentric cylinders. The annular space between the cylinders is to be filled with oil. The drive must transmit power, $\mathcal{P} = 10$ W. Other dimensions and properties are as shown. Neglect any bearing friction and end effects. Assume the minimum practical gap clearance δ for the device is $\delta = 0.25$ mm. Dow manufactures silicone fluids with viscosities as high as 10^6 centipoise. Determine the viscosity that should be specified to satisfy the requirement for this device.



Given: Shock-free coupling assembly

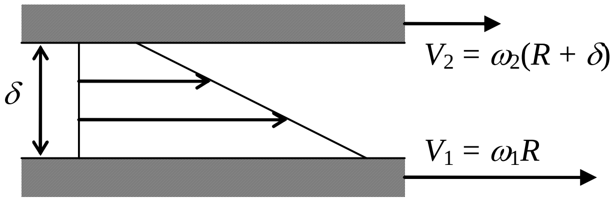
Find: Required viscosity

Solution:

Basic equation $\tau_{r\theta} = \mu \cdot \frac{du}{dr}$ Shear force $F = \tau \cdot A$ Torque $T = F \cdot R$ Power $P = T \cdot \omega$

Assumptions: Newtonian fluid, linear velocity profile

$$\tau_{r\theta} = \mu \cdot \frac{du}{dr} = \mu \cdot \frac{\Delta V}{\Delta r} = \mu \cdot \frac{\left[\omega_1 \cdot R - \omega_2 \cdot (R + \delta)\right]}{\delta}$$



$$\tau_{r\theta} = \mu \cdot \frac{\left(\omega_1 - \omega_2\right) \cdot R}{\delta}$$ Because δ << R

Then $$P = T \cdot \omega_2 = F \cdot R \cdot \omega_2 = \tau \cdot A_2 \cdot R \cdot \omega_2 = \frac{\mu \cdot \left(\omega_1 - \omega_2\right) \cdot R}{\delta} \cdot 2 \cdot \pi \cdot R \cdot L \cdot R \cdot \omega_2$$

$$P = \frac{2 \cdot \pi \cdot \mu \cdot \omega_2 \cdot \left(\omega_1 - \omega_2\right) \cdot R^3 \cdot L}{\delta}$$

Hence $$\mu = \frac{P \cdot \delta}{2 \cdot \pi \cdot \omega_2 \cdot \left(\omega_1 - \omega_2\right) \cdot R^3 \cdot L}$$

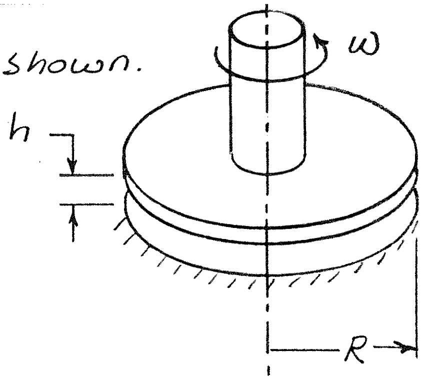
$$\mu = \frac{10 \cdot W \times 2.5 \times 10^{-4} \cdot m}{2 \cdot \pi} \times \frac{1}{9000} \cdot \frac{min}{rev} \times \frac{1}{1000} \cdot \frac{min}{rev} \times \frac{1}{(.01 \cdot m)^3} \times \frac{1}{0.02 \cdot m} \times \frac{N \cdot m}{s \cdot W} \times \left(\frac{rev}{2 \cdot \pi \cdot rad}\right)^2 \times \left(\frac{60 \cdot s}{min}\right)^2$$

$\mu = 0.202 \cdot \frac{N \cdot s}{m^2}$ $\mu = 2.02$ poise which corresponds to SAE 30 oil at 30ºC.

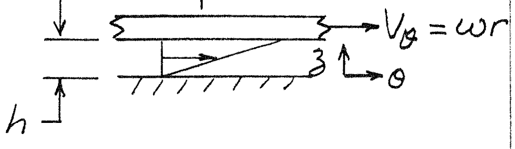
Problem 2.59

[4]

Given: Parallel-disk apparatus as shown.



Find: (a) Algebraic expression for shear stress at any radial location.
(b) Expression for the torque needed to turn the upper disk.



Solution: Use r, θ, z coordinates at right:

Basic equations: $\tau_{z\theta} = \mu \frac{dV_\theta}{dz}$

$$dT = r\,dF = r\,\tau_{z\theta}\,dA$$

Assumptions: (1) Newtonian fluid
(2) No-slip condition
(3) Linear velocity profile (in narrow gap)

The velocity at any radial location on the rotating disk is $V_\theta = \omega r$.

Since the velocity profile is linear, then

$$\tau_{z\theta} = \mu \frac{dV_\theta}{dz} = \mu \frac{\Delta V}{\Delta z} = \mu \frac{(\omega r - 0)}{(h - 0)} = \frac{\mu \omega r}{h} \longleftarrow \quad \tau_{z\theta}$$

and

$$dT = r\,\tau_{z\theta}\,dA = r\,\mu \frac{\omega r}{h}\,2\pi r\,dr = \frac{2\pi \mu \omega r^3}{h}\,dr$$

Integrating

$$T = \int_A dT = \int_0^R \frac{2\pi \mu \omega r^3}{h}\,dr = \left.\frac{\pi \mu \omega r^4}{2h}\right]_0^R$$

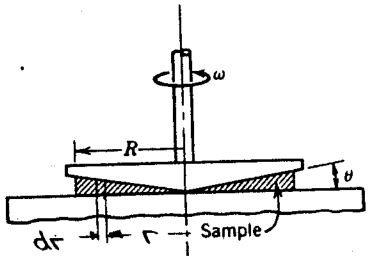
$$T = \frac{\pi \mu \omega R^4}{2h} \longleftarrow \quad T$$

The device could not be used to measure the viscosity of a non-Newtonian fluid because the applied shear stress is not uniform. It varies from zero at the center of the disks to $\mu\omega R/h$ at the edge

Problem 2.60

[4]

Given: Cone and plate viscometer shown.
Apex of cone just touches the plate,
θ is very small



Find: (a) Derive an expression for the shear rate in the liquid that fills the gap
(b) Evaluate the torque on the driven cone in terms of the shear stress and geometry of the system.

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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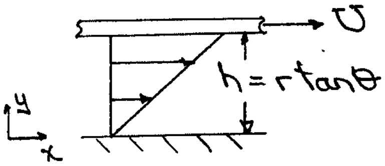
Solution:

Since the angle θ is very small, the average gap width is also very small.

It is reasonable to assume a linear velocity profile across the gap and to neglect end effects.

The shear (deformation) rate is given by



$$\dot{\gamma} = \frac{du}{dy} = \frac{\Delta u}{\Delta y}$$

At any radius, r,

the velocity $U = \omega r$ and

the gap width $h = r\tan\theta$

$$\therefore \dot{\gamma} = \frac{\omega r}{r\tan\theta} = \frac{\omega}{\tan\theta}$$

Since θ is very small, $\tan\theta \simeq \theta$ and

$$\dot{\gamma} = \frac{\omega}{\theta} \longleftarrow \qquad \dot{\gamma}$$

Note: The shear rate is independent of r. The entire sample is subjected to the same shear rate.

The torque on the driven cone is given by

$$T = \int r\,dF \qquad \text{where} \qquad dF = \tau_{yx}\,dA$$

Since $\dot{\gamma}$ is a constant (for a given ω) then τ_{yx} = constant

and

$$T = \int r\,dF = \int_A r\,\tau_{yx}\,dA = \tau_{yx}\int_0^R r\,2\pi r\,dr$$

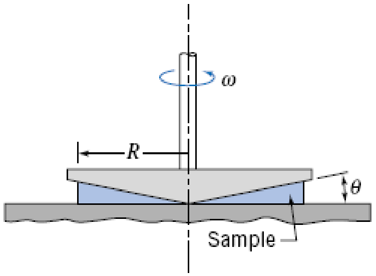
$$T = \frac{2\pi}{3}R^3\,\tau_{yx} \longleftarrow \qquad T$$

Problem 2.61

[4]

2.61 The viscometer of Problem 2.60 is used to measure the apparent viscosity of a fluid. The data below are obtained. What kind of non-Newtonian fluid is this? Find the values of k and n used in Eqs. 2.16 and 2.17 in defining the apparent viscosity of a fluid. (Assume θ is 0.5 degrees.) Predict the viscosity at 90 and 100 rpm.

Speed (rpm)	10	20	30	40	50	60	70	80
μ (N • s/m^2)	0.121	0.139	0.153	0.159	0.172	0.172	0.183	0.185



Given: Data on the viscometer

Find: The values of coefficients k and n; determine the kind of non-Newtonial fluid it is; estimate viscosity at 90 and 100 rpm

Solution:

The velocity gradient at any radius r is

$$\frac{du}{dy} = \frac{r \cdot \omega}{r \cdot \tan(\theta)}$$

where ω (rad/s) is the angular velocity

$$\omega = \frac{2 \cdot \pi \cdot N}{60}$$ where N is the speed in rpm

For small θ, tan(θ) can be replace with θ, so

$$\frac{du}{dy} = \frac{\omega}{\theta}$$

From Eq 2.11.

$$k \cdot \left(\left| \frac{du}{dy} \right| \right)^{n-1} \frac{du}{dy} = \eta \cdot \frac{du}{dy}$$

where η is the apparent viscosity. Hence

$$\eta = k \cdot \left(\frac{du}{dy} \right)^{n-1} = k \cdot \left(\frac{\omega}{\theta} \right)^{n-1}$$

The data is

N (rpm)	μ (N·s/m^2)
10	0.121
20	0.139
30	0.153
40	0.159
50	0.172
60	0.172
70	0.183
80	0.185

The computed data is

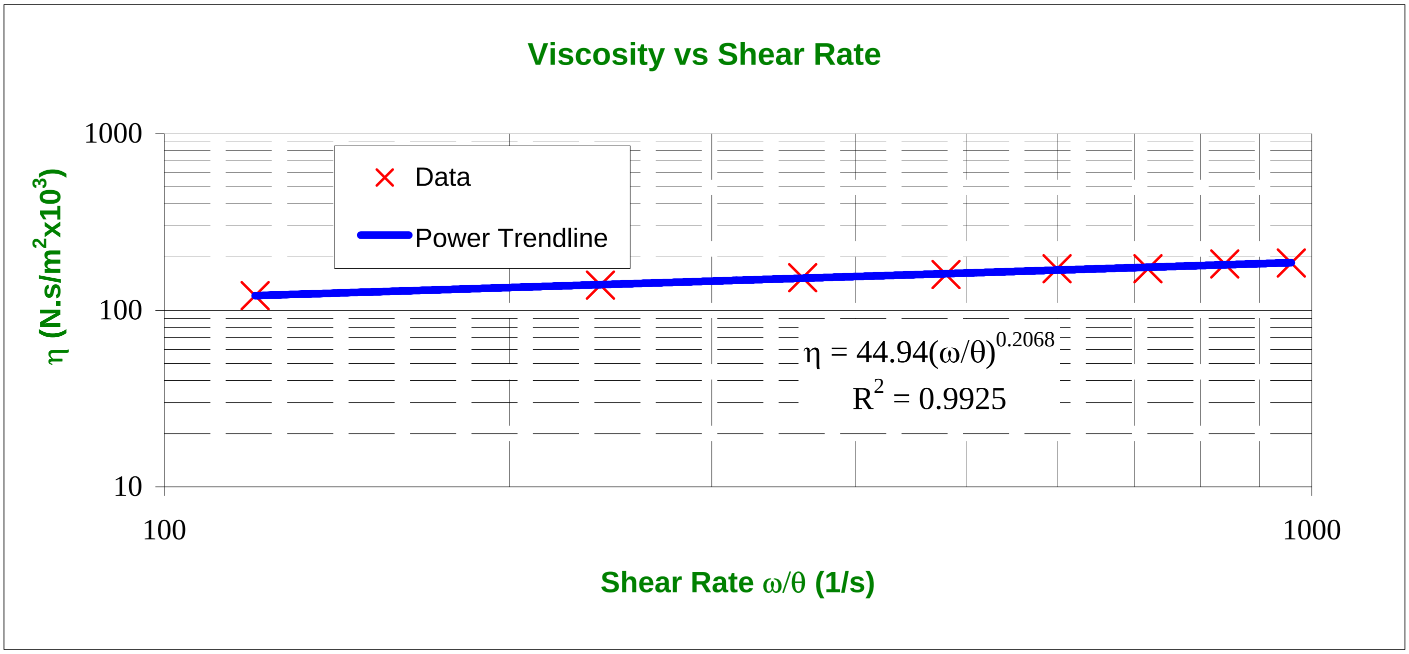
ω (rad/s)	ω/θ (1/s)	η (N·s/m^{2}x10^3)
1.047	120	121
2.094	240	139
3.142	360	153
4.189	480	159
5.236	600	172
6.283	720	172
7.330	840	183
8.378	960	185

From the *Trendline* analysis

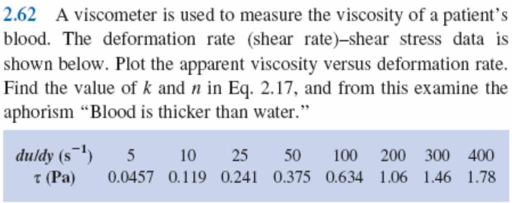
k = 0.0449
n - 1 = 0.2068
n = 1.21 The fluid is dilatant

The apparent viscosities at 90 and 100 rpm can now be computed

N (rpm)	ω (rad/s)	ω/θ (1/s)	η (N·s/m^{2}x10^3)
90	9.42	1080	191
100	10.47	1200	195

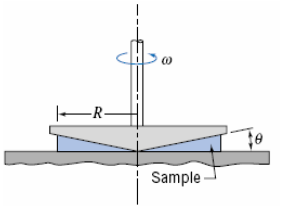


Problem 2.62 (In Excel)



2.62 A viscometer is used to measure the viscosity of a patient's blood. The deformation rate (shear rate)–shear stress data is shown below. Plot the apparent viscosity versus deformation rate. Find the value of k and n in Eq. 2.17, and from this examine the aphorism "Blood is thicker than water."

du/dy (s^{-1})	5	10	25	50	100	200	300	400
τ (Pa)	0.0457	0.119	0.241	0.375	0.634	1.06	1.46	1.78



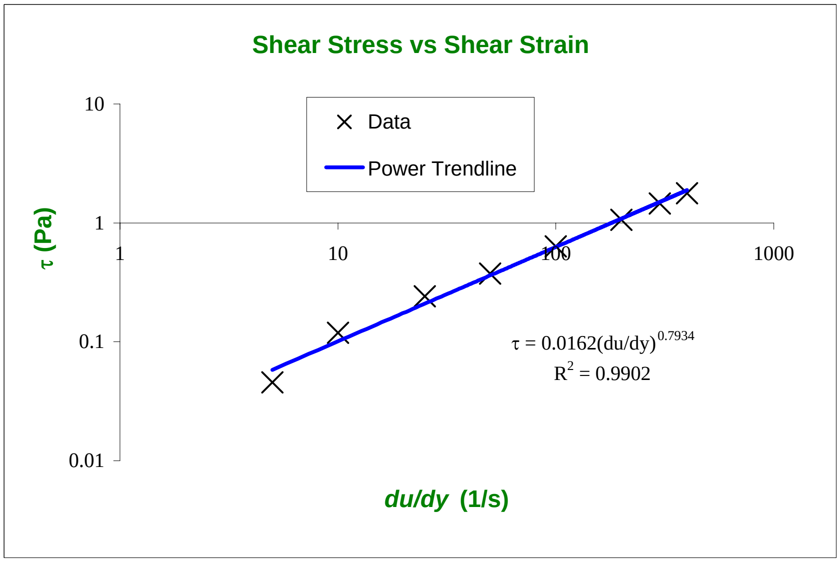
Given: Viscometer data

Find: Value of k and n in Eq. 2.17

Solution:

The data is

τ (Pa)	du/dy (s^{-1})
0.0457	5
0.119	10
0.241	25
0.375	50
0.634	100
1.06	200
1.46	300
1.78	400



Hence we have $k = 0.0162$

$n = 0.7934$ Blood is pseudoplastic (shear thinning)

We can compute the apparent viscosity fron $\eta = k\,(du/dy)^{n-1}$

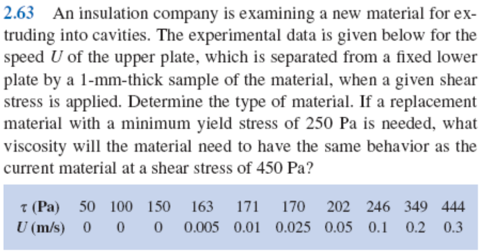
du/dy (s^{-1})	η ($N{\cdot}s/m^2$)
5	0.0116
10	0.0101
25	0.0083
50	0.0072
100	0.0063
200	0.0054
300	0.0050
400	0.0047

$\mu_{water} = 0.001\ N{\cdot}s/m^2$ at 20^oC

Hence, blood is "thicker" than water!

Problem 2.63 (In Excel)

[4]



2.63 An insulation company is examining a new material for extruding into cavities. The experimental data is given below for the speed U of the upper plate, which is separated from a fixed lower plate by a 1-mm-thick sample of the material, when a given shear stress is applied. Determine the type of material. If a replacement material with a minimum yield stress of 250 Pa is needed, what viscosity will the material need to have the same behavior as the current material at a shear stress of 450 Pa?

τ (Pa)	50	100	150	163	171	170	202	246	349	444
U (m/s)	0	0	0	0.005	0.01	0.025	0.05	0.1	0.2	0.3

Given: Data on insulation material

Find: Type of material; replacement material

Solution:

The velocity gradient is

$du/dy = U/\delta$ where $\delta =$ 0.001 m

Data and computations

τ (Pa)	U (m/s)	du/dy (s^{-1})
50	0.000	0
100	0.000	0
150	0.000	0
163	0.005	5
171	0.01	10
170	0.03	25
202	0.05	50
246	0.1	100
349	0.2	200
444	0.3	300

Hence we have a Bingham plastic, with $\tau_y =$ 154 Pa

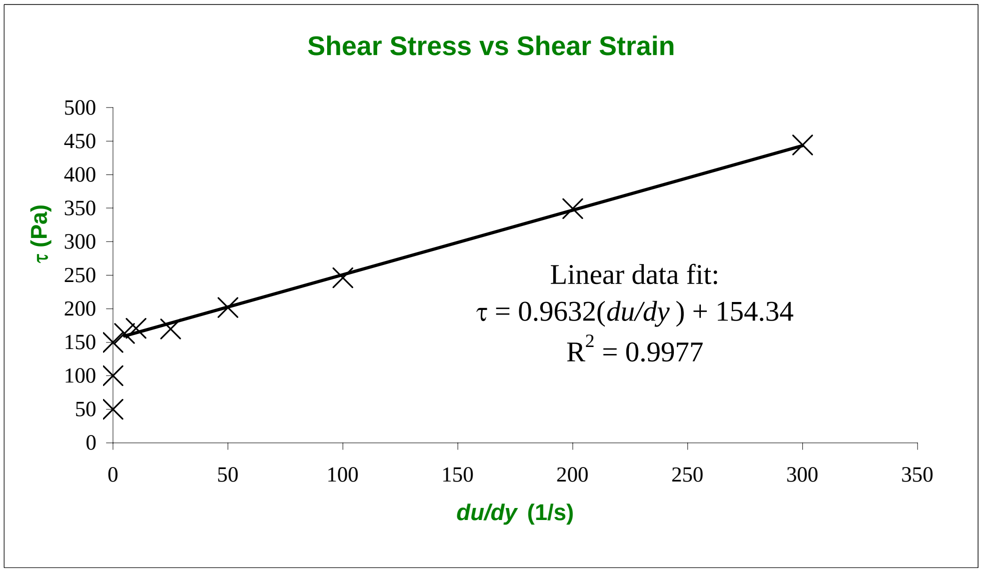
$\mu_p =$ 0.963 $N{\cdot}s/m^2$

At τ = 450 Pa, based on the linear fit $du/dy =$ 307 s^{-1}

For a fluid with $\tau_y =$ 250 Pa

we can use the Bingham plastic formula to solve for μ_p given τ, τ_y and du/dy from above

$\mu_p =$ 0.652 $N{\cdot}s/m^2$



Problem 2.64

[5]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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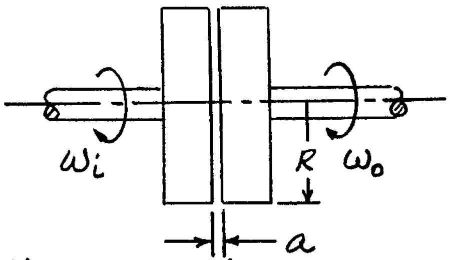
Given: Viscous clutch made from pair of closely spaced disks.

Input speed, ω_i

Output speed, ω_o

Viscous oil in gap, μ



Find algebraic expressions in terms of μ, R, a, ω_i, and ω_o for:
- (a) Torque transmitted, T
- (b) Power transmitted
- (c) Slip ratio, $s = \Delta\omega / \omega_i$, in terms of T
- (d) Efficiency, η, in terms of s, ω_i, and T

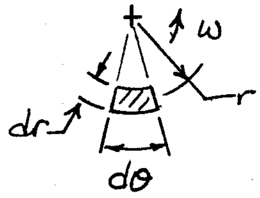
Solution: Apply Newton's law of viscosity

Basic equations: $\tau = \mu \frac{du}{dy}$ $\quad dF = \tau dA \quad dT = r dF$

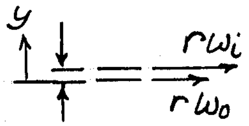
Assumptions: (1) Newtonian liquid
(2) Narrow gap so velocity profile is linear

Consider a segment of plates:

$$\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y} = \mu \frac{r(\omega_i - \omega_o)}{a}$$



End View



Bottom View

$$dA = r\,dr\,d\theta$$

$$dF = \tau dA = \frac{\mu r \Delta\omega}{a} r\,dr\,d\theta = \frac{\mu \Delta\omega}{a} r^2\,dr\,d\theta \; ; \; dT = r dF = \frac{\mu\Delta\omega}{a} r^3 dr\,d\theta$$

Integrating

$$T = \int_0^{2\pi}\int_0^R dT = \frac{\mu\Delta\omega}{a}\int_0^{2\pi}\int_0^R r^3 dr\,d\theta = \frac{2\pi\mu\Delta\omega}{a}\int_0^R r^3 dr = \frac{\pi\mu\Delta\omega R^4}{2a}$$ T

$$P_o = T\omega_o = \frac{\pi\mu\omega_o\Delta\omega R^4}{2a}$$ (power transmitted) P

$$s = \frac{\Delta\omega}{\omega_i} = \frac{2aT}{\pi\mu R^4 \omega_i}$$ s

Efficiency is $\eta = \frac{\text{Power out}}{\text{Power in}} = \frac{T\omega_o}{T\omega_i} = \frac{\omega_o}{\omega_i}$. But $\omega_o = \omega_i - \Delta\omega$, so

$$\eta = \frac{\omega_i - \Delta\omega}{\omega_i} = 1 - \frac{\Delta\omega}{\omega_i} = 1 - s$$ η

Problem 2.65

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
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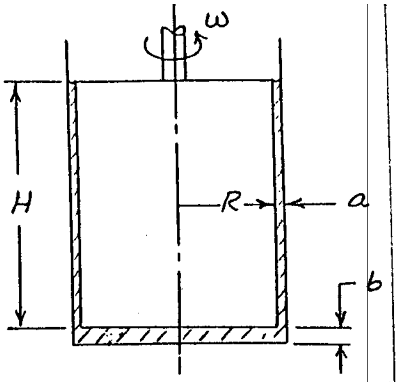
Given: Concentric-cylinder viscometer shown. When inner cylinder rotates at angular speed ω viscous retarding torque arises around circumference of inner cylinder and on cylinder bottom.

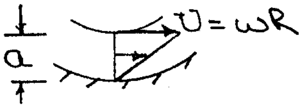


Find: (a) expression for viscous torque due to gap of width, a
(b) expression for viscous torque on bottom due to gap of width b
(c) For $T_{bottom}/T_{annulus} \le 0.01$, plot b/a vs geometric variables.
(d) What are design implications?
(e) What design modifications can you recommend?

Solution: Basic equation $\tau_{yx} = \mu \frac{du}{dy}$

Assumptions: (1) linear velocity profile (2) Newtonian liquid

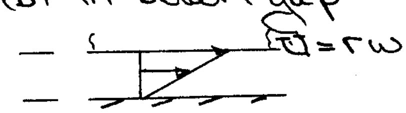
(a) in annular gap



$$\tau = \mu\frac{du}{dr} = \mu\frac{\Delta u}{\Delta r} = \mu\frac{U}{a} = \mu\frac{\omega R}{a}$$

$$\text{Torque} = RF_t = R\tau A = R\mu\frac{\omega R}{a}(2\pi R H) = \frac{2\pi\mu\omega R^3 H}{a} \longleftarrow (a)$$

(b) in bottom gap



$$\tau = \mu\frac{du}{dz} = \mu\frac{\Delta u}{\Delta z} = \mu\frac{U}{a} = \mu\frac{\omega r}{b}$$

(varies with r)

$$\text{Torque} = \int dT = \int r\,dF = \int r\tau\,dA = \int_0^R r\mu\frac{\omega r}{b}2\pi r\,dr$$

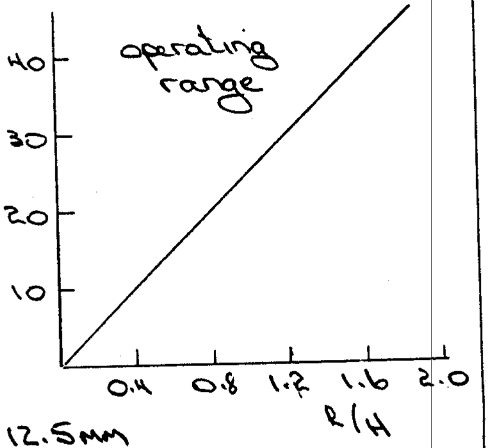
$$\text{Torque} = \frac{2\pi\mu\omega}{b}\int_0^R r^3\,dr = \frac{2\pi\mu\omega}{b}\left[\frac{r^4}{4}\right]_0^R = \frac{\pi\mu\omega}{2b}R^4 \longleftarrow (b)$$

(c) For $T_{bottom}/T_{annulus} \le \frac{1}{100}$, then

$$\frac{T_{bot}}{T_{an}} = \frac{\pi\mu\omega}{2b}R^4 \times \frac{a}{2\pi\mu\omega R^3 H} \le \frac{1}{100}$$

$$\frac{aR}{4bH} \le \frac{1}{100}$$

or $$\frac{b}{a} \ge 25\frac{R}{H}$$



(d) The plot shows the operating range. Specific design would depend on other constraints.

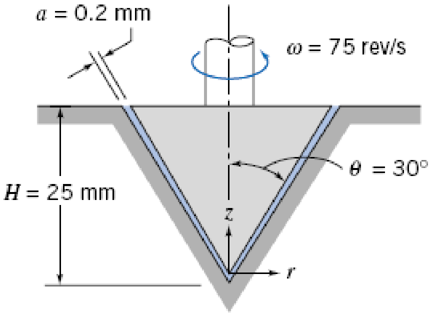
For a = 1mm with R/H = 1/2 gives b = 12.5mm

(e) For a given value of R/H, the dimension b could be effectively increased by "hollowing out" the inner cylinder as shown by the dashed lines in the diagram above.

Problem 2.66

[4]

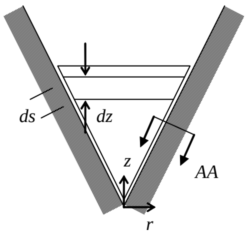
2.66 A conical pointed shaft turns in a conical bearing. The gap between shaft and bearing is filled with heavy oil having the viscosity of SAE 30 at 30°C. Obtain an algebraic expression for the shear stress that acts on the surface of the conical shaft. Calculate the viscous torque that acts on the shaft.



Given: Conical bearing geometry

Find: Expression for shear stress; Viscous torque on shaft

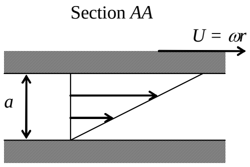
Solution:



Basic equation $\tau = \mu \cdot \frac{du}{dy}$ $dT = r \cdot \tau \cdot dA$ Infinitesimal shear torque

Assumptions: Newtonian fluid, linear velocity profile (in narrow clearance gap), no slip condition

$$\tan(\theta) = \frac{r}{z} \quad \text{so} \quad r = z \cdot \tan(\theta)$$



Then $\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{\Delta u}{\Delta y} = \mu \cdot \frac{(\omega \cdot r - 0)}{(a - 0)} = \frac{\mu \cdot \omega \cdot z \cdot \tan(\theta)}{a}$

As we move up the device, shear stress increases linearly (because rate of shear strain does)

But from the sketch $dz = ds \cdot \cos(\theta)$ $dA = 2 \cdot \pi \cdot r \cdot ds = 2 \cdot \pi \cdot r \cdot \frac{dz}{\cos(\theta)}$

The viscous torque on the element of area is $dT = r \cdot \tau \cdot dA = r \cdot \frac{\mu \cdot \omega \cdot z \cdot \tan(\theta)}{a} \cdot 2 \cdot \pi \cdot r \cdot \frac{dz}{\cos(\theta)}$ $dT = \frac{2 \cdot \pi \cdot \mu \cdot \omega \cdot z^3 \cdot \tan(\theta)^3}{a \cdot \cos(\theta)} \cdot dz$

Integrating and using limits z = H and z = 0 $T = \frac{\pi \cdot \mu \cdot \omega \cdot \tan(\theta)^3 \cdot H^4}{2 \cdot a \cdot \cos(\theta)}$

Using given data, and $\mu = 0.2 \cdot \frac{N \cdot s}{m^2}$ from Fig. A.2

$$T = \frac{\pi}{2} \times 0.2 \cdot \frac{N \cdot s}{m^2} \times 75 \cdot \frac{rev}{s} \times \tan(30 \cdot deg)^3 \times (0.025 \cdot m)^4 \times \frac{1}{0.2 \times 10^{-3} \cdot m} \times \frac{1}{\cos(30 \cdot deg)} \times \frac{2 \cdot \pi \cdot rad}{rev}$$

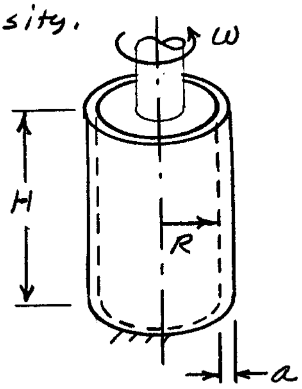
$T = 0.0643 \cdot N \cdot m$

Problem 2.67

[5]

Given: Concentric-cylinder viscometer, liquid similar to water. Goal is to obtain ±1 percent accuracy in viscosity value.

Specify: Configuration and dimensions to achieve ±1% measurement. Parameter to be measured to compute viscosity.

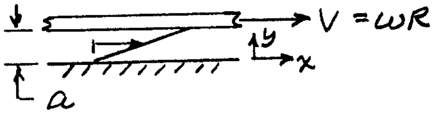


Solution: Apply definition of Newtonian fluid

Computing equation: $\tau = \mu \frac{du}{dy}$

Assumptions:
(1) Steady
(2) Newtonian liquid
(3) Narrow gap, so "unroll" it
(4) Linear velocity profile in gap
(5) Neglect end effects

Flow model:



$u = V\frac{y}{a} = \omega R \frac{y}{a}\;;\; \frac{du}{dy} = \frac{\omega R}{a}$

Thus $\tau = \mu \frac{du}{dy} = \mu \frac{\omega R}{a}$ and torque on rotor is $T = R\tau A$, where $A = 2\pi R H$

Consequently $T = R\mu \frac{\omega R}{a} 2\pi R H = \frac{2\pi\mu\omega R^3 H}{a}$, or

$$\mu = \frac{Ta}{2\pi\omega R^3 H}$$ μ

From this equation the uncertainty in μ is (see Appendix F),

$$u_\mu = \pm\left[u_T^2 + u_a^2 + u_\omega^2 + (3u_R)^2 + u_H^2\right]^{\frac{1}{2}} = \pm\left[13\, u^2\right]^{\frac{1}{2}} = \pm 3.61\, u$$

if the uncertainty of each parameter equals u. Thus

$$u = \pm\frac{u_\mu}{3.61} = \pm\frac{1 \text{ percent}}{3.61} = \pm 0.277 \text{ percent}$$ u

Typical dimensions for a bench-top unit might be

$H = 200$ mm, $R = 75$ mm, $a = 0.02$ mm, and $\omega = 10.5$ rad/s (100 rpm)

From Appendix A, Table A.8, water has $\mu = 1.00\times10^{-3}$ N·s/m² at $T = 20°$C.

The corresponding torque would be

$$T = 2\pi \times 1.00\times10^{-3}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times\frac{10.5}{\text{s}}\times(0.075)^3\text{m}^3\times 0.2\,\text{m}\times\frac{1}{0.00002\,\text{m}} = 0.278 \text{ N}\cdot\text{m}$$ T

It should be possible to measure this torque quite accurately.

{Many details would need to be considered (e.g. bearings, temperature rise, etc.) to produce a workable device.}

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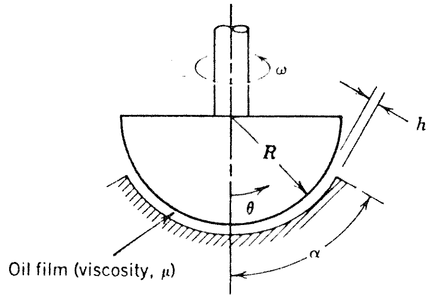
Problem 2.68

[5]

Given: Spherical thrust bearing shown:



Find: Obtain and plot an algebraic expression for the torque on the spherical member, as a function of α.

Solution: Apply definitions

Computing equations: $\tau = \mu \frac{du}{dy}$ $\qquad T = \int_A r\tau \, dA$

Assumptions: (1) Newtonian fluid, (2) Narrow gap, (3) Laminar flow

From the figure, $r = R\sin\theta \qquad u = \omega r = \omega R \sin\theta$

$$\tau = \mu \frac{du}{dy} = \mu\left(\frac{u-0}{h}\right) = \mu \frac{u}{h} = \mu \frac{\omega R \sin\theta}{h}$$

$$dA = 2\pi r R\, d\theta = 2\pi R^2 \sin\theta \, d\theta$$

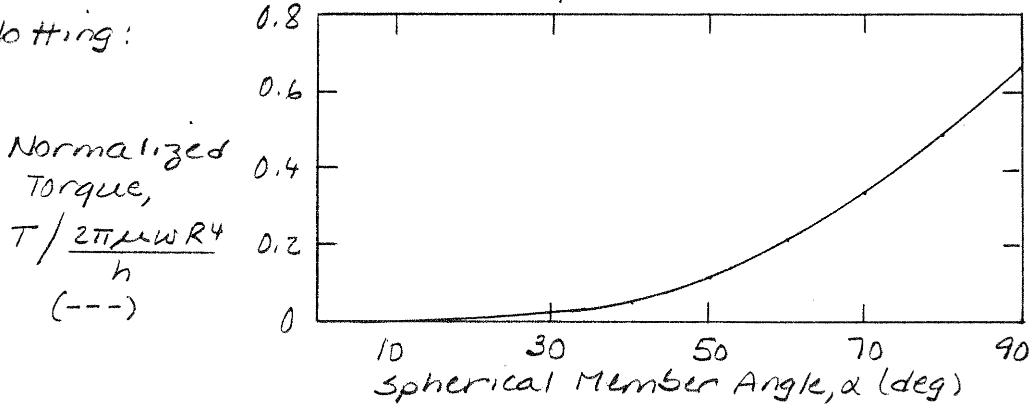
Thus

$$T = \int_0^\alpha R\sin\theta \left(\frac{\mu \omega R \sin\theta}{h}\right) 2\pi R^2 \sin\theta \, d\theta = \frac{2\pi \mu \omega R^4}{h} \int_0^\alpha \sin^3\theta \, d\theta$$

$$T = \frac{2\pi\mu\omega R^4}{h}\left[\frac{\cos^3\theta}{3} - \cos\theta\right]_0^\alpha = \frac{2\pi\mu\omega R^4}{h}\left[\frac{\cos^3\alpha}{3} - \cos\alpha + \frac{2}{3}\right]$$ T

To plot, normalize to $\left[T \Big/ \frac{2\pi\mu\omega R^4}{h}\right] = \left[\frac{\cos^3\alpha}{3} - \cos\alpha + \frac{2}{3}\right]$

Plotting:



$\left\{ \text{Check dimensions: } \left[\frac{\mu\omega R^4}{h}\right] = \frac{Ft}{L^2} \times \frac{1}{t} \times L^4 \times \frac{1}{L} = FL \ \checkmark\checkmark \right\}$

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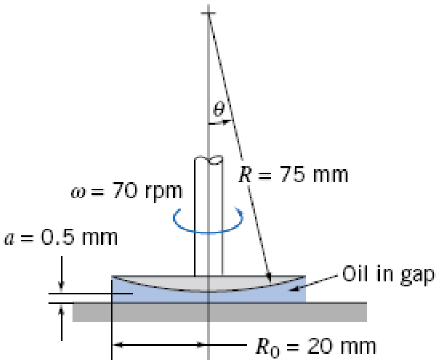
Problem 2.69

[5]

2.69 A cross section of a rotating bearing is shown. The spherical member rotates with angular speed ω, a small distance, a, above the plane surface. The narrow gap is filled with viscous oil, having $\mu = 1250$ cp. Obtain an algebraic expression for the shear stress acting on the spherical member. Evaluate the maximum shear stress that acts on the spherical member for the conditions shown. (Is the maximum necessarily located at the maximum radius?) Develop an algebraic expression (in the form of an integral) for the total viscous shear torque that acts on the spherical member. Calculate the torque using the dimensions shown.



Given: Geometry of rotating bearing

Find: Expression for shear stress; Maximum shear stress; Expression for total torque; Total torque

Solution:

Basic equation $\tau = \mu\cdot\frac{du}{dy}$ $\quad dT = r\cdot\tau\cdot dA$

Assumptions: Newtonian fluid, narrow clearance gap, laminar motion

From the figure $r = R\cdot\sin(\theta)$ $\quad u = \omega\cdot r = \omega\cdot R\cdot\sin(\theta)$ $\quad \frac{du}{dy} = \frac{u-0}{h} = \frac{u}{h}$

$h = a + R\cdot(1-\cos(\theta))$ $\quad dA = 2\cdot\pi\cdot r\cdot dr = 2\cdot\pi R\cdot\sin(\theta)\cdot R\cdot\cos(\theta)\cdot d\theta$

Then $\tau = \mu\cdot\frac{du}{dy} = \frac{\mu\cdot\omega\cdot R\cdot\sin(\theta)}{a + R\cdot(1-\cos(\theta))}$

To find the maximum τ set $\frac{d}{d\theta}\left[\frac{\mu\cdot\omega\cdot R\cdot\sin(\theta)}{a+R\cdot(1-\cos(\theta))}\right] = 0$ so $\frac{R\cdot\mu\cdot\omega\cdot(R\cdot\cos(\theta) - R + a\cdot\cos(\theta))}{(R + a - R\cdot\cos(\theta))^2} = 0$

$R\cdot\cos(\theta) - R + a\cdot\cos(\theta) = 0$ $\quad \theta = \mathrm{acos}\left(\frac{R}{R+a}\right) = \mathrm{acos}\left(\frac{75}{75+0.5}\right)$ $\quad \theta = 6.6\cdot\mathrm{deg}$

$$\tau = 12.5\cdot\mathrm{poise}\times 0.1\cdot\frac{\frac{\mathrm{kg}}{\mathrm{m\cdot s}}}{\mathrm{poise}}\times 2\cdot\pi\cdot\frac{70}{60}\cdot\frac{\mathrm{rad}}{\mathrm{s}}\times 0.075\cdot\mathrm{m}\times\sin(6.6\cdot\mathrm{deg})\times\frac{1}{[0.0005 + 0.075\cdot(1-\cos(6.6\cdot\mathrm{deg}))]\cdot\mathrm{m}}\times\frac{\mathrm{N\cdot s^2}}{\mathrm{m\cdot kg}}$$

$$\tau = 79.2\cdot\frac{\mathrm{N}}{\mathrm{m}^2}$$

The torque is $T = \int r\cdot\tau\cdot A\,d\theta = \int_0^{\theta_{max}} \frac{\mu\cdot\omega\cdot R^4\cdot\sin(\theta)^2\cdot\cos(\theta)}{a + R\cdot(1-\cos(\theta))}\,d\theta$ where $\theta_{max} = \mathrm{asin}\left(\frac{R_0}{R}\right)$ $\quad \theta_{max} = 15.5\cdot\mathrm{deg}$

This integral is best evaluated numerically using Excel, Mathcad, or a good calculator $\quad T = 1.02\times 10^{-3}\cdot\mathrm{N\cdot m}$

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Problem 2.70

[2]

Given: Small gas bubbles form in soda when opened; $D = 0.1$ mm.

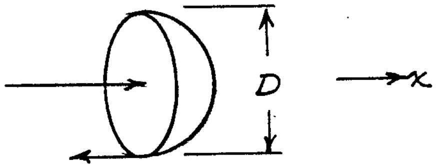
Find: Estimate pressure difference from inside to outside such a bubble.

Solution: Consider a free-body diagram of half a bubble:

Two forces act:

Pressure: $F_p = \Delta p \dfrac{\pi D^2}{4}$

Surface tension: $F_\sigma = \sigma \pi D$



Summing forces for equilibrium

$$\Sigma F_x = F_p - F_\sigma = \Delta p \frac{\pi D^2}{4} - \sigma \pi D = 0$$

so $\dfrac{\Delta p D}{4} - \sigma = 0$ or $\Delta p = \dfrac{4\sigma}{D}$

Assuming soda-gas interface is similar to water-air, then $\sigma = 72.8$ mN/m, and

$$\Delta p = 4 \times 72.8 \times 10^{-3} \frac{N}{m} \times \frac{1}{0.1 \times 10^{-3} m} = 2.91 \times 10^3 \frac{N}{m^2} = 2.91 \text{ kPa} \longleftarrow$$

Δp

Problem 2.71

Slowly fill a glass with water to the maximum possible level. Observe the water level closely. Explain how it can be higher than the rim of the glass.

Open-Ended Problem Statement: Slowly fill a glass with water to the maximum possible level before it overflows. Observe the water level closely. Explain how it can be higher than the rim of the glass.

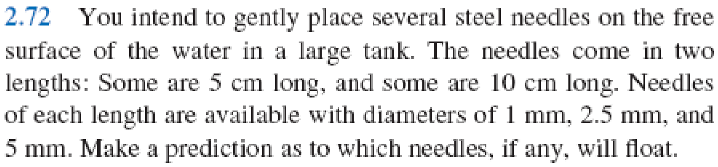
Discussion: Surface tension can cause the maximum water level in a glass to be higher than the rim of the glass. The same phenomenon causes an isolated drop of water to "bead up" on a smooth surface.

Surface tension between the water/air interface and the glass acts as an invisible membrane that allows trapped water to rise above the level of the rim of the glass. The mechanism can be envisioned as forces that act in the surface of the liquid above the rim of the glass. Thus the water appears to defy gravity by attaining a level higher than the rim of the glass.

To experimentally demonstrate that this phenomenon is the result of surface tension, set the liquid level nearly as far above the glass rim as you can get it, using plain water. Add a drop of liquid detergent (the detergent contains additives that reduce the surface tension of water). Watch as the excess water runs over the side of the glass.

Problem 2.72

[2]



2.72 You intend to gently place several steel needles on the free surface of the water in a large tank. The needles come in two lengths: Some are 5 cm long, and some are 10 cm long. Needles of each length are available with diameters of 1 mm, 2.5 mm, and 5 mm. Make a prediction as to which needles, if any, will float.

Given: Data on size of various needles

Find: Which needles, if any, will float

Solution:

For a steel needle of length *L*, diameter *D*, density ρ_s, to float in water with surface tension σ and contact angle θ, the vertical force due to surface tension must equal or exceed the weight

$$2\cdot L\cdot\sigma\cdot\cos(\theta) \ge W = m\cdot g = \frac{\pi\cdot D^2}{4}\cdot\rho_s\cdot L\cdot g \qquad \text{or} \qquad D \le \sqrt{\frac{8\cdot\sigma\cdot\cos(\theta)}{\pi\cdot\rho_s\cdot g}}$$

From Table A.4 $\quad \sigma = 72.8\times10^{-3}\cdot\frac{N}{m} \quad \theta = 0\cdot deg \quad$ and for water $\quad \rho = 1000\cdot\frac{kg}{m^3}$

From Table A.1, for steel $\quad SG = 7.83$

Hence $$\sqrt{\frac{8\cdot\sigma\cdot\cos(\theta)}{\pi\cdot SG\cdot\rho\cdot g}} = \sqrt{\frac{8}{\pi\cdot 7.83}\times72.8\times10^{-3}\cdot\frac{N}{m}\times\frac{m^3}{999\cdot kg}\times\frac{s^2}{9.81\cdot m}\times\frac{kg\cdot m}{N\cdot s^2}} = 1.55\times10^{-3}\cdot m = 1.55\cdot mm$$

Hence *D* < 1.55 mm. Only the 1 mm needles float (needle length is irrelevant)

Problem 2.73

[5]

Plan an experiment to measure the surface tension of a liquid similar to water. If necessary, review the NCFMF video *Surface Tension* for ideas. Which method would be most suitable for use in an undergraduate laboratory? What experimental precision could be expected?

Open-Ended Problem Statement: Plan an experiment to measure the surface tension of a liquid similar to water. If necessary, review the NCFMF video *Surface Tension* for ideas. Which method would be most suitable for use in an undergraduate laboratory? What experimental precision could be expected?

Discussion: Two basic kinds of experiment are possible for an undergraduate laboratory:

1. Using a clear small-diameter tube, compare the capillary rise of the unknown liquid with that of a known liquid (compare with water, because it is similar to the unknown liquid).

   This method would be simple to set up and should give fairly accurate results. A vertical traversing optical microscope could be used to increase the precision of measuring the liquid height in each tube.

   A drawback to this method is that the specific gravity and contact angle of the two liquids must be the same to allow the capillary rises to be compared.

   The capillary rise would be largest and therefore easiest to measure accurately in a tube with the smallest practical diameter. Tubes of several diameters could be used if desired.

2. Dip an object into a pool of test liquid and measure the vertical force required to pull the object from the liquid surface.

   The object might be made rectangular (e.g., a sheet of plastic material) or circular (e.g., a metal ring). The net force needed to pull the same object from each liquid should be proportional to the surface tension of each liquid.

   This method would be simple to set up. However, the force magnitudes to be measured would be quite small.

   A drawback to this method is that the contact angles of the two liquids must be the same.

The first method is probably best for undergraduate laboratory use. A quantitative estimate of experimental measurement uncertainty is impossible without knowing details of the test setup. It might be reasonable to expect results accurate to within ± 10% of the true surface tension.

---

*Net force is the total vertical force minus the weight of the object. A buoyancy correction would be necessary if part of the object were submerged in the test liquid.

Problem 2.74

[2]

Given: Water, with bulk modulus assumed constant.

Find: (a) Percent change in density at 100 atm
(b) Plot percent change vs. p/p_{atm} up to 50,000 psi.
(c) Comment on assumption of constant density.

Solution: By definition, $E_v = \dfrac{dp}{d\rho/\rho}$. Assume E_v = Constant. Then

$$\frac{d\rho}{\rho} = \frac{dp}{E_v}$$

Integrating, from ρ_0 to ρ gives $\ln\dfrac{\rho}{\rho_0} = \dfrac{p-p_0}{E_v} = \dfrac{\Delta p}{E_v}$, so $\dfrac{\rho}{\rho_0} = e^{\Delta p/E_v}$

The relative change in density is

$$\frac{\Delta\rho}{\rho_0} = \frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = e^{\Delta p/E_v} - 1$$

From Table A.2, $E_v = 2.24$ GPa for water at 20°C.

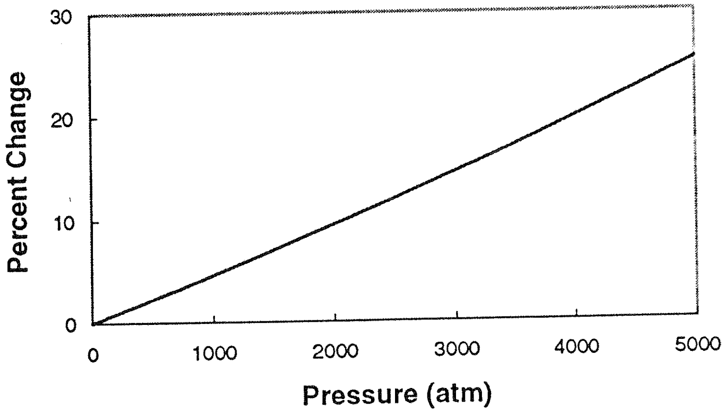
For $p = 100$ atm (gage), $\Delta p = 100$ atm, so

$$\frac{\Delta\rho}{\rho_0} = \exp\left(100\,\text{atm} \times \frac{1}{2.24\times10^9\,\text{Pa}} \times 101.325\times10^3\,\frac{\text{Pa}}{\text{atm}}\right) - 1 = 0.00453, \text{ or } 0.453\%$$

For $\Delta p = 50{,}000$ psi,

$$\frac{\Delta\rho}{\rho_0} = \exp\left(50{,}000\,\text{psi} \times \frac{1}{2.24\times10^9\,\text{Pa}} \times \frac{101.325\times10^3\,\text{Pa}}{14.696\,\text{psi}}\right) - 1 = 0.166 \text{ or } 16.6\%$$

Thus constant density is not a reasonable assumption for a cutting jet operating at 50,000 psi. Constant density (5% change) would be reasonable up to $\Delta p \approx 16{,}000$ psi.



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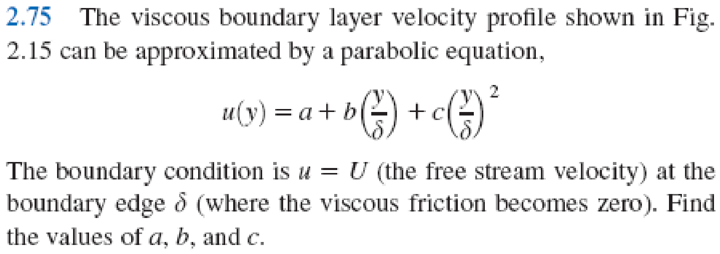
Problem 2.75

[2]



2.75 The viscous boundary layer velocity profile shown in Fig. 2.15 can be approximated by a parabolic equation,

$$u(y) = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$$

The boundary condition is $u = U$ (the free stream velocity) at the boundary edge δ (where the viscous friction becomes zero). Find the values of a, b, and c.

Given: Boundary layer velocity profile in terms of constants a, b and c

Find: Constants a, b and c

Solution:

Basic equation $\quad u = a + b\cdot\left(\frac{y}{\delta}\right) + c\cdot\left(\frac{y}{\delta}\right)^2$

Assumptions: No slip, at outer edge u = U and $\tau = 0$

At $y = 0$ $\quad 0 = a \qquad a = 0$

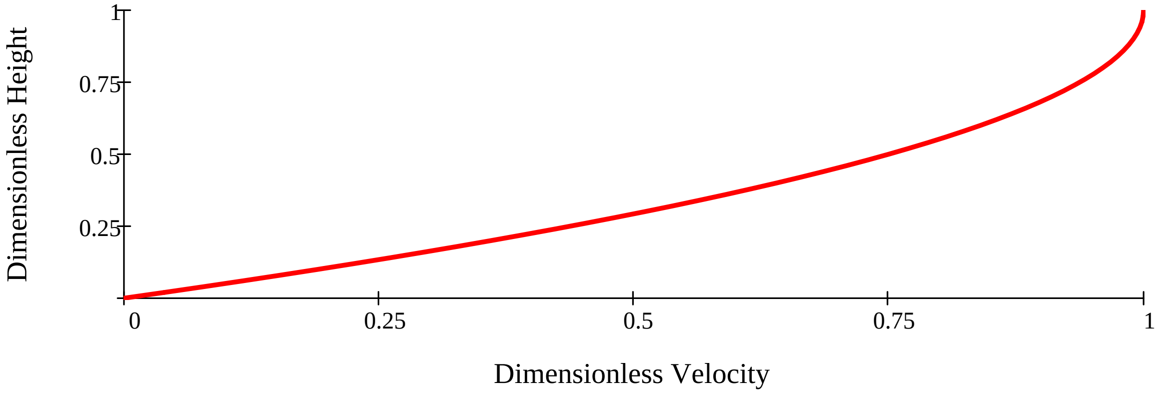
At $y = \delta$ $\quad U = a + b + c \qquad b + c = U \qquad (1)$

At $y = \delta$ $\quad \tau = \mu\cdot\frac{du}{dy} = 0$

$$0 = \frac{d}{dy}\left[a + b\cdot\left(\frac{y}{\delta}\right) + c\cdot\left(\frac{y}{\delta}\right)^2\right] = \frac{b}{\delta} + 2\cdot c\cdot\frac{y}{\delta^2} = \frac{b}{\delta} + 2\cdot\frac{c}{\delta} \qquad b + 2\cdot c = 0 \qquad (2)$$

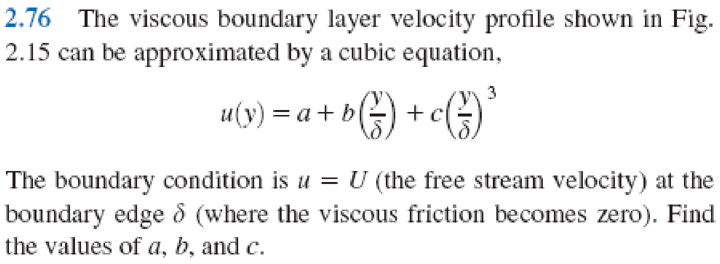
From 1 and 2 $\quad c = -U \qquad b = 2\cdot U$

Hence $\quad u = 2\cdot U\cdot\left(\frac{y}{\delta}\right) - U\cdot\left(\frac{y}{\delta}\right)^2 \qquad \frac{u}{U} = 2\cdot\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$



Problem 2.76

[2]



2.76 The viscous boundary layer velocity profile shown in Fig. 2.15 can be approximated by a cubic equation,

$$u(y) = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^3$$

The boundary condition is $u = U$ (the free stream velocity) at the boundary edge δ (where the viscous friction becomes zero). Find the values of a, b, and c.

Given: Boundary layer velocity profile in terms of constants a, b and c

Find: Constants a, b and c

Solution:

Basic equation $$u = a + b\cdot\left(\frac{y}{\delta}\right) + c\cdot\left(\frac{y}{\delta}\right)^3$$

Assumptions: No slip, at outer edge u = U and τ = 0

At y = 0 $\quad 0 = a \qquad a = 0$

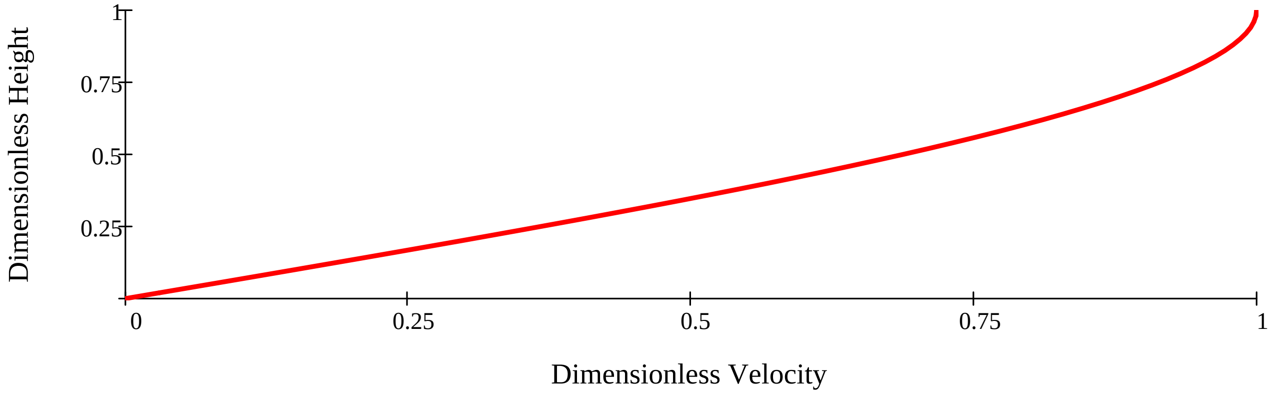
At y = δ $\quad U = a + b + c \qquad b + c = U \qquad (1)$

At y = δ $\quad \tau = \mu\cdot\frac{du}{dy} = 0$

$$0 = \frac{d}{dy}a + b\cdot\left(\frac{y}{\delta}\right) + c\cdot\left(\frac{y}{\delta}\right)^3 = \frac{b}{\delta} + 3\cdot c\cdot\frac{y^2}{\delta^3} = \frac{b}{\delta} + 3\cdot\frac{c}{\delta} \qquad b + 3\cdot c = 0 \qquad (2)$$

From 1 and 2 $\quad c = -\frac{U}{2} \qquad b = \frac{3}{2}\cdot U$

Hence $$u = \frac{3\cdot U}{2}\cdot\left(\frac{y}{\delta}\right) - \frac{U}{2}\cdot\left(\frac{y}{\delta}\right)^3 \qquad \frac{u}{U} = \frac{3}{2}\cdot\left(\frac{y}{\delta}\right) - \frac{1}{2}\cdot\left(\frac{y}{\delta}\right)^3$$



Problem 2.77

[1]

2.77 At what minimum speed (in mph) would an automobile have to travel for compressibility effects to be important? Assume the local air temperature is 60°F.

Given: Local temperature

Find: Minimum speed for compressibility effects

Solution:

Basic equation $V = M\cdot c$ and $M = 0.3$ for compressibility effects

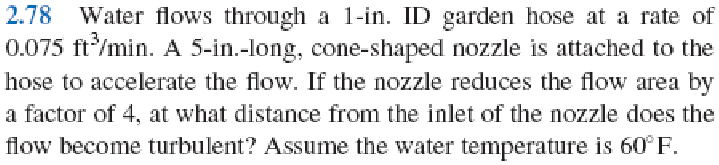
$c = \sqrt{k\cdot R\cdot T}$ For air at STP, k = 1.40 and R = 286.9J/kg.K (53.33 ft.lbf/lbmºR).

Hence $V = M\cdot c = M\cdot\sqrt{k\cdot R\cdot T}$

$$V = 0.3\times\left[1.4\times 53.33\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot\text{R}}\times\frac{32.2\cdot\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\times(60+460)\cdot\text{R}\right]^{\frac{1}{2}}\cdot\frac{60\cdot\text{mph}}{88\cdot\frac{\text{ft}}{\text{s}}} \qquad V = 229\cdot\text{mph}$$

Problem 2.78

[2]



2.78 Water flows through a 1-in. ID garden hose at a rate of 0.075 ft^3/min. A 5-in.-long, cone-shaped nozzle is attached to the hose to accelerate the flow. If the nozzle reduces the flow area by a factor of 4, at what distance from the inlet of the nozzle does the flow become turbulent? Assume the water temperature is 60°F.

NOTE: Flow rate should be $0.75 \cdot \frac{\text{ft}^3}{\text{min}}$

Given: Geometry of and flow rate through garden hose

Find: At which point becomes turbulent

Solution:

Basic equation For pipe flow (Section 2-6) $Re = \frac{\rho \cdot V \cdot D}{\mu} = 2300$ for transition to turbulence

Also flow rate Q is given by $Q = \frac{\pi \cdot D^2}{4} \cdot V$

We can combine these equations and eliminate V to obtain an expression for Re in terms of D

$$Re = \frac{\rho \cdot V \cdot D}{\mu} = \frac{\rho \cdot D}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D^2} = \frac{4 \cdot Q \cdot \rho}{\pi \cdot \mu \cdot D} = 2300$$

Hence $D = \frac{4 \cdot Q \cdot \rho}{2300 \cdot \pi \cdot \mu}$ From Appendix A: $\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ (Approximately)

$$\mu = 1.25 \times 10^{-3} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2} \times \frac{0.209 \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}}{1 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}}$$

(Approximately, from Fig. A.2) $\mu = 2.61 \times 10^{-4} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$

Hence $$D = \frac{4}{2300 \cdot \pi} \times \frac{0.75 \cdot \text{ft}^3}{\text{min}} \times \frac{1 \cdot \text{min}}{60 \cdot \text{s}} \times \frac{1.94 \cdot \text{slug}}{\text{ft}^3} \times \frac{\text{ft}^2}{2.61 \cdot 10^{-4} \cdot \text{lbf} \cdot \text{s}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}}$$ $D = 0.617 \cdot \text{in}$

NOTE: For wrong flow rate, will be 1/10th of this!

The nozzle is tapered: $D_{in} = 1 \cdot \text{in}$ $D_{out} = \frac{D_{in}}{\sqrt{4}}$ $D_{out} = 0.5 \cdot \text{in}$ $L = 5 \cdot \text{in}$

Linear ratios leads to the distance from D_{in} at which D = 0.617 in $\frac{L_{turb}}{L} = \frac{D - D_{in}}{D_{out} - D_{in}}$

$$L_{turb} = L \cdot \frac{D - D_{in}}{D_{out} - D_{in}}$$ $L_{turb} = 3.83 \cdot \text{in}$

NOTE: For wrong flow rate, this does not apply! Flow will not become turbulent.

Problem 2.79

[3]

2.79 A supersonic aircraft travels at 2700 km/hr at an altitude of 27 km. What is the Mach number of the aircraft? At what approximate distance measured from the leading edge of the aircraft's wing does the boundary layer change from laminar to turbulent?

Given: Data on supersonic aircraft

Find: Mach number; Point at which boundary layer becomes turbulent

Solution:

Basic equation $V = M \cdot c$ and $c = \sqrt{k \cdot R \cdot T}$ For air at STP, k = 1.40 and R = 286.9J/kg.K (53.33 ft.lbf/lbm^{o}R).

Hence $M = \frac{V}{c} = \frac{V}{\sqrt{k \cdot R \cdot T}}$

At 27 km the temperature is approximately (from Table A.3) $T = 223.5 \cdot K$

$$M = \left(2700 \times 10^{3} \cdot \frac{m}{hr} \times \frac{1 \cdot hr}{3600 \cdot s}\right) \cdot \left(\frac{1}{1.4} \times \frac{1}{286.9} \cdot \frac{kg \cdot K}{N \cdot m} \times \frac{1 \cdot N \cdot s^{2}}{kg \cdot m} \times \frac{1}{223.5} \cdot \frac{1}{K}\right)^{\frac{1}{2}} \qquad M = 2.5$$

For boundary layer transition, from Section 2-6 $Re_{trans} = 500000$

Then $Re_{trans} = \frac{\rho \cdot V \cdot x_{trans}}{\mu}$ so $x_{trans} = \frac{\mu \cdot Re_{trans}}{\rho \cdot V}$

We need to find the viscosity and density at this altitude and pressure. The viscosity depends on temperature only, but at 223.5 K = - 50^{o}C, it is off scale of Fig. A.3. Instead we need to use formulas as in Appendix A

At this altitude the density is (Table A.3) $\rho = 0.02422 \times 1.225 \cdot \frac{kg}{m^{3}}$ $\rho = 0.0297 \frac{kg}{m^{3}}$

For μ $\mu = \frac{b \cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}}$ where $b = 1.458 \times 10^{-6} \cdot \frac{kg}{m \cdot s \cdot K^{\frac{1}{2}}}$ $S = 110.4 \cdot K$

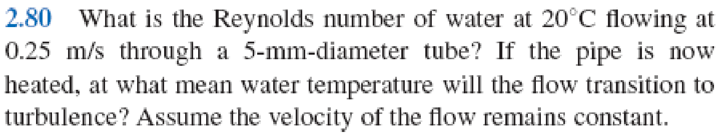
$$\mu = 1.459 \times 10^{-5} \frac{kg}{m \cdot s} \qquad \mu = 1.459 \times 10^{-5} \cdot \frac{N \cdot s}{m^{2}}$$

Hence

$$x_{trans} = 1.459 \times 10^{-5} \cdot \frac{kg}{m \cdot s} \times 500000 \times \frac{1}{0.0297} \cdot \frac{m^{3}}{kg} \times \frac{1}{2700} \times \frac{1}{10^{3}} \cdot \frac{hr}{m} \times \frac{3600 \cdot s}{1 \cdot hr} \qquad x_{trans} = 0.327 m$$

Problem 2.80

[2]



2.80 What is the Reynolds number of water at 20°C flowing at 0.25 m/s through a 5-mm-diameter tube? If the pipe is now heated, at what mean water temperature will the flow transition to turbulence? Assume the velocity of the flow remains constant.

Given: Data on water tube

Find: Reynolds number of flow; Temperature at which flow becomes turbulent

Solution:

Basic equation For pipe flow (Section 2-6) $\mathrm{Re} = \dfrac{\rho \cdot V \cdot D}{\mu} = \dfrac{V \cdot D}{\nu}$

At 20ºC, from Fig. A.3 $\nu = 9 \times 10^{-7} \cdot \dfrac{\mathrm{m}^2}{\mathrm{s}}$ and so $\mathrm{Re} = 0.25 \cdot \dfrac{\mathrm{m}}{\mathrm{s}} \times 0.005 \cdot \mathrm{m} \times \dfrac{1}{9 \times 10^{-7}} \cdot \dfrac{\mathrm{s}}{\mathrm{m}^2}$ $\mathrm{Re} = 1389$

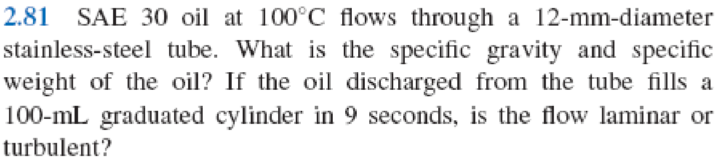
For the heated pipe $\mathrm{Re} = \dfrac{V \cdot D}{\nu} = 2300$ for transition to turbulence

Hence $\nu = \dfrac{V \cdot D}{2300} = \dfrac{1}{2300} \times 0.25 \cdot \dfrac{\mathrm{m}}{\mathrm{s}} \times 0.005 \cdot \mathrm{m}$ $\nu = 5.435 \times 10^{-7} \dfrac{\mathrm{m}^2}{\mathrm{s}}$

From Fig. A.3, the temperature of water at this viscosity is approximately $T = 52 \cdot C$

Problem 2.81

[2]



2.81 SAE 30 oil at 100°C flows through a 12-mm-diameter stainless-steel tube. What is the specific gravity and specific weight of the oil? If the oil discharged from the tube fills a 100-mL graduated cylinder in 9 seconds, is the flow laminar or turbulent?

Given: Type of oil, flow rate, and tube geometry

Find: Whether flow is laminar or turbulent

Solution:

Data on SAE 30 oil SG or density is limited in the Appendix. We can Google it or use the following $\nu = \frac{\mu}{\rho}$ so $\rho = \frac{\mu}{\nu}$

At 100ºC, from Figs. A.2 and A.3 $\mu = 9\times 10^{-3}\cdot\frac{N\cdot s}{m^2}$ $\nu = 1\times 10^{-5}\cdot\frac{m^2}{s}$

$$\rho = 9\times 10^{-3}\cdot\frac{N\cdot s}{m^2}\times\frac{1}{1\times 10^{-5}}\cdot\frac{s}{m^2}\times\frac{kg\cdot m}{s^2\cdot N} \qquad \rho = 900\,\frac{kg}{m^3}$$

Hence $SG = \frac{\rho}{\rho_{water}}$ $\rho_{water} = 1000\cdot\frac{kg}{m^3}$ $SG = 0.9$

The specific weight is $\gamma = \rho\cdot g$ $\gamma = 900\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}$ $\gamma = 8.829\times 10^3\cdot\frac{N}{m^3}$

For pipe flow (Section 2-6) $Q = \frac{\pi\cdot D^2}{4}\cdot V$ so $V = \frac{4\cdot Q}{\pi\cdot D^2}$

$$Q = 100\cdot mL\times\frac{10^{-6}\cdot m^3}{1\cdot mL}\times\frac{1}{9}\cdot\frac{1}{s} \qquad Q = 1.111\times 10^{-5}\,\frac{m^3}{s}$$

Then $V = \frac{4}{\pi}\times 1.11\times 10^{-5}\cdot\frac{m^3}{s}\times\left(\frac{1}{12}\cdot\frac{1}{mm}\times\frac{1000\cdot mm}{1\cdot m}\right)^2$ $V = 0.0981\,\frac{m}{s}$

Hence $Re = \frac{\rho\cdot V\cdot D}{\mu}$

$$Re = 900\cdot\frac{kg}{m^3}\times 0.0981\cdot\frac{m}{s}\times 0.012\cdot m\times\frac{1}{9\times 10^{-3}}\cdot\frac{m^2}{N\cdot s}\times\frac{N\cdot s^2}{kg\cdot m} \qquad Re = 118$$

Flow is laminar

Problem 2.82

[2]

2.82 A seaplane is flying at 100 mph through air at 45°F. At what distance from the leading edge of the underside of the fuselage does the boundary layer transition to turbulence? How does this boundary layer transition change as the underside of the fuselage touches the water during landing? Assume the water temperature is also 45°F.

Given: Data on seaplane

Find: Transition point of boundary layer

Solution:

For boundary layer transition, from Section 2-6 $Re_{trans} = 500000$

Then
$$Re_{trans} = \frac{\rho \cdot V \cdot x_{trans}}{\mu} = \frac{V \cdot x_{trans}}{\nu} \qquad \text{so} \qquad x_{trans} = \frac{\nu \cdot Re_{trans}}{V}$$

At 45ºF = 7.2ºC (Fig A.3)
$$\nu = 0.8 \times 10^{-5} \cdot \frac{m^2}{s} \times \frac{10.8 \cdot \frac{ft^2}{s}}{1 \cdot \frac{m^2}{s}} \qquad \nu = 8.64 \times 10^{-5} \cdot \frac{ft^2}{s}$$

$$x_{trans} = 8.64 \times 10^{-5} \cdot \frac{ft^2}{s} \cdot 500000 \times \frac{1}{100 \cdot mph} \times \frac{60 \cdot mph}{88 \cdot \frac{ft}{s}} \qquad x_{trans} = 0.295 \cdot ft$$

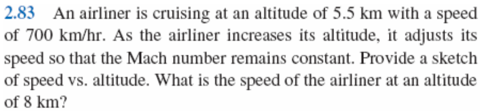
As the seaplane touches down:

At 45ºF = 7.2ºC (Fig A.3)
$$\nu = 1.5 \times 10^{-5} \cdot \frac{m^2}{s} \times \frac{10.8 \cdot \frac{ft^2}{s}}{1 \cdot \frac{m^2}{s}} \qquad \nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$$

$$x_{trans} = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s} \cdot 500000 \times \frac{1}{100 \cdot mph} \times \frac{60 \cdot mph}{88 \cdot \frac{ft}{s}} \qquad x_{trans} = 0.552 \cdot ft$$

Problem 2.83 (In Excel)

[3]



2.83 An airliner is cruising at an altitude of 5.5 km with a speed of 700 km/hr. As the airliner increases its altitude, it adjusts its speed so that the Mach number remains constant. Provide a sketch of speed vs. altitude. What is the speed of the airliner at an altitude of 8 km?

Given: Data on airliner

Find: Sketch of speed versus altitude (M = const)

Solution:

Data on temperature versus height can be obtained from Table A.3

At 5.5 km the temperature is approximate 252 K

The speed of sound is obtained from $c = \sqrt{k \cdot R \cdot T}$

where $k = 1.4$

$R = 286.9$ J/kg·K (Table A.6)

$c = 318$ m/s

We also have

$V = 700$ km/hr

or $V = 194$ m/s

Hence $M = V/c$ or

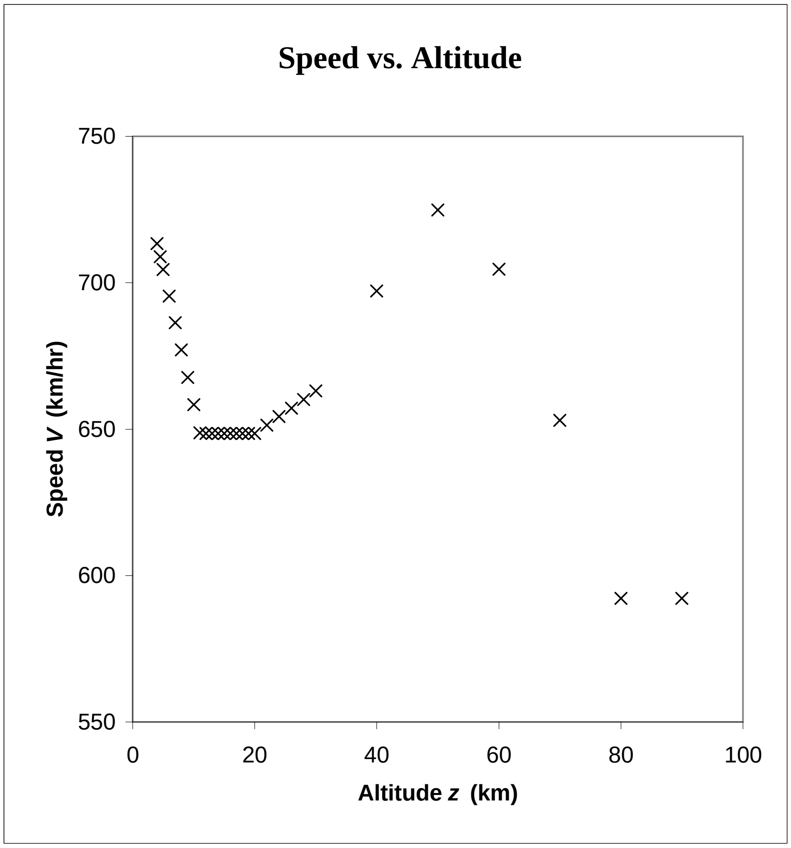
$M = 0.611$

To compute V for constant M, we use $V = M \cdot c = 0.611 \cdot c$

At a height of 8 km $V = 677$ km/hr

NOTE: Realistically, the aiplane will fly to a maximum height of about 10 km!

z (km)	T (K)	c (m/s)	V (km/hr)
4	262	325	713
5	259	322	709
5	256	320	704
6	249	316	695
7	243	312	686
8	236	308	677
9	230	304	668
10	223	299	658
11	217	295	649
12	217	295	649
13	217	295	649
14	217	295	649
15	217	295	649
16	217	295	649
17	217	295	649
18	217	295	649
19	217	295	649
20	217	295	649
22	219	296	651
24	221	298	654
26	223	299	657
28	225	300	660
30	227	302	663
40	250	317	697
50	271	330	725
60	256	321	705
70	220	297	653
80	181	269	592
90	181	269	592

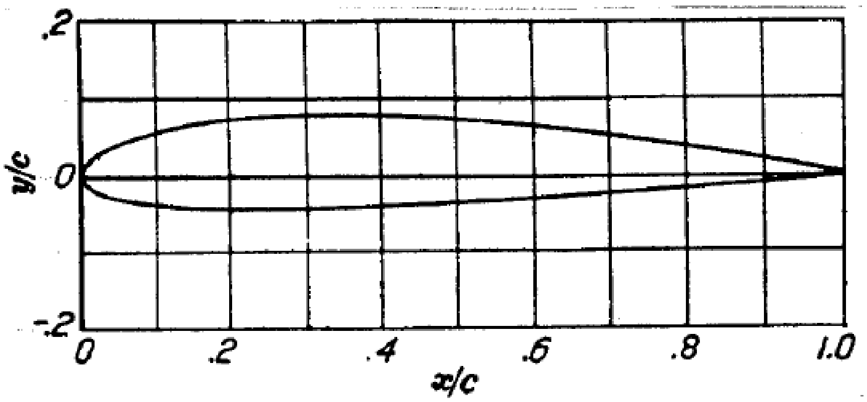


Problem 2.84

How does an airplane wing develop lift?

Open-Ended Problem Statement: How does an airplane wing develop lift?

Discussion: The sketch shows the cross-section of a typical airplane wing. The airfoil section is rounded at the front, curved across the top, reaches maximum thickness about a third of the way back, and then tapers slowly to a fine trailing edge. The bottom of the airfoil section is relatively flat. (The discussion below also applies to a symmetric airfoil at an angle of incidence that produces lift.)



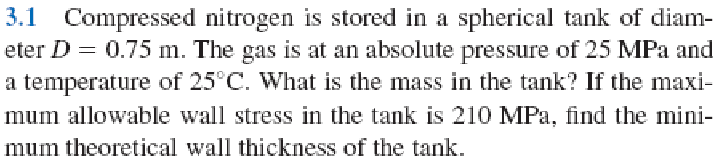
NACA 2412 Wing Section

It is both a popular expectation and an experimental fact that air flows more rapidly over the curved top surface of the airfoil section than along the relatively flat bottom. In the NCFMF video *Flow Visualization*, timelines placed in front of the airfoil indicate that fluid flows more rapidly along the top of the section than along the bottom.

In the absence of viscous effects (this is a valid assumption outside the boundary layers on the airfoil) pressure falls when flow speed increases. Thus the pressures on the top surface of the airfoil where flow speed is higher are lower than the pressures on the bottom surface where flow speed does not increase. (Actual pressure profiles measured for a lifting section are shown in the NCFMF video *Boundary Layer Control*.) The unbalanced pressures on the top and bottom surfaces of the airfoil section create a net force that tends to develop lift on the profile.

Problem 3.1

[2]



3.1 Compressed nitrogen is stored in a spherical tank of diameter $D = 0.75$ m. The gas is at an absolute pressure of 25 MPa and a temperature of 25°C. What is the mass in the tank? If the maximum allowable wall stress in the tank is 210 MPa, find the minimum theoretical wall thickness of the tank.

Given: Data on nitrogen tank

Find: Mass of nitrogen; minimum required wall thickness

Solution:

Assuming ideal gas behavior: $p \cdot V = M \cdot R \cdot T$

where, from Table A.6, for nitrogen $R = 297 \cdot \frac{J}{kg \cdot K}$

Then the mass of nitrogen is $M = \frac{p \cdot V}{R \cdot T} = \frac{p}{R \cdot T} \cdot \left(\frac{\pi \cdot D^3}{6}\right)$

$$M = \frac{25 \cdot 10^6 \cdot N}{m^2} \times \frac{kg \cdot K}{297 \cdot J} \times \frac{1}{298 \cdot K} \times \frac{J}{N \cdot m} \times \frac{\pi \cdot (0.75 \cdot m)^3}{6}$$

$$M = 62.4\,kg$$

To determine wall thickness, consider a free body diagram for one hemisphere:

$$\Sigma F = 0 = p \cdot \frac{\pi \cdot D^2}{4} - \sigma_c \cdot \pi \cdot D \cdot t$$

where σ_c is the circumferential stress in the container

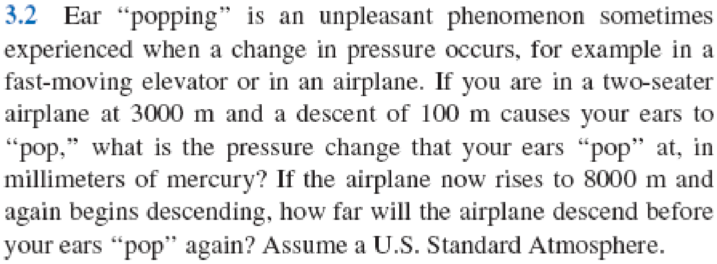
Then $t = \frac{p \cdot \pi \cdot D^2}{4 \cdot \pi \cdot D \cdot \sigma_c} = \frac{p \cdot D}{4 \cdot \sigma_c}$

$$t = 25 \cdot 10^6 \cdot \frac{N}{m^2} \times \frac{0.75 \cdot m}{4} \times \frac{1}{210 \cdot 10^6} \cdot \frac{m^2}{N}$$

$t = 0.0223\,m$ $\qquad$ $t = 22.3\,mm$

Problem 3.2

[2]



3.2 Ear "popping" is an unpleasant phenomenon sometimes experienced when a change in pressure occurs, for example in a fast-moving elevator or in an airplane. If you are in a two-seater airplane at 3000 m and a descent of 100 m causes your ears to "pop," what is the pressure change that your ears "pop" at, in millimeters of mercury? If the airplane now rises to 8000 m and again begins descending, how far will the airplane descend before your ears "pop" again? Assume a U.S. Standard Atmosphere.

Given: Data on flight of airplane

Find: Pressure change in mm Hg for ears to "pop"; descent distance from 8000 m to cause ears to "pop."

Solution:

Assume the air density is approximately constant constant from 3000 m to 2900 m.
From table A.3

$$\rho_{SL} = 1.225 \cdot \frac{kg}{m^3} \qquad \rho_{air} = 0.7423 \cdot \rho_{SL} \qquad \rho_{air} = 0.909 \frac{kg}{m^3}$$

We also have from the manometer equation, Eq. 3.7

$$\Delta p = -\rho_{air} \cdot g \cdot \Delta z \qquad \text{and also} \qquad \Delta p = -\rho_{Hg} \cdot g \cdot \Delta h_{Hg}$$

Combining $\Delta h_{Hg} = \frac{\rho_{air}}{\rho_{Hg}} \cdot \Delta z = \frac{\rho_{air}}{SG_{Hg} \cdot \rho_{H2O}} \cdot \Delta z$ $\qquad SG_{Hg} = 13.55$ from Table A.2

$$\Delta h_{Hg} = \frac{0.909}{13.55 \times 999} \times 100 \cdot m \qquad \Delta h_{Hg} = 6.72\,mm$$

For the ear popping descending from 8000 m, again assume the air density is approximately constant constant, this time at 8000 m.
From table A.3

$$\rho_{air} = 0.4292 \cdot \rho_{SL} \qquad \rho_{air} = 0.526 \frac{kg}{m^3}$$

We also have from the manometer equation

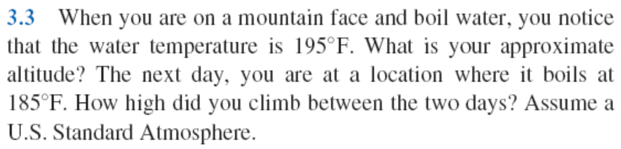
$$\rho_{air8000} \cdot g \cdot \Delta z_{8000} = \rho_{air3000} \cdot g \cdot \Delta z_{3000}$$

where the numerical subscripts refer to conditions at 3000m and 8000m.
Hence

$$\Delta z_{8000} = \frac{\rho_{air3000} \cdot g}{\rho_{air8000} \cdot g} \cdot \Delta z_{3000} = \frac{\rho_{air3000}}{\rho_{air8000}} \cdot \Delta z_{3000} \qquad \Delta z_{8000} = \frac{0.909}{0.526} \times 100 \cdot m \qquad \Delta z_{8000} = 173\,m$$

Problem 3.3

[3]



3.3 When you are on a mountain face and boil water, you notice that the water temperature is 195°F. What is your approximate altitude? The next day, you are at a location where it boils at 185°F. How high did you climb between the two days? Assume a U.S. Standard Atmosphere.

Given: Boiling points of water at different elevations

Find: Change in elevation

Solution:

From the steam tables, we have the following data for the boiling point (saturation temperature) of water

T_{sat} (^oF)	p (psia)
195	10.39
185	8.39

The sea level pressure, from Table A.3, is

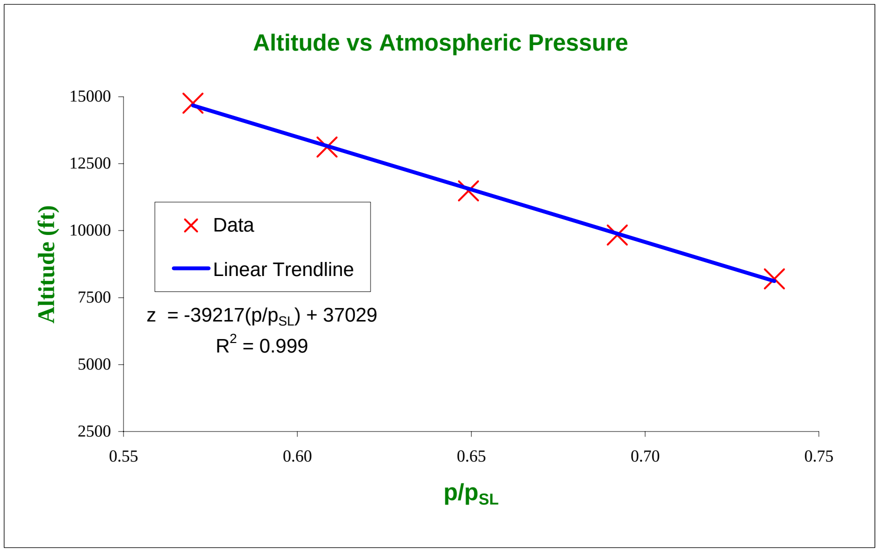
p_{SL} = 14.696 psia

Hence

T_{sat} (^oF)	p/p_{SL}
195	0.707
185	0.571

From Table A.3

p/p_{SL}	Altitude (m)	Altitude (ft)
0.7372	2500	8203
0.6920	3000	9843
0.6492	3500	11484
0.6085	4000	13124
0.5700	4500	14765



Then, any one of a number of *Excel* functions can be used to interpolate
(Here we use *Excel*'s *Trendline* analysis)

p/p_{SL}	Altitude (ft)
0.707	9303
0.571	14640

Current altitude is approximately 9303 ft

The change in altitude is then 5337 ft

Alternatively, we can interpolate for each altitude by using a linear regression between adjacent data points

For

p/p_{SL}	Altitude (m)	Altitude (ft)
0.7372	2500	8203
0.6920	3000	9843

p/p_{SL}	Altitude (m)	Altitude (ft)
0.6085	4000	13124
0.5700	4500	14765

Then

0.7070	2834	9299

0.5730	4461	14637

The change in altitude is then 5338 ft

Problem 3.4

[2]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Pure water on a standard day

Find: Boiling temperature at (a) 1000 m, and (b) 2000 m. Compare with sea level value

Solution

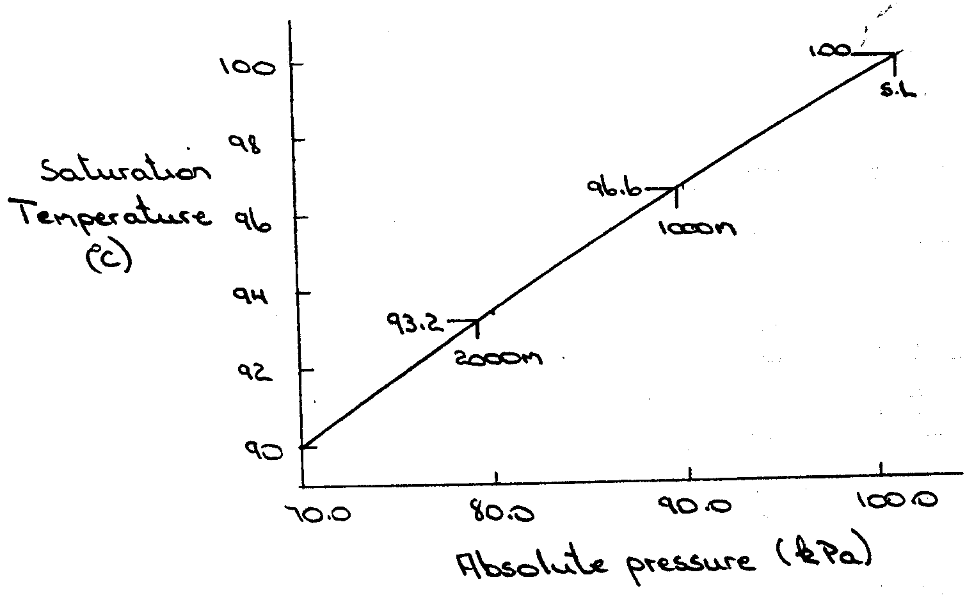
We can determine the atmospheric pressure at the given altitudes from table A.3, Appendix A

Elevation (m)	$\frac{p}{p_0}$	p (kPa)	T_{sat}^* (°C)
0	1.000	101	100
1000	0.887	89.6	96.6
2000	0.785	79.3	93.2

* T_{sat} obtained from plot of T_{sat} vs p given below

Data from Steam Tables gives T_{sat}

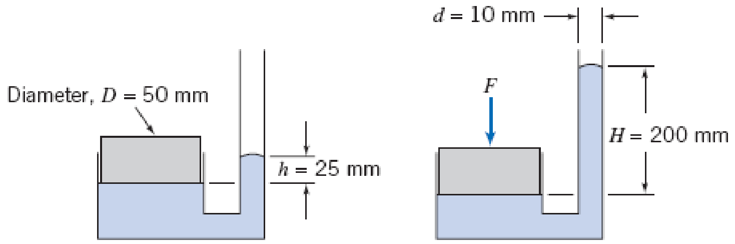
p (kPa)	T_{sat} (°C)
70	90.0
80	93.5
90	96.7
101.325	100.0



{ These data show that T_{sat} drops about 3.4°C / 1000 m }

Problem 3.5

3.5 The tube shown is filled with mercury at 20°C. Calculate the force applied to the piston.



Given: Data on system before and after applied force

Find: Applied force

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $p = p_{atm} - \rho \cdot g \cdot (y - y_0)$ with $p(y_0) = p_{atm}$

For initial state $p_1 = p_{atm} + \rho \cdot g \cdot h$ and $F_1 = p_1 \cdot A = \rho \cdot g \cdot h \cdot A$ (Gage; F_1 is hydrostatic upwards force)

For the initial FBD $\Sigma F_y = 0$ $F_1 - W = 0$ $W = F_1 = \rho \cdot g \cdot h \cdot A$

For final state $p_2 = p_{atm} + \rho \cdot g \cdot H$ and $F_2 = p_2 \cdot A = \rho \cdot g \cdot H \cdot A$ (Gage; F_2 is hydrostatic upwards force)

For the final FBD $\Sigma F_y = 0$ $F_2 - W - F = 0$ $F = F_2 - W = \rho \cdot g \cdot H \cdot A - \rho \cdot g \cdot h \cdot A = \rho \cdot g \cdot A \cdot (H - h)$

$$F = \rho_{H2O} \cdot SG \cdot g \cdot \frac{\pi \cdot D^2}{4} \cdot (H - h)$$

From Fig. A.1 $SG = 13.54$

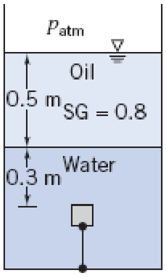
$$F = 1000 \cdot \frac{kg}{m^3} \times 13.54 \times 9.81 \cdot \frac{m}{s^2} \times \frac{\pi}{4} \times (0.05 \cdot m)^2 \times (0.2 - 0.025) \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$

$F = 45.6 N$

Problem 3.6

[2]

3.6 A 125-mL cube of solid oak is held submerged by a tether as shown. Calculate the actual force of the water on the bottom surface of the cube and the tension in the tether.



Given: Data on system

Find: Force on bottom of cube; tension in tether

Solution:

Basic equation $\frac{dp}{dy} = -\rho\cdot g$ or, for constant ρ $\Delta p = \rho\cdot g\cdot h$ where h is measured downwards

The absolute pressure at the interface is $p_{interface} = p_{atm} + SG_{oil}\cdot\rho\cdot g\cdot h_{oil}$

Then the pressure on the lower surface is $p_L = p_{interface} + \rho\cdot g\cdot h_L = p_{atm} + \rho\cdot g\cdot\left(SG_{oil}\cdot h_{oil} + h_L\right)$

For the cube $V = 125\cdot mL$ $V = 1.25\times 10^{-4}\cdot m^3$

Then the size of the cube is $d = V^{\frac{1}{3}}$ $d = 0.05\,m$ and the depth in water to the upper surface is $h_U = 0.3\cdot m$

Hence $h_L = h_U + d$ $h_L = 0.35\,m$ where h_L is the depth in water to the lower surface

The force on the lower surface is $F_L = p_L\cdot A$ where $A = d^2$ $A = 0.0025\,m^2$

$$F_L = \left[p_{atm} + \rho\cdot g\cdot\left(SG_{oil}\cdot h_{oil} + h_L\right)\right]\cdot A$$

$$F_L = \left[101\times 10^3\cdot\frac{N}{m^2} + 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times(0.8\times 0.5\cdot m + 0.35\cdot m)\times\frac{N\cdot s^2}{kg\cdot m}\right]\times 0.0025\cdot m^2$$

$F_L = 270.894\,N$ Note: Extra decimals needed for computing T later!

For the tension in the tether, an FBD gives $\Sigma F_y = 0$ $F_L - F_U - W - T = 0$ or $T = F_L - F_U - W$

where $F_U = \left[p_{atm} + \rho\cdot g\cdot\left(SG_{oil}\cdot h_{oil} + h_U\right)\right]\cdot A$

Note that we could instead compute $\Delta F = F_L - F_U = \rho \cdot g \cdot SG_{oil} \cdot \left(h_L - h_U\right) \cdot A$ and $T = \Delta F - W$

Using F_U

$$F_U = \left[101 \times 10^3 \cdot \frac{N}{m^2} + 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (0.8 \times 0.5 \cdot m + 0.3 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m}\right] \times 0.0025 \cdot m^2$$

$F_U = 269.668\,N$ Note: Extra decimals needed for computing T later!

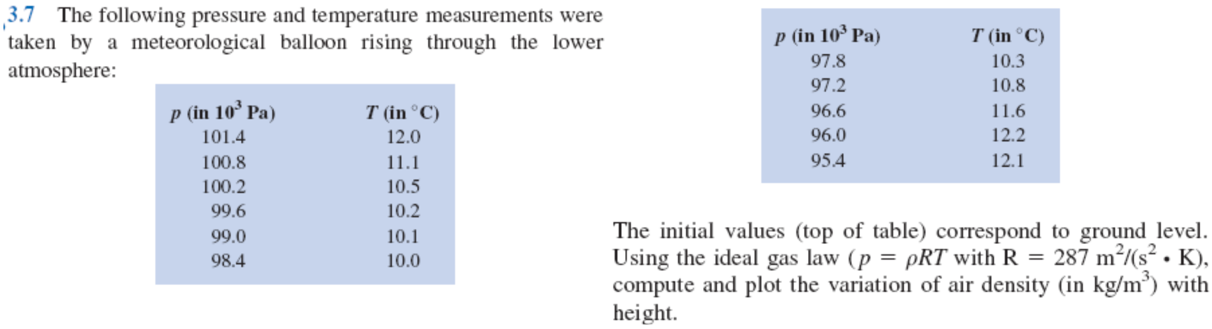
For the oak block (Table A.1) $SG_{oak} = 0.77$ so $W = SG_{oak} \cdot \rho \cdot g \cdot V$

$$W = 0.77 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 1.25 \times 10^{-4} \cdot m^3 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W = 0.944\,N$$

$T = F_L - F_U - W$ $T = 0.282\,N$

Problem 3.7

[1]



3.7 The following pressure and temperature measurements were taken by a meteorological balloon rising through the lower atmosphere:

p (in 10^3 Pa)	*T* (in °C)
101.4	12.0
100.8	11.1
100.2	10.5
99.6	10.2
99.0	10.1
98.4	10.0
97.8	10.3
97.2	10.8
96.6	11.6
96.0	12.2
95.4	12.1

The initial values (top of table) correspond to ground level. Using the ideal gas law ($p = \rho RT$ with $R = 287\ m^2/(s^2 \cdot K)$), compute and plot the variation of air density (in kg/m^3) with height.

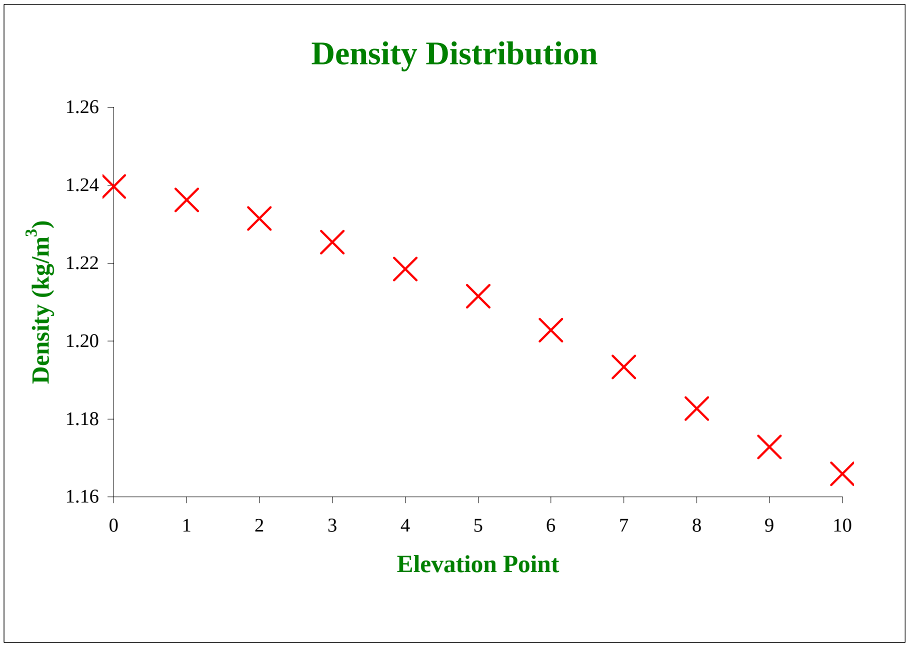
Given: Pressure and temperature data from balloon

Find: Plot density change as a function of elevation

Solution:

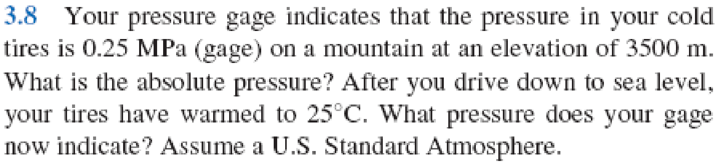
Using the ideal gas equation, $\rho = p/RT$

p (kPa)	T (^{o}C)	ρ (kg/m^3)
101.4	12.0	1.240
100.8	11.1	1.236
100.2	10.5	1.231
99.6	10.2	1.225
99.0	10.1	1.218
98.4	10.0	1.212
97.8	10.3	1.203
97.2	10.8	1.193
96.6	11.6	1.183
96.0	12.2	1.173
95.4	12.1	1.166



Problem 3.8

[2]



3.8 Your pressure gage indicates that the pressure in your cold tires is 0.25 MPa (gage) on a mountain at an elevation of 3500 m. What is the absolute pressure? After you drive down to sea level, your tires have warmed to 25°C. What pressure does your gage now indicate? Assume a U.S. Standard Atmosphere.

Given: Data on tire at 3500 m and at sea level

Find: Absolute pressure at 3500 m; pressure at sea level

Solution:

At an elevation of 3500 m, from Table A.3:

$p_{SL} = 101 \cdot kPa \qquad p_{atm} = 0.6492 \cdot p_{SL} \qquad p_{atm} = 65.6 \cdot kPa$

and we have $\quad p_g = 0.25 \cdot MPa \qquad p_g = 250 \cdot kPa \qquad p = p_g + p_{atm} \qquad p = 316 \cdot kPa$

At sea level $\quad p_{atm} = 101 \cdot kPa$

Meanwhile, the tire has warmed up, from the ambient temperature at 3500 m, to 25ºC.

At an elevation of 3500 m, from Table A.3 $\quad T_{cold} = 265.4 \cdot K \quad$ and $\quad T_{hot} = (25 + 273) \cdot K \qquad T_{hot} = 298\,K$

Hence, assuming ideal gas behavior, *pV* = *mRT*, and that the tire is approximately a rigid container, the absolute pressure of the hot tire is

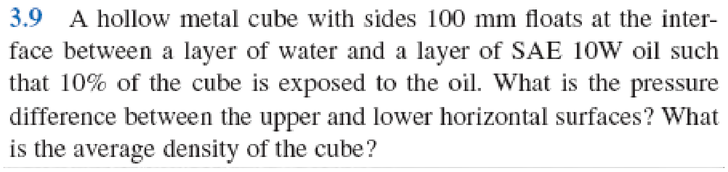
$$p_{hot} = \frac{T_{hot}}{T_{cold}} \cdot p \qquad p_{hot} = 354 \cdot kPa$$

Then the gage pressure is

$$p_g = p_{hot} - p_{atm} \qquad p_g = 253 \cdot kPa$$

Problem 3.9

[2]



3.9 A hollow metal cube with sides 100 mm floats at the interface between a layer of water and a layer of SAE 10W oil such that 10% of the cube is exposed to the oil. What is the pressure difference between the upper and lower horizontal surfaces? What is the average density of the cube?

Given: Properties of a cube floating at an interface

Find: The pressures difference between the upper and lower surfaces; average cube density

Solution:

The pressure difference is obtained from two applications of Eq. 3.7

$$p_U = p_0 + \rho_{SAE10} \cdot g \cdot (H - 0.1 \cdot d) \qquad p_L = p_0 + \rho_{SAE10} \cdot g \cdot H + \rho_{H2O} \cdot g \cdot 0.9 \cdot d$$

where p_U and p_L are the upper and lower pressures, p_0 is the oil free surface pressure, H is the depth of the interface, and d is the cube size

Hence the pressure difference is

$$\Delta p = p_L - p_U = \rho_{H2O} \cdot g \cdot 0.9 \cdot d + \rho_{SAE10} \cdot g \cdot 0.1 \cdot d \qquad \Delta p = \rho_{H2O} \cdot g \cdot d \cdot \left(0.9 + SG_{SAE10} \cdot 0.1\right)$$

From Table A.2 $\quad SG_{SAE10} = 0.92$

$$\Delta p = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.1 \cdot m \times (0.9 + 0.92 \times 0.1) \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 972\,Pa$$

For the cube density, set up a free body force balance for the cube

$$\Sigma F = 0 = \Delta p \cdot A - W$$

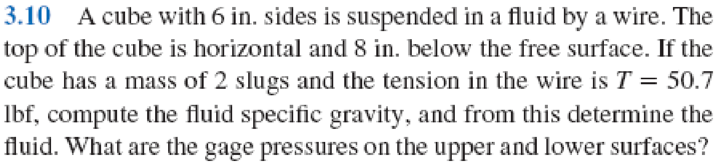
Hence $\quad W = \Delta p \cdot A = \Delta p \cdot d^2$

$$\rho_{cube} = \frac{m}{d^3} = \frac{W}{d^3 \cdot g} = \frac{\Delta p \cdot d^2}{d^3 \cdot g} = \frac{\Delta p}{d \cdot g}$$

$$\rho_{cube} = 972 \cdot \frac{N}{m^2} \times \frac{1}{0.1 \cdot m} \times \frac{s^2}{9.81 \cdot m} \times \frac{kg \cdot m}{N \cdot s^2} \qquad \rho_{cube} = 991 \frac{kg}{m^3}$$

Problem 3.10

[2]



3.10 A cube with 6 in. sides is suspended in a fluid by a wire. The top of the cube is horizontal and 8 in. below the free surface. If the cube has a mass of 2 slugs and the tension in the wire is $T = 50.7$ lbf, compute the fluid specific gravity, and from this determine the fluid. What are the gage pressures on the upper and lower surfaces?

Given: Properties of a cube suspended by a wire in a fluid

Find: The fluid specific gravity; the gage pressures on the upper and lower surfaces

Solution:

From a free body analysis of the cube: $\Sigma F = 0 = T + \left(p_L - p_U\right)\cdot d^2 - M\cdot g$

where *M* and *d* are the cube mass and size and p_L and p_U are the pressures on the lower and upper surfaces

For each pressure we can use Eq. 3.7 $p = p_0 + \rho\cdot g\cdot h$

Hence $p_L - p_U = \left[p_0 + \rho\cdot g\cdot(H + d)\right] - \left(p_0 + \rho\cdot g\cdot H\right) = \rho\cdot g\cdot d = SG\cdot\rho_{H2O}\cdot d$

where *H* is the depth of the upper surface

Hence the force balance gives $SG = \dfrac{M\cdot g - T}{\rho_{H2O}\cdot g\cdot d^3}$

$$SG = \frac{2\cdot slug\times 32.2\cdot\frac{ft}{s^2}\times\frac{lbf\cdot s^2}{slug\cdot ft} - 50.7\cdot lbf}{1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times\frac{lbf\cdot s^2}{slug\cdot ft}\times(0.5\cdot ft)^3} \qquad SG = 1.75$$

From Table A.1, the fluid is Meriam blue.

The individual pressures are computed from Eq 3.7

$$p = p_0 + \rho\cdot g\cdot h \quad \text{or} \quad p_g = \rho\cdot g\cdot h = SG\cdot\rho_{H2O}\cdot h$$

For the upper surface $p_g = 1.754\times 1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times\frac{2}{3}\cdot ft\times\frac{lbf\cdot s^2}{slug\cdot ft}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2$ $\qquad p_g = 0.507\,psi$

For the lower surface $p_g = 1.754\times 1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times\left(\frac{2}{3} + \frac{1}{2}\right)\cdot ft\times\frac{lbf\cdot s^2}{slug\cdot ft}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2$ $\qquad p_g = 0.888\,psi$

Note that the SG calculation can also be performed using a buoyancy approach (discussed later in the chapter):

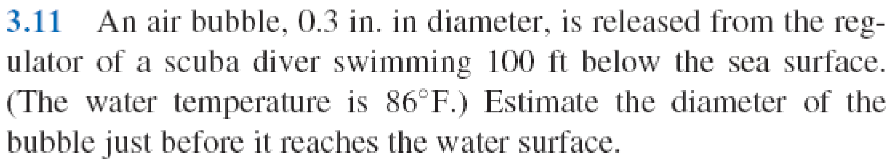
Consider a free body diagram of the cube: $\Sigma F = 0 = T + F_B - M\cdot g$

where *M* is the cube mass and F_B is the buoyancy force $F_B = SG\cdot\rho_{H2O}\cdot L^3\cdot g$

Hence $T + SG\cdot\rho_{H2O}\cdot L^3\cdot g - M\cdot g = 0$ or $SG = \dfrac{M\cdot g - T}{\rho_{H2O}\cdot g\cdot L^3}$ as before $\qquad SG = 1.75$

Problem 3.11

[2]



3.11 An air bubble, 0.3 in. in diameter, is released from the regulator of a scuba diver swimming 100 ft below the sea surface. (The water temperature is 86°F.) Estimate the diameter of the bubble just before it reaches the water surface.

Given: Data on air bubble

Find: Bubble diameter as it reaches surface

Solution:

Basic equation $\frac{dp}{dy} = -\rho_{sea} \cdot g$ and the ideal gas equation $p = \rho \cdot R \cdot T = \frac{M}{V} \cdot R \cdot T$

We assume the temperature is constant, and the density of sea water is constant

For constant sea water density $p = p_{atm} + SG_{sea} \cdot \rho \cdot g \cdot h$ where p is the pressure at any depth h

Then the pressure at the initial depth is $p_1 = p_{atm} + SG_{sea} \cdot \rho \cdot g \cdot h_1$

The pressure as it reaches the surface is $p_2 = p_{atm}$

For the bubble $p = \frac{M \cdot R \cdot T}{V}$ but M and T are constant $M \cdot R \cdot T = const = p \cdot V$

Hence $p_1 \cdot V_1 = p_2 \cdot V_2$ or $V_2 = V_1 \cdot \frac{p_1}{p_2}$ or $D_2{}^3 = D_1{}^3 \cdot \frac{p_1}{p_2}$

Then the size of the bubble at the surface i $D_2 = D_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1}{3}} = D_1 \cdot \left[\frac{\left(p_{atm} + \rho_{sea} \cdot g \cdot h_1\right)}{p_{atm}}\right]^{\frac{1}{3}} = D_1 \cdot \left(1 + \frac{\rho_{sea} \cdot g \cdot h_1}{p_{atm}}\right)^{\frac{1}{3}}$

From Table A.2 $SG_{sea} = 1.025$ (This is at 68ºF)

$$D_2 = 0.3 \cdot in \times \left[1 + 1.025 \times 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \times \frac{ft}{s^2} \times 100 \cdot ft \times \frac{in^2}{14.7 \cdot lbf} \times \left(\frac{1 \cdot ft}{12 \cdot in}\right)^2 \times \frac{lbf \cdot s^2}{slug\, ft}\right]^{\frac{1}{3}}$$

$$D_2 = 0.477 \cdot in$$

Problem 3.12

[4]

Given: Model behavior of seawater by assuming constant bulk modulus

Find: (a) expression density as a function of depth, h.
(b) Show that result may be written as
$\rho \approx \rho_0 + bh$
(c) evaluate the constant b.
(d) use results of (b) to obtain equation for $p(h)$
(e) determine percent error in predicted pressure at $h = 1000\,m$

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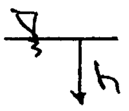
Solution: From Table A.2, App. A, $SG)_0 = 1.025$, $E_v = 2.42\ GN/m^2$

Basic equation: $\frac{dp}{dh} = \rho g$ Definition: $E_v = \frac{dp}{d\rho/\rho}$



Then, $dp = \rho g\, dh = E_v \frac{d\rho}{\rho}$ and $\frac{d\rho}{\rho^2} = \frac{g}{E_v} dh$

Integrating, $\int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = \int_0^h \frac{g}{E_v} dh$ and $-\frac{1}{\rho}\Big]_{\rho_0}^{\rho} = \frac{gh}{E_v}$

then, $\frac{gh}{E_v} = -\frac{1}{\rho} + \frac{1}{\rho_0} = \frac{-\rho_0 + \rho}{\rho\rho_0}$ or $\rho - \rho_0 = \rho\rho_0 \frac{gh}{E_v}$

$\therefore \rho\left(1 - \rho_0 \frac{gh}{E_v}\right) = \rho_0$ and $\frac{\rho}{\rho_0} = \frac{1}{\left\{1 - \frac{\rho_0 g h}{E_v}\right\}} \longleftarrow \rho(h)$

For $\frac{\rho_0 g}{E_v} \ll 1$, $\frac{\rho}{\rho_0} \approx 1 + \frac{\rho_0 g}{E_v} h$

Thus, $\rho \approx \rho_0 + \frac{\rho_0^2 g}{E_v} h = \rho_0 + bh$ where $b = \frac{\rho_0^2 g}{E_v} \longleftarrow$ Q.E.D.

Since $dp = \rho g\, dh$, then an approximate expression for $p(h)$ is

$$p - p_{atm} = \int_{p_{atm}}^{p} dp = \int_0^h (\rho_0 + bh) g\, dh = \left(\rho_0 h + \frac{bh^2}{2}\right) g$$

$$p_{approx} = p_{atm} + \left(\rho_0 h + \frac{\rho_0^2 g h^2}{E_v}\right) g = p_{atm} + \rho_0 h g \left[1 + \frac{\rho_0 g h}{E_v}\right] \longleftarrow p_{approx}$$

The exact solution for $p(h)$ is obtained by utilizing the exact equation for $\rho(h)$. Thus,

$$p - p_{atm} = \int_{p_{atm}}^{p} dp = \int_{\rho_0}^{\rho} E_v \frac{d\rho}{\rho} = E_v \ln \frac{\rho}{\rho_0}$$

$$p = p_{atm} + E_v \ln \left\{1 - \frac{\rho_0 g h}{E_v}\right\}^{-1} \longleftarrow - - - - - - - p_{exact}$$

$$\frac{\rho_0 g h}{E_v} = (1.025) 1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 10^3 m \times \frac{m^2}{2.42 \times 10^9 N} \times \frac{N \cdot s^2}{kg \cdot m} = 4.16 \times 10^{-3}$$

Substituting numerical values, $p_{approx} = p_{atm} + 9.851\ MPa$
$p_{exact} = p_{atm} + 10.076\ MPa$

$$\text{error} = \frac{p_{exact} - p_{app}}{p_{exact}} = \frac{10.076 - 9.851}{10.076} = 0.0224 = 2.24\% \longleftarrow \text{error}$$

Problem 3.13

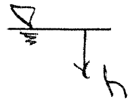
Given: Behavior of seawater to be modeled by assuming constant bulk modulus

Find: The percent deviations in (a) density, and (b) pressure, at depth h = 10 km, as compared to values obtained assuming constant density.

Plot: the results over range of $0 \le h \le 10$ km

Solution

Basic equation: $\frac{dp}{dh} = \rho g$ Definition: $E_v = \frac{dp}{d\rho/\rho}$



Then, $dp = \rho g\, dh = \frac{d\rho}{\rho} E_v$ and $\int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2} = \int_0^h \frac{g\,dh}{E_v}$

We obtain

$$-\frac{1}{\rho}\Big]_{\rho_0}^{\rho} = -\frac{1}{\rho}+\frac{1}{\rho_0} = \frac{-\rho_0+\rho}{\rho\rho_0} = \frac{gh}{E_v} \quad \text{or} \quad \rho-\rho_0 = \rho\rho_0\frac{gh}{E_v}$$

Then

$$\rho\left(1-\frac{\rho_0 gh}{E_v}\right) = \rho_0 \quad \text{and} \quad \frac{\rho}{\rho_0} = \frac{1}{\left(1-\frac{\rho_0 gh}{E_v}\right)}$$

Finally, $\frac{\Delta\rho}{\rho_0} = \frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0}-1 = \frac{\rho_0 gh/E_v}{(1-\rho_0 gh/E_v)}$ ---- (1)

To determine an expression for the percent deviation in pressure we write $\int_{p_{atm}}^{p} dp = E_v \int_{\rho_0}^{\rho}\frac{d\rho}{\rho}$

Then $p - p_{atm} = E_v \ln \rho/\rho_0$

For ρ = constant, $\int_{p_{atm}}^{p} dp = \rho_0 g\int_0^h dh$ and $p_{\rho=c} - p_{atm} = \rho_0 gh$

Then $\frac{p-p_{\rho=c}}{p_{\rho=c}} = \frac{\Delta p}{p_{\rho=c}} = \frac{E_v \ln \rho/\rho_0 - \rho_0 gh}{\rho_0 gh} = \frac{E_v}{\rho_0 gh}\ln\frac{\rho}{\rho_0} - 1$ — (2)

From Table A.2 for seawater SG = 1.025, $E_v = 2.42\ \mathrm{GN/m^2}$. Then

$$\frac{E_v}{\rho_0 g} = 2.42\times10^{9}\,\frac{N}{m^2}\times\frac{1}{(1000)(1.025)}\frac{m^3}{kg}\times\frac{1}{9.81}\frac{s^2}{m}\times\frac{kg\cdot m}{N\cdot s^2}\times\frac{km}{10^3\,m} = 240.7\ \mathrm{km}$$

Substituting into eqs (1) and (2)

$$\frac{\Delta\rho}{\rho_0} = \frac{4.155\times10^{-3}h}{1-4.155\times10^{-3}h} \quad \text{....(1a)}$$

$$\frac{\Delta p}{p_0} = \frac{240.7}{h}\ln\left[\frac{1}{1-4.155\times10^{-3}h}\right]-1 \quad \text{.... (2a)}$$

At h = 10 km, $\frac{\Delta\rho}{\rho_0} = 0.0434$ or 4.34% ________ $\frac{\Delta\rho}{\rho}\Big)_{h=10\,km}$

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Problem 3.13

$\frac{\Delta p}{p_o} = 0.0215$ or $2.15\,\%$ $\longleftarrow$ $\left.\frac{\Delta p}{p_o}\right)_{h=10\,km}$

Both $\Delta\rho/\rho_o$ and $\Delta p/p_o$ are plotted as a function of depth h (in km) below.

The computing equations are

$$\Delta\rho/\rho_o = \frac{\rho_o g h / E_v}{(1 - \rho_o g h / E_v}$$

$$\Delta p/p_o = \frac{E_v}{\rho_o g h} \ln \frac{\rho}{\rho_o} - 1$$

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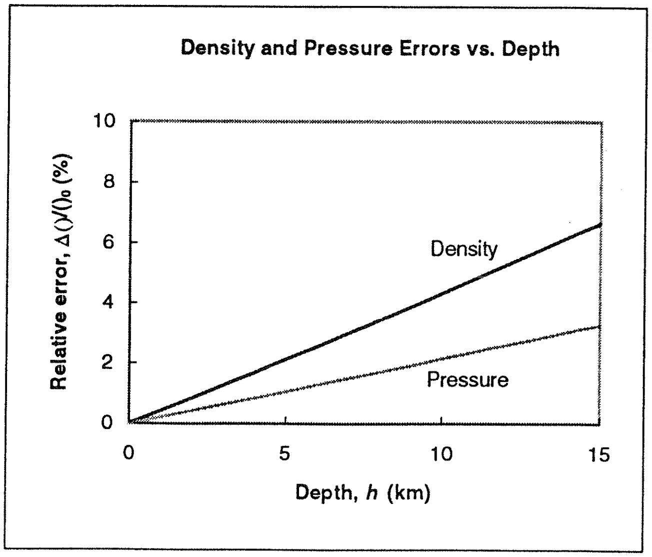
Density and pressure variation of seawater:

E_v = 2.42 GN/m^2 Bulk modulus of seawater

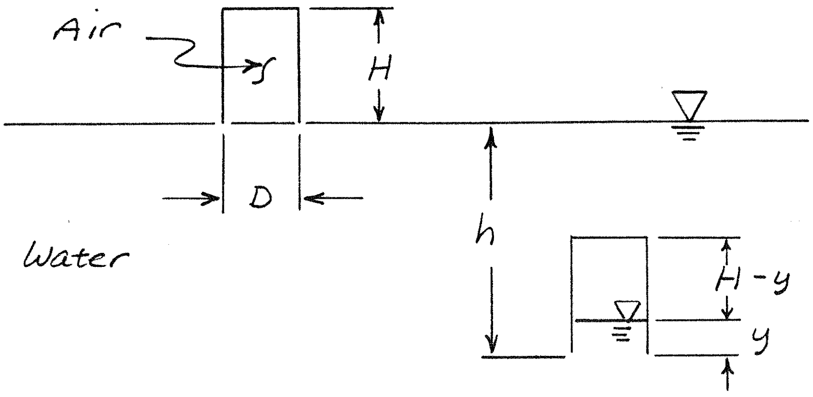
Depth, h (km)	Density Error, $\Delta\rho/\rho_0$ (---)	Pressure Error, $\Delta p/p_0$ (---)
0	0	0
1	0.417	0.219
2	0.838	0.429
3	1.26	0.639
4	1.69	0.851
5	2.12	1.06
6	2.56	1.28
7	3.00	1.49
8	3.44	1.71
9	3.88	1.93
10	4.34	2.15
11	4.79	2.37
12	5.25	2.59
13	5.71	2.81
14	6.18	3.04
15	6.65	3.26



Problem 3.14

[3]

Given: Cylindrical cup lowered slowly beneath pool surface.



Find: Expression for y in terms of h and H. Plot: y/H vs. h/H.

Solution: Apply ideal gas and hydrostatic equations.

Basic equations: $p\forall = mRT \qquad \frac{dp}{dh} = \rho g$

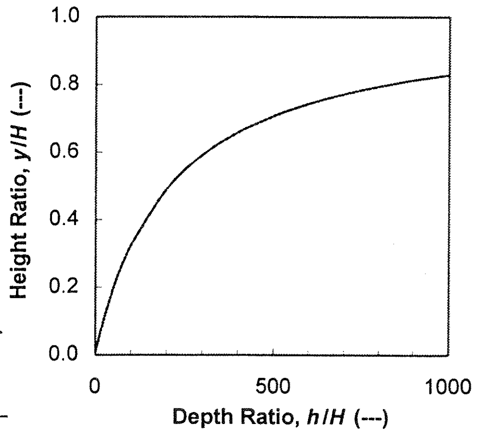
Assumptions: (1) T = constant
(2) Static liquid
(3) Incompressible liquid

Using (1), $p\forall = p_a \frac{\pi D^2}{4} H = p \frac{\pi D^2}{4}(H-y)$; or $\quad p_a H = p(H-y)$

Integrating $\frac{dp}{dh} = \rho g$ gives $p - p_a = \rho g (h-y)$ in container.

Thus

$$p_a H = [p_a + \rho g (h-y)](H-y) = p_a H - p_a y + \rho g (h-y)(H-y)$$



Expanding,

$$0 = \rho g h H - \rho g h y - \rho g y H + \rho g y^2 - p_a y$$

or

$$0 = hH - \left[(h+H) + \frac{p_a}{\rho g}\right] y + y^2$$

Using the quadratic equation

$$y = \frac{h + H + p_a/\rho g - \sqrt{\left[h + H + \frac{p_a}{\rho g}\right]^2 - 4hH}}{2}$$

y

(Note $y \leq H$, so the minus sign must be used.) In terms of y/H, this becomes

$$\frac{y}{H} = \frac{\frac{h}{H} + 1 + \frac{p_a}{\rho g H} - \sqrt{\left[\frac{h}{H} + 1 + \frac{p_a}{\rho g H}\right]^2 - 4\frac{h}{H}}}{2}$$

y/H

(see plot above.)

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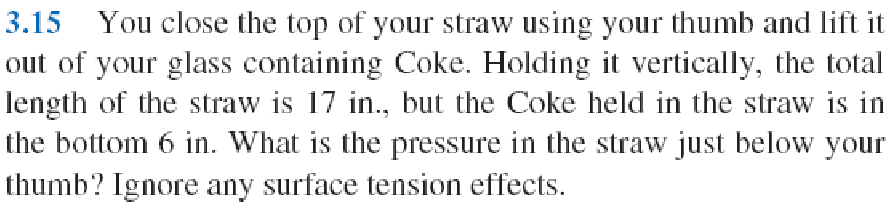
Problem 3.15

[1]



3.15 You close the top of your straw using your thumb and lift it out of your glass containing Coke. Holding it vertically, the total length of the straw is 17 in., but the Coke held in the straw is in the bottom 6 in. What is the pressure in the straw just below your thumb? Ignore any surface tension effects.

Given: Geometry of straw

Find: Pressure just below the thumb

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot h$ where h is measured downwards

This equation only applies in the 6 in of coke in the straw - in the other 11 inches of air the pressure is essentially constant.

The gage pressure at the coke surface is $p_{coke} = \rho \cdot g \cdot h_{coke}$ assuming coke is about as dense as water (it's actually a bit dens

Hence, with $h_{coke} = -6 \cdot in$ because h is measured downwards

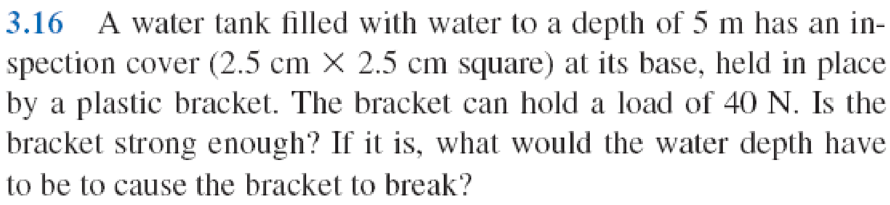
$$p_{coke} = -1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times 6 \cdot in \times \frac{1 \cdot ft}{12 \cdot in} \times \frac{lbf \cdot s^2}{slug\,ft}$$

$p_{coke} = -31.2 \cdot \frac{lbf}{ft^2}$ $p_{coke} = -0.217 \cdot psi$ gage

$p_{coke} = 14.5 \cdot psi$

Problem 3.16

[2]



3.16 A water tank filled with water to a depth of 5 m has an inspection cover (2.5 cm × 2.5 cm square) at its base, held in place by a plastic bracket. The bracket can hold a load of 40 N. Is the bracket strong enough? If it is, what would the water depth have to be to cause the bracket to break?

Given: Data on water tank and inspection cover

Find: If the support bracket is strong enough; at what water depth would it fail

Solution:

Basic equation $\frac{dp}{dy} = -\rho\cdot g$ or, for constant ρ $\Delta p = \rho\cdot g\cdot h$ where h is measured downwards

The absolute pressure at the base is $p_{base} = p_{atm} + \rho\cdot g\cdot h$ where $h = 5\cdot m$

The gage pressure at the base is $p_{base} = \rho\cdot g\cdot h$ This is the pressure to use as we have p_{atm} on the outside of the cover.

The force on the inspection cover is $F = p_{base}\cdot A$ where $A = 2.5\cdot cm \times 2.5\cdot cm$ $A = 6.25 \times 10^{-4} m^2$

$$F = \rho\cdot g\cdot h\cdot A$$

$$F = 1000\cdot\frac{kg}{m^3} \times 9.81\cdot\frac{m}{s^2} \times 5\cdot m \times 6.25 \times 10^{-4}\cdot m^2 \times \frac{N\cdot s^2}{kg\cdot m}$$

$$F = 30.7\,N$$

The bracket is strong enough (it can take 40 N). To find the maximum depth we start wi $F = 40\cdot N$

$$h = \frac{F}{\rho\cdot g\cdot A}$$

$$h = 40\cdot N \times \frac{1}{1000}\cdot\frac{m^3}{kg} \times \frac{1}{9.81}\cdot\frac{s^2}{m} \times \frac{1}{6.25 \times 10^{-4}}\cdot\frac{1}{m^2} \times \frac{kg\cdot m}{N\cdot s^2}$$

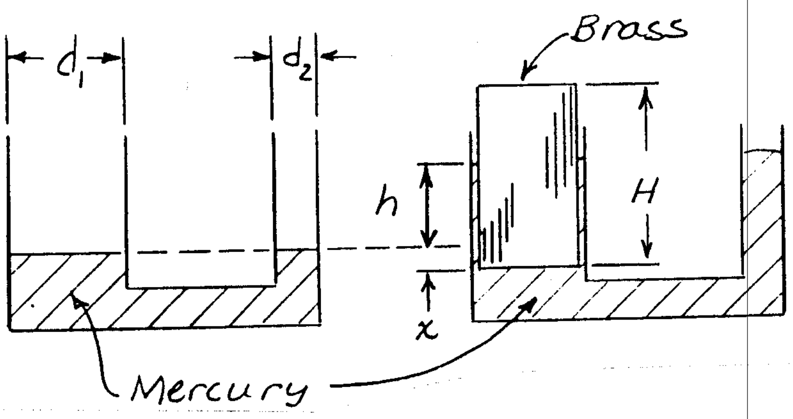
$$h = 6.52\,m$$

Problem 3.17

[4]

Given: Container of mercury with vertical tubes $d_1 = 39.5\ mm$ and $d_2 = 12.7\ mm$.

Brass cylinder with $D = 37.5\ mm$ and $H = 76.2\ mm$ is introduced into larger tube, where it floats.



Find: (a) Pressure on bottom of cylinder.
(b) New equilibrium level, h, of mercury.

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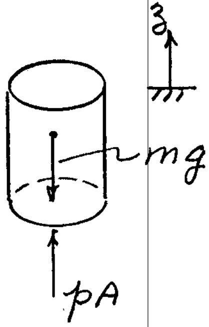
Solution: Analyze free-body diagram of cylinder, apply hydrostatics.

Computing equations: $\Sigma F_z = 0\,;\ \frac{dp}{dz} = -\rho g\,;\ \rho = SG\,\rho_{H_2O}$

Assumptions: (1) Static liquid
(2) Incompressible liquid



For the cylinder $\Sigma F_z = p\frac{\pi D^2}{4} - \rho_{brass}\, g\frac{\pi D^2}{4}H = 0$

Thus $p = \rho_{brass}\, g H = SG_{brass}\,\rho_{H_2O}\, g H$

From Table A.1, $SG_{brass} = 8.55$ at 20°C, so

$$p = 8.55 \times 1000\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 0.0762\ m \times \frac{N\cdot s^2}{kg\cdot m} = 6.39\ kPa\ (gage)$$

This pressure must be produced by a column of mercury $h+x$ in height. Thus, using SG_{Hg} from Table A.1,

$$p = \rho_{Hg}\, g(h+x) = SG_{Hg}\,\rho_{H_2O}\, g\,(h+x) = SG_{brass}\,\rho_{H_2O}\, g\, H$$

Thus $h + x = \frac{SG_{brass}}{SG_{Hg}}H = \frac{8.55}{13.55}H = 0.631\ H$ (1)

But the volume of mercury must remain constant. Therefore

$$\frac{\pi D^2}{4}x = \frac{\pi(d_1^2 - D^2)}{4}h + \frac{\pi d_2^2}{4}h \quad \text{or} \quad x\left[\left(\frac{d_1}{D}\right)^2 - 1 + \left(\frac{d_2}{D}\right)^2\right] = 0.224\ h$$

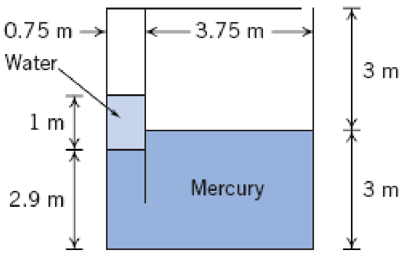
Substituting into Eq. 1,

$$h + x = h + 0.224\,h = 1.224\,h = 0.631\ H \quad \text{or} \quad h = \frac{0.631}{1.224}H = 0.516\ H$$

h = 39.3 mm

Problem 3.18

3.18 A partitioned tank as shown contains water and mercury. What is the gage pressure in the air trapped in the left chamber? What pressure would the air on the left need to be pumped to in order to bring the water and mercury free surfaces level?



Given: Data on partitioned tank

Find: Gage pressure of trapped air; pressure to make water and mercury levels equal

Solution:

The pressure difference is obtained from repeated application of Eq. 3.7, or in other words, from Eq. 3.8. Starting from the right air chamber

$$p_{gage} = SG_{Hg} \times \rho_{H2O} \times g \times (3 \cdot m - 2.9 \cdot m) - \rho_{H2O} \times g \times 1 \cdot m$$

$$p_{gage} = \rho_{H2O} \times g \times \left(SG_{Hg} \times 0.1 \cdot m - 1.0 \cdot m\right)$$

$$p_{gage} = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (13.55 \times 0.1 \cdot m - 1.0 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{gage} = 3.48 \cdot kPa$$

If the left air pressure is now increased until the water and mercury levels are now equal, Eq. 3.8 leads to

$$p_{gage} = SG_{Hg} \times \rho_{H2O} \times g \times 1.0 \cdot m - \rho_{H2O} \times g \times 1.0 \cdot m$$

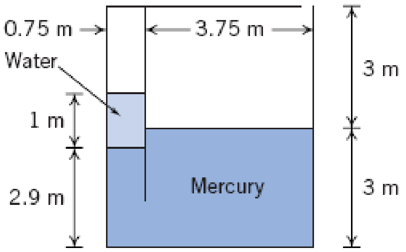
$$p_{gage} = \rho_{H2O} \times g \times \left(SG_{Hg} \times 1 \cdot m - 1.0 \cdot m\right)$$

$$p_{gage} = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (13.55 \times 1 \cdot m - 1.0 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{gage} = 123 \cdot kPa$$

Problem 3.19

[2]

3.19 In the tank of Problem 3.18, if the opening to atmosphere on the right chamber is first sealed, what pressure would the air on the left now need to be pumped to in order to bring the water and mercury free surfaces level? (Assume the air trapped in the right chamber behaves isothermally.)



Given: Data on partitioned tank

Find: Pressure of trapped air required to bring water and mercury levels equal if right air opening is sealed

Solution:

First we need to determine how far each free surface moves.

In the tank of Problem 3.15, the ratio of cross section areas of the partitions is 0.75/3.75 or 1:5. Suppose the water surface (and therefore the mercury on the left) must move down distance *x* to bring the water and mercury levels equal. Then by mercury volume conservation, the mercury free surface (on the right) moves up (0.75/3.75)*x* = *x*/5. These two changes in level must cancel the original discrepancy in free surface levels, of (1m + 2.9m) - 3 m = 0.9 m. Hence *x* + *x*/5 = 0.9 m, or *x* = 0.75 m. The mercury level thus moves up *x*/5 = 0.15 m.

Assuming the air (an ideal gas, *pV*=*RT*) in the right behaves isothermally, the new pressure there will be

$$p_{right} = \frac{V_{rightold}}{V_{rightnew}} \cdot p_{atm} = \frac{A_{right} \cdot L_{rightold}}{A_{right} \cdot L_{rightnew}} \cdot p_{atm} = \frac{L_{rightold}}{L_{rightnew}} \cdot p_{atm}$$

where *V*, *A* and *L* represent volume, cross-section area, and vertical length
Hence

$$p_{right} = \frac{3}{3 - 0.15} \times 101 \cdot kPa \qquad p_{right} = 106\,kPa$$

When the water and mercury levels are equal application of Eq. 3.8 gives:

$$p_{left} = p_{right} + SG_{Hg} \times \rho_{H2O} \times g \times 1.0 \cdot m - \rho_{H2O} \times g \times 1.0 \cdot m$$

$$p_{left} = p_{right} + \rho_{H2O} \times g \times \left(SG_{Hg} \times 1.0 \cdot m - 1.0 \cdot m\right)$$

$$p_{left} = 106 \cdot kPa + 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (13.55 \cdot 1.0 \cdot m - 1.0 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{left} = 229\,kPa$$

$$p_{gage} = p_{left} - p_{atm} \qquad p_{gage} = 229 \cdot kPa - 101 \cdot kPa \qquad p_{gage} = 128\,kPa$$

Problem 3.20

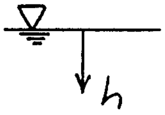
[2]

Given: U-tube manometer, partially filled with water, then $\forall_{oil} = 3.25\ cm^3$ of Meriam red oil is added to the left side.

Find: Equilibrium height, H, when both legs are open to atmosphere.

Solution: Apply basic pressure-height relation.

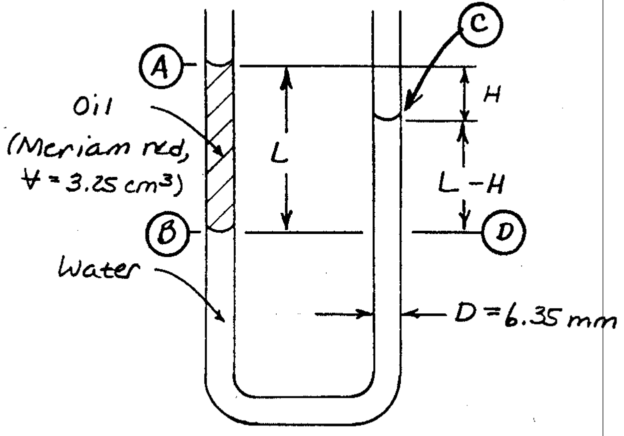
Basic equation: $\frac{dp}{dh} = +\rho g$



Assumptions: (1) Incompressible liquid
(2) h measured down

Integration gives

$$p_2 - p_1 = \rho g (h_2 - h_1)$$



Thus

$$p_B = p_A + \rho_{oil}\, g\, L$$

$$p_D = p_C + \rho_{water}\, g (L - H)$$

Since $p_A = p_C = p_{atm}$, then

$$\rho_{oil}\, g\, L = \rho_{water}\, g\, (L - H)$$

or

$$SG_{oil}\, L = L - H$$

Thus

$$H = L\,(1 - SG_{oil})$$

From the volume of oil, $\forall = \frac{\pi D^2}{4} L$, so

$$L = \frac{4\forall}{\pi D^2} = \frac{4}{\pi} \times 3.25\ cm^3 \times \frac{1}{(6.35)^2 mm^2} \times (10)^3 \frac{mm^3}{cm^3} = 103\ mm$$

Finally, since $SG = 0.827$ (Table A.1, Appendix A), then

$$H = 103\ mm\,(1 - 0.827) = 17.8\ mm$$

H

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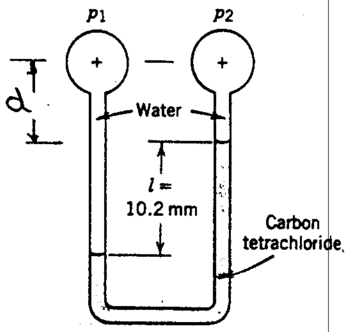
Problem 3.21

[2]

Given: Two-fluid manometer shown.

Find: Pressure difference, $p_1 - p_2$



Solution:

Basic equation: $\frac{dp}{dh} = \rho g$ $\quad \nabla\ \downarrow h$

Assumptions: (1) static liquid
(2) incompressible
(3) $g =$ constant

Then, $dp = \rho g\, dh$ and $\Delta p = \rho g \Delta h$

Starting at point ① and progressing to point ② we have

$$p_1 + \rho_{H_2O}\, g(d+l) - \rho_{c.t.}\, g l - \rho_{H_2O}\, g d = p_2$$

$$\therefore p_1 - p_2 = \rho_{ct}\, g l - \rho_{H_2O}\, g l = SG_{ct}\, \rho_{H_2O}\, g l - \rho_{H_2O}\, g l$$

$$p_1 - p_2 = \rho_{H_2O}\, g l\, (SG_{ct} - 1)$$

From Table A.2, Appendix A, $SG_{ct} = 1.595$

$$\therefore p_1 - p_2 = 1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 10.2\,mm \times \frac{m}{10^3\,mm} (1.595 - 1) \frac{N \cdot s^2}{kg \cdot m}$$

$$p_1 - p_2 = 59.5\ N/m^2 \longleftarrow \quad p_1 - p_2$$

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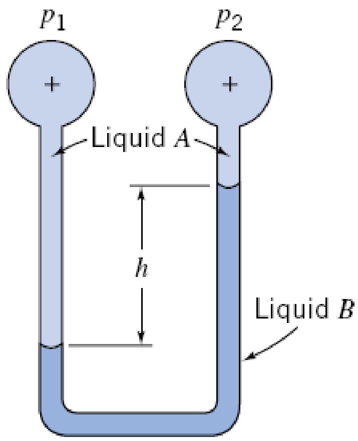
Problem 3.22

[2]

3.22 The manometer shown contains two liquids. Liquid *A* has SG = 0.88 and liquid *B* has SG = 2.95. Calculate the deflection, h, when the applied pressure difference is $p_1 - p_2 = 18\ \text{lbf/ft}^2$.



Given: Data on manometer

Find: Deflection due to pressure difference

Solution:

Basic equation $\dfrac{dp}{dy} = -\rho\cdot g$ or, for constant ρ $\Delta p = \rho\cdot g\cdot\Delta h$ where h is measured downwards

Starting at p_1 $p_A = p_1 + SG_A\cdot\rho\cdot g\cdot(h + l)$ where l is the (unknown) distance from the level of the right interface

Next, from A to B $p_B = p_A - SG_B\cdot\rho\cdot g\cdot h$

Finally, from A to the location of p_2 $p_2 = p_B - SG_A\cdot\rho\cdot g\cdot l$

Combining the three equations $p_2 = \left(p_A - SG_B\cdot\rho\cdot g\cdot h\right) - SG_A\cdot\rho\cdot g\cdot l = \left[p_1 + SG_A\cdot\rho\cdot g\cdot(h + l) - SG_B\cdot\rho\cdot g\cdot h\right] - SG_A\cdot\rho\cdot g\cdot l$

$$p_2 - p_1 = \left(SG_A - SG_B\right)\cdot\rho\cdot g\cdot h$$

$$h = \frac{p_1 - p_2}{\left(SG_B - SG_A\right)\cdot\rho\cdot g}$$

$$h = 18\cdot\frac{\text{lbf}}{\text{ft}^2}\times\frac{1}{(2.95 - 0.88)}\times\frac{1}{1.94}\cdot\frac{\text{ft}^3}{\text{slug}}\times\frac{1}{32.2}\cdot\frac{\text{s}^2}{\text{ft}}\times\frac{\text{slug}\cdot\text{ft}}{\text{s}^2\cdot\text{lbf}}$$

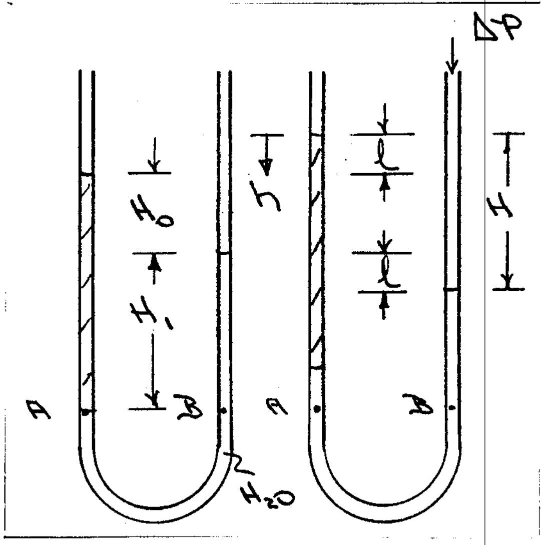
$h = 0.139\cdot\text{ft}$ $h = 1.67\cdot\text{in}$

Problem 3.23

[2]

Given: Two fluid manometer contains water and kerosene. With both tubes open to atmosphere, the free surface elevations differ by $H_o = 20.0\,mm$

Find: Elevation difference, H, between free-surface of fluids when a gage pressure of 98.0 Pa is applied to the right tube.



Solution:

Basic equation: $\frac{dp}{dh} = \rho g$; $\Delta p = \rho g \Delta h$

Assumptions: (1) static fluid
(2) gravity is the only body force

When the gage pressure $\Delta p = 98.0\,Pa$ is applied to the right tube, the water in the right tube is displaced downward a distance, l; the kerosene in the left tube is displaced upward the same distance, l.

Under the applied gage pressure, Δp, the elevation difference, H, is

$$H = H_o + 2l$$

Since points A & B are at the same elevation in the same fluid $p_A = p_B$.

Initially (left diagram), $p_A = \rho_k g (H_o + H_1)$, $p_B = \rho g H_1$ and hence

$$\rho_k g (H_o + H_1) = \rho g H_1$$

or

$$H_1 = \frac{\rho_k H_o}{\rho - \rho_k} = \frac{SG_k H_o}{(1 - SG_k)}$$. From table A.2, $SG_k = 0.82$

$$\therefore H_1 = \frac{0.82}{(1-0.82)} 20\,mm = 91.1\,mm \leftarrow --$$

Under the applied pressure Δp (right diagram),

$$p_A = \rho_k g (H_o + H_1) + \rho g l \quad , \quad p_B = \Delta p + \rho g (H_1 - l).$$

$$\therefore SG_k (H_o + H_1) + l = \frac{\Delta p}{\rho g} + (H_1 - l)$$

Solving for l.

$$l = \frac{1}{2}\left[H_1 + \frac{\Delta p}{\rho g} - SG_k (H_o + H_1) \right]$$

$$= \frac{1}{2}\left[91.1\,mm + \frac{98\,N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{s^2}{9.81\,m} \times \frac{kg \cdot m}{N \cdot s^2} \times \frac{10^3\,mm}{m} - 0.82(20 + 91.1)\,mm \right]$$

$$l = 5\,mm$$

$$H = H_o + 2l = 30\,mm \longleftarrow$$

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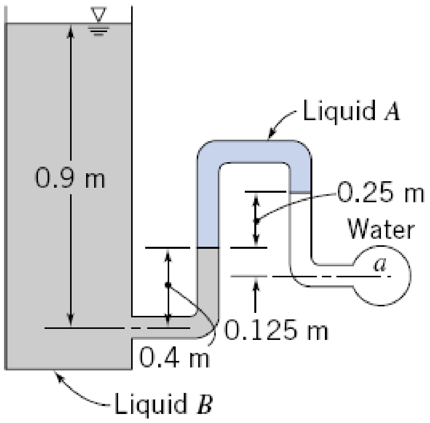
Problem 3.24

3.24 Determine the gage pressure in psig at point *a*, if liquid *A* has SG = 0.75 and liquid *B* has SG = 1.20. The liquid surrounding point *a* is water and the tank on the left is open to the atmosphere.



Given: Data on manometer

Find: Gage pressure at point a

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot \Delta h$ where Δh is height difference

Starting at point a $p_1 = p_a - \rho \cdot g \cdot h_1$ where $h_1 = 0.125 \cdot m + 0.25 \cdot m$ $h_1 = 0.375\,m$

Next, in liquid A $p_2 = p_1 + SG_A \cdot \rho \cdot g \cdot h_2$ where $h_2 = 0.25 \cdot m$

Finally, in liquid B $p_{atm} = p_2 - SG_B \cdot \rho \cdot g \cdot h_3$ where $h_3 = 0.9 \cdot m - 0.4 \cdot m$ $h_3 = 0.5\,m$

Combining the three equations $p_{atm} = \left(p_1 + SG_A \cdot \rho \cdot g \cdot h_2\right) - SG_B \cdot \rho \cdot g \cdot h_3 = p_a - \rho \cdot g \cdot h_1 + SG_A \cdot \rho \cdot g \cdot h_2 - SG_B \cdot \rho \cdot g \cdot h_3$

$$p_a = p_{atm} + \rho \cdot g \cdot \left(h_1 - SG_A \cdot h_2 + SG_B \cdot h_3\right)$$

or in gage pressures $p_a = \rho \cdot g \cdot \left(h_1 - SG_A \cdot h_2 + SG_B \cdot h_3\right)$

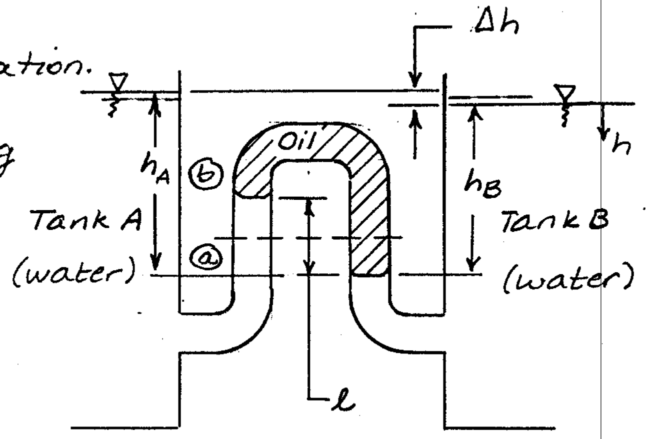
$$p_a = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times [0.375 - (0.75 \times 0.25) + (1.20 \times 0.5)] \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$

$p_a = 7.73 \times 10^3\,Pa$ $p_a = 7.73 \cdot kPa$ (gage)

Problem 3.25

Given: Two-fluid manometer; oil is second fluid.

Find: SG needed for 10 to 1 amplification.



Solution: Basic equation $\frac{dp}{dz} = -\rho g$

Assumptions: (1) Static liquid
(2) Incompressible

Then $dp = \rho g dh$

$$p = p_0 + \rho g h$$

For left leg, $p_a = p_{atm} + \rho_{H_2O} g h_A$

$$p_b = p_a - \rho_{H_2O} g \ell = p_{atm} + \rho_{H_2O} g (h_A - \ell) \qquad (1)$$

For right leg, $p_a = p_{atm} + \rho_{H_2O} g h_B$

$$p_b = p_a - SG_{oil} \rho_{H_2O} g \ell = p_{atm} + \rho_{H_2O} g (h_B - SG_{oil} \ell) \qquad (2)$$

Combining,

$$\cancel{p_{atm}} + \rho_{H_2O} g (h_A - \ell) = \cancel{p_{atm}} + \rho_{H_2O} g (h_B - SG_{oil} \ell)$$

or

$$h_A - \ell = h_B - SG_{oil} \ell \;;\; h_A - h_B = \Delta h = \ell (1 - SG_{oil})$$

Finally

$$SG_{oil} = 1 - \frac{\Delta h}{\ell} = 1 - \frac{1}{10} = 0.900 \longleftarrow$$

SG

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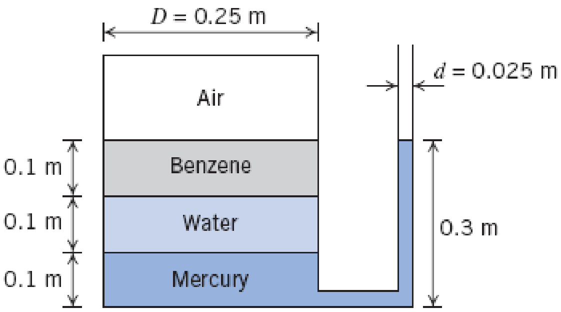
Problem 3.26

[2]

3.26 Consider a tank containing mercury, water, benzene, and air as shown. Find the air pressure (gage). If an opening is made in the top of the tank, find the equilibrium level of the mercury in the manometer.



Given:
Data on fluid levels in a tank

Find:
Air pressure; new equilibrium level if opening appears

Solution:

Using Eq. 3.8, starting from the open side and working in gage pressure

$$p_{air} = \rho_{H2O} \times g \times \left[SG_{Hg} \times (0.3 - 0.1) \cdot m - 0.1 \cdot m - SG_{Benzene} \times 0.1 \cdot m \right]$$

Using data from Table A.2 $\quad p_{air} = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (13.55 \times 0.2 \cdot m - 0.1 \cdot m - 0.879 \times 0.1 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{air} = 24.7 \cdot kPa$

To compute the new level of mercury in the manometer, assume the change in level from 0.3 m is an increase of x. Then, because the volume of mercury is constant, the tank mercury level will fall by distance $(0.025/0.25)^2 x$. Hence, the gage pressure at the bottom of the tank can be computed from the left and the right, providing a formula for x

$$SG_{Hg} \times \rho_{H2O} \times g \times (0.3 \cdot m + x) = SG_{Hg} \times \rho_{H2O} \times g \times \left[0.1 \cdot m - x \cdot \left(\frac{0.025}{0.25} \right)^2 \right] \cdot m \ldots$$
$$+ \rho_{H2O} \times g \times 0.1 \cdot m + SG_{Benzene} \times \rho_{H2O} \times g \times 0.1 \cdot m$$

Hence $\quad x = \dfrac{[0.1 \cdot m + 0.879 \times 0.1 \cdot m + 13.55 \times (0.1 - 0.3) \cdot m]}{\left[1 + \left(\frac{0.025}{0.25} \right)^2 \right] \times 13.55} \qquad x = -0.184 m$

(The negative sign indicates the manometer level actually fell)

The new manometer height is $\quad h = 0.3 \cdot m + x \qquad h = 0.116 m$

Problem 3.27

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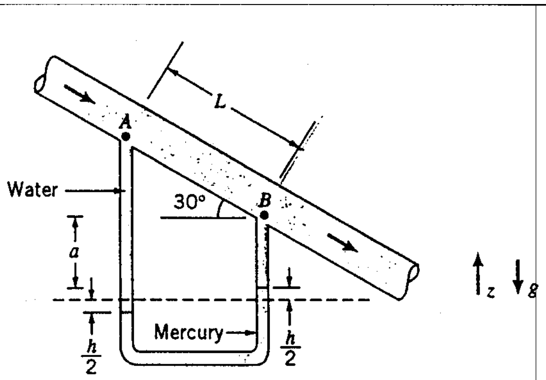


Given: Water flow in an inclined pipe as shown. Pressure difference, $p_A - p_B$, measured with two-fluid manometer
$L = 5\,ft$, $h = 6\,in.$

Find: Pressure difference, $p_A - p_B$.

Solution:

Basic equation: $\frac{dp}{dh} = \rho g$ where h is measured positive down

Assumptions: (1) static liquid
(2) incompressible
(3) g = constant

Then,

$$dp = \rho g\, dh \quad \text{and} \quad \Delta p = \rho g \Delta h$$

Start at p_A and progress through manometer to p_B

$$p_A + \rho_{H_2O} g L \sin 30° + \cancel{\rho_{H_2O} g} a + \rho_{H_2O} g h - \rho_{Hg} g h - \cancel{\rho_{H_2O} g} a = p_B$$

$$p_A - p_B = \rho_{Hg} g h - \rho_{H_2O} g h - \rho_{H_2O} g L \sin 30°$$

$$= SG_{Hg} \rho_{H_2O} g h - \rho_{H_2O} g h - \rho_{H_2O} g L \sin 30°$$

$$p_A - p_B = \rho_{H_2O}\, g \left[h (SG_{Hg} - 1) - L \sin 30° \right]$$

From Table A.2, $SG_{Hg} = 13.55$

Then,

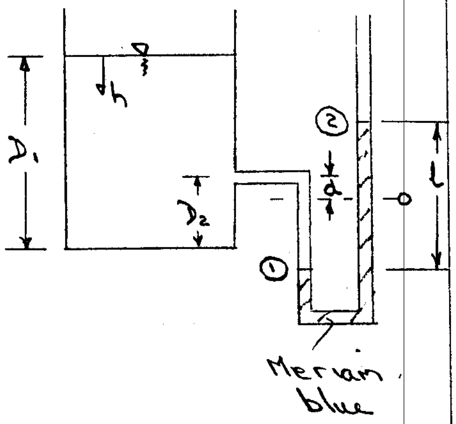
$$p_A - p_B = 1.94 \frac{slug}{ft^3} \times 32.2 \frac{ft}{s^2} \left[0.5\,ft\,(13.55 - 1) - 5\,ft \sin 30° \right] \times \frac{lbf \cdot s^2}{ft \cdot slug}$$

$$p_A - p_B = 236\ lbf/ft^2 \quad (1.64\ psi)$$ ______ $p_A - p_B$

Problem 3.28

[2]

Given: A U-tube manometer is connected to the open tank filled with water as shown (manometer fluid is Merian blue)



$D_1 = 2.5\,m$, $D_2 = 0.7\,m$, $d = 0.2\,m$

Find: The manometer deflection, l.

Solution:

Basic equation: $\frac{dP}{dh} = \rho g$

For γ = constant $\Delta P = \rho g \Delta h$

Then, beginning at the free surface and accounting for the changes in pressure with elevation,

$$P_{atm} + (P_1 - P_{atm}) + (P_2 - P_1) = P_2 = P_{atm}$$

$$\rho_{H_2O}\cancel{g}\left[(D_1 - D_2) + d + \frac{l}{2}\right] - \rho_{mb}\cancel{g}\,l = 0$$

$$(D_1 - D_2) + d + \frac{l}{2} = \frac{\rho_{mb}}{\rho_{H_2O}}\,l = (S.G)_{mb}\,l$$

and $$l = \frac{(D_1 - D_2) + d}{\left[(S.G)_{mb} - \frac{1}{2}\right]}$$ (From Table A.1, Appendix A, SG = 1.75.)

$$l = \frac{(2.5 - 0.7)m + 0.2m}{(1.75 - 0.5)}$$

$$l = 1.6\,m \longleftarrow$$ l

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Problem 3.29

[2]

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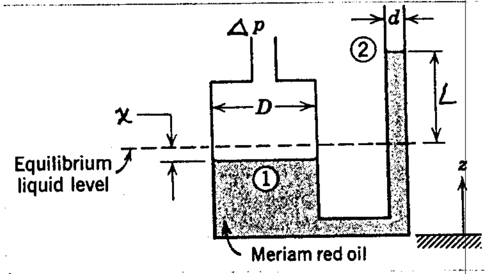
Given: Reservoir manometer with vertical tubes $D = 18$ mm and $d = 6$ mm diameter. Gage liquid is Meriam red oil.

Find: (a) Algebraic expression for deflection L in small tube when gage pressure Δp is applied to the reservoir.
(b) Evaluate L when Δp is equivalent to 25 mm H_2O (gage).

Solution: Use the diagram of Example Problem 3.2, apply hydrostatics.

Computing equations: $\frac{dp}{dh} = +\rho g$; $\Delta p = \rho g \Delta h$; $\rho = SG\,\rho_{H_2O}$



Assumptions: (1) Static liquid
(2) Incompressible liquid

Then $\Delta p = \rho_{oil}\, g\,(x + L)$

From conservation of volume,

$$\frac{\pi D^2}{4} x = \frac{\pi d^2}{4} L \;;\; x = \left(\frac{d}{D}\right)^2 L$$

so

$$\Delta p = \rho_{water}\, g\, \Delta h = \rho_{oil}\, g\left[\left(\frac{d}{D}\right)^2 L + L\right] = \rho_{oil}\, g L\left[1 + \left(\frac{d}{D}\right)^2\right]$$

Solving for L,

$$L = \frac{\Delta p}{\rho_{oil}\, g\left[1 + (d/D)^2\right]}$$ L

Substituting $\Delta p = \rho_{water}\, g\, \Delta h$,

$$L = \frac{\rho_{water}\, g\, \Delta h}{SG_{oil}\,\rho_{water}\, g\left[1 + (d/D)^2\right]} = \frac{\Delta h}{SG_{oil}\left[1 + (d/D)^2\right]}$$

Evaluating, with $SG_{oil} = 0.827$ (Table A.1),

$$L = \frac{25.0 \text{ mm}}{0.827\left[1 + (6/18)^2\right]} = 27.2 \text{ mm}$$ L

$\left\{ \text{Note: } s \equiv \frac{L}{\Delta h_e} = \frac{27.2 \text{ mm}}{25.0 \text{ mm}} = 1.09 \text{ for this manometer.} \right\}$

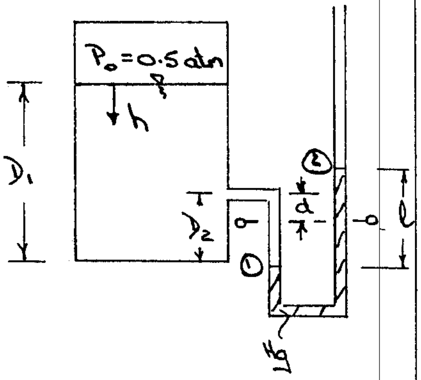
Problem 3.30

[2]

Given: A U-tube manometer is connected to a closed tank filled with water as shown. The manometer fluid is Hg.

$D_1 = 2.5\,m$, $D_2 = 0.7\,m$, $d = 0.2\,m$

At the water surface $P_0 = 0.5$ atm (gage)



Find: The manometer deflection l.

Solution

Basic equation $\quad \frac{dP}{dh} = \rho g$

For γ = constant $\quad \Delta P = \rho g \Delta h$

Then, beginning at the free surface and accounting for pressure changes with elevation,

$$P_0 + (P_1 - P_0) + (P_2 - P_1) = P_2 = P_{atm}$$

$$P_0 + \rho_{H_2O}\, g\left[(D_1 - D_2) + d + \frac{l}{2}\right] - \rho_{Hg}\, g\, l = P_{atm}$$

$$\frac{P_0 - P_{atm}}{\rho_{H_2O}\, g} + (D_1 - D_2) + d + \frac{l}{2} = \frac{\rho_{Hg}\, g}{\rho_{H_2O}\, g}\, l = (S.G)_{Hg}\, l$$

and

$$l = \frac{(P_0 - P_{atm})/\rho_{H_2O}\, g + (D_1 - D_2) + d}{(S.G)_{Hg} - 0.5}$$

$$= \frac{0.5\,atm \times 1.01 \times 10^5 \frac{N}{m^2 \cdot atm} \times \frac{m^3}{999\,kg} \times \frac{s^2}{9.81\,m} \times \frac{kg \cdot m}{N \cdot s^2} + (2.5 - 0.7)\,m + 0.2\,m}{13.6 - 0.5}$$

$l = 0.546\,m$ ⟵ l

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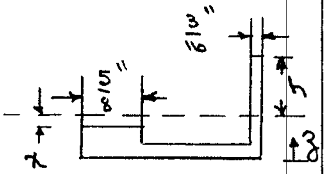
Problem 3.31

[2]

Given: Reservoir manometer with dimensions shown
Manometer fluid SG = 0.827

Find: required distance between marks on vertical scale for 1 in of water ΔP



Solution:

Basic equation: $\frac{dP}{dz} = -\gamma$

Assumptions: (1) static fluid
(2) gravity is only body force
(3) z axis directed vertically

$dP = -\gamma \, dz$

For constant γ, $\Delta P = P_1 - P_2 = -\gamma(z_1 - z_2)$

Under applied pressure $\Delta P = \gamma_{oil}(x+h)$

But conditions of problem require $\Delta P = \gamma_{H_2O}\, l$ where $l = 1$ in

$$\therefore \gamma_{oil}(x+h) = \gamma_{H_2O}\, l$$

Since the volume of the oil must remain constant

$$x A_{res} = h A_{tube}$$

$$\therefore x = h \frac{A_{tube}}{A_{res}}$$

and $\gamma_{oil}\left(h \frac{A_t}{A_r} + h\right) = \gamma_{H_2O}\, l$

$$\therefore \frac{h}{l} = \frac{\gamma_{H_2O}}{\gamma_{oil}} \left(\frac{1}{\frac{A_t}{A_r} + 1} \right) = \frac{1}{SG_{oil}\left[\left(\frac{D_t}{D_r}\right)^2 + 1\right]}$$

$$\frac{h}{l} = \frac{1}{0.827\left[\left(\frac{3}{16} \times \frac{8}{5}\right)^2 + 1\right]} = \frac{1}{0.827\left[(0.3)^2 + 1\right]}$$

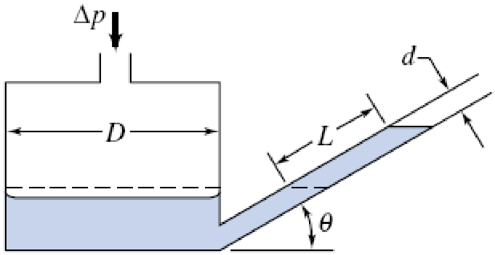
$$\frac{h}{l} = 1.11$$

For $l = 1.0$ in as given, then $h = 1.11$ in.

Problem 3.32

[3]

3.32 The inclined-tube manometer shown has $D = 76$ mm and $d = 8$ mm, and is filled with Meriam red oil. Compute the angle, θ, that will give a 15-cm oil deflection along the inclined tube for an applied pressure of 25 mm of water (gage). Determine the sensitivity of this manometer.



Given: Data on inclined manometer

Find: Angle θ for given data; find sensitivity

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot \Delta h$ where Δh is height difference

Under applied pressure $\Delta p = SG_{Mer} \cdot \rho \cdot g \cdot (L \cdot \sin(\theta) + x)$ (1)

From Table A.1 $SG_{Mer} = 0.827$

and Δp = 1 in. of water, or $\Delta p = \rho \cdot g \cdot h$ where $h = 25 \cdot mm$ $h = 0.025\,m$

$$\Delta p = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.025 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 245\,Pa$$

The volume of liquid must remain constant, so $x \cdot A_{res} = L \cdot A_{tube}$ $x = L \cdot \frac{A_{tube}}{A_{res}} = L \cdot \left(\frac{d}{D}\right)^2$ (2)

Combining Eqs 1 and 2 $\Delta p = SG_{Mer} \cdot \rho \cdot g \cdot \left[L \cdot \sin(\theta) + L \cdot \left(\frac{d}{D}\right)^2\right]$

Solving for θ $\sin(\theta) = \frac{\Delta p}{SG_{Mer} \cdot \rho \cdot g \cdot L} - \left(\frac{d}{D}\right)^2$

$$\sin(\theta) = 245 \cdot \frac{N}{m^2} \times \frac{1}{0.827} \times \frac{1}{1000} \cdot \frac{m^3}{kg} \times \frac{1}{9.81} \cdot \frac{s^2}{m} \times \frac{1}{0.15} \cdot \frac{1}{m} \times \frac{kg \cdot m}{s^2 \cdot N} - \left(\frac{8}{76}\right)^2 = 0.186$$

$\theta = 11 \cdot deg$

The sensitivity is the ratio of manometer deflection to a vertical water manometer

$$s = \frac{L}{h} = \frac{0.15 \cdot m}{0.025 \cdot m} \qquad s = 6$$

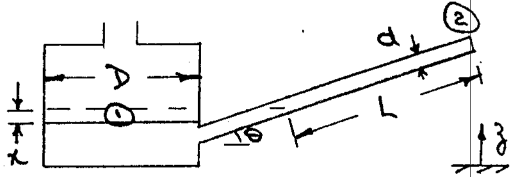
Problem 3.33

[3]

Given: Inclined manometer as shown

$D = 96\,\text{mm}$, $d = 8\,\text{mm}$

Angle θ is such that liquid deflection is five times that of U-tube manometer under same applied pressure difference



Find : angle , θ and manometer sensitivity

Solution:

Basic equation $\frac{dP}{dz} = -\rho g$

Then $dP = -\rho g\,dz$ and for constant ρ

$$\Delta P = P_1 - P_2 = -\rho g(z_1 - z_2)$$

For the inclined manometer,

$$P_1 - P_{atm} = \rho g\,(L\sin\theta + x)$$

Since the volume of the oil must remain constant,

$$x A_{res} = L A_{tube}$$

$$x = L\,\frac{A_{tube}}{A_{res}} = L\left(\frac{d}{D}\right)^2$$

Then

$$P_1 - P_{atm} = \rho g(L\sin\theta + x) = \rho g\left(L\sin\theta + L\left(\frac{d}{D}\right)^2\right) = \rho g L\left(\sin\theta + \left(\frac{d}{D}\right)^2\right)$$

For a U-tube manometer

$$P_1 - P_{atm} = -\rho g(z_1 - z_2) = \rho g h$$

Hence,

$$\frac{(P_1 - P_{atm})_{incl}}{(P_1 - P_{atm})_{U\text{-tube}}} = \frac{\rho g L\left[\sin\theta + \left(\frac{d}{D}\right)^2\right]}{\rho g h}$$

For same applied pressure and $L/h = 5$

$$1 = 5\left[\sin\theta + \left(\frac{d}{D}\right)^2\right]$$

$$\theta = \sin^{-1}\left[0.2 - \left(\frac{d}{D}\right)^2\right] = \sin^{-1}\left[0.2 - \left(\frac{8}{96}\right)^2\right] = 11.1^\circ \longleftarrow \theta$$

$s = L/\Delta h_e = L/(SG\ h) = 5/SG$

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Problem 3.34

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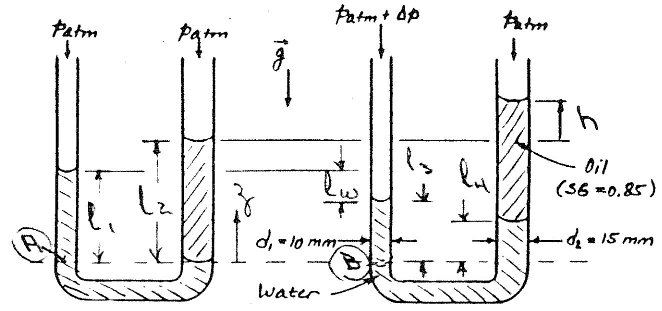
Given: U-tube manometer with tubes of different diameter and two liquids, as shown.



Find: (a) the deflection, h, for $\Delta p = 250\ \text{N/m}^2$.
(b) the sensitivity of the manometer.

Plot: the manometer sensitivity as a function of d_2/d_1

Solution:

Basic equation: $\dfrac{dp}{dz} = -\rho g$

Assumptions: (1) static liquid (2) incompressible

Integrating the basic equation from reference state at z_0 to general state at z gives

$$p - p_0 = -\rho g(z - z_0) = \rho g(z_0 - z)$$

From the left diagram: $p_A - p_{atm} = \rho_w g l_1 = \rho_o g l_2$ ......(1)

From the right diagram $p_B - (p_{atm} + \Delta p) = \rho_w g l_3$ ......(2)

$p_B - p_{atm} = \rho_w g l_4 + \rho_o g l_2$ ......(3)

Subtracting Eq.2 from Eq.3 and then employing Eq.1 gives

$$\Delta p = \rho_w g(l_4 - l_3) + \rho_o g l_2 = \rho_w g(l_4 + l_1 - l_3)$$

Define $l_w = l_1 - l_3$. Note $l_4 = h$. Then $\Delta p = \rho_w g(h + l_w)$ ......(4)

We can relate l_w to h by recognizing the volume of water must be conserved

$$\therefore \pi \frac{d_1^2}{4} l_w = \pi \frac{d_2^2}{4} h \quad \text{and} \quad l_w = h\left(\frac{d_2}{d_1}\right)^2$$

Substituting into Eq.4 gives

$$\Delta p = \rho_w g\left[h + h\left(\frac{d_2}{d_1}\right)^2\right] = \rho_w g h\left[1 + \left(\frac{d_2}{d_1}\right)^2\right]$$

Solving for h,

$$h = \frac{\Delta p}{\rho_w g\left[1 + (d_2/d_1)^2\right]} = 250\frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{999\,\text{kg}} \times \frac{\text{s}^2}{9.81\,\text{m}} \times \frac{1}{\left[1 + (15/10)^2\right]} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} \times \frac{10^3\,\text{mm}}{\text{m}}$$

$h = 7.85$ mm ← h

(b) The sensitivity of the manometer is defined as

$$s = \frac{h}{\Delta h_e} = \frac{\text{actual deflection}}{\text{equivalent } \Delta h_{H_2O}} \qquad \text{where } \Delta p = \rho_{H_2O}\, g\, \Delta h_e$$

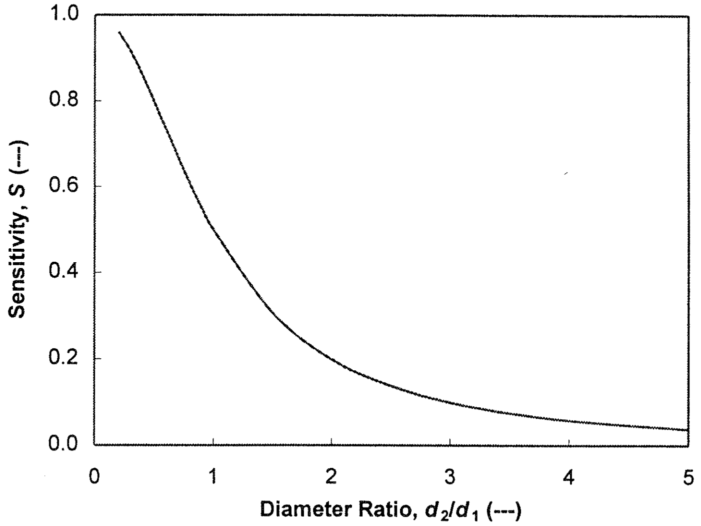
$$\therefore s = \frac{h}{\Delta h_e} = \frac{1}{\left[1 + (d_2/d_1)^2\right]} = \frac{1}{\left[1 + (1.5)^2\right]} = 0.308$$ ← s

The design is a poor one. The sensitivity could be improved by interchanging d_2 and d_1, i.e. having $d_2/d_1 < 1.0$ as shown in the plot below

Problem 3.34

$$S = \frac{1}{[1 + (d_2/d_1)^2]}$$

The manometer sensitivity, as a function of diameter ratio d_2/d_1 is shown below.



Problem 3.35

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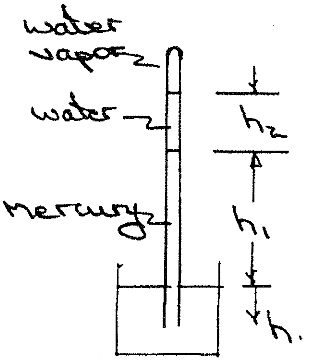
Given: Barometer with 6.5 in. of water on top of the mercury column of height 28.35 in.; Temperature $T = 70°F$.

Find: (a) Barometric pressure in psia.
(b) Effect of increase in ambient temperature (to $T_a = 85°F$) on length of mercury column for same barometric pressure.

Solution:



Basic equation: $\frac{dp}{dh} = \rho g$

Assumptions: (1) static liquid
(2) incompressible
(3) g = constant

Then, $dp = \rho g \, dh$ and $\Delta p = \rho g \Delta h$

Start at the free surface of the mercury ($p = p_{atm}$) and progress through the barometer to p_v (vapor pressure of the water)

$$p_{atm} - \rho_{Hg} g h_1 - \rho_{H_2O} g h_2 = p_v$$

$$p_{atm} = \rho_{Hg} g h_1 + \rho_{H_2O} g h_2 + p_v = \rho_{H_2O} SG_{Hg} g h_1 + \rho_{H_2O} g h_2 + p_v$$

$$p_{atm} = \rho_{H_2O} g \left[SG_{Hg} h_1 + h_2 \right] + p_v$$

From Table A.2, $SG_{Hg} = 13.55$
Table A.7 $\rho_{H_2O} = 1.93 \text{ slug/ft}^3$, $p_v = 0.363$ psia.

Evaluating,

$$p_{atm} = 1.93 \frac{\text{slug}}{\text{ft}^3} \times 32.2 \frac{\text{ft}}{\text{s}^2} \left[13.55 \times 28.35 \text{ in} + 6.5 \text{ in} \right] \frac{\text{ft}}{12 \text{ in}} \times \frac{\text{ft}^2}{144 \text{ in}^2} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{ft} \cdot \text{slug}} + 0.363 \text{ psia}$$

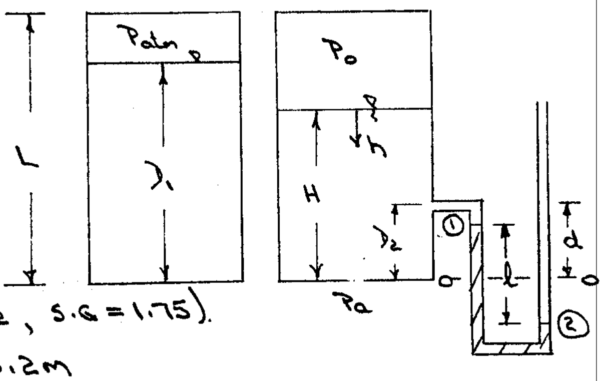
$$p_{atm} = 14.4 \text{ psia} \longleftarrow$$

At $T = 85°F$, the vapor pressure of water is estimated (from Table A.7) to be ≈ 0.60 psia. For the same barometric pressure the length of the mercury column would be shorter at the higher ambient temperature.

Problem 3.36

[4]

Given: Sealed tank of cross-section A and height, L = 3.0m is filled with water to a depth, $D_1 = 2.5m$.
Water drains slowly from the tank until system attains equilibrium.
U-tube manometer is connected to tank as shown.
(manometer fluid is merian blue, S.G = 1.75).



$D_1 = 2.5m$, $D_2 = 0.7m$, $d = 0.2m$

Find: The manometer deflection, l, under equilibrium conditions

Solution:

Basic equations: $\frac{dP}{dh} = \rho g$ $\qquad P\forall = MRT$

For γ = constant $\Delta P = \rho g \Delta h$

To determine the surface pressure P_0 under equilibrium conditions treat air above water as an ideal gas

$$\frac{P_a \forall_a}{P_0 \forall_0} = \frac{MRT_a}{MRT_0}$$ Assuming $T_a = T_0$, then

$$P_0 = \frac{\forall_a}{\forall_0} P_a = \frac{A(L - D_1)}{A(L - H)} P_a = \frac{(L - D_1)}{(L - H)} P_a$$

Under equilibrium conditions, $P_0 + \rho_{H_2O} g H = P_a$

Hence, $\frac{(L - D_1)}{(L - H)} P_a + \rho_{H_2O} g H = P_a$ or $\rho_{H_2O} g H^2 - H(P_a + \rho_{H_2O} g L) + D_1 P_a = 0$

and

$$H = \frac{(P_a + \rho_{H_2O} g L) \pm \sqrt{(P_a + \rho_{H_2O} g L)^2 - 4 \rho_{H_2O} g D_1 P_a}}{2 \rho_{H_2O} g}$$

$$H = \frac{\left[1.01 \times 10^5 \frac{N}{m^2} + \frac{999\,kg}{m^3} \times 9.81 \frac{m}{sec^2} \times 3m \times \frac{N \cdot sec^2}{kg \cdot m}\right] \pm \sqrt{[\;]^2 - 4 \times \frac{999\,kg}{m^3} \times 9.81 \frac{m}{sec^2} \times 2.5m \times 1.01 \times 10^5 \frac{N}{m^2} \times \frac{N \cdot sec^2}{kg \cdot m}}}{2 \times 999 \frac{kg}{m^3} \times 9.81 \frac{m}{sec^2} \times \frac{N \cdot sec^2}{kg \cdot m}}$$

H = 10.9m or 2.36m. From physical considerations H = 2.36m

$$P_0 = \frac{(L - D_1)}{(L - H)} P_a = \frac{(3.0 - 2.5)}{(3.0 - 2.36)} \times 1.01 \times 10^5 \, N/m^2 = 7.89 \times 10^4 \, N/m^2$$

For the manometer, $P_0 + (P_1 - P_0) + (P_2 - P_1) = P_2 = P_{atm}$

$$P_0 + \rho_{H_2O} g \left(H - D_2 + d - \frac{l}{2}\right) + \rho_{mb} g l = P_{atm}$$

$$\frac{P_{atm} - P_0}{\rho_{H_2O} g} - H + D_2 - d = (S.G)_{mb} l - \frac{l}{2} = l\left[(S.G)_{mb} - 0.5\right]$$

$$l = \frac{(P_{atm} - P_0)/\rho_{H_2O} g - H + D_2 - d}{(S.G)_{mb} - 0.5} = \frac{(10.1 - 7.89) \times 10^4 \frac{N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{s^2}{9.81\,m} \times \frac{kg\,m}{N \cdot s^2} - 2.36m + 0.7m - 0.2}{1.75 - 0.5}$$

$l = 0.316m \longleftarrow$ $\qquad l$

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42-382 100 SHEETS 5 SQUARE
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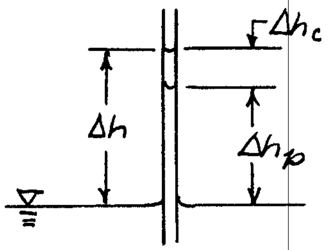
Problem 3.37

Given: Water column standing at $\Delta h = 50$ mm in $D = 2.5$ mm glass tube.

Find: (a) Column height if surface tension were zero.
(b) Column height in $D = 1$ mm tube.

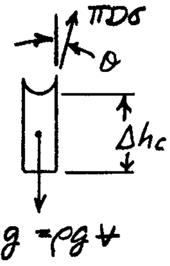


Solution: Assume column height is sum of capillary rise and rise caused by pressure difference,

$$\Delta h = \Delta h_c + \Delta h_p$$

Choose a free-body diagram of Δh_c for analysis:

$$\Sigma F_z = \pi D \sigma \cos\theta - \frac{\pi D^2}{4} \rho g \, \Delta h_c = 0$$



Assumptions: (1) Neglect volume under meniscus
(2) Δh_p remains constant

Then $\Delta h_c = \frac{4\sigma}{\rho g D} \cos\theta$

For water (Table A.4), $\sigma = 72.8$ mN/m and $\theta \approx 0$, so $\cos\theta = 1$, and

$$\Delta h_c = \frac{4\sigma}{\rho g D}$$

For the $D = 2.5$ mm tube,

$$\Delta h_c = 4 \times 72.8 \times 10^{-3} \frac{N}{m} \times \frac{m^3}{999 \, kg} \times \frac{s^2}{9.81 \, m} \times \frac{1}{0.0025 \, m} \times \frac{kg \cdot m}{N \cdot s^2} = 0.0119 \text{ m or } 11.9 \text{ mm}$$

Then

$$\Delta h_p = \Delta h - \Delta h_c = (50.0 - 11.9) \text{ mm} = 38.1 \text{ mm} \quad (\sigma = 0)$$

Δh

For the $D = 1.0$ mm tube,

$$\Delta h_c = 4 \times 72.8 \times 10^{-3} \frac{N}{m} \times \frac{m^3}{999 \, kg} \times \frac{s^2}{9.81 \, m} \times \frac{1}{0.001 \, m} \times \frac{kg \cdot m}{N \cdot s^2} = 0.0297 \text{ m or } 29.7 \text{ mm}$$

So

$$\Delta h = \Delta h_c + \Delta h_p = (29.7 + 38.1) \text{ mm} = 67.8 \text{ mm} \quad (D = 1.0 \text{ mm tube})$$

Δ

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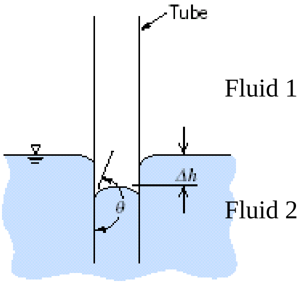
Problem 3.38

[2]

3.38 Consider a small diameter open-ended tube inserted at the interface between two immiscible fluids of different densities. Derive an expression for the height difference Δh between the interface level inside and outside the tube in terms of tube diameter D, the two fluid densities, ρ_1 and ρ_2, and the surface tension σ and angle θ for the two fluids' interface. If the two fluids are water and mercury, find the tube diameter such that $\Delta h < 10$ mm.



Given: Two fluids inside and outside a tube

Find: An expression for height *h*; find diameter for *h* < 10 mm for water/mercury

Solution:

A free-body vertical force analysis for the section of fluid 1 height Δ*h* in the tube below the "free surface" of fluid 2 leads to

$$\sum F = 0 = \Delta p \cdot \frac{\pi \cdot D^2}{4} - \rho_1 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} + \pi \cdot D \cdot \sigma \cdot \cos(\theta)$$

where Δ*p* is the pressure difference generated by fluid 2 over height Δ*h*, $\Delta p = \rho_2 \cdot g \cdot \Delta h$

Assumption: Neglect meniscus curvature for column height and volume calculations

Hence $\Delta p \cdot \frac{\pi \cdot D^2}{4} - \rho_1 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} = \rho_2 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} - \rho_1 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} = -\pi \cdot D \cdot \sigma \cdot \cos(\theta)$

Solving for Δ*h* $\Delta h = -\frac{4 \cdot \sigma \cdot \cos(\theta)}{g \cdot D \cdot \left(\rho_2 - \rho_1\right)}$

For fluids 1 and 2 being water and mercury (for mercury σ = 375 mN/m and θ = 140º, from Table A.4), solving for D to make Δh = 10 mm

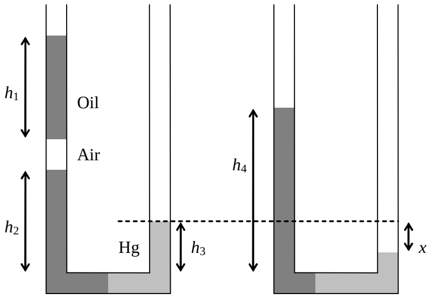
$$D = -\frac{4 \cdot \sigma \cdot \cos(\theta)}{g \cdot \Delta h \cdot \left(\rho_2 - \rho_1\right)} = -\frac{4 \cdot \sigma \cdot \cos(\theta)}{g \cdot \Delta h \cdot \rho_{H2O} \cdot \left(SG_{Hg} - 1\right)}$$

$$D = -\frac{4 \times 0.375 \cdot \frac{N}{m} \times \cos(140 \cdot deg)}{9.81 \cdot \frac{m}{s^2} \times 0.01 \cdot m \times 1000 \cdot \frac{kg}{m^3} \times (13.6 - 1)} \times \frac{kg \cdot m}{N \cdot s^2} \qquad D = 0.93\,mm \qquad D \geq 1 \cdot mm$$

Problem 3.39

[2]

3.39 You have a manometer consisting of a tube that is 1.1-cm ID. On one side the manometer leg contains mercury, 10 cc of an oil (SG = 1.67), and 3 cc of air as a bubble in the oil. The other leg just contains mercury. Both legs are open to the atmosphere and are in a static condition. An accident occurs in which 3 cc of the oil and the air bubble are removed from the one leg. How much do the mercury height levels change?



Given: Data on manometer before and after an "accident"

Find: Change in mercury level

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot \Delta h$ where Δh is height difference

For the initial state, working from right to left $p_{atm} = p_{atm} + SG_{Hg} \cdot \rho \cdot g \cdot h_3 - SG_{oil} \cdot \rho \cdot g \cdot \left(h_1 + h_2\right)$

$$SG_{Hg} \cdot \rho \cdot g \cdot h_3 = SG_{oil} \cdot \rho \cdot g \cdot \left(h_1 + h_2\right) \qquad (1)$$

Note that the air pocket has no effect!

For the final state, working from right to left $p_{atm} = p_{atm} + SG_{Hg} \cdot \rho \cdot g \cdot \left(h_3 - x\right) - SG_{oil} \cdot \rho \cdot g \cdot h_4$

$$SG_{Hg} \cdot \rho \cdot g \cdot \left(h_3 - x\right) = SG_{oil} \cdot \rho \cdot g \cdot h_4 \qquad (2)$$

The two unknowns here are the mercury levels before and after (i.e., h_3 and x)

Combining Eqs. 1 and 2 $SG_{Hg} \cdot \rho \cdot g \cdot x = SG_{oil} \cdot \rho \cdot g \cdot \left(h_1 + h_2 - h_4\right)$ $\quad x = \frac{SG_{oil}}{SG_{Hg}} \cdot \left(h_1 + h_2 - h_4\right) \qquad (3)$

From Table A.1 $SG_{Hg} = 13.55$

The term $h_1 + h_2 - h_4$ is the difference between the total height of oil before and after the accident

$$h_1 + h_2 - h_4 = \frac{\Delta V}{\left(\frac{\pi \cdot d^2}{4}\right)} = \frac{4}{\pi} \times \left(\frac{1}{0.011} \cdot \frac{1}{m}\right)^2 \times 3 \cdot cc \times \left(\frac{1 \cdot m}{100 \cdot cm}\right)^3 = 0.0316 \cdot m$$

Then from Eq. 3 $x = \frac{1.67}{13.55} \times 0.0316 \cdot m$ $\qquad x = 3.895 \times 10^{-3} m$ $\qquad x = 0.389 \cdot cm$

Problem 3.40

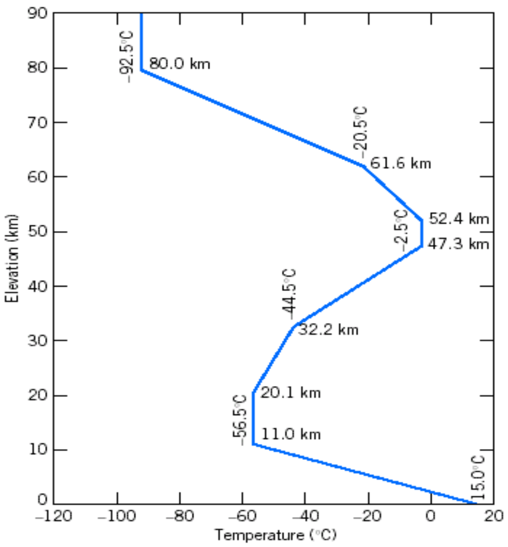
[3]

3.40 Based on the atmospheric temperature data of the U.S. Standard Atmosphere of Fig. 3.3, compute and plot the pressure variation with altitude, and compare with the pressure data of Table A.3.

Given: Atmospheric temperature data

Find: Pressure variation; compare to Table A.3

Solution:



From Section 3-3: $\frac{dp}{dz} = -\rho \cdot z$

For a linear temperature variation $m = -\frac{dT}{dz} = \text{const}$ $\quad p = p_0 \cdot \left(\frac{T}{T_0}\right)^{\frac{g}{m \cdot R}}$

For isothermal conditions (Example 3.4) $\quad p = p_0 \cdot e^{-\frac{g \cdot (z - z_0)}{R \cdot T}}$

In these equations p_0, T_0, and z_0 are reference conditions

p_{SL} = 101 kPa

R = 286.9 J/kg.K

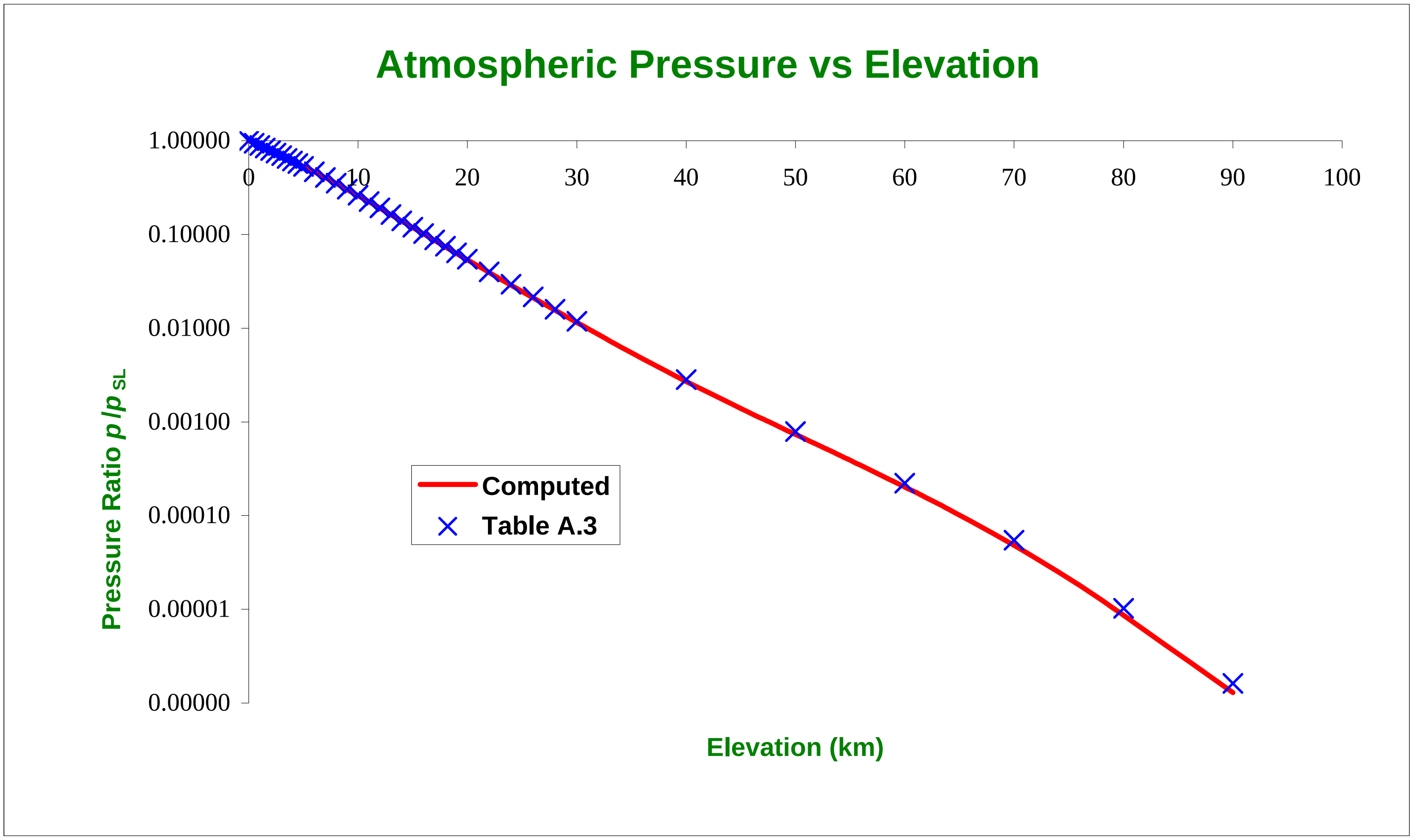
ρ = 999 kg/m^3

The temperature can be computed from the data in the figure
The pressures are then computed from the appropriate equation

z (km)	**T (°C)**	**T (K)**		**p/p_{SL}**
0.0	15.0	288.0	m = 0.0065 (K/m)	1.000
2.0	2.0	275.00		0.784
4.0	-11.0	262.0		0.608
6.0	-24.0	249.0		0.465
8.0	-37.0	236.0		0.351
11.0	-56.5	216.5		0.223
12.0	-56.5	216.5	T = const	0.190
14.0	-56.5	216.5		0.139
16.0	-56.5	216.5		0.101
18.0	-56.5	216.5		0.0738
20.1	-56.5	216.5		0.0530
22.0	-54.6	218.4	m = -0.000991736 (K/m)	0.0393
24.0	-52.6	220.4		0.0288
26.0	-50.6	222.4		0.0211
28.0	-48.7	224.3		0.0155
30.0	-46.7	226.3		0.0115
32.2	-44.5	228.5		0.00824
34.0	-39.5	233.5	m = -0.002781457 (K/m)	0.00632
36.0	-33.9	239.1		0.00473
38.0	-28.4	244.6		0.00356
40.0	-22.8	250.2		0.00270
42.0	-17.2	255.8		0.00206
44.0	-11.7	261.3		0.00158
46.0	-6.1	266.9		0.00122
47.3	-2.5	270.5		0.00104
50.0	-2.5	270.5	T = const	0.000736
52.4	-2.5	270.5		0.000544
54.0	-5.6	267.4	m = 0.001956522 (K/m)	0.000444
56.0	-9.5	263.5		0.000343
58.0	-13.5	259.5		0.000264
60.0	-17.4	255.6		0.000202
61.6	-20.5	252.5		0.000163
64.0	-29.9	243.1	m = 0.003913043 (K/m)	0.000117
66.0	-37.7	235.3		0.0000880
68.0	-45.5	227.5		0.0000655
70.0	-53.4	219.6		0.0000482
72.0	-61.2	211.8		0.0000351
74.0	-69.0	204.0		0.0000253
76.0	-76.8	196.2		0.0000180
78.0	-84.7	188.3		0.0000126
80.0	-92.5	180.5	T = const	0.00000861
82.0	-92.5	180.5		0.00000590
84.0	-92.5	180.5		0.00000404
86.0	-92.5	180.5		0.00000276
88.0	-92.5	180.5		0.00000189
90.0	-92.5	180.5		0.00000130

From Table A.3

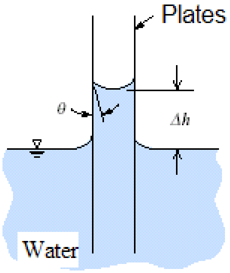
z (km)	**p/p_{SL}**
0.0	1.000
0.5	0.942
1.0	0.887
1.5	0.835
2.0	0.785
2.5	0.737
3.0	0.692
3.5	0.649
4.0	0.609
4.5	0.570
5.0	0.533
6.0	0.466
7.0	0.406
8.0	0.352
9.0	0.304
10.0	0.262
11.0	0.224
12.0	0.192
13.0	0.164
14.0	0.140
15.0	0.120
16.0	0.102
17.0	0.0873
18.0	0.0747
19.0	0.0638
20.0	0.0546
22.0	0.0400
24.0	0.0293
26.0	0.0216
28.0	0.0160
30.0	0.0118
40.0	0.00283
50.0	0.000787
60.0	0.000222
70.0	0.0000545
80.0	0.0000102
90.0	0.00000162



Agreement between calculated and tabulated data is very good (as it should be, considering the table data is also computed!)

Problem 3.41

3.41 Two vertical glass plates 300 mm × 300 mm are placed in an open tank containing water. At one end the gap between the plates is 0.1 mm, and at the other it is 2 mm. Plot the curve of water height between the plates from one end of the pair to the other.



Given: Geometry of vertical plates

Find: Curve of water height due to capillary action

Solution:

Parallel Plates: A free-body vertical force analysis for the section of water height Δh above the "free surface" between plates arbitrary width w (similar to the figure above), leads to

$$\sum F = 0 = 2 \cdot w \cdot \sigma \cdot \cos(\theta) - \rho \cdot g \cdot \Delta h \cdot w \cdot a$$

Solving for Δh

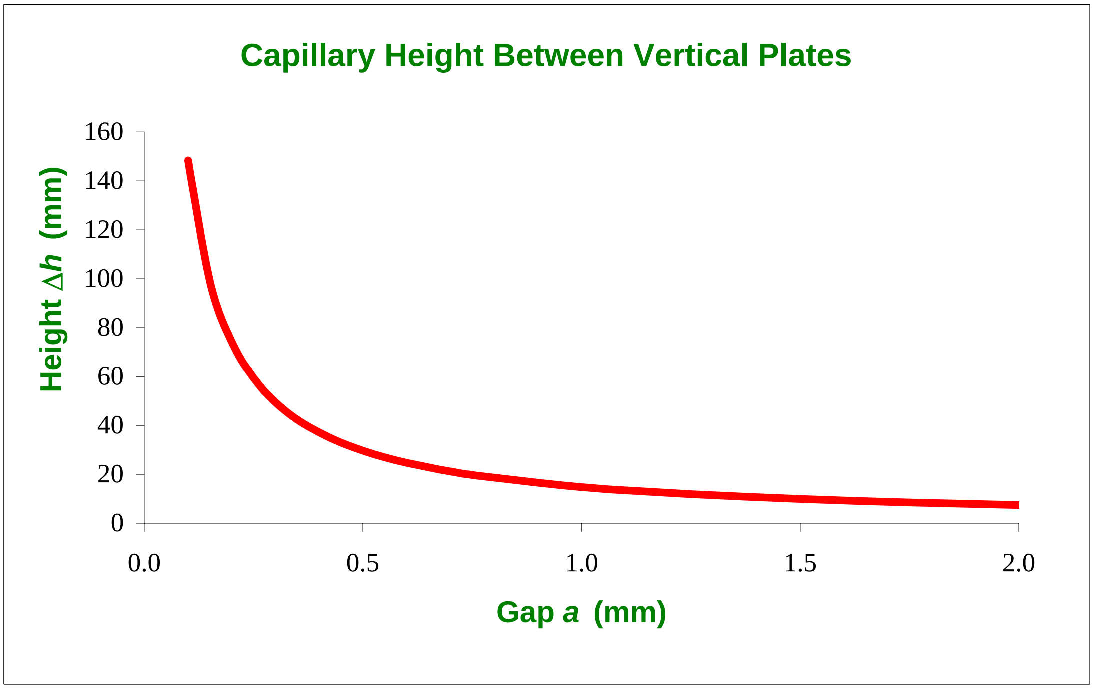
$$\Delta h = \frac{2 \cdot \sigma \cdot \cos(\theta)}{\rho \cdot g \cdot a}$$

For water σ = 72.8 mN/m and $\theta = 0^o$ (Table A.4), so

$\sigma =$ 72.8 mN/m
$\rho =$ 1000 kg/m^3

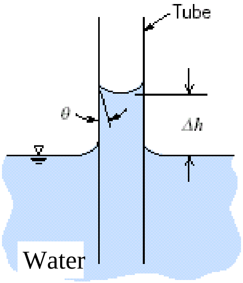
Using the formula above

a (mm)	Δ*h* (mm)
0.10	148
0.15	98.9
0.20	74.2
0.25	59.4
0.30	49.5
0.35	42.4
0.40	37.1
0.45	33.0
0.50	29.7
0.55	27.0
0.60	24.7
0.65	22.8
0.70	21.2
0.75	19.8
1.00	14.8
1.25	11.9
1.50	9.89
1.75	8.48
2.00	7.42



Problem 3.42

3.42 Compare the height due to capillary action of water exposed to air in a circular tube of diameter $D = 0.5$ mm, and between two infinite vertical parallel plates of gap $a = 0.5$ mm.



Given: Water in a tube or between parallel plates

Find: Height Δ*h* for each system

Solution:

a) Tube: A free-body vertical force analysis for the section of water height Δ*h* above the "free surface" in the tube, as shown in the figure, leads to

$$\sum F = 0 = \pi \cdot D \cdot \sigma \cdot \cos(\theta) - \rho \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4}$$

Assumption: Neglect meniscus curvature for column height and volume calculations

Solving for Δ*h* $$\Delta h = \frac{4 \cdot \sigma \cdot \cos(\theta)}{\rho \cdot g \cdot D}$$

b) Parallel Plates: A free-body vertical force analysis for the section of water height Δ*h* above the "free surface" between plates arbitrary width *w* (similar to the figure above), leads to

$$\sum F = 0 = 2 \cdot w \cdot \sigma \cdot \cos(\theta) - \rho \cdot g \cdot \Delta h \cdot w \cdot a$$

Solving for Δ*h* $$\Delta h = \frac{2 \cdot \sigma \cdot \cos(\theta)}{\rho \cdot g \cdot a}$$

For water σ = 72.8 mN/m and θ = 0º (Table A.4), so

a) Tube $$\Delta h = \frac{4 \times 0.0728 \cdot \frac{N}{m}}{999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.005 \cdot m} \times \frac{kg \cdot m}{N \cdot s^2} \qquad \Delta h = 5.94 \times 10^{-3} m \qquad \Delta h = 5.94 mm$$

b) Parallel Plates $$\Delta h = \frac{2 \times 0.0728 \cdot \frac{N}{m}}{999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.005 \cdot m} \times \frac{kg \cdot m}{N \cdot s^2} \qquad \Delta h = 2.97 \times 10^{-3} m \qquad \Delta h = 2.97 mm$$

Problem 3.43

[3]

3.43 On a certain calm day, a mild inversion causes the atmospheric temperature to remain constant at 85°F between sea level and 16,000 ft altitude. Under these conditions, (a) calculate the elevation change for which a 2 percent reduction in air pressure occurs, (b) determine the change of elevation necessary to effect a 10 percent reduction in density, and (c) plot p_2/p_1 and ρ_2/ρ_1 as a function of Δz.

Given: Data on isothermal atmosphere

Find: Elevation changes for 2% and 10% density changes; plot of pressure and density versus elevation

Solution:

Basic equation $\frac{dp}{dz} = -\rho \cdot g$ and $p = \rho \cdot R \cdot T$

Assumptions: static, isothermal fluid,; g = constant; ideal gas behavior

Then $\frac{dp}{dz} = -\rho \cdot g = -\frac{p \cdot g}{R_{air} \cdot T}$ and $\frac{dp}{p} = -\frac{g}{R_{air} \cdot T} \cdot dz$

Integrating $\Delta z = -\frac{R_{air} \cdot T_0}{g} \cdot \ln\left(\frac{p_2}{p_1}\right)$ where $T = T_0$

For an ideal with T constant $\frac{p_2}{p_1} = \frac{\rho_2 \cdot R_{air} \cdot T}{\rho_1 \cdot R_{air} \cdot T} = \frac{\rho_2}{\rho_1}$ so $\Delta z = -\frac{R_{air} \cdot T_0}{g} \cdot \ln\left(\frac{\rho_2}{\rho_1}\right) = -C \cdot \ln\left(\frac{\rho_2}{\rho_1}\right)$ (1)

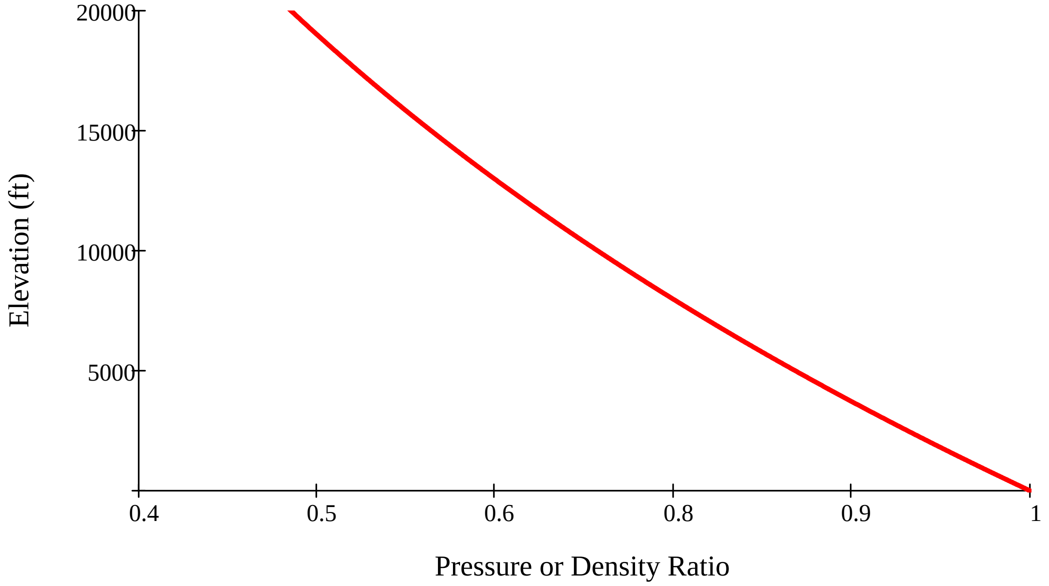
From Table A.6 $R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Evaluating $C = \frac{R_{air} \cdot T_0}{g} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R} \times (85 + 460) \cdot R \times \frac{1}{32.2} \cdot \frac{s^2}{ft} \times \frac{32.2 \cdot lbm \cdot ft}{s^2 \cdot lbf}$ $C = 29065 \cdot ft$

For a 2% reduction in density $\frac{\rho_2}{\rho_1} = 0.98$ so from Eq. 1 $\Delta z = -29065 \cdot ft \cdot \ln(0.98)$ $\Delta z = 587 \cdot ft$

For a 10% reduction in density $\frac{\rho_2}{\rho_1} = 0.9$ so from Eq. 1 $\Delta z = -29065 \cdot ft \cdot \ln(0.9)$ $\Delta z = 3062 \cdot ft$

To plot $\frac{p_2}{p_1}$ and $\frac{\rho_2}{\rho_1}$ we rearrange Eq. 1 $\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-\frac{\Delta z}{C}}$



This plot can be plotted in Excel

Problem 3.44

Given: Martian atmosphere behaves as an ideal gas, T = constant
$M_m = 32.0$, $T = 200\,K$, $g = 3.92\,m/s^2$, $\rho_0 = 0.015\,kg/m^3$

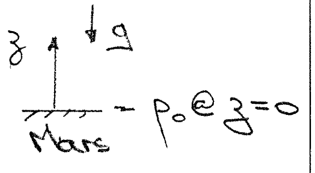
Find: Density at $z = 20\,km$.

Plot: the ratio ρ/ρ_0 (ratio of density to surface density) vs z; compare with earth's atmosphere

Solution:

Basic equations: $\frac{dp}{dz} = -\rho g$; $p = \rho R T$; $R = R_u/M_m$

Assumptions: (1) static fluid
(2) g constant
(3) ideal gas



Since T = constant, $dp = d(\rho R T) = RT\,d\rho$

$$\frac{dp}{dz} = RT\frac{d\rho}{dz} = -\rho g \quad \text{and} \quad \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = -\int_0^z \frac{g}{RT}\,dz$$

$$\ln\frac{\rho_z}{\rho_0} = -gz/RT \quad \text{and} \quad \frac{\rho}{\rho_0} = e^{-gz/RT} \quad \text{----} (1)$$

Evaluating

$$R = \frac{R_u}{M_m} = 8314.3\frac{N\cdot m}{kgmole\cdot K} \times \frac{kgmole}{32.0\,kg} = 260\frac{N\cdot m}{kg\cdot K}$$

$$\rho = 0.015\frac{kg}{m^3} \times \exp\left[-3.92\frac{m}{s^2} \times 20\times10^3 m \times \frac{kg\cdot K}{260\,N\cdot m} \times \frac{1}{200\,K} \times \frac{N\cdot s^2}{kg\cdot m}\right]$$

$$\rho = 0.00332\,kg/m^3 \qquad \rho_{z=20km}$$

For the Martian atmosphere, Eq. 1 gives $\rho/\rho_0 = e^{-0.0754\,z\,(km)}$

For the earth's atmosphere, ρ/ρ_0 is given in Table A.3

Both ρ/ρ_0 variations are plotted below

Note from the plot:

- on Mars $\rho/\rho_0 = 0.221$ at $z = 20\,km$, whereas
- on Earth, $\rho/\rho_0 = 0.073$ at $z = 20\,km$

The difference is caused by (a) the larger gravity on Earth, and (b) temperature decrease with altitude in our atmosphere.

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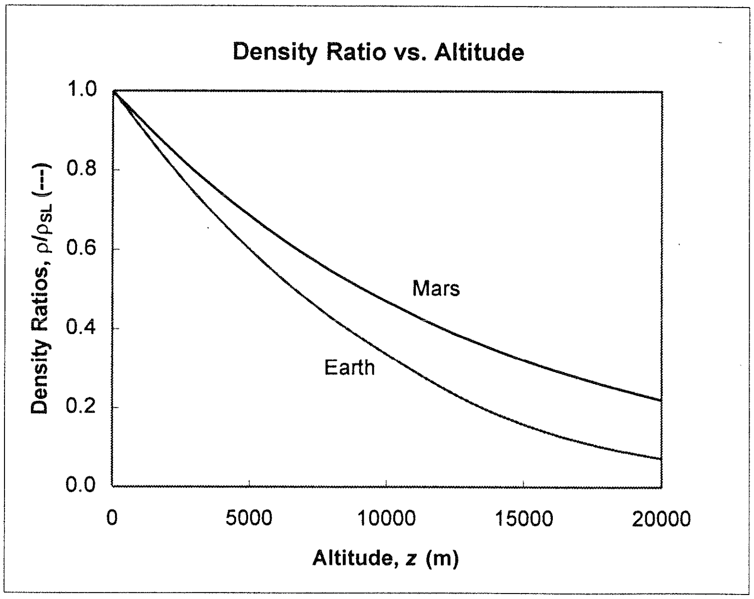
Problem 3.44

Density vs. elevation in Martian and Earth atmospheres:

Elevation Change Δz (m)	Density Ratio (Earth) ρ/ρ_{SL} (---)	Density Ratio (Mars) ρ/ρ_{SL} (---)
0	1.000	1.00
2000	0.8217	0.860
4000	0.6689	0.740
6000	0.5389	0.636
8000	0.4292	0.547
10000	0.3376	0.470
12000	0.2546	0.405
14000	0.1860	0.348
16000	0.1359	0.299
18000	0.09930	0.257
20000	0.07258	0.221



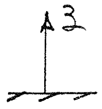
Problem 3.45

Given: Atmospheric conditions at ground level ($z=0$) in Denver, Colorado are $p_0 = 83.2$ kPa, $T_0 = 25°C$. Pike's peak is at elevation $z = 2690$ m

Find: Pressure on Pike's peak assuming (a) an incompressible, and (b) an adiabatic atmosphere.

Plot: p/p_0 vs z for both cases.

Solution:



Basic equations: $dp/dz = -\rho g$; $p = \rho R T$

Assumptions: (1) static fluid (2) g = constant
(3) ideal gas behavior

(a) For an incompressible atmosphere $\int_{p_0}^{p} dp = -\int_{0}^{z} \rho g \, dz$

$$p - p_0 = \rho g z = \rho_0 g z = \frac{p_0}{RT_0} g z \quad \text{and} \quad p = p_0\left[1 - \frac{gz}{RT_0}\right] \text{ ------(1)}$$

At $z = 2690$ m

$$p = 83.2\,\text{kPa}\left[1 - 9.81\frac{\text{m}}{\text{s}^2} \times 2690\,\text{m} \times \frac{\text{kg}\cdot\text{K}}{287\,\text{N}\cdot\text{m}} \times \frac{1}{298\,\text{K}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right] = 57.5\,\text{kPa} \longleftarrow p_{p=c}$$

(b) For an adiabatic atmosphere p/ρ^k = constant, $\rho = \rho_0 \left(p/p_0\right)^{1/k}$

$$\frac{dp}{dz} = -\rho g = -g\rho_0\left(\frac{p}{p_0}\right)^{1/k} dz \quad \text{or} \quad \int_{p_0}^{p} \frac{dp}{p^{1/k}} = -\int_{0}^{z} \rho_0 p_0^{-1/k} g \, dz$$

Then

$$\frac{k}{(k-1)} p^{-\frac{1}{k}+1}\Big]_{p_0}^{p} = -\rho_0 p_0^{-1/k} g\, dz \quad \text{or} \quad \frac{k}{(k-1)}\left[p^{(k-1)/k} - p_0^{(k-1)/k}\right] = -\rho_0 p_0^{-1/k} g z$$

and

$$\left(\frac{k}{k-1}\right) p_0^{(k-1)/k}\left[\left(\frac{p}{p_0}\right)^{(k-1)/k} - 1\right] = -\rho_0 p_0^{-1/k} g z$$

$$\left(\frac{p}{p_0}\right)^{(k-1)/k} = 1 - \frac{(k-1)}{k} p_0^{-1/k-(k-1)/k} \rho_0 g z = 1 - \frac{(k-1)}{k} p_0^{-1} \rho_0 g z$$

and

$$\frac{p}{p_0} = \left[1 - \frac{(k-1)}{k}\frac{\rho_0}{p_0} g z\right]^{k/k-1} = \left[1 - \frac{(k-1)}{k}\frac{gz}{RT_0}\right]^{k/k-1} \text{ ------(2)}$$

Evaluating at $z = 2690$ m

$$p = 83.2\,\text{kPa}\left[1 - \frac{0.4}{1.4} \times 9.81\frac{\text{m}}{\text{s}^2} \times 2690\,\text{m} \times \frac{\text{kg}\cdot\text{K}}{287\,\text{N}\cdot\text{m}} \times \frac{1}{298\,\text{K}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right]^{\frac{1.4}{0.4}}$$

$$p = 60.2\,\text{kPa} \longleftarrow p_{\text{adiab}}$$

The pressure ratio p/p_0 vs z is plotted for an incompressible atmosphere (Eq. 1) and an adiabatic atmosphere (Eq. 2) below

Incompressible case $p/p_0 = \left[1 - 0.115 z\right]$ (z in km).

Adiabatic case $p/p_0 = \left[1 - 0.0328 z\right]^{3.5}$ (z in km)

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Problem 3.45

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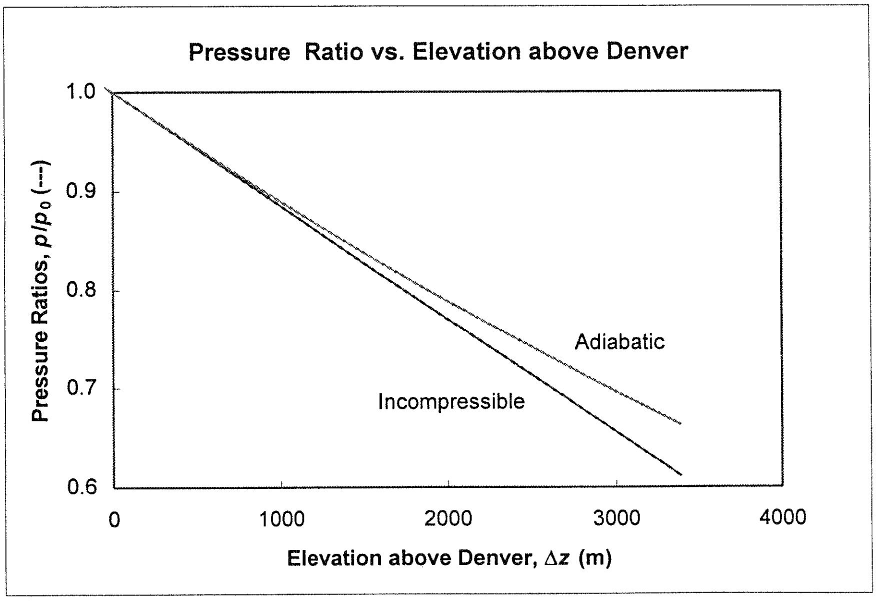
Pressure ratio vs. elevation above Denver:

Elevation z (m)	Elevation above Denver z (m)	Pressure Ratio (T = C) p/p_0 (---)	Pressure Ratio (adiabatic) p/p_0 (---)
0	-1610	1.185	1.20
500	-1110	1.127	1.13
1000	-610	1.070	1.07
1500	-110	1.013	1.01
2000	390	0.955	0.956
2500	890	0.898	0.902
3000	1390	0.841	0.849
3500	1890	0.783	0.800
4000	2390	0.726	0.752
4300	2690	0.691	0.724
4500	2890	0.669	0.706
5000	3390	0.611	0.662



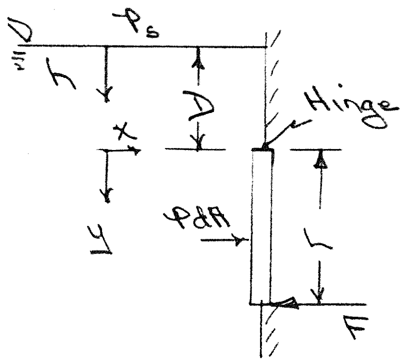
Problem 3.46

[2] Part 1/2

Given: Door, of width $b = 1\,m$, located in plane vertical wall of water tank is hinged along upper edge.

$D = 1\,m$, $L = 1.5\,m$

Atmospheric pressure acts on outer surface of door; force F is applied at lower applied at lower edge to keep door closed



Find: (a) Force F, if $p_s = p_{atm}$.

(b) Force F, if $p_s = 0.5$ atm.

Plot: F/F_o over range of p_s/p_{atm}. (F_o is force required when $p_s = p_{atm}$)

Solution:

Basic equations: $\frac{dp}{dh} = \rho g$; $F_R = \int p\,dA$; $\sum M_z = 0$

Assumptions: (1) static fluid (2) ρ = constant

(3) door is in equilibrium

Since $\sum M_z = 0$ for equilibrium, taking moments about the hinge.

$$\sum M_z = 0 = FL - \int y\,p\,dA = F.L - \int_0^L y\,p\,b\,dy$$

and

$$F = \frac{1}{L}\int_0^L y\,p\,b\,dy \quad \text{------------} (1)$$

Note: We will obtain a general expression for F (needed for the plot) and then simplify for cases (a) and (b)

Since $dp = \rho g\,dh$, then $p = p_s + \rho g h$

$h = D + y$ and hence $p = p_s + \rho g(D+y)$.

Because p_{atm} acts on the outside of the door, p_s is the surface gage pressure.

From Eq. (1), $F = \frac{1}{L}\int_0^L y\left[p_{s_g} + \rho g(D+y)\right]b\,dy$

$$F = \frac{b}{L}\left[p_{s_g}\frac{y^2}{2} + \rho g\left(\frac{Dy^2}{2} + \frac{y^3}{3}\right)\right]_0^L$$

$$F = \frac{b}{L}\left[p_{s_g}\frac{L^2}{2} + \rho g\left(\frac{DL^2}{2} + \frac{L^3}{3}\right)\right] = b\left[p_{s_g}\frac{L}{2} + \rho g L\left(\frac{D}{2} + \frac{L}{3}\right)\right] \quad \text{-----} (2)$$

(a) For $p_s = p_{atm}$, $p_{s_g} = 0$

$$F_o = \rho g b L\left(\frac{D}{2} + \frac{L}{3}\right) \quad \text{------------} (3)$$

$$F_o = 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 1m \times 1.5m\left(\frac{1m}{2} + \frac{1.5m}{3}\right) \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{kN}{10^3 N} = 14.7\,kN \longleftarrow F_o$$

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Problem 3.46

[2] Part 2/2

(b) For $p_{sg} = 0.5$ atm (50.6 kPa), from Eq (2)

$$F = p_{sg}\frac{bL}{2} + \rho g L b\left(\frac{D}{2}+\frac{L}{3}\right)$$

$$F = 50.6\,\frac{kN}{m^2}\times 1\,m\times\frac{1.5\,m}{2} + 14.7\,kN = 52.7\,kN \longleftarrow F)_{p_s=0.5\,atm}$$

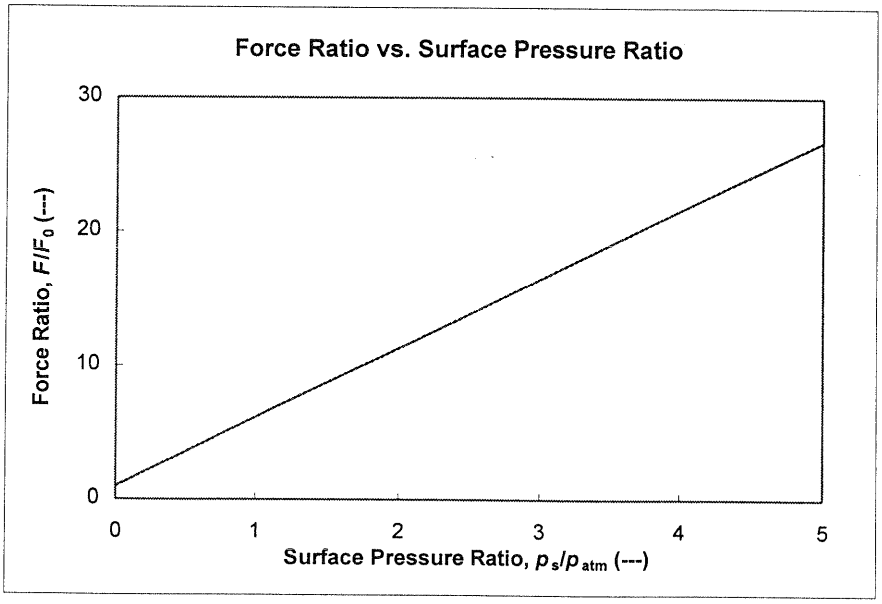
From Eqs (2) and (3) we can write

$$\frac{F}{F_0} = \frac{\cancel{b}\left[p_{sg}\frac{L}{2} + \rho g L\left(\frac{D}{2}+\frac{L}{3}\right)\right]}{\rho g \cancel{b} L\left(\frac{D}{2}+\frac{L}{3}\right)} = 1 + \frac{p_{sg}}{2\rho g\left(\frac{D}{2}+\frac{L}{3}\right)}$$

Substituting values

$$\frac{F}{F_0} = 1 + \frac{p_{sg}}{0.194} \quad \text{(with } p_{sg} \text{ in atmospheres).} \quad \text{------ (4)}$$

F/F_0 is plotted as a function of $p_{s\,gage}/p_{atm}$



Problem 3.47

Open-Ended Problem Statement: A hydropneumatic elevator consists of a piston-cylinder assembly to lift the elevator cab. Hydraulic oil, stored in an accumulator tank pressurized by air, is valved to the piston as needed to lift the elevator. When the elevator descends, oil is returned to the accumulator. Design the least expensive accumulator that can satisfy the system requirements. Assume the lift is 3 floors, the maximum load is 10 passengers, the maximum system pressure is 800 kPa (gage). For column bending strength, the piston diameter must be at least 150 mm. The elevator cab and piston have a combined mass of 3,000 kg, and are to be purchased. Perform the analysis needed to define, as a function of system operating pressure, the piston diameter, the accumulator volume and diameter, and the wall thickness. Discuss safety features that your company should specify for the complete elevator system. Would it be preferable to use a completely pneumatic design or a completely hydraulic design? Why?

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Discussion: The design requirements are specified, except that a typical floor height is about 12 ft, making the total required lift about 36 ft.)

A spreadsheet was used to calculate the system properties for various pressures. Results are presented on the next page, followed by a sample calculation.

Total cost dropped quickly as system pressure was increased. A shallow minimum was reached in the 100-110 psig range.

The lowest-cost solution was obtained at a system pressure of about 100 psig. At this pressure, the reservoir of 140 gal required a 3.30 ft diameter pressure sphere with a 0.250 in. wall thickness. The welding cost was $311 and the material cost $433, for a total cost of $744.

Accumulator wall thickness was constrained at 0.250 in. for pressures below 100 psi; it increased for higher pressures (this caused the discontinuity in slope of the curve at 100 psig). The mass of steel became constant above 110 psig.

No allowance was made for the extra volume needed to pressurize the accumulator.

Fail-safe design is essential for an elevator to be used by the public. The control circuitry should be redundant. Failures must be easy to spot. For this reason, hydraulic actuation is good: leaks will be readily apparent. The final design must be reviewed, approved, and stamped by a professional engineer since the design involves public safety.

The terminology used in the solution is defined in Table 1.

Table 1. Symbols, definitions, and units

Symbol	Definition	Units
p	system pressure	psig
A_p	area of lift piston	in.^{2}
$\not{V}_{\text{oil}}$	volume of oil	gal
D_s	diameter of (spherical) accumulator	ft
t	wall thickness of spherical accumulator	in.
A_w	area of weld	in.^{2}
C_w	cost of weld	$
M_s	mass of (steel) accumulator	lbm
C_s	cost of steel	$
C_t	total cost	$

Results of the system simulation and sample calculations are presented on the next page.

Problem 3.47

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Table 2. Results of system simulation

Input Data:

Cab and piston weight:	W_{cab} =	6,000	lbf
Passenger weight:	W_{pax} =	1,500	lbf
Total weight:	W_{tot} =	7,500	lbf
Allowable stress:	σ =	4,000	psi
Minimum wall thickness:	t =	0.250	in.
Welding cost factor:	cf_w =	5.00	\$/in.^{2}
Steel cost factor:	cf_s =	1.25	\$/pound

Results:

p (psig)	A_p (in.^{2})	$\mathcal{V}_{oil}$ (gal)	D_s (ft)	t (in.)	A_w (in.^{2})	C_w (\$)	M_s (lbm)	C_s (\$)	C_t (\$)
20	375	701	5.64	0.250	106.2	\$531	1012	\$1,265	\$1,796
30	250	468	4.92	0.250	92.8	\$464	772	\$965	\$1,429
40	188	351	4.47	0.250	84.3	\$422	638	\$797	\$1,218
50	150	281	4.15	0.250	78.3	\$391	549	\$687	\$1,078
60	125	234	3.91	0.250	73.7	\$368	487	\$608	\$976
70	107	200	3.71	0.250	70.0	\$350	439	\$549	\$899
80	93.8	175	3.55	0.250	66.9	\$335	402	\$502	\$837
90	83.3	156	3.41	0.250	64.4	\$322	371	\$464	\$786
100	75.0	140	3.30	0.250	62.1	\$311	346	\$433	\$743
110	68.2	128	3.19	0.263	63.4	\$317	342	\$428	\$745
120	62.5	117	3.10	0.279	65.3	\$326	342	\$428	\$754
130	57.7	108	3.02	0.294	67.1	\$335	342	\$428	\$763

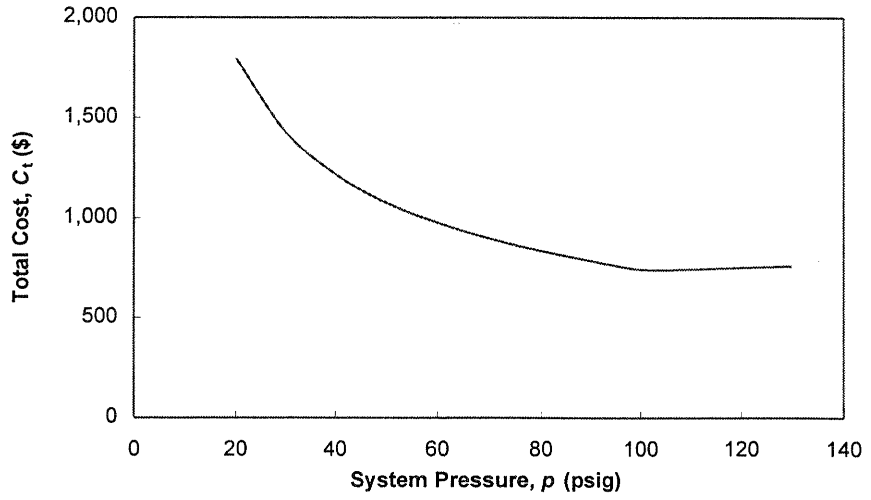


Fig. 1 Total cost versus system pressure

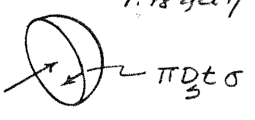
Sample Calculation (p = 20 psig):

$$W_t = p A_p \; ; \; A_p = \frac{W_t}{p} = 7500 \text{ lbf} \times \frac{\text{in.}^2}{20 \text{ lbf}} = 375 \text{ in.}^2$$

$$\forall_{oil} = A_p L = 375 \text{ in.}^2 \times \frac{1}{36 \text{ ft}} \times \frac{\text{ft}^2}{144 \text{ in.}^2} \times 7.48 \frac{\text{gal}}{\text{ft}^3} = 701 \text{ gal}$$

$$\forall_{oil} = \forall_s = \frac{4\pi R_s^3}{3} = \frac{\pi D_s^3}{6} \; ; \; D_s = \left(\frac{6\forall_{oil}}{\pi}\right)^{1/3} = \left(\frac{6}{\pi} \times 701 \text{ gal} \times \frac{\text{ft}^3}{7.48 \text{ gal}}\right)^{1/3} = 5.64 \text{ ft}$$

From a force balance on the sphere:



Problem 3.47

Thus $p\frac{\pi D_s^2}{4} = \pi D_s t \sigma$, so $t = \frac{p}{\sigma}\frac{D_s}{4} = \frac{1}{4} \times 20\frac{lbf}{in.^2} \times \frac{in.^2}{4000\,lbf} \times 5.64\,ft \times 12\frac{in.}{ft} = 0.0846\ in.$

Therefore $t = t_{min} = 0.250\ in.$

$$A_W = \pi D_s t = \pi \times 5.64\,ft \times 0.25\,in. \times 12\frac{in.}{ft} = 106\ in.^2$$

$$C_W = \frac{\$5.00}{in.^2} \times 106\,in.^2 = \$531$$

$$M_s = 4\pi R_s^2 t \rho_s = \pi D_s^2 t\, SG_s\, \rho_{H_2O} = \pi \times (5.64)^2 ft^2 \times 0.25\,in. \times 7.8 \times 62.4\frac{lbm}{ft^3} \times \frac{ft}{12\,in.} = 1012\,lbm$$

$$C_s = \frac{\$1.25}{lbm} \times 1012\,lbm = \$1265$$

and

$$C_t = C_W + C_s = \$531 + \$1265 = \$1,796 \longleftarrow C_t$$

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Problem 3.48

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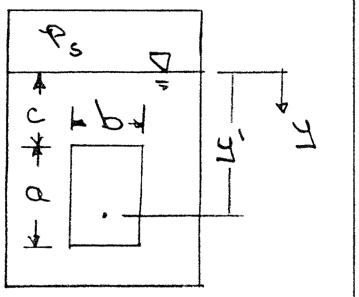


Given: Door located in plane vertical wall of water tank as shown
$a = 1.5\,m$, $b = 1\,m$, $c = 1\,m$.
Atmospheric pressure acts on outer surface of door

Find: (a) For $p_s = p_{atm}$, resultant force on door and line of action of force
(b) Resultant force and line of action if $p_s = 0.3\,atm\,(gage)$

Plot: F/F_o and y'/y_c over range of p_s/p_{atm}. (F_o is resultant force when $p_s = p_{atm}$; y_c is y coordinate of centroid)

Solution:

Basic equations: $\frac{dp}{dy} = \rho g$; $F_R = \int_A p\,dA$; $y' F_R = \int y p\,dA$

Assumptions: (1) static liquid
(2) incompressible liquid

Note: We will obtain a general expressions for F and y' (needed for the plot) and then simply for cases (a) & (b).

Since $dp = \rho g\,dy$ then $p = p_s + \rho g y$

Because p_{atm} acts on the outside of the door, then p_s is the surface gage pressure

$$F_R = \int p\,dA = \int_c^{(c+a)} p\,b\,dy = \int_c^{c+a} (p_s + \rho g y)\,b\,dy = b\left[p_s y + \rho g \frac{y^2}{2}\right]_c^{c+a}$$

$$F_R = b\left[p_s a + \frac{\rho g}{2}\left\{(c+a)^2 - c^2\right\}\right] = b\left[p_s a + \frac{\rho g}{2}(a^2 + 2ac)\right] \text{ ------ (1)}$$

$$y' F_R = \int y p\,dA \quad \text{and} \quad y' = \frac{1}{F_R}\int_c^{c+a} y(p_s + \rho g y)\,b\,dy$$

$$y' = \frac{b}{F_R}\left[p_s \frac{y^2}{2} + \rho g \frac{y^3}{3}\right]_c^{c+a}$$

$$y' = \frac{b}{F_R}\Big[\frac{p_s}{2}\left\{(c+a)^2 - c^2\right\} + \frac{\rho g}{3}\left\{(c+a)^3 - c^3\right\} \text{ ---------- (2)}$$

(a) For $p_s = 0$ (gage) then

from Eq. 1 $\quad F_R = \frac{\rho g b}{2}(a^2 + 2ac)$.

$$F_R = \frac{1}{2} \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 1\,m\left[(1.5\,m)^2 + 2(1.5\,m)(1\,m)\right]\frac{N \cdot s^2}{kg \cdot m} = 25.7\,kN \longleftarrow F_{R_o}$$

From Eq. 2

$$y' = \frac{b}{F_{R_o}}\frac{\rho g}{3}\left[(c+a)^3 - c^3\right]$$

$$y' = \frac{1\,m}{25.7\,kN} \times \frac{999}{3}\frac{kg}{m^3} \times 9.81\frac{m}{s^2}\left[(2.5)^3 - 1\right]m^3\,\frac{N \cdot s^2}{kg \cdot m} \times \frac{kN}{10^3\,N} = 1.86\,m \longleftarrow y'_o$$

(b) For $p_s = 0.3$ atm (gage) then

from Eq. 1 $\quad F_R = b\left[p_s a + \frac{\rho g}{2}(a^2+2ac)\right]$

$$F_R = 1\,m\left[0.3\,atm \times 1.01\times10^5\,\frac{N}{m^2\cdot atm}(1.5\,m) + \frac{1}{2}\times 999\,\frac{kg}{m^3}\times 9.81\,\frac{m}{s^2}\left\{(1.5)^2+2(1.5)(1)\right\}m^2\cdot\frac{N\cdot s^2}{kg\cdot m}\right.$$

$F_R = 71.2\,kN$ $\quad F_R$

$$y' = \frac{b}{F_R}\left[\frac{p_s}{2}\left\{(c+a)^2-c^2\right\}+\frac{\rho g}{3}\left\{(c+a)^3-c^3\right\}\right.$$

$$y' = \frac{1\,m}{71.2\,kN}\left[\frac{1}{2}\times 0.3\,atm\times1.01\times10^5\,\frac{N}{m^2\cdot atm}\left\{(2.5)^2-1\right\}+\frac{1}{3}\times 999\,\frac{kg}{m^3}\times 9.81\,\frac{m}{s^2}\times\left\{(2.5)^3-1\right\}m^3 \times\frac{N\cdot s^2}{kg\cdot m}\right]\frac{kN}{10^3\,N}$$

$y' = 1.79\,m$ $\quad y'$

The value of F/F_0 is obtained from Eq. 1 and $F_{R_0} = 25.7\,kN$.

$$\frac{F}{F_0} = \frac{1}{25.7\,kN}\,b\left[p_s a + \frac{\rho g}{2}(a^2+2ac)\right] = 0.0389\left[151.5\,p_s + 25.7\right]$$

with p_s in atm. $\quad F/F_0$

For the gate $y_c = c + \frac{a}{2} = 1.75\,m$. Then from Eq. 2

$$\frac{y'}{y_c} = \frac{b}{F_R(1.75)}\left[\frac{p_s}{2}\left\{(c+a)^2-c^2\right\}+\frac{\rho g}{3}\left\{(c+a)^3-c^3\right\}\right] = \frac{0.571}{F}\left[265\,p_s + 47.8\right]$$

with F in kN, p_s in atm $\quad y'/y_c$

The plots are shown below

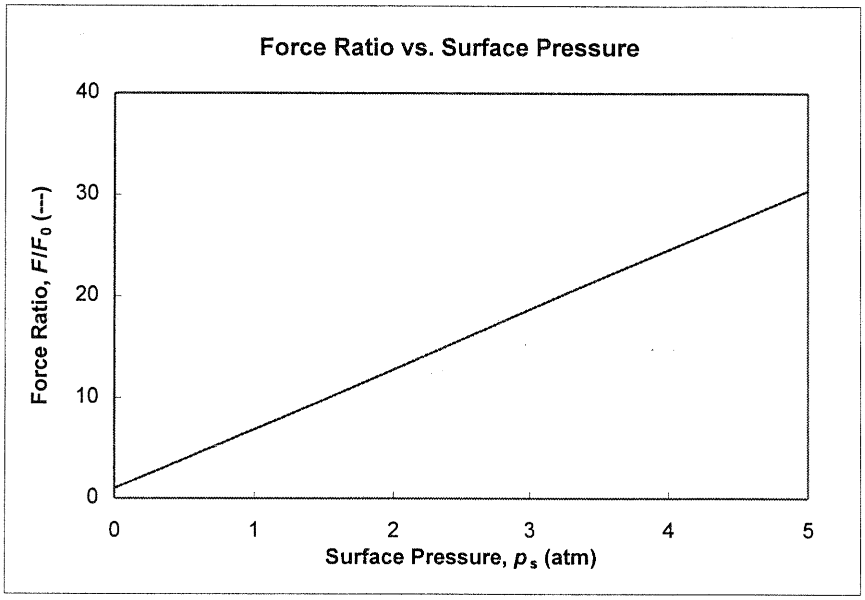
Note: The force on the gate increases linearly with increase in surface pressure.

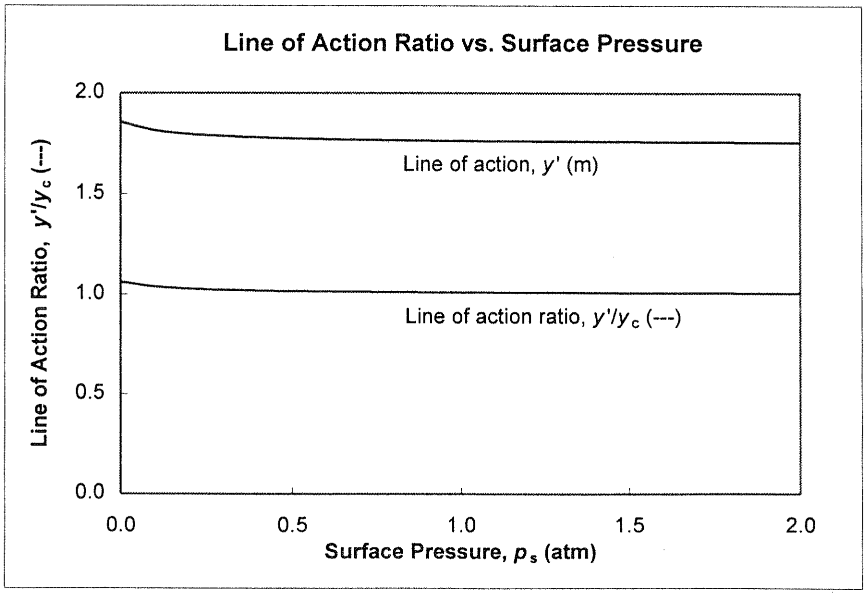
The line of action of the resultant force is always below the centroid of the gate; y'/y_c approaches unity as the surface pressure is increased.

Problem 3.48

Force ratio and line of action ratio vs. surface pressure:

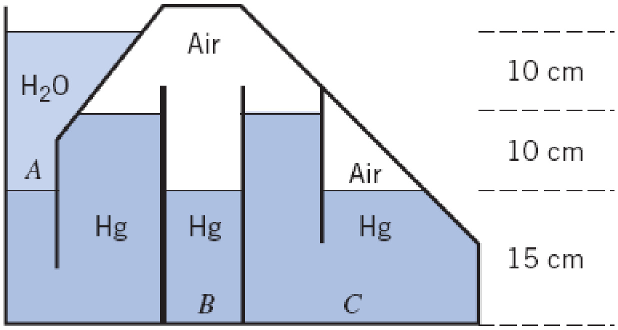
Surface Pressure, p_s (atm)	Force Ratio, F/F_0 (---)	Force, F_0 (kN)	Line of Action Ratio, y'/y_c (---)	Line of Action, y' (m)
0	1.00	25.7	1.0623	1.86
0.1	1.59	40.8	1.0388	1.82
0.2	2.18	56.0	1.0281	1.80
0.3	2.77	71.1	1.0219	1.79
0.5	3.95	101	1.0151	1.78
1.0	6.89	177	1.00822	1.76
2.0	12.8	329	1.00399	1.76
3.0	18.7	480		
4.0	24.6	632		
5.0	30.5	783		





Problem 3.49

3.49 Find the pressures at points A, B, and C, as shown, and in the two air cavities.



Given: Geometry of chamber system

Find: Pressure at various locations

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot \Delta h$ where Δh is height difference

For point A $p_A = p_{atm} + \rho \cdot g \cdot h_1$ or in gage pressure $p_A = \rho \cdot g \cdot h_1$

Here we have $h_1 = 20 \cdot cm$ $h_1 = 0.2\,m$

$$p_A = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.2 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_A = 1962\,Pa \qquad p_A = 1.96 \cdot kPa \quad \text{(gage)}$$

For the air cavity $p_{air} = p_A - SG_{Hg} \cdot \rho \cdot g \cdot h_2$ where $h_2 = 10 \cdot cm$ $h_2 = 0.1\,m$

From Table A.1 $SG_{Hg} = 13.55$

$$p_{air} = 1962 \cdot \frac{N}{m^2} - 13.55 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.1 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{air} = -11.3 \cdot kPa \quad \text{(gage)}$$

Note that p = constant throughout the air pocket

For point B $p_B = p_{atm} + SG_{Hg} \cdot \rho \cdot g \cdot h_3$ where $h_3 = 15 \cdot cm$ $h_3 = 0.15\,m$

$$p_B = -11300 \cdot \frac{N}{m^2} + 13.55 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.15 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_B = 8.64 \cdot kPa \quad \text{(gage)}$$

For point C $p_C = p_{atm} + SG_{Hg} \cdot \rho \cdot g \cdot h_4$ where $h_4 = 25 \cdot cm$ $h_4 = 0.25\,m$

$$p_C = -11300 \cdot \frac{N}{m^2} + 13.55 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.25 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_C = 21.93 \cdot kPa \quad \text{(gage)}$$

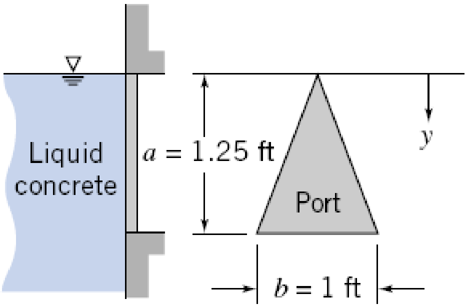
For the second air cavity $p_{air} = p_C - SG_{Hg} \cdot \rho \cdot h_5$ where $h_5 = 15 \cdot cm$ $h_5 = 0.15\,m$

$$p_{air} = 21930 \cdot \frac{N}{m^2} - 13.55 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.15 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{air} = 1.99 \cdot kPa \quad \text{(gage)}$$

Problem 3.50

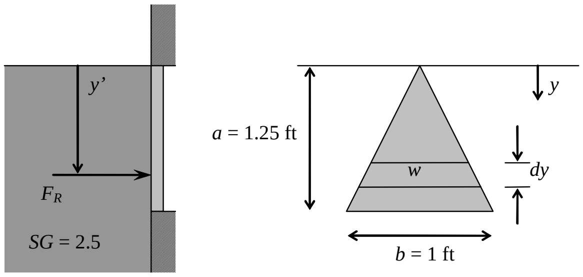
[2]

3.50 A triangular access port must be provided in the side of a form containing liquid concrete. Using the coordinates and dimensions shown, determine the resultant force that acts on the port and its point of application.



Given: Geometry of access port

Find: Resultant force and location



Solution:

Basic equation $F_R = \int p\,dA \qquad \frac{dp}{dy} = \rho\cdot g \qquad \Sigma M_s = y'\cdot F_R = \int y\,dF_R = \int y\cdot p\,dA$

or, use computing equations $F_R = p_c\cdot A \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c}$

We will show both methods

Assumptions: static fluid; ρ = constant; p_{atm} on other side

$$F_R = \int p\,dA = \int SG\cdot\rho\cdot g\cdot y\,dA \quad \text{but} \quad dA = w\cdot dy \quad \text{and} \quad \frac{w}{b} = \frac{y}{a} \qquad w = \frac{b}{a}\cdot y$$

Hence $F_R = \int_0^a SG\cdot\rho\cdot g\cdot y\cdot\frac{b}{a}\cdot y\,dy = \int_0^a SG\cdot\rho\cdot g\cdot\frac{b}{a}\cdot y^2\,dy = \frac{SG\cdot\rho\cdot g\cdot b\cdot a^2}{3}$

Alternatively $F_R = p_c\cdot A$ and $p_c = SG\cdot\rho\cdot g\cdot y_c = SG\cdot\rho\cdot g\cdot\frac{2}{3}\cdot a$ with $A = \frac{1}{2}\cdot a\cdot b$

Hence $F_R = \frac{SG\cdot\rho\cdot g\cdot b\cdot a^2}{3}$

For y' $y'\cdot F_R = \int y\cdot p\,dA = \int_0^a SG\cdot\rho\cdot g\cdot\frac{b}{a}\cdot y^3\,dy = \frac{SG\cdot\rho\cdot g\cdot b\cdot a^3}{4} \qquad y' = \frac{SG\cdot\rho\cdot g\cdot b\cdot a^3}{4\cdot F_R} = \frac{3}{4}\cdot a$

Alternatively $y' = y_c + \frac{I_{xx}}{A\cdot y_c}$ and $I_{xx} = \frac{b\cdot a^3}{36}$ (Google it!)

$$y' = \frac{2}{3}\cdot a + \frac{b\cdot a^3}{36}\cdot\frac{2}{a\cdot b}\cdot\frac{3}{2\cdot a} = \frac{3}{4}\cdot a$$

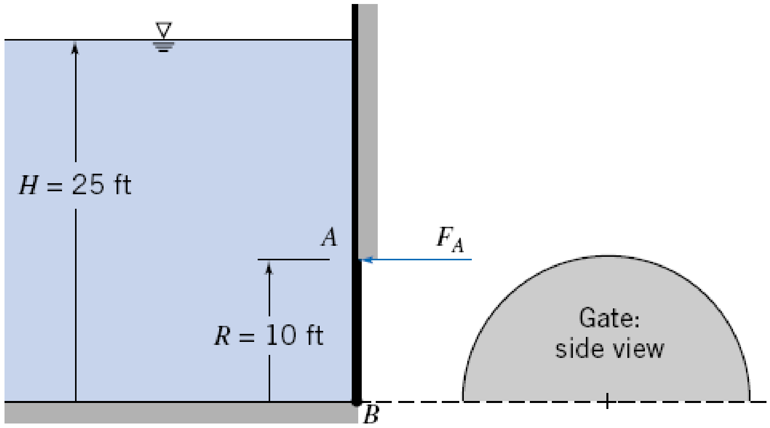
Using given data, and SG = 2.5 (Table A.1) $F_R = \frac{2.5}{3}\cdot 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times 32.2\cdot\frac{\text{ft}}{\text{s}^2}\times 1\cdot\text{ft}\times(1.25\cdot\text{ft})^2\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \qquad F_R = 81.3\cdot\text{lbf}$

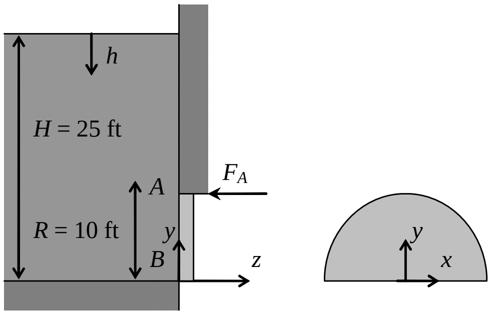
and $y' = \frac{3}{4}\cdot a \qquad y' = 0.938\cdot\text{ft}$

Problem 3.51

[3]

3.51 Semicircular plane gate *AB* is hinged along *B* and held by horizontal force F_A applied at *A*. The liquid to the left of the gate is water. Calculate the force F_A required for equilibrium.





Given: Geometry of gate

Find: Force F_A for equilibrium

Solution:

Basic equation $$F_R = \int p\,dA \qquad \frac{dp}{dh} = \rho \cdot g \qquad \Sigma M_z = 0$$

or, use computing equations $$F_R = p_c \cdot A \qquad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$$ where y would be measured from the free surface

Assumptions: static fluid; ρ = constant; p_{atm} on other side; door is in equilibrium

Instead of using either of these approaches, we note the following, using y as in the sketch

$$\Sigma M_z = 0 \qquad F_A \cdot R = \int y \cdot p\,dA \quad \text{with} \quad p = \rho \cdot g \cdot h$$

(Gage pressure, since p = p_{atm} on other side)

$$F_A = \frac{1}{R} \cdot \int y \cdot \rho \cdot g \cdot h\,dA \quad \text{with} \quad dA = r \cdot dr \cdot d\theta \quad \text{and} \quad y = r \cdot \sin(\theta) \quad h = H - y$$

Hence $$F_A = \frac{1}{R} \cdot \int_0^{\pi} \int_0^{R} \rho \cdot g \cdot r \cdot \sin(\theta) \cdot (H - r \cdot \sin(\theta)) \cdot r\,dr\,d\theta = \frac{\rho \cdot g}{R} \cdot \int_0^{\pi} \left(\frac{H \cdot R^3}{3} \cdot \sin(\theta) - \frac{R^4}{4} \cdot \sin(\theta)^2 \right) d\theta$$

$$F_R = \frac{\rho \cdot g}{R} \cdot \left(\frac{2 \cdot H \cdot R^3}{3} - \frac{\pi \cdot R^4}{8} \right) = \rho \cdot g \cdot \left(\frac{2 \cdot H \cdot R^2}{3} - \frac{\pi \cdot R^3}{8} \right)$$

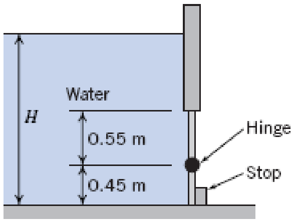
Using given data $$F_R = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times \left[\frac{2}{3} \times 25 \cdot \text{ft} \times (10 \cdot \text{ft})^2 - \frac{\pi}{8} \times (10 \cdot \text{ft})^3 \right] \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$$F_R = 7.96 \times 10^4 \cdot \text{lbf}$$

Problem 3.52

[3]

3.52 A rectangular gate (width $w = 2$ m) is hinged as shown, with a stop on the lower edge. At what depth H will the gate tip?



Given: Gate geometry

Find: Depth *H* at which gate tips

Solution:

This is a problem with atmospheric pressure on both sides of the plate, so we can first determine the location of the center of pressure with respect to the free surface, using Eq.3.11c (assuming depth *H*)

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} \qquad \text{and} \qquad I_{xx} = \frac{w \cdot L^3}{12} \qquad \text{with} \qquad y_c = H - \frac{L}{2}$$

where $L = 1$ m is the plate height and w is the plate width

Hence $$y' = \left(H - \frac{L}{2}\right) + \frac{w \cdot L^3}{12 \cdot w \cdot L \cdot \left(H - \frac{L}{2}\right)} = \left(H - \frac{L}{2}\right) + \frac{L^2}{12 \cdot \left(H - \frac{L}{2}\right)}$$

But for equilibrium, the center of force must always be at or below the level of the hinge so that the stop can hold the gate in place. Hence we must have

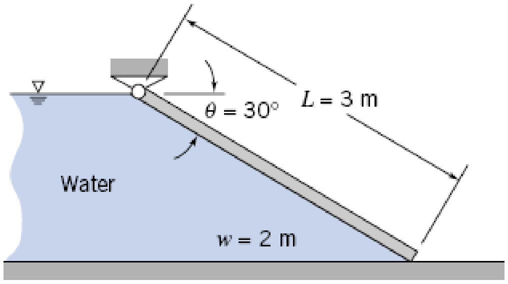
$$y' > H - 0.45 \cdot m$$

Combining the two equations $$\left(H - \frac{L}{2}\right) + \frac{L^2}{12 \cdot \left(H - \frac{L}{2}\right)} \geq H - 0.45 \cdot m$$

Solving for *H* $$H \leq \frac{L}{2} + \frac{L^2}{12 \cdot \left(\frac{L}{2} - 0.45 \cdot m\right)} \qquad H \leq \frac{1 \cdot m}{2} + \frac{(1 \cdot m)^2}{12 \times \left(\frac{1 \cdot m}{2} - 0.45 \cdot m\right)} \qquad H \leq 2.17 \cdot m$$

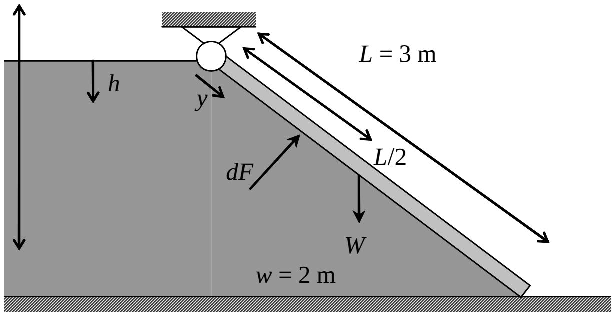
Problem 3.53

3.53 A plane gate of uniform thickness holds back a depth of water as shown. Find the minimum weight needed to keep the gate closed.



Given: Geometry of plane gate

Find: Minimum weight to keep it closed



Solution:

Basic equation $\quad F_R = \int p\, dA \qquad \frac{dp}{dh} = \rho \cdot g \qquad \Sigma M_O = 0$

or, use computing equations $\quad F_R = p_c \cdot A \qquad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$

Assumptions: static fluid; ρ = constant; p_{atm} on other side; door is in equilibrium

Instead of using either of these approaches, we note the following, using y as in the sketch

$$\Sigma M_O = 0 \qquad W \cdot \frac{L}{2} \cdot \cos(\theta) = \int y\, dF$$

We also have $\quad dF = p \cdot dA \quad$ with $\quad p = \rho \cdot g \cdot h = \rho \cdot g \cdot y \cdot \sin(\theta) \qquad$ (Gage pressure, since $p = p_{atm}$ on other side)

Hence
$$W = \frac{2}{L \cdot \cos(\theta)} \cdot \int y \cdot p\, dA = \frac{2}{L \cdot \cos(\theta)} \cdot \int y \cdot \rho \cdot g \cdot y \cdot \sin(\theta) \cdot w\, dy$$

$$W = \frac{2}{L \cdot \cos(\theta)} \cdot \int y \cdot p\, dA = \frac{2 \cdot \rho \cdot g \cdot w \cdot \tan(\theta)}{L} \cdot \int_0^L y^2\, dy = \frac{2}{3} \cdot \rho \cdot g \cdot w \cdot L^2 \cdot \tan(\theta)$$

Using given data
$$W = \frac{2}{3} \cdot 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 2 \cdot m \times (3 \cdot m)^2 \times \tan(30 \cdot deg) \times \frac{N \cdot s^2}{kg \cdot m} \qquad W = 68 \cdot kN$$

Problem 3.54

13-782 500 SHEETS FILLER 5 SQUARE
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Given: Semi-cylindrical trough, partly filled with water to depth d.

Find: (a) General expressions for F_R and y' on end of trough, if open to atmosphere.
(b) Plots of results vs. d/R for $0 \le d/R \le 1$.

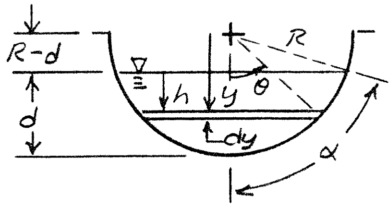
Solution: Apply basic equations for hydrostatics of incompressible liquid.

Computing equations: $p = \rho g h \quad F_R = \int_A p\,dA \quad y'F_R = \int_A y\,p\,dA$

Assumptions: (1) Static liquid
(2) ρ = constant

$$p = \rho g h = \rho g[y-(R-d)]$$

$$= \rho g R\left[\frac{y}{R} - \left(1-\frac{d}{R}\right)\right] = \rho g R(\cos\theta - \cos\alpha)$$



$h = y - (R-d)$

$\cos\alpha = \frac{R-d}{R} = 1 - \frac{d}{R}$

$w = 2R\sin\theta$

$dA = w\,dy = 2R\sin\theta\,dy \; ; \; y = R\cos\theta$

$dy = -R\sin\theta\,d\theta$

$$F_R = \int_{R-d}^{R} p\,w\,dy = \int_{R-d}^{R} \rho g R(\cos\theta - \cos\alpha)\,2R\sin\theta\,(-R\sin\theta)\,d\theta$$

The new limits are $y = R \rightarrow \theta = 0$ and $y = R-d \rightarrow \theta = \alpha$, so

$$F_R = 2\rho g R^3 \int_\alpha^0 (-\sin^2\theta\cos\theta + \sin^2\theta\cos\alpha)\,d\theta = 2\rho g R^3 \int_0^\alpha (\sin^2\theta\cos\theta - \sin^2\theta\cos\alpha)\,d\theta$$

$$= 2\rho g R^3\left[\frac{\sin^3\theta}{3} - \cos\alpha\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_0^\alpha = 2\rho g R^3\left[\frac{\sin^3\theta}{3} - \cos\alpha\left(\frac{\theta}{2} - \frac{\sin\theta\cos\theta}{2}\right)\right]_0^\alpha$$

$$F_R = 2\rho g R^3\left[\frac{\sin^3\alpha}{3} - \cos\alpha\left(\frac{\alpha}{2} - \frac{\sin\alpha\cos\alpha}{2}\right)\right]$$

F_R

$$y'F_R = \int_{R-d}^{R} y\,p\,w\,dy = \int_{R-d}^{R} R\cos\theta\,\rho g R\,(\cos\theta - \cos\alpha)\,2R\sin\theta(-R\sin\theta)\,d\theta$$

$$= 2\rho g R^4 \int_0^\alpha \sin^2\theta\cos\theta(\cos\theta - \cos\alpha)\,d\theta = 2\rho g R^4 \int_0^\alpha (\sin^2\theta\cos^2\theta - \cos\alpha\sin^2\theta\cos\theta)\,d\theta$$

$$= 2\rho g R^4\left[\frac{1}{8}\left(\theta - \frac{\sin 4\theta}{4}\right) - \cos\alpha\,\frac{\sin^3\theta}{3}\right]_0^\alpha$$

$$y'F_R = 2\rho g R^4\left[\frac{1}{8}\left(\alpha - \frac{\sin 4\alpha}{4}\right) - \cos\alpha\,\frac{\sin^3\alpha}{3}\right]$$

$y'F_R$

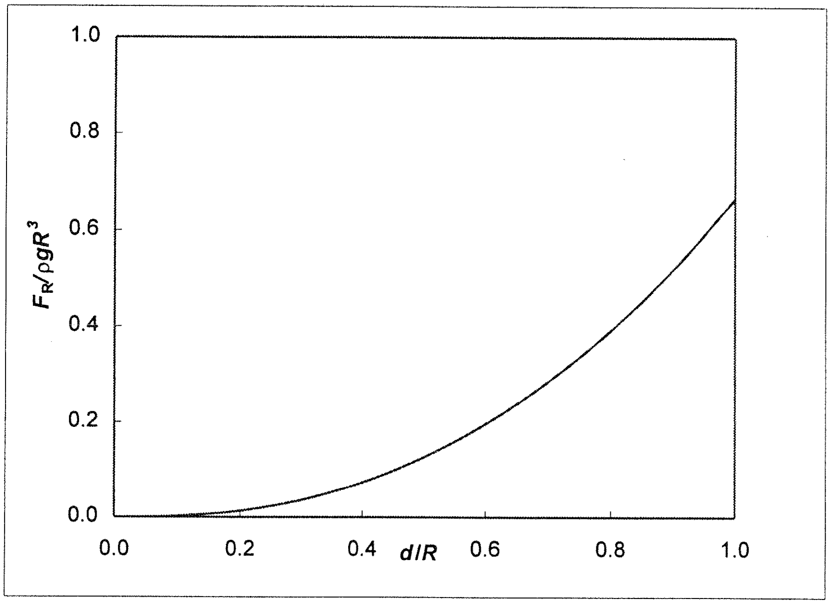
and $y' = \frac{y'F_R}{F_R}$ or $y'/R = \frac{y'F_R}{R F_R}$

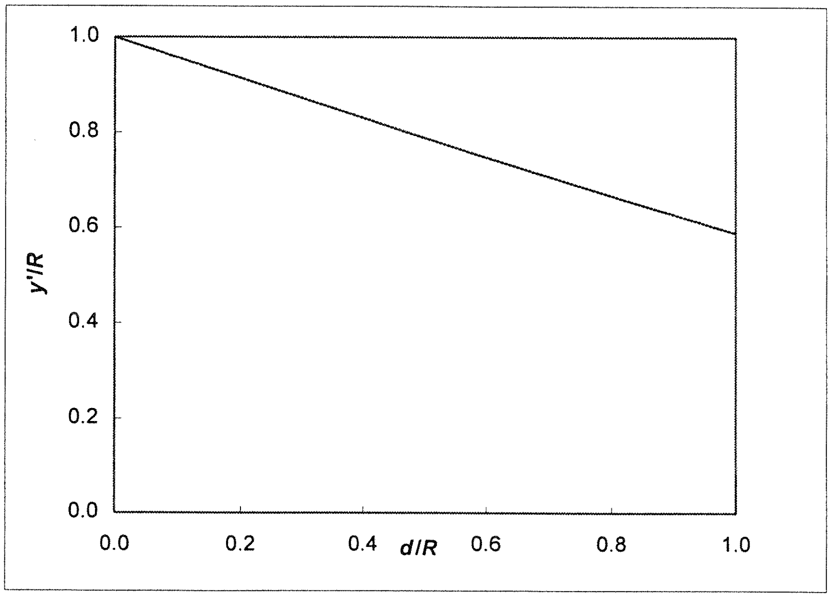
y'

Problem 3.54

Resultant force and line of action on end of semi-cylindrical water trough:

d/R	α (rad)	α (deg)	$F_R/\rho g R^3$	$y'F_R/\rho g R^4$	y'/R
0	0.001	0.08	7.54E-16	7.54E-16	1.000
0.05	0.318	18.2	0.000419	0.000410	0.979
0.1	0.451	25.8	0.00236	0.00226	0.957
0.2	0.644	36.9	0.0132	0.0121	0.915
0.3	0.795	45.6	0.0360	0.0314	0.873
0.4	0.927	53.1	0.0730	0.0606	0.831
0.5	1.05	60.0	0.126	0.0994	0.790
0.6	1.16	66.4	0.196	0.147	0.749
0.7	1.27	72.5	0.285	0.202	0.708
0.8	1.37	78.5	0.392	0.262	0.668
0.9	1.47	84.3	0.520	0.326	0.628
1.0	1.57	90.0	0.667	0.393	0.589





Problem 3.55

3.55 For a mug of tea (2½ in. diameter), imagine it cut symmetrically in half by a vertical plane. Find the force that each half experiences due to a 3-in. depth of tea.

Given: Geometry of cup

Find: Force on each half of cup

Solution:

Basic equation $F_R = \int p\,dA \qquad \frac{dp}{dh} = \rho \cdot g$

or, use computing equation $F_R = p_c \cdot A$

Assumptions: static fluid; ρ = constant; p_{atm} on other side; cup does not crack!

The force on the half-cup is the same as that on a rectangle of size $h = 3 \cdot in$ and $w = 2.5 \cdot in$

$$F_R = \int p\,dA = \int \rho \cdot g \cdot y\,dA \qquad \text{but} \qquad dA = w \cdot dy$$

Hence $F_R = \int_0^h \rho \cdot g \cdot y \cdot w\,dy = \frac{\rho \cdot g \cdot w \cdot h^2}{2}$

Alternatively $F_R = p_c \cdot A$ and $F_R = p_c \cdot A = \rho \cdot g \cdot y_c \cdot A = \rho \cdot g \cdot \frac{h}{2} \cdot h \cdot w = \frac{\rho \cdot g \cdot w \cdot h^2}{2}$

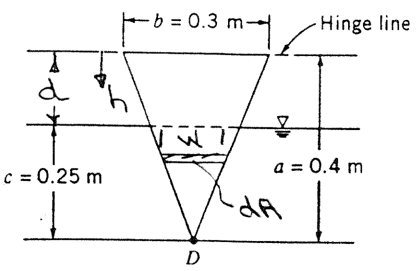
Using given data $F_R = \frac{1}{2} \cdot 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times 2.5 \cdot in \times (3 \cdot in)^2 \times \left(\frac{1 \cdot ft}{12 \cdot in}\right)^3 \times \frac{lbf \cdot s^2}{slug \cdot ft} \qquad F_R = 0.407 \cdot lbf$

Hence a teacup is being forced apart by about 0.4 lbf: not much of a force, so a paper cup works!

Problem 3.56

[4] Part 1/2

Given: Window, in shape of isosceles triangle and hinged at the top is located in the vertical wall of a form that contains concrete.



Find: the minimum force applied at D needed to keep the window closed.

Plot: the results over the range of concrete depth $0 \le c \le a$

Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $F = \int p\,dA$, $\Sigma M = 0$

Assumptions: (1) static fluid (2) ρ = constant

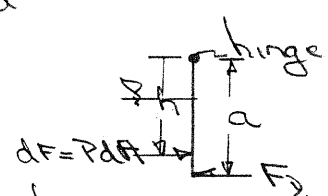
(3) p_{atm} acts at the free surface and on the outside of the window.

Then $dp = \rho g\,dh$ gives $p = \rho g(h-d)$ for $h > d$

and $p = 0$ for $h < d$

where $d = a - c$

Summing moments about the hinge



$$F_D = \frac{1}{a}\int h\,p\,dA = \frac{1}{a}\int_d^a h\,\rho g(h-d)\,w\,dh$$

From the law of similar triangles $\frac{w}{b} = \frac{a-h}{a}$; $w = \frac{b}{a}(a-h)$

$$F_D = \frac{b}{a^2}\rho g\int_d^a h(h-d)(a-h)\,dh$$ $\{\rho = SG_{concrete}\,\rho_{H_2O}\}$

$$F_D = \frac{b}{a^2}\rho g\int_d^a \left[-h^3 + h^2(a+d) - adh\right] dh$$

$$F_D = \frac{b}{a^2}\rho g\left[-\frac{h^4}{4} + \frac{h^3}{3}(a+d) - \frac{1}{2}adh^2\right]_d^a$$

$$F_D = \frac{b}{a^2}\rho g\left[-\frac{1}{4}(a^4-d^4) + \frac{1}{3}(a^3-d^3)(a+d) - \frac{1}{2}ad(a^2-d^2)\right]$$

$$F_D = b\rho g a^2\left[-\frac{1}{4}\left(1-\frac{d^4}{a^4}\right) + \frac{1}{3}\left(1-\frac{d^3}{a^3}\right)\left(1+\frac{d}{a}\right) - \frac{1}{2}\frac{d}{a}\left(1-\frac{d^2}{a^2}\right)\right] \quad (1)$$

Evaluating with $\rho = SG_{conc}\,\rho_{H_2O}$ (SG = 2.5 - Table A.1)

$$b\rho g a^2 = 0.3\,m \times 2.5\times 10^3\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (0.4)^2\,m^2 \times \frac{N\cdot s^2}{kg\cdot m} = 1177\,N$$

For $a = 0.4\,m$, $c = 0.25\,m$, $d = a - c = 0.15\,m$, $\frac{d}{a} = 0.375$

The term [ ] in Eq. 1 has a value of 0.0280

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Problem 3.56

[4] Part 2/2

Then for the conditions given

$F_D = 1177\,N \times 0.0280 = 33.0\,N$

To plot F_D vs c/a for $0 \le c \le a$, recognize

· Since $d = a - c$, then $\frac{d}{a} = 1 - \frac{c}{a}$

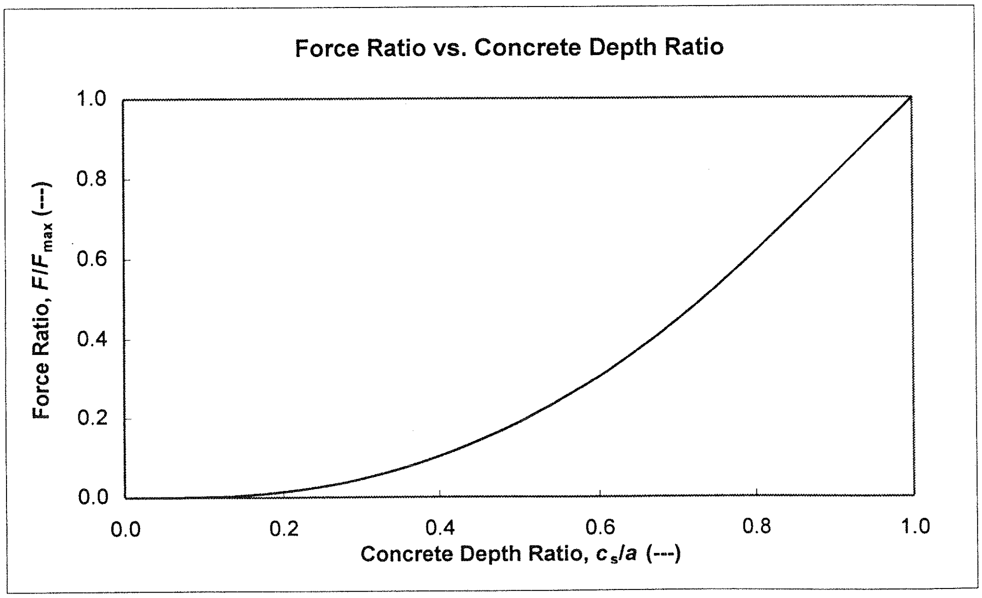
and

$$F_D = 1177\,N\left[-\frac{1}{4}\left\{1-\left(\frac{d}{a}\right)^4\right\} + \frac{1}{3}\left\{1-\left(\frac{d}{a}\right)^3\right\}\left(1+\frac{d}{a}\right) - \frac{1}{2}\frac{d}{a}\left\{1-\left(\frac{d}{a}\right)^2\right\}\right]$$

The results are plotted below

Hinge force vs. concrete depth ratio:

Depth Ratio, c/a (---)	Depth Ratio, d/a (---)	Force Ratio, F/F_{max} (---)
0	1.0	0.0000
0.1	0.9	0.0019
0.2	0.8	0.0144
0.3	0.7	0.0459
0.4	0.6	0.102
0.5	0.5	0.187
0.6	0.4	0.302
0.625	0.375	0.336
0.7	0.3	0.446
0.8	0.2	0.614
0.9	0.1	0.802
1.0	0.0	1.000



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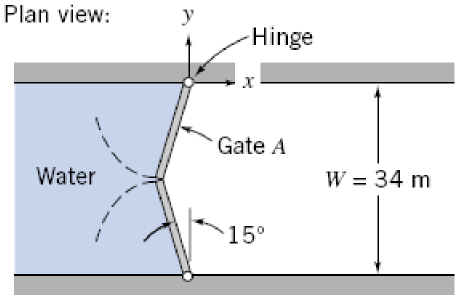
Problem 3.57

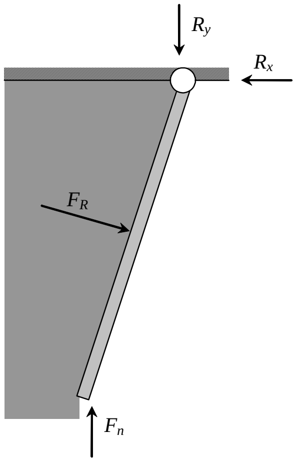
[3]

3.57 Gates in the Poe Lock at Sault Ste. Marie, Michigan, close a channel $W = 34$ m wide, $L = 360$ m long, and $D = 10$ m deep. The geometry of one pair of gates is shown; each gate is hinged at the channel wall. When closed, the gate edges are forced together at the center of the channel by water pressure. Evaluate the force exerted by the water on gate A. Determine the magnitude and direction of the force components exerted by the gate on the hinge. (Neglect the weight of the gate.)



Given: Geometry of lock system

Find: Force on gate; reactions at hinge



Solution:

Basic equation $\quad F_R = \int p\,dA \qquad \dfrac{dp}{dh} = \rho\cdot g$

or, use computing equation $\quad F_R = p_c\cdot A$

Assumptions: static fluid; ρ = constant; p_{atm} on other side

The force on each gate is the same as that on a rectangle of size $\quad h = D = 10\cdot m \quad$ and $\quad w = \dfrac{W}{2\cdot\cos(15\cdot deg)}$

$$F_R = \int p\,dA = \int \rho\cdot g\cdot y\,dA \qquad \text{but} \qquad dA = w\cdot dy$$

Hence $\quad F_R = \int_0^h \rho\cdot g\cdot y\cdot w\,dy = \dfrac{\rho\cdot g\cdot w\cdot h^2}{2}$

Alternatively $\quad F_R = p_c\cdot A \quad$ and $\quad F_R = p_c\cdot A = \rho\cdot g\cdot y_c\cdot A = \rho\cdot g\cdot\dfrac{h}{2}\cdot h\cdot w = \dfrac{\rho\cdot g\cdot w\cdot h^2}{2}$

Using given data $\quad F_R = \dfrac{1}{2}\cdot 1000\cdot\dfrac{kg}{m^3}\times 9.81\cdot\dfrac{m}{s^2}\times\dfrac{34\cdot m}{2\cdot\cos(15\cdot deg)}\times(10\cdot m)^2\times\dfrac{N\cdot s^2}{kg\cdot m} \qquad F_R = 8.63\cdot MN$

For the force components R_x and R_y we do the following

$$\Sigma M_{hinge} = 0 = F_R\cdot\frac{w}{2} - F_n\cdot w\cdot\sin(15\cdot deg) \qquad F_n = \frac{F_R}{2\cdot\sin(15\cdot deg)} \qquad F_n = 16.7\cdot MN$$

$$\Sigma F_x = 0 = F_R\cdot\cos(15\cdot deg) - R_x = 0 \qquad R_x = F_R\cdot\cos(15\cdot deg) \qquad R_x = 8.34\cdot MN$$

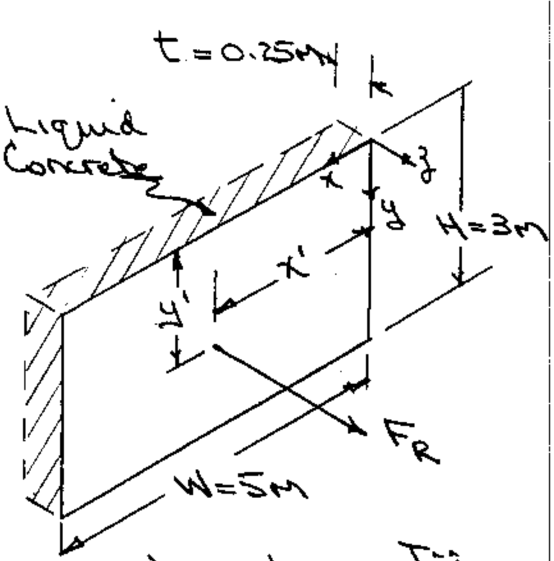
$$\Sigma F_y = 0 = -R_y - F_R\cdot\sin(15\cdot deg) + F_n = 0 \qquad R_y = F_n - F_R\cdot\sin(15\cdot deg) \qquad R_y = 14.4\cdot MN$$

$$R = (8.34\cdot MN, 14.4\cdot MN) \qquad R = 16.7\cdot MN$$

Problem 3.58

[2]

Given: Liquid concrete poured between vertical forms as shown



Find: (a) Resultant force on form
(b) Line of application

Solution:

Basic equation: $\frac{dp}{dy} = \rho g$

Computing equations:

$F_R = p_c A$ (3.14) ; $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c}$ (3.15a) ; $x' = x_c + \frac{I_{\hat{x}\hat{y}}}{A y_c}$

For the rectangular plate: $x_c = 2.5\,m$, $y_c = 1.5\,m$.

$I_{\hat{x}\hat{x}} = \frac{1}{12} W H^3$, $I_{\hat{x}\hat{y}} = 0$

Assumptions: (1) static liquid (2) incompressible liquid
(3) p_{atm} acts at free surface and on the vertical form.

Then on integrating $dp = \rho g\,dy$, we obtain $p = \rho g y$

$F_R = p_c A = \rho g y_c A = \rho g y_c W H = SG_{conc}\, \rho_{H_2O}\, y_c W H$

$F_R = 2.5 \times 10^3 \frac{\cancel{kg}}{\cancel{m^3}} \times 9.81 \frac{m}{s^2} \times 1.5\,m \times 5\,m \times 3\,m \times \frac{N \cdot s^2}{\cancel{kg} \cdot m}$ {SG = 2.5, Table A.1}

$F_R = 552\,kN \longleftarrow F_R$

$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c} = y_c + \frac{1}{12}\frac{W H^3}{W H y_c} = y_c + \frac{1}{12}\frac{H^2}{y_c} = 1.5\,m + \frac{1}{12}\frac{(3\,m)^2}{1.5\,m} = 2.0\,m$

$x' = x_c = 2.5\,m$

Line of application is through $(x', y') = (2.5, 2.0)\,m \longleftarrow (x', y')$

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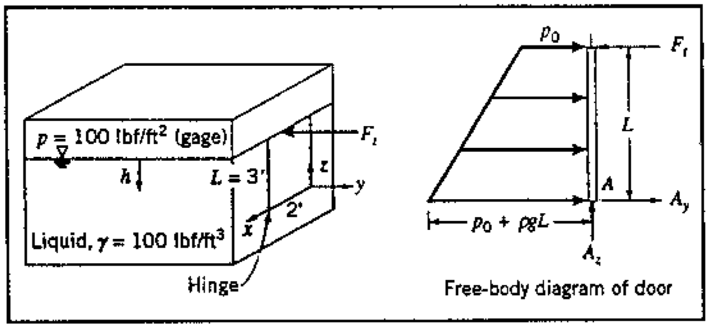
Problem 3.59

[2]

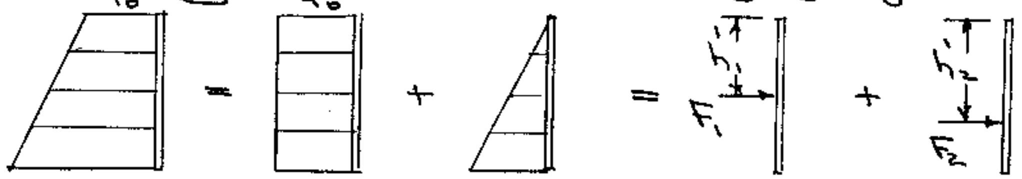
Given: Door as shown in the figure; x axis is along the hinge
From Ex. Prob 3.6, pressure in liquid is $p = p_{0\,gage} + \rho g h$.


$p = 100$ lbf/ft^2 (gage)
h
$L = 3'$
z
y
x
$2'$
F_t
Liquid, $\gamma = 100$ lbf/ft^3
Hinge
p_0
F_t
L
A
A_y
$p_0 + \rho g L$
A_z
Free-body diagram of door

Find: Force required to keep door shut (by considering the distributed force to be the sum of a force F_1 caused by uniform gage pressure, and force F_2 caused by the liquid)

Solution:

Computing equations: $F_R = p_c A$; $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A}$; $I_{\hat{x}\hat{x}} = \frac{bL^3}{12}$



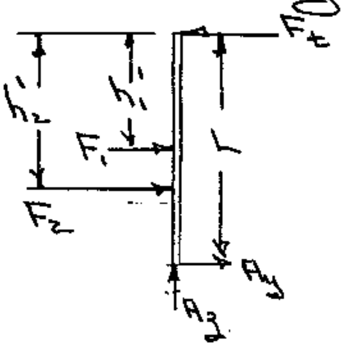
$F_1 = p_0 A = 100\,\frac{\text{lbf}}{\text{ft}^2} \times 3\,\text{ft} \times 2\,\text{ft} = 600\,\text{lb}$ $\{$ applied at $(x', z') = (1.0, 1.5)$ ft

$F_2 = p_c A = \rho g h_c L b = \gamma h_c L b = 100\,\frac{\text{lb}}{\text{ft}^3} \times 1.5\,\text{ft} \times 3\,\text{ft} \times 2\,\text{ft} = 900\,\text{lbf}$.

For the rectangular door $I_{\hat{x}\hat{x}} = \frac{1}{12} bL^3$

$h_2' = h_c + \frac{I_{\hat{x}\hat{x}}}{A h_c} = h_c + \frac{1}{12}\frac{bL^3}{bL h_c} = h_c + \frac{1}{12}\frac{L^2}{h_c} = 1.5\,\text{m} + \frac{1}{12}\frac{(3\,\text{m})^2}{1.5\,\text{m}} = 2.0\,\text{m}$

The free-body diagram of the door is then



$\Sigma M_{Ax} = 0 = L F_t - F_1 (L - h_1') - F_2 (L - h_2')$.

$F_t = F_1 \left(1 - \frac{h_1'}{L}\right) + F_2 \left(1 - \frac{h_2'}{L}\right)$

$= 600\,\text{lb}\left(1 - \frac{1.5}{3.0}\right) + 900\,\text{lb}\left(1 - \frac{2}{3}\right)$

$F_t = 600\,\text{lb}$ $\underline{\qquad\qquad\qquad\qquad}$ F_t

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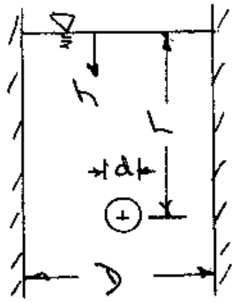
Problem 3.60

[2]



Given: Circular access port, of diameter d = 0.6m, in side of water standpipe, of diameter D = 7m, is held in place by eight bolts evenly spaced around circumference of the port.
Center of the port is located at distance L = 12m below the free surface of the water

Find: (a) Total force on the port
(b) Appropriate bolt diameter

Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $\sigma = \frac{F}{A}$

Computing equation: $F_R = p_c A$

Assumptions:
(1) static fluid
(2) incompressible
(3) force distributed uniformly over the bolts
(4) appropriate working stress for steel bolts is $\sigma = 100$ MPa
(5) p_{atm} acts at free surface and on the outside of the port

Then on integrating $dp = \rho g dh$ we obtain $p = \rho g h$

$$F_R = p_c A = \rho g h_c \pi R^2 = \rho g L \pi R^2$$

$$F_R = 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 12m \times \pi \times (0.3m)^2 \times \frac{N \cdot s^2}{kg \cdot m} = 33.3 \text{ kN} \quad F_R$$

$\sigma = \frac{F}{A}$ where A (total area of bolts) $= 8 \times \frac{\pi d_b^2}{4}$

Then
$$\sigma = \frac{F}{2\pi d_b^2}$$

$$d_b = \left[\frac{F}{2\pi\sigma}\right]^{1/2} = \left[\frac{33.3 \times 10^3 N}{2\pi} \times 10^8 \frac{m^2}{N} \times 10^6 \frac{mm^2}{m^2}\right]^{1/2} = 7.28 \text{ mm} \quad d_b$$

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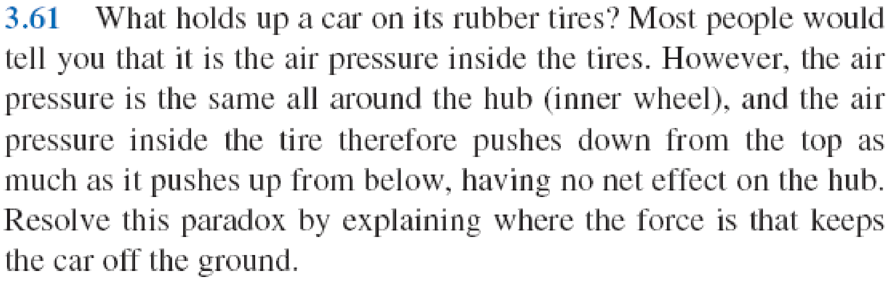
Problem 3.61

[1]



3.61 What holds up a car on its rubber tires? Most people would tell you that it is the air pressure inside the tires. However, the air pressure is the same all around the hub (inner wheel), and the air pressure inside the tire therefore pushes down from the top as much as it pushes up from below, having no net effect on the hub. Resolve this paradox by explaining where the force is that keeps the car off the ground.

Given: Description of car tire

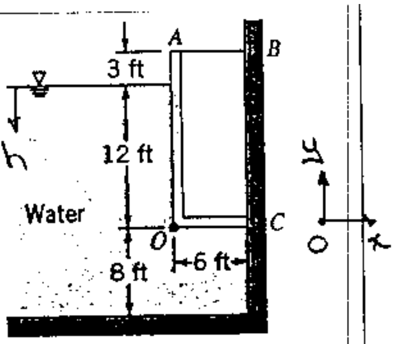
Find: Explanation of lift effect

Solution:

The explanation is as follows: It is true that the pressure in the entire tire is the same everywhere. However, the tire at the top of the hub will be essentially circular in cross-section, but at the bottom, where the tire meets the ground, the cross section will be approximately a flattened circle, or elliptical. Hence we can explain that the lower cross section has greater upward force than the upper cross section has downward force (providing enough lift to keep the car up) two ways. First, the horizontal projected area of the lower ellipse is larger than that of the upper circular cross section, so that net pressure times area is upwards. Second, any time you have an elliptical cross section that's at high pressure, that pressure will always try to force the ellipse to be circular (thing of a round inflated balloon - if you squeeze it it will resist!). This analysis ignores the stiffness of the tire rubber, which also provides a little lift.

Problem 3.62

[3]



Given: Gate AOC, hinged along O, has width $b = 6$ ft; weight of gate may be neglected. Gate is sealed at C

Find: Force in bar AB

Solution:

Basic equations: $\frac{dp}{dh} = \rho g$; $\sum M_z = 0$

Computing equations: $F_R = p_c A$; $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A}$; $I_{\hat{x}\hat{x}} = \frac{bL^3}{12}$

Assumptions: (1) static liquid (2) ρ = constant

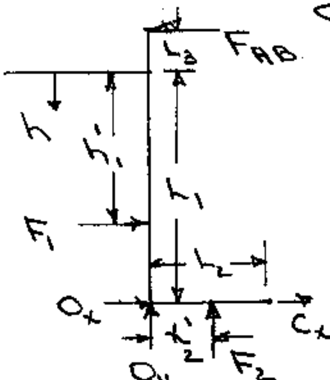
(3) p_{atm} acts at free surface and on outside of gate

(4) no resisting moment in hinge along O

(5) no vertical resisting force at C

Then on integrating $dp = \rho g dh$, we obtain $p = \rho g h$

The free body diagram of the gate is as shown.



F_1 is resultant of distributed force on L_1

F_2 " " " uniform force on L_2

F_{AB} is force of bar

C_x is force from seal at C

$F_1 = p_c A_1 = \rho g h_{c_1} b L_1$

$F_1 = 1.94 \frac{slug}{ft^3} \times 32.2 \frac{ft}{s^2} \times 6 ft \times 6 ft \times 12 ft \times \frac{lbf \cdot s^2}{ft \cdot slug} = 27.0 \times 10^3 \, lbf$

$h_1' = h_{c_1} + \frac{bL_1^3}{12 h_c b L_1} = \frac{L_1}{2} + \frac{L_1^2}{12} \frac{2}{\times L_1} = \frac{L_1}{2} + \frac{L_1}{6} = \frac{2}{3} L_1 - \frac{2}{3} \times 12 ft = 8 ft$

$F_2 = p_c A_2 = \rho g h_c b L_2 = \rho g L_1 b L_2$

$F_2 = 1.94 \frac{slug}{ft^3} \times 32.2 \frac{ft}{s^2} \times 12 ft \times 6 ft \times 6 ft = 27.0 \times 10^3 \, lbf$

Since the pressure is uniform over surface ⑤, the force F_2 acts at the centroid of the surface, i.e $x_2' = L_2/2 = 3 ft$

Then summing moments about O gives

$\sum M_{O_z} = 0 = (L_1 + L_3) F_{AB} + x_2' F_2 - (L_1 - h_1') F_1$

$F_{AB} = \frac{1}{(L_1 + L_3)} \left[(L_1 - h_1') F_1 - x_2' F_2 \right] = \frac{1}{15 ft} \left[(12 - 8) ft \times 27{,}000 \, lbf - 3 ft \times 27{,}000 \, lbf \right]$

$F_{AB} = 1800 \, lbf$

F_{AB}

Thus bar AB [illegible]

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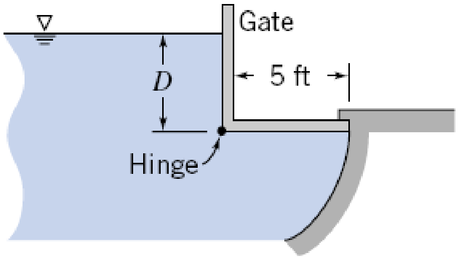
Problem 3.63

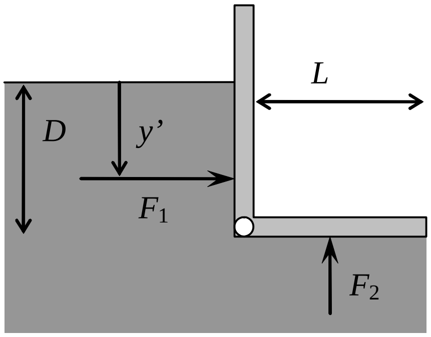
[3]

3.63 As water rises on the left side of the rectangular gate, the gate will open automatically. At what depth above the hinge will this occur? Neglect the mass of the gate.



Given: Geometry of rectangular gate

Find: Depth for gate to open



Solution:

Basic equation $\frac{dp}{dh} = \rho \cdot g$ $\quad \Sigma M_z = 0$

Computing equations $F_R = p_c \cdot A$ $\quad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$ $\quad I_{xx} = \frac{b \cdot D^3}{12}$

Assumptions: static fluid; ρ = constant; p_{atm} on other side; no friction in hinge

For incompressible fluid $p = \rho \cdot g \cdot h$ where p is gage pressure and h is measured downwards

The force on the vertical gate (gate 1) is the same as that on a rectangle of size h = D and width w

Hence
$$F_1 = p_c \cdot A = \rho \cdot g \cdot y_c \cdot A = \rho \cdot g \cdot \frac{D}{2} \cdot D \cdot w = \frac{\rho \cdot g \cdot w \cdot D^2}{2}$$

The location of this force is
$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \frac{D}{2} + \frac{w \cdot D^3}{12} \times \frac{1}{w \cdot D} \times \frac{2}{D} = \frac{2}{3} \cdot D$$

The force on the horizontal gate (gate 2) is due to constant pressure, and is at the centroid

$$F_2 = p(y = D) \cdot A = \rho \cdot g \cdot D \cdot w \cdot L$$

Summing moments about the hinge
$$\Sigma M_{hinge} = 0 = -F_1 \cdot (D - y') + F_2 \cdot \frac{L}{2} = -F_1 \cdot \left(D - \frac{2}{3} \cdot D\right) + F_2 \cdot \frac{L}{2}$$

$$F_1 \cdot \frac{D}{3} = \frac{\rho \cdot g \cdot w \cdot D^2}{2} \cdot \frac{D}{3} = F_2 \cdot \frac{L}{2} = \rho \cdot g \cdot D \cdot w \cdot L \cdot \frac{L}{2}$$

$$\frac{\rho \cdot g \cdot w \cdot D^3}{6} = \frac{\rho \cdot g \cdot D \cdot w \cdot L^2}{2}$$

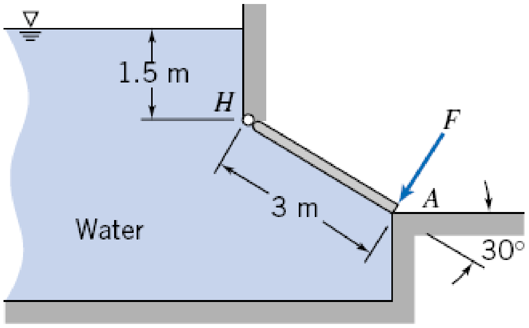
$$D = \sqrt{3} \cdot L = \sqrt{3} \times 5\text{ft}$$

$$D = 8.66 \cdot \text{ft}$$

Problem 3.64

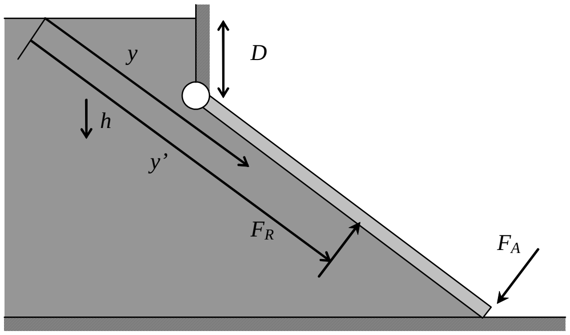
[3]

3.64 The gate shown is hinged at *H*. The gate is 3 m wide normal to the plane of the diagram. Calculate the force required at *A* to hold the gate closed.



Given: Geometry of gate

Find: Force at A to hold gate closed



Solution:

Basic equation $\frac{dp}{dh} = \rho \cdot g$ $\qquad \Sigma M_z = 0$

Computing equations $F_R = p_c \cdot A$ $\qquad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$ $\qquad I_{xx} = \frac{w \cdot L^3}{12}$

Assumptions: static fluid; ρ = constant; p_{atm} on other side; no friction in hinge

For incompressible fluid $p = \rho \cdot g \cdot h$ where p is gage pressure and h is measured downwards

The hydrostatic force on the gate is that on a rectangle of size L and width w.

Hence

$$F_R = p_c \cdot A = \rho \cdot g \cdot h_c \cdot A = \rho \cdot g \cdot \left(D + \frac{L}{2} \cdot \sin(30 \cdot deg) \right) \cdot L \cdot w$$

$$F_R = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \left(1.5 + \frac{3}{2} \sin(30 \cdot deg) \right) \cdot m \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_R = 199 \cdot kN$$

The location of this force is given by $y' = y_c + \frac{I_{xx}}{A \cdot y_c}$ where y' and y_c are measured along the plane of the gate to the free surface

$$y_c = \frac{D}{\sin(30 \cdot deg)} + \frac{L}{2} \qquad y_c = \frac{1.5 \cdot m}{\sin(30 \cdot deg)} + \frac{3 \cdot m}{2} \qquad y_c = 4.5\,m$$

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = y_c + \frac{w \cdot L^3}{12} \cdot \frac{1}{w \cdot L} \cdot \frac{1}{y_c} = y_c + \frac{L^2}{12 \cdot y_c} = 4.5 \cdot m + \frac{(3 \cdot m)^2}{12 \cdot 4.5 \cdot m} \qquad y' = 4.67\,m$$

Taking moments about the hinge $\Sigma M_H = 0 = F_R \cdot \left(y' - \frac{D}{\sin(30 \cdot deg)} \right) - F_A \cdot L$

$$F_A = F_R \cdot \frac{\left(y' - \frac{D}{\sin(30 \cdot deg)} \right)}{L} \qquad F_A = 199 \cdot kN \cdot \frac{\left(4.67 - \frac{1.5}{\sin(30 \cdot deg)} \right)}{3} \qquad F_A = 111 \cdot kN$$

Problem 3.65

13-782 500 SHEETS, FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
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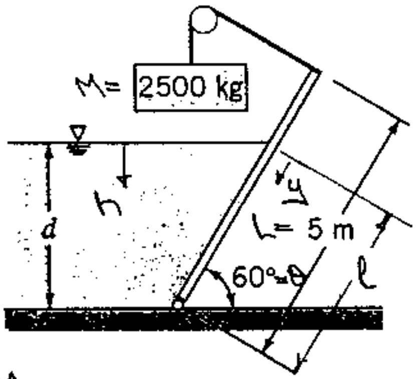


Given: Gate shown has width $b = 3\,m$; mass of gate is negligible.
Gate is in equilibrium

Find: Water depth, d

Solution:

Basic equation: $\frac{dp}{dh} = \rho g$ $\quad \sum M_z = 0$

Computing equations: $F_R = p_c A$; $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A}$; $I_{\hat{x}\hat{x}} = \frac{bL^3}{12}$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts at free surface and on underside of gate.

Then on integrating $dp = \rho g\, dh$, we obtain $p = \rho g h$

$$F_R = p_c A = \rho g h_c A \qquad h_c = \frac{d}{2}, \quad A = b \times \frac{d}{\sin\theta}$$

$$F_R = \rho g \frac{d}{2}\frac{db}{\sin\theta} = \frac{\rho g b d^2}{2\sin\theta}$$

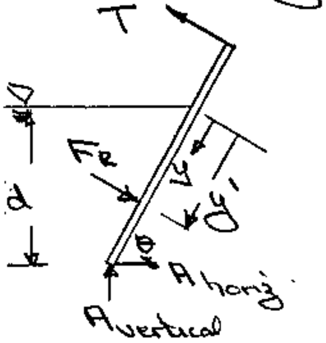
$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A} = y_c + \frac{1}{12}\frac{b l^3}{y_c l b}$$

where l is length of gate in contact with the water

$$y' = y_c + \frac{l^2}{12 y_c} \qquad l = \frac{d}{\sin\theta}, \quad y_c = \frac{l}{2} = \frac{d}{2\sin\theta}$$

$$y' = \frac{d}{2\sin\theta} + \frac{1}{12}\left(\frac{d}{\sin\theta}\right)^2 \frac{2\sin\theta}{d} = \frac{d}{2\sin\theta} + \frac{d}{6\sin\theta} = \frac{2d}{3\sin\theta}$$

The free body diagram of the gate is as shown.



Summing moments about A

$$\sum M_z = 0 = TL - (l - y')F_R \qquad T = Mg$$

$$MgL = (l - y')F_R = \left(\frac{d}{\sin\theta} - \frac{2d}{3\sin\theta}\right)\frac{\rho g b d^2}{2\sin\theta}$$

$$MgL = \frac{1}{3}\frac{d}{\sin\theta} \times \frac{\rho g b d^2}{2\sin\theta} = \frac{\rho g b d^3}{6 \sin^2\theta}$$

$$d^3 = \frac{6\sin^2\theta\, ML}{\rho b}$$

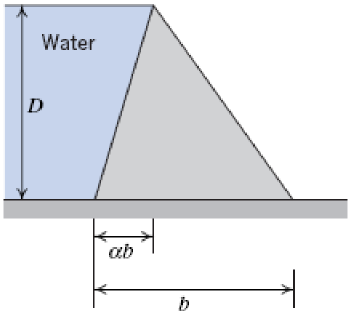
$$d = \left[6 \times \sin^2 60° \times 2500\,kg \times 5\,m \times \frac{m^3}{999\,kg} \times \frac{1}{3\,m}\right]^{1/3} = 2.66\,m$$

d

Problem 3.66

[4]

3.66 A solid concrete dam is to be built to hold back a depth *D* of water. For ease of construction the walls of the dam must be planar. Your supervisor asks you to consider the following dam cross-sections: a rectangle, a right triangle with the hypotenuse in contact with the water, and a right triangle with the vertical in contact with the water. She wishes you to determine which of these would require the least amount of concrete. What will your report say? You decide to look at one more possibility: a nonright triangle, as shown. Develop and plot an expression for the cross-section area *A* as a function of *α*, and find the minimum cross-sectional area.



Given: Various dam cross-sections

Find: Which requires the least concrete; plot cross-section area *A* as a function of α

Solution:

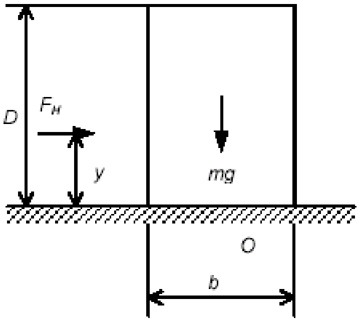
For each case, the dam width *b* has to be large enough so that the weight of the dam exerts enough moment to balance the moment due to fluid hydrostatic force(s). By doing a moment balance this value of *b* can be found

a) Rectangular dam

Straightforward application of the computing equations of Section 3-5 yields

$$F_H = p_c\cdot A = \rho\cdot g\cdot\frac{D}{2}\cdot w\cdot D = \frac{1}{2}\cdot\rho\cdot g\cdot D^2\cdot w$$

$$y' = y_c + \frac{I_{xx}}{A\cdot y_c} = \frac{D}{2} + \frac{w\cdot D^3}{12\cdot w\cdot D\cdot\frac{D}{2}} = \frac{2}{3}\cdot D$$



so $$y = D - y' = \frac{D}{3}$$

Also $$m = \rho_{cement}\cdot g\cdot b\cdot D\cdot w = SG\cdot\rho\cdot g\cdot b\cdot D\cdot w$$

Taking moments about *O* $$\sum M_{0.} = 0 = -F_H\cdot y + \frac{b}{2}\cdot m\cdot g$$

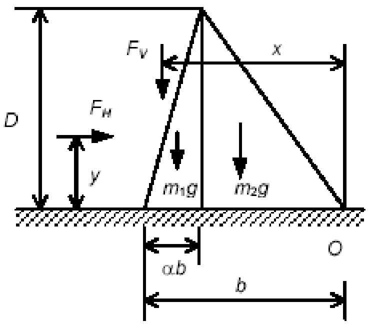
so $$\left(\frac{1}{2}\cdot\rho\cdot g\cdot D^2\cdot w\right)\cdot\frac{D}{3} = \frac{b}{2}\cdot(SG\cdot\rho\cdot g\cdot b\cdot D\cdot w)$$

Solving for *b* $$b = \frac{D}{\sqrt{3\cdot SG}}$$

The minimum rectangular cross-section area is $$A = b\cdot D = \frac{D^2}{\sqrt{3\cdot SG}}$$

For concrete, from Table A.1, SG = 2.4, so $$A = \frac{D^2}{\sqrt{3\cdot SG}} = \frac{D^2}{\sqrt{3\times 2.4}}$$ $$A = 0.373\cdot D^2$$

a) Triangular dams



Instead of analysing right-triangles, a general analysis is made, at the end of which right triangles are analysed as special cases by setting $\alpha = 0$ or 1.

Straightforward application of the computing equations of Section 3-5 yields

$$F_H = p_c \cdot A = \rho \cdot g \cdot \frac{D}{2} \cdot w \cdot D = \frac{1}{2} \cdot \rho \cdot g \cdot D^2 \cdot w$$

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \frac{D}{2} + \frac{w \cdot D^3}{12 \cdot w \cdot D \cdot \frac{D}{2}} = \frac{2}{3} \cdot D$$

so $$y = D - y' = \frac{D}{3}$$

Also $$F_V = \rho \cdot V \cdot g = \rho \cdot g \cdot \frac{\alpha \cdot b \cdot D}{2} \cdot w = \frac{1}{2} \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w \qquad x = (b - \alpha \cdot b) + \frac{2}{3} \cdot \alpha \cdot b = b \cdot \left(1 - \frac{\alpha}{3}\right)$$

For the two triangular masses

$$m_1 = \frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w \qquad x_1 = (b - \alpha \cdot b) + \frac{1}{3} \cdot \alpha \cdot b = b \cdot \left(1 - \frac{2 \cdot \alpha}{3}\right)$$

$$m_2 = \frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot (1 - \alpha) \cdot b \cdot D \cdot w \qquad x_2 = \frac{2}{3} \cdot b(1 - \alpha)$$

Taking moments about *O*

$$\sum M_{0.} = 0 = -F_H \cdot y + F_V \cdot x + m_1 \cdot g \cdot x_1 + m_2 \cdot g \cdot x_2$$

so $$\begin{aligned} &-\left(\frac{1}{2} \cdot \rho \cdot g \cdot D^2 \cdot w\right) \cdot \frac{D}{3} + \left(\frac{1}{2} \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w\right) \cdot b \cdot \left(1 - \frac{\alpha}{3}\right) \ldots \qquad = 0 \\ &+ \left(\frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w\right) \cdot b \cdot \left(1 - \frac{2 \cdot \alpha}{3}\right) + \left[\frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot (1 - \alpha) \cdot b \cdot D \cdot w\right] \cdot \frac{2}{3} \cdot b(1 - \alpha) \end{aligned}$$

Solving for *b* $$b = \frac{D}{\sqrt{\left(3 \cdot \alpha - \alpha^2\right) + SG \cdot (2 - \alpha)}}$$

For a right triangle with the hypotenuse in contact with the water, $\alpha = 1$, and

$$b = \frac{D}{\sqrt{3 - 1 + SG}} = \frac{D}{\sqrt{3 - 1 + 2.4}} \qquad b = 0.477 \cdot D$$

The cross-section area is $$A = \frac{b \cdot D}{2} = 0.238 \cdot D^2 \qquad A = 0.238 \cdot D^2$$

For a right triangle with the vertical in contact with the water, $\alpha = 0$, and

$$b = \frac{D}{\sqrt{2 \cdot SG}} = \frac{D}{\sqrt{2 \cdot 2.4}} \qquad b = 0.456 \cdot D$$

The cross-section area is $$A = \frac{b \cdot D}{2} = 0.228 \cdot D^2 \qquad A = 0.228 \cdot D^2$$

For a general triangle $$A = \frac{b \cdot D}{2} = \frac{D^2}{2 \cdot \sqrt{\left(3 \cdot \alpha - \alpha^2\right) + SG \cdot (2 - \alpha)}} \qquad A = \frac{D^2}{2 \cdot \sqrt{\left(3 \cdot \alpha - \alpha^2\right) + 2.4 \cdot (2 - \alpha)}}$$

The final result is $$A = \frac{D^2}{2 \cdot \sqrt{4.8 + 0.6 \cdot \alpha - \alpha^2}}$$

From the corresponding Excel workbook, the minimum area occurs at α = 0.3

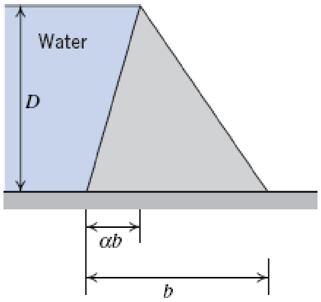
$$A_{min} = \frac{D^2}{2 \cdot \sqrt{4.8 + 0.6 \times 0.3 - 0.3^2}} \qquad A = 0.226 \cdot D^2$$

The final results are that a triangular cross-section with α = 0.3 uses the least concrete; the next best is a right triangle with the vertical in contact with the water; next is the right triangle with the hypotenuse in contact with the water; and the cross-section requiring the most concrete is the rectangular cross-section.

Problem 3.66

[4]

3.66 A solid concrete dam is to be built to hold back a depth D of water. For ease of construction the walls of the dam must be planar. Your supervisor asks you to consider the following dam cross-sections: a rectangle, a right triangle with the hypotenuse in contact with the water, and a right triangle with the vertical in contact with the water. She wishes you to determine which of these would require the least amount of concrete. What will your report say? You decide to look at one more possibility: a nonright triangle, as shown. Develop and plot an expression for the cross-section area A as a function of a, and find the minimum cross-sectional area.



Given: Various dam cross-sections

Find: Which requires the least concrete; plot cross-section area A as a function of α

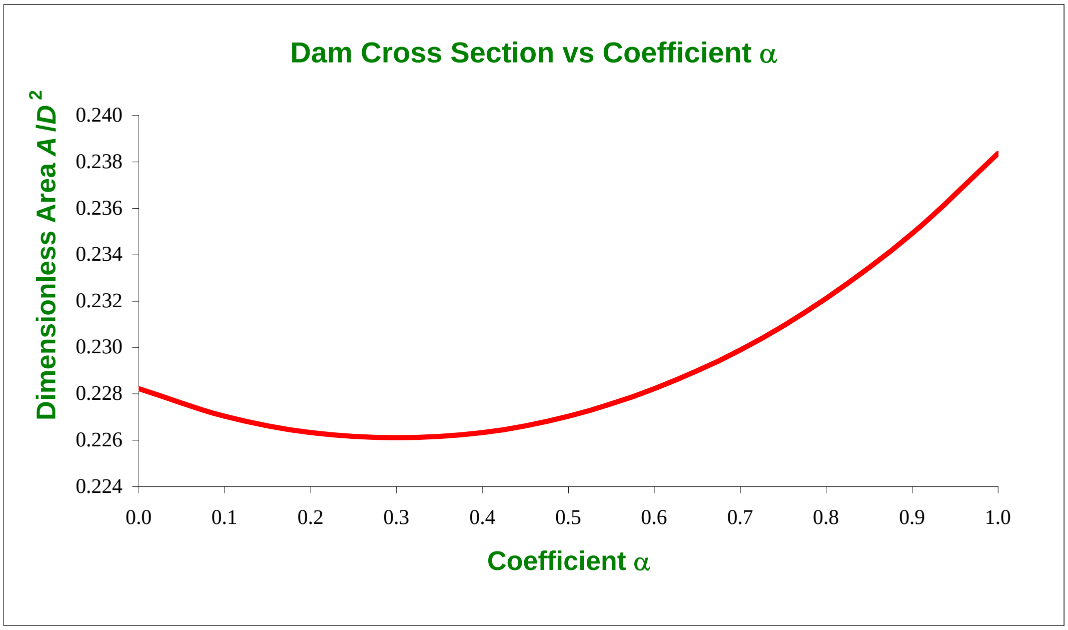
Solution:

The triangular cross-sections are considered in this workbook

The final result is
$$A = \frac{D^2}{2\cdot\sqrt{4.8 + 0.6\cdot\alpha - \alpha^2}}$$

The dimensionless area, A/D^2, is plotted

α	A/D^2
0.0	0.2282
0.1	0.2270
0.2	0.2263
0.3	0.2261
0.4	0.2263
0.5	0.2270
0.6	0.2282
0.7	0.2299
0.8	0.2321
0.9	0.2349
1.0	0.2384

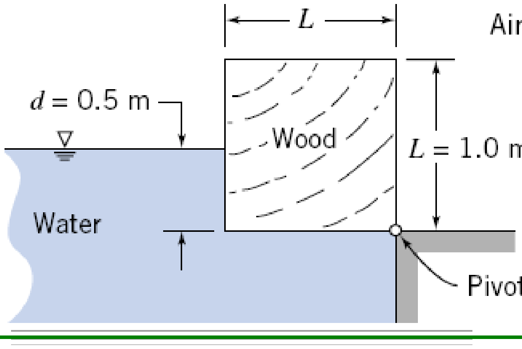


Solver can be used to find the minimum area

α	A/D^2
0.30	0.2261

Problem 3.67

3.67 A long, square wooden block is pivoted along one edge. The block is in equilibrium when immersed in water to the depth shown. Evaluate the specific gravity of the wood, if friction in the pivot is negligible.



Given: Block hinged and floating

Find: SG of the wood

Solution:

Basic equation $\frac{dp}{dh} = \rho \cdot g$ $\Sigma M_z = 0$

Computing equations $F_R = p_c \cdot A$ $y' = y_c + \frac{I_{xx}}{A \cdot y_c}$

Assumptions: static fluid; ρ = constant; p_{atm} on other side; no friction in hinge

For incompressible fluid $p = \rho \cdot g \cdot h$ where p is gage pressure and h is measured downwards

The force on the vertical section is the same as that on a rectangle of height d and width L

Hence $F_1 = p_c \cdot A = \rho \cdot g \cdot y_c \cdot A = \rho \cdot g \cdot \frac{d}{2} \cdot d \cdot L = \frac{\rho \cdot g \cdot L \cdot d^2}{2}$

The location of this force is $y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \frac{d}{2} + \frac{L \cdot d^3}{12} \times \frac{1}{L \cdot d} \times \frac{2}{d} = \frac{2}{3} \cdot d$

The force on the horizontal section is due to constant pressure, and is at the centroid

$$F_2 = p(y = d) \cdot A = \rho \cdot g \cdot d \cdot L \cdot L$$

Summing moments about the hinge $\Sigma M_{hinge} = 0 = -F_1 \cdot (d - y') - F_2 \cdot \frac{L}{2} + M \cdot g \cdot \frac{L}{2}$

Hence $F_1 \cdot \left(d - \frac{2}{3} \cdot d\right) + F_2 \cdot \frac{L}{2} = SG \cdot \rho \cdot L^3 \cdot g \cdot \frac{L}{2}$

$$\frac{SG \cdot \rho \cdot g \cdot L^4}{2} = \frac{\rho \cdot g \cdot L \cdot d^2}{2} \cdot \frac{d}{3} + \rho \cdot g \cdot d \cdot L^2 \cdot \frac{L}{2}$$

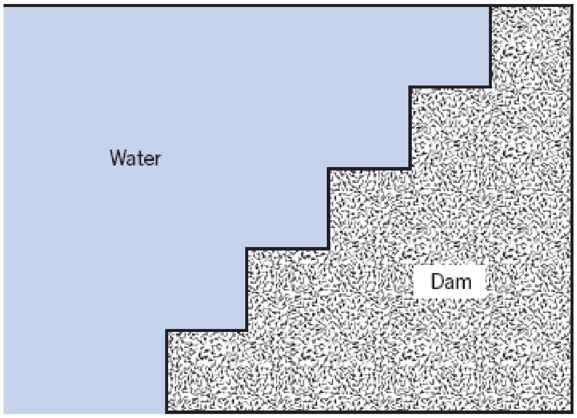
$$SG = \frac{1}{3} \cdot \left(\frac{d}{L}\right)^3 + \frac{d}{L}$$

$$SG = \frac{1}{3} \cdot \left(\frac{0.5}{1}\right)^3 + \frac{0.5}{1}$$ $SG = 0.542$

Problem 3.68

[2]

3.68 For the geometry shown, what is the vertical force on the dam? The steps are 1 ft high, 1 ft deep, and 10 ft wide.



Given: Geometry of dam

Find: Vertical force on dam

Solution:

Basic equation $\frac{dp}{dh} = \rho \cdot g$

Assumptions: static fluid; ρ = constant

For incompressible fluid $p = p_{atm} + \rho \cdot g \cdot h$ where h is measured downwards from the free surface

The force on each horizontal section (depth d = 1 ft and width w = 10 ft) is

$$F = p \cdot A = \left(p_{atm} + \rho \cdot g \cdot h\right) \cdot d \cdot w$$

Hence the total force is $F_T = \left[p_{atm} + \left(p_{atm} + \rho \cdot g \cdot h\right) + \left(p_{atm} + \rho \cdot g \cdot 2 \cdot h\right) + \left(p_{atm} + \rho \cdot 3 \cdot g \cdot h\right) + \left(p_{atm} + \rho \cdot g \cdot 4 \cdot h\right)\right] \cdot d \cdot w$

where we have used h as the height of the steps

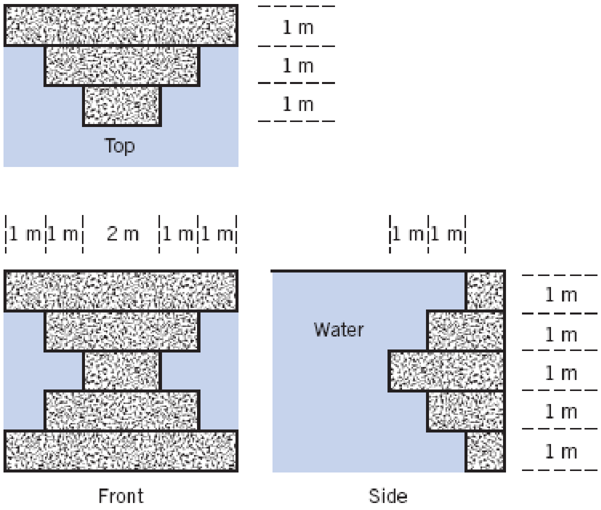
$$F_T = d \cdot w \cdot \left(5 \cdot p_{atm} + 10 \cdot \rho \cdot g \cdot h\right)$$

$$F_T = 1 \cdot ft \times 10 \cdot ft \times \left[5 \times 14.7 \cdot \frac{lbf}{in^2} \times \left(\frac{12 \cdot in}{1 \cdot ft}\right)^2 + 10 \times 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times 1 \cdot ft \times \frac{lbf \cdot s^2}{slug \cdot ft}\right]$$

$$F_T = 1.12 \times 10^5 \cdot lbf$$

Problem 3.69

3.69 For the dam shown, what is the vertical force of the water on the dam?



Given: Geometry of dam

Find: Vertical force on dam

Solution:

Basic equation $\frac{dp}{dh} = \rho \cdot g$

Assumptions: static fluid; ρ = constant; since we are asked for the force of water, we use gage pressures

For incompressible fluid $p = \rho \cdot g \cdot h$ where p is gage pressure and h is measured downwards from the free surface

The force on each horizontal section (depth d and width w) is

$$F = p \cdot A = \rho \cdot g \cdot h \cdot d \cdot w$$

Hence the total force is (allowing for the fact that some faces experience an upwards (negative) force)

$$F_T = p \cdot A = \Sigma \rho \cdot g \cdot h \cdot d \cdot w = \rho \cdot g \cdot d \cdot \Sigma h \cdot w$$

Starting with the top and working downwards

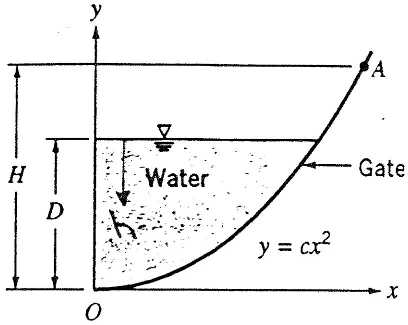
$$F_T = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 1 \cdot m \times [(1 \cdot m \times 4 \cdot m) + (2 \cdot m \times 2 \cdot m) - (3 \cdot m \times 2 \cdot m) - (4 \cdot m \times 4 \cdot m)] \times \frac{N \cdot s^2}{kg \cdot m}$$

$$F_T = -137 \cdot kN$$

The negative sign indicates a net upwards force (it's actually a buoyancy effect on the three middle sections)

Problem 3.70

Given: Parabolic gate, hinged at O, has width $b = 2m$.
$c = 0.25 m^{-1}$, $D = 2m$, $H = 3m$



Find: (a) Magnitude and line of action of vertical force on gate due to water
(b) Horizontal force applied at A needed for equilibrium
(c) Vertical force applied at A needed for equilibrium

Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $\Sigma M_{O_z} = 0$, $F_V = \int p\, dA_y$, $x' F_V = \int x\, dF_V$

Computing equations $F_H = p_c A$, $h' = h_c + \frac{I_{xx}}{h_c A}$

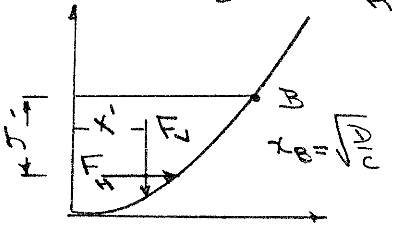
Assumptions: (1) static liquid (2) ρ = constant
(3) Patm acts on the surface of the water and along the outside surface of the gate

Then on integrating $dp = \rho g\, dh$, we obtain $p = \rho g h$

(a) $F_V = \int p\, dA_y = \int_0^{\sqrt{D/c}} \rho g h\, b\, dx = \int_0^{\sqrt{D/c}} \rho g (D-y)\, b\, dx = \int_0^{\sqrt{D/c}} \rho g (D - cx^2)\, b\, dx$

$F_V = \rho g b \left[Dx - \frac{cx^3}{3} \right]_0^{\sqrt{D/c}} = \rho g b \left[\frac{D^{3/2}}{c^{1/2}} - \frac{c}{3}\left(\frac{D}{c}\right)^{3/2} \right] = \frac{2}{3} \frac{\rho g b}{c^{1/2}} D^{3/2}$ —— (1)

$F_V = \frac{2}{3} \times 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 2m \times (2m)^{3/2} \left(\frac{m}{0.25}\right)^{0.5} \times \frac{N \cdot s^2}{kg \cdot m} = 73.9\, kN \longleftarrow F_V$



$x' = \frac{1}{F_V} \int x\, dF_V = \frac{1}{F_V} \int x p\, dA_y = \frac{1}{F_V} \int_0^{\sqrt{D/c}} x \rho g h b\, dx$

$x' = \frac{1}{F_V} \int_0^{\sqrt{D/c}} x \rho g (D - cx^2)\, b\, dx$

$x' = \frac{b\rho g}{F_V} \left[\frac{Dx^2}{2} - \frac{cx^4}{4} \right]_0^{\sqrt{D/c}} = \frac{b\rho g}{F_V} \left[\frac{D}{2} \cdot \frac{D}{c} - \frac{c}{4} \frac{D^2}{c^2} \right] = \frac{b\rho g}{F_V} \frac{D^2}{4c}$

Substituting for F_V from Eq. 1

$x' = \frac{\cancel{b} \cancel{\rho g} D^2}{4c} \times \frac{3}{2} \frac{c^{1/2}}{\cancel{\rho g} \cancel{b} D^{3/2}} = \frac{3}{8} \left(\frac{D}{c}\right)^{1/2} = \frac{3}{8} \left[2m \times \frac{m}{0.25} \right]^{1/2} = 1.06m \longleftarrow x'$

In order to sum moments about point O to find the required force at A required for equilibrium, we need to find the horizontal force of the water on the gate and its line of action

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Problem 3.70

[3] Part 2/2

$F_H = p_c A = \rho g h_c b D = \rho g b \frac{D^2}{2}$ $\{h_c = D/2\}$

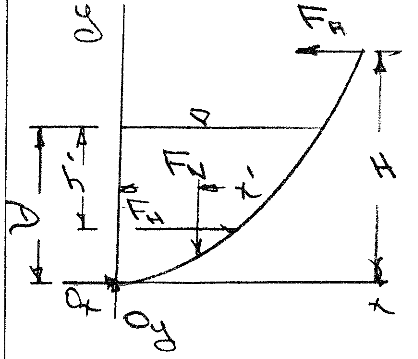
$F_H = 999\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times 2m \times \frac{(2m)^2}{2}\times\frac{N\cdot s^2}{kg\cdot m} = 39.2\ kN$

$h' = h_c + \frac{I_{\hat{x}\hat{x}}}{A h_c} = h_c + \frac{D^2}{12 h_c}$ $\{I_{\hat{x}\hat{x}} = \frac{bD^3}{12}$ and $A = bD\}$.

$h' = \frac{D}{2} + \frac{D^2}{12}\times\frac{2}{D}$ $\{h_c = \frac{D}{2}\}$

$h' = \frac{2}{3}D = \frac{4}{3}m$

(b) Horizontal force applied at A for equilibrium



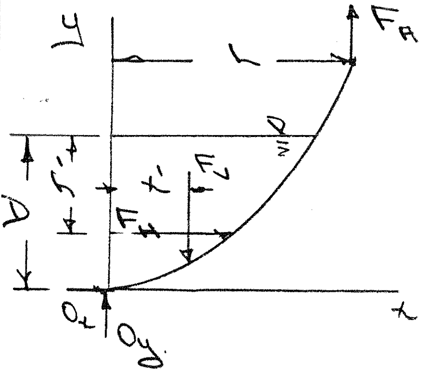
$\Sigma M_O = 0 = F_A H - F_V x' - F_H(D-h')$

$F_A = \frac{1}{H}\left[F_V x' + F_H(D-h')\right]$

$= \frac{1}{3m}\left[73.9\ kN \times 1.06m + 39.2\ kN \times \left(2-\frac{4}{3}\right)m\right]$

$F_{A_H} = 34.8\ kN$ ← F_{AH}

(c) Vertical force applied at A for equilibrium



$\Sigma M_O = 0 = F_A L - F_V x' - F_H(D-h')$.

$F_A = \frac{1}{L}\left[F_V x' + F_H(D-h')\right]$

$L = x$ @ $y = H$. Since $y = cx^2$

$L = \sqrt{\frac{H}{c}} = \left[3m \times \frac{m}{0.25}\right]^{1/2} = 3.46m$

$F_A = \frac{1}{3.46m}\left[73.9\ kN \times 1.06m + 39.2\ kN \times \left(2-\frac{4}{3}\right)m\right]$

$F_{A_V} = 30.2\ kN$ ← F_{AV}

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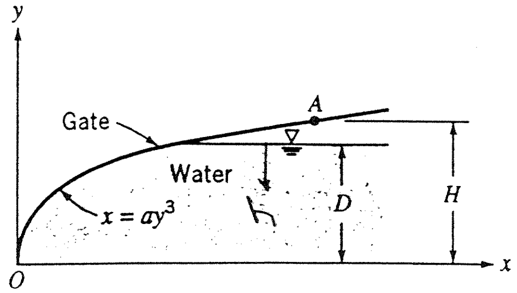
Problem 3.71

Given: Gate, hinged at O, has width $b = 1.5\text{ m}$

$a = 1.0\text{ m}^{-2}$, $D = 1.20\text{ m}$,

$H = 1.40\text{ m}$



Find: (a) Magnitude and moment about O of vertical force on gate due to water

(b) Horizontal force applied at A needed for equilibrium

Solution

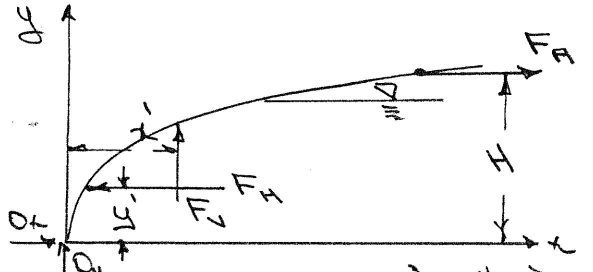
Basic equations: $\frac{dp}{dh} = \rho g$, $F_V = \int p\,dA_y$, $x' F_V = \int x\,dF_V$

$y' F_H = \int y\,dF_H$, $F_H = \int p\,dA_x$, $\sum M_{O_z} = 0$

Assumptions: (1) static liquid (2) ρ = constant

(3) p_{atm} acts on the surface of the water and along the top surface of the gate

Then on integrating $dp = \rho g\,dh$, we obtain $p = \rho g h$



$F_V = \int p\,dA_y = \int \rho g h\,b\,dx$

$h = D - y$ $\quad x = ay^3$ $\quad dx = 3ay^2\,dy$

$F_V = \int_0^D \rho g (D-y)\,b\,3ay^2\,dy$

$$F_V = 3\rho g b a \left[D\frac{y^3}{3} - \frac{y^4}{4} \right]_0^D = 3\rho g b a \frac{D^4}{12} = \rho g b a \frac{D^4}{4}$$

$$F_V = 999\frac{\text{kg}}{\text{m}^3} \times 9.81\frac{\text{m}}{\text{s}^2} \times 1.5\text{ m} \times \frac{1.0}{\text{m}^2} \times \frac{(1.20\text{ m})^4}{4} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 7.62\text{ kN} \qquad F_V$$

The moment of F_V about O is given by

$$x'F_V = \int x\,dF_V = \int x\,p\,dA_y = \int x\,\rho g h\,b\,dx$$

$$= \rho g b \int_0^D ay^3 (D-y)\,3ay^2\,dy = 3\rho g b a^2 \int_0^D y^5 (D-y)\,dy$$

$$= 3\rho g b a^2 \left[D\frac{y^6}{6} - \frac{y^7}{7} \right]_0^D = \rho g b \frac{a^2 D^7}{14}$$

$$x'F_V = 999\frac{\text{kg}}{\text{m}^3} \times 9.81\frac{\text{m}}{\text{s}^2} \times 1.5\text{ m} \times \frac{(1.0)^2}{\text{m}^4} \times \frac{(1.20\text{ m})^7}{14} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$x'F_V = 3.76\text{ kN}\cdot\text{m}$ {counterclockwise} $\qquad x'F_V$

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Problem 3.71

From the free body diagram of the gate

$$\Sigma M_{oz} = x' F_V + y' F_H - H F_A$$

$$y' F_H = \int y \, dF_H = \int y \, p \, dA_x = \int y \, \rho g h \, b \, dy = \rho g b \int_0^D y(D-y) \, dy$$

$$= \rho g b \left[\frac{D y^2}{2} - \frac{y^3}{3} \right]_0^D = \rho g b \frac{D^3}{6}$$

$$y' F_H = \frac{1}{6} \times 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 1.5\,m \times (1.20\,m)^3 \times \frac{N \cdot s^2}{kg \cdot m} = 4.23\ kN \cdot m$$

(counterclockwise)

Then

$$F_A = \frac{1}{H}\left[x' F_V + y' F_H \right] = \frac{1}{1.40\,m}\left[3.76 + 4.23 \right] kN \cdot m$$

$$F_A = 5.71\ kN \quad \longleftarrow F_A$$

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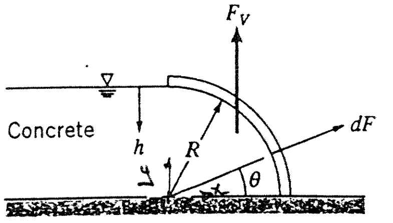
Problem 3.72

[2]

Given: Liquid concrete is poured into form shown; width $w = 4.25\,m$

Find: Magnitude and line of action of vertical force on form



Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $F_V = \int p\,dA_y$, $x' F_V = \int x\,dF_V$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts on the liquid surface and along the outside of the form.

Then on integrating $dp = \rho g\,dh$, we obtain $p = \rho g h$

$$F_V = \int p\,dA_y = \int \rho g h\,dA \sin\theta \qquad dA = wR\,d\theta\ ,\ h = R - y = R - R\sin\theta$$

$$F_V = \int_0^{\pi/2} \rho g R(1-\sin\theta)\sin\theta\, wR\,d\theta = \rho g R^2 w \int_0^{\pi/2} (\sin\theta - \sin^2\theta)\,d\theta$$

$$F_V = \rho g R^2 w \left[-\cos\theta - \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/2} = \rho g R^2 w \left[-0 + 1 - \frac{\pi}{4} + 0 + 0 - 0\right]$$

$$F_V = \rho g R^2 w \left(1 - \frac{\pi}{4}\right) \qquad \{\ \rho = SG\,\rho_{H_2O}\ ;\ SG = 2.5\ (\text{Table A.1})$$

$$F_V = 2.5 \times 1000\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times (0.313\,m)^2 \times 4.25\,m\left(1 - \frac{\pi}{4}\right) \times \frac{N \cdot s^2}{kg \cdot m}$$

$F_V = 2.19$ kN. ← F_V

$$x' F_V = \rho g R^2 w \int_0^{\pi/2} x(\sin\theta - \sin^2\theta)\,d\theta = \rho g R^2 w \int_0^{\pi/2} R\cos\theta(\sin\theta - \sin^2\theta)\,d\theta$$

$$= \rho g R^3 w \int_0^{\pi/2} (\sin\theta\cos\theta - \sin^2\theta\cos\theta)\,d\theta = \rho g R^3 w \left[\frac{\sin^2\theta}{2} - \frac{\sin^3\theta}{3}\right]_0^{\pi/2}$$

$$x' F_V = \rho g R^3 w \left[\frac{1}{2} - \frac{1}{3}\right] = \frac{\rho g R^3 w}{6}$$

$$x' = \frac{\rho g R^3 w}{6 F_V} = \frac{\rho g R^3 w}{6} \times \frac{1}{\rho g R^2 w (1 - \frac{\pi}{4})} = \frac{R}{6(1 - \frac{\pi}{4})} = \frac{0.313\,m}{6(1 - \pi/4)}$$

$x' = 0.243$ m ← x'

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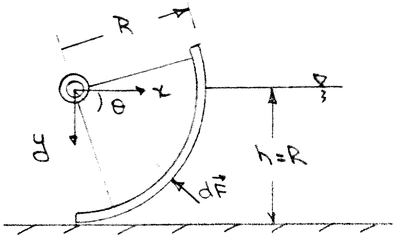
Problem 3.73

[2]

Given: Gate formed in the shape of a circular arc has width of w meters. Liquid is water; depth h = R



Find: (a) magnitude and direction of the net vertical force component due to fluids acting on the gate

(b) line of action of vertical component of the force.

Solution

Basic equations: $\vec{F}_R = -\int P\,d\vec{A}$ $\qquad \frac{dP}{dy} = \rho g$ $\qquad x' F_{R_y} = \int x\,dF$

Assumptions: (1) static fluid

(2) ρ = constant

(3) y is measured positive downward from free surface

$$\vec{F}_{R_y} = \vec{F}_R \cdot \hat{j} = \int d\vec{F} \cdot \hat{j} = -\int P\,d\vec{A} \cdot \hat{j} = -\int P\,dA \sin\theta = -\int_0^{\pi/2} P \sin\theta\, wR\, d\theta$$

We can obtain an expression for P as a function of y

$$\frac{dP}{dy} = \rho g \qquad dP = \rho g\, dy \qquad \text{and} \qquad P - P_0 = \int_{P_0}^{P} dP = \int_0^y \rho g\, dy = \rho g y$$

Since atmospheric pressure acts at the free surface and on the back surface of the gate, then the appropriate expression for P is $P = \rho g y$

Along the surface of the gate,

$$y = R\sin\theta \quad \text{and hence} \quad P = \rho g R \sin\theta$$

Thus,

$$F_{R_y} = -\int_0^{\pi/2} P \sin\theta\, wR\, d\theta = -\rho g w R^2 \int_0^{\pi/2} \sin^2\theta\, d\theta = -\rho g w R^2 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$

$$F_{R_y} = -\frac{\rho g w R^2 \pi}{4} \quad \{F_{R_y} \text{ acts upward}\} \longleftarrow F_{R_y}$$

For any element of surface area, $d\vec{A}$, the force, $d\vec{F}$, acts normal to the surface. Thus each $d\vec{F}$ has a line of action through the origin. Consequently, the line of action of $\vec{F}_R$ must also be through the origin.

We can find the line of action of F_{R_y} by recognizing that the moment of F_{R_y} about an axis through the origin must be equal to the sum of the moments of dF_y about the same axis.

$$x' F_{R_y} = \int x\, dF_y = \int x(-P\,dA \sin\theta) = -\int x\,P\,dA \sin\theta$$

$$x' F_{R_y} = -\int_0^{\pi/2} R\cos\theta\, \rho g R \sin\theta\, wR\, d\theta \sin\theta = -\rho g w R^3 \int_0^{\pi/2} \sin^2\theta \cos\theta\, d\theta$$

$$x' = \frac{-\rho g w R^3}{F_{R_y}} \int_0^{\pi/2} \sin^2\theta \cos\theta\, d\theta = \frac{-\rho g w R^3}{-\frac{\rho g w R^2 \pi}{4}} \left[\frac{1}{3}\sin^3\theta\right]_0^{\pi/2}$$

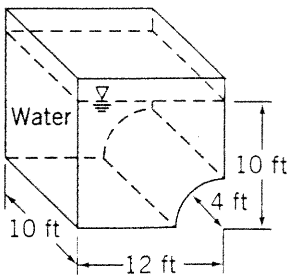
$$x' = \frac{4R}{3\pi}$$

Problem 3.74

[2]

Given: Open tank as shown
width of curved surface $b = 10\,ft$

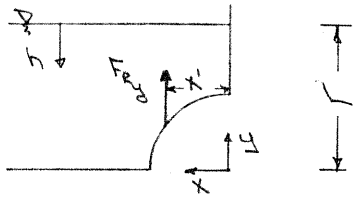
Find: (a) vertical force component, F_{R_y}, on curved surface
(b) line of action of F_{R_y}



Solution:

Basic equations: $\vec{F}_R = -\int P\,d\vec{A}$ $\quad \frac{dP}{dh} = \gamma$ $\quad \vec{r}' \times \vec{F}_R = \int \vec{r} \times d\vec{F} = -\int \vec{r} \times P\,d\vec{A}$

Assumptions: (1) static fluid
(2) gravity is only body force
(3) $\gamma = \text{constant} = 62.4\,lbf/ft^3$
(4) h is measured positive downward from free surface



$F_{R_y} = \vec{F}_R \cdot \hat{j} = -\int P\,d\vec{A} \cdot \hat{j} = -\int P\,dA_y = -\int P b\,dx$

We can obtain an expression for P as a function of y

$\frac{dP}{dh} = \gamma \quad dP = \gamma\,dh \quad P - P_0 = \int_{P_0}^{P} dP = \int_0^h \gamma\,dh = \gamma h$

Since atmospheric pressure acts at the free surface and on the underside of the curved surface, then the appropriate expression for P is $P = \gamma h$

Now, $h = L - y \quad \therefore P = \gamma(L-y)$

$F_{R_y} = -\int P b\,dx = -\int \gamma(L-y) b\,dx$. Along the surface $y = (R^2 - x^2)^{1/2}$ and so

$$F_{R_y} = -\gamma b \int_0^R \left\{L - (R^2-x^2)^{1/2}\right\} dx = -\gamma b \left[Lx - \frac{1}{2}\left(x\sqrt{R^2-x^2} + R^2 \arcsin\frac{x}{R}\right)\right]_0^R$$

$$= -\gamma b \left\{LR - \frac{1}{2}(R^2 \arcsin 1) + \frac{1}{2}R^2 \arcsin 0\right\} = \gamma b R \left\{L - \frac{R}{2}\arcsin 1\right\}$$

$$= -\gamma b R \left\{L - R\frac{\pi}{4}\right\}$$

$$F_{R_y} = -62.4\,\frac{lbf}{ft^3} \times 10\,ft \times 4\,ft \times \left\{10\,ft - 4\,ft \times \frac{\pi}{4}\right\} = -17{,}100\,lbf \quad \text{(ie acts downward)}$$

F_{R_y}

$$x'\hat{i} \times F_{R_y}\hat{j} = \int x\hat{i} \times dF_{R_y}\hat{j} = \int x\hat{i} \times (-P\,dA_y\hat{j}) = -\int x\hat{i} \times P b\,dx\,\hat{j}$$

$$x' F_{R_y}\hat{k} = -\hat{k}\int x P b\,dx$$

$$x' = -\frac{1}{F_{R_y}}\int_0^R x P b\,dx = -\frac{1}{F_{R_y}}\int_0^R x\gamma(L-y) b\,dx = -\frac{\gamma b}{F_{R_y}}\int_0^R x\left\{L - (R^2-x^2)^{1/2}\right\} dx$$

$$= -\frac{\gamma b}{F_{R_y}}\left[L\frac{x^2}{2} + \frac{1}{3}\sqrt{(R^2-x^2)^3}\right]_0^R = -\frac{\gamma b}{F_{R_y}}\left[L\frac{R^2}{2} - \frac{1}{3}R^3\right] = -\frac{\gamma b R^2}{F_{R_y}}\left[\frac{L}{2} - \frac{R}{3}\right]$$

$$x' = -62.4\,\frac{lbf}{ft^3} \times 10\,ft \times (4)^2\,ft^2 \times \frac{1}{(-17{,}100)\,lbf}\left[\frac{10\,ft}{2} - \frac{4\,ft}{3}\right]$$

$x' = 2.14\,ft$

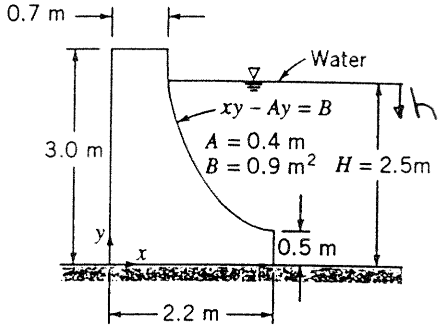
x'

Problem 3.75

Given: Dam with cross-section shown (width b = 50m)

Find: (a) Magnitude and line of action of vertical force on dam due to water.

(b) If it is possible for water force to overturn the dam



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Solution:

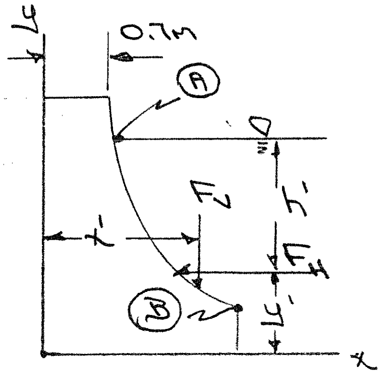
Basic equations: $\frac{dp}{dh} = \rho g$, $F_v = \int p\, dA_y$, $x' F_v = \int x\, dF_v$, $\sum M_z = 0$

Computing equations: $F_H = p_c A$, $h' = h_c + \frac{I_{\hat{x}\hat{x}}}{h_c A}$

Assumptions: (1) static fluid (2) ρ = constant

(3) p_{atm} acts on the surface of the water and on the back side of the dam

Then on integrating $dp = \rho g\, dh$ we obtain $p = \rho g h$



$$F_v = \int p\, dA_y = \int_{x_A}^{x_B} \rho g h b\, dx = \rho g b \int_{x_A}^{x_B} (H - y)\, dx$$

$$y(x - A) = B \quad \text{so} \quad y = \frac{B}{(x-A)}$$

$$F_v = \rho g b \int_{x_A}^{x_B} \left(H - \frac{B}{(x-A)} \right) dx$$

$$= \rho g b \left[Hx - B \ln(x-A) \right]_{x_A}^{x_B}$$

$$F_v = \rho g b \left[H(x_B - x_A) - B \ln \frac{(x_B - A)}{(x_A - A)} \right] \longleftarrow \text{---} (1)$$

$$F_v = 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 50m \left[2.5m(2.2 - 0.76)m - 0.9m^2 \ln \frac{(2.2-0.4)}{(0.76-0.4)} \right] \frac{N \cdot s^2}{kg \cdot m}$$

$$F_v = 1.05 \times 10^6 \text{ N} \longleftarrow \text{———} F_v$$

$$x' F_v = \int x\, dF_v = \int_{x_A}^{x_B} x \rho g b \left(H - \frac{B}{(x-A)} \right) dx = \rho g b \int_{x_A}^{x_B} \left[Hx - \frac{Bx}{(x-A)} \right] dx$$

$$x' F_v = \rho g b \left[H \frac{x^2}{2} - Bx - BA \ln(x-A) \right]_{x_A}^{x_B}$$

$$x' F_v = \rho g b \left[\frac{H}{2}(x_B^2 - x_A^2) - B(x_B - x_A) - BA \ln \frac{(x_B - A)}{(x_A - A)} \right] \text{— — — —} (2)$$

$$x' = 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 50m \left\{ \frac{2.5m}{2} \left[(2.2)^2 m^2 - (0.76)^2 m^2 - 0.9m^2 (2.2 - 0.76) m \right. \right.$$

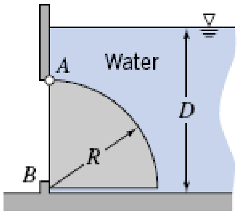
$$\left. - 0.9m^2 \times 0.4m \ln \frac{2.2 - 0.4}{0.76 - 0.4} \right\} \frac{N \cdot s^2}{kg \cdot m} \times \frac{1}{1.05 \times 10^6 \text{ N}}$$

$$x' = 1.61 m \longleftarrow \text{———} x'$$

Problem 3.76

[4]

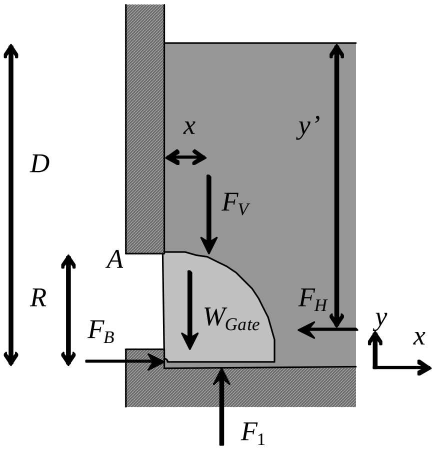
3.76 A gate, in the shape of a quarter-cylinder, hinged at *A* and sealed at *B*, is 3 m wide. The bottom of the gate is 4.5 m below the water surface. Determine the force on the stop at *B* if the gate is made of concrete; $R = 3$ m.

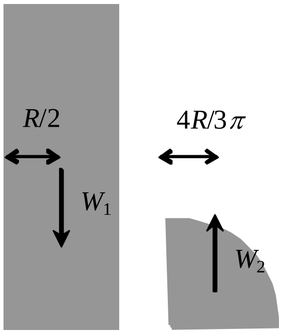


Given: Gate geometry

Find: Force on stop B

Solution:





Weights for computing F_V

Basic equations $$\frac{dp}{dh} = \rho \cdot g \qquad \Sigma M_A = 0$$

Assumptions: static fluid; ρ = constant; p_{atm} on other side

For incompressible fluid $$p = \rho \cdot g \cdot h$$ where p is gage pressure and h is measured downwards

We need to compute force (including location) due to water on curved surface and underneath. For curved surface we could integrate pressure, but here we use the concepts that F_V (see sketch) is equivalent to the weight of fluid above, and F_H is equivalent to the force on a vertical flat plate. Note that the sketch only shows forces that will be used to compute the moment at A

For F_V $$F_V = W_1 - W_2$$

with $$W_1 = \rho \cdot g \cdot w \cdot D \cdot R = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3 \cdot m \times 4.5 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad W_1 = 397 \cdot kN$$

$$W_2 = \rho \cdot g \cdot w \cdot \frac{\pi \cdot R^2}{4} = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3 \cdot m \times \frac{\pi}{4} \times (3 \cdot m)^2 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W_2 = 208 \cdot kN$$

$$F_V = W_1 - W_2 \qquad F_V = 189 \cdot kN$$

with x given by $$F_V \cdot x = W_1 \cdot \frac{R}{2} - W_2 \cdot \frac{4 \cdot R}{3 \cdot \pi}$$ or $$x = \frac{W_1}{F_V} \cdot \frac{R}{2} - \frac{W_2}{F_V} \cdot \frac{4 \cdot R}{3 \cdot \pi}$$

$$x = \frac{397}{189} \times \frac{3 \cdot m}{2} - \frac{208}{189} \times \frac{4}{3 \cdot \pi} \times 3 \cdot m \qquad x = 1.75\,m$$

For F_H Computing equations $$F_H = p_c \cdot A \qquad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$$

Hence $$F_H = p_c \cdot A = \rho \cdot g \cdot \left(D - \frac{R}{2}\right) \cdot w \cdot R$$

$$F_H = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \left(4.5 \cdot m - \frac{3 \cdot m}{2}\right) \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_H = 265 \cdot kN$$

The location of this force is

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \left(D - \frac{R}{2}\right) + \frac{w \cdot R^3}{12} \times \frac{1}{w \cdot R \cdot \left(D - \frac{R}{2}\right)} = D - \frac{R}{2} + \frac{R^2}{12 \cdot \left(D - \frac{R}{2}\right)}$$

$$y' = 4.5 \cdot m - \frac{3 \cdot m}{2} + \frac{(3 \cdot m)^2}{12 \times \left(4.5 \cdot m - \frac{3 \cdot m}{2}\right)} \qquad y' = 3.25 m$$

The force F_1 on the bottom of the gate is $F_1 = p \cdot A = \rho \cdot g \cdot D \cdot w \cdot R$

$$F_1 = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 4.5 \cdot m \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_1 = 397 \cdot kN$$

For the concrete gate (SG = 2.4 from Table A.2)

$$W_{Gate} = SG \cdot \rho \cdot g \cdot w \cdot \frac{\pi \cdot R^2}{4} = 2.4 \cdot 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3 \cdot m \times \frac{\pi}{4} \times (3 \cdot m)^2 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W_{Gate} = 499 \cdot kN$$

Hence, taking moments about A $$F_B \cdot R + F_1 \cdot \frac{R}{2} - W_{Gate} \cdot \frac{4 \cdot R}{3 \cdot \pi} - F_V \cdot x - F_H \cdot [y' - (D - R)] = 0$$

$$F_B = \frac{4}{3 \cdot \pi} \cdot W_{Gate} + \frac{x}{R} \cdot F_V + \frac{[y' - (D - R)]}{R} \cdot F_H - \frac{1}{2} \cdot F_1$$

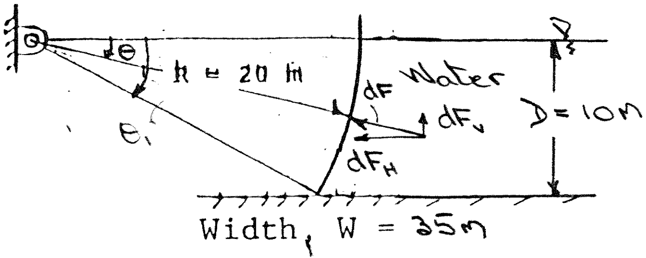
$$F_B = \frac{4}{3 \cdot \pi} \times 499 \cdot kN + \frac{1.75}{3} \times 189 \cdot kN + \frac{[3.25 - (4.5 - 3)]}{3} \times 265 \cdot kN - \frac{1}{2} \times 397 \cdot kN$$

$$F_B = 278 \cdot kN$$

Problem 3.77

Given: Tainter gate as shown

Find: Force of the water acting on the gate



Solution:

Basic equations: $dF = p\,dA$; $\frac{dp}{dh} = \rho g$

Assumptions: (1) static fluid
(2) ρ = constant
(3) p_{atm} acts at free surface and on surface of gate

For ρ = const, $\int dp = \int \rho g\,dh$ yields $p - \overset{(3)}{\cancel{p_{atm}}} = \rho g h = \rho g R \sin\theta$

$dF_H = dF\cos\theta = p\,dA\cos\theta = \rho g R\sin\theta\, wR\,d\theta\cos\theta$ $\{dA = wR\,d\theta\}$

$F_H = \int dF_H = \int_0^{\theta_1} \rho g w R^2 \sin\theta\cos\theta\,d\theta$ where $\theta_1 = \sin^{-1}\frac{10}{20} = 30^\circ$

$F_H = \rho g w R^2 \int_0^{30^\circ} \sin\theta\cos\theta\,d\theta = \rho g w R^2\left[\frac{\sin^2\theta}{2}\right]_0^{30^\circ} = \frac{\rho g w R^2}{8}$

$F_H = \frac{1}{8} \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{sec^2} \times 35m \times (20m)^2 \times \frac{N\cdot sec^2}{kg\cdot m} = 1.72 \times 10^7\,N \leftarrow$ - - -

$dF_V = dF\sin\theta = p\,dA\sin\theta = \rho g R\sin\theta\, wR\,d\theta\sin\theta$

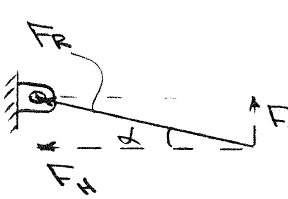
$F_V = \int dF_V = \rho g w R^2 \int_0^{30^\circ} \sin^2\theta\,d\theta = \rho g w R^2\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/6}$

$F_V = \rho g w R^2\left[\frac{\pi}{12} - \frac{0.866}{4}\right] = 0.0453\,\rho g w R^2$

$F_V = 0.0453 \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 35m \times (20m)^2 \times \frac{N\cdot s^2}{kg\cdot m} = 6.22 \times 10^6\,N \leftarrow$ - - -

Since the gate surface in contact with the water is a circular arc, all elements dF, of the force and hence the line of action of the resultant force must pass through the pivot. Thus

$F_R = \left[F_H^2 + F_V^2\right]^{1/2} = \left[(17.2\times10^6)^2 + (6.22\times10^6)^2\right]^{1/2} = 1.83 \times 10^7\,N \leftarrow$ F_R

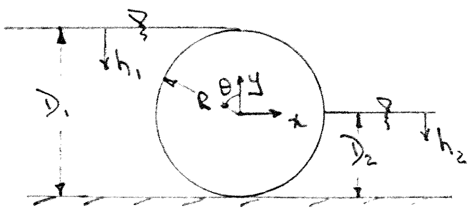


$\alpha = \tan^{-1}\frac{F_V}{F_R} = \tan^{-1}\frac{6.22}{17.2}$

$\alpha = 19.9^\circ \leftarrow$ α

F_R passes through pivot at angle α to the horizontal

Problem 3.78



Given: Cylindrical weir of radius, $R = 1.5\,m$ and length, $L = 6\,m$ as shown

Liquid is water

$D_1 = 3\,m \quad D_2 = 1.5\,m$

Find: Magnitude and direction of resultant force of water on the weir

Solution:

Basic equations: $\vec{F}_R = -\int P\,d\vec{A} \qquad \frac{dP}{dh} = \rho g$

Assumptions: (1) static fluid

(2) ρ = constant

(3) h is measured positive down from free surface

$$F_{R_x} = \int dF_x = \vec{F}_R \cdot \hat{i} = \int d\vec{F} \cdot \hat{i} = -\int P\,d\vec{A} \cdot \hat{i} = -\int P\,dA \cos(90+\theta) = \int P\,dA \sin\theta$$

$$F_{R_y} = \int dF_y = \vec{F}_R \cdot \hat{j} = \int d\vec{F} \cdot \hat{j} = -\int P\,d\vec{A} \cdot \hat{j} = -\int P\,dA \cos\theta$$

Since $dA = LR\,d\theta$,

$$F_{R_x} = \int_0^{3\pi/2} PLR \sin\theta\,d\theta \quad \text{and} \quad F_{R_y} = -\int_0^{3\pi/2} PLR\cos\theta\,d\theta$$

We can obtain an expression for P as a function of h

$$\frac{dP}{dh} = \rho g \qquad dP = \rho g\,dh \qquad \text{and} \quad P - P_0 = \int_{P_0}^{P} dP = \int_0^h \rho g\,dh = \rho g h$$

Since atmospheric pressure acts over the first quadrant of the cylinder and both free surfaces, the appropriate expression for P is $P = \rho g h$.

For

$0 \le \theta \le \pi$, $h_1 = R - R\cos\theta = R(1-\cos\theta)$ and hence $P_1 = \rho g R(1-\cos\theta)$

$\pi \le \theta \le \frac{3\pi}{2}$, $h_2 = -R\cos\theta$ and hence $P_2 = -\rho g R\cos\theta$

$$F_{R_x} = \int_0^{3\pi/2} PLR\sin\theta\,d\theta = \int_0^{\pi} \rho g R(1-\cos\theta) LR\sin\theta\,d\theta + \int_{\pi}^{3\pi/2} (-\rho g R\cos\theta) LR\sin\theta\,d\theta$$

$$= \rho g R^2 L \int_0^{\pi} (1-\cos\theta)\sin\theta\,d\theta - \rho g R^2 L \int_{\pi}^{3\pi/2} \cos\theta\sin\theta\,d\theta$$

$$= \rho g R^2 L \left[-\cos\theta - \frac{1}{2}\sin^2\theta\right]_0^{\pi} - \rho g R^2 L \left[\frac{1}{2}\sin^2\theta\right]_{\pi}^{3\pi/2} = \rho g R^2 L \left[2 - \frac{1}{2}\right] = \frac{3}{2}\rho g R^2 L$$

$$F_{R_x} = \frac{3}{2} \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (1.5)^2\,m^2 \times 6\,m \times \frac{N \cdot s^2}{kg \cdot m} = 198\ kN$$

$$F_{R_y} = -\int_0^{3\pi/2} PLR\cos\theta = -\int_0^{\pi} \rho g R(1-\cos\theta) LR\cos\theta\,d\theta - \int_{\pi}^{3\pi/2} (-\rho g R\cos\theta) LR\cos\theta\,d\theta$$

$$= -\rho g R^2 L \int_0^{\pi} (1-\cos\theta)\cos\theta\,d\theta + \rho g R^2 L \int_{\pi}^{3\pi/2} \cos^2\theta\,d\theta$$

$$= -\rho g R^2 L \left[\sin\theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi} + \rho g R^2 L \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{\pi}^{3\pi/2} = \rho g R^2 L \left[\frac{\pi}{2} + \frac{3\pi}{4} - \frac{\pi}{2}\right] = \frac{3\pi}{4}\rho g R^2 L$$

$$F_{R_y} = \frac{3\pi}{4} \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (1.5)^2\,m^2 \times 6\,m \times \frac{N \cdot s^2}{kg \cdot m} = 312\ kN$$

$$\vec{F}_R = \hat{i} F_{R_x} + \hat{j} F_{R_y} = 198\hat{i} + 312\hat{j}\ kN$$

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \left[(198)^2 + (312)^2\right]^{1/2} kN = 370\ kN \longleftarrow$$

F_R

Since all elements of force $d\vec{F}$ are normal to the surface, the direction α,

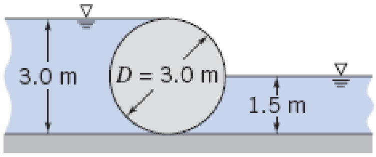
$$\alpha = \tan^{-1} F_{R_y}/F_{R_x} = \tan^{-1} 312/198 = 57.6^\circ \longleftarrow$$

α

Problem 3.79

[4]

3.79 Consider the cylindrical weir of diameter 3 m and length 6 m. If the fluid on the left has a specific gravity of 1.6, and on the right has a specific gravity of 0.8, find the magnitude and direction of the resultant force.



Given: Sphere with different fluids on each side

Find: Resultant force and direction

Solution:

The horizontal and vertical forces due to each fluid are treated separately. For each, the horizontal force is equivalent to that on a vertical flat plate; the vertical force is equivalent to the weight of fluid "above".

For horizontal forces, the computing equation of Section 3-5 is $F_H = p_c \cdot A$ where A is the area of the equivalent vertical plate.

For vertical forces, the computing equation of Section 3-5 is $F_V = \rho \cdot g \cdot V$ where V is the volume of fluid above the curved surface.

The data is For water $\rho = 999 \cdot \frac{kg}{m^3}$

For the fluids $SG_1 = 1.6$ $SG_2 = 0.8$

For the weir $D = 3 \cdot m$ $L = 6 \cdot m$

(a) Horizontal Forces

For fluid 1 (on the left) $F_{H1} = p_c \cdot A = \left(\rho_1 \cdot g \cdot \frac{D}{2}\right) \cdot D \cdot L = \frac{1}{2} \cdot SG_1 \cdot \rho \cdot g \cdot D^2 \cdot L$

$$F_{H1} = \frac{1}{2} \cdot 1.6 \cdot 999 \cdot \frac{kg}{m^3} \cdot 9.81 \cdot \frac{m}{s^2} \cdot (3 \cdot m)^2 \cdot 6 \cdot m \cdot \frac{N \cdot s^2}{kg \cdot m} \qquad F_{H1} = 423\,kN$$

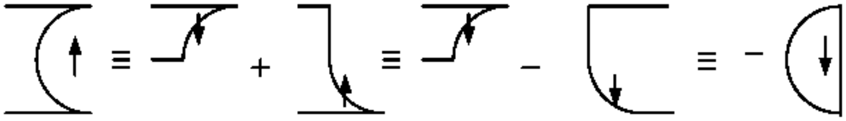
For fluid 2 (on the right) $F_{H2} = p_c \cdot A = \left(\rho_2 \cdot g \cdot \frac{D}{4}\right) \cdot \frac{D}{2} \cdot L = \frac{1}{8} \cdot SG_2 \cdot \rho \cdot g \cdot D^2 \cdot L$

$$F_{H2} = \frac{1}{8} \cdot 0.8 \cdot 999 \cdot \frac{kg}{m^3} \cdot 9.81 \cdot \frac{m}{s^2} \cdot (3 \cdot m)^2 \cdot 6 \cdot m \cdot \frac{N \cdot s^2}{kg \cdot m} \qquad F_{H2} = 52.9\,kN$$

The resultant horizontal force is $F_H = F_{H1} - F_{H2}$ $\qquad F_H = 370\,kN$

(b) Vertical forces

For the left geometry, a "thought experiment" is needed to obtain surfaces with fluid "above"



Hence $$F_{V1} = SG_1 \cdot \rho \cdot g \cdot \frac{\frac{\pi \cdot D^2}{4}}{2} \cdot L$$

$$F_{V1} = 1.6 \times 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \frac{\pi \cdot (3 \cdot m)^2}{8} \times 6 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_{V1} = 333\,kN$$

(Note: Use of buoyancy leads to the same result!)

For the right side, using a similar logic

$$F_{V2} = SG_2 \cdot \rho \cdot g \cdot \frac{\frac{\pi \cdot D^2}{4}}{4} \cdot L$$

$$F_{V2} = 0.8 \times 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \frac{\pi \cdot (3 \cdot m)^2}{16} \times 6 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_{V2} = 83.1\,kN$$

The resultant vertical force is $$F_V = F_{V1} + F_{V2} \qquad F_V = 416\,kN$$

Finally the resultant force and direction can be computed

$$F = \sqrt{{F_H}^2 + {F_V}^2} \qquad F = 557\,kN$$

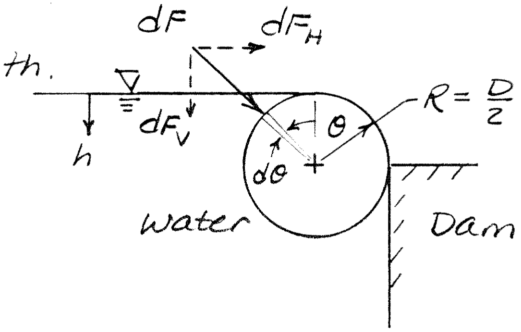
$$\alpha = \operatorname{atan}\left(\frac{F_V}{F_H}\right) \qquad \alpha = 48.3\,deg$$

Problem 3.80

Given: Cylindrical log floating against dam.

Find: (a) Mass per unit length

(b) Contact force per unit length.



Solution: Use hydrostatic equations

Basic equations: $\frac{dp}{dh} = \rho g \qquad dF = p\,dA$

Assumptions: (1) Static liquid
(2) Incompressible
(3) Neglect patm (it acts everywhere)

Then

$$p - \overset{(3)}{\cancel{p_0}} = \rho g h = \rho g R(1 - \cos\theta)$$

$$dF = p\,dA = p w R\,d\theta, \quad dF_H = dF \sin\theta, \quad dF_V = dF\cos\theta$$

$$F_H = \int_0^{3\pi/2} \rho g R(1-\cos\theta) w R \sin\theta\, d\theta = \rho g w R^2\left[-\cos\theta - \frac{\sin^2\theta}{2}\right]_0^{3\pi/2} = \rho g w R^2\left[-\frac{(-1)^2}{2} - (-1)\right]$$

$$F_H = \frac{1}{2}\rho g w R^2 \qquad \frac{F_H}{w} = \frac{1}{2}\rho g R^2 \longleftarrow \frac{F_H}{w}$$

$$F_V = \int_0^{3\pi/2} \rho g R(1-\cos\theta) w R\cos\theta\, d\theta = \int_0^{3\pi/2} \rho g w R^2\left(\cos\theta - \frac{1+\cos 2\theta}{2}\right) d\theta$$

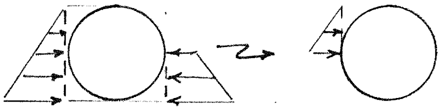
$$F_V = \rho g w R^2\left[\sin\theta - \frac{\theta + \frac{1}{2}\sin 2\theta}{2}\right]_0^{3\pi/2} = \rho g w R^2\left[-1 - \frac{3\pi}{4}\right] = -\rho g w R^2\left[1 + \frac{3\pi}{4}\right]$$

From a free-body diagram of the log

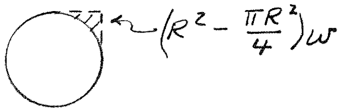
$$\Sigma F_y = -mg - F_V = 0 \qquad m = -\frac{F_V}{g} = \rho w R^2\left[1 + \frac{3\pi}{4}\right]$$

$$\frac{m}{w} = \rho R^2\left[1 + \frac{3\pi}{4}\right] \longleftarrow \frac{m}{w}$$

Check: $F_H = p_c A = \rho g \frac{R}{2} w R = \frac{1}{2}\rho g w R^2$ ✓



$$F_V = -\rho g \forall - \rho g\left[R^2 - \frac{\pi R^2}{4}\right] w = -\rho g w\left[-\pi R^2 - R^2 + \frac{\pi R^2}{4}\right] = -\rho g w R^2\left[1 + \frac{3\pi}{4}\right] ✓$$



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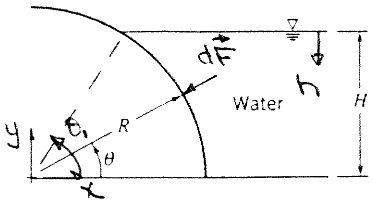
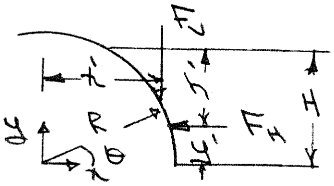
Problem 3.81

[3] Part 1/2

Given: Curved surface, in shape of quarter cylinder, with radius $R = 0.750\,m$ and width $w = 3.55\,m$; water stands to depth $H = 0.650\,m$



Find: Magnitude and line of action of:
(a) vertical force, and
(b) horizontal force
on the curved surface.



Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $F_V = \int p\,dA_y$, $x' F_V = \int x\,dF_V$

Computing equations: $F_H = p_c A$, $h' = h_c + \frac{I_{xx}}{h_c A}$

Assumptions: (1) static liquid (2) $\rho = \text{constant}$
(3) p_{atm} acts at free surface of the water

Then on integrating $dp = \rho g\,dh$, we obtain $p = \rho g h$

From the geometry $h = H - R\sin\theta$, $y = R\sin\theta$, $x = R\cos\theta$
$\theta_1 = \sin^{-1} H/R$, $dA = wR\,d\theta$

$$F_V = \int p\,dA_y = \int \rho g h\,dA\sin\theta = \int_0^{\theta_1} \rho g (H - R\sin\theta)\sin\theta\,wR\,d\theta$$

$$F_V = \rho g w R \int_0^{\theta_1} (H\sin\theta - R\sin^2\theta)\,d\theta = \rho g w R\left[-H\cos\theta - R\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_0^{\theta_1}$$

$$F_V = \rho g w R\left[H(1-\cos\theta_1) - R\left(\frac{\theta_1}{2} - \frac{\sin 2\theta_1}{4}\right)\right] \qquad (1)$$

Evaluating for $\theta_1 = \sin^{-1}\frac{H}{R} = \sin^{-1}\frac{0.650}{0.750} = 60^\circ\ (\pi/3)$.

$$F_V = 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 3.55\,m \times 0.75\,m\left[0.65\,m(1-\cos 60^\circ) - 0.75\,m\left(\frac{\pi}{6} - \frac{\sin(120^\circ)}{4}\right)\right]\frac{N\cdot s^2}{kg\cdot m}$$

$F_V = 2.47\,kN \longleftarrow F_V$

$$x'F_V = \rho g w R \int_0^{\theta_1} R\cos\theta(H\sin\theta - R\sin^2\theta)\,d\theta = \rho g w R^2 \int_0^{\theta_1}(H\sin\theta\cos\theta - R\sin^2\theta\cos\theta)\,d\theta$$

$$x'F_V = \rho g w R^2\left[H\frac{\sin^2\theta}{2} - R\frac{\sin^3\theta}{3}\right]_0^{\theta_1}$$

$$x' = \frac{\rho g w R^2}{F_V}\left[\frac{H}{2}\sin^2\theta_1 - \frac{R}{3}\sin^3\theta_1\right] \qquad (2)$$

$$x' = 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 3.55\,m \times (0.75\,m)^2 \times \frac{1}{2.47\times10^3\,N}\left[\frac{0.650\,m}{2}\sin^2 60^\circ - \frac{0.750\,m}{3}\sin^3 60^\circ\right]\frac{N\cdot s^2}{kg\cdot m}$$

$x' = 0.645\,m \longleftarrow x'$

$$F_H = p_c A = \rho g h_c H w = \rho g \frac{H}{2} H w = \frac{\rho g H^2 w}{2} \qquad (3)$$

$F_H = \frac{1}{2} \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (0.65\,m)^2 \times 3.55\,m \times \frac{N\cdot s^2}{kg\cdot m} = 7.35\,kN \longleftarrow F_H$

Problem 3.81

$$h' = h_c + \frac{I_{xx}}{h_c A} = h_c + \frac{1}{12}\frac{WH^3}{h_c A} = \frac{H}{2} + \frac{1}{12}\frac{WH^3}{\frac{H}{2}HW} = \frac{H}{2} + \frac{H}{6} = \frac{2}{3}H$$

$$y' = H - h' = H - \frac{2}{3}H = \frac{1}{3}H \quad \text{\_\_\_\_\_\_\_\_} (4)$$

$$y' = \frac{1}{3}H = \frac{1}{3} \times 0.650\,\text{m} = 0.217\,\text{m} \leftarrow \text{\_\_\_\_\_\_\_\_} \; y'$$

The computing equations for the plot are:

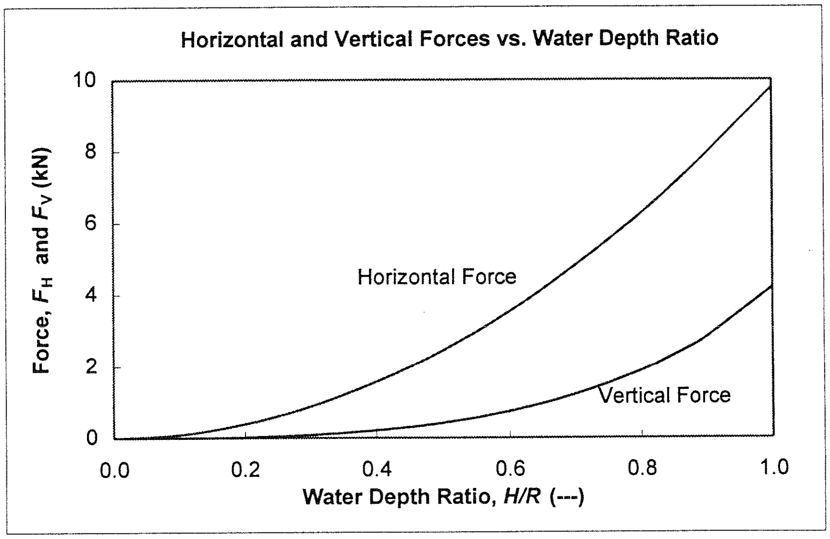
$$\theta_1 = \sin^{-1} H/R$$

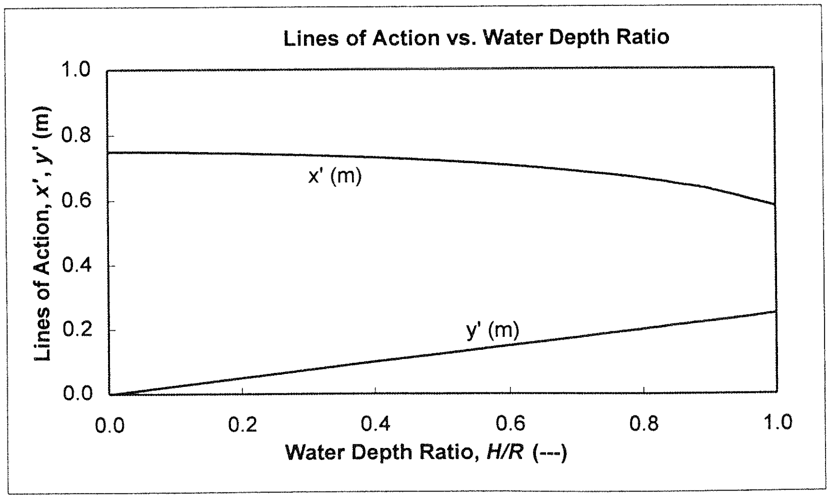
$$F_V = \rho g W R^2 \left[\frac{H}{R}(1-\cos\theta_1) - \frac{\theta_1}{2} + \frac{\sin 2\theta_1}{4} \right]$$

$$x' = \frac{\rho g W R^3 \sin^2\theta_1}{F_V} \left[\frac{1}{2}\frac{H}{R} - \frac{1}{3}\sin\theta_1 \right]$$

$$F_H = \frac{\rho g H^2 W}{2}$$

$$y' = \frac{H}{3}$$





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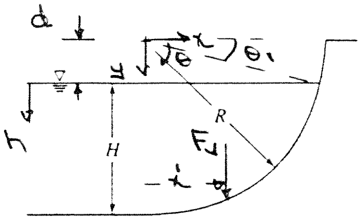
Problem 3.82



Given: Curved surface, in shape of quarter cylinder, with radius $R = 0.3\,m$ and width $w = 1.25\,m$ is filled to depth $H = 0.24\,m$ with liquid concrete.

Find: (a) Magnitude, and (b) line of action, of the vertical force on the form from the concrete.

Plot: F_V and x' over the range of depth $0 \le H \le R$

Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $F_V = \int p\, dA_y$, $x' F_V = \int x\, dF_V$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts at surface of concrete

Then on integrating $dp = \rho g\, dh$, we obtain $p = \rho g h$

$$F_V = \int p\, dA_y = \int \rho g h\, dA \sin\theta \qquad dA = wR\,d\theta$$

From the geometry: $y = R\sin\theta$, $h = y - d$, $d = R - H$

$$F_V = \int_{\theta_1}^{\pi/2} \rho g (R\sin\theta - d)\sin\theta\, wR\, d\theta \qquad \text{where } \theta_1 = \sin^{-1}\frac{d}{R}$$

$$F_V = \rho g R w \int_{\theta_1}^{\pi/2} (R\sin^2\theta - d\sin\theta)\, d\theta = \rho g R w \left[R\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right) + d\cos\theta \right]_{\theta_1}^{\pi/2}$$

$$F_V = \rho g R w \left[R\left(\frac{\pi}{4} - \frac{\theta_1}{2} + \frac{\sin 2\theta_1}{4}\right) - d\cos\theta_1 \right] \qquad (1)$$

Evaluating, $\theta_1 = \sin^{-1}\frac{d}{R} = \sin^{-1}\frac{0.3 - 0.24}{0.30} = 11.5°$

$\rho = SG\, \rho_{H_2O}$ $\quad$ { $SG = 2.50$, Table A.1)

$$F_V = 1000\,\frac{kg}{m^3} \times 2.5 \times 9.81\,\frac{m}{s^2} \times 0.3\,m \times 1.25\,m \times \frac{N \cdot s^2}{kg \cdot m}\left[0.3\,m\left(\frac{\pi}{4} - 0.0639\frac{\pi}{2} + \frac{\sin 23°}{4}\right) - 0.06\,m \cos 11.5° \right]$$

$$F_V = 1.62\,kN \qquad F_V$$

$$x' F_V = \rho g R w \int_{\theta_1}^{\pi/2} x (R\sin^2\theta - d\sin\theta)\, d\theta = \rho g R^2 w \int_{\theta_1}^{\pi/2} (R\sin^2\theta\cos\theta - d\sin\theta\cos\theta)\, d\theta$$

$$= \rho g R^2 w \left[R\,\frac{\sin^3\theta}{3} + d\,\frac{\cos^2\theta}{2} \right]_{\theta_1}^{\pi/2}$$

$$x' F_V = \rho g R^2 w \left[\frac{R}{3}(1 - \sin^3\theta_1) - \frac{d}{2}\cos^2\theta_1 \right]$$

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Problem 3.82

$$x' = SG\,\rho_{H_2O}\,\frac{gR^2W}{F_V}\left[\frac{R}{3}(1-\sin^3\theta_1)-\frac{d}{2}\cos^2\theta_1\right] \text{ - - - - - - - - - } (2)$$

$$x' = 2.5\times1000\frac{kg}{m^3}\times9.81\frac{m}{s^2}\times(0.3m)^2\times1.25m\times\frac{1}{1.62\times10^3N}\times\frac{N\cdot s^2}{kg\cdot m}\times$$

$$\left[\frac{0.3m}{3}(1-\sin^3 11.5^\circ)-\frac{0.06m}{2}\cos^2 11.5^\circ\right]$$

$$x' = 0.120\,m \longleftarrow x'$$

The computing equations for the required plots are:

$$\theta_1 = \sin^{-1}\frac{R-H}{R} = \sin^{-1}\left(1-\frac{H}{R}\right) \text{ - - - - - - - - } (3)$$

$$F_V = SG\,\rho_{H_2O}\,gR^2W\left[\frac{\pi}{4}-\frac{\theta_1}{2}+\frac{\sin2\theta_1}{4}-\left(1-\frac{H}{R}\right)\cos\theta_1\right] \text{ - - - - } (1a)$$

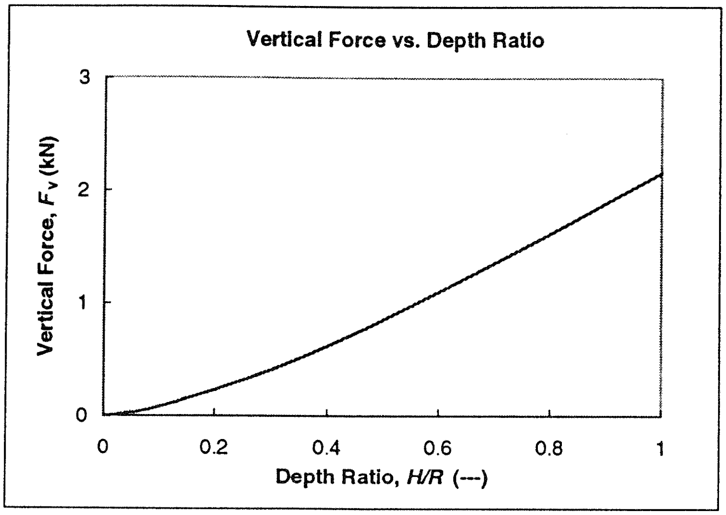
$$x' = SG\,\frac{\rho_{H_2O}\,gR^3W}{F_V}\left[\frac{1}{3}(1-\sin^3\theta_1)-\frac{1}{2}\left(1-\frac{H}{R}\right)\cos^2\theta_1\right] \text{ - - - - } (2a)$$

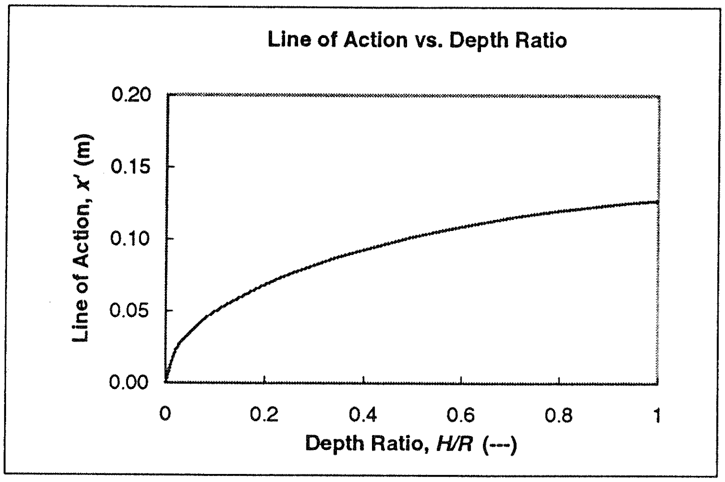
Force and line of action vs. liquid concrete depth:

Radius:	R =	0.3	m
Specific gravity:	SG =	2.5	---
Width:	W =	1.25	m

Depth Ratio, H/R (---)	Concrete Depth, H (m)	Angle, θ_1 (deg)	Vertical Force, F_V (kN)	Line of Action, x' (m)
0	0	90.0	0	0
0.02	0.006	78.5	0.00734	0.0224
0.05	0.015	71.8	0.0289	0.0352
0.1	0.03	64.2	0.0810	0.0494
0.2	0.06	53.1	0.226	0.0685
0.3	0.09	44.4	0.408	0.0822
0.4	0.12	36.9	0.617	0.0930
0.5	0.15	30.0	0.847	0.102
0.6	0.18	23.6	1.09	0.109
0.7	0.21	17.5	1.35	0.115
0.8	0.24	11.5	1.62	0.120
0.9	0.27	5.7	1.89	0.124
1.0	0.30	0.0	2.17	0.127

Problem 3.82

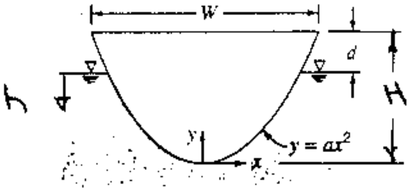

Vertical Force vs. Depth Ratio
Vertical Force, F_V (kN)
3
2
1
0
0
0.2
0.4
0.6
0.8
1
Depth Ratio, H/R (---)


Line of Action vs. Depth Ratio
Line of Action, x' (m)
0.20
0.15
0.10
0.05
0.00
0
0.2
0.4
0.6
0.8
1
Depth Ratio, H/R (---)

Problem 3.83

[3]

Given: Model cross section of canoe by $y = ax^2$, where $a = 3.89\ m^{-1}$; coordinates are in meters. Assume constant width $W = 0.6m$ over entire length $L = 5.25m$.



Find: Expression relating total mass of canoe and contents to distance d; determine maximum allowable total mass without swamping the canoe.

Solution:

At any value of d the weight of the canoe and its contents is balanced by the net vertical force of the water on the canoe.

Basic equations: $\frac{dp}{dh} = \rho g$, $F_v = \int p\,dA_y$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts at free surface of the water and on inner surface of canoe.

Then on integrating $dp = \rho g\,dh$, we obtain $p = \rho g h$

$F_v = \int p\,dA_y = \int \rho g h L\,dx$ where $h = (H-d) - y$

$y = ax^2$, At surface $y = H-d$ $\therefore$ $x = \sqrt{\frac{H-d}{a}}$

$$F_v = 2\int_0^{\sqrt{\frac{(H-d)}{a}}} \rho g\left[(H-d) - ax^2\right]L\,dx = 2\rho g L\left[(H-d)x - a\frac{x^3}{3}\right]_0^{\sqrt{\frac{H-d}{a}}}$$

$$F_v = 2\rho g L\left[\frac{(H-d)^{3/2}}{\sqrt{a}} - \frac{a}{3}\frac{(H-d)^{3/2}}{a^{3/2}}\right] = \frac{2\rho g L}{\sqrt{a}}(H-d)^{3/2}\left[1 - \frac{1}{3}\right]$$

$$F_v = \frac{4}{3}\frac{\rho g L}{\sqrt{a}}(H-d)^{3/2} = Mg$$

$$\therefore M = \frac{4\rho L(H-d)^{3/2}}{3\sqrt{a}}$$

At $d = 0$, $x = W/2$, $y = H = 0.35m$

For $d = 0$, $M = \frac{4}{3} \times 999\frac{kg}{m^3} \times 5.25m \times (0.35m)^{3/2} \times \left(\frac{m}{3.89}\right)^{1/2} = 734\ kg$

This does not provide any cushion from swamping

Set $d = 0.050\ m$

$M = \frac{4}{3} \times 999\frac{kg}{m^3} \times 5.25m \times (0.30m)^{3/2} \times \left(\frac{m}{3.89}\right)^{1/2} = 583\ kg \longleftarrow M$

The answer clearly depends on the allowed risk of swamping!

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Problem 3.84

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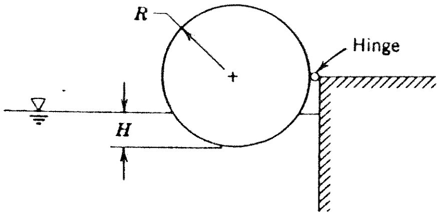
Given: Cylinder, of mass M, length L, and radius R, is hinged along it's length and immersed in an incompressible liquid to depth H.

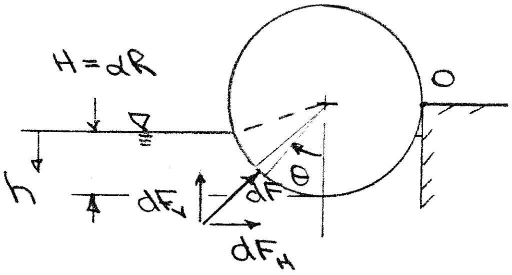
Find: a general expression for the cylinder specific gravity as a function of $\alpha = H/R$ needed to hold the cylinder in equilibrium for $0 \le \alpha \le 1$.



Solution: Apply fluid statics.

Basic eqs.: $\frac{dp}{dh} = \rho g$, $F = \int p\,dA$, $\Sigma M = 0$

Assumptions: (1) static liquid
(2) ρ = constant
$\therefore p = \rho g h$



For $0 \le \alpha \le 1$, F_H causes no net moment about O.

$$dF_v = dF\cos\theta = p\,dA\cos\theta = \rho g h\, wR\,d\theta\cos\theta$$

$$h + R(1-\cos\theta) = H \quad , \quad \therefore\ h = H - R(1-\cos\theta).$$

$$dF_v = \rho g[H - R(1-\cos\theta)]wR\cos\theta\,d\theta = \rho g w R^2\left[\frac{H}{R} - (1-\cos\theta)\right]\cos\theta\,d\theta$$

$$dF_v = \rho g w R^2[(\alpha-1)\cos\theta + \cos^2\theta]d\theta = \rho g w R^2\left[(\alpha-1)\cos\theta + \frac{1+\cos 2\theta}{2}\right]$$

For $\alpha \le 1$, $F_H = 0$, and

$$F_v = \int_{-\theta_{max}}^{\theta_{max}} dF_v = 2\int_0^{\theta_{max}} dF_v \quad \text{where } \cos\theta_{max} = \frac{R-H}{R} = 1-\alpha$$

$$\theta_{max} = \cos^{-1}(1-\alpha)$$

$$F_v = 2\rho g w R^2 \int_0^{\theta_{max}}\left[(\alpha-1)\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right]d\theta$$

$$F_v = 2\rho g w R^2\left[(\alpha-1)\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\theta_{max}}$$

$$\sin\theta_{max} = \sqrt{1-\cos^2\theta_{max}} = [1-(1-\alpha)^2]^{1/2} = [1-1+2\alpha-\alpha^2]^{1/2} = \sqrt{\alpha(2-\alpha)}.$$

$$\sin 2\theta_{max} = 2\sin\theta_{max}\cos\theta_{max} = 2\sqrt{\alpha(2-\alpha)}\,(1-\alpha)$$

Then,

$$F_v = 2\rho g w R^2\left[(\alpha-1)\sqrt{\alpha(2-\alpha)} + \frac{1}{2}\cos^{-1}(1-\alpha) + \frac{1}{2}(1-\alpha)\sqrt{\alpha(2-\alpha)}\right]$$

$$F_v = 2\rho g w R^2[\ \frac{1}{2}\cos^{-1}(1-\alpha) - \frac{1}{2}(1-\alpha)\sqrt{\alpha(2-\alpha)}$$

$$F_v = \rho g w R^2\left[\cos^{-1}(1-\alpha) - (1-\alpha)\sqrt{\alpha(2-\alpha)}\right] - - -$$

Problem 3.84

The line of action of the vertical force due to the liquid is through the centroid of the displaced liquid, i.e. through the center of the cylinder

The weight of the cylinder is given by

$$W = mg = \rho_c \forall g = SG\,\rho\,\pi R^2 w g$$

where $SG = \rho_c/\rho$ and the gravity force acts through the center of the cylinder

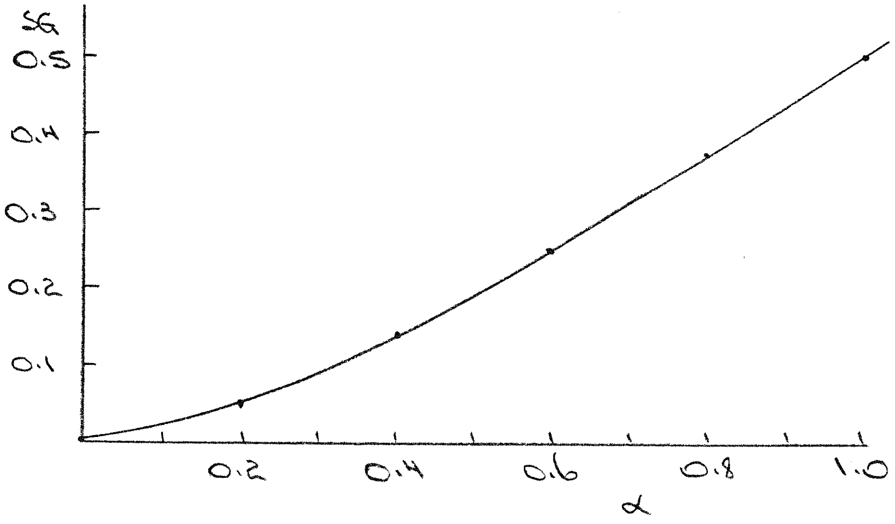
$$\Sigma M_o = WR - F_V R = 0 \qquad \therefore \; W = F_V \text{ and}$$

$$SG\,\cancel{\rho}\,\pi \cancel{R^2}\, w\,\cancel{g} = \cancel{\rho}\cancel{g} w \cancel{R^2}\left[\cos^{-1}(1-\alpha) - (1-\alpha)\sqrt{\alpha(2-\alpha)}\right.$$

$$SG = \frac{1}{\pi}\left[\cos^{-1}(1-\alpha) + (\alpha-1)\sqrt{\alpha(2-\alpha)}\right. \longleftarrow SG(0 \le \alpha \le 1)$$

Tabulating values.

α	SG
0	0
0.2	0.052
0.4	0.142
0.6	0.252
0.8	0.374
1.0	0.500



Problem 3.85

[4] Part 1/2

Given: Canoe, modelled as a right circular semi-cylindrical shell, floats in water of depth, d. The shell has outer radius, $R = 0.35$ m and length, $L = 5.25$ m.

Find: (a) a general algebraic expression for the maximum total mass that can be floated, as a function of depth and

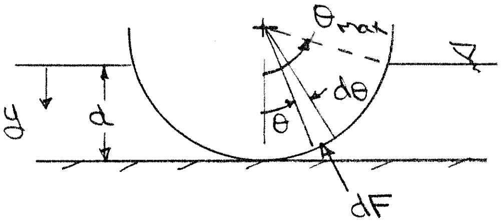
(b) evaluate for the given conditions with $d = 0.245$ m

Plot: the results over the range of water depth $0 \le d \le R$.

Solution:

Basic equations: $\frac{dp}{dy} = \rho g$; $p = p_{atm} + \rho g y$; $F_R = \int p\, dA$

End view of canoe



Assumptions: (1) static liquid
(2) p_{atm} acts on both inside & outside surfaces.

Geometry $y = y(\theta)$ for given d

$$y = d - (R - R\cos\theta) = d - R + R\cos\theta$$

$$\theta_{max} = \cos^{-1}\frac{R-d}{R}$$

A fbd of the canoe gives $\Sigma F_y = 0 = Mg - F_V$

where F_V is the vertical force of the water on the canoe

$$F_V = \int dF_V = \int dF\cos\theta = \int_A p\, dA\cos\theta = \int_{-\theta_{max}}^{\theta_{max}} \rho g y\, LR\, d\theta \cos\theta$$

$$F_V = 2\int_0^{\theta_{max}} \rho g LR\left[(d-R)\cos\theta + R\cos^2\theta\right] d\theta$$

$$F_V = 2\rho g LR\left[(d-R)\sin\theta + R\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\right]_0^{\theta_{max}}$$

$$F_V = 2\rho g LR\left[(d-R)\sin\theta_{max} + R\left(\frac{\theta_{max}}{2} + \frac{\sin 2\theta_{max}}{4}\right)\right]$$

where $\theta_{max} = \cos^{-1}\frac{(R-d)}{R}$.

Since $M = F_V/g$

$$M = 2\rho LR\left[(d-R)\sin\theta_{max} + R\left(\frac{\theta_{max}}{2} + \frac{\sin 2\theta_{max}}{4}\right)\right] \longleftarrow \quad M(d)$$

For $R = 0.35$ m, $L = 5.25$ m and $d = 0.245$ m,

$$\theta_{max} = \cos^{-1}\frac{(R-d)}{R} = \cos^{-1}\frac{(0.35 - 0.245)}{0.35} = \cos^{-1} 0.30 = 72.5°$$

$$\theta_{max} = 0.403\,\pi$$

$$M = 2 \times 999\,\frac{kg}{m^3} \times 5.25\,m \times 0.35\,m\left[(0.245 - 0.35)\sin 72.5 + 0.35\left(\frac{0.403\pi}{2} + \frac{1}{4}\sin 145\right)\right] m$$

$$M = 631\ kg \longleftarrow \quad M$$

Problem 3.85

[4] Part 2/2

The computing equations for the plot are.

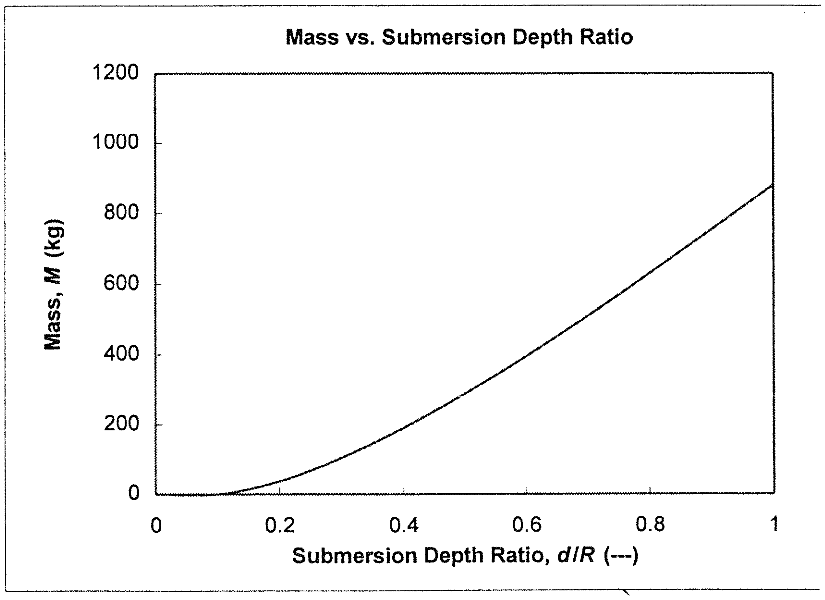
$$\theta_{max} = \cos^{-1}\left(1 - \frac{d}{R}\right)$$

$$M = 2\rho L R^2 \left[\frac{\theta_{max}}{2} + \frac{\sin 2\theta_{max}}{4} - \left(1 - \frac{d}{R}\right)\sin\theta_{max} \right]$$

Mass of canoe vs. depth of submersion ratio:

Density: ρ = 999 kg/m^3
Length: L = 5.25 m
Radius: R = 0.35 m

d (m)	d/R (---)	θ_{max} (rad)	θ_{max} (deg)	Mass (kg)
0	0	0	0	0
0.035	0.10	0.45	25.8	37.7
0.070	0.20	0.64	36.9	105
0.105	0.30	0.80	45.6	190
0.140	0.40	0.93	53.1	287
0.175	0.50	1.05	60.0	395
0.210	0.60	1.16	66.4	509
0.245	0.70	1.27	72.5	630
0.280	0.80	1.37	78.5	754
0.315	0.90	1.47	84.3	881
0.350	1.00	1.57	90.0	1009



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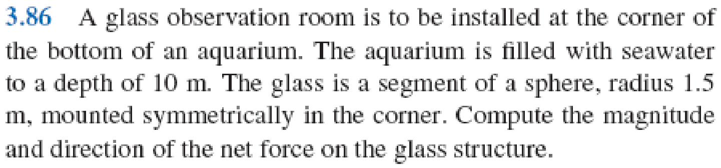
Problem 3.86

[4]



3.86 A glass observation room is to be installed at the corner of the bottom of an aquarium. The aquarium is filled with seawater to a depth of 10 m. The glass is a segment of a sphere, radius 1.5 m, mounted symmetrically in the corner. Compute the magnitude and direction of the net force on the glass structure.

Given: Geometry of glass observation room

Find: Resultant force and direction

Solution:

The *x*, *y* and *z* components of force due to the fluid are treated separately. For the *x*, *y* components, the horizontal force is equivalent to that on a vertical flat plate; for the *z* component, (vertical force) the force is equivalent to the weight of fluid above.

For horizontal forces, the computing equation of Section 3-5 is $F_H = p_c \cdot A$ where A is the area of the equivalent vertical plate.

For the vertical force, the computing equation of Section 3-5 is $F_V = \rho \cdot g \cdot V$ where V is the volume of fluid above the curved surface.

The data is For water $\rho = 999 \cdot \frac{kg}{m^3}$

For the fluid (Table A.2) $SG = 1.025$

For the aquarium $R = 1.5 \cdot m$ $H = 10 \cdot m$

(a) Horizontal Forces

Consider the *x* component

The center of pressure of the glass is $y_c = H - \frac{4 \cdot R}{3 \cdot \pi}$ $y_c = 9.36\,m$

Hence $F_{Hx} = p_c \cdot A = \left(SG \cdot \rho \cdot g \cdot y_c\right) \cdot \frac{\pi \cdot R^2}{4}$

$$F_{Hx} = 1.025 \times 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 9.36 \cdot m \times \frac{\pi \cdot (1.5 \cdot m)^2}{4} \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_{Hx} = 166\,kN$$

The *y* component is of the same magnitude as the *x* component

$$F_{Hy} = F_{Hx} \qquad F_{Hy} = 166\,kN$$

The resultant horizontal force (at 45° to the *x* and *y* axes) is

$$F_H = \sqrt{F_{Hx}^2 + F_{Hy}^2} \qquad F_H = 235\,kN$$

(b) Vertical forces

The vertical force is equal to the weight of fluid above (a volume defined by a rectangular column minus a segment of a sphere)

The volume is $$V = \frac{\pi \cdot R^2}{4} \cdot H - \frac{\frac{4 \cdot \pi \cdot R^3}{3}}{8} \qquad V = 15.9\,\text{m}^3$$

Then $$F_V = SG \cdot \rho \cdot g \cdot V \qquad F_V = 1.025 \times 999 \cdot \frac{\text{kg}}{\text{m}^3} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times 15.9 \cdot \text{m}^3 \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \qquad F_V = 160\,\text{kN}$$

Finally the resultant force and direction can be computed

$$F = \sqrt{F_H^{\,2} + F_V^{\,2}} \qquad F = 284\,\text{kN}$$

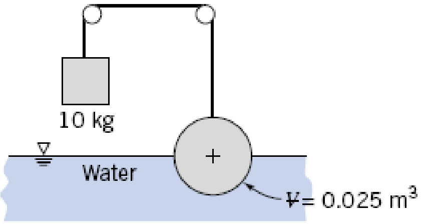
$$\alpha = \operatorname{atan}\left(\frac{F_V}{F_H}\right) \qquad \alpha = 34.2\,\text{deg}$$

Note that α is the angle the resultant force makes with the horizontal

Problem *3.87

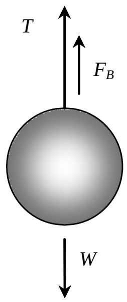
[3]

*3.87 Find the specific weight of the sphere shown if its volume is 0.025 m^3. State all assumptions. What is the equilibrium position of the sphere if the weight is removed?



Given: Data on sphere and weight

Find: SG of sphere; equilibrium position when freely floating



Solution:

Basic equation $F_B = \rho\cdot g\cdot V$ and $\Sigma F_z = 0$ $\Sigma F_z = 0 = T + F_B - W$

where $T = M\cdot g$ $M = 10\cdot kg$ $F_B = \rho\cdot g\cdot\frac{V}{2}$ $W = SG\cdot\rho\cdot g\cdot V$

Hence $M\cdot g + \rho\cdot g\cdot\frac{V}{2} - SG\cdot\rho\cdot g\cdot V = 0$ $SG = \frac{M}{\rho\cdot V} + \frac{1}{2}$

$$SG = 10\cdot kg \times \frac{m^3}{1000\cdot kg} \times \frac{1}{0.025\cdot m^3} + \frac{1}{2} \qquad SG = 0.9$$

The specific weight is $\gamma = \frac{Weight}{Volume} = \frac{SG\cdot\rho\cdot g\cdot V}{V} = SG\cdot\rho\cdot g$ $\gamma = 0.9 \times 1000\cdot\frac{kg}{m^3} \times 9.81\cdot\frac{m}{s^2} \times \frac{N\cdot s^2}{kg\cdot m}$ $\gamma = 8829\cdot\frac{N}{m^3}$

For the equilibriul position when floating, we repeat the force balance with T = 0

$$F_B - W = 0 \qquad W = F_B \quad \text{with} \quad F_B = \rho\cdot g\cdot V_{submerged}$$

From references (trying Googling "partial sphere volume") $V_{submerged} = \frac{\pi\cdot h^2}{3}\cdot(3\cdot R - h)$

where h is submerged depth and R is the sphere radius $R = \left(\frac{3\cdot V}{4\cdot\pi}\right)^{\frac{1}{3}}$ $R = \left(\frac{3}{4\cdot\pi}\cdot 0.025\cdot m^3\right)^{\frac{1}{3}}$ $R = 0.181\,m$

Hence $W = SG\cdot\rho\cdot g\cdot V = F_B = \rho\cdot g\cdot\frac{\pi\cdot h^2}{3}\cdot(3\cdot R - h)$ $h^2\cdot(3\cdot R - h) = \frac{3\cdot SG\cdot V}{\pi}$

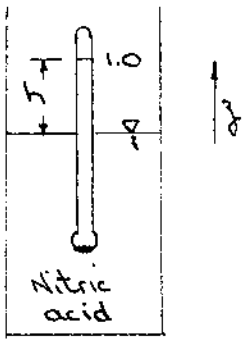
$$h^2\cdot(3\cdot 0.181\cdot m - h) = \frac{3\cdot 0.9\cdot .025\cdot m^3}{\pi} \qquad h^2\cdot(0.544 - h) = 0.0215$$

This is a cubic equation for h. We can keep guessing h values, manually iterate, or use Excel's Goal Seek to find $h = 0.292\cdot m$

Problem 3.88

[2]

Given: Hydrometer, as shown, submerged in nitric acid, S.G = 1.5
When immersed in water, h = 0 and immersed volume is 15 cm³.
Stem diameter d = 6 mm.



Find: The distance, h

Solution:

Basic equation: $\sum \vec{F} = m\vec{a} = 0$

Computing equation: $\vec{F}_{buoyancy} = \rho g \forall_d \hat{k}$

Assumptions: (1) static conditions
(2) ρ = constant

$$\sum \vec{F} = 0 = M\vec{g} + \vec{F}_{buoyancy}$$

Using the data given for water, we can calculate M

$$-Mg + F_b = 0 \qquad M = \frac{F_b}{g} = \rho_{H_2O} \forall_{H_2O}$$

When immersed in nitric acid

$$M = \rho_{n.a} \forall_{n.a} \qquad \text{where } \forall_{n.a} = \forall_{H_2O} - \frac{\pi d^2 h}{4}$$

Since the mass is the same in both cases

$$M = \rho_{H_2O} \forall_{H_2O} = \rho_{n.a}\left(\forall_{H_2O} - \frac{\pi d^2 h}{4}\right)$$

$$\frac{\pi d^2 h}{4} = \forall_{H_2O} - \frac{\rho_{H_2O}}{\rho_{n.a}} \forall_{H_2O} = \forall_{H_2O}\left(1 - \frac{1}{S.G_{n.a}}\right)$$

$$h = \frac{4 \forall_{H_2O}}{\pi d^2}\left(1 - \frac{1}{S.G_{n.a}}\right)$$

$$h = \frac{4}{\pi} \times 15\,cm^3 \times \frac{1}{6^2\,mm^2}\left(1 - \frac{1}{1.5}\right) \times \frac{1000\,mm^3}{cm^3} = 177\,mm \longleftarrow$$

h

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem *3.89

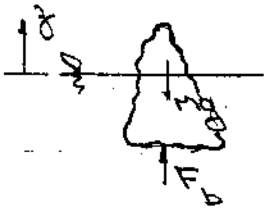
[2]

Given: Iceberg floating in sea water

Find: Quantify the statement "only the tip of an iceberg shows".

Solution:

A floating body is buoyed up by a force equal to the weight of the displaced liquid.



$$\Sigma F_z = 0 = F_b - mg$$

$$F_b = \rho_s \forall_{sub}\, g \qquad m = \rho \forall_{total}.$$

$$\therefore \rho_s \forall_{sub}\, g = \rho \forall_{tot}\, g$$

$$\therefore \forall_{sub} = \forall_{tot} \frac{\rho}{\rho_{sw}} = \forall_{tot} \frac{\rho/\rho^*}{\rho_{sw}/\rho^*}$$

where $\rho^* = \rho_{H_2O}$ at 4C.

$$\forall_{sub} = \forall_{tot} \frac{SG_{ice}}{SG_{sw}}$$

$$\forall_{not\,sub} = \forall_{tot} - \forall_{sub} = \forall_{tot}\left(1 - \frac{SG_{ice}}{SG_{sw}}\right) \quad \left\{\begin{array}{l}\text{Table A.1, } SG_{ice} = 0.917 \\ \text{Table A.2, } SG_{sw} = 1.025\end{array}\right\}$$

$$\therefore \frac{\forall_{not\,sub}}{\forall_{tot}} = 1 - \frac{SG_{ice}}{SG_s} = 1 - \frac{0.917}{1.025}$$

$$\frac{\forall_{notsub}}{\forall_{tot}} = 0.105 \qquad (10\% \text{ shows}) \longleftarrow$$

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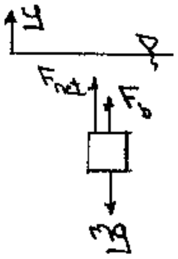
Problem *3.90

Given: Specific gravity of a person is to be determined from measurements of weight in air and the net weight when totally immersed in water.

Find: Expression for the specific gravity of a person from the measurements.

Solution:



For equilibrium $\Sigma F_y = 0$

$$F_{net} = mg - F_b$$

$$F_b = \rho_{H_2O}\, g\, ∀ \qquad F_{air} = mg$$

$$\therefore F_{net} = F_{air} - \rho_{H_2O}\, g\, ∀ \qquad \text{and } ∀ = \frac{F_{air} - F_{net}}{\rho_{H_2O}\, g}$$

$$F_{air} = mg = \rho ∀ g = \frac{\rho}{\rho_{H_2O}}\left(F_{air} - F_{net}\right)$$

Let $\rho^* = \rho_{H_2O}$ at 4C. Then

$$F_{air} = \frac{\rho/\rho^*}{\rho_{H_2O}/\rho^*}\left(F_{air} - F_{net}\right) = \frac{SG}{SG_{H_2O}}\left(F_{air} - F_{net}\right).$$

Solving for SG,

$$SG = SG_{H_2O}\,\frac{F_{air}}{\left(F_{air} - F_{net}\right)} \qquad SG$$

Problem *3.91

[2]

Given: Experiment performed by Archimedes to identify the material content of King Hero's crown.

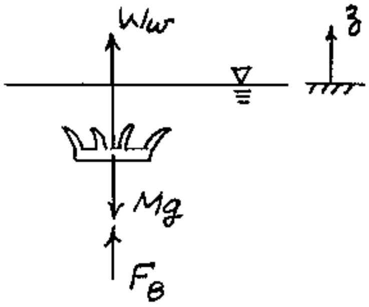
Measured weight of crown in air, W_a, and in water, W_w.

Find: Expression for specific gravity of crown as function of W_a and W_w.

Solution: Apply principle of bouyancy to free-body of crown:

Computing equation: $F_B = \rho_{H_2O} g \forall$

Assumptions: (1) Static liquid
(2) Incompressible liquid



Free-body diagram of crown in water:

$$\Sigma F_z = W_w - Mg + F_B = ma_z = 0$$

or

$$W_w - Mg + \rho_{H_2O} \forall g = 0$$

For the crown in air, $W_a = Mg$

Combining, $W_w - W_a + \rho_{H_2O} g \forall$, so $\forall = \frac{W_a - W_w}{\rho_{H_2O} g}$

The crown's density is $\rho_c = \frac{M}{\forall} = \frac{W_a}{g\forall} = \rho_{H_2O} \frac{W_a}{W_a - W_w}$

The crown's specific gravity is $SG = \frac{\rho_c}{\rho_{H_2O}} = \frac{W_a}{W_a - W_w}$ ← SG

{Note: by definition, $SG = \rho / \rho_{H_2O}(4°C)$, so the measured temperature of water and data from Table A.7 or A.8 may be used to correct the density to 4°C.}

Problem *3.92

***3.92** An open tank is filled to the top with water. A steel cylindrical container, wall thickness $\delta = 1$ mm, outside diameter $D = 100$ mm, and height $H = 1$ m, with an open top, is gently placed in the water. What is the volume of water that overflows from the tank? How many 1 kg weights must be placed in the container to make it sink? Neglect surface tension effects.

Given: Geometry of steel cylinder

Find: Volume of water displaced; number of 1 kg wts to make it sink

Solution:

The data is For water $\rho = 999 \cdot \frac{kg}{m^3}$

For steel (Table A.1) $SG = 7.83$

For the cylinder $D = 100 \cdot mm$ $H = 1 \cdot m$ $\delta = 1 \cdot mm$

The volume of the cylinder is $V_{steel} = \delta \cdot \left(\frac{\pi \cdot D^2}{4} + \pi \cdot D \cdot H \right)$ $V_{steel} = 3.22 \times 10^{-4} m^3$

The weight of the cylinder is $W = SG \cdot \rho \cdot g \cdot V_{steel}$

$$W = 7.83 \times 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3.22 \times 10^{-4} \cdot m^3 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W = 24.7 N$$

At equilibium, the weight of fluid displaced is equal to the weight of the cylinder

$$W_{displaced} = \rho \cdot g \cdot V_{displaced} = W$$

$$V_{displaced} = \frac{W}{\rho \cdot g} = 24.7 \cdot N \times \frac{m^3}{999 \cdot kg} \times \frac{s^2}{9.81 \cdot m} \times \frac{kg \cdot m}{N \cdot s^2} \qquad V_{displaced} = 2.52 L$$

To determine how many 1 kg wts will make it sink, we first need to find the extra volume that will need to be dsiplaced

Distance cylinder sank $x_1 = \frac{V_{displaced}}{\left(\frac{\pi \cdot D^2}{4} \right)}$ $x_1 = 0.321 m$

Hence, the cylinder must be made to sink an additional distance $x_2 = H - x_1$ $x_2 = 0.679 m$

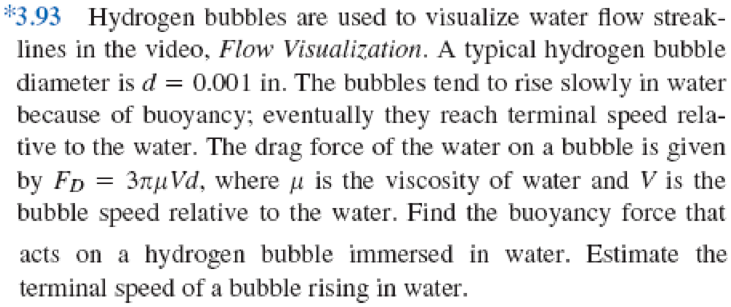
We deed to add n weights so that $1 \cdot kg \cdot n \cdot g = \rho \cdot g \cdot \frac{\pi \cdot D^2}{4} \cdot x_2$

$$n = \frac{\rho \cdot \pi \cdot D^2 \cdot x_2}{4 \times 1 \cdot kg} = 999 \cdot \frac{kg}{m^3} \times \frac{\pi}{4} \times (0.1 \cdot m)^2 \times 0.679 \cdot m \times \frac{1}{1 \cdot kg} \times \frac{N \cdot s^2}{kg \cdot m} \qquad n = 5.33$$

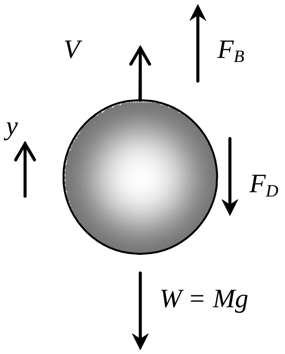
Hence we need $n = 6$ weights to sink the cylinder

Problem *3.93

[2]



*3.93 Hydrogen bubbles are used to visualize water flow streaklines in the video, *Flow Visualization*. A typical hydrogen bubble diameter is $d = 0.001$ in. The bubbles tend to rise slowly in water because of buoyancy; eventually they reach terminal speed relative to the water. The drag force of the water on a bubble is given by $F_D = 3\pi\mu V d$, where μ is the viscosity of water and V is the bubble speed relative to the water. Find the buoyancy force that acts on a hydrogen bubble immersed in water. Estimate the terminal speed of a bubble rising in water.



Given: Data on hydrogen bubbles

Find: Buoyancy force on bubble; terminal speed in water

Solution:

Basic equation $F_B = \rho\cdot g\cdot V = \rho\cdot g\cdot\frac{\pi}{6}\cdot d^3$ and $\Sigma F_y = M\cdot a_y$ $\Sigma F_y = 0 = F_B - F_D - W$ for terminal speed

$$F_B = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times 32.2\cdot\frac{\text{ft}}{\text{s}^2}\times\frac{\pi}{6}\times\left(0.001\cdot\text{in}\times\frac{1\cdot\text{ft}}{12\cdot\text{in}}\right)^3\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \qquad F_B = 1.89\times 10^{-11}\cdot\text{lbf}$$

For terminal speed $F_B - F_D - W = 0$ $F_D = 3\cdot\pi\cdot\mu\cdot V\cdot d = F_B$ where we have ignored W, the weight of the bubble (a
STP most gases are about 1/1000 the density of water)

Hence $V = \frac{F_B}{3\cdot\pi\cdot\mu\cdot d}$ with $\mu = 2.10\times 10^{-5}\cdot\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}$ from Table A.7 at 68ºF

$$V = 1.89\times 10^{-11}\cdot\text{lbf}\times\frac{1}{3\cdot\pi}\times\frac{1}{2.10\times 10^{-5}}\cdot\frac{\text{ft}^2}{\text{lbf}\cdot\text{s}}\times\frac{1}{0.001\cdot\text{in}}\times\frac{12\cdot\text{in}}{1\cdot\text{ft}}$$

$$V = 1.15\times 10^{-3}\cdot\frac{\text{ft}}{\text{s}} \qquad V = 0.825\cdot\frac{\text{in}}{\text{min}}$$

As noted by Professor Kline in the film "Flow Visualization", bubbles rise slowly!

Problem *3.94

[2]

Gas bubbles are released from the regulator of a submerged scuba diver. What happens to the bubbles as they rise through the seawater? Explain.

Open-Ended Problem Statement: Gas bubbles are released from the regulator of a submerged Scuba diver. What happens to the bubbles as they rise through the seawater?

Discussion: Air bubbles released by a submerged diver should be close to ambient pressure at the depth where the diver is swimming. The bubbles are small compared to the depth of submersion, so each bubble is exposed to essentially constant pressure. Therefore the released bubbles are nearly spherical in shape.

The air bubbles are buoyant in water, so they begin to rise toward the surface. The bubbles are quite light, so they reach terminal speed quickly. At low speeds the spherical shape should be maintained. At higher speeds the bubble shape may be distorted.

As the bubbles rise through the water toward the surface, the hydrostatic pressure decreases. Therefore the bubbles expand as they rise. As the bubbles grow larger, one would expect the tendency for distorted bubble shape to be exaggerated.

Problem *3.95

Given: Balloons with hot air, helium, and hydrogen. Claim lift per cubic foot of 0.018, 0.066, and 0.071 lbf/ft^3 for respective gases, with air heated to 150°F over ambient.

Find: (a) Evaluate claims
(b) Compare air at 250°F above ambient.

Solution: Assume ambient conditions are STP, $p_{gas} = p_{air}$, and apply ideal gas equation of state.

(Use data from Table A.6.)

Basic equations: $\text{Lift} = \rho_{air} g \forall - \rho_{gas} g \forall, \quad p = \rho R T$

Then

$$\text{Lift}/\forall = g(\rho_a - \rho_g) = \rho_a g \left(1 - \frac{\rho_g}{\rho_a}\right) = \rho_a g \left(1 - \frac{R_a T_a}{R_g T_g}\right); \; \rho_a g = 0.0765 \frac{\text{lbf}}{\text{ft}^3}$$

For helium

$$\frac{L}{\forall} = 0.0765 \frac{\text{lbf}}{\text{ft}^3}\left[1 - 53.33 \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot R} \times (460+59) R \times \frac{\text{lbm}\cdot R}{386.1 \text{ ft}\cdot\text{lbf}} \times \frac{1}{(460+59 F)}\right]$$

$$\frac{L}{\forall} = 0.0659 \text{ lbf/ft}^3 \quad (\text{rounds to } 0.066)$$

He

For hydrogen

$$\frac{L}{\forall} = 0.0765 \frac{\text{lbf}}{\text{ft}^3}\left(1 - \frac{53.33}{766.5}\right) = 0.0712 \text{ lbf/ft}^3 \quad (\text{rounds to } 0.071)$$

H_2

For air at 150°F above ambient,

$$\frac{L}{\forall} = 0.0765 \frac{\text{lbf}}{\text{ft}^3}\left[1 - \frac{53.33(460+59)}{53.33(460+59+150)}\right] = 0.0172 \text{ lbf/ft}^3$$

Air
$\Delta T = 150$

For air at 250°F above ambient,

$$\frac{L}{\forall} = 0.0765 \frac{\text{lbf}}{\text{ft}^3}\left[1 - \frac{53.33(460+59)}{53.33(460+59+250)}\right] = 0.0249 \text{ lbf/ft}^3$$

Air
$\Delta T = 250$ F

Agreement with claims is good.

Air at $\Delta T = 250$°F gives 45 percent more lift than at $\Delta T = 150$°F.

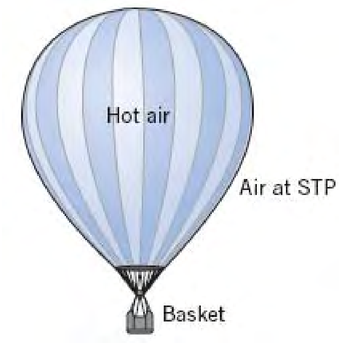
{ Hot air balloon needs 40.2 ft^3/lbf of lift at $\Delta T = 250$°F ! }

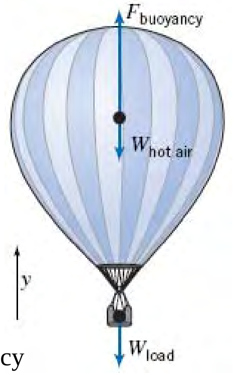
Problem *3.96

[3]

---

*3.96 A hot air balloon is designed to lift a basket, two people, three gallons of fuel, a pair of binoculars, a camera, a GPS, a cell phone, a pair of blankets, twelve candy bars, and the components of the balloon itself (fabric, ropes, and torch). The total mass is estimated at 450 kg. The rides are planned in summer morning hours when the air temperature is about 9°C. The torch will warm the air inside the balloon to a temperature of 70°C. Both inside and outside pressures will be "standard" (101 kPa). What volume of hot air should the balloon hold to create neutral buoyancy? What additional volume will ensure a vertical take-off acceleration of 0.8 m/s^2? For this, consider that both balloon and inside air have to be accelerated, as well as some of the surrounding air (to make way for the balloon). The rule of thumb is that the total mass subject to acceleration is the mass of the balloon, all its appurtenances, and twice its volume of air. Given that the volume of hot air is fixed during flight, what can the balloonists do when they want to go down?





Given: Data on hot air balloon

Find: Volume of balloon for neutral buoyancy; additional volume for initial acceleration of 0.8 m/s^2.

Solution:

Basic equation $F_B = \rho_{atm} \cdot g \cdot V$ and $\Sigma F_y = M \cdot a_y$

Hence $\Sigma F_y = 0 = F_B - W_{hotair} - W_{load} = \rho_{atm} \cdot g \cdot V - \rho_{hotair} \cdot g \cdot V - M \cdot g$ for neutral buoyancy

$$V = \frac{M}{\rho_{atm} - \rho_{hotair}} = \frac{M}{\dfrac{p_{atm}}{R \cdot T_{atm}} - \dfrac{p_{atm}}{R \cdot T_{hotair}}} = \frac{M \cdot R}{p_{atm}} \cdot \left(\frac{1}{\dfrac{1}{T_{atm}} - \dfrac{1}{T_{hotair}}} \right)$$

$$V = 450 \cdot kg \times 286.9 \cdot \frac{N \cdot m}{kg \cdot K} \times \frac{1}{101 \times 10^3} \cdot \frac{m^2}{N} \times \left[\frac{1}{\dfrac{1}{(9 + 273) \cdot K} - \dfrac{1}{(70 + 273) \cdot K}} \right] \qquad V = 2027 \cdot m^3$$

Initial acceleration $\Sigma F_y = F_B - W_{hotair} - W_{load} = \left(\rho_{atm} - \rho_{hotair}\right) \cdot g \cdot V_{new} - M \cdot g = M_{accel} \cdot a = \left(M + 2 \cdot \rho_{hotair} \cdot V_{new}\right) \cdot a$

Solving for V_{new} $\left(\rho_{atm} - \rho_{hotair}\right) \cdot g \cdot V_{new} - M \cdot g = \left(M + 2 \cdot \rho_{hotair} \cdot V_{new}\right) \cdot a$

$$V_{new} = \frac{M \cdot g + M \cdot a}{\left(\rho_{atm} - \rho_{hotair}\right) \cdot g - 2 \cdot \rho_{hotair} \cdot a} = \frac{M \cdot \left(1 + \dfrac{a}{g}\right) \cdot R}{p_{atm} \cdot \left[\left(\dfrac{1}{T_{atm}} - \dfrac{1}{T_{hotair}} \right) - \dfrac{2}{T_{hotair}} \cdot \dfrac{a}{g} \right]}$$

$$V_{new} = 450 \cdot kg \times \left(1 + \frac{0.8}{9.81}\right) \times 286.9 \cdot \frac{N \cdot m}{kg \cdot K} \times \frac{1}{101 \times 10^3} \cdot \frac{m^2}{N} \times \frac{1}{\left(\dfrac{1}{9 + 273} - \dfrac{1}{70 + 273} - \dfrac{2}{70 + 273} \cdot \dfrac{0.8}{9.81} \right)} \cdot K$$

$V_{new} = 8911 \cdot m^3$ Hence $\Delta V = V_{new} - V$ $\Delta V = 6884 \cdot m^3$

To make the balloon move up or down during flight, the air needs to be heated to a higher temperature, or let cool (or let in ambient air).

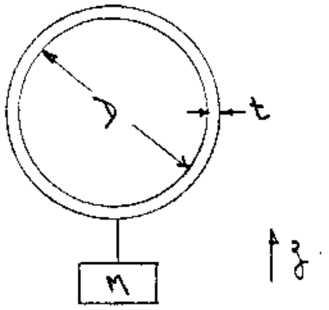
Problem *3.97

[4]

Given: Spherical balloon of diameter D, and skin thickness, $t = 0.013$ mm, filled with helium lifted a payload of mass $M = 230$ kg to an altitude of 49 km. At altitude,

$$p = 0.95 \text{ mbar} \quad \text{and} \quad T = -20°C.$$

The helium temperature is $-10°C$. The specific gravity of the skin material is 1.28



Find: The diameter and mass of the balloon.

Solution: Basic equation $\Sigma \vec{F} = m\vec{a} = 0$

Assumptions: (1) static equilibrium at altitude of 49 km
(2) air and helium exhibit ideal gas behavior.

$$\Sigma F_z = 0 = F_{buoy} - M_{He}g - M_s g - Mg = \rho_{air} g \forall_b - \rho_{He} g \forall_b - \rho_s \forall_s - Mg$$

$$0 = \forall_b(\rho_{air} - \rho_{He}) - \rho_s A_s t - M = \frac{4}{3}\pi R^3(\rho_{air} - \rho_{He}) - \rho_s 4\pi R^2 t - M$$

$$0 = \frac{\pi D^3}{6}(\rho_{air} - \rho_{He}) - \rho_s \pi D^2 t - M$$

This is a cubic equation which requires an iterative solution

$$\pi D^2\left[\frac{D}{6}(\rho_{air} - \rho_{He}) - \rho_s t\right] - M = 0 \qquad \text{Solving for } D,$$

$$D = \frac{6}{(\rho_{air} - \rho_{He})}\left[\frac{M}{\pi D^2} + \rho_s t\right] = 6\left[\frac{M}{\pi D^2(\rho_{air} - \rho_{He})} + \frac{\rho_s t}{(\rho_{air} - \rho_{He})}\right]$$

From the ideal gas law,

$$\rho_{air} = \frac{p}{RT} = 0.95 \times 10^{-3}\,\text{bar} \times \frac{\text{kg}\cdot\text{K}}{287\,\text{J}} \times \frac{1}{253\,\text{K}} \times \frac{10^5\,\text{Pa}}{\text{bar}} \times \frac{\text{N}}{\text{Pa}\cdot\text{m}^2} \times \frac{\text{J}}{\text{N}\cdot\text{m}} = 1.31 \times 10^{-3}\,\frac{\text{kg}}{\text{m}^3}$$

$$\rho_{He} = \frac{p}{RT} = 0.95 \times 10^{-3}\,\text{bar} \times \frac{\text{kg}\cdot\text{K}}{2080} \times \frac{1}{263\,\text{K}} \times \frac{10^5\,\text{Pa}}{\text{bar}} \times \frac{\text{N}}{\text{Pa}\cdot\text{m}^2} \times \frac{\text{J}}{\text{N}\cdot\text{m}} = 1.74 \times 10^{-4}\,\frac{\text{kg}}{\text{m}^3}$$

Substituting into the expression for D

$$D = 6\left[\frac{1}{\pi D^2} \times 230\,\text{kg} \times \frac{\text{m}^3}{11.4 \times 10^{-4}\,\text{kg}} + (1.28)999\,\frac{\text{kg}}{\text{m}^3} \times 1.3 \times 10^{-5}\,\text{m} \times \frac{\text{m}^3}{11.4 \times 10^{-4}\,\text{kg}}\right]$$

$$D = \left[\frac{38.5 \times 10^4}{D^2} + 87.5\right] \qquad \text{where } D \text{ is in meters}$$

Organizing Calculations: Guess D (m) = 100 120 116
R.H.S = 126 114 116.1

$$\therefore D = 116\,\text{m} \longleftarrow$$

D

$$M_b = \rho_s \forall_s = \rho_s A_s t = \rho_s \pi D^2 t = 1.28 \times 999\,\frac{\text{kg}}{\text{m}^3} \times \pi(116)^2\,\text{m}^2 \times 1.3 \times 10^{-5}\,\text{m}$$

$$M_b = 703\,\text{kg} \longleftarrow$$

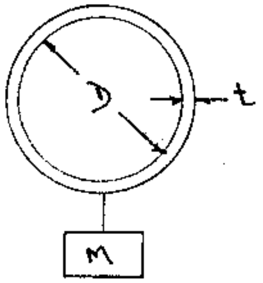
M_b

Problem *3.98

[3]

Given: A pressurized helium balloon is to be designed to lift a payload of mass, M, to an altitude of 40 km, where
$p = 3.0$ mbar and $T = -25°C$.
The balloon skin has a specific gravity, $S.G = 1.28$ and thickness, $t = 0.015$ mm
The gage pressure of the helium is 0.45 mbar. The allowable tensile stress in the balloon skin is $\sigma = 62\ MN/m^2$

Find: (a) Maximum balloon diameter
(b) Payload, M

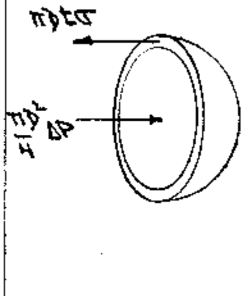


Solution:

Basic equation: $\Sigma \vec{F} = m\vec{a} = 0$

Assumptions: (1) static equilibrium at altitude
(2) air and helium exhibit ideal gas behavior

The balloon diameter is limited by tensile stress



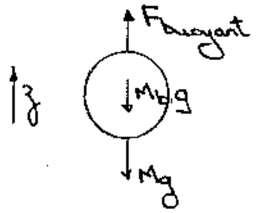
$$\Sigma F = 0 = \frac{\pi D^2}{4} \Delta P - \pi D t \sigma$$

$$D_{max} = \frac{4 t \sigma}{\Delta P}$$

$$D_{max} = 4 \times 1.50 \times 10^{-5}\ m \times 62 \times 10^6\ \frac{N}{m^2} \times \frac{1}{0.45 \times 10^{-3}\ bar} \times \frac{bar\ m^2}{10^5\ N}$$

$$D_{max} = 82.7\ m \longleftarrow$$

D_{max}



$$\Sigma F_z = 0 = F_{buoy} - M_{He} g - M_b g - Mg \qquad M_{He} = \rho_{He} \forall$$

$$F_{buoy} - M_{He} g = (\rho_{air} - \rho_{He}) g \forall = (\rho_{air} - \rho_{He}) g \frac{\pi D^3}{6}$$

$$M_b = \rho_s \forall_s = \rho_s A_s t_s = \rho_s \pi D^2 t$$

$$\therefore M = \frac{F_{buoy}}{g} - M_b = (\rho_{air} - \rho_{He}) \frac{\pi D^3}{6} - \rho_s \pi D^2 t$$

$$M = \pi D^2 \left[(\rho_{air} - \rho_{He}) \frac{D}{6} - \rho_s t \right]$$

From ideal gas law

$$\rho_{air} = \frac{P}{RT} = 3.0 \times 10^{-3}\ bar \times \frac{kg \cdot K}{287\ J} \times \frac{1}{248\ K} \times \frac{10^5\ Pa}{bar} \times \frac{N}{Pa\ m^2} \times \frac{J}{N \cdot m} = 4.21 \times 10^{-3}\ kg/m^3$$

$$\rho_{He} = \frac{(P + \Delta P)}{RT} = 3.45 \times 10^{-3}\ bar \times \frac{kg \cdot K}{2080\ J} \times \frac{1}{248\ K} \times \frac{10^5\ Pa}{bar} \times \frac{N}{Pa \cdot m^2} \times \frac{J}{N \cdot m} = 6.69 \times 10^{-4}\ kg/m^3$$

Then,

$$M = \pi (82.7)^2\ m^2 \left[(42.1 - 6.69) \times 10^{-4}\ \frac{kg}{m^3} \times \frac{82.7\ m}{6} - 1.28 \times 999\ \frac{kg}{m^3} \times 1.5 \times 10^{-5}\ m \right]$$

$$M = 637\ kg \longleftarrow$$

M

42-381 50 SHEETS 5 SQUARE
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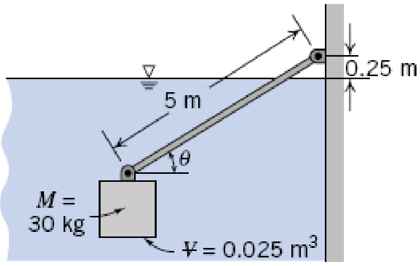
Problem 3.99

[3]

---

*3.99 A block of mass 30 kg and volume 0.025 m^3 is allowed to sink in water as shown. A circular rod 5 m long and 20 cm^2 in cross section is attached to the weight and also to the wall. If the rod mass is 1.25 kg, what will be the angle, θ, for equilibrium?

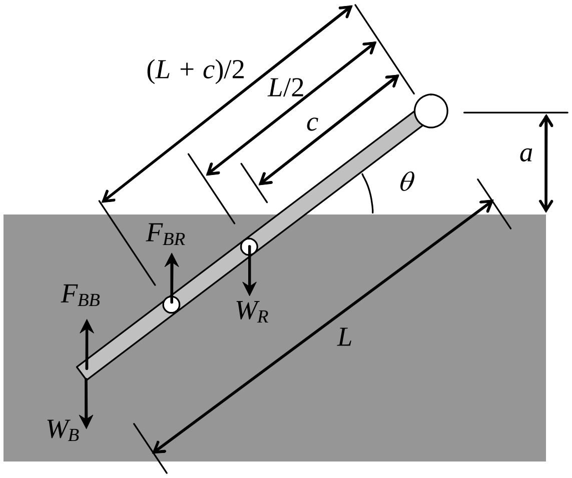


NEW PROBLEM STATEMENT NEEDED

NOTE: Cross section is 25 cm^2

Given: Geometry of block and rod

Find: Angle for equilibrium



Solution:

Basic equations $\quad \Sigma M_{Hinge} = 0 \qquad F_B = \rho \cdot g \cdot V \quad$ (Buoyancy)

The free body diagram is as shown. F_{BB} and F_{BR} are the buoyancy of the block and rod, respectively; c is the (unknown) exposed length of the rod

Taking moments about the hinge

$$\left(W_B - F_{BB}\right) \cdot L \cdot \cos(\theta) - F_{BR} \cdot \frac{(L + c)}{2} \cdot \cos(\theta) + W_R \cdot \frac{L}{2} \cdot \cos(\theta) = 0$$

with $\quad W_B = M_B \cdot g \qquad F_{BB} = \rho \cdot g \cdot V_B \qquad F_{BR} = \rho \cdot g \cdot (L - c) \cdot A \qquad W_R = M_R \cdot g$

Combining equations $\quad \left(M_B - \rho \cdot V_B\right) \cdot L - \rho \cdot A \cdot (L - c) \cdot \frac{(L + c)}{2} + M_R \cdot \frac{L}{2} = 0$

We can solve for c $\quad \rho \cdot A \cdot \left(L^2 - c^2\right) = 2 \cdot \left(M_B - \rho \cdot V_B + \frac{1}{2} \cdot M_R\right) \cdot L$

$$c = \sqrt{L^2 - \frac{2 \cdot L}{\rho \cdot A} \cdot \left(M_B - \rho \cdot V_B + \frac{1}{2} \cdot M_R\right)}$$

$$c = \sqrt{(5 \cdot m)^2 - 2 \times 5 \cdot m \times \frac{m^3}{1000 \cdot kg} \times \frac{1}{25} \cdot \frac{1}{cm^2} \times \left(\frac{100 \cdot cm}{1 \cdot m}\right)^2 \times \left[30 \cdot kg - \left(1000 \cdot \frac{kg}{m^3} \times 0.025 \cdot m^3\right) + \frac{1}{2} \times 1.25 \cdot kg\right]}$$

$$c = 1.58\,m$$

Then $\quad \sin(\theta) = \frac{a}{c} \quad$ with $\quad a = 0.25 \cdot m \qquad \theta = \operatorname{asin}\left(\frac{a}{c}\right) \qquad \theta = 9.1 \cdot deg$

Problem *3.100

[3]

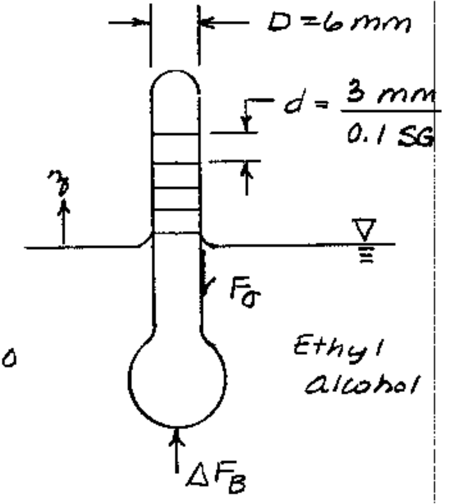
Given: Glass hydrometer used to measure SG of liquids.

Stem has $D = 6\text{ mm}$; distance between marks on stem is $d = 3\text{ mm}$ per 0.1 SG

Hydrometer floats in ethyl alcohol (assume contact angle is 0°).

Find: Magnitude of error introduced by surface tension.

Solution: Consider a free-body diagram of the floating hydrometer



Surface tension will cause the hydrometer to sink Δh lower into the liquid. Thus for this change,

$$\sum F_z = \Delta F_B - F_\sigma = m a_z = 0$$

Computing equation: $\Delta F_B = \rho g \Delta V$

Assumptions: (1) Static liquid (3) $\theta \approx 0$
(2) Incompressible liquid

Then $\Delta V = \frac{\pi D^2}{4} \Delta h$ and $\Delta F_B = \rho g \frac{\pi D^2}{4} \Delta h$

and $F_\sigma = \pi D \sigma \cos\theta = \pi D \sigma$

Combining $\rho g \frac{\pi D^2}{4} \Delta h = \pi D \sigma$ or $\Delta h = \frac{4\sigma}{\rho g D} = \frac{4\sigma}{SG \rho_{H_2O} g D}$

From Table A.2, $SG = 0.789$ and from Table A.4, $\sigma = 22.3\text{ mN/m}$ for ethanol, so

$$\Delta h = \frac{4}{0.789} \times 22.3 \times 10^{-3} \frac{N}{m} \times \frac{m^3}{1000\,kg} \times \frac{s^2}{9.81\,m} \times \frac{1}{0.006\,m} \times \frac{kg \cdot m}{N \cdot s^2} = 1.92 \times 10^{-3}\,m$$

Thus the change in SG will be

$$\Delta SG = 1.92 \times 10^{-3}\,m \times \frac{0.1\,SG}{3\,mm} \times 1000 \frac{mm}{m} = 0.0640$$

ΔS

{ From the diagram, surface tension acts to cause the hydrometer to float lower in the liquid. Therefore surface tension results in an indicated SG smaller than the actual SG. }

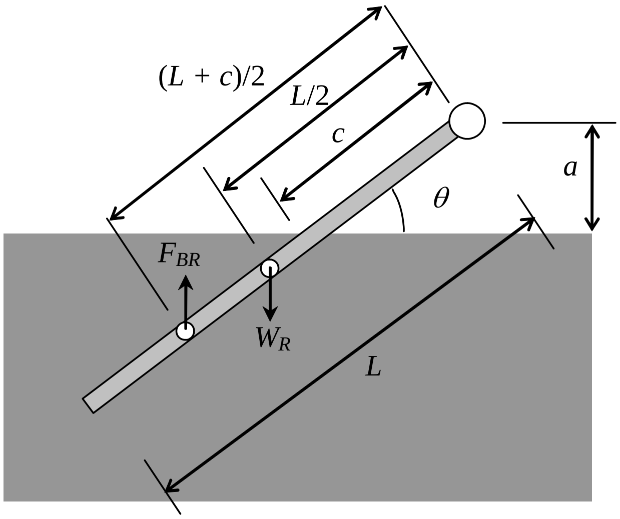
Problem 3.101

[2]

*3.101 If the mass M in Problem 3.99 is released from the rod, at equilibrium how much of the rod will remain submerged? What will be the minimum required upward force at the tip of the rod to just lift it out of the water?

Given: Geometry of rod

Find: How much of rod is submerged; force to lift rod out of water



Solution:

Basic equations $\Sigma M_{Hinge} = 0$ $F_B = \rho \cdot g \cdot V$ (Buoyancy)

The free body diagram is as shown. F_{BR} is the buoyancy of the rod; c is the (unknown) exposed length of the rod

Taking moments about the hinge

$$-F_{BR} \cdot \frac{(L+c)}{2} \cdot \cos(\theta) + W_R \cdot \frac{L}{2} \cdot \cos(\theta) = 0$$

with $F_{BR} = \rho \cdot g \cdot (L - c) \cdot A$ $W_R = M_R \cdot g$

Hence $-\rho \cdot A \cdot (L - c) \cdot \frac{(L+c)}{2} + M_R \cdot \frac{L}{2} = 0$

We can solve for c $\rho \cdot A \cdot \left(L^2 - c^2\right) = M_R \cdot L$

$$c = \sqrt{L^2 - \frac{L \cdot M_R}{\rho \cdot A}}$$

$$c = \sqrt{(5 \cdot m)^2 - 5 \cdot m \times \frac{m^3}{1000 \cdot kg} \times \frac{1}{25} \cdot \frac{1}{cm^2} \times \left(\frac{100 \cdot cm}{1 \cdot m}\right)^2 \times 1.25 \cdot kg}$$

$$c = 4.74 m$$

Then the submerged length is $L - c = 0.257 m$

To lift the rod out of the water requires a force equal to half the rod weight (the reaction also takes half the weight)

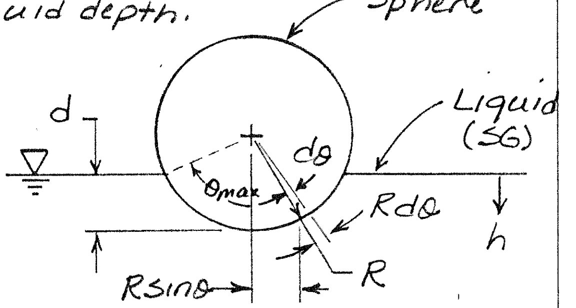
$$F = \frac{1}{2} \cdot M_R \cdot g = \frac{1}{2} \times 1.25 \cdot kg \times 9.81 \cdot \frac{m}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \qquad F = 6.1 N$$

Problem *3.102

[4]

Given: Sphere partially immersed in liquid of specific gravity, SG.

Find: (a) Formula, algebraic expression for buoyancy force, as a function of submersion depth, d, for $0 \le d \le R$.
(b) Plot of results over range of liquid depth.



__Solution:__ Apply fluid statics

Basic equations: $\dfrac{dp}{dh} = \rho g$

$dF = p\,dA$

Assumptions: (1) Static liquid
(2) Incompressible, so $p = p_0 + \rho g h$
(3) Neglect p_{atm} since it acts everywhere

Then $dF_V = \cos\theta\, p\, dA$; $p = \rho g h$; $d = h + R(1-\cos\theta)$; $h = d - R(1-\cos\theta)$

$$dA = 2\pi (R\sin\theta) R\,d\theta = 2\pi R^2 \sin\theta\, d\theta$$

$$dF_V = \cos\theta\, \rho g [d - R(1-\cos\theta)]\, 2\pi R^2 \sin\theta\, d\theta = 2\pi R^3 \left[\frac{d}{R} - (1-\cos\theta)\right] \sin\theta \cos\theta\, d\theta\, \rho g$$

Now

$$F_V = \int_A dF_V = \int_0^{\theta_{max}} 2\pi R^3 \left[\frac{d}{R} - (1-\cos\theta)\right] \sin\theta\cos\theta\, d\theta\, \rho g$$

$$F_V = 2\pi R^3 \left[(1 - d/R)\frac{\cos^2\theta}{2} - \frac{\cos^3\theta}{3}\right]_0^{\theta_{max}} \rho g \quad ; \ \rho = SG\,\rho_{H_2O}$$

At θ_{max}, $\cos\theta_{max} = \dfrac{R-d}{R} = 1 - d/R$, so

$$F_V = 2\pi\rho g R^3 \left\{(1 - d/R)\left[(1-d/R)^2/2 - 1/2\right] - \frac{(1-d/R)^3}{3} + \frac{1}{3}\right\}$$

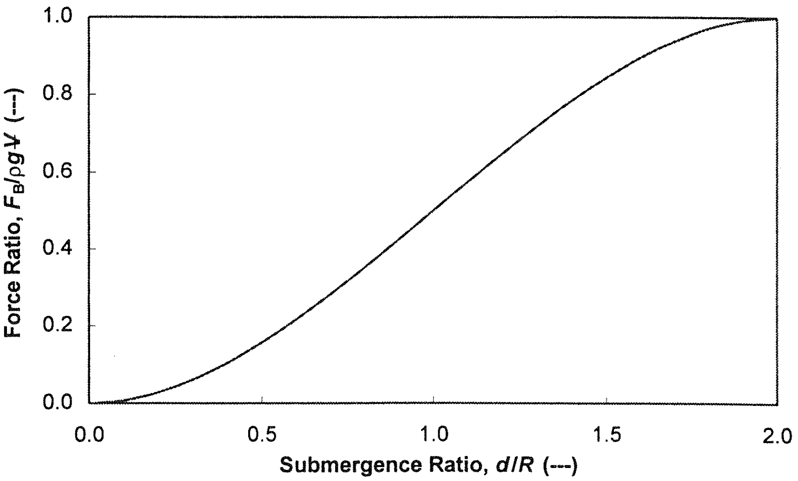
$$F_V = 2\pi\rho g R^3 \left[\frac{1}{6}\left(1-\frac{d}{R}\right)^3 - \frac{1}{2}\left(1-\frac{d}{R}\right) + \frac{1}{3}\right]$$

F_V

Dividing both sides by the vertical force on a fully submerged sphere,

$$\frac{F_V}{\rho g \frac{4\pi R^3}{3}} = \frac{3}{2}\left[\frac{1}{6}()^3 - \frac{1}{2}() + \frac{1}{3}\right]$$

where $() = (1 - \frac{d}{R})$.



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42 389 200 SHEETS 5 SQUARE
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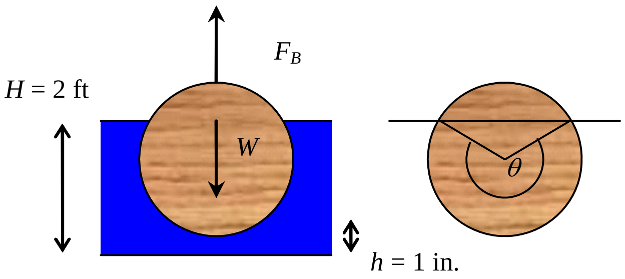
Problem *3.103

[2]

***3.103** In a logging operation, timber floats downstream to a lumber mill. It is a dry year, and the river is running low, as low as 2 feet in some locations. What is the largest diameter log that may be transported in this fashion (leaving a minimum 1 in. clearance between the log and the bottom of the river)? For the wood, SG = 0.8.



Given: Data on river

Find: Largest diameter of log that will be transported

Solution:

Basic equation $F_B = \rho \cdot g \cdot V_{sub}$ and $\Sigma F_y = 0$ $\Sigma F_y = 0 = F_B - W$

where $F_B = \rho \cdot g \cdot V_{sub} = \rho \cdot g \cdot A_{sub} \cdot L$ $W = SG \cdot \rho \cdot g \cdot V = SG \cdot \rho \cdot g \cdot A \cdot L$

From references (trying Googling "segment of a circle") $A_{sub} = \frac{R^2}{2} \cdot (\theta - \sin(\theta))$ where R is the radius and θ is the included angle

Hence $\rho \cdot g \cdot \frac{R^2}{2} \cdot (\theta - \sin(\theta)) \cdot L = SG \cdot \rho \cdot g \cdot \pi \cdot R^2 \cdot L$

$$\theta - \sin(\theta) = 2 \cdot SG \cdot \pi = 2 \times 0.8 \times \pi$$

This equation can be solved by manually iterating, or by using a good calculator, or by using Excel's Goal Seek

$$\theta = 239 \cdot \text{deg}$$

From geometry the submerged amount of a log is $H - h$ and also $R + R \cdot \cos\left(\pi - \frac{\theta}{2}\right)$

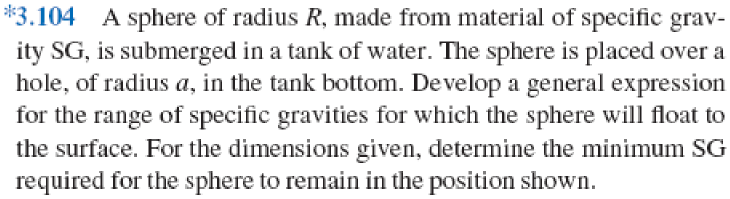
Hence $H - h = R + R \cdot \cos\left(\pi - \frac{\theta}{2}\right)$

Solving for R $R = \frac{H - h}{1 + \cos\left(180\text{deg} - \frac{\theta}{2}\right)}$ $R = \frac{\left(2 - \frac{1}{12}\right) \cdot \text{ft}}{1 + \cos\left[\left(180 - \frac{239}{2}\right) \cdot \text{deg}\right]}$ $R = 1.28 \cdot \text{ft}$

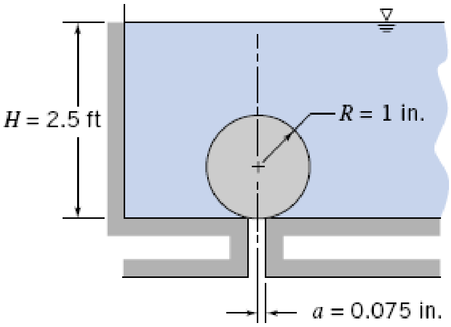
$D = 2 \cdot R$ $D = 2.57 \cdot \text{ft}$

Problem *3.104

[4]

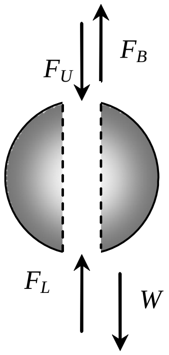


***3.104** A sphere of radius R, made from material of specific gravity SG, is submerged in a tank of water. The sphere is placed over a hole, of radius a, in the tank bottom. Develop a general expression for the range of specific gravities for which the sphere will float to the surface. For the dimensions given, determine the minimum SG required for the sphere to remain in the position shown.



Given: Data on sphere and tank bottom

Find: Expression for SG of sphere at which it will float to surface; minimum SG to remain in position



Solution:

Basic equations $\quad F_B = \rho\cdot g\cdot V \quad$ and $\quad \Sigma F_y = 0 \qquad \Sigma F_y = 0 = F_L - F_U + F_B - W$

where $\quad F_L = p_{atm}\cdot\pi\cdot a^2 \qquad F_U = \left[p_{atm} + \rho\cdot g\cdot(H - 2\cdot R)\right]\cdot\pi\cdot a^2$

$$F_B = \rho\cdot g\cdot V_{net} \qquad V_{net} = \frac{4}{3}\cdot\pi\cdot R^3 - \pi\cdot a^2\cdot 2\cdot R$$

$W = SG\cdot\rho\cdot g\cdot V \quad$ with $\quad V = \frac{4}{3}\cdot\pi\cdot R^3$

Note that we treat the sphere as a sphere with SG, and for fluid effects a sphere minus a cylinder (buoyancy) and cylinder with hydrostatic pressures

Hence

$$p_{atm}\cdot\pi\cdot a^2 - \left[p_{atm} + \rho\cdot g\cdot(H - 2\cdot R)\right]\cdot\pi\cdot a^2 + \rho\cdot g\cdot\left(\frac{4}{3}\cdot\pi\cdot R^3 - 2\cdot\pi\cdot R\cdot a^2\right) - SG\cdot\rho\cdot g\cdot\frac{4}{3}\cdot\pi\cdot R^3 = 0$$

Solving for SG

$$SG = \frac{3}{4\cdot\pi\cdot\rho\cdot g\cdot R^3}\cdot\left[-\pi\cdot\rho\cdot g\cdot(H - 2\cdot R)\cdot a^2 + \rho\cdot g\cdot\left(\frac{4}{3}\cdot\pi\cdot R^3 - 2\cdot\pi\cdot R\cdot a^2\right)\right]$$

$$SG = 1 - \frac{3}{4}\cdot\frac{H\cdot a^2}{R^3}$$

$$SG = 1 - \frac{3}{4}\times 2.5\cdot ft\times\left(0.075\cdot in\times\frac{1\cdot ft}{12\cdot in}\right)^2\times\left(\frac{1}{1\cdot in}\times\frac{12\cdot in}{1\cdot ft}\right)^3 \qquad SG = 0.873$$

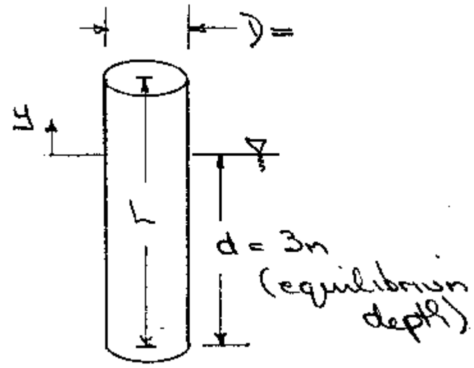
This is the minimum SG to remain submerged; any SG above this and the sphere remains on the bottom; any SG less than this and the sphere rises to the surface

Problem *3.105

Given: Cylindrical timber, $D = 0.3\,m$ and $L = 4\,m$ is weighted on lower end so it floats vertically with $3\,m$ submerged in sea water.
When displaced vertically from equilibrium position the timber oscillates in a vertical direction upon release

Find: Estimate frequency of oscillation. (Neglect any viscous effects or water motion)

Solution:



At equilibrium

$$\Sigma F_y = 0 = F_b - mg = \rho A d - mg$$

$$\therefore m = \frac{\rho A d}{g}$$

For displacement y

$$\Sigma F_y = m \frac{d^2 y}{dt^2} = m\ddot{y}$$

$$F_b - mg = m\ddot{y} \quad \text{where } F_b = \rho A(d-y)$$

$$\therefore \rho A(d-y) - mg = m\ddot{y}$$

$$\rho A d - \rho A y - \frac{\rho A d}{g} g = m\ddot{y}$$

or

$$m\ddot{y} + \rho A y = 0$$

$$\ddot{y} + \frac{\rho A}{m} y = 0 = \ddot{y} + \omega^2 y = 0$$

$$\text{where } \omega^2 = \frac{\rho A}{m} = \frac{\rho A\, g}{\rho A d} = \frac{g}{d}$$

$$\omega = \left(\frac{g}{d}\right)^{1/2} = \left[9.81\,\frac{m}{s^2} \times \frac{1}{3\,m}\right]^{1/2} = 1.81 \text{ rad/s}$$

ω

$$f = \frac{\omega}{2\pi} = 1.81\,\frac{rad}{s} \times \frac{cycle}{2\pi\,rad} = 0.288 \text{ cycle/s}$$

$$\tau = \frac{1}{f} = 3.47\,s$$

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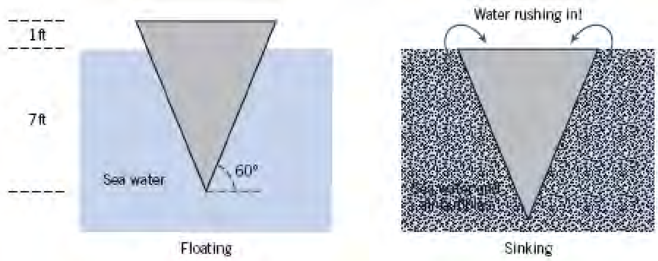
Problem *3.106

[3]

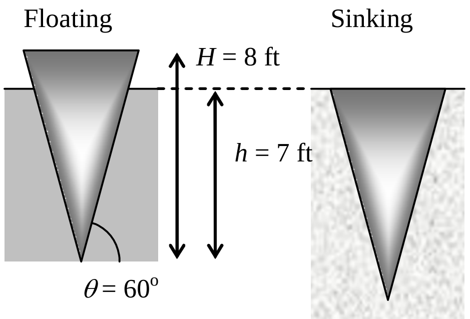
***3.106** You are in the Bermuda Triangle when you see a bubble plume eruption (a large mass of air bubbles, similar to a foam) off to the side of the boat. Do you want to head toward it and be part of the action? What is the effective density of the water and air bubbles in the drawing on the right that will cause the boat to sink? Your boat is 10 ft long, and weight is the same in both cases.



Floating Sinking

Given: Data on boat

Find: Effective density of water/air bubble mix if boat sinks



Solution:

Basic equations $F_B = \rho \cdot g \cdot V$ and $\Sigma F_y = 0$

We can apply the sum of forces for the "floating" free body

$$\Sigma F_y = 0 = F_B - W \qquad \text{where} \qquad F_B = SG_{sea} \cdot \rho \cdot g \cdot V_{subfloat}$$

$$V_{subfloat} = \frac{1}{2} \cdot h \cdot \left(\frac{2 \cdot h}{\tan \cdot \theta}\right) \cdot L = \frac{L \cdot h^2}{\tan(\theta)} \qquad SG_{sea} = 1.024 \qquad \text{(Table A.2)}$$

Hence $$W = \frac{SG_{sea} \cdot \rho \cdot g \cdot L \cdot h^2}{\tan(\theta)} \qquad (1)$$

We can apply the sum of forces for the "sinking" free body

$$\Sigma F_y = 0 = F_B - W \qquad \text{where} \qquad F_B = SG_{mix} \cdot \rho \cdot g \cdot V_{sub} \qquad V_{subsink} = \frac{1}{2} \cdot H \cdot \left(\frac{2 \cdot H}{\tan \cdot \theta}\right) \cdot L = \frac{L \cdot H^2}{\tan(\theta)}$$

Hence $$W = \frac{SG_{mix} \cdot \rho \cdot g \cdot L \cdot H^2}{\tan(\theta)} \qquad (2)$$

Comparing Eqs. 1 and 2 $$W = \frac{SG_{sea} \cdot \rho \cdot g \cdot L \cdot h^2}{\tan(\theta)} = \frac{SG_{mix} \cdot \rho \cdot g \cdot L \cdot H^2}{\tan(\theta)}$$

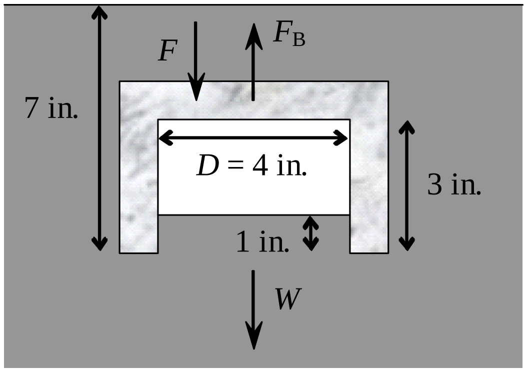
$$SG_{mix} = SG_{sea} \cdot \left(\frac{h}{H}\right)^2 \qquad SG_{mix} = 1.024 \times \left(\frac{7}{8}\right)^2 \qquad SG_{mix} = 0.784$$

The density is $$\rho_{mix} = SG_{mix} \cdot \rho \qquad \rho_{mix} = 0.784 \times 1.94 \cdot \frac{slug}{ft^3} \qquad \rho_{mix} = 1.52 \frac{slug}{ft^3}$$

Problem *3.107

[2]

***3.107** A bowl is inverted symmetrically and held in BXYB fluid, SG = 15.6, to a depth of 7 in. measured along the centerline of the bowl from the bowl rim. The bowl height is 3 in., and the BXYB fluid rises 1 in. inside the bowl. The bowl is unique: Its base is 4 in. inside diameter, and it is made from an old clay recipe, SG = 5.7. The volume of the bowl is about 56 in.^{3}. What is the force required to hold it in place?



Given: Data on inverted bowl and BXYB fluid

Find: Force to hold in place

Solution:

Basic equation $F_B = \rho \cdot g \cdot V$ and $\Sigma F_y = 0$ $\quad \Sigma F_y = 0 = F_B - F - W$

Hence $F = F_B - W$

For the buoyancy force $F_B = SG_{BXYB} \cdot \rho \cdot g \cdot V_{sub}$ with $V_{sub} = V_{bowl} + V_{air}$

For the weight $W = SG_{bowl} \cdot \rho \cdot g \cdot V_{bowl}$

Hence $F = SG_{BXYB} \cdot \rho \cdot g \cdot \left(V_{bowl} + V_{air}\right) - SG_{bowl} \cdot \rho \cdot g \cdot V_{bowl}$

$$F = \rho \cdot g \cdot \left[SG_{BXYB} \cdot \left(V_{bowl} + V_{air}\right) - SG_{bowl} \cdot V_{bowl}\right]$$

$$F = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times \left[15.6 \times \left[56 \cdot \text{in}^3 + (3-1) \cdot \text{in} \cdot \frac{\pi \cdot (4 \cdot \text{in})^2}{4}\right] - 5.7 \times 56 \cdot \text{in}^3\right] \times \left(\frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^3 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$$F = 34.2 \cdot \text{lbf}$$

Problem *3.108

[4]

Consider a conical funnel held upside down and submerged slowly in a container of water. Discuss the force needed to submerge the funnel if the spout is open to the atmosphere. Compare with the force needed to submerge the funnel when the spout opening is blocked by a rubber stopper.

Open-Ended Problem Statement: Consider a conical funnel held upside down and submerged slowly in a container of water. Discuss the force needed to submerge the funnel if the spout is open to the atmosphere. Compare with the force needed to submerge the funnel when the spout opening is blocked by a rubber stopper.

Discussion: Let the weight of the funnel in air be W_a. Assume the funnel is held with its spout vertical and the conical section down. Then W_a will also be vertical.

Two possible cases are with the funnel spout open to atmosphere or with the funnel spout sealed.
With the funnel spout open to atmosphere, the pressures inside and outside the funnel are equal, so no net pressure force acts on the funnel. The force needed to support the funnel will remain constant until it first contacts the water. Then a buoyancy force will act vertically upward on every element of volume located beneath the water surface.

The first contact of the funnel with the water will be at the widest part of the conical section. The buoyancy force will be caused by the volume formed by the funnel thickness and diameter as it begins to enter the water. The buoyancy force will reduce the force needed to support the funnel. The buoyancy force will increase as the depth of submergence of the funnel increases until the funnel is fully submerged. At that point the buoyancy force will be constant and equal to the weight of water displaced by the volume of the material from which the funnel is made.

If the funnel material is less dense than water, it would tend to float partially submerged in the water. The force needed to support the funnel would decrease to zero and then become negative (i.e., down) to fully submerge the funnel.

If the funnel material were denser than water it would not tend to float even when fully submerged. The force needed to support the funnel would decrease to a minimum when the funnel became fully submerged, and then would remain constant at deeper submersion depths.
With the funnel spout sealed, air will be trapped inside the funnel. As the funnel is submerged gradually below the water surface, it will displace a volume equal to the volume of the funnel material plus the volume of trapped air. Thus its buoyancy force will be much larger than when the spout is open to atmosphere. Neglecting any change in air volume (pressures caused by submersion should be small compared to atmospheric pressure) the buoyancy force would be from the entire volume encompassed by the outside of the funnel. Finally, when fully submerged, the volume of the rubber stopper (although small) will also contribute to the total buoyancy force acting on the funnel.

Problem *3.109

In the ‘‘Cartesian diver’’ child’s toy, a miniature ‘‘diver’’ is immersed in a column of liquid. When a diaphragm at the top of the column is pushed down, the diver sinks to the bottom. When the diaphragm is released, the diver again rises. Explain how the toy might work.

Open-Ended Problem Statement: In the “Cartesian diver” child's toy, a miniature “diver” is immersed in a column of liquid. When a diaphragm at the top of the column is pushed down, the diver sinks to the bottom. When the diaphragm is released, the diver again rises. Explain how the toy might work.

Discussion: A possible scenario is for the toy to have a flexible bladder that contains air. Pushing down on the diaphragm at the top of the liquid column would increase the pressure at any point in the liquid. The air in the bladder would be compressed slightly as a result. The volume of the bladder, and therefore its buoyancy, would decrease, causing the diver to sink to the bottom of the liquid column.

Releasing the diaphragm would reduce the pressure in the water column. This would allow the bladder to expand again, increasing its volume and therefore the buoyancy of the diver. The increased buoyancy would permit the diver to rise to the top of the liquid column and float in a stable, partially submerged position, on the surface of the liquid.

Problem *3.110

A proposed ocean salvage scheme involves pumping air into ‘‘bags’’ placed within and around a wrecked vessel on the sea bottom. Comment on the practicality of this plan, supporting your conclusions with analyses.

Open-Ended Problem Statement: A proposed ocean salvage scheme involves pumping air into “bags” placed within and around a wrecked vessel on the sea bottom. Comment on the practicality of this plan, supporting your conclusions with analyses.

Discussion: This plan has several problems that render it impractical. First, pressures at the sea bottom are very high. For example, *Titanic* was found in about 12,000 ft of seawater. The corresponding pressure is nearly 6,000 psi. Compressing air to this pressure is possible, but would require a multi-stage compressor and very high power.

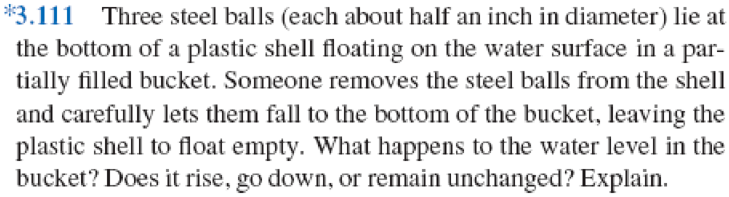
Second, it would be necessary to manage the buoyancy force after the bag and object are broken loose from the sea bed and begin to rise toward the surface. Ambient pressure would decrease as the bag and artifact rise toward the surface. The air would tend to expand as the pressure decreases, thereby tending to increase the volume of the bag. The buoyancy force acting on the bag is directly proportional to the bag volume, so it would increase as the assembly rises. The bag and artifact thus would tend to accelerate as they approach the sea surface. The assembly could broach the water surface with the possibility of damaging the artifact or the assembly.

If the bag were of constant volume, the pressure inside the bag would remain essentially constant at the pressure of the sea floor, e.g., 6,000 psi for *Titanic*. As the ambient pressure decreases, the pressure differential from inside the bag to the surroundings would increase. Eventually the difference would equal sea floor pressure. This probably would cause the bag to rupture.

If the bag permitted some expansion, a control scheme would be needed to vent air from the bag during the trip to the surface to maintain a constant buoyancy force just slightly larger than the weight of the artifact in water. Then the trip to the surface could be completed at low speed without danger of broaching the surface or damaging the artifact.

Problem *3.111

[2]



*3.111 Three steel balls (each about half an inch in diameter) lie at the bottom of a plastic shell floating on the water surface in a partially filled bucket. Someone removes the steel balls from the shell and carefully lets them fall to the bottom of the bucket, leaving the plastic shell to float empty. What happens to the water level in the bucket? Does it rise, go down, or remain unchanged? Explain.

Given: Steel balls resting in floating plastic shell in a bucket of water

Find: What happens to water level when balls are dropped in water

Solution: Basic equation $F_B = \rho \cdot V_{disp} \cdot g = W$ for a floating body weight W

When the balls are in the plastic shell, the shell and balls displace a volume of water equal to their own weight - a large volume because the balls are dense. When the balls are removed from the shell and dropped in the water, the shell now displaces only a small volume of water, and the balls sink, displacing only their own volume. Hence the difference in displaced water before and after moving the balls is the difference between the volume of water that is equal to the weight of the balls, and the volume of the balls themselves. The amount of water displaced is significantly reduced, so the water level in the bucket drops.

Volume displaced before moving balls: $V_1 = \dfrac{W_{plastic} + W_{balls}}{\rho \cdot g}$

Volume displaced after moving balls: $V_2 = \dfrac{W_{plastic}}{\rho \cdot g} + V_{balls}$

Change in volume displaced $\Delta V = V_2 - V_1 = V_{balls} - \dfrac{W_{balls}}{\rho \cdot g} = V_{balls} - \dfrac{SG_{balls} \cdot \rho \cdot g \cdot V_{balls}}{\rho \cdot g}$

$$\Delta V = V_{balls} \cdot \left(1 - SG_{balls}\right)$$

Hence initially a large volume is displaced; finally a small volume is displaced ($\Delta V < 0$ because $SG_{balls} > 1$)

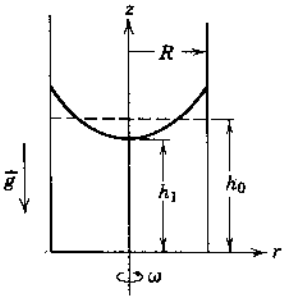
Problem *3.112

[3]

Given: Cylindrical container rotating as in Example 3.10

$R = 0.5\ \text{ft}$

$h_o = 4\ \text{in.}$



Determine: (a) value of ω such that $h_1 = 0$

(b) if solution is dependent on ρ

Solution:

In order to obtain the solution we need an expression for the shape of the free surface in terms of ω, r, and h_o

The required expression was derived in Example 3.10. The equation is

$$z = h_o - \frac{(\omega R)^2}{2g}\left[\frac{1}{2} - \left(\frac{r}{R}\right)^2\right]$$

Since $h_1 = 0$ corresponds to $z = 0$ and $r = 0$ we must determine ω such that

$$0 = h_o - \frac{(\omega R)^2}{4g}$$

Solving for ω,

$$\omega = \frac{2}{R}\sqrt{g h_o}$$

$$= \frac{2}{0.5\ \text{ft}}\left(32.2\ \frac{\text{ft}}{\text{s}^2} \times 4\ \text{in} \times \frac{\text{ft}}{12\ \text{in}}\right)^{1/2}$$

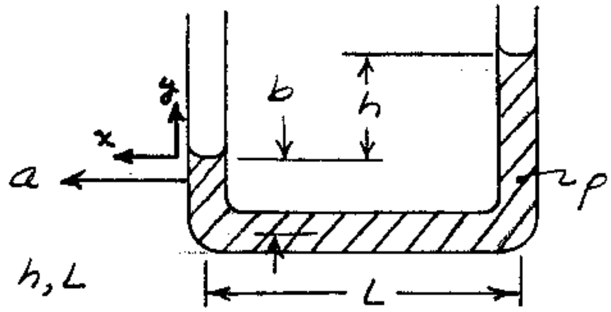
$$= 4 \times 3.28\ \frac{1}{\text{s}}$$

$$\omega = 13.1\ \text{rad/s} \longleftarrow \omega$$

The solution is independent of ρ since the equation of the free surface is independent of ρ.

Problem *3.113

Given: U-tube accelerometer



Find: Acceleration in terms of h, L

Solution: Apply x, y components of hydrostatic equation.

Basic equations:

$$-\frac{\partial p}{\partial x} + \rho g_x = \rho a_x \qquad a_x = a \qquad g_x = 0$$

$$-\frac{\partial p}{\partial y} + \rho g_y = \rho a_y \qquad a_y = 0 \qquad g_y = -g$$

Assumptions: (1) Neglect sloshing
(2) Ignore corners

Then $\frac{\partial p}{\partial x} = -\rho a$, $\frac{\partial p}{\partial y} = -\rho g$. Evaluate Δp from left leg to right:

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$$

$$\Delta p = \frac{\partial p}{\partial x}\Delta x + \frac{\partial p}{\partial y}\Delta y$$

$$= (-\rho g)(-b) + (-\rho a)(-L) + (-\rho g)(b+h)$$

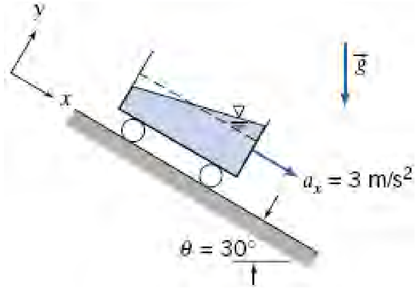
$$\Delta p = \rho a L - \rho g h = 0$$

Solving,

$$a = g\left(\frac{h}{L}\right) \longleftarrow a$$

Problem *3.114

***3.114** A rectangular container of water undergoes constant acceleration down an incline as shown. Determine the slope of the free surface using the coordinate system shown.



Given: Rectangular container with constant acceleration

Find: Slope of free surface

Solution: Basic equation $-\nabla p + \rho\vec{g} = \rho\vec{a}$

In components $-\frac{\partial}{\partial x}p + \rho\cdot g_x = \rho\cdot a_x$ $-\frac{\partial}{\partial y}p + \rho\cdot g_y = \rho\cdot a_y$ $-\frac{\partial}{\partial z}p + \rho\cdot g_z = \rho\cdot a_z$

We have $a_y = a_z = 0$ $g_x = g\cdot\sin(\theta)$ $g_y = -g\cdot\cos(\theta)$ $g_z = 0$

Hence $-\frac{\partial}{\partial x}p + \rho\cdot g\cdot\sin(\theta) = \rho\cdot a_x$ (1) $-\frac{\partial}{\partial y}p - \rho\cdot g\cdot\cos(\theta) = 0$ (2) $-\frac{\partial}{\partial z}p = 0$ (3)

From Eq. 3 we can simplify from $p = p(x,y,z)$ to $p = p(x,y)$

Hence a change in pressure is given by $dp = \frac{\partial}{\partial x}p\cdot dx + \frac{\partial}{\partial y}p\cdot dy$

At the free surface p = const., so $dp = 0 = \frac{\partial}{\partial x}p\cdot dx + \frac{\partial}{\partial y}p\cdot dy$ or $\frac{dy}{dx} = -\frac{\frac{\partial}{\partial x}p}{\frac{\partial}{\partial y}p}$ at the free surface

Hence at the free surface, using Eqs 1 and 2

$$\frac{dy}{dx} = -\frac{\frac{\partial}{\partial x}p}{\frac{\partial}{\partial y}p} = \frac{\rho\cdot g\cdot\sin(\theta) - \rho\cdot a_x}{\rho\cdot g\cdot\cos(\theta)} = \frac{g\cdot\sin(\theta) - a_x}{g\cdot\cos(\theta)}$$

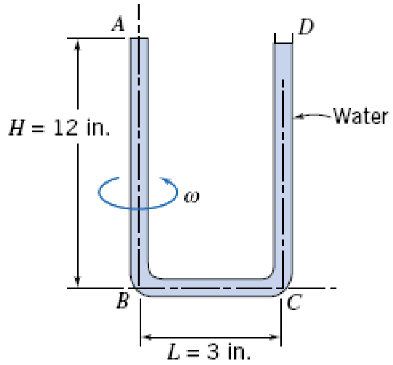
$$\frac{dy}{dx} = \frac{9.81\cdot(0.5)\cdot\frac{m}{s^2} - 3\cdot\frac{m}{s^2}}{9.81\cdot(0.866)\cdot\frac{m}{s^2}}$$

At the free surface, the slope is $\frac{dy}{dx} = 0.224$

Problem *3.115

[2]

***3.115** The U-tube shown is filled with water at $T = 68°F$. It is sealed at A and open to the atmosphere at D. The tube is rotated about vertical axis AB. For the dimensions shown, compute the maximum angular speed if there is to be no cavitation.



Given: Spinning U-tube sealed at one end

Find: Maximum angular speed for no cavitation

Solution: Basic equation $-\nabla p + \rho\vec{g} = \rho\vec{a}$

In components $$-\frac{\partial}{\partial r}p = \rho \cdot a_r = -\rho \cdot \frac{V^2}{r} = -\rho \cdot \omega^2 \cdot r \qquad \frac{\partial}{\partial z}p = -\rho \cdot g$$

Between D and C, r = constant, so $\frac{\partial}{\partial z}p = -\rho \cdot g$ and so $p_D - p_C = -\rho \cdot g \cdot H$ (1)

Between B and A, r = constant, so $\frac{\partial}{\partial z}p = -\rho \cdot g$ and so $p_A - p_B = -\rho \cdot g \cdot H$ (2)

Between B and C, z = constant, so $\frac{\partial}{\partial r}p = \rho \cdot \omega^2 \cdot r$ and so $\int_{p_B}^{p_C} 1\, dp = \int_0^L \rho \cdot \omega^2 \cdot r\, dr$

Integrating $p_C - p_B = \rho \cdot \omega^2 \cdot \frac{L^2}{2}$ (3)

Since $p_D = p_{atm}$, then from Eq 1 $p_C = p_{atm} + \rho \cdot g \cdot H$

From Eq. 3 $p_B = p_C - \rho \cdot \omega^2 \cdot \frac{L^2}{2}$ so $p_B = p_{atm} + \rho \cdot g \cdot H - \rho \cdot \omega^2 \cdot \frac{L^2}{2}$

From Eq. 2 $p_A = p_B - \rho \cdot g \cdot H$ so $p_A = p_{atm} - \rho \cdot \omega^2 \cdot \frac{L^2}{2}$

Thus the minimum pressure occurs at point A (not B)

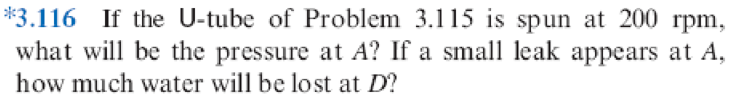
At 68^{o}F from steam tables, the vapor pressure of water is $p_v = 0.339 \cdot psi$

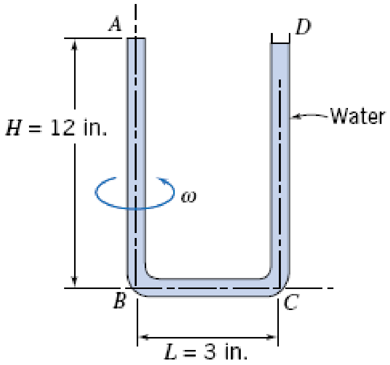
Solving for ω with $p_A = p_v$, we obtain $$\omega = \sqrt{\frac{2 \cdot (p_{atm} - p_v)}{\rho \cdot L^2}} = \left[2 \cdot (14.7 - 0.339) \cdot \frac{lbf}{in^2} \times \frac{ft^3}{1.94 \cdot slug} \times \frac{1}{(3 \cdot in)^2} \times \left(\frac{12 \cdot in}{1 \cdot ft}\right)^4 \times \frac{slug\, ft}{s^2 \cdot lbf}\right]^{\frac{1}{2}}$$

$$\omega = 185 \cdot \frac{rad}{s} \qquad \omega = 1764 \cdot rpm$$

Problem *3.116

[2]


***3.116** If the U-tube of Problem 3.115 is spun at 200 rpm, what will be the pressure at *A*? If a small leak appears at *A*, how much water will be lost at *D*?



Given: Spinning U-tube sealed at one end

Find: Pressure at A; water loss due to leak

Solution: Basic equation $-\nabla p + \rho\vec{g} = \rho\vec{a}$

From the analysis of Example Problem 3.10, solving the basic equation, the pressure *p* at any point (*r*,*z*) in a continuous rotating fluid is given by

$$p = p_0 + \frac{\rho\cdot\omega^2}{2}\cdot\left(r^2 - r_0^2\right) - \rho\cdot g\cdot\left(z - z_0\right) \qquad (1)$$

where p_0 is a reference pressure at point (r_0,z_0)

In this case $p = p_A$ $\quad p_0 = p_D$ $\quad z = z_A = z_D = z_0 = H$ $\quad r = 0$ $\quad r_0 = r_D = L$

The speed of rotation is $\omega = 200\cdot\text{rpm}$ $\quad \omega = 20.9\cdot\frac{\text{rad}}{\text{s}}$

The pressure at *D* is $p_D = 0\cdot\text{kPa}$ (gage)

Hence $p_A = \frac{\rho\cdot\omega^2}{2}\cdot\left(-L^2\right) - \rho\cdot g\cdot(0) = -\frac{\rho\cdot\omega^2\cdot L^2}{2} = -\frac{1}{2}\times 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left(20.9\cdot\frac{\text{rad}}{\text{s}}\right)^2\times(3\cdot\text{in})^2\times\left(\frac{1\cdot\text{ft}}{12\cdot\text{in}}\right)^4\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$

$p_A = -0.18\cdot\text{psi}$ (gage)

When the leak appears,the water level at *A* will fall, forcing water out at point *D*. Once again, from the analysis of Example Problem 3.10, we can use Eq 1

In this case $p = p_A = 0$ $\quad p_0 = p_D = 0$ $\quad z = z_A$ $\quad z_0 = z_D = H$ $\quad r = 0$ $\quad r_0 = r_D = L$

Hence $0 = \frac{\rho\cdot\omega^2}{2}\cdot\left(-L^2\right) - \rho\cdot g\cdot\left(z_A - H\right)$

$$z_A = H - \frac{\omega^2\cdot L^2}{2\cdot g} = 12\text{in} - \frac{1}{2}\times\left(20.9\cdot\frac{\text{rad}}{\text{s}}\right)^2\times(3\cdot\text{in})^2\times\frac{\text{s}^2}{32.2\cdot\text{ft}}\times\frac{1\cdot\text{ft}}{12\cdot\text{in}} \qquad z_A = 6.91\cdot\text{in}$$

The amount of water lost is $\Delta h = H - z_A = 12\cdot\text{in} - 6.91\cdot\text{in}$ $\qquad \Delta h = 5.09\cdot\text{in}$

Problem *3.117

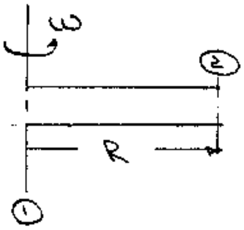
[2]

Given: Centrifugal micromanometer consists of pair of parallel disks that rotate to develop a radial pressure difference. There is no flow between the disks.

Find: (a) An expression for the pressure difference, Δp, as a function of ω, R, and ρ

(b) Find ω if $\Delta p = 8\ \mu m\ H_2O$ and $R = 50\ mm$.

Solution:



Basic equation: $-\nabla p + \rho\vec{g} = \rho\vec{a}$

(r component) $-\frac{\partial p}{\partial r} + \rho g_r = \rho a_r$

Assumptions: (1) standard air between disks

(2) r horizontal, so $g_r = 0$

(3) rigid body motion, so $a_r = -\frac{V^2}{r} = -\frac{(r\omega)^2}{r} = -r\omega^2$

Then

$$\frac{\partial p}{\partial r} = \rho r \omega^2 \qquad (\rho \text{ is a constant})$$

Separating variables and integrating, we obtain

$$\int_p^{p+\Delta p} dp = \rho\omega^2 \int_0^R r\,dr$$

$$\Delta p = \frac{\rho\omega^2 R^2}{2} \longleftarrow \Delta p$$

Then

$$\omega^2 = \frac{2\Delta p}{\rho R^2}$$

where $\Delta p = \rho_{H_2O}\, g\, \Delta h$ and $\Delta h = 8\times10^{-6}\ m$

$$\omega^2 = \frac{2\rho_{H_2O}\, g\, \Delta h}{\rho R^2}$$

$$= 2 \times \frac{999\ kg/m^3}{1.225\ kg/m^3} \times 9.81\ \frac{m}{s^2} \times 8\times10^{-6}\ m \times \frac{1}{(0.05)^2\ m^2}$$

$$\omega^2 = 51.2\ s^{-2}$$

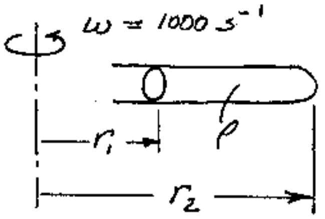
$$\omega = 7.16\ rad/s \longleftarrow \omega$$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE

Problem *3.118

[2]

Given: Test tube with water



$r_1 = 50\ mm$

$r_2 = 130\ mm$

Find: (a) Radial acceleration

(b) Radial pressure gradient, $\partial p/\partial r$

(c) Maximum pressure on bottom.

Solution: Apply equation for rigid-body motion

Basic equation: $-\nabla p + \rho\vec{g} = \rho\vec{a}$

(r component) $-\frac{\partial p}{\partial r} + \rho g_r = \rho a_r$

Assumptions: (1) Rigid-body motion, so $a_r = -\frac{V^2}{r} = -\frac{(r\omega)^2}{r} = -r\omega^2$ ← a_r

(2) r horizontal, so $g_r = 0$

Then $\frac{\partial p}{\partial r} = -\rho a_r = -\rho(-r\omega^2) = \rho r\omega^2$ ← $\frac{\partial p}{\partial r}$

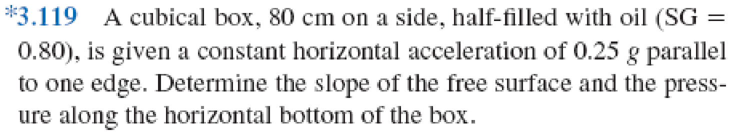
Integrating, $p_2 - p_1 = \int_1^2 \frac{\partial p}{\partial r}dr = \int_{r_1}^{r_2} \rho r\omega^2 = \rho\frac{r^2\omega^2}{2}\Big]_{r_1}^{r_2} = \frac{1}{2}\rho\omega^2(r_2^2 - r_1^2)$

$p_{max} = p_2 - p_1 = \frac{1}{2} \times 999\frac{kg}{m^3} \times \frac{(1000)^2}{s^2} \times [(0.130)^2 - (0.050)^2]m^2 \times \frac{N \cdot s^2}{kg \cdot m} = 7.19\ MPa$ ← p_{ma}

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem *3.119

[3]



*3.119 A cubical box, 80 cm on a side, half-filled with oil (SG = 0.80), is given a constant horizontal acceleration of 0.25 *g* parallel to one edge. Determine the slope of the free surface and the pressure along the horizontal bottom of the box.

Given: Cubical box with constant acceleration

Find: Slope of free surface; pressure along bottom of box

Solution: Basic equation $-\nabla p + \rho\vec{g} = \rho\vec{a}$

In components $-\frac{\partial}{\partial x}p + \rho\cdot g_x = \rho\cdot a_x$ $\quad -\frac{\partial}{\partial y}p + \rho\cdot g_y = \rho\cdot a_y$ $\quad -\frac{\partial}{\partial z}p + \rho\cdot g_z = \rho\cdot a_z$

We have $a_x = a_x \quad g_x = 0$ $\quad a_y = 0 \quad g_y = -g$ $\quad a_z = 0 \quad g_z = 0$

Hence $\frac{\partial}{\partial x}p = -SG\cdot\rho\cdot a_x$ (1) $\quad \frac{\partial}{\partial y}p = -SG\cdot\rho\cdot g$ (2) $\quad \frac{\partial}{\partial z}p = 0$ (3)

From Eq. 3 we can simplify from $p = p(x,y,z)$ to $p = p(x,y)$

Hence a change in pressure is given by $dp = \frac{\partial}{\partial x}p\cdot dx + \frac{\partial}{\partial y}p\cdot dy$ (4)

At the free surface p = const., so $dp = 0 = \frac{\partial}{\partial x}p\cdot dx + \frac{\partial}{\partial y}p\cdot dy$ or $\frac{dy}{dx} = -\frac{\frac{\partial}{\partial x}p}{\frac{\partial}{\partial y}p} = -\frac{a_x}{g} = -\frac{0.25\cdot g}{g}$

Hence at the free surface $\frac{dy}{dx} = -0.25$

The equation of the free surface is then $y = -\frac{x}{4} + C$ and through volume conservation the fluid rise in the rear balances the fluid fall in the front, so at the midpoint the free surface has not moved from the rest position

For size $L = 80\cdot cm$ at the midpoint $x = \frac{L}{2}$ $\quad y = \frac{L}{2}$ (box is half filled) $\frac{L}{2} = -\frac{1}{4}\cdot\frac{L}{2} + C$ $\quad C = \frac{5}{8}\cdot L$ $\quad y = \frac{5}{8}\cdot L - \frac{x}{4}$

Combining Eqs 1, 2, and 4 $dp = -SG\cdot\rho\cdot a_x\cdot dx - SG\cdot\rho\cdot g\cdot dy$ or $p = -SG\cdot\rho\cdot a_x\cdot x - SG\cdot\rho\cdot g\cdot y + c$

We have $p = p_{atm}$ when $x = 0$ $\quad y = \frac{5}{8}\cdot L$ so $p_{atm} = -SG\cdot\rho\cdot g\cdot\frac{5}{8}\cdot L + c$ $\quad c = p_{atm} + SG\cdot\rho\cdot g\cdot\frac{5}{8}\cdot L$

$$p(x,y) = p_{atm} + SG\cdot\rho\cdot\left(\frac{5}{8}\cdot g\cdot L - a_x\cdot x - g\cdot y\right) = p_{atm} + SG\cdot\rho\cdot g\cdot\left(\frac{5}{8}\cdot L - \frac{x}{4} - y\right)$$

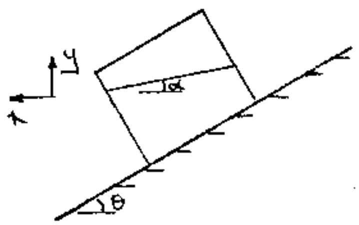
On the bottom y = 0 so $p(x,0) = p_{atm} + SG\cdot\rho\cdot g\cdot\left(\frac{5}{8}\cdot L - \frac{x}{4}\right) = 101 + 0.8\times 1000\cdot\frac{kg}{m^3}\times\frac{N\cdot s^2}{kg\cdot m}\times 9.81\cdot\frac{m}{s^2}\times\left(\frac{5}{8}\times 0.8\cdot m - \frac{x}{4}\right)\times\frac{kPa}{10^3\cdot Pa}$

$p(x,0) = 105 - 1.96\cdot x$ (p in kPa, x in m)

Problem *3.120

[3]

Given: Rectangular container of base dimensions 0.4m × 0.2m and height 0.4m is filled with water to a depth, $d = 0.2\,m$
Mass of empty container is $M_c = 10\,kg$
Container slides down an incline, $\theta = 30°$
Coefficient of sliding friction is 0.30



Find: The angle of the water surface relative to the horizontal

Solution:

Basic equations: $-\nabla P + \rho\vec{g} = M\vec{a}$ $\quad \Sigma\vec{F} = M\vec{a}$

Assumptions: (1) fluid moves as solid body, ie no sloshing

Writing component equations,

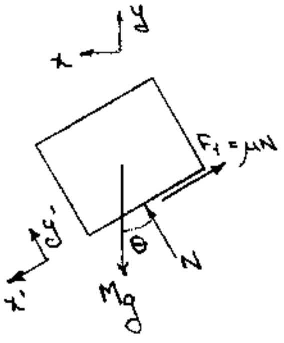
$$-\frac{\partial P}{\partial x} = \rho a_x \qquad \frac{\partial P}{\partial x} = -\rho a_x$$

$$-\frac{\partial P}{\partial y} - \rho g = \rho a_y \qquad \frac{\partial P}{\partial y} = -\rho(g + a_y)$$

$P = P(x,y)$ $\quad dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$. Along the water surface, $dP = 0$

$$\frac{dy}{dx} = -\frac{\partial P/\partial x}{\partial P/\partial y} = -\frac{a_x}{g + a_y}$$

To determine a_x and a_y consider the container and contents



$$M = M_c + M_{H_2O} = M_c + \rho V = 10\,kg + 999\frac{kg}{m^3} \times 0.4m \times 0.2m \times 0.2m$$

$$M = 26\,kg$$

$$\Sigma F_{y'} = 0 = N - Mg\cos\theta$$

$$N = Mg\cos\theta = 26\,kg \times 9.81\frac{m}{s^2} \times \cos 30° \times \frac{N \cdot s^2}{kg \cdot m} = 221\,N$$

$$\Sigma F_{x'} = Ma_{x'} = Mg\sin 30° - F_f = Mg\sin 30° - \mu N$$

$$a_{x'} = g\sin 30° - \mu\frac{N}{M} = 9.81\frac{m}{s^2}\sin 30° - 0.3 \times 221\,N \times \frac{1}{26\,kg} \times \frac{kg \cdot m}{N \cdot s^2}$$

$$a_{x'} = 2.36\,m/sec^2$$

Then
$$a_x = a_{x'}\cos\theta = 2.36\frac{m}{sec^2} \times \cos 30° = 2.04\,m/s^2$$
$$a_y = -a_{x'}\sin\theta = -2.36\frac{m}{sec^2} \times \sin 30° = -1.18\,m/s^2$$

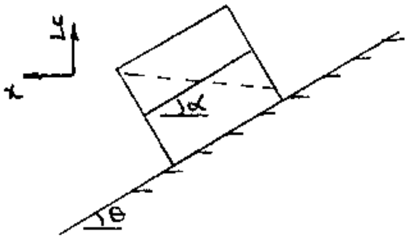
and
$$\frac{dy}{dx} = \frac{-a_x}{g + a_y} = -\frac{2.04}{9.81 - 1.18} = -0.236$$

$$\alpha = \tan^{-1} 0.236 = 13.3°$$ ←———— α

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem *3.121

[3]



Given: Rectangular container of base dimensions 0.4m × 0.2m and height 0.4m is filled with water to a depth, d = 0.2m
Mass of empty container is $M_c = 10\text{ kg}$
Container slides down an incline, $\theta = 30°$ without friction

Find: (a) The angle of the water surface relative to the horizontal.
(b) Slope of the free surface for the same acceleration up the plane

Solution:

Basic equations: $-\nabla P + \rho\vec{g} = M\vec{a}$ $\qquad \Sigma\vec{F} = M\vec{a}$

Assumptions: (1) fluid moves as solid body, ie. no sloshing

Writing component equations,

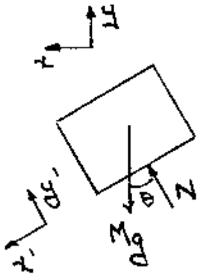
$$-\frac{\partial P}{\partial x} = \rho a_x \qquad \frac{\partial P}{\partial x} = -\rho a_x$$

$$-\frac{\partial P}{\partial y} - \rho g = \rho a_y \qquad \frac{\partial P}{\partial y} = -\rho(g + a_y)$$

$P = P(x,y) \qquad dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy \qquad$ Along the water surface, $dP = 0$

$$\frac{dy}{dx} = \frac{-\partial P/\partial x}{\partial P/\partial y} = -\frac{a_x}{(g + a_y)}$$

For motion without friction



$$\Sigma F_{x'} = Ma_{x'} = Mg\sin\theta \qquad \therefore a_{x'} = g\sin\theta$$

$$a_x = a_{x'}\cos\theta = g\sin\theta\cos\theta$$

$$a_y = -a_{x'}\sin\theta = -g\sin^2\theta$$

$$\frac{dy}{dx} = -\frac{a_x}{(g + a_y)} = -\frac{g\sin\theta\cos\theta}{(g - g\sin^2\theta)} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

$$\frac{dy}{dx} = -\tan 30° = -0.577$$

$$\alpha = \tan^{-1} 0.577 = 30°$$ ← α

For the same acceleration up the incline, $a_x = -g\sin\theta\cos\theta \quad a_y = g\sin^2\theta$

$$\frac{dy}{dx} = \frac{-a_x}{(g + a_y)} = \frac{g\sin\theta\cos\theta}{(g + g\sin^2\theta)} = \frac{\sin\theta\cos\theta}{1 + \sin^2\theta} = \frac{\sin 30\cos 30}{1 + \sin^2 30}$$

$$\frac{dy}{dx} = 0.346$$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem *3.122

[3]

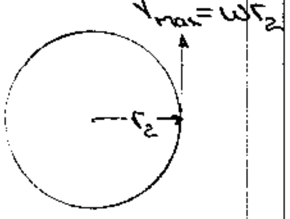
Given: Gas centrifuge, with maximum peripheral speed, $V_{max} = 300$ m/sec contains uranium hexafluoride gas ($M = 352$ kg/kgmol) at 325C.

Find: (a) Develop an expression for ratio of maximum pressure to pressure at centrifuge axis
(b) Evaluate for given conditions.

Solution:

Basic equation: $-\nabla p + \rho\vec{g} = \rho\vec{a}$ $\qquad p = \rho RT$

(r component) $-\frac{\partial p}{\partial r} + \rho g_r = \rho a_r$



Assumptions: (1) ideal gas behavior, T = constant
(2) r horizontal, so $g_r = 0$
(3) rigid body motion, so
$$a_r = -\frac{V^2}{r} = -\frac{(r\omega)^2}{r} = -r\omega^2$$

Then $\frac{\partial p}{\partial r} = -\rho a_r = \rho r\omega^2 = \frac{p}{RT} r\omega^2$

Separating variables and integrating, we obtain

$$\int_{p_1}^{p_2} \frac{dp}{p} = \frac{\omega^2}{RT}\int_{r_1=0}^{r_2} r\,dr = \frac{\omega^2}{RT}\frac{r_2^2}{2} \qquad V_{max} = \omega r_2$$

$$\ln\frac{p_2}{p_1} = \frac{V_{max}^2}{2RT}$$

$$\frac{p_2}{p_1} = e^{\frac{V_{max}^2}{2RT}} \longleftarrow \frac{p_2}{p_1}$$

To evaluate, $R = \frac{R_u}{M} = 8314 \frac{\text{N}\cdot\text{m}}{\text{kgmol}\cdot\text{K}} \times \frac{\text{kgmole}}{352\,\text{kg}} = 23.62 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}$

$$\frac{V_{max}^2}{2RT} = \frac{(300)^2}{2}\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{kg}\cdot\text{K}}{23.62\,\text{N}\cdot\text{m}} \times \frac{1}{598\,\text{K}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 3.186$$

$$\therefore \frac{p_2}{p_1} = e^{3.186} = 24.2 \longleftarrow$$

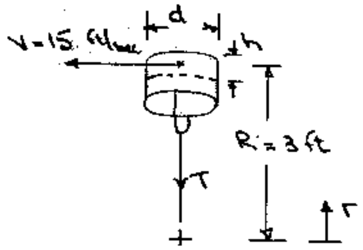
Problem *3.123

Given: Pail, 1 ft in diameter and 1 ft deep, weighs 3 lbf and contains 8 in. of water.
Pail is swung in a vertical circle of 3 ft radius and a speed of 15. ft/s.
Water moves as solid body
Point of interest is top of trajectory

Determine: (a) tension in string
(b) pressure on pail bottom from water

Solution

Assumption: center of mass of bucket and of water are located at $r = 3$ ft where $V = r\omega = 15$. ft/s



Summing forces in radial direction

$$\Sigma F_r \hat{e}_r = M_b a_{b_r} \hat{e}_r + M_w a_{w_r} \hat{e}_r$$

$$-T - (M_b + M_w) g = M_b a_{b_r} + M_w a_{w_r}$$

But $a_{b_r} = a_{w_r} = -\omega^2 r = -\frac{V^2}{r}$

$$\therefore T = \left(\frac{V^2}{r} - g\right)(M_b + M_w)$$

where

$$M_w = \rho_w \forall_w = \rho_w \frac{\pi d^2}{4} h = 1.94 \frac{\text{slug}}{\text{ft}^3} \pi \times \frac{1 \text{ ft}^2}{4} \times 8 \text{ in} \times \frac{\text{ft}}{12 \text{ in}} = 1.02 \text{ slug}$$

Then

$$T = \left((15)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{1}{3 \text{ ft}} - 32.2 \frac{\text{ft}}{\text{s}^2}\right)\left(3 \text{ lbf} \times \frac{1}{32.2} \frac{\text{s}^2}{\text{ft}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} + 1.02 \text{ slug}\right) \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}}$$

$$T = 47.6 \text{ lbf} \longleftarrow$$ T

In the water $-\nabla p + \rho \vec{g} = \rho \vec{a}$

Writing the component in the r direction

$$-\frac{\partial p}{\partial r} - \rho g = \rho a_r = -\rho \frac{V^2}{r}$$

$$\frac{\partial p}{\partial r} = \rho\left(\frac{V^2}{r} - g\right) = 1.94 \frac{\text{slug}}{\text{ft}^3}\left((15)^2 \frac{\text{ft}^2}{\text{sec}^2} \times \frac{1}{3 \text{ ft}} - 32.2 \frac{\text{ft}}{\text{sec}^2}\right) \times \frac{\text{lbf}\cdot\text{sec}^2}{\text{ft}\cdot\text{slug}}$$

$$\frac{\partial p}{\partial r} = 83.0 \text{ lbf}/\text{ft}^3$$

Assuming that $\partial p/\partial r$ is constant throughout the water then

$$p_{\text{bottom}} \cong p_{\text{surface}} + \frac{\partial p}{\partial r} \Delta r$$

$$p_{\text{bottom}} = p_{\text{atm}} + 83.0 \frac{\text{lbf}}{\text{ft}^3} \times 8 \text{ in} \times \frac{\text{ft}}{12 \text{ in}} = p_{\text{atm}} + 55.3 \frac{\text{lbf}}{\text{ft}^2}$$

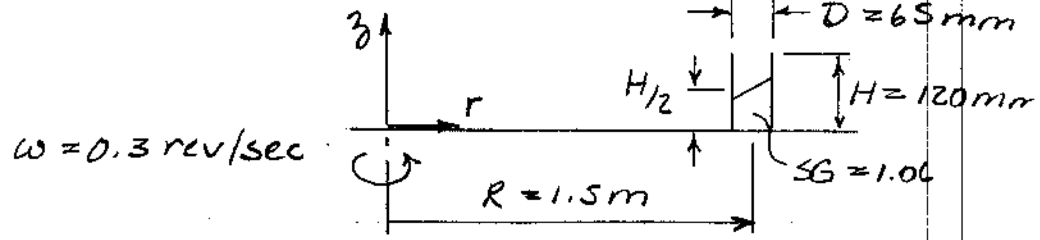
$$p_{\text{bottom}} - p_{\text{atm}} = 55.3 \text{ lbf}/\text{ft}^2 \text{ (gage)} \longleftarrow$$

Problem *3.124

[3]

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Given: Soft drink can at outer edge of merry-go-round.



Find: (a) Slope of free surface

(b) Spin rate to spill

(c) Likelihood of spilling vs. slipping

Solution: Assume rigid-body motion

Basic equation: $-\nabla p + \rho\vec{g} = \rho\vec{a}_r$ $a_r = -\frac{V^2}{r} = -\frac{(r\omega)^2}{r} = -r\omega^2$

$$\left.\begin{aligned} -\frac{\partial p}{\partial r} + \overset{=0(2)}{\cancel{\rho g_r}} &= \rho a_r \\ -\frac{\partial p}{\partial z} + \rho g_z &= \overset{=0(3)}{\cancel{\rho a_z}} \end{aligned}\right\} \quad \begin{aligned} \frac{\partial p}{\partial r} &= -\rho a_r = \rho r\omega^2 \\ \frac{\partial p}{\partial z} &= +\rho g_z = -\rho g \end{aligned}$$

Assumptions: (1) Rigid-body motion, (2) $g_r = 0$, (3) $a_z = 0$, (4) $g_z = -g$

Then $p = p(r,z)$ so $dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$

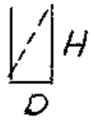
$dp = 0$ along free surface, so $\frac{dz}{dr} = -\frac{\partial p/\partial r}{\partial p/\partial z} = -\frac{\rho r\omega^2}{-\rho g} = \frac{r\omega^2}{g}$

$\omega = 0.3\,\frac{\text{rev}}{\text{sec}} \times 2\pi\,\frac{\text{rad}}{\text{rev}} = 1.88\ \text{rad/s}$

$\left.\frac{dz}{dr}\right)_{\text{surface}} = 1.5\,\text{m} \times (1.88)^2\,\frac{\text{rad}^2}{\text{s}^2} \times \frac{\text{s}^2}{9.81\,\text{m}} = 0.540$

Slope

To spill, slope must be



$H/D = 120/65 = 1.85$

Thus $\omega = \left[\frac{g}{r}\frac{dz}{dr}\right]^{1/2} = \left[9.81\,\frac{\text{m}}{\text{s}^2} \times 1.85 \times \frac{1}{1.5\,\text{m}}\right]^{1/2} = 3.48\ \text{rad/s}$

Spill

This is nearly double the speed.

The coefficient of static friction between the can and surface is probably $\mu_s \le 0.5$.

Thus the can would likely not spill or tip: it would slide off!

Problem *3.125

Open-Ended Problem Statement: When a water polo ball is submerged below the surface in a swimming pool and released from rest, it is observed to pop out of the water. How would you expect the height to which it rises above the water to vary with depth of submersion below the surface? Would you expect the same results for a beach ball? For a table-tennis ball?

Discussion: Separate the problem into two parts: (1) motion of the ball in water below the pool surface, and (2) motion of the ball in air above the pool surface.

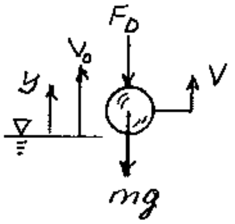
Below the pool water surface the motion of each ball is controlled by buoyancy force and inertia. For small depths of submersion ball speed upon reaching the pool surface will be small. As depth is increased, ball speed will increase until terminal speed in water is approached. For large depths, the actual depth will be irrelevant because the ball will reach terminal speed before reaching the pool water surface. All three balls are relatively light for their diameters, so terminal speed in water should be reached quickly. The depth of submersion needed to reach terminal speed should be fairly small, perhaps 1 meter or less.[1]

Buoyancy is proportional to volume and inertia is proportional to mass. The ball with the largest volume per unit mass should accelerate most quickly to terminal speed. This probably will be the beach ball, followed by the table-tennis ball and the water polo ball.

The ball with the largest diameter has the smallest frontal area per unit volume; the terminal speed should be highest for this ball. The beach ball should have the highest terminal speed, followed by the water polo ball and the table-tennis ball.

Above the pool water surface the motion of each ball is controlled by aerodynamic drag force, gravity force, and inertia (see equation below). Without aerodynamic drag, the height above the pool water surface reached by each ball would depend only its initial speed.[2] Aerodynamic drag reduces the height reached by each ball.

Aerodynamic drag force is proportional to frontal area. The heaviest ball per unit frontal area (probably the water polo ball) should reach the maximum height and the lightest ball per unit frontal area (probably the beach ball) should reach the minimum height.



$$\Sigma F_y = -F_D - mg = ma_y = m\frac{dV}{dt}$$

$$-C_D A \tfrac{1}{2}\rho V^2 - mg = m\frac{dV}{dt}, \quad \text{since } F_D = C_D A \tfrac{1}{2}\rho V^2$$

$$-\frac{C_D A \frac{1}{2}\rho V^2}{m} - g = \frac{dV}{dt} = V\frac{dV}{dy} \qquad (1)$$

Separating variables $$\frac{V\,dV}{1 + \frac{C_D A \rho}{mg}\frac{V^2}{2}} = -g\,dy$$

Integrating, $$\frac{mg}{\rho C_D A}\ln\left[1 + \frac{\rho C_D A}{mg}\frac{V^2}{2}\right]_{V_0}^{0} = -\frac{mg}{\rho C_D A}\ln\left[1 + \frac{\rho C_D A}{mg}\frac{V_0^2}{2}\right] = -g y_{max}$$

---

[1] The initial water depth required to reach terminal speed may be calculated using the methods of Chapter 9.

[2] The maximum height reached by a ball in air with aerodynamic drag may be calculated using the methods of Chapter 9.

Problem *3.125

13-782 500 SHEETS FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
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Thus $$y_{max} = \frac{m}{\rho C_D A} \ln\left[1 + \frac{\rho C_D A}{mg}\frac{V_0^2}{2}\right] = \frac{m}{\rho C_D A} \ln\left[1 + \frac{F_{D_0}}{mg}\right] \longleftarrow \qquad (2)$$

y_{max}

With no aerodynamic drag, Eq. 1 reduces to

$$-mg = mV\frac{dV}{dy} \quad \text{or} \quad V\,dV = -g\,dy$$

Integrating from V_0 to 0, $$\left.\frac{V^2}{2}\right]_{V_0}^{0} = -g\,y_{max}$$

$$y_{max} = \frac{V_0^2}{2g} \longleftarrow \qquad (3)$$

y_{max}
$(C_D =$

Check the limiting value predicted by Eq. 2 as $C_D \to 0$:

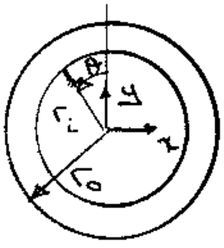
$$\lim_{C_D \to 0} y_{max} = \lim_{C_D \to 0} \frac{m}{\rho C_D A}\,\frac{\rho C_D A}{mg}\,\frac{V_0^2}{2} = \frac{V_0^2}{2g} \quad ✓✓$$

Problem *3.126

[4]

Given: A steel liner of length $L = 2\,m$, outer radius $r_o = 0.15\,m$, and inner radius $r_i = 0.10\,m$ is to be formed in a spinning horizontal mold. To insure uniform thickness the minimum radial acceleration should be $10g$. For steel, $S.G = 7.8$.

Find: (a) The required angular velocity
(b) the maximum and minimum pressures on the surface of the mold.



Solution:

Basic equation: $\nabla p + \rho\vec{g} = \rho\vec{a}$

Writing component equations,

$$-\frac{\partial p}{\partial r} + \rho g_r = \rho a_r \quad \text{and} \quad \frac{\partial p}{\partial r} = \rho g_r - \rho a_r = \rho(-g\cos\theta) - \rho(-r\omega^2) = \rho r\omega^2 - \rho g\cos\theta$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta = 0 \quad \text{and} \quad \frac{\partial p}{\partial \theta} = \rho g_\theta r = \rho g \sin\theta\, r$$

Then, $dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial \theta}d\theta = (\rho r\omega^2 - \rho g\cos\theta)dr + \rho g r\sin\theta\, d\theta$

$\left.\frac{\partial p}{\partial r}\right)_\theta = \text{const} = \rho r\omega^2 - \rho g\cos\theta$. Since $p = p_{atm}$ at $r = r_i$, then

$p - p_{atm} = \int_{r_i}^{r}(\rho r\omega^2 - \rho g\cos\theta)\,dr + f(\theta)$ where, $f(\theta)$ is an arbitrary function

$\therefore\ p = p_{atm} + \rho\omega^2\frac{(r^2 - r_i^2)}{2} - \rho g\cos\theta(r - r_i) + f(\theta)$. Then,

$\frac{\partial p}{\partial \theta} = \rho g\sin\theta(r - r_i) + \frac{df}{d\theta} = \rho g\sin\theta\, r$

Hence, $\frac{df}{d\theta} = \rho g\sin\theta\, r_i$ and $f = -\rho g r_i\cos\theta + c$

$\therefore\ p = p_{atm} + \rho\omega^2\frac{(r^2 - r_i^2)}{2} - \rho g\cos\theta(r - r_i) - \rho g r_i\cos\theta + c$

At $r = r_i$, $p = p_{atm}$ for any value of θ. Hence, $c = \rho g r_i\cos\theta$ and

$$p = p_{atm} + \rho\omega^2\frac{(r^2 - r_i^2)}{2} - \rho g\cos\theta(r - r_i)$$

Minimum value of $a_r = 10g = r\omega^2$ occurs at r_i for given ω. Hence,

$$\omega_{min} = \left[\frac{10g}{r_i}\right]^{1/2} = \left[10 \times 9.81\frac{m}{s^2} \times \frac{1}{0.10m}\right]^{1/2} = 31.3\ rad/s \longleftarrow$$

ω

p_{max} on the surface of the mold $(r = r_o)$ occurs at $\theta = \pi$

$p_{max} - p_{atm} = \frac{\rho\omega^2}{2}(r_o^2 - r_i^2) - \rho g\cos\theta(r - r_i)$.

$$p_{max} - p_{atm} = \frac{1}{2} \times 7.8 \times 999\frac{kg}{m^3} \times \frac{(31.3)^2}{s^2} \times [(0.15)^2 - (0.10)^2]m^2 \times \frac{N\cdot s^2}{kg\cdot m} - 7.8 \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2}(-1)[0.15 - 0.10]\frac{N\cdot s^2}{kg\cdot m}$$

$p_{max} = 51.5\ kPa\ (gage) \longleftarrow$

p_{ma}

p_{min} on the surface of the mold $(r = r_o)$ occurs at $\theta = 0$

$p_{min} - p_{atm} = \frac{\rho\omega^2}{2}(r_o^2 - r_i^2) - \rho g\cos\theta(r - r_i)$

$$p_{min} - p_{atm} = \frac{1}{2} \times 7.8 \times 999\frac{kg}{m^3} \times \frac{(31.3)^2}{s^2}[(0.15)^2 - (0.10)^2]m^2 \times \frac{N\cdot s^2}{kg\cdot m} - 7.8 \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2}(+1)[0.15 - 0.10]\frac{N\cdot s^2}{kg\cdot m}$$

$p_{min} = 43.9\ kPa\ (gage) \longleftarrow$

p_{mi}

Problem *3.127

[4]

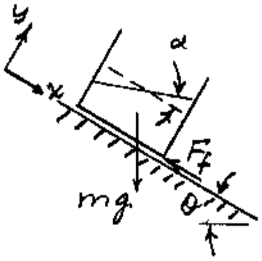
Open-Ended Problem Statement: The analysis of problem 3.120 suggests that it maybe possible to determine the coefficient of sliding friction between two surfaces by measuring the slope of the free surface in a liquid-filled container sliding down an inclined surface. Investigate the feasibility of this idea.

Discussion: A certain minimum angle of inclination would be needed to overcome static friction and start the container into motion down the incline. Once the container is in motion, the retarding force would be provided by sliding (dynamic) friction. The coefficient of dynamic friction usually is smaller than the static friction coefficient. Thus the container would continue to accelerate as it moved down the incline. This acceleration would provide a non-zero slope to the free surface of the liquid in the container.

In principle the slope could be measured and the coefficient of dynamic friction calculated. In practice several problems would arise.

To calculate dynamic friction coefficient one must assume the liquid moves as a solid body (i.e., that there is no sloshing). This condition could only be achieved if there were minimum initial disturbance and the sliding distance were long.

It would be difficult to measure the slope of the free surface of liquid in the moving container. Images made with a video camera or digital still camera might be processed to obtain the required slope information.



$$\Sigma F_y = N - mg\cos\theta \;;\; N = mg\cos\theta$$

$$\Sigma F_x = mg\sin\theta - F_f = ma_x \;;\; F_f = \mu_k N = \mu_k mg\cos\theta$$

$$a_x = g\sin\theta - \mu_k g\cos\theta = g(\sin\theta - \mu_k\cos\theta)$$

For static liquid $-\nabla p + \rho\vec{g} = \rho\vec{a}$

$$-\frac{\partial p}{\partial x} + \cancel{\rho g\sin\theta} = \rho a_x = \rho g(\cancel{\sin\theta} - \mu_k\cos\theta) \;;\; \frac{\partial p}{\partial x} = \rho g\mu_k\cos\theta$$

$$-\frac{\partial p}{\partial y} - \rho g\cos\theta = \rho \cancelto{0}{a_y} \qquad ; \frac{\partial p}{\partial y} = -\rho g\cos\theta$$

For the free surface, $dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy = 0$, so $\frac{dy}{dx} = -\frac{\partial p/\partial x}{\partial p/\partial y}$

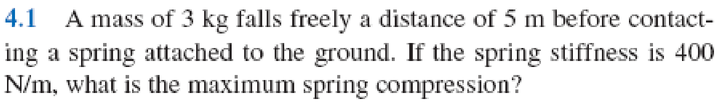
Thus $\frac{dy}{dx} = -\frac{\rho g\mu_k\cos\theta}{-\rho g\cos\theta} = \mu_k \;;\; \alpha = \tan^{-1}(\mu_k)$

Since it was necessary to make the container slip on the surface,

$$\theta > \tan^{-1}(\mu_s) > \tan^{-1}(\mu_k) = \alpha$$ $\longleftarrow$ α

Thus $\alpha < \theta$, as shown in the sketch above.

Problem 4.1



4.1 A mass of 3 kg falls freely a distance of 5 m before contacting a spring attached to the ground. If the spring stiffness is 400 N/m, what is the maximum spring compression?

Given:

Data on mass and spring

Find:

Maximum spring compression

Solution:

The given data is $M = 3 \cdot \text{kg}$ $h = 5 \cdot \text{m}$ $k = 400 \cdot \frac{\text{N}}{\text{m}}$

Apply the First Law of Thermodynamics: for the system consisting of the mass and the spring (the spring has gravitional potential energy and the spring elastic potential energy)

Total mechanical energy at initial state $E_1 = M \cdot g \cdot h$

Total mechanical energy at instant of maximum compression *x* $E_2 = M \cdot g \cdot (-x) + \frac{1}{2} \cdot k \cdot x^2$

Note: The datum for zero potential is the top of the uncompressed spring

But $E_1 = E_2$

so $M \cdot g \cdot h = M \cdot g \cdot (-x) + \frac{1}{2} \cdot k \cdot x^2$

Solving for x $x^2 - \frac{2 \cdot M \cdot g}{k} \cdot x - \frac{2 \cdot M \cdot g \cdot h}{k} = 0$

$$x = \frac{M \cdot g}{k} + \sqrt{\left(\frac{M \cdot g}{k}\right)^2 + \frac{2 \cdot M \cdot g \cdot h}{k}}$$

$$x = 3 \cdot \text{kg} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times \frac{\text{m}}{400 \cdot \text{N}} + \sqrt{\left(3 \cdot \text{kg} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times \frac{\text{m}}{400 \cdot \text{N}}\right)^2 + 2 \times 3 \cdot \text{kg} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times 5 \cdot \text{m} \times \frac{\text{m}}{400 \cdot \text{N}}}$$

$$x = 0.934\,\text{m}$$

Note that ignoring the loss of potential of the mass due to spring compression x gives

$x = \sqrt{\frac{2 \cdot M \cdot g \cdot h}{k}}$ $x = 0.858\,\text{m}$

Note that the deflection if the mass is dropped from immediately above the spring is

$x = \frac{2 \cdot M \cdot g}{k}$ $x = 0.147\,\text{m}$

Problem 4.2

[1]

Given: Six-pack cooled from 25°C to 5°C in freezer.

Find: Change in specific entropy.

Solution: Apply the $T\,ds$ equation.

Basic equation: $$T\,ds = du + p\,\cancel{dv}^{\simeq 0(1)}$$

Assumptions: (1) Neglect volume change
(2) Liquid properties are similar to water

Then

$$T\,ds = du = c_v\,dT$$

or

$$ds = c_v \frac{dT}{T}$$

Integrating,

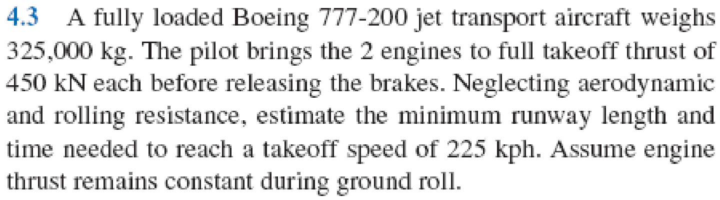
$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right)$$

$$= 1\,\frac{\text{kcal}}{\text{kg}\cdot\text{K}} \times \ln\left(\frac{273+5}{273+25}\right) \times 4190\,\frac{\text{J}}{\text{kcal}}$$

$$s_2 - s_1 = -0.291\ \text{kJ/kg}\cdot\text{K}$$ Δs

Problem 4.3

[2]



4.3 A fully loaded Boeing 777-200 jet transport aircraft weighs 325,000 kg. The pilot brings the 2 engines to full takeoff thrust of 450 kN each before releasing the brakes. Neglecting aerodynamic and rolling resistance, estimate the minimum runway length and time needed to reach a takeoff speed of 225 kph. Assume engine thrust remains constant during ground roll.

Given: Data on Boeing 777-200 jet

Find: Minimum runway length for takeoff

Solution:

Basic equation $\Sigma F_x = M\cdot\frac{dV}{dt} = M\cdot V\cdot\frac{dV}{dx} = F_t = \text{constant}$ Note that the "weight" is already in mass units!

Separating variables $M\cdot V\cdot dV = F_t\cdot dx$

Integrating $x = \frac{M\cdot V^2}{2\cdot F_t}$

$$x = \frac{1}{2}\times 325\times 10^3 kg\times\left(225\frac{km}{hr}\times\frac{1\cdot km}{1000\cdot m}\times\frac{1\cdot hr}{3600\cdot s}\right)^2\times\frac{1}{2\times 425\times 10^3}\cdot\frac{1}{N}\times\frac{N\cdot s^2}{kg\cdot m} \qquad x = 747\,m$$

For time calculation $M\cdot\frac{dV}{dt} = F_t$ $\qquad dV = \frac{F_t}{M}\cdot dt$

Integrating $t = \frac{M\cdot V}{F_t}$

$$t = 325\times 10^3 kg\times 225\frac{km}{hr}\times\frac{1\cdot km}{1000\cdot m}\times\frac{1\cdot hr}{3600\cdot s}\times\frac{1}{2\times 425\times 10^3}\cdot\frac{1}{N}\times\frac{N\cdot s^2}{kg\cdot m} \qquad t = 23.9\,s$$

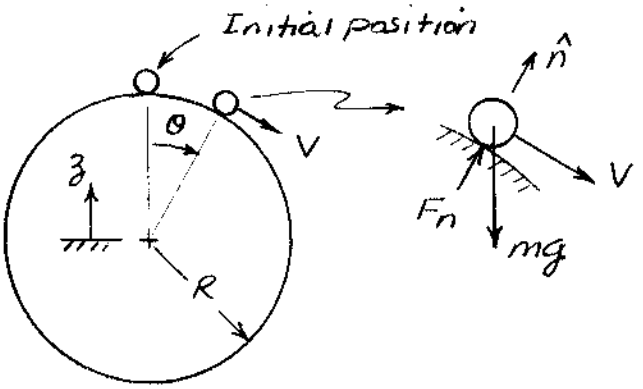
Aerodynamic and rolling resistances would significantly increase both these results

Problem 4.4

Given: Small steel ball of radius, r, atop large sphere of radius, R, begins to roll. Neglect rolling and air resistance.

Find: Location where ball loses contact and becomes a projectile.

Solution: Sum forces in n direction



$$\Sigma F_n = F_n - mg\cos\theta = ma_n$$

$$a_n = -\frac{V^2}{(R+r)}$$

Contact is lost when $F_n \rightarrow 0$, or

$$-mg\cos\theta = -m\frac{V^2}{(R+r)}$$

or

$$V^2 = (R+r)g\cos\theta \qquad (1)$$

Energy must be conserved if there is no resistance. Thus

$$E = mgz + m\frac{V^2}{2} = mg(R+r)\cos\theta + m\frac{V^2}{2} = E_0 = mg(R+r)$$

Thus from energy considerations

$$V^2 = 2g(R+r)(1-\cos\theta) \qquad (2)$$

Combining Eqs 1 and 2,

$$V^2 = 2g(R+r)(1-\cos\theta) = (R+r)g\cos\theta$$

or $\quad 2(1-\cos\theta) = 2 - 2\cos\theta = \cos\theta$

Thus $\cos\theta = \frac{2}{3}$ and $\theta = \cos^{-1}(\frac{2}{3}) = 48.2$ degrees $\leftarrow \theta$

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42-389 200 SHEETS 5 SQUARE
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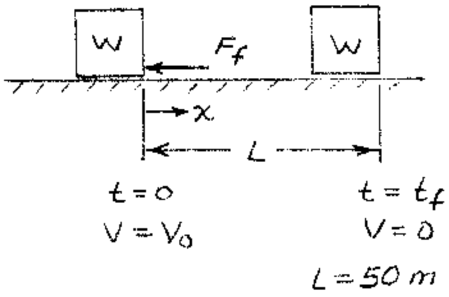
Problem 4.5

[2]

Given: Auto skids to stop in 50 meters on level road with $\mu = 0.6$.

Find: Initial speed.

Solution: Apply Newton's second law to a system (auto).



Basic equations: $\Sigma F_x = ma_x = \frac{W}{g}\frac{d^2x}{dt^2}$

Assumptions: (1) $F_f = \mu W$

(2) Neglect air resistance

Then $\Sigma F_x = -F_f = -\mu W = \frac{W}{g}\frac{d^2x}{dt^2}$

or $\frac{d^2x}{dt^2} = -\mu g$

Integrating,

$$\frac{dx}{dt} = -\mu g t + C_1 = -\mu g t + V_0 \qquad (1)$$

since $V = V_0$ at $t = 0$. Integrating again,

$$x = -\frac{1}{2}\mu g t^2 + V_0 t + c_2 = -\frac{1}{2}\mu g t^2 + V_0 t \qquad (2)$$

since $x = 0$ at $t = 0$.

Now at $x = L$, $\frac{dx}{dt} = 0$, and $t = t_f$. From Eq. 1,

$$0 = -\mu g t_f + V_0 \quad \text{or} \quad t_f = \frac{V_0}{\mu g}$$

Substituting into Eq. 2, evaluated at $t = t_f$,

$$L = -\frac{1}{2}\mu g t_f^2 + V_0 t_f = -\frac{1}{2}\mu g \frac{V_0^2}{(\mu g)^2} + V_0 \frac{V_0}{\mu g}$$

$$L = -\frac{1}{2}\frac{V_0^2}{\mu g} + \frac{V_0^2}{\mu g} = \frac{1}{2}\frac{V_0^2}{\mu g}$$

Solving, $V_0 = \sqrt{2\mu g L} = \sqrt{2(0.6)\,9.81\frac{m}{s^2} \times 50\,m} = 24.3\ m/s$

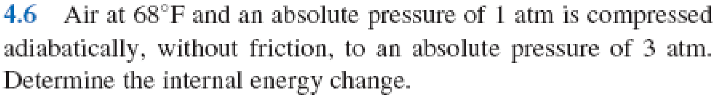
or

$$V_0 = 24.3\frac{m}{s} \times \frac{km}{1000\,m} \times 3600\frac{s}{hr} = 87.5\ km/hr$$

V_0

Problem 4.6

[2]


4.6 Air at 68°F and an absolute pressure of 1 atm is compressed adiabatically, without friction, to an absolute pressure of 3 atm. Determine the internal energy change.

Given: Data on air compression process

Find: Internal energy change

Solution:

Basic equation $\delta Q - \delta W = dE$

Assumptions: 1) Adiabatic so $\delta Q = 0$ 2) Stationary system $dE = dU$ 3) Frictionless process $\delta W = pdV = Mpdv$

Then $dU = -\delta W = -M\cdot p\cdot dv$

Before integrating we need to relate p and v. An adiabatic frictionless (reversible) process is isentropic, which for an ideal gas gives

$$p\cdot v^{k} = C \qquad \text{where} \qquad k = \frac{c_p}{c_v}$$

Hence $v = C^{\frac{1}{k}}\cdot p^{-\frac{1}{k}}$ and $dv = C^{\frac{1}{k}}\cdot\frac{1}{k}\cdot p^{-\frac{1}{k}-1}\cdot dp$

Substituting $$du = \frac{dU}{M} = -p\cdot dv = -p\cdot C^{\frac{1}{k}}\cdot\frac{1}{k}\cdot p^{-\frac{1}{k}-1}\cdot dp = \frac{-C^{\frac{1}{k}}}{k}\cdot p^{-\frac{1}{k}}\cdot dp$$

Integrating between states $$\Delta u = \frac{C^{\frac{1}{k}}}{k-1}\cdot\left(p_2^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}}\right) = \frac{C^{\frac{1}{k}}\cdot p_1^{\frac{k-1}{k}}}{k-1}\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$

But $$C^{\frac{1}{k}}\cdot p^{\frac{k-1}{k}} = C^{\frac{1}{k}}\cdot p^{-\frac{1}{k}}\cdot p = p\cdot v = R_{air}\cdot T$$

Hence $$\Delta u = \frac{R_{air}\cdot T_1}{k-1}\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$

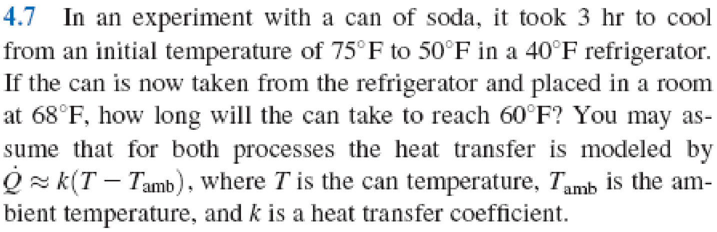
From Table A.6 $R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$ and $k = 1.4$

$$\Delta u = \frac{1}{0.4}\times 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}\times(68+460)R\times\left[\left(\frac{3}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right] \qquad \Delta u = 2.6\times 10^{4}\cdot\frac{ft\cdot lbf}{lbm}$$

$$\Delta u = 33.4\cdot\frac{Btu}{lbm} \qquad \Delta u = 1073\cdot\frac{Btu}{slug}$$ (Using conversions from Table G.2)

Problem 4.7

[2]



4.7 In an experiment with a can of soda, it took 3 hr to cool from an initial temperature of 75°F to 50°F in a 40°F refrigerator. If the can is now taken from the refrigerator and placed in a room at 68°F, how long will the can take to reach 60°F? You may assume that for both processes the heat transfer is modeled by $\dot{Q} \approx k(T - T_{\text{amb}})$, where T is the can temperature, T_{amb} is the ambient temperature, and k is a heat transfer coefficient.

Given:

Data on cooling of a can of soda in a refrigerator

Find:

How long it takes to warm up in a room

Solution:

The First Law of Thermodynamics for the can (either warming or cooling) is

$$M \cdot c \cdot \frac{dT}{dt} = -k \cdot \left(T - T_{amb}\right) \qquad \text{or} \qquad \frac{dT}{dt} = -A \cdot \left(T - T_{amb}\right) \qquad \text{where} \qquad A = \frac{k}{M \cdot c}$$

where *M* is the can mass, *c* is the average specific heat of the can and its contents, *T* is the temperature, and T_{amb} is the ambient temperature

Separating variables $\quad \dfrac{dT}{T - T_{amb}} = -A \cdot dt$

Integrating $\quad T(t) = T_{amb} + \left(T_{init} - T_{amb}\right) \cdot e^{-A\,t}$

where T_{init} is the initial temperature. The available data from the coolling can now be used to obtain a value for constant *A*

Given data for cooling $\quad T_{init} = (25 + 273) \cdot K \qquad T_{init} = 298\,K \qquad T_{amb} = (5 + 273) \cdot K \qquad T_{amb} = 278\,K$

$T = (10 + 273) \cdot K \qquad T = 283\,K \qquad$ when $\qquad t = \tau = 10 \cdot hr$

Hence
$$A = \frac{1}{\tau} \cdot \ln\left(\frac{T_{init} - T_{amb}}{T - T_{amb}}\right) = \frac{1}{3 \cdot hr} \times \frac{1 \cdot hr}{3600 \cdot s} \times \ln\left(\frac{298 - 278}{283 - 278}\right) \qquad A = 1.284 \times 10^{-4}\,s^{-1}$$

Then, for the warming up process

$T_{init} = (10 + 273) \cdot K \qquad T_{init} = 283\,K \qquad T_{amb} = (20 + 273) \cdot K \qquad T_{amb} = 293\,K$

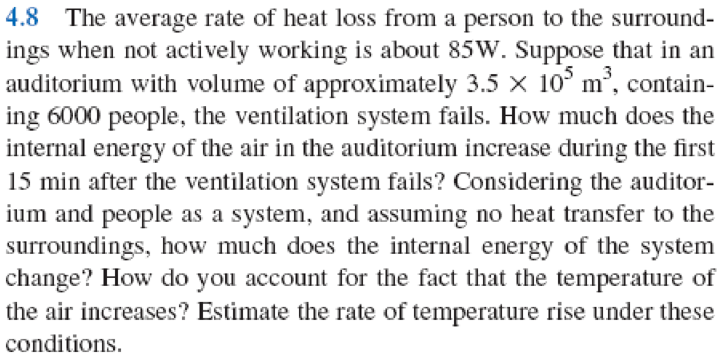
$T_{end} = (15 + 273) \cdot K \qquad T_{end} = 288\,K$

with $\quad T_{end} = T_{amb} + \left(T_{init} - T_{amb}\right) \cdot e^{-A\,\tau}$

Hence the time τ is
$$\tau = \frac{1}{A} \cdot \ln\left(\frac{T_{init} - T_{amb}}{T_{end} - T_{amb}}\right) = \frac{s}{1.284 \cdot 10^{-4}} \cdot \ln\left(\frac{283 - 293}{288 - 293}\right) \qquad \tau = 5.40 \times 10^{3}\,s \qquad \tau = 1.50\,hr$$

Problem 4.8

[2]



4.8 The average rate of heat loss from a person to the surroundings when not actively working is about 85W. Suppose that in an auditorium with volume of approximately 3.5×10^5 m^3, containing 6000 people, the ventilation system fails. How much does the internal energy of the air in the auditorium increase during the first 15 min after the ventilation system fails? Considering the auditorium and people as a system, and assuming no heat transfer to the surroundings, how much does the internal energy of the system change? How do you account for the fact that the temperature of the air increases? Estimate the rate of temperature rise under these conditions.

Given: Data on heat loss from persons, and people-filled auditorium

Find: Internal energy change of air and of system; air temperature rise

Solution:

Basic equation $Q - W = \Delta E$

Assumptions: 1) Stationary system dE =dU 2) No work W = 0

Then for the air $\Delta U = Q = 85 \cdot \frac{W}{\text{person}} \times 6000 \cdot \text{people} \times 15 \cdot \text{min} \times \frac{60 \cdot s}{\text{min}}$ $\Delta U = 459\,\text{MJ}$

For the air and people $\Delta U = Q_{\text{surroundings}} = 0$

The increase in air energy is equal and opposite to the loss in people energy

For the air $\Delta U = Q$ but for air (an ideal gas) $\Delta U = M \cdot c_v \cdot \Delta T$ with $M = \rho \cdot V = \frac{p \cdot V}{R_{air} \cdot T}$

Hence $\Delta T = \frac{Q}{M \cdot c_v} = \frac{R_{air} \cdot Q \cdot T}{c_v \cdot p \cdot V}$

From Table A.6 $R_{air} = 286.9 \cdot \frac{J}{kg \cdot K}$ and $c_v = 717.4 \cdot \frac{J}{kg \cdot K}$

$$\Delta T = \frac{286.9}{717.4} \times 459 \times 10^6 \cdot J \times (20 + 273)K \times \frac{1}{101 \times 10^3} \cdot \frac{m^2}{N} \times \frac{1}{3.5 \times 10^5} \cdot \frac{1}{m^3} \qquad \Delta T = 1.521\,K$$

This is the temperature change in 15 min. The rate of change is then $\frac{\Delta T}{15 \cdot \text{min}} = 6.09 \frac{K}{hr}$

Problem 4.9

[3] Part 1/2

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Given: Aluminum beverage can, $m_c = 20\,g$, $D = 65\,mm$, $H = 120\,mm$.

Maximum contents level is h_{max}, when $\forall_b = 354\,mL$ of beverage.

SG of beverage is 1.05.

Find: (a) Center of mass, y_c, vs. level, h.
(b) Level for least tendency to tip.
(c) Minimum coefficient of friction, μ_s, for full can to tip, not slide.
(d) Plot $\mu_s)$ minimum for can to tip (not slide) as a function of beverage level in can.

Solution: $M_b = SG\rho\forall_b = 1.05 \times 1.0\,\frac{g}{cm^3} \times 354\,mL \times \frac{cm^3}{mL} = 372\,g$ (max)

$$h_{max} = \frac{\forall_b}{A} = \frac{4\forall_b}{\pi D^2} = \frac{4}{\pi} \times 354\,mL \times \frac{1}{(6.5)^2\,cm^2} \times \frac{cm^3}{mL} \times \frac{10\,mm}{cm} = 107\,mm$$

At any level, $m_b = \frac{h}{h_{max}} M_b$; $m_b(g) = \frac{h(mm)}{107\,mm} \times 372\,g = 3.47\,h(mm)$

From moment considerations,

$$y_c M = \frac{h}{2} m_b + \frac{H}{2} m_c = \frac{1}{2}\left[h(3.47h) + 120(20)\right] = \frac{1}{2}(3.47h^2 + 2400)$$

$$M = m_b + m_c = 3.47h + 20$$

$$y_c = \frac{3.47\,h^2 + 2400}{6.94\,h + 40} \quad (h \text{ in mm})$$

y_c

Tendency to tip will be least when y_c is a minimum. Thus

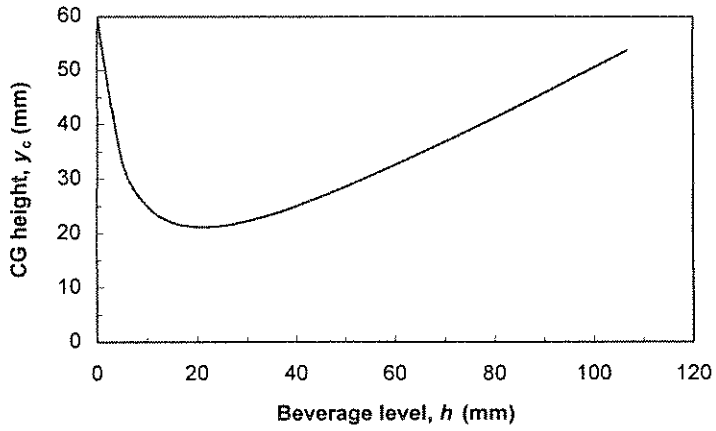
$$\frac{dy_c}{dh} = \frac{2(3.47h)}{6.94h + 40} + (-1)(6.95)\frac{3.47h^2 + 2400}{(6.94\,h + 40)^2} = \frac{24.1h^2 + 278\,h - 16{,}700}{(6.94\,h + 40)^2} = 0$$

Using the quadratic formula,

$$h\,(\text{at } y_c \text{ min}) = \frac{-278 \pm \sqrt{(278)^2 + 4(24.1)16{,}700}}{2(24.1)} = 21.2\,mm$$

h (y_c min)

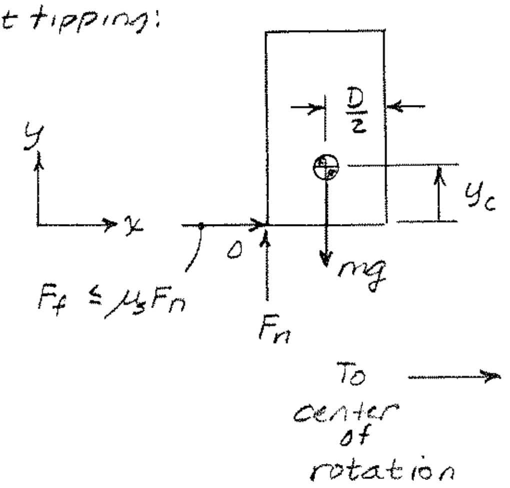
Plotting,



Next page →

Problem 4.9

Draw a free-body diagram of the can at tipping:



$$\sum F_x = F_f = ma_x$$

$$\sum F_y = F_n - mg = ma_y = 0$$

$$F_n = mg$$

Since $F_f \le \mu_s F_n$, then $\mu_s F_n \ge ma_x$

Summing moments about point O:

$$\overset{+}{\sum M_O} = y_c\, ma_x - \frac{D}{2} F_n = 0$$

or $$y_c\, ma_x = \frac{D}{2} F_n$$

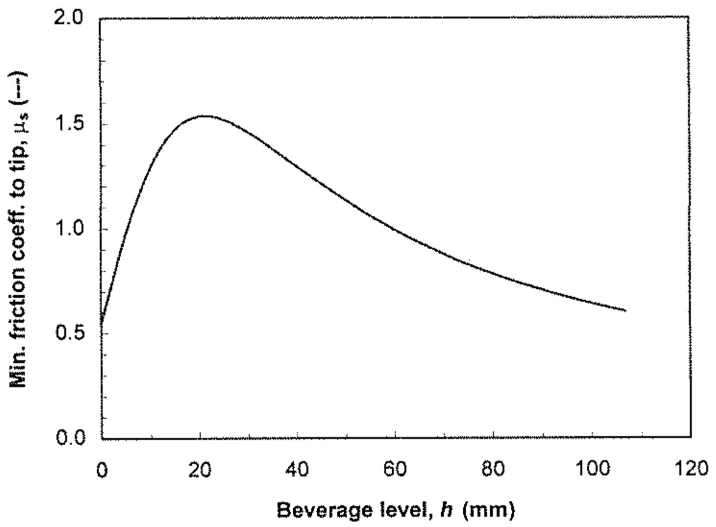
But $ma_x \le \mu_s F_n$, so

$$y_c \mu_s F_n \ge \frac{D}{2} F_n$$

Thus to tip

$$\mu_s \ge \frac{D}{2y_c}$$ μ_s

Plotting,



For the full can with $y_c = 53.8$ mm,

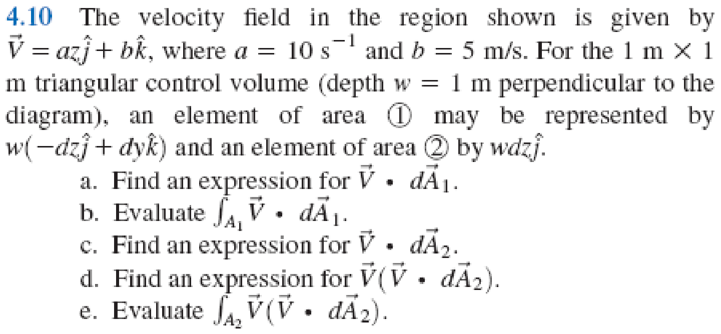
$$\mu_s \ge \frac{1}{2} \times 65\,\text{mm} \times \frac{1}{53.8\,\text{mm}} = 0.604$$ μ_s

This value is much higher than the can could develop. Therefore the can will not tip; it will slide.

The corresponding acceleration is $a_x = \mu_s g = 0.593\ \text{m/s}^2$

Problem 4.10

[3]



4.10 The velocity field in the region shown is given by $\vec{V} = az\hat{j} + b\hat{k}$, where $a = 10\ \text{s}^{-1}$ and $b = 5$ m/s. For the 1 m × 1 m triangular control volume (depth $w = 1$ m perpendicular to the diagram), an element of area ① may be represented by $w(-dz\hat{j} + dy\hat{k})$ and an element of area ② by $wdz\hat{j}$.

a. Find an expression for $\vec{V} \cdot d\vec{A}_1$.

b. Evaluate $\int_{A_1} \vec{V} \cdot d\vec{A}_1$.

c. Find an expression for $\vec{V} \cdot d\vec{A}_2$.

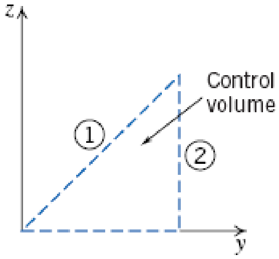
d. Find an expression for $\vec{V}(\vec{V} \cdot d\vec{A}_2)$.

e. Evaluate $\int_{A_2} \vec{V}(\vec{V} \cdot d\vec{A}_2)$.

Given: Data on velocity field and control volume geometry

Find: Several surface integrals

Solution:



$$d\vec{A}_1 = -wdz\hat{j} + wdy\hat{k} \qquad d\vec{A}_1 = -dz\hat{j} + dy\hat{k}$$

$$d\vec{A}_2 = wdz\hat{j} \qquad d\vec{A}_2 = dz\hat{j}$$

$$\vec{V} = \left(az\hat{j} + b\hat{k}\right) \qquad \vec{V} = \left(10z\hat{j} + 5\hat{k}\right)$$

(a) $$\vec{V} \cdot dA_1 = \left(10z\hat{j} + 5\hat{k}\right) \cdot \left(-dz\hat{j} + dy\hat{k}\right) = -10zdz + 5dy$$

(b) $$\int_{A_1} \vec{V} \cdot dA_1 = -\int_0^1 10zdz + \int_0^1 5dy = -5z^2\Big|_0^1 + 5y\Big|_0^1 = 0$$

(c) $$\vec{V} \cdot dA_2 = \left(10z\hat{j} + 5\hat{k}\right) \cdot \left(dz\hat{j}\right) = 10zdz$$

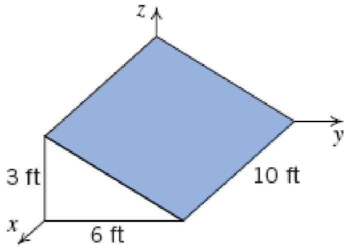
(d) $$\vec{V}\left(\vec{V} \cdot dA_2\right) = \left(10z\hat{j} + 5\hat{k}\right)10zdz$$

(e) $$\int_{A_2} \vec{V}\left(\vec{V} \cdot dA_2\right) = \int_0^1 \left(10z\hat{j} + 5\hat{k}\right)10zdz = \frac{100}{3}z^3\hat{j}\Big|_0^1 + 25z^2\hat{k}\Big|_0^1 = 33.3\hat{j} + 25\hat{k}$$

Problem 4.11

[3]

4.11 The shaded area shown is in a flow where the velocity field is given by $\vec{V} = ax\hat{i} - by\hat{j}$; $a = b = 1\ \text{s}^{-1}$, and the coordinates are measured in meters. Evaluate the volume flow rate and the momentum flux through the shaded area.



Given: Geometry of 3D surface

Find: Volume flow rate and momentum flux through area

Solution:

$$d\vec{A} = dxdz\hat{j} + dxdy\hat{k}$$

$$\vec{V} = ax\hat{i} - by\hat{j} \qquad \vec{V} = x\hat{i} - y\hat{j}$$

We will need the equation of the surface: $z = 3 - \frac{1}{2}y$ or $y = 6 - 2z$

a) Volume flow rate

$$Q = \int_A \vec{V} \cdot dA = \int_A \left(x\hat{i} - y\hat{j}\right) \cdot \left(dxdz\hat{j} + dxdy\hat{k}\right)$$

$$= \int_0^{10}\int_0^{3} - ydzdx = \int_0^{3} -10ydz = \int_0^{3} -10(6-2z)dz = -60z + 10z^2\Big|_0^3$$

$$Q = (-180 + 90)\frac{\text{ft}^3}{\text{s}}$$

$$Q = -90\frac{\text{ft}^3}{\text{s}}$$

b) Momentum flux

$$\rho\int_A \vec{V}\left(\vec{V} \cdot d\vec{A}\right) = \rho\int_A \left(x\hat{i} - y\hat{j}\right)\left(-ydxdz\right)$$

$$= \rho\int_0^{10}\int_0^{3}(-xy)dzdx\hat{i} + \rho\int_0^{3}10y^2dz\hat{j}$$

$$= -\rho\int_0^{10}xdx\int_0^{3}(6-2z)dz\hat{i} + \rho\int_0^{3}10(6-2z)^2dz\hat{j}$$

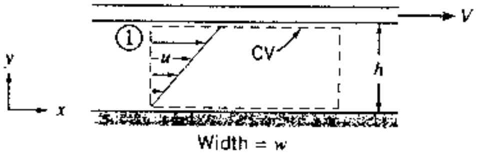
$$= \rho\left(-\frac{x^2}{2}\Big|_0^{10}\right)\left(6z - z^2\Big|_0^3\right)\hat{i} + \rho\left(10\left(36z - 12z^2 + \frac{4}{3}z^3\right)\Big|_0^3\right)\hat{j}$$

$$= \rho(-50)(18-9)\hat{i} + \rho(10(108 - 108 + 36))\hat{j}$$

$$= -450\rho\hat{i} + 360\rho\hat{j} \qquad \left(\frac{\text{slug}\cdot\text{ft}/\text{s}}{\text{s}} \text{ if } \rho \text{ is in } \frac{\text{slug}}{\text{ft}^3}\right)$$

Problem 4.12

Given: Control volume with linear velocity distribution across surface ① as shown; width = w.



Find: (a) Volume flow rate, and
(b) Momentum flux,
through surface ①

Solution:

The volume flow rate is $Q = \int \vec{V} \cdot d\vec{A}$

At surface ①, $\vec{V} = \frac{V}{h} y \hat{i}$ and $dA = -w\,dy\,\hat{i}$

Thus

$$Q = \int_{y=0}^{h} \frac{V}{h} y \hat{i} \cdot (-w\,dy\,\hat{i}) = -\frac{Vw}{h}\int_{y=0}^{h} y\,dy = -\frac{Vw}{h}\frac{y^2}{2}\Big]_0^h$$

$$Q = -\frac{1}{2} Vhw \longleftarrow \text{Volume flow rate}$$

The momentum flux is given by $m.f. = \int \vec{V}(\rho \vec{V} \cdot d\vec{A})$

Thus,

$$m.f. = \int_0^h \frac{V}{h} y \hat{i} \left(-\rho \frac{Vw}{h} y\,dy\right) = -\rho \frac{V^2 w}{h^2} \hat{i} \int_0^h y^2\,dy = -\rho \frac{V^2 w}{h^2} \hat{i}\, \frac{y^3}{3}\Big]_0^h$$

$$m.f. = -\frac{1}{3} \rho V^2 wh \hat{i} \longleftarrow \text{Momentum flux}$$

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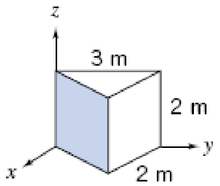
Problem 4.13

4.13 The area shown shaded is in a flow where the velocity field is given by $\vec{V} = -ax\hat{i} + by\hat{j} + c\hat{k}$; $a = b = 2\ \text{s}^{-1}$ and $c = 2.5$ m/s. Write a vector expression for an element of the shaded area. Evaluate the integrals $\int \vec{V} \cdot d\vec{A}$ and $\int \vec{V}(\vec{V} \cdot d\vec{A})$ over the shaded area.



Given: Geometry of 3D surface

Find: Surface integrals

Solution:

$$d\vec{A} = dydz\hat{i} - dxdz\hat{j}$$

$$\vec{V} = -ax\hat{i} + by\hat{j} + c\hat{k} \qquad \vec{V} = -2x\hat{i} + 2y\hat{j} + 2.5\hat{k}$$

We will need the equation of the surface: $y = \frac{3}{2}x$ or $x = \frac{2}{3}y$

$$\int_A \vec{V} \cdot dA = \int_A \left(-ax\hat{i} + by\hat{j} + c\hat{k}\right) \cdot \left(dydz\hat{i} - dxdz\hat{j}\right)$$

$$= \int_0^2\int_0^3 -axdydz - \int_0^2\int_0^2 bydxdz = -a\int_0^2 dz\int_0^3 \frac{2}{3}ydy - b\int_0^2 dz\int_0^2 \frac{3}{2}xdx = -2a\frac{1}{3}y^2\Big|_0^3 - 2b\frac{3}{4}x^2\Big|_0^2$$

$$Q = (-6a - 6b)$$

$$Q = -24\frac{\text{m}^3}{\text{s}}$$

We will again need the equation of the surface: $y = \frac{3}{2}x$ or $x = \frac{2}{3}y$, and also $dy = \frac{3}{2}dx$ and $a = b$

$$\int_A \vec{V}\left(\vec{V} \cdot d\vec{A}\right) = \int_A \left(-ax\hat{i} + by\hat{j} + c\hat{k}\right)\left(-ax\hat{i} + by\hat{j} + c\hat{k}\right) \cdot \left(dydz\hat{i} - dxdz\hat{j}\right)$$

$$= \int_A \left(-ax\hat{i} + by\hat{j} + c\hat{k}\right)\left(-axdydz - bydxdz\right)$$

$$= \int_A \left(-ax\hat{i} + \frac{3}{2}ax\hat{j} + c\hat{k}\right)\left(-ax\frac{3}{2}dxdz - a\frac{3}{2}xdxdz\right)$$

$$= \int_A \left(-ax\hat{i} + \frac{3}{2}ax\hat{j} + c\hat{k}\right)\left(-3axdxdz\right)$$

$$= 3\int_0^2\int_0^2 a^2x^2dxdz\hat{i} - \frac{9}{2}\int_0^2\int_0^2 a^2x^2dxdz\hat{j} - 3\int_0^2\int_0^2 acxdxdz\hat{k}$$

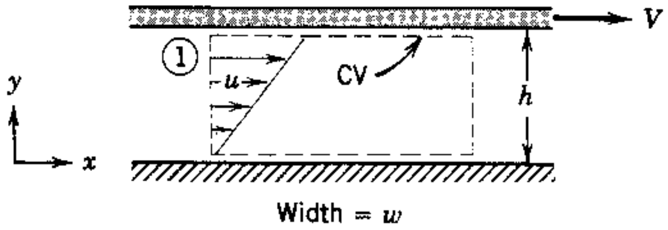
$$= (6)\left(a^2\frac{x^3}{3}\Big|_0^2\right)\hat{i} - (9)\left(a^2\frac{x^3}{3}\Big|_0^2\right)\hat{j} - (6)\left(ac\frac{x^2}{2}\Big|_0^2\right)$$

$$= 16a^2\hat{i} - 24a^2\hat{j} - 12ac\hat{k}$$

$$= 64\hat{i} - 96\hat{j} - 60\hat{k} \quad \frac{\text{m}^4}{\text{s}^2}$$

Problem 4.14

Given: Flow and CV of Problem 4.12, as shown



Find: Expression for kinetic energy flux through cross-section ① of CV.

Solution: Kinetic energy flux is defined as $kef = \int_A \frac{V^2}{2} \rho \vec{V} \cdot d\vec{A}$

Model the velocity profile as $u = V\frac{y}{h}$. Then

$$\vec{V} = u\hat{\imath} = V\frac{y}{h}\hat{\imath}\ ;\quad V^2 = V^2\left(\frac{y}{h}\right)^2$$

Since flow is into the CV, $\vec{V} \cdot d\vec{A} = -u\,dA = -V\frac{y}{h}w\,dy$

Substituting,

$$kef = \int_A \frac{V^2}{2}\left(\frac{y}{h}\right)^2\left\{-\rho V\frac{y}{h}w\,dy\right\} = -\frac{\rho V^3 w}{2h^3}\int_0^h y^3\,dy$$

$$= -\frac{\rho V^3 w}{2h^3}\left[\frac{y^4}{4}\right]_0^h$$

$$kef = -\frac{\rho V^3 w h}{8}$$

kef

Check dimensions:

$$[kef] = \frac{M}{L^3}\left(\frac{L}{t}\right)^3 L L = \frac{ML^2}{t^3} \times \frac{Ft^2}{ML} = \frac{FL}{t} = \frac{\text{Energy}}{\text{Time}} \quad \checkmark\checkmark$$

Problem 4.15

Given: Velocity distribution for laminar flow in a long circular tube

$$\vec{V} = u\hat{i} = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]\hat{i}$$

where R is the tube radius.

Evaluate: (a) The volume flow rate, and (b) the momentum flux, through a section normal to the pipe axis.

Solution: The volume flow rate is given by

$$\int_{A_{tube}} \vec{V}\cdot d\vec{A} = \int_0^R u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}\cdot 2\pi r\,dr\,\hat{i} \qquad \{A=\pi r^2,\ dA = 2\pi r\,dr\}$$

$$= u_{max}2\pi\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right]r\,dr = u_{max}2\pi\int_0^R\left[r-\frac{r^3}{R^2}\right]dr$$

$$= u_{max}2\pi\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R = u_{max}2\pi\left[\frac{R^2}{2}-\frac{R^2}{4}\right]$$

$$\int_{A_{tube}} \vec{V}\cdot d\vec{A} = \frac{1}{2}u_{max}\pi R^2 \longleftarrow \underline{\text{volume flow rate}}$$

The momentum flux is given by

$$\int_{A_{tube}} \vec{V}(\vec{V}\cdot d\vec{A}) = \int_0^R u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}\left\{u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}\cdot 2\pi r\,dr\,\hat{i}\right\}$$

$$= \int_0^R u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}\left\{u_{max}2\pi\left[r-\frac{r^3}{R^2}\right]dr\right\}$$

$$= u_{max}^2 2\pi\int_0^R\left(r-\frac{2r^3}{R^2}+\frac{r^5}{R^4}\right)dr\,\hat{i}$$

$$= u_{max}^2 2\pi\left[\frac{r^2}{2}-\frac{r^4}{2R^2}+\frac{r^6}{6R^4}\right]_0^R\hat{i}$$

$$= u_{max}^2 2\pi R^2\left[\frac{1}{2}-\frac{1}{2}+\frac{1}{6}\right]\hat{i}$$

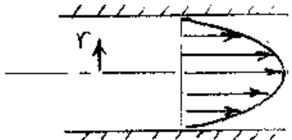
$$\int_{A_{tube}} \vec{V}(\vec{V}\cdot d\vec{A}) = \frac{1}{3}u_{max}^2\pi R^2\hat{i} \qquad \underline{\text{momentum flux}}$$

Problem 4.16

[2]

Given: Velocity profile in a circular tube,

$$\vec{V} = u\hat{i} = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]\hat{i}$$



Find: Expression for kinetic energy flux, $kef = \int \frac{V^2}{2}\rho\vec{V}\cdot d\vec{A}$

Solution: $V^2 = \vec{V}\cdot\vec{V} = u_{max}^2\left[1-\left(\frac{r}{R}\right)^2\right]^2 = u_{max}^2\left[1-2\left(\frac{r}{R}\right)^2+\left(\frac{r}{R}\right)^4\right]$

$$d\vec{A} = 2\pi r\,dr\,\hat{i}$$

$$\vec{V}\cdot d\vec{A} = 2\pi r\,u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Then

$$kef = \int_0^R \frac{u_{max}^2}{\cancel{2}}\left[1-2\left(\frac{r}{R}\right)^2+\left(\frac{r}{R}\right)^4\right]\rho\,\cancel{2}\pi r\,u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]dr$$

$$= \pi\rho u_{max}^3\int_0^R\left[1-3\left(\frac{r}{R}\right)^2+3\left(\frac{r}{R}\right)^4-\left(\frac{r}{R}\right)^6\right]r\,dr$$

$$= \pi\rho u_{max}^3 R^2\int_0^1\left[1-3\left(\frac{r}{R}\right)^2+3\left(\frac{r}{R}\right)^4-\left(\frac{r}{R}\right)^6\right]\frac{r}{R}\,d\left(\frac{r}{R}\right)$$

$$= \pi\rho u_{max}^3 R^2\left[\frac{1}{2}\left(\frac{r}{R}\right)^2-\frac{3}{4}\left(\frac{r}{R}\right)^4+\frac{1}{2}\left(\frac{r}{R}\right)^6-\frac{1}{8}\left(\frac{r}{R}\right)^8\right]_0^1$$

$$= \pi R^2\rho u_{max}^3\left[\frac{1}{2}-\frac{3}{4}+\frac{1}{2}-\frac{1}{8}\right]$$

$$kef = \frac{\pi R^2\rho u_{max}^3}{8} \longleftarrow kef$$

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Problem 4.17

4.17 A farmer is spraying a liquid through 10 nozzles, ⅛th in. ID, at an average exit velocity of 10 ft/s. What is the average velocity in the 1-in. ID head feeder? What is the system flow rate, in gpm?

Given: Data on flow through nozzles

Find: Average velocity in head feeder; flow rate

Solution:

Basic equation $$\sum_{CS} \left(\vec{V}\cdot\vec{A}\right) = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Then for the nozzle flow $$\sum_{CS} \left(\vec{V}\cdot\vec{A}\right) = -V_{feeder}\cdot A_{feeder} + 10\cdot V_{nozzle}\cdot A_{nozzle} = 0$$

Hence $$V_{feeder} = V_{nozzle}\cdot\frac{10\cdot A_{nozzle}}{A_{feeder}} = V_{nozzle}\cdot 10\cdot\left(\frac{D_{nozzle}}{D_{feeder}}\right)^2$$

$$V_{feeder} = 10\cdot\frac{ft}{s}\times 10\times\left(\frac{\frac{1}{8}}{1}\right)^2$$

$$V_{feeder} = 1.56\cdot\frac{ft}{s}$$

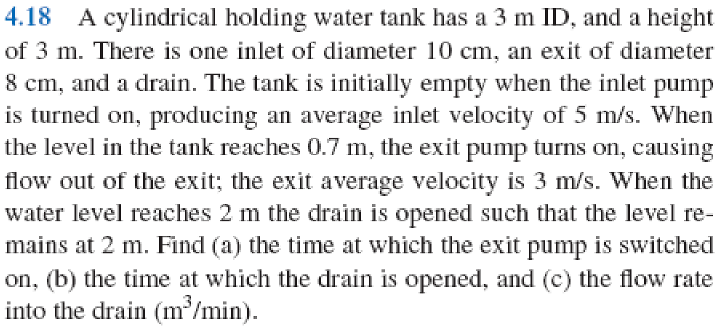
The flow rate is $$Q = V_{feeder}\cdot A_{feeder} = V_{feeder}\cdot\frac{\pi\cdot D_{feeder}^{\ 2}}{4}$$

$$Q = 1.56\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(1\cdot in\times\frac{1\cdot ft}{12\cdot in}\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min}$$

$$Q = 3.82\cdot gpm$$

Problem 4.18

[3]



4.18 A cylindrical holding water tank has a 3 m ID, and a height of 3 m. There is one inlet of diameter 10 cm, an exit of diameter 8 cm, and a drain. The tank is initially empty when the inlet pump is turned on, producing an average inlet velocity of 5 m/s. When the level in the tank reaches 0.7 m, the exit pump turns on, causing flow out of the exit; the exit average velocity is 3 m/s. When the water level reaches 2 m the drain is opened such that the level remains at 2 m. Find (a) the time at which the exit pump is switched on, (b) the time at which the drain is opened, and (c) the flow rate into the drain (m^3/min).

Given: Data on flow into and out of tank

Find: Time at which exit pump is switched on; time at which drain is opened; flow rate into drain

Solution:

Basic equation $$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$$

Assumptions: 1) Uniform flow 2) Incompressible flow

After inlet pump is on $$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \frac{\partial}{\partial t}M_{tank} - \rho\cdot V_{in}\cdot A_{in} = 0 \qquad \frac{\partial}{\partial t}M_{tank} = \rho\cdot A_{tank}\cdot\frac{dh}{dt} = \rho\cdot V_{in}\cdot A_{in}$$ where h is the level of water in the tank

$$\frac{dh}{dt} = V_{in}\cdot\frac{A_{in}}{A_{tank}} = V_{in}\cdot\left(\frac{D_{in}}{D_{tank}}\right)^2$$

Hence the time to reach h_{exit} = 0.7 m is $$t_{exit} = \frac{h_{exit}}{\frac{dh}{dt}} = \frac{h_{exit}}{V_{in}}\cdot\left(\frac{D_{tank}}{D_{in}}\right)^2 \qquad t_{exit} = 0.7\cdot m\times\frac{1}{5}\cdot\frac{s}{m}\times\left(\frac{3\cdot m}{0.1\cdot m}\right)^2 \qquad t_{exit} = 126\,s$$

After exit pump is on $$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \frac{\partial}{\partial t}M_{tank} - \rho\cdot V_{in}\cdot A_{in} + \rho\cdot V_{exit}\cdot A_{exit} = 0 \qquad A_{tank}\cdot\frac{dh}{dt} = V_{in}\cdot A_{in} - V_{exit}\cdot A_{exit}$$

$$\frac{dh}{dt} = V_{in}\cdot\frac{A_{in}}{A_{tank}} - V_{exit}\cdot\frac{A_{exit}}{A_{tank}} = V_{in}\cdot\left(\frac{D_{in}}{D_{tank}}\right)^2 - V_{exit}\cdot\left(\frac{D_{exit}}{D_{tank}}\right)^2$$

Hence the time to reach h_{drain} = 2 m is $$t_{drain} = t_{exit} + \frac{\left(h_{drain} - h_{exit}\right)}{\frac{dh}{dt}} = \frac{\left(h_{drain} - h_{exit}\right)}{V_{in}\cdot\left(\frac{D_{in}}{D_{tank}}\right)^2 - V_{exit}\cdot\left(\frac{D_{exit}}{D_{tank}}\right)^2}$$

$$t_{drain} = 126\cdot s + (2 - 0.7)\cdot m\times\frac{1}{5\cdot\frac{m}{s}\times\left(\frac{0.1\cdot m}{3\cdot m}\right)^2 - 3\cdot\frac{m}{s}\times\left(\frac{0.08\cdot m}{3\cdot m}\right)^2} \qquad t_{drain} = 506\,s$$

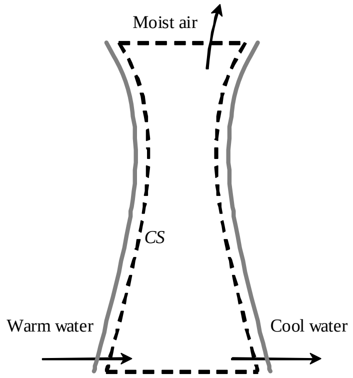
The flow rate into the drain is equal to the net inflow (the level in the tank is now constant)

$$Q_{drain} = V_{in}\cdot\frac{\pi\cdot D_{in}^2}{4} - V_{exit}\cdot\frac{\pi\cdot D_{exit}^2}{4} \qquad Q_{drain} = 5\cdot\frac{m}{s}\times\frac{\pi}{4}\times(0.1\cdot m)^2 - 3\cdot\frac{m}{s}\times\frac{\pi}{4}\times(0.08\cdot m)^2 \qquad Q_{drain} = 0.0242\frac{m^3}{s}$$

Problem 4.19

[4]

4.19 A wet cooling tower cools warm water by spraying it into a forced dry-air flow. Some of the water evaporates in this air and is carried out of the tower into the atmosphere; the evaporation cools the remaining water droplets, which are collected at the exit pipe (6 in. ID) of the tower. Measurements indicate the warm water mass flow rate is 250,000 lb/hr, and the cool water (70°F) flows at an average speed of 5.55 ft/s in the exit pipe. The flow rate of the moist air is to be obtained from measurements of the velocity at four points, each representing 1/4 of the air stream cross-sectional area of 13.2 ft^2. The moist air density is 0.066 $\mathrm{lb/ft}^3$. Find (a) the volume and mass flow rates of the cool water, (b) the mass flow rate of the moist air, and (c) the mass flow rate of the dry air.



Given: Data on flow into and out of cooling tower

Find: Volume and mass flow rate of cool water; mass flow rate of moist and dry air

Solution:

Basic equation $\sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$ and at each inlet/exit $Q = V\cdot A$

Assumptions: 1) Uniform flow 2) Incompressible flow

At the cool water exit $Q_{cool} = V\cdot A$ $\quad Q_{cool} = 5.55\cdot\frac{ft}{s}\times\frac{\pi}{4}\times(0.5\cdot ft)^2$ $\quad Q_{cool} = 1.09\frac{ft^3}{s}$ $\quad Q_{cool} = 489\,gpm$

The mass flow rate is $m_{cool} = \rho\cdot Q_{cool}$ $\quad m_{cool} = 1.94\cdot\frac{slug}{ft^3}\times 1.09\cdot\frac{ft^3}{s}$ $\quad m_{cool} = 2.11\frac{slug}{s}$ $\quad m_{cool} = 2.45\times 10^5\frac{lb}{hr}$

NOTE: Software does not allow dots over terms, so m represents mass flow rate, not mass!

For the air flow we need to use $\sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$ to balance the water flow

We have $-m_{warm} + m_{cool} + m_v = 0$ $\quad m_v = m_{warm} - m_{cool}$ $\quad m_v = 5073\frac{lb}{hr}$

This is the mass flow rate of water vapor. We need to use this to obtain air flow rates. From psychrometrics $x = \frac{m_v}{m_{air}}$

where x is the relative humidity. It is also known (try Googling "density of moist air") that $\frac{\rho_{moist}}{\rho_{dry}} = \frac{1+x}{1+x\cdot\frac{R_{H2O}}{R_{air}}}$

We are given $\rho_{moist} = 0.066\cdot\frac{lb}{ft^3}$

For dry air we could use the ideal gas equation $\rho_{dry} = \frac{p}{R\cdot T}$ but here we use atmospheric air density (Table A.3)

$\rho_{dry} = 0.002377\cdot\frac{slug}{ft^3}$ $\quad \rho_{dry} = 0.002377\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{lb}{slug}$ $\quad \rho_{dry} = 0.0765\frac{lb}{ft^3}$

Note that moist air is less dense than dry air!

Hence $\frac{0.066}{0.0765} = \frac{1+x}{1+x\cdot\frac{85.78}{53.33}}$ using data from Table A.6

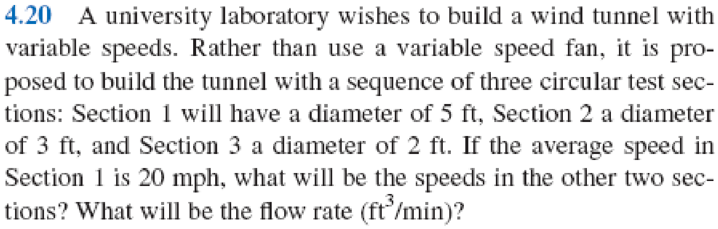
$x = \frac{0.0765 - 0.066}{0.066\cdot\frac{85.78}{53.33} - .0765}$ $x = 0.354$

Hence $\frac{m_v}{m_{air}} = x$ leads to $m_{air} = \frac{m_v}{x}$ $m_{air} = 5073\cdot\frac{lb}{hr}\times\frac{1}{0.354}$ $m_{air} = 14331\,\frac{lb}{hr}$

Finally, the mass flow rate of moist air is $m_{moist} = m_v + m_{air}$ $m_{moist} = 19404\,\frac{lb}{hr}$

Problem 4.20

[1]



4.20 A university laboratory wishes to build a wind tunnel with variable speeds. Rather than use a variable speed fan, it is proposed to build the tunnel with a sequence of three circular test sections: Section 1 will have a diameter of 5 ft, Section 2 a diameter of 3 ft, and Section 3 a diameter of 2 ft. If the average speed in Section 1 is 20 mph, what will be the speeds in the other two sections? What will be the flow rate (ft^3/min)?

Given: Data on wind tunnel geometry

Find: Average speeds in wind tunnel

Solution:

Basic equation $Q = V \cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Between sections 1 and 2 $Q = V_1 \cdot A_1 = V_1 \cdot \frac{\pi \cdot D_1^{\,2}}{4} = V_2 \cdot A_2 = V_2 \cdot \frac{\pi \cdot D_2^{\,2}}{4}$

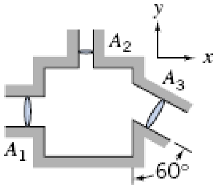
Hence $V_2 = V_1 \cdot \left(\frac{D_1}{D_2}\right)^2$ $\qquad V_2 = 20 \cdot \text{mph} \cdot \left(\frac{5}{3}\right)^2$ $\qquad V_2 = 55.6\,\text{mph}$

Similarly $V_3 = V_1 \cdot \left(\frac{D_1}{D_3}\right)^2$ $\qquad V_3 = 20 \cdot \text{mph} \cdot \left(\frac{5}{2}\right)^2$ $\qquad V_3 = 125\,\text{mph}$

Problem 4.21

[1]

4.21 Fluid with 65 lbm/ft^3 density is flowing steadily through the rectangular box shown. Given $A_1 = 0.5\ \text{ft}^2$, $A_2 = 0.1\ \text{ft}^2$, $A_3 = 0.6\ \text{ft}^2$, $\vec{V}_1 = 10\hat{i}\ \text{ft/s}$, and $\vec{V}_2 = 20\hat{j}\ \text{ft/s}$, determine velocity $\vec{V}_3$.



Given: Data on flow through box

Find: Velocity at station 3

Solution:

Basic equation $$\sum_{CS} \left(\vec{V} \cdot \vec{A}\right) = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Then for the box $$\sum_{CS} \left(\vec{V} \cdot \vec{A}\right) = -V_1 \cdot A_1 + V_2 \cdot A_2 + V_3 \cdot A_3 = 0$$

Note that the vectors indicate that flow is in at location 1 and out at location 2; we assume outflow at location 3

Hence $$V_3 = V_1 \cdot \frac{A_1}{A_3} - V_2 \cdot \frac{A_2}{A_3} \qquad V_3 = 10 \cdot \frac{\text{ft}}{\text{s}} \times \frac{0.5}{0.6} - 20 \cdot \frac{\text{ft}}{\text{s}} \times \frac{0.1}{0.6} \qquad V_3 = 5\frac{\text{ft}}{\text{s}}$$

Based on geometry $$V_x = V_3 \cdot \sin(60 \cdot \text{deg}) \qquad V_x = 4.33\frac{\text{ft}}{\text{s}}$$

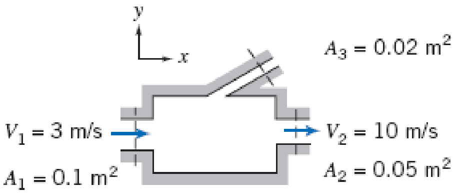
$$V_y = -V_3 \cdot \cos(60 \cdot \text{deg}) \qquad V_y = -2.5\frac{\text{ft}}{\text{s}}$$

$$\vec{V_3} = \left(4.33 \cdot \frac{\text{ft}}{\text{s}}, -2.5 \cdot \frac{\text{ft}}{\text{s}}\right)$$

Problem 4.22

[1]

4.22 Consider steady, incompressible flow through the device shown. Determine the magnitude and direction of the volume flow rate through port 3.



Given: Data on flow through device

Find: Volume flow rate at port 3

Solution:

Basic equation $\displaystyle\sum_{CS} \left(\vec{V}\cdot\vec{A}\right) = 0$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Then for the box $\displaystyle\sum_{CS} \left(\vec{V}\cdot\vec{A}\right) = -V_1\cdot A_1 + V_2\cdot A_2 + V_3\cdot A_3 = -V_1\cdot A_1 + V_2\cdot A_2 + Q_3$

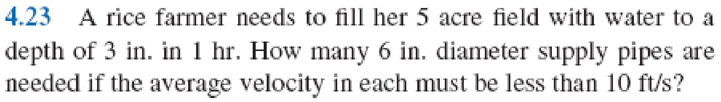
Note we assume outflow at port 3

Hence $Q_3 = V_1\cdot A_1 - V_2\cdot A_2$ $\qquad Q_3 = 3\cdot\frac{m}{s}\times 0.1\cdot m^2 - 10\cdot\frac{m}{s}\times 0.05\cdot m^2$ $\qquad Q_3 = -0.2\cdot\frac{m^3}{s}$

The negative sign indicates the flow at port 3 is inwards. Flow rate at port 3 is 0.2 m^3/s inwards

Problem 4.23

[1]



4.23 A rice farmer needs to fill her 5 acre field with water to a depth of 3 in. in 1 hr. How many 6 in. diameter supply pipes are needed if the average velocity in each must be less than 10 ft/s?

Given: Water needs of farmer

Find: Number of 6 in. pipes needed

Solution:

Basic equation $Q = V \cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Then $Q = n \cdot V \cdot \frac{\pi \cdot D^2}{4}$ where n is the number of pipes, V is the average velocity in the pipes, and D is the pipe diameter

The flow rate is given by $Q = \frac{5 \cdot \text{acre} \cdot 0.25 \cdot \text{ft}}{1 \cdot \text{hr}} = \frac{5 \cdot \text{acre} \cdot 0.25 \cdot \text{ft}}{1 \cdot \text{hr}} \times \frac{43560 \cdot \text{ft}^2}{1 \cdot \text{acre}} \times \frac{1 \cdot \text{hr}}{3600 \cdot \text{s}}$ Data on acres from Googling!

$$Q = 15.1 \cdot \frac{\text{ft}^3}{\text{s}}$$

Hence $n = \frac{4 \cdot Q}{\pi \cdot V \cdot D^2}$ $n = \frac{4}{\pi} \times \frac{\text{s}}{10 \cdot \text{ft}} \times \left(\frac{1}{0.5 \cdot \text{ft}}\right)^2 \times 15.1 \cdot \frac{\text{ft}^3}{\text{s}}$ $n = 7.69$

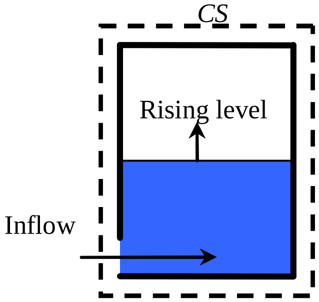
Hence we need at least eight pipes

Problem 4.24

4.24 You are filling your car with gasoline at a rate of 5.3 gals/min. Although you can't see it, the gasoline is rising in the tank at a rate of 4.3 in. per minute. What is the horizontal cross-sectional area of your gas tank? Is this a realistic answer?

Given:
Data on filling of gas tank

Find:
Cross-section area of tank



Solution:

We can treat this as a steady state problem if we choose a CS as the original volume of gas in the tank, so that additional gas "leaves" the gas as the gas level in the tank rises, OR as an unsteady problem if we choose the CS as the entire gas tank. We choose the latter

Basic equation $$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$$

Assumptions: 1) Incompressible flow 2) Uniform flow

Hence $$\frac{\partial}{\partial t}M_{CV} = \rho\cdot A\cdot\frac{dh}{dt} = -\sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \rho\cdot Q$$

where Q is the gas fill rate, A is the tank cross-section area, and h is the rate of rise in the gas tank

Hence $$A = \frac{Q}{\frac{dh}{dt}} \qquad A = 5.3\cdot\frac{gal}{min}\times\frac{1\cdot ft^3}{7.48\cdot gal}\times\frac{1}{4.3}\cdot\frac{min}{in}\times\frac{12\cdot in}{1\cdot ft}$$ Data on gals from Table G.2

$A = 1.98\,ft^2$ $A = 285\,in^2$ This seems like a reasonable area e.g., 1 ft x 2 ft

Problem 4.25

[1]

4.25 For your sink at home, the flow rate in is 5000 units/hr. Accumulation is 2500 units. What is the accumulation rate if the outflow is 60 units/min? Suddenly, the outflow becomes 13 units/min: What is the accumulation rate? At another time, the flow rate in is 5 units/sec. The accumulation is 50 units. The accumulation rate is −4 units/sec. What is the flow rate out?

Given: Data on filling of a sink

Find: Accumulation rate under various circumstances

Solution:

This is an unsteady problem if we choose the CS as the entire sink

Basic equation
$$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$$

Assumptions: 1) Incompressible flow

Hence
$$\frac{\partial}{\partial t}M_{CV} = \text{Accumulationrate} = -\sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \text{Inflow} - \text{Outflow}$$

$$\text{Accumulationrate} = \text{Inflow} - \text{Outflow}$$

For the first case
$$\text{Accumulationrate} = 5000\cdot\frac{\text{units}}{\text{hr}} - 60\cdot\frac{\text{units}}{\text{min}}\times\frac{60\cdot\text{min}}{\text{hr}} \qquad \text{Accumulationrate} = 1400\cdot\frac{\text{units}}{\text{hr}}$$

For the second case
$$\text{Accumulationrate} = 5000\cdot\frac{\text{units}}{\text{hr}} - 13\cdot\frac{\text{units}}{\text{min}}\times\frac{60\cdot\text{min}}{\text{hr}} \qquad \text{Accumulationrate} = 4220\cdot\frac{\text{units}}{\text{hr}}$$

For the third case
$$\text{Outflow} = \text{Inflow} - \text{Accumulationrate}$$

$$\text{Outflow} = 5\cdot\frac{\text{units}}{\text{s}} - (-4)\cdot\frac{\text{units}}{\text{s}} \qquad \text{Outflow} = 9\cdot\frac{\text{units}}{\text{s}}$$

Problem 4.26

4.26 You are trying to pump storm water out of your basement during a storm. The pump can extract 10 gpm. The water level in the basement is now sinking about 1 in./hr. What is the flow rate (gpm) from the storm into the basement? The basement is 25 ft by 20 ft.

Given: Data on filling of a basement during a storm

Find: Flow rate of storm into basement

Solution:

This is an unsteady problem if we choose the CS as the entire basement

Basic equation $$\frac{\partial}{\partial t}M_{CV} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = 0$$

Assumptions: 1) Incompressible flow

Hence $$\frac{\partial}{\partial t}M_{CV} = \rho\cdot A\cdot\frac{dh}{dt} = -\sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \rho\cdot Q_{storm} - \rho\cdot Q_{pump}$$

where A is the basement area and dh/dt is the rate at which the height of water in the basement changes.

or $$Q_{storm} = Q_{pump} - A\cdot\frac{dh}{dt}$$

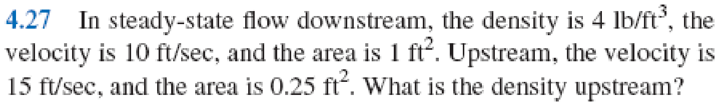
$$Q_{storm} = 10\cdot\frac{gal}{min} - 25\cdot ft \times 20\cdot ft \times \left(-\frac{1}{12}\cdot\frac{ft}{hr}\right) \times \frac{7.48\cdot gal}{ft^3} \times \frac{1\cdot hr}{60\cdot min}$$

Data on gals from Table G.2

$$Q_{storm} = 15.2\,gpm$$

Problem 4.27

[1]



4.27 In steady-state flow downstream, the density is 4 lb/ft^3, the velocity is 10 ft/sec, and the area is 1 ft^2. Upstream, the velocity is 15 ft/sec, and the area is 0.25 ft^2. What is the density upstream?

Given: Data on flow through device

Find: Volume flow rate at port 3

Solution:

Basic equation

$$\sum_{CS} \left(\rho \cdot \vec{V} \cdot \vec{A}\right) = 0$$

Assumptions: 1) Steady flow 2) Uniform flow

Then for the box

$$\sum_{CS} \left(\rho \cdot \vec{V} \cdot \vec{A}\right) = -\rho_u \cdot V_u \cdot A_u + \rho_d \cdot V_d \cdot A_d = 0$$

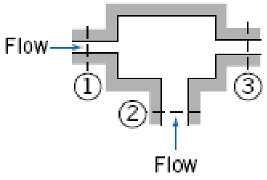
Hence

$$\rho_u = \rho_d \cdot \frac{V_d \cdot A_d}{V_u \cdot A_u} \qquad \rho_u = 4 \cdot \frac{lb}{ft^3} \times \frac{10}{15} \times \frac{1}{0.25} \qquad \rho_u = 10.7 \frac{lb}{ft^3}$$

Problem 4.28

[2]

4.28 In the incompressible flow through the device shown, velocities may be considered uniform over the inlet and outlet sections. The following conditions are known: $A_1 = 0.1\ \text{m}^2$, $A_2 = 0.2\ \text{m}^2$, $A_3 = 0.15\ \text{m}^2$, $V_1 = 10e^{-t/2}$ m/s, and $V_2 = 2\cos(2\pi t)$ m/s (t in seconds). Obtain an expression for the velocity at section ③, and plot V_3 as a function of time. At what instant does V_3 first become zero? What is the total mean volumetric flow at section ③?



Given: Data on flow through device

Find: Velocity V_3; plot V_3 against time; find when V_3 is zero; total mean flow

Solution:

Governing equation: For incompressible flow (Eq. 4.13) and uniform flow

$$\int \vec{V}\,d\vec{A} = \sum \vec{V}\cdot\vec{A} = 0$$

Applying to the device (assuming V_3 is out)

$$-V_1\cdot A_1 - V_2\cdot A_2 + V_3\cdot A_3 = 0$$

$$V_3 = \frac{V_1\cdot A_1 + V_2\cdot A_2}{A_3} = \frac{10\cdot e^{-\frac{t}{2}}\cdot\frac{\text{m}}{\text{s}} \times 0.1\cdot\text{m}^2 + 2\cdot\cos(2\cdot\pi\cdot t)\cdot\frac{\text{m}}{\text{s}} \times 0.2\cdot\text{m}^2}{0.15\cdot\text{m}^2}$$

The velocity at A_3 is

$$V_3 = 6.67\cdot e^{-\frac{t}{2}} + 2.67\cdot\cos(2\cdot\pi\cdot t)$$

The total mean volumetric flow at A_3 is

$$Q = \int_0^\infty V_3\cdot A_3\,dt = \int_0^\infty \left(6.67\cdot e^{-\frac{t}{2}} + 2.67\cdot\cos(2\cdot\pi\cdot t)\right)\cdot 0.15\,dt\cdot\left(\frac{\text{m}}{\text{s}}\cdot\text{m}^2\right)$$

$$Q = \lim_{t\to\infty}\left(-2\cdot e^{-\frac{t}{2}} + \frac{1}{5\cdot\pi}\cdot\sin(2\cdot\pi\cdot t)\right) - (-2) = 2\cdot\text{m}^3$$

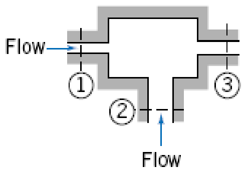
$$Q = 2\cdot\text{m}^3$$

The time at which V_3 first is zero, and the plot of V_3 is shown in the corresponding *Excel* workbook $t = 2.39\cdot\text{s}$

Problem 4.28

[2]

4.28 In the incompressible flow through the device shown, velocities may be considered uniform over the inlet and outlet sections. The following conditions are known: $A_1 = 0.1$ m^2, $A_2 = 0.2$ m^2, $A_3 = 0.15$ m^2, $V_1 = 10e^{-t/2}$ m/s, and $V_2 = 2\cos(2\pi t)$ m/s (t in seconds). Obtain an expression for the velocity at section ③, and plot V_3 as a function of time. At what instant does V_3 first become zero? What is the total mean volumetric flow at section ③?



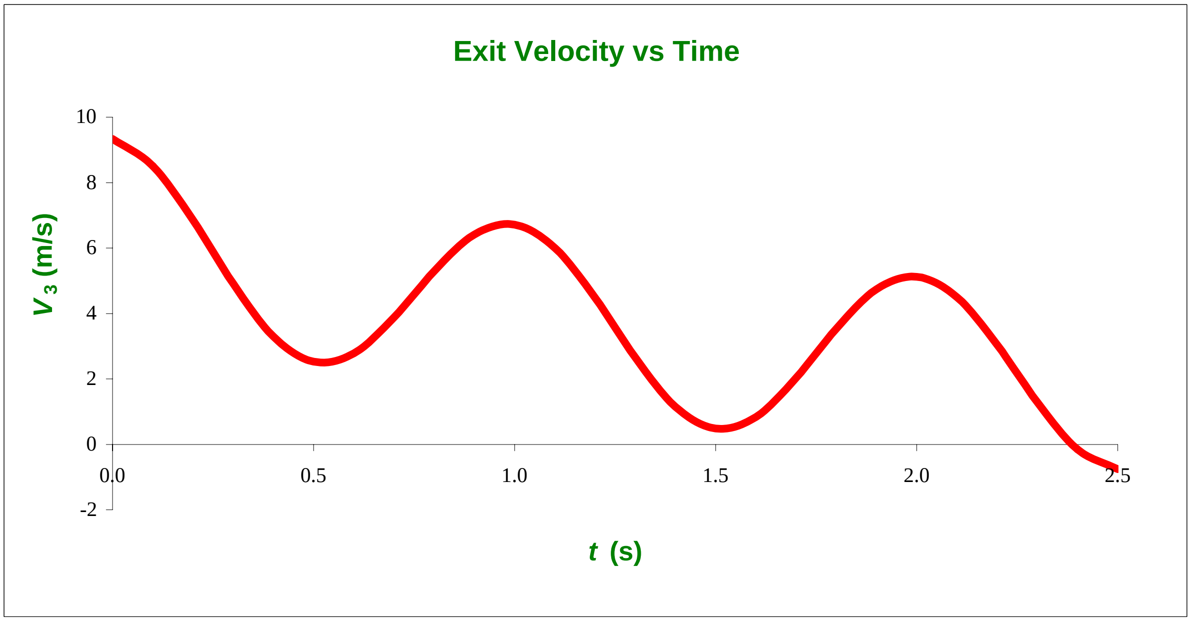
Given: Data on flow through device

Find: Velocity V_3; plot V_3 against time; find when V_3 is zero; total mean flow

Solution:

The velocity at A_3 is $$V_3 = 6.67 \cdot e^{-\frac{t}{2}} + 2.67 \cdot \cos(2 \cdot \pi \cdot t)$$

t (s)	*V*$_3$ (m/s)
0.00	9.33
0.10	8.50
0.20	6.86
0.30	4.91
0.40	3.30
0.50	2.53
0.60	2.78
0.70	3.87
0.80	5.29
0.90	6.41
1.00	6.71
1.10	6.00
1.20	4.48
1.30	2.66
1.40	1.15
1.50	0.48
1.60	0.84
1.70	2.03
1.80	3.53
1.90	4.74
2.00	5.12
2.10	4.49
2.20	3.04
2.30	1.29
2.40	-0.15
2.50	-0.76



The time at which V_3 first becomes zero can be found using *Goal Seek*

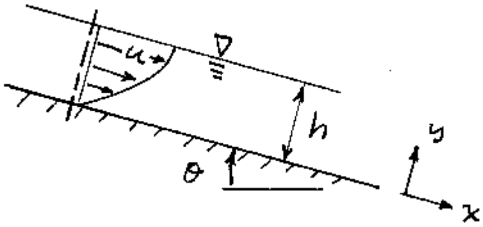
t (s)	*V*$_3$ (m/s)
2.39	0.00

Problem 4.29

[2]

Given: Oil flow down inclined plane.

$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right)$$



Find: Mass flow rate per unit width.

Solution: At the dashed cross-section, $\dot{m} = \int \rho u \, dA$

$dA = w\,dy$, where w = width

$$\dot{m} = \int_0^h \rho \, \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) w \, dy = \frac{\rho^2 g \sin\theta}{\mu}\int_0^h \left(hy - \frac{y^2}{2}\right) w \, dy$$

$$\dot{m} = \frac{\rho^2 g \sin\theta}{\mu} w \left[\frac{hy^2}{2} - \frac{y^3}{6}\right]_0^h = \frac{\rho^2 g \sin\theta \, w}{\mu} \, \frac{h^3}{3} = \frac{\rho^2 g \sin\theta \, w h^3}{3\mu}$$

Thus

$$\dot{m}/w = \frac{\rho^2 g \sin\theta \, h^3}{3\mu} \longleftarrow \dot{m}/w$$

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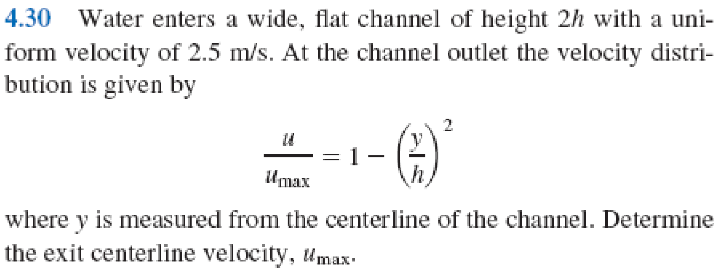
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 4.30

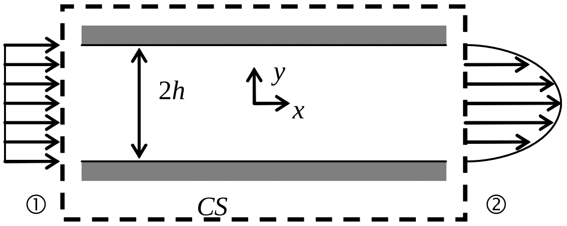
[2]



4.30 Water enters a wide, flat channel of height $2h$ with a uniform velocity of 2.5 m/s. At the channel outlet the velocity distribution is given by

$$\frac{u}{u_{\max}} = 1 - \left(\frac{y}{h}\right)^2$$

where y is measured from the centerline of the channel. Determine the exit centerline velocity, $u_{\max}$.



Given: Data on flow at inlet and outlet of channel

Find: Find u_{max}

Solution:

Basic equation $\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at 1 and 2 $-\rho \cdot U \cdot 2 \cdot h \cdot w + \int_{-h}^{h} \rho \cdot u(y)\, dy = 0$ $\int_{-h}^{h} u_{max} \cdot \left[1 - \left(\frac{y}{h}\right)^2\right] dy = 2 \cdot h \cdot U$

$$u_{max} \cdot \left[[h - (-h)] - \left[\frac{h^3}{3 \cdot h^2} - \left(-\frac{h^3}{3 \cdot h^2}\right)\right]\right] = 2 \cdot h \cdot U \qquad u_{max} \cdot \frac{4}{3} \cdot h = 2 \cdot h \cdot U$$

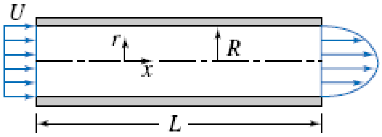
Hence $u_{max} = \frac{3}{2} \cdot U = \frac{3}{2} \times 2.5 \cdot \frac{m}{s}$ $u_{max} = 3.75 \cdot \frac{m}{s}$

Problem 4.31

4.31 Water flows steadily through a pipe of length L and radius $R = 75$ mm. Calculate the uniform inlet velocity, U, if the velocity distribution across the outlet is given by

$$u = u_{\max}\left[1 - \frac{r^2}{R^2}\right]$$

and $u_{\max} = 3$ m/s.



Given: Data on flow at inlet and outlet of pipe

Find: Find U

Solution:

Basic equation $\int_{CS} \rho\vec{V}\cdot d\vec{A} = 0$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at inlet and exit $-\rho\cdot U\cdot\pi\cdot R^2 + \int_0^R \rho\cdot u(r)\cdot 2\cdot\pi\cdot r\,dr = 0$

$$\int_0^R u_{\max}\cdot\left[1 - \left(\frac{r}{R}\right)^2\right]\cdot 2\cdot r\,dr = R^2\cdot U$$

$$u_{\max}\cdot\left(R^2 - \frac{1}{2}\cdot R^2\right) = R^2\cdot U$$

$$U = \frac{1}{2}\cdot u_{\max}$$

Hence $U = \frac{1}{2}\times 3\cdot\frac{m}{s}$ $\qquad U = 1.5\cdot\frac{m}{s}$

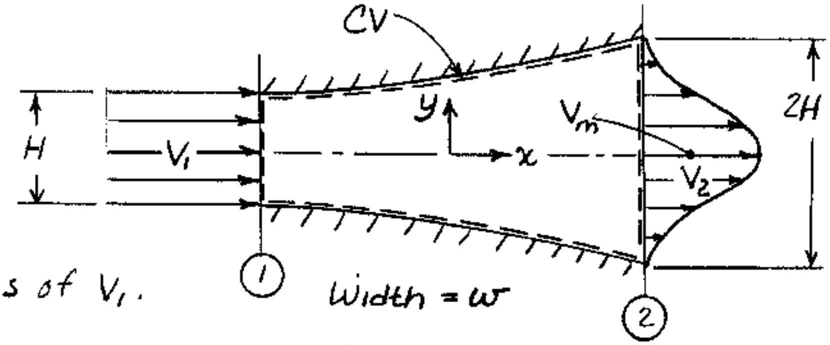
Problem 4.32

[2]

Given: Incompressible flow in a diverging channel, as shown.

$V_1 = \text{constant}$

$V_2 = V_m \cos\left(\frac{\pi y}{2H}\right)$



Find: Express V_m in terms of V_1.

Solution: Apply conservation of mass using the CV shown.

Basic equation: $$0 = \overset{=0\,(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Uniform flow at section 1
(3) Incompressible flow

Then $$0 = \left\{-|\cancel{\rho} V_1 A_1|\right\} + \int_{-H}^{H}\cancel{\rho} V_2\, w\, dy$$

Since $A_1 = wH$, then $$V_1 w H = \int_{-H}^{H} V_m \cos\left(\frac{\pi}{2}\frac{y}{H}\right) w\, dy = 2\int_0^H V_m \cos\left(\frac{\pi}{2}\frac{y}{H}\right) w\, dy$$

So $$V_1 H = 2V_m\left(\frac{2H}{\pi}\right)\int_0^H \cos\left(\frac{\pi}{2}\frac{y}{H}\right) d\left(\frac{\pi}{2}\frac{y}{H}\right) = \frac{4V_m H}{\pi}\left[\sin\left(\frac{\pi}{2}\frac{y}{H}\right)\right]_0^H = \frac{4V_m H}{\pi}$$

Thus $$V_m = \frac{\pi}{4} V_1$$ V_m

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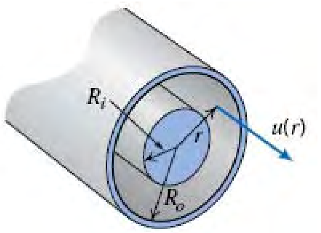
Problem 4.33

[3]

4.33 The velocity profile for laminar flow in an annulus is given by

$$u(r) = -\frac{\Delta p}{4\mu L}\left[R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln(R_i/R_o)}\ln\frac{R_o}{r}\right]$$

where $\Delta p/L = -10$ kPa/m is the pressure gradient, μ is the viscosity (SAE 10 oil at 20°C), and $R_o = 5$ mm and $R_i = 1$ mm are the outer and inner radii. Find the volume flow rate, the average velocity, and the maximum velocity. Plot the velocity distribution.



Given: Velocity distribution in annulus

Find: Volume flow rate; average velocity; maximum velocity; plot velocity distribution

Solution:

Governing equation: For the flow rate (Eq. 4.14a) and average velocity (Eq. 4.14b) $Q = \int \vec{V}\,d\vec{A}$ $\quad V_{av} = \frac{Q}{A}$

The given data is $R_o = 5\cdot mm$ $\quad R_i = 1\cdot mm$ $\quad \frac{\Delta p}{L} = -10\cdot\frac{kPa}{m}$ $\quad \mu = 0.1\cdot\frac{N\cdot s}{m^2}$ (From Fig. A.2)

$$u(r) = \frac{-\Delta p}{4\cdot\mu\cdot L}\cdot\left(R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)}\cdot\ln\left(\frac{R_o}{r}\right)\right)$$

The flow rate is $Q = \int_{R_i}^{R_o} u(r)\cdot 2\cdot\pi\cdot r\,dr$

Considerable mathematical manipulation leads to $Q = \frac{\Delta p\cdot\pi}{8\cdot\mu\cdot L}\cdot\left(R_o^2 - R_i^2\right)\cdot\left[\frac{\left(R_o^2 - R_i^2\right)}{\ln\left(\frac{R_o}{R_i}\right)} - \left(R_i^2 + R_o^2\right)\right]$

Substituting values $Q = \frac{\pi}{8}\cdot\left(-10\cdot 10^3\right)\cdot\frac{N}{m^2\cdot m}\cdot\frac{m^2}{0.1\cdot N\cdot s}\cdot\left(5^2 - 1^2\right)\cdot\left(\frac{m}{1000}\right)^2\cdot\left[\frac{5^2 - 1^2}{\ln\left(\frac{5}{1}\right)} - \left(5^2 + 1^2\right)\right]\cdot\left(\frac{m}{1000}\right)^2$

$Q = 1.045\times 10^{-5}\,\frac{m^3}{s}$ $\quad Q = 10.45\cdot\frac{mL}{s}$

The average velocity is $V_{av} = \frac{Q}{A} = \frac{Q}{\pi\cdot\left(R_o^2 - R_i^2\right)}$ $\quad V_{av} = \frac{1}{\pi}\times 1.045\times 10^{-5}\cdot\frac{m^3}{s}\times\frac{1}{5^2 - 1^2}\cdot\left(\frac{1000}{m}\right)^2$ $\quad V_{av} = 0.139\,\frac{m}{s}$

The maximum velocity occurs when $\frac{du}{dr} = 0 = \frac{d}{dx}\left[\frac{-\Delta p}{4\cdot\mu\cdot L}\cdot\left(R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln\left(\frac{R_i}{R_o}\right)}\cdot\ln\left(\frac{R_o}{r}\right)\right)\right] = -\frac{\Delta p}{4\cdot\mu\cdot L}\cdot\left[-2\cdot r - \frac{\left(R_o^2 - R_i^2\right)}{\ln\left(\frac{R_i}{R_o}\right)\cdot r}\right]$

$r = \sqrt{\frac{R_i^2 - R_o^2}{2\cdot\ln\left(\frac{R_i}{R_o}\right)}}$ $\quad r = 2.73\cdot mm$ $\quad$ Substituting in u(r) $\quad u_{max} = u(2.73\cdot mm) = 0.213\cdot\frac{m}{s}$

The maximum velocity using Solver instead, and the plot, are also shown in the corresponding *Excel* workbook

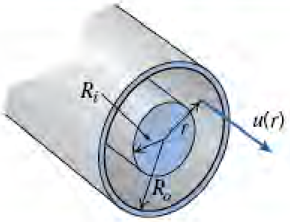
Problem 4.33

[3]

4.33 The velocity profile for laminar flow in an annulus is given by

$$u(r) = -\frac{\Delta p}{4\mu L}\left[R_o^2 - r^2 + \frac{R_o^2 - R_i^2}{\ln(R_i/R_o)}\ln\frac{R_o}{r}\right]$$

where $\Delta p/L = -10$ kPa/m is the pressure gradient, μ is the viscosity (SAE 10 oil at 20°C), and $R_o = 5$ mm and $R_i = 1$ mm are the outer and inner radii. Find the volume flow rate, the average velocity, and the maximum velocity. Plot the velocity distribution.



Given: Velocity distribution in annulus

Find: Volume flow rate; average velocity; maximum velocity; plot velocity distribution

Solution:

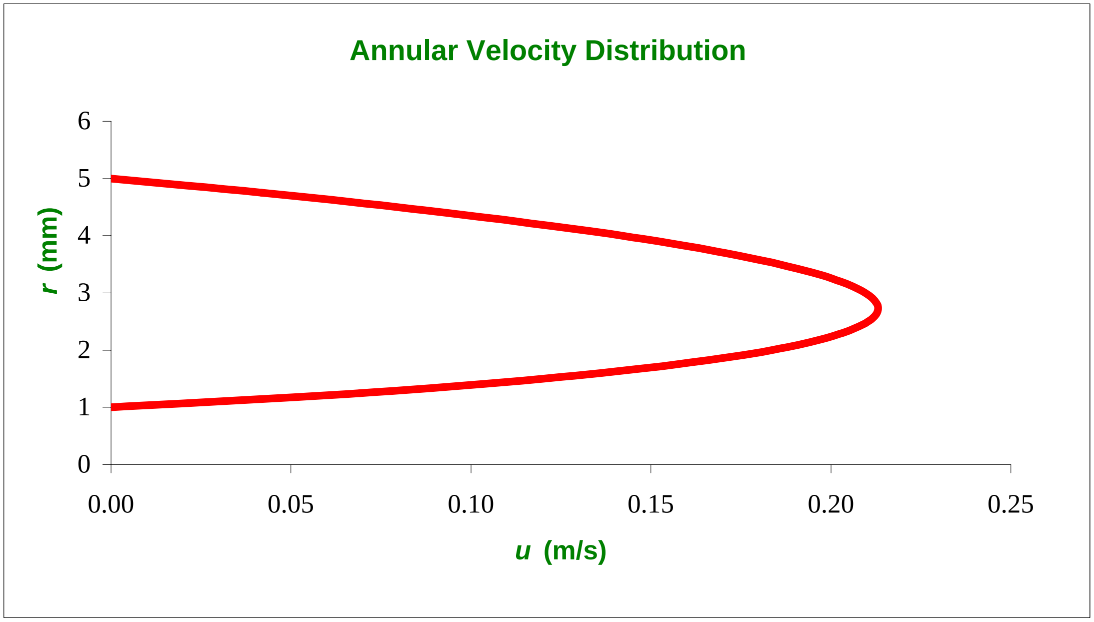
R_o = 5 mm

R_i = 1 mm

$\Delta p/L$ = -10 kPa/m

μ = 0.1 $N.s/m^2$

r (mm)	*u* (m/s)
1.00	0.000
1.25	0.069
1.50	0.120
1.75	0.157
2.00	0.183
2.25	0.201
2.50	0.210
2.75	0.213
3.00	0.210
3.25	0.200
3.50	0.186
3.75	0.166
4.00	0.142
4.25	0.113
4.50	0.079
4.75	0.042
5.00	0.000



The maximum velocity can be found using *Solver*

r (mm)	*u* (m/s)
2.73	0.213

Problem 4.34

[2]

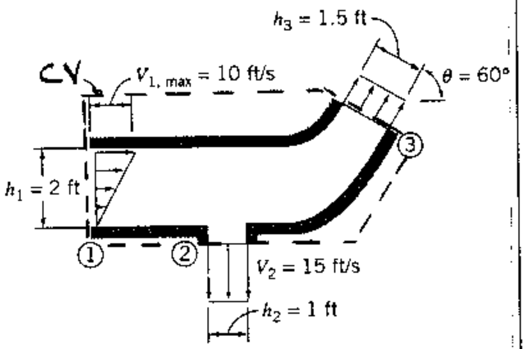
Given: Two-dimensional reducing bend as shown.

Find: Magnitude and direction of uniform velocity at section ③.

Solution: Apply conservation of mass using CV shown.

Basic equation:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$



Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at ② and ③

Then

$$0 = \int_{CS}\vec{V}\cdot d\vec{A} = \int_{A_1}\vec{V}_1\cdot d\vec{A}_1 + \vec{V}_2\cdot\vec{A}_2 + \vec{V}_3\cdot\vec{A}_3$$

or

$$\vec{V}_3\cdot\vec{A}_3 = -\int_{A_1}\vec{V}_1\cdot d\vec{A}_1 - \vec{V}_2\cdot\vec{A}_2 = +\int_0^{h_1} V_{1,max}\frac{y}{h_1}\,w\,dy - V_2 w h_2$$

$$\vec{V}_3\cdot\vec{A}_3 = V_{1,max}\,w\left[\frac{y^2}{2h_1}\right]_0^{h_1} - V_2 w h_2 = \frac{V_{1,max}\,w h_1}{2} - V_2 w h_2$$

so

$$\frac{\vec{V}_3\cdot\vec{A}_3}{w} = \frac{1}{2}\times 10\,\frac{ft}{s}\times 2\,ft - 15\,\frac{ft}{s}\times 1\,ft = -5\,ft^2/s$$

Since $\vec{V}_3\cdot\vec{A}_3 < 0$, flow at ③ is into the CV ← Direction

Thus

$$\frac{\vec{V}_3\cdot\vec{A}_3}{w} = -\frac{V_3 A_3}{w} = -\frac{V_3 w h_3}{w} = -V_3 h_3 = -5\,ft^2/s$$

$$V_3 = \frac{1}{h_3}\times 5\,\frac{ft^2}{s} = \frac{1}{1.5\,ft}\times 5\,\frac{ft^2}{s} = 3.33\,ft/s \quad (\underline{into}\ CV)$$ ← V_3

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Problem 4.35

[2]

Given: Water flow in the two-dimensional square channel shown.

$v_{max} = 2v_{min}$, $U = 7.5$ m/s , $h = 75.5$ mm

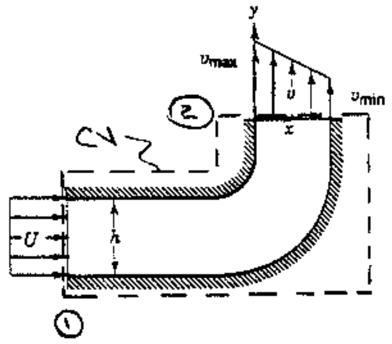
Find: v_{min}

Solution: Apply conservation of mass to the CV shown.

Basic equation:

$$0 = \overset{0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) steady flow
(2) incompressible flow
(3) uniform flow at section ①



Then

$$0 = \vec{V}_1\cdot\vec{A}_1 + \int\vec{V}_2\cdot dA_2$$

$$0 = -Uwh + \int_0^h v\,w\,dx$$

The velocity distribution across the exit at ② is linear

$$v_2 = v_{max} - (v_{max}-v_{min})\frac{x}{h} = 2v_{min} - v_{min}\frac{x}{h} = v_{min}\left(2-\frac{x}{h}\right)$$

$$\therefore Uwh = \int_0^h v_{min}\left(2-\frac{x}{h}\right)w\,dx = v_{min}w\left[2x-\frac{x^2}{2h}\right]_0^h$$

$$U\cancel{w}\cancel{h} = v_{min}w\left[2h-\frac{h}{2}\right] = \frac{3}{2}v_{min}\cancel{w}\cancel{h}$$

$$\therefore v_{min} = \frac{2}{3}U = \frac{2}{3}\times 7.5\frac{m}{s} = 5.0\ m/s \longleftarrow v_{min}$$

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Problem 4.36

[2]

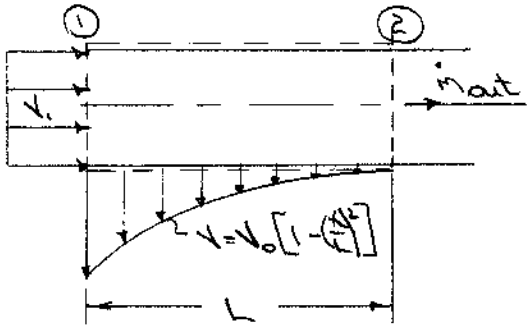
Given: Water flows in a porous round tube of diameter $D = 60\,\text{mm}$. At the pipe inlet the flow is uniform with $V_1 = 7.0\,\text{m/sec}$. Flow out through the porous wall is radial and axisymmetric with velocity distribution

$$v = V_0\left[1-\left(\frac{x}{L}\right)^2\right]$$

where $V_0 = 0.03\,\text{m/s}$ and $L = 0.950\,\text{m}$.

Find: the mass flow rate, $\dot{m}_2$, inside the tube at $x=L$

Solution:



Basic equation: $0 = \cancel{\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall}^{\,0(1)} + \int_{CS} \rho \vec{V}\cdot d\vec{A}$

Assumptions: (1) steady flow
(2) ρ = constant

Then

$$0 = \int_{A_1} \rho\vec{V}\cdot d\vec{A} + \int_{A_2} \rho\vec{V}\cdot d\vec{A} + \int_{A_{surf}} \rho\vec{V}\cdot d\vec{A}$$

$$= -|\rho V_1 A_1| + \dot{m}_2 + \int_0^L \rho\, V_0\left[1-\left(\frac{x}{L}\right)^2\right] 2\pi R\, dx$$

$$\dot{m}_2 = \rho V_1 A_1 - 2\pi R \rho V_0 \int_0^L \left[1-\frac{x^2}{L^2}\right] dx$$

$$= \rho V_1 \frac{\pi D_1^2}{4} - 2\pi R \rho V_0 \left[x - \frac{x^3}{3L^2}\right]_0^L$$

$$= \frac{\pi}{4}\rho V_1 D_1^2 - \frac{4}{3}\pi R \rho V_0 L$$

$$\dot{m}_2 = \frac{\pi}{4} \times 999\,\frac{\text{kg}}{\text{m}^3} \times 7.0\,\frac{\text{m}}{\text{s}} \times (0.06)^2\,\text{m}^2 - \frac{4}{3}\pi \times 0.03\,\text{m} \times 999\,\frac{\text{kg}}{\text{m}^3} \times 0.03\,\frac{\text{m}}{\text{s}} \times 0.95\,\text{m}$$

$$\dot{m}_2 = 19.8\,\frac{\text{kg}}{\text{s}} - 3.6\,\frac{\text{kg}}{\text{s}} = 16.2\,\text{kg/s} \longleftarrow \dot{m}_{out}$$

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Problem 4.37

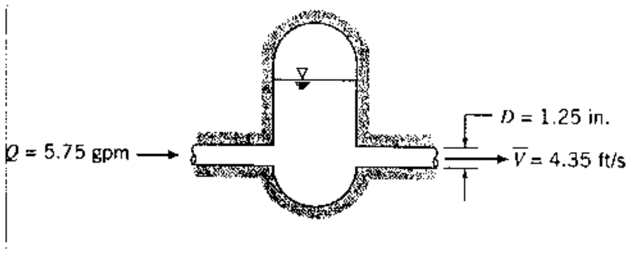
[2]

Given: A hydraulic accumulator, designed to reduce pressure pulsations in a hydraulic system, is operating under conditions shown, at a given instant.

Find: Rate at which accumulator gains or loses hydraulic oil.

Solution:

Use the control volume shown



Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) uniform flow at section ②

(2) ρ = constant

Then,

$$0 = \frac{\partial}{\partial t}(M_{cv}) + \int_{A_1} \{-|\rho V_1 dA_1|\} + \int_{A_2} \{|\rho V_2 dA_2\}$$

But $\int_{A_1} \rho V_1 dA_1 = \rho Q_1$ where Q = volume flowrate and $\rho = SG\, \rho_{H_2O}$

So $0 = \frac{\partial}{\partial t} M_{cv} - \rho Q_1 + \rho V_2 A_2$

or

$$\frac{\partial M_{cv}}{\partial t} = \rho(Q_1 - V_2 A_2)$$

$$= SG\, \rho_{H_2O}\left(Q_1 - V_2 \pi \frac{D_2^2}{4}\right)$$ where S.G = 0.88 (Table A.2)

$$= 0.88 \times 1.94 \frac{\text{slug}}{\text{ft}^3}\left[5.75 \frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{min}}{60\,\text{s}} - 4.35 \frac{\text{ft}}{\text{s}} \times \frac{\pi}{4} \times (1.25)^2 \text{in}^2 \times \frac{\text{ft}^2}{144\,\text{in}^2}\right]$$

$$\frac{\partial M_{cv}}{\partial t} = -4.14 \times 10^{-2} \frac{\text{slug}}{\text{s}} \quad \text{or} \quad -1.33\ \text{lbm/s} \longleftarrow \frac{\partial M_{cv}}{\partial t}$$

(mass is decreasing in the c$\forall$)

Since $M_{cv} = \rho_{oil} \forall_{oil}$

$$\frac{\partial M_{cv}}{\partial t} = \frac{\partial}{\partial t}(\rho_{oil} \forall_{oil}) = \rho_{oil} \frac{\partial \forall_{oil}}{\partial t} = SG_{oil}\, \rho_{H_2O} \frac{\partial \forall_{oil}}{\partial t}$$

$$\frac{\partial \forall_{oil}}{\partial t} = \frac{1}{SG_{oil}\, \rho_{H_2O}} \frac{\partial M_{cv}}{\partial t} = \frac{1}{0.88} \times \frac{1}{1.94} \frac{\text{ft}^3}{\text{slugs}} \times (-4.14) \times 10^{-2} \frac{\text{slug}}{\text{s}}$$

$$\frac{\partial \forall_{oil}}{\partial t} = -2.43 \times 10^{-2} \frac{\text{ft}^3}{\text{s}} \quad \text{or} \quad 0.181\ \text{gal/s} \longleftarrow \frac{\partial \forall_{oil}}{\partial t}$$

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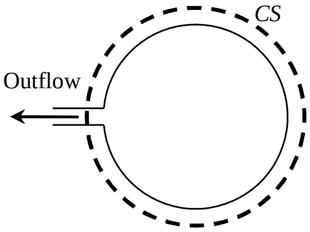
Problem 4.38

[2]

4.38 A tank of 0.4 m^3 volume contains compressed air. A valve is opened and air escapes with a velocity of 250 m/s through an opening of 100 mm^2 area. Air temperature passing through the opening is −20°C and the absolute pressure is 300 kPa. Find the rate of change of density of the air in the tank at this moment.



Given: Data on airflow out of tank

Find: Find rate of change of density of air in tank

Solution:

Basic equation

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$$

Assumptions: 1) Density in tank is uniform 2) Uniform flow 3) Air is an ideal gas

Hence

$$V_{tank}\cdot\frac{d\rho_{tank}}{dt} + \rho_{exit}\cdot V\cdot A = 0 \qquad \frac{d\rho_{tank}}{dt} = -\frac{\rho_{exit}\cdot V\cdot A}{V_{tank}} = -\frac{p_{exit}\cdot V\cdot A}{R_{air}\cdot T_{exit}\cdot V_{tank}}$$

$$\frac{d\rho_{tank}}{dt} = -300\times10^{3}\cdot\frac{N}{m^{2}}\times250\cdot\frac{m}{s}\times100\cdot mm^{2}\times\left(\frac{1\cdot m}{1000\cdot mm}\right)^{2}\times\frac{1}{286.9}\cdot\frac{kg\cdot K}{N\cdot m}\times\frac{1}{(-20+273)\cdot K}\times\frac{1}{0.4\cdot m^{3}}$$

Hence

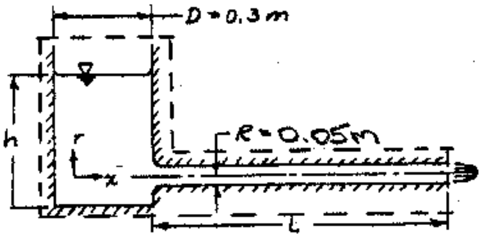
$$\frac{d\rho_{tank}}{dt} = -0.258\cdot\frac{\frac{kg}{m^{3}}}{s}$$

The mass in the tank is decreasing, as expected

Problem 4.39

Given: Liquid drains from a tank through a long circular tube. Flow is laminar; velocity profile at tube discharge is given by

$$u = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]$$



Find: (a) Show that $\bar{V} = 0.5\, u_{max}$ at any instant
(b) rate of change of liquid level in tank when $u_{max} = 0.155$ m/s

Solution:

(a) The average velocity $\bar{V}$ is defined as Q/A.

Since $Q = \int u\,dA$, $dA = 2\pi r\,dr$ and $A = \pi R^2$, then

$$\bar{V} = \frac{Q}{A} = \frac{1}{\pi R^2}\int_0^R u_{max}\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r\,dr = \frac{2u_{max}}{R^2}\int_0^R\left[1-\left(\frac{r}{R}\right)^2\right]r\,dr$$

$$\bar{V} = \frac{2u_{max}}{R^2}R^2\int_0^1\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) = 2u_{max}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2-\frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^1$$

$$\bar{V} = \frac{1}{2}u_{max} \longleftarrow \quad \bar{V}$$

(b) Apply conservation of mass to the CV shown

Basic equation: $$0 = \frac{\partial}{\partial t}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) neglect air entering the CV
(2) incompressible flow

Then

$$0 = \rho_\ell\frac{\partial}{\partial t}\forall_{CV} + \left\{\left|\rho_\ell\bar{V}A_e\right|\right\} = \rho_\ell\frac{\partial}{\partial t}\left[\frac{\pi D^2}{4}h + L\pi R^2\right] + \rho\bar{V}\pi R^2$$

$$0 = \frac{\pi D^2}{4}\frac{dh}{dt} + \bar{V}\pi R^2 \qquad \left(\text{note } \frac{dL}{dt} = 0\right)$$

$$\therefore \frac{dh}{dt} = -4\bar{V}\left(\frac{R}{D}\right)^2$$

But $\bar{V} = \frac{1}{2}u_{max}$ and hence

$$\frac{dh}{dt} = -2u_{max}\left(\frac{R}{D}\right)^2 = -2\times 0.155\frac{m}{s}\times\left(\frac{0.05\,m}{0.30\,m}\right)^2\times 1000\frac{mm}{m}$$

$$\frac{dh}{dt} = -8.61 \text{ mm/s} \quad \text{(level is falling)} \longleftarrow \quad \frac{dh}{dt}$$

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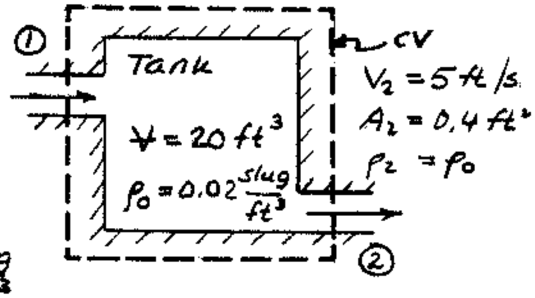
Problem 4.40

[2]

Given: Air flow through tank with conditions shown at time, t_0.



$V_1 = 15\ ft/s$
$A_1 = 0.2\ ft^2$
$\rho_1 = 0.03\ \frac{slug}{ft^3}$

Find: $\frac{\partial \rho}{\partial t}$ in tank at time, t_0.

Solution: Apply conservation of mass, using CV shown.

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) Density is uniform in tank, so $\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall = \frac{\partial}{\partial t}(\rho_0 \forall)$

(2) Flow is uniform at inlet and outlet sections.

Then

$$0 = \frac{\partial}{\partial t}(\rho_0 \forall) + \rho_1 \vec{V}_1 \cdot \vec{A}_1 + \rho_0 \vec{V}_2 \cdot \vec{A}_2$$

$$0 = \rho_0 \overset{=0}{\cancel{\frac{\partial \forall}{\partial t}}} + \forall \frac{\partial \rho_0}{\partial t} - |\rho_1 V_1 A_1| + |\rho_0 V_2 A_2|$$

or

$$\frac{\partial \rho_0}{\partial t} = \frac{|\rho_1 V_1 A_1| - |\rho_0 V_2 A_2|}{\forall}$$

Substituting magnitudes

$$\frac{\partial \rho_0}{\partial t} = \frac{1}{20\ ft^3}\left[\frac{0.03\ slug}{ft^3} \times \frac{15\ ft}{s} \times 0.2\ ft^2 - \frac{0.02\ slug}{ft^3} \times \frac{5\ ft}{s} \times 0.4\ ft^2\right]$$

$$\frac{\partial \rho_0}{\partial t} = 2.50 \times 10^{-3}\ slug/(ft^3 \cdot s)$$

$\frac{\partial \rho_0}{\partial t}$

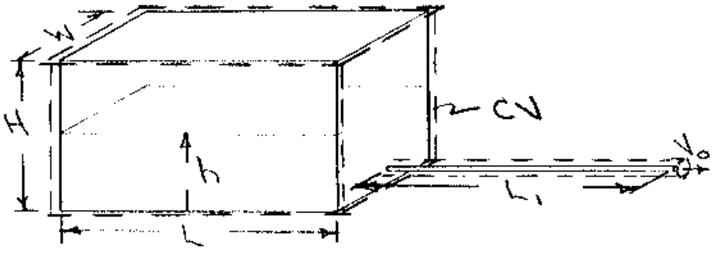
{Note since $\frac{\partial \rho_0}{\partial t} > 0$, mass in tank increases.}

Problem 4.41

[2]

Given: Rectangular tank with dimensions $H = 230\,mm$, $W = 150\,mm$, $L = 230\,mm$, supplies water to an outlet tube of diameter, $D = 6.35\,mm$. When the tank is half full the flow in the tube is at Reynolds number $Re = 2000$. At this instant there is no water flow into the tank.

Find: the rate of change of water level in the tank at this instant.



Solution:

Apply conservation of mass to CV which includes tank and tube.

Basic equation: $$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

Definition: $Re = \frac{\rho D \bar{V}}{\mu} = \frac{D\bar{V}}{\nu}$

Assumptions: (1) uniform flow at exit of tube
(2) incompressible flow
(3) neglect air entering the control volume

Then,

$$0 = \frac{\partial}{\partial t}\left[\cancel{\rho} W L h + \cancel{\rho}\,\pi \frac{D^2}{4} L_1\right] + \left\{+\left|\cancel{\rho} \bar{V}_0 \frac{\pi D^2}{4}\right|\right\}$$

$$0 = WL\frac{dh}{dt} + \bar{V}_0 \pi \frac{D^2}{4} \qquad (\text{note } L_1 = \text{constant})$$

$$\therefore \quad \frac{dh}{dt} = -\bar{V}_0 \frac{\pi D^2}{4\,WL}$$

To find $\bar{V}$ use the definition of Re

$$\bar{V}_0 = \frac{Re\,\nu}{D}$$

For water at 20C $\nu = 1\times10^{-6}\ m^2/sec$ (Table A.8)

$$V_0 = 2000 \times 1\times10^{-6}\,\frac{m^2}{sec} \times \frac{1}{6.35\times10^{-3}\,m} = 0.315\ m/sec$$

$$\frac{dh}{dt} = -\bar{V}_0 \frac{\pi D^2}{4WL} = -\frac{0.315}{4}\,\frac{m}{sec} \times \frac{\pi (6.35)^2\,mm^2}{150\,mm \times 230\,mm} \times 10^3\,\frac{mm}{m}$$

$$\frac{dh}{dt} = -0.289\ mm/sec \quad (\text{falling})$$

$\frac{dh}{dt}$

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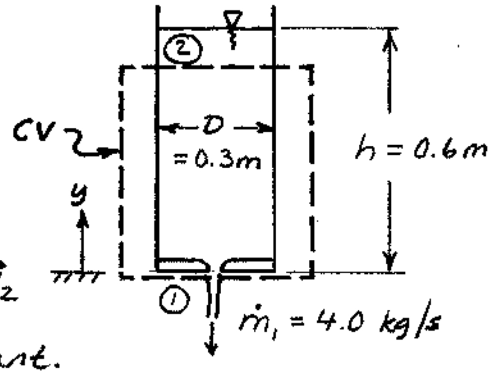
Problem 4.42

[2]

Given: Circular tank, with $D = 1$ ft draining through a hole in its bottom. Fluid is water

Find: Rate of change of water level at the instant shown.



Solution: Apply conservation of mass to CV shown. Note section ② cuts below free surface, so $\vec{V}_2$ corresponds to free surface velocity; volume of CV is constant.

$$\text{Basic equation:} \quad 0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}\int_{CV} \rho\, d\forall} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Incompressible flow, so unsteady term is zero, since volume of CV is fixed
(2) Uniform flow at each section

Then

$$0 = \rho \vec{V}_1 \cdot \vec{A}_1 + \rho \vec{V}_2 \cdot \vec{A}_2 = \dot{m}_1 + \rho \vec{V}_2 \cdot \vec{A}_2$$

and

$$\vec{V}_2 \cdot \vec{A}_2 = -\frac{\dot{m}_1}{\rho} = -4.0\,\frac{\text{kg}}{\text{s}} \times \frac{\text{m}^3}{999\text{ kg}} = -0.004\ \text{m}^3/\text{s}$$

Since $\vec{V}_2 \cdot \vec{A}_2 < 0$, flow at section ② is into CV. Therefore

$$V_2 = \frac{|V_2 A_2|}{A_2} = 0.004\,\frac{\text{m}^3}{\text{s}} \times \frac{4}{\pi} \times \frac{1}{(0.3)^2\text{m}^2} = 0.0566\ \text{m/s}$$

The water level is falling at 56.6 mm/s.

$$\vec{V}_s = -V_2\,\hat{j} = -56.6\,\hat{j}\ \text{mm/s}$$

$\vec{V}_s$

Problem 4.43

Given: Lake being drained at 2,000 cubic feet per second (cfs). Level falls at 1 ft per 8 hr. Normal flow rate is 290 cfs.

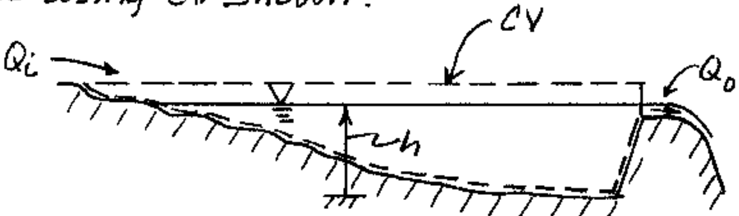
Find: (a) Actual flow rate during draining (gal/s).
(b) Estimate surface area of lake.

Solution: Convert units

$$Q = 2000\,\frac{ft^3}{s} = 2000\,\frac{ft^3}{s} \times 7.48\,\frac{gal}{ft^3} = 1.50 \times 10^4\ gal/s$$

Q

Apply conservation of mass using CV shown:



Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumption: (1) ρ = constant

Then $\frac{d\forall}{dt} = A\frac{dh}{dt} = -\int_{CS} \vec{V} \cdot d\vec{A} = -Q_0 + Q_i$

$$A = -\frac{Q_0 - Q_i}{dh/dt} = -\frac{\Delta Q}{dh/dt}\ ;\ \Delta Q = Q_0 - Q_i$$

But $\Delta Q = 1{,}710\ ft^3/s$ and $dh/dt = -1\ ft/8\ hr$, since decreasing.

Thus

$$A = -1{,}710\,\frac{ft^3}{s} \times \frac{8\,hr}{-1\,ft} \times 3600\,\frac{s}{hr} = 4.92 \times 10^7\ ft^2$$

A

Since 1 acre = 43,600 ft^2,

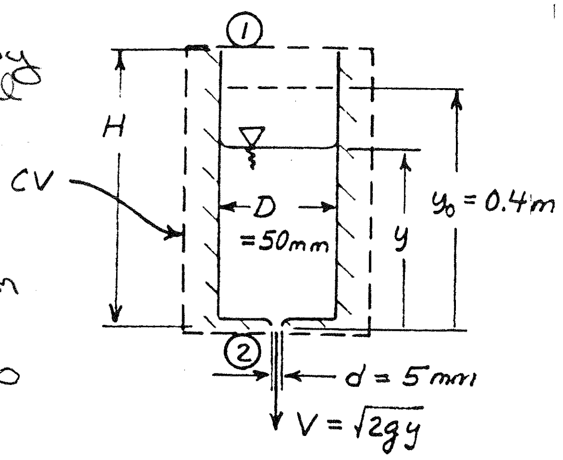
$$A = 4.92 \times 10^7\ ft^2 \times \frac{acre}{43{,}600\ ft^2} \approx 1{,}130\ acres$$

Since 1 square mile = 640 acres, the lake surface area is slightly less than 2 square miles!

Problem 4.44

[3] Part 1/2

Given: Cylindrical tank, draining by gravity as shown; initial depth is y_0



Find: Water depth at t = 12 s

Plot: (a) y/y_0 vs t for $0.1 \le y_0 \le 1$m and $D/d = 10$

(b) y/y_0 vs t for $2 \le D/d \le 10$ and $y_0 = 0.4$m

Solution:

Apply conservation of mass using CV shown.

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) incompressible flow
(2) uniform flow at each section
(3) neglect ρ_{air} compared to ρ_{H_2O}

For the CV, $d\forall = A_t\, dy$, so

$$0 = \frac{\partial}{\partial t}\int_0^y \rho_{H_2O} A_t\, dy + \cancelto{\approx 0 (3)}{\frac{\partial}{\partial t}\int_y^H \rho_{air} A_t\, dy} + \cancelto{\approx 0 (3)}{\{-|\rho_{air} V_1 A_t|\}} + \{|\rho_{H_2O} V_2 A_2|\}$$

or

$$0 = \rho A_t \frac{dy}{dt} + \rho A_2 V_2 = A_t \frac{dy}{dt} + A_2 \sqrt{2gy}$$

Separating variables, $\frac{dy}{y^{1/2}} = -\sqrt{2g}\,\frac{A_2}{A_t}\, dt$

Integrating from y_0 at $t = 0$ to y at t

$$\int_{y_0}^{y} y^{-1/2}\, dy = 2\left[y^{1/2} - y_0^{1/2}\right] = -\sqrt{2g}\,\frac{A_2}{A_t}\, t$$

$$\frac{y^{1/2}}{y_0^{1/2}} = 1 - \sqrt{\frac{g}{2y_0}}\,\frac{A_2}{A_t}\, t \quad \text{or} \quad y = y_0\left[1 - \sqrt{\frac{g}{2y_0}}\left(\frac{d}{D}\right)^2 t\right]^2 \text{ --- } (1)$$

At t = 12 sec

$$y = 0.4\text{m}\left[1 - \left(\frac{9.81}{2}\frac{\text{m}}{\text{s}^2} \times \frac{1}{0.4\text{m}}\right)^{1/2}\left(\frac{5\text{mm}}{50\text{mm}}\right)^2 12\text{s}\right]^2 = 0.134\text{m} \longleftarrow y_{t=12s}$$

For D/d = 10, Eq. 1 gives

$$\frac{y}{y_0} = \left[1 - 2.215 \times 10^{-2}\, y_0^{-1/2}\, t\right]^2$$

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Problem 4.44

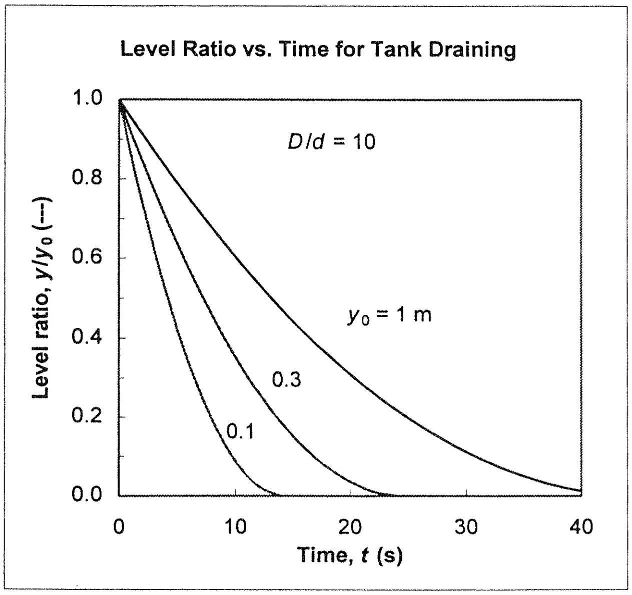
For $y_0 = 0.4\,m$, Eq. 1 gives

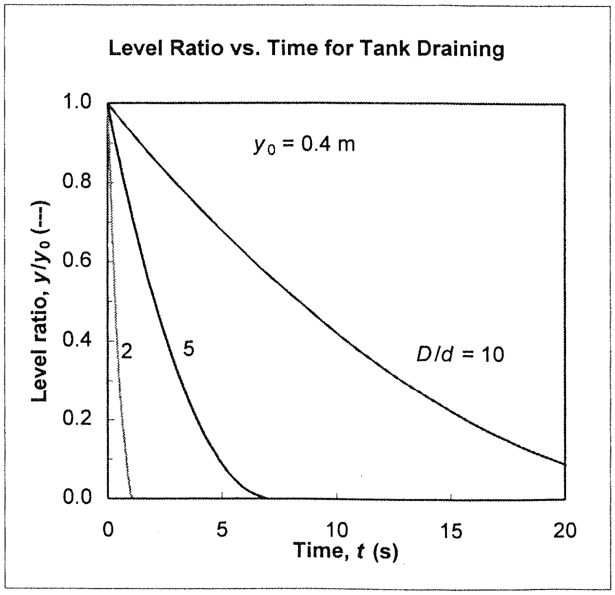
$$\frac{y}{y_0} = \left[1 - \frac{3.502}{(D/d)^2}\,t\right]^2$$

The variation of y/y_0 with t is plotted below for:

- $D/d = 10$ and $0.1 \leq y_0 \leq 1.0\,m$
- $y_0 = 0.4\,m$ and $2 \leq D/d \leq 10$

y_0 (m) =	0.1	0.3	1	D/d (---) =	2	5	10
Time, t (s)	**y/y_0 (---)**	**y/y_0 (---)**	**y/y_0 (---)**	**Time, t (s)**	**y/y_0 (---)**	**y/y_0 (---)**	**y/y_0 (---)**
0	1.000	1.000	1.000	0	1.000	1.000	1.000
2	0.739	0.845	0.913	0.5	0.316	0.865	0.965
4	0.518	0.703	0.831	1	0.016	0.739	0.931
6	0.336	0.574	0.752	1.1	0.001	0.716	0.924
8	0.193	0.458	0.677	2		0.518	0.865
10	0.090	0.355	0.606	3		0.336	0.801
12	0.025	0.265	0.539	4		0.193	0.739
14	0.000	0.188	0.476	5		0.090	0.680
16		0.125	0.417	6		0.025	0.624
18		0.074	0.362	7		0.000	0.570
20		0.037	0.310	10			0.422
22		0.012	0.263	12			0.336
24		0.001	0.219	14			0.260
26			0.180	16			0.193
28			0.144	18			0.137
30			0.113	20			0.090
32			0.085	22			0.053
34			0.061	24			0.025
36			0.041	26			0.008
38			0.025	28			0.000
40			0.013				
45			0.000				

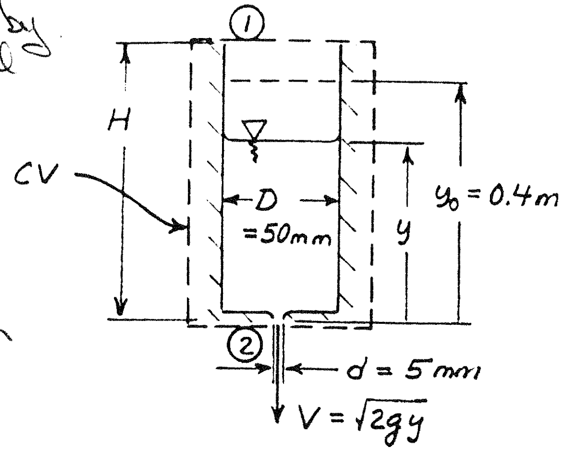




Given: Cylindrical tank, draining by gravity as shown; initial depth is y_0.

Find: Time to drain tank to depth $y = 20$ mm

Plot: Time t to drain the tank (to $y = 20$ mm) as a function of y_0 for $0.1 \le y_0 \le 1$ m with d/D as a parameter for $0.1 \le d/D \le 0.5$.



Solution:

Apply conservation of mass using CV shown.

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) incompressible flow
(2) uniform flow at each section.
(3) neglect ρ_{air} compared to ρ_{H_2O}

For the CV, $d\forall = A_t \, dy$, so

$$0 = \frac{\partial}{\partial t}\int_0^y \rho_{H_2O} A_t \, dy + \cancelto{\approx 0(3)}{\frac{\partial}{\partial t}\int_y^H \rho_{air} A_t \, dy} + \cancelto{\approx 0(3)}{\{-|\rho_{air} V_1 A_t|\}} + \{|\rho_{H_2O} V_2 A_2|\}$$

or $$0 = \frac{\partial}{\partial t}\int_0^y \rho_{H_2O} A_t \, dy + \rho_{H_2O} V_2 A_2 = A_t \frac{dy}{dt} + A_2 \sqrt{2gy}$$

Separating variables, $$\frac{dy}{y^{1/2}} = -\sqrt{2g}\,\frac{A_2}{A_t}\,dt$$

Integrating from y_0 at $t=0$ to y at t

$$\int_{y_0}^{y} \frac{dy}{y^{1/2}} = 2\left[y^{1/2} - y_0^{1/2}\right] = -\sqrt{2g}\,\frac{A_2}{A_t}\,t$$

$$-\sqrt{2g}\,\frac{A_2}{A_t}\,t = 2y_0^{1/2}\left[\left(\frac{y}{y_0}\right)^{1/2} - 1\right] \quad \text{or} \quad t = \sqrt{\frac{2y_0}{g}}\left(\frac{D}{d}\right)^2\left[1 - \left(\frac{y}{y_0}\right)^{1/2}\right] \text{—(1)}$$

Evaluating at $y = 20$ mm

$$t = \left[2 \times 0.4\,m \times \frac{s^2}{9.81\,m}\right]^{1/2}\left[\frac{50\,mm}{5\,mm}\right]^2\left[1 - \left(\frac{0.02\,m}{0.40\,m}\right)^{1/2}\right] = 22.2\,s \longleftarrow t_{y=20mm}$$

Time t is plotted as a function of y/y_0 ($y = 20$ mm) with d/D as a parameter.

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Problem 4.45

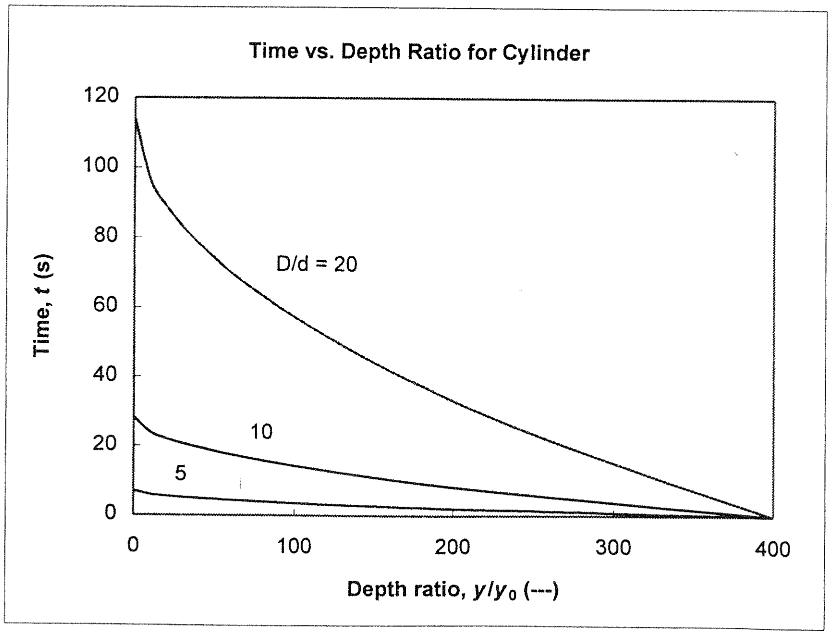
Draining of a cylindrical liquid tank:

Input Data:

Initial height:	y_0	0.4	m		
Diameter ratio:	D/d	20	10	5	---

Calculated Results:

Level, y (mm)	D/d =	Time, t (s) 20	10	5
400		0	0	0
380		2.89	0.723	0.181
360		5.86	1.47	0.366
340		8.91	2.23	0.557
320		12.1	3.01	0.754
300		15.3	3.83	0.96
280		18.7	4.66	1.17
260		22.1	5.53	1.38
240		25.7	6.44	1.61
220		29.5	7.38	1.84
200		33.5	8.36	2.09
180		37.6	9.40	2.35
160		42.0	10.5	2.62
140		46.6	11.7	2.92
120		51.7	12.9	3.23
100		57.1	14.3	3.57
80		63.1	15.8	3.95
60		70.0	17.5	4.37
40		78.1	19.5	4.88
30		82.9	20.7	5.18
20		88.7	22.2	5.54
10		96.2	24.0	6.01
0		114	28.6	7.14



Problem 4.46

[3]

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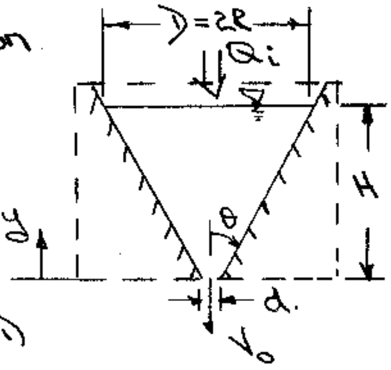
Given: Water flows into the top of a conical flask at a constant rate of $Q = 3.75 \times 10^{-7}\ m^3/hr$. Water drains out through the round opening of diameter $d = 7.35$ mm at the apex of the cone; the flow speed at the exit is $V = (2gy)^{1/2}$ where y is the water depth above the exit plane. At the instant of interest, the water depth $H = 36.8$ mm and the corresponding diameter $D = 29.4$ mm

Find: At the instant of interest:
(a) find the volume flow rate from the bottom of the flask
(b) evaluate the direction and rate of change of water surface level

Solution: Apply continuity to the CV shown.



Basic eq.: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) uniform flow at each section
(2) neglect mass of air.

Then

$$0 = \rho \left.\frac{d\forall}{dt}\right)_{water} + \rho Q_{out} - \rho Q_{in} \quad \ldots\ldots(1)$$

$$Q_{out} = V_o A_o = (2gH)^{1/2}\,\frac{\pi d^2}{4}$$

$$Q_{out} = \left[2 \times 9.81 \frac{m}{s^2} \times 0.0368\,m\right]^{1/2} \frac{\pi}{4} \times (0.00735)^2\,m^2$$

$$Q_{out} = 3.61 \times 10^{-5}\ m^3/s \quad (0.130\ m^3/hr) \longleftarrow$$

Q_{out}

From eq.(1)

$$\left.\frac{d\forall}{dt}\right)_{water} = Q_{in} - Q_{out}$$

$\forall = \frac{1}{3}$ area of base × altitude $= \frac{1}{3}\pi R^2 y$

Since $R = y\tan\theta$, $\forall = \frac{1}{3}\pi y^3 \tan^2\theta$

$$\frac{d\forall}{dt} = \frac{1}{3}\pi \tan^2\theta \times 3y^2 \frac{dy}{dt} = \pi y^2 \tan^2\theta \frac{dy}{dt} = \pi R^2 \frac{dy}{dt}$$

$$\therefore \frac{dy}{dt} = \frac{Q_{in} - Q_{out}}{\pi R^2} = \frac{4}{\pi D^2}(Q_{in} - Q_{out})$$

$$= \frac{4}{\pi} \times (0.0294)^2\,m^2 \left(3.75 \times 10^{-7} - 0.130\right)\frac{m^3}{hr} \times \frac{hr}{3600\,s}$$

$$\frac{dy}{dt} = -0.0532\ m/s \quad \text{(surface moves downward)} \longleftarrow$$

$\frac{dy}{dt}$

Problem 4.47

[3]

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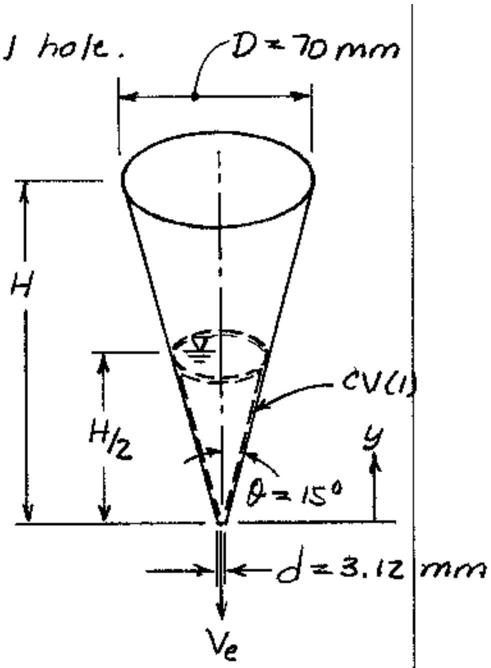
Given: Conical funnel draining through small hole.

$$V_e = \sqrt{2gy}$$



Find: Rate of change of surface level when $y = H/2$.

Solution: Apply conservation of mass.

(1) Choose CV with top just below surface level.

Basic equation: $0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

Assumptions: (1) ρ = constant, $\forall$ = const, so $\partial/\partial t = 0$
(2) Uniform flow at each section.

For CV(1): $0 = \{\underset{in}{-}|\cancel{\rho}V_sA_s|\} + \{\underset{out}{+}|\cancel{\rho}V_eA_e|\}$ or $V_s = V_e\frac{A_e}{A_s}$

Thus

$$V_s = V_e\left(\frac{d}{D/2}\right)^2 = \sqrt{2g\frac{H}{2}}\;4\left(\frac{d}{D}\right)^2 = 4\sqrt{gH}\left(\frac{d}{D}\right)^2 = -\frac{dy}{dt} \text{ (since } y \text{ decreases)}$$

But $\tan\theta = \frac{D/2}{H}$ so $H = \frac{D}{2\tan\theta} = \frac{0.070\text{ m}}{2\tan 15°} = 0.131\text{ m}$

Substituting,

$$\frac{dy}{dt} = -4\sqrt{9.81\frac{m}{s^2}\times 0.131\,m}\left(\frac{0.00312\,m}{0.070\,m}\right)^2 1000\frac{mm}{m} = -9.01\text{ mm/s}$$

$\frac{dy}{dt}$

Alternate solution: Choose CV(2) enclosing entire funnel.

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

Assumptions: (1) ρ = constant, but $\forall$ changes (Note: $\forall = \frac{\pi}{3}r^2h$ for a cone.)
(2) Neglect air
(3) Uniform flow at outlet section

Then

$$0 = \cancel{\rho}\frac{\partial}{\partial t}\forall_{H_2O} + \{\underset{out}{+}|\cancel{\rho}V_eA_e|\} \quad\text{or}\quad \frac{d\forall}{dt} = -V_eA_e$$

The volume of water is $\forall = \frac{\pi}{3}r^2h = \frac{\pi}{3}(y\tan\theta)^2y = \frac{\pi y^3\tan^2\theta}{3}$

so $\frac{d\forall}{dt} = \pi y^2\tan^2\theta\frac{dy}{dt} = \pi\left(\frac{D}{4}\right)^2\frac{dy}{dt}$ and $\frac{\pi D^2}{16}\frac{dy}{dt} = -V_eA_e = -\sqrt{2gy}\,\frac{\pi d^2}{4}$

Finally, since $y = H/2$, $\frac{dy}{dt} = -4\sqrt{2gH}\left(\frac{d}{D}\right)^2$ as before.

{Note: Flow is not steady in either CV. The $\partial/\partial t$ term vanishes for CV(1) because there is no change in mass inside the CV.}

Problem 4.48

[3]

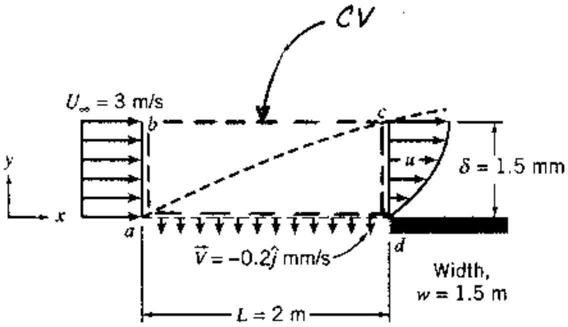
Given: Steady flow of water past a porous flat plate. Suction is constant. Velocity profile at section cd is

$$\frac{u}{U_\infty} = 3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}$$

Find: Mass flow rate across section bc.

Solution: Apply conservation of mass using the CV shown.

Basic equation:



$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) $\vec{V} = -v_0 \hat{j}$ along da

Then

$$0 = \int_{CS} \rho \vec{V} \cdot d\vec{A} = \int_{ab} \rho \vec{V} \cdot d\vec{A} + \dot{m}_{bc} + \int_{cd} \rho \vec{V} \cdot d\vec{A} + \int_{da} \rho \vec{V} \cdot d\vec{A}$$

or

$$0 = -\rho U_\infty w \delta + \dot{m}_{bc} + \int_0^\delta \rho U_\infty \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] w \, dy + \rho v_0 w L$$

Thus

$$\dot{m}_{bc} = \rho U_\infty w \delta - \rho U_\infty w \delta \int_0^1 \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] d\left(\frac{y}{\delta}\right) - \rho v_0 w L$$

$$= \rho w \left\{ U_\infty \delta - U_\infty \delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right)^2 - \frac{2}{2.5}\left(\frac{y}{\delta}\right)^{1.5}\right]_0^1 - v_0 L \right\}$$

$$= \rho w \left[U_\infty \delta - U_\infty \delta \left(\frac{3}{2} - \frac{2}{2.5}\right) - v_0 L \right] = \rho w \left(0.3 U_\infty \delta - v_0 L\right)$$

$$= 999 \frac{kg}{m^3} \times 1.5\,m \left(0.3 \times 3 \frac{m}{s} \times 0.0015\,m - 0.0002 \frac{m}{s} \times 2\,m\right)$$

$$\dot{m}_{bc} = 1.42 \text{ kg/s} \quad (\dot{m} > 0, \text{ so } \underline{\text{out}} \text{ of CV})$$

$\dot{m}_{bc}$

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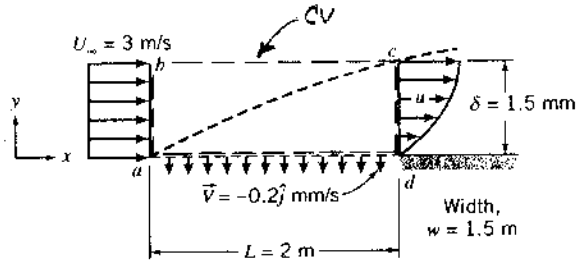
Problem 4.49

[3]

Given: Steady incompressible flow of air on porous surface shown in Fig. P4.48. Velocity profile at downstream end is parabolic. Uniform suction is applied along ad.

Find: (a) Volume flow rate across cd.
(b) Volume flow rate through porous surface (ad).
(c) Volume flow rate across bc.



Solution: Apply conservation of mass to CV shown.

Basic equation:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Incompressible flow
(2) Parabolic profile at section cd: $\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Then $0 = \int_{CS}\vec{V}\cdot d\vec{A} = Q_{ab} + Q_{bc} + Q_{cd} + Q_{da}$ (1)

$$Q_{cd} = \int_{cd}\vec{V}\cdot d\vec{A} = \int_0^\delta u w\, dy = wU_\infty\delta\int_0^1 \frac{u}{U}\, d\left(\frac{y}{\delta}\right) = wU_\infty\delta\int_0^1\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]d\frac{y}{\delta}$$

$$= wU_\infty\delta\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]_0^1 = \frac{2}{3}w\delta U_\infty$$

$Q_{cd} = \frac{2}{3}\times 1.5\,m \times 0.0015\,m \times 3\frac{m}{s} = 4.50\times10^{-3}\,m^3/s$ (out of CV) $\quad Q_{cd}$

Flow across ad is uniform, so

$$Q_{ad} = \vec{V}\cdot\vec{A} = v\hat{j}\cdot wL(-\hat{j}) = -vwL$$

$Q_{ad} = -0.2\frac{mm}{s}\times 1.5\,m\times 2\,m\times\frac{m}{1000\,mm} = 6.00\times10^{-4}\,m/s$ (out of CV) $\quad Q_a$

Finally, from Eq. 1,

$$Q_{bc} = -Q_{ab} - Q_{cd} - Q_{da} \qquad (2)$$

But $Q_{ab} = \vec{U}_\infty\cdot\vec{A}_{ab} = U_\infty\hat{i}\cdot w\delta(-\hat{i}) = -w\delta U_\infty$

$Q_{ab} = -1.5\,m\times0.0015\,m\times3\frac{m}{s} = -6.75\times10^{-3}\,m^3/s$ (into CV)

Substituting into Eq. 2,

$$Q_{bc} = \left[-\left(-6.75\times10^{-3}\right) - 4.50\times10^{-3} - 6.00\times10^{-4}\right]m^3/s$$

$Q_{bc} = 1.65\times10^{-3}\,m^3/s$ (out of CV) $\quad Q_{bc}$

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Problem 4.50

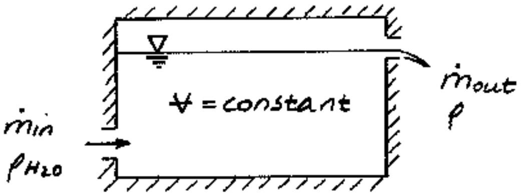
Given: Tank containing brine with steady inlet stream of water. Initial density is $\rho_i > \rho_{H_2O}$.

Find: (a) Rate of change of liquid density in tank.
(b) Time required to reach density, ρ_f, where $\rho_i > \rho_f > \rho_{H_2O}$.

Solution: Apply conservation of mass using the CV shown.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$



Assumptions: (1) $\forall_{tank}$ = constant
(2) ρ uniform in tank
(3) Uniform flows at inlet and outlet sections

Then $V_1 A_1 = V_2 A_2$ since tank volume is constant, and

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \rho VA - \rho_{H_2O} VA = \frac{\partial}{\partial t}\rho\forall + (\rho - \rho_{H_2O})VA = \forall \frac{d\rho}{dt} + (\rho - \rho_{H_2O})VA$$

So that

$$\frac{d\rho}{dt} = -\frac{(\rho - \rho_{H_2O})\,VA}{\forall}$$

$\frac{d\rho}{dt}$

Separating variables,

$$\frac{d\rho}{\rho - \rho_{H_2O}} = -\frac{VA}{\forall}\, dt$$

Integrating from ρ_i at $t = 0$ to ρ_f at t,

$$\int_{\rho_i}^{\rho_f} \frac{d\rho}{\rho - \rho_{H_2O}} = \ln(\rho - \rho_{H_2O})\Big]_{\rho_i}^{\rho_f} = \ln\left(\frac{\rho_f - \rho_{H_2O}}{\rho_i - \rho_{H_2O}}\right) = \int_0^t -\frac{VA}{\forall}\, dt = -\frac{VA}{\forall}\, t$$

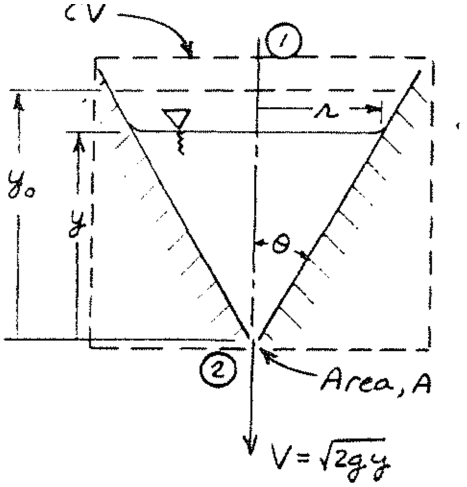
Finally,

$$t = -\frac{\forall}{VA} \ln\left(\frac{\rho_f - \rho_{H_2O}}{\rho_i - \rho_{H_2O}}\right)$$

t

{ Note that $\rho_f \to \rho_{H_2O}$ asymptotically as $t \to \infty$. }

Problem 4.51



Given: Funnel of liquid draining through a small hole of diameter d = 5mm (area A) as shown; y_0 is initial depth.

Find: (a) Expression for time to drain
(b) Expression for result in terms of
· initial volume $\forall_0$, and
· initial volume flow rate $Q_0 = AV_0 = A\sqrt{2gy_0}$

Plot: t as a function of y_0 ($0.1 \le y_0 \le 1$ m) with angle θ as a parameter for $15° \le \theta \le 45°$.

Solution

Apply conservation of mass using CV shown.

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) Incompressible flow
(2) Uniform flow at each section
(3) Neglect ρ_{air} compared to ρ_{H_2O}

Then,

$$0 = \cancelto{\approx 0(3)}{\frac{\partial}{\partial t}\int_{\forall_{air}} \rho_{air}\, d\forall} + \frac{\partial}{\partial t}\int_{\forall_{H_2O}} \rho_{H_2O}\, d\forall + \cancelto{\approx 0(3)}{\{-|\rho_{air} V_1 A_1|\}} + \{|\rho_{H_2O} V A|\}$$

For the CV,

$$d\forall = A_s\, dy = \pi r^2 dy = \pi (y\tan\theta)^2 dy \; ; \; \forall = \pi \tan^2\theta \frac{y^3}{3}$$

Thus

$$0 = \rho_{H_2O} \frac{\partial}{\partial t}\left(\pi \tan^2\theta \frac{y^3}{3}\right) + \rho_{H_2O} A\sqrt{2gy}$$

$$0 = \pi \tan^2\theta \, y^2 \frac{dy}{dt} + A\sqrt{2g}\, y^{1/2}$$

Separating variables, $y^{3/2}\, dy = \frac{-\sqrt{2g}\,A}{\pi \tan^2\theta} dt$

Integrating from y_0 at $t=0$ to 0 at t,

$$\int_{y_0}^{0} y^{3/2}\, dy = \frac{2}{5}\left(-y_0^{5/2}\right) = -\frac{\sqrt{2g}\,A}{\pi \tan^2\theta} t$$

or

$$t = \frac{2}{5} \frac{\pi \tan^2\theta \, y_0^{5/2}}{\sqrt{2g}\,A} \longleftarrow t$$

Problem 4.51

But $\mathcal{V}_0 = \pi \tan^2\theta \frac{y_0^3}{3}$ and $Q_0 = AV_0 = A\sqrt{2gy_0}$, so

$$t = \frac{2}{5}\frac{\pi \tan^2\theta\, y_0^{5/2}}{\sqrt{2g}\, A} \times \frac{3}{3}\cdot\frac{\cancel{y_0^{1/2}}}{y_0^{1/2}} = \frac{6}{5}\frac{\mathcal{V}_0}{Q_0} \longleftarrow t$$

Since $A = \frac{\pi d^2}{4}$, we can write

$$t = \frac{2}{5}\frac{\cancel{\pi}\tan^2\theta\, y_0^{5/2}}{\sqrt{2g}\,\frac{\cancel{\pi} d^2}{4}} = \frac{8}{5}\frac{\tan^2\theta\, y_0^{5/2}}{d^2\sqrt{2g}}$$

t is plotted as a function of y_0 with θ as a parameter

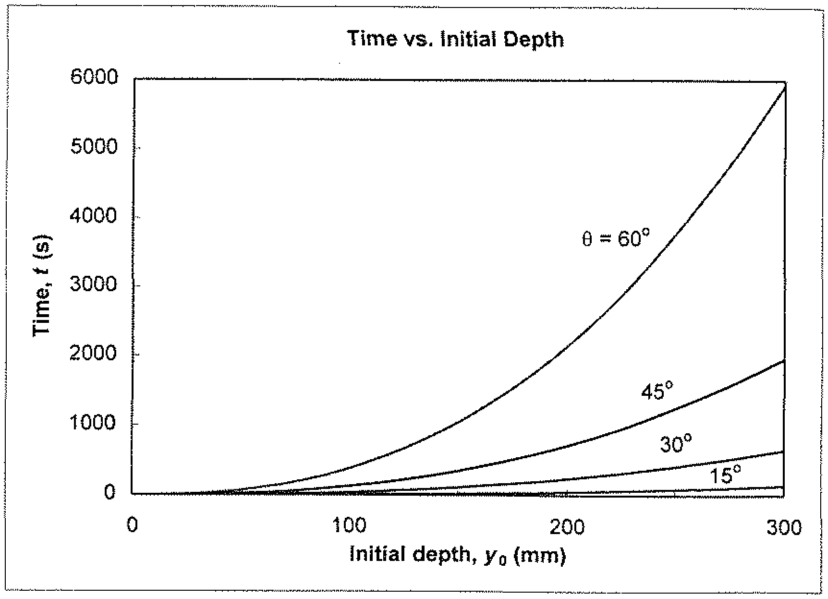
Draining of a conical liquid tank:

Input Data:

Orifice diameter: $d =$ 3 mm

Calculated Results:

Initial Height, y_0 (mm)	Cone Half Angle, θ (deg)	Drain Time, t (s)			
		60	**45**	**30**	**15**
300		5935	1978	659	142
275		4775	1592	531	114
250		3763	1254	418	90.0
225		2891	964	321	69.2
200		2154	718	239	51.5
175		1543	514	171	36.9
150		1049	350	117	25.1
125		665	222	74	15.9
100		381	127	42	9.11
75		185	62	21	4.44
50		67	22	7	1.61
25		12	4	1	0.285
0		0	0	0	0



Problem 4.52

Given: The instantaneous leakage mass flow rate $\dot{m}$ from a bicycle tire is proportional to the air density ρ in the tire and to the gage pressure p_g in the tire.

Air in the tire is nearly isothermal (because the leakage rate is slow).

The initial air pressure is $p_0 = 0.60$ MPa (gage) and the initial rate of pressure loss is 1 psi/day

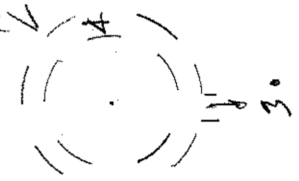
Find: (a) Pressure in the tire after 30 days

(b) Accuracy of rule of thumb which says a tire loses pressure at the rate of "a pound [1 psi] a day".

Plot: the pressure as a function of time over the 30 days; show rule of thumb results for comparison.

Solution:

Apply conservation of mass to tire as the CV



Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Assumptions: (1) uniform properties in tire

(2) air inside CV behaves as ideal gas

(3) T = constant $\cdot$ $\forall$ = constant

(4) $\dot{m} = c(p - p_{atm})\rho$

Then we can write

$$0 = \forall \frac{\partial \rho}{\partial t} + \dot{m} = \forall \frac{\partial \rho}{\partial t} + c(p - p_{atm})\rho \quad \ldots\ldots\ldots(1)$$

But $\rho = p/RT$ and $\frac{\partial \rho}{\partial t} = \frac{1}{RT}\frac{dp}{dt}$, so

$$0 = \frac{\forall}{RT}\frac{dp}{dt} + \frac{cp}{RT}(p - p_{atm}).$$

At $t = 0$, $p = p_0$ and $dp/dt = dp/dt)_0$. Thus

$$0 = \forall \frac{dp}{dt}\Big)_0 + c p_0 (p_0 - p_{atm}) \quad \text{and} \quad c = -\frac{\forall}{p_0(p_0 - p_{atm})}\frac{dp}{dt}\Big)_0$$

Substituting into Eq. 1 we obtain

$$0 = \frac{dp}{dt} - \frac{p(p - p_{atm})}{p_0(p_0 - p_{atm})}\frac{dp}{dt}\Big)_0$$

Separating variables and integrating

$$\int_{p_0}^{p} \frac{dp}{p(p - p_{atm})} = \frac{dp/dt)_0}{p_0(p_0 - p_{atm})}\int_0^t dt$$

$$\frac{1}{p_{atm}}\left[\ln \frac{p_0(p - p_{atm})}{p(p_0 - p_{atm})}\right] = \frac{dp/dt)_0}{p_0(p_0 - p_{atm})}\, t$$

$$\ln\left[\frac{1 - p_{atm}/p}{1 - p_{atm}/p_0}\right] = \frac{dp/dt)_0}{p_0(p_0/p_{atm} - 1)}\, t$$

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Problem 4.52

[4] Part 2/2

Taking antilogs,

$$1-\frac{p_{atm}}{p} = \left(1-\frac{p_{atm}}{p_0}\right) e^{\left\{\frac{\left.dp/dt\right)_0}{p_0(p_0/p_{atm}-1)}t\right\}} = \left(1-\frac{p_{atm}}{p_0}\right)e^{kt}$$

where

$$k = \frac{\left.dp/dt\right)_0}{p_0(p_0/p_{atm}-1)} = -1\,psi \times \frac{6.895\,kPa}{psi} \times \frac{1}{701\,kPa}\,\frac{1}{(701/101-1)}$$

$$k = -0.00166\ day^{-1}$$

Then

$$\frac{p_{atm}}{p} = 1-\left(\frac{p_0-p_{atm}}{p_0}\right)e^{kt}$$

and

$$p = \frac{p_{atm}}{1-\left(\frac{p_0-p_{atm}}{p_0}\right)e^{kt}} \quad \text{— — — — — — — (2)}$$

Evaluating at $t = 30$ days.

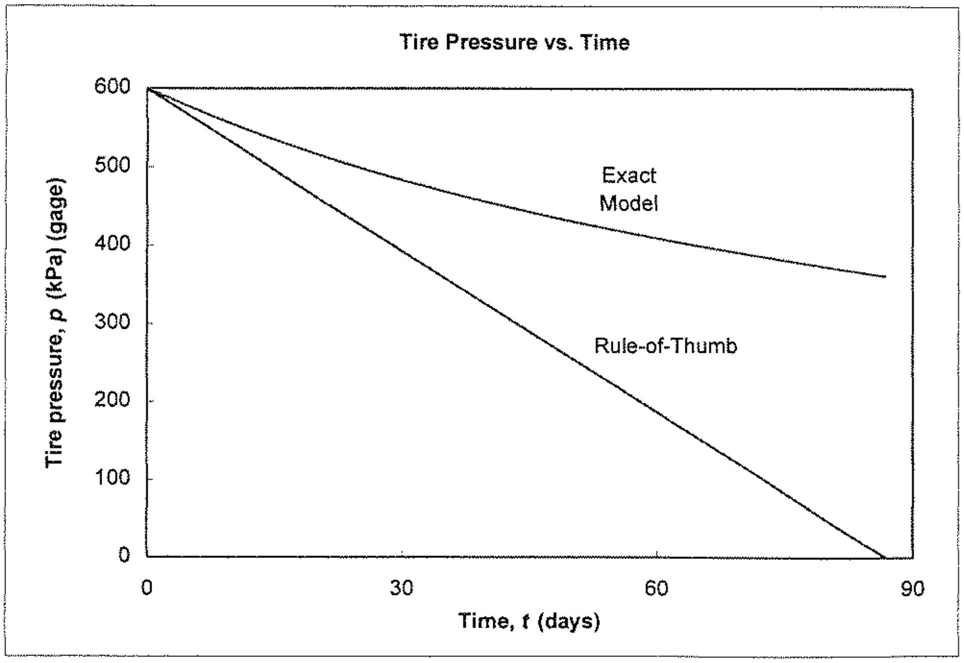
$$p = \frac{101\,kPa}{1-\frac{600}{701}e^{-30(0.00166)}} = 544\,kPa \longleftarrow \underline{p_{t=30\,days}}$$

Rule of thumb gives $p = p_0 - 6.895\frac{kPa}{day}\,t$ ------ (3)

At $t = 30$ days $p = 600\,kPa - 207\,kPa = 493\,kPa \;\_\; \underline{p_{ROT.}}$

The rule of thumb predicts a larger pressure loss

Results for both models are presented below



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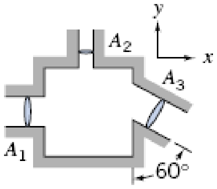
Problem 4.53

[3]

4.53 Evaluate the net rate of flux of momentum out through the control surface of Problem 4.21.



Given: Data on flow through a control surface

Find: Net rate of momentum flux

Solution:

Basic equation: We need to evaluate $\int_{CS} \vec{V}\rho\vec{V} \cdot dA$

Assumptions: 1) Uniform flow at each section

From Problem 4.21 $V_1 = 10\cdot\frac{ft}{s}$ $A_1 = 0.5\cdot ft^2$ $V_2 = 20\cdot\frac{ft}{s}$ $A_2 = 0.1\cdot ft^2$ $A_3 = 0.6\cdot ft^2$ $V_3 = 5\cdot\frac{ft}{s}$ It is an outlet

Then for the control surface

$$\int_{CS} \vec{V}\rho\vec{V} \cdot dA = \vec{V}_1\rho\vec{V}_1 \cdot \vec{A}_1 + \vec{V}_2\rho\vec{V}_2 \cdot \vec{A}_2 + \vec{V}_3\rho\vec{V}_3 \cdot \vec{A}_3$$

$$= V_1\hat{i}\rho\left(\vec{V}_1 \cdot \vec{A}_1\right) + V_2\hat{j}\rho\left(\vec{V}_2 \cdot \vec{A}_2\right) + \left[V_3\sin(60)\hat{i} - V_3\cos(60)\hat{j}\right]\rho\left(\vec{V}_3 \cdot \vec{A}_3\right)$$

$$= -V_1\hat{i}\rho V_1 A_1 + V_2\hat{j}\rho V_2 A_2 + \left[V_3\sin(60)\hat{i} - V_3\cos(60)\hat{j}\right]\rho V_3 A_3$$

$$= \rho\left[-V_1^2 A_1 + V_3^2 A_3\sin(60)\right]\hat{i} + \rho\left[V_2^2 A_2 - V_3^2 A_3\cos(60)\right]\hat{j}$$

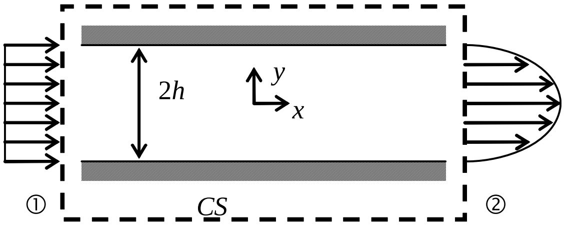
Hence the x component is $\rho\left[-V_1^2 A_1 + V_3^2 A_3\sin(60)\right] =$ $65\cdot\frac{lbm}{ft^3} \times \left(-10^2 \times 0.5 + 5^2 \times 0.6 \times \sin(60\cdot deg)\right)\cdot\frac{ft^4}{s^2} \times \frac{lbf\cdot s^2}{lbm\cdot ft} = -2406\,lbf$

and the y component is $\rho\left[V_2^2 A_2 - V_3^2 A_3\cos(60)\right] =$ $65\cdot\frac{lbm}{ft^3} \times \left(20^2 \times 0.1 - 5^2 \times 0.6 \times \cos(60\cdot deg)\right)\cdot\frac{ft^4}{s^2} \times \frac{lbf\cdot s^2}{lbm\cdot ft} = 2113\,lbf$

Problem 4.54

[3]

4.54 For the conditions of Problem 4.30, evaluate the ratio of the x-direction momentum flux at the channel outlet to that at the inlet.



Given:

Data on flow at inlet and outlet of channel

Find:

Ratio of outlet to inlet momentum flux

Solution:

Basic equation: Momentum flux in x direction at a section $\mathrm{mf}_x = \int_A u\rho\vec{V}\cdot dA$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at 1 and 2 $\quad \mathrm{mf}_{x1} = U\cdot\rho\cdot(-U\cdot 2\cdot h)\cdot w \qquad \left|\mathrm{mf}_{x1}\right| = 2\cdot\rho\cdot w\cdot U^2\cdot h$

Hence

$$\mathrm{mf}_{x2} = \int_{-h}^{h}\rho\cdot u^2\cdot w\,dy = \rho\cdot w\cdot u_{max}^{2}\cdot\int_{-h}^{h}\left[1-\left(\frac{y}{h}\right)^2\right]^2 dy = \rho\cdot w\cdot u_{max}^{2}\cdot\int_{-h}^{h}\left[1-2\cdot\left(\frac{y}{h}\right)^2+\left(\frac{y}{h}\right)^4\right]dy$$

$$\left|\mathrm{mf}_{x2}\right| = \rho\cdot w\cdot u_{max}^{2}\cdot\left(2\cdot h-\frac{4}{3}\cdot h+\frac{2}{5}\cdot h\right) = \rho\cdot w\cdot u_{max}^{2}\cdot\frac{16}{15}\cdot h$$

Then the ratio of momentum fluxes is

$$\frac{\left|\mathrm{mf}_{x2}\right|}{\left|\mathrm{mf}_{x1}\right|} = \frac{\frac{16}{15}\cdot\rho\cdot w\cdot u_{max}^{2}\cdot h}{2\cdot\rho\cdot w\cdot U^2\cdot h} = \frac{8}{15}\cdot\left(\frac{u_{max}}{U}\right)^2$$

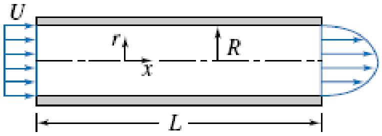
But, from Problem 4.30 $\quad u_{max} = \frac{3}{2}\cdot U \qquad \frac{\left|\mathrm{mf}_{x2}\right|}{\left|\mathrm{mf}_{x1}\right|} = \frac{8}{15}\cdot\left(\frac{\frac{3}{2}\cdot U}{U}\right)^2 = \frac{6}{5} = 1.2$

Hence the momentum increases as it flows in the entrance region of the channel. This appears to contradict common sense, as friction should reduce flow momentum. What happens is the pressure drops significantly along the channel so the net force on the CV is to the right.

Problem 4.55

[3]

4.55 For the conditions of Problem 4.31, evaluate the ratio of the x-direction momentum flux at the pipe outlet to that at the inlet.



Given: Data on flow at inlet and outlet of pipe

Find: Ratio of outlet to inlet momentum flux

Solution:

Basic equation: Momentum flux in x direction at a section $\mathrm{mf}_x = \int_A u\rho\vec{V}\cdot dA$

Assumptions: 1) Steady flow 2) Incompressible flow

Evaluating at 1 and 2 $\quad \mathrm{mf}_{x1} = U\cdot\rho\cdot\left(-U\cdot\pi\cdot R^2\right) \qquad \left|\mathrm{mf}_{x1}\right| = \rho\cdot\pi\cdot U^2\cdot R^2$

Hence

$$\mathrm{mf}_{x2} = \int_0^R \rho\cdot u^2\cdot 2\cdot\pi\cdot r\,dr = 2\cdot\rho\cdot\pi\cdot u_{max}^{\ 2}\cdot\int_0^R r\cdot\left[1-\left(\frac{r}{R}\right)^2\right]^2 dr = 2\cdot\rho\cdot\pi\cdot u_{max}^{\ 2}\cdot\int_0^R \left(r - 2\cdot\frac{r^3}{R^2} + \frac{r^5}{R^4}\right) dy$$

$$\left|\mathrm{mf}_{x2}\right| = 2\cdot\rho\cdot\pi\cdot u_{max}^{\ 2}\cdot\left(\frac{R^2}{2} - \frac{R^2}{2} + \frac{R^2}{6}\right) = \rho\cdot\pi\cdot u_{max}^{\ 2}\cdot\frac{R^2}{3}$$

Then the ratio of momentum fluxes is

$$\frac{\left|\mathrm{mf}_{x2}\right|}{\left|\mathrm{mf}_{x1}\right|} = \frac{\frac{1}{3}\cdot\rho\cdot\pi\cdot u_{max}^{\ 2}\cdot R^2}{\rho\cdot\pi\cdot U^2\cdot R^2} = \frac{1}{3}\cdot\left(\frac{u_{max}}{U}\right)^2$$

But, from Problem 4.31 $\quad u_{max} = 2\cdot U \qquad \dfrac{\left|\mathrm{mf}_{x2}\right|}{\left|\mathrm{mf}_{x1}\right|} = \dfrac{1}{3}\cdot\left(\dfrac{2\cdot U}{U}\right)^2 = \dfrac{4}{3} = 1.33$

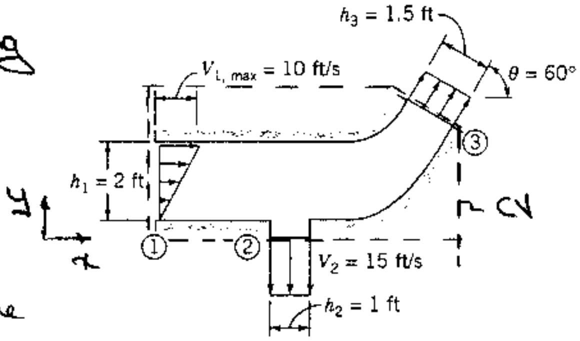
Hence the momentum increases as it flows in the entrance region of the pipe This appears to contradict common sense, as friction should reduce flow momentum. What happens is the pressure drops significantly along the pipe so the net force on the CV is to the right.

Problem 4.56

[2]

Given: Two-dimensional reducing bend shown has width $w = 3$ ft.

$V_3 = 3.33$ ft/s into CV (from Problem 4.24)



Find: Momentum flux through the bend.

Solution:

The momentum flux is defined as $m.f = \int \vec{V}(\rho\vec{V}\cdot d\vec{A})$

The net momentum flux through the CV is

$$m.f = \int_{A_1}\vec{V}(\rho\vec{V}\cdot d\vec{A}) + \int_{A_2}\vec{V}(\rho\vec{V}\cdot d\vec{A}) + \int_{A_3}\vec{V}(\rho\vec{V}\cdot d\vec{A})$$

where $\vec{V}_1 = V_{1\max}\frac{y}{h_1}\hat{i}$, $\vec{V}_2 = -V_2\hat{j}$, $\vec{V}_3 = -V_3(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$V_{1\max} = 10$ ft/s, $V_2 = 15$ ft/s, $V_3 = 3.33$ ft/s

Assumptions: (1) incompressible flow
(2) fluid is water
(3) uniform flow at ② and ③ (given)

$$\int_{A_1}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = \int_0^{h_1} V_{1\max}\frac{y}{h_1}\hat{i}\,\rho\left\{-V_{1\max}\frac{y}{h_1}\right\}w\,dy = -\hat{i}\rho V_{1\max}^2\frac{w}{h_1^2}\int_0^{h_1} y^2\,dy$$

$$\int_{A_1}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = -\hat{i}\rho V_{1\max}^2\frac{w}{h_1^2}\,\frac{y^3}{3}\Big]_0^{h_1} = -\hat{i}\rho V_{1\max}^2\frac{wh_1}{3} \quad \text{----(1)}$$

$$\int_{A_2}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = \vec{V}_2|\rho V_2h_2w| = -V_2\hat{j}|\rho V_2h_2w| = -\hat{j}\rho V_2^2h_2w \quad \text{----(2)}$$

$$\int_{A_3}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = \vec{V}_3(-|\rho V_3h_3w|) = -V_3(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})(-|\rho V_3h_3w|)$$

$$\int_{A_3}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = \rho V_3^2h_3w(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \quad \text{---------(3)}$$

$$m.f = \hat{i}\left[\rho V_3^2h_3w\cos\theta - \rho V_{1\max}^2\frac{wh_1}{3}\right] + \hat{j}\left[\rho V_3^2h_3w\sin\theta - \rho V_2^2h_2w\right]$$

$$m.f = \rho w\left\{\left[V_3^2h_3\cos\theta - V_{1\max}^2\frac{h_1}{3}\right]\hat{i} + \left[V_3^2h_3\sin\theta - V_2^2h_2\right]\hat{j}\right\}$$

Evaluating

$$m.f = 1.94\frac{\text{slug}}{\text{ft}^3}\times 3\text{ft}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}}\left\{\begin{array}{l}\left[(3.33)^2\frac{\text{ft}^2}{\text{s}^2}\times 1.5\text{ ft}\times\cos 60° - (10)^2\frac{\text{ft}^2}{\text{s}^2}\times\frac{2\text{ ft}}{3}\right]\hat{i} \\ +\left[(3.33)^2\frac{\text{ft}^2}{\text{s}^2}\times 1.5\text{ft}\times\sin 60° - (15)^2\frac{\text{ft}^2}{\text{s}^2}\times 1\text{ ft}\right]\hat{j}\end{array}\right\}$$

$m.f = -340\,\hat{i} - 1230\,\hat{j}$ lbf $\longleftarrow$ m.f.

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Problem 4.57

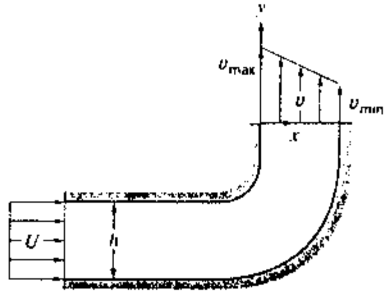
[2]

Given: Water flow in the two-dimensional square channel shown

$\bar{U} = 7.5\ \mathrm{m/s}$, $h = w = 75.5\ \mathrm{mm}$

$v_{max} = 2\,v_{min}$

$v_{min} = 5.0\ \mathrm{m/s}$
(from Problem 4.25)



Find: Momentum flux through the channel; comment on expected outlet pressure (relative to pressure at the inlet.

Solution:

The momentum flux is defined as $m.f. = \int \vec{V}(\rho\vec{V}\cdot d\vec{A})$
The net momentum flux through the CV is

$$m.f. = \int_{A_1}\vec{V}(\rho\vec{V}\cdot d\vec{A}) + \int_{A_2}\vec{V}(\rho\vec{V}\cdot d\vec{A})$$

where $\vec{V}_1 = \bar{U}\hat{i}$, $\vec{V}_2 = \left\{v_{max} - (v_{max}-v_{min})\frac{x}{h}\right\}\hat{j}$

$$\vec{V}_2 = \left\{2v_{min} - v_{min}\frac{x}{h}\right\}\hat{j} = v_{min}\left(2-\frac{x}{h}\right)\hat{j}$$

Assumptions: (1) incompressible flow
(2) uniform flow at ① (given).

$$\int_{A_1}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = \vec{V}_1\left\{-|\rho V_1 A_1|\right\} = -\rho\bar{U}^2h^2\hat{i} \quad \text{.......(1)}$$

$$\int_{A_2}\vec{V}(\rho\vec{V}\cdot d\vec{A}) = \int_0^h v_{min}\left(2-\frac{x}{h}\right)\hat{j}\,\rho v_{min}\left(2-\frac{x}{h}\right)h\,dx$$

$$= \hat{j}\rho v_{min}^2 h\int_0^h\left(4-4\frac{x}{h}+\frac{x^2}{h^2}\right)dx$$

$$= \hat{j}\rho v_{min}^2 h\left[4x-2\frac{x^2}{h}+\frac{x^3}{3h^2}\right]_0^h = \hat{j}\rho v_{min}^2 h\left[4h-2h+\frac{h}{3}\right]$$

$$= \hat{j}\,\frac{7}{3}\rho v_{min}^2 h^2$$

$$\therefore\ m.f. = -\rho\bar{U}^2h^2\hat{i} + \frac{7}{3}\rho v_{min}^2h^2\hat{j} = \rho h^2\left[-\bar{U}^2\hat{i} + \frac{7}{3}v_{min}^2\hat{j}\right]$$

Evaluating

$$m.f. = 999\,\frac{\mathrm{kg}}{\mathrm{m^3}} \times (0.0755)^2\,\mathrm{m^2}\left[-(7.5)^2\,\frac{\mathrm{m^2}}{\mathrm{s^2}}\hat{i} + \frac{7}{3}(5)^2\,\frac{\mathrm{m^2}}{\mathrm{s^2}}\hat{j}\right] \times \frac{\mathrm{N\cdot s^2}}{\mathrm{kg\cdot m}}$$

$m.f. = -320\,\hat{i} + 332\,\hat{j}\ \mathrm{N}$ ← m.f.

For viscous (real) flow friction causes a pressure drop in the direction of flow (Chapter 8)
For flow in a bend streamline curvature results in a pressure gradient normal to the flow (Chapter 6)

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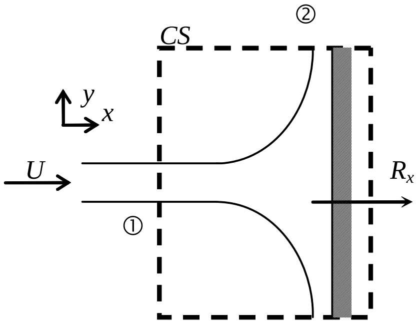
Problem 4.58

4.58 What force (lbf) will a horizontal 2-in.-diameter stream of water moving at 20 ft/s generate upon hitting a vertical flat plate?



Given: Water jet hitting wall

Find: Force generated on wall

Solution:

Basic equation: Momentum flux in x direction

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow 5) Water leaves vertically

Hence $R_x = u_1\cdot\rho\cdot\left(-u_1\cdot A_1\right) = -\rho\cdot U^2\cdot A = -\rho\cdot U^2\cdot\frac{\pi\cdot D^2}{4}$

$$R_x = -1.94\cdot\frac{slug}{ft^3}\times\left(20\cdot\frac{ft}{s}\right)^2\times\frac{\pi\cdot\left(\frac{1}{6}\cdot ft\right)^2}{4}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad R_x = -16.9\cdot lbf$$

Problem 4.59

4.59 Considering that in the fully developed region of a pipe, the integral of the axial momentum is the same at all cross sections, explain the reason for the pressure drop along the pipe.

Given:

Fully developed flow in pipe

Find:

Why pressure drops if momentum is constant

Solution:

Basic equation: Momentum flux in x direction

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!/ + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Fully developed flow

Hence $F_x = \frac{\Delta p}{L} - \tau_w \cdot A_s = 0$ $\Delta p = L \cdot \tau_w \cdot A_s$

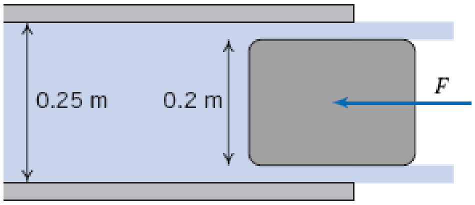
where Δp is the pressure drop over length L, τ_w is the wall friction and As is the pipe surface area

The sum of forces in the x direction is zero. The friction force on the fluid is in the negative x direction, so the net pressure force must be in the positive direction. Hence pressure drops in the x direction so that pressure and friction forces balance

Problem 4.60

[2]

4.60 Find the force required to hold the plug in place at the exit of the water pipe. The flow rate is 1.5 m^3/s, and the upstream pressure is 3.5 MPa.



Given:
Data on flow and system geometry

Find:
Force required to hold plug

Solution:

The given data is $D_1 = 0.25\cdot m$ $\quad D_2 = 0.2\cdot m$ $\quad Q = 1.5\cdot\frac{m^3}{s}$ $\quad p_1 = 3500\cdot kPa$ $\quad \rho = 999\cdot\frac{kg}{m^3}$

Then $A_1 = \frac{\pi\cdot D_1^2}{4}$ $\qquad A_1 = 0.0491\,m^2$

$A_2 = \frac{\pi}{4}\cdot\left(D_1^2 - D_2^2\right)$ $\qquad A_2 = 0.0177\,m^2$

$V_1 = \frac{Q}{A_1}$ $\qquad V_1 = 30.6\,\frac{m}{s}$

$V_2 = \frac{Q}{A_2}$ $\qquad V_2 = 84.9\,\frac{m}{s}$

Governing equation:

Momentum
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A} \qquad (4.18a)$$

Applying this to the current system

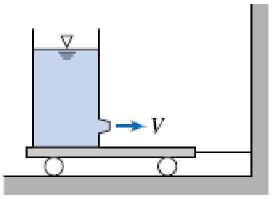
$$-F + p_1\cdot A_2 - p_2\cdot A_2 = 0 + V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$$ and $p_2 = 0$ (gage)

Hence $$F = p_1\cdot A_1 + \rho\cdot\left(V_1^2\cdot A_1 - V_2^2\cdot A_2\right)$$

$$F = 3500\times\frac{kN}{m^2}\cdot 0.0491\cdot m^2 + 999\cdot\frac{kg}{m^3}\times\left[\left(30.6\cdot\frac{m}{s}\right)^2\cdot 0.0491\cdot m^2 - \left(84.9\cdot\frac{m}{s}\right)^2\cdot 0.0177\cdot m^2\right]$$ $F = 90.4\,kN$

Problem 4.61

4.61 A large tank of height $h = 1$ m and diameter $D = 0.75$ m is affixed to a cart as shown. Water issues from the tank through a nozzle of diameter $d = 15$ mm. The speed of the liquid leaving the tank is approximately $V = \sqrt{2gy}$ where y is the height from the nozzle to the free surface. Determine the tension in the wire when $y = 0.9$ m. Plot the tension in the wire as a function of water depth for $0 \le y \le 0.9$ m.



Given:

Large tank with nozzle and wire

Find:

Tension in wire; plot for range of water depths

Solution:

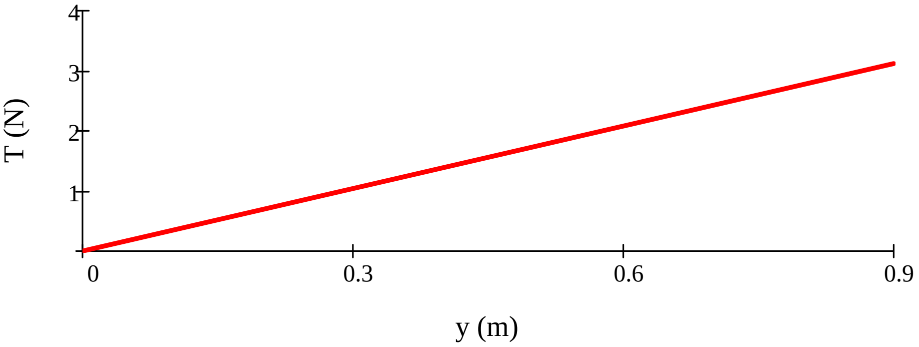
Basic equation: Momentum flux in x direction for the tank $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Hence $R_x = T = V\cdot\rho\cdot(V\cdot A) = \rho\cdot V^2\cdot A = \rho\cdot(2\cdot g\cdot y)\cdot\frac{\pi\cdot d^2}{4}$ $\qquad T = \frac{1}{2}\cdot\rho\cdot g\cdot y\cdot\pi\cdot d^2 \qquad (1)$

When y = 0.9 m $T = \frac{\pi}{2}\times 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 0.9\cdot m\times(0.015\cdot m)^2\times\frac{N\cdot s^2}{kg\cdot m}$ $\qquad T = 3.12\,N$

From Eq 1

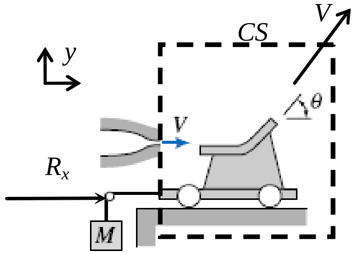


This graph can be plotted in *Excel*

Problem 4.62

[2]

4.62 A jet of water issuing from a stationary nozzle at 10 m/s ($A_j = 0.1\ \text{m}^2$) strikes a turning vane mounted on a cart as shown. The vane turns the jet through angle $\theta = 40°$. Determine the value of M required to hold the cart stationary. If the vane angle θ is adjustable, plot the mass, M, needed to hold the cart stationary versus θ for $0 \le \theta \le 180°$.



Given: Nozzle hitting stationary cart

Find: Value of M to hold stationary; plot M versu θ

Solution:

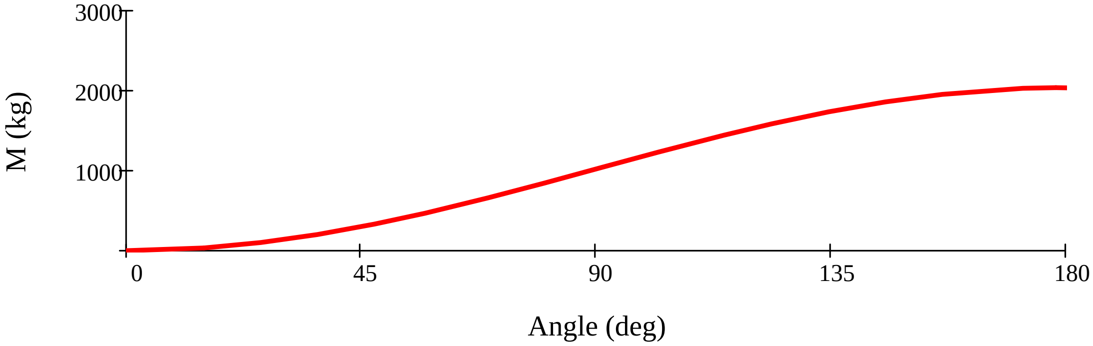
Basic equation: Momentum flux in x direction for the tank $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow 5) Exit velocity is V

Hence $R_x = -M\cdot g = V\cdot\rho\cdot(-V\cdot A) + V\cdot\cos(\theta)\cdot(V\cdot A) = \rho\cdot V^2\cdot A\cdot(\cos(\theta) - 1)$ $\qquad M = \frac{\rho\cdot V^2\cdot A}{g}\cdot(1 - \cos(\theta))$ (1)

When θ = 40º $\quad M = \frac{s^2}{9.81\cdot m} \times 1000\cdot\frac{kg}{m^3} \times \left(10\cdot\frac{m}{s}\right)^2 \times 0.1\cdot m^2 \times (1 - \cos(40\cdot deg))$ $\qquad M = 238\,kg$

From Eq 1

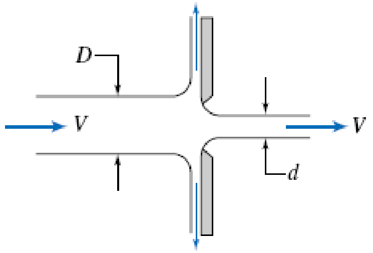


This graph can be plotted in *Excel*

Problem 4.63

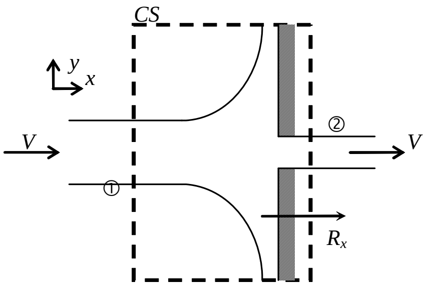
[3]

4.63 A vertical plate has a sharp-edged orifice at its center. A water jet of speed V strikes the plate concentrically. Obtain an expression for the external force needed to hold the plate in place, if the jet leaving the orifice also has speed V. Evaluate the force for $V = 15$ ft/s, $D = 4$ in., and $d = 1$ in. Plot the required force as a function of diameter ratio for a suitable range of diameter d.



Given: Water jet hitting plate with opening

Find: Force generated on plate; plot force versus diameter d



Solution:

Basic equation: Momentum flux in x direction

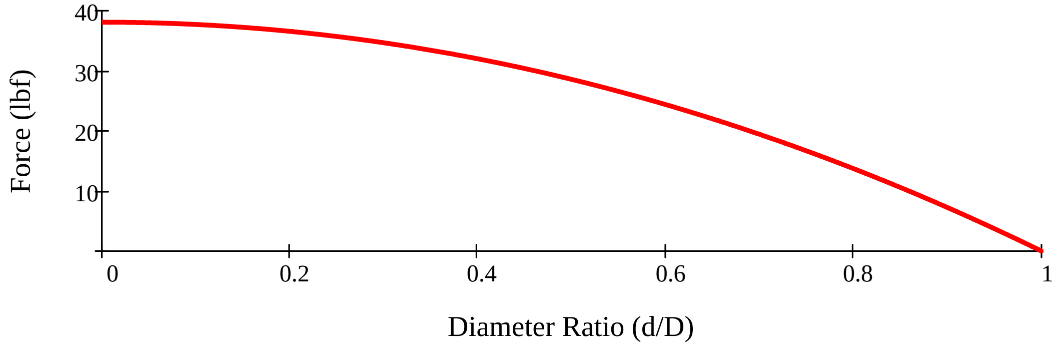
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\mathcal{V} + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Hence $$R_x = u_1\cdot\rho\cdot\left(-u_1\cdot A_1\right) + u_2\cdot\rho\cdot\left(u_2\cdot A_2\right) = -\rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4} + \rho\cdot V^2\cdot\frac{\pi\cdot d^2}{4} \qquad R_x = -\frac{\pi\cdot\rho\cdot V^2\cdot D^2}{4}\cdot\left[1-\left(\frac{d}{D}\right)^2\right] \quad (1)$$

For given data $$R_x = -\frac{\pi}{4}\cdot 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left(15\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\left(\frac{1}{3}\cdot\text{ft}\right)^2\times\left[1-\left(\frac{1}{4}\right)^2\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \qquad R_x = -35.7\cdot\text{lbf}$$

From Eq 1 (using the absolute value of R_x)

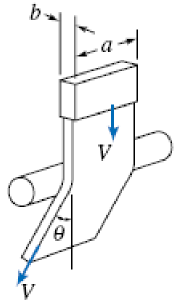


This graph can be plotted in *Excel*

Problem 4.64

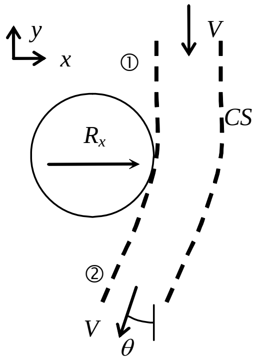
[3]

4.64 A circular cylinder inserted across a stream of flowing water deflects the stream through angle θ, as shown. (This is termed the "Coanda effect.") For $a = 12.5$ mm, $b = 2.5$ mm, $V = 3$ m/s, and $\theta = 20°$, determine the horizontal component of the force on the cylinder caused by the flowing water.



Given: Water flowing past cylinder

Find: Horizontal force on cylinder



Solution:

Basic equation: Momentum flux in x direction

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!-\; + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Hence $R_x = u_1 \cdot \rho \cdot \left(-u_1 \cdot A_1\right) + u_2 \cdot \rho \cdot \left(u_2 \cdot A_2\right) = 0 + \rho \cdot (-V \cdot \sin(\theta)) \cdot (V \cdot a \cdot b)$ $\quad R_x = -\rho \cdot V^2 \cdot a \cdot b \cdot \sin(\theta)$

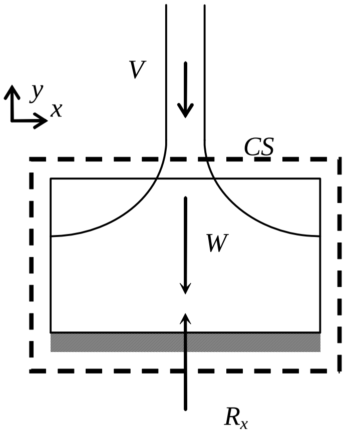
For given data $R_x = -1000 \cdot \frac{kg}{m^3} \times \left(3 \cdot \frac{m}{s}\right)^2 \times 0.0125 \cdot m \times 0.0025 \cdot m \times \sin(20 \cdot deg) \times \frac{N \cdot s^2}{kg \cdot m}$ $\quad R_x = -0.0962\,N$

This is the force on the fluid (it is to the left). Hence the force on the cylinder is $R_x = -R_x$ $\quad R_x = 0.0962\,N$

Problem 4.65

[5]

4.65 In a laboratory experiment, the water flow rate is to be measured catching the water as it vertically exits a pipe into an empty open cylindrical (3-ft diameter) tank that is on a zeroed balance. The tank bottom is 5 ft directly below the pipe exit, and the pipe diameter is 2 in. One student obtains a flow rate by noting that after 30 seconds the volume of water (at 50°F) in the tank was 15 ft^3. Another student obtains a flow rate by reading the instantaneous weight of 960 lb indicated at the 30-second point. Find the mass flow rate each student computes. Why do they disagree? Which one is more accurate? Show that the magnitude of the discrepancy can be explained by any concept you may have.



Given: Water flowing into tank

Find: Mass flow rates estimated by students. Explain discrepancy

Solution:

Basic equation: Momentum flux in y direction

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,dV + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

For the first student $m_1 = \frac{\rho \cdot V}{t}$ where m_1 represents mass flow rate (software cannot render a dot above it!)

$$m_1 = 1.94\cdot\frac{slug}{ft^3}\times 15\cdot ft^3 \times \frac{1}{30\cdot s} \qquad m_1 = 0.97\cdot\frac{slug}{s} \qquad m_1 = 31.2\cdot\frac{lbm}{s}$$

For the second student $m_2 = \frac{M}{t}$ where m_2 represents mass flow rate

$$m_2 = 960\cdot lb \times \frac{1}{30\cdot s} \qquad m_2 = 0.995\cdot\frac{slug}{s} \qquad m_2 = 32\cdot\frac{lbm}{s}$$

There is a discrepancy because the second student is measuring instantaneous weight PLUS the force generated as the pipe flow momentum is "killed".

To analyse this we first need to find the speed at which the water stream enters the tank, 5 ft below the pipe exit. This would be a good place to use the Bernoulli equation, but this problem is in the set before Bernoulli is covered. Instead we use the simple concept that the fluid is falling under gravity (a conclusion supported by the Bernoulli equation). From the equations for falling under gravity:

$$V_{tank}^2 = V_{pipe}^2 + 2\cdot g\cdot h$$

where V_{tank} is the speed entering the tank, V_{pipe} is the speed at the pipe, and h = 5 ft is the distance traveled. V_{pipe} is obtained from

$$V_{pipe} = \frac{m_1}{\rho\cdot\frac{\pi\cdot d_{pipe}^2}{4}} = \frac{4\cdot m_1}{\pi\cdot\rho\cdot d_{pipe}^2}$$

$$V_{pipe} = \frac{4}{\pi}\times 31.2\cdot\frac{lbm}{s}\times\frac{ft^3}{1.94\cdot slug}\times\frac{1\cdot slug}{32.2\cdot lbm}\times\left(\frac{1}{\frac{1}{6}\cdot ft}\right)^2 \qquad V_{pipe} = 22.9\frac{ft}{s}$$

Then $$V_{tank} = \sqrt{V_{pipe}^2 + 2\cdot g\cdot h} \qquad V_{tank} = \sqrt{\left(22.9\cdot\frac{ft}{s}\right)^2 + 2\times 32.2\cdot\frac{ft}{s^2}\times 5ft} \qquad V_{tank} = 29.1\frac{ft}{s}$$

We can now use the y momentum equation for the CS shown above

$$R_y - W = -V_{tank}\cdot\rho\cdot\left(-V_{tank}\cdot A_{tank}\right)$$

where A_{tank} is the area of the water flow as it enters the tank. But for the water flow $V_{tank}\cdot A_{tank} = V_{pipe}\cdot A_{pipe}$

Hence $\Delta W = R_y - W = \rho\cdot V_{tank}\cdot V_{pipe}\cdot\frac{\pi\cdot d_{pipe}^{\,2}}{4}$

This equation indicate the instantaneous difference ΔW between the scale reading (R_y) and the actual weight of water (W) in the tank

$$\Delta W = 1.94\cdot\frac{slug}{ft^3}\times 29.1\cdot\frac{ft}{s}\times 22.9\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{6}\cdot ft\right)^2\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad \Delta W = 28.2\,lbf$$

Hence the scale overestimates the weight of water by 28.2 lbf, or a mass of 28.2 lbm

For the second student $M = 960\cdot lbm - 28.2\cdot lbm = 932\cdot lbm$

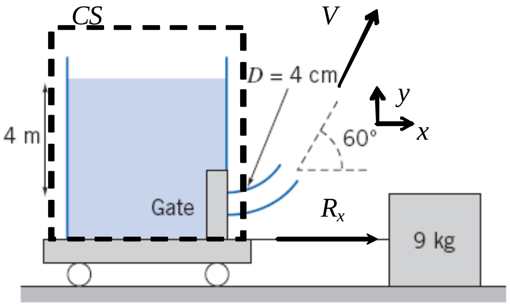
Hence $m_2 = \frac{M}{t}$ where m_2 represents mass flow rate

$$m_2 = 932\cdot lb\times\frac{1}{30\cdot s} \qquad m_2 = 0.966\cdot\frac{slug}{s} \qquad m_2 = 31.1\cdot\frac{lbm}{s}$$

Comparing with the answer obtained from student 1, we see the students now agree! The discrepancy was entirely caused by the fact that t second student was measuring the weight of tank water PLUS the momentum lost by the water as it entered the tank!

Problem 4.66

4.66 A tank of water sits on a cart with frictionless wheels as shown. The cart is attached using a cable to a 9 kg mass, and the coefficient of static friction of the mass with the ground is 0.5. At time $t = 0$, a second cable is used to remove a gate blocking the tank exit. Will the resulting exit flow be sufficient to start the tank moving? (Assume the water flow is frictionless.)



Given: Water tank attached to mass

Find: Whether tank starts moving

Solution:

Basic equation: Momentum flux in x direction for the tank $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure at exit 4) Uniform flow

Hence $R_x = V\cdot\cos(\theta)\cdot\rho\cdot(V\cdot A) = \rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}\cdot\cos(\theta)$

We need to find V. We could use the Bernoulli equation, but here it is known that $V = \sqrt{2\cdot g\cdot h}$ where h = 4 m is the height of fluid in the tank

$$V = \sqrt{2\times 9.81\cdot\frac{m}{s^2}\times 4\cdot m} \qquad V = 8.86\frac{m}{s}$$

Hence $R_x = 1000\cdot\frac{kg}{m^3}\times\left(8.86\cdot\frac{m}{s}\right)^2\times\frac{\pi}{4}\times(0.04\cdot m)^2\times\cos(60\cdot deg)$ $\quad R_x = 49.3\,N$

This force is equal to the tension T in the wire $T = R_x$ $\quad T = 49.3\,N$

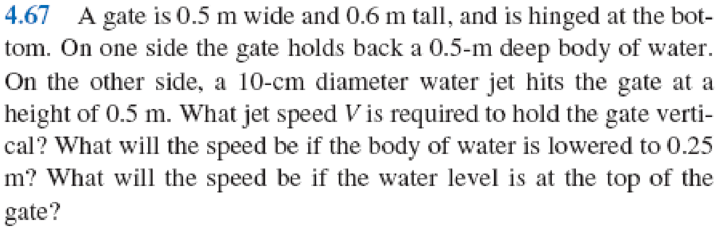
For the block, the maximum friction force a mass of M = 9 kg can generate is $F_{max} = M\cdot g\cdot\mu$ where μ is static friction

$$F_{max} = 9\cdot kg\times 9.81\cdot\frac{m}{s^2}\times 0.5\times\frac{N\cdot s^2}{kg\cdot m} \qquad F_{max} = 44.1\,N$$

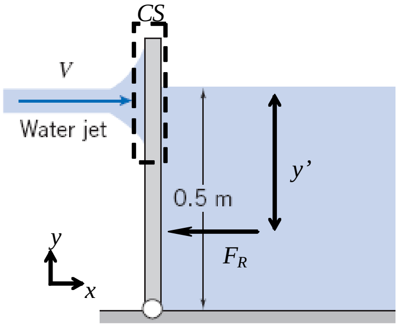
Hence the tension T created by the water jet is larger than the maximum friction F_{max}; the tank starts to move

Problem 4.67

[4]



4.67 A gate is 0.5 m wide and 0.6 m tall, and is hinged at the bottom. On one side the gate holds back a 0.5-m deep body of water. On the other side, a 10-cm diameter water jet hits the gate at a height of 0.5 m. What jet speed V is required to hold the gate vertical? What will the speed be if the body of water is lowered to 0.25 m? What will the speed be if the water level is at the top of the gate?



Given: Gate held in place by water jet

Find: Required jet speed for various water depths

Solution:

Basic equation: Momentum flux in x direction for the wall $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!/ + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Note: We use this equation ONLY for the jet impacting the wall. For the hydrostatic force and location we use computing equations

$$F_R = p_c \cdot A \qquad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $$R_x = V \cdot \rho \cdot \left(-V \cdot A_{jet}\right) = -\rho \cdot V^2 \cdot \frac{\pi \cdot D^2}{4}$$

This force is the force generated by the wall on the jet; the force of the jet hitting the wall is then

$$F_{jet} = -R_x = \rho \cdot V^2 \cdot \frac{\pi \cdot D^2}{4}$$ where D is the jet diameter

For the hydrostatic force $$F_R = p_c \cdot A = \rho \cdot g \cdot \frac{h}{2} \cdot h \cdot w = \frac{1}{2} \cdot \rho \cdot g \cdot w \cdot h^2 \qquad y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \frac{h}{2} + \frac{\frac{w \cdot h^3}{12}}{w \cdot h \cdot \frac{h}{2}} = \frac{2}{3} \cdot h$$

where h is the water depth and w is the gate width

For the gate, we can take moments about the hinge to obtain $$-F_{jet} \cdot h_{jet} + F_R \cdot (h - y') = -F_{jet} \cdot h_{jet} + F_R \cdot \frac{h}{3} = 0$$

where h_{jet} is the height of the jet from the ground

Hence $$F_{jet} = \rho \cdot V^2 \cdot \frac{\pi \cdot D^2}{4} \cdot h_{jet} = F_R \cdot \frac{h}{3} = \frac{1}{2} \cdot \rho \cdot g \cdot w \cdot h^2 \cdot \frac{h}{3} \qquad V = \sqrt{\frac{2 \cdot g \cdot w \cdot h^3}{3 \cdot \pi \cdot D^2 \cdot h_j}}$$

For the first case (h = 0.5 m) $$V = \sqrt{\frac{2}{3 \cdot \pi} \times 9.81 \cdot \frac{m}{s^2} \times 0.5 \cdot m \times (0.5 \cdot m)^3 \times \left(\frac{1}{0.01 \cdot m}\right)^2 \times \frac{1}{0.5 \cdot m}} \qquad V = 51 \frac{m}{s}$$

For the second case (h = 0.25 m) $$V = \sqrt{\frac{2}{3 \cdot \pi} \times 9.81 \cdot \frac{m}{s^2} \times 0.5 \cdot m \times (0.25 \cdot m)^3 \times \left(\frac{1}{0.01 \cdot m}\right)^2 \times \frac{1}{0.5 \cdot m}} \qquad V = 18 \frac{m}{s}$$

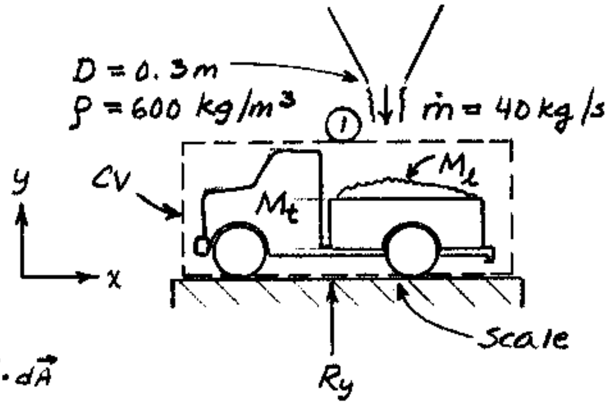
For the first case (h = 0.6 m) $$V = \sqrt{\frac{2}{3 \cdot \pi} \times 9.81 \cdot \frac{m}{s^2} \times 0.5 \cdot m \times (0.6 \cdot m)^3 \times \left(\frac{1}{0.01 \cdot m}\right)^2 \times \frac{1}{0.5 \cdot m}} \qquad V = 67.1 \frac{m}{s}$$

Problem 4.68

[2]

Given: Farmer purchases 675 kg of bulk grain. The grain is loaded into a pickup truck from a hopper as shown. Grain flow is terminated when the scale reading reaches the desired gross value.

Find: The true payload.



Solution: Apply the y component of momentum equation using CV shown.

Basic equation:

$$F_{S_y} + F_{B_y} = \overset{\simeq 0(2)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} v\,\rho\, d\forall + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: (1) No net pressure force; $F_{S_y} = R_y$

(2) Neglect v inside CV

(3) Uniform flow of grain at inlet section ①

Then

$$R_y - (M_t + M_\ell)g = v_1\{-|\dot{m}|\}$$

$$v_1 = -V_1 = -\frac{\dot{m}}{\rho A}$$

or

$$R_y = (M_t + M_\ell)g + \frac{\dot{m}^2}{\rho A} \quad \text{(indicated during grain flow)}$$

Loading is terminated when

$$\frac{R_y}{g} - M_t = M_\ell + \frac{\dot{m}^2}{\rho g A} = 675 \text{ kg}$$

Thus

$$M_\ell = 675\text{ kg} - \frac{\dot{m}^2}{\rho g A}$$

$$= 675\text{ kg} - (40)^2\frac{\text{kg}^2}{\text{s}^2} \times \frac{\text{m}^3}{600\text{ kg}} \times \frac{\text{s}^2}{9.81\text{ m}} \times \frac{4}{\pi}\frac{1}{(0.3)^2\text{ m}^2}$$

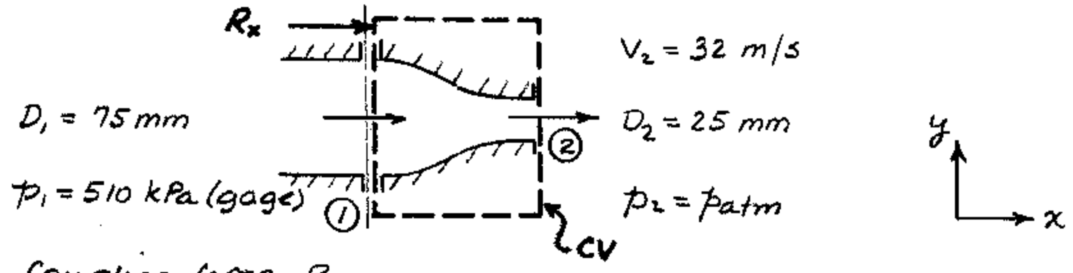
$$M_\ell = 671\text{ kg}$$

M_ℓ

Problem 4.69

[2]

Given: Water flow through a fire hose and nozzle.



Find: (a) Coupling force, R_x

(b) Indicate if in tension or compression.

Solution: Apply continuity and x component of momentum equation to inertial CV shown; use gage pressures to cancel p_{atm}.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho\, dV + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(4)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, dV + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow

(2) Uniform flow at each section

(3) Incompressible flow

(4) $F_{B_x} = 0$

Then

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} = -\rho V_1 A_1 + \rho V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \left(\frac{D_2}{D_1}\right)^2 = 32\,\frac{m}{s} \times \left(\frac{25\text{ mm}}{75\text{ mm}}\right)^2 = 3.56\ m/s$$

and

$$R_x + p_{1g} A_1 = u_1\{-|\rho V_1 A_1|\} + u_2\{|\rho V_2 A_2|\}$$

$$u_1 = V_1 \qquad u_2 = V_2$$

$$R_x = -p_{1g}A_1 - V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = -p_{1g}A_1 + \rho V_2 A_2 (V_2 - V_1)$$

$$= -510\times 10^3 \frac{N}{m^2} \times \frac{\pi}{4}(0.075)^2 m^2 + 999\,\frac{kg}{m^3} \times 32\,\frac{m}{s} \times \frac{\pi}{4}(0.025)^2 m^2 (32.0 - 3.56)\frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m}$$

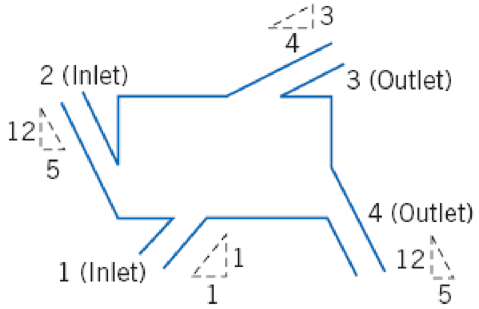
$R_x = -1.81$ kN (i.e. force on CV is to the left) $\longleftarrow$ R_x

Thus the coupling must be in tension.

Problem 4.70

[3]

4.70 Obtain expressions for the rate of change in mass of the control volume shown, as well as the horizontal and vertical forces required to hold it in place, in terms of p_1, A_1, V_1, p_2, A_2, V_2, p_3, A_3, V_3, p_4, A_4, V_4, and the constant density ρ.



Given: Flow into and out of CV

Find: Expressions for rate of change of mass, and force

Solution:

Basic equations: Mass and momentum flux

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV\!\!\!\!- + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV}u\,\rho\, dV\!\!\!\!- + \int_{CS}u\,\rho\vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV}v\,\rho\, dV\!\!\!\!- + \int_{CS}v\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Incompressible flow 2) Uniform flow

For the mass equation

$$\frac{dM_{CV}}{dt} + \sum_{CS}\left(\rho\cdot\vec{V}\cdot\vec{A}\right) = \frac{dM_{CV}}{dt} + \rho\cdot\left(-V_1\cdot A_1 - V_2\cdot A_2 + V_3\cdot A_3 + V_4\cdot A_4\right) = 0$$

$$\frac{dM_{CV}}{dt} = \rho\cdot\left(V_1\cdot A_1 + V_2\cdot A_2 - V_3\cdot A_3 - V_4\cdot A_4\right)$$

For the x momentum

$$F_x + \frac{p_1\cdot A_1}{\sqrt{2}} + \frac{5}{13}\cdot p_2\cdot A_2 - \frac{4}{5}\cdot p_3\cdot A_3 - \frac{5}{13}\cdot p_4\cdot A_4 = 0 + \frac{V_1}{\sqrt{2}}\cdot\left(-\rho\cdot V_1\cdot A_1\right) + \frac{5}{13}\cdot V_2\cdot\left(-\rho\cdot V_2\cdot A_2\right)\ \ldots$$
$$+ \frac{4}{5}\cdot V_3\cdot\left(\rho\cdot V_3\cdot A_3\right) + \frac{5}{13}\cdot V_3\cdot\left(\rho\cdot V_3\cdot A_3\right)$$

$$F_x = -\frac{p_1\cdot A_1}{\sqrt{2}} - \frac{5}{13}\cdot p_2\cdot A_2 + \frac{4}{5}\cdot p_3\cdot A_3 + \frac{5}{13}\cdot p_4\cdot A_4 + \rho\cdot\left(-\frac{1}{\sqrt{2}}\cdot V_1^2\cdot A_1 - \frac{5}{13}\cdot V_2^2\cdot A_2 + \frac{4}{5}\cdot V_3^2\cdot A_3 + \frac{5}{13}\cdot V_3^2\cdot A_3\right)$$

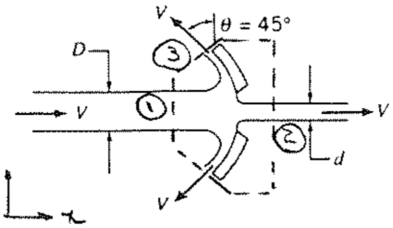
For the y momentum

$$F_y + \frac{p_1\cdot A_1}{\sqrt{2}} - \frac{12}{13}\cdot p_2\cdot A_2 - \frac{3}{5}\cdot p_3\cdot A_3 + \frac{12}{13}\cdot p_4\cdot A_4 = 0 + \frac{V_1}{\sqrt{2}}\cdot\left(-\rho\cdot V_1\cdot A_1\right) - \frac{12}{13}\cdot V_2\cdot\left(-\rho\cdot V_2\cdot A_2\right)\ \ldots$$
$$+ \frac{3}{5}\cdot V_3\cdot\left(\rho\cdot V_3\cdot A_3\right) - \frac{12}{13}\cdot V_3\cdot\left(\rho\cdot V_3\cdot A_3\right)$$

$$F_y = -\frac{p_1\cdot A_1}{\sqrt{2}} + \frac{12}{13}\cdot p_2\cdot A_2 + \frac{3}{5}\cdot p_3\cdot A_3 - \frac{12}{13}\cdot p_4\cdot A_4 + \rho\cdot\left(-\frac{1}{\sqrt{2}}\cdot V_1^2\cdot A_1 - \frac{12}{13}\cdot V_2^2\cdot A_2 + \frac{3}{5}\cdot V_3^2\cdot A_3 - \frac{12}{13}\cdot V_3^2\cdot A_3\right)$$

Problem 4.71

Given: Circular dish with central orifice struck concentrically by water jet as shown



Find: (a) Expression for force needed to hold the dish in place.
(b) Value of force for V = 5 m/s, D = 100 mm, and d = 20 mm

Plot: required force as a function of θ ($0 \le \theta \le 90^\circ$) with d/D as a parameter.

Solution:

Apply the x component of the momentum equation to the inertial CV shown.

$$\text{Basic equation:}\quad F_{S_x} + \cancelto{=0(2)}{F_{B_x}} = \cancelto{=0(3)}{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \int_{CS} u(\rho\vec{V}\cdot d\vec{A})$$

Assumptions: (1) atmospheric pressure acts on all CV surfaces
(2) $F_{B_x} = 0$
(3) steady flow
(4) uniform flow at each section
(5) incompressible flow
(6) no change in jet speed on dish: $V_1 = V_2 = V_3 = V$

Then,

$$R_x = u_1\{-|\rho V_1 A_1|\} + u_2\{|\rho V_2 A_2|\} + u_3\{|\rho V_3 A_3|\}$$

$$u_1 = V \quad A_1 = \frac{\pi D^2}{4} \quad u_2 = V \quad A_2 = \frac{\pi d^2}{4} \quad u_3 = -V\sin\theta \quad A_3 = A_1 - A_2$$

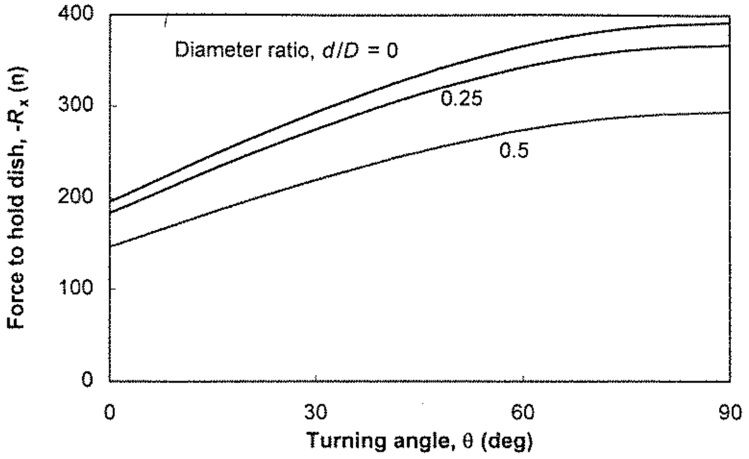
$$R_x = -\rho V^2\frac{\pi D^2}{4} + \rho V^2\frac{\pi d^2}{4} - \rho V^2\sin\theta\,\frac{\pi}{4}(D^2-d^2) = \rho V^2\frac{\pi}{4}(1+\sin\theta)(d^2-D^2)$$

$$R_x = -\rho V^2\frac{\pi D^2}{4}(1+\sin\theta)\left[1-\left(\frac{d}{D}\right)^2\right] \longleftarrow R_x$$

Evaluating for d = 25 mm

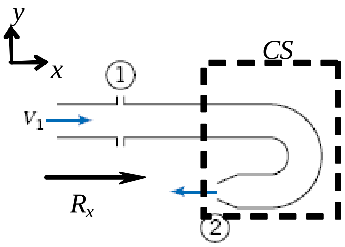
$$R_x = -\frac{\pi}{4}\times 999\frac{kg}{m^3}\times(5)^2\frac{m^2}{s^2}\times(0.10)^2 m^2\,(1+\sin 45^\circ)\left[1-\left(\frac{25}{100}\right)^2\right]\frac{N\cdot s^2}{kg\cdot m} = -314\,N \longleftarrow R_x$$

Since $R_x < 0$, it must be applied to the left. R_x is plotted as a function of θ for different values of d/D



Problem 4.72

4.72 Water is flowing steadily through the 180° elbow shown. At the inlet to the elbow the gage pressure is 15 psi. The water discharges to atmospheric pressure. Assume properties are uniform over the inlet and outlet areas: $A_1 = 4 \text{ in.}^2$, $A_2 = 1 \text{ in.}^2$, and $V_1 = 10$ ft/s. Find the horizontal component of force required to hold the elbow in place.



Given: Water flow through elbow

Find: Force to hold elbow

Solution:

Basic equation: Momentum flux in x direction for the elbow $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\mathcal{V} + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure at exit 4) Uniform flow

Hence $R_x + p_{1g}\cdot A_1 = V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) - V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$ $\qquad R_x = -p_{1g}\cdot A_1 - \rho\cdot\left(V_1^{\,2}\cdot A_1 + V_2^{\,2}\cdot A_2\right)$

From continuity $V_2\cdot A_2 = V_1\cdot A_1$ so $V_2 = V_1\cdot\frac{A_1}{A_2}$ $\qquad V_2 = 10\cdot\frac{\text{ft}}{\text{s}}\cdot\frac{4}{1}$ $\qquad V_2 = 40\,\frac{\text{ft}}{\text{s}}$

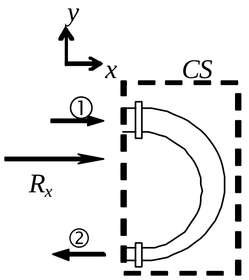
Hence $R_x = -15\cdot\frac{\text{lbf}}{\text{in}^2}\times 4\cdot\text{in}^2 - 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left[\left(10\cdot\frac{\text{ft}}{\text{s}}\right)^2\cdot 4\cdot\text{in}^2 + \left(40\cdot\frac{\text{ft}}{\text{s}}\right)^2\cdot 1\cdot\text{in}^2\right]\times\left(\frac{1\cdot\text{ft}}{12\cdot\text{in}}\right)^2\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\,\text{ft}}$ $\qquad R_x = -86.9\cdot\text{lbf}$

The force is to the left: It is needed to hold the elbow on against the high pressure, plus it generates the large change in x momentum

Problem 4.73

[2]

4.73 A 180° elbow takes in water at an average velocity of 0.8 m/s and a pressure of 350 kPa (gage) at the inlet, where the diameter is 0.2 m. The exit pressure is 75 kPa, and the diameter is 0.04 m. What is the force required to hold the elbow in place?



Given: Water flow through elbow

Find: Force to hold elbow

Solution:

Basic equation: Momentum flux in x direction for the elbow $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $R_x + p_{1g}\cdot A_1 + p_{2g}\cdot A_2 = V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) - V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$ $R_x = -p_{1g}\cdot A_1 - p_{2g}\cdot A_2 - \rho\cdot\left(V_1^2\cdot A_1 + V_2^2\cdot A_2\right)$

From continuity $V_2\cdot A_2 = V_1\cdot A_1$ so $V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\left(\frac{D_1}{D_2}\right)^2$ $V_2 = 0.8\cdot\frac{m}{s}\cdot\left(\frac{0.2}{0.04}\right)^2$ $V_2 = 20\frac{m}{s}$

Hence

$$R_x = -350\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.2\cdot m)^2}{4} - 75\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.04\cdot m)^2}{4}\ \ldots$$

$$+ -1000\cdot\frac{kg}{m^3}\times\left[\left(0.8\cdot\frac{m}{s}\right)^2\times\frac{\pi\cdot(0.2\cdot m)^2}{4} + \left(20\cdot\frac{m}{s}\right)^2\times\frac{\pi\cdot(.04\cdot m)^2}{4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

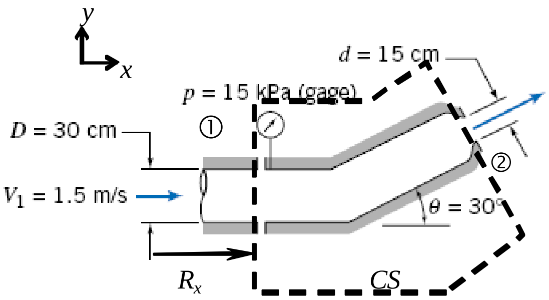
$R_x = -11.6\cdot kN$

The force is to the left: It is needed to hold the elbow on against the high pressures, plus it generates the large change in x momentum

Problem 4.74

[2]

4.74 Water flows steadily through the nozzle shown, discharging to atmosphere. Calculate the horizontal component of force in the flanged joint. Indicate whether the joint is in tension or compression.



Given: Water flow through nozzle

Find: Force to hold nozzle

Solution:

Basic equation: Momentum flux in x direction for the elbow

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $R_x + p_{1g}\cdot A_1 + p_{2g}\cdot A_2 = V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + V_2\cdot\cos(\theta)\cdot\left(\rho\cdot V_2\cdot A_2\right)$ $\qquad R_x = -p_{1g}\cdot A_1 + \rho\cdot\left(V_2^{\,2}\cdot A_2\cdot\cos(\theta) - V_1^{\,2}\cdot A_1\right)$

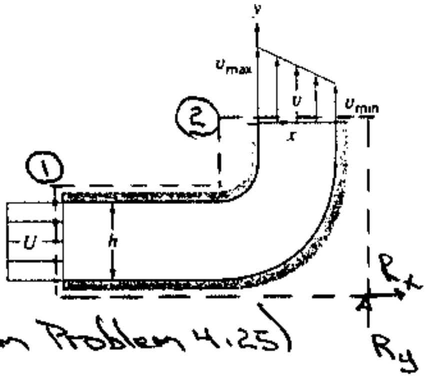
From continuity $V_2\cdot A_2 = V_1\cdot A_1$ so $V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\left(\frac{D_1}{D_2}\right)^2$ $\qquad V_2 = 1.5\cdot\frac{m}{s}\cdot\left(\frac{30}{15}\right)^2$ $\qquad V_2 = 6\cdot\frac{m}{s}$

Hence $$R_x = -15\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.3\cdot m)^2}{4} + 1000\cdot\frac{kg}{m^3}\times\left[\left(6\cdot\frac{m}{s}\right)^2\times\frac{\pi\cdot(0.15\cdot m)^2}{4}\cdot\cos(30\cdot deg) - \left(1.5\cdot\frac{m}{s}\right)^2\times\frac{\pi\cdot(.3\cdot m)^2}{4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -668\cdot N$ The joint is in tension: It is needed to hold the elbow on against the high pressure, plus it generates the large change in x momentum

Problem 4.75

[2]



Given: Two-dimensional square bend shown is a segment of a larger channel, lies in horizontal plane.

$U = 7.5\ m/s$, $h = w = 75.5\ mm$

$p_1 = 170\ kPa(abs)$, $p_2 = 130\ kPa(abs)$

$v_{max} = 2v_{min}$; $v_{min} = 5.0\ m/s$ (from Problem 4.25)

Find: Force required to hold the band in place.

Solution:

Basic equation: $\vec{F}_S + \vec{F}_B = \cancelto{0(1)}{\frac{\partial}{\partial t}\int_{CV}\vec{V}\rho\, d\forall} + \int_{CS}\vec{V}(\rho\vec{V}\cdot d\vec{A})$

Assumptions:
(1) steady flow
(2) $F_{B_x} = F_{B_y} = 0$
(3) incompressible flow
(4) atmospheric pressure acts on outside surfaces.

The x-momentum equation becomes

$$R_x + p_1A_1 + \cancelto{0(2)}{F_{B_x}} = \int_{CS} u(\rho\vec{V}\cdot d\vec{A}) = U\{-|\rho U A_1|\}$$

$$R_x = -p_1A_1 - \rho U^2 A_1 = -h^2(p_1 + \rho U^2)$$

$$R_x = -(0.0755)^2 m^2\left[(170-101)10^3\frac{N}{m^2} + 999\frac{kg}{m^3}\times(7.5)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}\right] = -714\ N$$

The y-momentum equation becomes

$$R_y - p_2A_2 + \cancelto{=0(2)}{F_{B_y}} = \int_{CS} v(\rho\vec{V}\cdot d\vec{A})$$

$$v_2 = V_2 = v_{max} - (v_{max} - v_{min})\frac{x}{h} = 2v_{min} - v_{min}\frac{x}{h} = v_{min}\left(2 - \frac{x}{h}\right)$$

$$R_y - p_2A_2 = \int_0^h v_{min}\left(2-\frac{x}{h}\right)\rho v_{min}\left(2-\frac{x}{h}\right)h\,dx$$

$$R_y = p_2A_2 + \rho v_{min}^2 h\int_0^h\left(4 - 4\frac{x}{h} + \frac{x^2}{h^2}\right)dx$$

$$= p_2A_2 + \rho v_{min}^2 h\left[4x - 2\frac{x^2}{h} + \frac{x^3}{3h^2}\right]_0^h$$

$$R_y = p_2A_2 + \rho v_{min}^2 h\left[4h - 2h + \frac{h}{3}\right] = p_2A_2 + \frac{7}{3}\rho v_{min}^2 h^2$$

$$R_y = h^2\left(p_2 + \frac{7}{3}\rho v_{min}^2\right)$$

$$= (0.0755)^2 m^2\left[(130-101)10^3\frac{N}{m^2} + \frac{7}{3}\times 999\frac{kg}{m^3}\times(5.0)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}\right]$$

$R_y = 498\ N$

$\therefore \vec{R} = -714\hat{i} + 498\hat{j}\ N$ $\vec{R}$

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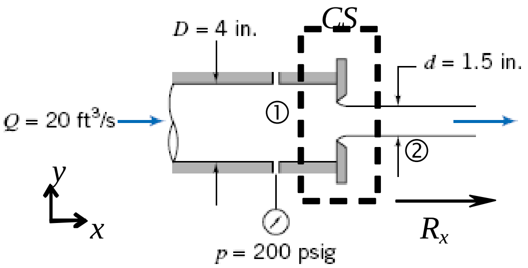
Problem 4.76

[2]

4.76 A flat plate orifice of 2 in. diameter is located at the end of a 4-in. diameter pipe. Water flows through the pipe and orifice at 20 ft^3/s. The diameter of the water jet downstream from the orifice is 1.5 in. Calculate the external force required to hold the orifice in place. Neglect friction on the pipe wall.



Given: Water flow through orifice plate

Find: Force to hold plate

Solution:

Basic equation: Momentum flux in x direction for the elbow

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\mathcal{V} + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence $R_x + p_{1g}\cdot A_1 - p_{2g}\cdot A_2 = V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$ $\quad R_x = -p_{1g}\cdot A_1 + \rho\cdot\left(V_2^{\,2}\cdot A_2 - V_1^{\,2}\cdot A_1\right)$

From continuity $Q = V_1\cdot A_1 = V_2\cdot A_2$

so $V_1 = \dfrac{Q}{A_1} = 20\cdot\dfrac{ft^3}{s}\times\dfrac{4}{\pi\cdot\left(\frac{1}{3}\cdot ft\right)^2} = 229\cdot\dfrac{ft}{s}$ and $V_2 = V_1\cdot\dfrac{A_1}{A_2} = V_1\cdot\left(\dfrac{D}{d}\right)^2 = 229\cdot\dfrac{ft}{s}\times\left(\dfrac{4}{1.5}\right)^2 = 1628\cdot\dfrac{ft}{s}$

NOTE: problem has an error: Flow rate should be 2 ft^3/s not 20 ft^3/s! We will provide answers to both

Hence $R_x = -200\cdot\dfrac{lbf}{in^2}\times\dfrac{\pi\cdot(4\cdot in)^2}{4} + 1.94\cdot\dfrac{slug}{ft^3}\times\left[\left(1628\cdot\dfrac{ft}{s}\right)^2\times\dfrac{\pi\cdot(1.5\cdot in)^2}{4} - \left(229\cdot\dfrac{ft}{s}\right)^2\times\dfrac{\pi\cdot(4\cdot in)^2}{4}\right]\times\left(\dfrac{1\cdot ft}{12\cdot in}\right)^2\times\dfrac{lbf\cdot s^2}{slug\,ft}$

$R_x = 51707\cdot lbf$

With more realistic velocities

Hence $R_x = -200\cdot\dfrac{lbf}{in^2}\times\dfrac{\pi\cdot(4\cdot in)^2}{4} + 1.94\cdot\dfrac{slug}{ft^3}\times\left[\left(163\cdot\dfrac{ft}{s}\right)^2\times\dfrac{\pi\cdot(1.5\cdot in)^2}{4} - \left(22.9\cdot\dfrac{ft}{s}\right)^2\times\dfrac{\pi\cdot(4\cdot in)^2}{4}\right]\times\left(\dfrac{1\cdot ft}{12\cdot in}\right)^2\times\dfrac{lbf\cdot s^2}{slug\cdot ft}$

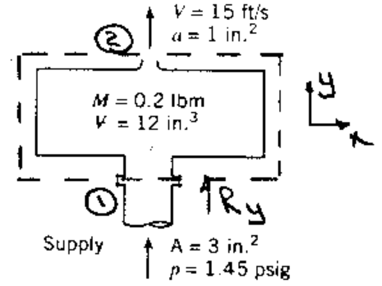
$R_x = -1970\cdot lbf$

Problem 4.77

[2]

Given: Spray system, of mass M = 0.200 lbm and internal volume $\forall = 12\text{ in.}^3$ operates under steady state conditions shown.

Find: the vertical force exerted on the supply pipe by the spray system



Solution:

Apply the y component of the momentum equation to the fixed control volume shown.

Basic Equation:

$$F_{S_y} + F_{B_y} = \overset{0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} v\,\rho\, d\forall}} + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A} \quad \text{-----} \; (1)$$

Assumptions:
- (1) steady flow
- (2) incompressible flow
- (3) uniform flow at each section
- (4) calculation of surface forces is simplified through use of gage pressures

From continuity, $0 = \cancel{\frac{\partial}{\partial t}\int \rho\, d\forall} + \int_{CS} \rho \vec{V}\cdot d\vec{A}$, for given conditions

$$0 = -|\rho V_1 A_1| + |\rho V_2 A_2| \quad \text{and} \quad V_1 = V_2\frac{A_2}{A_1} = V\frac{a}{A}$$

The momentum flux is

$$\int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A} = v_1\{-|\rho V_1 A_1|\} + v_2\{|\rho V_2 A_2|\} = V_1(-\rho V_1 A_1) + V(\rho V a)$$

$$= V\frac{a}{A}(-\rho V a) + V(\rho V a) = \rho V^2 a\left(1 - \frac{a}{A}\right)$$

Then from eq (1) we can write

$$R_y + p_{1g}A - \rho\forall g - Mg = \rho V^2 a\left(1 - \frac{a}{A}\right). \quad \text{Solving for } R_y,$$

$$R_y = -p_{1g}A + \rho\forall g + Mg + \rho V^2 a\left(1 - \frac{a}{A}\right)$$

$$= -1.45\,\frac{\text{lbf}}{\text{in}^2}\times 3\,\text{in}^2 + 1.94\,\frac{\text{slug}}{\text{ft}^3}\times 12\,\text{in}^3\times 32.2\,\frac{\text{ft}}{\text{s}^2}\times\frac{\text{ft}^3}{1728\,\text{in}^3}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$+ 0.2\,\text{lbm}\times 32.2\,\frac{\text{ft}}{\text{s}^2}\times\frac{\text{slug}}{32.2\,\text{lbm}}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$+ 1.94\,\frac{\text{slug}}{\text{ft}^3}\times(15)^2\,\frac{\text{ft}^2}{\text{s}^2}\times 1\,\text{in}^2\times\frac{\text{ft}^2}{144\,\text{in}^2}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\left(1 - \frac{1\,\text{in}^2}{3\,\text{in}^2}\right)$$

$$R_y = -1.70\ \text{lbf}$$

The force of the spray system on the supply pipe is

$K_y = -R_y = 1.70$ lbf (upward) ←

K_y

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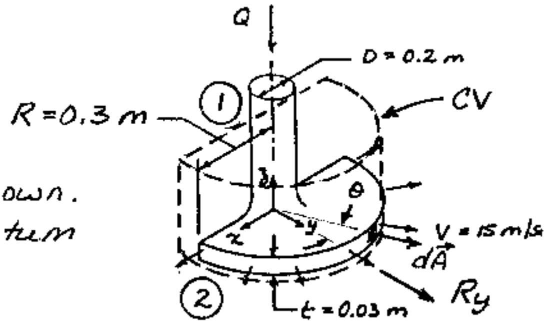
Problem 4.78

[2]

Given: Flow through semi-circular nozzle, as shown.

Find: (a) Volume flow rate

(b) y-component of force required to hold in place



Solution: Choose CV and coordinates shown. Apply continuity and momentum equation in y-direction.

Basic equations: $Q = \int_A \vec{V}\cdot d\vec{A}$

$$F_{Sy} + \overset{=0(2)}{\cancel{F_{By}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v\rho\, d\forall + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Flow uniform across exit section

(2) $F_{By} = 0$

(3) Steady flow

At section ②, $\vec{V}\cdot d\vec{A} = VRt\,d\theta$, since flow out of CV. Then

$$Q = \int_{-\pi/2}^{\pi/2} VRt\,d\theta = VRt\left[\theta\right]_{-\pi/2}^{\pi/2} = VRt\pi$$

$$Q = 15\frac{m}{s}\times 0.3\,m\times 0.03\,m\times\pi = 0.424\ m^3/s$$ Q

From momentum

$$R_y = \int_{CS} v\rho\vec{V}\cdot d\vec{A} = \int_{A_1} v_1\left\{-|\rho V_1 dA_1|\right\} + \int_{A_2} v_2\left\{+|\rho V_2 dA_2|\right\}$$

with $v_1 = 0$ $v_2 = V\cos\theta$

$$R_y = \int_{-\pi/2}^{\pi/2} V\cos\theta\,\rho VRt\,d\theta = \rho V^2Rt\left[\sin\theta\right]_{-\pi/2}^{\pi/2} = 2\rho V^2Rt$$

$$R_y = 2\times 999\frac{kg}{m^3}\times(15)^2\frac{m^2}{s^2}\times 0.3\,m\times 0.03\,m\times\frac{N\cdot s^2}{kg\cdot m} = 4.05\ kN$$ R_y

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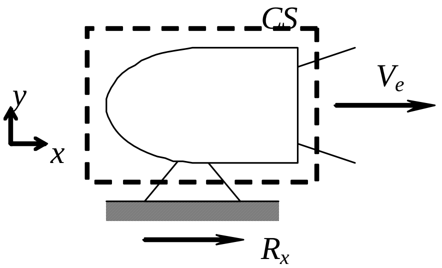
Problem 4.79

4.79 At rated thrust, a liquid-fueled rocket motor consumes 80 kg/s of nitric acid as oxidizer and 32 kg/s of aniline as fuel. Flow leaves axially at 180 m/s relative to the nozzle and at 110 kPa. The nozzle exit diameter is $D = 0.6\,\text{m}$. Calculate the thrust produced by the motor on a test stand at standard sea-level pressure.



Given: Data on rocket motor

Find: Thrust produced

Solution:

Basic equation: Momentum flux in x direction for the elbow

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Neglect change of momentum within CV 3) Uniform flow

Hence $R_x - p_{eg}\cdot A_e = V_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = m_e\cdot V_e$ $\quad R_x = p_{eg}\cdot A_e + m_e\cdot V_e$

where p_{eg} is the exit pressure (gage), m_e is the mass flow rate at the exit (software cannot render dot over m!) and V_e is the xit velocity

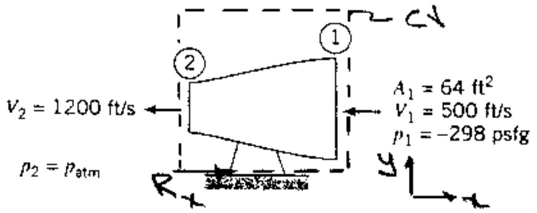
For the mass flow rate $m_e = m_{nitricacid} + m_{aniline} = 80\cdot\frac{kg}{s} + 32\cdot\frac{kg}{s}$ $\quad m_e = 112\cdot\frac{kg}{s}$

Hence $R_x = (110 - 101)\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.6\cdot m)^2}{4} + 112\cdot\frac{kg}{s}\times 180\cdot\frac{m}{s}\times\frac{N\cdot s^2}{kg\cdot m}$ $\quad R_x = 22.7\,kN$

Problem 4.80

[2]

Given: Jet engine on test stand.
Fuel enters vertically
at rate
$\dot m_{fuel} = 0.02\,\dot m_{air}$



Find: (a) Air flow rate
(b) Estimate of engine thrust.

Solution:

Apply x-component of the momentum equation to CV shown

Basic equations: $F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$

$\dot m_{air} = \rho_1 V_1 A_1 \quad , \quad \rho = p/RT$

Assumptions: (1) $F_{B_x} = 0$
(2) steady flow
(3) uniform flow at inlet and outlet sections.
(4) air behaves as ideal gas; $T = 70°F$
(5) fuel enters vertically (given).

$$\rho_1 = \frac{p_1}{RT_1} = \left(14.7\,\frac{\text{lbf}}{\text{in}^2}\times 144\,\frac{\text{in}^2}{\text{ft}^2} - 298\,\frac{\text{lbf}}{\text{ft}^2}\right)\times\frac{\text{lbm}\,°\text{R}}{53.3\,\text{ft}\cdot\text{lbf}}\times\frac{1}{530°\text{R}} = 0.0644\,\frac{\text{lbm}}{\text{ft}^3}$$

$$\dot m_{air} = \rho_1 V_1 A_1 = 0.0644\,\frac{\text{lbm}}{\text{ft}^3}\times 500\,\frac{\text{ft}}{\text{s}}\times 64\,\text{ft}^2 = 2060\,\text{lbm/s} \longleftarrow \dot m$$

From the x-momentum equation

$$R_x - p_{1g}A_1 + \overset{=0}{\cancel{p_{2g}A_2}} = u_1\{-\dot m_1\} + u_2\{\dot m_2\} + \overset{=0(5)}{\cancel{u_f\{-\dot m_f\}}}$$

$u_1 = -V_1 \quad , \quad u_2 = -V_2 \quad , \quad \dot m_2 = \dot m_1 + \dot m_f$

Also thrust $T = K_x$ (force of engine on surroundings) $= -R_x$

so

$$-T - p_{1g}A_1 = \dot m_1 V_1 - \dot m_2 V_2 = \dot m_1 V_1 - (1.02\,\dot m_1)V_2$$

$$T = \dot m_1(1.02\,V_2 - V_1) - p_{1g}A_1$$

$$T = 2060\,\frac{\text{lbm}}{\text{s}}\left[1.02\times 1200\,\frac{\text{ft}}{\text{s}} - 500\,\frac{\text{ft}}{\text{s}}\right]\times\frac{\text{slug}}{32.2\,\text{lbm}}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}} - \left(-298\,\frac{\text{lbf}}{\text{ft}^2}\right)64\,\text{ft}^2$$

$$T = 65{,}400\,\text{lbf} \longleftarrow T$$

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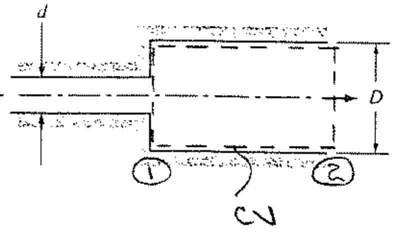
Problem 4.81

[3]

Given: Incompressible, frictionless flow through a sudden expansion as shown.

Show: Pressure rise, $\Delta p = p_2 - p_1$, is given by

$$\frac{\Delta p}{\frac{1}{2}\rho \bar{V}_1^2} = 2\left(\frac{d}{D}\right)^2\left[1-\left(\frac{d}{D}\right)^2\right]$$



Plot: the nondimensional pressure rise vs d/D to determine the optimum d/D and corresponding nondimensional pressure rise

Solution:

Apply x component of momentum equation, using fixed CV shown

Basic equation: $F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u \rho \, d\forall + \int_{CS} u \left(\rho \vec{V} \cdot d\vec{A}\right)$

Assumptions: (1) no friction, so surface force due to pressure only
(2) $F_{B_x} = 0$
(3) steady flow (4) incompressible flow (given).
(5) uniform flow at sections ① and ②
(6) uniform pressure p_1 on vertical surface of expansion

Then,

$$p_1 A_2 - p_2 A_2 = u_1\{-|\rho \bar{V}_1 A_1|\} + u_2\{|\rho \bar{V}_2 A_2|\} \qquad u_1 = \bar{V}_1, \; u_2 = \bar{V}_2$$

From continuity for uniform flow, $\dot{m} = \rho A_1 \bar{V}_1 = \rho A_2 \bar{V}_2$; $\bar{V}_2 = \bar{V}_1 \frac{A_1}{A_2}$

Thus,

$$p_2 - p_1 = \rho \bar{V}_1 \frac{A_1}{A_2} V_1 - \rho \bar{V}_1 \frac{A_1}{A_2} V_2 = \rho \bar{V}_1 \frac{A_1}{A_2}(\bar{V}_1 - \bar{V}_2)$$

$$p_2 - p_1 = \rho \bar{V}_1^2 \frac{A_1}{A_2}\left(1 - \frac{\bar{V}_2}{\bar{V}_1}\right) = \rho \bar{V}_1^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right).$$

and

$$\frac{p_2 - p_1}{\frac{1}{2}\rho \bar{V}_1^2} = 2\frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right) = 2\left(\frac{d}{D}\right)^2\left[1 - \left(\frac{d}{D}\right)^2\right]$$

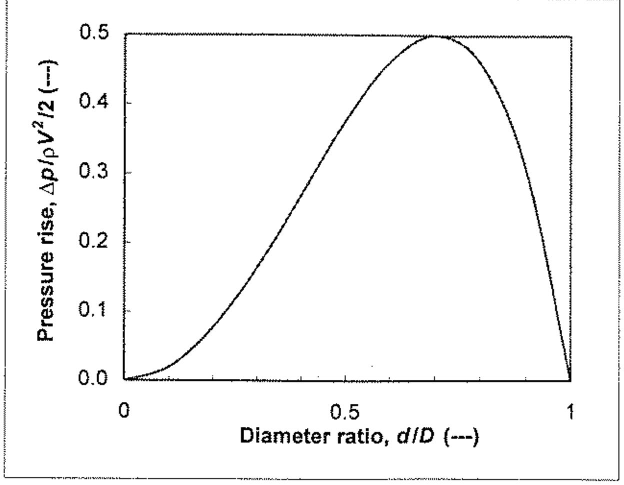
Q.E.D.

From the plot below we see that $\frac{\Delta p}{\frac{1}{2}\rho \bar{V}_1^2}$ has an optimum value of ≈ 0.5 at $d/D = 0.70$

Note: As expected

- for $d = D$, $\Delta p = 0$ for straight pipe
- for $\frac{d}{D} \to 0$, $\Delta p = 0$ for free jet

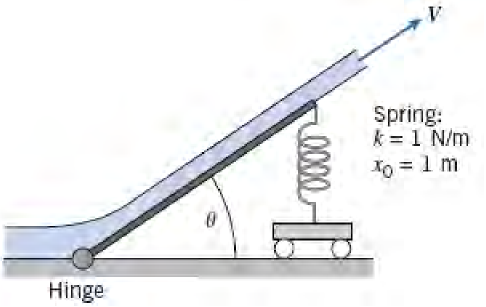
Also note that the location of section ② would have to be chosen with care to make assumption (5) reasonable.



Problem 4.82

[2]

4.82 A free jet of water with constant cross-section area 0.005 m^2 is deflected by a hinged plate of length 2 m supported by a spring with spring constant $k = 1$ N/m and uncompressed length $x_0 = 1$ m. Find and plot the deflection angle θ as a function of jet speed V. What jet speed has a deflection of 10°?



Given:

Data on flow and system geometry

Find:

Deflection angle as a function of speed; jet speed for 10º deflection

Solution:

The given data is $\rho = 999 \cdot \frac{kg}{m^3}$ $A = 0.005 \cdot m^2$ $L = 2 \cdot m$ $k = 1 \cdot \frac{N}{m}$ $x_0 = 1 \cdot m$

Governing equation:

y -momentum
$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,d\forall + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A} \qquad (4.18b)$$

Applying this to the current system in the vertical direction

$$F_{spring} = V \cdot \sin(\theta) \cdot (\rho \cdot V \cdot A)$$

But
$$F_{spring} = k \cdot x = k \cdot \left(x_0 - L \cdot \sin(\theta)\right)$$

Hence
$$k \cdot \left(x_0 - L \cdot \sin(\theta)\right) = \rho \cdot V^2 \cdot A \cdot \sin(\theta)$$

Solving for θ
$$\theta = \mathrm{asin}\left(\frac{k \cdot x_0}{k \cdot L + \rho \cdot A \cdot V^2}\right)$$

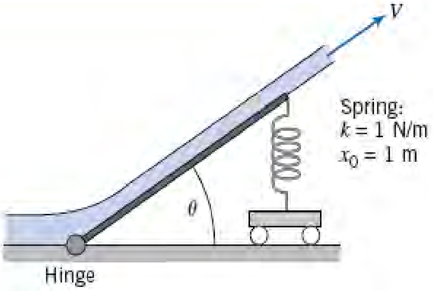
For the speed at which θ = 10º, solve
$$V = \sqrt{\frac{k \cdot \left(x_0 - L \cdot \sin(\theta)\right)}{\rho \cdot A \cdot \sin(\theta)}} \qquad V = \sqrt{\frac{1 \cdot \frac{N}{m} \cdot (1 - 2 \cdot \sin(10 \cdot deg)) \cdot m}{999 \cdot \frac{kg}{m^3} \cdot 0.005 \cdot m^2 \cdot \sin(10 \cdot deg)} \cdot \frac{kg \cdot m}{N \cdot s^2}} \qquad V = 0.867 \frac{m}{s}$$

The deflection is plotted in the corresponding *Excel* workbook, where the above velocity is obtained using *Goal Seek*

Problem 4.82

[2]

4.82 A free jet of water with constant cross-section area 0.005 m^2 is deflected by a hinged plate of length 2 m supported by a spring with spring constant $k = 1$ N/m and uncompressed length $x_0 = 1$ m. Find and plot the deflection angle θ as a function of jet speed V. What jet speed has a deflection of 10°?



Given: Data on flow and system geometry

Find: Deflection angle as a function of speed; jet speed for 10° deflection

Solution:

Solving for θ

$$\theta = \operatorname{asin}\left(\frac{k \cdot x_0}{k \cdot L + \rho \cdot A \cdot V^2}\right)$$

$\rho =$ 999 kg/m^3

$x_0 =$ 1 m

$L =$ 2 m

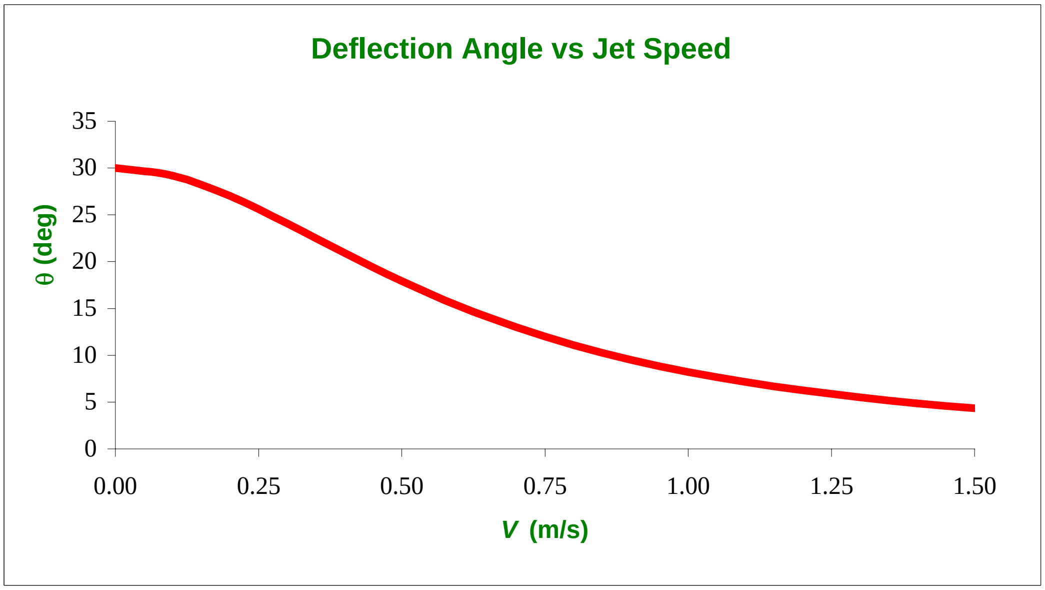
$k =$ 1 N/m

$A =$ 0.005 m^2

To find when $\theta = 10^\circ$, use *Goal Seek*

V (m/s)	θ (°)
0.867	10

V (m/s)	θ (°)
0.0	30.0
0.1	29.2
0.2	27.0
0.3	24.1
0.4	20.9
0.5	17.9
0.6	15.3
0.7	13.0
0.8	11.1
0.9	9.52
1.0	8.22
1.1	7.14
1.2	6.25
1.3	5.50
1.4	4.87
1.5	4.33

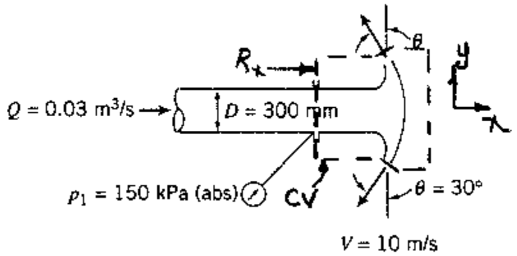


Problem 4.83

Given: Conical spray head discharging water, as shown.

Find: (a) Thickness of spray sheet at $R = 400$ mm radius.

(b) Axial force exerted on supply pipe.



Solution: Apply continuity and the x component of the momentum equation, using the CV, CS shown.

Basic equation:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) $F_{B_x} = 0$

(2) Steady flow

(3) Incompressible flow

(4) Uniform flow at each section

(5) Use gage pressure to cancel p_{atm}

From continuity,

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times 0.03\frac{m^3}{sec} \times \frac{1}{(0.3)^2\, m^2} = 0.424\, m/s$$

Assume velocity in jet sheet is constant at $V = 10$ m/s. Then

$$Q = 2\pi R t V\, ; \; t = \frac{Q}{2\pi R V} = \frac{1}{2\pi} \times 0.03\frac{m^3}{s} \times \frac{1}{0.4\, m} \times \frac{s}{10\, m} \times 1000\frac{mm}{m} = 1.19\, mm$$ t

From momentum,

$$R_x + p_{1g}A_1 = u_1\{-\rho Q\} + u_2\{+\rho Q\}$$

$$u_1 = V_1 \qquad u_2 = -V\sin\theta$$

$$R_x + p_{1g}A_1 = -(V_1 + V\sin\theta)\rho Q$$

or

$$R_x = -p_{1g}A_1 - (V_1 + V\sin\theta)\rho Q$$

$$= -(150-101)10^3\frac{N}{m^2} \times \frac{\pi}{4}(0.3)^2 m^2 - (0.424 + 10\sin 30°)\frac{m}{s} \times 999\frac{kg}{m^3} \times 0.03\frac{m^3}{s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$R_x = -3.63\, kN$$

But R_x is force on CV; force on supply pipe is K_x,

$$K_x = -R_x = 3.63\, kN \text{ (to the \underline{right})}$$ K_x

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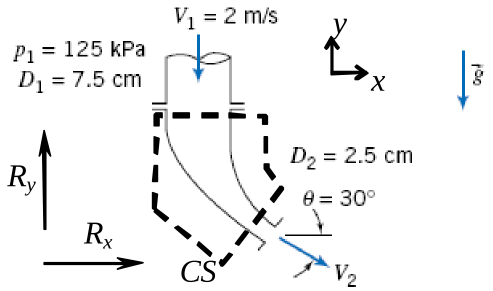
Problem 4.84

[2]

4.84 A curved nozzle assembly that discharges to the atmosphere is shown. The nozzle mass is 4.5 kg and its internal volume is 0.002 m^3. The fluid is water. Determine the reaction force exerted by the nozzle on the coupling to the inlet pipe.



Given: Data on nozzle assembly

Find: Reaction force

Solution:

Basic equation: Momentum flux in x and y directions

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,dV\!\!\!\!- + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow CV 3) Uniform flow

For x momentum
$$R_x = V_2\cdot\cos(\theta)\cdot\left(\rho\cdot V_2\cdot A_2\right) = \rho\cdot V_2^{\,2}\cdot\frac{\pi\cdot D_2^{\,2}}{4}\cdot\cos(\theta)$$

From continuity
$$A_1\cdot V_1 = A_2\cdot V_2 \qquad V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\left(\frac{D_1}{D_2}\right)^2 \qquad V_2 = 2\cdot\frac{m}{s}\times\left(\frac{7.5}{2.5}\right)^2 \qquad V_2 = 18\frac{m}{s}$$

Hence
$$R_x = 1000\cdot\frac{kg}{m^3}\times\left(18\cdot\frac{m}{s}\right)^2\times\frac{\pi}{4}\times(0.025\cdot m)^2\times\cos(30\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = 138\,N$$

For y momentum
$$R_y - p_1\cdot A_1 - W - \rho\cdot Vol\cdot g = -V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) - V_2\cdot\sin(\theta)\cdot\left(\rho\cdot V_2\cdot A_2\right)$$

$$R_y = p_1\cdot\frac{\pi\cdot D_1^{\,2}}{4} + W + \rho\cdot Vol\cdot g + \frac{\rho\cdot\pi}{4}\cdot\left(V_1^{\,2}\cdot D_1^{\,2} - V_2^{\,2}\cdot D_2^{\,2}\cdot\sin(\theta)\right)$$

where
$$W = 4.5\cdot kg\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad W = 44.1\,N \qquad Vol = 0.002\cdot m^3$$

Hence
$$R_y = 125\times 10^3\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.075\cdot m)^2}{4} + 44.1\cdot N + 1000\cdot\frac{kg}{m^3}\times 0.002\cdot m^3\times 9.81\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}\ \ldots$$
$$+1000\cdot\frac{kg}{m^3}\times\frac{\pi}{4}\times\left[\left(2\cdot\frac{m}{s}\right)^2\times(0.075\cdot m)^2 - \left(18\cdot\frac{m}{s}\right)^2\times(0.025\cdot m)^2\times\sin(30\cdot deg)\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

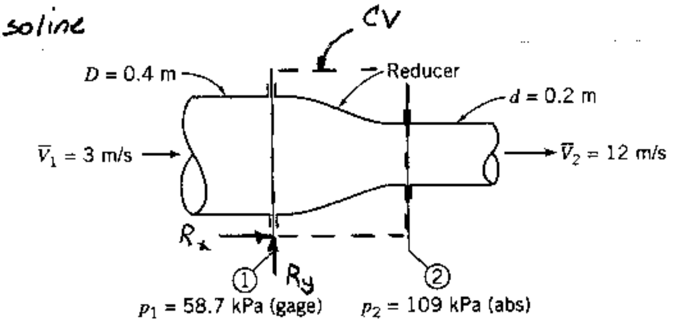
$$R_y = 554\,N$$

Problem 4.85

[3]

Given: Flow through reducer in gasoline piping system, as shown.

$M = 25\ \text{kg} \quad \forall = 0.2\ \text{m}^3$



Find: Force needed to hold reducer in place.

Solution: Apply the x and y components of the momentum equation, using the CV and coordinates shown. Use gage pressures to cancel patm.

Basic equations:

$$F_{Sx} + \overset{=0(1)}{\cancel{F_{Bx}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec V\cdot d\vec A$$

$$F_{Sy} + F_{By} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v\rho\, d\forall + \int_{CS} v\rho\vec V\cdot d\vec A$$

Assumptions: (1) $F_{Bx} = 0$

(2) Steady flow

(3) Uniform flow at each section

(4) Incompressible flow, SG = 0.72 {Table A.2, Appendix A}

From the x component of momentum,

$$R_x + p_{1g}A_1 - p_{2g}A_2 = u_1\{-|\rho V_1 A_1|\} + u_2\{+|\rho V_2 A_2|\} = (V_2 - V_1)\rho V_1 A_1$$

$$u_1 = V_1 \qquad u_2 = V_2$$

$$R_x = p_{2g}A_2 - p_{1g}A_1 + (\bar V_2 - \bar V_1)\rho \bar V_1 A_1$$

Note $\rho = SG\,\rho_{H_2O}$

$$= (109-101)10^3\,\frac{N}{m^2}\times\frac{\pi}{4}(0.2)^2 m^2 - 58.7\times10^3\,\frac{N}{m^2}\times\frac{\pi}{4}(0.4)^2 m^2$$
$$+ (12-3)\frac{m}{s}\times(0.72)1000\,\frac{kg}{m^3}\times 3\,\frac{m}{s}\times\frac{\pi}{4}(0.4)^2 m^2\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -4.68$ kN (force must be applied to left) R_x

From the y component of momentum,

$$R_y - Mg - \rho g\forall = \overset{=0}{\cancel{v_1}}\{-|\rho V_1 A_1|\} + \overset{=0}{\cancel{v_2}}\{+|\rho V_2 A_2|\}$$

$$R_y = Mg + \rho g \forall$$

$$= 25\ kg\times 9.81\,\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} + (0.72)1000\,\frac{kg}{m^3}\times 9.81\,\frac{m}{s^2}\times 0.2\ m^3\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_y = 1.66$ kN (force must be applied up) R_y

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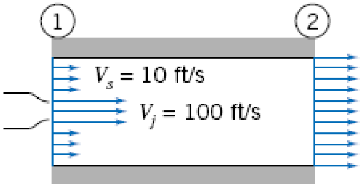
Problem 4.86

[3]

4.86 A water jet pump has jet area 0.1 ft^2 and jet speed 100 ft/s. The jet is within a secondary stream of water having speed V_s = 10 ft/s. The total area of the duct (the sum of the jet and secondary stream areas) is 0.75 ft^2. The water is thoroughly mixed and leaves the jet pump in a uniform stream. The pressures of the jet and secondary stream are the same at the pump inlet. Determine the speed at the pump exit and the pressure rise, $p_2 - p_1$.



Given:

Data on water jet pump

Find:

Speed at pump exit; pressure rise

Solution:

Basic equation: Continuity, and momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV\!\!\!\!- + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV\!\!\!\!- + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow CV 3) Uniform flow

From continuity $\quad -\rho\cdot V_s\cdot A_s - \rho\cdot V_j\cdot A_j + \rho\cdot V_2\cdot A_2 = 0 \qquad V_2 = V_s\cdot\frac{A_s}{A_2} + V_j\cdot\frac{A_j}{A_2} = V_s\cdot\left(\frac{A_2 - A_j}{A_2}\right) + V_j\cdot\frac{A_j}{A_2}$

$$V_2 = 10\cdot\frac{\text{ft}}{\text{s}}\times\left(\frac{0.75 - 0.1}{0.75}\right) + 100\cdot\frac{\text{ft}}{\text{s}}\times\frac{0.1}{0.75} \qquad V_2 = 22\,\frac{\text{ft}}{\text{s}}$$

For x momentum $\quad p_1\cdot A_2 - p_2\cdot A_2 = V_j\cdot\left(-\rho\cdot V_j\cdot A_j\right) + V_s\cdot\left(-\rho\cdot V_s\cdot A_s\right) + V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$

$$\Delta p = p_2 - p_1 = \rho\cdot\left(V_j^{\,2}\cdot\frac{A_j}{A_2} + V_s^{\,2}\cdot\frac{A_s}{A_2} - V_2^{\,2}\right)$$

$$\Delta p = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left[\left(100\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\frac{0.1}{0.75} + \left(10\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\frac{(0.75-0.1)}{0.75} - \left(22\cdot\frac{\text{ft}}{\text{s}}\right)^2\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

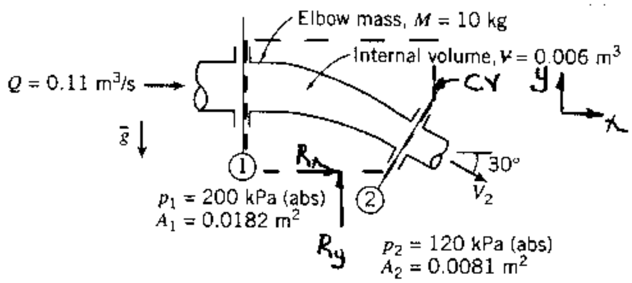
Hence $\quad \Delta p = 1816\,\frac{\text{lbf}}{\text{ft}^2} \qquad \Delta p = 12.6\,\text{psi}$

Problem 4.87

Given: Reducing elbow shown.

Fluid is water.

Find: Force components needed to keep elbow from moving.



Solution: Apply the x and y components of the momentum equation using the cs and CV shown.

Basic equations:
$$F_{S_x} + \overset{=0(4)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}\int_{CV} u\rho\, d\forall} + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$
$$F_{S_y} + F_{B_y} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}\int_{CV} v\rho\, d\forall} + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow (3) Use gage pressures
(2) Uniform flow at each section (4) x horizontal

x comp: $R_x + p_{1g}A_1 - p_{2g}A_2\cos\theta = u_1\{-|\rho Q|\} + u_2\{+|\rho Q|\}$

$u_1 = V_1$ $\quad u_2 = V_2\cos\theta$

$$R_x = (-V_1 + V_2\cos\theta)\rho Q - p_{1g}A_1 + p_{2g}A_2\cos\theta$$
$$= \left(-6.04\,\frac{\text{m}}{\text{s}} + 13.6\,\frac{\text{m}}{\text{s}}\times\cos 30°\right)999\,\frac{\text{kg}}{\text{m}^3}\times 0.11\,\frac{\text{m}^3}{\text{s}}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} - (200-101)10^3\,\frac{\text{N}}{\text{m}^2}\times 0.0182\,\text{m}^2 + (120-101)10^3\,\frac{\text{N}}{\text{m}^2}\times 0.0081\,\text{m}^2\times\cos 30°$$

$$V_1 = \frac{Q}{A_1} = 0.11\,\frac{\text{m}^3}{\text{s}}\times\frac{1}{0.0182\,\text{m}^2} = 6.04\,\frac{\text{m}}{\text{s}}$$
$$V_2 = \frac{Q}{A_2} = 0.11\,\frac{\text{m}^3}{\text{s}}\times\frac{1}{0.0081\,\text{m}^2} = 13.6\,\frac{\text{m}}{\text{s}}$$

$R_x = +631 - 1800 + 133\ \text{N} = -1040\ \text{N}$ $\qquad R_x$

y comp: $R_y + p_{2g}A_2\sin\theta - Mg - \rho\forall g = v_1\{-|\rho Q|\} + v_2\{+|\rho Q|\}$

$v_1 = 0$ $\quad v_2 = -V_2\sin\theta$

$$R_y = -V_2\sin\theta\,\rho Q + Mg + \rho\forall g - p_{2g}A_2\sin\theta$$
$$= -13.6\,\frac{\text{m}}{\text{s}}\times\sin 30°\times 999\,\frac{\text{kg}}{\text{m}^3}\times 0.11\,\frac{\text{m}^3}{\text{s}}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} + 10\,\text{kg}\times 9.81\,\frac{\text{m}}{\text{s}^2}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$
$$+ 999\,\frac{\text{kg}}{\text{m}^3}\times 0.006\,\text{m}^3\times 9.81\,\frac{\text{m}}{\text{s}^2}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} - (120-101)10^3\,\frac{\text{N}}{\text{m}^2}\times 0.0081\,\text{m}^2\times\sin 30°$$

$R_y = -747 + 98.1 + 58.8 - 77 = -667\ \text{N}$ $\qquad R_y$

{ R_x and R_y are the horizontal and vertical components of force that must be supplied by the adjacent pipes to keep the elbow (the control volume) from moving. }

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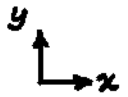
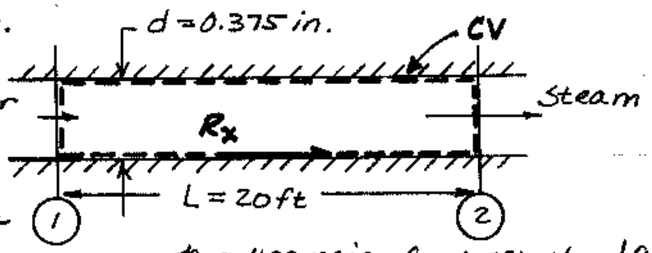
Problem 4.88

[3]

Given: Monotube boiler, as shown.





$\dot{m} = 0.3$ lbm/s $\quad p_1 = 500$ psia

$p_2 = 400$ psig, $\rho_2 = 0.024$ slug/ft³

Find: Magnitude and direction of force exerted by fluid on tube.

Solution: Apply the x component of the momentum equation, using the CV and coordinates shown.

Basic equation:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) $F_{B_x} = 0$

(2) Steady flow

(3) Uniform flow at each section

(4) Use gage pressures to cancel patm

From continuity,

$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$; A = constant, so $\rho_1 V_1 = \rho_2 V_2$. Thus

$$V_1 = \frac{\dot{m}}{\rho_1 A} = 0.3\,\frac{\text{lbm}}{\text{s}} \times \frac{\text{ft}^3}{1.94\,\text{slug}} \times \frac{4}{\pi}\frac{1}{(0.375)^2\,\text{in.}^2} \times \frac{\text{slug}}{32.2\,\text{lbm}} \times 144\frac{\text{in.}^2}{\text{ft}^2} = 6.26\ \text{ft/s}$$

and

$$V_2 = V_1\frac{\rho_1}{\rho_2} = 6.26\,\frac{\text{ft}}{\text{s}} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times \frac{\text{ft}^3}{0.024\,\text{slug}} = 506\ \text{ft/s}$$

From momentum,

$$R_x + p_{1g}A_1 - p_{2g}A_2 = u_1\{-\dot{m}\} + u_2\{+\dot{m}\} = (V_2 - V_1)\,\dot{m}$$

$$u_1 = V_1 \qquad u_2 = V_2$$

$$R_x = (p_{2g} - p_{1g})A + (V_2 - V_1)\,\dot{m}$$

$$= [400 - (500 - 14.7)]\,\frac{\text{lbf}}{\text{in.}^2} \times \frac{\pi}{4}(0.375)^2\,\text{in.}^2 + (506 - 6.26)\frac{\text{ft}}{\text{s}} \times 0.3\,\frac{\text{lbm}}{\text{s}} \times \frac{\text{slug}}{32.2\,\text{lbm}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$R_x = -4.77\ \text{lbf}$$

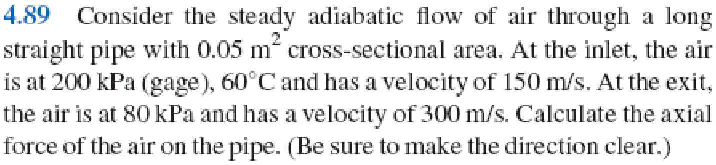
But R_x is force on CV; force on pipe is K_x,

$$K_x = -R_x = 4.77\ \text{lbf} \quad (\text{to } \underline{\text{right}})$$

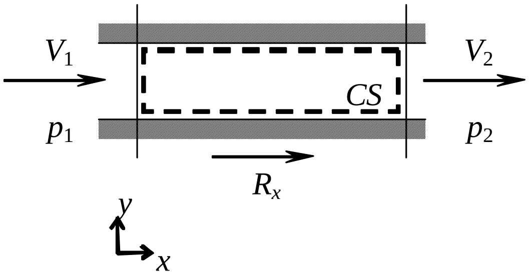
K_x

Problem 4.89

[3]



4.89 Consider the steady adiabatic flow of air through a long straight pipe with 0.05 m^2 cross-sectional area. At the inlet, the air is at 200 kPa (gage), 60°C and has a velocity of 150 m/s. At the exit, the air is at 80 kPa and has a velocity of 300 m/s. Calculate the axial force of the air on the pipe. (Be sure to make the direction clear.)



Given: Data on adiabatic flow of air

Find: Force of air on pipe

Solution:

Basic equation: Continuity, and momentum flux in x direction, plus ideal gas equation

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad p = \rho\cdot R\cdot T$$

Assumptions: 1) Steady flow 2) Ideal gas CV 3) Uniform flow

From continuity $-\rho_1\cdot V_1\cdot A_1 + \rho_2\cdot V_2\cdot A_2 = 0$ $\qquad \rho_1\cdot V_1\cdot A = \rho_2\cdot V_2\cdot A \qquad \rho_1\cdot V_1 = \rho_2\cdot V_2$

For x momentum $R_x + p_1\cdot A - p_2\cdot A = V_1\cdot\left(-\rho_1\cdot V_1\cdot A\right) + V_2\cdot\left(\rho_2\cdot V_2\cdot A\right) = \rho_1\cdot V_1\cdot A\cdot\left(V_2 - V_1\right)$

$$R_x = \left(p_2 - p_1\right)\cdot A + \rho_1\cdot V_1\cdot A\cdot\left(V_2 - V_1\right)$$

For the air $\rho_1 = \dfrac{P_1}{R_{air}\cdot T_1}$ $\qquad \rho_1 = (200 + 101)\times 10^3\cdot\dfrac{N}{m^2}\times\dfrac{kg\cdot K}{286.9\cdot N\cdot m}\times\dfrac{1}{(60 + 273)\cdot K}$ $\qquad \rho_1 = 3.15\dfrac{kg}{m^3}$

$$R_x = (80 - 200)\times 10^3\cdot\frac{N}{m^2}\times 0.05\cdot m^2 + 3.15\cdot\frac{kg}{m^3}\times 150\cdot\frac{m}{s}\times 0.05\cdot m^2\times(300 - 150)\cdot\frac{m}{s}\times\frac{N\cdot s^2}{kg\cdot m}$$

Hence $R_x = -2456\,N$

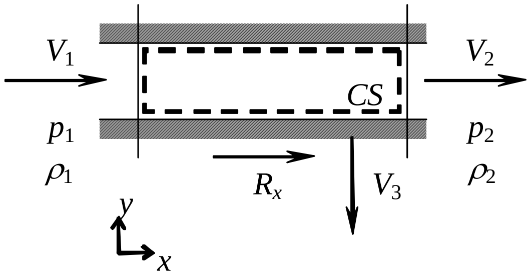
This is the force of the pipe on the air; the pipe is opposing flow. Hence the force of the air on the pipe is $F_{pipe} = -R_x$

$F_{pipe} = 2456\,N$ $\qquad$ The air is dragging the pipe to the right

Problem 4.90

[3]

4.90 A gas flows steadily through a heated porous pipe of constant 0.15 m^2 cross-sectional area. At the pipe inlet, the absolute pressure is 400 kPa, the density is 6 kg/m^3, and the mean velocity is 170 m/s. The fluid passing through the porous wall leaves in a direction normal to the pipe axis, and the total flow rate through the porous wall is 20 kg/s. At the pipe outlet, the absolute pressure is 300 kPa and the density is 2.75 kg/m^3. Determine the axial force of the fluid on the pipe.



Given: Data on heated flow of gas

Find: Force of gas on pipe

Solution:

Basic equation: Continuity, and momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\mathcal{V} + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\mathcal{V} + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad p = \rho\cdot R\cdot T$$

Assumptions: 1) Steady flow 2) Uniform flow

From continuity $-\rho_1\cdot V_1\cdot A_1 + \rho_2\cdot V_2\cdot A_2 + m_3 = 0$ $\qquad V_2 = V_1\cdot\frac{\rho_1}{\rho_2} - \frac{m_3}{\rho_2\cdot A}$ $\qquad$ where m_3 = 20 kg/s is the mass leaving through the walls (the software does not allow a dot)

$$V_2 = 170\cdot\frac{m}{s}\times\frac{6}{2.75} - 20\cdot\frac{kg}{s}\times\frac{m^3}{2.75\cdot kg}\times\frac{1}{0.15\cdot m^2} \qquad V_2 = 322\frac{m}{s}$$

For x momentum $R_x + p_1\cdot A - p_2\cdot A = V_1\cdot\left(-\rho_1\cdot V_1\cdot A\right) + V_2\cdot\left(\rho_2\cdot V_2\cdot A\right)$

$$R_x = \left[\left(p_2 - p_1\right) + \rho_2\cdot V_2^{\,2} - \rho_1\cdot V_1^{\,2}\right]\cdot A$$

$$R_x = \left[(300 - 400)\times 10^3\cdot\frac{N}{m^2} + \left[2.75\cdot\frac{kg}{m^3}\times\left(322\cdot\frac{m}{s}\right)^2 - 6\cdot\frac{kg}{m^3}\times\left(170\cdot\frac{m}{s}\right)^2\right]\times\frac{N\cdot s^2}{kg\cdot m}\right]\times 0.15\cdot m^2$$

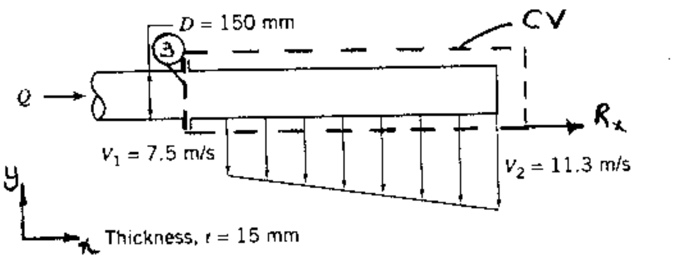
Hence $R_x = 1760\,N$

Problem 4.91

[3]

Given: Water flow discharging nonuniformly from slot, as shown.

$p_{1g} = 30$ kPa



Find: (a) Volume flow rate.
(b) Forces to hold pipe.

Solution: Apply x, y components of momentum, using the CV, CS shown.

Basic equations:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}\ ;\quad F_{S_y} + \overset{=0(1)}{\cancel{F_{B_y}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v\rho\, d\forall + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) $F_{Bx} = F_{By} = 0$
(2) Steady flow
(3) Uniform flow at inlet section
(4) Use gage pressures to cancel p_{atm}

From continuity,

$$Q = \bar{V}A = \tfrac{1}{2}(V_1 + V_2)Lt = \tfrac{1}{2}(7.5+11.3)\frac{m}{s}\times 1\,m\times 0.015\,m = 0.141\ m^3/s$$ Q

$$V_3 = \frac{Q}{A_3} = 0.141\frac{m^3}{s}\times\frac{4}{\pi}\frac{1}{(0.15)^2 m^2} = 7.98\ m/s$$

From x momentum, since flow leaves slot vertically ($u = 0$),

$$R_x + p_{3g}A_3 = u_3\{-\rho Q\} = -V_3\rho Q\ ;\quad R_x = -p_{3g}A_3 - V_3\rho Q$$

$$R_x = -30\times10^3\frac{N}{m^2}\times\frac{\pi}{4}(0.15)^2 m^2 - 7.98\frac{m}{s}\times 999\frac{kg}{m^3}\times 0.141\frac{m^3}{s}\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -1.65$ kN (to left) R_x

From y momentum, since $v_3 = 0$,

$$R_y = \overset{=0}{\cancel{v_3}}\{-\rho Q\} + \int_0^L v\rho V t\, dx = -\rho t\int_0^L \left(V_1 + \frac{V_2 - V_1}{L}x\right)^2 dx$$

$$= -\rho t\left[V_1^2 x + 2V_1\left(\frac{V_2-V_1}{L}\right)\frac{x^2}{2} + \left(\frac{V_2-V_1}{L}\right)^2\frac{x^3}{3}\right]_0^L$$

$$= -999\frac{kg}{m^3}\times 0.015\,m\left[(7.5)^2\frac{m^2}{s^2} + 7.5\frac{m}{s}\times(11.3-7.5)\frac{m}{s}\times\frac{1}{1m}\times(1)^2 m^2 + (11.3-7.5)^2\frac{m^2}{s^2}\times\frac{1}{(1)^2 m^2}\times\frac{(1)^3 m^3}{3}\right]$$

$R_y = -1.34$ kN (down) R_y

{A moment also would be required at the coupling.}

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Problem 4.92

[3]

Given: Steady flow of water through square channel shown

$v_{max} = 2v_{min}$, $U = 7.5\ m/s$, $p_1 = 185\ kPa\,(gage)$, $p_2 = p_{atm}$

$M_c = 2.05\ kg$, $\forall_c = 0.00355\ m^3$, $h = 75.5\ mm = w$

Find: Force exerted by channel assembly on the supply duct.

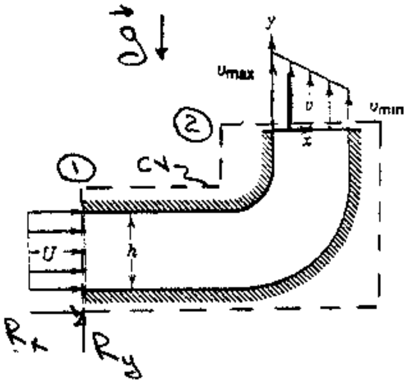
Solution: Apply conservation of mass + momentum equations to the CV shown.

Basic equations:

$$0 = \cancelto{0(1)}{\frac{\partial}{\partial t}\int_{CV}\rho\,d\forall} + \int_{CS}\rho\vec{V}\cdot d\vec{A} \qquad (1)$$

$$F_{S_x} + F_{B_x} = \cancelto{0(1)}{\frac{\partial}{\partial t}\int_{CV}u\,\rho\,d\forall} + \int_{CS}u\,\rho\vec{V}\cdot d\vec{A} \qquad (2)$$

$$F_{S_y} + F_{B_y} = \cancelto{0(1)}{\frac{\partial}{\partial t}\int_{CV}v\,\rho\,d\forall} + \int_{CS}v\,\rho\vec{V}\cdot d\vec{A} \qquad (3)$$



Assumptions:
(1) steady flow (2) incompressible flow
(3) uniform flow at inlet
(4) use gage pressures.

From continuity, $0 = \vec{V}_1\cdot\vec{A}_1 + \int\vec{V}_2\cdot d\vec{A}_2 = -Uwh + \int_0^h v\,w\,dx$

$\therefore Uh = \int_0^h v\,dx = \int_0^h v_{min}\left(2-\frac{x}{h}\right)dx = v_{min}\left[2x-\frac{x^2}{2h}\right]_0^h = \frac{3}{2}v_{min}h$

and $v_{min} = \frac{2}{3}U = \frac{2}{3}\times 7.5\frac{m}{s} = 5.0\ m/s$

From Eq. 2,

$R_x + p_{1g}A_1 = u_1\{-\rho U A_1\} + \int_0^h \cancelto{0}{u_2}\,\rho\,v_{min}\left(2-\frac{x}{h}\right)w\,dx = -\rho U^2 A_1$

$R_x = -p_{1g}A_1 - \rho U^2 A_1 = -(185-101)10^3\frac{N}{m^2}\times(0.0755)^2 m^2 - 999\frac{kg}{m^3}\times(7.5)^2\frac{m^2}{s^2}\times(0.0755)^2 m^2$

$R_x = -479\ N - 320\frac{kg\cdot m}{s^2}\cdot\frac{N\cdot s^2}{kg\cdot m} = -479\,N - 320\,N = -799\ N$

$K_x = -R_x = 799\ N$ (on supply duct to the right) $\longleftarrow K_x$

From Eq. 3,

$R_y - M_c g - \rho\forall g = \cancelto{v_1=0}{v_1}\{-\rho U A_1\} + \int_0^h v_2\{\rho v_2 w\,dx\}$

$R_y - M_c g - \rho\forall g = \int_0^h v_{min}\left(2-\frac{x}{h}\right)\rho\,v_{min}\left(2-\frac{x}{h}\right)w\,dx$

$= \rho v_{min}^2 w\int_0^h\left(4-4\frac{x}{h}+\frac{x^2}{h^2}\right)dx$

$= \rho v_{min}^2 w\left[4x-2\frac{x^2}{h}+\frac{x^3}{3h^2}\right]_0^h = \rho v_{min}^2 w h\,\frac{7}{3}$

$\therefore R_y = \left[2.05\,kg\times 9.81\frac{m}{s^2} + 999\frac{kg}{m^3}\times(0.00355\,m^3)\,9.81\frac{m}{s^2} + \frac{7}{3}\times 999\frac{kg}{m^3}\times(5.0)^2\frac{m^2}{s^2}\times(0.0755)^2 m^2\right]\frac{N\cdot s^2}{kg\cdot m}$

$R_y = (20.1 + 34.8 + 332)\,N = 387\ N$ (on CV)

$K_y = -R_y = -387\ N$ (on supply duct, down) $\longleftarrow K_y$

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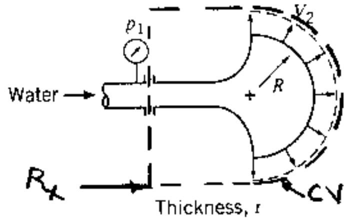
Problem 4.93

[3]

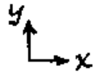
Given: Nozzle discharging flat, radial sheet of water, as shown.

Find: Axial force of nozzle
on coupling.

$D_1 = 35\ mm$



Solution: Apply the x component of momentum, using CV and coordinates shown.



Basic equation:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) $F_{B_x} = 0$
(2) Steady flow
(3) Uniform flow at each section
(4) Use gage pressure to cancel p_{atm}

From continuity

$$Q = V_1 A_1 = V_2 A_2 = V_2 \pi R t = \pi \times 10\frac{m}{sec} \times 0.05\,m \times 0.0015\,m = 0.00236\ m^3/s$$

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times 0.00236\frac{m^3}{sec} \times \frac{1}{(0.035)^2 m^2} = 2.45\ m/s$$

From momentum

$$\left\{ \text{Note } A_1 = \frac{\pi D_1^2}{4} = 0.000962\ m^2 \right\}$$

$$R_x + p_{1g} A_1 = u_1\{-\rho Q\} + \int_{A_2} u_2 \rho V_2\, dA_2$$

$$u_1 = V_1 \qquad u_2 = V_2 \cos\theta\,;\ dA_2 = Rt\,d\theta$$

$$\int_{A_2} = \int_{-\pi/2}^{\pi/2} V_2\cos\theta\,\rho V_2 R t\,d\theta = 2\rho V_2^2 R t \int_0^{\pi/2} \cos\theta\,d\theta = 2\rho V_2^2 R t$$

Thus

$$R_x = -p_{1g} A_1 - V_1 \rho Q + 2\rho V_2^2 R t$$

$$= -(150-101)10^3 \frac{N}{m^2} \times 0.000962\,m^2 - 2.45\frac{m}{sec} \times 999\frac{kg}{m^3} \times 0.00236\frac{m^3}{sec} \times \frac{N\cdot sec^2}{kg\cdot m}$$

$$+ 2 \times 999\frac{kg}{m^3} \times (10)^2 \frac{m^2}{sec^2} \times 0.05\,m \times 0.0015\,m \times \frac{N\cdot sec^2}{kg\cdot m}$$

$$R_x = -37.9\ N$$

But R_x is force on CV; force on coupling is K_x,

$K_x = -R_x = 37.9\ N$ (to right) $\qquad K_x$

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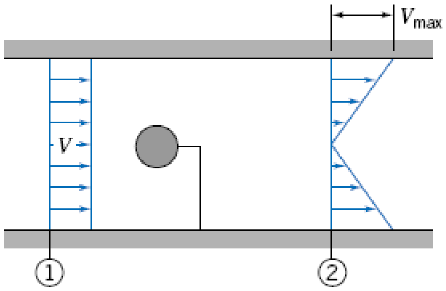
Problem 4.94

[4]

4.94 A small round object is tested in a 0.75-m diameter wind tunnel. The pressure is uniform across sections ① and ②. The upstream pressure is 30 mm H_2O (gage), the downstream pressure is 15 mm H_2O (gage), and the mean air speed is 12.5 m/s. The velocity profile at section ② is linear; it varies from zero at the tunnel centerline to a maximum at the tunnel wall. Calculate (a) the mass flow rate in the wind tunnel, (b) the maximum velocity at section ②, and (c) the drag of the object and its supporting vane. Neglect viscous resistance at the tunnel wall.



Given:

Data on flow in wind tunnel

Find:

Mass flow rate in tunnel; Maximum velocity at section 2; Drag on object

Solution:

Basic equations: Continuity, and momentum flux in x direction; ideal gas equation

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad p = \rho\cdot R\cdot T$$

Assumptions: 1) Steady flow 2) Uniform density at each section

From continuity $\quad m_{flow} = \rho_1\cdot V_1\cdot A_1 = \rho_1\cdot V_1\cdot\frac{\pi\cdot D_1^{\,2}}{4}$ $\quad$ where m_{flow} is the mass flow rate

We take ambient conditions for the air density $\quad \rho_{air} = \frac{p_{atm}}{R_{air}\cdot T_{atm}} \qquad \rho_{air} = 101000\cdot\frac{N}{m^2}\times\frac{kg\cdot K}{286.9\cdot N\cdot m}\times\frac{1}{293\cdot K} \qquad \rho_{air} = 1.2\frac{kg}{m^3}$

$$m_{flow} = 1.2\cdot\frac{kg}{m^3}\times 12.5\cdot\frac{m}{s}\times\frac{\pi\cdot(0.75\cdot m)^2}{4} \qquad m_{flow} = 6.63\frac{kg}{s}$$

Also

$$m_{flow} = \int \rho_2\cdot u_2\, dA_2 = \rho_{air}\cdot\int_0^R V_{max}\cdot\frac{r}{R}\cdot 2\cdot\pi\cdot r\, dr = \frac{2\cdot\pi\cdot\rho_{air}\cdot V_{max}}{R}\cdot\int_0^R r^2\, dr = \frac{2\cdot\pi\cdot\rho_{air}\cdot V_{max}\cdot R^2}{3}$$

$$V_{max} = \frac{3\cdot m_{flow}}{2\cdot\pi\cdot\rho_{air}\cdot R^2} \qquad V_{max} = \frac{3}{2\cdot\pi}\times 6.63\cdot\frac{kg}{s}\times\frac{m^3}{1.2\cdot kg}\times\left(\frac{1}{0.375\cdot m}\right)^2 \qquad V_{max} = 18.8\frac{m}{s}$$

For x momentum $\quad R_x + p_1\cdot A - p_2\cdot A = V_1\cdot\left(-\rho_1\cdot V_1\cdot A\right) + \int \rho_2\cdot u_2\cdot u_2\, dA_2$

$$R_x = \left(p_2 - p_1\right)\cdot A - V_1\cdot m_{flow} + \int_0^R \rho_{air}\cdot\left(V_{max}\cdot\frac{r}{R}\right)^2\cdot 2\cdot\pi\cdot r\, dr = \left(p_2 - p_1\right)\cdot A - V_1\cdot m_{flow} + \frac{2\cdot\pi\cdot\rho_{air}\cdot V_{max}^{\;2}}{R^2}\cdot\int_0^R r^3\, dr$$

$$R_x = \left(p_2 - p_1\right)\cdot A - V_1\cdot m_{flow} + \frac{\pi}{2}\cdot\rho_{air}\cdot V_{max}^{\;2}\cdot R^2$$

We also have $\quad p_1 = \rho\cdot g\cdot h_1 \qquad p_1 = 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 0.03\cdot m \qquad p_1 = 294\,Pa \qquad p_2 = \rho\cdot g\cdot h_2 \qquad p_2 = 147\cdot Pa$

Hence

$$R_x = (147 - 294)\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.75\cdot m)^2}{4} + \left[-6.63\cdot\frac{kg}{s}\times 12.5\cdot\frac{m}{s} + \frac{\pi}{2}\times 1.2\cdot\frac{kg}{m^3}\times\left(18.8\cdot\frac{m}{s}\right)^2\times(0.375\cdot m)^2\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -54\,N$ $\quad$ The drag on the object is equal and opposite $\quad F_{drag} = -R_x \qquad F_{drag} = 54.1\,N$

Problem 4.95

4.95 The horizontal velocity in the wake behind an object in an air stream of velocity U is given by

$$u(r) = U\left[1 - \cos^2\left(\frac{\pi r}{2}\right)\right] \qquad |r| \leq 1$$

$$u(r) = U \qquad |r| > 1$$

where r is the non-dimensional radial coordinate, measured perpendicular to the flow. Find an expression for the drag on the object.

Given:
Data on wake behind object

Find:
An expression for the drag

Solution:

Governing equation:

Momentum
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{\mathrm{CV}} u\,\rho\,d\forall + \int_{\mathrm{CS}} u\,\rho\,\vec{V}\cdot d\vec{A} \qquad (4.18a)$$

Applying this to the horizontal motion

$$-F = U\cdot\left(-\rho\cdot\pi\cdot 1^2\cdot U\right) + \int_0^1 u(r)\cdot\rho\cdot 2\cdot\pi\cdot r\cdot u(r)\,dr \qquad F = \pi\,\rho\cdot\left(U^2 - 2\cdot\int_0^1 r\cdot u(r)^2\,dr\right)$$

$$F = \pi\,\rho\cdot U^2\cdot\left[1 - 2\cdot\int_0^1 r\cdot\left(1 - \cos\left(\frac{\pi\cdot r}{2}\right)^2\right)^2 dr\right]$$

$$F = \pi\,\rho\cdot U^2\cdot\left(1 - 2\cdot\int_0^1 r - 2\cdot r\cdot\cos\left(\frac{\pi\cdot r}{2}\right)^2 + r\cdot\cos\left(\frac{\pi\cdot r}{2}\right)^4 dr\right)$$

Integrating and using the limits
$$F = \pi\,\rho\cdot U^2\cdot\left[1 - \left(\frac{3}{8} + \frac{2}{\pi^2}\right)\right] \qquad F = \left(\frac{5\cdot\pi}{8} - \frac{2}{\pi}\right)\cdot\rho\cdot U^2$$

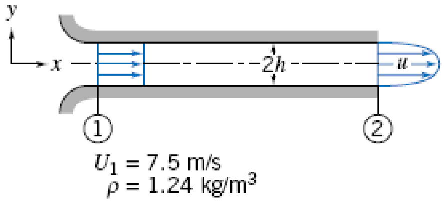
Problem 4.96

[4]

4.96 An incompressible fluid flows steadily in the entrance region of a two-dimensional channel of height $2h$. The uniform velocity at the channel entrance is $U_1 = 7.5$ m/s. The velocity distribution at a section downstream is

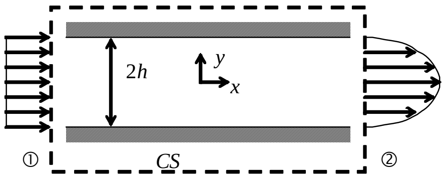
$$\frac{u}{u_{\max}} = 1 - \left[\frac{y}{h}\right]^2$$

Evaluate the maximum velocity at the downstream section. Calculate the pressure drop that would exist in the channel if viscous friction at the walls could be neglected.



Given: Data on flow in 2D channel

Find: Maximum velocity; Pressure drop



Solution:

Basic equations: Continuity, and momentum flux in x direction; ideal gas equation

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV\!\!\!\!- + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV\!\!\!\!- + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Neglect frition

From continuity $$-\rho\cdot U_1\cdot A_1 + \int \rho\cdot u_2\, dA = 0$$

$$U_1\cdot 2\cdot h\cdot w = w\cdot\int_{-h}^{h} u_{max}\cdot\left(1-\frac{y^2}{h^2}\right)dy = w\cdot u_{max}\cdot\left[[h-(-h)]-\left[\frac{h}{3}-\left(-\frac{h}{3}\right)\right]\right] = w\cdot u_{max}\cdot\frac{4}{3}\cdot h$$

Hence $u_{max} = \frac{3}{2}\cdot U_1$ $\qquad u_{max} = \frac{3}{2}\times 7.5\cdot\frac{m}{s}$ $\qquad u_{max} = 11.3\frac{m}{s}$

For x momentum $$p_1\cdot A - p_2\cdot A = V_1\cdot\left(-\rho_1\cdot V_1\cdot A\right) + \int \rho_2\cdot u_2\cdot u_2\, dA_2$$ Note that there is no R_x (no friction)

$$p_1 - p_2 = -\rho\cdot U_1^2 + \frac{w}{A}\cdot\int_{-h}^{h}\rho\cdot u_{max}^2\cdot\left(1-\frac{y^2}{h^2}\right)^2 dy = -\rho\cdot U_1^2 + \frac{\rho\cdot u_{max}^2}{h}\cdot\left[2\cdot h - 2\cdot\left(\frac{2}{3}\cdot h\right) + 2\cdot\left(\frac{1}{5}\cdot h\right)\right]$$

$$\Delta p = p_1 - p_2 = -\rho\cdot U_1^2 + \frac{8}{15}\cdot\rho\cdot u_{max}^2 = \rho\cdot U_1\cdot\left[\frac{8}{15}\cdot\left(\frac{3}{2}\right)^2 - 1\right] = \frac{1}{5}\cdot\rho\cdot U_1$$

Hence $\Delta p = \frac{1}{5}\times 1.24\cdot\frac{kg}{m^3}\times\left(7.5\cdot\frac{m}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m}$ $\qquad \Delta p = 14\,Pa$

Problem 4.97

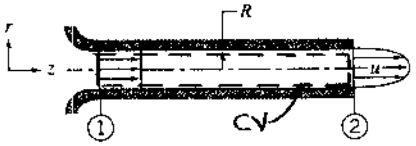
[3]___

Given: Incompressible flow in entrance region of circular tube of radius, R.

$$u_2 = u_{max}\Big[1-\Big(\frac{r}{R}\Big)^2\Big]$$

Find: (a) Maximum velocity at Section②.
(b) Pressure drop if viscous friction could be neglected.

Solution: Apply continuity and the x direction momentum equations. Use the CV and CS shown.



U_1 = 30 ft/s
ρ = 0.075 lbm/ft³

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(3)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}u\rho\, d\forall + \int_{CS}u\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Uniform flow at Section①
(3) $F_{B_x} = 0$
(4) Neglect friction at duct wall
(5) Incompressible flow

Then

$$0 = \{-|\rho U_1 \pi R^2|\} + \int_0^R \rho u_{max}\Big[1-\Big(\frac{r}{R}\Big)^2\Big]2\pi r\, dr$$

or $\pi\rho U_1 R^2 = 2\pi\rho u_{max}R^2\int_0^1\Big[1-\Big(\frac{r}{R}\Big)^2\Big]\Big(\frac{r}{R}\Big)d\Big(\frac{r}{R}\Big) = 2\pi\rho u_{max}R^2\Big[\frac{1}{2}\Big(\frac{r}{R}\Big)^2 - \frac{1}{4}\Big(\frac{r}{R}\Big)^4\Big]_0^1$

Thus $u_{max} = 2U_1 = 2\times 30\frac{ft}{s} = 60\ ft/s$ $\longleftarrow$ u_{ma}

From the momentum equation,

$$p_1\pi R^2 - p_2\pi R^2 = u_1\{-\rho U_1\pi R^2\} + \int_0^R u_2\rho u_2\, dA_2 = -\rho U_1\pi R^2 + \rho u_{max}^2 2\pi R^2\int_0^1\Big[1-\Big(\frac{r}{R}\Big)^2\Big]\Big(\frac{r}{R}\Big)d($$

$$u_1 = U_1 \qquad u_2 = u_{max}\Big[1-\Big(\frac{r}{R}\Big)^2\Big]$$

or

$$p_1 - p_2 = -\rho U_1^2 + 2\rho u_{max}^2\int_0^1(1-\eta^2)^2\eta\, d\eta\ ;\ \eta = \frac{r}{R}$$

But $\int_0^1(1-\eta^2)\eta\, d\eta = \int_0^1(1-2\eta^2+\eta^4)\eta\, d\eta = \frac{1}{2}\eta^2 - \frac{1}{2}\eta^4 + \frac{1}{6}\eta^6\Big]_0^1 = \frac{1}{6}$

and $u_{max}^2 = (2U_1)^2 = 4U_1^2$, so

$$p_1 - p_2 = -\rho U_1^2 + \frac{8}{6}\rho U_1^2 = \rho U_1^2\Big(\frac{4}{3}-1\Big) = \frac{1}{3}\rho U_1^2$$

$$= \frac{1}{3}\times 0.075\,\frac{lbm}{ft^3}\times(30)^2\frac{ft^2}{s^2}\times\frac{slug}{32.2\,lbm}\times\frac{lbf\cdot s^2}{slug\cdot ft}$$

$p_1 - p_2 = 0.699\ lbf/ft^2$ $\longleftarrow$ $p_1 - p_2$

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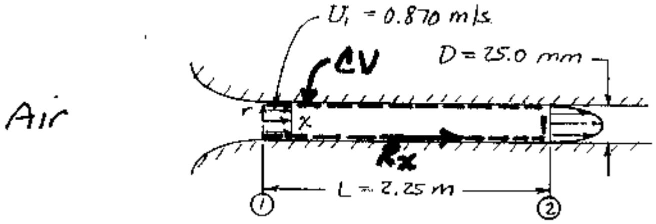
Problem 4.98

[3]

Given: Uniform flow into, fully developed flow from duct shown.



$\frac{u(r)}{U_c} = 1 - \left(\frac{r}{R}\right)^2$ at ②

$p_1 - p_2 = 1.92\ N/m^2$

Find: Total force exerted by tube on the flowing air.

Solution: Apply continuity and momentum to CV, CS shown.

Basic equations: $$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(4)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow (3) Uniform flow at inlet
(2) Incompressible flow (4) $F_{B_x} = 0$

Then $$0 = \{-|\rho U_1 A_1|\} + \int_{②}\rho u\, dA = -\rho U_1 \pi R^2 + \int_0^R \rho U_c\left[1-\left(\tfrac{r}{R}\right)^2\right]2\pi r\, dr$$

$$0 = -\rho U_1 \pi R^2 + 2\rho\pi R^2 U_c \int_0^1 (1-\lambda^2)\lambda\, d\lambda \quad \text{or} \quad 0 = -U_1 + 2U_c\left[\frac{\lambda^2}{2} - \frac{\lambda^4}{4}\right]_0^1$$

Thus $0 = -U_1 + \frac{1}{2}U_c$ or $U_c = 2U_1$ $(\lambda = r/R)$

From momentum $$R_x + p_1A_1 - p_2A_2 = u_1\{-|\rho U_1 A_1|\} + \int_{②} u_2\{+\rho u_2\, dA_2\}$$

$u_1 = U_1$ $u_2 = U_c\left[1-\left(\frac{r}{R}\right)^2\right]$

so $$\int_{②} = \int_0^R U_c\left[1-\left(\tfrac{r}{R}\right)^2\right]\rho U_c\left[1-\left(\tfrac{r}{R}\right)^2\right]2\pi r\, dr = 2\pi\rho U_c^2 R^2\int_0^1 (1-\lambda^2)(1-\lambda^2)\lambda\, d\lambda$$
$$= 2\pi\rho U_c^2 R^2 \int_0^1 (1-2\lambda^2+\lambda^4)\lambda\, d\lambda = 2\pi\rho U_c^2 R^2\left[\frac{\lambda^2}{2} - \frac{\lambda^4}{2} + \frac{\lambda^6}{6}\right]_0^1 = \frac{1}{3}\pi\rho U_c^2 R^2$$

Substituting,

$$R_x + (p_1 - p_2)\pi R^2 = -\pi\rho U_1^2 R^2 + \frac{1}{3}\pi\rho U_c^2 R^2 = -\pi\rho U_1^2 R^2 + \frac{1}{3}\pi\rho(2U_1)^2 R^2$$

$$R_x = -(p_1 - p_2)\frac{\pi D^2}{4} + \frac{1}{3}\rho U_1^2 \frac{\pi D^2}{4}$$

$$= -1.92\frac{N}{m^2}\times\frac{\pi}{4}(0.025)^2 m^2 + \frac{1}{3}\times 1.23\frac{kg}{m^3}\times(0.870)^2\frac{m^2}{s^2}\times\frac{\pi}{4}(0.025)^2 m^2\times\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -7.90\times10^{-4}$ N (to left on CV, since <0)

R_x

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Problem 4.99

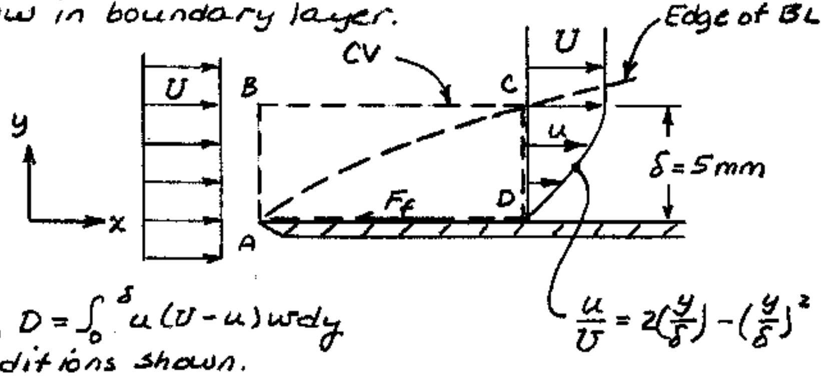
[3]

Given: Incompressible flow in boundary layer.

$w = 0.6\ m$

$U = 30\ m/s$

$\rho = 1.24\ kg/m^3$



Edge of BL
U
CV
U
B
C
y
x
u
$\delta = 5mm$
F_f
D
A
$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Find: (a) Show that drag, $D = \int_0^\delta u(U-u)w\,dy$

(b) Evaluate for conditions shown.

Solution: Apply continuity and x component of momentum using CV shown.

Basic equations: $0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

$$F_{S_x} + \overset{=0(3)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\,d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow

(2) No net pressure force; $F_{S_x} = -F_f$

(3) $F_{B_x} = 0$

(4) Uniform flow at section Ⓐⓑ

(5) Incompressible flow

Then from continuity

$$0 = \{-|\rho U w\delta|\} + \left\{\left|\int_0^\delta \rho u w\,dy\right|\right\} + \dot{m}_{BC}\ ;\ \delta = \int_0^\delta dy\ ;\ \dot{m}_{BC} = \rho\int_0^\delta (U-u)w\,dy$$

From momentum

$$-F_f = U\{-|\rho U w\delta|\} + \left\{\int_0^\delta \rho u^2 w\,dy\,\Big|\right\} + U\dot{m}_{BC} = \rho\int_0^\delta\left[-U^2 + u^2 + U(U-u)\right]w\,dy$$

$$\text{Drag} = F_f = \int_0^\delta \rho u(U-u)w\,dy \longleftarrow \text{Drag}$$

At CD, $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$; $dy = \delta\, d\left(\frac{y}{\delta}\right) = \delta\, d\eta$

$$\text{Drag} = \int_0^\delta \rho U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\left(U - U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right)w\,dy = \rho U^2 w\delta\int_0^1 (2\eta - \eta^2)(1 - 2\eta + \eta^2)\,d\eta$$

$$= \rho U^2 w\delta\int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)\,d\eta = \rho U^2 w\delta\left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1$$

$$= \frac{2}{15}\rho U^2 w\delta$$

$$\text{Drag} = \frac{2}{15} \times 1.24\,\frac{kg}{m^3} \times (30)^2\frac{m^2}{s^2} \times 0.6\,m \times 0.005\,m \times \frac{N\cdot s^2}{kg\cdot m} = 0.446\,N \longleftarrow \text{Drag}$$

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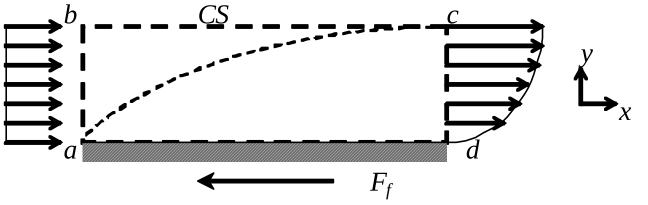
Problem 4.100

[4]

4.100 Air at standard conditions flows along a flat plate. The undisturbed freestream speed is $U_0 = 30$ ft/s. At $L = 6$ in. downstream from the leading edge of the plate, the boundary-layer thickness is $\delta = 0.1$ in. The velocity profile at this location is

$$\frac{u}{U_0} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left[\frac{y}{\delta}\right]^3$$

Calculate the horizontal component of force per unit width required to hold the plate stationary.



Given: Data on flow of boundary layer

Find: Force on plate per unit width

Solution:

Basic equations: Continuity, and momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\,dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV}u\,\rho\,dV + \int_{CS}u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No net pressure force

From continuity $\quad -\rho\cdot U_0\cdot w\cdot\delta + m_{bc} + \int_0^\delta \rho\cdot u\cdot w\,dy = 0 \quad$ where m_{bc} is the mass flow rate across bc (Note: sotware cannot render a dot!)

Hence $\quad m_{bc} = \int_0^\delta \rho\cdot\left(U_0 - u\right)\cdot w\,dy$

For x momentum $\quad -F_f = U_0\cdot\left(-\rho\cdot U_0\cdot w\cdot\delta\right) + U_0\cdot m_{bc} + \int_0^\delta u\cdot\rho\cdot u\cdot w\,dy = \int_0^\delta \left[-U_0^{\,2} + u^2 + U_0\cdot\left(U_0 - u\right)\right]\cdot w\,dy$

Then the drag force is $\quad F_f = \int_0^\delta \rho\cdot u\cdot\left(U_0 - u\right)\cdot w\,dy = \int_0^\delta \rho\cdot U_0^{\,2}\cdot\frac{u}{U_0}\cdot\left(1 - \frac{u}{U_0}\right)dy$

But we have $\quad \frac{u}{U_0} = \frac{3}{2}\cdot\eta - \frac{1}{2}\cdot\eta^3 \quad$ where we have used substitution $\quad y = \delta\cdot\eta$

$$\frac{F_f}{w} = \int_0^{\eta=1} \rho\cdot U_0^{\,2}\cdot\delta\cdot\frac{u}{U_0}\cdot\left(1 - \frac{u}{U_0}\right)d\eta = \rho\cdot U_0^{\,2}\cdot\delta\cdot\int_0^1 \left(\frac{3}{2}\cdot\eta - \frac{9}{4}\cdot\eta^2 - \frac{1}{2}\cdot\eta^3 + \frac{3}{2}\cdot\eta^4 - \frac{1}{4}\cdot\eta^6\right)d\eta$$

$$\frac{F_f}{w} = \rho\cdot U_0^{\,2}\cdot\delta\cdot\left(\frac{3}{4} - \frac{3}{4} - \frac{1}{8} + \frac{3}{10} - \frac{1}{28}\right) = 0.139\cdot\rho\cdot U_0^{\,2}\cdot\delta$$

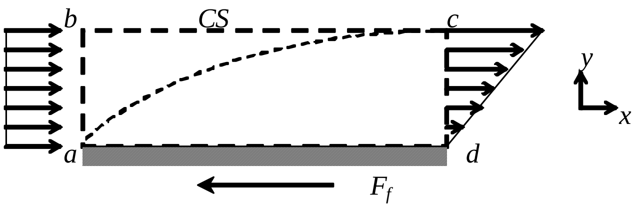
Hence $\quad \frac{F_f}{w} = 0.139 \times 0.002377\cdot\frac{slug}{ft^3} \times \left(30\cdot\frac{ft}{s}\right)^2 \times \frac{0.1}{12}\cdot ft \times \frac{lbf\cdot s^2}{slug\cdot ft} \quad$ (using standard atmosphere density)

$$\frac{F_f}{w} = 2.48 \times 10^{-3}\cdot\frac{lbf}{ft}$$

Problem 4.101

[4]

4.101 Air at standard conditions flows along a flat plate. The undisturbed freestream speed is $U_0 = 20$ m/s. At $L = 0.4$ m downstream from the leading edge of the plate, the boundary-layer thickness is $\delta = 2$ mm. The velocity profile at this location is approximated as $u/U_0 = y/\delta$. Calculate the horizontal component of force per unit width required to hold the plate stationary.



Given: Data on flow of boundary layer

Find: Force on plate per unit width

Solution:

Basic equations: Continuity, and momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No net pressure force

From continuity $$-\rho\cdot U_0\cdot w\cdot\delta + m_{bc} + \int_0^\delta \rho\cdot u\cdot w\,dy = 0$$ where m_{bc} is the mass flow rate across bc (Note: sotware cannot render a dot!)

Hence $$m_{bc} = \int_0^\delta \rho\cdot\left(U_0 - u\right)\cdot w\,dy$$

For x momentum $$-F_f = U_0\cdot\left(-\rho\cdot U_0\cdot w\cdot\delta\right) + U_0\cdot m_{bc} + \int_0^\delta u\cdot\rho\cdot u\cdot w\,dy = \int_0^\delta \left[-U_0^2 + u^2 + U_0\cdot\left(U_0 - u\right)\right]\cdot w\,dy$$

Then the drag force is $$F_f = \int_0^\delta \rho\cdot u\cdot\left(U_0 - u\right)\cdot w\,dy = \int_0^\delta \rho\cdot U_0^2\cdot\frac{u}{U_0}\cdot\left(1 - \frac{u}{U_0}\right)dy$$

But we have $\frac{u}{U_0} = \frac{y}{\delta}$ where we have used substitution $y = \delta\cdot\eta$

$$\frac{F_f}{w} = \int_0^{\eta=1} \rho\cdot U_0^2\cdot\delta\cdot\frac{u}{U_0}\cdot\left(1 - \frac{u}{U_0}\right)d\eta = \rho\cdot U_0^2\cdot\delta\cdot\int_0^1 \eta\cdot(1-\eta)\,d\eta$$

$$\frac{F_f}{w} = \rho\cdot U_0^2\cdot\delta\cdot\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}\cdot\rho\cdot U_0^2\cdot\delta$$

Hence $$\frac{F_f}{w} = \frac{1}{6}\times 1.225\cdot\frac{kg}{m^3}\times\left(20\cdot\frac{m}{s}\right)^2\times\frac{2}{1000}\cdot m\times\frac{N\cdot s^2}{kg\cdot m}$$ (using standard atmosphere density)

$$\frac{F_f}{w} = 0.163\cdot\frac{N}{m}$$

Problem 4.102

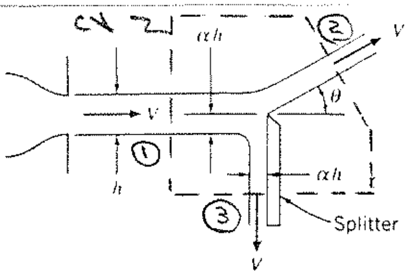
Given: Flow of flat jet over sharp-edged splitter plate, as shown.
Neglect friction force between water and plate;
$0 \le \alpha \le 0.5$.

Find: (a) Expression for angle θ as a function of α.
(b) Expression for force R_x needed to hold splitter plate in-place.

Plot: both θ and R_x as functions of θ.

Solution

Apply the x and y components of the momentum equation to the CV shown.



Basic equations:

$$F_{S_x} + \overset{=0(3)}{\cancel{F_{B_x}}} = \overset{=0(4)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}} + \int_{CS} u(\rho\vec{V}\cdot d\vec{A})$$

$$\overset{=0(2)}{\cancel{F_{S_y}}} + \overset{=0(3)}{\cancel{F_{B_y}}} = \overset{=0(4)}{\cancel{\frac{\partial}{\partial t}\int_{CV} v\rho\, d\forall}} + \int_{CS} v(\rho\vec{V}\cdot d\vec{A})$$

Assumptions: (1) no net pressure forces on CV.
(2) no friction in y direction, so $F_{S_y} = 0$
(3) neglect body forces
(4) steady flow
(5) no change in jet speed: $V_1 = V_2 = V_3 = V$
(6) uniform flow at each section

Then from the y equation

$$0 = v_1\{-|\rho V_1 A_1|\} + v_2\{|\rho V_2 A_2|\} + v_3\{|\rho V_3 A_3|\}$$

$v_1 = 0$ $\quad v_2 = V\sin\theta$ $\quad v_3 = -V$

{w is depth} $\quad A_1 = wh$ $\quad A_2 = w(1-\alpha)h$ $\quad A_3 = w\alpha h$

$$0 = 0 + \rho V^2 \sin\theta\, w(1-\alpha)h - \rho V^2 w\alpha h$$

Thus

$$\sin\theta = \frac{\rho V^2 w\alpha h}{\rho V^2 w(1-\alpha)h} = \frac{\alpha}{(1-\alpha)}\ ;\quad \theta = \sin^{-1}\left(\frac{\alpha}{1-\alpha}\right) \longleftarrow \theta(\alpha)$$

From the x equation

$$R_x = u_1\{-|\rho_1 V_1 A_1|\} + u_2\{|\rho_2 V_2 A_2|\} + u_3\{|\rho_3 V_3 A_3|\}$$

$u_1 = V$ $\quad u_2 = V\cos\theta$ $\quad u_3 = 0$

$$R_x = -\rho V^2 wh + \rho V^2 \cos\theta\, w(1-\alpha)h = \rho V^2 wh\left[\cos\theta(1-\alpha) - 1\right]$$

But
$$\cos\theta = (1-\sin^2\theta)^{1/2} = \left(1 - \frac{\alpha^2}{(1-\alpha)^2}\right)^{1/2} = \frac{(1-2\alpha)^{1/2}}{(1-\alpha)}$$

$$\therefore R_x = -\rho V^2 wh\left[1 - (1-2\alpha)^{1/2}\right] \longleftarrow (R_x < 0;\ \text{so to left})\quad R_x$$

{ Check: $\alpha = 0$, $R_x = 0$ ✓ ; $\alpha = \frac{1}{2}$, $R_x = -\rho V^2 wh$ ✓ }

Problem 4.102

Plots of: $\theta = \sin^{-1}\left(\frac{\alpha}{1-\alpha}\right)$ and

$$\frac{R_x}{R_{x\,\alpha=0.5}} = 1 - \sqrt{1-2\alpha}$$

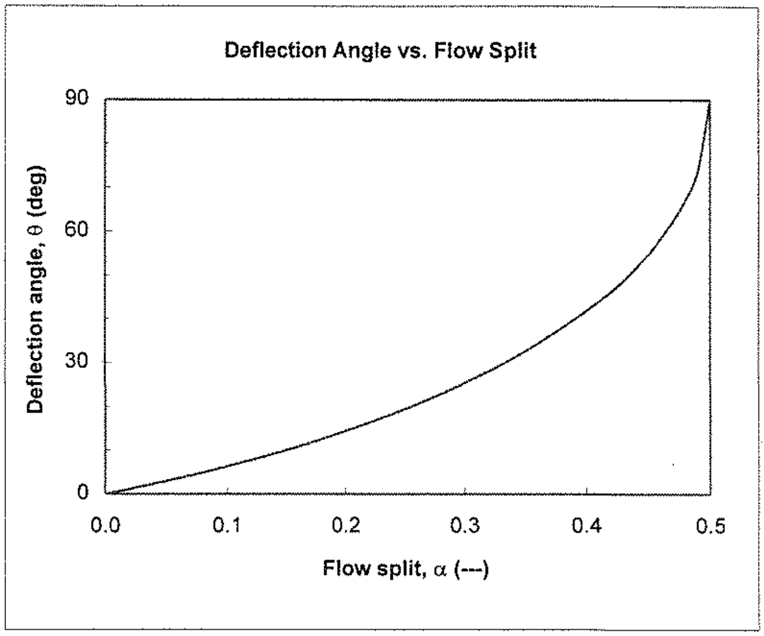
are presented below

Flow deflection by sharp-edged splitter:

α = fraction of jet intercepted by splitter

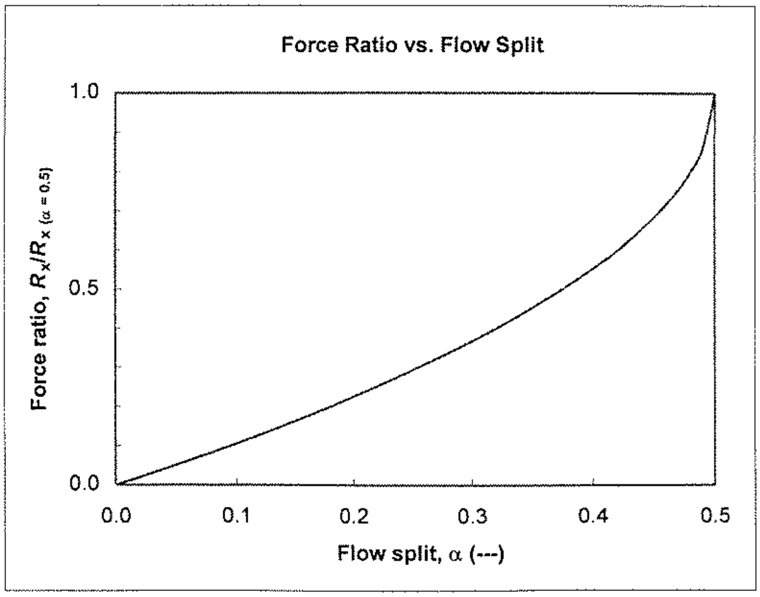
Calculated Results: Deflection angle

α (---)	θ (deg)
0	0
0.05	3.02
0.10	6.38
0.15	10.2
0.20	14.5
0.25	19.5
0.30	25.4
0.35	32.6
0.40	41.8
0.425	47.7
0.45	54.9
0.470	62.5
0.480	67.4
0.490	73.9
0.50	90.0



Calculated Results: Force over maximum force

α (---)	$R_x/R_{x\,(\alpha=0.5)}$
0	0
0.05	0.0513
0.10	0.106
0.15	0.163
0.20	0.225
0.25	0.293
0.30	0.368
0.35	0.452
0.40	0.553
0.425	0.613
0.45	0.684
0.470	0.755
0.480	0.800
0.490	0.859
0.50	1.00



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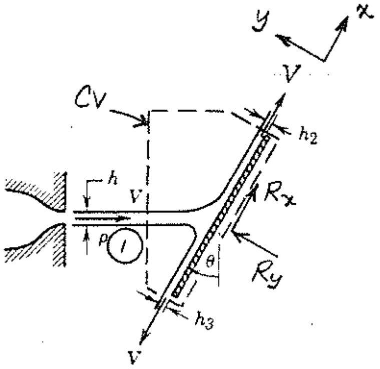
Problem 4.103

[4]

Given: Plane jet striking inclined plate, as shown. No frictional force along plate surface.

Find: (a) Expression for h_2/h as a function of θ.
(b) Plot of results.
(c) Comment on limiting cases, $\theta = 0$ and $\theta = 90°$

Solution: Apply the x component of the momentum equation using the CV and coordinates shown.



Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) No surface force on CV
(2) Neglect body forces
(3) Steady flow
(4) No change in jet speed: $V_1 = V_2 = V_3 = V$
(5) Uniform flow at each section

From continuity for uniform incompressible flow $0 = -\rho V w h + \rho V w h_2 + \rho V w h_3$
or

$$h = h_2 + h_3 = h_1 \quad \text{or} \quad h_3 = h_1 - h_2$$

From momentum

$$0 = u_1\{-|\rho V w h_1|\} + u_2\{+|\rho V w h_2|\} + u_3\{+|\rho V w h_3|\}$$

$$u_1 = V\sin\theta \qquad u_2 = V \qquad u_3 = -V$$

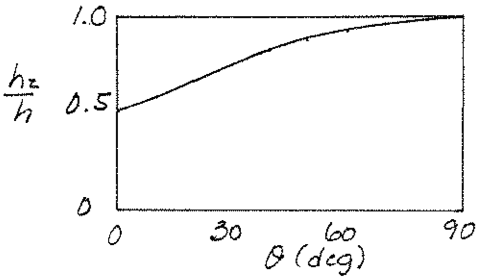
$$0 = -\rho V^2 \sin\theta\, w h_1 + \rho V^2 w h_2 - \rho V^2 w h_3$$

Substituting from continuity and simplifying

$$0 = -\sin\theta\, h_1 + h_2 - (h_1 - h_2) \quad \text{so} \quad \frac{h_2}{h} = \frac{h_2}{h_1} = \frac{1+\sin\theta}{2}$$

$\frac{h_2}{h}$

Plot:



At $\theta = 0$, $\frac{h_2}{h} = 0.5$; flow is equally split when plate is ⊥ to jet.

At $\theta = 90°$, $\frac{h_2}{h} = 1.0$; plate has no effect on flow.

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Problem 4.104

[4]

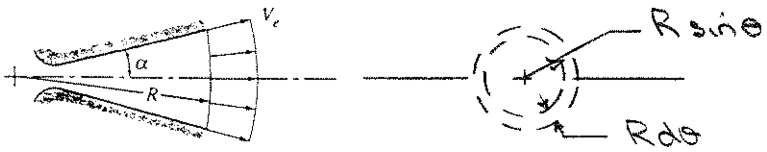
Given: Model gas flow in a propulsion nozzle as a spherical source; V_e = constant

Find: (a) Expression for axial thrust, T_a, and compare to the 1-D approximation, $T = \dot{m}V_e$
(b) Percent error for $\alpha = 15°$.

Plot: the percent error vs α for $0 \le \alpha \le 22.5°$.

Solution:

Apply definitions $\dot{m} = \int_A \rho V dA$, $T_a = \int_A u\rho V dA$. Use spherically symmetric flow.



The mass flow rate is [assuming $\rho_e \ne \rho_e(\theta)$]

$$\dot{m} = \int_A \rho V dA = \int_0^\alpha \rho_e V_e (2\pi R\sin\theta) R\,d\theta = 2\pi\rho_e V_e R^2 \Big[-\cos\theta\Big]_0^\alpha = 2\pi\rho_e V_e R^2(1-\cos\alpha)$$

The one-dimensional approximation for thrust is then

$$T = \dot{m}V_e = 2\pi\rho_e V_e^2 R^2(1-\cos\alpha) \longleftarrow T_{1\text{-}D}$$

The axial thrust is given by

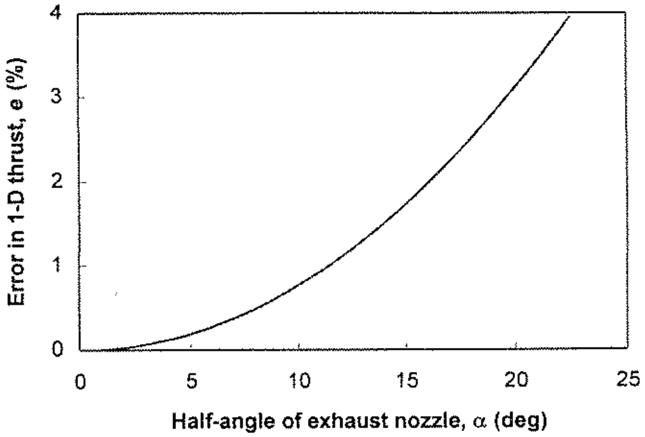
$$T_a = \int u\rho V dA = \int_0^\alpha V_e\cos\theta\,\rho_e V_e(2\pi R\sin\theta)R\,d\theta = 2\pi\rho_e V_e^2 R^2\int_0^\alpha \sin\theta\cos\theta\,d\theta$$

$$T_a = 2\pi\rho_e V_e^2 R^2\left[\frac{\sin^2\theta}{2}\right]_0^\alpha = \pi\rho_e V_e^2 R^2\sin^2\alpha \longleftarrow T_a$$

The error in the one-dimensional approximation is

$$e = \frac{T_{1\text{-}D}-T_a}{T_a} = \frac{T_{1\text{-}D}}{T_a}-1 = \frac{2\pi\rho_e V_e^2R^2(1-\cos\alpha)}{\pi\rho_e V_e^2R^2\sin^2\alpha}-1 = \frac{2(1-\cos\alpha)}{\sin^2\alpha}-1 \quad ---(1)$$

The percent error is plotted as a function of α



For $\alpha = 15°$

$$e_{15} = \frac{2(1-\cos 15)}{\sin^2 15°}-1$$

$$e_{15} = 0.0173 \text{ or } 1.73\% \longleftarrow e_{15}$$

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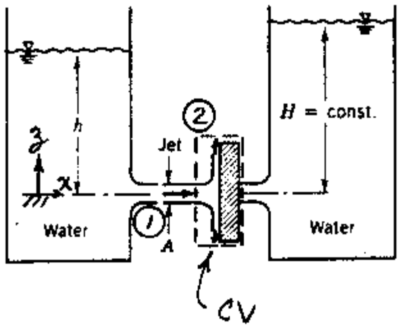
Problem *4.105

[4]

Given: Tanks and flat plate shown.



Find: Minimum height h needed to keep plate in place.

Solution: Apply Bernoulli and momentum equations. Use CV enclosing plate, as shown.

Basic equations: $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$

$$F_{S_x} + \overset{=0(5)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}} + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Flow along a streamline
(4) No friction
(5) $F_{Bx} = 0$

Apply Bernoulli from water surface to jet

$$\cancel{\frac{p}{\rho}} + \overset{\ll V^2}{\cancel{\frac{V_s^2}{2}}} + gh = \cancel{\frac{p}{\rho}} + \frac{V^2}{2} + g(0) \quad \text{so that} \quad V^2 = 2gh \quad \text{or} \quad V = \sqrt{2gh}$$

From fluid statics, $p_{3g} = \rho g H$

From momentum

$$-p_{3g}A = -\rho g H A = u_1\{-\rho V A\} + u_2\{+\rho V A\} = -\rho V^2 A$$

$$u_1 = V \qquad u_2 = 0$$

Thus, using Bernoulli,

$$\rho g H A = \rho V^2 A = \rho(2gh)A = 2\rho g h A$$

and

$$h = \frac{H}{2} \longleftarrow$$

h

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Problem *4.106

[4]

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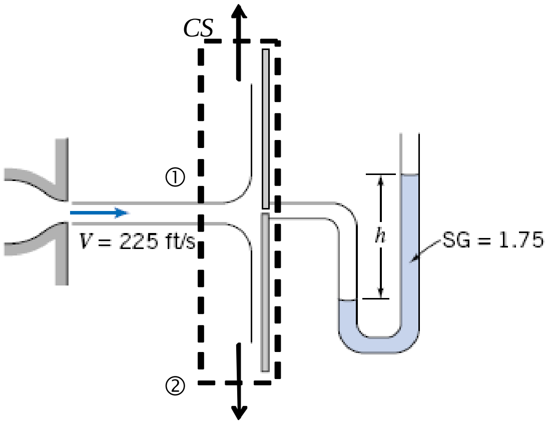
*4.106 A horizontal axisymmetric jet of air with 0.5 in. diameter strikes a stationary vertical disk of 8 in. diameter. The jet speed is 225 ft/s at the nozzle exit. A manometer is connected to the center of the disk. Calculate (a) the deflection, *h*, if the manometer liquid has SG = 1.75 and (b) the force exerted by the jet on the disk.



Given: Air jet striking disk

Find: Manometer deflection; Force to hold disk

Solution:

Basic equations: Hydrostatic pressure, Bernoulli, and momentum flux in x direction

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction 4) Flow along streamline 5) Uniform flow 6) Horizontal flow ($g_x = 0$)

Applying Bernoulli between jet exit and stagnation point

$$\frac{p}{\rho_{air}} + \frac{V^2}{2} = \frac{p_0}{\rho_{air}} + 0 \qquad p_0 - p = \frac{1}{2}\cdot\rho_{air}\cdot V^2$$

But from hydrostatics $\quad p_0 - p = SG\cdot\rho\cdot g\cdot\Delta h \quad$ so $\quad \Delta h = \dfrac{\frac{1}{2}\cdot\rho_{air}\cdot V^2}{SG\cdot\rho\cdot g} = \dfrac{\rho_{air}\cdot V^2}{2\cdot SG\cdot\rho\cdot g}$

$$\Delta h = 0.002377\cdot\frac{slug}{ft^3}\times\left(225\cdot\frac{ft}{s}\right)^2\times\frac{1}{2\cdot 1.75}\times\frac{ft^3}{1.94\cdot slug}\times\frac{s^2}{32.2\cdot ft} \qquad \Delta h = 0.55\cdot ft \qquad \Delta h = 6.6\cdot in$$

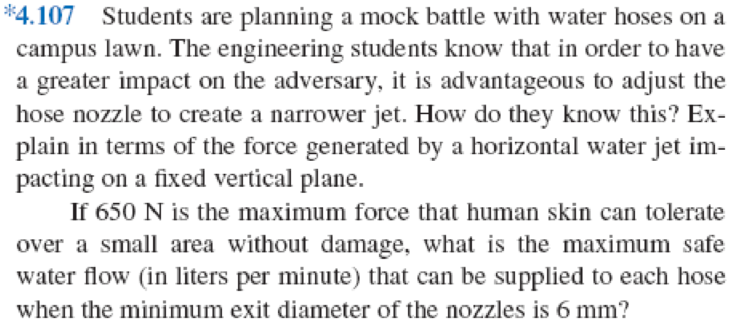
For x momentum $\quad R_x = V\cdot\left(-\rho_{air}\cdot A\cdot V\right) = -\rho_{air}\cdot V^2\cdot\dfrac{\pi\cdot D^2}{4}$

$$R_x = -0.002377\cdot\frac{slug}{ft^3}\times\left(225\cdot\frac{ft}{s}\right)^2\times\frac{\pi\cdot\left(\frac{0.5}{12}\cdot ft\right)^2}{4}\times\frac{lbf\cdot s^2}{slug\,ft} \qquad R_x = -0.164\cdot lbf$$

The force of the jet on the plate is then $\quad F = -R_x \qquad F = 0.164\cdot lbf$

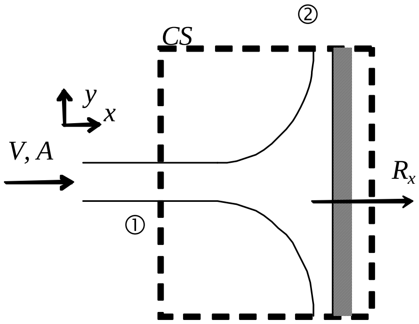
Problem *4.107

[2]



***4.107** Students are planning a mock battle with water hoses on a campus lawn. The engineering students know that in order to have a greater impact on the adversary, it is advantageous to adjust the hose nozzle to create a narrower jet. How do they know this? Explain in terms of the force generated by a horizontal water jet impacting on a fixed vertical plane.

If 650 N is the maximum force that human skin can tolerate over a small area without damage, what is the maximum safe water flow (in liters per minute) that can be supplied to each hose when the minimum exit diameter of the nozzles is 6 mm?



Given: Water jet striking surface

Find: Force on surface

Solution:

Basic equations: Momentum flux in x direction $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure throughout 4) Uniform flow

Hence $R_x = u_1\cdot\left(-\rho\cdot u_1\cdot A_1\right) = -\rho\cdot V^2\cdot A = -\rho\cdot\left(\frac{Q}{A}\right)^2\cdot A = -\frac{\rho\cdot Q^2}{A} = -\frac{4\cdot\rho\cdot Q^2}{\pi\cdot D^2}$ where Q is the flow rate

The force of the jet on the surface is then $F = -R_x = \frac{4\cdot\rho\cdot Q^2}{\pi\cdot D^2}$

For a fixed flow rate Q, the force of a jet varies as $\frac{1}{D^2}$: A smaller diameter leads to a larger force. This is because as the diameter decreases the speed increases, and the impact force varies as the square of the speed, but linearly with area

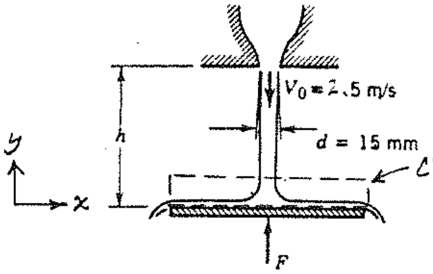
For a force of F = 650 N

$Q = \sqrt{\frac{\pi\cdot D^2\cdot F}{4\cdot\rho}}$ $\quad Q = \sqrt{\frac{\pi}{4}\times\left(\frac{6}{1000}\cdot m\right)^2\times 650\cdot N\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}}\times\frac{1\cdot L}{10^{-3}\cdot m^3}\times\frac{60\cdot s}{1\cdot min}$ $\quad Q = 257\cdot\frac{L}{min}$

Problem *4.108

Given: Jet flowing downward, striking horizontal disk, as shown.

Find: (a) Velocity in jet at h.
(b) Expression ~~for force~~ to hold disk.
(c) Evaluate for $h = 3.0\,m$.



Solution: Apply Bernoulli and momentum equations. Use CV shown.

Basic equations: $$\frac{\cancelto{(5)}{p}}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

$$F_{S_z} + \cancelto{=0(6)}{F_{B_z}} = \cancelto{=0(1)}{\frac{\partial}{\partial t}}\int_{CV} w\rho\, d\forall + \int_{CS} w\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Flow along a streamline
(4) Frictionless flow
(5) Atmospheric pressure along jet
(6) Neglect water on plate; $F_{B_z} = 0$
(7) Uniform flow at each section

The Bernoulli equation becomes

$$\frac{V_0^2}{2} + gh = \frac{V^2}{2} + g(0) \quad \text{or} \quad V^2 = V_0^2 + 2gh\;;\quad V = \sqrt{V_0^2 + 2gh}$$ V

From the momentum equation

$$R_z = w_1\{-\rho V A\} + w_2\{+\rho V_0 A_0\} = +\rho V^2 A$$

$$w_1 = -V \qquad w_2 = 0$$

But from continuity, $\dot{m} = \rho V_0 A_0 = \rho V A$. Thus $VA = V_0 A_0$, and

$$R_z = \rho V_0 A_0 V = \rho V_0 A_0 \sqrt{V_0^2 + 2gh}$$ R_z

At $h = 3.0\,m$,

$$R_z = 999\,\frac{kg}{m^3} \times 2.5\,\frac{m}{s} \times \frac{\pi}{4}(0.015)^2\,m^2 \left[(2.5)^2\frac{m^2}{s^2} + 2 \times 9.81\,\frac{m}{s^2} \times 3.0\,m\right]^{1/2} \frac{N\cdot s^2}{kg\cdot m}$$

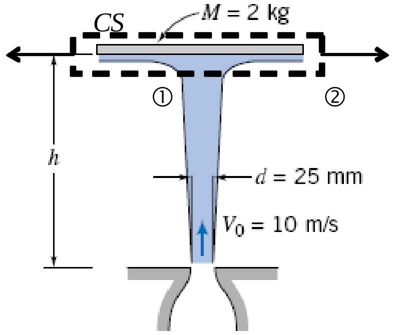
R_z = 3.56 N (upward force) R_z

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem *4.109

[3]

***4.109** A 2-kg disk is constrained horizontally but is free to move vertically. The disk is struck from below by a vertical jet of water. The speed and diameter of the water jet are 10 m/s and 25 mm at the nozzle exit. Obtain a general expression for the speed of the water jet as a function of height, *h*. Find the height to which the disk will rise and remain stationary.



Given: Water jet striking disk

Find: Expression for speed of jet as function of height; Height for stationary disk

Solution:

Basic equations: Bernoulli; Momentum flux in z direction

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_z = F_{S_z} + F_{B_z} = \frac{\partial}{\partial t}\int_{CV} w\,\rho\,d\forall + \int_{CS} w\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow

The Bernoulli equation becomes $\frac{V_0^2}{2} + g\cdot 0 = \frac{V^2}{2} + g\cdot h$ $\quad V^2 = V_0^2 - 2\cdot g\cdot h$ $\quad V = \sqrt{V_0^2 - 2\cdot g\cdot h}$

Hence $\quad -M\cdot g = w_1\cdot\left(-\rho\cdot w_1\cdot A_1\right) = -\rho\cdot V^2\cdot A$

But from continuity $\quad \rho\cdot V_0\cdot A_0 = \rho\cdot V\cdot A$ $\quad$ so $\quad V\cdot A = V_0\cdot A_0$

Hence we get $\quad M\cdot g = \rho\cdot V\cdot V\cdot A = \rho\cdot V_0\cdot A_0\cdot\sqrt{V_0^2 - 2\cdot g\cdot h}$

Solving for h

$$h = \frac{1}{2\cdot g}\cdot\left[V_0^2 - \left(\frac{M\cdot g}{\rho\cdot V_0\cdot A_0}\right)^2\right]$$

$$h = \frac{1}{2}\times\frac{s^2}{9.81\cdot m}\times\left[\left(10\cdot\frac{m}{s}\right)^2 - \left[2\cdot kg\times\frac{9.81\cdot m}{s^2}\times\frac{m^3}{1000\cdot kg}\times\frac{s}{10\cdot m}\times\frac{4}{\pi\cdot\left(\frac{25}{1000}\cdot m\right)^2}\right]^2\right]$$

$$h = 4.28\,m$$

Given: Water jet supporting conical object, as shown.

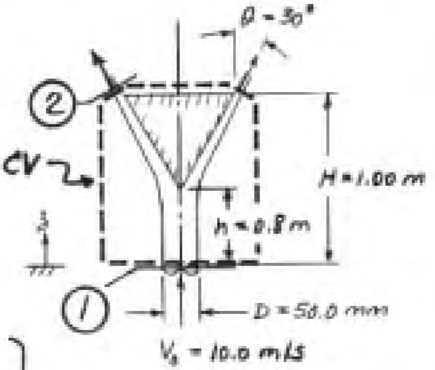
Find: (a) Combined mass of cone and water, M, supported.
(b) Estimate mass of water in CV.

Solution: Apply continuity, Bernoulli, and momentum equations using CV shown.

Basic equations: $0 = \cancelto{=0(1)}{\frac{\partial}{\partial t}}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$

$$\frac{\cancelto{(6)}{p_1}}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{\cancelto{(6)}{p_2}}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$\cancelto{=0(6)}{F_{S_z}} + F_{B_z} = \cancelto{=0(1)}{\frac{\partial}{\partial t}}\int_{CV} w\rho\, d\forall + \int_{CS} w\rho\vec{V}\cdot d\vec{A}$$



Assumptions: (1) Steady flow
(2) No friction
(3) Flow along a streamline
(4) Incompressible flow
} required for Bernoulli
(5) Uniform flow at each cross-section
(6) $F_{S_z} = 0$ since p_{atm} acts everywhere

Then
$$0 = \{-|\rho V_1 A_1|\} + \{+|\rho V_2 A_2|\} \quad \text{so} \quad V_1 A_1 = V_2 A_2$$

From Bernoulli
$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 = \frac{V_0^2}{2} = \frac{V_2^2}{2} + gH \;; \quad V_2^2 = V_0^2 - 2gH$$

From momentum
$$F_{B_z} = \int_{CS} w\rho\vec{V}\cdot d\vec{A} = -Mg = w_1\{-|\rho V_1 A_1|\} + w_2\{+|\rho V_2 A_2|\}$$
$$w_1 = V_0 \qquad w_2 = V_2\cos\theta$$

or
$$-Mg = -V_0\rho V_1 A_1 + V_2\cos\theta\,\rho V_2 A_2 = \rho V_0 A_1 (V_2\cos\theta - V_0)$$

so
$$M = \frac{(V_0 - V_2\cos\theta)\rho V_0 A_1}{g}$$

From Bernoulli
$$V_2 = (V_0^2 - 2gH)^{\frac{1}{2}} = \left[(10)^2\frac{m^2}{s^2} - 2\times 9.81\frac{m}{s^2}\times 1\,m\right]^{\frac{1}{2}} = 8.97\ m/s$$

Substituting
$$M = \left(10.0\frac{m}{s} - 8.97\frac{m}{s}\times\cos 30°\right)999\frac{kg}{m^3}\times 10\frac{m}{s}\times\frac{\pi}{4}(0.050)^2 m^2 \times \frac{s^2}{9.81\,m}$$

$M = 4.46\ kg \leftarrow$ (total mass in CV: water + object)   M

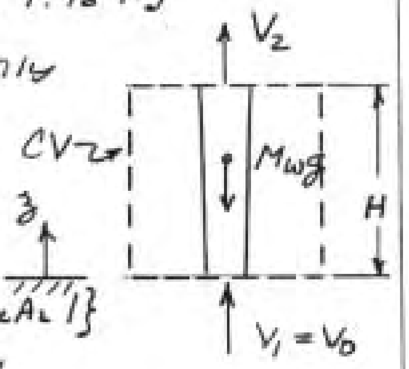
Problem *4.110

To find mass of water in CV, we have 3 options:

(1) assume area of jet is constant

$$M = \rho \forall \simeq \rho A_1 H = 999 \frac{kg}{m^3} \times \frac{\pi}{4}(0.05)^2 m^2 \times 1 m = 1.96 \, kg$$

(2) Use a CV that encloses the free jet only



Continuity $V_1 A_1 = V_2 A_2$

Bernoulli $V_2 = (V_1^2 - 2gH)^{1/2}$

Momentum $-M_w g = u_1\{-|\rho V_1 A_1|\} + u_2\{+|\rho V_2 A_2|\}$

$$u_1 = V_1 = V_0 \qquad u_2 = V_2$$

Substituting in momentum

$$-M_w g = V_0(-\rho V_0 A_1) + V_2(+\rho V_0 A_1) = \rho V_0 A_1 (V_2 - V_0)$$

$$M_w = \frac{\rho V_0 A_1 (V_0 - V_2)}{g}$$

$$= 999 \frac{kg}{m^3} \times 10 \frac{m}{s} \times \frac{\pi}{4}(0.05)^2 m^2 (10 - 8.97)\frac{m}{s} \times \frac{s^2}{9.81 m}$$

$$M_w = 2.06 \, kg$$

M_w

(3) Evaluate the area at each cross-section using Bernoulli and continuity, then integrate to find $\forall$.

$$VA = V_0 A_1 = (V_0^2 - 2gz)^{1/2} A = V_0 A_1 \quad \text{so} \quad A = \frac{V_0 A_1}{(V_0^2 - 2gz)^{1/2}}$$

$$\forall = \int_0^H A \, dz = \int_0^H \frac{V_0 A_1}{(V_0^2 - 2gz)^{1/2}} dz = A_1 \int_0^H \frac{V_0^2}{2g} \frac{1}{(1 - \frac{2gz}{V_0^2})^{1/2}} d\left(\frac{2gz}{V_0^2}\right)$$

This can be integrated. Let $u = 1 - 2gz/V_0^2$, so $\int = \int \frac{-du}{u^{1/2}}$

$$\text{Then} \quad \forall = A_1 \frac{V_0^2}{2g}\left[-2\left(1 - \frac{2gz}{V_0^2}\right)^{1/2}\right]_{z=0}^{z=h} = \frac{A_1}{g}\left[V_0^2 - V_0(V_0^2 - 2gh)^{1/2}\right]$$

and

$$M_w = \rho \forall = \frac{\rho A_1 V_0 (V_0 - V_2)}{g} = 2.06 \, kg \text{ (same as (2) above)}$$

Thus the mass of the cone is $M_c = M - M_w = 2.40 \, kg$.

M_c

{Note: If V_0 were smaller or H larger, V_2 would differ more from V_0 and the jet area would increase significantly. Option (2) would still give the correct result with little effort.}

Problem *4.111

Given: Stream of air at standard conditions strikes a curved vane. Stagnation tube with water-filled manometer in exit plane.

Find: (a) Speed of air leaving nozzle.
(b) Horizontal component of force exerted on vane by jet.
(c) Comment on each assumption used to solve this problem.

Solution: Apply the definition of stagnation pressure and the x component of the momentum equation.

By definition $p_0 = p + \frac{1}{2}\rho_{air} V^2$

From fluid statics, $p_0 - p = \rho_{water}\, g\, \Delta h$

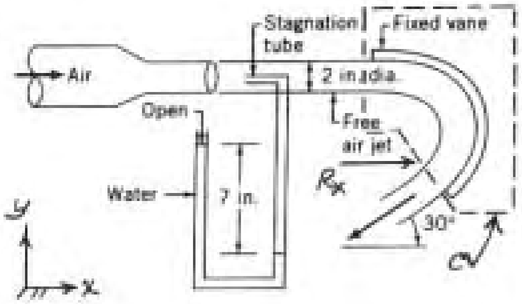
Combining, $\rho_{water}\, g\, \Delta h = \frac{1}{2}\rho_{air} V^2$ or $V = \sqrt{\dfrac{2\rho_{water}\, g\, \Delta h}{\rho_{air}}}$

$$V = \left[2 \times 1.94 \frac{\text{slug}}{\text{ft}^3} \times 32.2 \frac{\text{ft}}{\text{s}^2} \times 7 \text{ in.} \times \frac{\text{ft}^3}{0.00238 \text{ slug}} \times \frac{\text{ft}}{12 \text{ in.}} \right]^{\frac{1}{2}} = 175 \text{ ft/s}$$

V

The momentum equation is

$$F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u \rho\, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$



Assumptions: (1) No net pressure force
(2) $F_{B_x} = 0$
(3) Steady flow
(4) Uniform flow
(5) Constant speed on vane

Then

$$R_x = u_1\{-\rho V A\} + u_2\{\rho V A\} = -\rho V^2 A (1 + \cos\theta)$$

$$u_1 = V \qquad u_2 = -V\cos\theta$$

$$R_x = -0.00238 \frac{\text{slug}}{\text{ft}^3} \times (175)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\pi}{4}\left(\frac{2}{12}\right)^2 \text{ft}^2 (1 + \cos 30°) = -2.97 \text{ lbf}$$

Force of air on vane is $K_x = -R_x = +2.97$ lbf (to right)

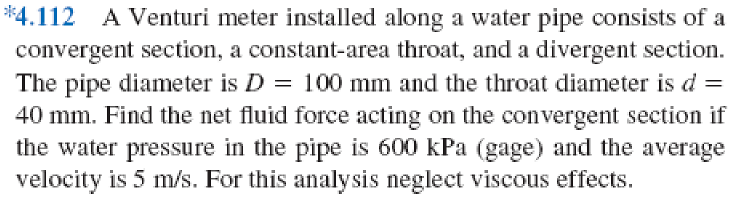
K_x

Comments on each assumption used to solve this problem:

- Frictionless flow in the nozzle is a good assumption.
- Incompressible flow is a good assumption for this low-speed flow.
- No horizontal component of body force is exact.
- No net pressure force on the control volume is exact.
- Frictionless flow along the vane is not realistic; air flow along the vane would be slowed by friction, reducing the momentum flux at the exit.

Problem *4.112

[2]



*4.112 A Venturi meter installed along a water pipe consists of a convergent section, a constant-area throat, and a divergent section. The pipe diameter is $D = 100$ mm and the throat diameter is $d = 40$ mm. Find the net fluid force acting on the convergent section if the water pressure in the pipe is 600 kPa (gage) and the average velocity is 5 m/s. For this analysis neglect viscous effects.

Given: Data on flow and venturi geometry

Find: Force on convergent section

Solution:

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $\quad D = 0.1\cdot m$ $\quad d = 0.04\cdot m$ $\quad p_1 = 600\cdot kPa$ $\quad V_1 = 5\cdot\frac{m}{s}$

Then $A_1 = \frac{\pi\cdot D^2}{4}$ $\quad A_1 = 0.00785\,m^2$ $\quad A_2 = \frac{\pi}{4}\cdot d^2$ $\quad A_2 = 0.00126\,m^2$

$Q = V_1\cdot A_1$ $\quad Q = 0.0393\,\frac{m^3}{s}$ $\quad V_2 = \frac{Q}{A_2}$ $\quad V_2 = 31.3\,\frac{m}{s}$

Governing equations:

Bernoulli equation $\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$ (4.24)

Momentum $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$ (4.18a)

Applying Bernoulli between inlet and throat $\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_2}{\rho} + \frac{V_2^2}{2}$

Solving for p_2 $\quad p_2 = p_1 + \frac{\rho}{2}\cdot\left(V_1^2 - V_2^2\right)$ $\quad p_2 = 600\cdot kPa + 999\cdot\frac{kg}{m^3}\times\left(5^2 - 31.3^2\right)\cdot\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}\times\frac{kN}{1000\cdot N}$ $\quad p_2 = 125\cdot kPa$

Applying the horizontal component of momentum

$-F + p_1\cdot A_2 - p_2\cdot A_2 = V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$ or $F = p_1\cdot A_1 - p_2\cdot A_2 + \rho\cdot\left(V_1^2\cdot A_1 - V_2^2\cdot A_2\right)$

$$F = 600\cdot\frac{kN}{m^2}\times 0.00785\cdot m^2 - 125\cdot\frac{kN}{m^2}\times 0.00126\cdot m^2 + 999\cdot\frac{kg}{m^3}\times\left[\left(5\cdot\frac{m}{s}\right)^2\cdot 0.00785\cdot m^2 - \left(31.3\cdot\frac{m}{s}\right)^2\cdot 0.00126\cdot m^2\right]\cdot\frac{N\cdot s^2}{kg\times m}$$

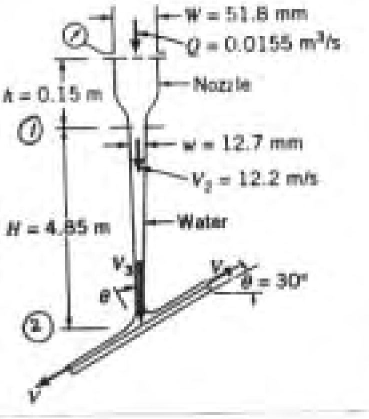
$F = 3.52\cdot kN$

Problem *4.113

[4]

Given: Plane nozzle discharging water steadily, striking an inclined plate.

Neglect friction in nozzle and along plate surface.

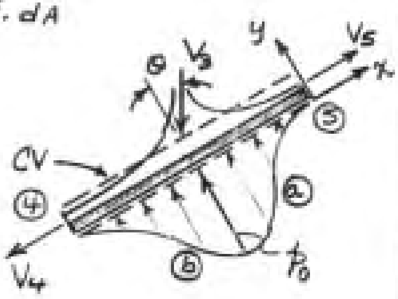


Find: (a) Minimum gage pressure at nozzle inlet.
(b) Magnitude and direction of force exerted by water stream on plate.
(c) Sketch pressure distribution on plate. Explain why the pressure distribution is shaped as you show it.

Solution: Apply continuity, Bernoulli, and momentum equations using the CV and coordinates shown.

Basic equations: $V_1A_1 = V_2A_2$ $\qquad \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$

$$R_y + \overset{=0(6)}{\cancel{F_{B_y}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}\int_{CV} v\rho\, d\forall}} + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$



Assumptions: (1) Frictionless flow
(2) Incompressible flow
(3) Steady flow
(4) Flow along a streamline
(5) Uniform flow at each section

Then from continuity $V_1 = \frac{A_2}{A_1}V_2 = \frac{w}{W}V_2 = \frac{12.7\text{ mm}}{51.8\text{ mm}} \times 12.2\frac{\text{m}}{\text{s}} = 2.99\text{ m/s}$

From Bernoulli $p_{1g} = \frac{\rho}{2}(V_2^2 - V_1^2) - \rho g(z_1 - z_2) + \overset{=0}{\cancel{p_{2g}}}$; $z_1 - z_2 = h$

$$p_{1g} = \left[\frac{1}{2}\times 999\frac{\text{kg}}{\text{m}^3}\times\left[(12.2)^2 - (2.99)^2\right]\frac{\text{m}^2}{\text{s}^2} - 999\frac{\text{kg}}{\text{m}^3}\times 9.81\frac{\text{m}}{\text{s}^2}\times 0.15\text{ m}\right]\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 68.4\text{ kPa (gage)}$$ p

Calculate V_3 in the absense of the plate using Bernoulli ($p_2 = p_3$)

$$V_3 = \sqrt{V_2^2 + 2gH} = \left[(12.2)^2\frac{\text{m}^2}{\text{s}^2} + 2\times 9.81\frac{\text{m}}{\text{s}^2}\times 4.85\text{ m}\right]^{\frac{1}{2}} = 15.6\text{ m/s}$$

From momentum: $R_x = 0$ since there is no friction on the plate surface.

Assumptions: (6) Neglect mass of plate and of water on plate.
(7) Atmospheric pressure acts on entire CV; $F_{S_y} = R_y$

Then $\quad R_y = v_3\{-\dot{m}_3\} + \overset{0}{\cancel{v_4}}\{+\dot{m}_4\} + \overset{0}{\cancel{v_5}}\{+\dot{m}_5\} = V_3\cos\theta\,\rho Q$, since $v_3 = -V_3\cos\theta$

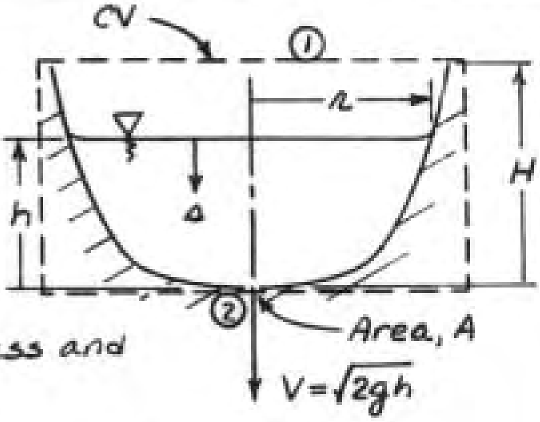
$$R_y = 15.6\frac{\text{m}}{\text{s}}\times\cos 30^\circ\times 999\frac{\text{kg}}{\text{m}^3}\times 0.0155\frac{\text{m}^3}{\text{s}}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 209\text{ N};\ K_y = -R_y = -209\text{ N}$$ K_y

Pressure is maximum at stagnation, minimum (p_{atm}) at ④ and ⑤.
Pressure at ⓐ is higher than at ⓑ because of streamline curvature.

Problem *4.114

Given: Egyptian water clock.
Surface level drops at rate, s = constant.

Find: (a) Expression for $r(h)$.
(b) Volume needed for n hours' operation.



Solution: Apply conservation of mass and the Bernoulli equation.

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Assumptions: (1) Quasi-steady flow; $\frac{\partial}{\partial t}$ small
(2) Incompressible flow
(3) Uniform flow at each cross section
(4) Flow along a streamline
(5) No friction
(6) $\rho_{air} \ll \rho_{H_2O}$

Writing Bernoulli from the liquid surface to the jet exit,

$$\cancel{\frac{p_{atm}}{\rho}} + \frac{s^2}{2} + gh = \cancel{\frac{p_{atm}}{\rho}} + \frac{V^2}{2} + g(0);$$

For $s \ll V$, then $V = \sqrt{2gh}$.

For the CV,

$$0 = \frac{\partial}{\partial t}\int_{\forall_{air}} \overset{\ll \rho_{H_2O}(6)}{\cancel{\rho_{air}}}\, d\forall + \frac{\partial}{\partial t}\int_{\forall_{H_2O}} \rho_{H_2O}\, d\forall + \left\{-|\overset{\ll \rho_{H_2O}(6)}{\cancel{\rho_{air}}} V_1 A_1|\right\} + \left\{|\rho_{H_2O} V A|\right\}$$

or

$$0 = \rho \frac{d\forall}{dt} + \rho V A = \rho \pi r^2 \frac{dh}{dt} + \rho\sqrt{2gh}\, A = 0$$

But h decreases, so $\frac{dh}{dt} = -s$. Thus

$$\pi r^2 s = \sqrt{2gh}\, A \quad \text{or} \quad r = \sqrt[4]{2g}\sqrt{\frac{A}{\pi s}}\, h^{1/4}$$

r

For n hours' operation, $H = ns$, and

$$\forall = \int_0^H \pi r^2\, dh = \int_0^{ns} \sqrt{2gh}\, \frac{A}{s}\, dh = \frac{2A}{3s}\sqrt{2g}\,(ns)^{3/2}$$

or

$$\forall = \frac{2A\sqrt{2g}\, n^{3/2} s^{1/2}}{3}$$

$\forall$

Check dimensions:

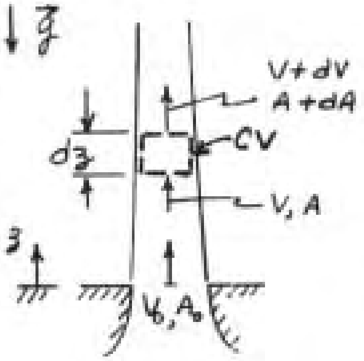
$$[\forall] = L^3 = \left[A\sqrt{g}\, n^{3/2} s^{1/2}\right] = L^2 \frac{L^{1/2}}{t}\, t^{3/2} \frac{L^{1/2}}{t^{1/2}} = L^3 \ ✓$$

Problem *4.115

[4]

Given: Low-speed jet of incompressible liquid moving upward from nozzle.

Find: Expressions for $V(z)$, $A(z)$.
Location where $V = 0$.



Solution: Apply continuity and momentum equation using CV shown.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

$$\overset{=0(4,5)}{\cancel{F_{S_z}}} + F_{B_z} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} w\rho\, d\forall + \int_{CS} w\rho \vec{V}\cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) p_{atm} acts everywhere
(5) No friction
} $F_{S_z} = 0$

Then

$$0 = \int_{CS} \vec{V}\cdot d\vec{A} = \{-VA\} + \{+(V+dV)(A+dA)\}\ ; \quad VA = V_0A_0 = \text{constant}$$

From momentum,

$$-\rho g\left(A + \frac{dA}{2}\right)dz = V\{-\rho VA\} + (V+dV)\{+\rho(V+dV)(A+dA)\} = \rho VA\,dV$$

since $dVdA \ll dA$. Also, since $dA\,dz \ll dz$, the left side is $-\rho g A\,dz$.
Thus

$$-\rho g A\,dz = \rho VA\,dV \quad \text{or} \quad VdV = -g\,dz$$

Integrating from V_0 at $z_0 = 0$ to V at z,

$$\int_{V_0}^{V} VdV = \frac{V^2}{2}\Big]_{V_0}^{V} = \frac{V^2}{2} - \frac{V_0^2}{2} = \int_{z_0}^{z} -g\,dz = -g(z - z_0) = -gz$$

Thus $V^2 = V_0^2 - 2gz$ or $V(z) = \sqrt{V_0^2 - 2gz}$ ← $V(z)$

Since $VA = V_0A_0$, then $A = A_0\dfrac{V_0}{V}$

$$A(z) = A_0\frac{V_0}{\sqrt{V_0^2 - 2gz}} = \frac{A_0}{\sqrt{1 - 2gz/V_0^2}}$$ ← $A(z)$

Solving for z at $V = 0$,

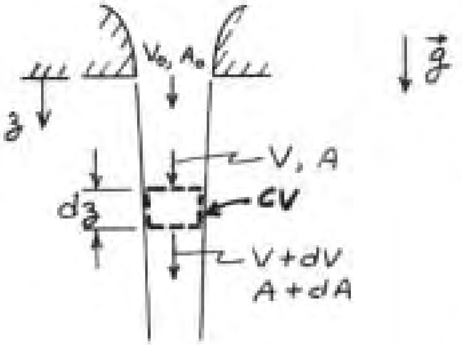
$$z = \frac{V_0^2}{2g}$$ ← z

Problem *4.116

[4]

Given: Low-speed jet of incompressible liquid moving downward from nozzle.

Find: Expressions for $V(z)$, $A(z)$.
Location where $A = A_0/2$.



Solution: Apply continuity and momentum equations using CV shown.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\overset{=0(4,5)}{\cancel{F_{S_z}}} + F_{B_z} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} w \rho \, d\forall + \int_{CS} w \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) p_{atm} acts everywhere $\}$ $F_{S_z} = 0$
(5) No friction

Then

$$0 = \int_{CS} \vec{V} \cdot d\vec{A} = \{-VA\} + \{+(V+dV)(A+dA)\} \; ; \; VA = V_0 A_0 = \text{constant}$$

From momentum,

$$\rho g \left(A + \frac{dA}{2}\right) dz = V\{-\rho V A\} + (V+dV)\{-\rho(V+dV)(A+dA)\} = \rho V A \, dV$$

since $dV dA \ll dA$. Also, since $dA dz \ll dz$, the left side is $\rho g A dz$.
Thus

$$\rho g A \, dz = \rho V A \, dV \quad \text{or} \quad V dV = g \, dz$$

Integrating from V_0 at $z_0 = 0$ to V at z,

$$\int_{V_0}^{V} V dV = \left.\frac{V^2}{2}\right]_{V_0}^{V} = \frac{V^2}{2} - \frac{V_0^2}{2} = \int_{z_0}^{z} g \, dz = g(z - z_0) = gz$$

Thus

$$V^2 = V_0^2 + 2gz \quad \text{or} \quad V(z) = \sqrt{V_0^2 + 2gz}$$

V(z)

Since $VA = V_0 A_0$, $A = A_0 \dfrac{V_0}{V}$

$$A(z) = A_0 \frac{V_0}{\sqrt{V_0^2 + 2gz}} = \frac{A_0}{\sqrt{1 + 2gz/V_0^2}}$$

A(z)

Solving for z,

$$z = \frac{V_0^2}{2g}\left[\left(\frac{A_0}{A}\right)^2 - 1\right] \; ; \text{ for } \frac{A}{A_0} = \frac{1}{2}, \; \frac{A_0}{A} = 2, \text{ and } \; z_{1/2} = \frac{3V_0^2}{2g}$$

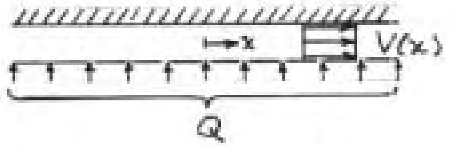
$z_{1/2}$

Problem *4.117

[4]

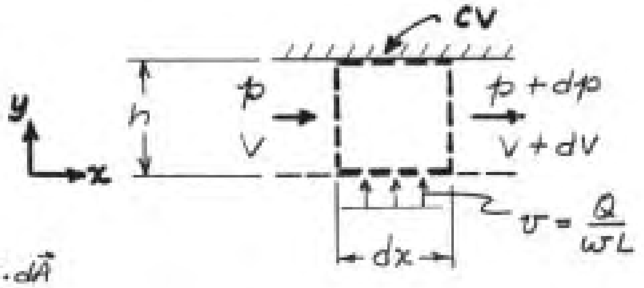
Given: Uniform flow in narrow gap between parallel plates, as shown.

Fluid in gap has only horizontal motion.



Find: Expression for $p(x)$.

Solution: Apply continuity and x component of momentum equation.



Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$F_{Sx} + \overset{=0(5)}{\cancel{F_{Bx}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}u\rho\,d\forall + \int_{CS}u\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) Neglect friction
(5) $F_{Bx} = 0$

Then

$$0 = \int_{CS}\vec{V}\cdot d\vec{A} = \{-Vwh\} + \left\{-\frac{Q}{wL}w\,dx\right\} + \{(V+dV)wh\}; \quad wh\,dV = \frac{Q}{L}dx$$

$$V = \frac{Q}{wh}\frac{x}{L} + c; \quad c = 0 \text{ since } V(0) = 0; \quad V(x) = \frac{Q}{wh}\frac{x}{L}$$

From momentum,

$$p\,wh - (p+dp)wh = u_x\{-\rho Vwh\} + u_{dx}\left\{-\rho\frac{Q}{wh}w\,dx\right\} + u_{x+dx}\{+\rho(V+dV)wh\}$$

$$u_x = V \qquad u_{dx} = 0 \qquad u_{x+dx} = V + dV$$

From continuity, $(V+dV)wh = Vwh + Q\frac{dx}{L}$, so

$$-dp\,wh = -\rho V^2wh + 0 + (V+dV)\left(Vwh + Q\frac{dx}{L}\right)\rho$$

$$= -\cancel{\rho V^2wh} + \cancel{\rho V^2wh} + \rho Vwh\,dV + V\rho Q\frac{dx}{L} + \rho Q\,dV\frac{dx}{L}$$

Neglecting products of differentials $(dV\,dx \ll dx)$, and with $dV = \frac{Q}{wh}\frac{dx}{L}$

$$-dp = \rho V\,dV + \frac{V\rho Q}{wh}\frac{dx}{L} = \rho V\frac{Q}{wh}\frac{dx}{L} + \frac{V\rho Q}{wh}\frac{dx}{L} = 2\rho\frac{Q}{wh}\frac{x}{L}\frac{Q}{wh}\frac{dx}{L}$$

$$-dp = 2\rho\left(\frac{Q}{whL}\right)^2x\,dx \qquad p(x) = -\rho\left(\frac{Q}{whL}\right)^2x^2 + C$$

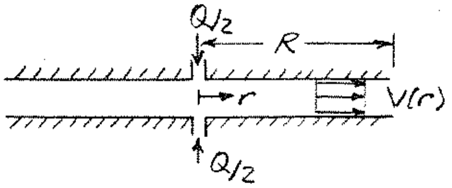
If $p(0) = p_0$, then $p(x) = p_0 - \rho\left(\frac{Q}{wh}\right)^2\left(\frac{x}{L}\right)^2$

$p(x$

Problem *4.118

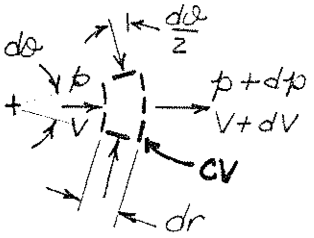
Given: Uniform flow in narrow gap between parallel disks, as shown.

Liquid in gap has only
radial motion.



Find: Expression for $p(r)$; plot

Solution: Apply continuity and
momentum equations
to the differential CV shown.



Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall}} + \int_{CS}\rho \vec{V}\cdot d\vec{A}$$

$$F_{S_r} + \overset{=0(5)}{\cancel{F_{B_r}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} V_r \rho\, d\forall}} + \int_{CS} V_r \rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) Neglect friction
(5) $F_{B_r} = 0$
(6) No flow in θ direction
(7) $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

Then

$$0 = \int_{CS}\vec{V}\cdot d\vec{A} = \{-\rho V h r d\theta\} + \{\rho(V+dV)h(r+dr)d\theta\}\ ;\ Vr = \text{constant}$$

For $r = R$, $Q = V_R\, 2\pi r h$, so $V_R = Q/2\pi R h$

From momentum,

$$p\,h r d\theta + 2\left(p + \frac{dp}{2}\right) h\, dr \sin\frac{d\theta}{2} - (p+dp)h(r+dr)d\theta$$

$$= V\{-\rho V h r d\theta\} + (V+dV)\{\rho(V+dV)h(r+dr)d\theta\}$$

$$\cancel{phr}d\theta + \cancel{phdr}d\theta + \frac{1}{2}dp\,h\,dr\,d\theta - (\cancel{pr} + \cancel{pdr} + r\,dp + dr\,dp)h\,d\theta$$

$$= dV(\rho V h r d\theta) \quad \{\text{Note terms in braces are equal.}\}$$

Assuming products of differentials are much smaller than single differentials,

$$-r\,dp\,h\,d\theta = dV(\rho V h r d\theta) \quad \text{or} \quad dp = -\rho V dV$$

Integrating, $p(r) - p(R) = -\rho\frac{V^2}{2} + \rho\frac{V_R^2}{2}$ or $p(r) - p_{atm} = \frac{\rho}{2}(V_R^2 - V^2)$

$$= \rho\frac{V_R^2}{2}\left[1 - \left(\frac{V}{V_R}\right)^2\right]$$

Since $V_R = \frac{Q}{2\pi R h}$, and Vr = constant, $\frac{V}{V_R} = \frac{R}{r}$, so

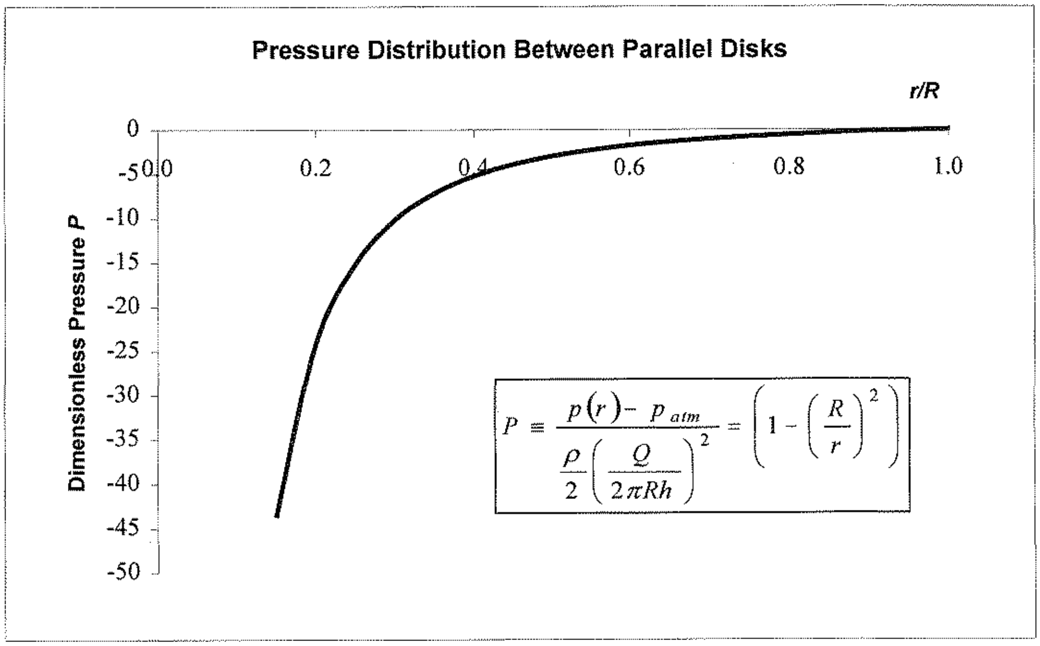
$$p(r) - p_{atm} = \frac{\rho}{2}\left(\frac{Q}{2\pi R h}\right)^2\left[1 - \left(\frac{R}{r}\right)^2\right] \longleftarrow p(r)$$

Note since $r < R$, that $p(r) < p_{atm}$ between the disks.

Problem *4.118

The pressure distribution is computed and plotted in *Excel*:

r/R	*P*
0.15	-43.4
0.20	-24.0
0.25	-15.0
0.30	-10.1
0.35	-7.16
0.40	-5.25
0.45	-3.94
0.50	-3.00
0.55	-2.31
0.60	-1.78
0.65	-1.37
0.70	-1.04
0.75	-0.78
0.80	-0.563
0.85	-0.384
0.90	-0.235
0.95	-0.108
1.00	0.000



$$P \equiv \frac{p(r) - p_{atm}}{\frac{\rho}{2}\left(\frac{Q}{2\pi R h}\right)^2} = \left(1 - \left(\frac{R}{r}\right)^2\right)$$

Problem *4.119

[5]

Given: Narrow gap between parallel disks filled with liquid.

At $t = 0^+$, upper disk begins to move downward at V_0.

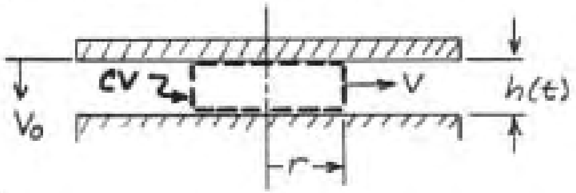
Neglect viscous effects; flow uniform in horizontal direction.

Find: Expression for velocity field, $V(r)$. Note flow is not steady.

Solution: Apply continuity, using the deformable CV shown.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$



Assumptions: (1) Incompressible flow
(2) Uniform flow at each cross section

Then

$$0 = \frac{\partial}{\partial t}\int_{CV} d\forall + \int_{CS} \vec{V}\cdot d\vec{A} = \frac{\partial}{\partial t}\int_{CV} d\forall + V\,2\pi r h$$

But

$$\int_{CV} d\forall = \pi r^2 h, \quad \text{so} \quad \frac{\partial}{\partial t}\int_{CV} d\forall = \frac{\partial}{\partial t}(\pi r^2 h) = \pi r^2 \frac{dh}{dt}$$

Thus

$$0 = \pi r^2 \frac{dh}{dt} + V 2\pi r h = \pi r^2(-V_0) + V\,2\pi r h$$

so

$$V(r) = V_0 \frac{r}{2h}$$

V(r)

If V_0 is constant, so $h = h_0 - V_0 t$, and

$$V(r,t) = \frac{V_0 r}{2(h_0 - V_0 t)} \qquad \text{for } t < \frac{h_0}{V_0}$$

V(r,t)

Problem *4.120

[5] Part 1/2

Given: Liquid falling vertically into short, horizontal, rectangular open channel. Neglect viscous effects.

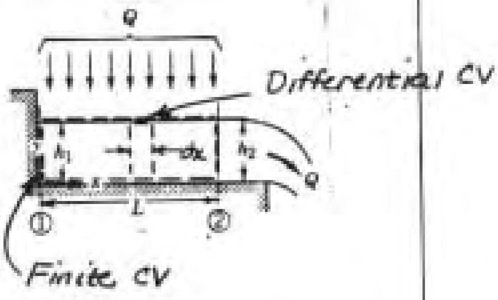
Find: (a) Expression for h_1 in terms of h_2, Q, and b.
(b) Sketch surface profile, $h(x)$.

Solution: Apply continuity and momentum equations to (i) finite CV, and (ii) differential CV, as shown.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(6)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}u\rho\, d\forall + \int_{CS}u\rho\vec{V}\cdot d\vec{A}$$



Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) Hydrostatic pressure distribution; $F_p(h) = \rho g b \frac{h^2}{2}$
(5) No friction on bed
(6) Horizontal bed; $F_{B_x} = 0$

Then for finite CV shown,

$$0 = \int_{CS}\vec{V}\cdot d\vec{A} = -Q + V_2 b h_2\ ; \quad V_2 = \frac{Q}{bh_2}$$

From momentum

$$\rho g b\frac{h_1^2}{2} - \rho g b\frac{h_2^2}{2} = u_1\{0\} + u_2\{+\rho Q\} + u_3\{-\rho Q\}$$

$$u_2 = V_2 \qquad u_3 = 0$$

$$\frac{\rho g b}{2}(h_1^2 - h_2^2) = V_2\rho Q = \frac{Q}{bh_2}\rho Q = \frac{\rho Q^2}{bh_2}\ ; \quad h_1 = \sqrt{h_2^2 + \frac{2Q^2}{gb^2h_2}}$$

$\longleftarrow$ h_1

For differential CV shown,

$$0 = \int_{CS}\vec{V}\cdot d\vec{A} = \{-Vbh\} + \left\{-\frac{Q}{bL}b\,dx\right\} + \{+(V+dV)b(h+dh)\}$$

$$0 = -\frac{Q}{L}dx + b(h\,dV + V\,dh) = -\frac{Q}{L}dx + b\,d(hV)\ ; \quad \frac{d(hV)}{dx} = \frac{Q}{L}$$

From momentum,

$$\rho g b\frac{h^2}{2} - \rho g b\frac{(h+dh)^2}{2} = V\{-\rho Vbh\} + 0\left\{-\frac{\rho Q}{L}dx\right\} + (V+dV)\{+\rho(V+dV)b(h+dh)\}$$

Using continuity,

$$\frac{\rho g b}{2}(-2h\,dh + \overset{\ll dh}{\cancel{dh}}\,dh) = -\rho V^2bh + (V+dV)\left\{\rho Vbh + \frac{\rho Q}{L}dx\right\}$$

Cont'd. →

$$-\rho g b h\,dh = -\cancel{\rho V^2 bh} + \cancel{\rho V^2 bh} + \rho V b h\,dV + \frac{\rho Q}{L}\,V\,dx + \frac{\rho Q}{L}\,\overset{\ll dx}{\cancel{dV\,dx}}$$

or

$$-gh\,dh = Vh\,dV + \frac{Q}{bL}\,V\,dx$$

From continuity, $Vh\,dV = -V^2 dh + \frac{Q}{bL}V\,dx$, so

$$-gh\,dh = -V^2 dh + \frac{2Q}{bL}\,V\,dx$$

Solving,

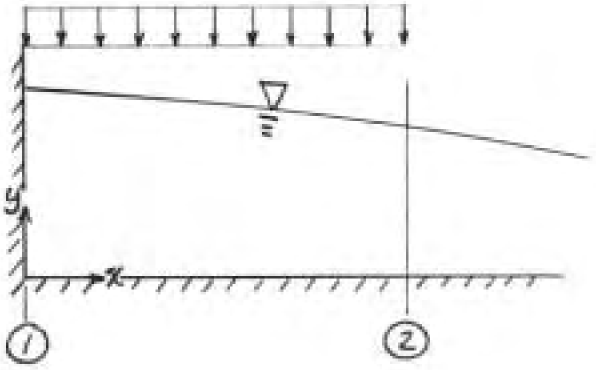
$$\frac{dh}{dx}\left(V^2 - gh\right) = \frac{2Q}{bL}\,V$$

$$\frac{dh}{dx} = \frac{2QV}{bL(V^2-gh)} = \frac{2QV}{bLgh\left(V^2/gh - 1\right)}$$

From finite CV analysis, $h_1 > h_2$, so $\frac{dh}{dx} < 0$. Thus $V^2/gh < 1$. As x increases, $V \uparrow$ and $h \downarrow$. Therefore

$$\frac{V^2}{gh}\uparrow,\quad \frac{V}{h}\uparrow,\quad \text{and}\quad \left|\frac{dh}{dx}\right|\uparrow.$$

Sketch:



Problem *4.121

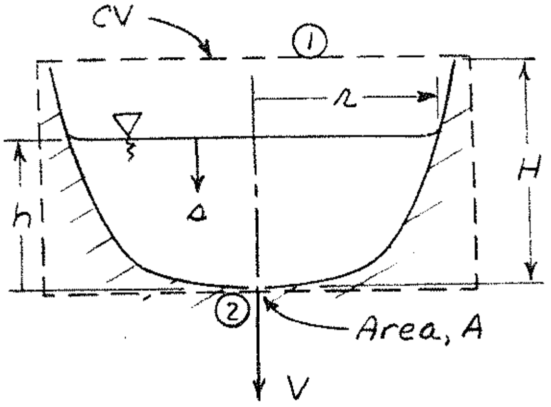
[4] Part 1/2

Open-Ended Problem Statement: Design a clepsydra (Egyptian water clock) — a vessel from which water drains by gravity through a hole in the bottom and time is indicated by the level of the remaining water. Specify the dimensions of the vessel and the size of the drain hole; indicate the amount of water needed to fill the vessel, and at what interval it must be filled. Plot the vessel shape. (This is an open-ended problem when choosing dimensions for a specific application.)

Discussion: The original Egyptian water clock was an open water-filled vessel with an orifice in the bottom. The vessel shape was designed so that the water level dropped at a constant rate during use.

Water leaves the orifice at higher speed when the water level within the vessel is high, and at lower speed when the water level within the vessel is low. The size of the orifice is constant. Thus the instantaneous volume flow rate depends on the water level in the vessel.

The rate at which the water level falls in the vessel depends on the volume flow rate and the area of the water surface. The surface area at each water level must be chosen so that the water level within the vessel decreases at a constant rate. The continuity and Bernoulli equations can be applied to determine the required vessel shape so that the water surface level drops at a constant rate.



Use the CV and notation shown (Problem 4.97):

Solution: Basic equations are

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Assumptions: (1) Quasi-steady flow
(2) Incompressible flow
(3) Uniform flow at each cross-section
(4) Flow along a streamline
(5) No friction
(6) $\rho_{air} \ll \rho_{H_2O}$

Writing Bernoulli from the liquid surface to the jet exit,

$$\cancel{\frac{p_{atm}}{\rho}} + \frac{\Delta^2}{2} + gh = \cancel{\frac{p_{atm}}{\rho}} + \frac{V^2}{2} + g(0)$$

For $\Delta \ll V$, then $V = \sqrt{2gh}$

For the CV,

$$0 = \frac{\partial}{\partial t}\int_{\forall_{air}} \cancelto{\ll \rho_{H_2O}\,(6)}{\rho_{air}}\, d\forall + \frac{\partial}{\partial t}\int_{\forall_{H_2O}} \rho_{H_2O}\, d\forall + \left\{-|\cancelto{\ll \rho_{H_2O}\,(6)}{\rho_{air}} V_1 A_1|\right\} + \left\{|\rho_{H_2O} V A|\right\}$$

or $$0=\rho\frac{d\forall}{dt}+\rho VA=\rho\pi r^2\frac{dh}{dt}+\rho\sqrt{2gh}\,A$$

But h decreases, so $\frac{dh}{dt}=-s$. Thus

$$\pi r^2 s=\sqrt{2gh}\,A \quad \text{or} \quad r=\sqrt[4]{2g}\sqrt{\frac{A}{\pi s}}\,h^{1/4}$$

r

For n hours operation, $H=ns$, and

$$\forall=\int_0^H \pi r^2\,dh=\int_0^{ns}\sqrt{2gh}\,\frac{A}{s}\,dh=\frac{2A}{3s}\sqrt{2g}\,(ns)^{3/2}$$

or $$\forall=\frac{2A\sqrt{2g}\,n^{3/2}}{3}s^{1/2}$$

$\forall$

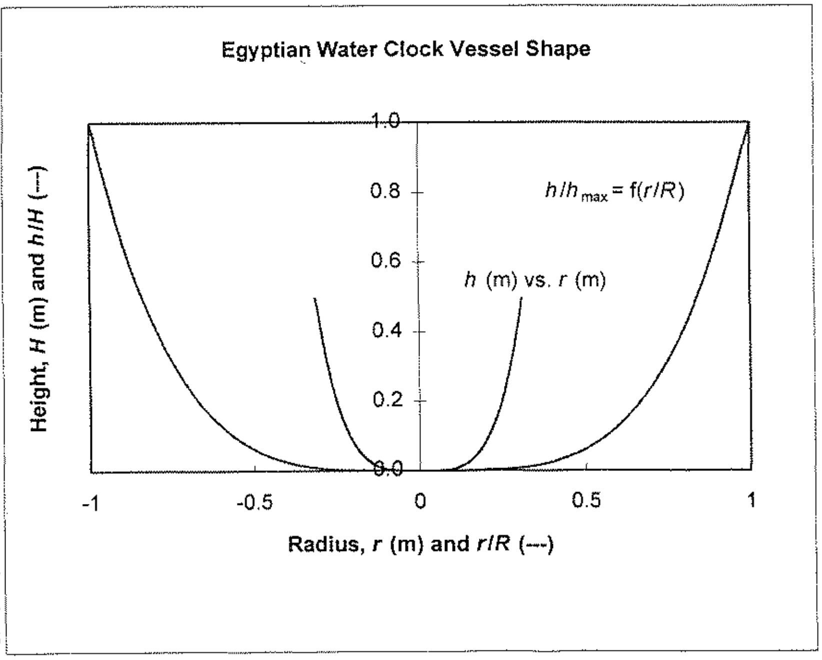
Evaluating and plotting:

Input Parameters:

Maximum water height: H = 0.5 m

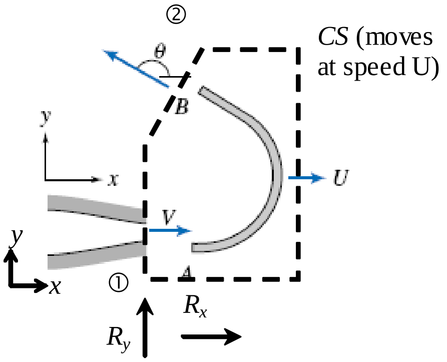
Number of hours' duration: n = 24 hr

Dimensionless Shape		Actual Shape	
r*/*R	***h*/*H***	***r* (m)**	***h* (m)**
-1	1.00	-0.309	0.500
-0.9	0.656	-0.278	0.328
-0.8	0.410	-0.247	0.205
-0.7	0.240	-0.216	0.120
-0.6	0.130	-0.185	0.065
-0.5	0.063	-0.155	0.031
-0.4	0.026	-0.124	0.013
-0.3	0.008	-0.093	0.004
-0.2	0.002	-0.062	0.001
-0.1	0.000	-0.031	0.000
0	0	0	0
0.1	0.000	0.031	0.000
0.2	0.002	0.062	0.001
0.3	0.008	0.093	0.004
0.4	0.026	0.124	0.013
0.5	0.063	0.155	0.031
0.6	0.130	0.185	0.065
0.7	0.240	0.216	0.120
0.8	0.410	0.247	0.205
0.9	0.656	0.278	0.328
1	1.000	0.309	0.500



Problem *4.122

4.122 A jet of water is directed against a vane, which could be a blade in a turbine or in any other piece of hydraulic machinery. The water leaves the stationary 40-mm diameter nozzle with a speed of 25 m/s and enters the vane tangent to the surface at A. The inside surface of the vane at B makes angle $\theta = 150°$ with the x direction. Compute the force that must be applied to maintain the vane speed constant at $U = 5$ m/s.



Given: Water jet striking moving vane

Find: Force needed to hold vane to speed U = 5 m/s

Solution:

Basic equations: Momentum flux in x and y directions

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\, d\forall + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Jet relative velocity is constant

Then $R_x = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

$R_x = \rho(V-U)^2\cdot A\cdot(\cos(\theta)-1)$ $\qquad A = \frac{\pi}{4}\cdot\left(\frac{40}{1000}\cdot m\right)^2$ $\qquad A = 1.26\times 10^{-3}\, m^2$

Using given data

$$R_x = 1000\cdot\frac{kg}{m^3}\times\left[(25-5)\cdot\frac{m}{s}\right]^2\times 1.26\times 10^{-3}\cdot m^2\times(\cos(150\cdot deg)-1)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = -940\,N$$

Then $R_y = v_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + v_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -0 + (V-U)\cdot\sin(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

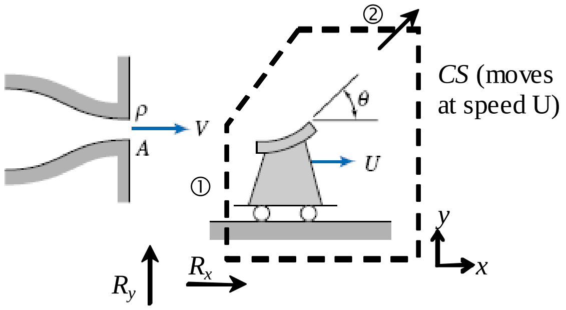
$$R_y = \rho(V-U)^2\cdot A\cdot\sin(\theta) \qquad R_y = 1000\cdot\frac{kg}{m^3}\times\left[(25-5)\cdot\frac{m}{s}\right]^2\times 1.26\times 10^{-3}\cdot m^2\times\sin(150\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_y = 252\,N$$

Hence the force required is 940 N to the left and 252 N upwards to maintain motion at 5 m/s

Problem 4.123

[3]

4.123 Water from a stationary nozzle impinges on a moving vane with turning angle $\theta = 120°$. The vane moves away from the nozzle with constant speed, $U = 10$ m/s, and receives a jet that leaves the nozzle with speed $V = 30$ m/s. The nozzle has an exit area of 0.004 m^2. Find the force that must be applied to maintain the vane speed constant.



Given: Water jet striking moving vane

Find: Force needed to hold vane to speed U = 10 m/s

Solution:

Basic equations: Momentum flux in x and y directions

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\mathcal{V} + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,d\mathcal{V} + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Jet relative velocity is constant

Then $R_x = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

$$R_x = \rho(V-U)^2\cdot A\cdot(\cos(\theta)-1)$$

Using given data

$$R_x = 1000\cdot\frac{kg}{m^3}\times\left[(30-10)\cdot\frac{m}{s}\right]^2\times 0.004\cdot m^2\times(\cos(120\cdot deg)-1)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = -2400\,N$$

Then $R_y = v_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + v_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -0 + (V-U)\cdot\sin(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

$$R_y = \rho(V-U)^2\cdot A\cdot\sin(\theta) \qquad R_y = 1000\cdot\frac{kg}{m^3}\times\left[(30-10)\cdot\frac{m}{s}\right]^2\times 0.004\cdot m^2\times\sin(120\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_y = 1386\,N$$

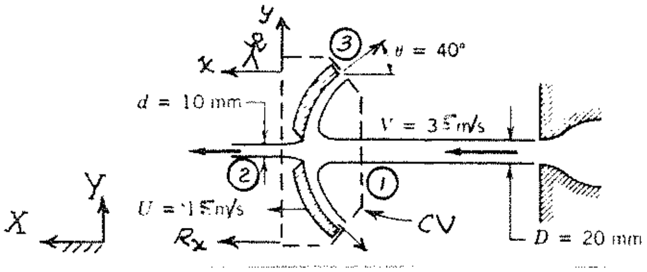
Hence the force required is 2400 N to the left and 1390 N upwards to maintain motion at 10 m/s

Problem 4.124

[2]

NATIONAL 42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE

Given: Circular dish and jet
moving as shown.



Find: Force required to maintain
dish motion.

Solution: Apply continuity and x
momentum equation to CV
moving with dish as shown.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(3)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\, \rho\, d\forall + \int_{CS} u_{xyz}\, \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Steady flow w.r.t. CV
(2) No pressure forces on CV
(3) Horizontal; $F_{B_x} = 0$
(4) Uniform flow at each section
(5) No change in speed of jet relative to vane
(6) Incompressible flow

Then

$$0 = \int_{CS} \vec{V}_{xyz} \cdot d\vec{A} = (V-U)\left(-\frac{\pi D^2}{4} + \frac{\pi d^2}{4} + A_{3,4}\right)$$

$$A_{3,4} = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}\left[(0.020)^2 - (0.010)^2\right] m^2 = 2.36 \times 10^{-4}\, m^2$$

From the momentum equation

$$R_x = u_1\left\{-\rho(V-U)\frac{\pi D^2}{4}\right\} + u_2\left\{+\rho(V-U)\frac{\pi d^2}{4}\right\} + u_3\left\{+\rho(V-U)A_{3,4}\right\}$$

$$u_1 = V - U \qquad u_2 = V - U \qquad u_3 = -(V-U)\cos 40°$$

$$R_x = -\rho(V-U)^2\frac{\pi D^2}{4} + \rho(V-U)^2\frac{\pi d^2}{4} - \rho(V-U)^2\frac{\pi}{4}(D^2 - d^2)\cos 40°$$

$$= -\rho(V-U)^2\frac{\pi}{4}(D^2 - d^2)(1 + \cos 40°)$$

$$= -999\,\frac{kg}{m^3} \times (35-15)^2\,\frac{m^2}{s^2} \times 2.36 \times 10^{-4}\, m^2 (1 + \cos 40°) \times \frac{N \cdot s^2}{kg \cdot m}$$

$R_x = -167$ N (force must be applied to right) R_x

{ Note: $R_y = Mg$, since there is no net momentum flux in the
y-direction. }

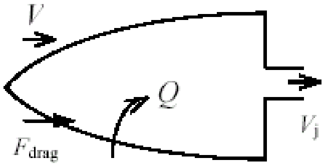
Problem 4.125

[2]

4.125 A jet boat takes in water at a constant volumetric rate Q through side vents and ejects it at a high jet speed V_j at the rear. A variable-area exit orifice controls the jet speed. The drag on the boat is given by $F_{drag} \approx kV^2$, where V is the boat speed. Find an expression for the steady speed V. If a jet speed $V_j = 25$ m/s produces a boat speed of 10 m/s, what jet speed will be required to double the boat speed?

Given: Data on jet boat

Find: Formula for boat speed; jet speed to double boat speed



CV in boat coordinates

Solution:

Governing equation:

Momentum $$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, d\forall + \int_{CS} \vec{V}_{xyz}\,\rho\, \vec{V}_{xyz}\cdot d\vec{A} \qquad (4.26)$$

Applying the horizontal component of momentum

$$F_{drag} = V\cdot(-\rho\cdot Q) + V_j\cdot(\rho\cdot Q)$$ or, with $F_{drag} = k\cdot V^2$ $\quad k\cdot V^2 = \rho\cdot Q\cdot V_j - \rho\cdot Q\cdot V$

$$k\cdot V^2 + \rho\cdot Q\cdot V - \rho\cdot Q\cdot V_j = 0$$

Solving for *V* $$V = -\frac{\rho\cdot Q}{2\cdot k} + \sqrt{\left(\frac{\rho\cdot Q}{2\cdot k}\right)^2 + \frac{\rho\cdot Q\cdot V_j}{k}}$$

Let $$\alpha = \frac{\rho\cdot Q}{2\cdot k}$$

$$V = -\alpha + \sqrt{\alpha^2 + 2\cdot\alpha\cdot V_j}$$

We can use given data at *V* = 10 m/s to find α $\quad V = 10\cdot\frac{m}{s} \quad V_j = 25\cdot\frac{m}{s}$

$$10\cdot\frac{m}{s} = -\alpha + \sqrt{\alpha^2 + 2\cdot 25\cdot\frac{m}{s}\cdot\alpha} \qquad \alpha^2 + 50\cdot\alpha = (10+\alpha)^2 = 100 + 20\cdot\alpha + \alpha^2 \qquad \alpha = \frac{10}{3}\cdot\frac{m}{s}$$

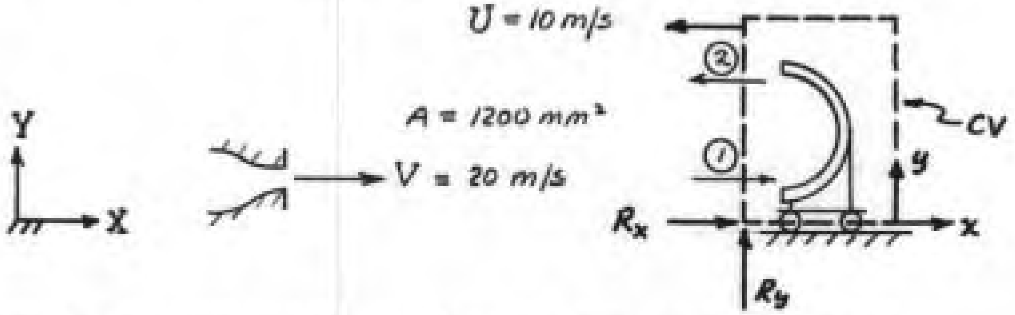
Hence $$V = -\frac{10}{3} + \sqrt{\frac{100}{9} + \frac{20}{3}\cdot V_j}$$

For *V* = 20 m/s $$20 = -\frac{10}{3} + \sqrt{\frac{100}{9} + \frac{20}{3}\cdot V_j} \qquad \frac{100}{9} + \frac{20}{3}\cdot V_j = \frac{70}{3} \qquad V_j = 80\cdot\frac{m}{s}$$

Problem 4.126

[2]

Given: Jet of oil (SG = 0.8) striking moving vane.



Find: Force needed to maintain vane speed constant.

Solution: Apply x component of momentum equation to moving CV shown.

$$\text{Basic equation:}\quad F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) No net pressure force on CV; $F_{S_x} = R_x$
(2) $F_{B_x} = 0$
(3) Steady flow
(4) Flow uniform at each section
(5) Jet area and speed relative to vane are constant

The subscript xyz is a reminder that all velocities must be evaluated relative to the CV. Then

$$R_x = u_1\{-|\rho(V+U)A|\} + u_2\{|\rho(V+U)A|\}$$

$$u_1 = V + U \qquad\qquad u_2 = -(V+U)$$

and $R_x = -\rho(V+U)^2A - \rho(V+U)^2A = -2\rho(V+U)^2A = -2SG\,\rho_{H_2O}(V+U)^2A$

$$R_x = -2(0.8)\,999\,\frac{kg}{m^3}\,(20+10)^2\frac{m^2}{s^2}\times 1200\,mm^2\times\frac{m^2}{10^6\,mm^2}\times\frac{N\cdot s^2}{kg\cdot m} = -1.73\ kN$$

R_x ←

This force must be applied to the left on the vane.

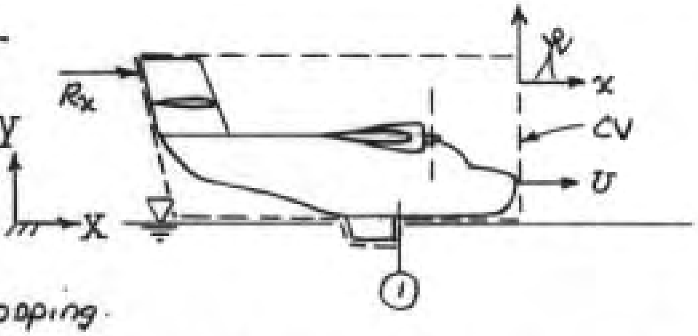
{Note $R_y = mg$, since there are no vertical components of velocity.}

Problem 4.127

[2]

Given: Aircraft scooping water from lake:

1620 gal in 12 sec



Find: Added thrust needed to maintain steady aircraft speed during scooping.

Solution: Use CV moving with aircraft, as shown. Apply momentum.

Basic equation: $$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{\approx 0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall}} + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) Horizontal motion, so $F_{B_x} = 0$
(2) Neglect u_{xyz} within the CV
(3) Uniform flow at inlet cross-section
(4) Neglect hydrostatic pressure

Then

$$R_x = u_1\left\{-|\rho Q_1|\right\} = -U(-\rho Q) = +U\rho Q$$

$$u_1 = -U$$

From data given

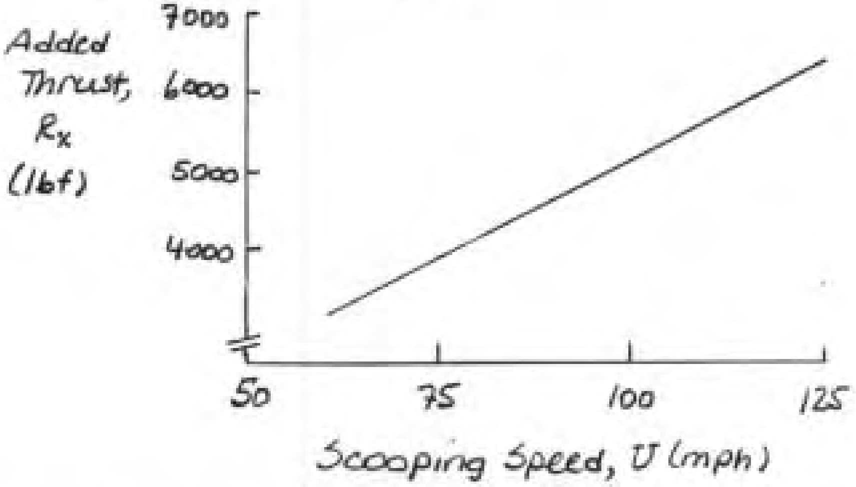
$$Q = \frac{\Delta\forall}{\Delta t} = \frac{1620\ \text{gal}}{12\ \text{sec}} \times \frac{\text{ft}^3}{7.48\ \text{gal}} = 18.0\ \text{ft}^3/\text{s}$$

For an aircraft speed of $U = 75$ mph (110 ft/s)

$$R_x = 110\,\frac{\text{ft}}{\text{s}} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times 18.0\,\frac{\text{ft}^3}{\text{s}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 3{,}840\ \text{lbf}$$

R_x

For a range of aircraft speeds:

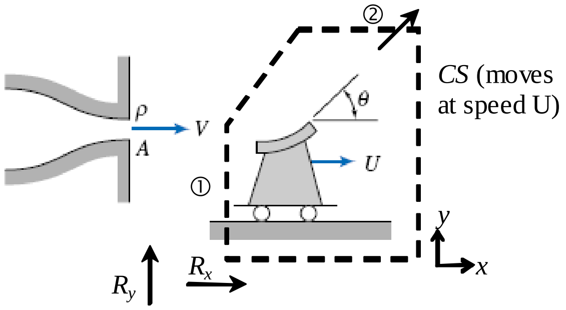


{Thus at 60 mph the added thrust is 3,070 lbf, while at 125 mph the added thrust is 6,400 lbf.}

Problem 4.128

[3]

4.128 Consider a single vane, with turning angle θ, moving horizontally at constant speed, U, under the influence of an impinging jet as in Problem 4.123. The absolute speed of the jet is V. Obtain general expressions for the resultant force and power that the vane could produce. Show that the power is maximized when $U = V/3$.



Given: Water jet striking moving vane

Find: Expressions for force and power; Show that maximum power is when U = V/3

Solution:

Basic equation: Momentum flux in x direction

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Jet relative velocity is constant

Then

$$R_x = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$$

$$R_x = \rho(V-U)^2\cdot A\cdot(\cos(\theta)-1)$$

This is force on vane; Force exerted by vane is equal and opposite $F_x = \rho\cdot(V-U)^2\cdot A\cdot(1-\cos(\theta))$

The power produced is then $P = U\cdot F_x = \rho\cdot U\cdot(V-U)^2\cdot A\cdot(1-\cos(\theta))$

To maximize power wrt to U

$$\frac{dP}{dU} = \rho\cdot(V-U)^2\cdot A\cdot(1-\cos(\theta)) + \rho\cdot(2)\cdot(-1)\cdot(V-U)\cdot U\cdot A\cdot(1-\cos(\theta)) = 0$$

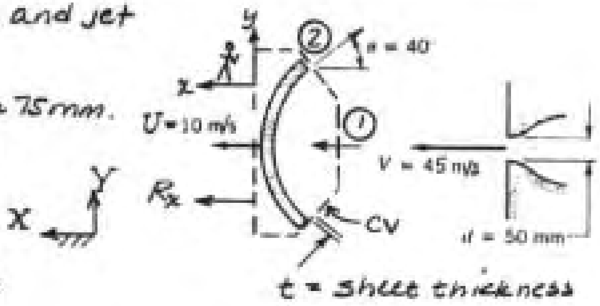
Hence $V - U - 2\cdot U = V - 3\cdot U = 0$ $\quad U = \frac{V}{3}$ for maximum power

Note that there is a vertical force, but it generates no power

Problem 4.129

Given: Circular dish with $D = 0.15$ m and jet as shown.

Find: (a) Thickness of jet sheet at $R = 75$ mm.
(b) Horizontal force required to maintain dish motion.



Solution: Apply the momentum equation to a CV moving with the dish, as shown.

Basic equation:

$$F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) No pressure forces
(2) Horizontal; $F_{B_x} = 0$
(3) Steady flow w.r.t. CV
(4) Uniform flow at each section
(5) Use relative velocities
(6) No change in relative velocity on the dish

Then

$$R_x = u_1\{-\rho(V-U)A\} + u_2\{+\rho(V-U)A\}$$

$$u_1 = V - U \qquad u_2 = -(V-U)\cos\theta$$

$$R_x = -\rho(V-U)^2A - \rho(V-U)^2A\cos\theta = -\rho(V-U)^2A(1+\cos\theta)$$

$$= -999\,\frac{kg}{m^3}\times(45-10)^2\frac{m^2}{s^2}\times\frac{\pi}{4}(0.050)^2m^2\,(1+\cos 40°)\frac{N\cdot s^2}{kg\cdot m}$$

$R_x = -4.24$ kN (force must act to right) $\qquad R_x$

Apply conservation of mass to determine the jet sheet thickness:

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

Using the above assumptions, then

$$0 = -\rho V_1A_1 + \rho V_2A_2$$

$$V_1 = V - U \;;\; V_2 = V - U \;;\; A_1 = \frac{\pi d^2}{4} \;;\; A_2 = 2\pi Rt$$

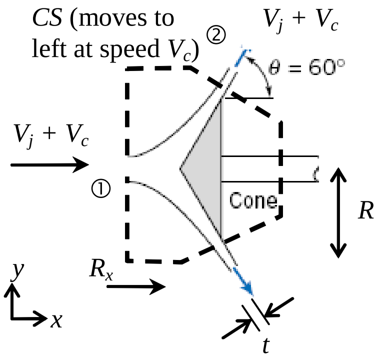
Therefore $A_1 = A_2 = \frac{\pi d^2}{4} = 2\pi Rt$, and $t = \frac{d^2}{8R}$

$t = \frac{1}{8}\times(0.050)^2m^2\times\frac{1}{0.075\,m} = 4.17\times10^{-3}$ m or 4.17 mm $\qquad t$

Problem 4.130

[3]

4.130 Water, in a 4-in. diameter jet with speed of 100 ft/s to the right, is deflected by a cone that moves to the left at 45 ft/s. Determine (a) the thickness of the jet sheet at a radius of 9 in. and (b) the external horizontal force needed to move the cone.



Given:
Water jet striking moving cone

Find:
Thickness of jet sheet; Force needed to move cone

Solution:

Basic equations: Mass conservation; Momentum flux in x direction

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Jet relative velocity is constant

Then $-\rho\cdot V_1\cdot A_1 + \rho\cdot V_2\cdot A_2 = 0$ $\qquad -\rho\cdot\left(V_j + V_c\right)\cdot\frac{\pi\cdot D_j^2}{4} + \rho\cdot\left(V_j + V_c\right)\cdot 2\cdot\pi\cdot R\cdot t = 0$ (Refer to sketch)

Hence $t = \frac{D_j^2}{8\cdot R}$ $\qquad t = \frac{1}{8}\times(4\cdot in)^2\times\frac{1}{9\cdot in}$ $\qquad t = 0.222\,in$

Using relative velocities, x momentum is

$$R_x = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -\left(V_j + V_c\right)\cdot\left[\rho\cdot\left(V_j + V_c\right)\cdot A_j\right] + \left(V_j + V_c\right)\cdot\cos(\theta)\cdot\left[\rho\cdot\left(V_j + V_c\right)\cdot A_j\right]$$

$$R_x = \rho\left(V_j + V_c\right)^2\cdot A_j\cdot(\cos(\theta) - 1)$$

Using given data

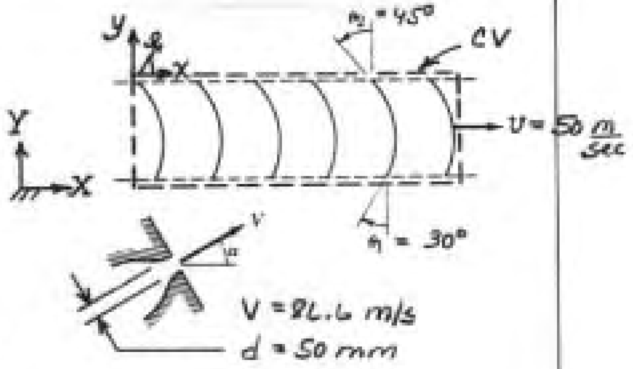
$$R_x = 1.94\cdot\frac{slug}{ft^3}\times\left[(100 + 45)\cdot\frac{ft}{s}\right]^2\times\frac{\pi\cdot\left(\frac{4}{12}\cdot ft\right)^2}{4}\times(\cos(60\cdot deg) - 1)\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad R_x = -1780\cdot lbf$$

Hence the force is 1780 lbf to the left; the upwards equals the weight

Problem 4.131

[3]

Given: Series of vanes struck by continuous jet, as shown.



Find: (a) Nozzle angle, α.
(b) Force to hold vane speed constant.

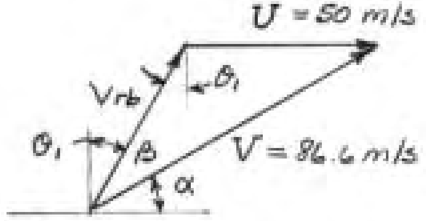
Solution: Apply momentum equation using CV moving with vanes, as shown.

Basic equation:

$$F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u_{xyz}\,\rho\, dV\!\!\!\!- + \int_{CS} u_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions:
(1) No pressure forces
(2) Horizontal; $F_{B_x} = 0$
(3) Steady flow w.r.t. CV
(4) Uniform flow at each section
(5) No change in relative velocity on vane
(6) Flow enters and leaves tangent to vanes

The nozzle angle may be obtained from trigonometry. The inlet velocity relationship is shown in the sketch:



From the law of sines,

$$\frac{\sin\alpha}{V_{rb}} = \frac{\sin(90+\theta_1)}{V} = \frac{\sin\beta}{U}$$

$$\beta = \sin^{-1}\left[\frac{U}{V}\sin(90+\theta_1)\right] = \sin^{-1}\left[\frac{50}{86.6}\sin(120°)\right] = 30°$$

From the sketch, $90° = \alpha + \beta + \theta_1$, so $\alpha = 90° - \beta - \theta_1 = 90° - 30° - 30° = 30°$ α

Also $V_{rb}\cos\theta_1 = V\sin\alpha$; $V_{rb} = V\frac{\sin\alpha}{\cos\theta_1} = 86.6\frac{m}{s} \times \frac{\sin 30°}{\cos 30°} = 50.0\ m/s$

From momentum equation (note all of $\dot{m}$ flows across vanes)

$$R_x = u_1\{-\dot{m}\} + u_2\{\dot{m}\} = V_{rb}\sin\theta(-\dot{m}) - V_{rb}\sin\theta_2(\dot{m}) = V_{rb}\dot{m}(-\sin\theta_1 - \sin\theta_2)$$

$$u_1 = V_{rb}\sin\theta_1 \quad u_2 = -V_{rb}\sin\theta_2\ ; \qquad R_y = \dot{m}V_{rb}(-\cos\theta_1 + \cos\theta_2)$$

Thus, since $\dot{m} = \rho Q$,

$$R_x = V_{rb}\rho Q(-\sin\theta_1 - \sin\theta_2)$$

$$= 50\frac{m}{s} \times 999\frac{kg}{m^3} \times 0.170\frac{m^3}{s}(-\sin 30° - \sin 45°) \times \frac{N\cdot s^2}{kg\cdot m}$$

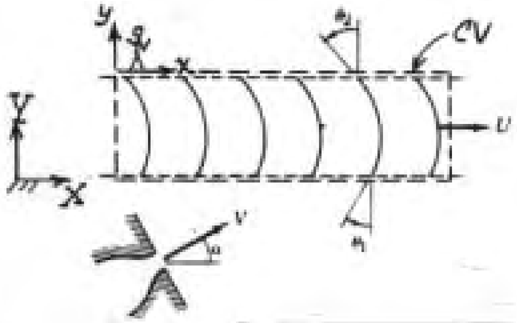
$R_x = -10.3$ kN (to left) R_x

{Note: The net force on the CV in the y-direction is $R_y = -1.35$ kN.}

Problem 4.132

[3]

Given: Series of vanes struck by continuous jet, as shown.



Find: For $\alpha \approx 0$ ($\theta_1 \approx 90°$), vane speed, U, to maximize power produced by vane.

Solution: Apply momentum equation using CV moving with vanes, as shown.

Basic equation:

$$F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions:
(1) No pressure forces
(2) Horizontal; $F_{B_x} = 0$
(3) Steady flow w.r.t. CV
(4) Uniform flow at each section
(5) No change in relative velocity on vane
(6) Flow enters and leaves tangent to vanes

For $\alpha \approx 0$, $V_{rb} \approx V - U$; the momentum equation becomes

$$R_x = u_1\{-\dot{m}\} + u_2\{+\dot{m}\} = -\dot{m}(V-U) - \dot{m}(V-U)\sin\theta_2 = -\dot{m}(V-U)(1+\sin\theta_2)$$

$$u_1 \approx V_{rb} \approx V-U \,;\; u_2 \approx -V_{rb}\sin\theta_2 \approx -(V-U)\sin\theta_2$$

The vane system produces force, $K_x = -R_x$, and power $\mathcal{P} = K_x U$. Thus

$$\mathcal{P} = K_x U = -R_x U = \dot{m}(V-U)U(1+\sin\theta_2) \qquad (1)$$

To find maximum power, set $\dfrac{d\mathcal{P}}{dU} = 0$

$$\frac{d\mathcal{P}}{dU} = \dot{m}(-1)U(1+\sin\theta_2) + \dot{m}(V-U)(1)(1+\sin\theta_2) = \dot{m}(V-2U)(1+\sin\theta_2)$$

Thus power is maximized when $V - 2U = 0$, or $U = \dfrac{V}{2}$ (for $\mathcal{P}_{max}$) $\qquad U$

{Note from Eq. 1 that $\theta_2 \to 90°$ increases power also.}

{Note also that $K_y = -R_y = -\dot{m}V_{rb}\cos\theta_2$ but this force does not produce power.}

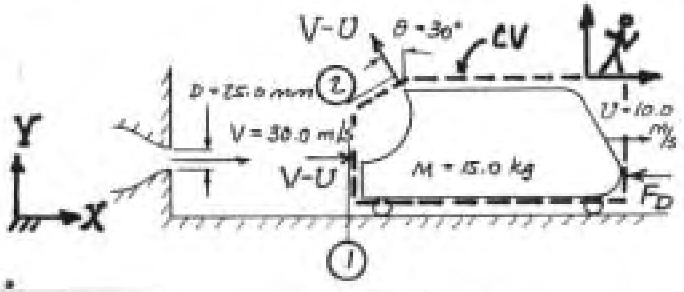
Problem 4.133

[2]

Given: Cart propelled by steady water jet, as shown. Total resistance to motion is

$$F_D = kU^2$$

Where $k = 0.92 \ \frac{N \cdot s^2}{m^2}$



Find: Acceleration of cart at instant when $U = 10$ m/s.

Solution: Apply the momentum equation using CV and CS shown.

Basic equation: $$F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x} \rho \, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u_{xyz} \rho \, d\forall + \int_{CS} u_{xyz} \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Only resistance is F_D; $F_{S_x} = -F_D = -kU^2$
(2) Horizontal; $F_{B_x} = 0$
(3) Neglect $\partial u / \partial t$ of mass of water in CV
(4) No change in speed w.r.to vane
(5) Uniform flow at each cross-section

Then

$$-kU^2 - a_{rf_x} M_{cv} = u_1 \{-\rho(V-U)A\} + u_2 \{+\rho(V-U)A\}$$

Measure u w.r.to CV: $u_1 = V - U$ $\qquad u_2 = -(V-U)\sin\theta$

$$-kU^2 - a_{rf_x} M_{cv} = -\rho(V-U)^2 A - \rho(V-U)^2 A \sin\theta = -\rho(V-U)^2 A(1+\sin\theta)$$

so

$$a_{rf_x} = \frac{1}{M}\left[\rho(V-U)^2 A(1+\sin\theta) - kU^2\right]$$

$$= \frac{1}{15\,kg}\left[999 \frac{kg}{m^3} (30-10)^2 \frac{m^2}{s^2} \frac{\pi}{4}(0.025)^2 m^2 (1+\sin 30°) - 0.92 \frac{N \cdot s^2}{m^2} \times (10)^2 \frac{m^2}{sec^2} \times \frac{kg \cdot m}{N \cdot s^2}\right]$$

$a_{rf_x} = 13.5 \ m/s^2$ (to right)

a_{rf_x}

Problem 4.134

Given: Splitter dividing flow into two flat streams, as shown.

Find: (a) Mass flow rate ratio, $\dot{m}_2/\dot{m}_3$, so net vertical force is zero.

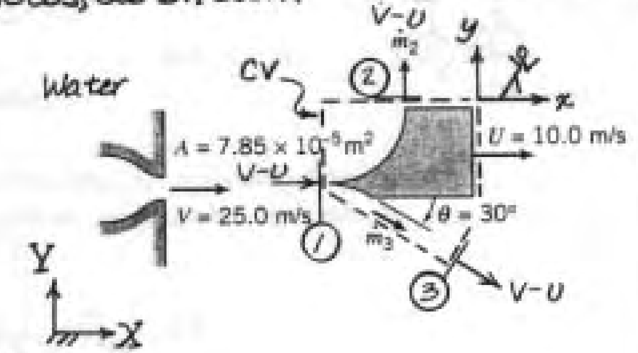
(b) Horizontal force need to maintain constant speed.

Solution: Apply x and y components of momentum to CV drawn with boundaries ⊥ to flows, as shown.

Basic equations:

$$\cancelto{=0(1)}{F_{S_y}} + \cancelto{=0(2)}{F_{B_y}} = \cancelto{=0(3)}{\frac{\partial}{\partial t}}\int_{CV} v_{xyz}\,\rho\, dV\!\!\!\!- + \int_{CS} v_{xyz}\,\rho\,\vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \cancelto{=0}{F_{B_x}} = \cancelto{=0(3)}{\frac{\partial}{\partial t}}\int_{CV} u_{xyz}\,\rho\, dV\!\!\!\!- + \int_{CS} u_{xyz}\,\rho\,\vec{V}\cdot d\vec{A}$$



Assumptions: (1) No pressure forces

(2) Neglect mass of water on vane

(3) Steady flow w.r.to vane

(4) Uniform flow at each section

(5) No change in speed w.r.to vane

Then

$$0 = \int_{CS} v\rho\vec{V}\cdot d\vec{A} = v_1\{-\dot{m}_1\} + v_2\{+\dot{m}_2\} + v_3\{+\dot{m}_3\}$$

Measure w.r.to CV: $\quad v_1 = 0 \quad v_2 = V-U \quad v_3 = -(V-U)\sin\theta$

so $\quad 0 = (V-U)\dot{m}_2 - (V-U)\sin\theta\,\dot{m}_3 \; ; \; \dfrac{\dot{m}_2}{\dot{m}_3} = \sin\theta = \dfrac{1}{2}$

$\dot{m}_2/\dot{m}_3$

and

$$F_{S_x} = \int_{CS} u\rho\vec{V}\cdot d\vec{A} = R_x = u_1\{-\dot{m}_1\} + u_2\{+\dot{m}_2\} + u_3\{+\dot{m}_3\}$$

Measure w.r.to CV: $\quad u_1 = V-U \quad u_2 = 0 \quad u_3 = (V-U)\cos\theta$

$$R_x = (V-U)(-\dot{m}_1) + (V-U)\cos\theta(\dot{m}_3) = (V-U)(\dot{m}_3\cos\theta - \dot{m}_1)$$

From continuity $\; 0 = -\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = -\dot{m}_1 + \dfrac{\dot{m}_3}{2} + \dot{m}_3 \; ; \; \dot{m}_3 = \dfrac{2}{3}\dot{m}_1$

$$R_x = (V-U)\left(\frac{2}{3}\dot{m}_1\cos\theta - \dot{m}_1\right) = (V-U)\dot{m}_1\left(\frac{2\cos\theta}{3} - 1\right)$$

$$R_x = (25-10)\frac{\mathrm{m}}{\mathrm{s}} \times 999\,\frac{\mathrm{kg}}{\mathrm{m}^3} \times (25-10)\frac{\mathrm{m}}{\mathrm{s}} \times 7.85\times10^{-5}\,\mathrm{m}^2\left(\frac{2}{3}\cos30^\circ - 1\right)\frac{\mathrm{N}\cdot\mathrm{s}^2}{\mathrm{kg}\cdot\mathrm{m}}$$

$$R_x = -7.46\ \mathrm{N}\ \text{(to left)}$$

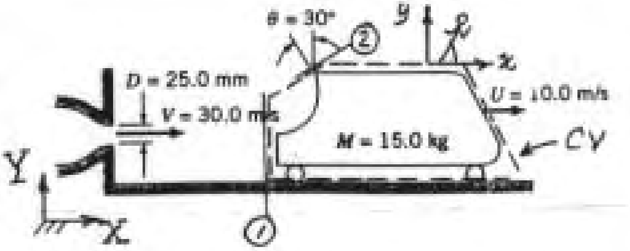
R_x

{ Force must be applied to left to maintain vane speed constant; if R_x were zero, vane would accelerate. }

Problem 4.135

[2]

Given: Hydraulic catapult of Problem 4.133, rolling on level track with negligible resistance, speed U.



Find: Time required to accelerate from rest to $U = V/2$.

Solution: Apply x component of momentum equation to accelerating CV.

Basic equation:
$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) $F_{S_x} = 0$, since no pressure forces, no resistance
(2) $F_{B_x} = 0$, since horizontal
(3) Neglect mass of water on vane
(4) Uniform flow in jet
(5) No change in relative velocity on vane

Then
$$-a_{rf_x} M_{cv} = u_1\{-\rho(V-U)A\} + u_2\{+\rho(V-U)A\} = -(1+\sin\theta)\rho(V-U)^2 A$$
$$u_1 = V-U \qquad u_2 = -(V-U)\sin\theta$$

so
$$\frac{dU}{dt} = \frac{\rho A(1+\sin\theta)}{M}(V-U)^2$$

To integrate, note since V = constant, $d(V-U) = -dU$, so
$$-\int_0^{V/2}\frac{d(V-U)}{(V-U)^2} = \int_0^t \frac{\rho A(1+\sin\theta)}{M}dt$$

or
$$\left.\frac{1}{V-U}\right]_{U=0}^{U=V/2} = \frac{2}{V} - \frac{1}{V} = \frac{1}{V} = \rho\frac{A(1+\sin\theta)}{M}t$$

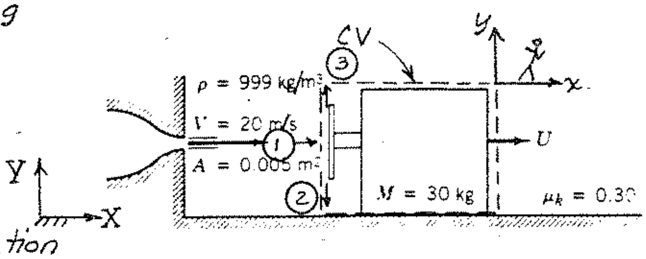
Thus
$$t = \frac{M}{\rho V A(1+\sin\theta)}$$
$$= 15.0\,\text{kg} \times \frac{\text{m}^3}{999\,\text{kg}} \times \frac{\text{s}}{30.0\,\text{m}} \times \frac{4}{\pi(0.025)^2\,\text{m}^2} \times \frac{1}{(1+\sin 30°)}$$

$$t = 0.680\ \text{s}$$ ← t

Problem 4.136

[2]

Given: Vane/slider assembly moving under influence of jet.



Find: Terminal speed.

Solution: Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u_{xyz}\rho\, d\forall}} + \int_{CS} u_{xyz}\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) Horizontal motion, so $F_{B_x} = 0$

(2) Neglect mass of liquid on vane, $u \approx 0$ on vane

(3) Uniform flow at each section

(4) Measure velocities relative to CV

Then

$$-Mg\mu_k - a_{rf_x}M = u_1\{-|\rho(V-U)A|\} + u_2\{+\dot{m}_2\} + u_3\{+\dot{m}_3\}$$

$$u_1 = V - U \qquad u_2 = 0 \qquad u_3 = 0$$

$$-Mg\mu_k - M\frac{dU}{dt} = -\rho(V-U)^2 A$$

or

$$\frac{dU}{dt} = \frac{\rho(V-U)^2 A}{M} - g\mu_k$$

At terminal speed, $dU/dt = 0$ and $U = U_t$, so

$$0 = \frac{\rho(V-U_t)^2 A}{M} - g\mu_k \quad \text{or} \quad V - U_t = \sqrt{\frac{Mg\mu_k}{\rho A}}$$

and

$$U_t = V - \sqrt{\frac{Mg\mu_k}{\rho A}}$$

$$= 20\,\frac{m}{s} - \left[30\,kg \times 9.81\,\frac{m}{s^2} \times 0.3 \times \frac{m^3}{999\,kg} \times \frac{1}{0.005\,m^2}\right]^{1/2}$$

$$U_t = 15.8\ m/s$$

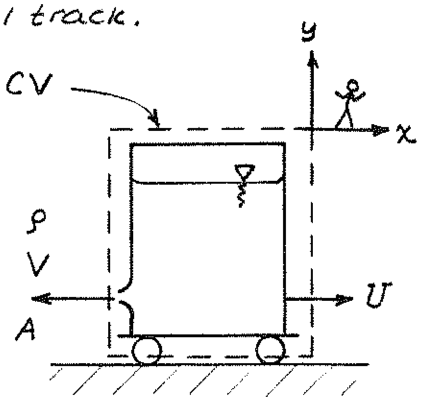
U_t

Problem 4.137

[2]

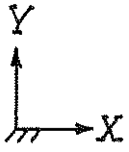
Given: Cart propelled by a horizontal liquid jet of constant speed. Neglect resistance along horizontal track.

Initial mass is M_0.



Find: (a) A general expression for speed, U, as cart accelerates from rest.

(b) V for $U = 1.5$ m/s @ $t = 30$ s



Solution:

a) Apply x component of momentum equation using linearly accelerating CV shown.

Basic equation:

$$\overset{=0(1)}{F_{S_x}} + \overset{=0(2)}{F_{B_x}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(3)}{\frac{\partial}{\partial t}}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions:
(1) No resistance
(2) $F_{B_x} = 0$ since track is horizontal
(3) Neglect u_{xyz} within CV
(4) Uniform flow at jet exit

Then

$$-a_{rf_x} M = u\{|\rho V A|\} = -\rho V^2 A$$

$$u = -V$$

From continuity, $M = M_0 - \dot{m}t = M_0 - \rho V A t$. Using $a_{rf_x} = \frac{dU}{dt}$,

$$\frac{dU}{dt} = \frac{\rho V^2 A}{M_0 - \rho V A t}$$

Separating variables and integrating,

$$\int_0^U dU = U = \int_0^t \frac{\rho V^2 A\, dt}{M_0 - \rho V A t} = -V \ln(M_0 - \rho V A t)\Big|_0^t = V \ln\left(\frac{M_0}{M_0 - \rho V A t}\right)$$

or

$$\frac{U}{V} = \ln\left(\frac{M_0}{M_0 - \rho V A t}\right) \longleftarrow \frac{U}{V}$$

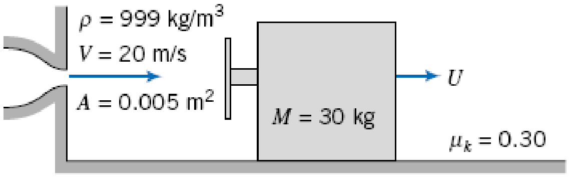
Check dimensions: $[\rho V A t] = \frac{M}{L^3}\frac{L}{t}L^2 t = M$ ✓

b) Using the given data in Excel (with Solver) the jet speed for $U = 1.5$ m/s @ $t = 30$ s is $V = 0.61$ m/s

Problem 4.138

[4]

4.138 For the vane/slider problem of Problem 4.136, find and plot expressions for the acceleration, speed, and position of the slider as a function of time.



Given: Data on vane/slider

Find: Formula for acceleration, speed, and position; plot

Solution:

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $\quad M = 30\cdot kg$ $\quad A = 0.005\cdot m^2$ $\quad V = 20\cdot\frac{m}{s}$ $\quad \mu_k = 0.3$

The equation of motion, from Problem 4.136, is $\frac{dU}{dt} = \frac{\rho\cdot(V-U)^2\cdot A}{M} - g\cdot\mu_k$

The acceleration is thus $a = \frac{\rho\cdot(V-U)^2\cdot A}{M} - g\cdot\mu_k$

Separating variables

$$\frac{dU}{\frac{\rho\cdot(V-U)^2\cdot A}{M} - g\cdot\mu_k} = dt$$

Substitute $u = V - U$ $\quad dU = -du$ $\quad \frac{du}{\frac{\rho\cdot A\cdot u^2}{M} - g\cdot\mu_k} = -dt$

$$\int \frac{1}{\left(\frac{\rho\cdot A\cdot u^2}{M} - g\cdot\mu_k\right)}\,du = -\sqrt{\frac{M}{g\cdot\mu_k\cdot\rho\cdot A}}\cdot\operatorname{atanh}\left(\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot u\right)$$

and u = V - U so

$$-\sqrt{\frac{M}{g\cdot\mu_k\cdot\rho\cdot A}}\cdot\operatorname{atanh}\left(\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot u\right) = -\sqrt{\frac{M}{g\cdot\mu_k\cdot\rho\cdot A}}\cdot\operatorname{atanh}\left[\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot(V-U)\right]$$

Using initial conditions

$$-\sqrt{\frac{M}{g\cdot\mu_k\cdot\rho\cdot A}}\cdot\operatorname{atanh}\left[\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot(V-U)\right] + \sqrt{\frac{M}{g\cdot\mu_k\cdot\rho\cdot A}}\cdot\operatorname{atanh}\left(\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot V\right) = -t$$

$$V - U = \sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot V\right)\right)$$

$$U = V - \sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot V\right)\right)$$

Note that $$\operatorname{atanh}\left(\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot V\right) = 0.213 - \frac{\pi}{2}\cdot i$$

which is complex and difficult to handle in *Excel*, so we use the identity $\operatorname{atanh}(x) = \operatorname{atanh}\left(\frac{1}{x}\right) - \frac{\pi}{2}\cdot i$ for x > 1

so $$U = V - \sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\frac{1}{\sqrt{\frac{\rho\cdot A}{g\cdot\mu_k\cdot M}}\cdot V}\right) - \frac{\pi}{2}\cdot i\right)$$

and finally the identity $$\tanh\left(x - \frac{\pi}{2}\cdot i\right) = \frac{1}{\tanh(x)}$$

to obtain $$U = V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}$$

For the position x $$\frac{dx}{dt} = V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}$$

This can be solved analytically, but is quite messy. Instead, in the corresponding *Excel* workbook, it is solved numerically using a simple Euler method. The complete set of equations is

$$U = V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}$$

$$a = \frac{\rho\cdot(V-U)^2\cdot A}{M} - g\cdot\mu_k$$

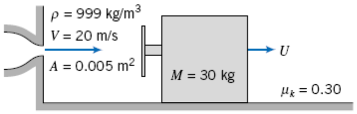
$$x(n+1) = x(n) + \left(V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}\right)\cdot\Delta t$$

The plots are presented in the *Excel* workbook

Problem 4.138

[4]

4.138 For the vane/slider problem of Problem 4.136, find and plot expressions for the acceleration, speed, and position of the slider as a function of time.



Given: Data on vane/slider

Find: Formula for acceleration, speed, and position; plot

Solution:

The equations are

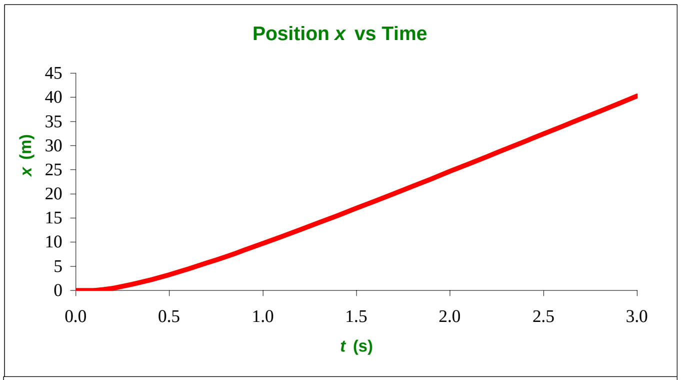
$$U = V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}$$

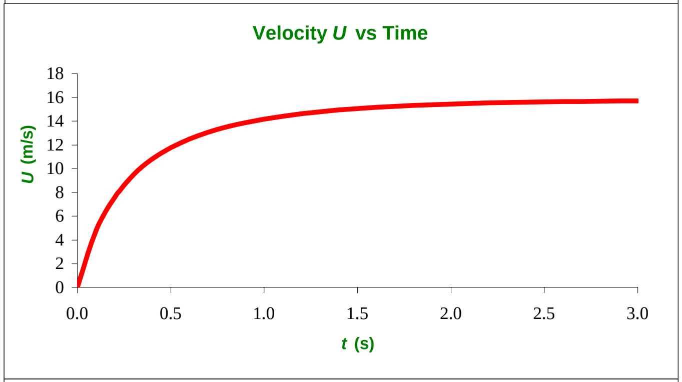
$$a = \frac{\rho\cdot(V-U)^2\cdot A}{M} - g\cdot\mu_k$$

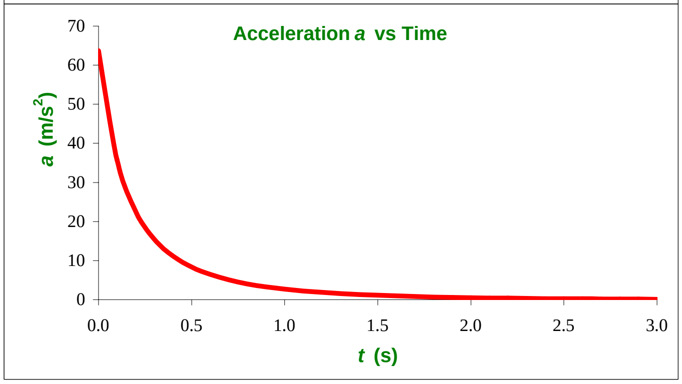
$$x(n+1) = x(n) + \left(V - \frac{\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}}{\tanh\left(\sqrt{\frac{g\cdot\mu_k\cdot\rho\cdot A}{M}}\cdot t + \operatorname{atanh}\left(\sqrt{\frac{g\cdot\mu_k\cdot M}{\rho\cdot A}}\cdot\frac{1}{V}\right)\right)}\right)\cdot\Delta t$$

ρ = 999 kg/m^3
μ_k = 0.3
A = 0.005 m^2
V = 20 m/s
M = 30 kg
Δt = 0.1 s

t (s)	x (m)	U (m/s)	a (m/s^2)
0.0	0.0	0.0	63.7
0.1	0.0	4.8	35.7
0.2	0.5	7.6	22.6
0.3	1.2	9.5	15.5
0.4	2.2	10.8	11.2
0.5	3.3	11.8	8.4
0.6	4.4	12.5	6.4
0.7	5.7	13.1	5.1
0.8	7.0	13.5	4.0
0.9	8.4	13.9	3.3
1.0	9.7	14.2	2.7
1.1	11.2	14.4	2.2
1.2	12.6	14.6	1.9
1.3	14.1	14.8	1.6
1.4	15.5	14.9	1.3
1.5	17.0	15.1	1.1
1.6	18.5	15.2	0.9
1.7	20.1	15.3	0.8
1.8	21.6	15.3	0.7
1.9	23.1	15.4	0.6
2.0	24.7	15.4	0.5
2.1	26.2	15.5	0.4
2.2	27.8	15.5	0.4
2.3	29.3	15.6	0.3
2.4	30.9	15.6	0.3
2.5	32.4	15.6	0.2
2.6	34.0	15.6	0.2
2.7	35.6	15.7	0.2
2.8	37.1	15.7	0.2
2.9	38.7	15.7	0.1
3.0	40.3	15.7	0.1


Position x vs Time

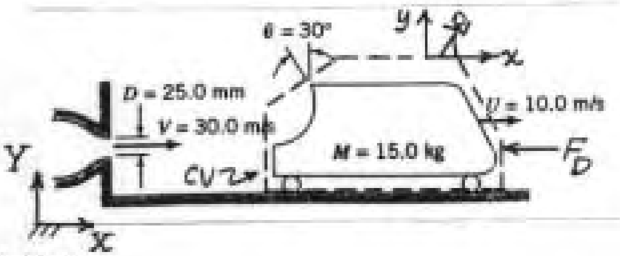

Velocity U vs Time


Acceleration a vs Time

Problem 4.139

[3]

Given: Hydraulic catapult of Problem 4.133 rolling on level track with resistance, $F_D = kU^2$, speed U, starting from rest at $t=0$.



Find: (a) when acceleration is maximum

(b) Sketch of acceleration vs. time

(c) Value of θ to maximize acceleration, why?

(d) If U will ever reach V; explanation

Solution: Apply x component of momentum equation to accelerating CV

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\, \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) $F_{S_x} = -F_D = -kU^2$, where $k = 0.92\ \mathrm{N\cdot s^2/m^2}$

(2) $F_{B_x} = 0$, since horizontal

(3) Neglect mass of water on vane

(4) Uniform flow in jet

(5) No change in relative velocity on vane

Then

$$-kU^2 - a_{rf_x} M_{CV} = u_1\{-\rho(V-U)A\} + u_2\{+\rho(V-U)A\} = -(1+\sin\theta)\rho(V-U)^2 A$$

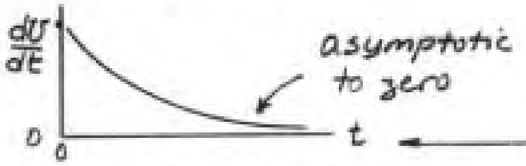
$$u_1 = V-U \qquad u_2 = -(V-U)\sin\theta$$

so

$$\frac{dU}{dt} = \frac{\rho A(1+\sin\theta)}{M}(V-U)^2 - kU^2/M \qquad (1)$$

(a) Acceleration is maximum at $t=0$, when $U=0$ ⟵

(b) Acceleration vs. time will be



⟵

(c) From Eq. 1, dU/dt is maximum when $\theta = \pi/2$ and $\sin\theta = 1$ ⟵

(d) From Eq. 1, $\frac{dU}{dt}$ will go to zero when $U < V$; this will be the terminal speed for the cart, U_t. From Eq. 1, $\frac{dU}{dt} = 0$ when

$$\rho A(1+\sin\theta)(V-U)^2 = kU^2$$

or

$$U = \frac{\left[\frac{\rho A(1+\sin\theta)}{k}\right]^{1/2}}{1+\left[\frac{\rho A(1+\sin\theta)}{k}\right]^{1/2}} V = 0.472V$$

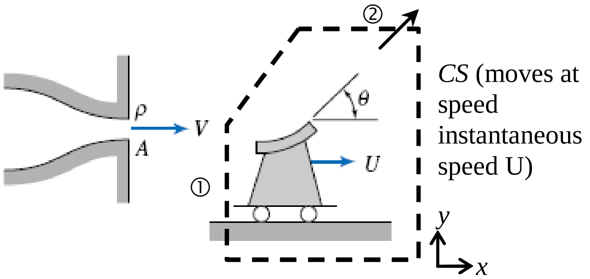
⟵

U will be asymptotic to V.

Problem 4.140

[4]

4.140 The acceleration of the vane/cart assembly of Problem 4.123 is to be controlled as it accelerates from rest by changing the vane angle, θ. A constant acceleration, $a = 1.5$ m/s^2, is desired. The water jet leaves the nozzle of area $A = 0.025$ m^2, with speed $V = 15$ m/s. The vane/cart assembly has a mass of 55 kg; neglect friction. Determine θ at $t = 5$ s. Plot $\theta(t)$ for the given constant acceleration over a suitable range of t.



Given: Water jet striking moving vane/cart assembly

Find: Angle θ at t = 5 s; Plot θ(t)

Solution:

Basic equation: Momentum flux in x direction for accelerating CV

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) cahnges in CV 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow 5) Constant jet relative velocity

Then $-M\cdot a_{rfx} = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$

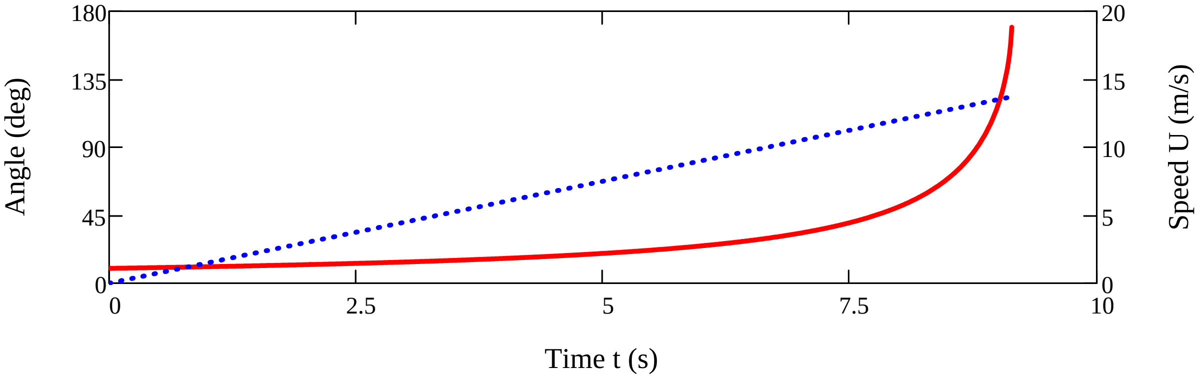
$-M\cdot a_{rfx} = \rho(V-U)^2\cdot A\cdot(\cos(\theta)-1)$ or $\cos(\theta) = 1 - \dfrac{M\cdot a_{rfx}}{\rho\cdot(V-U)^2\cdot A}$

Since $a_{rfx} = \text{constant}$ then $U = a_{rfx}\cdot t$ $\cos(\theta) = 1 - \dfrac{M\cdot a_{rfx}}{\rho\cdot\left(V - a_{rfx}\cdot t\right)^2\cdot A}$

$$\theta = \operatorname{acos}\left[1 - \frac{M\cdot a_{rfx}}{\rho\cdot\left(V - a_{rfx}\cdot t\right)^2\cdot A}\right]$$

Using given data

$$\theta = \operatorname{acos}\left[1 - 55\cdot\text{kg}\times 1.5\cdot\frac{\text{m}}{\text{s}^2}\times\frac{\text{m}^3}{1000\cdot\text{kg}}\times\frac{1}{\left(15\cdot\frac{\text{m}}{\text{s}} - 1.5\cdot\frac{\text{m}}{\text{s}^2}\times 5\cdot\text{s}\right)^2}\times\frac{1}{0.025\cdot\text{m}^2}\right] \qquad \theta = 19.7\,\text{deg} \quad \text{at t = 5 s}$$



The solution is only valid for θ up to 180º (when t = 9.14 s). This graph can be plotted in *Excel*

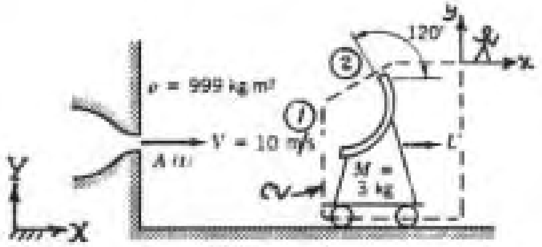
Problem 4.141

[3]

Given: Vaned cart rolling with negligible resistance.

$a_{rf_x} = 2\ m/s^2 =$ constant

Jet area is $A(t)$, programmed.



Find: (a) Expression for $A(t)$ at cart.
(b) Sketch for $t \leq 4s$.
(c) Evaluate at $t = 2s$.

Solution: Apply x momentum to CV with linear acceleration.

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x} \rho \, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}\int_{CV}}} u_{xyz} \rho \, d\forall + \int_{CS} u_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) No resistance to motion
(2) Horizontal motion, so $F_{B_x} = 0$
(3) Neglect mass of liquid in CV
(4) Uniform flow at each section
(5) All velocities measured relative to CV
(6) No change in stream area or speed on vane

Then (with $a_{rf_x} = a$)

$$-aM = u_1\{-|\rho(V-U)A|\} + u_2\{+|\rho(V-U)A|\} = -\frac{3}{2}\rho(V-U)^2 A$$

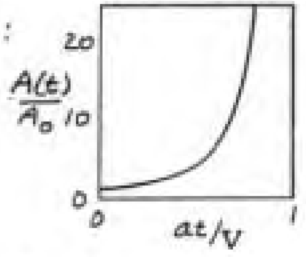
$$u_1 = V - U \qquad u_2 = (V-U)\cos 120° = -\frac{1}{2}(V-U)$$

Since a = constant, $U = at$, and

$$A = A(t) = \frac{2aM}{3\rho(V-at)^2}$$ A(t)

At $t = 0$, $A(0) = A_0 = \frac{2aM}{3\rho V^2}$. Thus $\frac{A}{A_0} = \frac{1}{(1-at/V)^2}$.

Sketch:



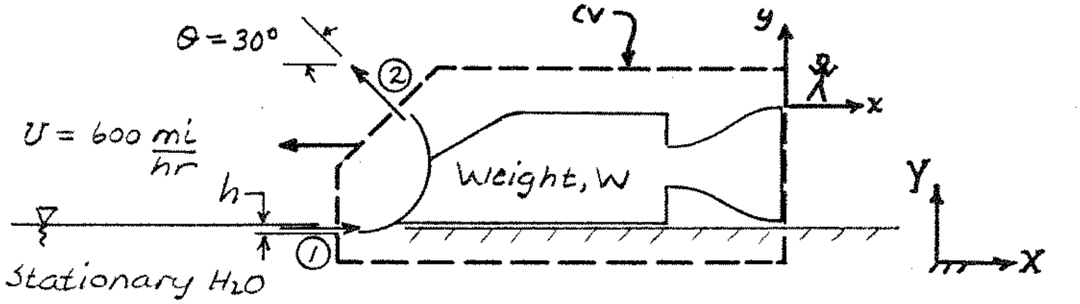
At $t = 2$ sec,

$$A = \frac{2}{3} \times 2\frac{m}{s^2} \times 3\ kg \times \frac{m^3}{999\ kg} \frac{1}{\left[10\frac{m}{s} - \frac{2m}{s^2} \times 2s\right]^2} \times 10^6 \frac{mm^2}{m^2} = 111\ mm^2$$ A(2)

Problem 4.142

[3] Part 1/2

Given: Rocket sled with water scoop brake. $W = 10{,}000\ \text{lbf}$



Scoop immersed in trough is $w = 6$ in. wide, $h = 3$ in. deep.

Find: Time needed to decelerate to 20 mph. Plot: Speed vs. time.

Solution: Apply x component of momentum equation to linearly accelerating CV. Basic equation is

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\, \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) $F_{S_x} = 0$

(2) $F_{B_x} = 0$

(3) Neglect u_{xyz} and its rate of change in CV

(4) Uniform flow at each section

(5) Speed of water relative to sled is constant

Then

$$-a_{rf_x} M = u_1\{-|\rho U w h|\} + u_2\{|\rho U w h|\}\ ;\ u_1 = U,\ u_2 = -U\cos\theta$$

$$-a_{rf_x}\frac{W}{g} = -\rho U^2 wh(1+\cos\theta), \text{ or } \quad a_{rf_x} = \frac{\rho g U^2 wh(1+\cos\theta)}{W}$$

Now $a_{rf_x} = -\frac{dU}{dt}$, because of coordinate choice. Thus

$$\frac{dU}{U^2} = -\frac{\gamma wh}{W}(1+\cos\theta)\,dt$$

and

$$\int_{U_i}^{U}\frac{dU}{U^2} = -\frac{1}{U} + \frac{1}{U_i} = -\frac{\gamma wh}{W}(1+\cos\theta)\,t \qquad (1)$$

Solving for t,

$$t = \left[\frac{1}{U} - \frac{1}{U_i}\right]\frac{W}{\gamma wh(1+\cos\theta)}$$

$$= \left[\frac{1}{20} - \frac{1}{600}\right]\frac{hr}{mi}\times\frac{mi}{5280\ ft}\times 3600\frac{s}{hr}\times\frac{ft^3}{62.4\ lbf}\times\frac{1}{6\ in.}\times\frac{1}{3\ in.}\times\frac{144\ in.^2}{ft^2}\times\frac{10{,}000\ lbf}{1+\cos 30°}$$

$t = 22.6\ s \quad \longleftarrow \quad t$

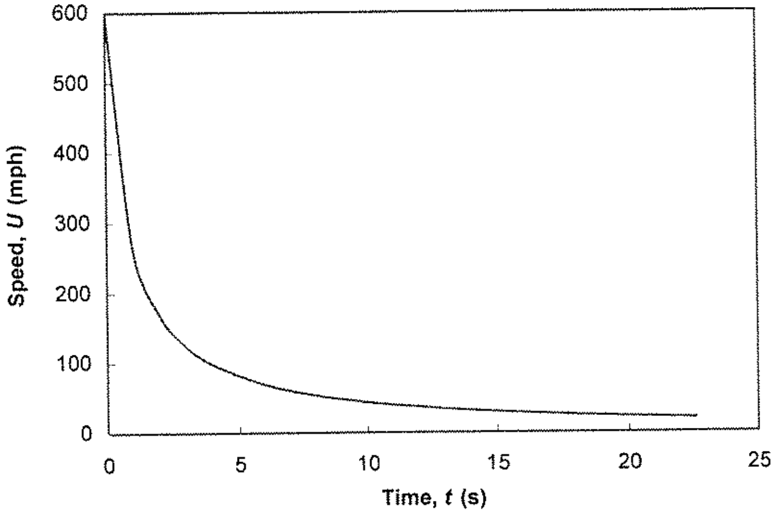
The plot is presented on the next page.

Solving Eq. 1 for U,

$$\frac{1}{U} = \frac{1}{U_i} + \frac{\gamma \omega h}{W}(1+\cos\theta)t = \frac{W + \gamma \omega h U_i (1+\cos\theta)t}{W U_i}$$

or $$U = \frac{W U_i}{W + \gamma \omega h U_i (1+\cos\theta)t} \qquad (2)$$

Plotting,

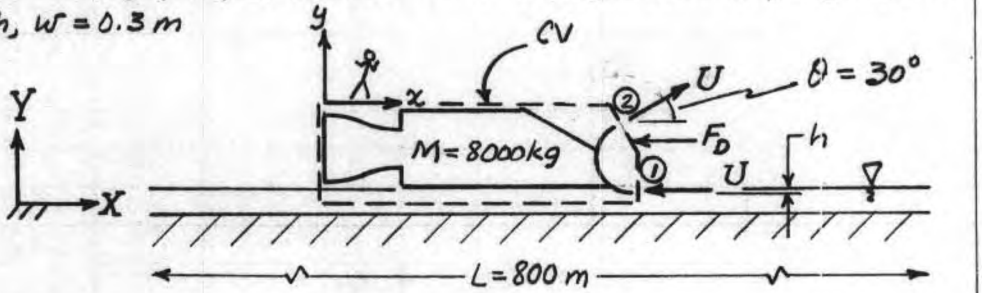


Problem 4.143

[3]

Given: Rocket sled slowed by scoop in water trough.

Aerodynamic drag proportional to U^2. At $U_0 = 300\,m/s$, $F_D = 90\,kN$.
Scoop width, $w = 0.3\,m$



Find: Depth of scoop immersion to slow to 100 m/s in trough length, L.

Solution: Apply x component of momentum equation using linearly accelerating CV shown.

Basic equation: $$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\simeq 0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\rho\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) $F_{B_x} = 0$
(2) Neglect rate of change of u in CV
(3) Uniform flow at each section
(4) No change in relative speed of liquid crossing scoop

Then
$$-F_D - Ma_{rf_x} = u_1\{-|\rho U w h|\} + u_2\{|\rho U w h|\}\ ;\ h = \text{scoop immersion}$$

$$u_1 = -U \qquad u_2 = U\cos\theta$$

But $F_D = kU^2$; $k = \frac{F_{D_0}}{U_0^2} = 90\,kN \times \frac{s^2}{(300)^2 m^2} \times \frac{10^3 N}{kN} \times \frac{kg\cdot m}{N\cdot s^2} = 1.00\,kg/m$

$$-kU^2 - M\frac{dU}{dt} = \rho U^2 w h(1+\cos\theta)\ ,\ \text{since } a_{rf_x} = dU/dt,$$

Thus $-M\frac{dU}{dt} = [k + \rho w h(1+\cos\theta)]U^2 = -MU\frac{dU}{dX}$

or $\frac{dU}{U} = -c\,dX$, where $c = \frac{k + \rho w h(1+\cos\theta)}{M}$

Integrating, $\ln\frac{U}{U_0} = -cX$, so $c = -\frac{1}{X}\ln\frac{U}{U_0}$

$$c = -\frac{1}{800\,m}\ln\left(\frac{100}{300}\right) = 1.37\times10^{-3}\,m^{-1}$$

Solving for h, $h = \frac{Mc - k}{\rho w(1+\cos\theta)}$

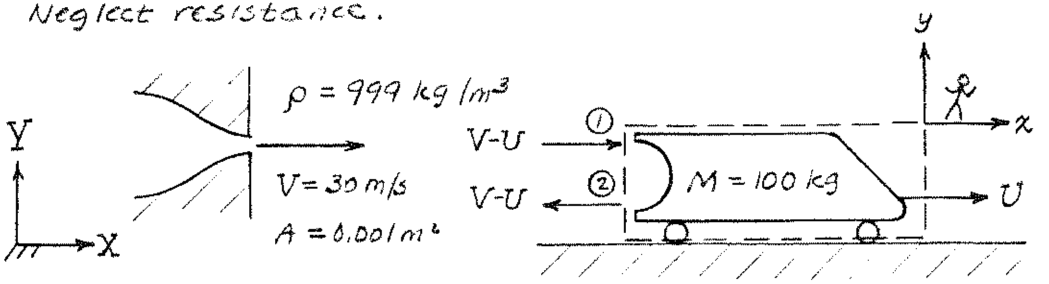
$$h = \left[8000\,kg \times \frac{1.37\times10^{-3}}{m} - 1.00\,\frac{kg}{m}\right]\frac{m^3}{999\,kg} \times \frac{1}{0.3\,m} \times \frac{1}{(1+\cos 30°)} = 0.0179\,m$$

$h = 17.9$ mm ⟵ h

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 4.144

Given: Vehicle accelerated from rest by a hydraulic catapult. Neglect resistance.



Find: Vehicle speed at $t = 5$ sec. Plot: Vehicle speed vs. time.

Solution: Apply x component of momentum equation using the linearly accelerating CV shown above.

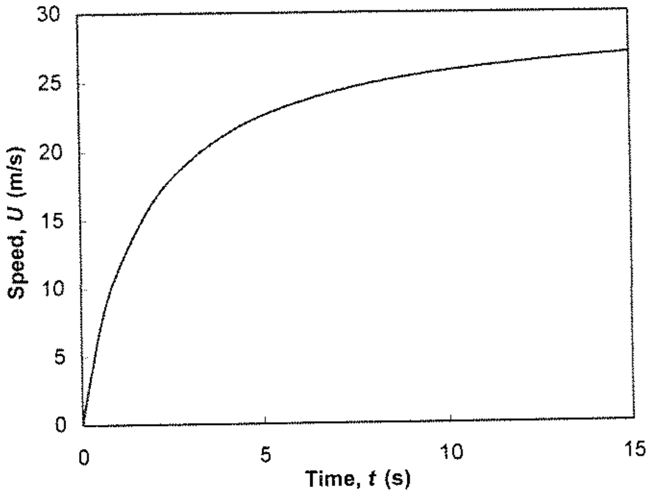
Basic equation: $$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x} \rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u_{xyz} \rho\, d\forall + \int_{CS} u_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) $F_{S_x} = 0$
(2) $F_{B_x} = 0$
(3) Neglect mass of liquid and rate of change of u in CV
(4) Uniform flow at each section
(5) Jet area and speed with respect to vehicle are constant

Then
$$-M a_{rf_x} = -M\frac{dU}{dt} = u_1\{-|\rho(V-U)A|\} + u_2\{|\rho(V-U)A|\}$$
$$u_1 = V-U \qquad u_2 = -(V-U)$$
or
$$\frac{dU}{dt} = \frac{2\rho(V-U)^2 A}{M}$$
Note that $dU = -d(V-U)$, and separate variables to obtain
$$-\frac{d(V-U)}{(V-U)^2} = \frac{2\rho A}{M}\,dt$$
Integrate from $U=0$ at $t=0$ to U at t,
$$\int_{V-U=V}^{V-U} -\frac{d(V-U)}{(V-U)^2} = \frac{1}{V-U}\Bigg]_{V}^{V-U} = \frac{1}{V-U} - \frac{1}{V} = \frac{V-(V-U)}{V(V-U)} = \frac{U}{V(V-U)} = \frac{2\rho A}{M}t$$
Solving,
$$U = (V-U)\frac{2\rho VA}{M}t \quad \text{or} \quad U = V\left[\frac{\frac{2\rho VA}{M}t}{1+\frac{2\rho VA}{M}t}\right] \qquad (1)$$
For the given conditions at $t = 5$ s,
$$\frac{2\rho VA}{M}t = 2 \times 999\frac{kg}{m^3} \times 30\frac{m}{s} \times 0.001\,m^2 \times 5\,s \times \frac{1}{100\,kg} = 3.00$$
$$U = 30\frac{m}{s}\left[\frac{3.00}{1+3.00}\right] = 22.5\ m/s$$ U

The plot is on the next page.

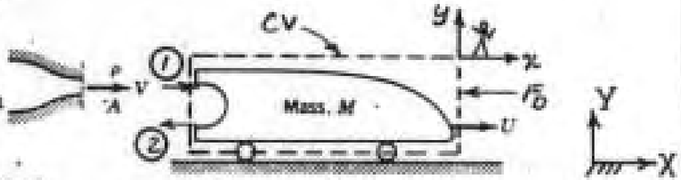
The speed vs. time plot is



Problem 4.145

Given: Cart accelerated from rest by hydraulic catapult.

$F_D = kU^2$; $k = 2.0\ \mathrm{N \cdot s^2/m^2}$



$\rho = 999\ \mathrm{kg/m^3}$
$V = 30\ \mathrm{m/s}$ $\quad M = 100\ \mathrm{kg}$
$A = 0.001\ \mathrm{m^2}$

Find:
- (a) Expression for acceleration in terms of speed, U.
- (b) Evaluate at $U = 10\ \mathrm{m/s}$.
- (c) Fraction of U_t.

Solution: Apply x momentum for CV with linear acceleration.

Basic equation:

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions:
1. (1) Horizontal, $F_{B_x} = 0$
2. (2) Neglect mass of liquid in CV (components of u cancel)
3. (3) Uniform flow at each section
4. (4) Measure all velocities relative to the CV
5. (5) No change in stream area or speed on vane

Then

$$-kU^2 - a_{rf_x}M = u_1\{-|\rho(V-U)A|\} + u_2\{+|\rho(V-U)A|\} = -2\rho(V-U)^2A$$

$$u_1 = V - U \qquad u_2 = -(V-U)$$

or

$$a_{rf_x} = \frac{dU}{dt} = \frac{2\rho(V-U)^2A - kU^2}{M}$$ a(U)

At $U = 10\ \mathrm{m/sec}$

$$a_{rf_x} = \frac{2 \times 999\,\frac{\mathrm{kg}}{\mathrm{m^3}}(30-10)^2\frac{\mathrm{m^2}}{\mathrm{s^2}} \times 0.001\ \mathrm{m^2} - 2.0\,\frac{\mathrm{N\cdot s^2}}{\mathrm{m^2}} \times (10)^2\frac{\mathrm{m^2}}{\mathrm{s^2}} \times \frac{\mathrm{kg\cdot m}}{\mathrm{N\cdot s^2}}}{100\ \mathrm{kg}} = 5.99\ \frac{\mathrm{m}}{\mathrm{s^2}}$$ a_{rf_x}

At terminal speed, $a_{rf_x} = 0$. Then $2\rho(V-U_t)^2A = kU_t^2$, or

$$V - U_t = U_t\sqrt{\frac{k}{2\rho A}}$$

solving,

$$U_t = \frac{V}{1 + \sqrt{k/2\rho A}}$$

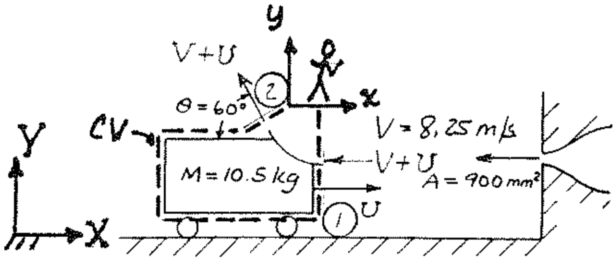
$$U_t = 30\,\frac{\mathrm{m}}{\mathrm{s}} \times \frac{1}{1 + \left[\frac{1}{2} \times 2.0\,\frac{\mathrm{N\cdot s^2}}{\mathrm{m^2}} \times \frac{\mathrm{m^3}}{999\ \mathrm{kg}} \times \frac{1}{0.001\ \mathrm{m^2}} \times \frac{\mathrm{kg\cdot m}}{\mathrm{N\cdot s^2}}\right]^{1/2}} = 15.0\ \mathrm{m/s}$$

Finally,

$$\text{Fraction} = \frac{U}{U_t} = 10.0\,\frac{\mathrm{m}}{\mathrm{s}} \times \frac{\mathrm{s}}{15.0\ \mathrm{m}} = 0.667$$ Fractio

Given: Small vaned cart rolling on level track, struck by a water jet, as shown. At $t=0$, $U_0 = 12.5$ m/sec. Neglect air resistance and rolling resistance.

Find: (a) Time and (b) distance needed to bring cart to rest, and (c) Plot of $U(t)$, $x(t)$.



Solution: Apply x component of momentum using CS and CV shown.

Basic equation: $$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) No resistance; $F_{S_x}=0$
(2) Horizontal; $F_{B_x}=0$
(3) Neglect mass of water on vane; $\partial/\partial t \approx 0$
(4) No change in speed w.r.to vane
(5) Uniform flow at each cross-section

Then

$$-a_{rf_x}M_{CV} = u_1\{-|\rho(V+U)A|\} + u_2\{+|\rho(V+U)A|\}$$

$$a_{rf_x} = \frac{dU}{dt} \qquad u_1 = -(V+U) \qquad u_2 = -(V+U)\cos\theta \quad (\text{w.r.to CV})$$

So $$-\frac{dU}{dt}M = \rho(V+U)^2A - \rho(V+U)^2A\cos\theta = \rho(V+U)^2A(1-\cos\theta) \qquad (1)$$

Note V = constant, so $dU = d(V+U)$. Substituting

$$-\frac{d(V+U)}{(V+U)^2} = \frac{\rho A(1-\cos\theta)}{M}dt \qquad (2)$$

Integrate from U_0 at $t=0$ to stop, when $U=0$

$$\left.\frac{1}{V+U}\right]_{U=U_0}^{U=0} = \frac{1}{V} - \frac{1}{V+U_0} = \frac{V+U_0-V}{V(V+U_0)} = \frac{U_0}{V(V+U_0)} = \frac{\rho A(1-\cos\theta)t}{M}$$

Thus $$t = \frac{U_0 M}{\rho(V+U_0)VA(1-\cos\theta)}$$

$$= 12.5\frac{\text{m}}{\text{sec}} \times 10.5\,\text{kg} \times \frac{\text{m}^3}{999\,\text{kg}} \times \frac{\text{sec}}{(12.5+8.25)\,\text{m}} \times \frac{\text{sec}}{8.25\,\text{m}} \times \frac{1}{900\times10^{-6}\,\text{m}^2} \times \frac{1}{(1-\cos 60°)}$$

$t = 1.71$ sec (to stop) ← t

To find distance note $\frac{dU}{dt} = \frac{dU}{ds}\frac{ds}{dt} = \frac{dU}{ds}U = U\frac{dU}{ds}$; so from Eq. 1

$$-U\frac{dU}{ds}M = \rho(V+U)^2A(1-\cos\theta)$$

Separating variables $$\frac{U\,dU}{(V+U)^2} = -\frac{\rho A(1-\cos\theta)}{M}ds \qquad (3)$$

Equation 3 may be integrated. Using tables, and integrating from U_0 at $t=0$ to stop (when $U=0$),

$$\int_{U_0}^{0} \frac{U\,dU}{(V+U)^2} = \left[\ln(V+U) + \frac{V}{V+U}\right]_{U_0}^{0} = \ln\left(\frac{V}{V+U_0}\right) + \frac{V}{V} - \frac{V}{V+U_0} = -\frac{\rho A(1-\cos\theta)}{M}s$$

Simplifying and solving for s,

$$s = -\frac{M}{\rho A(1-\cos\theta)}\ln\left(\frac{V}{V+U_0}\right) + 1 - \frac{V}{V+U_0}\Big)$$

$$= -10.5\,\text{kg} \times \frac{\text{m}^3}{999\,\text{kg}} \times \frac{1}{900\times10^{-6}\,\text{m}^2} \times \frac{1}{(1-\cos 60^\circ)}\left[\ln\left(\frac{8.25}{8.25+12.5}\right) + 1 - \frac{8.25}{8.25+12.5}\right]$$

$s = 7.47$ m (to stop) s

From Eq. 2 the general solution is

$$\int_{U_0}^{U} -\frac{d(V+U)}{(V+U)^2} = \frac{1}{V+U}\Bigg]_{U_0}^{U} = \frac{1}{V+U} - \frac{1}{V+U_0} = \frac{(V+U_0)-(V+U)}{(V+U)(V+U_0)} = \frac{\rho A(1-\cos\theta)t}{M} = at$$

Thus $U_0 - U = a(V+U)(V+U_0)t = aV(V+U_0)t + aU(V+U_0)t$ $\{\text{Let } b = V+U_0\}$

Simplifying, $U = \dfrac{U_0 - aVbt}{1+abt}$ (4) $U(t)$

Acceleration is found from Eq. 1

$$a_x = \frac{dU}{dt} = \frac{\rho A(1-\cos\theta)(V+U)^2}{M} = a(V+U)^2$$ $a_x(U$

Integrate Eq. 4 to get $X(t)$:

$$U = \frac{dX}{dt} = \frac{U_0 - abVt}{1+abt}$$

$$dX = \frac{U_0}{1+abt}dt - \frac{abVt}{1+abt}dt$$

Integrating

$$X = \frac{U_0}{ab}\ln(1+abt)\Big]_0^t - \frac{V}{ab}\int_0^t \frac{x}{1+x}dx = \left[\frac{U_0}{ab}\ln(1+abt) - \frac{V}{ab}\left(1+abt - \ln(1+abt)\right)\right]_0^t$$

$$X = \frac{U_0}{ab}\ln(1+abt) - \frac{V}{ab}\left[abt - \ln(1+abt)\right]$$ $X(t)$

Numerical values and plots are on the next page.

Problem 4.146

Acceleration, Velocity, and Position of Cart vs. Time:

Input Parameters:

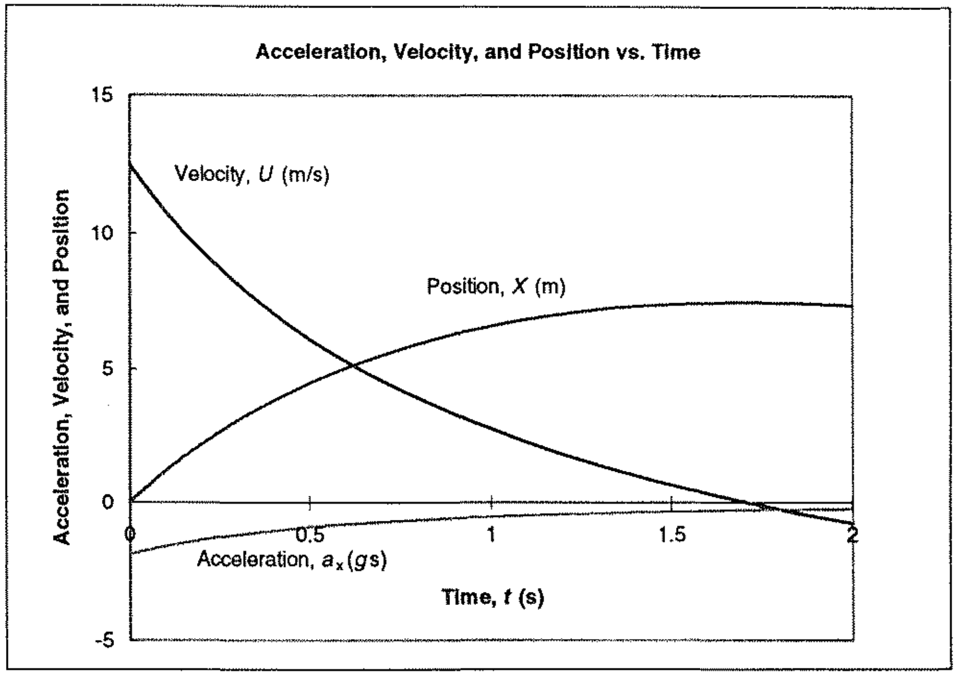
$A =$	900	mm^2	9.00E-04	m^2
$M =$	10.5	kg		
$U_0 =$	12.5	m/s		
$V =$	8.25	m/s		
$\theta =$	60	degrees	1.047	rad
$\rho =$	999	kg/m^3		

Calculated Parameters:

$a =$	0.0428	m^{-1}
$b =$	20.75	m/s

Calculated Results:

Time, t (s)	Velocity, U (m/s)	Accel., a_x (m/s)	Accel., a_x (gs)	Position, X (m)
0	12.5	-18.4	-1.88	0.00
0.1	10.8	-15.5	-1.58	1.16
0.2	9.37	-13.3	-1.35	2.17
0.3	8.13	-11.5	-1.17	3.04
0.4	7.06	-10.0	-1.02	3.80
0.5	6.12	-8.84	-0.901	4.46
0.6	5.29	-7.84	-0.800	5.03
0.7	4.54	-7.01	-0.714	5.52
0.8	3.88	-6.30	-0.642	5.94
0.9	3.28	-5.69	-0.580	6.30
1.0	2.74	-5.17	-0.527	6.60
1.1	2.24	-4.72	-0.481	6.85
1.2	1.79	-4.32	-0.440	7.05
1.3	1.38	-3.97	-0.405	7.21
1.4	0.998	-3.66	-0.373	7.33
1.5	0.646	-3.39	-0.345	7.41
1.6	0.319	-3.14	-0.320	7.46
1.7	0.0160	-2.93	-0.298	7.47
1.705	0.00000	-2.91	-0.297	7.47
1.8	-0.267	-2.73	-0.278	7.46
1.9	-0.530	-2.55	-0.260	7.42
2.0	-0.777	-2.39	-0.244	7.35

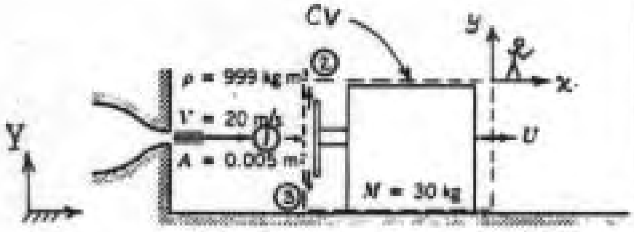


Problem 4.147

[3]

Given: Vane/slider assembly moving under influence of jet.

$F_R = kU; \; k = 7.5\, N\cdot s/m$



Find: (a) Acceleration at instant when $U = 10\, m/s$.
(b) Terminal speed of slider.

Solution: Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$F_{S_x} + \cancel{F_{B_x}}^{=0(1)} - \int_{CV} a_{rf_x}\rho\, d\forall = \cancel{\frac{\partial}{\partial t}}^{\approx 0(2)}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) Horizontal, so $F_{B_x} = 0$
(2) Neglect mass of liquid on vane, $u \approx 0$ on vane
(3) Uniform flow at each section
(4) Measure velocities relative to CV

Then

$$-kU - a_{rf_x}M = u_1\{-|\rho(V-U)A|\} + u_2\{+\dot m_2\} + u_3\{+\dot m_3\}$$

$$u_1 = V - U \qquad u_2 = 0 \qquad u_3 = 0$$

$$-kU - M\frac{dU}{dt} = -\rho(V-U)^2A$$

or

$$\frac{dU}{dt} = \rho\frac{(V-U)^2A}{M} - \frac{kU}{M}$$

$$= 999\frac{kg}{m^3}(20-10)^2\frac{m^2}{s^2}\times 0.005\, m^2\times\frac{1}{30\, kg} - 7.5\frac{N\cdot s}{m}\times 10\frac{m}{s}\times\frac{1}{30\, kg}\times\frac{kg\cdot m}{N\cdot s^2}$$

$$\frac{dU}{dt} = 14.2\, m/s^2 \quad (\text{at } U = 10\, m/s)$$

$\frac{dU}{dt}$

At terminal speed, $U = U_t$ and $dU/dt = 0$, so

$$0 = \rho\frac{(V-U)^2A}{M} - \frac{kU}{M} \quad \text{or} \quad V^2 - 2UV + U^2 - \frac{k}{\rho A}U = 0$$

$$U^2 - \left(2V + \frac{k}{\rho A}\right)U + V^2 = 0$$

$$U = \frac{2V + k/\rho A \pm\sqrt{(2V + k/\rho A)^2 - 4V^2}}{2} = V\left\{\left(1 + \frac{k}{2\rho VA}\right) \pm\sqrt{\left(1 + \frac{k}{2\rho VA}\right)^2 - 1}\right\}$$

$$1 + \frac{k}{2\rho VA} = 1 + \frac{1}{2}\times 7.5\frac{N\cdot s}{m}\times\frac{m^3}{999\, kg}\times\frac{s}{20\, m}\times\frac{1}{0.005\, m^2}\times\frac{kg\cdot m}{N\cdot s^2} = 1.0375$$

$$U = V\left\{1.0375 \pm\sqrt{(1.0375)^2 - 1}\right\} = 0.761\, V = 0.761\times 20\frac{m}{s} = 15.2\, m/s$$

U_t

{The negative root was chosen so $U_t < V$, as required.}

Problem 4.148

4.148 For the vane/slider problem of Problem 4.147, find and plot expressions for the acceleration, speed, and position of the slider as functions of time. (Consider numerical integration.)

Given: Data on vane/slider

Find: Formula for acceleration, speed, and position; plot

Solution:

The given data is $\rho = 999 \cdot \frac{kg}{m^3}$ $M = 30 \cdot kg$ $A = 0.005 \cdot m^2$ $V = 20 \cdot \frac{m}{s}$ $k = 7.5 \cdot \frac{N \cdot s}{m}$

The equation of motion, from Problem 4.147, is $\frac{dU}{dt} = \frac{\rho \cdot (V - U)^2 \cdot A}{M} - \frac{k \cdot U}{M}$

The acceleration is thus $a = \frac{\rho \cdot (V - U)^2 \cdot A}{M} - \frac{k \cdot U}{M}$

The differential equation for *U* can be solved analytically, but is quite messy. Instead we use a simple numerical method - Euler's method

$$U(n + 1) = U(n) + \left[\frac{\rho \cdot (V - U)^2 \cdot A}{M} - \frac{k \cdot U}{M}\right] \cdot \Delta t$$ where Δt is the time step

For the position x $\frac{dx}{dt} = U$

so $x(n + 1) = x(n) + U \cdot \Delta t$

The final set of equations is

$$U(n + 1) = U(n) + \left[\frac{\rho \cdot (V - U)^2 \cdot A}{M} - \frac{k \cdot U}{M}\right] \cdot \Delta t$$

$$a = \frac{\rho \cdot (V - U)^2 \cdot A}{M} - \frac{k \cdot U}{M}$$

$$x(n + 1) = x(n) + U \cdot \Delta t$$

The results are plotted in the corresponding *Excel* workbook

Problem 4.148

[4]

4.148 For the vane/slider problem of Problem 4.147, find and plot expressions for the acceleration, speed, and position of the slider as functions of time. (Consider numerical integration.)

Given: Data on vane/slider

Find: Formula for acceleration, speed, and position; plot

Solution:

The final set of equations is

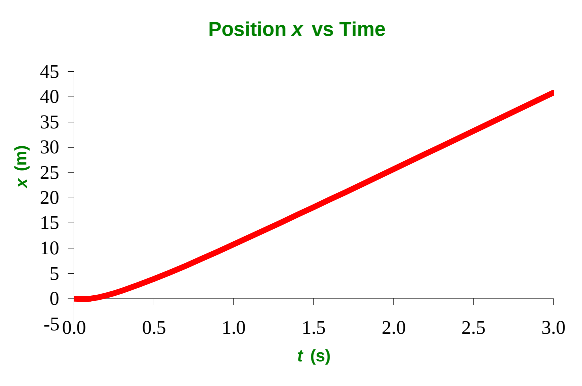
$$U(n+1) = U(n) + \left[\frac{\rho\cdot(V-U)^2\cdot A}{M} - \frac{k\cdot U}{M}\right]\cdot\Delta t$$

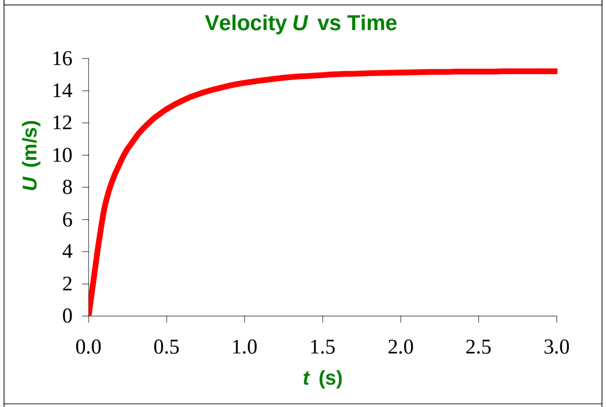
$$a = \frac{\rho\cdot(V-U)^2\cdot A}{M} - \frac{k\cdot U}{M}$$

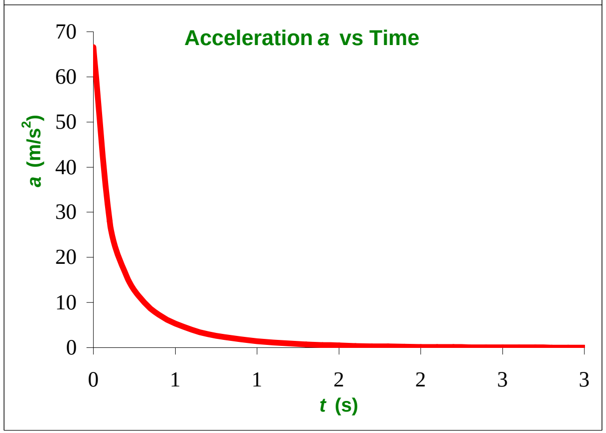
$$x(n+1) = x(n) + U\cdot\Delta t$$

ρ =	999	kg/m^3
k =	7.5	N.s/m
A =	0.005	m^2
V =	20	m/s
M =	30	kg
Δt =	0.1	s

t (s)	*x* (m)	*U* (m/s)	*a* (m/s^2)
0.0	0.0	0.0	66.6
0.1	0.0	6.7	28.0
0.2	0.7	9.5	16.1
0.3	1.6	11.1	10.5
0.4	2.7	12.1	7.30
0.5	3.9	12.9	5.29
0.6	5.2	13.4	3.95
0.7	6.6	13.8	3.01
0.8	7.9	14.1	2.32
0.9	9.3	14.3	1.82
1.0	10.8	14.5	1.43
1.1	12.2	14.6	1.14
1.2	13.7	14.7	0.907
1.3	15.2	14.8	0.727
1.4	16.6	14.9	0.585
1.5	18.1	15.0	0.472
1.6	19.6	15.0	0.381
1.7	21.1	15.1	0.309
1.8	22.6	15.1	0.250
1.9	24.1	15.1	0.203
2.0	25.7	15.1	0.165
2.1	27.2	15.1	0.134
2.2	28.7	15.2	0.109
2.3	30.2	15.2	0.0889
2.4	31.7	15.2	0.0724
2.5	33.2	15.2	0.0590
2.6	34.8	15.2	0.0481
2.7	36.3	15.2	0.0392
2.8	37.8	15.2	0.0319
2.9	39.3	15.2	0.0260
3.0	40.8	15.2	0.0212


Position *x* vs Time


Velocity *U* vs Time

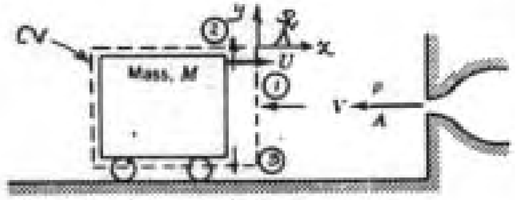

Acceleration *a* vs Time

Problem 4.149

[3]

Given: Block and jet as shown.

Jet strikes block at $t > 0$.



Find: (a) Expression for acceleration.
(b) Time at which $U = 0$.

Solution: Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x} \rho \, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u_{xyz} \rho \, d\forall + \int_{CS} u_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) No pressure or friction forces, so $F_{S_x} = 0$
(2) Horizontal, so $F_{B_x} = 0$
(3) Neglect mass of liquid in CV, $u \approx 0$ in CV
(4) Uniform flow at each section
(5) Measure velocities relative to CV

Then

$$-Ma_{rf_x} = -M\frac{dU}{dt} = u_1\{-|\rho(V+U)A|\} + u_2\{+\dot{m}_2\} + u_3\{+\dot{m}_3\}$$

$$u_1 = -(V+U) \qquad u_2 = 0 \qquad u_3 = 0$$

or

$$\frac{dU}{dt} = -\frac{\rho(V+U)^2 A}{M}$$

$\frac{dU}{dt}$

But, since V = constant, $dU = d(V+U)$, so

$$\frac{d(V+U)}{(V+U)^2} = -\frac{\rho A}{M}dt$$

Integrating from U_0 at $t = 0$ to $U = 0$ at t

$$\int_{V+U_0}^{V} \frac{d(V+U)}{(V+U)^2} = -\frac{1}{(V+U)}\Bigg]_{V+U_0}^{V} = -\frac{1}{V} + \frac{1}{V+U_0} = \frac{-U_0}{V(V+U_0)} = -\frac{\rho A t}{M}$$

Solving, $t = \dfrac{MU_0}{\rho VA(V+U_0)} = \dfrac{M}{\rho VA(1+V/U_0)}$

t

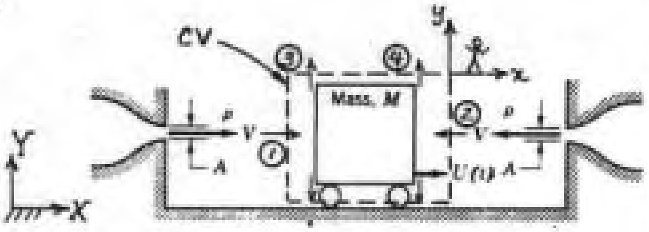
Problem 4.150

[3]

Given: Block rolling between opposing jets, as shown.

Speed is U_0 at $t=0$.

There is no resistance for $t>0$.



Find: (a) Expression for acceleration, $a(t)$.
(b) Expression for speed, $U(t)$.

Solution: Apply x momentum to linearly accelerating CV.

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\rho\, d\forall + \int_{CS} u_{xyz}\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) No pressure or friction forces, so $F_{S_x}=0$
(2) Horizontal, so $F_{B_x}=0$
(3) Neglect mass of liquid in CV; $u \approx 0$ in CV
(4) Uniform flow at each section
(5) Measure velocities relative to CV

Then

$$-a_{rf_x}M = -M\frac{dU}{dt} = u_1\{-|\rho(V-U)A|\} + u_2\{-|\rho(V+U)A|\} + u_3\{\dot m_3\} + u_4\{\dot m_4\}$$

$$u_1 = V-U \qquad u_2 = -(V+U) \qquad u_3 = 0 \qquad u_4 = 0$$

or

$$-M\frac{dU}{dt} = \rho A\left[-(V-U)^2 + (V+U)^2\right] = \rho A\left[4UV\right] = 4\rho VAU$$

Thus $\displaystyle \frac{dU}{U} = -\frac{4\rho VA}{M}\,dt$

Integrating $\displaystyle \int_{U_0}^{U}\frac{dU}{U} = \ln U\Big]_{U_0}^{U} = \ln\frac{U}{U_0} = -\frac{4\rho VA}{M}t$

or

$$U(t) = U_0 e^{-\frac{4\rho VA}{M}t}$$

U(t)

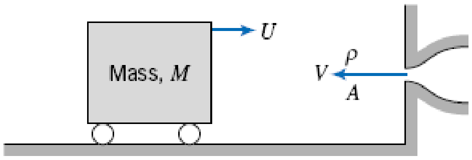
Also

$$a(t) = \frac{dU}{dt} = -\frac{4\rho VA}{M}U_0 e^{-\frac{4\rho VA}{M}t}$$

a(t)

Problem 4.151

4.151 Consider the diagram of Problem 4.149. If $M = 100$ kg, $\rho = 999$ kg/m^3, and $A = 0.01$ m^2, find the jet speed V required for the cart to be brought to rest after one second if the initial speed of the cart is $U_0 = 5$ m/s. For this condition, plot the speed U and position x of the cart as functions of time. What is the maximum value of x, and how long does the cart take to return to its initial position?



Given:

Data on system

Find:

Jet speed to stop cart after 1 s; plot speed & position; maximum x; time to return to origin

Solution:

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $M = 100\cdot kg$ $A = 0.01\cdot m^2$ $U_0 = 5\cdot\frac{m}{s}$

The equation of motion, from Problem 4.149, is $\frac{dU}{dt} = -\frac{\rho\cdot(V+U)^2\cdot A}{M}$

which leads to $\frac{d(V+U)}{(V+U)^2} = -\left(\frac{\rho\cdot A}{M}\cdot dt\right)$

Integrating and using the IC $U = U_0$ at $t = 0$ $U = -V + \frac{V+U_0}{1+\frac{\rho\cdot A\cdot\left(V+U_0\right)}{M}\cdot t}$

To find the jet speed V to stop the cart after 1 s, solve the above equation for V, with $U = 0$ and $t = 1$ s. (The equation becomes a quadratic in V). Instead we use *Excel*'s *Goal Seek* in the associated workbook

From *Excel* $V = 5\cdot\frac{m}{s}$

For the position x we need to integrate $\frac{dx}{dt} = U = -V + \frac{V+U_0}{1+\frac{\rho\cdot A\cdot\left(V+U_0\right)}{M}\cdot t}$

The result is $x = -V\cdot t + \frac{M}{\rho\cdot A}\cdot\ln\left[1+\frac{\rho\cdot A\cdot\left(V+U_0\right)}{M}\cdot t\right]$

This equation (or the one for U with $U = 0$) can be easily used to find the maximum value of x by differentiating, as well as the time for x to be zero again. Instead we use *Excel*'s *Goal Seek* and *Solver* in the associated workbook

From *Excel* $x_{max} = 1.93\cdot m$ $t(x = 0) = 2.51\cdot s$

The complete set of equations is

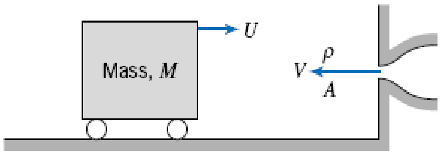
$$U = -V + \frac{V+U_0}{1+\frac{\rho\cdot A\cdot\left(V+U_0\right)}{M}\cdot t} \qquad x = -V\cdot t + \frac{M}{\rho\cdot A}\cdot\ln\left[1+\frac{\rho\cdot A\cdot\left(V+U_0\right)}{M}\cdot t\right]$$

The plots are presented in the *Excel* workbook

Problem 4.151

[3]

4.151 Consider the diagram of Problem 4.149. If $M = 100$ kg, $\rho = 999$ kg/m^3, and $A = 0.01$ m^2, find the jet speed V required for the cart to be brought to rest after one second if the initial speed of the cart is $U_0 = 5$ m/s. For this condition, plot the speed U and position x of the cart as functions of time. What is the maximum value of x, and how long does the cart take to return to its initial position?



Given:

Data on system

Find:

Jet speed to stop cart after 1 s; plot speed & position; maximum x; time to return to origin

Solution:

The complete set of equations is

$$U = -V + \frac{V + U_0}{1 + \frac{\rho \cdot A \cdot (V + U_0)}{M} \cdot t} \qquad x = -V \cdot t + \frac{M}{\rho \cdot A} \cdot \ln\left[1 + \frac{\rho \cdot A \cdot (V + U_0)}{M} \cdot t\right]$$

$M =$ 100 kg

$\rho =$ 999 kg/m^3

$A =$ 0.01 m^2

$U_o =$ 5 m/s

t (s)	*x* (m)	*U* (m/s)
0.0	0.00	5.00
0.2	0.82	3.33
0.4	1.36	2.14
0.6	1.70	1.25
0.8	1.88	0.56
1.0	1.93	0.00
1.2	1.88	-0.45
1.4	1.75	-0.83
1.6	1.56	-1.15
1.8	1.30	-1.43
2.0	0.99	-1.67
2.2	0.63	-1.88
2.4	0.24	-2.06
2.6	-0.19	-2.22
2.8	-0.65	-2.37
3.0	-1.14	-2.50

To find *V* for *U* = 0 in 1 s, use *Goal Seek*

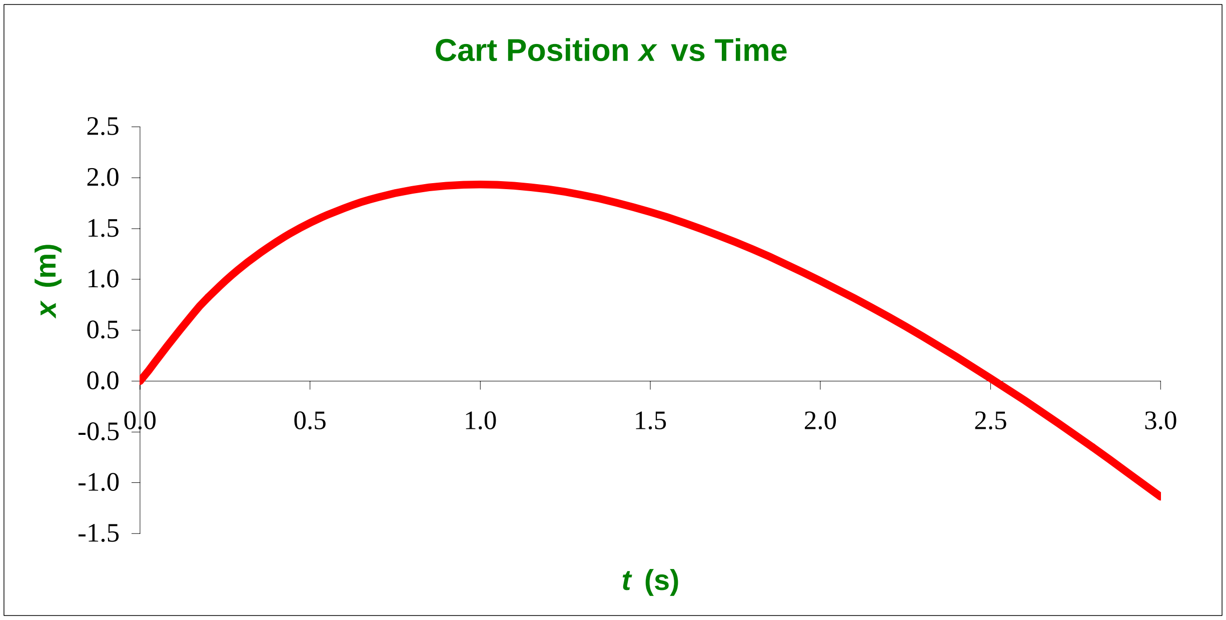
t (s)	*U* (m/s)	*V* (m/s)
1.0	0.00	5.00

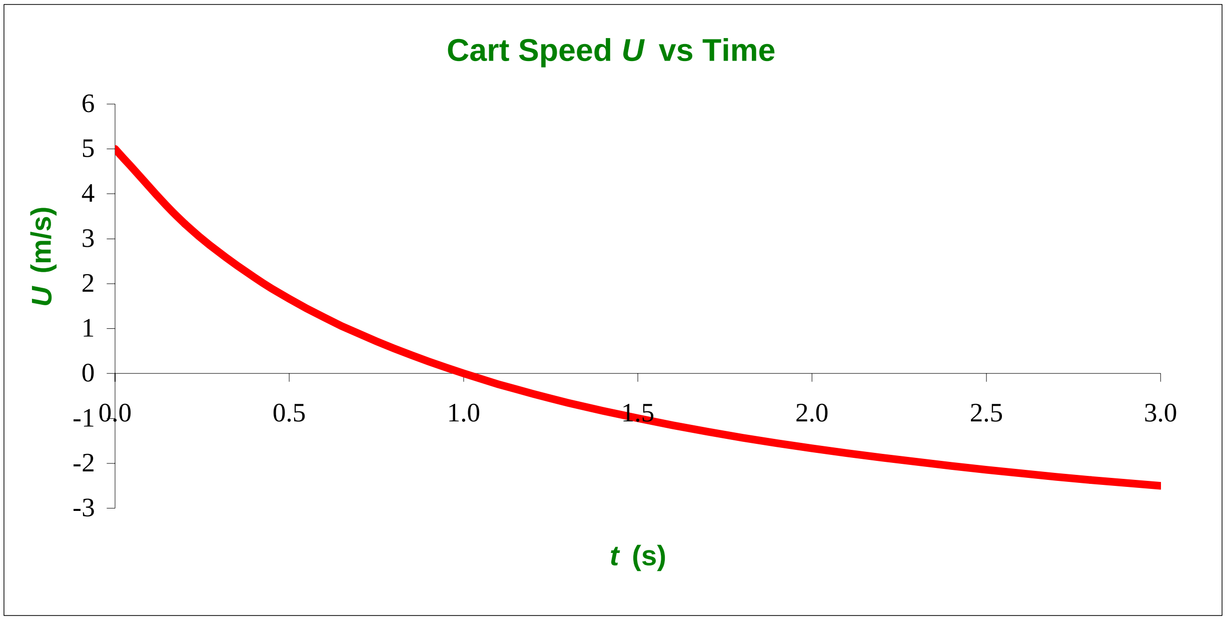
To find the maximum *x*, use *Solver*

t (s)	*x* (m)
1.0	1.93

To find the time at which *x* = 0 use *Goal Seek*

t (s)	*x* (m)
2.51	0.00


Cart Position x vs Time
x (m)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t (s)


Cart Speed U vs Time
U (m/s)
6
5
4
3
2
1
0
-1
-2
-3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t (s)

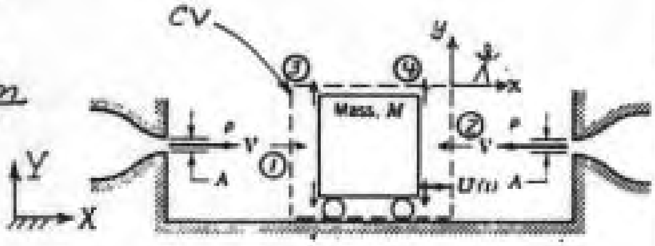
Problem 4.152

[3]

Given: Block rolling between opposing jets, as shown.

At $t = 0$, block moves at $U_0 = 10\,\frac{m}{s}$

starting from $X = 0$.



Find: (a) Time to reduce speed to $U = 0.5$ m/s.

(b) Position at that instant.

Solution: Apply x momentum equation to linearly accelerating CV.

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall}} + \int_{CS} u_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) No pressure or friction forces, so $F_{S_x} = 0$
(2) Horizontal, so $F_{B_x} = 0$
(3) Neglect mass of liquid in CV; $u \approx 0$ in CV
(4) Uniform flow at each section
(5) Measure velocities relative to CV

Then

$$-a_{rf_x} M = -M\frac{dU}{dt} = u_1\{-|\rho(V-U)A|\} + u_2\{-|\rho(V+U)A|\} + u_3\{\dot{m}_3\} + u_4\{\dot{m}_4\}$$

$$u_1 = V - U \qquad u_2 = -(V+U) \qquad u_3 = 0 \qquad u_4 = 0$$

or

$$-M\frac{dU}{dt} = \rho A\left[-(V-U)^2 + (V+U)^2\right] = \rho A\left[4UV\right] = 4\rho V A U$$

Thus

$$\frac{dU}{U} = -\frac{4\rho V A}{M}\,dt$$

Integrating, $$\int_{U_0}^{U}\frac{dU}{U} = \ln U\Big]_{U_0}^{U} = \ln\frac{U}{U_0} = -\frac{4\rho V A}{M}\,t \qquad (1)$$

Thus $$t = -\frac{M}{4\rho V A}\ln\frac{U}{U_0} = -\frac{1}{4}\cdot\frac{M}{\rho V A}\ln\frac{0.5}{10} = 0.750\,\frac{M}{\rho V A}$$ t

From Eq. 1, $$U(t) = \frac{dX}{dt} = U_0\,e^{-\frac{4\rho V A}{M}t}$$

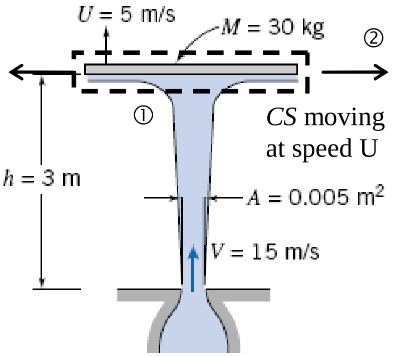
Integrating, $$X = \int_0^X dX = \int_0^t U_0\,e^{-\frac{4\rho V A}{M}t}\,dt = -\frac{M U_0}{4\rho V A}\,e^{-\frac{4\rho V A}{M}t}\Big]_0^t$$

$$X = \frac{M U_0}{4\rho V A}\left[1 - e^{-\frac{4\rho V A}{M}t}\right] = \frac{0.95}{4}\,\frac{M U_0}{\rho V A} = 0.238\,\frac{M U_0}{\rho V A}$$ X

Problem *4.153

[3]

***4.153** A vertical jet of water impinges on a horizontal disk as shown. The disk assembly mass is 30 kg. When the disk is 3 m above the nozzle exit, it is moving upward at $U = 5$ m/s. Compute the vertical acceleration of the disk at this instant.



Given: Water jet striking moving disk

Find: Acceleration of disk when at a height of 3 m

Solution:

Basic equations: Bernoulli; Momentum flux in z direction (treated as upwards) for linear accelerating CV

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_{S_z} + F_{B_z} - \int_{CV} a_{rf_z}\,\rho\, dV\!\!\!\!- = \frac{\partial}{\partial t}\int_{CV} w_{xyz}\,\rho\, dV\!\!\!\!- + \int_{CS} w_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow (All in jet)

The Bernoulli equation becomes $\dfrac{V_0^{\,2}}{2} + g\cdot 0 = \dfrac{V_1^{\,2}}{2} + g\cdot\left(z - z_0\right)$ $\qquad V_1 = \sqrt{V_0^{\,2} + 2\cdot g\cdot\left(z_0 - z\right)}$

$$V_1 = \sqrt{\left(15\cdot\frac{m}{s}\right)^2 + 2\times 9.81\cdot\frac{m}{s^2}\cdot(0-3)\cdot m} \qquad V_1 = 12.9\,\frac{m}{s}$$

The momentum equation becomes

$$-W - M\cdot a_{rfz} = w_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + w_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = \left(V_1 - U\right)\cdot\left[-\rho\cdot\left(V_1 - U\right)\cdot A_1\right] + 0$$

Hence $\quad a_{rfz} = \dfrac{\rho\cdot\left(V_1 - U\right)^2\cdot A_1 - W}{M} = \dfrac{\rho\cdot\left(V_1 - U\right)^2\cdot A_1}{M} - g = \dfrac{\rho\cdot\left(V_1 - U\right)^2\cdot A_0\cdot\dfrac{V_0}{V_1}}{M} - g \qquad$ using $\quad V_1\cdot A_1 = V_0\cdot A_0$

$$a_{rfz} = 1000\cdot\frac{kg}{m^3}\times\left[(12.9 - 5)\cdot\frac{m}{s}\right]^2\times 0.005\cdot m^2\times\frac{15}{12.9}\times\frac{1}{30\cdot kg} - 9.81\cdot\frac{m}{s^2} \qquad a_{rfz} = 2.28\,\frac{m}{s^2}$$

Problem *4.154

[4]

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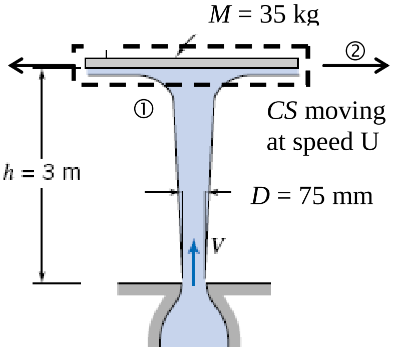
*4.154 A vertical jet of water leaves a 75-mm diameter nozzle. The jet impinges on a horizontal disk (see Problem 4.153). The disk is constrained horizontally but is free to move vertically. The mass of the disk is 35 kg. Plot disk mass versus flow rate to determine the water flow rate required to suspend the disk 3 m above the jet exit plane.



Given: Water jet striking disk

Find: Plot mass versus flow rate to find flow rate for a steady height of 3 m

Solution:

Basic equations: Bernoulli; Momentum flux in z direction (treated as upwards)

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_z = F_{S_z} + F_{B_z} = \frac{\partial}{\partial t}\int_{CV} w\,\rho\,d\Psi + \int_{CS} w\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure in jet 4) Uniform flow (All in jet)

The Bernoulli equation becomes $\frac{V_0^2}{2} + g\cdot 0 = \frac{V_1^2}{2} + g\cdot h$ $\qquad V_1 = \sqrt{V_0^2 - 2\cdot g\cdot h}$

The momentum equation becomes

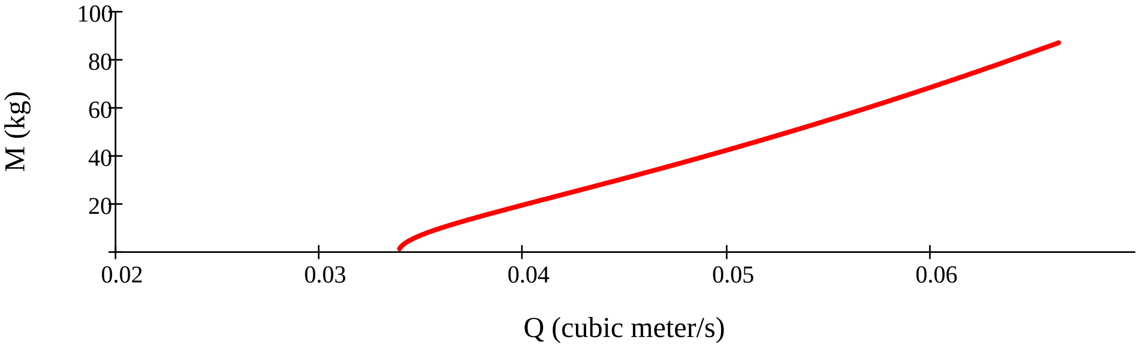
$$-M\cdot g = w_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + w_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = V_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + 0$$

Hence $M = \frac{\rho\cdot V_1^2\cdot A_1}{g}$ $\qquad$ but from continuity $\qquad V_1\cdot A_1 = V_0\cdot A_0$

$$M = \frac{\rho\cdot V_1\cdot V_0\cdot A_0}{g} = \frac{\pi}{4}\cdot\frac{\rho\cdot V_0\cdot D_0^2}{g}\cdot\sqrt{V_0^2 - 2\cdot g\cdot h}$$

and also $\qquad Q = V_0\cdot A_0$

This equation is difficult to solve for V_0 for a given M. Instead we plot first:



This graph can be parametrically plotted in *Excel*. The Goal Seek or Solver feature can be used to find Q when M = 35 kg

$$Q = 0.0469\cdot\frac{m^3}{s}$$

Problem 4.155

[3]

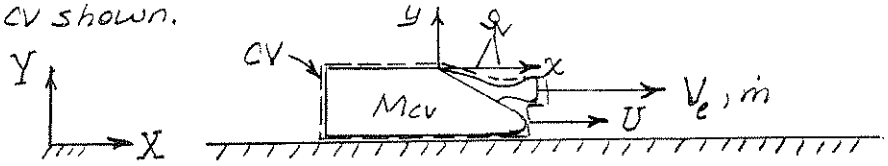
Given: Rocket sled on horizontal track, slowed by retro-rocket.

Initial mass $M_0 = 1500\ kg$ Initial speed $U_0 = 500\ m/s$
Mass flow rate $\dot{m} = 7.75\ kg/s$ Exhaust speed $V_e = 2500\ m/s$
Firing time $t_{bo} = 20.0\ s$

Neglect aerodynamic drag and rolling resistance.

Find: (a) Algebraic expression for sled speed U as a function of t.
(b) Speed at end of retro-rocket firing.

Solution: Apply x-component of momentum equation to the linearly accelerating CV shown.



From continuity,

$$M_{cv} = M_0 - \dot{m}t, \quad t < t_{bo}$$

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x} \rho \, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u_{xyz} \rho \, d\forall}} + \int_{CS} u_{xyz} \, \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) No pressure, drag, or rolling resistance, so $F_{S_x} = 0$
(2) Horizontal motion, so $F_{B_x} = 0$
(3) Neglect unsteady effects within CV
(4) Uniform flow at nozzle exit plane
(5) $p_e = p_{atm}$

Then $$-a_{rf_x} M_{cv} = u_e\{+\dot{m}\} = +V_e\dot{m} \quad \text{or} \quad \frac{dU}{dt} = -\frac{V_e\dot{m}}{M_{cv}} = -\frac{V_e\dot{m}}{M_0 - \dot{m}t}$$

$$u_e = V_e$$

Thus $$dU = V_e\left(\frac{-\dot{m}\,dt}{M_0 - \dot{m}t}\right) \quad \text{and} \quad U - U_0 = V_e \ln(M_0 - \dot{m}t)\Big]_0^t = V_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right)$$

$$U(t) = U_0 + V_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right); \quad t < t_{bo}$$

U(t)

At t_{bo}, $$U(t_{bo}) = 500\,\frac{m}{s} + 2500\,\frac{m}{s} \times \ln\left(1 - 7.75\,\frac{kg}{s} \times 20.0\,s \times \frac{1}{1500\,kg}\right)$$

$$U(t_{bo}) = 227\ m/s$$

$U(t_{bo})$

Problem 4.156

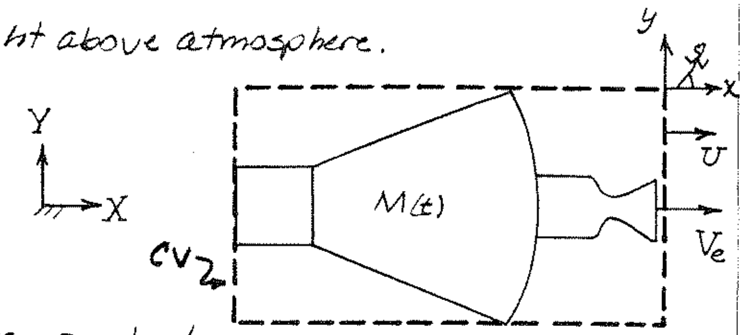
Given: Space capsule in level flight above atmosphere.

$U_0 = 8.0$ km/s

$M_0 = 1600$ kg

$\dot m = 8$ kg/s

$V_e = 3000$ m/s



Find: Time to reduce speed to $U = 5.00$ km/s.

Solution: Apply x component of momentum to CV with linear acceleration.

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} \overset{\approx 0(4)}{\cancel{u_{xyz}}}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho \vec V_{xyz}\cdot d\vec A$$

Assumptions: (1) No resistance; $F_{S_x} = 0$

(2) Horizontal; $F_{B_x} = 0$

(3) Use velocities measured relative to CV

(4) Neglect velocity within CV

(5) Uniform flow at exit plane with negligible p_e (given)

From continuity,
$$\frac{dM}{dt} = \frac{\partial}{\partial t}\int_{CV}\rho\, d\forall = -\int_{CS}\rho \vec V_{xyz}\cdot d\vec A = -\dot m\, ; \quad M(t) = M_0 - \dot m t$$

From momentum,
$$-a_{rf_x} M = -\frac{dU}{dt}(M_0 - \dot m t) = u_e\{+\dot m\} = V_e \dot m$$

Thus
$$\frac{dU}{dt} = -\frac{V_e \dot m}{M_0 - \dot m t} \qquad u_e = V_e$$

Integrating, $$U - U_0 = V_e\int_0^t \frac{-\dot m\, dt}{M_0 - \dot m t} = V_e \ln(M_0 - \dot m t)\Big]_0^t = V_e \ln\left(\frac{M_0 - \dot m t}{M_0}\right)$$

Solving for t,
$$\frac{M_0 - \dot m t}{M_0} = e^{\frac{U-U_0}{V_e}}\ ; \quad M_0 - \dot m t = M_0\, e^{U-U_0/V_e}$$

$$t = \frac{M_0}{\dot m}\left(1 - e^{U-U_0/V_e}\right)$$

$$= 1600\ \text{kg} \times \frac{\text{s}}{8\ \text{kg}}\left\{1 - e^{\left[(5.00-8.0)\frac{\text{km}}{\text{s}} \times \frac{\text{s}}{3000\ \text{m}} \times 1000\frac{\text{m}}{\text{km}}\right]}\right\}$$

$t = 126$ s $\longleftarrow$ t

Problem 4.157

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Given: Rocket sled accelerates from rest on a level track. Initial mass, $M_0 = 600$ kg, includes fuel - $M_f = 150$ kg. The rocket motor burns fuel at rate $\dot{m} = 15$ kg/s. Exhaust gases leave nozzle uniformly and axially at atmospheric pressure with $V_e = 2900$ m/s relative to the nozzle. Neglect air and rolling resistance.

Find: (a) Maximum speed reached by the sled.
(b) Maximum acceleration of sled during the run.

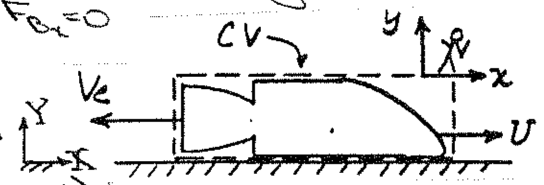
Plot: the sled speed and acceleration as functions of time.

Solution:

Apply the momentum equation to linearly accelerating CV shown.

Basic equation: $\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf}\,\rho\,d\forall = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\,\rho\,d\forall + \int_{CS} u_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$

Assumptions: (1) no net pressure forces ($p_e = p_{atm}$, given)
(2) horizontal motion, $F_{B_x} = 0$
(3) neglect $\partial/\partial t$ in CV
(4) uniform axial jet



From continuity, $M = M_0 - \dot{m}t$. Then

$$-a_{rf_x} M = -\frac{dU}{dt}(M_0 - \dot{m}t) = u_e\{\dot{m}\} = -V_e\dot{m} \quad \ldots\ (1)$$

Separating variables,

$$dU = V_e \frac{\dot{m}\,dt}{M_0 - \dot{m}t}$$

Integrating from $U = 0$ at $t = 0$ to U at t gives

$$U = -V_e \ln(M_0 - \dot{m}t)\Big]_0^t = -V_e \ln\frac{(M_0 - \dot{m}t)}{M_0} = V_e \ln\frac{M_0}{(M_0 - \dot{m}t)} \quad \ldots\ (2)$$

The speed is a maximum at burnout. At burnout $M_f = 0$ and $M = M_0 - \dot{m}t = 450$ kg

At burnout, $t = \frac{M_f)_{initial}}{\dot{m}_{fuel}} = 150\,\text{kg} \times \frac{s}{15\,\text{kg}} = 10\,\text{s}$

Then from Eq. 2

$$U_{max} = 2900\,\frac{m}{s} \ln\frac{600\,\text{kg}}{450\,\text{kg}} = 834\,\text{m/s} \longleftarrow U_{max}$$

From Eq. 1 the acceleration is $\frac{dU}{dt} = \frac{\dot{m}V_e}{M_0 - \dot{m}t}$

The maximum acceleration occurs at the instant prior to burn out

$$\left.\frac{dU}{dt}\right)_{max} = 15\,\frac{kg}{s} \times 2900\,\frac{m}{s} \times \frac{1}{450\,kg} = 96.7\,\text{m/s}^2 \longleftarrow \left.\frac{dU}{dt}\right)_{max}$$

Problem 4.157

The sled speed as a function of time is

$$U = V_e \ln \frac{M_0}{(M_0 - \dot{m}t)} \quad \text{for } 0 \le t \le 10\text{s}$$

$$U = \text{constant} = 834 \text{ m/s} \quad \text{for } t > 10 \text{ (neglecting resistance)}$$

The sled acceleration is given by

$$\frac{dU}{dt} = \frac{\dot{m}V_e}{(M_0 - \dot{m}t)} \quad \text{for } 0 \le t \le 10\text{s}$$

$$\frac{dU}{dt} = 0 \quad \text{for } t \ge 10\text{s}.$$

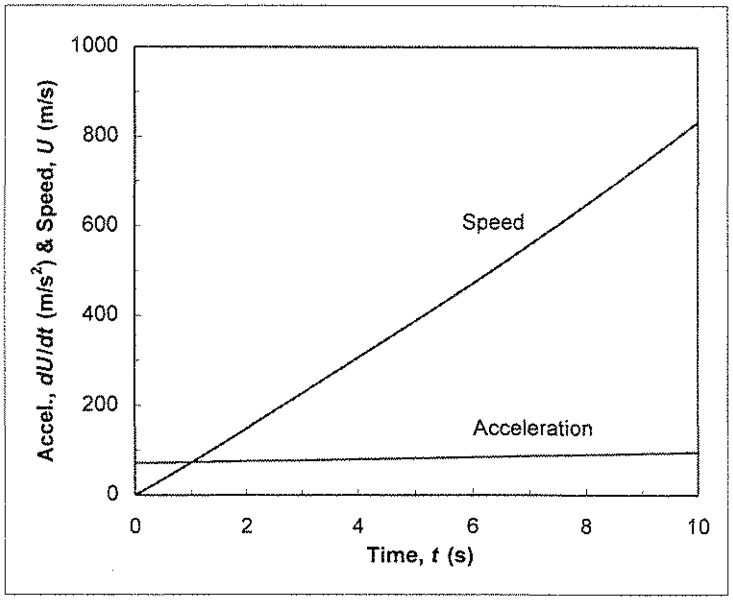
Acceleration and Velocity vs. Time for Rocket Sled:

Input Data:

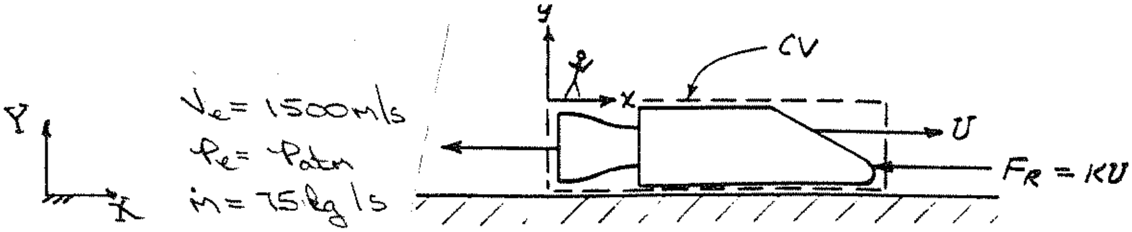
M_0 =	600	kg
m(dot) =	15	kg/s
V_e =	2900	m/s

Calculated Results:

Time, t (s)	Acceleration, dU/dt (m/s^2)	Velocity, U (m/s)
0	72.5	0
1	74.4	73.4
2	76.3	149
3	78.4	226
4	80.6	306
5	82.9	387
6	85.3	471
7	87.9	558
8	90.6	647
9	93.5	739
10	96.7	834



Given: Rocket sled with initial mass of 4 metric tons, including 1 ton of fuel. Motion resistance is given by kU where $k = 75\ \text{N}/\text{m}/\text{s}$.



Find: Sled speed 10 s after starting from rest, & U_{max}

Plot: sled speed and acceleration as functions of time.

Solution:

Apply the x component of the momentum equation to linearly accelerating CV shown

Basic equation: $F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,(\rho \vec{V}_{xyz}\cdot d\vec{A})$

Assumptions: (1) $p_e = p_{atm}$ (given) so $F_{S_x} = -F_R$

(2) $F_{B_x} = 0$

(3) neglect unsteady effects within CV

(4) uniform flow at exit plane

Then,

$$-F_R - a_{rf_x} M = u_e\{+|\dot{m}|\} = -V_e \dot{m} \qquad \{F_R = kU,\ u_e = -V_e\}$$

From continuity, $M = M_0 - \dot{m}t$. Substituting with $a_{rf_x} = \frac{dU}{dt}$

$$-kU - (M_0 - \dot{m}t)\frac{dU}{dt} = -V_e\dot{m}$$

$$\frac{dU}{dt} = \frac{V_e\dot{m} - kU}{M_0 - \dot{m}t} \qquad \text{or} \qquad \frac{dU}{V_e\dot{m} - kU} = \frac{dt}{M_0 - \dot{m}t}$$

Integrating, $\left.\frac{1}{k}\ln(V_e\dot{m} - kU)\right]_0^U = \left.\frac{1}{\dot{m}}\ln(M_0 - \dot{m}t)\right]_0^t$

and $\ln\frac{(V_e\dot{m} - kU)}{V_e\dot{m}} = \ln\left(1 - \frac{kU}{V_e\dot{m}}\right) = \frac{k}{\dot{m}}\ln\frac{(M_0 - \dot{m}t)}{M_0} = \frac{k}{\dot{m}}\ln\left(1 - \frac{\dot{m}t}{M_0}\right)$

Then $1 - \frac{kU}{V_e\dot{m}} = \left(1 - \frac{\dot{m}t}{M_0}\right)^{k/\dot{m}}$ and

$$U = \frac{V_e\dot{m}}{k}\left[1 - \left(1 - \frac{\dot{m}t}{M_0}\right)^{k/\dot{m}}\right] \qquad (1)$$

At $t = 10\ \text{s}$

$$U = 1500\frac{\text{m}}{\text{s}} \times 75\frac{\text{kg}}{\text{s}} \times \frac{\text{m}}{75\ \text{N}\cdot\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\left[1 - \left(1 - 75\frac{\text{kg}}{\text{s}} \times 10\ \text{s} \times \frac{1}{4000\ \text{kg}}\right)^{75\frac{\text{N}\cdot\text{s}}{\text{m}} \times \frac{\text{s}}{75\ \text{kg}} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}}\right]$$

$U = 281\ \text{m/s}$ $\qquad U$

Problem 4.158

Note that all fuel would be expended at $t_{bo} = \frac{M_f}{\dot{M}} = 1000\,kg \times \frac{s}{75\,kg}$
i.e. at $t = 13.3\,s$

The sled speed as a function of time is then

$$U = \frac{V_e \dot{M}}{k}\left[1 - \left(1 - \frac{\dot{M}t}{M_0}\right)^{k/\dot{M}}\right] \quad \text{for } 0 \le t \le 13.3\,s$$

The speed reaches a maximum at $t = 13.3\,s$ and decays with time due to the motion resistance. $U_{max} = 375\,m/s$

The sled acceleration is given by

$$\frac{dU}{dt} = \frac{V_e \dot{M} - kU}{M_0 - \dot{M}t} \quad \text{for } 0 \le t \le 11.1\,s$$

At $t \ge 13.3\,s$ $V_e = 0$ and

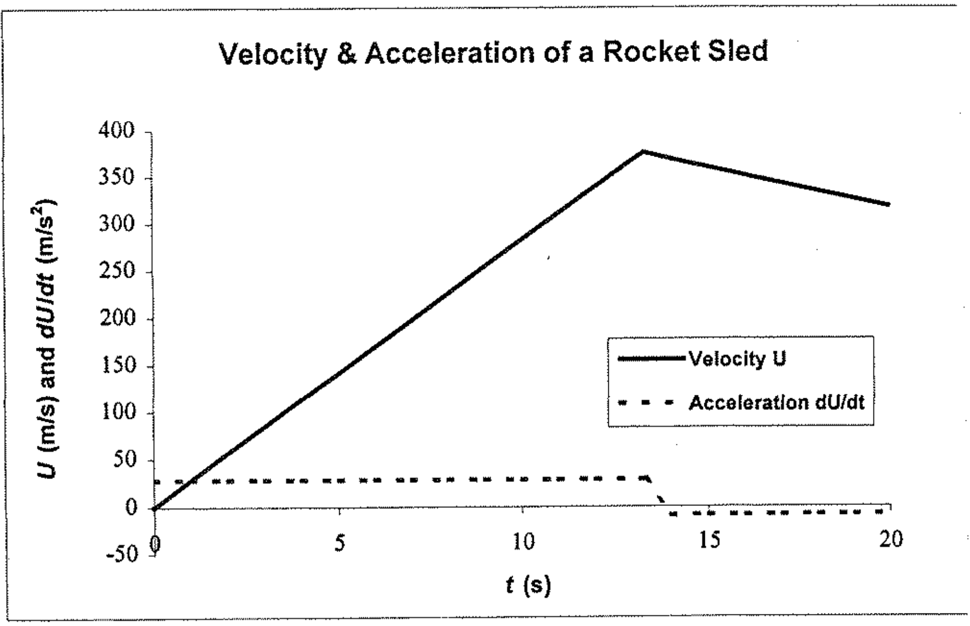
$$\frac{dU}{dt} = \frac{-kU}{M_0 - M_{fuel}}$$

Note that for $t > t_{bo} = 13.3\,s$, $\frac{dU}{dt} = -\frac{kU}{M_{bo}}$ and

$$\frac{dU}{U} = -\frac{k}{M_{bo}}dt \quad , \ln\frac{U}{U_{bo}} = -\frac{k(t - t_{bo})}{M_{bo}}$$

and $U = U_{bo}\, e^{-k(t - t_{bo})/M_{bo}}$

t (s)	*U* (m/s)	*dU/dt* (m/s^2)
0.0	0.0	28.1
1.0	28.1	28.1
2.0	56.3	28.1
3.0	84.4	28.1
4.0	113	28.1
5.0	141	28.1
6.0	169	28.1
7.0	197	28.1
8.0	225	28.1
9.0	253	28.1
10.0	281	28.1
11.0	309	28.1
12.0	338	28.1
13.2	371	28.1
13.3	375	28.1
14.0	369	-9.22
15.0	360	-8.99
16.0	351	-8.77
17.0	342	-8.55
18.0	334	-8.34
19.0	325	-8.14
20.0	317	-7.94



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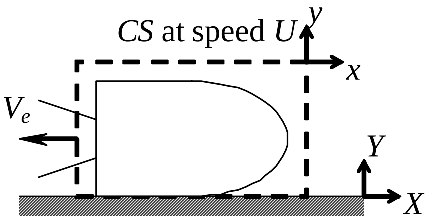
Problem 4.159

[3]

4.159 A rocket sled with initial mass of 900 kg is to be accelerated on a level track. The rocket motor burns fuel at constant rate $\dot{m} = 13.5$ kg/s. The rocket exhaust flow is uniform and axial. Gases leave the nozzle at 2750 m/s relative to the nozzle, and the pressure is atmospheric. Determine the minimum mass of rocket fuel needed to propel the sled to a speed of 265 m/s before burnout occurs. As a first approximation, neglect resistance forces.



Given: Data on rocket sled

Find: Minimum fuel to get to 265 m/s

Solution:

Basic equation: Momentum flux in x direction $$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, d\mathcal{V} = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\mathcal{V} + \int_{CS} u_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities

From continuity $\frac{dM}{dt} = m_{rate} = \text{constant}$ so $M = M_0 - m_{rate}\cdot t$ (Note: Software cannot render a dot!)

Hence from momentum $$-a_{rfx}\cdot M = -\frac{dU}{dt}\cdot\left(M_0 - m_{rate}\cdot t\right) = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate}$$

Separating variables $$dU = \frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t}\cdot dt$$

Integrating $$U = V_e\cdot\ln\left(\frac{M_0}{M_0 - m_{rate}\cdot t}\right) = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right)$$ or $$t = \frac{M_0}{m_{rate}}\cdot\left(1 - e^{-\frac{U}{V_e}}\right)$$

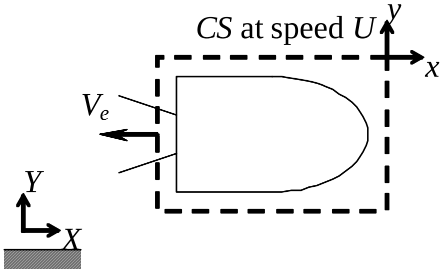
The mass of fuel consumed is $$m_f = m_{rate}\cdot t = M_0\cdot\left(1 - e^{-\frac{U}{V_e}}\right)$$

Hence $$m_f = 900\cdot\text{kg}\times\left(1 - e^{-\frac{265}{2750}}\right)$$ $$m_f = 82.7\,\text{kg}$$

Problem 4.160

[3]

4.160 A rocket motor is used to accelerate a kinetic energy weapon to a speed of 3500 mph in horizontal flight. The exit stream leaves the nozzle axially and at atmospheric pressure with a speed of 6000 mph relative to the rocket. The rocket motor ignites upon release of the weapon from an aircraft flying horizontally at $U_0 = 600$ mph. Neglecting air resistance, obtain an algebraic expression for the speed reached by the weapon in level flight. Determine the minimum fraction of the initial mass of the weapon that must be fuel to accomplish the desired acceleration.



Given: Data on rocket weapon

Find: Expression for speed of weapon; minimum fraction of mass that must be fuel

Solution:

Basic equation: Momentum flux in x direction

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, dV\!\!\!/ = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, dV\!\!\!/ + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities 5) Constant mass flow rate

From continuity $\quad \dfrac{dM}{dt} = m_{rate} = \text{constant} \quad$ so $\quad M = M_0 - m_{rate}\cdot t \qquad$ (Note: Software cannot render a dot!)

Hence from momentum $\quad -a_{rfx}\cdot M = -\dfrac{dU}{dt}\cdot\left(M_0 - m_{rate}\cdot t\right) = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate}$

Separating variables $\quad dU = \dfrac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t}\cdot dt$

Integrating from $U = U_0$ at $t = 0$ to $U = U$ at $t = t$

$$U - U_0 = -V_e\cdot\left(\ln\left(M_0 - m_{rate}\cdot t\right) - \ln\left(M_0\right)\right) = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right)$$

$$U = U_0 - V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right)$$

Rearranging $\quad \text{MassFractionConsumed} = \dfrac{m_{rate}\cdot t}{M_0} = 1 - e^{-\frac{\left(U-U_0\right)}{V_e}} = 1 - e^{-\frac{(3500-600)}{6000}} = 0.383$

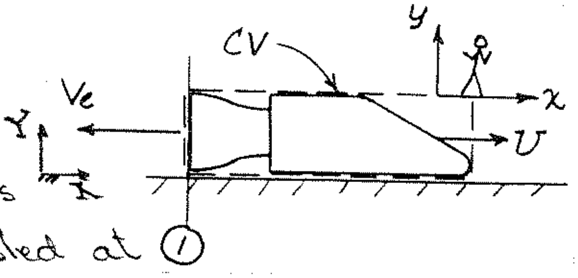
Hence 38.3% of the mass must be fuel to accomplish the task. In reality, a much higher percentage would be needed due to drag effects

Given: Rocket sled moving on level track without resistance

Inital mass, $M_0 = 3000$ kg (includes $M_{fuel} = 1000$ kg)

$V_e = 2500$ m/s ; $p_e = p_{atm}$

Fuel consumption, $\dot{m} = 75$ kg/s



Find: Acceleration and speed of sled at ① t = 10 s.

Plot: sled speed and acceleration as functions of time.

Solution:

Apply x component of momentum to linearly accelerating CV; use continuity to find M(t)

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}_{xyz} \cdot d\vec{A}$

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf}\, \rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u_{xyz}\, \rho\, d\forall + \int_{CS} u_{xyz} (\rho \vec{V}_{xyz} \cdot d\vec{A})$$

Assumptions:
(1) $F_{S_x} = 0$, no resistance (given)
(2) $F_{B_x} = 0$, horizontal
(3) neglect $\partial/\partial t$ inside CV
(4) uniform flow at nozzle exit
(5) $p_e = p_{atm}$ (given)

From continuity, $0 = \frac{\partial M}{\partial t} + \{+|\dot{m}|\} = \frac{dM}{dt} + \dot{m}$ or $dM = -\dot{m}\,dt$

Integrating, $\int_{M_0}^{M} dM = M - M_0 = \int_0^t -\dot{m}\,dt = -\dot{m}t$ or $M = M_0 - \dot{m}t$

From the momentum equation

$$-a_{rf_x} M = -a_{rf_x}(M_0 - \dot{m}t) = u_1\{+|\dot{m}|\} = -V_e \dot{m} \qquad \{u_1 = -V_e\}$$

Thus

$$a_{rf_x} = \frac{dU}{dt} = \frac{V_e \dot{m}}{(M_0 - \dot{m}t)} \quad \text{— — — — — — — — (1)}$$

At t = 10 s

$$\frac{dU}{dt} = 2500\,\frac{m}{s} \times 75\,\frac{kg}{s} \times \frac{1}{3000\,kg - 75\,\frac{kg}{s} \times 10\,s} = 83.3\ m/s^2 \longleftarrow a_{rf_x}$$

From Eq. 1, $dU = V_e \frac{\dot{m}\,dt}{M_0 - \dot{m}t}$

Integrating from U = 0 at t = 0 to U at t gives

$$U = -V_e \ln(M_0 - \dot{m}t)\Big]_0^t = -V_e \ln\frac{(M_0 - \dot{m}t)}{M_0}$$

$$U = V_e \ln\frac{M_0}{(M_0 - \dot{m}t)} \quad \text{— — — — — — — — (2)}$$

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Problem 4.161

At t = 10 s

$$U = 2500 \tfrac{m}{s} \ln \frac{3000\,kg}{3000\,kg - 75\,\frac{kg}{s} \times 10\,s} = 719\ m/s \qquad U.$$

Note that all fuel would be expended at $t_{b0} = \frac{M_f}{\dot m} = 1000\,kg \times \frac{s}{75\,kg}$

i.e. at $t_{b,o} = 13.3$ s.

The sled speed as a function of time is then

$$U = V_e \ln \frac{M_0}{(M_0 - \dot m t)} \quad \text{for } t \le 13.3\,s$$

$$U = U_{max} = 1010\ m/s \quad \text{for } t \ge 13.3\,s$$

The sled acceleration is given by

$$\frac{dU}{dt} = \frac{\dot m V_e}{(M_0 - \dot m t)} \quad \text{for } 0 \le t \le 13.3\,s$$

$$\frac{dU}{dt} = 0 \quad \text{for } t \ge 13.3\,s$$

Acceleration and Speed vs. Time for Rocket Sled:

Input Data:

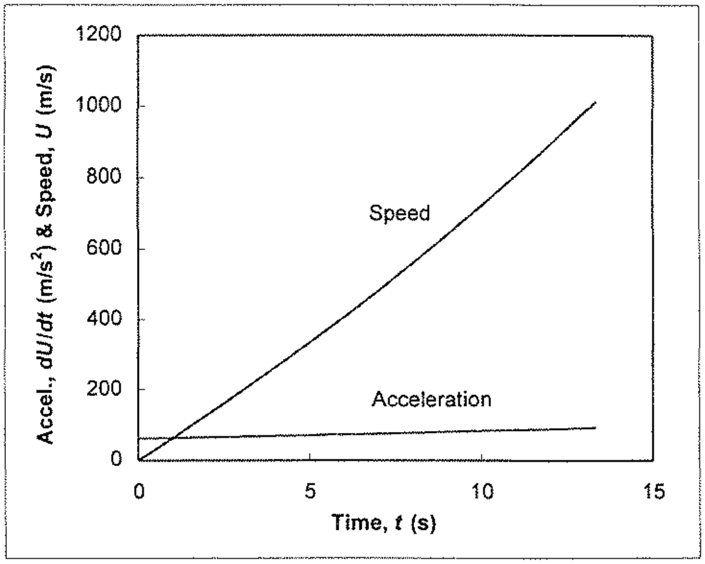
M_0 = 3000 kg

m(dot) = 75 kg/s

V_e = 2500 m/s

Calculated Results:

Time, t (s)	Acceleration, dU/dt (m/s^2)	Speed, U (m/s)
0	62.5	0
1	64.1	63.3
2	65.8	128
3	67.6	195
4	69.4	263
5	71.4	334
6	73.5	406
7	75.8	481
8	78.1	558
9	80.6	637
10	83.3	719
11	86.2	804
12	89.3	892
13	92.6	983
13.33	93.8	1014



Problem 4.162

[3]

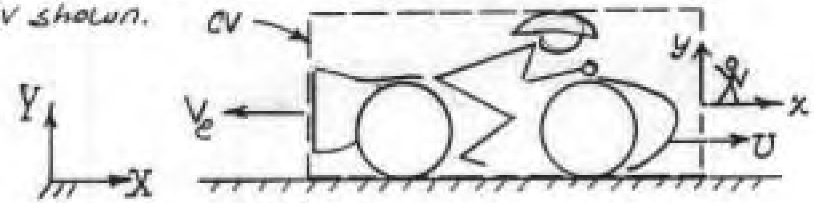
Given: Rocket-propelled motorcycle, to jump, standing start, level.

Speed needed $U_j = 875\ \text{km/hr}$ Rocket exhaust speed $V_e = 2510\ \text{m/s}$

Total mass $M_B = 375\ \text{kg}$ (without fuel)

Find: Minimum fuel mass needed to reach V_j.

Solution: Apply x-component of momentum equation to linearly accelerating CV shown.



From continuity,

$$M_{cv} = M_0 - \dot{m}t$$

Basic equation:

$$\overset{\approx 0(1)}{F_{S_x}} + \overset{\approx 0(2)}{F_{B_x}} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \overset{\approx 0(3)}{\frac{\partial}{\partial t}}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) Neglect air and rolling resistance

(2) Level track, so $F_{B_x} = 0$

(3) Neglect unsteady effects within CV

(4) Uniform flow at nozzle exit plane

(5) $p_e = p_{atm}$

Then

$$-a_{rf_x} M_{cv} = u_e\{+\dot{m}\} = -V_e\dot{m} \quad \text{or} \quad \frac{dU}{dt} = \frac{V_e\dot{m}}{M_{cv}} = \frac{V_e\dot{m}}{M_0 - \dot{m}t}$$

$$u_e = -V_e$$

Separating variables and integrating,

$$dU = -V_e\left(\frac{-\dot{m}dt}{M_0 - \dot{m}t}\right) \quad \text{or} \quad U_j = -V_e \ln(M_0 - \dot{m}t)\Big]_0^t = V_e \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right)$$

But $M_0 = M_B + M_F$ and $M_F = \dot{m}t$, so

$$\frac{U_j}{V_e} = \ln\left(\frac{M_B + M_F}{M_B}\right) = \ln\left(1 + \frac{M_F}{M_B}\right)\ ;\ 1 + \frac{M_F}{M_B} = e^{U_j/V_e}\ ;\ \frac{M_F}{M_B} = e^{U_j/V_e} - 1$$

Finally, $M_F = M_B\left(e^{U_j/V_e} - 1\right)$

$$M_F = 375\ \text{kg} \times \exp\left[875\,\frac{\text{km}}{\text{hr}} \times \frac{\text{s}}{2510\ \text{m}} \times 1000\,\frac{\text{m}}{\text{km}} \times \frac{\text{hr}}{3600\ \text{s}} - 1\right]$$

$$M_F = 38.1\ \text{kg}$$ M_F

The fuel mass required is about 10 percent of the mass of the motorcycle and rider.

Given: Home made rocket launched vertically from rest.

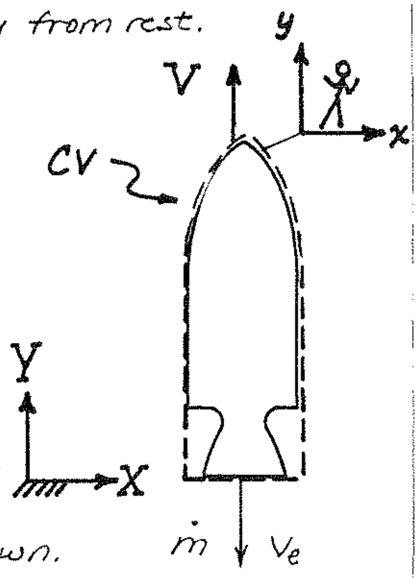
$M_0 = 20$ lbm, of which 15 lbm is fuel

$\dot{m} = 0.5$ lbm/s

$V_e = 6500$ ft/s (relative to rocket)

$p_e = p_{atm}$

Neglect aerodynamic drag.



Find: (a) Speed at $t = 20$ s.
(b) Height at $t = 20$ s.

Plot: Speed and height as functions of time.

Solution: Apply y-component of momentum equation to accelerating CV using CS shown.

Basic equation:

$$\overset{=0(1)}{\cancel{F_{S_y}}} + F_{B_y} - \int_{CV} a_{rf_y}\rho\, d\forall = \overset{\approx 0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v_{xyz}\rho\, d\forall + \int_{CS} v_{xyz}\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) Neglect air resistance; $p_e = p_{atm}$ (given)
(2) Neglect v_{xyz} and $\partial/\partial t$ within CV
(3) Uniform flow at nozzle exit section

Then

$$F_{B_y} - a_{rf_y}M = -Mg - Ma_{rf_y} = v_e\{+\dot{m}\} = -V_e\dot{m}$$

and $\qquad v_e = -V_e$

$$a_{rf_y} = \frac{dV}{dt} = \frac{V_e\dot{m}}{M} - g$$

Introducing $M = M_0 - \dot{m}t$ and separating variables,

$$dV = \left(\frac{V_e\dot{m}}{M_0 - \dot{m}t} - g\right)dt$$

Integrating from rest at $t = 0$

$$V = \int_0^t \left(\frac{V_e\dot{m}}{M_0 - \dot{m}t} - g\right)dt = -V_e \ln(M_0 - \dot{m}t)\Big]_0^t - gt$$

or

$$V = V_e \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - gt \qquad (1)$$

At $t = 20$ sec,

$$V = 6500\,\frac{\text{ft}}{\text{s}}\ln\left(\frac{20\text{ lbm}}{20\text{ lbm} - 0.5\,\frac{\text{lbm}}{\text{s}}\times 20\text{ s}}\right) - 32.2\,\frac{\text{ft}}{\text{s}^2}\times 20\text{ s}$$

$$V(20\text{ s}) = 3{,}860\text{ ft/s} \longleftarrow \qquad V_{20}$$

To find height, note $V = \frac{dY}{dt}$. Substitute into Eq. 1 to obtain

Problem 4.163

$$\frac{dY}{dt} = V_e \ln\left(\frac{M_0}{M_0 - \dot{m}t}\right) - gt = -V_e \ln\left(1 - \frac{\dot{m}t}{M_0}\right) - gt$$

Let $r = 1 - \frac{\dot{m}t}{M_0}$, and $dr = -\frac{\dot{m}}{M_0}dt$, then

$$dY = -V_e \ln r \, dt - gt\,dt = +\frac{V_e M_0}{\dot{m}} \ln r \, dr - gt\,dt$$

Integrating from $Y=0$ at $t=0$,

$$Y = \int_0^t \frac{V_e M_0}{\dot{m}} \ln r \, dr - \frac{1}{2}gt^2 = \frac{V_e M_0}{\dot{m}}\Big[r \ln r - r\Big]_0^t - \frac{1}{2}gt^2$$

$$= \frac{V_e M_0}{\dot{m}}\left\{\left(1 - \frac{\dot{m}t}{M_0}\right)\left[\ln\left(1 - \frac{\dot{m}t}{M_0}\right) - 1\right]\right\}\Bigg|_0^t - \frac{1}{2}gt^2$$

$$Y = \frac{V_e M_0}{\dot{m}}\left\{\left(1 - \frac{\dot{m}t}{M_0}\right)\left[\ln\left(1 - \frac{\dot{m}t}{M_0}\right) - 1\right] + 1\right\} - \frac{1}{2}gt^2$$

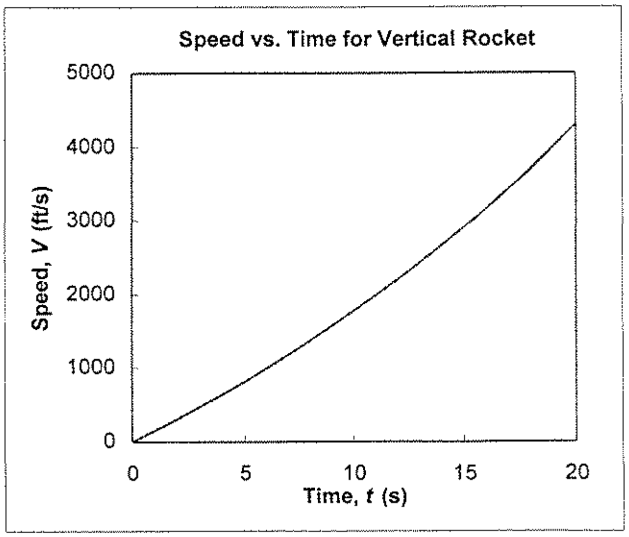
At $t = 20$ s,

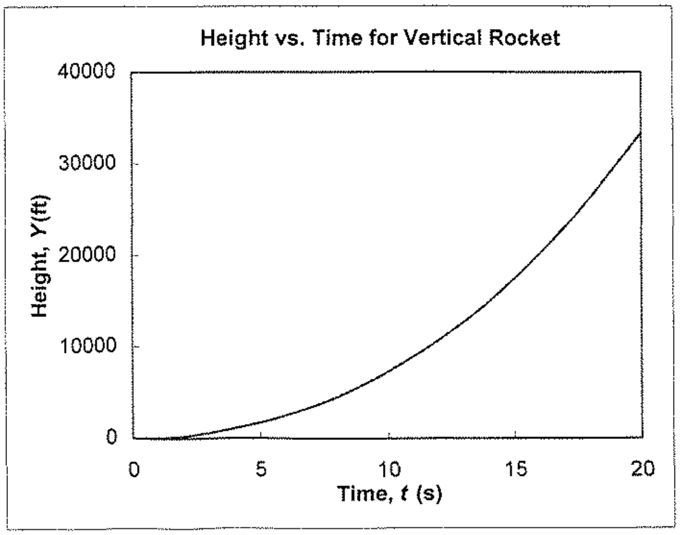
$$1 - \frac{\dot{m}t}{M_0} = 1 - 0.5\,\frac{\text{lbm}}{\text{s}} \times 20\,\text{s} \times \frac{1}{20\,\text{lbm}} = \frac{1}{2}$$

so

$$Y = 6500\,\frac{\text{ft}}{\text{s}} \times 20\,\text{lbm} \times \frac{\text{s}}{0.5\,\text{lbm}}\left\{\left(\frac{1}{2}\right)\left[\ln\left(\frac{1}{2}\right) - 1\right] + 1\right\} - \frac{1}{2} \times 32.2\,\frac{\text{ft}}{\text{s}^2}\,(20)^2\,\text{s}^2$$

$$Y = 33{,}500\ \text{ft} \longleftarrow \qquad Y$$





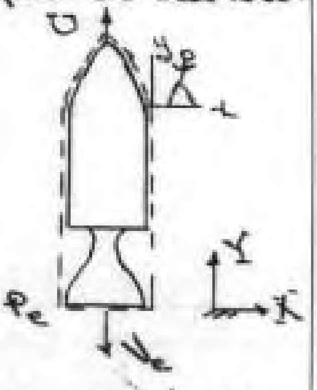
Problem 4.164

Given: Liquid-fueled rocket launched from pad at sea level

$M_0 = 30{,}000\ \text{kg}$ $\quad \dot{m} = 2450\ \text{kg/s}$

$V_e = 2270\ \text{m/s}$ $\quad p_e = 166\ \text{kPa (abs)}$

Exit plane diameter, $D_e = 2.6\ \text{m}$



Find: acceleration at lift-off

expression for rocket speed, $U(t)$

Solution: Apply y component of momentum equation to CV with linear acceleration

Basic equation: $$F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\, dV = \overset{\approx 0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, dV}} + \int_{CS} v_{xyz}\,\rho\, \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) F_{S_y} due to pressure, p_{atm} assumed constant, neglect air resistance

(2) neglect rate of change of momentum inside CV

(3) uniform flow at exit

Then, $$(-p_e - p_{atm})A_e - Mg - a_{rf_y} M = v_e\{+\dot{m}\} = -\dot{m}V_e$$

Solving for a_{rf_y}, $$a_{rf_y} = \frac{dU}{dt} = \frac{1}{M}\left[\dot{m}V_e + (p_e - p_{atm})A_e\right] - g \quad \ldots(1)$$

$M = M(t)$. From conservation of mass $\frac{\partial}{\partial t}\int_{CV} \rho\, dV + \int_{CS} \rho\vec{V}\cdot d\vec{A} = 0$

Then $$\frac{\partial}{\partial t}\int_{CV} \rho\, dV = \frac{dM}{dt} = -\int_{CS} \rho\vec{V}\cdot d\vec{A} = -\dot{m}_e \quad \text{(constant)}$$

Hence $M(t) = M_0 - \dot{m}t$, and

$$a_{rf_y} = \frac{dU}{dt} = \frac{\dot{m}V_e}{M_0 - \dot{m}t} + \frac{(p_e - p_{atm})A_e}{M_0 - \dot{m}t} - g$$

$$U = \int_0^U dU = \int_0^t \frac{\dot{m}V_e}{M_0 - \dot{m}t}\,dt + \int_0^t \frac{(p_e - p_{atm})A_e}{M_0 - \dot{m}t}\,dt - \int_0^t g\,dt$$

$$U = -V_e \ln\left[\frac{M_0 - \dot{m}t}{M_0}\right] - \frac{(p_e - p_{atm})A_e}{\dot{m}}\ln\left[\frac{M_0 - \dot{m}t}{M_0}\right] - gt$$

$$U = -\left[V_e + \frac{(p_e - p_{atm})A_e}{\dot{m}}\right]\ln\left[\frac{M_0 - \dot{m}t}{M_0}\right] - gt \longleftarrow U(t)$$

At lift-off, $t = 0$, $M = M_0$

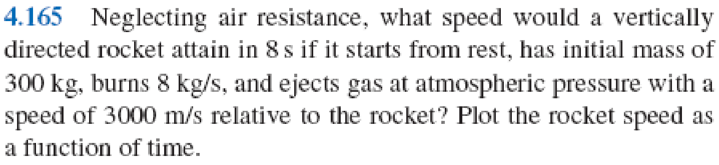
$$a_{rf_y} = \frac{1}{M}\left[\dot{m}V_e + (p_e - p_{atm})A_e\right] - g$$

$$= \frac{1}{3\times10^4\ \text{kg}}\left[2450\frac{\text{kg}}{\text{s}}\times 2270\frac{\text{m}}{\text{s}} + (166-101)10^3\frac{\text{N}}{\text{m}^2}\times\frac{\pi}{4}(2.6)^2\text{m}^2\cdot\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right] - 9.81\frac{\text{m}}{\text{s}^2}$$

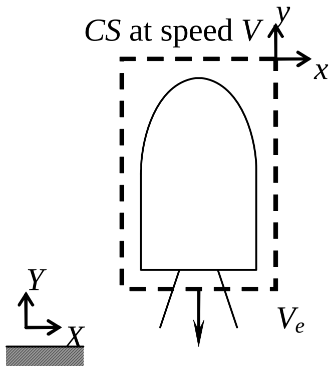
$$a_{rf_y} = 169\ \text{m/s}^2 \longleftarrow a_{rf_y}$$

Problem 4.165

[3]



4.165 Neglecting air resistance, what speed would a vertically directed rocket attain in 8 s if it starts from rest, has initial mass of 300 kg, burns 8 kg/s, and ejects gas at atmospheric pressure with a speed of 3000 m/s relative to the rocket? Plot the rocket speed as a function of time.



Given: Data on rocket

Find: Speed after 8 s; Plot of speed versus time

Solution:

Basic equation: Momentum flux in y direction $F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\,dV\!\!\!\!- = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\,dV\!\!\!\!- + \int_{CS} v_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A}$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities 5) Constant mass flow rate

From continuity $\frac{dM}{dt} = m_{rate} = \text{constant}$ so $M = M_0 - m_{rate}\cdot t$ (Note: Software cannot render a dot!)

Hence from momentum $-M\cdot g - a_{rfy}\cdot M = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate}$

Hence $a_{rfy} = \frac{dV}{dt} = \frac{V_e\cdot m_{rate}}{M} - g = \frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g$ (1)

Separating variables $dV = \left(\frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g\right)\cdot dt$

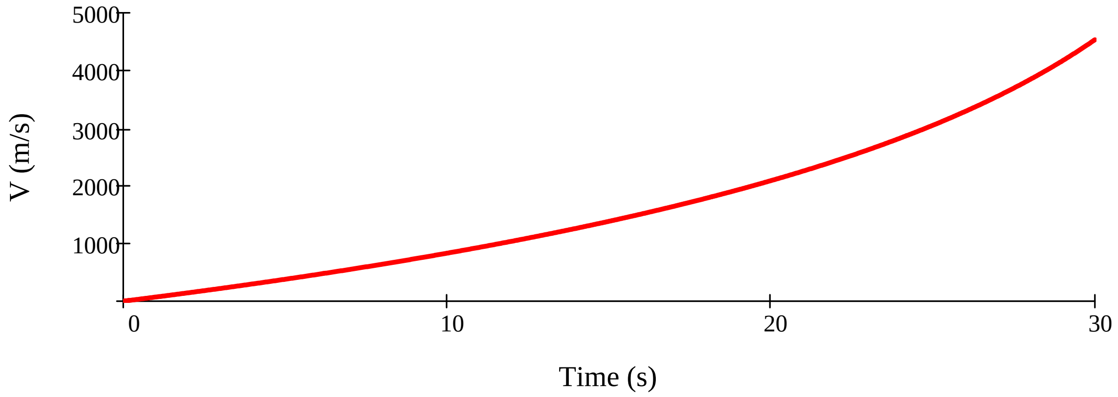
Integrating from V = at t = 0 to V = V at t = t

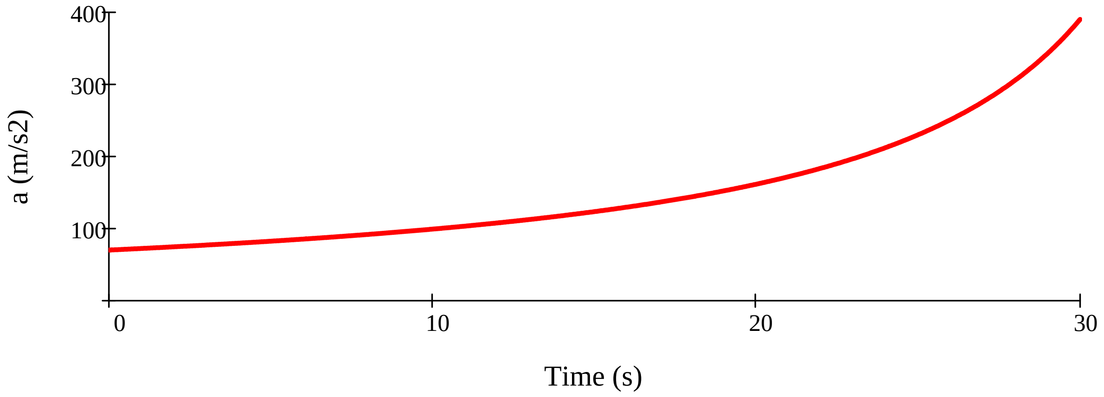
$$V = -V_e\cdot\left(\ln\left(M_0 - m_{rate}\cdot t\right) - \ln\left(M_0\right)\right) - g\cdot t = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t$$

$$V = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t \qquad (2)$$

At t = 8 s $V = -3000\cdot\frac{m}{s}\cdot\ln\left(1 - 8\cdot\frac{kg}{s}\times\frac{1}{300\cdot kg}\times 8\cdot s\right) - 9.81\cdot\frac{m}{s^2}\times 8\cdot s$ $V = 641\,\frac{m}{s}$

The speed and acceleration as functions of time are plotted below. These are obtained from Eqs 2 and 1, respectively, and can be plotted in *Excel*


5000
4000
3000
2000
1000
V (m/s)
0
10
20
30
Time (s)


400
300
200
100
a (m/s2)
0
10
20
30
Time (s)

Problem 4.166

Open-Ended Problem Statement: Inflate a toy balloon with air and release it. Watch as the balloon darts about the room. Explain what causes the phenomena you see.

Discussion: Air blown into a balloon to inflate it must be compressed to overcome the skin's resistance to stretching. (Remember how hard it is to create enough pressure to "start" the inflation process!) After decreasing briefly, the required pressure seems to increase as inflation of the balloon continues.

As the balloon is inflated, the skin stretches and stores energy. When the inflated balloon is released, the stored energy in the skin forces the compressed air out the open mouth of the balloon. The expansion of the compressed air to the lower surrounding atmospheric pressure creates a high-speed jet of air, which propels the relatively light balloon initially at a high speed.

The moving balloon is unstable because it has a poor aerodynamic shape. Therefore it darts about in a random pattern. The balloon keeps moving as long as it contains pressurized air to act as a propulsion jet. However, it is not long before the energy stored in the skin is exhausted and the air in the balloon is reduced to atmospheric pressure.

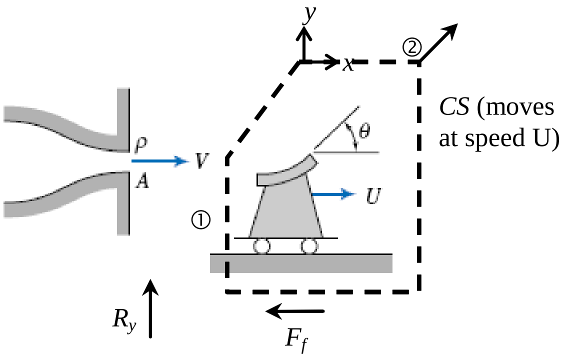
When the balloon reaches atmospheric pressure it is slowed by aerodynamic drag. Finally the empty, wrinkled balloon simply falls to the floor.

Some toys that use a balloon for propulsion are available. Most have stabilizing surfaces. It is instructive to study these toys carefully to understand how each works, and why each toy is shaped the way it is.

Problem 4.167

[4]

4.167 The vane/cart assembly of mass $M = 30$ kg, shown in Problem 4.123, is driven by a water jet. The water leaves the stationary nozzle of area $A = 0.02\ \text{m}^2$, with a speed of 20 m/s. The coefficient of kinetic friction between the assembly and the surface is 0.10. Plot the terminal speed of the assembly as a function of vane turning angle, θ, for $0 \le \theta \le \pi/2$. At what angle does the assembly begin to move if the coefficient of static friction is 0.15?



Given: Water jet striking moving vane

Find: Plot of terminal speed versus turning angle; angle to overcome static friction

Solution:

Basic equations: Momentum flux in x and y directions

$$F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

$$F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, d\forall + \int_{CS} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) Incompressible flow 2) Atmospheric pressure in jet 3) Uniform flow 4) Jet relative velocity is constant

Then $$-F_f - M\cdot a_{rfx} = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = -(V-U)\cdot[\rho\cdot(V-U)\cdot A] + (V-U)\cdot\cos(\theta)\cdot[\rho\cdot(V-U)\cdot A]$$

$$a_{rfx} = \frac{\rho(V-U)^2\cdot A\cdot(1-\cos(\theta)) - F_f}{M} \qquad (1)$$

Also $$R_y - M\cdot g = v_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + v_2\cdot\rho\cdot V_2\cdot A_2 = 0 + (V-U)\cdot\sin(\theta)\cdot[\rho\cdot(V-U)\cdot A]$$

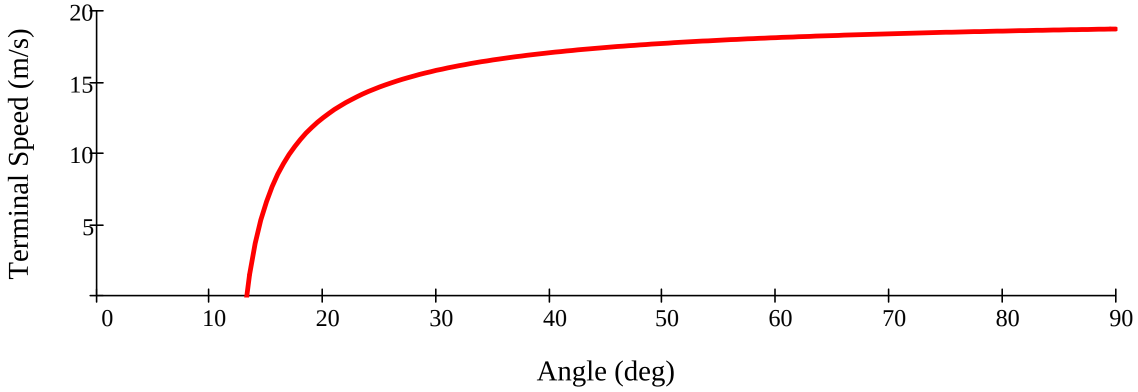
$$R_y = M\cdot g + \rho(V-U)^2\cdot A\cdot\sin(\theta)$$

At terminal speed $a_{rfx} = 0$ and $F_f = \mu_k R_y$. Hence in Eq 1

$$0 = \frac{\rho\cdot\left(V-U_t\right)^2\cdot A\cdot(1-\cos(\theta)) - \mu_k\cdot\left[M\cdot g + \rho\cdot\left(V-U_t\right)^2\cdot A\cdot\sin(\theta)\right]}{M} = \frac{\rho\cdot\left(V-U_t\right)^2\cdot A\cdot\left(1-\cos(\theta)-\mu_k\cdot\sin(\theta)\right)}{M} - \mu_k\cdot g$$

or $$V - U_t = \sqrt{\frac{\mu_k\cdot M\cdot g}{\rho\cdot A\cdot\left(1-\cos(\theta)-\mu_k\cdot\sin(\theta)\right)}} \qquad U_t = V - \sqrt{\frac{\mu_k\cdot M\cdot g}{\rho\cdot A\cdot\left(1-\cos(\theta)-\mu_k\cdot\sin(\theta)\right)}}$$

The terminal speed as a function of angle is plotted below; it can be generated in *Excel*



For the static case $F_f = \mu_s \cdot R_y$ and $a_{rfx} = 0$ (the cart is about to move, but hasn't)

Substituting in Eq 1, with U = 0

$$0 = \frac{\rho \cdot V^2 \cdot A \cdot \left[1 - \cos(\theta) - \mu_s \cdot \left(\rho \cdot V^2 \cdot A \cdot \sin(\theta) + M \cdot g\right)\right.}{M}$$

or
$$\cos(\theta) + \mu_s \cdot \sin(\theta) = 1 - \frac{\mu_s \cdot M \cdot g}{\rho \cdot V^2 \cdot A}$$

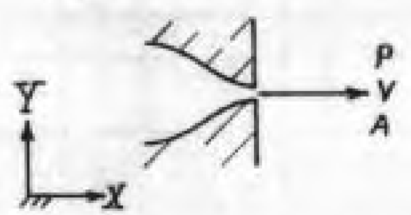
We need to solve this for θ! This can be done by hand or by using Excel's Goal Seek or Solver $\theta = 19\,\text{deg}$

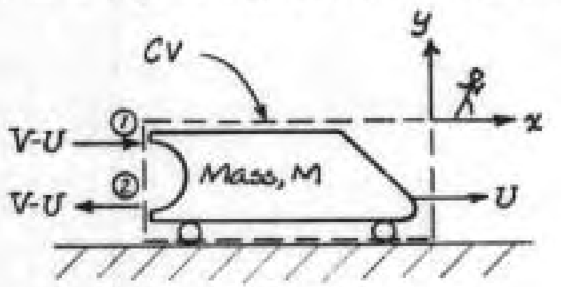
Note that we need θ = 19°, but once started we can throttle back to about θ = 12.5° and still keep moving!

Problem 4.168

[4]

Given: Vehicle accelerated from rest by a hydraulic catapult. Neglect resistance.





Find: (a) Expression for acceleration at any time, t.
(b) Time required to reach $U = V/2$.

Solution: Apply x component of momentum equation using linearly accelerating CV shown above.

Basic equation: $\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\,\rho\, d\forall = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}\int_{CV}}} u_{xyz}\,\rho\, d\forall + \int_{CS} u_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$

Assumptions: (1) $F_{S_x} = 0$
(2) $F_{B_x} = 0$
(3) Neglect mass of liquid and rate of change of u in CV
(4) Uniform flow at each section
(5) Jet area and speed with respect to vehicle are constant

Then

$$-Ma_{rf_x} = -M\frac{dU}{dt} = u_1\{-|\rho(V-U)A|\} + u_2\{|\rho(V-U)A|\}$$

$$u_1 = V-U \qquad u_2 = -(V-U)$$

or

$$a_{rf_x} = \frac{dU}{dt} = \frac{2\rho(V-U)^2A}{M} \quad ; \quad \frac{dU}{(V-U)^2} = \frac{2\rho A}{M}dt \quad ; -\frac{d(V-U)}{(V-U)^2} = \frac{2\rho A}{M}dt$$

To obtain $a_{rf_x}(t)$, we must first find $U(t)$. Integrating from $U=0$ at $t=0$ to U at t,

$$\int_{V-U=V}^{V-U} -\frac{d(V-U)}{(V-U)^2} = \left.\frac{1}{V-U}\right]_V^{V-U} = \frac{1}{V-U} - \frac{1}{V} = \frac{V-(V-U)}{V(V-U)} = \frac{2\rho A}{M}t \; ; \; \frac{U}{V-U} = \frac{2\rho VA}{M}t$$

Solving,

$$U = (V-U)\frac{2\rho VA}{M}t \, , \quad U = V\frac{\frac{2\rho VA}{M}t}{1+\frac{2\rho VA}{M}t} \quad \text{and} \quad V-U = V\left[1-\frac{\frac{2\rho VA}{M}t}{1+\frac{2\rho VA}{M}t}\right]$$

Substituting,

$$a_{rf_x} = \frac{2\rho V^2A}{M}\left[1-\frac{\frac{2\rho VA}{M}t}{1+\frac{2\rho VA}{M}t}\right]^2 = \frac{2\rho V^2A}{M}\left[\frac{1}{1+\frac{2\rho VA}{M}t}\right]^2$$

$a_{rf_x}(t)$

The time to reach $U = V/2$ is

$$\frac{U}{V} = \frac{1}{2} = \frac{\frac{2\rho VA}{M}t}{1+\frac{2\rho VA}{M}t} \quad \text{or} \quad t = \frac{M}{2\rho VA}$$

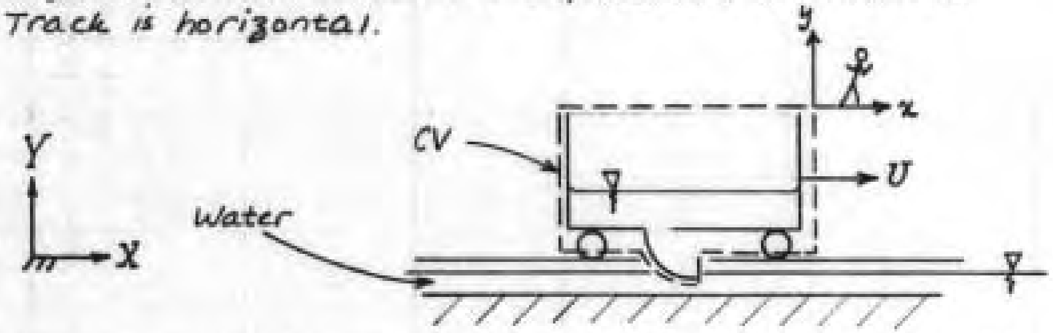
$t(\frac{V}{2})$

Check: $\left[\frac{M}{\rho VA}\right] = M\frac{L^3}{M}\frac{t}{L}\frac{1}{L^2} = t$ ✓ ; $\left[\frac{\rho V^2 A}{M}\right] = \frac{M}{L^3}\frac{L^2}{t^2}L^2\frac{1}{M} = \frac{L}{t^2}$ ✓

Problem 4.169

[4]

Given: Moving tank slowed by lowering scoop into water trough.
Initial mass and speed are M_0 and U_0, respectively.
Neglect external forces due to pressure or friction.
Track is horizontal.



Find: (a) Apply continuity and momentum to show $U = U_0 M_0 / M$.
(b) Obtain a general expression for $U(t)$.

Solution: Apply continuity and momentum equations to linearly accelerating CV shown.

Basic equations: $$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

$$\overset{=0(1)}{\cancel{F_{S_x}}} + \overset{=0(2)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x}\, \rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} \overset{\approx 0(3)}{\cancel{u_{xyz}}}\, \rho\, d\forall + \int_{CS} u_{xyz}\, \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) $F_{S_x} = 0$
(2) $F_{B_x} = 0$
(3) Neglect u within CV
(4) Uniform flow across inlet section

From continuity

$$0 = \frac{\partial}{\partial t} M_{cv} + \{-|\rho U A|\} \quad \text{or} \quad \frac{dM}{dt} = \rho U A$$

From momentum

$$-a_{rf_x} M = -\frac{dU}{dt} M = u\{-|\rho U A|\} = U \rho U A, \text{ since } u = -U$$

But from continuity, $\rho U A = \frac{dM}{dt}$, so

$$M\frac{dU}{dt} + U\frac{dM}{dt} = 0 \quad \text{or} \quad UM = \text{constant} = U_0 M_0\,; \quad U = U_0 M_0 / M$$

U

Substituting $M = M_0 U_0 / U$ into momentum, $-\frac{dU}{dt}\frac{M_0 U_0}{U} = \rho U^2 A$, or

$$\frac{dU}{U^3} = -\frac{\rho A}{U_0 M_0}\, dt$$

Integrating, $$\int_{U_0}^{U} \frac{dU}{U^3} = -\frac{1}{2}\frac{1}{U^2}\Bigg]_{U_0}^{U} = -\frac{1}{2}\left(\frac{1}{U^2} - \frac{1}{U_0^2}\right) = -\int_0^t \frac{\rho A}{U_0 M_0}\, dt = -\frac{\rho A}{U_0 M_0} t$$

Solving for U,

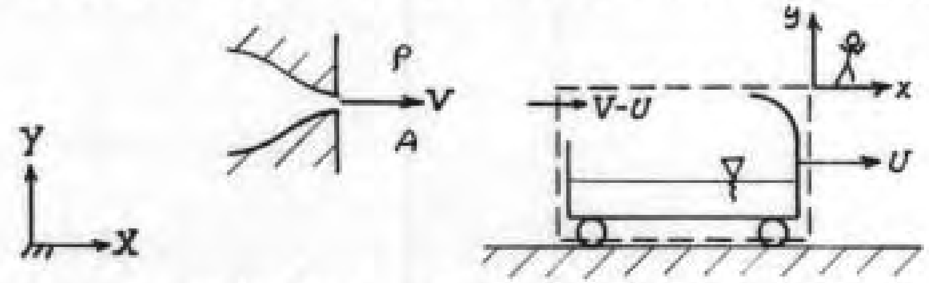
$$U = \frac{U_0}{\left[1 + \frac{2\rho U_0 A}{M_0} t\right]^{\frac{1}{2}}}$$

U(t)

Problem 4.170

[4]

Given: Tank driven by jet along horizontal track. Neglect resistance. Acceleration is from rest. Initial mass is M_0. Track horizontal.



Find: (a) Apply continuity and momentum to show $M = M_0V/(V-U)$
(b) General expression for U/V as a function of time.

Solution: Apply continuity and x component of momentum equation to linearly accelerating CV shown.

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}_{xyz} \cdot d\vec{A}$

$$\overset{=0(1)}{F_{S_x}} + \overset{=0(2)}{F_{B_x}} - \int_{CV} a_{rf_x} \rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} \overset{\approx 0(3)}{u_{xyz}} \rho\, d\forall + \int_{CS} u_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) $F_{S_x} = 0$
(2) $F_{B_x} = 0$
(3) Neglect u within CV
(4) Uniform flow in jet

From continuity

$$0 = \frac{\partial}{\partial t} M_{cv} + \{-|\rho(V-U)A|\} \quad \text{or} \quad \frac{dM}{dt} = \rho(V-U)A$$

From momentum

$$-a_{rf_x} M = -\frac{dU}{dt} M = u\{-|\rho(V-U)A|\} = (V-U)[-\rho(V-U)A]; \; u = V-U$$

But from continuity, $\rho(V-U)A = \frac{dM}{dt}$, and $dU = -d(V-U)$, so

$$-\frac{dU}{dt} M = \frac{d(V-U)}{dt} M = -(V-U)\frac{dM}{dt} \quad \text{or} \quad M(V-U) = \text{constant} = M_0V$$

Thus $M = M_0V/(V-U)$ ← M

Substituting into momentum, $-\frac{dU}{dt} M = \frac{d(V-U)}{dt}\frac{M_0V}{(V-U)} = -\rho(V-U)^2A$, or

$$\frac{d(V-U)}{(V-U)^3} = -\frac{\rho A}{VM_0}\, dt$$

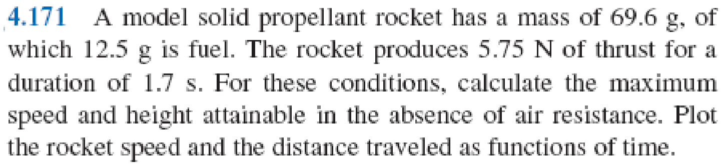
Integrating, $\int_V^{V-U} \frac{d(V-U)}{(V-U)^3} = -\frac{1}{2}\left[\frac{1}{(V-U)^2} - \frac{1}{V^2}\right] = -\int_0^t \frac{\rho A}{VM_0}\, dt = -\frac{\rho A}{VM_0} t$

Solving,

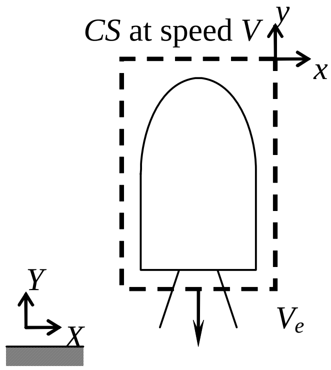
$$\frac{U}{V} = \left\{1 - \frac{1}{\left[1 + \frac{2\rho VA}{M_0} t\right]^{1/2}}\right\}$$ ← $\frac{U}{V}$

Problem 4.171

[5]



4.171 A model solid propellant rocket has a mass of 69.6 g, of which 12.5 g is fuel. The rocket produces 5.75 N of thrust for a duration of 1.7 s. For these conditions, calculate the maximum speed and height attainable in the absence of air resistance. Plot the rocket speed and the distance traveled as functions of time.



Given: Data on rocket

Find: Maximum speed and height; Plot of speed and distance versus time

Solution:

Basic equation: Momentum flux in y direction $F_{S_y} + F_{B_y} - \int_{CV} a_{rf_y}\,\rho\, dV\!\!\!/ = \frac{\partial}{\partial t}\int_{CV} v_{xyz}\,\rho\, dV\!\!\!/ + \int_{CS} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$

Assumptions: 1) No resistance 2) $p_e = p_{atm}$ 3) Uniform flow 4) Use relative velocities 5) Constant mass flow rate

From continuity $\frac{dM}{dt} = m_{rate} = \text{constant}$ so $M = M_0 - m_{rate}\cdot t$ (Note: Software cannot render a dot!)

Hence from momentum $-M\cdot g - a_{rfy}\cdot M = u_e\cdot\left(\rho_e\cdot V_e\cdot A_e\right) = -V_e\cdot m_{rate}$

Hence $a_{rfy} = \frac{dV}{dt} = \frac{V_e\cdot m_{rate}}{M} - g = \frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g$

Separating variables $dV = \left(\frac{V_e\cdot m_{rate}}{M_0 - m_{rate}\cdot t} - g\right)\cdot dt$

Integrating from V = at t = 0 to V = V at t = t

$$V = -V_e\cdot\left(\ln\left(M_0 - m_{rate}\cdot t\right) - \ln\left(M_0\right)\right) - g\cdot t = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t$$

$$V = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t}{M_0}\right) - g\cdot t \qquad \text{for} \qquad t \le t_b \quad \text{(burn time)} \qquad (1)$$

To evaluate at t_b = 1.7 s, we need V_e and m_{rate} $\quad m_{rate} = \frac{m_f}{t_b} \qquad m_{rate} = \frac{12.5\cdot gm}{1.7\cdot s} \qquad m_{rate} = 7.35\times 10^{-3}\frac{kg}{s}$

Also note that the thrust F_t is due to momentum flux from the rocket $\quad F_t = m_{rate}\cdot V_e \qquad V_e = \frac{F_t}{m_{rate}} \qquad V_e = \frac{5.75\cdot N}{7.35\times 10^{-3}\cdot\frac{kg}{s}}\times\frac{kg\cdot m}{s^2\cdot N} \qquad V_e = 782\frac{m}{s}$

Hence $V_{max} = -V_e\cdot\ln\left(1 - \frac{m_{rate}\cdot t_b}{M_0}\right) - g\cdot t_b$

$$V_{max} = -782\cdot\frac{m}{s}\cdot\ln\left(1 - 7.35\times 10^{-3}\cdot\frac{kg}{s}\times\frac{1}{0.0696\cdot kg}\times 1.7\cdot s\right) - 9.81\cdot\frac{m}{s^2}\times 1.7\cdot s \qquad V_{max} = 138\frac{m}{s}$$

To obtain Y(t) we set V = dY/dt in Eq 1, and integrate to find

$$Y = \frac{V_e\cdot M_0}{m_{rate}}\cdot\left[\left(1-\frac{m_{rate}\cdot t}{M_0}\right)\cdot\left(\ln\left(1-\frac{m_{rate}\cdot t}{M_0}\right)-1\right)+1\right]-\frac{1}{2}\cdot g\cdot t^2 \qquad t \le t_b \qquad t_b = 1.7\cdot s \qquad (2)$$

At $t = t_b$

$$Y_b = 782\cdot\frac{m}{s}\times 0.0696\cdot kg\times\frac{s}{7.35\times 10^{-3}\cdot kg}\cdot\left[\left(1-\frac{0.00735\cdot 1.7}{0.0696}\right)\left(\ln\left(1-\frac{.00735\cdot 1.7}{.0696}\right)-1\right)+1\right]\ldots$$

$$+ -\frac{1}{2}\times 9.81\cdot\frac{m}{s^2}\times(1.7\cdot s)^2$$

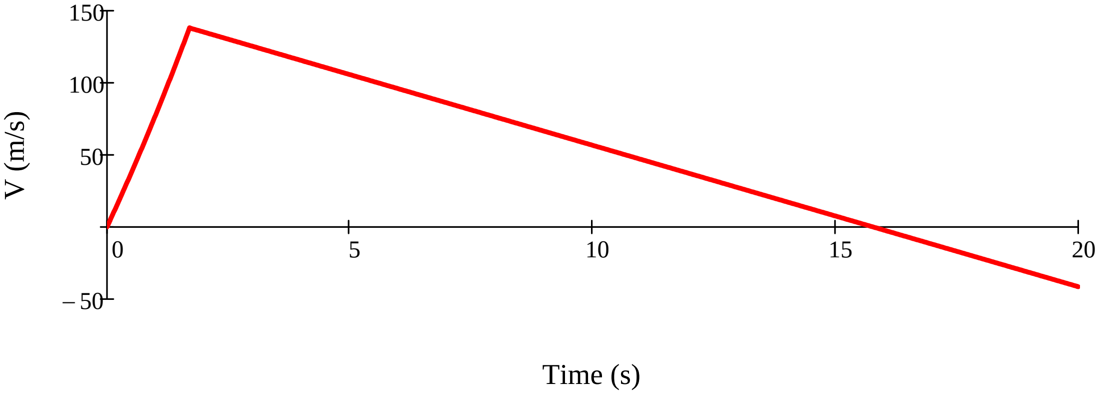
$$Y_b = 113\,m$$

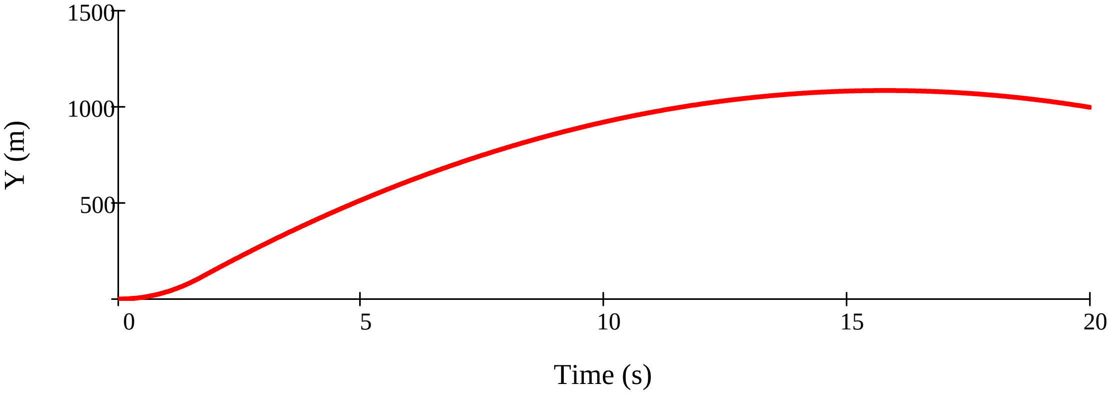
After burnout the rocket is in free assent. Ignoring drag

$$V(t) = V_{max} - g\cdot\left(t - t_b\right) \qquad (3)$$

$$Y(t) = Y_b + V_{max}\cdot\left(t-t_b\right) - \frac{1}{2}\cdot g\cdot\left(t-t_b\right)^2 \qquad t > t_b \qquad (4)$$

The speed and position as functions of time are plotted below. These are obtained from Eqs 1 through 4, and can be plotted in *Excel*





Using Solver, or by differentiating y(t) and setting to zero, or by setting V(t) = 0, we find for the maximum $t = 15.8\,s$ $\qquad y_{max} = 1085\,m$

Problem 4.172

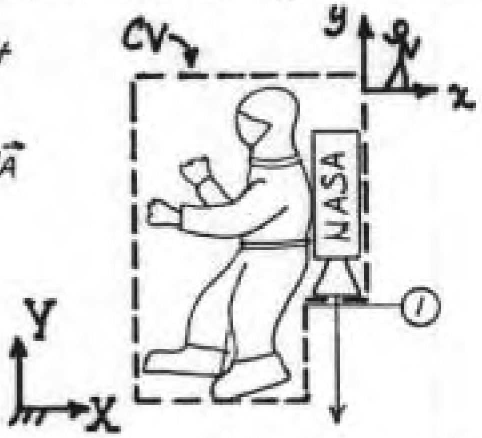
[4]

Given: Small rocket "jet pack" used to lift astronaut above Earth. Exhaust jet speed is constant but mass flow rate varies.

Find: (a) Algebraic expression for mass flow rate needed to hover.
(b) Maximum hover time.

Solution: Apply continuity and momentum using CV & CS shown.

Basic equation: $\overset{=0(1)}{F_{Sy}} + F_{By} - \int_{CV} \overset{=0(2)}{a_{rf_y}}\, \rho\, d\forall = \overset{\approx 0(3)}{\frac{\partial}{\partial t}}\int_{CV} v_{xyz}\, \rho\, d\forall + \int_{CS} v_{xyz}\, \rho \vec{V} \cdot d\vec{A}$



Assumptions: (1) Hover; $F_{Sy} = 0$
(2) $a_{rf_y} = 0$
(3) Neglect $\partial/\partial t$ in CV
(4) Uniform flow exhaust

$V_e = 2940$ m/s
$M_0 = 130$ kg
$M_f = 40$ kg

Then

$$-Mg = v_1 \{+\dot{m}\}$$

$$v_1 = -V_e$$

$$-Mg = -V_e \dot{m}$$

so

$$\dot{m} = \frac{Mg}{V_e}$$

$\dot{m}(t)$

From conservation of mass, $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = \frac{dM}{dt} + \dot{m}$

so $\frac{dM}{dt} = -\dot{m} = -\frac{Mg}{V_e}$ or $\frac{dM}{M} = -\frac{g}{V_e}\, dt$

Integrating from M_0 at $t = 0$ to $M_0 - M_f$ at t,

$$\int_{M_0}^{M_0-M_f} \frac{dM}{M} = \ln M \Big]_{M_0}^{M_0-M_f} = \ln\left(\frac{M_0 - M_f}{M_0}\right) = \ln\left(1 - \frac{M_f}{M_0}\right) = -\frac{gt}{V_e}$$

Solving for t,

$$t = -\frac{V_e}{g} \ln\left(1 - \frac{M_f}{M_0}\right) = -2940\,\frac{m}{s} \times \frac{s^2}{9.81\,m} \ln\left(1 - \frac{40\,kg}{130\,kg}\right)$$

$t = 110$ s (hover time)

t

Problem *4.173

Open-Ended Problem Statement: Several toy manufacturers sell water "rockets" that consist of plastic tanks to be partially filled with water and then pressurized with air. Upon release, the compressed air forces water out the nozzle rapidly, propelling the rocket. You are asked to help specify optimum conditions for this water-jet propulsion system. To simplify the analysis, consider horizontal motion only. Perform the analysis and design needed to define the acceleration performance of the compressed air/water-propelled rocket. Identify the fraction of tank volume that initially should be filled with compressed air to achieve optimum performance (i.e., maximum speed from the water charge). Describe the effect of varying the initial air pressure in the tank.

Discussion: The process may be modeled as a polytropic expansion of the trapped air which forces water out the jet nozzle, causing the "rocket" to accelerate. The polytropic exponent may be varied to model anything from an isothermal expansion process ($n = 1$) to an adiabatic expansion process ($n = k$), which is more likely to be an accurate model for the sudden expansion of the air.

Speed of the water jet leaving the "rocket" is proportional to the square root of the pressure difference between the tank and atmosphere.

Qualitatively it is apparent that the smaller the initial volume fraction of trapped air, the larger will be the expansion ratio of the air, and the more rapid will be the pressure reduction as the air expands. This will cause the water jet speed to drop rapidly. The combination of low water jet speed and relatively large mass of water will produce sluggish acceleration.

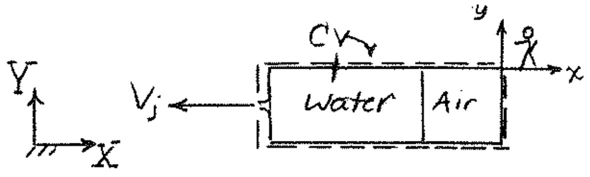
Increasing the initial volume fraction of air will reduce the expansion ratio, so higher pressure will be maintained longer in the tank and the water jet will maintain higher speed longer. This combined with the relatively small mass of water in the tank will produce rapid acceleration.

If the initial volume fraction of air is too large, all water will be expended before the air pressure is reduced significantly. In this situation, some of the stored energy of the air will be dissipated in a relatively ineffective air jet. Consequently, for any initial pressure in the tank, there is an optimum initial air fraction.

This problem cannot be solved in closed form because of the varying air pressure, mass flow rate, and mass of water in the tank;. it can only be solved numerically. One possible integration scheme is to increment time and solve for all properties of the system at each instant. The drawback to this scheme is that the water is unlikely to be exhausted at an even increment of time. A second scheme is to increment the volume of water remaining and solve for properties using the average flow rate during the interval. This scheme is outlined below.

Model the air/water jet-propelled "rocket" using the CV and coordinates shown.

First choose dimensions and mass of "rocket" to be simulated:



Input Data:

Jet diameter:	$D_j =$	0.003	m
Tank diameter:	$D_t =$	0.035	m
Tank length:	$L =$	0.1	m
Tank mass:	$M_t =$	0.01	kg
Polytropic exponent:	$n =$	1.4	---

Next choose initial conditions for the simulation (see sample calculations below):

Problem *4.173

Initial Conditions:

Air fraction in tank: $\alpha =$ 0.5 ---
Tank pressure: $p_0 =$ 200 kPa (gage)
Volume increment: $\Delta\alpha =$ 0.02 ---

Compute reference parameters:

Calculated Parameters:

Jet area: $A_j =$ 7.07E-06 m^2
Tank volume: $V_t =$ 9.62E-05 m^3
Initial air volume: $V_0 =$ 4.81E-05 m^3
Initial water mass: $M_0 =$ 0.0481 kg

(These are used in the spreadsheet below.)

Then decrease the water fraction in the tank by $\Delta\alpha$:

Calculated Results:

Water Fraction, V_w/V_t (---)	Gage Pressure, p (kPa)	Water Mass, M_w (kg)	Jet Speed, V_j (m/s)	Flow Rate, dm/dt (kg/s)	Time Interval, Δt (s)	Current Time, t (s)	"Rocket" Accel., a (m/s^2)	"Rocket" Speed, U (m/s)
0.50	200	0.0481	20.0	0.141	0	0	48.7	0
0.48	184	0.0461	19.2	0.135	0.0139	0.0139	47.5	0.668

The computation is made as follows:

(1) Decrease α by $\Delta\alpha$

(2) Compute p from $p = p_0\left(\frac{V_0}{V}\right)^n$

$$p = (200+101.325)\,kPa\left(\frac{0.50}{0.52}\right)^{1.4} - 101.325 = 183.9\ kPa\,(gage)$$

(3) Use Bernoulli to calculate jet speed

$$V_j = \sqrt{\frac{2\Delta p}{\rho}} = \left[2 \times 183.9\times10^3\,\frac{N}{m^2}\times\frac{m^3}{999\,kg}\times\frac{kg\cdot m}{N\cdot s^2}\right]^{\frac{1}{2}} = 19.10\ m/s^*$$

(4) Calculate water mass using α.

(5) Use conservation of mass to compute mass flow rate

$$\dot{m} = \rho V_j A_j = 999\,\frac{kg}{m^3}\times 19.10\,\frac{m}{s}\times 7.07\times10^{-6}\,m^2 = 0.1349\ kg/s$$

(6) Use the average mass flow rate during the interval to approximate Δt:

$$\Delta t = \frac{\Delta m}{dm/dt} = \frac{\Delta m}{\bar{\dot{m}}} = (0.0481-0.0461)\,kg \times \frac{s}{0.138\,kg} = 0.01449\ s^*$$

(7) Use momentum to compute acceleration (note $M = M_w + M_t$):

$$a_{rf_x} = \frac{\dot{m}V_j}{M} = 0.135\,\frac{kg}{s}\times 19.2\,\frac{m}{s}\times\frac{1}{0.0461+0.0100\,kg} = 46.2\ m/s^{2\,*}$$

(8) Finally: use average acceleration to get speed

$$U = U_0 + \bar{a}\Delta t = 0 + 48.1\,\frac{m}{s^2}\times 0.0139\,s = 0.669\ m/s^*$$

---

* Note effect of roundoff error.

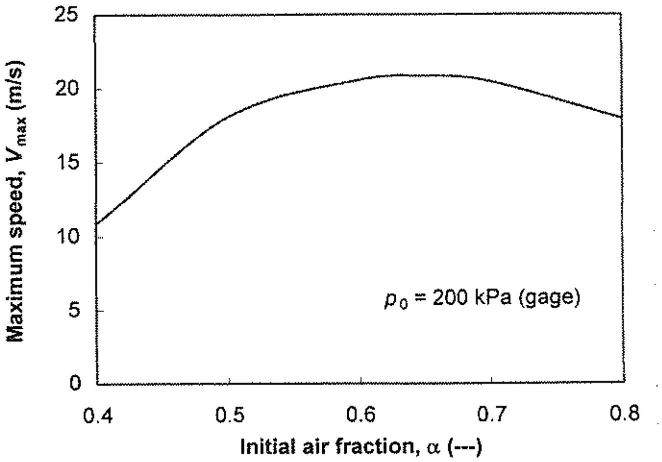
Problem *4.173

Repeat these calculations until water is depleted or air pressure falls to zero, as shown below:

Water Fraction, V_w/V_t (---)	Gage Pressure, p (kPa)	Water Mass, M_w (kg)	Jet Speed, V_j (m/s)	Flow Rate, dm/dt (kg/s)	Time Interval, Δt (s)	Current Time, t (s)	"Rocket" Accel., a (m/s^2)	"Rocket" Speed, U (m/s)
0.50	200	0.0481	20.0	0.141	0	0	48.7	0
0.48	184	0.0461	19.2	0.135	0.0139	0.0139	47.5	0.668
0.46	169	0.0442	18.4	0.130	0.0145	0.0284	45.2	1.34
0.44	156	0.0423	17.7	0.125	0.0151	0.0435	43.1	2.01
0.42	143	0.0404	16.9	0.120	0.0157	0.0592	41.2	2.67
0.40	132	0.0384	16.3	0.115	0.0164	0.0756	39.4	3.33
0.38	122	0.0365	15.6	0.110	0.0171	0.0927	37.8	3.99
0.36	112	0.0346	15.0	0.106	0.0178	0.110	36.2	4.65
0.34	103	0.0327	14.4	0.101	0.0186	0.129	34.8	5.31
0.32	94.6	0.0308	13.8	0.0972	0.0194	0.148	33.5	5.97
0.30	86.8	0.0288	13.2	0.0931	0.0202	0.169	32.2	6.63
0.28	79.5	0.0269	12.6	0.0891	0.0211	0.190	31.0	7.30
0.26	72.7	0.0250	12.1	0.0852	0.0221	0.212	29.9	7.97
0.24	66.3	0.0231	11.5	0.0814	0.0231	0.235	28.9	8.65
0.22	60.4	0.0211	11.0	0.0776	0.0242	0.259	27.9	9.34
0.20	54.7	0.0192	10.5	0.0739	0.0254	0.284	26.9	10.0
0.18	49.4	0.0173	9.95	0.0702	0.0267	0.311	26.0	10.7
0.16	44.4	0.0154	9.43	0.0666	0.0281	0.339	25.2	11.5
0.14	39.7	0.0135	8.92	0.0630	0.0297	0.369	24.3	12.2
0.12	35.2	0.0115	8.40	0.0593	0.0314	0.400	23.5	12.9
0.10	31.0	0.00961	7.88	0.0556	0.0334	0.434	22.7	13.7
0.08	27.0	0.00769	7.35	0.0519	0.0357	0.469	22.0	14.5
0.06	23.2	0.00577	6.81	0.0481	0.0384	0.508	21.2	15.3
0.04	19.6	0.00384	6.26	0.0442	0.0416	0.550	20.4	16.2
0.02	16.1	0.00192	5.68	0.0401	0.0456	0.595	19.5	17.1
0.00	12.9	0.0000	5.07	0.0358	0.0506	0.646	18.6	18.1

In this simulation, the water is depleted when $t \approx 0.65$ s; $V_{max} = 18.1$ m/s.

Varying the initial air fraction produces the following:



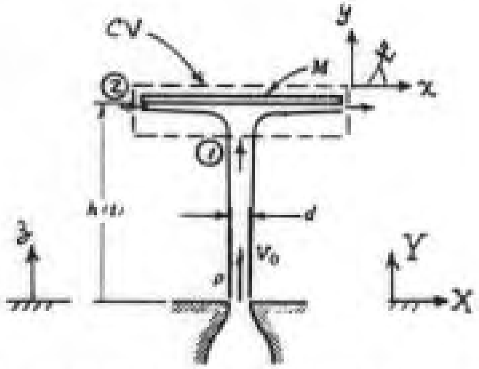
For this combination of parameters, a peak speed of about 20.8 m/s is attained with an initial air fraction of about 0.66.

Problem *4.174

[5] Part 1/2

Given: Vertical jet impinging on disk.

Disk is unconstrained vertically.



Find: (a) Differential equation for $h(t)$, if disk released from $H > h_0$, where h_0 is equilibrium height.

(b) Sketch $h(t)$ and explain.

Solution: Apply Bernoulli equation to jet, then y momentum equation to CV with linear acceleration.

Basic equations:

$$\cancel{\frac{p_0}{\rho}} + \frac{V_0^2}{2} + g\overset{=0}{\cancel{z_0}} = \cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + gz_1$$

$$\overset{=0(6)}{\cancel{F_{S_y}}} + F_{B_y} - \int_{CV} a_{rf_y}\rho\, d\forall = \overset{\approx 0(7)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v_{xyz}\rho\, d\forall + \int_{CS} v_{xyz}\rho\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions:
- (1) Steady flow
- (2) Incompressible flow
- (3) No friction
- (4) Flow along a streamline

} in jet

- (5) $p_1 = p_0 = p_{atm}$
- (6) No pressure force on CV, so $F_{S_y} = 0$
- (7) Neglect mass of liquid in CV and $v \approx 0$ in CV
- (8) Uniform flow at each section
- (9) Measure velocities relative to CV

From momentum

$$-(M + \overset{\approx 0(7)}{\cancel{m_{cv}}})g - a_{rf_y}(M + \overset{\approx 0(7)}{\cancel{m_{cv}}}) = v_1\{-|\rho(V_1 - U)A_1|\} + \cancel{v_2}\{\dot{m}_2\}$$

$$v_1 = V_1 - U \qquad\qquad v_2 \approx 0$$

With $a_{rf_y} = \frac{d^2h}{dt^2}$, $U = \frac{dh}{dt}$, then

$$-Mg - M\frac{d^2h}{dt^2} = -\rho\left(V_1 - \frac{dh}{dt}\right)^2 A_1$$

But from Bernoulli, $\frac{V_1^2}{2} = \frac{V_0^2}{2} - gz_1$, so $V_1 = \sqrt{V_0^2 - 2gh}$, since $z_1 = h(t)$

Also from continuity, $V_1A_1 = V_0A_0$, so $A_1 = A_0V_0/V_1$. Substituting

$$\frac{d^2h}{dt^2} = \rho\left(\sqrt{V_0^2 - 2gh} - \frac{dh}{dt}\right)^2 \frac{A_0V_0}{M\sqrt{V_0^2 - 2gh}} - g$$

$\ddot{h}(t)$

At equilibrium height, $h = h_0$, $\frac{dh}{dt} = 0$, and $\frac{d^2h}{dt^2} = 0$. Then

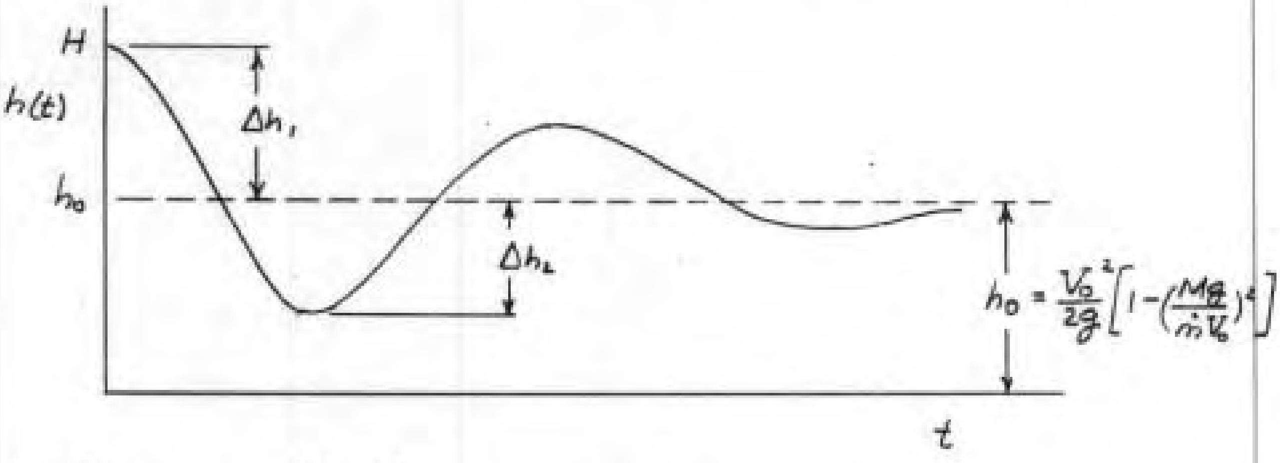
$$\rho\sqrt{V_0^2 - 2gh_0}\,A_0V_0 - Mg = 0$$

Thus $$V_0^2 - 2gh_0 = \left(\frac{Mg}{\rho V_0 A_0}\right)^2$$

This may be solved to obtain

$$h_0 = \frac{V_0^2}{2g}\left[1-\left(\frac{Mg}{\rho V_0^2 A_0}\right)^2\right] = \frac{V_0^2}{2g}\left[1-\left(\frac{Mg}{\dot{m} V_0}\right)^2\right]$$

When released, $H > h_0$, and $dh/dt = 0$. Because the equation for d^2h/dt^2 is nonlinear, oscillations will occur. The expected behavior is sketched below:

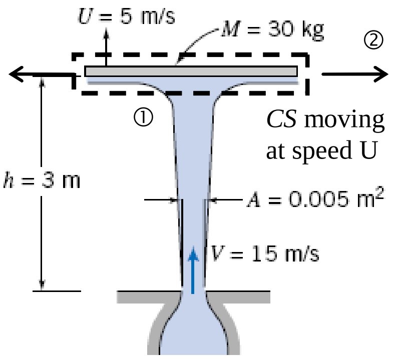


Notes: (1) Expect oscillations
(2) $\Delta h_3 < \Delta h_2 < \Delta h_1$ due to nonlinear equation

Problem *4.175

[5]

***4.175** Consider the configuration of the vertical jet impinging on a horizontal disk shown in Problem 4.153. Assume the disk is released from rest at an initial height of 2 m above the jet exit plane. Solve for the subsequent motion of this disk. Identify the steady-state height of the disk.



Given: Water jet striking moving disk

Find: Motion of disk; steady state height

Solution:

Basic equations: Bernoulli; Momentum flux in z direction (treated as upwards) for linear accelerating CV

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_{S_z} + F_{B_z} - \int_{CV} a_{rf_z}\,\rho\, d\mathcal{V} = \frac{\partial}{\partial t}\int_{CV} w_{xyz}\,\rho\, d\mathcal{V} + \int_{CS} w_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Atmospheric pressure 4) Uniform flow 5) velocities wrt CV (All in jet)

The Bernoulli equation becomes $\frac{V_0^{\,2}}{2} + g\cdot 0 = \frac{V_1^{\,2}}{2} + g\cdot h$ $\qquad V_1 = \sqrt{V_0^{\,2} - 2\cdot g\cdot h}$ (1)

$$V_1 = \sqrt{\left(15\cdot\frac{m}{s}\right)^2 + 2\times 9.81\cdot\frac{m}{s^2}\cdot(0-3)\cdot m} \qquad V_1 = 12.9\,\frac{m}{s}$$

The momentum equation becomes

$$-M\cdot g - M\cdot a_{rfz} = w_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + w_2\cdot\left(\rho\cdot V_2\cdot A_2\right) = \left(V_1 - U\right)\cdot\left[-\rho\cdot\left(V_1 - U\right)\cdot A_1\right] + 0$$

With $a_{rfz} = \frac{d^2h}{dt^2}$ and $U = \frac{dh}{dt}$ we get $-M\cdot g - M\cdot\frac{d^2h}{dt^2} = -\rho\cdot\left(V_1 - \frac{dh}{dt}\right)^2\cdot A_1$

Using Eq 1, and from continuity $V_1\cdot A_1 = V_0\cdot A_0$

$$\frac{d^2h}{dt^2} = \left(\sqrt{V_0^{\,2} - 2\cdot g\cdot h} - \frac{dh}{dt}\right)^2\cdot\frac{\rho\cdot A_0\cdot V_0}{M\cdot\sqrt{V_0^{\,2} - 2\cdot g\cdot h}} - g \qquad (2)$$

This must be solved numerically! One approach is to use Euler's method (see the *Excel* solution)

At equilibrium $h = h_0$ $\quad \frac{dh}{dt} = 0 \quad \frac{d^2h}{dt^2} = 0$ so

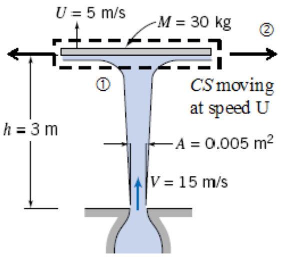
$$\sqrt{\left(V_0^{\,2} - 2\cdot g\cdot h_0\right)}\cdot\rho\cdot A_0\cdot V_0 - M\cdot g = 0 \qquad \text{and} \qquad h_0 = \frac{V_0^{\,2}}{2\cdot g}\cdot\left[1 - \left(\frac{M\cdot g}{\rho\cdot V_0^{\,2}\cdot A_0}\right)^2\right]$$

Hence

$$h_0 = \frac{1}{2}\times\left(15\cdot\frac{m}{s}\right)^2\times\frac{s^2}{9.81\cdot m}\times\left[1 - \left[30\cdot kg\times 9.81\cdot\frac{m}{s^2}\times\frac{m^3}{1000\cdot kg}\times\left(\frac{s}{15\cdot m}\right)^2\times\frac{1}{.005\cdot m^2}\right]^2\right] \qquad h_0 = 10.7\,m$$

Problem *4.175 (In Excel)

*4.175 Consider the configuration of the vertical jet impinging on a horizontal disk shown in Problem 4.153. Assume the disk is released from rest at an initial height of 2 m above the jet exit plane. Solve for the subsequent motion of this disk. Identify the steady-state height of the disk.



$\Delta t =$ 0.05 s

$A_0 =$ 0.005 m^2

$g =$ 9.81 $\mathrm{m/s^2}$

$V =$ 15 m/s

$M =$ 30 kg

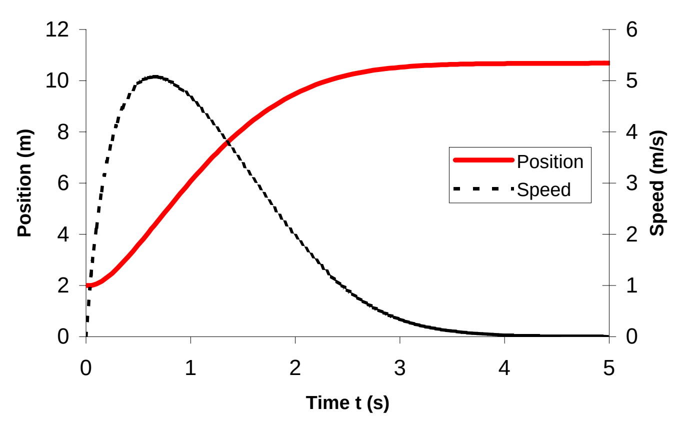
$\rho =$ 1000 $\mathrm{kg/m^3}$

$$h_{i+1} = h_i + \Delta t \cdot \left.\frac{dh}{dt}\right|_i$$

$$\left(\frac{dh}{dt}\right)_{i+1} = \left(\frac{dh}{dt}\right)_i + \Delta t \cdot \left.\frac{d^2h}{dt^2}\right|_i$$

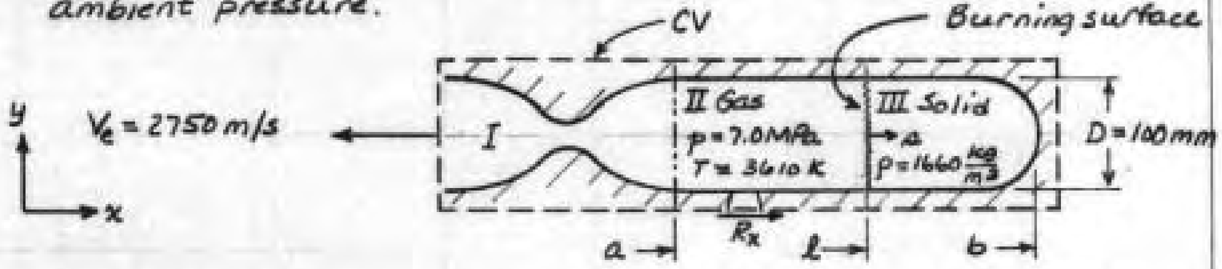
$$\frac{d^2h}{dt^2} = \left(\sqrt{V_0^2 - 2\cdot g\cdot h} - \frac{dh}{dt}\right)^2 \cdot \frac{\rho\cdot A_0\cdot V_0}{M\cdot\sqrt{V_0^2 - 2\cdot g\cdot h}} - g$$

t **(s)**	***h*** **(m)**	***dh/dt*** **(m/s)**	d^2h/dt^2 **(m/s²)**
0.000	2.000	0.000	24.263
0.050	2.000	1.213	18.468
0.100	2.061	2.137	14.311
0.150	2.167	2.852	11.206
0.200	2.310	3.412	8.811
0.250	2.481	3.853	6.917
0.300	2.673	4.199	5.391
0.350	2.883	4.468	4.140
0.400	3.107	4.675	3.100
0.450	3.340	4.830	2.227
0.500	3.582	4.942	1.486
0.550	3.829	5.016	0.854
0.600	4.080	5.059	0.309
0.650	4.333	5.074	-0.161
0.700	4.587	5.066	-0.570
0.750	4.840	5.038	-0.926
0.800	5.092	4.991	-1.236
0.850	5.341	4.930	-1.507
0.900	5.588	4.854	-1.744
0.950	5.830	4.767	-1.951
1.000	6.069	4.669	-2.130
1.050	6.302	4.563	-2.286
1.100	6.530	4.449	-2.420
1.150	6.753	4.328	-2.535
1.200	6.969	4.201	-2.631
1.250	7.179	4.069	-2.711
1.300	7.383	3.934	-2.776
1.350	7.579	3.795	-2.826
1.400	7.769	3.654	-2.864
1.450	7.952	3.510	-2.889
1.500	8.127	3.366	-2.902
1.550	8.296	3.221	-2.904
1.600	8.457	3.076	-2.896
1.650	8.611	2.931	-2.878
1.700	8.757	2.787	-2.850
1.750	8.896	2.645	-2.814
1.800	9.029	2.504	-2.769
1.850	9.154	2.365	-2.716
1.900	9.272	2.230	-2.655
1.950	9.384	2.097	-2.588
2.000	9.488	1.967	-2.514



Problem 4.176

Given: Small solid fuel rocket motor on test stand. The fuel burns uniformly at $r = 12.7$ mm/s. Exhaust gases leave at ambient pressure.



Treat combustion products as ideal gas with molecular mass, $M_m = 25.8$.

Find: (a) Evaluate rate of change of mass and of linear momentum within rocket motor.
(b) Express rate of change of momentum as a percentage of thrust.

Solution: Apply continuity and x component of momentum equations using fixed CV shown.

Basic equations: $$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(2)}{\cancel{F_{B_x}}} = \frac{\partial}{\partial t}\int_{CV} u \rho\, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) No net pressure force; $F_{S_x} = R_x$
(2) $F_{B_x} = 0$
(3) All properties constant at each point, except at surface where combustion takes place
(4) Uniform flow at exit section

The continuity equation becomes

$$0 = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{I} \rho\, d\forall + \frac{\partial}{\partial t}\int_a^{\ell} \rho_g A\, dx + \frac{\partial}{\partial t}\int_{\ell}^{b} \rho_f A\, dx + \{|\rho_e V_e A_e|\}$$

$$0 = \frac{\partial}{\partial t}[\rho_g A(\ell - a)] + \frac{\partial}{\partial t}[\rho_f A(b - \ell)] + \dot{m}_e = (\rho_g - \rho_f) A \frac{d\ell}{dt} + \dot{m}_e$$

or

$$\dot{m}_e = (\rho_f - \rho_g) A \frac{d\ell}{dt} = (\rho_f - \rho_g) A r$$

For an ideal gas,

$$\rho_g = \frac{p_g}{R T_g} = \frac{p_g M_m}{R_u T_g} = 7.0 \times 10^6 \frac{N}{m^2} \times \frac{25.8\ kg}{mol} \times \frac{mol \cdot K}{8314\ N \cdot m} \times \frac{1}{3610\ K} = 6.02\ kg/m^3$$

so

$$\dot{m}_e = (1660 - 6)\frac{kg}{m^3} \times \frac{\pi}{4}(0.1)^2 m^2 \times 0.0127 \frac{m}{s} = 0.165\ kg/s$$

Mass flow is out, so $$\frac{\partial M_{cv}}{\partial t} = -0.165\ kg/s$$

$\frac{\partial M_{cv}}{\partial t}$

From the momentum equation,

Problem 4.176

$$R_x = \overset{\approx 0(3)}{\cancel{\frac{\partial}{\partial t}\int_I u\rho\, dV}} + \frac{\partial}{\partial t}\int_a^l u_g \rho_g A\, dx + \frac{\partial}{\partial t}\int_l^b \overset{\approx 0}{\cancel{u_f}}\rho_f A\, dx + u_e\{|\rho_e V_e A_e|\}$$

$$= \frac{\partial}{\partial t}[u_g \rho_g A(l-a)] + u_e \dot{m}_e \quad ; \; u_g = -V_g \text{ and } u_e = -V_e$$

$$R_x = -\rho_g V_g A \frac{dl}{dt} - V_e \dot{m}_e = -\rho_g V_g A s - V_e \dot{m}_e$$

But from continuity, $\rho_g V_g A = \dot{m}_e$, since no mass accumulates in region I of the CV. Thus

$$R_x = -\dot{m}_e(V_e + s)$$

R_x is the force on the CV. The thrust is

$$K_x = \text{Thrust} = -R_x = \dot{m}_e(V_e + s)$$

$$K_x = 0.165\,\frac{kg}{s}\,(2750 + 0.0127)\frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m} = 454\ N$$

The rate of change of linear momentum within the CV is

$$\frac{\partial P_{x_{cv}}}{\partial t} = -\dot{m}_e s = -0.165\,\frac{kg}{s} \times 0.0127\,\frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m} = -2.10\ mN$$

$\frac{\partial P_{x_{cv}}}{\partial t}$

The ratio of rate of change of linear momentum to thrust is

$$\frac{\frac{\partial P_{x_{cv}}}{\partial t}}{K_x} = \frac{-\dot{m}_e s}{\dot{m}_e(V_e + s)} = -\frac{s}{(V_e + s)} = -\frac{0.0127\,\frac{m}{s}}{(2750 + 0.0127)\frac{m}{s}} = -4.62 \times 10^{-6}$$

or

$$\frac{\frac{\partial P_{x_{cv}}}{\partial t}}{K_x} = -4.62 \times 10^{-4}\ \text{percent}$$

Ratio

{ Neglecting the unsteady momentum term in the analysis of this rocket motor would cause an error of approximately 1 part in 217,000. The assumption that $\partial P_{x_{cv}}/\partial t \simeq 0$ is certainly justified for engineering work. }

Problem *4.177

Open-Ended Problem Statement: The capability of the Aircraft Landing Loads and Traction Facility at NASA's Langley Research Center is to be upgraded. The facility consists of a rail-mounted carriage propelled by a water jet issuing from a pressurized tank. (The setup is identical in concept to the hydraulic catapult of Problem 4.133 The 49,000 kg carriage must accelerate to 220 knots in 122 m. (The vane turning angle is 170°.) Identify a range of water jet sizes and speeds needed to accomplish this performance. Specify the recommended operating pressure for the water jet system and determine the shape and estimated size of tankage to contain the pressurized water.

Discussion: The analysis of Example 4.11 forms the basis for the solution outlined below. Use a control volume attached to and moving with the carriage to analyze the motion. Neglect aerodynamic and rolling resistance to obtain a best-case solution. Solve the resulting differential equation of motion for carriage speed and position as functions of time, and for speed as a function of position along the rails.

Computing equations are summarized and results tabulated below. As shown in Example 4.11, analysis of the carriage motion results in the differential equation

$$\frac{dU}{dt} = \frac{\rho (V_j - U)^2 (1-\cos\theta)}{M} \qquad (1)$$

Integrating with respect to time gives carriage speed versus time

$$U = V_j \frac{bt}{1+bt} \qquad (2)$$

where parameter *b* is

$$b = \frac{\rho V_j A_j (1-\cos\theta)}{M} \qquad (3)$$

Equation 2 is integrated to obtain carriage position versus time

$$x = V_j \left[t - \frac{\ln(1+bt)}{b} \right] \qquad (4)$$

Substitute *dU/dt* = *UdU/dx* and integrate Eq. 1 for distance traveled versus carriage speed

$$x = \frac{V_j}{b} \left[\ln(1 - U/V_j) + \frac{1}{1 - U/V_j} - 1 \right] \qquad (5)$$

Relate jet speed to water tank pressure using the Bernoulli equation

$$V_j = \sqrt{2\Delta p/\rho} \qquad (6)$$

The required volume of water is computed as follows:

1. Assume a range of tank pressures.
2. Compute the jet speed corresponding to each tank pressure from Eq. 6.
3. Solve for parameter *b* from Eq. 5 using the known maximum speed and specified distance.
4. Obtain jet area from Eq. 3.
5. Compute the time required to accelerate the carriage from Eq. 2.
6. Calculate jet diameter from jet area.
7. Compute the required volume of water from the product of mass flow rate and acceleration time.

Problem *4.177

The optimum operating pressure requires the least costly tankage. (Assume the most efficient spherical shape for pressurized tankage and constant tank pressure during acceleration.) Tankage calculations are organized as follows:

1. Obtain tank diameter from tank volume.
2. Calculate wall thickness from a force balance on the thin wall of the tank.
3. Calculate steel volume from tank surface area and wall thickness.
4. Assume steel cost is proportional to steel volume.

Sample calculation: assume $p = 6000$ psig

$$V_j = \left[2 \times 6000 \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{1.94\ \text{slug}} \times 144 \frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{\frac{1}{2}} = 944\ \text{ft/s} \ ; \ \frac{U}{V_j} = \frac{371}{944} = 0.393$$

$$b = 944 \frac{\text{ft}}{\text{s}} \times \frac{1}{400\ \text{ft}} \left[\ln(1-0.393) + \frac{1}{1-0.393} - 1\right] = 0.350\ \text{s}^{-1}$$

$$A_j = \frac{bM}{\rho V_j (1-\cos\theta)} = \frac{0.350}{\text{s}} \times 3350\ \text{slug} \times \frac{\text{ft}^3}{1.94\ \text{slug}} \times \frac{\text{s}}{944\ \text{ft}} \times \frac{1}{(1-\cos 170°)} = 0.323\ \text{ft}^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \left[\frac{4}{\pi} \times 0.323\ \text{ft}^2 \times 144 \frac{\text{in.}^2}{\text{ft}^2}\right]^{\frac{1}{2}} = 7.69\ \text{in.}$$

$$t = \frac{1}{b}\left(\frac{U/V_j}{1-U/V_j}\right) = \frac{\text{s}}{0.350} \times \frac{0.393}{1-0.393} = 1.85\ \text{s}$$

$$Q = V_j A = 944 \frac{\text{ft}}{\text{s}} \times 0.323\ \text{ft}^2 \times 7.48 \frac{\text{gal}}{\text{ft}^3} = 2280\ \text{gal/s}$$

$$\forall = Qt = 2280 \frac{\text{gal}}{\text{s}} \times 1.85\ \text{s} = 4220\ \text{gal}$$

$$D = (6\forall/\pi)^{1/3} = \left(\frac{6}{\pi} \times 4220\ \text{gal} \times \frac{\text{ft}^3}{7.48\ \text{gal}}\right)^{1/3} = 10.3\ \text{ft}$$

$$\Delta p \frac{\pi D^2}{4} = \pi D t \ ; \ t = \frac{pD}{4\sigma} = \frac{1}{4} \times 6000 \frac{\text{lbf}}{\text{in.}^2} \times 10.3\ \text{ft} \times \frac{\text{in.}^2}{40{,}000\ \text{lbf}} \times 12 \frac{\text{in.}}{\text{ft}} = 4.64\ \text{in.}$$

$$\forall_{steel} = \pi D^2 t = \pi \times (10.3)^2\ \text{ft}^2 \times 4.64\ \text{in.} \times \frac{\text{ft}}{12\ \text{in.}} = 129\ \text{ft}^3$$

Discussion: The results show the steel volume plummets as tank pressure is raised, with a broad minimum between 3,000 and 4,000 psig.

Problem *4.177

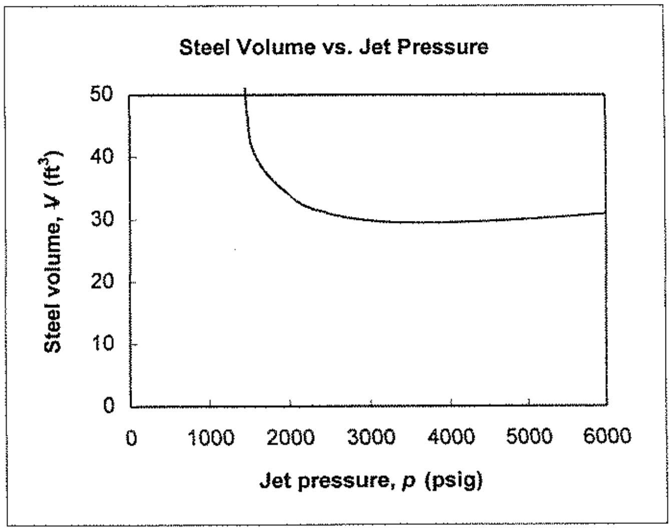
Input Data:

M =	49000	kg	3355	slug
U =	220	kt	371.3	ft/s
X =	122	m	400.3	ft
θ =	170	degrees		

Calculated Results:

Jet Pressure (psig)	Jet Speed (ft/s)	Parameter b (s^{-1})	Jet Area (ft^2)	Jet Diameter (in.)	Flow Rate (gal/s)	Flow Time (s)	Water Volume (gal)
6000	944	0.351	0.324	7.70	2285	1.85	4227
5500	904	0.380	0.367	8.20	2477	1.84	4546
5000	862	0.417	0.421	8.79	2715	1.82	4936
4500	817	0.463	0.494	9.51	3019	1.80	5426
4000	771	0.525	0.593	10.4	3419	1.77	6061
3500	721	0.610	0.737	11.6	3973	1.74	6924
3000	667	0.736	0.961	13.3	4797	1.70	8174
2500	609	0.944	1.35	15.7	6155	1.65	10172
2000	545	1.35	2.17	19.9	8830	1.58	13942
1500	472	2.53	4.67	29.3	16490	1.46	24061
1000	385	22.4	50.6	96.3	145835	1.19	173113

Jet Pressure (psig)	Water Volume (gal)	Tank Diameter (ft)	Wall Thickness (in.)	Steel Volume (ft^3)	Steel Mass (ton)
6000	4227	10.3	4.6	127.2	30.9
5500	4546	10.5	4.3	125.4	30.5
5000	4936	10.8	4.1	123.7	30.1
4500	5426	11.1	3.8	122.4	29.8
4000	6061	11.6	3.5	121.5	29.6
3500	6924	12.1	3.2	121.5	29.6
3000	8174	12.8	2.9	122.9	29.9
2500	10172	13.7	2.6	127.5	31.0
2000	13942	15.3	2.3	139.8	34.0
1500	24061	18.3	2.1	180.9	44.0
1000	173113	35.4	2.7	867.9	211.2



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Problem *4.178

Open-Ended Problem Statement: A classroom demonstration of linear momentum is planned, using a water-jet propulsion system for a cart traveling on a horizontal linear air track. The track is 5 m long, and the cart mass is 155 g. The objective of the design is to obtain the best performance for the cart, using 1 L of water contained in an open cylindrical tank made from plastic sheet with density of 0.0819 g/cm^2. For stability, the maximum height of the water tank cannot exceed 0.5 m. The diameter of the smoothly rounded water jet may not exceed 10 percent of the tank diameter. Determine the best dimensions for the tank and the water jet by modeling the system performance. Plot acceleration, velocity, and distance as functions of time. Find the optimum dimensions of the water tank and jet opening from the tank. Discuss the limitations on your analysis. Discuss how the assumptions affect the predicted performance of the cart. Would the actual performance of the cart be better or worse than predicted? Why? What factors account for the difference(s)?

Discussion: This solution is an extension of Problem *4.179. The analyses for tank level, acceleration, and velocity are identical; please refer to the solution for Problem *4.179 for equations describing each of these variables as functions of time.

One new feature of this problem is computation of distance traveled. Equation 7 of Problem *4.179 could be integrated in closed form to provide an equation for distance traveled as a function of time. However, the integral would be messy, and it would provide little insight into the dependence on key parameters. Consequently, a numerical analysis has been chosen in this problem. The results are presented in the plots and spreadsheet on the next page.

We have chosen to define velocity as the output to be maximized.

A second new feature of this problem is the geometric constraints: the maximum track length is 5 m. Intuitively jet diameter should be chosen as the largest possible fraction of tank diameter for optimum performance. Using the spreadsheet to vary $\beta = d/D$ verifies that this is the case. Therefore we have used the maximum allowable ratio, $\beta = 0.1$, for all computations.

Tank height should be a factor in performance. Intuition suggests that increasing tank height should improve performance. Using the spreadsheet shows a very weak dependence on tank height. Performance is best at smaller tank heights, corresponding to the minimum tank mass.

As tank height is decreased, diameter increases because tank volume is held constant. Since diameter ratio is constant, then jet diameter increases with decreasing tank height. This effect almost overshadows the effect of tank height.

The principal limitations on the analysis are the assumptions of negligible motion resistance and no slope to the free surface of water in the tank. Actual performance of the cart would likely be less than predicted because of motion resistance.

Distance is modeled as

$$x_{i+1} = x_i + U_i \Delta t + \frac{1}{2} a_{x,i} \Delta t^2$$

The accuracy of this model for position is consistent with the accuracy of modeling the water-jet propulsion system.

Problem *4.178

Analysis of Cart Propelled by Gravity-Driven Water Jet:

Input Data:

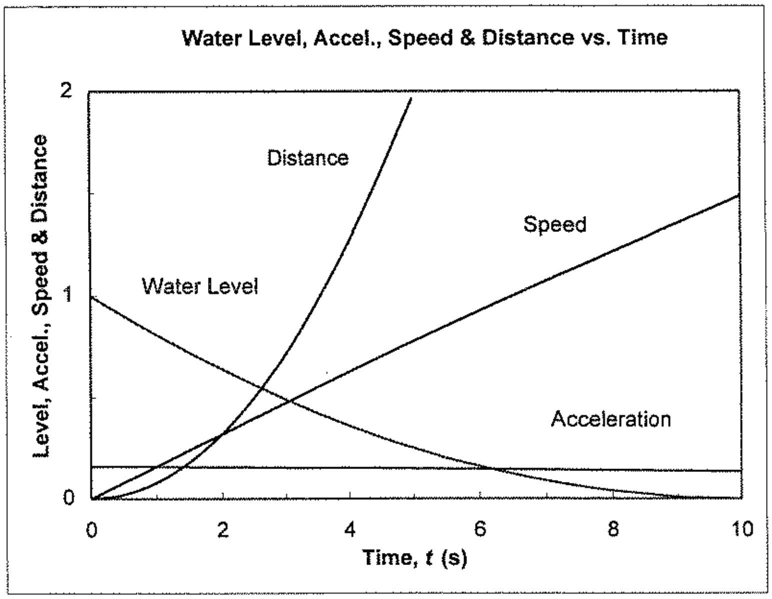
g =	9.81	$\mathrm{m/s^2}$	Acceleration of gravity
H =	500	mm	Height of tank
M_c =	0.155	kg	Mass of cart
V =	1.00	L	Tank volume
β =	0.100	(---)	Ratio of jet diameter to tank diameter
ρ =	999	$\mathrm{kg/m^3}$	Density of water
ρ'' =	0.819	$\mathrm{kg/m^2}$	(Area) density of tank material

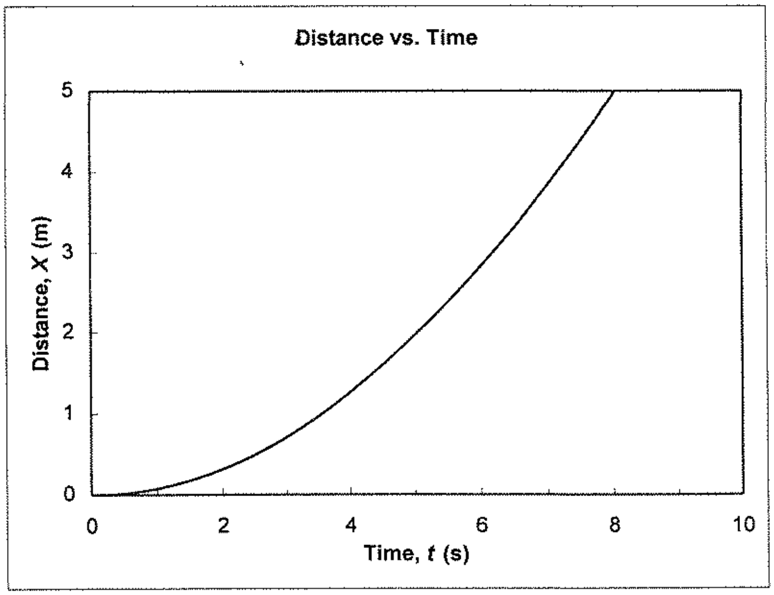
Calculated Parameters:

a =	0.471	(---)	(a^2 =) Ratio of mass of tank to initial mass of water
b =	0.0313	$\mathrm{s^{-1}}$	Geometric parameter of solution
d =	5.05	mm	Diameter of water jet
D =	50.5	mm	Diameter of tank
M_0 =	1.00	kg	Initial mass of water in tank
M_p =	0.0666	kg	Mass of plastic in tank
M_t =	0.222	kg	Mass of plastic tank plus cart

Calculated Results:

Time, t (s)	Level, y/H (---)	Accel., a_x ($\mathrm{m/s^2}$)	Velocity, U (m/s)	Position, X (m)
0	1	0.161	0	0
0.5	0.903	0.160	0.080	0.0201
1.0	0.810	0.159	0.160	0.080
1.5	0.723	0.158	0.239	0.180
2.0	0.640	0.157	0.317	0.319
2.5	0.563	0.156	0.395	0.497
3.0	0.490	0.154	0.473	0.714
3.5	0.423	0.153	0.550	0.97
4.0	0.360	0.152	0.626	1.26
4.5	0.303	0.151	0.702	1.60
5.0	0.250	0.150	0.777	1.97
5.5	0.203	0.148	0.852	2.37
6.0	0.160	0.147	0.925	2.82
6.5	0.123	0.145	1.00	3.30
7.0	0.0900	0.144	1.07	3.82
7.5	0.0625	0.142	1.14	4.37
8.0	0.0400	0.141	1.21	4.96
8.03	0.0388	0.141	1.22	5.00
9.0	0.0100	0.137	1.35	
9.5	0.0025	0.135	1.42	
10.0	0.0000	0.133	1.49	





Problem *4.179

Open-Ended Problem Statement: Analyze the design and optimize the performance of a cart propelled along a horizontal track by a water jet that issues under gravity from an open cylindrical tank carried on board the cart. (A water-jet-propelled cart is shown in the diagram for Proble 4.137.) Neglect any change in slope of the liquid free surface in the tank during acceleration. Analyze the motion of the cart along a horizontal track, assuming it starts from rest and begins to accelerate when water starts to flow from the jet. Derive algebraic equations or solve numerically for the acceleration and speed of the cart as functions of time. Present results as plots of acceleration and speed versus time, neglecting the mass of the tank. Determine the dimensions of a tank of minimum mass required to accelerate the cart from rest along a horizontal track to a specified speed in a specified time interval.

Discussion: This problem solution consists of two parts. The first is to analyze the acceleration and velocity of a cart propelled by a gravity-driven water jet. The second is to optimize the dimensions of the cart and jet to accelerate to a specified speed in a specified time interval.

To analyze the problem, apply conservation of mass and the Bernoulli equation to the draining of the tank, then apply the x component of the momentum equation for a control volume to analyze the resulting linear acceleration. A representative plot of the results is presented below.

To optimize the performance of the water-jet-propelled cart, manipulate the solution dimensions until the best performance is attained.

Input Data:

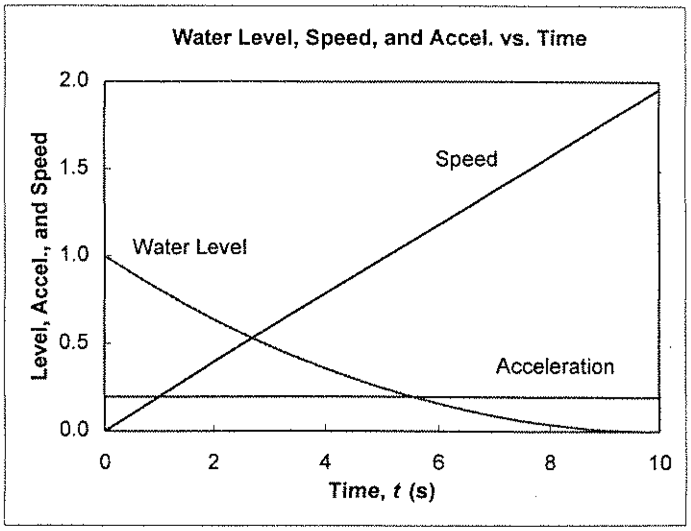
$d =$	10	mm	Diameter of water jet
$D =$	100	mm	Diameter of tank
$g =$	9.81	ft/s^2	Acceleration of gravity
$H =$	150	mm	Height of tank
$M_t =$	0.001	kg	Mass of tank
$\rho =$	999	kg/m^3	Density of water

Calculated Parameters:

$a =$	0.029	(---)	($a^2 =$) Ratio of mass of tank to initial mass of water
$b =$	0.0572	s^{-1}	Geometric parameter of solution
$M_0 =$	1.18	kg	Initial mass of water in tank
$\beta =$	0.1	(---)	Ratio of jet diameter to tank diameter

Calculated Results:

Time, t (s)	Level Ratio, y/H (---)	Accel., a_x (m/s^2)	Velocity, U (m/s)
0	1	0.196	0
1	0.810	0.196	0.196
2	0.640	0.196	0.392
3	0.490	0.196	0.588
4	0.360	0.196	0.784
5	0.250	0.196	0.980
6	0.160	0.196	1.176
7	0.0900	0.196	1.37
8	0.0400	0.196	1.57
9	0.0100	0.196	1.76
10	0	0.195	1.96



Problem *4.179

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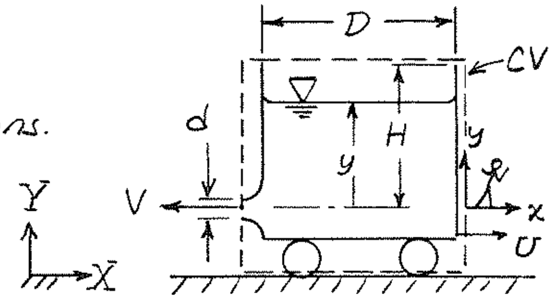
Given: Cart, propelled by water jet, accelerates along horizontal track.

Find: (a) Analyze motion, derive algebraic equations for acceleration and speed of cart as functions of time
(b) Plot acceleration and speed vs. time.

Solution: Apply conservation of mass, Bernoulli, and momentum equations.



M_t = mass of tank, cart

$$\beta = \frac{d}{D}$$

Basic equations:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\frac{\cancel{p_j}}{\rho} + \frac{V_j^2}{2} + g\overset{=0(8)}{\cancel{y_j}} = \frac{\cancel{p}}{\rho} + \frac{V^2}{2} + gy$$

$$\overset{=0(9)}{\cancel{F_{S_x}}} + \overset{=0(10)}{\cancel{F_{B_x}}} - \int_{CV} a_{rf_x} \rho\, d\forall = \overset{\approx 0(11)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u \rho\, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Uniform flow from exit jet (2) Neglect air in CV

$$0 = \frac{\partial}{\partial t}(\rho A_t y) + \{+|\rho V_j A_j|\} = \rho A_t \frac{dy}{dt} + \rho V_j A_j = -\rho A_t V + \rho V_j A_j \qquad (1)$$

Thus $V = V_j \frac{A_j}{A_t} = V_j \left(\frac{d}{D}\right)^2 = \beta^2 V_j$ (2)

(3) No slope to free surface (given)
(4) Quasi-steady flow
(5) Frictionless flow
(6) Incompressible flow
(7) Flow along a streamline
(8) $p = p_j = p_{atm}$
(9) $y_j = 0$

From Bernoulli, $\frac{V_j^2}{2} = \frac{V^2}{2} + gy$ or $V_j^2 - V^2 = 2gy$

Substituting from (2), $V_j^2 - \beta^4 V_j^2 = V_j^2(1-\beta^4) = 2gy$; $V_j^2 = \frac{2gy}{(1-\beta^4)}$ (3)

Substituting into (1), $\frac{dy}{dt} = -\beta^2 V_j = -\beta^2 \frac{\sqrt{2gy}}{(1-\beta^4)}$ or $\frac{dy}{y^{1/2}} = -\frac{\beta^2\sqrt{2g}}{1-\beta^4}\,dt$

Integrating, $2y^{1/2}\Big]_{y_0}^{y} = -\frac{\beta^2\sqrt{2g}}{(1-\beta^4)}\,t$ or $y^{1/2} - y_0^{1/2} = -\frac{\beta^2\sqrt{2g}}{2(1-\beta^4)}\,t$

Thus $\left(\frac{y}{y_0}\right)^{1/2} = 1 - \left[\frac{g\beta^4}{2y_0(1-\beta^4)}\right]^{1/2} t = 1 - bt$; $b = \left[\frac{g\beta^4}{2y_0(1-\beta^4)}\right]^{1/2}$ (4)

From momentum (10) $F_{Sx} = 0$; no resistance
(11) $F_{Bx} = 0$; horizontal motion
(12) $u \approx 0$ in CV, so $\partial/\partial t \approx 0$

Then

$$-a_{rfx} M(t) = u_j \{+|\rho V_j A_j|\} = -\rho V_j^2 A_j \qquad (5)$$

$$a_{rfx} = \frac{dU}{dt} \qquad u_j = -V_j$$

But from (4), $M(t) = M_t + \rho A_t y = M_t + \rho A_t y_0 (1-bt)^2$

From (3), $V_j^2 = \frac{2gy}{1-\beta^4} = \frac{2g}{1-\beta^4} y_0 (1-bt)^2$

Substituting into (5)

$$\frac{dU}{dt}\left[M_t + \rho A_t y_0 (1-bt)^2\right] = \rho A_j \frac{2g}{1-\beta^4} y_0 (1-bt)^2 = \rho A_t y_0 \frac{2g\beta^2}{1-\beta^4}(1-bt)^2$$

Define M_0 = initial mass of water $= \rho A_t y_0$. Then

$$\frac{dU}{dt}\left[M_t + M_0 (1-bt)^2\right] = M_0 \frac{2g\beta^2}{1-\beta^4}(1-bt)^2$$

or

$$\frac{dU}{dt} = \frac{2g\beta^2}{1-\beta^4} \frac{M_0 (1-bt)^2}{M_t + M_0 (1-bt)^2} \qquad (6) \qquad \frac{dU}{dt}(t)$$

To integrate, let $r = 1-bt$, $dr = -b\,dt$, and $a^2 = M_t/M_0$. Then

$$U = \int_0^U dU = \frac{2g\beta^2}{1-\beta^4}\left(-\frac{1}{b}\right)\int_0^t \frac{r^2}{a^2+r^2}\,dr = -\frac{2g\beta^2}{1-\beta^4}\frac{1}{b}\left[r - a\tan^{-1}\left(\frac{r}{a}\right)\right]_0^t$$

$$= -\frac{2g\beta^2}{1-\beta^4}\frac{1}{b}\left[(1-bt) - a\tan^{-1}\left(\frac{1-bt}{a}\right)\right]_0^t$$

$$U = -\frac{2g\beta^2}{1-\beta^4}\frac{1}{b}\left[(1-bt) - a\tan^{-1}\left(\frac{1-bt}{a}\right) - 1 + a\tan^{-1}\left(\frac{1}{a}\right)\right]$$

Simplifying, then

$$U = \frac{2g\beta^2}{1-\beta^4}\left\{t + \frac{a}{b}\left[\tan^{-1}\left(\frac{1-bt}{a}\right) - \tan^{-1}\left(\frac{1}{a}\right)\right]\right\} \qquad (7) \qquad U(t)$$

$$a^2 = \frac{M_t}{M_0}\ ; \quad b = \left[\frac{g\beta^4}{2y_0(1-\beta^4)}\right]^{1/2}$$

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Problem *4.179

[5] Part 4/4

Given: Cart, propelled by water jet, accelerating on horizontal track.

$$\frac{dU}{dt} = \frac{2g\beta^2}{1-\beta^4}\,\frac{(1-bt)^2}{a^2+(1-bt)^2} \qquad (1)$$

$$U(t) = \frac{2g\beta^2}{1-\beta^4}\left\{t + \frac{a}{b}\left[\tan^{-1}\left(\frac{1-bt}{a}\right) - \tan^{-1}\left(\frac{1}{a}\right)\right]\right\} \qquad (2)$$

$$\beta = \frac{d}{D}\,,\quad a^2 = \frac{M_t}{M_0}\,,\quad b = \left[\frac{g\beta^4}{2y_0(1-\beta^4)}\right]^{1/2}$$

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Find: (a) Shape for tank of minimum mass for given volume.
(b) Minimum water volume to reach $U = 2.5$ m/sec in $t = 25$ sec.

Solution: Mass of tank is $M = \rho_t A_s t$, where t = thickness of wall

$$A_s = A_{bottom} + A_{cylinder} = \pi\frac{D^2}{4} + \pi DH$$

Since volume is $\forall = \frac{\pi D^2}{4}H$, then $H = \frac{4\forall}{\pi D^2}$, and

$$A_s = \frac{\pi D^2}{4} + \pi D\left(\frac{4\forall}{\pi D^2}\right) = \frac{\pi D^2}{4} + \frac{4\forall}{D}$$

To minimize, set $dA_s/dD = 0$

$$\frac{dA_s}{dD} = \frac{\pi D}{2} + (-1)\frac{4\forall}{D^2} = 0 \quad \text{so} \quad D^3 = \frac{8\forall}{\pi} \quad \text{or} \quad D = \left(\frac{8\forall}{\pi}\right)^{1/3} \qquad (3) \longleftarrow D_{opt}$$

$$\text{Then} \quad \forall = \frac{\pi D^2 H}{4} = \frac{\pi D^3}{8} \quad \text{so} \quad \frac{H}{D} = \frac{1}{2} \qquad (4) \longleftarrow \left.\frac{H}{D}\right)_{opt}$$

The tank mass per volume for optimum H/D is

$$m = \frac{M}{\forall} = \frac{\rho_t\left(\frac{\pi D^2}{4} + \pi DH\right)t}{\frac{\pi D^2}{4}H} = \rho_t\left(\frac{t}{H} + \frac{4t}{D}\right) = \rho_t\frac{t}{H}\left(1 + 4\frac{H}{D}\right) = 3\rho_t\frac{t}{H}$$

Therefore mass depends on $\rho_t t$ for a given volume. The minimum mass is achieved for the smallest combination of ρ_t and t.

$$a^2 = \frac{M_t}{M_0} = \frac{M_t}{\rho\forall} = \frac{3\rho_t}{\rho}\frac{t}{H} = 3\,SG\left(\frac{t}{H}\right) \qquad (5)$$

which still depends on Volume, since it contains H.

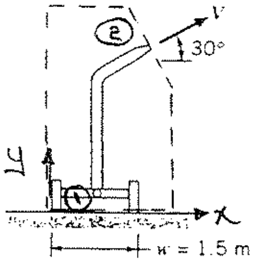
The best solution strategy seems to be: pick $\forall$, calculate H, D, β, a, and b, then plot $U(t)$.

Given: Irrigation sprinkler mounted on cart

$V = 40\,\text{m/s}$ $\theta = 30^\circ$

$D = 50\,\text{mm}$ Flow is water

$h = 3\,\text{m}$ $M = 350\,\text{kg}$



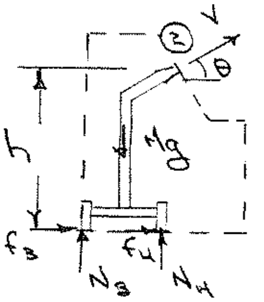
Find: (a) Magnitude of moment which tends to overturn the cart

(b) Value of V to cause impending motion; nature of impending motion.

(c) Effect on jet inclination on results

Plot: Jet velocity as a function of θ for the case of impending motion

Solution:



Apply moment of momentum equation, using fixed CV shown at left. Origin of coordinates is on ground at left wheel of cart. With this coordinate system counterclockwise moments are positive (about the z axis).

Basic equation:

$$\vec r\times\vec F_s+\int_{CV}\vec r\times\vec g\,\rho\,dV+\overset{=0(1)}{\cancel{\vec T_s}}=\overset{=0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV}}}\vec r\times\vec V\rho\,dV+\int_{CS}\vec r\times\vec V(\rho\vec V\cdot d\vec A)$$

Assumptions: (1) $T_s = 0$
(2) steady flow
(3) uniform flow at nozzle outlet
(4) neglect $\vec r\times\vec V$ of inlet flow
(5) center of mass located at $x = w/2$
(6) nozzle length is short; coordinates of nozzle exit at $(x_2, y_2) = (w/2, h)$

Then $$\vec r\times\vec F_s+\vec r\times M\vec g=\overset{=0(4)}{\cancel{\vec r_1\times\vec V_1}}\{-|\rho V_1A_1|\}+\vec r_2\times\vec V_2\{|\rho_2V_2A_2|\}$$

$$r_2=\frac{w}{2}\hat i+h\hat j \qquad V_2=V(\cos\theta\hat i+\sin\theta\hat j).$$

and $$wN_4\hat k-\frac{w}{2}Mg\hat k=\frac{w}{2}V\sin\theta\,\dot m_2\hat k-hV\cos\theta\,\dot m\hat k$$

$$wN_4-\frac{w}{2}Mg=\dot m_2V\left[\frac{w}{2}\sin\theta-h\cos\theta\right] \quad\text{------(1)}$$

Rewriting Eq. 1 in the form $\Sigma M_3 = 0$ {for static equilibrium}

$$wN_4-\frac{w}{2}Mg+\dot m_2V\left[h\cos\theta-\frac{w}{2}\sin\theta\right]=0 \quad\text{---(2)}$$

The last term in Eq 2 is the moment (due to the jet) which tends to overturn the cart.

Problem *4.180

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42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
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Evaluating, $\dot{m}_2 = \rho A_2 V_2 = \rho \frac{\pi D^2}{4} V_2$

$$\dot{m}_2 = 999 \frac{kg}{m^3} \times \frac{\pi}{4} (0.05)^2 m^2 \times 40 \frac{m}{s} = 78.5 \, kg/s$$

then with $V_2 = 40 m/s$

$$\text{Moment from jet} = 78.5 \frac{kg}{s} \times 40 \frac{m}{s} \times \frac{N \cdot s^2}{kg \cdot m} \left[3m \cos 30^\circ - \frac{1.5m}{2} \sin 30^\circ \right]$$

$$\text{Moment}_{jet} = 6.98 \, kN \cdot m \longleftarrow \text{Moment}_{jet}$$

For the case of impending tipping (about point 3) $N_4 \to 0$ and from Eq. 2

$$-\frac{W}{2} Mg + \dot{m}_2 V \left[h\cos\theta - \frac{W}{2} \sin\theta \right] = 0$$

To solve for V_2, write $\dot{m} = \rho A_2 V_2$

$$V_2^2 = \frac{WMg}{2\rho A_2 \left[h\cos\theta - \frac{W}{2}\sin\theta \right]} \qquad (3)$$

$$V_2^2 = \frac{1.5m}{2} \times 350 kg \times 9.81 \frac{m}{s^2} \times \frac{m^3}{999 kg} \times \frac{1}{1.96 \times 10^{-3} m^2} \times \frac{1}{(3\cos 30^\circ - 0.75 \sin 30^\circ) m}$$

$$V_2^2 = 592 \, m^2 s^2 \qquad \therefore V_2 = 24.3 \, m/s \longleftarrow V_2$$

Thus, the maximum speed allowable without tipping is less than the value suggested.

The impending motion will be tipping since $f_3 < \mu N_3$

From the x momentum equation $f_3 = \dot{m} V_2 \cos\theta$

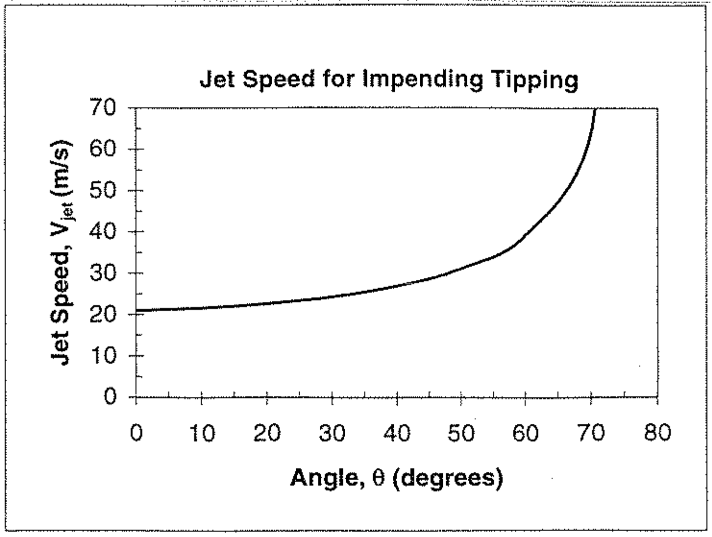
From the y momentum equation $N_3 = Mg + \dot{m} V_2 \sin\theta$

For tipping $\mu > 0.377$

From Eq. 2 we see that as θ increases the tendency to tip decreases

For impending motion from Eq. 3.

$$V = \left\{ \frac{WMg}{2\rho A_2 \left[h\cos\theta - \frac{W}{2}\sin\theta \right]} \right\}^{1/2}$$

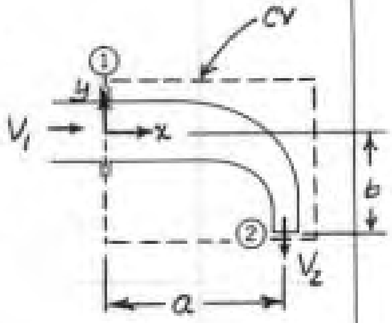


Problem *4.181

Given: The 90° reducing elbow of Example 4.6 discharges to atmosphere. Section ② is located 0.3 m to the right of Section ①.

Find: Estimate the moment exerted by the flange on the elbow.

Solution: Apply moment of momentum, using the CV and CS shown.



From Example Problem 4.7, $\vec{V}_2 = -16\hat{j}$ m/s, $A_1 = 0.01\ m^2$

Steady flow, $A_2 = 0.0025\ m^2$

Basic equation (fixed CV):

$$\vec{r}\times\vec{F}_s + \overset{\approx 0(1)}{\cancel{\int_{CV}\vec{r}\times\vec{g}\rho\, d\forall}} + \overset{=0(2)}{\cancel{\vec{T}_{shaft}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\vec{r}\times\vec{V}\rho\, d\forall + \int_{CS}\vec{r}\times\vec{V}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Neglect body forces
(2) No shafts, so $\vec{T}_{shaft} = 0$
(3) Steady flow (given)
(4) Uniform flow at each cross section
(5) Incompressible flow

Then

$$\vec{M}_{flange} = \vec{r}\times\vec{F}_s\Big)_{flange} = \vec{r}_1\times\vec{V}_1\left\{-\rho V_1 A_1\right\} + \vec{r}_2\times\vec{V}_2\left\{+\rho V_2 A_2\right\} \qquad (1)$$

$$\vec{r}_1 = 0 \qquad \left.\begin{array}{l}\vec{r}_2 = a\hat{i} - b\hat{j}\\ \vec{V}_2 = -V_2\hat{j}\end{array}\right\}\ \vec{r}_2\times\vec{V}_2 = -aV_2\hat{k} + 0$$

Substituting into Eq. 1,

$$\vec{M}_{flange} = -aV_2\hat{k}\left\{+\rho V_2 A_2\right\} = -a\rho V_2^2 A_2\hat{k}$$

$$= 0.3\ m \times 999\frac{kg}{m^3}\times(16)^2\frac{m^2}{s^2}\times 0.0025\ m^2\times\frac{N\cdot s^2}{kg\cdot m}(-\hat{k})$$

$$\vec{M}_{flange} = -192\ \hat{k}\ N\cdot m$$

$\vec{M}_{flan}$

This is the torque that must be exerted on the CV by the flange.

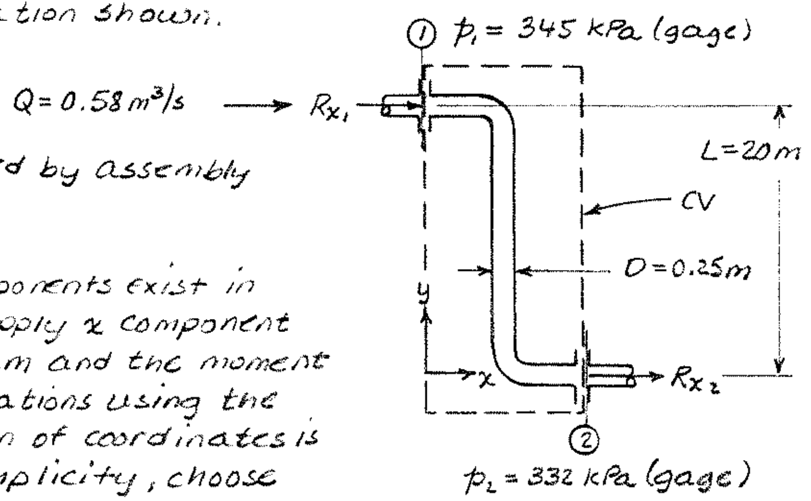
{ Since $\vec{M}_{flange}$ is in the $-\hat{k}$ direction, it must act cw in the xy-plane. }

Problem *4.182

[3]

Given: Crude oil (SG = 0.95) flow through a pipe assembly in the horizontal configuration shown.

Find: Force and torque exerted by assembly on its supports.



Solution: No momentum components exist in the y direction. Apply x component of linear momentum and the moment of momentum equations using the CV shown. Location of coordinates is arbitrary; for simplicity, choose as shown.

$$\text{Basic equations: } F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV}}} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

$$\vec{r}\times\vec{F}_s + \overset{=0(4)}{\cancel{\int_{CV} \vec{r}\times\vec{g}\,\rho\, d\forall}} + \overset{=0(5)}{\cancel{\vec{T}_{shaft}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV}}} \vec{r}\times\vec{V}\rho\, d\forall + \int_{CS} \vec{r}\times\vec{V}\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) $F_{B_x} = 0$; $\vec{g}$ acts in z direction
(2) Steady flow
(3) Uniform flow at each section
(4) No z component of $\vec{r}\times\vec{g}$
(5) $\vec{T}_{shaft} = 0$

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.25)^2 m^2 = 0.049\ m^2$$

From momentum equation,

$$R_{x_1} + R_{x_2} + p_1 A - p_2 A = u_1\{-\dot{m}\} + u_2\{\dot{m}\} = 0\ ;\quad R_{x_1} + R_{x_2} = (p_2 - p_1)A$$

From moment of momentum,

$$\vec{r}_1\times(R_{x_1} + p_1 A)\hat{\imath} + \overset{=0}{\cancel{\vec{r}_2}}\times(R_{x_2} - p_2 A)\hat{\imath} = \vec{r}_1\times V_1\hat{\imath}\{-\dot{m}\} + \overset{=0}{\cancel{\vec{r}_2}}\times V_2\hat{\imath}\{\dot{m}\}\ ;\ \vec{r}_1 = L\hat{\jmath},\ \vec{r}_1\times\hat{\imath} = -L\hat{k}$$

$$-L(R_{x_1} + p_1 A)\hat{k} = -LV_1(-\dot{m})\hat{k} = LV_1\dot{m}\,\hat{k} = L\frac{Q}{A}(\rho Q)\hat{k} = L\frac{\rho Q^2}{A}\hat{k}$$

$$R_{x_1} = -\frac{\rho Q^2}{A} - p_1 A = -0.95\times 999\frac{kg}{m^3}\times(0.58)^2\frac{m^6}{s^2}\times\frac{1}{0.049\,m^2}\times\frac{N\cdot s^2}{kg\cdot m} - 3.45\times10^5\frac{N}{m^2}\times 0.049\,m^2 = -23.4\ kN$$

$$R_{x_2} = (p_2 - p_1)A - R_{x_1} = p_2 A - p_1 A + \frac{\rho Q^2}{A} + p_1 A = p_2 A + \frac{\rho Q^2}{A}$$

$$= 3.32\times10^5\frac{N}{m^2}\times 0.049\,m^2 + 0.95\times 999\frac{kg}{m^3}\times(0.58)^2\frac{m^6}{s^2}\times\frac{1}{0.049\,m^2}\times\frac{N\cdot s^2}{kg\cdot m} = 22.8\ kN$$

$$\vec{r}\times\vec{F}_s = \vec{r}_1\times R_{x_1}\hat{\imath} = L\hat{\jmath}\times R_{x_1}\hat{\imath} = -LR_{x_1}\hat{k} = -20\,m\times(-46.0)\,kN\,\hat{k} = 468\,\hat{k}\ kN\cdot m$$

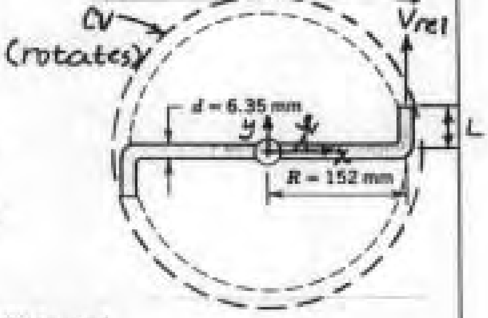
These are forces and torque on CV. The corresponding reactions are:

$$K_{x_1} = -R_{x_1} = 23.4\ kN\ ,\quad K_{x_2} = -R_{x_2} = -22.8\ kN \longleftarrow$$ Force

$$\vec{M} = -\vec{r}\times\vec{F}_s = -468\,\hat{k}\ kN\cdot m\quad \text{(i.e. clockwise)} \longleftarrow$$ Torque

Problem *4.183

[3]



Given: Simplified lawn sprinkler rotating in horizontal plane, $Q = 4.5$ gal/min.

Water discharges horizontally from jets.

Neglect pivot friction, inertia of sprinkler.

Find: (a) Torque needed to hold at $\omega = 0$.
(b) Angular acceleration when torque is removed.

Solution: Choose rotating CV. Apply angular momentum principle, Eq. 4.53.

Basic equation: $\vec{r} \times \overset{=0(1)}{\cancel{\vec{F}_s}} + \int_{CV} \overset{=0(2)}{\cancel{\vec{r} \times \vec{g}}}\, \rho\, d\forall + \vec{T}_{shaft}$

$$- \int_{CV} \vec{r} \times \left[2\overset{=0(3)}{\cancel{\vec{\omega}}} \times \vec{V}_{xyz} + \overset{=0(3)}{\cancel{\vec{\omega}}} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r} \right] \rho\, d\forall$$

$$= \overset{=0(4)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} \vec{r} \times \vec{V}_{xyz}\, \rho\, d\forall + \int_{CS} \vec{r} \times \vec{V}_{xyz}\, \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) No surface forces
(2) Body torques cancel
(3) sprinkler stationary, $\vec{\omega} = 0$
(4) Steady flow
(5) Uniform flow at each section
(6) $L \ll R$

Analyze right arm of sprinkler. From geometry $\vec{r} = r\hat{i}$ in CV, $\vec{r} = R\hat{i}$ at jet. Then

$$T\hat{k} - \int_{CV} \vec{r} \times (\dot{\vec{\omega}} \times \vec{r})\rho\, d\forall = R\hat{i} \times V\hat{j}\, \rho \frac{Q}{2} = \rho \frac{QRV}{2}\hat{k} = \frac{\dot{m}RV}{2}\hat{k}$$

$$r\hat{i} \times (\dot{\omega}\hat{k} \times r\hat{i}) = r\hat{i} \times \dot{\omega} r\hat{j} = \dot{\omega} r^2 \hat{k}\, ; \int_{CV} = \dot{\omega}\frac{R^3}{3}\rho A\, \hat{k}$$

Dropping $\hat{k}$, $T - \frac{\dot{\omega}\rho A R^3}{3} = \frac{\dot{m}RV}{2}$. When arm is stationary, $\dot{\omega} = 0$, and

$$T = \frac{\dot{m}RV}{2} \qquad \dot{m} = \rho Q = 999\,\frac{\text{kg}}{\text{m}^3} \times 4.5\,\frac{\text{gal}}{\text{min}} \times 231\,\frac{\text{in.}^3}{\text{gal}} \times (0.0254)^3\frac{\text{m}^3}{\text{in.}^3} \times \frac{\text{min}}{60\,\text{s}} = 0.284\,\frac{\text{kg}}{\text{s}}$$

$$V = \frac{Q}{2A} = \frac{2Q}{\pi d^2} = \frac{2}{\pi} \times 2.84 \times 10^{-4}\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{(0.00635)^2\,\text{m}^2} = 4.48\ \text{m/s}$$

$$T = \frac{1}{2} \times 0.284\,\frac{\text{kg}}{\text{sec}} \times 0.152\,\text{m} \times 4.48\,\frac{\text{m}}{\text{sec}} = 0.0967\ \text{N}\cdot\text{m (per arm)}$$

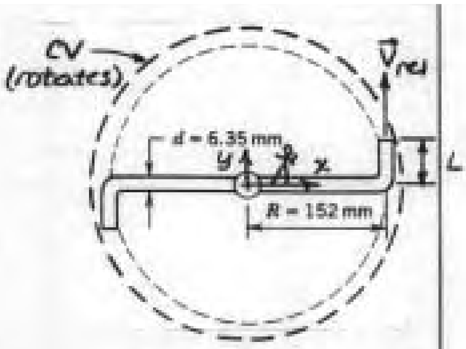
For two arms, $T_2 = 2T = 2 \times 0.0967\ \text{N}\cdot\text{m} = 0.193\ \text{N}\cdot\text{m}$ T_2

When torque is removed, angular acceleration would be the same for each arm. Thus

$$\dot{\omega} = \frac{\dot{m}RV}{2} \times \frac{3}{\rho A R^3} = \frac{3\dot{m}V}{2\rho A R^2}$$

$$\dot{\omega} = \frac{3}{2} \times 0.284\,\frac{\text{kg}}{\text{s}} \times 4.48\,\frac{\text{m}}{\text{s}} \times \frac{\text{m}^3}{999\,\text{kg}} \times \frac{4}{\pi(0.00635)^2\,\text{m}^2} \times \frac{1}{(0.152)^2\,\text{m}^2} = 2610\ \text{rad/s}^2$$ $\dot{\omega}$

Problem *4.184



Given: Simplified lawn sprinkler rotating in horizontal plane, Q = 4.5 gal/min.

Water discharges horizontally from jets.

Neglect pivot friction, inertia of sprinkler.

Find: (a) Derive a differential equation for angular speed as a function of time.
(b) Evaluate steady-state angular speed.

Solution: Choose rotating CV. Apply angular momentum principle, Eq. 4.53.

Basic equation: $\vec r \times \overset{=0(1)}{\vec F_s} + \int_{CV} \vec r \times \overset{=0(2)}{\vec g}\,\rho\, d\forall + \overset{=0(3)}{\vec T_{shaft}}$

$$-\int_{CV} \vec r \times \left[2\vec\omega \times \vec V_{xyz} + \vec\omega \times (\overset{=0(4)}{\vec\omega \times \vec r}) + \dot{\vec\omega} \times \vec r\right]\rho\, d\forall$$

$$= \overset{=0(5)}{\frac{\partial}{\partial t}}\int_{CV} \vec r \times \vec V_{xyz}\,\rho\, d\forall + \int_{CS} \vec r \times \vec V_{xyz}\,\rho \vec V_{xyz} \cdot d\vec A$$

Assumptions: (1) $\vec F_S = 0$, (2) Body torques cancel, (3) $\vec T_{shaft} = 0$, (4) No $\hat k$ component of centripetal acceleration, (5) steady flow, (6) $L \ll R$

Analyze right arm of sprinkler. From geometry, $\vec r = r\hat i$ in CV, $\vec r = R\hat i$ at jet.
Then

$$-\int_{CV} r\hat i \times \left[2\omega\hat k \times V\hat i + \dot\omega\hat k \times r\hat i\right]\rho A\,dr = R\hat i \times V\hat j\,\rho\frac{Q}{2} = \rho\frac{QRV}{2}\hat k$$

$$r\hat i \times \left[2\omega V(+\hat j) + \dot\omega r(+\hat j)\right] = (2\omega r V + \dot\omega r^2)(+\hat k);\quad -\int_{CV} = -\left(\omega R^2 V + \dot\omega\frac{R^3}{3}\right)\rho A$$

Dropping $\hat k$,

$$-\omega\rho VAR^2 - \frac{\dot\omega\rho AR^3}{3} = \rho\frac{QRV}{2} \quad \text{or} \quad \dot\omega = \frac{3}{\rho AR^3}\left[-\omega\rho VAR^2 - \rho\frac{QRV}{2}\right]$$ O.D.E.

Thus $\frac{d\omega}{dt} = -a - b\omega$, where $a = \frac{3}{\rho AR^3}\rho\frac{QRV}{2} = \frac{3}{2}\frac{QV}{AR^2} = \frac{3V^2}{R^2}$†, $b = \frac{3\rho VAR^2}{\rho AR^3} = \frac{3V}{R}$

$\frac{d\omega}{dt} = 0$ when $-a - b\omega_{max} = 0$, i.e., when $\omega_{max} = -a/b$. †Note $V = \frac{Q}{2A}$ (one arm)

$$Q = 4.5\,\frac{gal}{min} \times 231\,\frac{in^3}{gal} \times (0.0254)^3\,\frac{m^3}{in.^3} \times \frac{min}{60\,s} = 2.84 \times 10^{-4}\ m^3/s$$

$$\omega_{max} = -\frac{a}{b} = -\frac{3V^2}{R^2} \times \frac{R}{3V} = -\frac{V}{R} = -4.48\,\frac{m}{s} \times \frac{1}{0.152\,m} = -29.5\ rad/s \quad (-281\ rpm)$$ ω_{max}

{Note it is not necessary to solve the differential equation to find ω_{max}.}

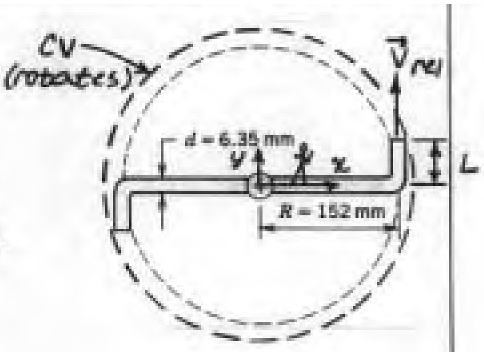
Problem *4.185

[3]

Given: Simplified lawn sprinkler rotating in horizontal plane, $Q = 4.5$ gal/min.

Water discharges horizontally from jets.

Neglect inertia of sprinkler; $T_f = 0.045$ ft·lbf



Find: (a) Derive a differential equation for angular speed as a function of time.
(b) Evaluate steady-state angular speed.

Solution: Choose rotating CV. Apply angular momentum principle, Eq. 4.53.

Basic equation:

$$\vec r\times\overset{=0(1)}{\cancel{\vec F_s}} + \int_{CV}\vec r\times\overset{=0(2)}{\cancel{\vec g}}\,\rho\,dV + \vec T_{shaft}$$

$$-\int_{CV}\vec r\times\left[2\vec\omega\times\vec V_{xyz} + \vec\omega\times(\overset{=0(4)}{\cancel{\vec\omega\times\vec r}}) + \dot{\vec\omega}\times\vec r\right]\rho\,dV$$

$$= \overset{=0(5)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\vec r\times\vec V_{xyz}\,\rho\,dV + \int_{CS}\vec r\times\vec V_{xyz}\,\rho\vec V_{xyz}\cdot d\vec A$$

Assumptions: (1) $\vec F_s = 0$, (2) Body torques cancel, (3) $\vec T_{shaft} = 0.045$ ft·lbf, (4) No $\hat k$ component of centripetal acceleration, (5) Steady flow, (6) $L \ll R$.

Analyze right arm of sprinkler. From geometry, $\vec r = r\hat\imath$ in CV, $\vec r = R\hat\imath$ at jet. Then

$$-\int_{CV} r\hat\imath\times\left[2\omega\hat k\times V\hat\imath + \dot\omega\hat k\times r\hat\imath\right]\rho A\,dr = R\hat\imath\times V\hat\jmath\,\rho\frac{Q}{2} = \rho\frac{QRV}{2}\hat k$$

$$r\hat\imath\times\left[2\omega V\hat\jmath + \dot\omega r\hat\jmath\right] = (2\omega Vr + \dot\omega r^2)\hat k\,;\quad -\int_{CV} = -\left(\omega VR^2 + \frac{\dot\omega R^3}{3}\right)\rho A\,\hat k$$

For both arms, dropping $\hat k$, $\{T = 0.045 \text{ ft·lbf} = 0.0610 \text{ N·m}\}$

$$T - 2\omega\rho VAR^2 - \frac{2\dot\omega\rho AR^3}{3} = \rho QRV \quad\text{or}\quad \dot\omega = \frac{3}{2\rho AR^3}\left[T - \rho QRV - 2\omega\rho VAR^2\right]$$

O.D.E.

Thus $\frac{d\omega}{dt} = a - b\omega$, where $a = \frac{3}{2\rho AR^3}(T - \rho QRV)$; $b = \frac{3}{2\rho AR^3}\times 2\rho VAR^2 = 3\frac{V}{R}$

The steady-state speed occurs when $\frac{d\omega}{dt} = 0$, i.e. when $\omega_{max} = \frac{a}{b}$

$$Q = 4.5\frac{\text{gal}}{\text{min}}\times 231\frac{\text{in}^3}{\text{gal}}\times(0.0254)^3\frac{\text{m}^3}{\text{in}^3}\times\frac{\text{min}}{60\text{ s}} = 2.84\times10^{-4}\ \text{m}^3/\text{s}\,;\quad A = \frac{\pi d^2}{4} = 3.17\times10^{-5}\ \text{m}^2$$

From the O.D.E., $\omega_{max} = \frac{T - \rho QRV}{2\rho VAR^2}$

$$\omega_{max} = \frac{1}{2}\left[0.0610\ \text{N·m}\times\frac{\text{kg·m}}{\text{N·s}^2} - 999\frac{\text{kg}}{\text{m}^3}\times 2.84\times10^{-4}\frac{\text{m}^3}{\text{s}}\times 0.152\ \text{m}\times 4.48\frac{\text{m}}{\text{s}}\right]\frac{\text{m}^3}{999\ \text{kg}}\times\frac{\text{s}}{4.48\ \text{m}}$$

$$\times\frac{1}{3.17\times10^{-5}\ \text{m}^2}\times\frac{1}{(0.152)^2\ \text{m}^2}$$

$$\omega_{max} = -20.2\ \text{rad/s}\quad(-193\ \text{rpm})$$

ω_{max}

Problem *4.186

[3]

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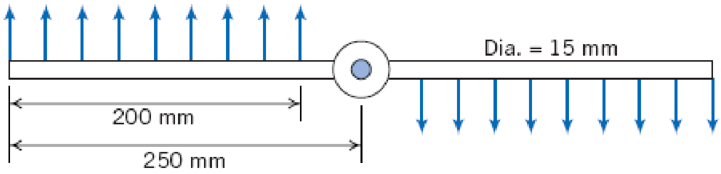
*4.186 Water flows in a uniform flow out of the 5 mm slots of the rotating spray system as shown. The flow rate is 15 kg/s. Find the torque required to hold the system stationary, and the steady-state speed of rotation after it is released.



Given: Data on rotating spray system

Find: Torque required to hold stationary; steady-state speed

Solution:

The given data is $\rho = 999\cdot\frac{kg}{m^3}$ $m_{flow} = 15\cdot\frac{kg}{s}$ $D = 0.015\cdot m$ $r_o = 0.25\cdot m$ $r_i = 0.05\cdot m$ $\delta = 0.005\cdot m$

Governing equation: Rotating CV

$$\vec{r}\times\vec{F}_s + \int_{CV}\vec{r}\times\vec{g}\,\rho\,d\forall + \vec{T}_{shaft} - \int_{CV}\vec{r}\times[2\vec{\omega}\times\vec{V}_{xyz} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \dot{\vec{\omega}}\times\vec{r}]\,\rho\,d\forall = \frac{\partial}{\partial t}\int_{CV}\vec{r}\times\vec{V}_{xyz}\,\rho\,d\forall + \int_{CS}\vec{r}\times\vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A} \qquad (4.52)$$

For no rotation (ω = 0) this equation reduces to a single scalar equation

$$T_{shaft} = \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A} \qquad \text{or} \qquad T_{shaft} = 2\cdot\delta\cdot\int_{r_i}^{r_o} r\cdot V\cdot\rho\cdot V\,dr = 2\cdot\rho\cdot V^2\cdot\delta\cdot\int_{r_i}^{r_o} r\,dr = \rho\cdot V^2\cdot\delta\cdot\left(r_o^{\ 2} - r_i^{\ 2}\right)$$

where *V* is the exit velocity with respect to the CV

$$V = \frac{\frac{m_{flow}}{\rho}}{2\cdot\delta\cdot\left(r_o - r_i\right)}$$

Hence

$$T_{shaft} = \rho\cdot\left[\frac{\frac{m_{flow}}{\rho}}{2\cdot\delta\cdot\left(r_o - r_i\right)}\right]^2\cdot\delta\cdot\left(r_o^{\ 2} - r_i^{\ 2}\right) \qquad T_{shaft} = \frac{m_{flow}^{\ 2}}{4\cdot\rho\cdot\delta}\cdot\frac{\left(r_o + r_i\right)}{\left(r_o - r_i\right)}$$

$$T_{shaft} = \frac{1}{4}\times\left(15\cdot\frac{kg}{s}\right)^2\times\frac{m^3}{999\cdot kg}\times\frac{1}{0.005\cdot m}\times\frac{(0.25 + 0.05)}{(0.25 - 0.05)} \qquad T_{shaft} = 16.9\,N\cdot m$$

For the steady rotation speed the equation becomes

$$-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\,dV = \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A}$$

The volume integral term $-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\,dV$ must be evaluated for the CV. The velocity in the CV varies with *r*. This variation can be found from mass conservation

For an infinitesimal CV of length *dr* and cross-section *A* at radial position *r*, if the flow in is *Q*, the flow out is *Q* + *dQ*, and the loss through the slot is *V*δ*dr*. Hence mass conservation leads to

$$(Q + dQ) + V\cdot\delta\cdot d\,dQ = -V\cdot\delta\cdot dr \qquad Q(r) = -V\cdot\delta\cdot r + \text{const}$$

At the inlet ($r = r_i$) $\quad Q = Q_i = \dfrac{m_{flow}}{2\cdot\rho}$

Hence $\quad Q = Q_i + V\cdot\delta\cdot\left(r_i - r\right) = \dfrac{m_{flow}}{2\cdot\rho} + \dfrac{m_{flow}}{2\cdot\rho\cdot\delta\cdot\left(r_o - r_i\right)}\cdot\delta\cdot\left(r_i - r\right) \qquad Q = \dfrac{m_{flow}}{2\cdot\rho}\cdot\left(1 + \dfrac{r_i - r}{r_o - r_i}\right) = \dfrac{m_{flow}}{2\cdot\rho}\cdot\left(\dfrac{r_o - r}{r_o - r_i}\right)$

and along each rotor the water speed is $\quad v(r) = \dfrac{Q}{A} = \dfrac{m_{flow}}{2\cdot\rho\cdot A}\cdot\left(\dfrac{r_o - r}{r_o - r_i}\right)$

Hence the term - $\displaystyle\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\,dV$ becomes

$$-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\,dV = 4\cdot\rho\cdot A\cdot\omega\cdot\int_{r_i}^{r_o} r\cdot v(r)\,dr = 4\cdot\rho\cdot\omega\cdot\int_{r_i}^{r_o} r\cdot\frac{m_{flow}}{2\cdot\rho}\cdot\left(\frac{r_o - r}{r_o - r_i}\right)dr$$

or

$$-\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\,dV = 2\cdot m_{flow}\cdot\omega\cdot\int_{r_i}^{r_o} r\cdot\left(\frac{r_o - r}{r_o - r_i}\right)dr = m_{flow}\cdot\omega\cdot\frac{r_o^{\,3} + r_i^{\,2}\cdot\left(2\cdot r_i - 3\cdot r_o\right)}{3\cdot\left(r_o - r_i\right)}$$

Recall that $\quad\displaystyle\int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A} = \rho\cdot V^2\cdot\delta\cdot\left(r_o^{\,2} - r_i^{\,2}\right)$

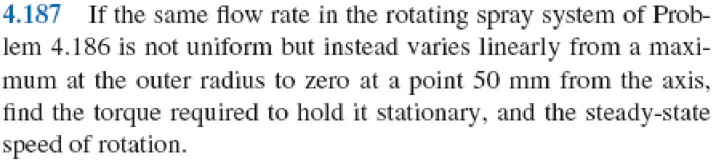
Hence equation $\quad\displaystyle -\int \vec{r}\times\left(2\cdot\vec{\omega}\times\overrightarrow{V_{xyz}}\right)\cdot\rho\,dV = \int \vec{r}\times\overrightarrow{V_{xyz}}\cdot\rho\cdot\overrightarrow{V_{xyz}}\,d\vec{A}$ $\quad$ becomes

$$m_{flow}\cdot\omega\cdot\frac{r_o^{\,3} + r_i^{\,2}\cdot\left(2\cdot r_i - 3\cdot r_o\right)}{3\cdot\left(r_o - r_i\right)} = \rho\cdot V^2\cdot\delta\cdot\left(r_o^{\,2} - r_i^{\,2}\right)$$

Solving for ω $\quad \omega = \dfrac{3\cdot\left(r_o - r_i\right)\cdot\rho\cdot V^2\cdot\delta\cdot\left(r_o^{\,2} - r_i^{\,2}\right)}{m_{flow}\cdot\left[r_o^{\,3} + r_i^{\,2}\cdot\left(2\cdot r_i - 3\cdot r_o\right)\right]} \qquad \omega = 461\,\text{rpm}$

Problem *4.187

[3]



4.187 If the same flow rate in the rotating spray system of Problem 4.186 is not uniform but instead varies linearly from a maximum at the outer radius to zero at a point 50 mm from the axis, find the torque required to hold it stationary, and the steady-state speed of rotation.

Given: Data on rotating spray system

Find: Torque required to hold stationary; steady-state speed

Solution:

The given data is $\rho = 999 \cdot \frac{kg}{m^3}$ $\quad m_{flow} = 15 \cdot \frac{kg}{s}$ $\quad D = 0.015 \cdot m$ $\quad r_o = 0.25 \cdot m$ $\quad r_i = 0.05 \cdot m$ $\quad \delta = 0.005 \cdot m$

Governing equation: Rotating CV

$$\vec{r} \times \vec{F}_s + \int_{CV} \vec{r} \times \vec{g}\,\rho\, d\forall + \vec{T}_{shaft} - \int_{CV} \vec{r} \times [2\vec{\omega} \times \vec{V}_{xyz} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r}]\,\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} \vec{r} \times \vec{V}_{xyz}\,\rho\, d\forall + \int_{CS} \vec{r} \times \vec{V}_{xyz}\,\rho \vec{V}_{xyz} \cdot d\vec{A} \qquad (4.52)$$

For no rotation (ω = 0) this equation reduces to a single scalar equation

$$T_{shaft} = \int \vec{r} \times \overrightarrow{V_{xyz}} \cdot \rho \cdot \overrightarrow{V_{xyz}}\, d\vec{A} \qquad \text{or} \qquad T_{shaft} = 2 \cdot \delta \cdot \int_{r_i}^{r_o} r \cdot V \cdot \rho \cdot V\, dr$$

where *V* is the exit velocity with respect to the CV. We need to find *V*(*r*). To do this we use mass conservation, and the fact that the distribution is linear

$$V(r) = V_{max} \cdot \frac{(r - r_i)}{(r_o - r_i)} \qquad \text{and} \qquad 2 \cdot \frac{1}{2} \cdot V_{max} \cdot (r_o - r_i) \cdot \delta = \frac{m_{flow}}{\rho}$$

so

$$V(r) = \frac{m_{flow}}{\rho \cdot \delta} \cdot \frac{(r - r_i)}{(r_o - r_i)^2}$$

Hence

$$T_{shaft} = 2 \cdot \rho \cdot \delta \cdot \int_{r_i}^{r_o} r \cdot V^2\, dr = 2 \cdot \frac{m_{flow}^2}{\rho \cdot \delta} \cdot \int_{r_i}^{r_o} r \cdot \left[\frac{(r - r_i)}{(r_o - r_i)^2}\right]^2 dr \qquad T_{shaft} = \frac{m_{flow}^2 \cdot (r_i + 3 \cdot r_o)}{6 \cdot \rho \cdot \delta \cdot (r_o - r_i)}$$

$$T_{shaft} = \frac{1}{6} \times \left(15 \cdot \frac{kg}{s}\right)^2 \times \frac{m^3}{999 \cdot kg} \times \frac{1}{0.005 \cdot m} \times \frac{(0.05 + 3 \cdot 0.25)}{(0.25 - 0.05)} \qquad T_{shaft} = 30 \cdot N \cdot m$$

For the steady rotation speed the equation becomes

$$-\int \vec{r} \times \left(2 \cdot \vec{\omega} \times \overrightarrow{V_{xyz}}\right) \cdot \rho\, dV = \int \vec{r} \times \overrightarrow{V_{xyz}} \cdot \rho \cdot \overrightarrow{V_{xyz}}\, d\vec{A}$$

The volume integral term $-\int \vec{r} \times \left(2 \cdot \vec{\omega} \times \overrightarrow{V_{xyz}}\right) \cdot \rho \, dV$ must be evaluated for the CV. The velocity in the CV varies with *r*. This variation can be found from mass conservation

For an infinitesmal CV of length *dr* and cross-section *A* at radial position *r*, if the flow in is *Q*, the flow out is *Q* + *dQ*, and the loss through the slot is *V*δdr Hence mass conservation leads to

$$(Q + dQ) + V \cdot \delta \cdot dr - Q = 0 \qquad dQ = -V \cdot \delta \cdot dr \qquad Q(r) = Q_i - \delta \cdot \int_{r_i}^{r} \frac{m_{flow}}{\rho \cdot \delta} \cdot \frac{\left(r - r_i\right)}{\left(r_o - r_i\right)^2} \, dr = Q_i - \int_{r_i}^{r} \frac{m_{flow}}{\rho} \cdot \frac{\left(r - r_i\right)}{\left(r_o - r_i\right)^2} \, dr$$

At the inlet ($r = r_i$) $\qquad Q = Q_i = \frac{m_{flow}}{2 \cdot \rho}$

Hence $\qquad Q(r) = \frac{m_{flow}}{2 \cdot \rho} \cdot \left[1 - \frac{\left(r - r_i\right)^2}{\left(r_o - r_i\right)^2}\right]$

and along each rotor the water speed is $\qquad v(r) = \frac{Q}{A} = \frac{m_{flow}}{2 \cdot \rho \cdot A} \cdot \left[1 - \frac{\left(r - r_i\right)^2}{\left(r_o - r_i\right)^2}\right]$

Hence the term - $\int \vec{r} \times \left(2 \cdot \vec{\omega} \times \overrightarrow{V_{xyz}}\right) \cdot \rho \, dV$ $\qquad 4 \cdot \rho \cdot A \cdot \omega \cdot \left(\int_{r_i}^{r_o} r \cdot v(r) \, dr\right) = 4 \cdot \rho \cdot \omega \cdot \int_{r_i}^{r_o} \frac{m_{flow}}{2 \cdot \rho} \cdot r \cdot \left[1 - \frac{\left(r - r_i\right)^2}{\left(r_o - r_i\right)^2}\right] dr$

becomes

or $\qquad 2 \cdot m_{flow} \cdot \omega \cdot \int_{r_i}^{r_o} r \cdot \left[1 \cdot - \frac{\left(r_o - r\right)^2}{\left(r_o - r_i\right)^2}\right] dr = m_{flow} \cdot \omega \cdot \left(\frac{1}{6} \cdot r_o^{\,2} + \frac{1}{3} \cdot r_i \cdot r_o - \frac{1}{2} \cdot r_i^{\,2}\right)$

Recall that $\qquad \int \vec{r} \times \overrightarrow{V_{xyz}} \cdot \rho \cdot \overrightarrow{V_{xyz}} \, d\vec{A} = \frac{m_{flow}^{\,2} \cdot \left(r_i + 3 \cdot r_o\right)}{6 \cdot \left(r_o - r_i\right) \cdot \rho \cdot \delta}$

Hence equation $\qquad -\int \vec{r} \times \left(2 \cdot \vec{\omega} \times \overrightarrow{V_{xyz}}\right) \cdot \rho \, dV = \int \vec{r} \times \overrightarrow{V_{xyz}} \cdot \rho \cdot \overrightarrow{V_{xyz}} \, d\vec{A}$

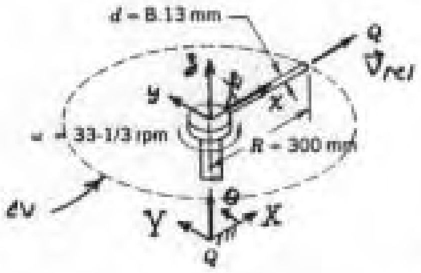
becomes $\qquad m_{flow} \cdot \omega \cdot \left(\frac{1}{6} \cdot r_o^{\,2} + \frac{1}{3} \cdot r_i \cdot r_o - \frac{1}{2} \cdot r_i^{\,2}\right) = \frac{m_{flow}^{\,2} \cdot \left(r_i + 3 \cdot r_o\right)}{6 \cdot \left(r_o - r_i\right) \cdot \rho \cdot \delta}$

Solving for ω $\qquad \omega = \frac{m_{flow} \cdot \left(r_i + 3 \cdot r_o\right)}{\left(r_o^{\,2} + 2 \cdot r_i \cdot r_o - 3 \cdot r_i^{\,2}\right) \cdot \left(r_o - r_i\right) \cdot \rho \cdot \delta} \qquad \omega = 1434 \cdot rpm$

Problem *4.188

Given: Single rotating tube with water.

$Q = 13.8$ L/min



Find: Torque that must be applied to maintain steady rotation using:
(a) Rotating control volume.
(b) Fixed control volume.

Solution: Apply angular momentum principle. $\left\{\omega = 33\tfrac{1}{3}\,\frac{\text{rev}}{\text{min}} = 3.49\text{ rad/s}\right\}$

(a) Rotating CV: use relative velocities, Eq. 4.53:

Basic equation:
$$\vec r\times\overset{=0(1)}{\cancel{\vec F_s}} + \int_{CV}\vec r\times\overset{=0(2)}{\cancel{\vec g}}\,\rho\,d\forall + \vec T_{shaft} - \int_{CV}\vec r\times\left[2\vec\omega\times\vec V_{xyz} + \vec\omega\times\overset{=0(3)}{\cancel{(\vec\omega\times\vec r)}} + \overset{=0(4)}{\cancel{\dot{\vec\omega}}}\times\vec r\right]\rho\,d\forall = \overset{=0(5)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\vec r\times\vec V_{xyz}\,\rho\,d\forall + \int_{CS}\vec r\times\overset{=0(6)}{\cancel{\vec V_{xyz}}}\,\rho\vec V_{xyz}\cdot d\vec A$$

Assumptions: (1) $\vec F_S = 0$, (2) Body torques cancel, (3) No $\hat k$ in centripetal accel, (4) $\dot{\vec\omega} = 0$, (5) steady flow, (6) $\vec r\times\vec V = 0$

Then
$$T_{shaft}\hat k = \int_{CV}\vec r\times(2\vec\omega\times\vec V)\rho\,d\forall = \int_0^R r\hat i\times(2\omega\hat k\times V\hat i)\rho A\,dr = \omega\rho VAR^2\hat k = \omega\rho QR^2\hat k$$

$$T_{shaft} = 3.49\,\frac{\text{rad}}{\text{s}}\times 999\,\frac{\text{kg}}{\text{m}^3}\times 13.8\times10^{-3}\,\frac{\text{m}^3}{\text{min}}\times(0.3)^2\,\text{m}^2\times\frac{\text{min}}{60\text{ s}}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 0.0722\text{ N}\cdot\text{m}$$ T

(b) Fixed control volume: use absolute velocities, Eq. 4.47:

Basic equation:
$$\vec r\times\overset{=0(1)}{\cancel{\vec F_s}} + \int_{CV}\vec r\times\overset{=0(2)}{\cancel{\vec g}}\,\rho\,d\forall + \vec T_{shaft} = \frac{\partial}{\partial t}\int_{CV}\vec r\times\vec V\rho\,d\forall + \int_{CS}\vec r\times\vec V\rho\vec V_{xyz}\cdot d\vec A$$

Relative to fixed coordinates XY, $\vec r = r(\cos\theta\hat i + \sin\theta\hat j)$

$$\vec V = V(\cos\theta\hat i + \sin\theta\hat j) + r\omega(-\sin\theta\hat i + \cos\theta\hat j)$$

$$\vec r\times\vec V = \begin{vmatrix}\hat i & \hat j & \hat k\\ r\cos\theta & r\sin\theta & 0\\ V\cos\theta - r\omega\sin\theta & V\sin\theta + r\omega\cos\theta & 0\end{vmatrix} = \hat k\,(rV\sin\theta\cos\theta + \omega^2r^2\cos^2\theta - rV\sin\theta\cos\theta + \omega^2r^2\sin^2\theta) = \omega r^2\hat k$$

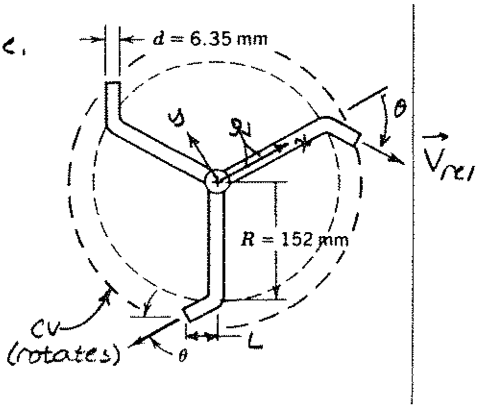
Thus $\partial/\partial t = 0$ and $\int_{CS}\vec r\times\vec V\rho\vec V_{xyz}\cdot d\vec A = \omega R^2\hat k\{+\rho Q\} = \omega\rho QR^2\hat k$ and

$T_{shaft}\hat k = \omega\rho QR^2\hat k$ (as before); $T = 0.0722$ N·m T

{Note that when applied correctly, either choice of CV produces the same result.}

Problem *4.189

[3]



Given: Lawn sprinkler rotating in horizontal plane.

Neglect friction. $Q = 68$ L/min

Find: Steady-state angular speed for $\theta = 30°$.

Plot: Steady-state angular speed for $0 \le \theta \le 90°$.

Solution: Choose rotating CV. Apply angular momentum principle, Eq. 4.53.

Basic equation:

$$\overset{=0(1)}{\cancel{\vec{r}\times\vec{F}_s}} + \overset{=0(2)}{\cancel{\int_{CV}\vec{r}\times\vec{g}\,\rho\,d\forall}} + \overset{=0(3)}{\cancel{\vec{T}_{shaft}}}$$

$$-\int_{CV}\vec{r}\times\left[2\vec{\omega}\times\vec{V}_{xyz} + \vec{\omega}\times(\overset{=0(5)}{\cancel{\vec{\omega}\times\vec{r}}}) + \overset{=0(6)}{\cancel{\dot{\vec{\omega}}}}\times\vec{r}\right]\rho\,d\forall$$

$$= \overset{=0(6)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\vec{r}\times\vec{V}_{xyz}\,\rho\,d\forall + \int_{CS}\vec{r}\times\vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) $\vec{F}_s = 0$, (2) Body torques cancel, (3) $\vec{T}_{shaft} = 0$, (4) Neglect aerodynamic drag, (5) No $\hat{k}$ component of centripetal acceleration, (6) Steady flow, (7) $L \ll R$

Analyze one arm of sprinkler. From geometry, $\vec{r} = r\hat{i}$ in CV, $\vec{r} = R\hat{i}$ at jet. Then

$$-\int_{CV}\vec{r}\times[2\vec{\omega}\times\vec{V}_{xyz}]\rho\,d\forall = R\hat{i}\times(-V\sin\theta\,\hat{j})\,\rho\frac{Q}{3} = -\rho\frac{QRV}{3}\sin\theta\,\hat{k}$$

$$r\hat{i}\times(2\omega\hat{k}\times V\hat{i}) = 2\omega V r\,\hat{k}\;;\; -\int_{CV} = -\omega V R^2 \rho A\,\hat{k}$$

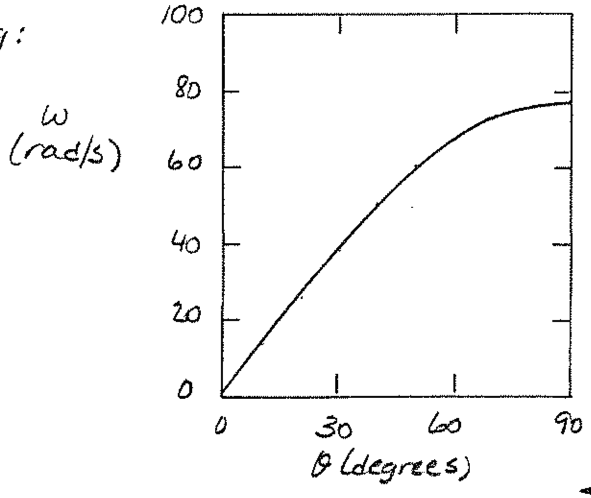
Dropping $\hat{k}$, $\quad -\omega V R^2 \rho A = -\rho\frac{QRV}{3}\sin\theta$, so with $VA = Q/3$,

$$\omega = \frac{V}{R}\sin\theta \quad ; \quad V = \frac{Q}{3A} = \frac{4Q}{3\pi d^2} = \frac{4}{3\pi}\times 68\times10^{-3}\frac{m^3}{min}\times\frac{1}{(0.00635)^2\,m^2}\times\frac{min}{60\,s} = 11.9\ m/s$$

$$\omega = 11.9\,\frac{m}{s}\times\frac{1}{0.152\,m}\times\sin\theta = 78.3\sin\theta\ rad/s$$

ω

Plotting:



For $\theta = 30°$,

$\omega = 78.3\sin 30°$

$\omega = 39.1$ rad/s

ω
$(\theta = 30°)$

Plot

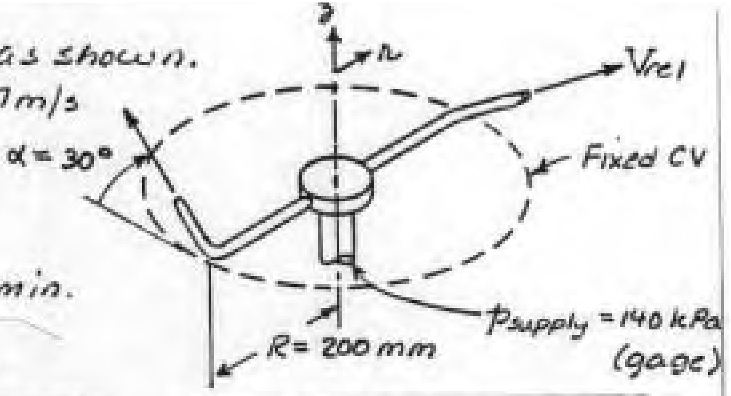
Problem *4.190

Given: Small lawn sprinkler as shown.

$V_{rel} = 17\ m/s$

Friction torque at pivot is $T_f = 0.18\ N\cdot m$.

Flowrate is $Q = 4.0$ liter/min.



Find: Torque to hold stationary.

Solution: Apply moment of momentum using fixed CV enclosing sprinkler arms.

Basic equation:

$$\overset{=0(1)}{\vec r \times \vec F_s} + \int_{CV} \overset{=0(2)}{\vec r \times \vec g}\, \rho\, dV + \vec T_{shaft} = \overset{=0(3)}{\frac{\partial}{\partial t}}\int_{CV} \vec r \times \vec V \rho\, dV + \int_{CS} \vec r \times \vec V \rho \vec V \cdot d\vec A$$

Assumptions: (1) Neglect torque due to surface forces

(2) Torques due to body forces cancel by symmetry

(3) Steady flow

(4) Uniform flow leaving each jet

Then

$$-T_f \hat k = (\vec r \times \vec V)_{in}\{-\rho Q\} + 2(\vec r \times \vec V)_{jet}\{\tfrac{1}{2}\rho Q\}$$

$(\vec r \times \vec V)_{in} \simeq 0$ $\qquad$ $\vec r = R\hat e_r$

$\vec V = (R\omega - V_{rel}\cos\alpha)\hat e_\theta + V_{rel}\sin\alpha\, \hat e_z$

The absolute velocity of the jet leaving sprinkler is $\vec V = V_{rel}[\cos\alpha(-\hat e_\theta) + \sin\alpha(\hat e_z)]$

Then $(\vec r \times \vec V)_z = \{R\hat e_r \times V_{rel}[\cos\alpha(-\hat e_\theta) + \sin\alpha(\hat e_z)]\}_z = \{RV_{rel}\cos\alpha(-\hat e_z) + RV_{rel}\sin\alpha(-\hat e_\theta)\}_z$

$(\vec r \times \vec V)_z = -RV_{rel}\cos\alpha$

Substituting, $T_{shaft} = T_{ext} - T_f = 2(-RV_{rel}\cos\alpha)(\tfrac{1}{2}\rho Q)$

Thus $T_{ext} = T_f - \rho Q R V_{rel}\cos\alpha$

$$= 0.18\ N\cdot m - 999\frac{kg}{m^3} \times 4\frac{L}{min} \times 0.2\,m \times 17\frac{m}{s} \times 0.866 \times \frac{m^3}{1000\,L} \times \frac{min}{60\,s} \times \frac{N\cdot s^2}{kg\cdot m}$$

$T_{ext} = -0.0161\ N\cdot m$ (to hold sprinkler stationary) $\qquad$ T_{ext}

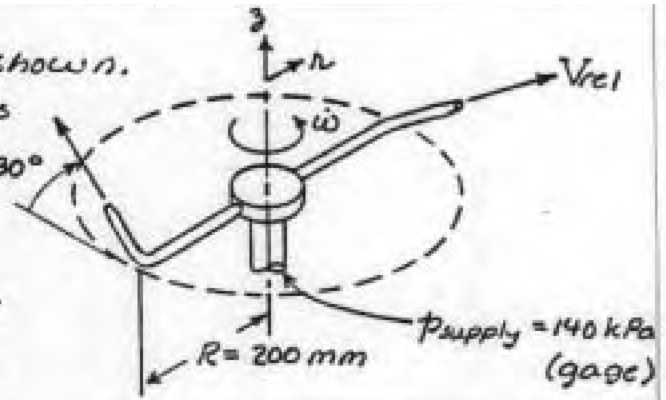
{ Since $T_{ext} < 0$, it must be applied in the minus z direction to oppose motion. }

Problem *4.191

Given: Small lawn sprinkler as shown.
$V_{rel} = 17\ m/s$

Friction torque at pivot is zero. $I = 0.1\ kg \cdot m^2$

Flowrate is $Q = 4.0$ liter/min.



Find: Initial angular acceleration from rest.

Solution: Apply moment of momentum using fixed CV enclosing sprinkler arms.

Basic equation:

$$\cancelto{\approx 0(1)}{\vec{r} \times \vec{F}_s} + \int_{CV} \cancelto{=0(2)}{\vec{r} \times \vec{g}}\, \rho\, d\forall + \vec{T}_{shaft} = \cancelto{=0(3)}{\frac{\partial}{\partial t}} \int_{CV} \vec{r} \times \vec{V} \rho\, d\forall + \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Neglect torque due to surface forces
(2) Torques due to body forces cancel by symmetry
(3) Steady flow
(4) Uniform flow leaving each jet

Then

$$-T_f \hat{k} = (\vec{r} \times \vec{V})_{in} \{-\rho Q\} + 2 (\vec{r} \times \vec{V})_{jet} \{\tfrac{1}{2} \rho Q\}$$

$$(\vec{r} \times \vec{V})_{in} \approx 0 \qquad \vec{r} = R\hat{e}_r$$
$$\vec{V} = (R\omega - V_{rel} \cos\alpha)\hat{e}_\theta + V_{rel} \sin\alpha\, \hat{e}_z$$

The jet leaves the sprinkler at $\vec{V}(abs) = V_{rel}[\cos\alpha(-\hat{e}_\theta) + \sin\alpha(\hat{e}_z)]$

Then $\vec{r} \times \vec{V} = R\hat{e}_r \times V_{rel}[\cos\alpha(-\hat{e}_\theta) + \sin\alpha(\hat{e}_z)] = RV_{rel}[\cos\alpha(-\hat{e}_z) + \sin\alpha(-\hat{e}_\theta)]$

Summing moments on the rotor, $\Sigma \vec{M} = I\dot{\vec{\omega}}$. Thus

$$\dot{\omega} = \frac{\Sigma T}{I} = \frac{\rho Q R V_{rel} \cos\alpha - T_f}{I}$$

$$= \left[\frac{999\ kg}{m^3} \times \frac{4\ L}{min} \times 0.2\ m \times \frac{17\ m}{s} \times 0.866 \times \frac{m^3}{1000\ L} \times \frac{min}{60\ s} - 0.18\ N \cdot m \times \frac{kg \cdot m}{N \cdot s^2} \right] \frac{1}{0.1\ kg \cdot m^2}$$

$\dot{\omega} = 0.161\ rad/s^2$ $\longleftarrow$ $\dot{\omega}$

{ It is not necessary to use a rotating CV, because at the instant considered, $\vec{\omega} = 0$ and I is known. }

Problem *4.192

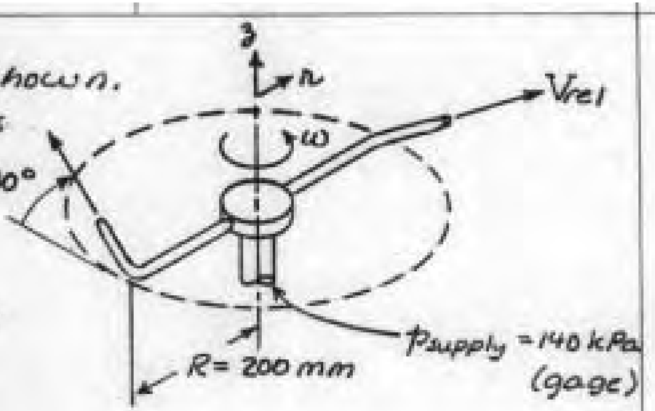
[4]

Given: Small lawn sprinkler as shown.

$V_{rel} = 17\,m/s$

Friction torque at pivot is $T_f = 0.18\ N\cdot m$.

Flowrate is $Q = 4.0\ liter/min$.



Find: (a) Steady speed of rotation.
(b) Area covered by spray.

Solution: Apply moment of momentum using fixed CV enclosing sprinkler arms.

Basic equation:

$$\overset{=0(1)}{\vec{r}\times\vec{F}_s} + \int_{CV} \overset{=0(2)}{\vec{r}\times\vec{g}}\,\rho\, dV + \vec{T}_{shaft} = \overset{=0(3)}{\frac{\partial}{\partial t}}\int_{CV} \vec{r}\times\vec{V}\,\rho\, dV + \int_{CS} \vec{r}\times\vec{V}\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Neglect torque due to surface forces
(2) Torques due to body forces cancel by symmetry
(3) Steady flow
(4) Uniform flow leaving each jet

Then

$$-T_f\,\hat{k} = (\vec{r}\times\vec{V})_{in}\{-\rho Q\} + 2(\vec{r}\times\vec{V})_{jet}\{\tfrac{1}{2}\rho Q\}$$

$$(\vec{r}\times\vec{V})_{in} \approx 0 \qquad \vec{r} = R\,\hat{\imath}_r$$

$$\vec{V} = (R\omega - V_{rel}\cos\alpha)\,\hat{\imath}_\theta + V_{rel}\sin\alpha\,\hat{\imath}_z$$

$$(\vec{r}\times\vec{V})_z = R(R\omega - V_{rel}\cos\alpha)$$

or

$$-T_f = R(R\omega - V_{rel}\cos\alpha)\,\rho Q$$

Thus

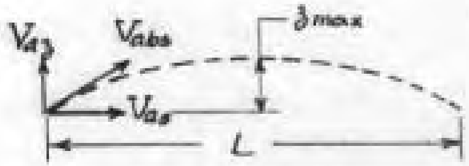
$$\omega = \frac{V_{rel}\cos\alpha}{R} - \frac{T_f}{\rho Q R^2}$$

$$= 17\frac{m}{s}\times\frac{\cos 30°}{0.2\,m} - 0.18\,N\cdot m\times\frac{m^3}{999\,kg}\times\frac{min}{4.0\,L}\times\frac{1}{(0.2)^2\,m^2}\times\frac{60\,s}{min}\times\frac{10^3\,L}{m^3}\times\frac{kg\cdot m}{N\cdot s^2}$$

$$\omega = 6.04\,\frac{rad}{s} \quad \text{or} \quad 57.7\,rpm$$

ω

Treat the spray outside each nozzle as moving without air resistance:



For each particle, $\frac{dV_{az}}{dt} = -g$, so $V_z = V_{z_0} - gt$

At z_{max}, $V_z = 0$, so $t = \frac{V_{z_0}}{g}$; flight time is $2t$.

$$L = 2t\,V_{a\theta} = \frac{2V_{az}}{g}V_{a\theta} = \frac{2V_{rel}\sin\alpha\,(V_{rel}\cos\alpha - R\omega)}{g}$$

$$L = 2\times 17\frac{m}{s}\times\sin 30°\left(17\frac{m}{s}\times\cos 30° - 0.2\,m\times 6.04\frac{rad}{s}\right)\frac{s^2}{9.81\,m} = 23.4\,m$$

$$R_{spray} = \sqrt{R^2 + L^2} = 23.4\,m\ ;\quad A_{spray} = \pi R_{spray}^2 = \pi\times(23.4)^2\,m^2 = 1720\,m^2$$

A_{spra}

Problem *4.193

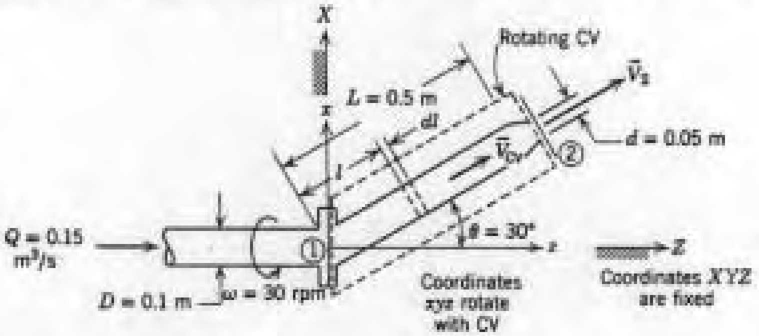
Open-Ended Problem Statement: When a garden hose is used to fill a bucket, water in the bucket may develop a swirling motion. Why does this happen? How could the amount of swirl be calculated approximately?

Discussion: Frequently when filling a bucket the hose is held so that the water stream entering the bucket is not vertical. If, in addition, the water stream is off-center in the bucket, then flow entering the bucket has a tangential component of velocity, a swirl component.

The tangential component of the water velocity entering the bucket has a moment-of-momentum (swirl) with respect to a control volume drawn around the stationary bucket. This entering swirl can only be reduced by a torque acting to oppose it. Viscous forces among fluid layers will tend to transfer swirl to other layers so that eventually all of the water in the bucket has a swirling motion.

Swirl in the bucket may be influenced by viscosity. The swirl may tend to nearly a rigid-body motion to minimize viscous forces between annular layers of water in the bucket. The rigid-body motion assumption may be a reasonable model to calculate the total angular momentum (moment-of-momentum) of the water in the bucket.

Given: Nozzle assembly rotating steadily, as shown in the sketch.



Find: (a) Torque required to drive the nozzle assembly
(b) Reaction torques at the flange.

Solution: Apply the moment of momentum equation to the rotating CV shown.

Basic equation:

$$\vec{r} \times \vec{F}_s + \int_{CV} \cancelto{\approx 0(2)}{\vec{r} \times \vec{g}}\, \rho\, d\forall + \vec{T}_{shaft}$$

$$-\int_{CV} \vec{r} \times \left[2\vec{\omega} \times \vec{V}_{xyz} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \cancelto{=0(3)}{\dot{\vec{\omega}} \times \vec{r}}\right] \rho\, d\forall = \cancelto{=0(5)}{\frac{\partial}{\partial t}\int_{CV} \vec{r} \times \vec{V}_{xyz}\, \rho\, d\forall} + \int_{CS} \cancelto{=0(7)}{\vec{r} \times \vec{V}_{xyz}}\, \rho \vec{V}_{xyz} \cdot d\vec{A}$$

Assumptions: (1) Let $\vec{T}_{cv}$ represent all torques acting on the CV
(2) Neglect torque due to body force
(3) Constant angular speed
(4) Neglect mass of arm compared to water inside
(5) Steady flow in CV
(6) Neglect nozzle length compared to L
(7) $\vec{r}$ colinear with $\vec{V}_1$ so $\vec{r} \times \vec{V}_{xyz} = 0$

Then

$$\vec{T}_{cv} = \int_{CV} \vec{r} \times \left[2\vec{\omega} \times \vec{V}_{xyz} + \vec{\omega} \times (\vec{\omega} \times \vec{r})\right] \rho\, d\forall$$

Since $\vec{\omega} = \omega \hat{k}$ and $\vec{r} = l(\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$, then

$$\vec{\omega} \times \vec{r} = \omega l \sin\theta\,\hat{j}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \omega\hat{k} \times \omega l \sin\theta\,\hat{j} = \omega^2 l \sin\theta\,(-\hat{i})$$

and $\vec{r} \times [\vec{\omega} \times (\vec{\omega} \times \vec{r})] = l(\sin\theta\,\hat{i} + \cos\theta\,\hat{k}) \times \omega^2 l \sin\theta\,(-\hat{i}) = \omega^2 l^2 \sin\theta \cos\theta\,(-\hat{j})$

Since $\vec{V}_{xyz} = V_{cv}(\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$, then

$$2\vec{\omega} \times \vec{V}_{xyz} = 2\omega\hat{k} \times V_{cv}(\sin\theta\,\hat{i} + \cos\theta\,\hat{k}) = 2\omega V_{cv} \sin\theta\,\hat{j}$$

and $\vec{r} \times [2\vec{\omega} \times \vec{V}_{xyz}] = l(\sin\theta\,\hat{i} + \cos\theta\,\hat{k}) \times 2\omega V_{cv} \sin\theta\,\hat{j} = 2\omega l V_{cv} \sin^2\theta\,\hat{k}$

$$+ 2\omega l V_{cv} \sin\theta \cos\theta\,(-\hat{i})$$

Substituting and introducing $d\forall = A\,dl$,

$$\vec{T}_{cv} = \int_0^L \left(-2\omega l V_{cv} \sin\theta \cos\theta\, \hat{i} - \omega^2 l^2 \sin\theta \cos\theta\, \hat{j} + 2\omega l V_{cv} \sin^2\theta\, \hat{k}\right) \rho A\, dl$$

$$\vec{T}_{cv} = \left[-\omega L^2 V_{cv} \sin\theta\cos\theta\, \hat{i} - \frac{\omega^2 L^3}{3} \sin\theta\cos\theta\, \hat{j} + \omega L^2 V_{cv} \sin^2\theta\, \hat{k}\right]\rho A$$

The shaft torque needed to maintain steady rotation of the assembly is

$$T_{shaft} = T_{cv_z} = \omega L^2 V_{cv} \sin^2\theta\, \rho A = \omega L^2 \frac{Q}{A} \sin^2\theta\, \rho A = \rho Q \omega L^2 \sin^2\theta$$

$$= 999\,\frac{kg}{m^3} \times 0.15\,\frac{m^3}{s} \times 30\,\frac{rev}{min} \times (0.5)^2 m^2 \times (0.5)^2 \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} \times \frac{N\cdot s^2}{kg\cdot m}$$

$T_{shaft} = 29.4\ N\cdot m$ ⟵ T_{shaft}

The reaction moments acting on the flange are

$$M_x = -T_{cv_x} = \omega L^2 V_{cv} \sin\theta\cos\theta\, \rho A = \rho Q \omega L^2 \sin\theta\cos\theta$$

$$= 999\,\frac{kg}{m^3} \times 0.15\,\frac{m^3}{s} \times 30\,\frac{rev}{min} \times (0.5)^2 m^2 \times (0.5)(0.866)\, 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} \times \frac{N\cdot s^2}{kg\cdot m}$$

$M_x = 51.0\ N\cdot m$ (applied to flange by CV) ⟵ M_x

$$M_y = -T_{cv_y} = \frac{1}{3}\rho\omega^2 L^3 A \sin\theta\cos\theta$$

$$= \frac{1}{3} \times 999\,\frac{kg}{m^3}\left[30\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s}\right]^2 (0.5)^3 m^3 \times \frac{\pi}{4}(0.1)^2 m^2 \times (0.5)(0.866) \times \frac{N\cdot s^2}{kg\cdot m}$$

$M_y = 1.40\ N\cdot m$ (applied to flange by CV) ⟵ M_y

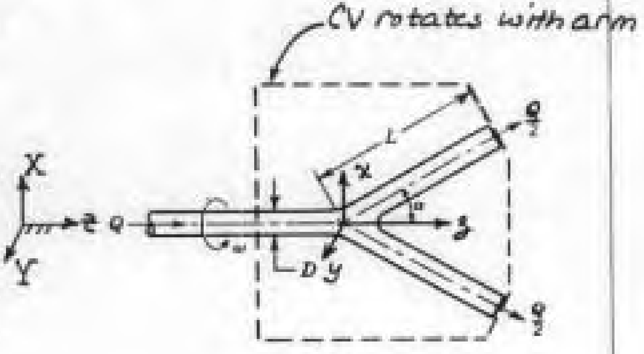
{ Torques due to the masses of water, tube, and nozzle must be considered in the overall design. }

Problem *4.195

Given: Branched pipe with symmetrical legs as shown.

Angular momentum zero at inlet, relative to nonrotating frame.

Find: (a) External torque expression
(b) Additional torque to produce angular acceleration of $\dot{\omega}$.



Solution: Apply moment of momentum equation using rotating CV.

Basic equation:

$$\overset{=0(1)}{\cancel{\vec{r}\times\vec{F}_s}} + \int_{CV} \overset{=0(2)}{\cancel{\vec{r}\times\vec{g}}}\,\rho\, d\forall + \vec{T}_{shaft}$$

$$-\int_{CV} \vec{r}\times\left[2\vec{\omega}\times\vec{V}_{xyz} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \dot{\vec{\omega}}\times\vec{r}\right]\rho\, d\forall = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\vec{r}\times\vec{V}_{xyz}\,\rho\, d\forall + \int_{CS}\overset{=0(4)}{\cancel{\vec{r}\times\vec{V}_{xyz}}}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A}$$

Assumptions: (1) No surface forces
(2) Body-forces produce no torque about axis (symmetry)
(3) Flow steady in rotating frame
(4) $\vec{r}$ and $\vec{V}_{xyz}$ are colinear: $\vec{r}\times\vec{V}_{xyz} = 0$

Then

$$\vec{T}_{shaft} = \int_{CV} \vec{r}\times\left[2\vec{\omega}\times\vec{V}_{xyz} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \dot{\vec{\omega}}\times\vec{r}\right]\rho\, d\forall$$

Using the coordinates above, $\vec{\omega} = \omega\hat{k}$ $\qquad \dot{\vec{\omega}} = \dot{\omega}\hat{k}$

$\vec{r} = r(\cos\alpha\,\hat{k} + \sin\alpha\,\hat{i})$ (upper tube)

$\vec{V}_{xyz} = \frac{Q}{2A}(\cos\alpha\,\hat{k} + \sin\alpha\,\hat{i})$ (upper tube); $A = \frac{\pi D^2}{4}$

and

$$\dot{\vec{\omega}}\times\vec{r} = \dot{\omega} r \sin\alpha\,\hat{j}$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}) = \omega\hat{k}\times\omega r\sin\alpha\,\hat{j} = -\omega^2 r\sin\alpha\,\hat{i}$$

$$2\vec{\omega}\times\vec{V}_{xyz} = 2\omega\frac{Q}{2A}\sin\alpha\,\hat{j} = \frac{\omega Q}{A}\sin\alpha\,\hat{j}$$

Thus for the upper tube,

$$\vec{T}_{shaft} = \int_0^L \left\{ r(\cos\alpha\,\hat{k} + \sin\alpha\,\hat{i})\times\left[\left(\frac{\omega Q}{A} + \dot{\omega} r\right)\sin\alpha\,\hat{j} - \omega^2 r\sin\alpha\,\hat{i}\right]\right\}\rho A\, dr$$

$$= \int_0^L \left[\left(\frac{r\omega Q}{A} + \dot{\omega} r^2\right)(\sin\alpha\cos\alpha)\,\hat{i} + \left(\frac{r\omega Q}{A} + \dot{\omega} r^2\right)\sin^2\alpha\,\hat{k} + \omega^2 r^2\sin\alpha\cos\alpha\,(-\hat{j})\right]\rho A\, dr$$

$$\vec{T}_{shaft}(upper) = \left(\frac{L^2\omega Q}{2A} + \frac{\dot{\omega} L^3}{3}\right)\sin\alpha\cos\alpha\,\hat{i} + \left(\frac{L^2\omega Q}{2A} + \frac{\dot{\omega} L^3}{3}\right)\sin^2\alpha\,\hat{k} + \frac{\omega^2 L^3}{3}\sin\alpha\cos\alpha\,(-\hat{j})\Big]\rho A$$

Problem *4.195

For the lower tube, $\vec{\omega} = \omega\hat{k}$ $\quad\quad \dot{\vec{\omega}} = \dot{\omega}\hat{k}$

$$\vec{r} = r(\cos\alpha\,\hat{k} - \sin\alpha\,\hat{i}) \quad \text{(lower tube)}$$

$$\vec{V}_{xyz} = \frac{Q}{2A}(\cos\alpha\,\hat{k} - \sin\alpha\,\hat{i}) \quad \text{(lower tube)}$$

and

$$\dot{\vec{\omega}} \times \vec{r} = -r\dot{\omega}\sin\alpha\,\hat{j}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \omega\hat{k} \times (-r\omega\sin\alpha\,\hat{j}) = r\omega^2\sin\alpha\,\hat{i}$$

$$2\vec{\omega} \times \vec{V}_{xyz} = 2\omega\frac{Q}{2A}(-\sin\alpha)(\hat{j}) = -\frac{\omega Q}{A}\sin\alpha\,\hat{j}$$

Thus for the lower tube,

$$\vec{T}_{shaft} = \int_0^L \left\{ r(\cos\alpha\,\hat{k} - \sin\alpha\,\hat{i}) \times \left[\left(\frac{\omega Q}{A} + r\dot{\omega}\right)\sin\alpha(-\hat{j}) + r\omega^2\sin\alpha \right] \right\} \rho A\,dr$$

$$= \int_0^L \left[\left(\frac{r\omega Q}{A} + r^2\dot{\omega}\right)\sin\alpha\cos\alpha(-\hat{i}) + \left(\frac{r\omega Q}{A} + r^2\dot{\omega}\right)\sin^2\alpha\,\hat{k} + r^2\omega^2\sin\alpha\cos\alpha\,\hat{j} \right] \rho A\,dr$$

$$\vec{T}_{shaft}(\text{lower}) = \left[\left(\frac{L^2\omega Q}{2A} + \frac{L^3\dot{\omega}}{3}\right)\sin\alpha\cos\alpha\,\hat{i} + \left(\frac{L^2\omega Q}{2A} + \frac{L^3\dot{\omega}}{3}\right)\sin^2\alpha\,\hat{k} + \frac{L^3\omega^2}{3}\sin\alpha\cos\alpha\,\hat{j} \right] \rho A$$

Summing these expressions gives

$$\vec{T}_{shaft}(\text{total}) = \left(\frac{L^2\omega Q}{A} + \frac{2L^3\dot{\omega}}{3}\right)\sin^2\alpha\,\rho A\,\hat{k}$$

Thus the steady-state portion of the torque is

$$\vec{T}_{shaft}(\text{steady state}) = \left(\frac{L^2\omega Q}{A}\right)\sin^2\alpha\,\rho A\,\hat{k} = L^2\rho\omega Q\sin^2\alpha\,\hat{k}$$

Stea

The additional torque needed to provide angular acceleration, $\dot{\omega}$, is

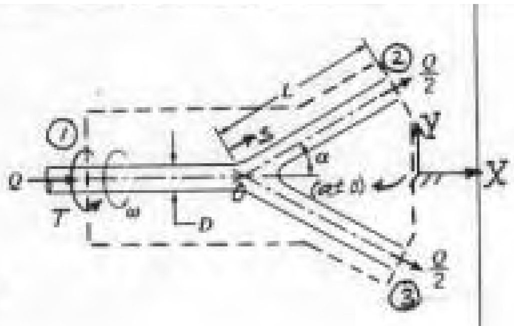
$$\vec{T}_{shaft}(\text{acceleration}) = \frac{2L^3\rho\dot{\omega}A}{3}\sin^2\alpha\,\hat{k}$$

Accel

{ Torques of individual tubes about the x and y axes are reacted internally; they must be considered in design of the tube. }

Problem *4.195

(b) Using fixed CV:



Basic equation: $\overset{=0(1)}{\cancel{\vec{r}\times\vec{F}_s}} + \int_{CV} \overset{=0(2)}{\cancel{\vec{r}\times\vec{g}}}\,\rho\, d\forall + \vec{T}_{shaft}$

$$= \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \vec{r}\times\vec{V}\rho\, d\forall + \int_{CS} \vec{r}\times\vec{V}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) No surface forces

(2) Body forces symmetric (no moment about X axis)

(3) No change in angular momentum within CV w.r.to time

(4) Symmetry in two branches

(5) Uniform flow at each cross-section

Then $\vec{T}_s = T\hat{I} = \cancel{\vec{r}_1}\times\vec{V}_1\{-\rho Q\} + \vec{r}_2\times\vec{V}_2\{+\rho\frac{Q}{2}\} + \vec{r}_3\times\vec{V}_3\{+\rho\frac{Q}{2}\} = 2\vec{r}_2\times\vec{V}_2\{\rho\frac{Q}{2}\}$

$\vec{r}_1 = 0$ $\quad\quad$ $\vec{r}_2 = L\sin\alpha\,\hat{J}$; $\vec{V}_2 = \omega r_2\,\hat{K}$; $\vec{r}_2\times\vec{V}_2 = \omega L^2\sin^2\alpha\,\hat{I}$

or

$T_{ss} = \rho\omega Q L^2 \sin^2\alpha$ (steady-state torque) $\longleftarrow$ T_{ss}

The torque required for acceleration is $T_{acc} = I\dot{\omega}$, where $I = \int r^2 dm$

For one leg of the branch, $I = \int r^2 dm = \int_0^L (s\sin\alpha)^2 \rho A\, ds = \frac{\rho A L^3}{3}\sin^2\alpha$

(6) Neglect mass of pipe

For both sides, $I = \frac{2\rho A L^3}{3}\sin^2\alpha$.

Thus

$T_{acc} = \frac{2\rho\dot{\omega}AL^3}{3}\sin^2\alpha$ (torque required for angular acceleration) $\longleftarrow$ T_{acc}

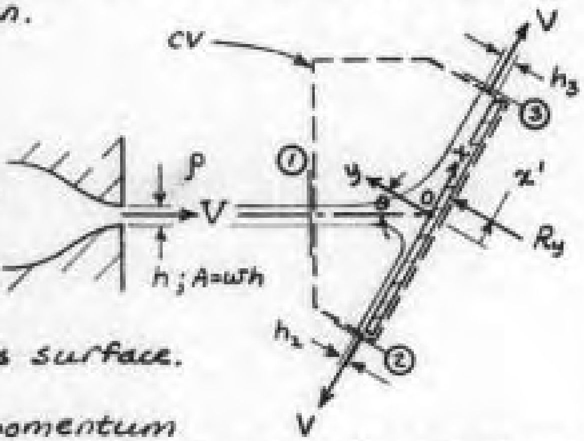
The total torque that must be applied is

$T = T_{ss} + T_{acc} = \rho\omega Q L^2\sin^2\alpha + \frac{2\rho\dot{\omega}A}{3}L^3\sin^2\alpha$ $\longleftarrow$ T_{total}

Problem *4.196

Given: Thin sheet of liquid, of width, w, and thickness, h, striking inclined flat plate, as shown.

Neglect any viscous effects.



Find: (a) Magnitude and line of action of resultant force as functions of θ.
(b) Equilibrium angle of plate if force is applied at point O, where jet centerline intersects surface.

Solution: Apply continuity, linear momentum and moment of momentum using CV and coordinates shown.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\overset{=0(4)}{\cancel{F_{S_x}}} + \overset{=0(5)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u \rho \, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

$$F_{S_y} + \overset{=0(5)}{\cancel{F_{B_y}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} v \rho \, d\forall + \int_{CS} v \rho \vec{V} \cdot d\vec{A}$$

$$\vec{r} \times \vec{F}_S + \overset{=0(5)}{\cancel{\int_{CV} \vec{r} \times \vec{g} \, \rho \, d\forall}} + \overset{=0(6)}{\cancel{\vec{T}_{shaft}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} \vec{r} \times \vec{V} \rho \, d\forall + \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} \cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Uniform flow at each section
(3) No net pressure forces; $F_{S_x} = R_x$, $F_{S_y} = R_y$
(4) No viscous effects; $R_x = 0$ and $V_1 = V_2 = V_3 = V$
(5) Neglect body forces and torques
(6) $\vec{T}_{shaft} = 0$
(7) Incompressible flow, ρ = constant

Then from continuity,

$$0 = \{-|\rho V w h_1|\} + \{|\rho V w h_2|\} + \{|\rho V w h_3|\} \quad \text{or} \quad h_1 = h_2 + h_3 = h \tag{1}$$

From x momentum

$$0 = u_1\{-|\rho V w h_1|\} + u_2\{|\rho V w h_2|\} + u_3\{|\rho V w h_3|\}$$

$$u_1 = V \sin\theta \qquad u_2 = -V \qquad u_3 = V$$

$$0 = \rho V^2 w (-h_1 \sin\theta - h_2 + h_3) \quad \text{or} \quad h_3 - h_2 = h_1 \sin\theta = h \sin\theta \tag{2}$$

Combining Eqs. 1 and 2,

$$h_2 = h\left(\frac{1 - \sin\theta}{2}\right) \tag{3}$$

$$h_3 = h\left(\frac{1 + \sin\theta}{2}\right) \tag{4}$$

Problem *4.196

[5] Part 2/2

From y momentum, $R_y = v_1\{-|\rho V w h_1|\} + v_2\{|\rho V w h_2|\} + v_3\{|\rho V w h_3|\}$

$$v_1 = -V\cos\theta \qquad v_2 = 0 \qquad v_3 = 0$$

$$R_y = \rho V^2 w h\cos\theta$$

(5) R_y

From moment of momentum,

$$\vec{r}\,' \times \vec{F}_s = \vec{r}_1 \times \vec{V}_1\{-|\rho V w h_1|\} + \vec{r}_2 \times \vec{V}_2\{|\rho V w h_2|\} + \vec{r}_3 \times \vec{V}_3\{|\rho V w h_3|\}$$

$\vec{r}\,' = x'\hat{i}$ $\qquad$ $\vec{r}_1 \times \vec{V}_1 = 0$ $\qquad$ $\vec{r}_2 = \frac{h_2}{2}\hat{j}$ $\qquad$ $\vec{r}_3 = \frac{h_3}{2}\hat{j}$

$\vec{F}_s = R_y\hat{j}$ $\qquad$ $\vec{V}_2 = -V\hat{i}$ $\qquad$ $\vec{V}_3 = V\hat{i}$

$\vec{r}\,' \times \vec{F}_s = x' R_y\hat{k}$ $\qquad$ $\vec{r}_2 \times \vec{V}_2 = \frac{h_2 V}{2}\hat{k}$ $\qquad$ $\vec{r}_3 \times \vec{V}_3 = -\frac{h_3 V}{2}\hat{k}$

Combining and dropping $\hat{k}$,

$$x' R_y = \frac{1}{2}\rho V^2 w h_2^2 - \frac{1}{2}\rho V^2 w h_3^2 = \frac{1}{2}\rho V^2 w (h_2^2 - h_3^2)$$

or

$$x' = \frac{\rho V^2 w (h_2^2 - h_3^2)}{2R_y} = \frac{\rho V^2 w (h_2 + h_3)(h_2 - h_3)}{2R_y}$$

Substituting from Eqs. 3, 4 and 5,

$$x' = \frac{\rho V^2 w h^2 \left(\frac{1-\sin\theta}{2} + \frac{1+\sin\theta}{2}\right)\left(\frac{1-\sin\theta}{2} - \frac{1+\sin\theta}{2}\right)}{2\rho V^2 w h\cos\theta} = \frac{h(-\sin\theta)}{2\cos\theta}$$

or

$$x' = -\frac{h}{2}\tan\theta$$

(6) x'

Note that $x' < 0$. This means that R_y must be applied below point 0.

If R_y is applied at point 0, then $x' = 0$. For equilibrium, from Eq. 6, $\theta = 0$. Thus if force is applied at point 0, plate will be in equilibrium when perpendicular to jet.

Problem *4.197

Given: The rotating lawn sprinkler of Example Problem 4.14.

Find:
- (a) Jet angle α for maximum speed of rotation.
- (b) What jet angle will provide the maximum area of coverage by the spray?
- (c) Draw a velocity diagram to show the absolute velocity of the water jet leaving the nozzle.
- (d) What governs the steady rotational speed of the sprinkler?
- (e) Does the rotational speed of the sprinkler affect the area covered by the spray?
- (f) How would you estimate the area of coverage?
- (g) For fixed α, what might be done to increase or reduce the area covered by the spray?

Solution: The results of Example Problem 4.14 were computed assuming steady flow of water and constant frictional retarding torque at the sprinkler pivot.

$$T_f = R(V_{rel}\cos\alpha - \omega R)\rho Q$$

From these results,

$$\omega = \frac{V_{rel}\cos\alpha}{R} - \frac{T_f}{\rho Q R^2}$$

Thus rotational speed of the sprinkler increases as cos α increases, i.e., as α decreases. The maximum rotational speed occurs when α = 0. Then cos α = 1 and the rotational speed is

$$\omega = \frac{V_{rel}}{R} - \frac{T_f}{\rho Q R^2}$$

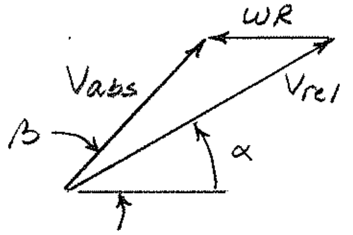
For the conditions of Example Problem 4.14 the maximum rotational speed is

$$\omega = 4.97\,\frac{m}{s} \times \frac{1}{0.150\,m} - 0.0718\,N\cdot m \times \frac{m^3}{999\,kg} \times \frac{min}{7.5\,L} \times \frac{1}{(0.150)^2\,m^2} \times \frac{1000\,L}{m^3} \times \frac{60\,s}{min} = 7.58\ rad/s$$

ω_{ma}

The steady rotation speed ω of the sprinkler is governed by torque T_f and angle α.

Maximum coverage by the spray occurs when the "carry" of each jet stream is the longest. When aerodynamic drag on the stream is neglected, maximum carry occurs when the absolute velocity of the stream leaves the sprinkler at β = 45°, as shown in the velocity diagram below.



Note $\vec{V}_{abs} = \vec{V}_{rel} - \omega R\,\hat{i}_\theta$

Both the magnitude and direction of $\vec{V}_{abs}$ vary with ω!

For ω = 0, the relative velocity angle α and absolute velocity angle β are equal. Therefore maximum carry occurs when α = 45° (see graph on next page).

Any rotation rate ω reduces the magnitude V_{abs} and increases the angle β of the absolute velocity leaving the sprinkler jet. When ω > 0, then β > α, so for maximum carry α must be less than 45°. Consequently rotation reduces the carry of the stream and the area of coverage; at specified α the area of coverage decreases with increasing ω.

For the conditions of Example Problem 4.14 (ω = 30 rpm), optimum carry occurs at α ≈ 42°, and the coverage area is reduced from approximately 20 m^2 with a fixed sprinkler to 15 m^2 with 30 rpm rotation. If the rotation speed is increased (by decreasing pivot friction or decreasing nozzle angle α), coverage area may be reduced still further, to 9 m^2 or less.

$$A \approx \pi (x_{max})^2$$

Problem *4.197

[5] Part 2/2

Analysis of Ground Area Covered by Rotating Lawn Sprinkler:

Variables:

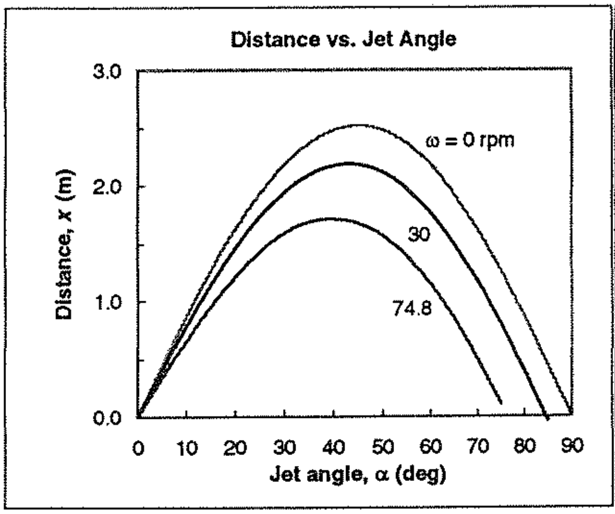
A = ground area covered by spray stream
x = ground distance reached by spray stream
α = angle of jet above ground plane
β = angle of absolute velocity above ground plane

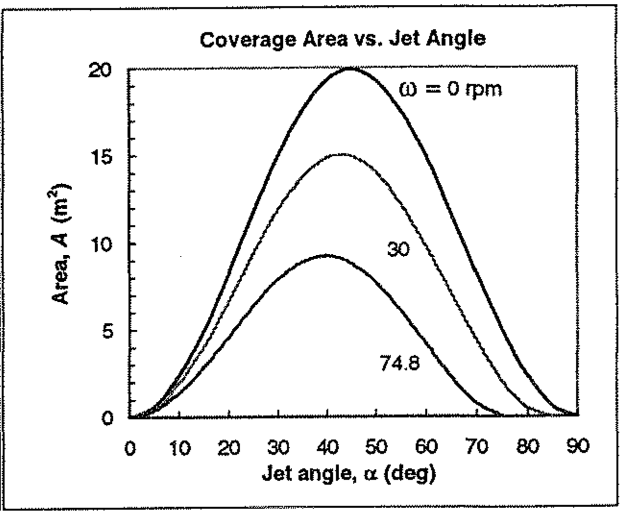
Input Data:

R = 0.150 m
V_{rel} = 4.97 m/s (Q = 7.5 L/min)

Results:

	ω (rpm) =	0		30		74.8
	ωR (m/s) =	0		0.471		1.17
α (deg)	x_{max} (m)	A (m^2)	x_{max} (m)	A (m^2)	x_{max} (m)	A (m^2)
0	0.00	0.00	0.00	0.00	0.00	0.00
5	0.437	0.601	0.396	0.492	0.333	0.349
10	0.861	2.33	0.778	1.90	0.654	1.35
15	1.26	4.98	1.14	4.05	0.951	2.84
20	1.62	8.23	1.46	6.65	1.21	4.61
25	1.93	11.7	1.73	9.37	1.43	6.39
30	2.18	14.9	1.94	11.8	1.59	7.90
35	2.37	17.6	2.09	13.8	1.68	8.90
40	2.48	19.3	2.17	14.8	1.71	9.23
45	2.52	19.9	2.18	14.9	1.68	8.83
50	2.48	19.3	2.11	14.0	1.57	7.72
55	2.37	17.6	1.97	12.3	1.39	6.08
60	2.18	14.9	1.77	9.81	1.15	4.15
65	1.93	11.7	1.50	7.03	0.850	2.269
70	1.62	8.23	1.17	4.30	0.500	0.785
75	1.26	4.98	0.798	2.00	0.109	0.037
78	1.02	3.30	0.557	0.975		
80	0.861	2.33	0.391	0.480		
85	0.437	0.601	-0.04	0.00		
90	0.00	0.00				


Distance vs. Jet Angle


Coverage Area vs. Jet Angle

Problem 4.198

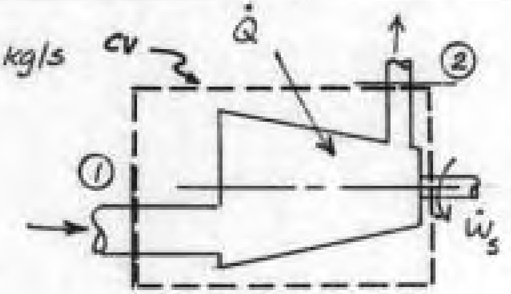
[2]

Given: Compressor, $\dot m = 1.0\ \text{kg/s}$

$p_1 = 101\ \text{kPa (abs)}$

$T_1 = 288\ \text{K}$

$V_1 = 75\ \text{m/s}$



$p_2 = 200\ \text{kPa (abs)}$

$T_2 = 345\ \text{K}$

$V_2 = 125\ \text{m/s}$

$\frac{dQ}{dm} = -18\ \text{kJ/kg}$

Find: Power required.

Solution: Apply first law of thermodynamics, using CV shown.

B.E. $$\dot Q - \dot W_s - \overset{=0(1)}{\cancel{\dot W_{shear}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} e\rho\, d\forall + \int_{CS}(e+pv)\rho\vec V\cdot d\vec A$$

Assume: (1) $\dot W_{shear} = 0$

(2) Steady flow

(3) Uniform flow at each section

(4) Neglect Δz

(5) Ideal gas, $p = \rho RT$, $\Delta h = c_p\Delta T$; $c_p = 1.00\ \text{kJ/kg}\cdot\text{K}$

(6) From continuity, $\dot m_1 = \dot m_2 = \dot m$

Then $$\dot Q - \dot W_s = \left(u_2 + \frac{V_2^2}{2} + \overset{\approx 0(4)}{\cancel{gz_2}} + p_2v_2\right)\{|\dot m|\} + \left(u_1 + \frac{V_1^2}{2} + \overset{\approx 0(4)}{\cancel{gz_1}} + p_1v_1\right)\{-|\dot m|\}$$

Note that $h = u + pv$, and $\dot Q = \dot m\frac{dQ}{dm}$, so

$$\dot W_{in} = -\dot W_s = \dot m\left(\frac{V_2^2 - V_1^2}{2} + h_2 - h_1 - \frac{dQ}{dm}\right) = \dot m\left[\frac{V_2^2 - V_1^2}{2} + c_p(T_2 - T_1) - \frac{dQ}{dm}\right]$$

or

$$\dot W_{in} = 1.0\frac{\text{kg}}{\text{s}}\left\{\frac{1}{2}\left[(125)^2 - (75)^2\right]\frac{\text{m}^2}{\text{s}^2}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\times\frac{\text{kJ}}{1000\,\text{N}\cdot\text{m}} + 1.00\frac{\text{kJ}}{\text{kg}\cdot\text{K}}(345 - 288)\,\text{K} - \left(-18\frac{\text{kJ}}{\text{kg}}\right)\right\}\frac{\text{kW}\cdot\text{s}}{\text{kJ}}$$

$\dot W_{in} = 80.0\ \text{kW}$ ← $\dot W_{in}$

Problem 4.199

4.199 Compressed air is stored in a pressure bottle with a volume of 0.5 m^3, at 20 MPa and 60°C. At a certain instant a valve is opened and mass flows from the bottle at $\dot{m} = 0.05$ kg/s. Find the rate of change of temperature in the bottle at this instant.

Given: Compressed air bottle

Find: Rate of temperature change

Solution:

Basic equations: Continuity; First Law of Thermodynamics for a CV

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad \dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\, d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Adiabatic 2) No work 3) Neglect KE 4) Uniform properties at exit 5) Ideal gas

From continuity $$\frac{\partial}{\partial t}M_{CV} + m_{exit} = 0$$ where m_{exit} is the mass flow rate at the exit (Note: Software does not allow a dot!)

$$\frac{\partial}{\partial t}M_{CV} = -m_{exit}$$

From the 1st law $$0 = \frac{\partial}{\partial t}\int u\, dM + \left(u + \frac{p}{\rho}\right)\cdot m_{exit} = u\cdot\left(\frac{\partial}{\partial t}M\right) + M\cdot\left(\frac{\partial}{\partial t}u\right) + \left(u + \frac{p}{\rho}\right)\cdot m_{exit}$$

Hence $$u\cdot\left(-m_{exit}\right) + M\cdot c_v\cdot\frac{dT}{dt} + u\cdot m_{exit} + \frac{p}{\rho}\cdot m_{exit} = 0 \qquad \frac{dT}{dt} = -\frac{m_{exit}\cdot p}{M\cdot c_v\cdot\rho}$$

But $M = \rho\cdot Vol$ so $$\frac{dT}{dt} = -\frac{m_{exit}\cdot p}{Vol\cdot c_v\cdot\rho^2}$$

For air $$\rho = \frac{p}{R\cdot T} \qquad \rho = 20\times 10^6\cdot\frac{N}{m^2}\times\frac{kg\cdot K}{286.9\cdot N\cdot m}\times\frac{1}{(60+273)\cdot K} \qquad \rho = 209\frac{kg}{m^3}$$

Hence $$\frac{dT}{dt} = -0.05\cdot\frac{kg}{s}\times 20\times 10^6\cdot\frac{N}{m^2}\times\frac{1}{0.5\cdot m^3}\times\frac{kg\cdot K}{717.4\cdot N\cdot m}\times\left(\frac{m^3}{209\cdot kg}\right)^2 = -0.064\cdot\frac{K}{s}$$

Problem 4.200

[3]

4.200 A centrifugal water pump with a 0.1-m diameter inlet and a 0.1-m diameter discharge pipe has a flow rate of 0.02 m^3/s. The inlet pressure is 0.2 m Hg vacuum and the exit pressure is 240 kPa. The inlet and outlet sections are located at the same elevation. The measured power input is 6.75 kW. Determine the pump efficiency.

Given:
Data on centrifugal water pump

Find:
Pump efficiency

Solution:

Basic equations:

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\,d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} \qquad (4.56)$$

$$\Delta p = SG_{Hg}\cdot\rho\cdot g\cdot\Delta h \qquad \eta = \frac{W_s}{P_{in}}$$

Available data:

$D_1 = 0.1\cdot m$ $\quad D_2 = 0.1\cdot m$ $\quad Q = 0.02\cdot\frac{m^3}{s}$ $\quad P_{in} = 6.75\cdot kW$

$\rho = 1000\frac{kg}{m^3}$ $\quad SG_{Hg} = 13.6$ $\quad h_1 = -0.2\cdot m$ $\quad p_2 = 240\cdot kPa$

Assumptions: 1) Adiabatic 2) Only shaft work 3) Steady 4) Neglect Δu 5) $\Delta z = 0$ 6) Incompressible 7) Uniform flow

Then $$-W_s = \left(p_1\cdot v_1 + \frac{V_1^2}{2}\right)\cdot\left(-m_{rate}\right) + \left(p_2\cdot v_2 + \frac{V_2^2}{2}\right)\cdot\left(m_{rate}\right)$$

Since $m_{rate} = \rho\cdot Q$ and $V_1 = V_2$ (from continuity)

$$-W_s = \rho\cdot Q\cdot\left(p_2\cdot v_2 - p_1\cdot v_1\right) = Q\cdot\left(p_2 - p_1\right)$$

$p_1 = \rho_{Hg}\cdot g\cdot h$ or $p_1 = SG_{Hg}\cdot\rho\cdot g\cdot h_1$ $\quad p_1 = -26.7\,kPa$

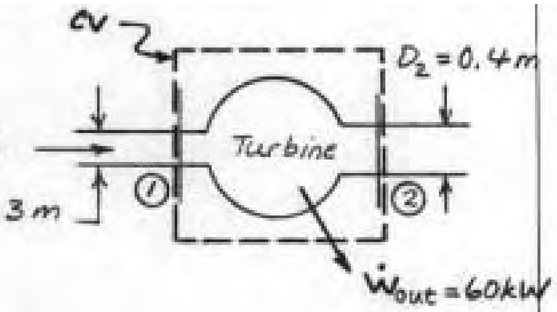
$W_s = Q\cdot\left(p_1 - p_2\right)$ $\quad W_s = -5.33\,kW$ The negative sign indicates work in

$\eta = \frac{\left|W_s\right|}{P_{in}}$ $\quad \eta = 79.0\,\%$

Problem 4.201

[2]

Given: Turbine operating on water.



Find: Pressure drop across turbine.

Solution: Apply continuity, energy equations, using CV shown.

Basic equations: $0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho\, dV + \int_{CS} \rho \vec{V}\cdot d\vec{A}$

$$\overset{=0(4)}{\cancel{\dot Q}} - \dot W_s - \overset{=0(5)}{\cancel{\dot W_{shear}}} - \overset{=0(5)}{\cancel{\dot W_{other}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} e\rho\, dV + \int_{CS}\left(\overset{\approx 0(6)}{\cancel{u}} + \frac{V^2}{2} + \overset{\approx 0(7)}{\cancel{gz}} + pv\right)\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Uniform flow at each section
(3) Incompressible flow
(4) $\dot Q = 0$
(5) $\dot W_{shear} = 0$ by choice of CV ; $\dot W_{other} = 0$
(6) Neglect Δu
(7) Neglect Δz

Then

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} \quad \text{or} \quad V_2 = V_1\frac{A_1}{A_2} = V_1\left(\frac{D_1}{D_2}\right)^2$$

and

$$-\dot W_s = \left(\frac{V_1^2}{2} + p_1 v\right)\{-|\rho V_1 A_1|\} + \left(\frac{V_2^2}{2} + p_2 v\right)\{|\rho V_2 A_2|\}$$

$$-\dot W_s = -\left[\frac{V_1^2 - V_2^2}{2} + (p_1 - p_2)v\right]\rho Q = -\left\{\frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] + (p_1 - p_2)v\right\}\rho Q$$

or

$$p_1 - p_2 = \frac{1}{v}\left\{\frac{\dot W_s}{\rho Q} - \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]\right\} = \frac{\dot W_s}{Q} - \frac{\rho V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

But $V_1 = \frac{Q}{A_1} = 0.6\frac{m^3}{s} \times \frac{4}{\pi}\frac{1}{(0.3)^2 m^2} = 8.49\ m/s$, and $\dot W_s = \dot W_{out} = 60\ kW$, so

$$p_1 - p_2 = (60\ kW)\, 10^3\frac{N\cdot m}{kW\cdot s} \times \frac{s}{0.6\,m^3} - \frac{1}{2} \times 999\frac{kg}{m^3} \times (8.49)^2\frac{m^2}{s^2}\left[1 - \left(\frac{0.3}{0.4}\right)^4\right]\frac{N\cdot s^2}{kg\cdot m}$$

$p_1 - p_2 = 75.4\ kPa$

$p_1 - p_2$

Problem 4.202

[2]

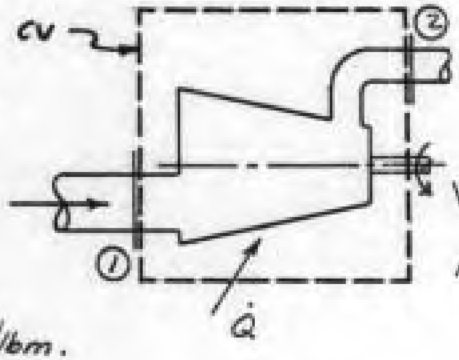
Given: Compressor operating at conditions shown

$\dot m = 20\ \frac{lbm}{s}$

$p_1 = 14\ psia$

$T_1 = 80°F$

$V_1 \simeq 0$



$p_2 = 70\ psia$

$T_2 = 500°F$

$V_2 = 500\ \frac{ft}{s}$

$\dot W_{in} = 3200\ hp$

Fluid is air.

Find: Heat transfer, in Btu/lbm.

Solution: Apply energy equation to CV shown.

Basic equations: $p = \rho RT$, $\Delta h = c_p \Delta T$

$$\dot Q - \dot W_s - \overset{=0(2)}{\cancel{\dot W_{shear}}} - \overset{=0(2)}{\cancel{\dot W_{other}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} e\rho\, d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + \overset{=0(5)}{\cancel{gz}}\right)\rho \vec V \cdot d\vec A$$

Assumptions:
(1) Ideal gas, constant specific heat
(2) $\dot W_{shear} = 0$ by choice of CV; $\dot W_{other} = 0$
(3) Steady flow
(4) Uniform flow at each section
(5) Neglect Δz
(6) $V_1 \approx 0$

By definition $h \equiv u + pv$, so

$$\dot Q - \dot W_s = \left(h_1 + \overset{\approx 0(6)}{\cancel{\frac{V_1^2}{2}}}\right)\{-|\dot m|\} + \left(h_2 + \frac{V_2^2}{2}\right)\{|\dot m|\} = \dot m\left[\frac{V_2^2}{2} + c_p(T_2 - T_1)\right]$$

or

$$\frac{\delta Q}{dm} = \frac{\dot Q}{\dot m} = \frac{\dot W_s}{\dot m} + \frac{V_2^2}{2} + c_p(T_2 - T_1)$$

Noting $\dot W_s = -3200\ hp$, so

$$\frac{\delta Q}{dm} = -3200\ hp \times \frac{2545\ Btu}{hp \cdot hr} \times \frac{s}{20\ lbm} \times \frac{hr}{3600\ s} + 0.240\ \frac{Btu}{lbm \cdot °F} \times (500 - 80)°F$$

$$+ \frac{(500)^2}{2}\frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{slug}{32.2\ lbm} \times \frac{Btu}{778\ ft \cdot lbf}$$

$$\frac{\delta Q}{dm} = -7.32\ Btu/lbm$$

$\frac{\delta Q}{dm}$

Therefore heat transfer is out of CV, since $\delta Q/dm < 0$. The rate of heat transfer is

$$\dot Q = -7.32\ \frac{Btu}{lbm} \times 20\ \frac{lbm}{s} = -146\ Btu/s$$

$\dot Q$

Problem 4.203

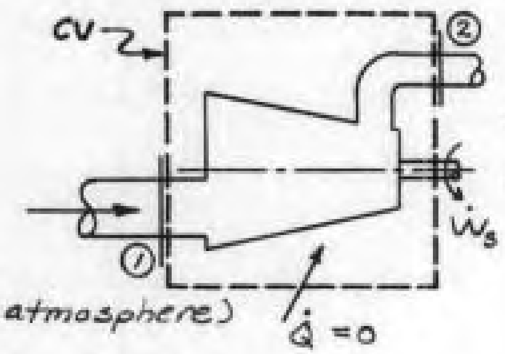
[2]

Given: Flow through turbomachine shown. Fluid is air.

$\dot m = 0.8\ \mathrm{kg/s}$

$T_1 = 288\ \mathrm{K}$

$p_1 = 101\ \mathrm{kPa\ (abs)}$



$T_2 = 130°\mathrm{C}$

$p_2 = 500\ \mathrm{kPa\ (gage)}$

$V_2 = 100\ \mathrm{m/s}$

$V_1 \simeq 0$ (from atmosphere)

Find: Shaft work interaction with surroundings.

Solution: Apply energy equation, using CV shown.

Basic equations: $p = \rho R T$, $\Delta h = c_p \Delta T$

$$\overset{=0(7)}{\cancel{\dot Q}} - \dot W_s - \overset{=0(2)}{\cancel{\dot W_{shear}}} - \overset{=0(2)}{\cancel{\dot W_{other}}} = \overset{=0(3)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} e\rho\, d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + \overset{\approx 0(5)}{\cancel{gz}}\right)\rho \vec V \cdot d\vec A$$

Assumptions: (1) Ideal gas, constant specific heat
(2) $\dot W_{shear} = 0$ by choice of CV; $\dot W_{other} = 0$
(3) Steady flow
(4) Uniform flow at each section
(5) Neglect Δz
(6) $V_1 \simeq 0$
(7) $\dot Q = 0$

By definition, $h \equiv u + pv$, so

$$-\dot W_s = \left(h_1 + \overset{\simeq 0(1)}{\cancel{\frac{V_1^2}{2}}}\right)\{-|\dot m|\} + \left(h_2 + \frac{V_2^2}{2}\right)\{|\dot m|\} = \dot m\left(h_2 - h_1 + \frac{V_2^2}{2}\right)$$

or

$$-\dot W_s = \dot m\left(h_2 - h_1 + \frac{V_2^2}{2}\right) = \dot m\left[c_p(T_2 - T_1) + \frac{V_2^2}{2}\right]$$

$$= 0.8\,\frac{\mathrm{kg}}{\mathrm{s}}\left[1.00\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}(403 - 288)\,\mathrm{K} + (100)^2\,\frac{\mathrm{m}^2}{\mathrm{s}^2} \times \frac{\mathrm{N\cdot s^2}}{\mathrm{kg\cdot m}} \times \frac{\mathrm{kJ}}{10^3\,\mathrm{N\cdot m}}\right]\frac{\mathrm{kW\cdot s}}{\mathrm{kJ}}$$

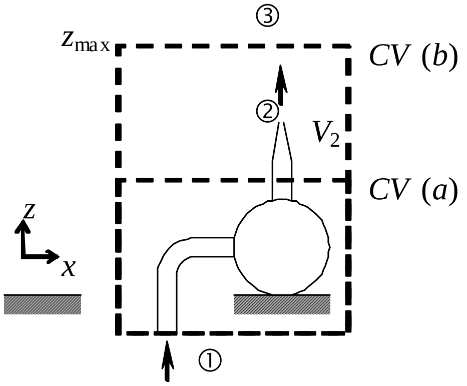
$-\dot W_s = 96.0\ \mathrm{kW}$ or $\dot W_s = -96.0\ \mathrm{kW}$ ←——— $\dot W_s$

{Power is into CV because $\dot W_s < 0$.}

Problem 4.204

[3]

4.204 All major harbors are equipped with fire boats for extinguishing ship fires. A 3-in. diameter hose is attached to the discharge of a 15-hp pump on such a boat. The nozzle attached to the end of the hose has a diameter of 1 in. If the nozzle discharge is held 10 ft above the surface of the water, determine the volume flow rate through the nozzle, the maximum height to which the water will rise, and the force on the boat if the water jet is directed horizontally over the stern.



Given: Data on fire boat hose system

Find: Volume flow rate of nozzle; Maximum water height; Force on boat

Solution:

Basic equation: First Law of Thermodynamics for a CV

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t}\int_{\text{CV}} e\,\rho\,d\forall + \int_{\text{CS}}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Neglect losses 2) No work 3) Neglect KE at 1 4) Uniform properties at exit 5) Incompressible 6) p_{atm} at 1 and 2

Hence for CV (a) $-W_s = \left(\frac{V_2^2}{2} + g\cdot z_2\right)\cdot m_{exit}$ $m_{exit} = \rho\cdot V_2\cdot A_2$ where m_{exit} is mass flow rate (Note: Software cannot render a dot!)

Hence, for V_2 (to get the flow rate) we need to solve $\left(\frac{1}{2}\cdot V_2^2 + g\cdot z_2\right)\cdot\rho\cdot V_2\cdot A_2 = -W_s$ which is a cubic for V_2!

To solve this we could ignore the gravity term, solve for velocity, and then check that the gravity term is in fact minor. Alternatively we could manually iterate, or use a calculator or Excel, to solve. The answer is $V_2 = 114\,\frac{ft}{s}$

Hence the flow rate is $Q = V_2\cdot A_2 = V_2\cdot\frac{\pi\cdot D_2^2}{4}$ $Q = 114\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.622\,\frac{ft^3}{s}$ $Q = 279\,gpm$

To find z_{max}, use the first law again to (to CV (b)) to get $-W_s = g\cdot z_{max}\cdot m_{exit}$

$$z_{max} = -\frac{W_s}{g\cdot m_{exit}} = -\frac{W_s}{g\cdot\rho\cdot Q} \qquad z_{max} = 15\cdot hp\times\frac{\frac{550\cdot ft\cdot lbf}{s}}{1\cdot hp}\times\frac{s^2}{32.2\cdot ft}\times\frac{ft^3}{1.94\cdot slug}\times\frac{s}{0.622\cdot ft^3}\times\frac{slug\cdot ft}{s^2\cdot lbf} \qquad z_{max} = 212\,ft$$

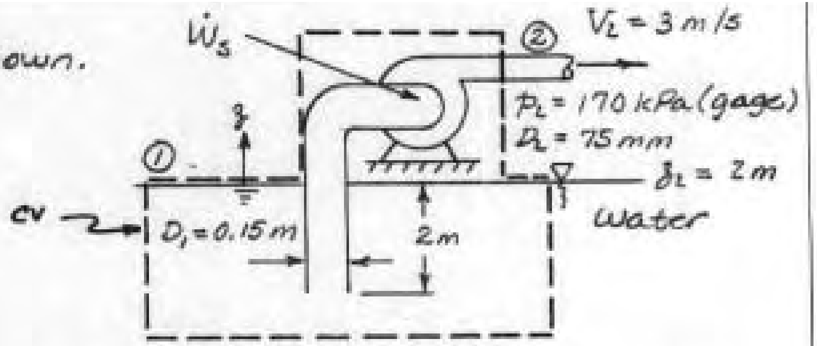
For the force in the x direction when jet is horizontal we need x momentum

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{\text{CV}} u\,\rho\,d\forall + \int_{\text{CS}} u\,\rho\vec{V}\cdot d\vec{A}$$

Then $R_x = u_1\cdot(-\rho\cdot V_1\cdot A_1) + u_2\cdot(\rho\cdot V_2\cdot A_2) = 0 + V_2\cdot\rho\cdot Q$ $R_x = \rho\cdot Q\cdot V_2$

$$R_x = 1.94\cdot\frac{slug}{ft^3}\times 0.622\cdot\frac{ft^3}{s}\times 114\cdot\frac{ft}{s}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad R_x = 138\,lbf$$

Problem 4.205



Given: Pump system as shown.

$\eta_{pump} = 0.75$

Find: Power required.

Solution: Apply first law to cv shown, noting that flow enters with negligible velocity at section ①.

Basic equation:

$$\dot{Q} - \dot{W}_{shaft} - \overset{=0(1)}{\dot{W}_{shear}} - \overset{=0(1)}{\dot{W}_{other}} = \overset{=0(2)}{\frac{\partial}{\partial t}}\int_{CV} e\rho\, d\forall + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho\vec{V}\cdot d\vec{A}$$

$$e = u + \frac{V^2}{2} + gz$$

Assumptions:
(1) $\dot{W}_{shear} = \dot{W}_{other} = 0$
(2) Steady flow
(3) $V_1 \simeq 0$
(4) $z_1 = 0$
(5) $p_1 = 0$ (gage)
(6) Uniform flow at each section
(7) Incompressible flow; $V_1 A_1 = V_2 A_2$

Then

$$\dot{Q} - \dot{W}_s = \left(u_1 + \overset{\simeq 0(3)}{\frac{V_1^2}{2}} + g\overset{=0(4)}{z_1} + \overset{=0(5)}{\frac{p_1}{\rho}}\right)\{-\dot{m}\} + \left(u_2 + \frac{V_2^2}{2} + gz_2 + \frac{p_2}{\rho}\right)\{\dot{m}\}$$

or

$$-\dot{W}_s = \dot{m}\left[\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \left(u_2 - u_1 - \frac{\delta Q}{dm}\right)\right]$$

Obtain the ideal or minimum power input by neglecting thermal effects. Thus

$$-\dot{W}_{s,ideal} = \dot{m}\left[\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2\right]$$

For the system,

$$\dot{m} = \rho V_2 A_2 = 999\,\frac{kg}{m^3} \times 3\,\frac{m}{s} \times \frac{\pi}{4}(0.075)^2\,m^2 = 13.2\ kg/s$$

and

$$-\dot{W}_{s,ideal} = 13.2\,\frac{kg}{s}\left[1.70\times10^5\,\frac{N}{m^2} \times \frac{m^3}{999\,kg} + \frac{1}{2}(3)^2\,\frac{m^2}{s^2} \times \frac{N\cdot s^2}{kg\cdot m} + 9.81\,\frac{m}{s^2} \times 2\,m \times \frac{N\cdot s^2}{kg\cdot m}\right]$$

$$\dot{W}_{s,ideal} = -2560\,\frac{N\cdot m}{s} \times \frac{kW\cdot s}{10^3\,N\cdot m} = -2.56\ kW$$

Finally

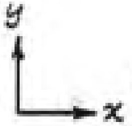
$$\dot{W}_{s,actual} = \frac{\dot{W}_{s,ideal}}{\eta} = \frac{-2.56\ kW}{0.75} = -3.41\ kW$$

$\dot{W}_{s,actua}$

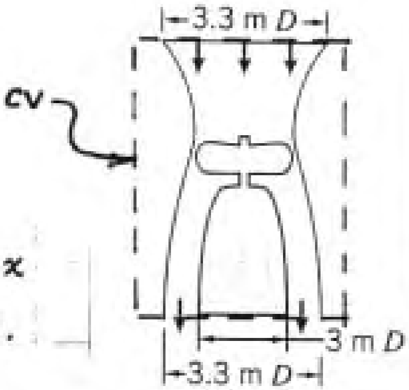
Given: Helicopter-type craft hovering

Mass, $M = 1500\text{ kg}$

Assume atmospheric pressure at outlet, and treat as steady, uniform, incompressible flow.



Assume air is at standard conditions.



Find: (a) Speed of air leaving craft.
(b) Minimum power required.

Solution: Use inertial CV and coordinates shown. Apply continuity and momentum to determine V_2, then apply energy to find power.

Basic equations: $p = \rho RT$; $\Delta h = c_p \Delta T$; $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$

$$0 = \overset{=0(2)}{\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall} + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_z} + F_{B_z} = \overset{=0(2)}{\frac{\partial}{\partial t}\int_{CV} w\rho\, d\forall} + \int_{CS} w\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Air is an ideal gas, $c_p = \text{constant}$
(2) Steady flow
(3) Incompressible flow
(4) Uniform flow at each section
(5) Uniform pressure at inlet; $F_{S_z} = (p_{atm} - p_1)A_1 = -p_{1g}A_1$

Then

$$\rho = \frac{p}{RT} = 1.01\times10^5\,\frac{N}{m^2}\times\frac{kg\cdot K}{287\,N\cdot m}\times\frac{1}{288\,K} = 1.22\text{ kg/m}^3$$

and from continuity

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} = \rho(V_2A_2 - V_1A_1) \quad \text{or} \quad V_1 = V_2\left(\frac{A_2}{A_1}\right)$$

Now $A_1 = \frac{\pi}{4}D_o^2 = \frac{\pi}{4}(3.3)^2 m^2 = 8.55\text{ m}^2$

$$A_2 = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}\left[(3.3)^2 - (3.0)^2\right]m^2 = 1.48\text{ m}^2$$

From momentum

$$-p_{1g}A_1 - Mg = w_1\{-|\rho V_1 A_1|\} + w_2\{|\rho V_2 A_2|\}$$

$$w_1 = -V_1 \qquad w_2 = -V_2 \qquad \text{and} \quad \rho V_1 A_1 = \rho V_2 A_2$$

$$-p_{1g}A_1 - Mg = V_1\rho V_2 A_2 - V_2\rho V_2 A_2 = -\rho V_2 A_2(V_2 - V_1)$$

Problem *4.206

[4] Part 2/2

For steady, incompressible flow without friction, along a streamline from atmosphere to ①, Bernoulli gives, neglecting Δz,

$$p_{atm} + \frac{1}{2}\rho \overset{\approx 0}{\cancel{V_0^2}} + g\cancel{z_0} = p_1 + \frac{1}{2}\rho V_1^2 + g\cancel{z_1} \quad \text{so} \quad p_{1g} = -\frac{1}{2}\rho V_1^2$$

Using continuity, $p_{1g}A_1 = -\frac{1}{2}\rho V_1^2 A_1 = -\frac{1}{2}\rho V_2 A_2 V_1 = -\frac{1}{2}\rho V_2^2 A_2 \frac{A_2}{A_1}$

Substituting into the momentum equation and using continuity,

$$\frac{1}{2}\rho V_2^2 A_2 \frac{A_2}{A_1} - Mg = -\rho V_2^2 A_2\left(1 - \frac{V_1}{V_2}\right) = -\rho V_2^2 A_2\left(1 - \frac{A_2}{A_1}\right) \quad \text{or} \quad Mg = \rho V_2^2 A_2\left(1 - \frac{1}{2}\frac{A_2}{A_1}\right)$$

Thus

$$V_2 = \sqrt{\frac{Mg}{\rho A_2\left(1 - \frac{1}{2}\frac{A_2}{A_1}\right)}} = \left[1500\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times \frac{\text{m}^3}{1.22\,\text{kg}} \times \frac{1}{1.48\,\text{m}^2} \times \frac{1}{\left(1 - \frac{1}{2}\,\frac{1.48}{8.55}\right)}\right]^{\frac{1}{2}} = 94.5\ \text{m/s}$$

V_2

Basic equation:

$$\dot{Q} - \dot{W}_s - \overset{=0(6)}{\cancel{\dot{W}_{shear}}} - \overset{=0(6)}{\cancel{\dot{W}_{other}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall}} + \int_{CS}\left(u + \overset{=0(7)}{\cancel{pv}} + \frac{V^2}{2} + \overset{\approx 0(8)}{\cancel{gz}}\right)\rho\vec{V}\cdot d\vec{A}$$

Additional assumptions: (6) $\dot{W}_{shear} = \dot{W}_{other} = 0$
(7) pv = constant
(8) Neglect Δz

Then

$$-\dot{W}_s = \left(u_1 + \frac{V_1^2}{2}\right)\{-|\dot{m}|\} + \left(u_2 + \frac{V_2^2}{2}\right)\{|\dot{m}|\} - \dot{Q}$$

$$-\dot{W}_s = \dot{m}\left(\frac{V_2^2 - V_1^2}{2}\right) + \dot{m}\left(u_2 - u_1 - \frac{dQ}{dm}\right)$$

The term $\left(u_2 - u_1 - \frac{dQ}{dm}\right)$ represents nonmechanical energy. The minimum possible work would be attained when the nonmechanical energy is zero. Thus

$$-\dot{W}_s\Big)_{min} = \dot{m}\left(\frac{V_2^2 - V_1^2}{2}\right) = \dot{m}\frac{V_2^2}{2}\left[1 - \left(\frac{V_1}{V_2}\right)^2\right] = \frac{\rho A_2 V_2^3}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

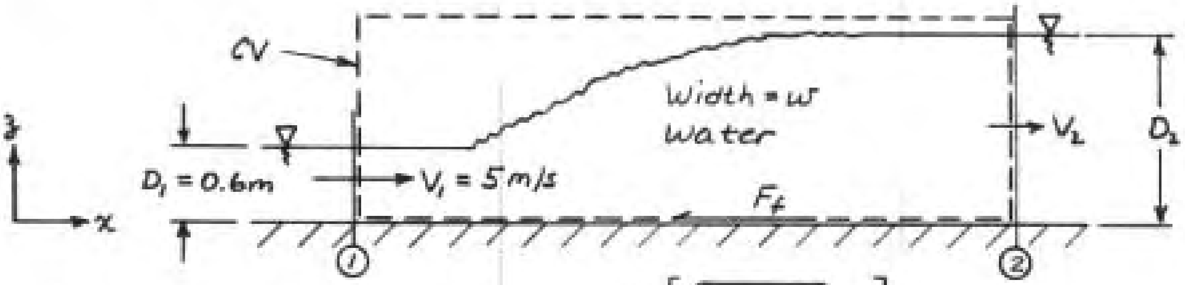
$$-\dot{W}_s = \frac{1}{2} \times 1.22\,\frac{\text{kg}}{\text{m}^3} \times 1.48\,\text{m}^2 \times (94.5)^3\,\frac{\text{m}^3}{\text{s}^3}\left[1 - \left(\frac{1.48}{8.55}\right)^2\right]\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \times \frac{\text{kW}\cdot\text{s}}{10^3\,\text{N}\cdot\text{m}}$$

$$\dot{W}_s\Big)_{min} = -739\ \text{kW} \quad (\text{input})$$

$\dot{W}_s$

{The power required for hovering in a real craft would be greater due to flow losses, nonuniformities, etc.}

Given: Liquid flow in a wide, horizontal open channel, as shown.



Find: (a) Show that in general, $D_2 = \frac{D_1}{2}\left[\sqrt{1+\frac{8V_1^2}{gD_1}}-1\right]$

(b) Change in mechanical energy across hydraulic jump.
(c) Temperature rise if no heat transfer.

Solution: Apply continuity, x component of momentum, and energy equations using CV shown.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(8)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} V_x\rho\, d\forall + \int_{CS} V_x\rho\vec{V}\cdot d\vec{A}$$

$$\overset{=0(6)}{\cancel{\dot{Q}}} - \overset{=0(7)}{\cancel{\dot{W}_s}} - \overset{=0(7)}{\cancel{\dot{W}_{shear}}} - \overset{=0(7)}{\cancel{\dot{W}_{other}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} e\rho\, d\forall + \int_{CS}(e+pv)\rho\vec{V}\cdot d\vec{A};$$

$$e = u + \frac{V^2}{2} + gz$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) Hydrostatic pressure distribution at sections ①, ②, so $p = \rho g(D-z)$
(5) Neglect friction force, F_f, on CV
(6) $\dot{Q} = 0$
(7) $\dot{W}_s = \dot{W}_{shear} = \dot{W}_{other} = 0$
(8) $F_{B_x} = 0$, since channel is horizontal

From continuity,

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_2|\} = -\rho V_1 w D_1 + \rho V_2 w D_2\ ;\ V_1 D_1 = V_2 D_2$$

From momentum,

$$F_{S_x} = \underbrace{\rho g\frac{D_1}{2}wD_1 - \rho g\frac{D_2}{2}wD_2}_{\text{hydrostatic forces}} = V_{x_1}\{-|\rho V_1 w D_1|\} + V_{x_2}\{|\rho V_2 w D_2|\}$$

$$V_{x_1} = V_1 \qquad V_{x_2} = V_2$$

or

$$\frac{g}{2}(D_1^2 - D_2^2) = V_1 D_1(V_2 - V_1) = V_1^2 D_1\left(\frac{V_2}{V_1} - 1\right) = V_1^2 D_1\left(\frac{D_1}{D_2} - 1\right)$$

or

$$\frac{g}{2}(D_1 + D_2)\cancel{(D_1 - D_2)} = V_1^2\frac{D_1}{D_2}\cancel{(D_1 - D_2)}$$

Problem 4.207

[4] Part 2/2

Thus $\frac{gD_1}{2}\left(1+\frac{D_2}{D_1}\right) = V_1^2\frac{D_1}{D_2}$ or $\frac{D_2}{D_1}\left(1+\frac{D_2}{D_1}\right) = \frac{2V_1^2}{gD_1}$ or $\left(\frac{D_2}{D_1}\right)^2+\frac{D_2}{D_1}-\frac{2V_1^2}{gD_1}=0$

Using the quadratic equation,

$$\frac{D_2}{D_1}=\frac{1}{2}\left[-1\pm\sqrt{1+\frac{8V_1^2}{gD_1}}\right] \quad \text{or} \quad D_2=\frac{D_1}{2}\left[\sqrt{1+\frac{8V_1^2}{gD_1}}-1\right] \longleftarrow$$

D_2

Solving for D_2

$$D_2=\frac{1}{2}\times 0.6\,m\left[\sqrt{1+8\times(5)^2\frac{m^2}{s^2}\times\frac{s^2}{9.81\,m}\times\frac{1}{0.6\,m}}-1\right]=1.47\,m$$

$$V_2=\frac{D_1}{D_2}V_1=\frac{0.6}{1.47}\times\frac{5\,m}{s}=2.04\,m/s$$

From the energy equation, with $e_{mech}=\frac{V^2}{2}+gz+\frac{p}{\rho}$, and $dA=w\,dz$, the mechanical energy fluxes are

$$mef_1=\int_0^{D_1}\left[\frac{V_1^2}{2}+gz+\frac{1}{\rho}\rho g(D-z)\right]\rho V_1 w\,dz=\left(\frac{V_1^2}{2}+gD_1\right)\rho V_1 w D_1$$

$$mef_2=\int_0^{D_2}\left[\frac{V_2^2}{2}+gz+\frac{1}{\rho}\rho g(D-z)\right]\rho V_2 w\,dz=\left(\frac{V_2^2}{2}+gD_2\right)\rho V_2 w D_2$$

and

$$\Delta mef = mef_2-mef_1=\left[\frac{V_2^2-V_1^2}{2}+g(D_2-D_1)\right]\rho V_1 w D_1, \text{ since } V_1D_1=V_2D_2$$

Thus $\frac{\Delta mef}{\dot{m}}=\frac{1}{2}\left[V_2^2-V_1^2+2g(D_2-D_1)\right]$

$$\frac{\Delta mef}{\dot{m}}=\frac{1}{2}\left[(2.04)^2\frac{m^2}{s^2}-(5)^2\frac{m^2}{s^2}+2\times 9.81\frac{m}{s^2}(1.47-0.6)m\right]\frac{N\cdot s^2}{kg\cdot m}=-1.88\ N\cdot m/kg \longleftarrow$$

$\frac{\Delta mef}{\dot{m}}$

From the energy equation,

$$0=\left[u_1+\frac{V_1^2}{2}+gz+\frac{1}{\rho}\rho g(D-z)\right]\left\{-|\rho V_1 w D_1|\right\}$$
$$+\left[u_2+\frac{V_2^2}{2}+gz+\frac{1}{\rho}\rho g(D-z)\right]\left\{|\rho V_2 w D_2|\right\}$$

or

$$0=(u_2-u_1)\dot{m}+\Delta mef$$

Thus

$$u_2-u_1=c_v(T_2-T_1)=-\frac{\Delta mef}{\dot{m}}$$

$$\Delta T=T_2-T_1=-\frac{\Delta mef}{\dot{m}c_v}=-\left(-1.88\frac{N\cdot m}{kg}\right)\frac{kg\cdot K}{1\,kcal}\times\frac{kcal}{4187\,J}=4.49\times10^{-4}K \longleftarrow$$

ΔT

{This small temperature change would be almost impossible to measure.}

Problem 5.1

[1]

Given: Velocity fields listed below

Find: Which are possible two-dimensional, incompressible flow cases?

Solution: Apply the continuity equation in differential form.

Basic equation: $\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \overset{=0(1)}{\cancel{\frac{\partial}{\partial z}\rho w}} + \overset{=0(2)}{\cancel{\frac{\partial \rho}{\partial t}}} = 0$

Assumptions: (1) Two-dimensional flow, $\vec{V} = \vec{V}(x,y)$, so $\frac{\partial}{\partial z} = 0$

(2) Incompressible flow

$\rho =$ constant, so $\frac{\partial \rho}{\partial t} = 0$, $\frac{\partial \rho}{\partial(\text{distance})} = 0$

Then, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ is criterion.

(a) $u = 2x^2 + y^2 - x^2 y$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (4x - 2xy) + x(2y - 2)$

$v = x^3 + x(y^2 - 2y)$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4x - 2xy + 2xy - 2x \neq 0$

so $\rho \neq$ constant

(b) $u = 2xy - x^2 + y$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (2y - 2x) + (2x - 2y) = 0$

$v = 2xy - y^2 + x^2$

so possible

(c) $u = xt + 2y$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = t - t = 0$, so possible

$v = xt^2 - yt$

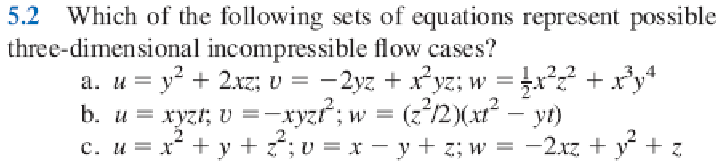
(d) $u = (x + 2y)xt$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (2xt + 2yt) + (-2xt - 2yt) = 0$

$v = -(2x + y)yt$

so possible

Problem 5.2

[2]



5.2 Which of the following sets of equations represent possible three-dimensional incompressible flow cases?

a. $u = y^2 + 2xz$; $v = -2yz + x^2yz$; $w = \frac{1}{2}x^2z^2 + x^3y^4$

b. $u = xyzt$; $v = -xyzt^2$; $w = (z^2/2)(xt^2 - yt)$

c. $u = x^2 + y + z^2$; $v = x - y + z$; $w = -2xz + y^2 + z$

Given: Velocity fields

Find: Which are 3D incompressible

Solution:

Basic equation: $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$

Assumption: Incompressible flow

a) $u(x,y,z,t) = y^2 + 2\cdot x\cdot z$ $\quad v(x,y,z,t) = -2\cdot y\cdot z + x^2\cdot y\cdot z$ $\quad w(x,y,z,t) = \frac{1}{2}\cdot x^2\cdot z^2 + x^3\cdot y^4$

$\frac{\partial}{\partial x}u(x,y,z,t) \rightarrow 2\cdot z$ $\quad \frac{\partial}{\partial y}v(x,y,z,t) \rightarrow x^2\cdot z - 2\cdot z$ $\quad \frac{\partial}{\partial z}w(x,y,z,t) \rightarrow x^2\cdot z$

Hence $\quad \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$ $\quad$ INCOMPRESSIBLE

b) $u(x,y,z,t) = x\cdot y\cdot z\cdot t$ $\quad v(x,y,z,t) = -x\cdot y\cdot z\cdot t^2$ $\quad w(x,y,z,t) = \frac{z^2}{2}\cdot\left(x\cdot t^2 - y\cdot t\right)$

$\frac{\partial}{\partial x}u(x,y,z,t) \rightarrow t\cdot y\cdot z$ $\quad \frac{\partial}{\partial y}v(x,y,z,t) \rightarrow -t^2\cdot x\cdot z$ $\quad \frac{\partial}{\partial z}w(x,y,z,t) \rightarrow z\cdot\left(t^2\cdot x - t\cdot y\right)$

Hence $\quad \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$ $\quad$ INCOMPRESSIBLE

c) $u(x,y,z,t) = x^2 + y + z^2$ $\quad v(x,y,z,t) = x - y + z$ $\quad w(x,y,z,t) = -2\cdot x\cdot z + y^2 + z$

$\frac{\partial}{\partial x}u(x,y,z,t) \rightarrow 2\cdot x$ $\quad \frac{\partial}{\partial y}v(x,y,z,t) \rightarrow -1$ $\quad \frac{\partial}{\partial z}w(x,y,z,t) \rightarrow 1 - 2\cdot x$

Hence $\quad \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$ $\quad$ INCOMPRESSIBLE

Problem 5.3

Given: Velocity field $u = Ax + By + Cz$

$$v = Dx + Ey + Fz$$

$$w = Gx + Hy + Jz$$

Find: The relationship among coefficients A thru J for this to be an incompressible flow field.

Solution: Flow must satisfy differential form of continuity.

Basic equation: $\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \cancelto{=0}{\frac{\partial \rho}{\partial t}} = 0$

Assumption: Incompressible flow, so $\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial x} = 0$

Then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

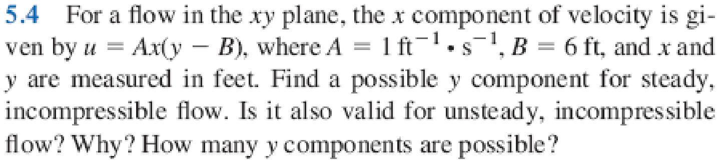
For the given flow field, $\frac{\partial u}{\partial x} = A$, $\frac{\partial v}{\partial y} = E$, $\frac{\partial w}{\partial z} = J$. Thus

$A + E + J = 0$, and

B, C, D, F, G, H are arbitrary ←

Problem 5.4

[2]



5.4 For a flow in the xy plane, the x component of velocity is given by $u = Ax(y - B)$, where $A = 1\ \text{ft}^{-1} \cdot \text{s}^{-1}$, $B = 6$ ft, and x and y are measured in feet. Find a possible y component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why? How many y components are possible?

Given: x component of velocity

Find: y component for incompressible flow; Valid for unsteady?; How many y components?

Solution:

Basic equation: $$\frac{\partial}{\partial x}(\rho \cdot u) + \frac{\partial}{\partial y}(\rho \cdot v) + \frac{\partial}{\partial z}(\rho \cdot w) + \frac{\partial}{\partial t}\rho = 0$$

Assumption: Incompressible flow; flow in x-y plane

Hence $$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0 \qquad \text{or} \qquad \frac{\partial}{\partial y}v = -\frac{\partial}{\partial x}u = -\frac{\partial}{\partial x}[A \cdot x \cdot (y - B)] = -A \cdot (y - B)$$

Integrating $$v(x,y) = -\int A \cdot (y - B)\,dy = -A \cdot \left(\frac{y^2}{2} - B \cdot y\right) + f(x)$$

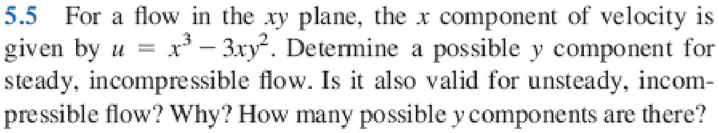
This basic equation is valid for steady and unsteady flow (t is not explicit)

There are an infinite number of solutions, since f(x) can be any function of x. The simplest is f(x) = 0

$$v(x,y) = -A \cdot \left(\frac{y^2}{2} - B \cdot y\right) \qquad v(x,y) = 6 \cdot y - \frac{y^2}{2}$$

Problem 5.5

[2]



5.5 For a flow in the *xy* plane, the *x* component of velocity is given by $u = x^3 - 3xy^2$. Determine a possible *y* component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why? How many possible *y* components are there?

Given: x component of velocity

Find: y component for incompressible flow; Valid for unsteady? How many y components?

Solution:

Basic equation: $$\frac{\partial}{\partial x}(\rho\cdot u) + \frac{\partial}{\partial y}(\rho\cdot v) + \frac{\partial}{\partial z}(\rho\cdot w) + \frac{\partial}{\partial t}\rho = 0$$

Assumption: Incompressible flow; flow in x-y plane

Hence $$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0 \qquad \text{or} \qquad \frac{\partial}{\partial y}v = -\frac{\partial}{\partial x}u = -\frac{\partial}{\partial x}\left(x^3 - 3\cdot x\cdot y^2\right) = -\left(3\cdot x^2 - 3\cdot y^2\right)$$

Integrating $$v(x,y) = -\int \left(3\cdot x^2 - 3\cdot y^2\right) dy = -3\cdot x^2\cdot y + y^3 + f(x)$$

This basic equation is valid for steady and unsteady flow (t is not explicit)

There are an infinite number of solutions, since f(x) can be any function of x. The simplest is f(x) = 0 $v(x,y) = y^3 - 3\cdot x^2\cdot y$

Given: Steady, incompressible flow field in the xy plane has an x component of velocity given by

$$u = \frac{A}{x} \text{ , where } A = 2\ m^2/s \text{ and } x \text{ is in meters.}$$

Find: the simplest y component of velocity for this flow field.

Solution:

Apply the continuity equation for the conditions given

Basic equation: $\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0$

For steady flow $\frac{\partial \rho}{\partial t} = 0$ and for two-dimensional flow in the xy plane, $\frac{\partial}{\partial z}() = 0$. Thus the basic equation reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Then

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{A}{x}\right) = \frac{A}{x^2}$$

and

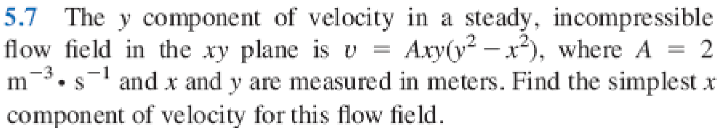
$$v = \int \frac{\partial v}{\partial y} dy + f(x) = \int \frac{A}{x^2} dy + f(x) = \frac{Ay}{x^2} + f(x)$$

The simplest y component of velocity is obtained with $f(x) = 0$

$$\therefore\ v = \frac{Ay}{x^2} \longleftarrow v$$

Problem 5.7

[2]



5.7 The y component of velocity in a steady, incompressible flow field in the xy plane is $v = Axy(y^2 - x^2)$, where $A = 2$ $\mathrm{m^{-3} \cdot s^{-1}}$ and x and y are measured in meters. Find the simplest x component of velocity for this flow field.

Given: y component of velocity

Find: x component for incompressible flow; Simplest x components?

Solution:

Basic equation: $$\frac{\partial}{\partial x}(\rho \cdot u) + \frac{\partial}{\partial y}(\rho \cdot v) + \frac{\partial}{\partial z}(\rho \cdot w) + \frac{\partial}{\partial t}\rho = 0$$

Assumption: Incompressible flow; flow in x-y plane

Hence $$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$$ or $$\frac{\partial}{\partial x}u = -\frac{\partial}{\partial y}v = -\frac{\partial}{\partial y}\left[A \cdot x \cdot y \cdot \left(y^2 - x^2\right)\right] = -\left[A \cdot x \cdot \left(y^2 - x^2\right) + A \cdot x \cdot y \cdot 2 \cdot y\right]$$

Integrating $$u(x,y) = -\int A \cdot \left(3 \cdot x \cdot y^2 - x^3\right) dx = -\frac{3}{2} \cdot A \cdot x^2 \cdot y^2 + \frac{1}{4} \cdot A \cdot x^4 + f(y)$$

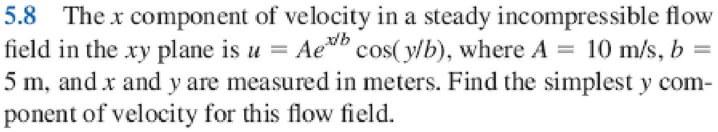
This basic equation is valid for steady and unsteady flow (t is not explicit)

There are an infinite number of solutions, since f(y) can be any function of y. The simplest is f(y) = 0

$$u(x,y) = \frac{1}{4} \cdot A \cdot x^4 - \frac{3}{2} \cdot A \cdot x^2 \cdot y^2 \qquad u(x,y) = \frac{1}{2} \cdot x^4 - 3 \cdot x^2 y^2$$

Problem 5.8

[2]



5.8 The x component of velocity in a steady incompressible flow field in the xy plane is $u = Ae^{x/b}\cos(y/b)$, where $A = 10$ m/s, $b = 5$ m, and x and y are measured in meters. Find the simplest y component of velocity for this flow field.

Given: x component of velocity

Find: y component for incompressible flow; Valid for unsteady? How many y components?

Solution:

Basic equation: $$\frac{\partial}{\partial x}(\rho\cdot u) + \frac{\partial}{\partial y}(\rho\cdot v) + \frac{\partial}{\partial z}(\rho\cdot w) + \frac{\partial}{\partial t}\rho = 0$$

Assumption: Incompressible flow; flow in x-y plane

Hence $$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0 \qquad \text{or} \qquad \frac{\partial}{\partial y}v = -\frac{\partial}{\partial x}u = -\frac{\partial}{\partial x}\left(A\cdot e^{\frac{x}{b}}\cdot\cos\left(\frac{y}{b}\right)\right) = -\left(\frac{A}{b}\cdot e^{\frac{x}{b}}\cdot\cos\left(\frac{y}{b}\right)\right)$$

Integrating $$v(x,y) = -\int \frac{A}{b}\cdot e^{\frac{x}{b}}\cdot\cos\left(\frac{y}{b}\right)dy = -A\cdot e^{\frac{x}{b}}\cdot\sin\left(\frac{y}{b}\right) + f(x)$$

This basic equation is valid for steady and unsteady flow (t is not explicit)

There are an infinite number of solutions, since f(x) can be any function of x. The simplest is f(x) = 0

$$v(x,y) = -A\cdot e^{\frac{x}{b}}\cdot\sin\left(\frac{y}{b}\right) \qquad v(x,y) = -10\cdot e^{\frac{x}{5}}\cdot\sin\left(\frac{y}{5}\right)$$

Problem 5.9

[3]

5.9 The y component of velocity in a steady incompressible flow field in the xy plane is

$$v = \frac{2xy}{(x^2+y^2)^2}$$

Show that the simplest expression for the x component of velocity is

$$u = \frac{1}{(x^2+y^2)} - \frac{2y^2}{(x^2+y^2)^2}$$

Given:

y component of velocity

Find:

x component for incompressible flow; Simplest x component

Solution:

Basic equation:
$$\frac{\partial}{\partial x}(\rho\cdot u) + \frac{\partial}{\partial y}(\rho\cdot v) + \frac{\partial}{\partial z}(\rho\cdot w) + \frac{\partial}{\partial t}\rho = 0$$

Assumption: Incompressible flow; flow in x-y plane

Hence
$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0 \qquad \text{or} \qquad \frac{\partial}{\partial x}u = -\frac{\partial}{\partial y}v = -\frac{\partial}{\partial y}\left[\frac{2\cdot x\cdot y}{\left(x^2+y^2\right)^2}\right] = -\left[\frac{2\cdot x\cdot\left(x^2-3\cdot y^2\right)}{\left(x^2+y^2\right)^3}\right]$$

Integrating
$$u(x,y) = -\int\left[\frac{2\cdot x\cdot\left(x^2-3\cdot y^2\right)}{\left(x^2+y^2\right)^3}\right]dx = \frac{x^2-y^2}{\left(x^2+y^2\right)^2} + f(y) = \frac{x^2+y^2-2\cdot y^2}{\left(x^2+y^2\right)^2} + f(y)$$

$$u(x,y) = \frac{1}{x^2+y^2} - \frac{2\cdot y^2}{\left(x^2+y^2\right)^2} + f(y)$$

The simplest form is
$$u(x,y) = \frac{1}{x^2+y^2} - \frac{2\cdot y^2}{\left(x^2+y^2\right)^2}$$

Note: Instead of this approach we could have verified that u and v satisfy continuity

$$\frac{\partial}{\partial x}\left[\frac{1}{x^2+y^2} - \frac{2\cdot y^2}{\left(x^2+y^2\right)^2}\right] + \frac{\partial}{\partial y}\left[\frac{2\cdot x\cdot y}{\left(x^2+y^2\right)^2}\right] \rightarrow 0$$

However, this does not verify the solution is the simplest

Problem 5.10

Given: Approximate profile for laminar boundary layer

$$u = cU\frac{y}{x^{1/2}}$$

Find: (a) Show simplest v is $v = \frac{U}{4}\frac{y}{x}$

(b) Evaluate maximum value of v/U where $\delta = 5mm$, $x = 0.5m$.

Solution: Apply continuity for incompressible flow

$$\text{Basic equation: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \overset{\text{2-D flow}}{\cancel{\frac{\partial w}{\partial z}}} = 0$$

Thus

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\left(-\frac{1}{2}\right)cU\frac{y}{x^{3/2}}$$

$$v = \int \frac{\partial v}{\partial y}dy + f(x) = \int \frac{1}{2}cU\frac{y}{x^{3/2}}dy + f(x) = \frac{1}{4}cU\frac{y^2}{x^{3/2}} + f(x)$$

or

$$v = \frac{U}{4}\frac{y}{x} \quad [f(x) = 0 \text{ since } v = 0 \text{ along } y = 0]$$

v

From

$$\frac{v}{U} = \frac{1}{4}\frac{y}{x}$$

maximum value occurs at $y = \delta$. At the location given,

$$\left.\frac{v}{U}\right)_{max} = \frac{1}{4}\frac{\delta}{x} = \frac{1}{4}\frac{0.005m}{0.5m} = 0.0025$$

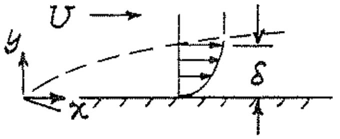
$\left.\frac{v}{U}\right)_m$

Problem 5.11

[3]

Given: Laminar boundary layer, parabolic approximate profile.

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \qquad \delta = c\,x^{1/2}$$



Find: Show $\frac{v}{U} = \frac{\delta}{x}\left[\frac{1}{2}\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]$ for incompressible flow.

Plot: $\frac{v}{U}$ vs. $\frac{y}{\delta}$, evaluate max. at $x = 0.5$ m, if $\delta = 5$ mm.

Solution: Apply conservation of mass for incompressible flow.

Basic equation: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \overset{=0(2)}{\cancel{\frac{\partial w}{\partial z}}} = 0$$

Assumptions: (1) Incompressible flow (ρ = const)

(2) $w = 0$

Then $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \;;\; \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \;;\; v = \int_0^y -\frac{\partial u}{\partial x}\,dy + \overset{=0(\text{simplest})}{\cancel{f(x)}}$$

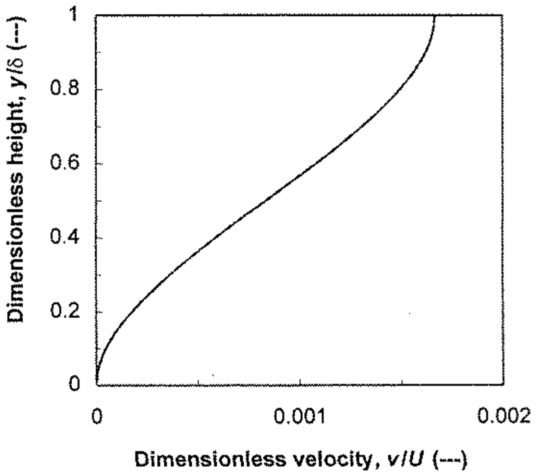
From the given profile

$$\frac{\partial u}{\partial x} = 2Uy(-1)\frac{1}{\delta^2}\frac{d\delta}{dx} - Uy^2(-2)\frac{1}{\delta^3}\frac{d\delta}{dx} = 2U\frac{d\delta}{dx}\left(\frac{y^2}{\delta^3} - \frac{y}{\delta^2}\right)$$

Since $\delta = cx^{1/2}$, $\frac{d\delta}{dx} = \frac{1}{2}cx^{-1/2} = \frac{cx^{1/2}}{2x} = \frac{\delta}{2x}$, so $\frac{\partial u}{\partial x} = \frac{U\delta}{x}\left(\frac{y^2}{\delta^3} - \frac{y}{\delta^2}\right)$

Integrating, $$\frac{v}{U} = \frac{\delta}{x}\int_0^y\left(\frac{y}{\delta^2} - \frac{y^2}{\delta^3}\right)dy = \frac{\delta}{x}\left[\frac{1}{2}\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]$$ $\frac{v}{U}$

Plotting shows:



Maximum occurs at $\left(\frac{y}{\delta}\right) = 1$

$$\left.\frac{v}{U}\right)_{max} = \left.\frac{v}{U}\right)_{\frac{y}{\delta}=1} = \frac{\delta}{x}\left[\frac{1}{2}(1)^2 - \frac{1}{3}(1)^2\right] = \frac{\delta}{6x}$$ $\left.\frac{v}{U}\right)_{max}$

Evaluating, $\left.\frac{v}{U}\right)_{max} = \frac{1}{6} \times 0.005\,\text{m} \times \frac{1}{0.5\,\text{m}} = 0.00167$ or 0.167 percent $\left.\frac{v}{U}\right)_{max}$

Problem 5.12

[3]

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
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Given: Approximation for x component of velocity in laminar boundary layer

$$u = U \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) \qquad \text{where } \delta = c x^{1/2}$$

Show: $\frac{v}{U} = \frac{\delta}{\pi x}\left[\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) + \frac{\pi}{2}\frac{y}{\delta}\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - 1\right]$ for incompressible flow.

Plot: u/U, v/U vs. y/δ to locate maximum value of v/U;
evaluate at location where $x = 0.5\text{ m}$ and $\delta = 5\text{ mm}$.

Solution: Apply differential continuity for incompressible flow.

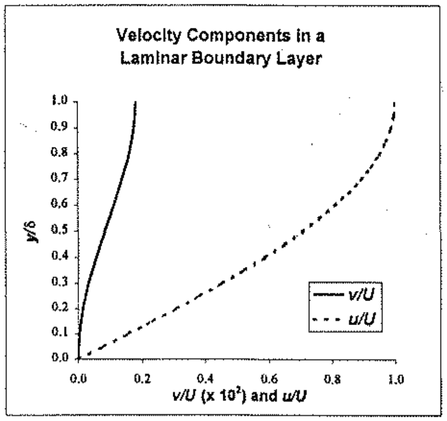
Basic equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \overset{=0\ (2\text{-D flow})}{\cancel{\frac{\partial w}{\partial z}}} = 0$

Thus $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial \delta}\frac{d\delta}{dx} = -\left(\frac{\pi y}{2}\right)\left(-\frac{1}{\delta^2}\right)\cos\left(\frac{\pi y}{2\delta}\right)\frac{U}{2}c x^{-1/2} = \frac{U}{2x}\left(\frac{\pi y}{2\delta}\right)\cos\left(\frac{\pi y}{2\delta}\right)$

Integrating, $v = \int_0^y \frac{\partial v}{\partial y}dy + f(x) = \int_0^y \frac{U}{2x}\left(\frac{\pi}{2}\frac{y}{\delta}\right)\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy + f(x)$

$$v = \frac{2\delta}{\pi}\frac{U}{2x}\int_0^{\frac{\pi}{2}\frac{y}{\delta}} \eta\cos\eta\, d\eta + f(x) = \frac{\delta}{\pi}\frac{U}{x}\Big[\cos\eta + \eta\sin\eta\Big]_0^{\frac{\pi}{2}\frac{y}{\delta}} + \overset{0}{\cancel{f(x)}}$$

$$\frac{v}{U} = \frac{1}{\pi}\frac{\delta}{x}\left[\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) + \left(\frac{\pi}{2}\frac{y}{\delta}\right)\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - 1\right]$$


Velocity Components in a Laminar Boundary Layer

This expression is a maximum at $y = \delta$, where

$$\frac{v}{U} = \frac{1}{\pi}\frac{\delta}{x}\left[\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) - 1\right] = \frac{\delta}{\pi x}\left(\frac{\pi}{2} - 1\right)$$

and

$$\left.\frac{v}{U}\right)_{max} = 0.182\,\frac{\delta}{x}$$

$\left.\frac{v}{U}\right)_{ma}$

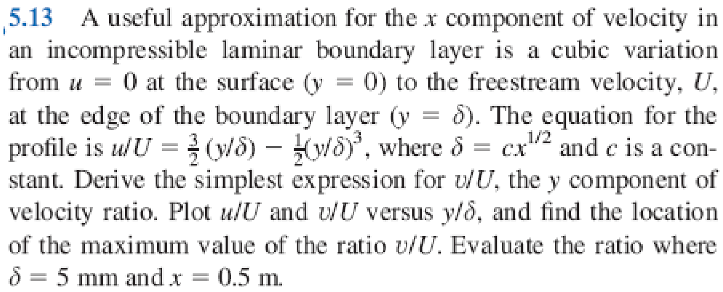
At the location given

$$\left.\frac{v}{U}\right)_{max} = 0.182 \times 0.005\text{ m} \times \frac{1}{0.5\text{ m}} = 0.00182 \quad \text{or} \quad 0.182 \text{ percent}$$

$\left.\frac{v}{U}\right)_{ma}$

Problem 5.13

[3]



5.13 A useful approximation for the x component of velocity in an incompressible laminar boundary layer is a cubic variation from $u = 0$ at the surface ($y = 0$) to the freestream velocity, U, at the edge of the boundary layer ($y = \delta$). The equation for the profile is $u/U = \frac{3}{2}(y/\delta) - \frac{1}{2}(y/\delta)^3$, where $\delta = cx^{1/2}$ and c is a constant. Derive the simplest expression for v/U, the y component of velocity ratio. Plot u/U and v/U versus y/δ, and find the location of the maximum value of the ratio v/U. Evaluate the ratio where $\delta = 5$ mm and $x = 0.5$ m.

Given: Data on boundary layer

Find: *y* component of velocity ratio; location of maximum value; plot velocity profiles; evaluate at particular point

Solution:

$$u(x,y) = U\cdot\left[\frac{3}{2}\cdot\left(\frac{y}{\delta(x)}\right) - \frac{1}{2}\cdot\left(\frac{y}{\delta(x)}\right)^3\right] \qquad \text{and} \qquad \delta(x) = c\cdot\sqrt{x}$$

so
$$u(x,y) = U\cdot\left[\frac{3}{2}\cdot\left(\frac{y}{c\cdot\sqrt{x}}\right) - \frac{1}{2}\cdot\left(\frac{y}{c\cdot\sqrt{x}}\right)^3\right]$$

For incompressible flow
$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$$

Hence
$$v(x,y) = -\int \frac{d}{dx}u(x,y)\,dy \qquad \text{and} \qquad \frac{du}{dx} = \frac{3}{4}\cdot U\cdot\left(\frac{y^3}{c^3\cdot x^{\frac{5}{2}}} - \frac{y}{c\cdot x^{\frac{3}{2}}}\right)$$

so
$$v(x,y) = -\int \frac{3}{4}\cdot U\cdot\left(\frac{y^3}{c^3}\cdot\frac{x^5}{2} - \frac{y}{c}\cdot\frac{x^3}{2}\right) dy$$

$$v(x,y) = \frac{3}{8}\cdot U\cdot\left(\frac{y^2}{c\cdot x^{\frac{3}{2}}} - \frac{y^4}{2\cdot c^3\cdot x^{\frac{5}{2}}}\right) \qquad v(x,y) = \frac{3}{8}\cdot U\cdot\frac{\delta}{x}\cdot\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{2}\cdot\left(\frac{y}{\delta}\right)^4\right]$$

The maximum occurs at $y = \delta$ as seen in the corresponding *Excel* workbook

$$v_{max} = \frac{3}{8}\cdot U\cdot\frac{\delta}{x}\cdot\left(1 - \frac{1}{2}\cdot 1\right)$$

At $\delta = 5\cdot mm$ and $x = 0.5\cdot m$, the maximum vertical velocity is $\frac{v_{max}}{U} = 0.00188$

Problem 5.13

[3]

5.13 A useful approximation for the *x* component of velocity in an incompressible laminar boundary layer is a cubic variation from $u = 0$ at the surface ($y = 0$) to the freestream velocity, U, at the edge of the boundary layer ($y = \delta$). The equation for the profile is $u/U = \frac{3}{2}(y/\delta) - \frac{1}{2}(y/\delta)^3$, where $\delta = cx^{1/2}$ and *c* is a constant. Derive the simplest expression for v/U, the *y* component of velocity ratio. Plot u/U and v/U versus y/δ, and find the location of the maximum value of the ratio v/U. Evaluate the ratio where $\delta = 5$ mm and $x = 0.5$ m.

Given: Data on boundary layer

Find: *y* component of velocity ratio; location of maximum value; plot velocity profiles; evaluate at particular point

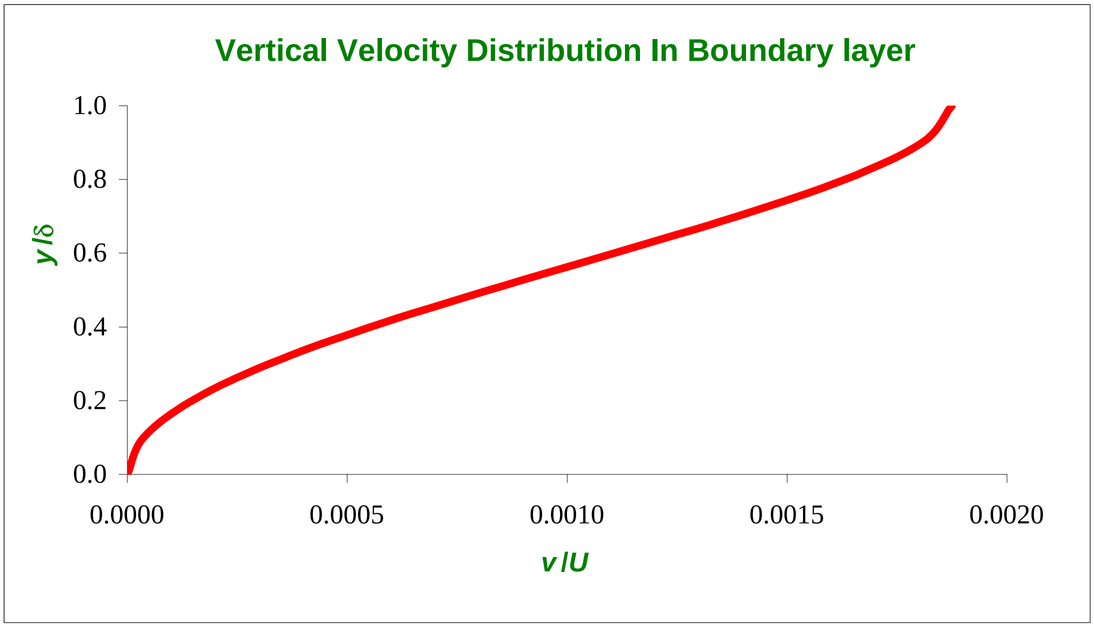
Solution:

$$v(x,y) = \frac{3}{8}\cdot U\cdot\frac{\delta}{x}\cdot\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{2}\cdot\left(\frac{y}{\delta}\right)^4\right]$$

To find when v/U is maximum, use *Solver*

v /*U*	*y* /d
0.00188	1.0

v /*U*	*y* /d
0.000000	0.0
0.000037	0.1
0.000147	0.2
0.000322	0.3
0.000552	0.4
0.00082	0.5
0.00111	0.6
0.00139	0.7
0.00163	0.8
0.00181	0.9
0.00188	1.0



Problem 5.14

[3]

Given: Flow in xy plane, $v = -Bxy^3$ where $B = 0.2\,m^{-3}\cdot s^{-1}$ and coordinates are measured in meters; steady, $\rho = c$.

Find: (a) Simplest x component of velocity.
(b) Equation of streamlines.

Plot: streamlines through points (1,4) and (2,4).

Solution:

Basic equation: $\nabla\cdot\rho\vec{V} + \frac{\partial\rho}{\partial t} = \frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \overset{=0(1)}{\cancel{\frac{\partial}{\partial z}\rho w}} + \overset{=0(2)}{\cancel{\frac{\partial\rho}{\partial t}}}$

Assumptions: (1) flow in the xy plane (given), $\frac{\partial}{\partial z} = 0$
(2) ρ = constant (given).

Then, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ or $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$.

and $\frac{\partial u}{\partial x} = -\frac{\partial}{\partial y}(-Bxy^3) = 3Bxy^2$

Integrating,

$$u = \int \frac{\partial u}{\partial x}\,dx = \int 3Bxy^2 = \frac{3}{2}Bx^2y^2 + f(y).$$

The simplest expression is obtained with $f(y) = 0$

$$\therefore\ u = \frac{3}{2}Bx^2y^2 \longleftarrow u$$

The equation of the streamlines is

$$\left.\frac{dy}{dx}\right)_{sl} = \frac{v}{u} = \frac{-Bxy^3}{\frac{3}{2}Bx^2y^2} = \frac{-2y}{3x}$$

Separating variables & integrating

$$\frac{3}{2}\frac{dy}{y} + \frac{dx}{x} = 0$$

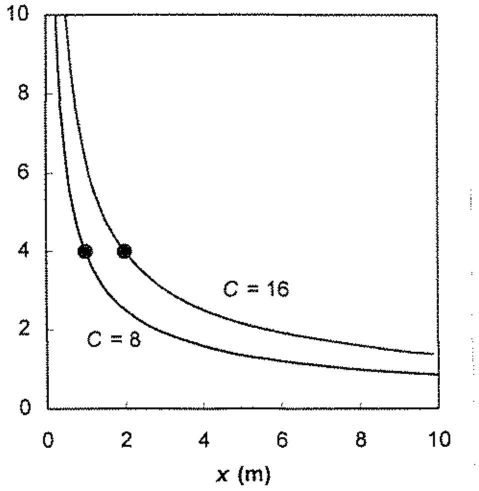
$$\frac{3}{2}\ln y + \ln x = \ln c$$

$$xy^{3/2} = c \quad \underline{\text{Streamline}}$$

pt (1,4) $xy^{3/2} = 8$

pt (2,4) $xy^{3/2} = 16$

Streamline Plot



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Problem 5.15

[3]

Given: Flow in xy plane, $u = Ax^2y^2$ where $A = 0.3\ m^{-3}\cdot s^{-1}$, and coordinates are measured in meters

Find: (a) Possible y component for steady, incompressible flow.
(b) If result is valid for unsteady, incompressible flow
(c) Number of possible y components.
(d) Equation of streamlines for simplest value of v.

Plot: streamlines through points (1,4) and (2,4)

Solution:

Basic equation: $\nabla\cdot\rho\vec{V} + \frac{\partial\rho}{\partial t} = 0 = \frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \overset{=0(1)}{\cancel{\frac{\partial}{\partial z}\rho w}} + \overset{=0(2)}{\cancel{\frac{\partial\rho}{\partial t}}}$

Assumptions: (1) flow in xy plane (given), $\frac{\partial}{\partial z} = 0$
(2) ρ = constant (given)

Then, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ or $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}(Ax^2y^2) = -2Axy^2$

Integrating

$$v = \int \frac{\partial v}{\partial y}\,dy = -\int 2Axy^2 = -\frac{2}{3}Axy^3 + f(x) \leftarrow v$$

The basic equation reduces to the same form for unsteady flow. Hence the result is also valid for unsteady flow. ← v (b)

There are an infinite number of possible y components, since $f(x)$ is arbitrary. The simplest is obtained with $f(x) = 0$. ← (c)

The equation of the streamline is

$$\left.\frac{dy}{dx}\right)_{s.l} = \frac{v}{u} = -\frac{2}{3}\frac{Axy^3}{Ax^2y^2} = -\frac{2y}{3x}$$

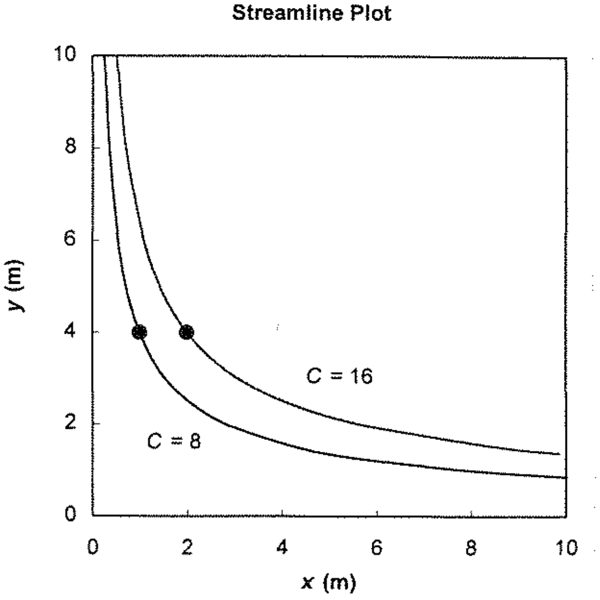
Separating variables & integrating

$$\frac{3}{2}\frac{dy}{y} + \frac{dx}{x} = 0$$

$$\ln y^{3/2} + \ln x = \ln c$$

$$xy^{3/2} = c \leftarrow \text{Streamline}$$

pt (1,4) $xy^{3/2} = 8$
(2,4) $xy^{3/2} = 16$



Problem 5.16

[4]

Given: Conservation of mass.

Find: Identical result to Eq. 5.1a by expanding products of density and velocity in Taylor series.

Solution: Use diagram of Fig. 5.1:

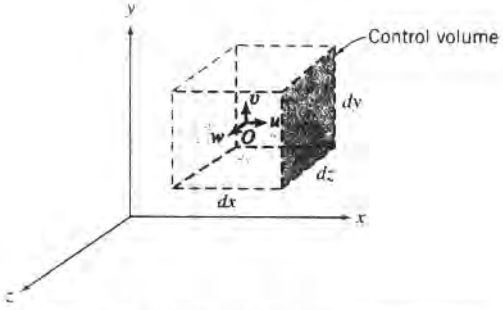


Fig. 5.1 Differential control volume in rectangular coordinates

Apply conservation of mass, using a Taylor series expansion of products. Evaluate derivatives at 0.

For the x direction the mass flux is

$$\dot{m}_x = \rho u\, dA = \rho u\, dx\, dy$$

At the right face

$$\dot{m}_{x+dx/2} = \rho u\, dy\, dz + \frac{\partial}{\partial x}\rho u \frac{dx}{2} dy\, dz \quad \text{(out of CV)}$$

At the left face

$$\dot{m}_{x-dx/2} = \rho u\, dy\, dz + \frac{\partial}{\partial x}\rho u \left(-\frac{dx}{2}\right) dy\, dz \quad \text{(into CV)}$$

The net mass flux is "out" minus "in," so

$$\dot{m}_x(\text{net}) = \dot{m}_{x+dx/2} - \dot{m}_{x-dx/2} = \frac{\partial}{\partial x}\rho u\, dx\, dy\, dz$$

Summing terms for x, y, and z, and including $\frac{\partial \rho}{\partial t} dx\, dy\, dz$, we get

$$0 = \frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w + \frac{\partial \rho}{\partial t}$$

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42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
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Problem 5.17

[5]

Consider a water stream from a jet of an oscillating lawn sprinkler. Describe the corresponding pathline and streakline.

Open-Ended Problem Statement: Consider a water stream from a jet of an oscillating lawn sprinkler. Describe the corresponding pathline and streakline.

Discussion: Refer back to the discussion of streamlines, pathlines, and streaklines in Section 2-2.

Because the sprinkler jet oscillates, this is an unsteady flow. Therefore pathlines and streaklines need not coincide.

A *pathline* is a line tracing the path of an individual fluid particle. The path of each particle is determined by the jet angle and the speed at which the particle leaves the jet.

Once a particle leaves the jet it is subject to gravity and drag forces. If aerodynamic drag were negligible, the path of each particle would be parabolic. The horizontal speed of the particle would remain constant throughout its trajectory. The vertical speed would be slowed by gravity until reaching peak height, and then it would become increasingly negative until the particle strikes the ground. The effect of aerodynamic drag is to reduce the particle speed. With drag the particle will not rise as high vertically nor travel as far horizontally. At each instant the particle trajectory will be lower and closer to the jet compared to the no-friction case. The trajectory after the particle reaches its peak height will be steeper than in the no-friction case.

A *streamline* is a line drawn in the flow that is tangent everywhere to the velocity vectors of the fluid motion. It is difficult to visualize the streamlines for an unsteady flow field because they move laterally. However, the streamline pattern may be drawn at an instant.

A *streakline* is the locus of the present locations of fluid particles that passed a reference point at previous times. As an example, choose the exit of a jet as the reference point. Imagine marking particles that pass the jet exit at a given instant and at uniform time intervals later. The first particle will travel farthest from the jet exit and on the lowest trajectory; the last particle will be located right at the jet exit. The curve joining the present positions of the particles will resemble a spiral whose radius increases with distance from the jet opening.

Problem 5.18

[2]_____

Given: Velocity fields listed below.

Find: Which are possible incompressible flow cases?

Solution: Apply the continuity equation in differential form.

Basic equation: $$\frac{1}{r}\frac{\partial r\rho V_r}{\partial r} + \frac{1}{r}\frac{\partial \rho V_\theta}{\partial \theta} + \overset{=0(1)}{\cancel{\frac{\partial \rho V_z}{\partial z}}} + \overset{=0(2)}{\cancel{\frac{\partial \rho}{\partial t}}} = 0$$

Assumptions: (1) Two-dimensional flow, so $\frac{\partial}{\partial z} = 0$
(2) Incompressible flow

$$\rho = \text{constant}, \text{ so } \frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial(\text{distance})} = 0$$

Then
$$\frac{1}{r}\frac{\partial r V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} = 0$$

or
$$\frac{\partial r V_r}{\partial r} + \frac{\partial V_\theta}{\partial \theta} = 0 \quad \text{is the criterion.}$$

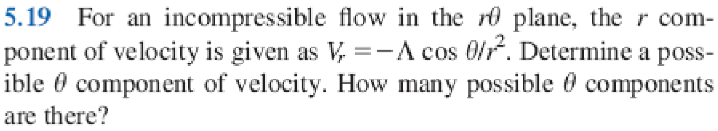
Field	V_r	V_θ	$\frac{\partial r V_r}{\partial r}$	$\frac{\partial V_\theta}{\partial \theta}$	$\frac{\partial r V_r}{\partial r} + \frac{\partial V_\theta}{\partial \theta}$	Possible?
(a)	$U\cos\theta$	$-U\sin\theta$	$U\cos\theta$	$-U\cos\theta$	0	Yes
(b)	$-\frac{q}{2\pi r}$	$\frac{K}{2\pi r}$	0	0	0	Yes
(c)	$U\cos\theta\left[1-\left(\frac{a}{r}\right)^2\right]^*$	$-U\sin\theta\left[1+\left(\frac{a}{r}\right)^2\right]$	$U\cos\theta\left[1+\left(\frac{a}{r}\right)^2\right]$	$-U\cos\theta\left[1+\left(\frac{a}{r}\right)^2\right]$	0	Yes

* Note if $V_r = U\cos\theta\left[1-\left(\frac{a}{r}\right)^2\right]$, then $rV_r = U\cos\theta\left[r - \frac{a^2}{r}\right]$

and $\frac{\partial r V_r}{\partial r} = U\cos\theta\left[1 + \frac{a^2}{r^2}\right] = U\cos\theta\left[1 + \left(\frac{a}{r}\right)^2\right]$

Problem 5.19

[3]



5.19 For an incompressible flow in the $r\theta$ plane, the r component of velocity is given as $V_r = -\Lambda \cos\theta / r^2$. Determine a possible θ component of velocity. How many possible θ components are there?

Given: r component of velocity

Find: θ component for incompressible flow; How many θ components

Solution:

Basic equation:
$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(\rho\cdot r\cdot V_r\right) + \frac{1}{r}\cdot\frac{\partial}{\partial \theta}\left(\rho\cdot V_\theta\right) + \frac{\partial}{\partial z}\left(\rho\cdot V_z\right) + \frac{\partial}{\partial t}\rho = 0$$

Assumption: Incompressible flow; flow in r-θ plane

Hence
$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot V_r\right) + \frac{1}{r}\cdot\frac{\partial}{\partial \theta}\left(V_\theta\right) = 0$$
or
$$\frac{\partial}{\partial \theta}V_\theta = -\frac{\partial}{\partial r}\left(r\cdot V_r\right) = -\frac{\partial}{\partial r}\left(-\frac{\Lambda\cdot\cos(\theta)}{r}\right) = -\frac{\Lambda\cdot\cos(\theta)}{r^2}$$

Integrating
$$V_\theta(r,\theta) = -\int \frac{\Lambda\cdot\cos(\theta)}{r^2}\,d\theta = -\frac{\Lambda\cdot\sin(\theta)}{r^2} + f(r)$$

$$V_\theta(r,\theta) = -\frac{\Lambda\cdot\sin(\theta)}{r^2} + f(r)$$

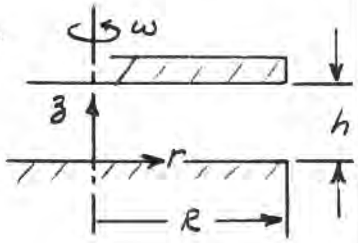
There are an infinite number of solutions as f(r) can be any function of r

The simplest form is
$$V_\theta(r,\theta) = -\frac{\Lambda\cdot\sin(\theta)}{r^2}$$

Problem 5.20

Given: Flow between parallel disks as shown.

Velocity is purely tangential.
No-slip condition is satisfied, so velocity varies linearly with z.



Find: Expression for velocity field.

Solution: A general velocity field would be

$$\vec{V} = V_r \hat{e}_r + V_\theta \hat{e}_\theta + V_z \hat{k}$$

but velocity is purely tangential, so $V_r = V_z = 0$. Then we seek

$$V_\theta = V_\theta(r, \theta, z)$$

By symmetry, $\frac{\partial V_\theta}{\partial \theta} = 0$, so

$$V_\theta = V_\theta(r, z)$$

Since the variation with z is linear, $V_\theta = z f(r) + c$ at most, that is

$$\frac{\partial V_\theta}{\partial z} = f(r)$$

at most.

Along the surface $z=0$, $V_\theta = 0$, so $c = 0$.

Along the surface $z = h$, $V_\theta = \omega r$, so

$$V_\theta(z=h) = \omega r = h f(r)$$

or

$$f(r) = \frac{\omega r}{h}$$

and

$$V_\theta = \omega r \frac{z}{h}$$

Thus

$$\vec{V} = \omega r \frac{z}{h} \hat{e}_\theta \longleftarrow \vec{V}$$

Problem 5.21

Given: Definition of ∇ in cylindrical coordinates.

Obtain: $\nabla\cdot\rho\vec{V}$ in cylindrical coordinates (use hint on page 169).

Show result is identical to Eq. 5.2c.

Solution: The definition of ∇ in cylindrical coordinates is

$$\nabla = \hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z} \qquad (3.19)$$

Note $\rho\vec{V} = \rho(\hat{e}_r V_r + \hat{e}_\theta V_\theta + \hat{k}V_z)$

Hint: $\frac{\partial\hat{e}_r}{\partial\theta} = \hat{e}_\theta$, and $\frac{\partial\hat{e}_\theta}{\partial\theta} = -\hat{e}_r$ (Page 169)

Substituting $\nabla\cdot\rho\vec{V} = \left(\hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\rho(\hat{e}_r V_r + \hat{e}_\theta V_\theta + \hat{k}V_z)$

$$\nabla\cdot\rho\vec{V} = \hat{e}_r\cdot\frac{\partial}{\partial r}\rho(\hat{e}_r V_r + \hat{e}_\theta V_\theta + \hat{k}V_z)$$

$$+ \hat{e}_\theta\cdot\frac{\partial}{\partial\theta}\rho(\hat{e}_r V_r + \hat{e}_\theta V_\theta + \hat{k}V_z)$$

$$+ \hat{k}\cdot\frac{\partial}{\partial z}\rho(\hat{e}_r V_r + \hat{e}_\theta V_\theta + \hat{k}V_z)$$

$$= \hat{e}_r\cdot\hat{e}_r\frac{\partial}{\partial r}\rho V_r + \hat{e}_\theta\cdot\cancelto{\hat{e}_\theta}{\frac{1}{r}\frac{\partial\hat{e}_r}{\partial\theta}}\rho V_r + \cancelto{0}{\hat{e}_\theta\cdot\hat{e}_r}\frac{\partial}{\partial\theta}\rho V_r$$

$$+ \hat{e}_\theta\cdot\cancelto{-\hat{e}_r}{\frac{\partial\hat{e}_\theta}{\partial\theta}}\rho V_\theta + \hat{e}_\theta\cdot\hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}\rho V_\theta + \hat{k}\cdot\hat{k}\frac{\partial}{\partial z}\rho V_z$$

$$\nabla\cdot\rho\vec{V} = \frac{\partial}{\partial r}\rho V_r + \rho\frac{V_r}{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\rho V_\theta + \frac{\partial}{\partial z}\rho V_z$$

Combining the first two terms, $\frac{\partial}{\partial r}\rho V_r + \rho\frac{V_r}{r} = \frac{1}{r}\frac{\partial}{\partial r}r\rho V_r$, as may be verified by differentiation. Substituting

$$\nabla\cdot\rho\vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) \longleftarrow$$

This result is identical to the corresponding terms in Eq. 5.2c.

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Problem 5.22

[3]

5.22 A velocity field in cylindrical coordinates is given as $\vec{V} = \hat{e}_r A/r + \hat{e}_\theta B/r$, where A and B are constants with dimensions of $\mathrm{m^2/s}$. Does this represent a possible incompressible flow? Sketch the streamline that passes through the point $r_0 = 1$ m, $\theta = 90°$ if $A = B = 1\ \mathrm{m^2/s}$, if $A = 1\ \mathrm{m^2/s}$ and $B = 0$, and if $B = 1\ \mathrm{m^2/s}$ and $A = 0$.

Given: The velocity field

Find: Whether or not it is a incompressible flow; sketch various streamlines

Solution:

$$V_r = \frac{A}{r} \qquad V_\theta = \frac{B}{r}$$

For incompressible flow $\quad \frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot V_r\right) + \frac{1}{r}\cdot\frac{d}{d\theta}V_\theta = 0 \qquad \frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot V_r\right) = 0 \qquad \frac{1}{r}\cdot\frac{d}{d\theta}V_\theta = 0$

Hence $\quad \frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot V_r\right) + \frac{1}{r}\cdot\frac{d}{d\theta}V_\theta = 0 \quad$ Flow is incompressible

For the streamlines $\quad \frac{dr}{V_r} = \frac{r\cdot d\theta}{V_\theta} \qquad \frac{r\cdot dr}{A} = \frac{r^2\cdot d\theta}{B}$

so $\quad \int \frac{1}{r}\,dr = \int \frac{A}{B}\,d\theta \quad$ Integrating $\quad \ln(r) = \frac{A}{B}\cdot\theta + \text{const}$

Equation of streamlines is $\quad r = C\cdot e^{\frac{A}{B}\cdot\theta}$

(a) For $A = B = 1\ \mathrm{m^2/s}$, passing through point (1m, $\pi/2$)

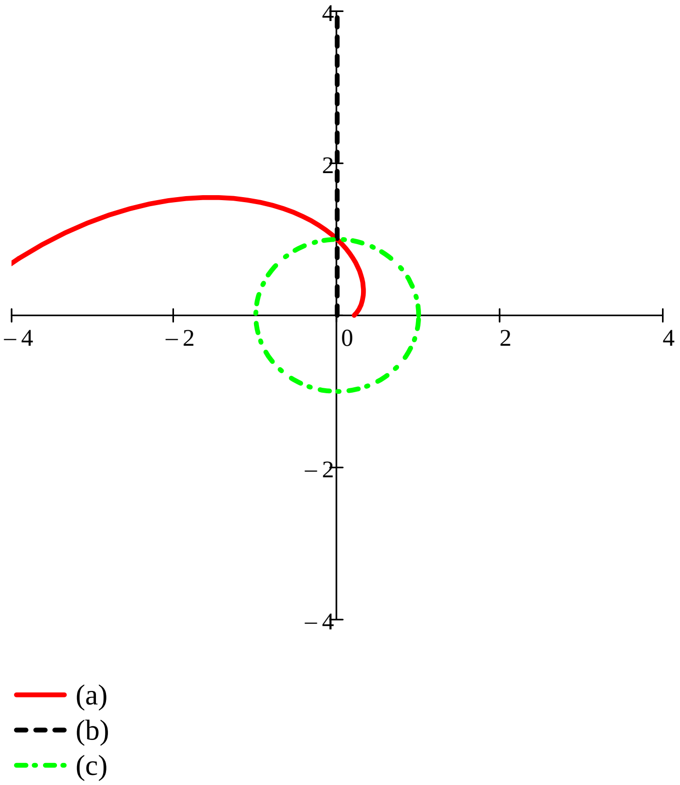
$$r = e^{\theta - \frac{\pi}{2}}$$

(b) For $A = 1\ \mathrm{m^2/s}$, $B = 0\ \mathrm{m^2/s}$, passing through point (1m, $\pi/2$)

$$\theta = \frac{\pi}{2}$$

(c) For $A = 0\ \mathrm{m^2/s}$, $B = 1\ \mathrm{m^2/s}$, passing through point (1m, $\pi/2$)

$$r = 1\cdot m$$



Problem *5.23

[2]

Given: Velocity field for viscometric flow of Example Problem 5.7

$$\vec{V} = U\frac{y}{h}\hat{i}$$

Find: (a) Stream function

(b) Locate streamline that divides flow rate equally.

Solution: Flow is incompressible, so stream function can be derived.

$$\frac{\partial\psi}{\partial y} = u = U\frac{y}{h}, \text{ so } \psi = \int\frac{\partial\psi}{\partial y}dy + f(x) = \int\frac{Uy}{h}dy + f(x) = \frac{Uy^2}{2h} + f(x)$$

Let $\psi = 0$ at $y = 0$, so $f(x) = 0$

$$\psi = \frac{Uy^2}{2h} \longleftarrow$$

ψ

Stream function is maximum at $y = h$.

$$\psi_{max} = \frac{Uh^2}{2h} = \frac{Uh}{2} \; ; \; \frac{Q}{w} = \psi_{max} - \psi_{min} = \frac{Uh}{2} - 0 = \frac{Uh}{2}$$

$$\psi_{Q/2} = \frac{1}{2}\psi_{max} = \frac{Uh}{4} = \frac{Uy^2}{2h}$$

Thus

$$y^2 = \frac{2h}{U}\frac{Uh}{4} = \frac{h^2}{2} \quad \text{so} \quad y = \frac{h}{\sqrt{2}} \longleftarrow$$

$\psi_{Q/2}$

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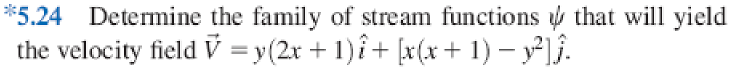
Problem *5.24

[3]



*5.24 Determine the family of stream functions ψ that will yield the velocity field $\vec{V} = y(2x+1)\hat{i} + [x(x+1) - y^2]\hat{j}$.

Given: Velocity field

Find: Stream function ψ

Solution:

Basic equation: $\frac{\partial}{\partial x}(\rho \cdot u) + \frac{\partial}{\partial y}(\rho \cdot v) + \frac{\partial}{\partial z}(\rho \cdot w) + \frac{\partial}{\partial t}\rho = 0$ $\qquad u = \frac{\partial}{\partial y}\psi \qquad v = -\frac{\partial}{\partial x}\psi$

Assumption: Incompressible flow; flow in x-y plane

Hence $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ or $\frac{\partial}{\partial x}[y\cdot(2x+2)] + \frac{\partial}{\partial y}\left[x\cdot(x+1) - y^2\right] \rightarrow 0$

Hence $u = y\cdot(2\cdot x+1) = \frac{\partial}{\partial y}\psi \qquad \psi(x,y) = \int y\cdot(2\cdot x+1)\,dy = x\cdot y^2 + \frac{y^2}{2} + f(x)$

and $v = x\cdot(x+1) - y^2 = -\frac{\partial}{\partial x}\psi \qquad \psi(x,y) = -\int \left[x\cdot(x+1) - y^2\right]dx = -\frac{x^3}{3} - \frac{x^2}{2} + x\cdot y^2 + g(y)$

Comparing these $f(x) = -\frac{x^3}{3} - \frac{x^2}{2}$ and $g(y) = \frac{y^2}{2}$

The stream function is $\psi(x,y) = \frac{y^2}{2} + x\cdot y^2 - \frac{x^2}{2} - \frac{x^3}{3}$

Checking $u(x,y) = \frac{\partial}{\partial y}\left(\frac{y^2}{2} + x\cdot y^2 - \frac{x^2}{2} - \frac{x^3}{3}\right) \rightarrow u(x,y) = y + 2\cdot x\cdot y$

$v(x,y) = -\frac{\partial}{\partial x}\left(\frac{y^2}{2} + x\cdot y^2 - \frac{x^2}{2} - \frac{x^3}{3}\right) \rightarrow v(x,y) = x^2 + x - y^2$

Problem *5.25

[2]

Given: Stream function for an incompressible flow field,

$$\psi = -Ur\sin\theta + \frac{q}{2\pi}\theta$$

Find: (a) An expression for the velocity field.
(b) Points where $|\vec{V}| = 0$.
(c) Show $\psi = 0$ where $|\vec{V}| = 0$.

Solution: The velocity components are given by

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = -U\cos\theta + \frac{q}{2\pi r}$$

$$V_\theta = -\frac{\partial\psi}{\partial r} = U\sin\theta$$

So $\vec{V} = V_r\hat{i}_r + V_\theta\hat{i}_\theta = \left(-U\cos\theta + \frac{q}{2\pi r}\right)\hat{i}_r + U\sin\theta\,\hat{i}_\theta$ $\qquad \vec{V}$

Now $|\vec{V}| = (V_r^2 + V_\theta^2)^{1/2} = 0$ only when both V_r and V_θ are zero.

From the component equations, $V_\theta = 0$ for $\theta = 0, \pi$. When $V_r = 0$,

$$r = \frac{q}{2\pi U\cos\theta}$$

For $r > 0$, then $V_r = 0$ for $\theta = 0$, and $r = \frac{q}{2\pi U}$.

Stagnation point ($|\vec{V}| = 0$) occurs at $(r,\theta) = \left(\frac{q}{2\pi U}, 0\right)$ $\qquad |\vec{V}| =$

Substituting, $\psi_{stagnation} = -Ur\sin\theta + \frac{q}{2\pi}\theta \Big]_{r = \frac{q}{2\pi U},\ \theta = 0}$

or $\psi_{stagnation} = 0$ $\qquad \psi_{st}$

Problem *5.26

***5.26** Does the velocity field of Problem 5.22 represent a possible incompressible flow case? If so, evaluate and sketch the stream function for the flow. If not, evaluate the rate of change of density in the flow field.

Given: The velocity field

Find: Whether or not it is a incompressible flow; sketch stream function

Solution:

$$V_r = \frac{A}{r} \qquad V_\theta = \frac{B}{r}$$

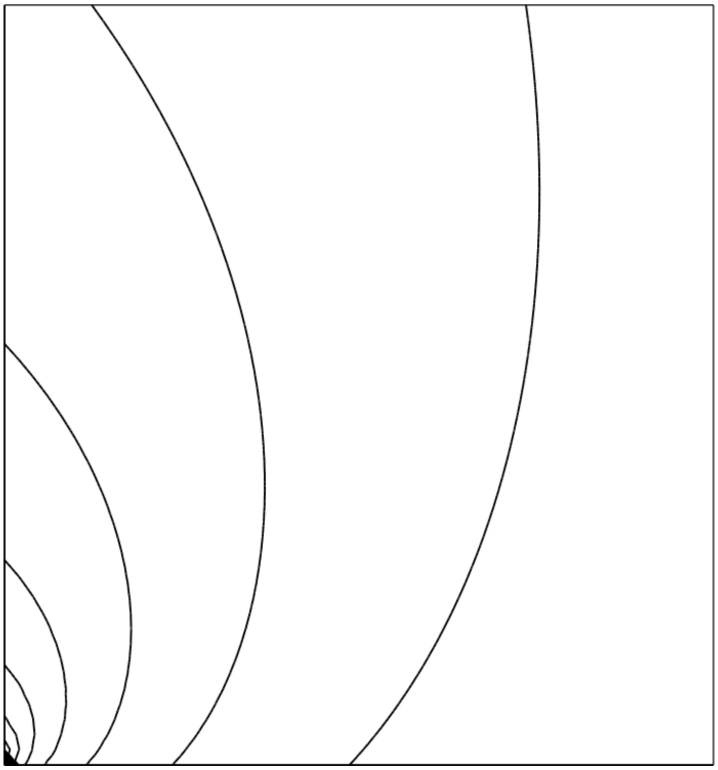
For incompressible flow $\frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot V_r\right) + \frac{1}{r}\cdot\frac{d}{d\theta}V_\theta = 0$ $\quad \frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot V_r\right) = 0$ $\quad \frac{1}{r}\cdot\frac{d}{d\theta}V_\theta = 0$

Hence $\frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot V_r\right) + \frac{1}{r}\cdot\frac{d}{d\theta}V_\theta = 0$ Flow is incompressible

For the stream function $\frac{\partial}{\partial\theta}\psi = r\cdot V_r = A$ $\quad \psi = A\cdot\theta + f(r)$

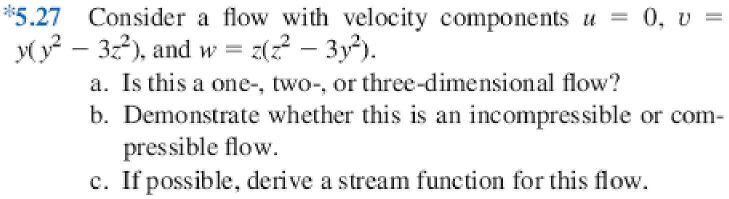
Integrating $\frac{\partial}{\partial r}\psi = -V_\theta = -\frac{B}{r}$ $\quad \psi = -B\cdot\ln(r) + g(\theta)$

Comparing, stream function is $\psi = A\cdot\theta - B\cdot\ln(r)$



Problem *5.27

[3]



***5.27** Consider a flow with velocity components $u = 0$, $v = y(y^2 - 3z^2)$, and $w = z(z^2 - 3y^2)$.

a. Is this a one-, two-, or three-dimensional flow?

b. Demonstrate whether this is an incompressible or compressible flow.

c. If possible, derive a stream function for this flow.

Given: Velocity field

Find: Whether it's 1D, 2D or 3D flow; Incompressible or not; Stream function ψ

Solution:

Basic equation: $\frac{\partial}{\partial x}(\rho\cdot u) + \frac{\partial}{\partial y}(\rho\cdot v) + \frac{\partial}{\partial z}(\rho\cdot w) + \frac{\partial}{\partial t}\rho = 0$ $\quad v = \frac{\partial}{\partial z}\psi \quad w = -\frac{\partial}{\partial y}\psi$

Assumption: Incompressible flow; flow in y-z plane (u = 0)

Velocity field is a function of y and z only, so is 2D

Check for incompressible $\frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$

$$\frac{\partial}{\partial y}\left[y\cdot\left(y^2 - 3\cdot z^2\right)\right] \rightarrow 3\cdot y^2 - 3\cdot z^2 \qquad \frac{\partial}{\partial z}\left[z\cdot\left(z^2 - 3\cdot y^2\right)\right] \rightarrow 3\cdot z^2 - 3\cdot y^2$$

Hence $\frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$ Flow is INCOMPRESSIBLE

Hence $v = y\cdot\left(y^2 - 3\cdot z^2\right) = \frac{\partial}{\partial z}\psi$ $\quad \psi(y,z) = \int y\cdot\left(y^2 - 3\cdot z^2\right)dz = y^3\cdot z - y\cdot z^3 + f(y)$

and $w = z\cdot\left(z^2 - 3\cdot y^2\right) = -\frac{\partial}{\partial y}\psi$ $\quad \psi(y,z) = -\int\left[z\cdot\left(z^2 - 3\cdot y^2\right)\right]dy = -y\cdot z^3 + z\cdot y^3 + g(z)$

Comparing these $f(y) = 0$ and $g(z) = 0$

The stream function is $\psi(y,z) = z\cdot y^3 - z^3\cdot y$

Checking $u(y,z) = \frac{\partial}{\partial z}\left(z\cdot y^3 - z^3\cdot y\right) \rightarrow u(y,z) = y^3 - 3\cdot y\cdot z^2$

$$w(y,z) = -\frac{\partial}{\partial y}\left(z\cdot y^3 - z^3\cdot y\right) \rightarrow w(y,z) = z^3 - 3\cdot y^2\cdot z$$

Problem *5.28

Given: An incompressible, frictionless flow specified by

$\psi = -2Ax - 5Ay$; x, y in meters, $A = 1\ \mathrm{m/s}$

Find: (a) Sketch streamlines $\psi = 0$ and $\psi = 5\ \mathrm{m^2/s}$

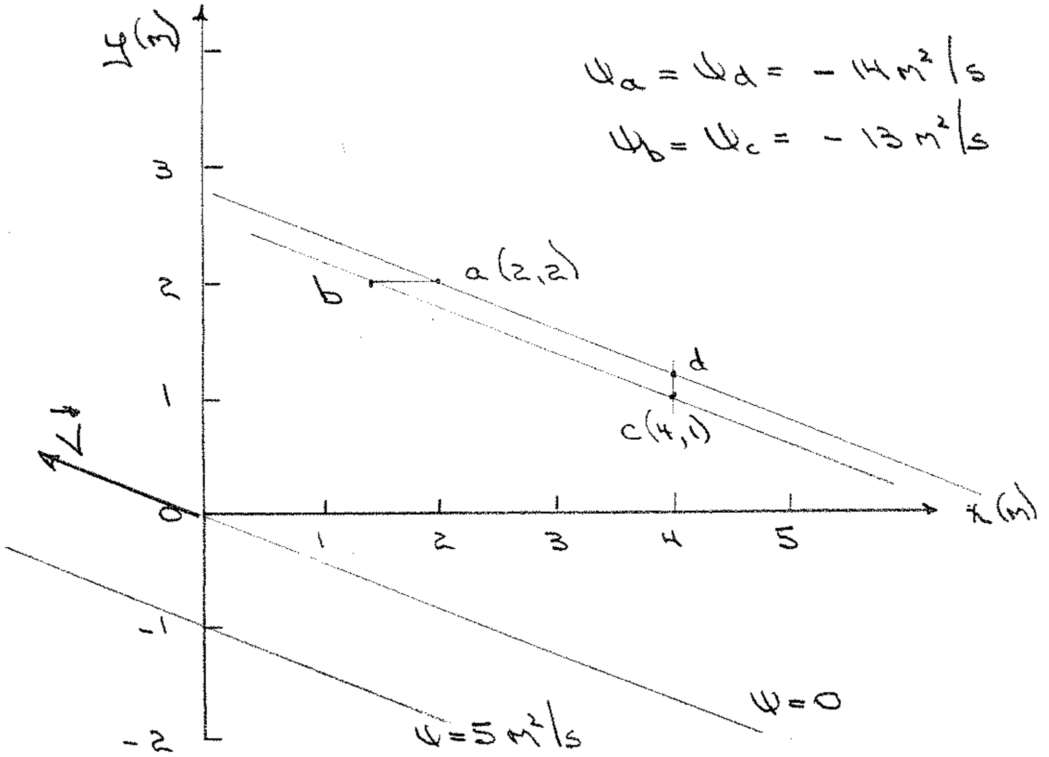
(b) Velocity vector at (0,0)

(c) Flow rate between streamlines passing through points (2,2) and (4,1)

Solution: Streamlines are lines ψ = constant

For $\psi = 0$, $0 = -2Ax - 5Ay$ or $y = -\frac{2}{5}x$

For $\psi = 5$, $5 = -2Ax - 5Ay$ or $y = -\frac{2}{5}x - \frac{1}{5} \times 5\frac{\mathrm{m}^2}{\mathrm{s}} \times \frac{\mathrm{s}}{1\,\mathrm{m}} = -\frac{2}{5}x - 1\mathrm{m}$



$\psi_a = \psi_d = -14\ \mathrm{m^2/s}$
$\psi_b = \psi_c = -13\ \mathrm{m^2/s}$
y(m)
x(m)
3
2
1
0
-1
-2
1
2
3
4
5
a (2,2)
b
d
c (4,1)
$\vec{V}$
$\psi = 0$
$\psi = 5\ \mathrm{m^2/s}$

$u = \frac{\partial\psi}{\partial y} = -5A$; $v = -\frac{\partial\psi}{\partial x} = 2A$, so $\vec{V} = -5\hat{i} + 2\hat{j}\ \mathrm{m/s}$ $\longleftarrow$ $\vec{V}$

$Q = \int_{x=b}^{x=a} v\,dx = \int_{x=b}^{x=a} -\frac{\partial\psi}{\partial x}\,dx = \int_{\psi_b}^{\psi_a} -d\psi = \psi_b - \psi_a = 1\ \mathrm{m^2/s}$, ie ↑

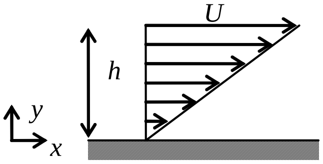
$Q = \int_{y=c}^{y=d} u\,dy = \int_{y=c}^{y=d} \frac{\partial\psi}{\partial y}\,dy = \int_{\psi_c}^{\psi_d} d\psi = \psi_d - \psi_c = -1\ \mathrm{m^2/s}$, ie ←

Thus $Q = 1\ \mathrm{m^3/s}$ per meter of depth. $\longleftarrow$ Q

Problem *5.29

[3]

***5.29** In a parallel one-dimensional flow in the positive x direction, the velocity varies linearly from zero at $y = 0$ to 30 m/s at $y = 1.5$ m. Determine an expression for the stream function, ψ. Also determine the y coordinate above which the volume flow rate is half the total between $y = 0$ and $y = 1.5$ m.



Given: Linear velocity profile

Find: Stream function ψ; y coordinate for half of flow

Solution:

Basic equations: $u = \frac{\partial}{\partial y}\psi$ $\quad v = -\frac{\partial}{\partial x}\psi$ and we have $u = U\cdot\left(\frac{y}{h}\right)$ $\quad v = 0$

Assumption: Incompressible flow; flow in x-y plane

Check for incompressible $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$

$$\frac{\partial}{\partial x}\left(U\cdot\frac{y}{h}\right) \rightarrow 0 \qquad \frac{\partial}{\partial y}0 \rightarrow 0$$

Hence $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ Flow is INCOMPRESSIBLE

Hence $u = U\cdot\frac{y}{h} = \frac{\partial}{\partial y}\psi$ $\qquad \psi(x,y) = \int U\cdot\frac{y}{h}\,dy = \frac{U\cdot y^2}{2\cdot h} + f(x)$

and $v = 0 = -\frac{\partial}{\partial x}\psi$ $\qquad \psi(x,y) = -\int 0\,dx = g(y)$

Comparing these $f(x) = 0$ and $g(y) = \frac{U\cdot y^2}{2\cdot h}$

The stream function is $\psi(x,y) = \frac{U\cdot y^2}{2\cdot h}$

For the flow (0 < y < h) $Q = \int_0^h u\,dy = \frac{U}{h}\cdot\int_0^h y\,dy = \frac{U\cdot h}{2}$

For half the flow rate $\frac{Q}{2} = \int_0^{h_{half}} u\,dy = \frac{U}{h}\cdot\int_0^{h_{half}} y\,dy = \frac{U\cdot h_{half}{}^2}{2\cdot h} = \frac{1}{2}\cdot\left(\frac{U\cdot h}{2}\right) = \frac{U\cdot h}{4}$

Hence $h_{half}{}^2 = \frac{1}{2}\cdot h^2$ $\qquad h_{half} = \frac{1}{\sqrt{2}}\cdot h = \frac{1.5\cdot m}{\sqrt{2}\cdot s} = 1.06\cdot\frac{m}{s}$

Problem *5.30

[3]

Given: Linear approximation to boundary layer velocity profile

$$u = U\frac{y}{\delta}$$

Find: (a) stream function for the flow field
(b) location of streamlines at one-quarter and one-half the total flow rate in the boundary layer.

Solution: For 2-D incompressible flow, ψ satisfies

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \frac{\partial \psi}{\partial y} = U\frac{y}{\delta} \qquad \therefore \psi = \int \frac{\partial \psi}{\partial y}\,dy + f(x) = \int U\frac{y}{\delta}\,dy + f(x)$$

Thus $\quad \psi = \dfrac{Uy^2}{2\delta} + f(x)$

Let $\psi = 0$ along $y = 0$, so $f(x) = 0$ and $\psi = \dfrac{U}{2\delta}y^2 \longleftarrow \psi$

The total flow rate within the boundary layer is

$$\frac{Q}{w} = \psi(\delta) - \psi(0) = \frac{1}{2}U\delta$$

At $\frac{1}{4}$ of total, $\psi - \psi_0 = \dfrac{U}{2\delta}y^2 = \dfrac{1}{4}\left(\dfrac{1}{2}U\delta\right)$

$$\therefore \left(\frac{y}{\delta}\right)^2 = \frac{1}{4} \qquad \text{and} \quad \frac{y}{\delta} = \frac{1}{2} \longleftarrow \frac{1}{4}\frac{Q}{w}$$

At $\frac{1}{2}$ of total, $\psi - \psi_0 = \dfrac{U}{2\delta}y^2 = \dfrac{1}{2}\left(\dfrac{1}{2}U\delta\right)$

$$\therefore \left(\frac{y}{\delta}\right)^2 = \frac{1}{2} \qquad \text{and} \quad \frac{y}{\delta} = \sqrt{\frac{1}{2}} = 0.707 \longleftarrow \frac{1}{2}\frac{Q}{w}$$

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Problem *5.31

[3]

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Given: Parabolic approximation to boundary layer velocity profile

$$u = U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]$$

Find: (a) stream function for the flow field
(b) location of streamlines at one-quarter and one-half the total flow rate in the boundary layer.

Solution: For 2-D incompressible flow, ψ satisfies

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \frac{\partial \psi}{\partial y} = U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]$$

$$\therefore \quad \psi = \int \frac{\partial \psi}{\partial y}\,dy + f(x) = U\int\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] dy + f(x).$$

$$\psi = U\left[\frac{y^2}{\delta} - \frac{y^3}{3\delta^2}\right] + f(x)$$

Let $\psi = 0$ along $y = 0$, so $f(x) = 0$ and $\psi = U\delta\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]$

The total flow rate within the boundary layer is

$$\frac{Q}{w} = \psi(\delta) - \psi(0) = U\delta\left[1 - \frac{1}{3}\right] = \frac{2}{3}U\delta$$

At $\frac{1}{4}$ of total, $\psi - \psi_0 = U\delta\left[\left(y/\delta\right)^2 - \frac{1}{3}\left(y/\delta\right)^3\right] = \frac{1}{4}\left(\frac{2}{3}U\delta\right)$

$$\therefore \left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3 = \frac{1}{6} = 0.167$$

Trial and error solution gives $\frac{y}{\delta} = 0.442 \longleftarrow \frac{1}{4}\frac{Q}{w}$

At $\frac{1}{2}$ of total, $\psi - \psi_0 = U\delta\left[\left(y/\delta\right)^2 - \frac{1}{3}\left(y/\delta\right)^3\right] = \frac{1}{2}\left(\frac{2}{3}U\delta\right)$

$$\therefore \left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3 = \frac{1}{3} = 0.333$$

Trial and error solution gives $\frac{y}{\delta} = 0.652 \longleftarrow \frac{1}{2}\frac{Q}{w}$

Problem *5.32

[3]

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Given: Sinusoidal approximation to boundary layer velocity profile

$$u = U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

Find: Locate streamlines at quarter and half total flow rate.

Solution: Flow is incompressible so ψ may be derived.

$$u = \frac{\partial\psi}{\partial y} = U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\ ;\ \psi = \int\frac{\partial\psi}{\partial y}dy + f(x) = \int U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy + f(x)$$

Thus $\psi = -\frac{2\delta U}{\pi}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) + f(x)$

Let $\psi = 0$ along $y = 0$, so $f(x) = 0$ $\qquad \psi = -\frac{2\delta U}{\pi}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)$ $\qquad \psi$

The total flow rate is $\frac{Q}{w} = \psi(\delta) - \psi(0) = -\frac{2\delta U}{\pi}\cos\left(\frac{\pi}{2}\right) + \frac{2\delta U}{\pi}\cos(0) = \frac{2\delta U}{\pi}$

At 1/4 of total, $\psi - \psi_0 = \frac{2\delta U}{\pi}\left[1 - \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] = \frac{1}{4}\frac{2\delta U}{\pi} = \frac{\delta U}{2\pi}$

$$1 - \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) = \frac{\pi}{2\delta U}\frac{\delta U}{2\pi} = \frac{1}{4}\ ;\ \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) = \frac{3}{4}\ ;\ \frac{y}{\delta} = 0.460$$

$\frac{1}{4}$

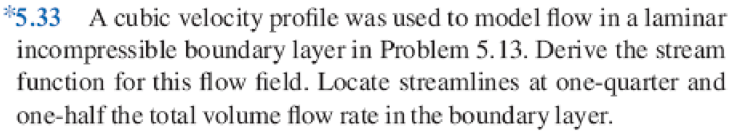
At 1/2 of total, $\psi - \psi_0 = \frac{2\delta U}{\pi}\left[1 - \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] = \frac{1}{2}\frac{2\delta U}{\pi} = \frac{\delta U}{\pi}$

$$1 - \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) = \frac{\pi}{2\delta U}\frac{\delta U}{\pi} = \frac{1}{2}\ ;\ \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) = \frac{1}{2}\ ;\ \frac{y}{\delta} = 0.667$$

$\frac{1}{2}$

Problem *5.33

[3]



*5.33 A cubic velocity profile was used to model flow in a laminar incompressible boundary layer in Problem 5.13. Derive the stream function for this flow field. Locate streamlines at one-quarter and one-half the total volume flow rate in the boundary layer.

Given: Data on boundary layer

Find: Stream function; locate streamlines at 1/4 and 1/2 of total flow rate

Solution:

$$u(x,y) = U\cdot\left[\frac{3}{2}\cdot\left(\frac{y}{\delta}\right) - \frac{1}{2}\cdot\left(\frac{y}{\delta}\right)^3\right] \qquad \text{and} \qquad \delta(x) = c\cdot\sqrt{x}$$

For the stream function $u = \frac{\partial}{\partial y}\psi = U\cdot\left[\frac{3}{2}\cdot\left(\frac{y}{\delta}\right) - \frac{1}{2}\cdot\left(\frac{y}{\delta}\right)^3\right]$

Hence $\psi = \int U\cdot\left[\frac{3}{2}\cdot\left(\frac{y}{\delta}\right) - \frac{1}{2}\cdot\left(\frac{y}{\delta}\right)^3\right] dy \qquad \psi = U\cdot\left(\frac{3}{4}\cdot\frac{y^2}{\delta} - \frac{1}{8}\cdot\frac{y^4}{\delta^3}\right) + f(x)$

Let ψ = 0 = 0 along *y* = 0, so f(*x*) = 0, so $\psi = U\cdot\delta\cdot\left[\frac{3}{4}\cdot\left(\frac{y}{\delta}\right)^2 - \frac{1}{8}\cdot\left(\frac{y}{\delta}\right)^4\right]$

The total flow rate in the boundary layer is

$$\frac{Q}{W} = \psi(\delta) - \psi(0) = U\cdot\delta\cdot\left(\frac{3}{4} - \frac{1}{8}\right) = \frac{5}{8}\cdot U\cdot\delta$$

At 1/4 of the total $\psi - \psi_0 = U\cdot\delta\cdot\left[\frac{3}{4}\cdot\left(\frac{y}{\delta}\right)^2 - \frac{1}{8}\cdot\left(\frac{y}{\delta}\right)^4\right] = \frac{1}{4}\cdot\left(\frac{5}{8}\cdot U\cdot\delta\right)$

$$24\cdot\left(\frac{y}{\delta}\right)^2 - 4\cdot\left(\frac{y}{\delta}\right)^4 = 5 \qquad \text{or} \qquad 4\cdot X^2 - 24\cdot X + 5 = 0 \qquad \text{where} \qquad X^2 = \frac{y}{\delta}$$

The solution to the quadratic is $X = \frac{24 - \sqrt{24^2 - 4\cdot 4\cdot 5}}{2\cdot 4}$ $X = 0.216$ Note that the other root is $\frac{24 + \sqrt{24^2 - 4\cdot 4\cdot 5}}{2\cdot 4} = 5.784$

Hence $\frac{y}{\delta} = \sqrt{X} = 0.465$

At 1/2 of the total flow $\psi - \psi_0 = U\cdot\delta\cdot\left[\frac{3}{4}\cdot\left(\frac{y}{\delta}\right)^2 - \frac{1}{8}\cdot\left(\frac{y}{\delta}\right)^4\right] = \frac{1}{2}\cdot\left(\frac{5}{8}\cdot U\cdot\delta\right)$

$$12\cdot\left(\frac{y}{\delta}\right)^2 - 2\cdot\left(\frac{y}{\delta}\right)^4 = 5 \qquad \text{or} \qquad 2\cdot X^2 - 12\cdot X + 5 = 0 \qquad \text{where} \qquad X^2 = \frac{y}{\delta}$$

The solution to the quadratic is $X = \frac{12 - \sqrt{12^2 - 4\cdot 2\cdot 5.}}{2\cdot 2}$ $X = 0.450$ Note that the other root is $\frac{12 + \sqrt{12^2 - 4\cdot 2\cdot 5}}{2\cdot 2} = 5.55$

Hence $\frac{y}{\delta} = \sqrt{X} = 0.671$

Problem *5.34

[3]

Given: Rigid-body motion in Example Problem 5.6

$$\vec{V} = r\omega\hat{e}_\theta \qquad \omega = 0.5\ \text{rad/s}$$

Find: (a) Obtain the stream function for this flow.
(b) Evaluate the volume flow rate per unit depth between $r_1 = 0.10$ m and $r_2 = 0.12$ m.
(c) Sketch the velocity profile along a line of constant θ.
(d) Check the volume flow rate calculated from the stream function by integrating the velocity profile along this line.

Solution: From the definition of ψ, $\frac{\partial\psi}{\partial r} = -V_\theta = -r\omega$

Thus $\psi = \int \frac{\partial\psi}{\partial r}dr + f(\theta) = \int -r\omega\,dr + f(\theta) = -\frac{1}{2}r^2\omega + f(\theta)$

But $V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}f'(\theta) = 0 \quad \therefore f(\theta) = c$

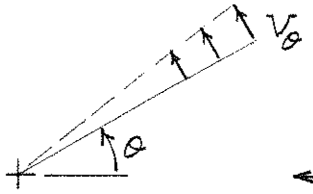
and $\psi = -\frac{1}{2}r^2\omega + c$ ←—— ψ

The volume flow rate per unit depth is

$$\frac{Q}{b} = \psi(r_2) - \psi(r_1) = -\frac{1}{2}r_2^2\omega + c - \left[-\frac{1}{2}r_1^2\omega + c\right] = \frac{\omega}{2}(r_1^2 - r_2^2)$$

$$\frac{Q}{b} = \frac{1}{2} \times 0.5\frac{\text{rad}}{\text{s}}\left[(0.10)^2 - (0.12)^2\right]\text{m}^2 = -0.0011 \quad \text{m}^3/\text{s}\,/\,\text{m}$$ ←—— Q/b

Because $Q/b < 0$, flow is in the direction of $\hat{e}_\theta$.



Along θ = constant, V_θ varies linearly: ←—— Plot

From the linear velocity variation, $V_\theta = \omega r$

Thus $\frac{Q}{b} = \int_{r_1}^{r_2} V_\theta\,dr = \int_{r_1}^{r_2} r\omega\,dr = \frac{1}{2}r^2\omega\Big]_{r_1}^{r_2} = \frac{\omega}{2}(r_2^2 - r_1^2)$ ←—— Q/b

From the sketch, this flow is in the direction of $\hat{e}_\theta$.

Comparing the expressions for Q/b shows they are the same except for sign.

Problem *5.35

[3]

Given: Velocity field for a free vortex from Example Problem 5.6:

$$\vec{V} = \frac{c}{r}\hat{e}_\theta \qquad c = 0.5\ m^2/sec$$

Find: (a) Obtain the stream function for this flow.
(b) Evaluate the volume flow rate per unit depth between $r_1 = 0.10$ m and $r_2 = 0.12$ m.
(c) Sketch the velocity profile along a line of constant θ.
(d) Check the volume flow rate calculated from the stream function by integrating the velocity profile along this line.

Solution: From the definition of ψ, $\frac{\partial \psi}{\partial r} = -V_\theta = -\frac{c}{r}$

Thus $\psi = \int \frac{\partial \psi}{\partial r} dr + f(\theta) = \int -\frac{c}{r} dr + f(\theta) = -c \ln r + f(\theta)$

But $V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r} f'(\theta) = 0$. Therefore $f(\theta) = \text{constant} = c_1$, and

$$\psi = -c \ln r + c_1$$

ψ

The volume flow rate per unit depth is

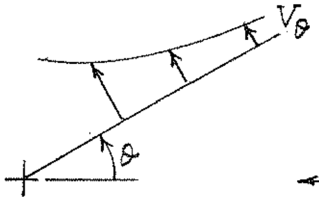
$$\frac{Q}{b} = \psi(r_2) - \psi(r_1) = -c \ln r_2 + c_1 - [-c \ln r_1 + c_1] = c(\ln r_1 - \ln r_2) = c \ln\left(\frac{r_1}{r_2}\right)$$

$$\frac{Q}{b} = 0.5\frac{m^2}{s} \times \ln\left(\frac{0.10\ m}{0.12\ m}\right) = -0.0912\ m^3/s\ /m$$

Q/b

Because $Q/b < 0$, flow is in the direction of $\hat{e}_\theta$.

Along $\theta = \text{constant}$, V_θ varies inversely with r:



Plot

From the expression for $\vec{V}$, $V_\theta = \frac{c}{r}$. Thus

$$\frac{Q}{b} = \int_{r_1}^{r_2} V_\theta dr = \int_{r_1}^{r_2} \frac{c}{r} dr = c \ln\left(\frac{r_2}{r_1}\right)$$

Q/b

From the sketch, this flow is in the direction of $\hat{e}_\theta$.

Comparing shows that the expressions for Q/b are the same except for sign.

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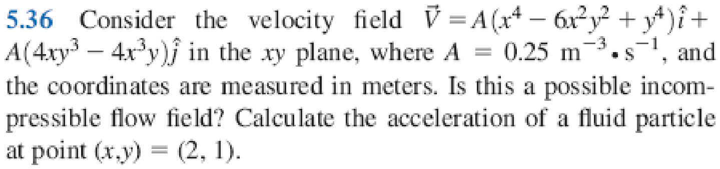
Problem 5.36

[3]



5.36 Consider the velocity field $\vec{V} = A(x^4 - 6x^2y^2 + y^4)\hat{i} + A(4xy^3 - 4x^3y)\hat{j}$ in the xy plane, where $A = 0.25\ \text{m}^{-3}\cdot\text{s}^{-1}$, and the coordinates are measured in meters. Is this a possible incompressible flow field? Calculate the acceleration of a fluid particle at point $(x,y) = (2, 1)$.

Given: Velocity field

Find: Whether flow is incompressible; Acceleration of particle at (2,1)

Solution:

Basic equations

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0 \qquad \vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

$$u(x,y) = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right) \qquad v(x,y) = A\cdot\left(4\cdot x\cdot y^3 - 4\cdot x^3\cdot y\right)$$

For incompressible flow

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$$

Checking

$$\frac{\partial}{\partial x}\left[A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\right] \rightarrow A\cdot\left(4\cdot x^3 - 12\cdot x\cdot y^2\right) \qquad \frac{\partial}{\partial y}\left[A\cdot\left(4\cdot x\cdot y^3 - 4\cdot x^3\cdot y\right)\right] \rightarrow -A\cdot\left(4\cdot x^3 - 12\cdot x\cdot y^2\right)$$

Hence

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$$

The acceleration is given by

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

For this flow

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u$$

$$a_x = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\cdot\frac{\partial}{\partial x}\left[A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\right] + A\cdot\left(4\cdot x\cdot y^3 - 4\cdot x^3\cdot y\right)\cdot\frac{\partial}{\partial y}\left[A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\right]$$

$$a_x = 4\cdot A^2\cdot x\cdot\left(x^2 + y^2\right)^3$$

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v$$

$$a_y = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\cdot\frac{\partial}{\partial x}\left[A\cdot\left(4\cdot x\cdot y^3 - 4\cdot x^3\cdot y\right)\right] + A\cdot\left(4\cdot x\cdot y^3 - 4\cdot x^3\cdot y\right)\cdot\frac{\partial}{\partial y}\left[A\cdot\left(4\cdot x\cdot y^3 - 4\cdot x^3\cdot y\right)\right]$$

$$a_y = 4\cdot A^2\cdot y\cdot\left(x^2 + y^2\right)^3$$

Hence at (2,1)

$$a_x = 4\times\left(\frac{1}{4}\cdot\frac{1}{m^3\cdot s}\right)^2\times 2\cdot m\times\left[(2\cdot m)^2 + (1\cdot m)^2\right]^3 \qquad a_x = 62.5\frac{m}{s^2}$$

$$a_y = 4\times\left(\frac{1}{4}\cdot\frac{1}{m^3\cdot s}\right)^2\times 1\cdot m\times\left[(2\cdot m)^2 + (1\cdot m)^2\right]^3 \qquad a_y = 31.3\frac{m}{s^2} \qquad a = \sqrt{a_x^2 + a_y^2} \qquad a = 69.9\frac{m}{s^2}$$

Problem 5.37

[2]

Given: Flow field $\vec{V} = xy^2\hat{i} - \frac{1}{3}y^3\hat{j} + xy\hat{k}$

Find: (a) Dimensions.
(b) If possible incompressible flow.
(c) Acceleration of particle at point $(x,y,z) = (1,2,3)$.

Solution: Apply continuity, use substantial derivative.

Basic equations: $$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \overset{=0(1)}{\cancel{\frac{\partial \rho w}{\partial z}}} + \overset{=0(2)}{\cancel{\frac{\partial \rho}{\partial t}}} = 0$$

$$\vec{a}_p = \frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \overset{=0(1)}{\cancel{w\frac{\partial \vec{V}}{\partial z}}} + \overset{=0(2)}{\cancel{\frac{\partial \vec{V}}{\partial t}}}$$

Assumptions: (1) Two-dimensional flow, $\vec{V} = \vec{v}(x,y)$, so $\partial/\partial z = 0$
(2) Incompressible flow
(3) Steady flow, $\vec{V} \neq \vec{V}(t)$

Then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = y^2 - y^2 = 0$ Flow is a possible incompressible case. $\rho =$

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} \; ; \quad \frac{\partial \vec{V}}{\partial x} = y^2\hat{i} + y\hat{k} \; ; \quad \frac{\partial \vec{V}}{\partial y} = 2xy\hat{i} - y^2\hat{j} + x\hat{k}$$

$$= (xy^2)(y^2\hat{i} + y\hat{k}) + (-\tfrac{1}{3}y^3)(2xy\hat{i} - y^2\hat{j} + x\hat{k})$$

$$= \hat{i}\,(xy^4 - \tfrac{2}{3}xy^4) + \hat{j}\,(\tfrac{1}{3}y^5) + \hat{k}\,(xy^3 - \tfrac{1}{3}xy^3)$$

$$\vec{a}_p = \hat{i}\,(\tfrac{1}{3}xy^4) + \hat{j}\,(\tfrac{1}{3}y^5) + \hat{k}\,(\tfrac{2}{3}xy^3)$$

At $(x,y,z) = (1,2,3)$

$$\vec{a}_p = \hat{i}\left[\tfrac{1}{3}(1)(16)\right] + \hat{j}\left[\tfrac{1}{3}(32)\right] + \hat{k}\left[\tfrac{2}{3}(1)(8)\right] = \frac{16}{3}\hat{i} + \frac{32}{3}\hat{j} + \frac{16}{3}\hat{k}$$ $\vec{a}_p$

($\vec{a}_p$ will be in m/s^2)

Problem 5.38

[2]

Given: Flow field $\vec{V} = ax^2y\,\hat{i} - by\,\hat{j} + cz^2\,\hat{k}$; $a = 1/m^2\cdot s$

$b = 3/s$

$c = 2/m\cdot s$

Find: (a) Dimensions of flow field.

(b) If possible incompressible flow.

(c) Acceleration of a particle at $(x,y,z) = (3,1,2)$.

Solution: Apply continuity, use substantial derivative.

Basic equations: $$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \overset{=0}{\cancel{\frac{\partial \rho}{\partial t}}} = 0$$

$$\vec{a}_p = \frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \overset{=0}{\cancel{\frac{\partial \vec{V}}{\partial t}}}$$

Assumption: Incompressible flow, ρ = constant

Then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ is criterion.

Note $\vec{V} = \vec{V}(x,y,z)$, so flow is three-dimensional, and ← 3-D

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2xy - 3 + 4z \neq 0$$

Flow cannot be incompressible. ← $\rho \neq$

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \ ; \ \frac{\partial \vec{V}}{\partial x} = 2axy\,\hat{i}\ ,\ \frac{\partial \vec{V}}{\partial y} = ax^2\,\hat{i} - b\,\hat{j}\ ,\ \frac{\partial \vec{V}}{\partial z} = 2cz\,\hat{k}$$

$$= (ax^2y)(2axy\,\hat{i}) + (-by)(ax^2\,\hat{i} - b\,\hat{j}) + (cz^2)(2cz\,\hat{k})$$

$$\vec{a}_p = \hat{i}\,(2a^2x^3y^2 - abx^2y) + \hat{j}\,(b^2y) + \hat{k}\,(2c^2z^3)$$

At $(x,y,z) = (3,1,2)$,

$$\vec{a}_p = \hat{i}\left[2\times\frac{(1)^2}{m^4\cdot s^2}\times(3)^3m^3\times(1)^2m^2 - \frac{1}{m^2\cdot s}\times\frac{3}{s}\times(3)^2m^2\times 1m\right] + \hat{j}\left[\frac{(3)^2}{s^2}\times 1m\right] + \hat{k}\left[2\times\frac{(2)^2}{m^2\cdot s^2}\times(2)^3m^3\right]$$

$$\vec{a}_p = 27\,\hat{i} + 9\,\hat{j} + 64\,\hat{k}\ \frac{m}{s^2}$$ ← $\vec{a}$

Problem 5.39

Given: Velocity field (within a laminar boundary layer) is given by $\vec{V} = A\frac{Uy}{x^{1/2}}\left(\hat{i} + \frac{y}{4x}\hat{j}\right)$

where $A = 141\ m^{-1/2}$

$U = 0.240\ m/s$

Find: (a) Show that this velocity field represents a possible incompressible flow

(b) Calculate $\vec{a}$ of particle at $(x,y) = (0.5m, 5mm)$

(c) Slope of streamline through point $(0.5m, 5mm)$

Solution:

From given velocity field $\vec{V} = \vec{V}(x,y)$, $w = 0$, flow is steady

(a) Check conservation of mass for ρ = constant

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancelto{0}{\frac{\partial w}{\partial z}} = 0$$

$$\left.\begin{aligned} u &= A\frac{Uy}{x^{1/2}} & \frac{\partial u}{\partial x} &= -\frac{1}{2}AU\frac{y}{x^{3/2}} \\ v &= AU\frac{y^2}{4x^{3/2}} & \frac{\partial v}{\partial y} &= \frac{1}{2}AU\frac{y}{x^{3/2}} \end{aligned}\right\} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$\therefore$ incompressible ← Q.E.D.

(b) $$\vec{a} = \frac{D\vec{V}}{Dt} = u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + \cancelto{0}{w\frac{\partial\vec{V}}{\partial z}} + \cancelto{0}{\frac{\partial\vec{V}}{\partial t}}$$

$$a_{p_x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad ; \quad \frac{\partial u}{\partial y} = AU\frac{1}{x^{1/2}}$$

$$a_{p_x} = A\frac{Uy}{x^{1/2}}\left(-\frac{1}{2}AU\frac{y}{x^{3/2}}\right) + AU\frac{y^2}{4x^{3/2}}\left(AU\frac{1}{x^{1/2}}\right)$$

$$a_{p_x} = -\frac{1}{2}A^2U^2\frac{y^2}{x^2} + A^2U^2\frac{y^2}{4x^2} = -\frac{1}{4}\left(\frac{AUy}{x}\right)^2$$

$$a_{p_x} = -\frac{1}{4}\left[\frac{141}{m^{1/2}} \times 0.240\frac{m}{s} \times \frac{0.005m}{0.5m}\right]^2 = -0.0286\ m/s^2$$ ←

$$a_{p_y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \quad ; \quad \frac{\partial v}{\partial x} = -\frac{3}{8}\frac{AUy^2}{x^{5/2}}$$

$$= AU\frac{y}{x^{1/2}}\left(-\frac{3}{8}AU\frac{y^2}{x^{5/2}}\right) + AU\frac{y^2}{4x^{3/2}}\left(\frac{1}{2}AU\frac{y}{x^{3/2}}\right)$$

$$= -\frac{3}{8}A^2U^2\frac{y^3}{x^3} + \frac{1}{8}A^2U^2\frac{y^3}{x^3} = -\frac{1}{4}A^2U^2\frac{y^3}{x^3}$$

$$a_{p_y} = -\frac{1}{4}\left(\frac{141}{m^{1/2}} \times 0.240\frac{m}{s}\right)^2\left(\frac{0.005m}{0.5m}\right)^3 = -2.86 \times 10^{-4}\ m/s^2$$ ←

$$\therefore \vec{a}_p = -2.86\left(10^{-2}\hat{i} + 10^{-4}\hat{j}\right)m/s^2$$ ← $\vec{a}_p$

The slope of the streamline is given by

$$\left.dy/dx\right)_s = \frac{v}{u} = \frac{y}{4x} = \frac{5 \times 10^{-3}m}{4 \times 0.5m} = 0.0025$$ ← $\left.\frac{dy}{dx}\right)_s$

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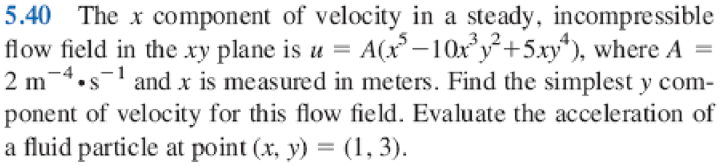
Problem 5.40

[3]



5.40 The x component of velocity in a steady, incompressible flow field in the xy plane is $u = A(x^5 - 10x^3y^2 + 5xy^4)$, where $A = 2\ \mathrm{m^{-4}\cdot s^{-1}}$ and x is measured in meters. Find the simplest y component of velocity for this flow field. Evaluate the acceleration of a fluid particle at point $(x, y) = (1, 3)$.

Given: x component of velocity field

Find: Simplest y component for incompressible flow; Acceleration of particle at (1,3)

Solution:

Basic equations

$$u = \frac{\partial}{\partial y}\psi \qquad v = -\frac{\partial}{\partial x}\psi \qquad \vec{a}_p = \underbrace{\frac{D\vec{V}}{Dt}}_{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\substack{\text{local}\\\text{acceleration}}}$$

We are given

$$u(x,y) = A\cdot\left(x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4\right)$$

Hence for incompressible flow

$$\psi(x,y) = \int u\,dy = \int A\cdot\left(x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4\right)dy = A\cdot\left(x^5\cdot y - \frac{10}{3}\cdot x^3\cdot y^3 + x\cdot y^5\right) + f(x)$$

$$v(x,y) = -\frac{\partial}{\partial x}\psi(x_y) = -\frac{\partial}{\partial x}\left[A\cdot\left(x^5\cdot y - \frac{10}{3}\cdot x^3\cdot y^3 + x\cdot y^5\right) + f(x)\right] = -A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right) + F(x)$$

Hence

$$v(x,y) = -A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right) + F(x)$$

where F(x) is an arbitrary function of x

The simplest is

$$v(x,y) = -A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right)$$

The acceleration is given by

$$\vec{a}_p = \underbrace{\frac{D\vec{V}}{Dt}}_{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\substack{\text{local}\\\text{acceleration}}}$$

For this flow

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u$$

$$a_x = A\cdot\left(x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4\right)\cdot\frac{\partial}{\partial x}\left[A\cdot\left(x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4\right)\right] - A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right)\cdot\frac{\partial}{\partial y}\left[A\cdot\left(x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4\right)\right]$$

$$a_x = 5\cdot A^2\cdot x\cdot\left(x^2 + y^2\right)^4$$

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v$$

$$a_y = A\cdot\left(x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4\right)\cdot\frac{\partial}{\partial x}\left[-A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right)\right] - A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right)\cdot\frac{\partial}{\partial y}\left[-A\cdot\left(5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5\right)\right]$$

$$a_y = 5\cdot A^2\cdot y\cdot\left(x^2 + y^2\right)^4$$

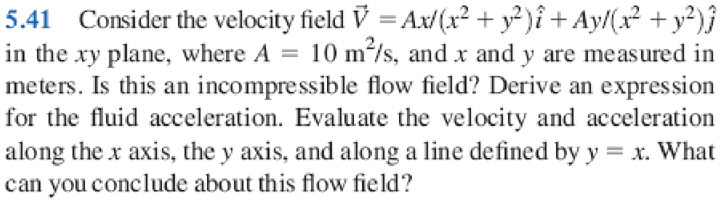
Hence at (1,3)

$$a_x = 5\times\left(\frac{1}{2}\cdot\frac{1}{m^4\cdot s}\right)^2\times 1\cdot m\times\left[(1\cdot m)^2 + (3\cdot m)^2\right]^4 \qquad a_x = 1.25\times 10^4\,\frac{m}{s^2}$$

$$a_y = 5\times\left(\frac{1}{2}\cdot\frac{1}{m^4\cdot s}\right)^2\times 3\cdot m\times\left[(1\cdot m)^2 + (3\cdot m)^2\right]^4 \qquad a_y = 3.75\times 10^4\,\frac{m}{s^2} \qquad a = \sqrt{a_x^2 + a_y^2} \qquad a = 3.95\times 10^4\,\frac{m}{s^2}$$

Problem 5.41

[2]



5.41 Consider the velocity field $\vec{V} = Ax/(x^2 + y^2)\hat{i} + Ay/(x^2 + y^2)\hat{j}$ in the xy plane, where $A = 10\ \text{m}^2/\text{s}$, and x and y are measured in meters. Is this an incompressible flow field? Derive an expression for the fluid acceleration. Evaluate the velocity and acceleration along the x axis, the y axis, and along a line defined by $y = x$. What can you conclude about this flow field?

Given: Velocity field

Find: Whether flow is incompressible; expression for acceleration; evaluate acceleration along axes and along $y = x$

Solution:

The given data is $A = 10 \cdot \frac{\text{m}^2}{\text{s}}$ $\qquad u(x,y) = \frac{A \cdot x}{x^2 + y^2}$ $\qquad v(x,y) = \frac{A \cdot y}{x^2 + y^2}$

For incompressible flow $\frac{\partial}{\partial x} u + \frac{\partial}{\partial y} v = 0$

Hence, checking $\frac{\partial}{\partial x} u + \frac{\partial}{\partial y} v = -A \cdot \frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} + A \cdot \frac{\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} = 0$ $\qquad$ Incompressible flow

The acceleration is given by

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

For the present steady, 2D flow

$$a_x = u \cdot \frac{du}{dx} + v \cdot \frac{du}{dy} = \frac{A \cdot x}{x^2 + y^2} \cdot \left[-\frac{A \cdot \left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} \right] + \frac{A \cdot y}{x^2 + y^2} \cdot \left[-\frac{2 \cdot A \cdot x \cdot y}{\left(x^2 + y^2\right)^2} \right] \qquad a_x = -\frac{A^2 \cdot x}{\left(x^2 + y^2\right)^2}$$

$$a_y = u \cdot \frac{dv}{dx} + v \cdot \frac{dv}{dy} = \frac{A \cdot x}{x^2 + y^2} \cdot \left[-\frac{2 \cdot A \cdot x \cdot y}{\left(x^2 + y^2\right)^2} \right] + \frac{A \cdot y}{x^2 + y^2} \cdot \left[\frac{A \cdot \left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} \right] \qquad a_y = -\frac{A^2 \cdot y}{\left(x^2 + y^2\right)^2}$$

Along the x axis $a_x = -\frac{A^2}{x^3} = -\frac{100}{x^3}$ $\qquad a_y = 0$

Along the y axis $a_x = 0$ $\qquad a_y = -\frac{A^2}{y^3} = -\frac{100}{y^3}$

Along the line $x = y$ $a_x = -\frac{A^2 \cdot x}{r^4} = -\frac{100 \cdot x}{r^4}$ $\qquad a_y = -\frac{A^2 \cdot y}{r^4} = -\frac{100 \cdot y}{r^4}$

where $r = \sqrt{x^2 + y^2}$

For this last case the acceleration along the line $x = y$ is

$$a = \sqrt{a_x^2 + a_y^2} = -\frac{A^2}{r^4} \cdot \sqrt{x^2 + y^2} = -\frac{A^2}{r^3} = -\frac{100}{r^3} \qquad a = -\frac{A^2}{r^3} = -\frac{100}{r^3}$$

In each case the acceleration vector points towards the origin, proportional to 1/distance^3, so the flow field is a radial decelerating flow

Problem 5.42

[2]

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Given: Incompressible, two-dimensional flow field with $w = 0$, has a y component of velocity given by

$$v = -Axy$$

where units of v are m/s; x and y are in meters and A is a dimensional constant

Find: (a) the dimensions of the constant A
(b) the simplest x component of velocity for this flow field
(c) the acceleration of a fluid particle at the point $(x, y) = (1, 2)$

Solution:

(a) Since $v = -Axy$, then the dimensions of A, $[A]$, are given by

$$[A] = \left[\frac{v}{xy}\right] = \frac{L}{t}\cdot\frac{1}{L}\cdot\frac{1}{L} = \frac{1}{Lt} \longleftarrow \boxed{A}$$

(b) Apply the continuity equation for the conditions given

Basic equation: $\nabla \cdot \rho\vec{V} + \dfrac{\partial\rho}{\partial t} = 0$

For incompressible flow, $\dfrac{\partial\rho}{\partial t} = 0$. Thus with $w = 0$, the basic equation reduces to $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

Then, $\dfrac{\partial u}{\partial x} = -\dfrac{\partial v}{\partial y} = -\dfrac{\partial}{\partial y}(-Axy) = Ax$

and

$$u = \int \frac{\partial u}{\partial x}\,dx + f(y) = \int Ax\,dx + f(y) = \frac{1}{2}Ax^2 + f(y)$$

The simplest x component of velocity is obtained with $f(y) = 0$

$$\therefore\ u = \frac{1}{2}Ax^2 \longleftarrow u$$

(c) The acceleration of a fluid particle is given by

$$\vec{a}_p = \frac{D\vec{V}}{Dt} = u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + \cancel{w\frac{\partial\vec{V}}{\partial z}}^{\,0} + \cancel{\frac{\partial\vec{V}}{\partial t}}^{\,0}$$

$$\vec{a}_p = \frac{1}{2}Ax^2\frac{\partial}{\partial x}\left[\frac{1}{2}Ax^2\hat{i} - Axy\hat{j}\right] - Axy\frac{\partial}{\partial y}\left[\frac{1}{2}Ax^2\hat{i} - Axy\hat{j}\right]$$

$$\vec{a}_p = \frac{1}{2}Ax^2\left[Ax\hat{i} - Ay\hat{j}\right] - Axy\left[-Ax\hat{j}\right] = \frac{1}{2}A^2x^3\hat{i} + \frac{1}{2}A^2x^2y\hat{j}$$

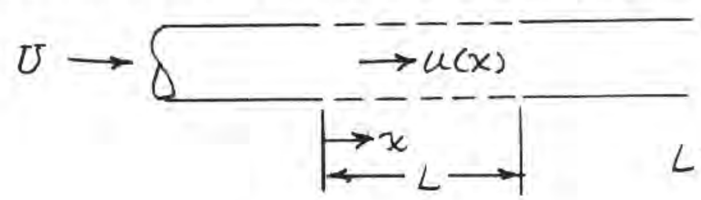
At the point $(x, y) = (1, 2)$

$$\vec{a}_p = \frac{1}{2}A^2(1)^3\hat{i} + \frac{1}{2}A^2(1)^2(2)\hat{j} = A^2\left[\frac{1}{2}\hat{i} + \hat{j}\right] \longleftarrow \vec{a}_p$$

Problem 5.43

[2]

Given: Duct flow with inviscid liquid, ρ = constant.



$L = 0.3\ m$

$u(x) = U(1 - x/2L)$ $\qquad$ $U = 5\ m/s$

Find: Expression for acceleration along ℄.

Solution: Computing equation

$$a_{p_x} = u\frac{\partial u}{\partial x} + \overset{=0(1)}{\cancel{v}}\frac{\partial u}{\partial y} + \overset{=0(1)}{\cancel{w}}\frac{\partial u}{\partial z} + \overset{=0(2)}{\cancel{\frac{\partial u}{\partial t}}}$$

Assumptions: (1) Along ℄ $v = w = 0$

(2) Steady flow

Then

$$a_{p_x} = u\frac{\partial u}{\partial x} = U\left(1 - \frac{x}{2L}\right)U\left(-\frac{1}{2L}\right) = -\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right) \longleftarrow a_{p_x}$$

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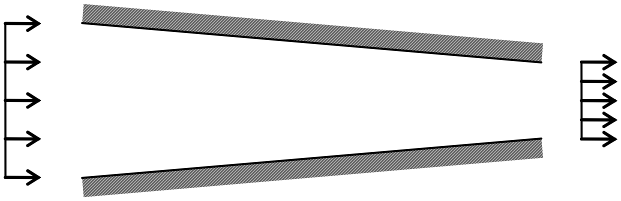
Problem 5.44

[4]

5.44 An incompressible liquid with negligible viscosity flows steadily through a horizontal pipe. The pipe diameter linearly varies from a diameter of 10 cm to a diameter of 2.5 cm over a length of 2 m. Develop an expression for the acceleration of a fluid particle along the pipe centerline. Plot the centerline velocity and acceleration versus position along the pipe, if the inlet centerline velocity is 1 m/s.



Given: Flow in a pipe with variable diameter

Find: Expression for particle acceleration; Plot of velocity and acceleration along centerline

Solution:

Assumptions: 1) Incompressible flow 2) Flow profile remains unchanged so centerline velocity can represent average velocity

Basic equations $Q = V\cdot A$

$$\vec{a}_p = \underset{\substack{\text{total}\\ \text{acceleration}\\ \text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\ \text{acceleration}}} + \underset{\substack{\text{local}\\ \text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

For the flow rate $Q = V\cdot A = V\cdot\frac{\pi\cdot D^2}{4}$

But $D = D_i + \frac{\left(D_o - D_i\right)}{L}\cdot x$ where D_i and D_o are the inlet and exit diameters, and x is distance along the pipe of length L: $D(0) = D_i$, $D(L) = D_o$.

Hence $V_i\cdot\frac{\pi\cdot D_i^2}{4} = V\cdot\frac{\pi\cdot\left[D_i + \frac{\left(D_o - D_i\right)}{L}\cdot x\right]^2}{4}$

$$V = V_i\cdot\frac{D_i^2}{\left[D_i + \frac{\left(D_o - D_i\right)}{L}\cdot x\right]^2} = \frac{V_i}{\left[1 + \frac{\left(\frac{D_o}{D_i} - 1\right)}{L}\cdot x\right]^2} \qquad V(x) = \frac{V_i}{\left[1 + \frac{\left(\frac{D_o}{D_i} - 1\right)}{L}\cdot x\right]^2}$$

Some representative values are $V(0\cdot m) = 1\frac{m}{s}$ $\quad V\left(\frac{L}{2}\right) = 2.56\frac{m}{s}$ $\quad V(L) = 16\frac{m}{s}$

The acceleration is given by

$$\vec{a}_p = \underset{\substack{\text{total}\\ \text{acceleration}\\ \text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\ \text{acceleration}}} + \underset{\substack{\text{local}\\ \text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

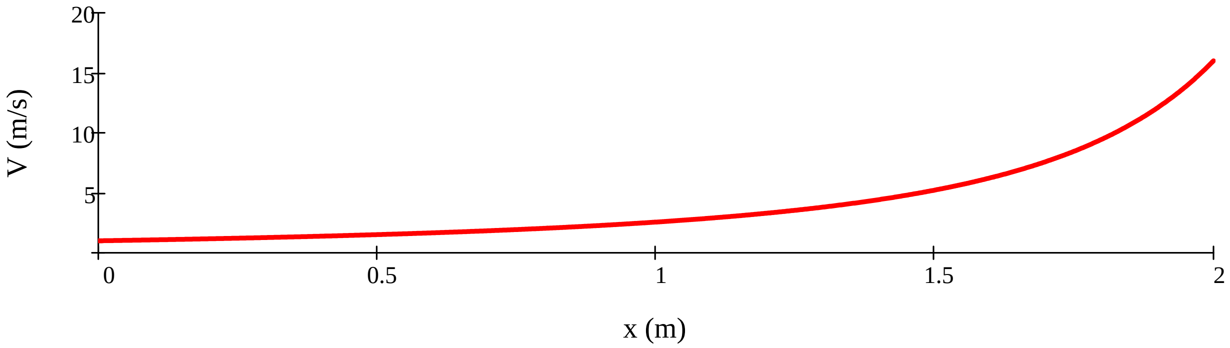
For this flow $a_x = V\cdot\frac{\partial}{\partial x}V$

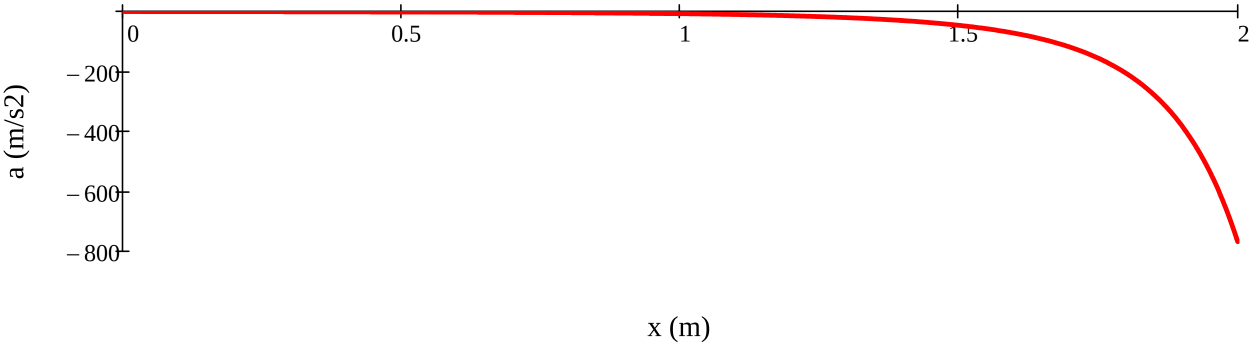
$$a_x = \frac{V_i}{\left[1 + \frac{\left(\frac{D_o}{D_i} - 1\right)}{L}\cdot x\right]^2}\cdot\frac{\partial}{\partial x}\left[\frac{V_i}{\left[1 + \frac{\left(\frac{D_o}{D_i} - 1\right)}{L}\cdot x\right]^2}\right] = -\frac{2\cdot V_i^2\cdot\left(\frac{D_o}{D_i} - 1\right)}{L\cdot\left[\frac{x\cdot\left(\frac{D_o}{D_i} - 1\right)}{L} + 1\right]^5}$$

$$a_x(x) = \frac{2 \cdot V_i^{2} \cdot \left(\frac{D_o}{D_i} - 1\right)}{L \cdot \left[\frac{x \cdot \left(\frac{D_o}{D_i} - 1\right)}{L} + 1\right]^{5}}$$

Some representative values are $a_x(0 \cdot m) = -0.75 \frac{m}{s^2}$ $a_x\left(\frac{L}{2}\right) = -7.864 \frac{m}{s^2}$ $a_x(L) = -768 \frac{m}{s^2}$

The following plots can be done in *Excel*





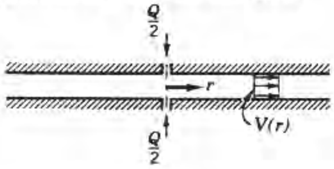
Problem 5.45

[2]

Given: Incompressible flow between parallel plates as shown.

Find: (a) Show $V_r = \frac{Q}{2\pi r h}$

(b) Acceleration in gap.



Solution: Apply conservation of mass

Basic equation: $$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{1}{r}\frac{\partial}{\partial \theta}\overset{=0(1)}{\cancel{(V_\theta)}}+\frac{\partial}{\partial z}\overset{=0(2)}{\cancel{V_z}}=0$$

Assumptions: (1) $V_\theta = 0$
(2) $V_z = 0$

Then

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)=0 \quad \text{or} \quad rV_r = C \quad \text{or} \quad V_r = \frac{C}{r} \text{ is form of solution.}$$

The volume flow rate is $Q = 2\pi r h V_r$, so $V_r = \frac{Q}{2\pi r h}$ ← V_r

Because $V_\theta = 0$, $a_\theta = 0$. The radial acceleration is

$$a_r = V_r\frac{\partial V_r}{\partial r} = \frac{Q}{2\pi r h}\left[(-1)\frac{Q}{2\pi r^2 h}\right] = -\left(\frac{Q}{2\pi h}\right)^2\frac{1}{r^3}$$

Thus

$$\vec{a}_p = -\left(\frac{Q}{2\pi h}\right)^2\frac{1}{r^3}\hat{e}_r$$ ← $\vec{a}_p$

The above expressions are valid only for $r>0$.

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Problem 5.46

[2]

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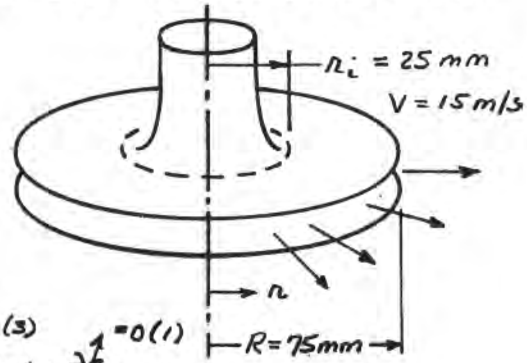
Given: Incompressible, inviscid flow of air between parallel disks.

Find: (a) Simplify continuity.
(b) Show $\vec{V} = V(R/r)\hat{e}_r$, $r_i < r < R$
(c) Calculate acceleration of a particle at $r = r_i, R$.



Solution: Apply continuity equation and substantial derivative

Basic equations:
$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r) + \overset{=0(2)}{\cancel{\frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta)}} + \overset{=0(3)}{\cancel{\frac{\partial}{\partial z}(\rho V_z)}} + \overset{=0(1)}{\cancel{\frac{\partial \rho}{\partial t}}} = 0$$

$$a_r = V_r\frac{\partial V_r}{\partial r} + \overset{=0(2)}{\cancel{\frac{V_\theta}{r}}}\frac{\partial V_r}{\partial \theta} + \overset{=0(3)}{\cancel{\frac{\partial V_r}{\partial z}}} - \overset{=0(2)}{\cancel{\frac{V_\theta}{r}}} + \overset{=0(4)}{\cancel{\frac{\partial V_r}{\partial t}}}$$

Assumptions: (1) Incompressible flow, ρ = constant
(2) Radial flow, $V_\theta = 0$
(3) Uniform flow at each radial location, $\partial/\partial z = 0$
(4) Steady flow

Then
$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r) = 0 \quad \text{or} \quad rV_r = \text{constant} = RV\ ; \quad V_r = V\frac{R}{r}$$

so that $\vec{V} = V\frac{R}{r}\hat{e}_r$ ⟵ $\vec{V}$

The radial acceleration of a fluid particle is

$$a_r = V_r\frac{\partial V_r}{\partial r} = V\frac{R}{r}(VR)\left(-\frac{1}{r^2}\right) = -\frac{V^2R^2}{r^3} = -\frac{V^2}{R}\left(\frac{R}{r}\right)^3$$

At $r = r_i = 25mm$,

$$a_r = -(15)^2\frac{m^2}{s^2} \times \frac{1}{0.075\,m}\left(\frac{75}{25}\right)^3 = -81.0\ \frac{km}{s^2}$$ ⟵ $a_r(r_i$

At $r = R = 75\ mm$

$$a_r = -(15)^2\frac{m^2}{s^2} \times \frac{1}{0.075\,m}\left(\frac{75}{75}\right)^3 = -3.00\ \frac{km}{s^2}$$ ⟵ $a_r(R)$

Problem 5.47

5.47 As part of a pollution study, a model concentration c as a function of position x has been developed,

$$c(x) = A(e^{-x/a} - e^{-x/2a})$$

where $A = 10^{-5}$ ppm (parts per million) and $a = 1$ m. Plot this concentration from $x = 0$ to $x = 10$ m. If a vehicle with a pollution sensor travels through this atmosphere at $u = U(U = 20$ m/s), develop an expression for the measured concentration rate of change of c with time, and plot using given data. At what location will the sensor indicate the most rapid rate of change, and what is the value of this rate of change?

Given: Data on pollution concentration

Find: Plot of concentration; Plot of concentration over time for moving vehicle; Location and value of maximum rate change

Solution:

Basic equation: Material derivative $$\frac{D}{Dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \frac{\partial}{\partial t}$$

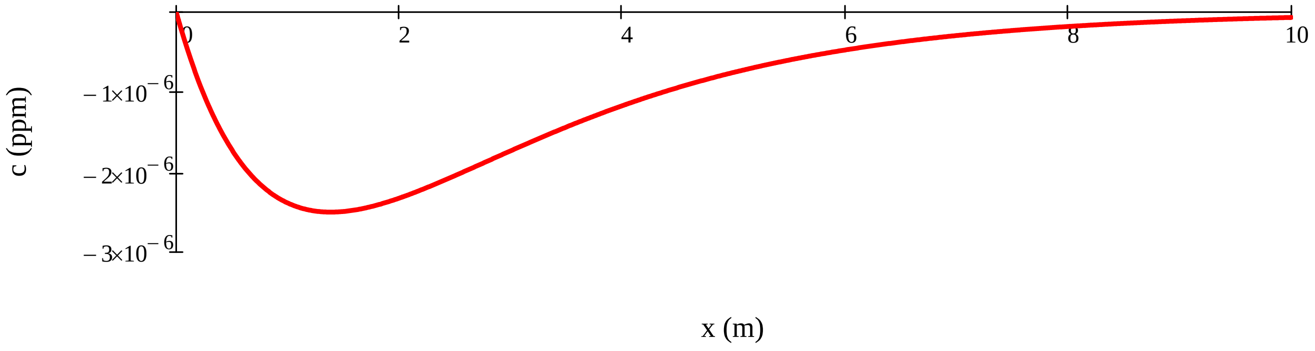
For this case we have $u = U \quad v = 0 \quad w = 0 \quad c(x) = A\cdot\left(e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}}\right)$

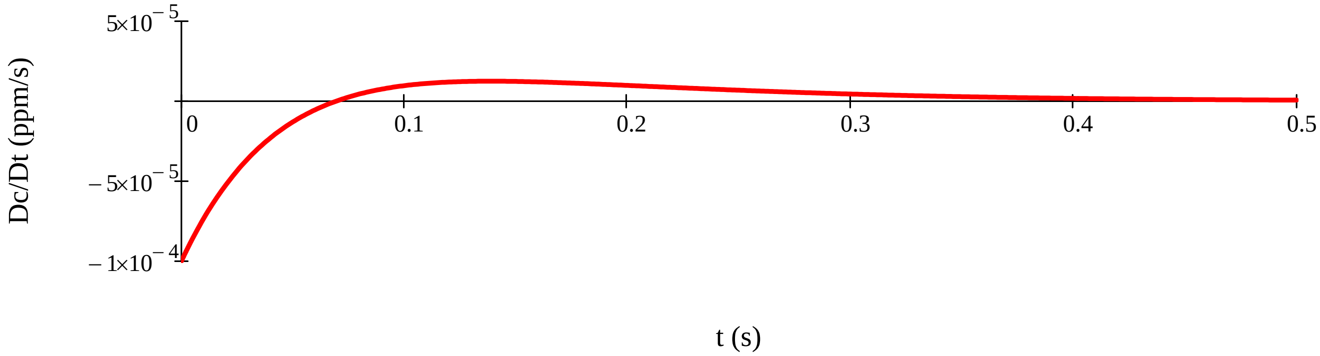
Hence $$\frac{Dc}{Dt} = u\cdot\frac{dc}{dx} = U\cdot\frac{d}{dx}\left[A\cdot\left(e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}}\right)\right] = \frac{U\cdot A}{a}\cdot\left(\frac{1}{2}\cdot e^{-\frac{x}{2\cdot a}} - e^{-\frac{x}{a}}\right)$$

We need to convert this to a function of time. For this motion u = U so $x = U\cdot t$

$$\frac{Dc}{Dt} = \frac{U\cdot A}{a}\cdot\left(\frac{1}{2}\cdot e^{-\frac{U\cdot t}{2\cdot a}} - e^{-\frac{U\cdot t}{a}}\right)$$

The following plots can be done in *Excel*





The maximum rate of change is when

$$\frac{d}{dx}\left(\frac{Dc}{Dt}\right) = \frac{d}{dx}\cdot\left[\frac{U\cdot A}{a}\cdot\left(\frac{1}{2}\cdot e^{-\frac{x}{2\cdot a}} - e^{-\frac{x}{a}}\right)\right] = 0$$

$$\frac{U\cdot A}{a^2}\cdot\left(e^{-\frac{x}{a}} - \frac{1}{4}\cdot e^{-\frac{x}{2\cdot a}}\right) = 0 \qquad \text{or} \qquad e^{-\frac{x}{2\cdot a}} = \frac{1}{4}$$

$$x_{max} = 2\cdot a\cdot \ln(4) = 2\times 1\cdot m\times \ln\left(\frac{1}{4}\right) \qquad x_{max} = 2.77\cdot m$$

$$t_{max} = \frac{x_{max}}{U} = 2.77\cdot m\times\frac{s}{20\cdot m} \qquad t_{max} = 0.138\cdot s$$

$$\frac{Dc_{max}}{Dt} = \frac{U\cdot A}{a}\cdot\left(\frac{1}{2}\cdot e^{-\frac{x_{max}}{2\cdot a}} - e^{-\frac{x_{max}}{a}}\right)$$

$$\frac{Dc_{max}}{Dt} = 20\cdot\frac{m}{s}\times 10^{-5}\cdot ppm\times\frac{1}{1\cdot m}\times\left(\frac{1}{2}\times e^{-\frac{2.77}{2\cdot 1}} - e^{-\frac{2.77}{1}}\right) \qquad \frac{Dc_{max}}{Dt} = 1.25\times 10^{-5}\cdot\frac{ppm}{s}$$

Note that there is another maximum rate, at t = 0 (x = 0)

$$\frac{Dc_{max}}{Dt} = 20\cdot\frac{m}{s}\times 10^{-5}\cdot ppm\times\frac{1}{1\cdot m}\cdot\left(\frac{1}{2} - 1\right) \qquad \frac{Dc_{max}}{Dt} = -1\times 10^{-4}\cdot\frac{ppm}{s}$$

Problem 5.48

[2]

Given: Aircraft flying north with velocity component $u = 300$ mph is climbing at rate, $v = 3000$ ft/min. The rate of temperature change with vertical distance y is $\partial T/\partial y = -3°F/1000$ ft. The variation of temperature with position x is $\partial T/\partial x = -1°F/\text{mile}$

Find: the rate of temperature change shown by a recorder on board the aircraft

Solution: Apply the substantial derivative concept

Basic equation: $\frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \cancel{\frac{\partial T}{\partial t}}^{\,0}$

Substituting numerical values,

$$\frac{DT}{Dt} = 300\,\frac{\text{mile}}{\text{hr}} \times -\frac{1°F}{\text{mile}} \times \frac{\text{hr}}{60\,\text{min}} + 3000\,\frac{\text{ft}}{\text{min}} \times -\frac{3°F}{1000\,\text{ft}}$$

$$\frac{DT}{Dt} = (-5-9)°F/\text{min} = -14°F/\text{min} \longleftarrow \frac{DT}{Dt}$$

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Problem 5.49

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Given: Instruments on board an aircraft flying through a cold front give the following information:

- rate of change of temperature is $-0.5\,F/min$
- air speed = 300 knots
- rate of climb = 3500 ft/min

Front is stationary and vertically uniform

Find: rate of change of temperature with respect to horizontal distance through the cold front

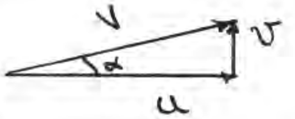
Solution: Apply the substantial derivative concept

Basic equation: $$\frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\cancel{\frac{\partial T}{\partial y}}^{\,0} + \cancel{\frac{\partial T}{\partial t}}^{\,0 \text{ (stationary front)}}$$

vertically uniform

$\frac{DT}{Dt} = -0.5\,F/min$. Need to find $\frac{\partial T}{\partial x}$

Velocity picture



$$V = 300\,\frac{nm}{hr} \times 6080\,\frac{ft}{nm} \times \frac{hr}{3600\,s} = 507\,\frac{ft}{s}$$

$$v = 3500\,\frac{ft}{min} \times \frac{min}{60\,s} = 58.3\,ft/s$$

Then $$\alpha = \sin^{-1}\frac{v}{V} = \sin^{-1}\frac{58.3}{507} = 6.60^\circ$$

and $$u = V\cos\alpha = 507\,\frac{ft}{sec}\cos 6.60^\circ = 504\,ft/s$$

$$\therefore \frac{\partial T}{\partial x} = \frac{1}{u}\frac{DT}{Dt} = \frac{s}{504\,ft} \times \frac{-0.5^\circ F}{min} \times \frac{min}{60\,s} \times \frac{5280\,ft}{mi}$$

$$\frac{\partial T}{\partial x} = -0.0873^\circ F/mile \longleftarrow \frac{\partial T}{\partial x}$$

Problem 5.50

[3]

Given: Sediment concentration rates in a river after a rainfall are:

$$\frac{\partial c}{\partial t} = 100\,\frac{ppm}{hr}\,,\quad \frac{\partial c}{\partial x} = 50\,\frac{ppm}{mi}\quad \text{(downstream)}$$

Stream speed is $u_s = 0.5$ mph, where a boat is used to survey concentration.

Boat speed is $V_b = 2.5$ mph.

Find: (a) Calculate rates of change of sediment concentration observed when boat travels upstream, drifts with the current, or travels downstream.

(b) Explain physically why the observed rates differ.

Solution: Apply substantial derivative concept

Basic equation: $\dfrac{Dc}{Dt} = u\dfrac{\partial c}{\partial x} + \dfrac{\partial c}{\partial t}$

To obtain rate of change seen from boat, set $u = u_B$.

(i) For travel upstream, $u_B = u_s - V_b = 0.5 - 2.5 = -2.0$ mph

$$\frac{Dc}{Dt}(up) = -2.0\,\frac{mi}{hr} \times 50\,\frac{ppm}{mi} + 100\,\frac{ppm}{hr} = 0.00\ ppm/hr$$ up

(ii) For drifting, $u_B = u_s + 0 = 0.5$ mph

$$\frac{Dc}{Dt}(drift) = 0.5\,\frac{mi}{hr} \times 50\,\frac{ppm}{mi} + 100\,\frac{ppm}{hr} = 125\ ppm/hr$$ drift

(iii) For travel downstream, $u_B = u_s + V_b = 0.5 + 2.5 = 3.0$ mph

$$\frac{Dc}{Dt}(down) = 3.0\,\frac{mi}{hr} \times 50\,\frac{ppm}{mi} + 100\,\frac{ppm}{hr} = 250\ ppm/hr$$ down

Physically the observed rates of change differ because the observer is convected through the flow. The convective change may add to or subtract from the local rate of change.

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Problem 5.51

[3]

Expand $(\vec{V}\cdot\nabla)\vec{V}$ in rectangular coordinates to obtain the convective acceleration of a fluid particle. Verify the results given in Eqs 5.11

Solution:

In rectangular coordinates $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$, $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$(\vec{V}\cdot\nabla)\vec{V} = \left[(u\hat{i} + v\hat{j} + w\hat{k})\cdot\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\right] u\hat{i} + v\hat{j} + w\hat{k}$$

$$= \left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right] u\hat{i} + v\hat{j} + w\hat{k}$$

$$(\vec{V}\cdot\nabla)\vec{V} = \left\{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\}^{①}\hat{i} + \left\{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right\}^{②}\hat{j}$$
$$+ \left\{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right\}^{③}\hat{k}$$

Term ① is x component of convective acceleration

Eq. 5.11a $\quad a_{x_p} = \left\{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right\} + \frac{\partial u}{\partial t}$

Term ② is the y component of convective acceleration

Eq 5.11b $\quad a_{y_p} = \left\{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right\} + \frac{\partial v}{\partial t}$

Term ③ is the z component of convective acceleration

Eq. 5.11c $\quad a_{z_p} = \left\{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right\} + \frac{\partial w}{\partial t}$

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Problem 5.52

[3]

Given: Velocity field represented by

$$\vec{V} = (Ax - B)\hat{\imath} + Cy\hat{\jmath} + Dt\,\hat{k} \qquad (x, y \text{ in m})$$

$$\text{where } A = 2\,s^{-1},\quad B = 4\,m/s,\quad \text{and } D = 5\,m/s^2$$

Find: (a) Proper value of C for incompressible flow.
(b) Acceleration of particle at $(x,y) = (3,2)$.
(c) Sketch streamlines in xy plane.

Solution: For incompressible flow, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$. Since $w = Dt$, $\partial w/\partial z = 0$, and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = C = -\frac{\partial u}{\partial x} = -A = -2\,s^{-1}$$

C

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

$$\vec{a}_p = (Ax - B)(A\hat{\imath}) + (Cy)(C\hat{\jmath}) + (Dt)(0) + D\hat{k}$$

$$\vec{a}_p(3,2) = \left(\frac{2}{s} \times 3\,m - \frac{4\,m}{s}\right)\left(\frac{2}{s}\right)\hat{\imath} + \left(\frac{-2}{s} \times 2\,m\right)\left(-\frac{2}{s}\right)\hat{\jmath} + 5\frac{m}{s^2}\,\hat{k}$$

$$\vec{a}_p(3,2) = 4\hat{\imath} + 8\hat{\jmath} + 5\hat{k} \quad m/s^2$$

$\vec{a}_p(3,2$

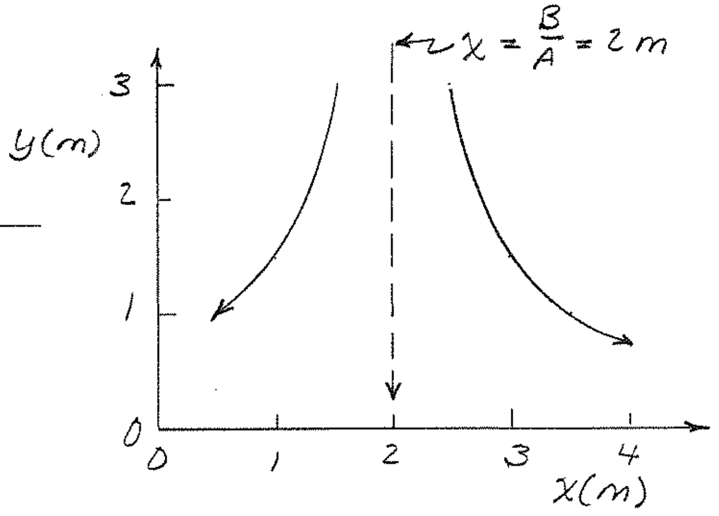
In the xy plane, streamlines are $\frac{dy}{dx} = \frac{v}{u} = \frac{Cy}{Ax - B}$. Thus

$$\frac{dx}{Ax - B} = \frac{dy}{Cy} \quad \text{or} \quad \frac{dx}{Ax - B} = -\frac{dy}{Ay} \quad \text{or} \quad \frac{dx}{x - B/A} + \frac{dy}{y} = 0$$

Integrating

$$\ln(x - B/A) + \ln y = \ln c_0$$

$$\left(x - \frac{B}{A}\right)y = \text{const}$$



Problem 5.53

[3]

Given: Steady, two-dimensional velocity field, $\vec{V} = Ax\hat{i} - Ay\hat{j}$;
$A = 1\,s^{-1}$, coordinates measured in meters

Show: that streamlines are hyperbolas, $xy = C$

Find: (a) Expression for acceleration.

(b) Particle acceleration at $(x,y) = (\frac{1}{2}, 2)$, $(1,1)$ and $(2, \frac{1}{2})$

Plot: streamlines corresponding to $C = 0, 1,$ and $2\,m^2$; show acceleration vectors on the plot

Solution:

Along a streamline, $\frac{dy}{dx} = \frac{v}{u} = \frac{-y}{x}$ or $\frac{dy}{y} + \frac{dx}{x} = 0$

Integrating we obtain $\ln y + \ln x = \ln c$ and $xy = c$ ← Streamline

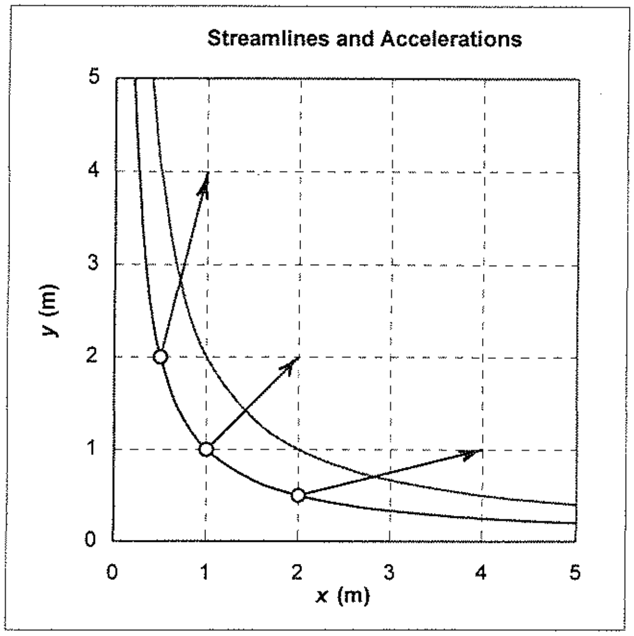
The acceleration of a particle is

$$\vec{a}_p = \frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \cancel{w\frac{\partial \vec{V}}{\partial z}} + \cancel{\frac{\partial \vec{V}}{\partial t}} \quad \{w = 0 \text{ and steady flow}\}$$

$$\vec{a}_p = Ax(A\hat{i}) - (Ay)(-A\hat{j}) = A^2(x\hat{i} + y\hat{j}) \longleftarrow \vec{a}_p$$

$$\left.\begin{array}{l} \vec{a}_p)_{1/2,2} = \frac{1}{2}\hat{i} + 2\hat{j}\ m/s^2 \qquad \vec{a}_p)_{1,1} = \hat{i} + \hat{j}\ m/s^2 \\ \vec{a}_p)_{2,1/2} = 2\hat{i} + \frac{1}{2}\hat{j}\ m/s^2 \end{array}\right\} \longleftarrow \vec{a}_p$$

Plot:



Problem 5.54

[3]

Given: Velocity field $\vec{V} = (Ax - B)\hat{i} - Ay\hat{j}$; $A = 0.2\ s^{-1}$, $B = 0.6\ s^{-1}$, x in m.

Find: (a) General expression for acceleration of a fluid particle.
(b) Acceleration at $(x,y) = (0, 4/3), (1, 2)$, and $(2, 4)$.
(c) Plot of streamlines.
(d) Acceleration vectors on plot.

Solution: Note $w = 0$ and flow is steady. Then

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} = (Ax - B)A\hat{i} + (-Ay)(-A)\hat{j} = (A^2x - AB)\hat{i} + A^2y\hat{j} \longleftarrow \vec{a}_p$$

At $(x, y) = (0, 4/3)$, $\vec{a}_p = -0.12\hat{i} + 0.0533\hat{j}\ m/s^2$

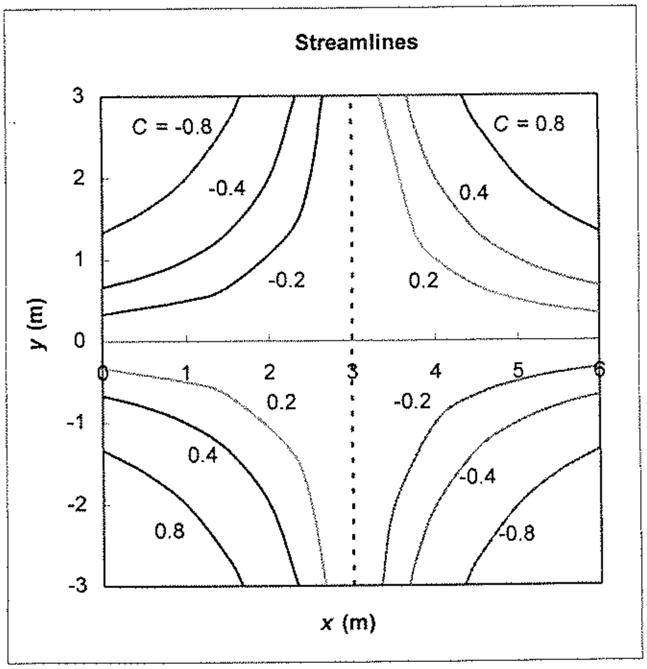
$(1, 2)$, $\vec{a}_p = -0.08\hat{i} + 0.0800\hat{j}\ m/s^2$

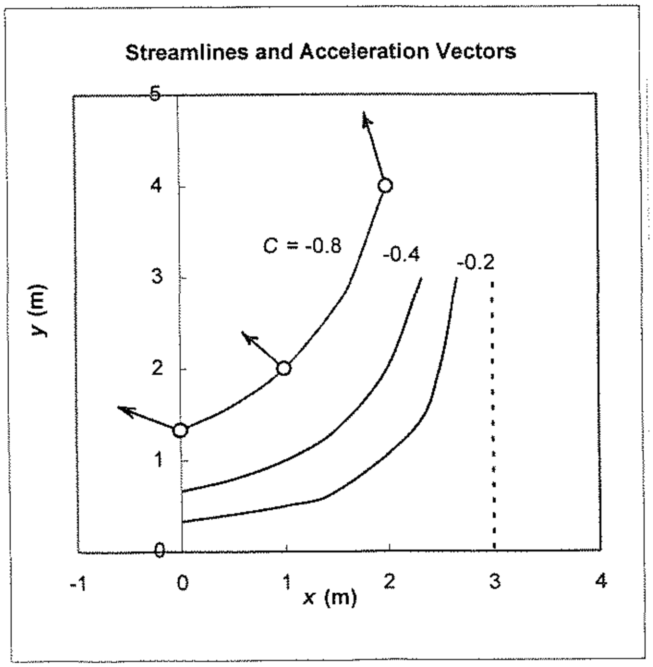
$(2, 4)$, $\vec{a}_p = -0.04\hat{i} + 0.160\hat{j}\ m/s^2 \longleftarrow \vec{a}_p$

streamlines are $\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{Ax - B} = \frac{dy}{-Ay}$. Integrating,

$$\frac{1}{A}\ln(Ax - B) + \frac{1}{A}\ln y = \frac{1}{A}\ln C \quad \text{or} \quad (Ax - B)y = C \longleftarrow$$

The plots are:





Problem 5.55

Given: Air flowing downward toward infinite horizontal flat plate.
Velocity field is $\vec{V} = (ax\hat{i} - ay\hat{j})(2+\cos\omega t)$; $a = 3\ s^{-1}$, $\omega = \pi\ s^{-1}$

Find: (a) Expression for streamline at $t = 1.5$ s.
(b) Plot of streamline through $(x,y) = (2,4)$ at this instant.
(c) Velocity vector
(d) Vectors representing local, convective, and total acceleration.

Solution: Streamline is $\frac{dx}{u} = \frac{dy}{v}$, or $\frac{dx}{x} + \frac{dy}{y} = 0$ or $xy = c$

At point $(x,y) = (2,4)$, $c = 2m \times 4m = 8m^2$; $xy = 8\ m^2$ Streamline

The plot is shown below. Note $u = ax\hat{i}[2+\cos\omega t]$, $v = -ay\hat{j}[2+\cos\omega t]$

At $(x,y,t) = (2m, 4m, 1.5s)$, $\vec{V} = (6\hat{i} - 12\hat{j})(2+0) = 12\hat{i} - 24\hat{j}$ $\vec{V}$

The local acceleration components at $(x,y,t) = (2m, 4m, 1.5s)$ are

$$a_{x,local} = \frac{\partial u}{\partial t} = ax\hat{i}(-\omega\sin\omega t) = \frac{3}{s} \times 2m \times \left(\frac{-\pi}{s}\right) \times \sin\left(\frac{3\pi}{2}\right) = 6\pi\hat{i}\ m/s^2$$

$$a_{y,local} = \frac{\partial v}{\partial t} = -ay\hat{j}(-\omega\sin\omega t) = \frac{3}{s} \times 4m \times \left(\frac{-\pi}{s}\right) \times \sin\left(\frac{3\pi}{2}\right) = -12\pi\hat{j}\ m/s^2$$ Local

The convective acceleration components at $(x,y,t) = (2m, 4m, 1.5s)$ are

$$a_{x,conv} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = ax(a\hat{i})\left[2+\cos\frac{3\pi}{2}\right]^2 = (3)(2)(3)[2]^2\hat{i} = 72\hat{i}$$

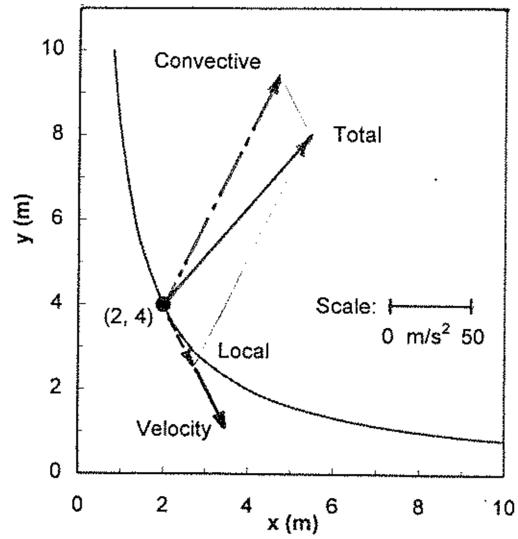
$$a_{y,conv} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (-ay)(-a\hat{j})\left[2+\cos\frac{3\pi}{2}\right]^2 = 4a^2y\hat{j} = 4(3)^2 4\hat{j} = 144\hat{j}$$ Convective

The total acceleration is the sum of the convective and local values:

$$a_{x,total} = a_{x,conv} + a_{x,local} = (72 + 6\pi)\hat{i} = 90.8\hat{i}\ m/s^2$$

$$a_{y,total} = a_{y,conv} + a_{y,local} = (144 - 12\pi)\hat{j} = 106\ \hat{j}\ m/s^2$$ Total

The plot is

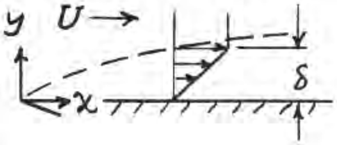


Problem 5.56

[3]

Given: Laminar boundary layer, linear approximate profile.

$$\frac{u}{U} = \frac{y}{\delta} \qquad \delta = cx^{1/2}$$



From Problem 5.7, $v = \frac{uy}{4x} = U\frac{y^2}{4x\delta}$

Find: (a) x and y components of acceleration of a fluid particle.

(b) Locate maximum values.

(c) Ratio, $a_{x,max}/a_{y,max}$.

Solution: Basic equations: $a_{p_x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \overset{=0(1)}{\cancel{w\frac{\partial u}{\partial z}}} + \overset{=0(2)}{\cancel{\frac{\partial u}{\partial t}}}$

$$a_{p_y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \overset{=0(1)}{\cancel{w\frac{\partial v}{\partial z}}} + \overset{=0(2)}{\cancel{\frac{\partial v}{\partial t}}}$$

Assumptions: (1) w and $\partial/\partial z$ zero, (2) steady flow. $\frac{d\delta}{dx} = \frac{1}{2}cx^{-1/2} = \frac{\delta}{2x}$

$$u = U\frac{y}{\delta}\ ;\ \frac{\partial u}{\partial x} = Uy\left(-\frac{1}{\delta^2}\right)\frac{d\delta}{dx} = -Uy\frac{1}{\delta^2}\frac{\delta}{2x} = -\frac{Uy}{2x\delta}\ ;\ \frac{\partial u}{\partial y} = \frac{U}{\delta}$$

$$v = U\frac{y^2}{4x\delta}\ ;\ \frac{\partial v}{\partial x} = \frac{Uy^2}{4}\left(-\frac{1}{x^2\delta} - \frac{1}{x\delta^2}\frac{d\delta}{dx}\right) = -\frac{3Uy^2}{8x^2\delta}\ ;\ \frac{\partial v}{\partial y} = \frac{Uy}{2x\delta}$$

Thus

$$a_{p_x} = \left(U\frac{y}{\delta}\right)\left(-\frac{Uy}{2x\delta}\right) + \left(U\frac{y^2}{4x\delta}\right)\left(\frac{U}{\delta}\right) = -\frac{U^2}{2x}\left(\frac{y}{\delta}\right)^2 + \frac{U^2}{4x}\left(\frac{y}{\delta}\right)^2 = -\frac{U^2}{4x}\left(\frac{y}{\delta}\right)^2$$

a_{px}

$$a_{p_y} = \left(U\frac{y}{\delta}\right)\left(-\frac{3Uy^2}{8x^2\delta}\right) + \left(U\frac{y^2}{4x\delta}\right)\left(U\frac{y}{2x\delta}\right) = -\frac{3U^2}{8x}\left(\frac{y}{x}\right)\left(\frac{y}{\delta}\right)^2 + \frac{U^2}{8x}\left(\frac{y}{x}\right)\left(\frac{y}{\delta}\right)^2$$

$$a_{p_y} = -\frac{U^2}{4x}\left(\frac{y}{x}\right)\left(\frac{y}{\delta}\right)^2$$

a_{py}

Maximum values are at $y = \delta$

$$a_{p_x,\,max} = -\frac{U^2}{4x}$$

(max; a_{px}

$$a_{p_y,\,max} = -\frac{U^2}{4x}\left(\frac{\delta}{x}\right)$$

(max a_{py}

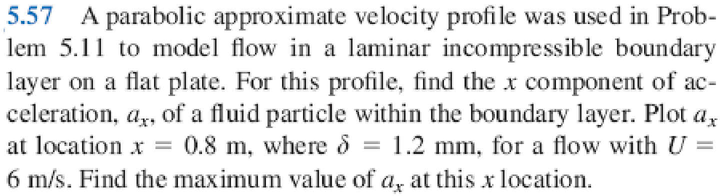
Thus $\frac{a_{p_x,\,max}}{a_{p_y,\,max}} = \frac{x}{\delta}$

At $x = 0.5\text{ m}, \delta = 5\text{ mm}$, $\frac{a_{p_x,\,max}}{a_{p_y,\,max}} = \frac{0.5\text{ m}}{0.005\text{ m}} = 100$

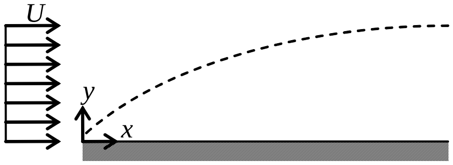
Ratio

Problem 5.57

[4]



5.57 A parabolic approximate velocity profile was used in Problem 5.11 to model flow in a laminar incompressible boundary layer on a flat plate. For this profile, find the x component of acceleration, a_x, of a fluid particle within the boundary layer. Plot a_x at location $x = 0.8$ m, where $\delta = 1.2$ mm, for a flow with $U = 6$ m/s. Find the maximum value of a_x at this x location.



Given: Flow in boundary layer

Find: Expression for particle acceleration a_x; Plot acceleration and find maximum at x = 0.8 m

Solution:

Basic equations

$$\frac{u}{U} = 2\cdot\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \qquad \frac{v}{U} = \frac{\delta}{x}\cdot\left[\frac{1}{2}\cdot\left(\frac{y}{\delta}\right) - \frac{1}{3}\cdot\left(\frac{y}{\delta}\right)^3\right] \qquad \delta = c\cdot\sqrt{x}$$

$$\underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\vec{a}_p = \frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

We need to evaluate

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u$$

First, substitute

$$\lambda(x,y) = \frac{y}{\delta(x)} \qquad \text{so} \qquad \frac{u}{U} = 2\cdot\lambda - \lambda^2 \qquad \frac{v}{U} = \frac{\delta}{x}\cdot\left(\frac{1}{2}\cdot\lambda - \frac{1}{3}\cdot\lambda^3\right)$$

Then

$$\frac{\partial}{\partial x}u = \frac{du}{d\lambda}\cdot\frac{d\lambda}{dx} = U\cdot(2 - 2\cdot\lambda)\cdot\left(-\frac{y}{\delta^2}\right)\cdot\frac{d\delta}{dx} \qquad \frac{d\delta}{dx} = \frac{1}{2}\cdot c\cdot x^{-\frac{1}{2}}$$

$$\frac{\partial}{\partial x}u = U\cdot(2 - 2\cdot\lambda)\cdot\left(-\frac{\lambda}{\delta}\right)\cdot\frac{1}{2}\cdot c\cdot x^{-\frac{1}{2}} = U\cdot(2 - 2\cdot\lambda)\cdot\left(-\frac{\lambda}{c\cdot x^{\frac{1}{2}}}\right)\cdot\frac{1}{2}\cdot c\cdot x^{-\frac{1}{2}}$$

$$\frac{\partial}{\partial x}u = -U\cdot(2 - 2\cdot\lambda)\cdot\frac{\lambda}{2\cdot x} = -\frac{U\cdot\left(\lambda - \lambda^2\right)}{x}$$

$$\frac{\partial}{\partial y}u = U\cdot\left(\frac{2}{\delta} - 2\cdot\frac{y}{\delta^2}\right) = \frac{2\cdot U}{\delta}\cdot\left[\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right] = \frac{2\cdot U\cdot\left(\lambda - \lambda^2\right)}{y}$$

Hence

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = U\cdot\left(2\cdot\lambda - \lambda^2\right)\left[\frac{U\cdot\left(\lambda - \lambda^2\right)}{x}\right] + U\cdot\frac{\delta}{x}\cdot\left(\frac{1}{2}\cdot\lambda - \frac{1}{3}\cdot\lambda^3\right)\cdot\left[\frac{2\cdot U\cdot\left(\lambda - \lambda^2\right)}{y}\right]$$

Collecting terms

$$a_x = \frac{U^2}{x}\cdot\left(-\lambda^2 + \frac{4}{3}\cdot\lambda^3 - \frac{1}{3}\cdot\lambda^4\right) = \frac{U^2}{x}\cdot\left[-\left(\frac{y}{\delta}\right)^2 + \frac{4}{3}\cdot\left(\frac{y}{\delta}\right)^3 - \frac{1}{3}\cdot\left(\frac{y}{\delta}\right)^4\right]$$

To find the maximum

$$\frac{da_x}{d\lambda} = 0 = \frac{U^2}{x}\cdot\left(-2\cdot\lambda + 4\cdot\lambda^2 - \frac{4}{3}\cdot\lambda^3\right) \qquad \text{or} \qquad -1 + 2\cdot\lambda - \frac{2}{3}\cdot\lambda^2 = 0$$

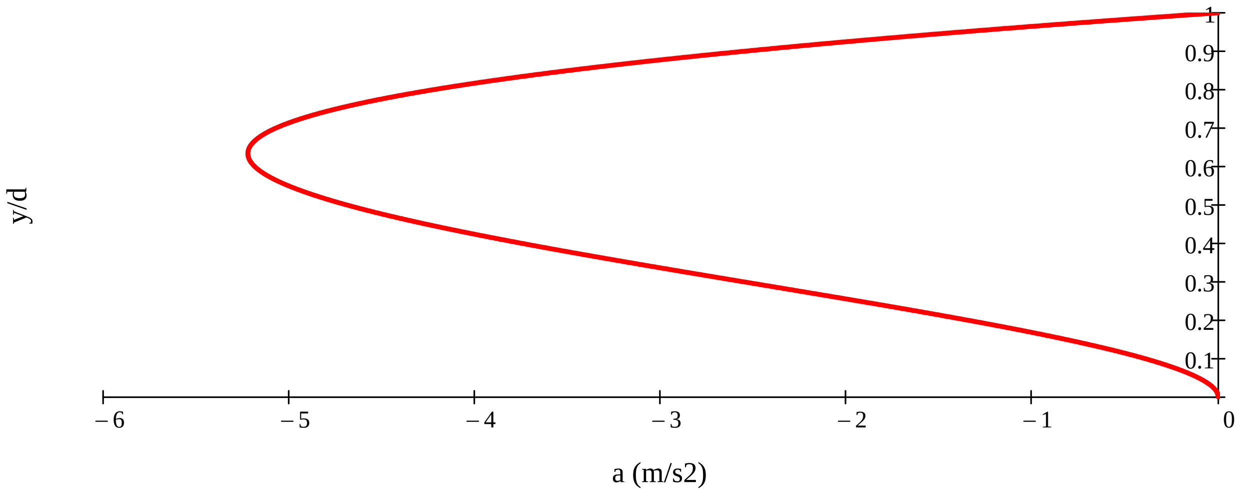
The solution of this quadratic ($\lambda < 1$) is

$$\lambda = \frac{3 - \sqrt{3}}{2} \qquad \lambda = 0.634 \qquad \frac{y}{\delta} = 0.634$$

At λ = 0.634 $$a_x = \frac{U^2}{x}\cdot\left(-0.634^2 + \frac{4}{3}\cdot 0.634^3 - \frac{1}{3}\cdot 0.634^4\right) = -0.116\cdot\frac{U^2}{x}$$

$$a_x = -0.116 \times \left(6\cdot\frac{m}{s}\right)^2 \times \frac{1}{0.8\cdot m} \qquad a_x = -5.22\frac{m}{s^2}$$

The following plot can be done in *Excel*



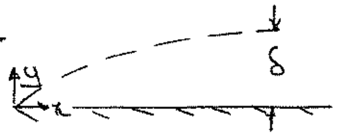
Problem 5.58

[3] Part 1/2

Given: Laminar boundary layer on a flat plate. (Problem 5.11)

$$\frac{u}{U} = \sin\frac{\pi y}{2\delta}\ , \quad \delta = c x^{1/2}$$

$$\frac{v}{U} = \frac{1}{\pi}\frac{\delta}{x}\left[\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) + \left(\frac{\pi}{2}\frac{y}{\delta}\right)\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - 1\right]$$



Find: Expression for a_{xp} and a_{yp}

Plot: a_x and a_y as functions of y/δ for $U = 5\,m/s$, $x = 1m$ & $\delta = 1mm$ determine maximum values at locations at which maxima occur.

Solution:

Basic equations:
$$a_{px} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}}^{\,0(2)} + \cancel{\frac{\partial u}{\partial t}}^{\,0(1)} \quad \text{-----}(1)$$
$$a_{py} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \cancel{w\frac{\partial v}{\partial z}}^{\,0(2)} + \cancel{\frac{\partial v}{\partial t}}^{\,0(1)} \quad \text{--------}(2)$$

Assumptions: (1) steady flow
(2) w and $\partial/\partial z = 0$

Let $\eta = \frac{\pi}{2}\frac{y}{\delta}$; $\eta = \eta(x,y)$. $\frac{\partial\eta}{\partial y} = \frac{\pi}{2\delta}$; $\delta = cx^{1/2}$, $\frac{d\delta}{dx} = \frac{1}{2}cx^{-1/2} = \frac{\delta}{2x}$

$$\frac{\partial\eta}{\partial x} = \frac{\partial\eta}{\partial\delta}\frac{d\delta}{dx} = \frac{\pi y}{2}\left(-\frac{1}{\delta^2}\right)\frac{\delta}{2x} = -\frac{\pi}{4x}\left(\frac{y}{\delta}\right)$$

$u = U\sin\eta$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial x} = U\cos\eta\left(-\frac{\pi}{4x}\frac{y}{\delta}\right) = -\frac{U}{2x}\left(\frac{\pi}{2}\frac{y}{\delta}\right)\cos\eta = -\frac{U}{2x}\eta\cos\eta \quad \text{----}(3)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial\eta}\frac{\partial\eta}{\partial y} = U\cos\eta\frac{\pi}{2\delta} = \frac{U\pi}{2\delta}\cos\eta \quad \text{--------------}(4)$$

$v = U\frac{1}{\pi}\frac{\delta}{x}(\cos\eta + \eta\sin\eta - 1)$. Differentiating using product rule

$$\frac{\partial v}{\partial x} = \frac{U}{\pi}\left(\frac{1}{x}\frac{d\delta}{dx} - \frac{\delta}{x^2}\right)(\cos\eta + \eta\sin\eta - 1) + \frac{U}{\pi}\frac{\delta}{x}\left(-\cancel{\sin\eta} + \eta\cos\eta + \cancel{\sin\eta}\right)\frac{\partial\eta}{\partial x}$$
$$= \frac{U}{\pi}\left(\frac{1}{x}\frac{\delta}{2x} - \frac{\delta}{x^2}\right)(\cos\eta + \eta\sin\eta - 1) + \frac{U}{\cancel{\pi}}\frac{\delta}{x}\eta\cos\eta\left(-\frac{\cancel{\pi}}{4x}\right)\frac{y}{\delta}$$

$$\frac{\partial v}{\partial x} = -\frac{U}{\pi}\frac{\delta}{2x^2}(\cos\eta + \eta\sin\eta - 1) - \frac{U\delta}{4x^2}\left(\frac{y}{\delta}\right)\eta\cos\eta \quad \text{-----------}(5)$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial\eta}\frac{\partial\eta}{\partial y} = \frac{U}{\pi}\frac{\delta}{x}\left(-\cancel{\sin\eta} + \eta\cos\eta + \cancel{\sin\eta}\right)\frac{\pi}{2\delta} = \frac{U}{2x}\eta\cos\eta \quad \text{------}(6)$$

Substituting into Eq. 1,

$$a_x = U\sin\eta\left(-\frac{U}{2x}\eta\cos\eta\right) + \frac{U}{\cancel{\pi}}\frac{\delta}{x}(\cos\eta - \eta\sin\eta - 1)\frac{U\cancel{\pi}}{2\cancel{\delta}}\cos\eta$$

$$a_x = -\frac{U^2}{2x}\eta\sin\eta\cos\eta + \frac{U^2}{2x}(\cos\eta - \eta\sin\eta - 1)\cos\eta$$

$$a_x = \frac{U^2}{2x}\cos\eta\left[\cos\eta - \cancel{\eta\sin\eta} - 1 - \cancel{\eta\sin\eta}\right]$$

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Problem 5.58

[3] Part 2/2

$$a_x = \frac{U^2}{2x}\cos\eta(\cos\eta-1) = -\frac{U^2}{2x}\cos\eta(1-\cos\eta) \longleftarrow a_x$$

Substituting into Eq. 2.

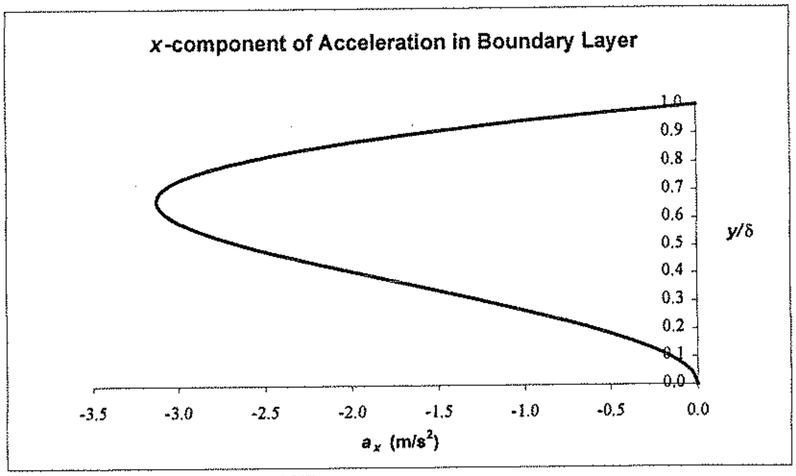
$$a_y = U\sin\eta\left[-\frac{U}{\pi}\frac{\delta}{2x^2}(\cos\eta+\eta\sin\eta-1) - \frac{U\delta}{4x^2}\left(\frac{y}{\delta}\right)\eta\cos\eta\right]$$
$$+\frac{U}{\pi}\frac{\delta}{x}(\cos\eta+\eta\sin\eta-1)\frac{U}{2x}\eta\cos\eta$$

$$a_y = \frac{U^2\delta}{\pi 2x^2}\left\{\left[-\sin\eta(\cos\eta+\eta\sin\eta-1) - \frac{\pi}{2}\left(\frac{y}{\delta}\right)\eta\cos\eta\sin\eta\right] + \eta\cos\eta(\cos\eta+\eta\sin\eta-1)\right\}$$

$$a_y = \frac{U^2\delta}{\pi 2x^2}\left\{-\sin\eta(\cos\eta+\eta\sin\eta-1) - \frac{\pi}{2}\left(\frac{y}{\delta}\right)\eta\cos\eta\sin\eta + \eta\cos\eta(\cos\eta+\eta\sin\eta-1)\right\} \longleftarrow a_y$$

***x* component**

y/δ	η	a_x (m/s^2)
0.00	0.000	0.000
0.05	0.0785	-0.0384
0.10	0.157	-0.152
0.15	0.236	-0.336
0.20	0.314	-0.582
0.25	0.393	-0.879
0.30	0.471	-1.21
0.35	0.550	-1.57
0.40	0.628	-1.93
0.45	0.707	-2.28
0.50	0.785	-2.59
0.55	0.864	-2.85
0.60	0.942	-3.03
0.65	1.02	-3.12
0.70	1.10	-3.10
0.75	1.18	-2.95
0.80	1.26	-2.67
0.85	1.34	-2.24
0.90	1.41	-1.65
0.95	1.49	-0.904
1.00	1.57	0.000

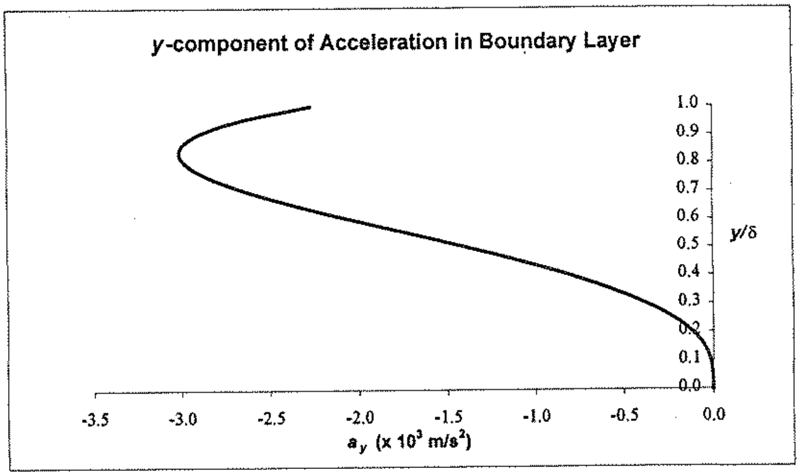


y/δ	η	a_x (m/s^2)
0.667	1.05	-3.12

(Maximum absolute value using *Solver*)

***y* component**

y/δ	η	a_y (x 10^3 m/s^2)
0.00	0.000	0.0000
0.05	0.0785	-0.00192
0.10	0.157	-0.0152
0.15	0.236	-0.0506
0.20	0.314	-0.117
0.25	0.393	-0.223
0.30	0.471	-0.372
0.35	0.550	-0.566
0.40	0.628	-0.803
0.45	0.707	-1.08
0.50	0.785	-1.39
0.55	0.864	-1.71
0.60	0.942	-2.04
0.65	1.02	-2.35
0.70	1.10	-2.62
0.75	1.18	-2.84
0.80	1.26	-2.98
0.85	1.34	-3.01
0.90	1.41	-2.91
0.95	1.49	-2.67
1.00	1.57	-2.27



y/δ	η	a_y (x 10^3 m/s^2)
0.839	1.32	-3.01

(Maximum absolute value using *Solver*)

Note: a_y is normalized with x^2/δ and a_x is normalized with x. Thus $a_y = O\left(\frac{\delta}{x}a_x\right) \approx 0.001\,a_x$

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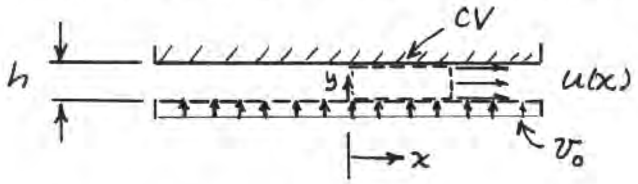
Problem 5.59

Given: Air flow through porous surface into narrow gap.



Find: (a) Show $u(x) = v_0 x/h$

(b) Component, v

(c) Acceleration of a fluid particle in the gap.

Solution: Apply conservation of mass to CV shown.

Basic equation: $$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow

(2) Incompressible flow

(3) Uniform flow at each section

Then
$$0 = \{-xwv_0\} + \{hwu(x)\} \quad \text{or} \quad u(x) = v_0\frac{x}{h}$$
u(x)

Apply differential form to find v:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \frac{\partial u}{\partial x} = \frac{v_0}{h}$$

$$v - v_0 = \int_0^y \frac{\partial v}{\partial y}dy + f(x) = \int_0^y -\frac{v_0}{h}dy + f(x) = -\frac{v_0 y}{h} + f(x)$$

or
$$v = v_0\left(1 - \frac{y}{h}\right) \qquad \left[f(x) = 0 \text{ since } v = v_0 = \text{const along } y = 0\right]$$
v(y)

$$a_{p_x} = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + \overset{=0(3)}{\cancel{v\frac{\partial u}{\partial y}}} + \overset{=0}{\cancel{w\frac{\partial u}{\partial z}}} + \overset{=0(1)}{\cancel{\frac{\partial u}{\partial t}}}$$

$$a_{p_x} = \left(v_0\frac{x}{h}\right)\left(\frac{v_0}{h}\right) = \frac{v_0^2 x}{h^2}$$
a_{p_x}

$$a_{p_y} = \cancel{u\frac{\partial v}{\partial x}} + v\frac{\partial v}{\partial y} + \cancel{w\frac{\partial v}{\partial z}} + \cancel{\frac{\partial v}{\partial t}}$$

$$a_{p_y} = v_0\left(1 - \frac{y}{h}\right)\left(-\frac{v_0}{h}\right) = \frac{v_0^2}{h}\left(\frac{y}{h} - 1\right)$$
a_{p_y}

Thus
$$\vec{a}_p = a_{p_x}\hat{\imath} + a_{p_y}\hat{\jmath} = \frac{v_0^2 x}{h^2}\hat{\imath} + \frac{v_0^2}{h}\left(\frac{y}{h} - 1\right)\hat{\jmath}$$
$\vec{a}_p$

Problem 5.60

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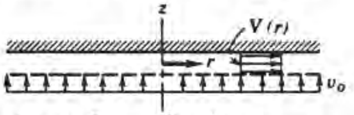
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Flow between parallel disks through porous surface.

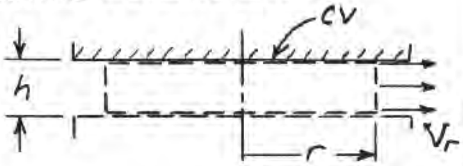


Find: (a) Show $V_r = v_0 r/2h$

(b) V_z, if $v_0 \ll V_r$

(c) Components of acceleration for a fluid particle in the gap.

Solution: Apply CV form of continuity to finite CV shown.



Basic equation:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) Steady flow

(2) Incompressible flow

(3) Uniform flow at each section

Then

$$0 = \left\{ -|\rho v_0 \pi r^2| \right\} + \left\{ +|\rho V_r \, 2\pi r h| \right\} \quad \text{or} \quad V_r = \frac{v_0 r}{2h}$$ V_r

Apply differential form of conservation of mass for incompressible flow.

Basic equation: $$\frac{1}{r}\frac{\partial}{\partial r}(r V_r) + \overset{=0(1)}{\cancel{\frac{1}{r}\frac{\partial}{\partial \theta}(V_\theta)}} + \frac{\partial}{\partial z} V_z = 0$$

Assumptions: (4) $V_\theta = 0$ by symmetry

(5) $V_r = v_0 r/2h$ from above

Then

$$\frac{\partial V_z}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}(r V_r) = -\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{v_0 r^2}{2h}\right) = -\frac{1}{r}\left(\frac{v_0 r}{h}\right) = -\frac{v_0}{h}$$

Integrating,

$$V_z = -\frac{v_0 z}{h} + f(r)$$

Boundary conditions are $V_z = v_0$ at $z = 0$, $V_z = 0$ at $z = h$

Thus from first BC, $f(r) = v_0$ = constant, so

$$V_z = v_0 \left(1 - \frac{z}{h}\right)$$ V_z

The r component of acceleration is

$$a_r = V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\cancel{\frac{\partial V_r}{\partial \theta}} + V_z \cancel{\frac{\partial V_r}{\partial z}} + \cancel{\frac{\partial V_r}{\partial t}} = \left(\frac{v_0 r}{2h}\right)\left(\frac{v_0}{2h}\right) = \left(\frac{v_0}{2h}\right)^2 r$$ a_r

The z component is

$$a_z = V_r \cancel{\frac{\partial V_z}{\partial r}} + \frac{V_\theta}{r}\cancel{\frac{\partial V_z}{\partial \theta}} + V_z \frac{\partial V_z}{\partial z} + \cancel{\frac{\partial V_z}{\partial t}} = v_0\left(1 - \frac{z}{h}\right)\left(-\frac{v_0}{h}\right) = \frac{v_0^2}{h}\left(\frac{z}{h} - 1\right)$$ a_z

Problem 5.61

Given: Steady, inviscid flow over a circular cylinder of radius R.

$$\vec{V} = U\cos\theta\left[1-\left(\frac{R}{r}\right)^2\right]\hat{e}_r - U\sin\theta\left[1+\left(\frac{R}{r}\right)^2\right]\hat{e}_\theta$$

Find: (a) Expression for acceleration of particle moving along $\theta=\pi$
(b) Expression for acceleration of particle moving along $r=R$
(c) Locations at which accelerations a_r and a_θ reach maximum and minimum values.

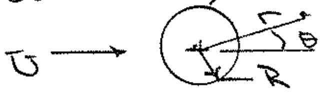
Plot: a_r as a function of R/r for $\theta=\pi$ and as a function of θ for $r=R$; plot a_θ as a function of θ for $r=R$

Solution:

Basic equations: $a_r = V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + \overset{=0(1)}{\cancel{\frac{\partial V_r}{\partial t}}}$

$a_\theta = V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + \overset{=0(1)}{\cancel{\frac{\partial V_\theta}{\partial t}}}$

Assumptions: (1) steady flow.



Along $\theta=\pi$

· $\cos\theta=-1$, $\sin\theta=0$, so $V_\theta=0$ and $V_r=-U\left[1-\left(\frac{R}{r}\right)^2\right]$

Then

$$a_r = V_r\frac{\partial V_r}{\partial r} = -U\left[1-\left(\frac{R}{r}\right)^2\right](-U)(-2)\left(-\frac{R^2}{r^3}\right)] = \frac{2U^2}{R}\left[1-\left(\frac{R}{r}\right)^2\right]\left(\frac{R}{r}\right)^3 \leftarrow a_r$$

$$a_\theta = 0 \leftarrow a_\theta$$

$\left.\begin{matrix}a_r\\a_\theta\end{matrix}\right\}\theta=\pi$

To determine location of maximum a_r, let $\frac{R}{r}=\eta$ and evaluate $\frac{da_r}{d\eta}$

$$a_r = \frac{2U^2}{R}\left[1-\eta^2\right]\eta^3 = \frac{2U^2}{R}\left[\eta^3-\eta^5\right]$$

$\frac{da_r}{d\eta} = \frac{2U^2}{R}\left[3\eta^2-5\eta^4\right]$. Thus $\frac{da_r}{d\eta}=0$ at $\eta^2=\frac{3}{5}$ or $\eta=0.775$

Thus, $a_{r\,max}$ occurs at $r = R/0.775 = 1.29R \leftarrow r_{a\,max}$

$a_{r\,max} = \frac{2U^2}{R}(0.775)^3\left[1-(0.775)^2\right] = 0.372\,\frac{U^2}{R}$ @ $r=1.29R$.

Since $a_\theta=0$, $\vec{a}_{max} = a_{r\,max}\hat{e}_r = 0.372\frac{U^2}{R}\hat{e}_r$ @ $r=1.29R$

Along $r=R$

· $r=R$, $V_r=0$ and $V_\theta=-2U\sin\theta$

$$a_r = -\frac{V_\theta^2}{r} = -\frac{(-2U\sin\theta)^2}{R} = -\frac{4U^2}{R}\sin^2\theta$$

$$a_\theta = \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} = \left(-\frac{2U\sin\theta}{R}\right)\left(-\frac{2U\cos\theta}{R}\right) = \frac{4U^2}{R^2}\sin\theta\cos\theta$$

$\left.\begin{matrix}a_r\\a_\theta\end{matrix}\right\}r=R$

a_r has maximum negative value at $\theta=\pm\pi/2$
has minimum value (of zero) at $\theta=0,\pi$

a_θ has maximum values at $\theta=\pm\pi/4,\ 3\pi/4$
has minimum values at $\theta=0,\ \pm\pi/2,\ \pi$

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Problem 5.61

The acceleration magnitude is

$$|\vec{a}| = \left[a_r^2 + a_\theta^2\right]^{1/2} = \left[\left(-4\frac{U^2}{R}\right)^2 \sin^4\theta + \left(\frac{4U^2}{R}\right)^2 \sin^2\cos^2\theta\right]^{1/2} = \frac{4U^2}{R}\sin\theta$$

. This is a maximum at $\theta = \pm \pi/2$.

Thus

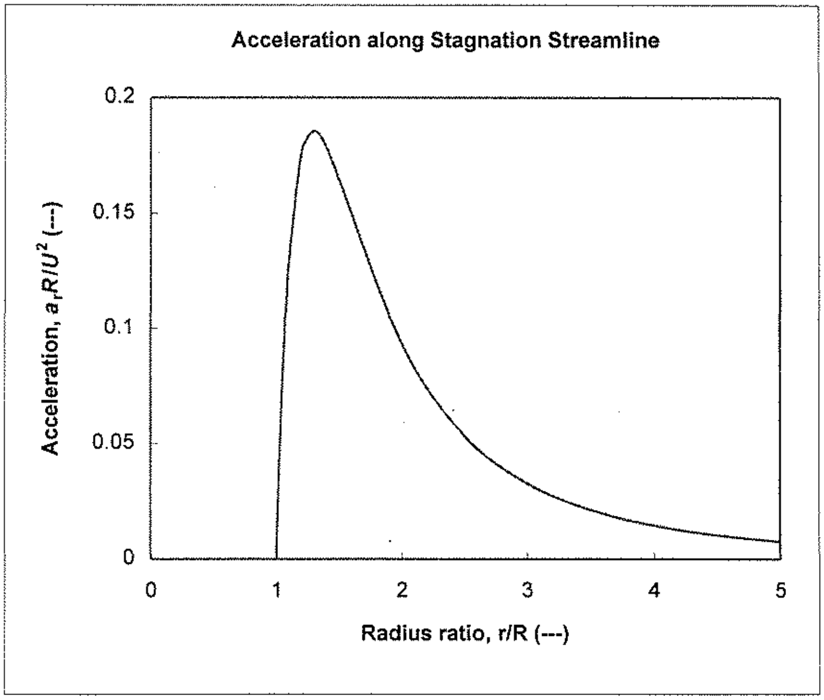
$$\vec{a}_{max} = \pm 4\frac{U^2}{R} \quad \text{at } \theta = \pm\pi/2.$$

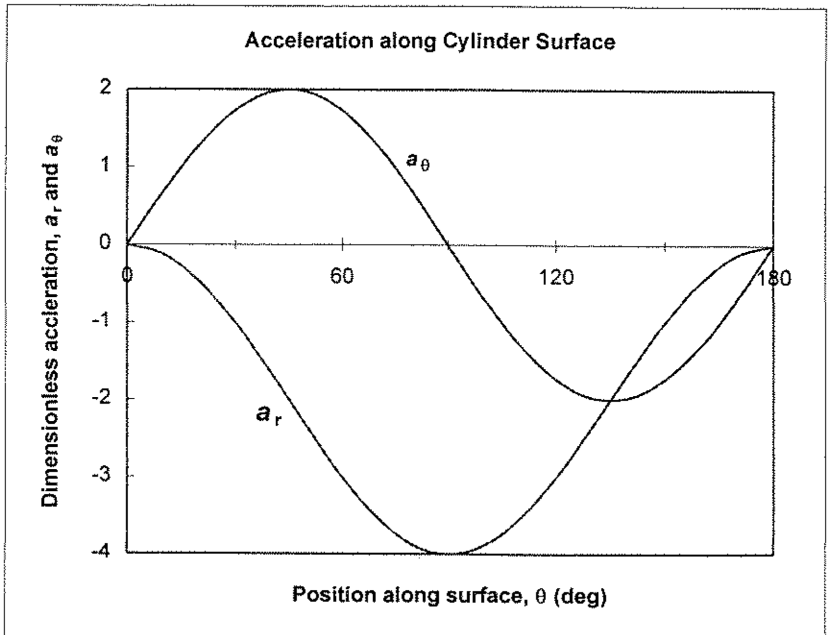
Plots:

(1) $\theta = \pi$ $\quad a_r = \frac{2U^2}{R}\left(\frac{R}{r}\right)^3\left[1-\left(\frac{R}{r}\right)^2\right]$; $\quad \frac{a_r}{U^2/R} = \left(\frac{R}{r}\right)^3\left[1-\left(\frac{R}{r}\right)^2\right]$

(2) $r = R$. $\quad a_r = -4\frac{U^2}{R}\sin^2\theta$; $\quad \frac{a_r}{U^2/R} = -4\sin^2\theta$

$a_\theta = 4\frac{U^2}{R}\sin\theta\cos\theta$; $\quad \frac{a_\theta}{U^2/R} = 4\sin\theta\cos\theta$

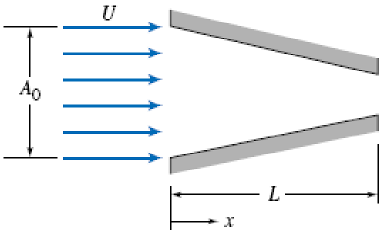




Problem 5.62

[3]

5.62 Consider the incompressible flow of a fluid through a nozzle as shown. The area of the nozzle is given by $A = A_0(1 - bx)$ and the inlet velocity varies according to $U = U_0(1 - e^{-\lambda t})$, where $A_0 = 0.5\ \text{m}^2$, $L = 5$ m, $b = 0.1\ \text{m}^{-1}$, $\lambda = 0.2\ \text{s}^{-1}$, and $U_0 = 5$ m/s. Find and plot the acceleration on the centerline, with time as a parameter.



Given: Velocity field and nozzle geometry

Find: Acceleration along centerline; plot

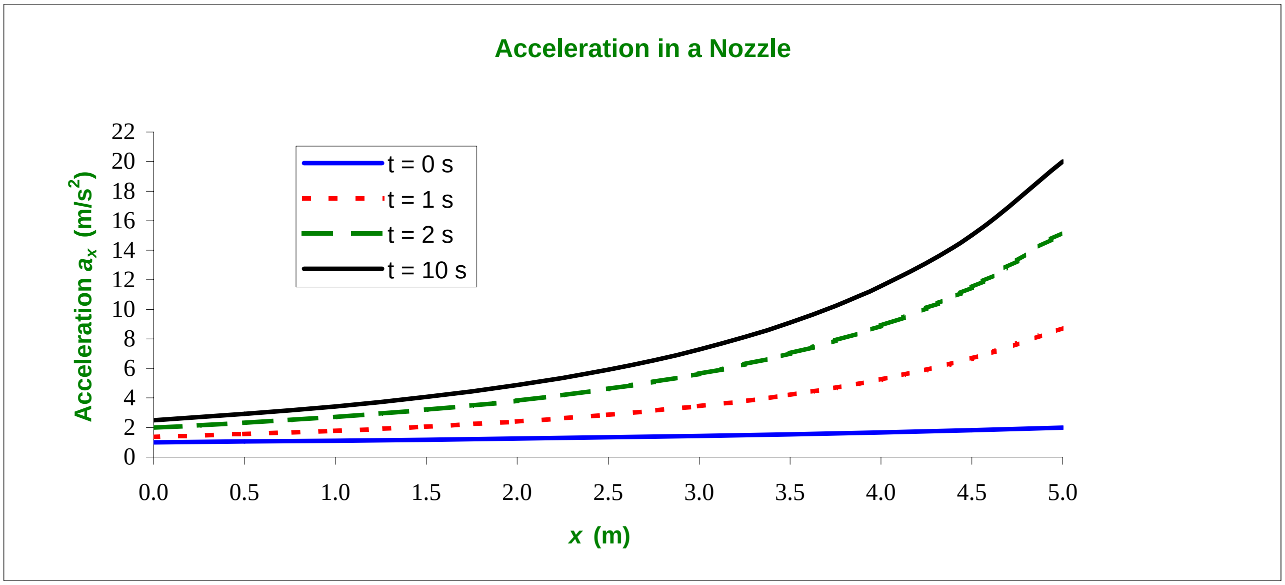
Solution:

$$a_x = \frac{U_0}{(1 - b \cdot x)} \cdot \left[\lambda \cdot e^{-\lambda \cdot t} + \frac{b \cdot U_0}{(1 - b \cdot x)^2} \cdot \left(1 - e^{-\lambda \cdot t}\right)^2\right]$$

$A_0 =$	0.5	m^2
$L =$	5	m
$b =$	0.1	m^{-1}
$\lambda =$	0.2	s^{-1}
$U_0 =$	5	m/s

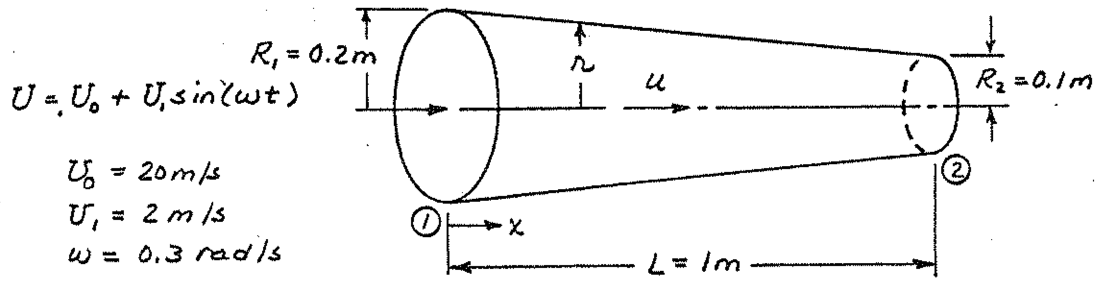
t **=**	**0**	**5**	**10**	**60**
x **(m)**	***a*** $_x$ **(m/s^2)**	***a*** $_x$ **(m/s^2)**	***a*** $_x$ **(m/s^2)**	***a*** $_x$ **(m/s^2)**
0.0	1.00	1.367	2.004	2.50
0.5	1.05	1.552	2.32	2.92
1.0	1.11	1.78	2.71	3.43
1.5	1.18	2.06	3.20	4.07
2.0	1.25	2.41	3.82	4.88
2.5	1.33	2.86	4.61	5.93
3.0	1.43	3.44	5.64	7.29
3.5	1.54	4.20	7.01	9.10
4.0	1.67	5.24	8.88	11.57
4.5	1.82	6.67	11.48	15.03
5.0	2.00	8.73	15.22	20.00

For large time (> 30 s) the flow is essentially steady-state



Problem 5.63

Given: One-dimensional, incompressible flow through circular channel.



Find: (a) The acceleration of a particle at the channel exit.
(b) Plot as a function of time for a complete cycle.
(c) On same plot, show acceleration if channel is constant area; explain

Solution: The acceleration of a particle in one-dimensional flow is

$$a_x = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

From continuity, $u = U\frac{A_1}{A} = U\frac{R_1^2}{r^2}$

From geometry, $r = R_1 - (R_1 - R_2)\frac{x}{L} = R_1 - \Delta R\frac{x}{L}$, so

$$u = U\frac{R_1^2}{(R_1 - \Delta R\frac{x}{L})^2} = \left[U_0 + U_1\sin(\omega t)\right]\frac{1}{\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right]^2}$$

Thus

$$a_x = \left[U_0 + U_1\sin(\omega t)\right]\frac{1}{\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right]^2}\left[U_0 + U_1\sin(\omega t)\right](-2)\left(-\frac{\Delta R}{R_1 L}\right)\frac{1}{\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right]^3}$$

$$+ \frac{\omega U_1\cos(\omega t)}{\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right]^2}$$

$$a_x = \frac{2\Delta R}{R_1 L}\frac{\left[U_0 + U_1\sin(\omega t)\right]^2}{\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right]^5} + \frac{\omega U_1\cos(\omega t)}{\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right]^2}$$

At $x/L = 1$, $\left[1 - \frac{\Delta R}{R_1}(\frac{x}{L})\right] = 1 - \frac{0.1m}{0.2m} = 0.5$, so

$$a_x = 2 \times 0.1m \times \frac{1}{0.2m} \times \frac{1}{1m}\left[20 + 2\sin(\omega t)\right]^2\frac{m^2}{s^2} \times \frac{1}{(0.5)^5} + 0.3\frac{rad}{s} \times 2\frac{m}{s} \times \cos(\omega t) \times \frac{1}{(0.5)^2}$$

or

$$a_x\,(m/sec^2) = 32\left[20 + 2\sin(\omega t)\right]^2 + 2.4\cos(\omega t) \quad (at\ x = L)$$

a_x

(See next page for plots)

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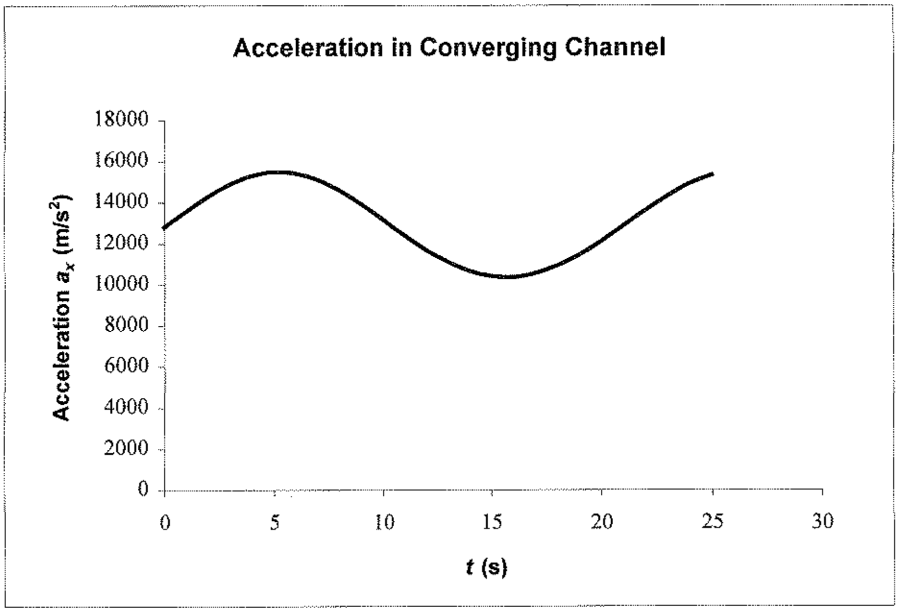
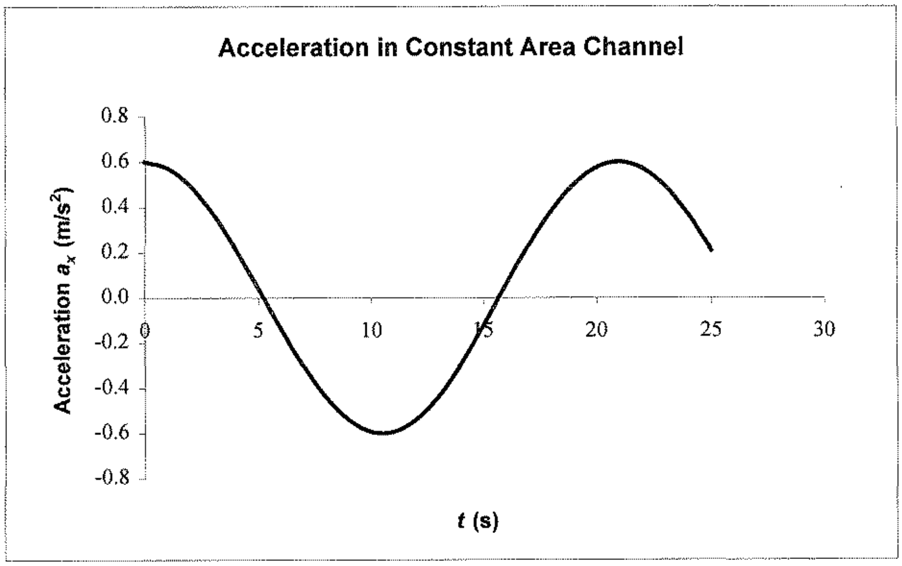
Problem 5.63

The acceleration in the channel and in a constant area are calculated and plotted below

t (s)	a_x (m/s^2) (Convergent)	a_x (m/s^2) (A = const.)
0	12802	0.600
1	13570	0.573
2	14288	0.495
3	14885	0.373
4	15298	0.217
5	15481	0.042
6	15414	-0.136
7	15104	-0.303
8	14586	-0.442
9	13915	-0.542
10	13161	-0.594
11	12397	-0.592
12	11690	-0.538
13	11098	-0.436
14	10665	-0.294
15	10419	-0.126
16	10377	0.052
17	10541	0.227
18	10900	0.381
19	11431	0.501
20	12097	0.576
21	12845	0.600
22	13612	0.570
23	14326	0.489
24	14914	0.365
25	15315	0.208





The acceleration in the convergent channel is massively larger than that in the constant area channel because very large convective acceleration is generated by the convergence (the constant area channel only has local acceleration)

Problem 5.64

[4]

Given: Steady, two-dimensional velocity field of Problem 5.53,

$$\vec{V} = Ax\hat{i} - Ay\hat{j} \quad ; \quad A = 1\,s^{-1}$$

Find: (a) Expressions for particle coordinates, $x_p = f_1(t)$ and $y_p = f_2(t)$.
(b) Time required for particle to travel from $(x_0, y_0) = (\frac{1}{2}, 2)$ to $(x, y) = (1, 1)$ and $(2, \frac{1}{2})$.
(c) Compare acceleration determined from $f_1(t)$ and $f_2(t)$ with those found in Problem 5.53.

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Solution: For the given flow, $u = Ax$ and $v = -Ay$. Thus

$$u_p = \frac{df_1}{dt} = Ax_p = Af_1 \text{ , or } \frac{df_1}{f_1} = A\,dt$$

Integrating from x_0 to f_1,

$$\int_{x_0}^{f_1} \frac{df_1}{f_1} = \ln f_1 \Big]_{x_0}^{f_1} = \ln\left(\frac{f_1}{x_0}\right) = At \text{ , or } f_1 = x_0 e^{At}$$

$f_1(t)$

Likewise $v_p = \frac{df_2}{dt} = -Ay_p = -Af_2$, or $\frac{df_2}{f_2} = -A\,dt$

Integrating from y_0 to f_2,

$$\int_{y_0}^{f_2} \frac{df_2}{f_2} = \ln f_2 \Big]_{y_0}^{f_1} = \ln\left(\frac{f_2}{y_0}\right) = -At \text{ or } f_2 = y_0 e^{-At}$$

$f_2(t)$

For a particle initially at $(\frac{1}{2}, 2)$, $x_0 = \frac{1}{2}$ and $y_0 = 2$

To reach the point $(x, y) = (1, 1)$, $e^{At} = \frac{x}{x_0} = 2$, so $t = \frac{\ln 2}{A} = 0.693$ sec

$e^{-At} = \frac{y}{y_0} = \frac{1}{2}$, so $t = \frac{-\ln\frac{1}{2}}{A} = 0.693$ sec

$t(1,$

To reach the point $(x, y) = (2, \frac{1}{2})$, $e^{At} = \frac{x}{x_0} = 4$, so $t = \frac{\ln 4}{A} = 1.39$ sec

$e^{-At} = \frac{y}{y_0} = \frac{1}{4}$, so $t = \frac{-\ln\frac{1}{4}}{A} = 1.39$ sec

$t(2,$

The acceleration components are

$$a_{p_x} = \frac{d^2 f_1}{dt^2} = x_0 A^2 e^{At} = x_0 A^2 \frac{f_1}{x_0} = A^2 f_1$$

$$a_{p_y} = \frac{d^2 f_2}{dt^2} = y_0 A^2 e^{-At} = y_0 A^2 \frac{f_2}{y_0} = A^2 f_2$$

At $(x, y) = (1, 1)$

$$\vec{a}_p = a_{p_x}\hat{i} + a_{p_y}\hat{j} = \frac{(1)^2}{s^2} \times 1\,m\,\hat{i} + \frac{(1)^2}{s^2} \times 1\,m\,\hat{j} = (\hat{i} + \hat{j})\frac{m}{s^2}$$

$\vec{a}_p(1,$

At $(x, y) = (2, \frac{1}{2})$

$$\vec{a}_p = \frac{(1)^2}{s^2} \times 2\,m\,\hat{i} + \frac{(1)^2}{s^2} \times \frac{1}{2}\,m\,\hat{j} = (2\hat{i} + \tfrac{1}{2}\hat{j})\frac{m}{s^2}$$

$\vec{a}_p(2,$

These are identical to the accelerations found in Problem 5.53.

Problem 5.65

Expand $(\vec{V}\cdot\nabla)\vec{V}$ in cylindrical coordinates to obtain the convective acceleration of a fluid particle. Verify the results given in Eqs. 5.12.

Recall $\partial\hat{e}_r/_{\partial\theta} = \hat{e}_\theta$ and $\partial\hat{e}_\theta/_{\partial\theta} = -\hat{e}_r$

Solution:

In cylindrical coordinates $\nabla = \hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z}$

$$\vec{V} = V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{k}$$

$$(\vec{V}\cdot\nabla)\vec{V} = \left[V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{k}\right]\cdot\left[\hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z}\right](V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{k})$$

$$= \left[V_r\frac{\partial}{\partial r} + \frac{V_\theta}{r}\frac{\partial}{\partial\theta} + V_z\frac{\partial}{\partial z}\right](V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{k})$$

$$= V_r\frac{\partial}{\partial r}V_r\hat{e}_r + \frac{V_\theta}{r}\frac{\partial}{\partial\theta}V_r\hat{e}_r + V_z\frac{\partial}{\partial z}(V_r\hat{e}_r)$$

$$+ V_r\frac{\partial}{\partial r}V_\theta\hat{e}_\theta + \frac{V_\theta}{r}\frac{\partial}{\partial\theta}V_\theta\hat{e}_\theta + V_z\frac{\partial}{\partial z}V_\theta\hat{e}_\theta$$

$$+ V_r\frac{\partial}{\partial r}V_z\hat{k} + \frac{V_\theta}{r}\frac{\partial}{\partial\theta}V_z\hat{k} + V_z\frac{\partial}{\partial z}V_z\hat{k}$$

$$= \hat{e}_r\left\{V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial\theta} + V_z\frac{\partial V_r}{\partial z}\right\}① + \frac{V_\theta V_r}{r}\left(\frac{\partial\hat{e}_r}{\partial\theta}\right)^{=\hat{e}_\theta}$$

$$+\hat{e}_\theta\left\{V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta} + V_z\frac{\partial V_\theta}{\partial z}\right\}② + \frac{V_\theta^2}{r}\left(\frac{\partial\hat{e}_\theta}{\partial\theta}\right) = -\hat{e}_r$$

$$+\hat{k}\left\{V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial\theta} + V_z\frac{\partial V_z}{\partial z}\right\}③$$

$$(\vec{V}\cdot\nabla)\vec{V} = \hat{e}_r\left\{V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial\theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}\right\}$$

$$+\hat{e}_\theta\left\{V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta} + \frac{V_\theta V_r}{r} + V_z\frac{\partial V_\theta}{\partial z}\right\}.$$

$$+\hat{k}\left\{V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial\theta} + V_z\frac{\partial V_z}{\partial z}\right\}.$$

Term ① is the r component of convective acceleration

Eq. 5.12a $a_{r_p} = \left\{V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial\theta} - \frac{V_\theta^2}{r} + V_z\frac{\partial V_r}{\partial z}\right\} + \frac{\partial V_r}{\partial t}$

Term ② is the θ component of convective acceleration

Eq. 5.12b $a_{\theta_p} = \left\{V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}\right\} + \frac{\partial V_\theta}{\partial t}$

Term ③ is the z component of convective acceleration

Eq. 5.12c $a_{z_p} = \left\{V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial\theta} + V_z\frac{\partial V_z}{\partial z}\right\} + \frac{\partial V_z}{\partial t}$

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Problem 5.66

[2]

5.66 Which, if any, of the flow fields of Problem 5.1 are irrotational?

Given: Velocity components

Find: Which flow fields are irrotational

Solution:

a. $u = 2x^2 + y^2 - x^2y;\ v = x^3 + x(y^2 - 2y)$
b. $u = 2xy - x^2 + y;\ v = 2xy - y^2 + x^2$
c. $u = xt + 2y;\ v = xt^2 - yt$
d. $u = (x + 2y)xt;\ v = -(2x + y)yt$

For a 2D field, the irrotionality the test is $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

(a) $$\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = \left[3\cdot x^2 + \left(y^2 - 2\cdot y\right)\right] - \left(2\cdot y - x^2\right) = 4\cdot x^2 + y^2 - 4\cdot y \neq 0$$ Not irrotional

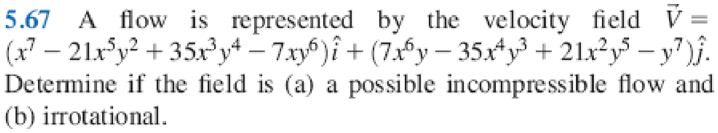
(b) $$\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = (2\cdot y + 2\cdot x) - (2\cdot y - 2\cdot x) = 4\cdot x \neq 0$$ Not irrotional

(c) $$\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = \left(t^2\right) - (2) = t^2 - 2 \neq 0$$ Not irrotional

(d) $$\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = (-2\cdot y\cdot t) - (2\cdot x\cdot t) = -2\cdot x\cdot t - 2\cdot y\cdot t \neq 0$$ Not irrotional

Problem 5.67

[3]



5.67 A flow is represented by the velocity field $\vec{V} = (x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6)\hat{i} + (7x^6y - 35x^4y^3 + 21x^2y^5 - y^7)\hat{j}$. Determine if the field is (a) a possible incompressible flow and (b) irrotational.

Given: Flow field

Find: If the flow is incompressible and irrotational

Solution:

Basic equations: Incompressibility $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ Irrotationality $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

a) $u(x,y) = x^7 - 21\cdot x^5\cdot y^2 + 35\cdot x^3\cdot y^4 - 7\cdot x\cdot y^6$ $v(x,y) = 7\cdot x^6\cdot y - 35\cdot x^4\cdot y^3 + 21\cdot x^2\cdot y^5 - y^7$

$\frac{\partial}{\partial x}u(x,y) \rightarrow 7\cdot x^6 - 105\cdot x^4\cdot y^2 + 105\cdot x^2\cdot y^4 - 7\cdot y^6$ $\frac{\partial}{\partial y}v(x,y) \rightarrow 7\cdot x^6 - 105\cdot x^4\cdot y^2 + 105\cdot x^2\cdot y^4 - 7\cdot y^6$

Hence $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v \neq 0$ COMPRESSIBLE

b) $u(x,y) = x^7 - 21\cdot x^5\cdot y^2 + 35\cdot x^3\cdot y^4 - 7\cdot x\cdot y^6$ $v(x,y) = 7\cdot x^6\cdot y - 35\cdot x^4\cdot y^3 + 21\cdot x^2\cdot y^5 - y^7$

$\frac{\partial}{\partial x}v(x,y) \rightarrow 42\cdot x^5\cdot y - 140\cdot x^3\cdot y^3 + 42\cdot x\cdot y^5$ $-\frac{\partial}{\partial y}u(x,y) \rightarrow 42\cdot x^5\cdot y - 140\cdot x^3\cdot y^3 + 42\cdot x\cdot y^5$

Hence $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u \neq 0$ ROTATIONAL

Note that if we define $v(x,y) = -\left(7\cdot x^6\cdot y - 35\cdot x^4\cdot y^3 + 21\cdot x^2\cdot y^5 - y^7\right)$ then the flow is incompressible and irrotational!

Problem 5.68

[2]

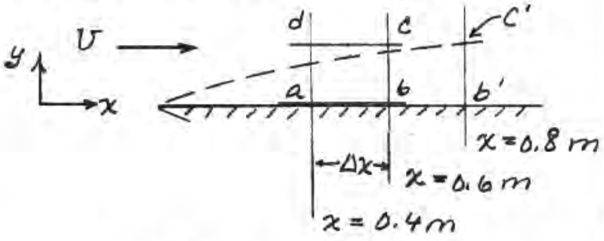
Given: Sinusoidal approximation to boundary-layer velocity profile,

$$u = U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) \quad \text{where } \delta = 5\text{ mm at } x = 0.5\text{ m (Problem 5.12)}$$

Neglect vertical component of velocity. $U = 0.5$ m/s.

Find: (a) Circulation about contour bounded by $x = 0.4$ m, $x = 0.6$ m, $y = 0$, and $y = 8$ mm.
(b) Result if evaluated $\Delta x = 0.2$ m further downstream?

Solution: Evaluate circulation



Defining equation:

$$\Gamma = \oint \vec{V}\cdot d\vec{s}$$

From the definition

$$\Gamma = \int_{ab} \overset{u=0}{\cancel{\vec{V}\cdot d\vec{s}}} + \int_{bc} \overset{u\perp dy}{\cancel{\vec{V}\cdot d\vec{s}}} + \int_{cd} \vec{V}\cdot d\vec{s} + \int_{da} \overset{u\perp dy}{\cancel{\vec{V}\cdot d\vec{s}}} = \int_0^{\Delta x} U\hat{i}\cdot dx(-\hat{i})$$

$$\Gamma = -U\Delta x = -5\,\frac{\text{m}}{\text{sec}} \times 0.2\text{ m} = -0.100\text{ m}^2/\text{sec} \longleftarrow \Gamma$$

At the downstream location, since $\delta = cx^{1/2}$

$$\delta' = \delta\left(\frac{x}{x'}\right)^{1/2} = 5\text{ mm}\left(\frac{0.8}{0.5}\right)^{1/2} = 6.32\text{ mm}$$

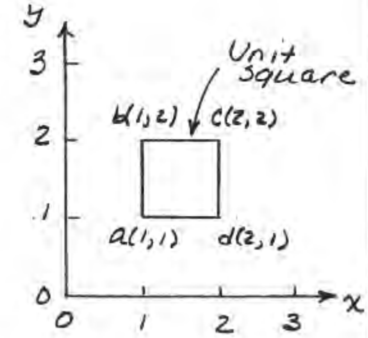
Point c' is also outside the boundary layer. Consequently the integral along c'c will be the same as along cd. Thus

$$\Gamma_{bb'c'c} = \Gamma_{abcd} \longleftarrow \Gamma$$

Given: Velocity field for flow in a rectangular "corner,"

$$\vec{V} = Ax\hat{i} - Ay\hat{j} \quad \text{with } A = 0.3\,s^{-1}$$

as in Example Problem 5.8.



Find: Circulation about unit square shown.

Solution: Evaluate circulation

Defining equation:

$$\Gamma = \oint \vec{V} \cdot d\vec{s}$$

The dot product is $\vec{V} \cdot d\vec{s} = (Ax\hat{i} - Ay\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = Ax\,dx - Ay\,dy$.

For the contour shown, $dy = 0$ along ad and cb, and $dx = 0$ along ba and dc. Thus

$$\Gamma = \int_a^d Ax\,dx + \int_d^c -Ay\,dy + \int_c^b Ax\,dx + \int_b^a -Ay\,dy$$

$$= \frac{Ax^2}{2}\Big]_{x_a}^{x_d} - \frac{Ay^2}{2}\Big]_{y_d}^{y_c} + \frac{Ax^2}{2}\Big]_{x_c}^{x_b} - \frac{Ay^2}{2}\Big]_{y_b}^{y_a}$$

$$= \frac{A}{2}(x_d^2 - x_a^2 + x_b^2 - x_c^2) - \frac{A}{2}(y_c^2 - y_d^2 + y_a^2 - y_b^2)$$

$\Gamma = 0$ (since $x_a = x_b$ and $x_c = x_d$

$y_a = y_d$ and $y_b = y_c$) $\longleftarrow$ Γ

$\Bigg\{$ This result is to be expected, since flow is irrotational ($\nabla \times \vec{V} = 0$). From Stokes' Theorem (Eq. 5.18),

$$\Gamma = \int_A (\nabla \times \vec{V})_z\,dA = 0$$ $\Bigg\}$

Problem 5.70

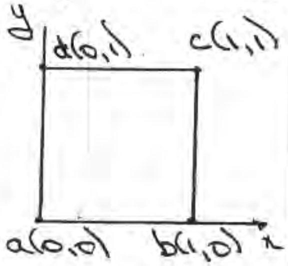
[2]

Given: Two dimensional flow field $\vec{V} = Axy\hat{i} + By^2\hat{j}$, where $A = 1\,m^{-1}\cdot s^{-1}$, $B = -\frac{1}{2}\,m^{-1}\cdot s^{-1}$ and coordinates are measured in meters

Show: velocity field represents a possible incompressible flow

Find: (a) Rotation at point $(x,y) = (1,1)$

(b) Circulation about curve shown



Solution:

For incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

For given flow field.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}(Axy) + \frac{\partial}{\partial y}(By^2) = Ay + 2By = (1)y + 2\left(-\frac{1}{2}\right)y = 0$$ ✓

The fluid rotation is defined as $\vec{\omega} = \frac{1}{2}\nabla\times\vec{V}$

$$\vec{\omega} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Axy & By^2 & 0 \end{vmatrix} = -\frac{1}{2}Ax\hat{k}$$

$$\vec{\omega}_{1,1} = -\frac{1}{2}\times\frac{1}{m\cdot s}\times 1m\,\hat{k} = -0.5\hat{k}\ \text{rad/s} \longleftarrow \vec{\omega}_{1,1}$$

The circulation is defined as $\Gamma = \oint \vec{V}\cdot d\vec{s}$

For the contour shown with $\vec{V} = Axy\hat{i} + By^2\hat{j}$

$$\Gamma = \int_a^b \cancel{u\,dx} + \int_b^c v\,dy + \int_c^d u(-dx) + \int_d^a v(-dy)$$

u=0 along ab.

$$\Gamma = \int_0^1 By^2\,dy + \int_c^d Axy\,dx + \int_1^0 By^2\,dy \qquad \{y=1 \text{ along } cd\}$$

$$\Gamma = \frac{1}{3}By^3\Big]_0^1 + \frac{1}{2}Ax^2\Big]_1^0 + \frac{1}{3}By^3\Big]_1^0$$

$$\Gamma = \frac{1}{3}B - \frac{1}{2}A - \frac{1}{3}B = -\frac{1}{2}A = -\frac{1}{2}\,m^2/s \longleftarrow \Gamma$$

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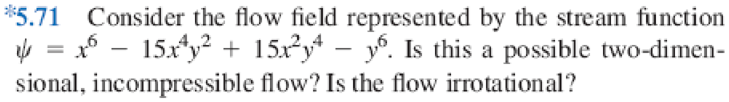
Problem *5.71

[3]



*5.71 Consider the flow field represented by the stream function $\psi = x^6 - 15x^4y^2 + 15x^2y^4 - y^6$. Is this a possible two-dimensional, incompressible flow? Is the flow irrotational?

Given: Stream function

Find: If the flow is incompressible and irrotational

Solution:

Basic equations: Incompressibility $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ Irrotationality $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

Note: The fact that ψ exists means the flow is incompressible, but we check anyway

$$\psi(x,y) = x^6 - 15\cdot x^4\cdot y^2 + 15\cdot x^2\cdot y^4 - y^6$$

Hence $u(x,y) = \frac{\partial}{\partial y}\psi(x,y) \rightarrow 60\cdot x^2\cdot y^3 - 30\cdot x^4\cdot y - 6\cdot y^5$ $v(x,y) = -\frac{\partial}{\partial x}\psi(x,y) \rightarrow 60\cdot x^3\cdot y^2 - 6\cdot x^5 - 30\cdot x\cdot y^4$

For incompressibility

$$\frac{\partial}{\partial x}u(x,y) \rightarrow 120\cdot x\cdot y^3 - 120\cdot x^3\cdot y \qquad \frac{\partial}{\partial y}v(x,y) \rightarrow 120\cdot x^3\cdot y - 120\cdot x\cdot y^3$$

Hence $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ INCOMPRESSIBLE

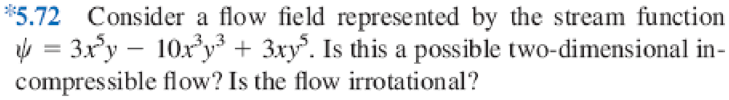
For irrotationality

$$\frac{\partial}{\partial x}v(x,y) \rightarrow 180\cdot x^2\cdot y^2 - 30\cdot x^4 - 30\cdot y^4 \qquad -\frac{\partial}{\partial y}u(x,y) \rightarrow 30\cdot x^4 - 180\cdot x^2\cdot y^2 + 30\cdot y^4$$

Hence $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$ IRROTATIONAL

Problem *5.72

[3]



*5.72 Consider a flow field represented by the stream function $\psi = 3x^5y - 10x^3y^3 + 3xy^5$. Is this a possible two-dimensional incompressible flow? Is the flow irrotational?

Given: Stream function

Find: If the flow is incompressible and irrotational

Solution:

Basic equations: Incompressibility $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ Irrotationality $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

Note: The fact that ψ exists means the flow is incompressible, but we check anyway

$$\psi(x,y) = 3\cdot x^5\cdot y - 10\cdot x^3\cdot y^3 + 3\cdot x\cdot y^5$$

Hence $u(x,y) = \frac{\partial}{\partial y}\psi(x,y) \rightarrow 3\cdot x^5 - 30\cdot x^3\cdot y^2 + 15\cdot x\cdot y^4$ $v(x,y) = -\frac{\partial}{\partial x}\psi(x,y) \rightarrow 30\cdot x^2\cdot y^3 - 15\cdot x^4\cdot y - 3\cdot y^5$

For incompressibility

$$\frac{\partial}{\partial x}u(x,y) \rightarrow 15\cdot x^4 - 90\cdot x^2\cdot y^2 + 15\cdot y^4 \qquad \frac{\partial}{\partial y}v(x,y) \rightarrow 90\cdot x^2\cdot y^2 - 15\cdot x^4 - 15\cdot y^4$$

Hence $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ INCOMPRESSIBLE

For irrotationality

$$\frac{\partial}{\partial x}v(x,y) \rightarrow 60\cdot x\cdot y^3 - 60\cdot x^3\cdot y \qquad -\frac{\partial}{\partial y}u(x,y) \rightarrow 60\cdot x^3\cdot y - 60\cdot x\cdot y^3$$

Hence $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$ IRROTATIONAL

Problem *5.73

[2]

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*5.73 Consider a flow field represented by the stream function $\psi = -A/2(x^2 + y^2)$, where A = constant. Is this a possible two-dimensional incompressible flow? Is the flow irrotational?

Given: The stream function

Find: Whether or not the flow is incompressible; whether or not the flow is irrotational

Solution:

The stream function is $$\psi = -\frac{A}{2\cdot\pi\left(x^2+y^2\right)}$$

The velocity components are $$u = \frac{d\psi}{dy} = \frac{A\cdot y}{\pi\left(x^2+y^2\right)^2} \qquad v = -\frac{d\psi}{dx} = -\frac{A\cdot x}{\pi\left(x^2+y^2\right)^2}$$

Because a stream function exists, the flow is: Incompressible

Alternatively, we can check with $$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$$

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = -\frac{4\cdot A\cdot x\cdot y}{\pi\left(x^2+y^2\right)^3} + \frac{4\cdot A\cdot x\cdot y}{\pi\left(x^2+y^2\right)^3} = 0$$ Incompressible

For a 2D field, the irrotionality the test is $$\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$$

$$\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = \frac{A\cdot\left(x^2-3\cdot y^2\right)}{\pi\cdot\left(x^2+y^2\right)^3} - \frac{A\cdot\left(3\cdot x^2-y^2\right)}{\pi\cdot\left(x^2+y^2\right)^3} = -\frac{2\cdot A}{\pi\cdot\left(x^2+y^2\right)^2} \neq 0$$ Not irrotational

Given: Velocity field for motion in x direction with constant shear.
The shear rate is

$$\frac{\partial u}{\partial y} = A \quad \text{where } A = 0.1\ s^{-1}$$

Find: (a) Expression for $\vec{V}$
(b) Rate of rotation
(c) Stream function.

Solution: The velocity field is

$$\vec{V} = u\hat{\imath} = \left[\int \frac{\partial u}{\partial y}dy + f(x)\right]\hat{\imath} = \left[Ay + f(x)\right]\hat{\imath} \longleftarrow \vec{V}$$

Fluid rotation is given by

$$\vec{\omega} = \frac{1}{2}\nabla \times \vec{V} = \frac{1}{2}\left(\cancelto{0}{\frac{\partial v}{\partial x}} - \frac{\partial u}{\partial y}\right)\hat{k} = -\frac{1}{2}\frac{\partial u}{\partial y}\hat{k} = -\frac{A}{2}\hat{k} = -0.05\ s^{-1}\ \hat{k} \longleftarrow \vec{\omega}$$

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \quad \text{so} \quad \frac{\partial \psi}{\partial y} = Ay + f(x) \quad \text{and} \quad \psi = \frac{1}{2}Ay^2 + f(x)y + g(x)$$

$$v = -\frac{\partial \psi}{\partial x} = f'(x)y + g'(x) = 0$$

Thus $f'(x) = 0$ and $g'(x) = 0$, and

$$\psi = \frac{1}{2}Ay^2 + c \longleftarrow c$$

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Given: Flow field represented by $\psi = x^2 - y^2$

Find: corresponding velocity field
Show: that flow field is irrotational
Plot: several streamlines and illustrate the velocity field

Solution:

Apply definition of ψ and irrotationality condition:

Computing equations: $u = \frac{\partial \psi}{\partial y}$ $v = -\frac{\partial \psi}{\partial x}$

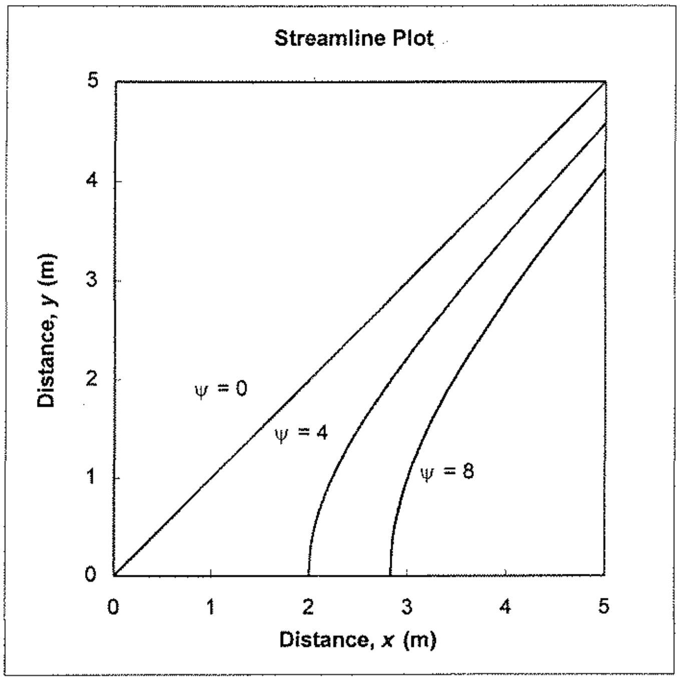
$\vec{\omega} = \frac{1}{2} \nabla \times \vec{V} = 0$

From the given $\psi = x^2 - y^2$

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2) = -2y \\ v &= -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(x^2 - y^2) = -2x \end{aligned}\right\} \vec{V} = u\hat{i} + v\hat{j} = -2y\hat{i} - 2x\hat{j} \longleftarrow \vec{V}$$

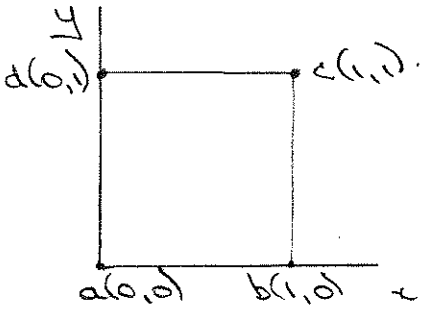
$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2y & -2x & 0 \end{vmatrix} = \hat{k}\left(-2 - (-2)\right) = 0$$

Since $\vec{\omega} = \frac{1}{2} \nabla \times \vec{V} = 0$ flow is irrotational $\longleftarrow$ $\vec{\omega} = 0$



Problem *5.76

Given: Velocity field $\vec{V} = Axy\hat{i} + By^2\hat{j}$, where $A = 4\,m^{-1}s^{-1}$, $B = -2\,m^{-1}s^{-1}$ and coordinates are in meters.



Find: (a) Fluid rotation
(b) Circulation about "curve" shown
(c) Stream function

Plot: several streamlines in first quadrant.

Solution:

(a) The fluid rotation is given by

$$\vec{\omega} = \frac{1}{2}\nabla\times\vec{V} = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ Axy & By^2 & 0\end{vmatrix} = \hat{k}\,\frac{1}{2}(-Ax) = -\frac{1}{2}\times\frac{4}{m\cdot s}\,x\,\hat{k} = -2x(m)\,\hat{k}\,\frac{rad}{s}$$

$\vec{\omega}$

(b) The circulation is defined as $\Gamma = \oint \vec{V}\cdot d\vec{s}$

For the contour shown with $\vec{V} = Axy\hat{i} + By^2\hat{j}$

$$\Gamma = \int_a^b \overset{y=0}{\cancel{Axy}}\,dx + \int_b^c By^2\,dy + \int_c^d \overset{y=1}{Axy}\,dx + \int_d^a By^2\,dy$$

$$\Gamma = \int_0^1 By^2\,dy + \int_1^0 Ax\,dx + \int_1^0 By^2\,dy = B\frac{y^3}{3}\Big]_0^1 + A\frac{x^2}{2}\Big]_1^0 + B\frac{y^3}{3}\Big]_1^0$$

$$\Gamma = \frac{1}{3}B - \frac{1}{2}A - \frac{1}{3}B = -\frac{1}{2}A = -2\,m^2/s$$

Γ

(c) For incompressible flow $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = Ay + 2By = 4y + 2(-2)y = 0 \checkmark \quad \therefore \text{incompressible}$$

Thus $u = Axy = \frac{\partial\psi}{\partial y}$ and

$\psi = \int Axy\,dy + f(x)$

$\psi = \frac{1}{2}Axy^2 + f(x)$

Then,

$v = -\frac{\partial\psi}{\partial x} = -\frac{1}{2}Ay^2 - \frac{df}{dx} = By^2$

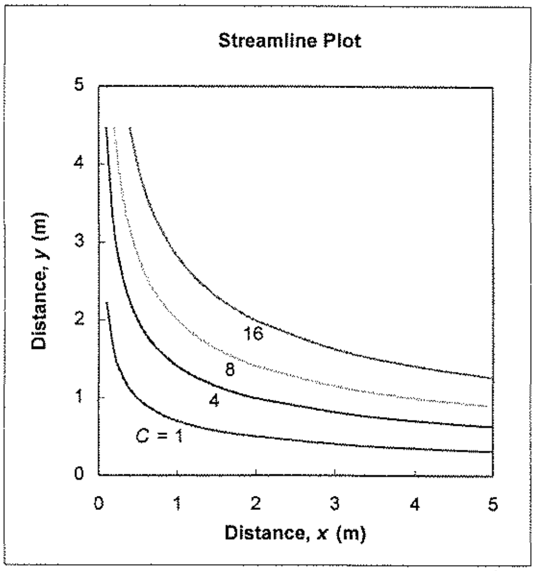
$\therefore \frac{df}{dx} = -\frac{1}{2}Ay^2 - By^2 = -2y^2 + 2y^2 = 0$

Hence f = constant.

Taking f=0 gives

$\psi = \frac{1}{2}Axy^2 = 2xy^2$

ψ



Problem *5.77

[2]

Given: Flow field represented by $\psi = Axy + Ay^2$; $A = 1\,s^{-1}$

Find: (a) Show that this represents a possible incompressible flow field.
(b) Evaluate the rotation of the flow.
(c) Plot a few streamlines in the upper half plane.

Solution: For incompressible flow, $\nabla \cdot \vec{V} = 0$

The velocity field is determined from the stream function

$$\left.\begin{aligned} u &= \partial\psi/\partial y = Ax + 2Ay \\ v &= -\partial\psi/\partial x = -Ay \end{aligned}\right\} \quad \vec{V} = A\{(x+2y)\hat{i} - y\hat{j}\}$$

Then

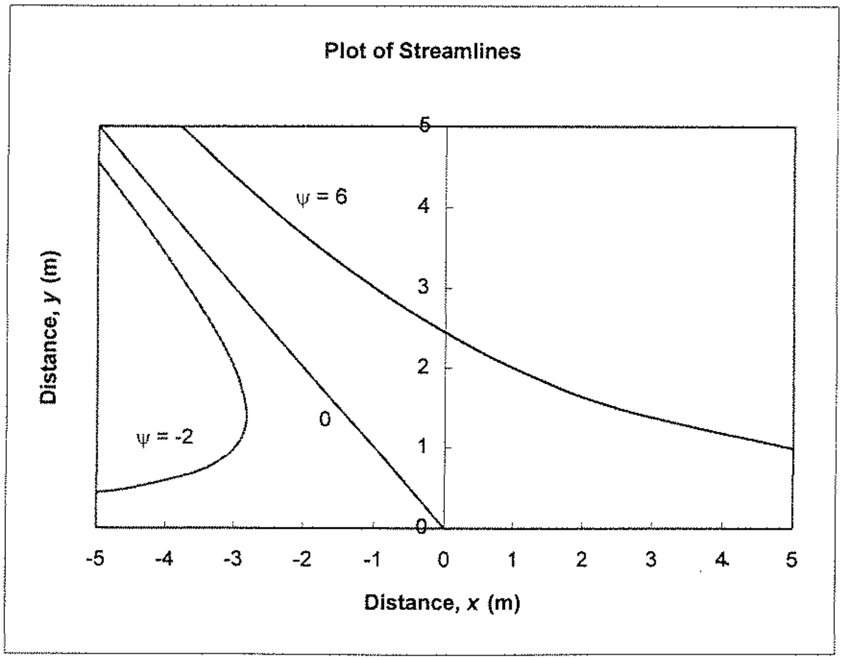
$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x}A(x+2y) - \frac{\partial}{\partial y}(Ay) = A - A = 0 \longleftarrow \text{Q.E.D.}$$

The rotation is given by $\vec{\omega} = \frac{1}{2}\nabla \times \vec{V} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$

$$\vec{\omega} = \frac{1}{2}\left[\frac{\partial}{\partial x}(-Ay) - \frac{\partial}{\partial y}A(x+2y)\right]\hat{k} = \frac{1}{2}\left[0 - 2A\right]\hat{k} = -A\hat{k}$$

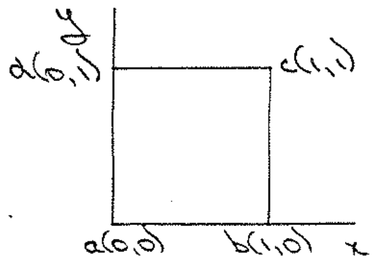
$$\vec{\omega} = -\hat{k}\ \text{rad/s} \longleftarrow \vec{\omega}$$

To plot a few streamlines, $\psi = Axy + Ay^2$, note that for a given streamline $x = \frac{\psi}{y} - y$.



Problem *5.78

Given: Velocity field, $\vec{V} = (Ay+B)\hat{i} + Ax\hat{j}$, where $A = 6\,s^{-1}$, $B = 3\,m/s$ and coordinates are in meters.



Find: (a) An expression for the stream function.
(b) Circulation about "curve" shown.

Plot: several streamlines (including stagnation streamline) in the first quadrant.

Solution

For incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}(Ay+B) + \frac{\partial}{\partial y}(Ax) = 0+0 = 0 \quad \therefore \text{incompressible.}$$

Then

$$u = Ay + B = \frac{\partial \psi}{\partial y} \quad \text{and } \psi = \int (Ay+B)\,dy + f(x) = \frac{1}{2}Ay^2 + By + f(x)$$

and

$$v = -\frac{\partial \psi}{\partial x} = -\frac{df}{dx} = Ax \quad \text{and } f(x) = -\frac{1}{2}Ax^2 + \cancel{\text{constant}} \xleftarrow{} \text{set} = 0$$

$$\therefore \psi = \frac{1}{2}A(y^2 - x^2) + By \quad \xleftarrow{\hspace{4cm}} \psi$$

Several streamlines are plotted below. The stagnation point (where $\vec{V} = 0$) is at $x = 0$, $y = -B/A = -0.5\,m$.

The circulation is defined as $\Gamma = \oint \vec{V} \cdot d\vec{s}$

For the contour shown with $\vec{V} = (Ay+B)\hat{i} + Ax\hat{j}$

$$\Gamma = \int_a^b u\,dx + \int_b^c v\,dy + \int_c^d u\,dx + \cancel{\int_d^a v\,dy} = 0$$

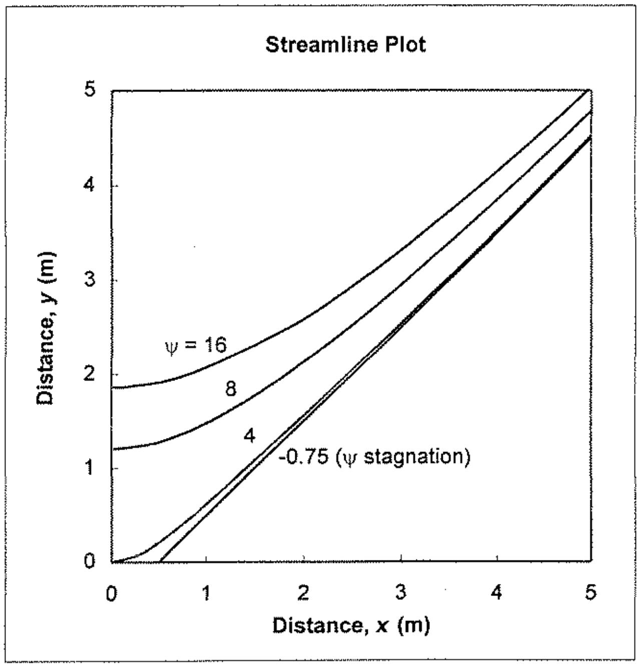
$$\Gamma = \int_0^1 B\,dx + \int_0^1 A\,dy + \int_1^0 (A+B)\,dx$$

$$\begin{Bmatrix} x = 1 \text{ from } b \text{ to } c \\ y = 1 \quad \text{"} \quad c \text{ to } d \end{Bmatrix}$$

$$\Gamma = Bx\Big]_0^1 + Ay\Big]_0^1 + (A+B)\Big]_1^0$$

$$\Gamma = B + A - (A+B)$$

$$\Gamma = 0 \quad \xleftarrow{\hspace{4cm}} \Gamma$$



Note: The flow is irrotational, i.e. $\vec{\omega} = \frac{1}{2}\nabla \times \vec{V} = 0$ and hence we would expect $\Gamma = 0$

At stagnation, $\psi(x,y) = \psi(0,-0.5)$

$$\psi(x,y) = 3[(-0.5)^2 - 0] + 3(-0.5) = -3/4$$

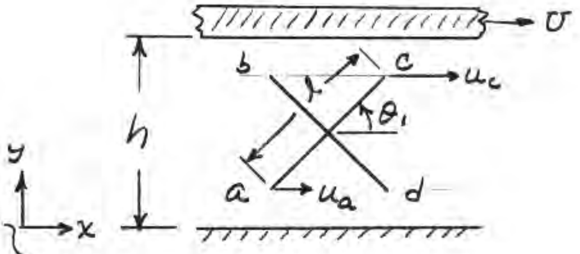
Problem 5.79

[3]

Given: Viscometric flow of Example Problem 5.7, $\vec{V} = U(y/h)\hat{i}$, where $U = 4$ mm/s and $h = 4$ mm.

Find: (a) Average rate of rotation of two line segments at ± 45°
(b) Show that this is the same as in the Example.

Solution: Consider lines shown:

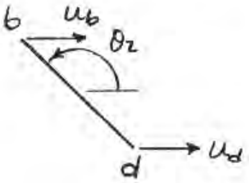


$$u_c = u_a + \frac{\partial u}{\partial y}(l\sin\theta_1)$$

$$-\omega_{ac} = \frac{(u_c - u_a)\sin\theta_1}{l} \quad \left\{\begin{array}{l}\text{Component } \perp \\ \text{to } l \text{ is } u\sin\theta_1.\end{array}\right\}$$

$$-\omega_{ac} = \frac{\frac{\partial u}{\partial y}(l\sin\theta_1)\sin\theta_1}{l} = \frac{\partial u}{\partial y}\sin^2\theta_1 = \frac{U}{h}\sin^2\theta_1$$

Sketch showing θ_2:



$$u_b = u_d + \frac{\partial u}{\partial y}(l\sin\theta_2)$$

$$-\omega_{bd} = \frac{(u_b - u_d)\sin\theta_2}{l} \quad \left\{\begin{array}{l}\text{Component } \perp \\ \text{to } l \text{ is } u\sin\theta_2.\end{array}\right\}$$

$$-\omega_{bd} = \frac{\frac{\partial u}{\partial y}(l\sin\theta_2)\sin\theta_2}{l} = \frac{\partial u}{\partial y}\sin^2\theta_2 = \frac{U}{h}\sin^2\theta_2$$

$$\omega\,(+\circlearrowleft) = \frac{1}{2}(\omega_{ac} + \omega_{bd}) = -\frac{1}{2}\frac{U}{h}(\sin^2\theta_1 + \sin^2\theta_2) = -\frac{1}{2}\frac{U}{h}(\sin^2 45° + \sin^2 135°)$$

$$= -\frac{1}{2}\frac{U}{h}\left[\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2\right] = -\frac{1}{2}\frac{U}{h} \longleftarrow$$ ω

$$\omega = -\frac{1}{2} \times \frac{4\,\text{mm}}{\text{sec}} \times \frac{1}{4\,\text{mm}} = -0.5\ \text{s}^{-1} \longleftarrow$$ ω

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Problem *5.80

[3]

Given: Velocity field $\vec{V} = -\frac{q}{2\pi r}\hat{e}_r + \frac{K}{2\pi r}\hat{e}_\theta$ approximates a tornado.

Is it irrotational? Obtain the stream function.

Solution: Apply irrotationality condition.

Basic equation: $\nabla \times \vec{V} = 0$ (if irrotational)

It makes sense to work in cylindrical coordinates, where

$$\nabla = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + \hat{k}\frac{\partial}{\partial z}$$

But flow is in the $r\theta$ plane, so $\frac{\partial}{\partial z} = 0$. Then

$$\nabla \times \vec{V} = \left(\hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\frac{\partial}{\partial \theta}\right) \times \left(V_r \hat{e}_r + V_\theta \hat{e}_\theta\right)$$

$$= \hat{e}_r \times \left(\frac{\partial V_r}{\partial r}\hat{e}_r + \frac{\partial V_\theta}{\partial r}\hat{e}_\theta\right)$$

$$+ \hat{e}_\theta \frac{1}{r} \times \left(\frac{\partial V_r}{\partial \theta}\hat{e}_r + V_r \overset{\hat{e}_\theta}{\cancel{\frac{\partial \hat{e}_r}{\partial \theta}}} + \frac{\partial V_\theta}{\partial \theta}\hat{e}_\theta + V_\theta \overset{-\hat{e}_r}{\cancel{\frac{\partial \hat{e}_\theta}{\partial \theta}}}\right)$$

$$\nabla \times \vec{V} = \hat{k}\left(\frac{\partial V_\theta}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta} + \frac{V_\theta}{r}\right) = \hat{k}\frac{1}{r}\left(\frac{\partial r V_\theta}{\partial r} - \frac{\partial V_r}{\partial \theta}\right)$$

For the given flow field, $\vec{V} = \vec{V}(r)$, so

$$\nabla \times \vec{V} = \hat{k}\,\frac{1}{r}\frac{\partial r V_\theta}{\partial r} = \hat{k}\,\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{K}{2\pi}\right) \equiv 0$$

Flow is irrotational.

$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$; $\frac{1}{r}\frac{\partial \psi}{\partial \theta} = -\frac{q}{2\pi r}$; $\frac{\partial \psi}{\partial \theta} = \frac{-q}{2\pi}$; $\psi = \frac{-q}{2\pi}\theta + f(r)$

$V_\theta = -\frac{\partial \psi}{\partial r}$; $\frac{\partial \psi}{\partial r} = -\frac{K}{2\pi r}$; $\psi = -\frac{K}{2\pi}\ln r + g(\theta)$

Comparing,

$$\psi = \frac{-q}{2\pi}\theta - \frac{K}{2\pi}\ln r$$

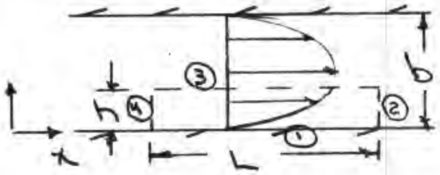
ψ

Problem 5.81

[3]

Given: Flow between parallel plates. Velocity field given by

$$u = U\left(\frac{y}{b}\right)\left[1 - \frac{y}{b}\right]$$



Find: (a) expression for circulation about a closed contour of height h and length L

(b) evaluate for h = b/2 and h = b.

(c) show that same result is obtained from area integral of Stokes Theorem (Eq. 5.18).

Solution:

Basic equations : $\Gamma = \oint \vec{V}\cdot d\vec{s} = \int_A (\nabla\times\vec{V})_z \, dA$

Then, $\Gamma = \cancelto{0}{\int_1 \vec{V}\cdot d\vec{s}} + \cancelto{0}{\int_2 \vec{V}\cdot d\vec{s}} + \int_3 \vec{V}\cdot d\vec{s} + \cancelto{0}{\int_4 \vec{V}\cdot d\vec{s}}$

$$= \int_L^0 U \frac{y}{b}\left(1 - \frac{y}{b}\right) dx$$

$$\Gamma = -UL\,\frac{h}{b}\left(1 - \frac{h}{b}\right) \longleftarrow \Gamma$$

For $h = y = \frac{b}{2}$, $\Gamma = -\frac{UL}{4}$

$h = y = b$, $\Gamma = 0$

From Stokes Theorem,

$$\Gamma = \int_A (\vec{\nabla}\times\vec{V})_z \, dA = \int_A \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dA = \int_A -U\left(\frac{1}{b} - \frac{2y}{b^2}\right) dA$$

$$\Gamma = -U\int\left(\frac{1}{b} - \frac{2y}{b^2}\right) L\,dy = -UL\left[\frac{y}{b} - \frac{y^2}{b^2}\right]_0^h$$

$$\Gamma = -UL\left[\frac{h}{b} - \frac{h^2}{b^2}\right] = -UL\,\frac{h}{b}\left(1 - \frac{h}{b}\right) \longleftarrow \Gamma$$

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Problem 5.82

[2]

Given: Velocity profile for fully developed flow in a circular tube is

$$V_z = V_{max}\left[1-\left(r/R\right)^2\right]$$

Find: (a) rates of linear and angular deformation for this flow.

(b) expression for the vorticit vector, $\vec{\zeta}$

Solution:

Computing equations: B.1 and B.2 of Appendix B

Volume dilation rate $= \nabla\cdot\vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial\theta} + \frac{\partial}{\partial z}V_z = 0$

Rates of linear deformation in each of the three coordinate directions r, θ, z are zero $\longleftarrow$ Linear Def

Angular deformation in the:

$r\theta$ plane is $r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right) + \frac{1}{r}\frac{\partial V_r}{\partial\theta} = 0$

θz plane is $\frac{\partial V_\theta}{\partial z} + \frac{1}{r}\frac{\partial V_z}{\partial\theta} = 0$

zr plane is $\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} = -V_{max}\frac{2r}{R^2}$ $\longleftarrow$ Angular Def.

The vorticity vector is given by $\vec{\zeta} = \nabla\times\vec{V}$

In cylindrical coordinates.

$$\nabla\times\vec{V} = \hat{e}_r\left(\cancel{\frac{1}{r}\frac{\partial V_z}{\partial\theta}} - \cancel{\frac{\partial V_\theta}{\partial z}}\right) + \hat{e}_\theta\left(\cancel{\frac{\partial V_r}{\partial z}} - \frac{\partial V_z}{\partial r}\right) + \hat{k}\left(\cancel{\frac{1}{r}\frac{\partial rV_\theta}{\partial r}} - \cancel{\frac{1}{r}\frac{\partial V_r}{\partial\theta}}\right) \quad (5.16)$$

$\vec{\zeta} = \nabla\times\vec{V} = \hat{e}_\theta\, V_{max}\frac{2r}{R^2}$ $\longleftarrow$ $\vec{\zeta}$

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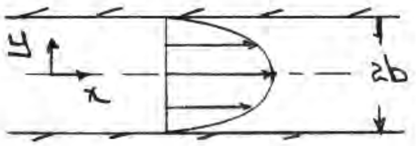
Problem 5.83

[3]

Given: Flow between parallel plates. Velocity field given by

$$u = U_{max}\left[1 - \left(\frac{y}{b}\right)^2\right]$$



Find: (a) rates of linear and angular deformation
(b) expression for the vorticity vector, $\vec{\zeta}$
(c) location of maximum vorticity

Solution:

The rate of linear deformation is zero since $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\partial w}{\partial z} = 0$

The rate of angular deformation in the xy plane is

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = -\frac{2y\,U_{max}}{b^2} \longleftarrow$$

The vorticity vector is given by $\vec{\zeta} = \nabla \times \vec{V}$

$$\vec{\zeta} = \hat{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \hat{j}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\vec{\zeta} = -\frac{\partial u}{\partial y}\hat{k} = \frac{2y\,U_{max}}{b^2}\hat{k} \longleftarrow \vec{\zeta}$$

The vorticity is a maximum at $y = \pm b$

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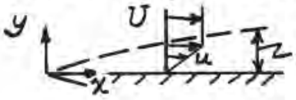
Problem 5.84

[3]

Given: Linear approximate velocity profile in boundary layer.



$\delta = cx^{1/2}$; $u = U\frac{y}{\delta} = \frac{Uy}{cx^{1/2}}$; $v = \frac{uy}{4x} = \frac{Uy^2}{4cx^{3/2}}$

Find: (a) Express rotation, find maximum.
(b) Express angular deformation, locate maximum.
(c) Express linear deformation, locate maximum.
(d) Express shear force per unit volume, locate maximum.

Solution: Work in xy plane.

Computing equations: $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ $\quad -\frac{d\gamma}{dt} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

Linear def: $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$

Evaluating partial derivatives,

$$\frac{\partial u}{\partial x} = -\frac{1}{2}\frac{Uy}{cx^{3/2}} \qquad \frac{\partial u}{\partial y} = \frac{U}{cx^{1/2}} \qquad \frac{\partial v}{\partial x} = -\frac{3}{8}\frac{Uy^2}{cx^{5/2}} \qquad \frac{\partial v}{\partial y} = \frac{1}{2}\frac{Uy}{cx^{3/2}}$$

Then

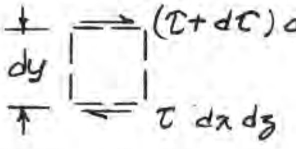
$$\omega_z = \frac{1}{2}\left[-\frac{3}{8}\frac{Uy^2}{cx^{5/2}} - \frac{U}{cx^{1/2}}\right] = -\frac{U}{2cx^{1/2}}\left[1 + \frac{3}{8}\left(\frac{y}{x}\right)^2\right] \quad (\text{max at } y = \delta) \quad \xleftarrow{\qquad} \omega_z$$

$$-\frac{d\gamma}{dt} = -\frac{3}{8}\frac{Uy^2}{cx^{5/2}} + \frac{U}{cx^{1/2}} = \frac{U}{cx^{1/2}}\left[1 - \frac{3}{8}\left(\frac{y}{x}\right)^2\right] \quad (\text{max at } y = 0) \quad \xleftarrow{\qquad} -\frac{d\gamma}{dt}$$

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= -\frac{1}{2}\frac{Uy}{cx^{3/2}} = -\frac{U}{2cx^{1/2}}\left(\frac{y}{x}\right) \quad (\text{max at } y = \delta) \\ \frac{\partial v}{\partial y} &= +\frac{1}{2}\frac{Uy}{cx^{3/2}} = +\frac{U}{2cx^{1/2}}\left(\frac{y}{x}\right) \quad (\text{max at } y = \delta) \end{aligned}\right\} \text{sum} = 0 \quad \xleftarrow{\qquad} \text{Lin Def}$$

Shear stress is $\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left(\frac{U}{cx^{1/2}} - \frac{3}{8}\frac{Uy^2}{cx^{5/2}}\right) = \frac{\mu U}{cx^{1/2}}\left[1 - \frac{3}{8}\left(\frac{y}{x}\right)^2\right]$

Net shear force on a fluid element is $d\tau\, dx dz$



$$d\tau = \frac{\partial\tau}{\partial y}dy = \frac{\mu U}{cx^{1/2}}\left(-\frac{3}{8}\frac{2y}{x^2}\right)dy = -\frac{3\mu Uy}{4cx^{5/2}}dy$$

Shear stress per volume is $\frac{dF}{d\forall} = -\frac{3\mu U}{4cx^{3/2}}\left(\frac{y}{x}\right)$ (max at $y = \delta$) $\xleftarrow{\qquad}$ $\frac{dF}{d\forall}$

Problem 5.85

[2]

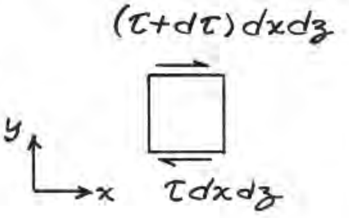
Given: x component of velocity in laminar boundary layer in water

$$u = U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) \qquad U = 3\ m/s, \quad \delta = 2\ mm$$

y component is much smaller than u.

Find: (a) Expression for net shear force per unit volume in x direction.
(b) Maximum value for this flow

Solution: Consider a small element of fluid



Then

$$dF_{shear,x} = (\tau+d\tau)dxdz - \tau dxdz$$

$$= d\tau\, dxdz = \frac{d\tau}{dy}dx\,dy\,dz$$

and

$$\frac{dF_{s,x}}{d\forall} = \frac{d\tau}{dy} = \frac{d}{dy}\left(\mu\frac{du}{dy}\right) = \mu\frac{d^2u}{dy^2}$$

From the given profile,

$$\frac{du}{dy} = \frac{\pi U}{2\delta}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

and

$$\frac{d^2u}{dy^2} = U\left(\frac{\pi}{2\delta}\right)^2\left(-\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right)$$

The maximum value occurs when $y=\delta$, when

$$\frac{dF_{sx,max}}{d\forall} = -\mu U\left(\frac{\pi}{2\delta}\right)^2 \longleftarrow \frac{dF_{sx}}{d\forall}$$

$$= -1\times10^{-3}\frac{N\cdot sec}{m^2}\times 3\frac{m}{sec}\left(\frac{\pi}{2}\frac{1}{0.002\ m}\right)^2 = -1.85\times10^{3}N/m^3$$

$$\frac{dF_{sx,max}}{d\forall} = -1.85\ kN/m^3 \longleftarrow \frac{dF_{sx}}{d\forall}$$

Problem 5.86

[2]

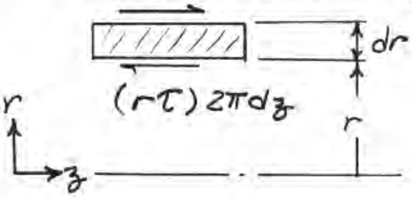
Given: Velocity profile for fully developed laminar flow in a tube

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2$$

where $u_{max} = 10$ ft/s, $R = 3$ in., fluid is water.

Find: (a) Expression for shear force per unit volume in z direction.
(b) Maximum value for these conditions.

Solution: Consider a differential element: $\left[r\tau + \frac{d}{dr}(r\tau)dr\right]2\pi dz$



Then

$$dF_{shear,z} = \left[r\tau + \frac{d}{dr}(r\tau)dr\right]2\pi dz - r\tau\, 2\pi dz$$

$$= \frac{d}{dr}(r\tau)\, 2\pi\, dr\, dz$$

Since $d\forall = 2\pi r\, dr\, dz$, then

$$\frac{dF_{s_z}}{d\forall} = \frac{1}{2\pi r\, dr\, dz}\frac{d}{dr}(r\tau)\, 2\pi\, dr\, dz = \frac{1}{r}\frac{d}{dr}(r\tau)$$

In cylindrical coordinates, $\tau_{rz} = \mu\frac{du}{dr}$. For the given profile

$$\tau = \tau_{rz} = \mu\frac{du}{dr} = -\mu u_{max}\frac{2r}{R^2}$$

Substituting

$$\frac{dF_{s_z}}{d\forall} = \frac{1}{r}\frac{d}{dr}\left[r\left(-\frac{2\mu u_{max} r}{R^2}\right)\right] = \frac{1}{r}\frac{d}{dr}\left[-\frac{2\mu u_{max} r^2}{R^2}\right] = \frac{1}{r}\left[\frac{-4\mu u_{max} r}{R^2}\right]$$

$$\frac{dF_{s_z}}{d\forall} = -\frac{4\mu u_{max}}{R^2} = \text{constant} \longleftarrow \frac{dF_{s_z}}{d\forall}$$

Evaluating,

$$\frac{dF_{s_z}}{d\forall} = -4\times 10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times 10\,\frac{\text{ft}}{\text{s}}\times\frac{1}{(0.25)^2\,\text{ft}^2}\times(0.3048)^2\,\frac{\text{m}^2}{\text{ft}^2}\times\frac{\text{lbf}}{4.448\,\text{N}}$$

$$\frac{dF_{s_z}}{d\forall} = -0.0134\ \text{lbf}/\text{ft}^3 \longleftarrow$$

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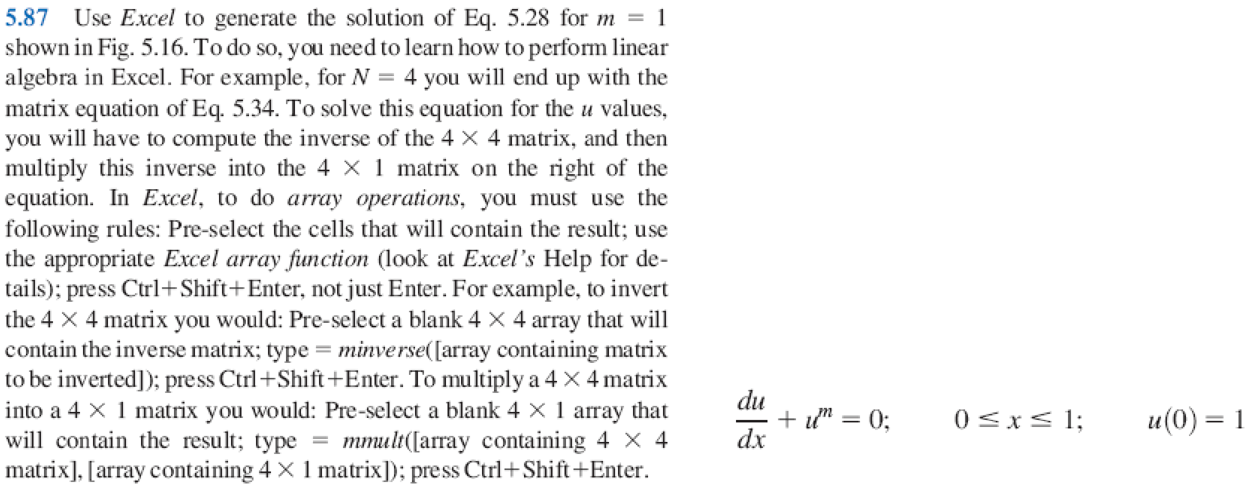
Problem 5.87



5.87 Use *Excel* to generate the solution of Eq. 5.28 for $m = 1$ shown in Fig. 5.16. To do so, you need to learn how to perform linear algebra in Excel. For example, for $N = 4$ you will end up with the matrix equation of Eq. 5.34. To solve this equation for the u values, you will have to compute the inverse of the 4 × 4 matrix, and then multiply this inverse into the 4 × 1 matrix on the right of the equation. In *Excel*, to do *array operations*, you must use the following rules: Pre-select the cells that will contain the result; use the appropriate *Excel array function* (look at *Excel's* Help for details); press Ctrl+Shift+Enter, not just Enter. For example, to invert the 4 × 4 matrix you would: Pre-select a blank 4 × 4 array that will contain the inverse matrix; type = *minverse*([array containing matrix to be inverted]); press Ctrl+Shift+Enter. To multiply a 4 × 4 matrix into a 4 × 1 matrix you would: Pre-select a blank 4 × 1 array that will contain the result; type = *mmult*([array containing 4 × 4 matrix], [array containing 4 × 1 matrix]); press Ctrl+Shift+Enter.

$$\frac{du}{dx} + u^m = 0; \qquad 0 \le x \le 1; \qquad u(0) = 1$$

N = **4**
Δx = **0.333**

Eq. 5.34 (LHS)				**(RHS)**
1.000	0.000	0.000	0.000	1
-1.000	1.333	0.000	0.000	0
0.000	-1.000	1.333	0.000	0
0.000	0.000	-1.000	1.333	0

x	**Inverse Matrix**				**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	1.000	1.000	0.000
0.333	0.750	0.750	0.000	0.000	0.750	0.717	0.000
0.667	0.563	0.563	0.750	0.000	0.563	0.513	0.001
1.000	0.422	0.422	0.563	0.750	0.422	0.368	0.001
							0.040

N = **8**
Δx = **0.143**

Eq. 5.34 (LHS)								**(RHS)**
1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
-1.000	1.143	0.000	0.000	0.000	0.000	0.000	0.000	0
0.000	-1.000	1.143	0.000	0.000	0.000	0.000	0.000	0
0.000	0.000	-1.000	1.143	0.000	0.000	0.000	0.000	0
0.000	0.000	0.000	-1.000	1.143	0.000	0.000	0.000	0
0.000	0.000	0.000	0.000	-1.000	1.143	0.000	0.000	0
0.000	0.000	0.000	0.000	0.000	-1.000	1.143	0.000	0
0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.143	0

	Inverse Matrix										
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
0.143	0.875	0.875	0.000	0.000	0.000	0.000	0.000	0.000	0.875	0.867	0.000
0.286	0.766	0.766	0.875	0.000	0.000	0.000	0.000	0.000	0.766	0.751	0.000
0.429	0.670	0.670	0.766	0.875	0.000	0.000	0.000	0.000	0.670	0.651	0.000
0.571	0.586	0.586	0.670	0.766	0.875	0.000	0.000	0.000	0.586	0.565	0.000
0.714	0.513	0.513	0.586	0.670	0.766	0.875	0.000	0.000	0.513	0.490	0.000
0.857	0.449	0.449	0.513	0.586	0.670	0.766	0.875	0.000	0.449	0.424	0.000
1.000	0.393	0.393	0.449	0.513	0.586	0.670	0.766	0.875	0.393	0.368	0.000
											0.019

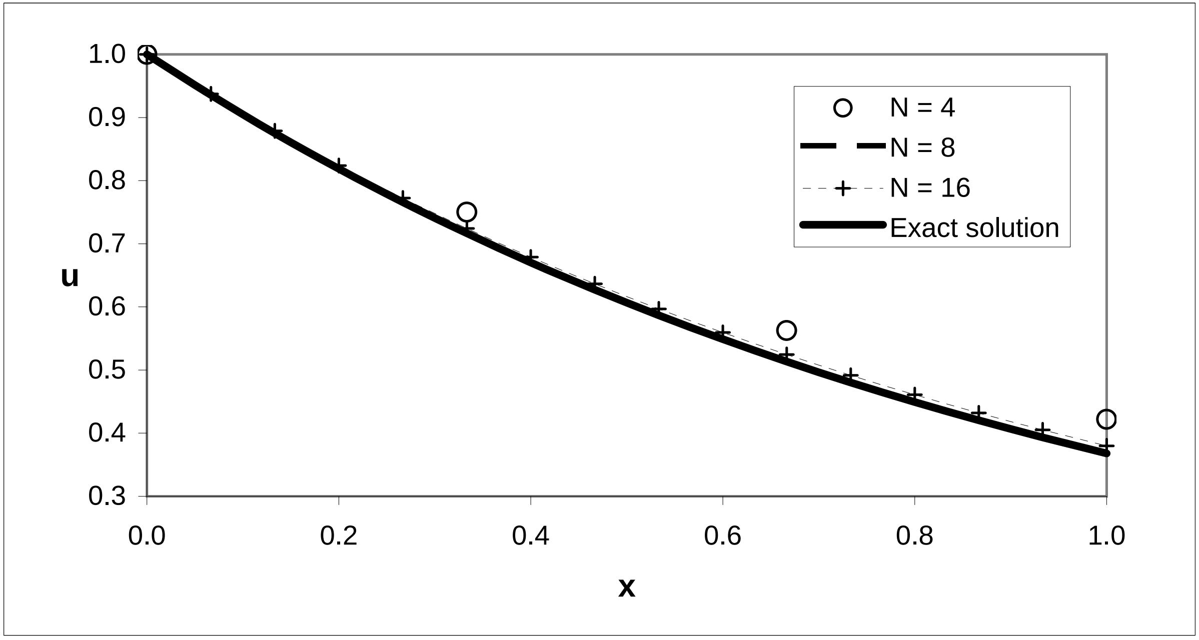
N = **16**

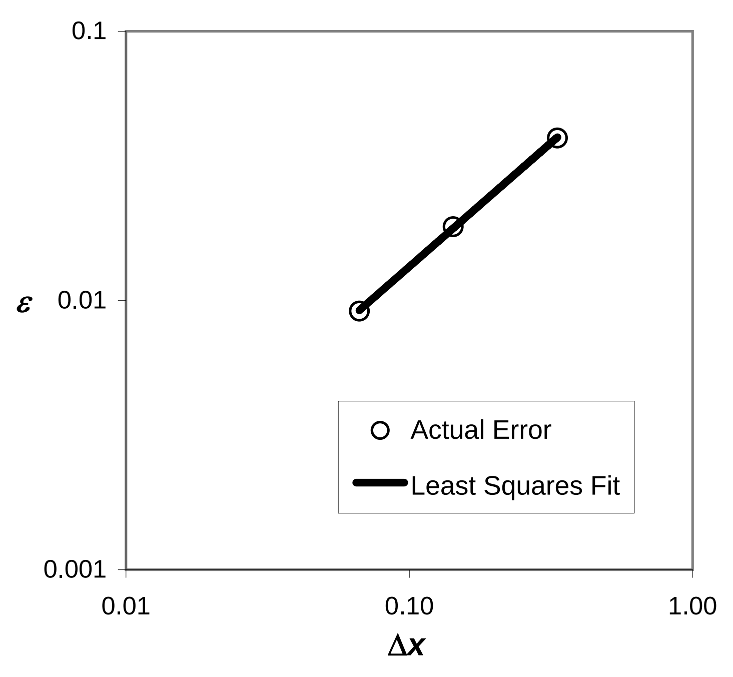
Δ*x* = **0.067** **Eq. 5.34 (LHS)**

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**(RHS)**
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
2	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
3	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
4	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
5	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
6	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
7	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
8	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0
13	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0

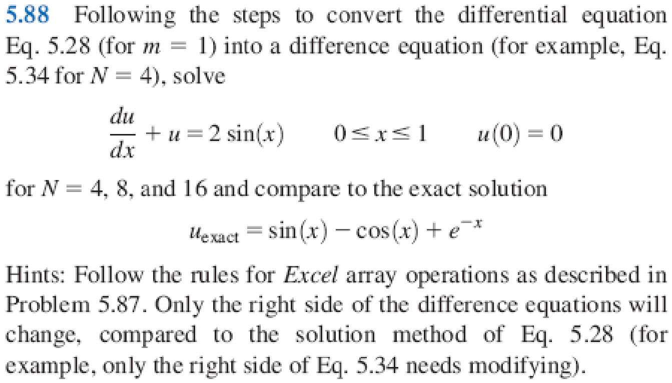
x	**Inverse Matrix**																**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
0.067	0.938	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.938	0.936	0.000
0.133	0.879	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.879	0.875	0.000
0.200	0.824	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.824	0.819	0.000
0.267	0.772	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.772	0.766	0.000
0.333	0.724	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.724	0.717	0.000
0.400	0.679	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.679	0.670	0.000
0.467	0.637	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.637	0.627	0.000
0.533	0.597	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.597	0.587	0.000
0.600	0.559	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.559	0.549	0.000
0.667	0.524	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.524	0.513	0.000
0.733	0.492	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.492	0.480	0.000
0.800	0.461	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.461	0.449	0.000
0.867	0.432	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.432	0.420	0.000
0.933	0.405	0.405	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.405	0.393	0.000
1.000	0.380	0.380	0.405	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.380	0.368	0.000
																			0.009

N	**Δ*x***	**Error**
4	0.333	0.040
8	0.143	0.019
16	0.067	0.009


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
u
0.0
0.2
0.4
0.6
0.8
1.0
x
N = 4
N = 8
N = 16
Exact solution


0.1
0.01
0.001
ε
0.01
0.10
1.00
Δx
Actual Error
Least Squares Fit

Problem 5.88



5.88 Following the steps to convert the differential equation Eq. 5.28 (for $m = 1$) into a difference equation (for example, Eq. 5.34 for $N = 4$), solve

$$\frac{du}{dx} + u = 2\sin(x) \qquad 0 \le x \le 1 \qquad u(0) = 0$$

for $N = 4$, 8, and 16 and compare to the exact solution

$$u_{\text{exact}} = \sin(x) - \cos(x) + e^{-x}$$

Hints: Follow the rules for *Excel* array operations as described in Problem 5.87. Only the right side of the difference equations will change, compared to the solution method of Eq. 5.28 (for example, only the right side of Eq. 5.34 needs modifying).

New Eq. 5.34: $$-u_{i-1} + (1+\Delta x)u_i = 2\Delta x \cdot \sin(x_i)$$

N = **4**

Δx = **0.333**

Eq. 5.34 (LHS)				**(RHS)**
1.000	0.000	0.000	0.000	0
-1.000	1.333	0.000	0.000	0.21813
0.000	-1.000	1.333	0.000	0.41225
0.000	0.000	-1.000	1.333	0.56098

x	**Inverse Matrix**				**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.333	0.750	0.750	0.000	0.000	0.164	0.099	0.001
0.667	0.563	0.563	0.750	0.000	0.432	0.346	0.002
1.000	0.422	0.422	0.563	0.750	0.745	0.669	0.001
							0.066

N = **8**

Δx = **0.143**

Eq. 5.34 (LHS)								**(RHS)**
1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
-1.000	1.143	0.000	0.000	0.000	0.000	0.000	0.000	0.04068
0.000	-1.000	1.143	0.000	0.000	0.000	0.000	0.000	0.08053
0.000	0.000	-1.000	1.143	0.000	0.000	0.000	0.000	0.11873
0.000	0.000	0.000	-1.000	1.143	0.000	0.000	0.000	0.15452
0.000	0.000	0.000	0.000	-1.000	1.143	0.000	0.000	0.18717
0.000	0.000	0.000	0.000	0.000	-1.000	1.143	0.000	0.21599
0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.143	0.24042

	Inverse Matrix									
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.143	0.875	0.875	0.000	0.000	0.000	0.000	0.000	0.000	0.036	0.019	0.000
0.286	0.766	0.766	0.875	0.000	0.000	0.000	0.000	0.000	0.102	0.074	0.000
0.429	0.670	0.670	0.766	0.875	0.000	0.000	0.000	0.000	0.193	0.157	0.000
0.571	0.586	0.586	0.670	0.766	0.875	0.000	0.000	0.000	0.304	0.264	0.000
0.714	0.513	0.513	0.586	0.670	0.766	0.875	0.000	0.000	0.430	0.389	0.000
0.857	0.449	0.449	0.513	0.586	0.670	0.766	0.875	0.000	0.565	0.526	0.000
1.000	0.393	0.393	0.449	0.513	0.586	0.670	0.766	0.875	0.705	0.669	0.000
											0.032

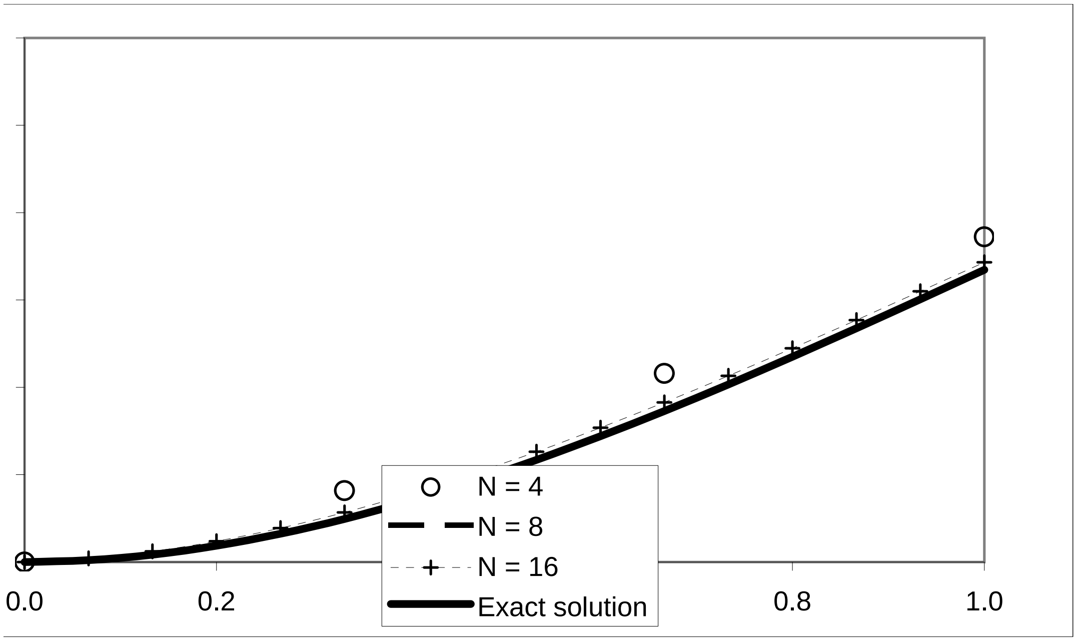
N = **16**

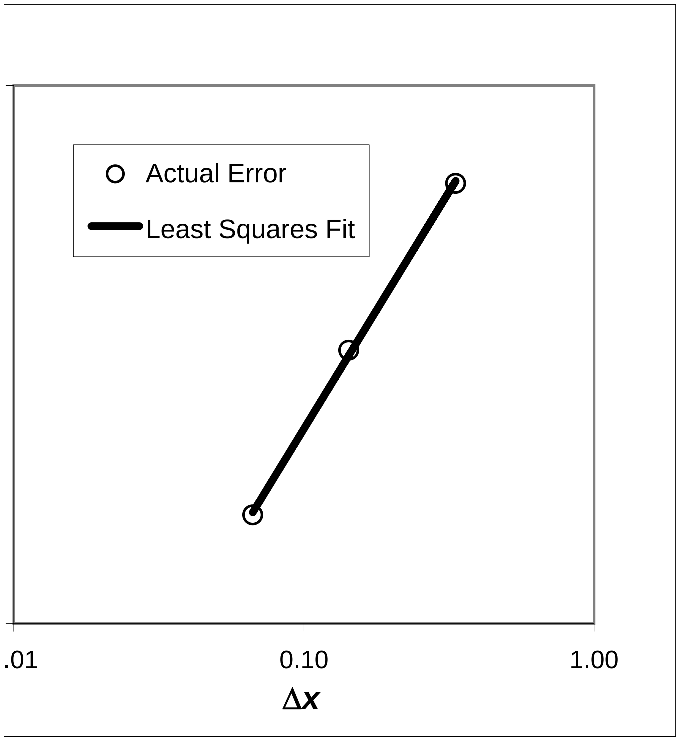
Δ*x* = **0.067** **Eq. 5.34 (LHS)**

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**(RHS)**
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
2	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00888
3	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.01773
4	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.02649
5	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.03514
6	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.04363
7	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.05192
8	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.05999
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.06779
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.07529
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.08245
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.08925
13	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.09565
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.10162
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.10715
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.1122

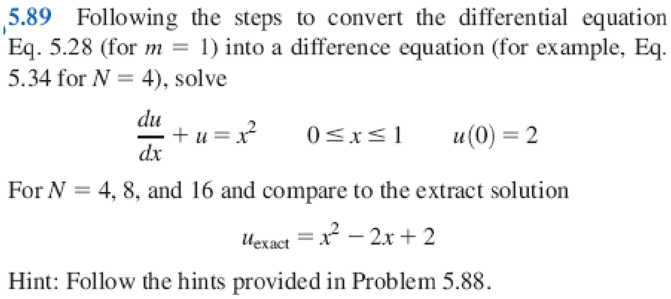
x	**Inverse Matrix**																**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.067	0.938	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.004	0.000
0.133	0.879	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.017	0.000
0.200	0.824	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.048	0.037	0.000
0.267	0.772	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.078	0.065	0.000
0.333	0.724	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.114	0.099	0.000
0.400	0.679	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.155	0.139	0.000
0.467	0.637	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.202	0.184	0.000
0.533	0.597	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.253	0.234	0.000
0.600	0.559	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.308	0.288	0.000
0.667	0.524	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.366	0.346	0.000
0.733	0.492	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.426	0.407	0.000
0.800	0.461	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.489	0.470	0.000
0.867	0.432	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.554	0.535	0.000
0.933	0.405	0.405	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.620	0.602	0.000
1.000	0.380	0.380	0.405	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.686	0.669	0.000
																			0.016

N	**Δ*x***	**Error**
4	0.333	0.066
8	0.143	0.032
16	0.067	0.016


0.0
0.2
0.8
1.0
N = 4
N = 8
N = 16
Exact solution


Actual Error
Least Squares Fit
.01
0.10
1.00
Δx

Problem 5.89



5.89 Following the steps to convert the differential equation Eq. 5.28 (for $m = 1$) into a difference equation (for example, Eq. 5.34 for $N = 4$), solve

$$\frac{du}{dx} + u = x^2 \qquad 0 \le x \le 1 \qquad u(0) = 2$$

For $N = 4$, 8, and 16 and compare to the extract solution

$$u_{\text{exact}} = x^2 - 2x + 2$$

Hint: Follow the hints provided in Problem 5.88.

New Eq. 5.34: $$-u_{i-1} + (1 + \Delta x)u_i = \Delta x \cdot x_i^2$$

N **= 4**
Δ*x* = 0.333

Eq. 5.34 (LHS)				**(RHS)**
1.000	0.000	0.000	0.000	2
-1.000	1.333	0.000	0.000	0.03704
0.000	-1.000	1.333	0.000	0.14815
0.000	0.000	-1.000	1.333	0.33333

x	**Inverse Matrix**				**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	2.000	2.000	0.000
0.333	0.750	0.750	0.000	0.000	1.528	1.444	0.002
0.667	0.563	0.563	0.750	0.000	1.257	1.111	0.005
1.000	0.422	0.422	0.563	0.750	1.193	1.000	0.009
							0.128

N **= 8**
Δ*x* = 0.143

Eq. 5.34 (LHS)								**(RHS)**
1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2
-1.000	1.143	0.000	0.000	0.000	0.000	0.000	0.000	0.00292
0.000	-1.000	1.143	0.000	0.000	0.000	0.000	0.000	0.01166
0.000	0.000	-1.000	1.143	0.000	0.000	0.000	0.000	0.02624
0.000	0.000	0.000	-1.000	1.143	0.000	0.000	0.000	0.04665
0.000	0.000	0.000	0.000	-1.000	1.143	0.000	0.000	0.07289
0.000	0.000	0.000	0.000	0.000	-1.000	1.143	0.000	0.10496
0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.143	0.14286

	Inverse Matrix										
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	0.000
0.143	0.875	0.875	0.000	0.000	0.000	0.000	0.000	0.000	1.753	1.735	0.000
0.286	0.766	0.766	0.875	0.000	0.000	0.000	0.000	0.000	1.544	1.510	0.000
0.429	0.670	0.670	0.766	0.875	0.000	0.000	0.000	0.000	1.374	1.327	0.000
0.571	0.586	0.586	0.670	0.766	0.875	0.000	0.000	0.000	1.243	1.184	0.000
0.714	0.513	0.513	0.586	0.670	0.766	0.875	0.000	0.000	1.151	1.082	0.001
0.857	0.449	0.449	0.513	0.586	0.670	0.766	0.875	0.000	1.099	1.020	0.001
1.000	0.393	0.393	0.449	0.513	0.586	0.670	0.766	0.875	1.087	1.000	0.001
											0.057

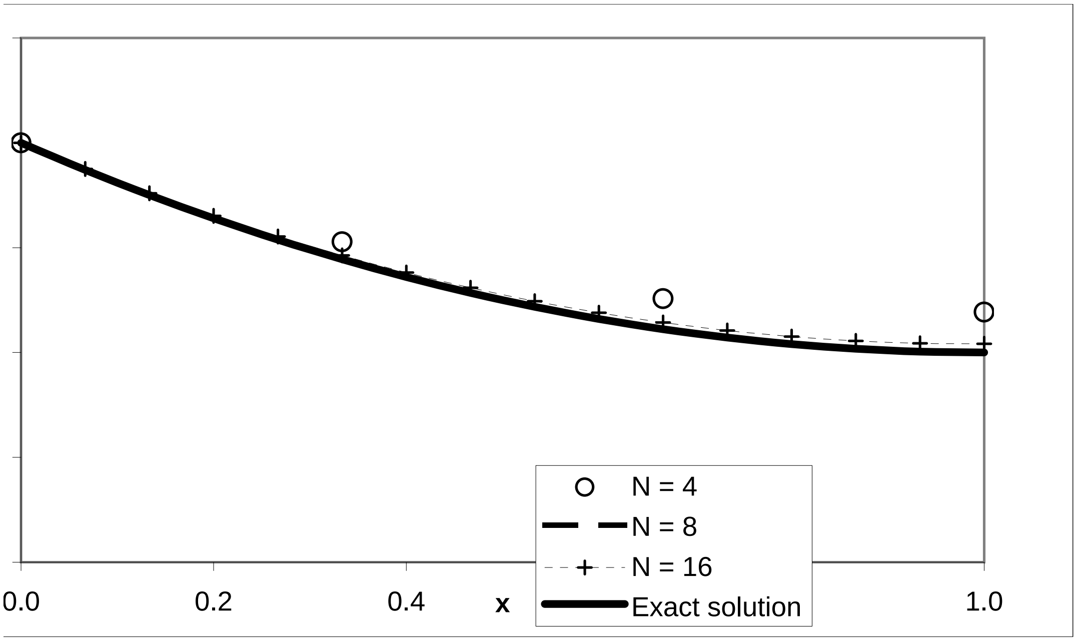
N = **16**

Δ*x* = **0.067** **Eq. 5.34 (LHS)**

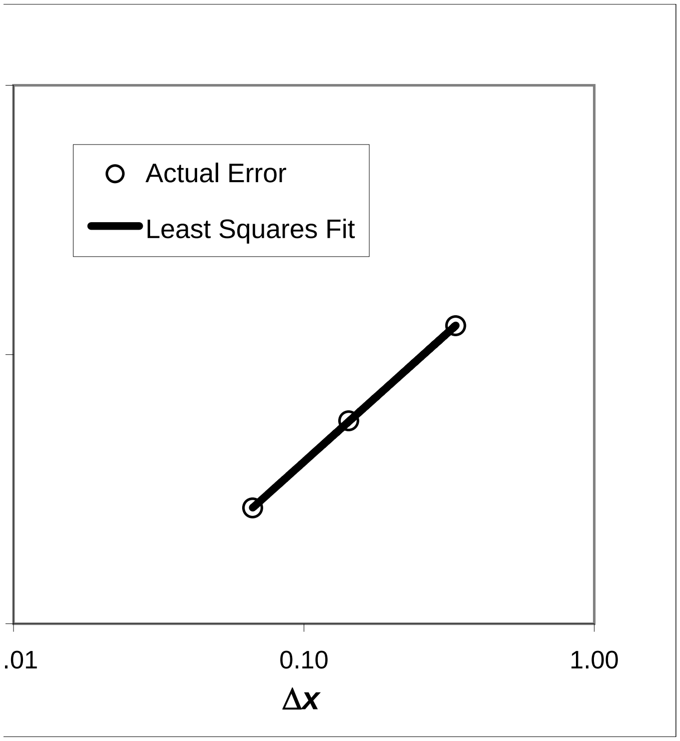
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**(RHS)**
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2
2	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.0003
3	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00119
4	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00267
5	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00474
6	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00741
7	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.01067
8	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.01452
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.01896
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.000	0.024
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.000	0.02963
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.000	0.03585
13	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.000	0.04267
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.000	0.05007
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.000	0.05807
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.067	0.06667

x	**Inverse Matrix**																**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	0.000
0.067	0.938	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.875	1.871	0.000
0.133	0.879	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.759	1.751	0.000
0.200	0.824	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.652	1.640	0.000
0.267	0.772	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.553	1.538	0.000
0.333	0.724	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.463	1.444	0.000
0.400	0.679	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.381	1.360	0.000
0.467	0.637	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.309	1.284	0.000
0.533	0.597	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.245	1.218	0.000
0.600	0.559	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	0.000	1.189	1.160	0.000
0.667	0.524	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	0.000	1.143	1.111	0.000
0.733	0.492	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	0.000	1.105	1.071	0.000
0.800	0.461	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	0.000	1.076	1.040	0.000
0.867	0.432	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	0.000	1.056	1.018	0.000
0.933	0.405	0.405	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	0.000	1.044	1.004	0.000
1.000	0.380	0.380	0.405	0.432	0.461	0.492	0.524	0.559	0.597	0.637	0.679	0.724	0.772	0.824	0.879	0.938	1.041	1.000	0.000
																			0.027

N	**Δ*x***	**Error**
4	0.333	0.128
8	0.143	0.057
16	0.067	0.027

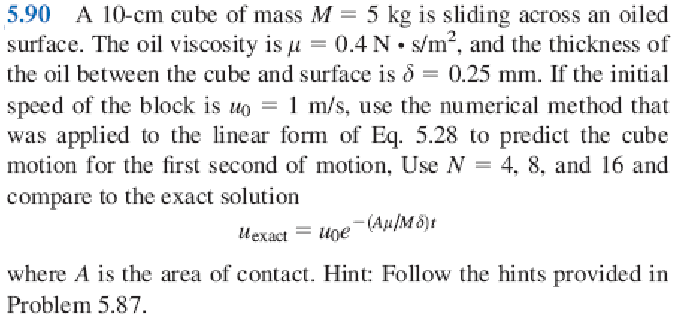


N = 4
N = 8
N = 16
Exact solution
0.0
0.2
0.4
x
1.0



Actual Error
Least Squares Fit
.01
0.10
1.00
Δx

Problem 5.90



5.90 A 10-cm cube of mass $M = 5$ kg is sliding across an oiled surface. The oil viscosity is $\mu = 0.4\ \mathrm{N\cdot s/m^2}$, and the thickness of the oil between the cube and surface is $\delta = 0.25$ mm. If the initial speed of the block is $u_0 = 1$ m/s, use the numerical method that was applied to the linear form of Eq. 5.28 to predict the cube motion for the first second of motion, Use $N = 4$, 8, and 16 and compare to the exact solution

$$u_{\mathrm{exact}} = u_0 e^{-(A\mu/M\delta)t}$$

where A is the area of contact. Hint: Follow the hints provided in Problem 5.87.

Equation of motion: $$M\frac{du}{dt} = -\mu\frac{du}{dy}A = -\mu A\frac{u}{\delta}$$

$$\frac{du}{dt} + \left(\frac{\mu A}{M\,\delta}\right)u = 0$$

$$\frac{du}{dt} + k\cdot u = 0$$

New Eq. 5.34: $$-u_{i-1} + (1 + k\cdot\Delta x)u_i = 0$$

A = 0.01 $\mathrm{m^2}$

δ = 0.25 mm

μ = 0.4 $\mathrm{N.s/m^2}$

M = 5 kg

k = 3.2 $\mathrm{s^{-1}}$

N **= 4**

Δ*t* **= 0.333**

Eq. 5.34 (LHS)				**(RHS)**
1.000	0.000	0.000	0.000	1
-1.000	2.067	0.000	0.000	0
0.000	-1.000	2.067	0.000	0
0.000	0.000	-1.000	2.067	0

t	**Inverse Matrix**				**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	1.000	1.000	0.000
0.333	0.484	0.484	0.000	0.000	0.484	0.344	0.005
0.667	0.234	0.234	0.484	0.000	0.234	0.118	0.003
1.000	0.113	0.113	0.234	0.484	0.113	0.041	0.001
							0.098

N **= 8**

Δ*t* **= 0.143**

Eq. 5.34 (LHS)								**(RHS)**
1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
-1.000	1.457	0.000	0.000	0.000	0.000	0.000	0.000	0
0.000	-1.000	1.457	0.000	0.000	0.000	0.000	0.000	0
0.000	0.000	-1.000	1.457	0.000	0.000	0.000	0.000	0
0.000	0.000	0.000	-1.000	1.457	0.000	0.000	0.000	0
0.000	0.000	0.000	0.000	-1.000	1.457	0.000	0.000	0
0.000	0.000	0.000	0.000	0.000	-1.000	1.457	0.000	0
0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.457	0

	Inverse Matrix										
t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
0.143	0.686	0.686	0.000	0.000	0.000	0.000	0.000	0.000	0.686	0.633	0.000
0.286	0.471	0.471	0.686	0.000	0.000	0.000	0.000	0.000	0.471	0.401	0.001
0.429	0.323	0.323	0.471	0.686	0.000	0.000	0.000	0.000	0.323	0.254	0.001
0.571	0.222	0.222	0.323	0.471	0.686	0.000	0.000	0.000	0.222	0.161	0.000
0.714	0.152	0.152	0.222	0.323	0.471	0.686	0.000	0.000	0.152	0.102	0.000
0.857	0.104	0.104	0.152	0.222	0.323	0.471	0.686	0.000	0.104	0.064	0.000
1.000	0.072	0.072	0.104	0.152	0.222	0.323	0.471	0.686	0.072	0.041	0.000
											0.052

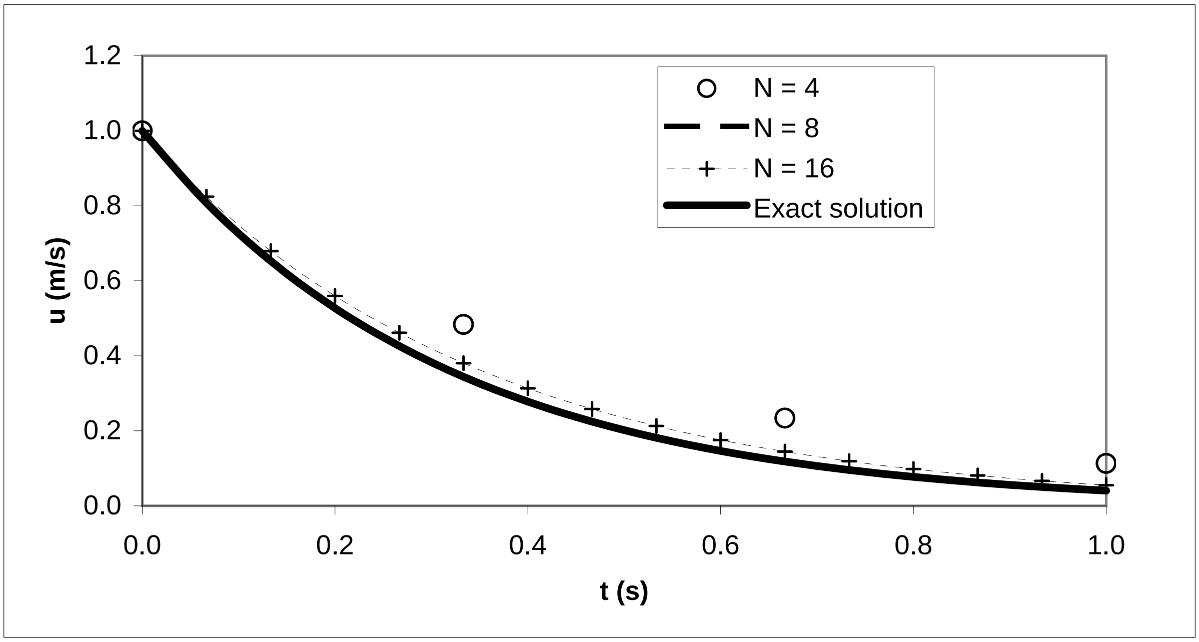
N = **16**

Δt = **0.067** **Eq. 5.34 (LHS)**

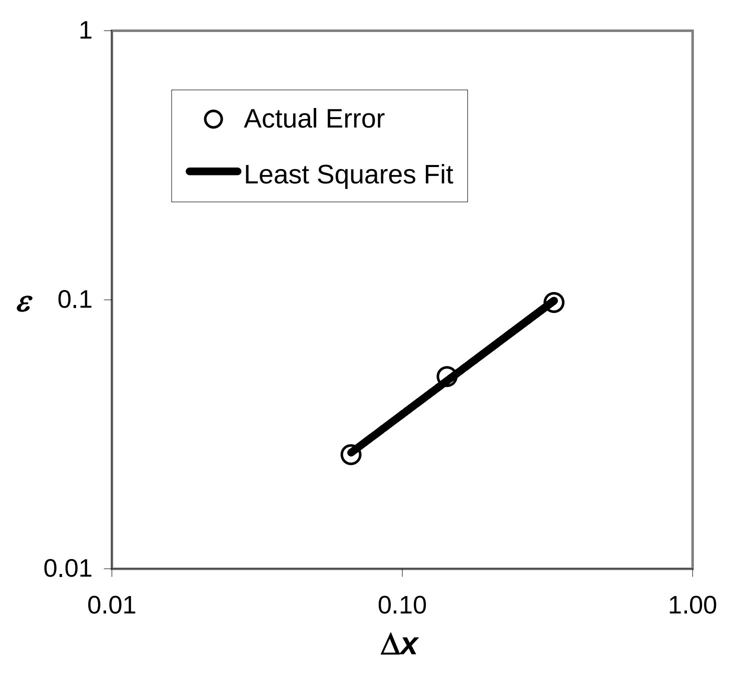
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**(RHS)**
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
2	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
3	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
4	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
5	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
6	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
7	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
8	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0.000	0
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0.000	0
12	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0.000	0
13	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0.000	0
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0.000	0
15	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0.000	0
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1.000	1.213	0

t	**Inverse Matrix**																**Result**	**Exact**	**Error**
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.000	0.000
0.067	0.824	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.824	0.808	0.000
0.133	0.679	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.679	0.653	0.000
0.200	0.560	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.560	0.527	0.000
0.267	0.461	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.461	0.426	0.000
0.333	0.380	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.380	0.344	0.000
0.400	0.313	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.313	0.278	0.000
0.467	0.258	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.258	0.225	0.000
0.533	0.213	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.213	0.181	0.000
0.600	0.175	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.000	0.175	0.147	0.000
0.667	0.145	0.145	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.000	0.145	0.118	0.000
0.733	0.119	0.119	0.145	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.000	0.119	0.096	0.000
0.800	0.098	0.098	0.119	0.145	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.000	0.098	0.077	0.000
0.867	0.081	0.081	0.098	0.119	0.145	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.000	0.081	0.062	0.000
0.933	0.067	0.067	0.081	0.098	0.119	0.145	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.000	0.067	0.050	0.000
1.000	0.055	0.055	0.067	0.081	0.098	0.119	0.145	0.175	0.213	0.258	0.313	0.380	0.461	0.560	0.679	0.824	0.055	0.041	0.000
																			0.027

N	**Δt**	**Error**
4	0.333	0.098
8	0.143	0.052
16	0.067	0.027



1.2
1.0
0.8
0.6
0.4
0.2
0.0
u (m/s)
0.0
0.2
0.4
0.6
0.8
1.0
t (s)
N = 4
N = 8
N = 16
Exact solution



1
0.1
0.01
ε
0.01
0.10
1.00
Δx
Actual Error
Least Squares Fit

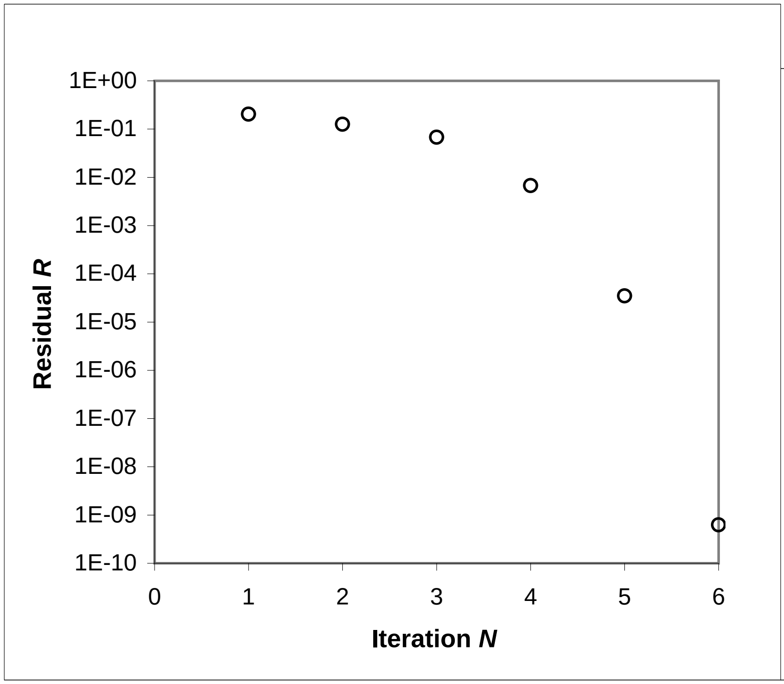
Problem 5.91

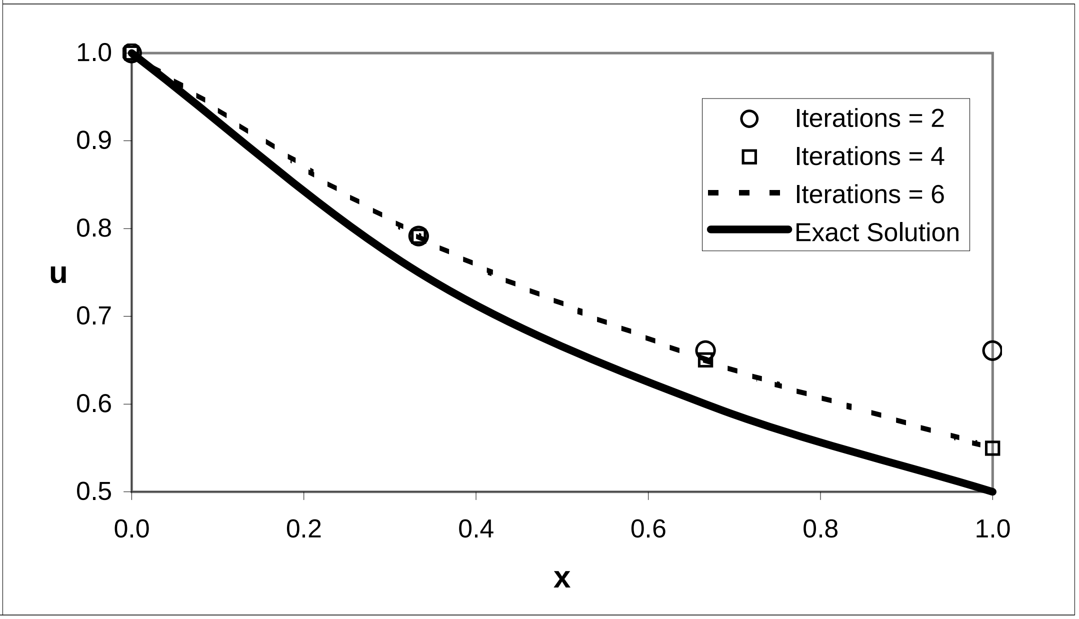
5.91 Use *Excel* to generate the solutions of Eq. 5.28 for $m = 2$ shown in Fig. 5.19.

$$u_i = \frac{u_{g_{i-1}} + \Delta x\, u_{g_i}^2}{1 + 2\Delta x\, u_{g_i}}$$

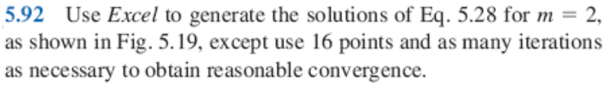
$\Delta x =$ 0.333

	x				
Iteration	0.000	0.333	0.667	1.000	**Residuals**
0	1.000	1.000	1.000	1.000	
1	1.000	0.800	0.800	0.800	0.204
2	1.000	0.791	0.661	0.661	0.127
3	1.000	0.791	0.650	0.560	0.068
4	1.000	0.791	0.650	0.550	0.007
5	1.000	0.791	0.650	0.550	0.000
6	1.000	0.791	0.650	0.550	0.000
Exact	**1.000**	**0.750**	**0.600**	**0.500**	





Problem 5.92

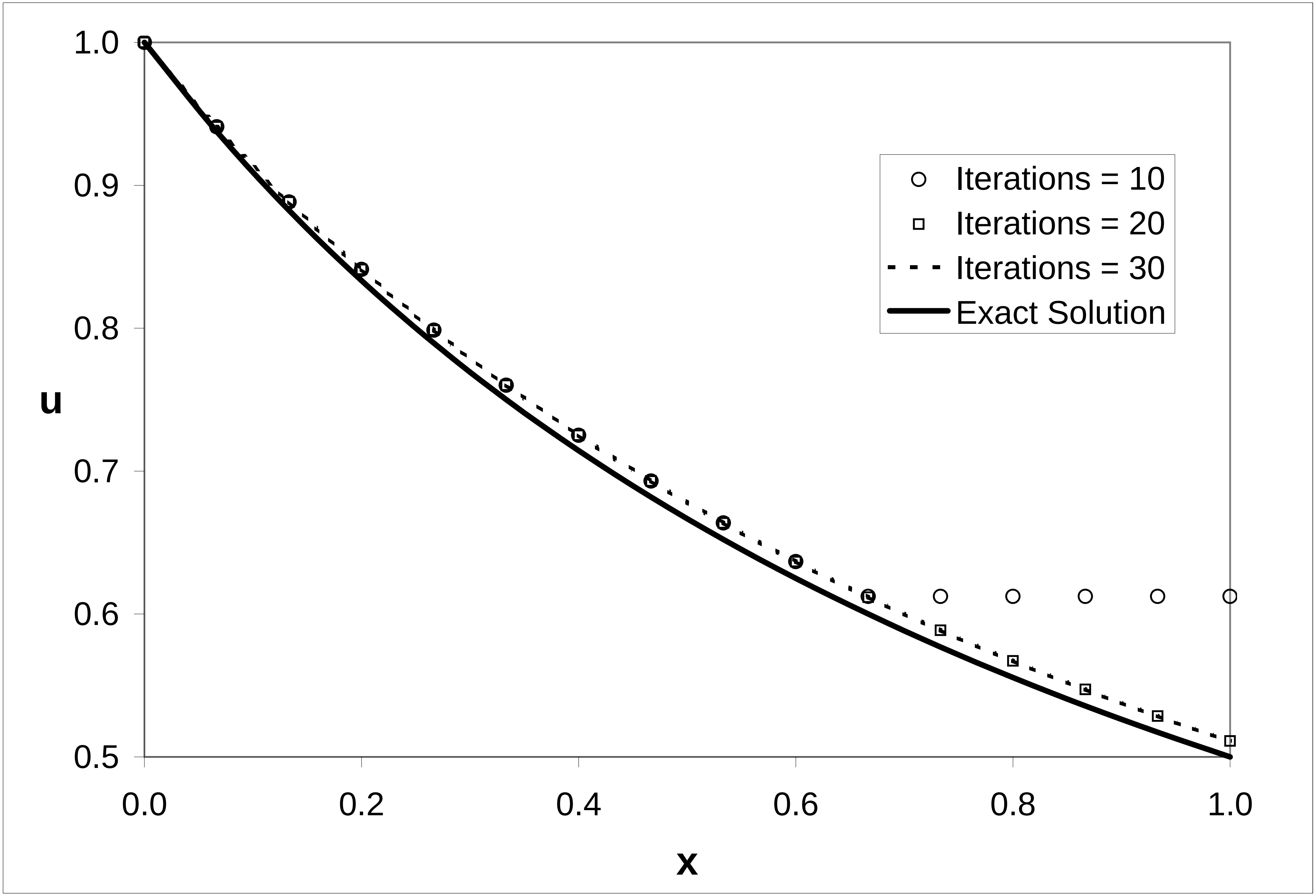


5.92 Use *Excel* to generate the solutions of Eq. 5.28 for $m = 2$, as shown in Fig. 5.19, except use 16 points and as many iterations as necessary to obtain reasonable convergence.

$$u_i = \frac{u_{g_{i-1}} + \Delta x\, u_{g_i}^2}{1 + 2\Delta x\, u_{g_i}}$$

$\Delta x =$ 0.0667

							x									
Iteration	0.000	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	0.667	0.733	0.800	0.867	0.933	1.000
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	1.000	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941	0.941
2	1.000	0.941	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889	0.889
3	1.000	0.941	0.888	0.842	0.842	0.842	0.842	0.842	0.842	0.842	0.842	0.842	0.842	0.842	0.842	0.842
4	1.000	0.941	0.888	0.841	0.799	0.799	0.799	0.799	0.799	0.799	0.799	0.799	0.799	0.799	0.799	0.799
5	1.000	0.941	0.888	0.841	0.799	0.761	0.761	0.761	0.761	0.761	0.761	0.761	0.761	0.761	0.761	0.761
6	1.000	0.941	0.888	0.841	0.799	0.760	0.726	0.726	0.726	0.726	0.726	0.726	0.726	0.726	0.726	0.726
7	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694	0.694
8	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.664	0.664	0.664	0.664	0.664	0.664	0.664
9	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.637	0.637	0.637	0.637	0.637	0.637
10	**1.000**	**0.941**	**0.888**	**0.841**	**0.799**	**0.760**	**0.725**	**0.693**	**0.664**	**0.637**	**0.612**	**0.612**	**0.612**	**0.612**	**0.612**	**0.612**
11	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.589	0.589	0.589	0.589
12	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.568	0.568	0.568	0.568
13	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.548	0.548	0.548
14	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.529
15	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.512
16	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
17	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
18	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
19	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
20	**1.000**	**0.941**	**0.888**	**0.841**	**0.799**	**0.760**	**0.725**	**0.693**	**0.664**	**0.637**	**0.612**	**0.589**	**0.567**	**0.547**	**0.529**	**0.511**
21	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
22	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
23	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
24	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
25	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
26	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
27	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
28	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
29	1.000	0.941	0.888	0.841	0.799	0.760	0.725	0.693	0.664	0.637	0.612	0.589	0.567	0.547	0.529	0.511
30	**1.000**	**0.941**	**0.888**	**0.841**	**0.799**	**0.760**	**0.725**	**0.693**	**0.664**	**0.637**	**0.612**	**0.589**	**0.567**	**0.547**	**0.529**	**0.511**
Exact	**1.000**	**0.938**	**0.882**	**0.833**	**0.789**	**0.750**	**0.714**	**0.682**	**0.652**	**0.625**	**0.600**	**0.577**	**0.556**	**0.536**	**0.517**	**0.500**


1.0
0.9
0.8
0.7
0.6
0.5
u
0.0
0.2
0.4
0.6
0.8
1.0
x
Iterations = 10
Iterations = 20
Iterations = 30
Exact Solution

Problem 5.93

5.93 Use *Excel* to generate the solutions of Eq. 5.28 for $m = -1$, with $u(0) = 2$, using four and 16 points, with sufficient iterations, and compare to the exact solution

$$u_{\text{exact}} = \sqrt{4 - 2x}$$

To do so, follow the steps described in "Dealing with Nonlinearity" section.

$$\Delta u_i = u_i - u_{g_i}$$

$$\frac{1}{u_i} = \frac{1}{u_{g_i} + \Delta u_i} \approx \frac{1}{u_{g_i}}\left(1 - \frac{\Delta u_i}{u_{g_i}}\right)$$

$$\frac{u_i - u_{i-1}}{\Delta x} + \frac{1}{u_i} = 0$$

$$\frac{u_i - u_{i-1}}{\Delta x} + \frac{1}{u_{g_i}}\left(1 - \frac{u_i - u_{g_i}}{u_{g_i}}\right) = 0$$

$$\frac{u_i - u_{i-1}}{\Delta x} + \frac{1}{u_{g_i}}\left(2 - \frac{u_i}{u_{g_i}}\right) = 0$$

$$u_i\left(1 - \frac{\Delta x}{u_{g_i}^2}\right) = u_{i-1} - \frac{2\Delta x}{u_{g_i}}$$

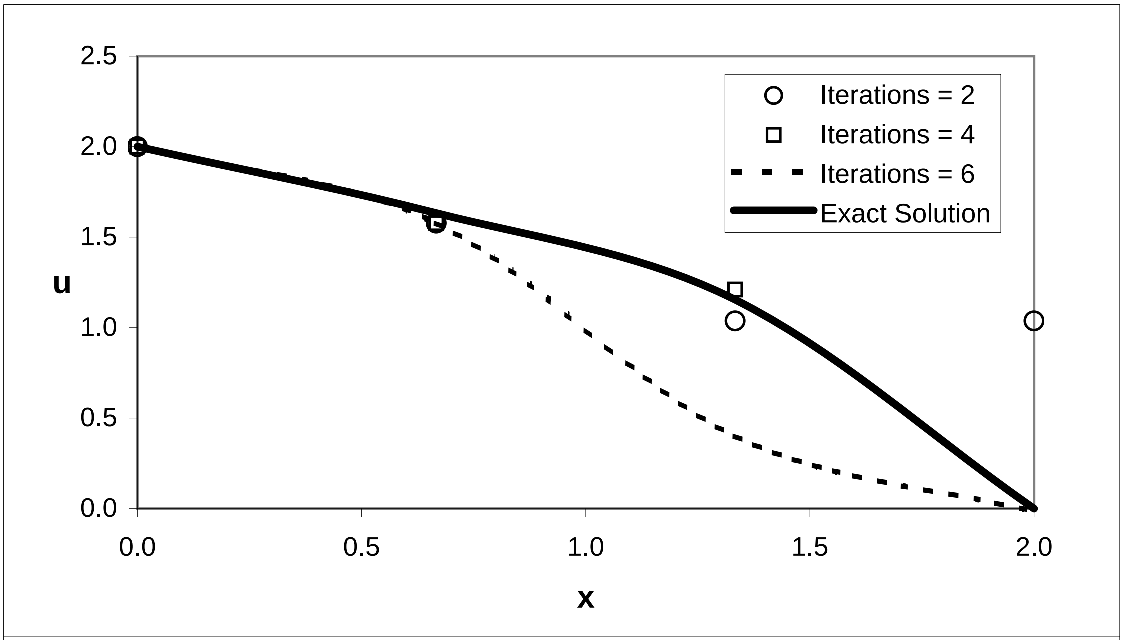
$$u_i = \frac{u_{i-1} - \dfrac{2\Delta x}{u_{g_i}}}{1 - \dfrac{\Delta x}{u_{g_i}^2}}$$

$\Delta x =$ 0.667

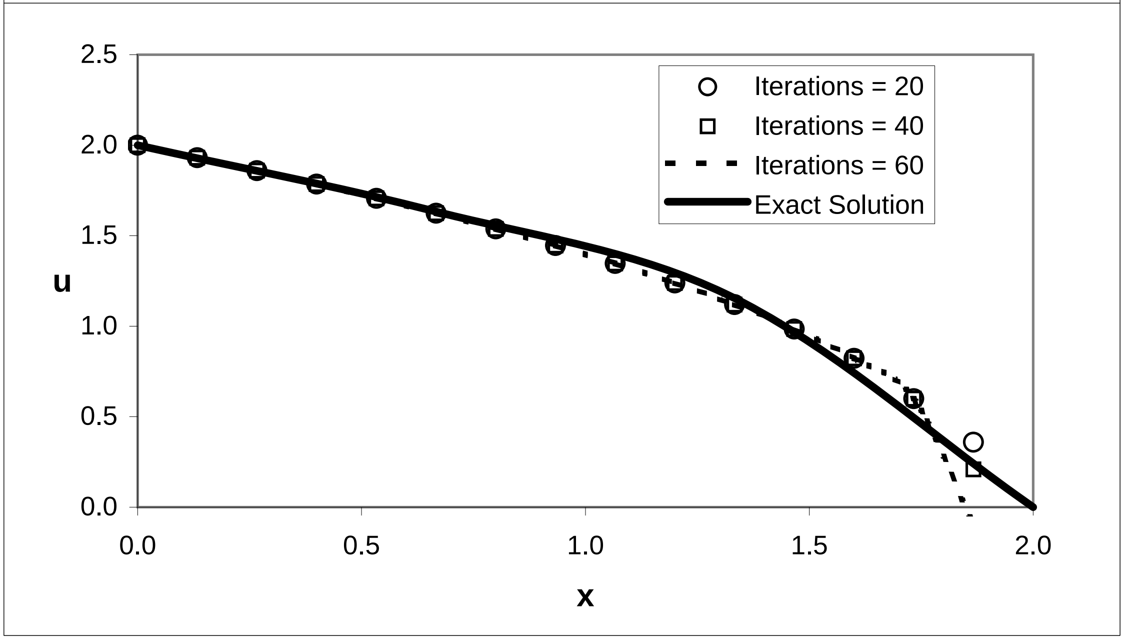
		x		
Iteration	0.000	0.667	1.333	2.000
0	2.000	2.000	2.000	2.000
1	2.000	1.600	1.600	1.600
2	2.000	1.577	1.037	1.037
3	2.000	1.577	0.767	-0.658
4	2.000	1.577	1.211	-5.158
5	2.000	1.577	0.873	1.507
6	2.000	1.577	0.401	-0.017
Exact	**2.000**	**1.633**	**1.155**	**0.000**

$\Delta x =$ 0.133

								x								
Iteration	0.000	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	1.333	1.467	1.600	1.733	1.867	2.000
0	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
1	2.000	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931	1.931
2	2.000	1.931	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859
3	2.000	1.931	1.859	1.785	1.785	1.785	1.785	1.785	1.785	1.785	1.785	1.785	1.785	1.785	1.785	1.785
4	2.000	1.931	1.859	1.785	1.707	1.707	1.707	1.707	1.707	1.707	1.707	1.707	1.707	1.707	1.707	1.707
5	2.000	1.931	1.859	1.785	1.706	1.625	1.625	1.625	1.625	1.625	1.625	1.625	1.625	1.625	1.625	1.625
6	2.000	1.931	1.859	1.785	1.706	1.624	1.539	1.539	1.539	1.539	1.539	1.539	1.539	1.539	1.539	1.539
7	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.447	1.447	1.447	1.447	1.447	1.447	1.447	1.447	1.447
8	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.348	1.348	1.348	1.348	1.348	1.348	1.348	1.348
9	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.242	1.242	1.242	1.242	1.242	1.242	1.242
10	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.124	1.124	1.124	1.124	1.124	1.124
11	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.991	0.991	0.991	0.991	0.991
12	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.836	0.836	0.836	0.836
13	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.639	0.639	0.639
14	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.601	0.329	0.329
15	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.899	2.061
16	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.363	0.795
17	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	9.602	0.034
18	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.572	-0.016
19	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.225	-0.034
20	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.359	-0.070
21	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	3.969	-0.160
22	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.537	-1.332
23	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.191	0.797
24	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.300	-0.182
25	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.600	-0.584
26	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.246	1.734
27	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.403	0.097
28	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.345	0.178
29	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-11.373	0.572
30	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.623	-19.981
31	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.261	0.637
32	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.442	-0.234
33	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.013	-1.108
34	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.027	0.255
35	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.059	1.023
36	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.136	-0.366
37	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.414	132.420
38	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	5.624	-0.416
39	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.554	27.391
40	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.209	0.545
41	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.329	-0.510
42	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.919	1.749
43	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.367	0.802
44	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-11.148	0.044
45	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.624	0.252
46	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.262	0.394
47	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.443	-2.929
48	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.010	0.542
49	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.019	-0.918
50	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.041	0.322
51	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.090	3.048
52	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.231	-0.180
53	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-1.171	-0.402
54	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.916	-2.886
55	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.366	1.025
56	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-18.029	0.122
57	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.614	2.526
58	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.256	0.520
59	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	0.426	-0.509
60	2.000	1.931	1.859	1.785	1.706	1.624	1.538	1.445	1.346	1.239	1.120	0.984	0.822	0.599	-0.097	1.962
Exact	**2.000**	**1.932**	**1.862**	**1.789**	**1.713**	**1.633**	**1.549**	**1.461**	**1.366**	**1.265**	**1.155**	**1.033**	**0.894**	**0.730**	**0.516**	**0.000**

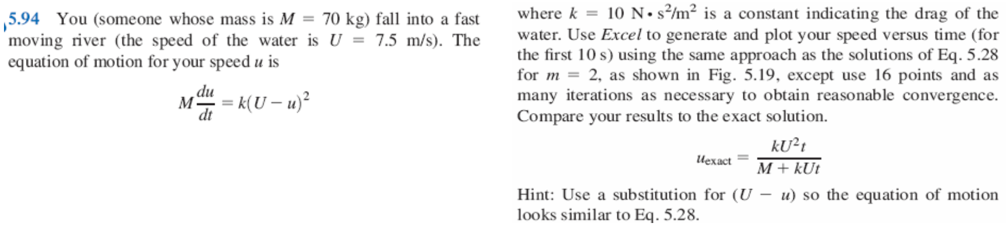


2.5
2.0
1.5
1.0
0.5
0.0
u
0.0
0.5
1.0
1.5
2.0
x
Iterations = 2
Iterations = 4
Iterations = 6
Exact Solution



2.5
2.0
1.5
1.0
0.5
0.0
u
0.0
0.5
1.0
1.5
2.0
x
Iterations = 20
Iterations = 40
Iterations = 60
Exact Solution

Problem 5.94



5.94 You (someone whose mass is $M = 70$ kg) fall into a fast moving river (the speed of the water is $U = 7.5$ m/s). The equation of motion for your speed u is

$$M\frac{du}{dt} = k(U - u)^2$$

where $k = 10\ \text{N}\cdot\text{s}^2/\text{m}^2$ is a constant indicating the drag of the water. Use *Excel* to generate and plot your speed versus time (for the first 10 s) using the same approach as the solutions of Eq. 5.28 for $m = 2$, as shown in Fig. 5.19, except use 16 points and as many iterations as necessary to obtain reasonable convergence. Compare your results to the exact solution.

$$u_{\text{exact}} = \frac{kU^2t}{M + kUt}$$

Hint: Use a substitution for $(U - u)$ so the equation of motion looks similar to Eq. 5.28.

$$M\frac{du}{dt} = k(U-u)^2$$

$$v = U - u$$

$$dv = -du$$

$$-M\frac{dv}{dt} = kv^2$$

$$\frac{dv}{dt} + \frac{k}{M}v^2 = 0$$

$$v_i^2 \approx 2v_{g_i}v_i - v_{g_i}^2$$

$$\frac{v_i - v_{i-1}}{\Delta t} + \frac{k}{M}\left(2v_{g_i}v_i - v_{g_i}^2\right) = 0$$

$$v_i = \frac{v_{g_{i-1}} + \frac{k}{M}\Delta t\, v_{g_i}^2}{1 + 2\frac{k}{M}\Delta t\, v_{g_i}}$$

$\Delta t =$ 1.000

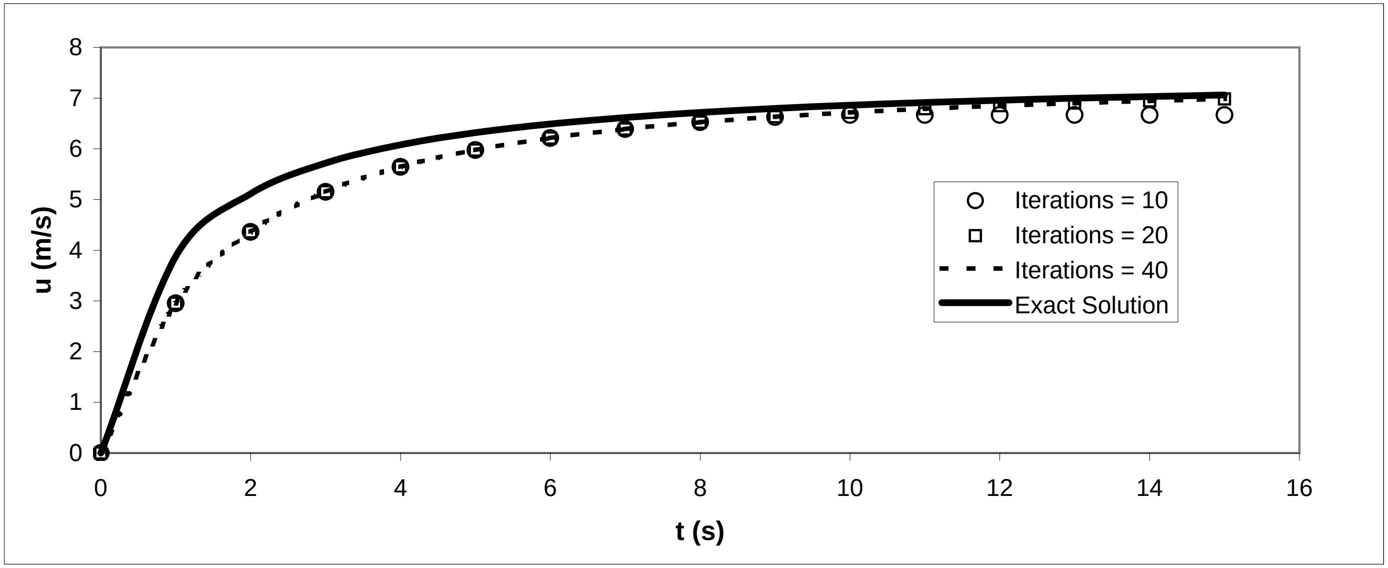
$k =$ 10 $\text{N.s}^2/\text{m}^2$

$M =$ 70 kg

	t															
Iteration	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500	7.500
1	7.500	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943	4.943
2	7.500	4.556	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496	3.496
3	7.500	4.547	3.153	2.623	2.623	2.623	2.623	2.623	2.623	2.623	2.623	2.623	2.623	2.623	2.623	2.623
4	7.500	4.547	3.139	2.364	2.061	2.061	2.061	2.061	2.061	2.061	2.061	2.061	2.061	2.061	2.061	2.061
5	7.500	4.547	3.139	2.350	1.870	1.679	1.679	1.679	1.679	1.679	1.679	1.679	1.679	1.679	1.679	1.679
6	7.500	4.547	3.139	2.350	1.857	1.536	1.407	1.407	1.407	1.407	1.407	1.407	1.407	1.407	1.407	1.407
7	7.500	4.547	3.139	2.350	1.857	1.525	1.297	1.205	1.205	1.205	1.205	1.205	1.205	1.205	1.205	1.205
8	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.119	1.051	1.051	1.051	1.051	1.051	1.051	1.051	1.051
9	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.982	0.930	0.930	0.930	0.930	0.930	0.930	0.930
10	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.874	0.832	0.832	0.832	0.832	0.832	0.832
11	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.786	0.752	0.752	0.752	0.752	0.752
12	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.713	0.686	0.686	0.686	0.686
13	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.653	0.629	0.629	0.629
14	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.601	0.581	0.581
15	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.557	0.540
16	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.519
17	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
18	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
19	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
20	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
21	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
22	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
23	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
24	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
25	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
26	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
27	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
28	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
29	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
30	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
31	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
32	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
33	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
34	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
35	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
36	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
37	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
38	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
39	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516
40	7.500	4.547	3.139	2.350	1.857	1.525	1.288	1.112	0.976	0.868	0.781	0.709	0.649	0.598	0.554	0.516

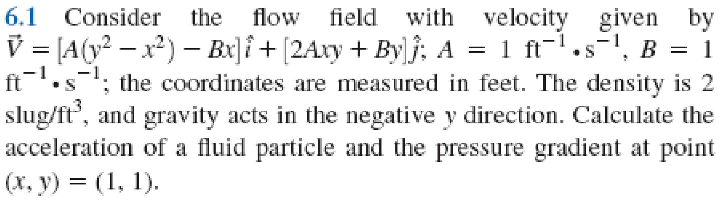
Above values are for v! To get u we compute u = U - v

Iteration																
10	0.000	2.953	4.361	5.150	5.643	5.975	6.212	6.388	6.524	6.626	6.668	6.668	6.668	6.668	6.668	6.668
20	0.000	2.953	4.361	5.150	5.643	5.975	6.212	6.388	6.524	6.632	6.719	6.791	6.851	6.902	6.946	6.984
40	0.000	2.953	4.361	5.150	5.643	5.975	6.212	6.388	6.524	6.632	6.719	6.791	6.851	6.902	6.946	6.984
Exact	**0.000**	**3.879**	**5.114**	**5.720**	**6.081**	**6.320**	**6.490**	**6.618**	**6.716**	**6.795**	**6.860**	**6.913**	**6.959**	**6.998**	**7.031**	**7.061**



Problem 6.1

[2]



6.1 Consider the flow field with velocity given by $\vec{V} = [A(y^2 - x^2) - Bx]\hat{i} + [2Axy + By]\hat{j}$; $A = 1\ \text{ft}^{-1} \cdot \text{s}^{-1}$, $B = 1\ \text{ft}^{-1} \cdot \text{s}^{-1}$; the coordinates are measured in feet. The density is 2 slug/ft^3, and gravity acts in the negative y direction. Calculate the acceleration of a fluid particle and the pressure gradient at point $(x, y) = (1, 1)$.

Given: Velocity field

Find: Acceleration of particle and pressure gradient at (1,1)

Solution:

NOTE: Units of B are s^{-1} not $ft^{-1}s^{-1}$

Basic equations

$$\vec{a}_p = \underset{\substack{\text{total}\\ \text{acceleration}\\ \text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\ \text{acceleration}}} + \underset{\substack{\text{local}\\ \text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}} \qquad \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

For this flow

$$u(x,y) = A\cdot\left(y^2 - x^2\right) - B\cdot x \qquad v(x,y) = 2\cdot A\cdot x\cdot y + B\cdot y$$

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = \left[A\cdot\left(y^2 - x^2\right) - B\cdot x\right]\cdot\frac{\partial}{\partial x}\left[A\cdot\left(y^2 - x^2\right) - B\cdot x\right] + (2\cdot A\cdot x\cdot y + B\cdot y)\cdot\frac{\partial}{\partial y}\left[A\cdot\left(y^2 - x^2\right) - B\cdot x\right]$$

$$a_x = (B + 2\cdot A\cdot x)\cdot\left(A\cdot x^2 + B\cdot x + A\cdot y^2\right)$$

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = \left[A\cdot\left(y^2 - x^2\right) - B\cdot x\right]\cdot\frac{\partial}{\partial x}(2\cdot A\cdot x\cdot y + B\cdot y) + (2\cdot A\cdot x\cdot y + B\cdot y)\cdot\frac{\partial}{\partial y}(2\cdot A\cdot x\cdot y + B\cdot y)$$

$$a_y = (B + 2\cdot A\cdot x)\cdot(B\cdot y + 2\cdot A\cdot x\cdot y) - 2\cdot A\cdot y\cdot\left[B\cdot x + A\cdot\left(x^2 - y^2\right)\right]$$

Hence at (1,1)

$$a_x = (1 + 2\cdot 1\cdot 1)\cdot\frac{1}{s}\times\left(1\cdot 1^2 + 1\cdot 1 + 1\cdot 1^2\right)\cdot\frac{ft}{s} \qquad a_x = 9\cdot\frac{ft}{s^2}$$

$$a_y = (1 + 2\cdot 1\cdot 1)\cdot\frac{1}{s}\times(1\cdot 1 + 2\cdot 1\cdot 1\cdot 1)\cdot\frac{ft}{s} - 2\cdot 1\cdot 1\cdot\frac{1}{s}\times\left[1\cdot 1 + 1\cdot\left(1^2 - 1^2\right)\right]\cdot\frac{ft}{s} \qquad a_y = 7\cdot\frac{ft}{s^2}$$

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} \qquad \theta = \operatorname{atan}\left(\frac{a_y}{a_x}\right) \qquad a = 11.4\cdot\frac{ft}{s^2} \qquad \theta = 37.9\cdot deg$$

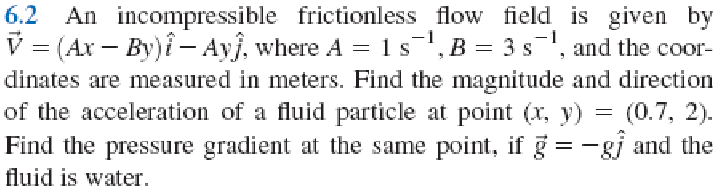
For the pressure gradient

$$\frac{\partial}{\partial x}p = \rho\cdot g_x - \rho\cdot a_x = -2\cdot\frac{slug}{ft^3}\times 9\cdot\frac{ft}{s^2}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad \frac{\partial}{\partial x}p = -18\cdot\frac{\frac{lbf}{ft^2}}{ft} = -0.125\cdot\frac{psi}{ft}$$

$$\frac{\partial}{\partial y}p = \rho\cdot g_y - \rho\cdot a_y = 2\cdot\frac{slug}{ft^3}\times(-32.2 - 7)\cdot\frac{ft}{s^2}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad \frac{\partial}{\partial y}p = -78.4\cdot\frac{\frac{lbf}{ft^2}}{ft} = -0.544\cdot\frac{psi}{ft}$$

Problem 6.2

[2]



6.2 An incompressible frictionless flow field is given by $\vec{V} = (Ax - By)\hat{i} - Ay\hat{j}$, where $A = 1\ \text{s}^{-1}$, $B = 3\ \text{s}^{-1}$, and the coordinates are measured in meters. Find the magnitude and direction of the acceleration of a fluid particle at point $(x, y) = (0.7, 2)$. Find the pressure gradient at the same point, if $\vec{g} = -g\hat{j}$ and the fluid is water.

Given: Velocity field

Find: Acceleration of particle and pressure gradient at (0.7,2)

Solution:

Basic equations

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}} \qquad \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

For this flow $\quad u(x,y) = A\cdot x - B\cdot y \qquad v(x,y) = -A\cdot y$

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = (A\cdot x - B\cdot y)\cdot\frac{\partial}{\partial x}(A\cdot x - B\cdot y) + (-A\cdot y)\cdot\frac{\partial}{\partial y}(A\cdot x - B\cdot y) \qquad a_x = A^2\cdot x$$

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = (A\cdot x - B\cdot y)\cdot\frac{\partial}{\partial x}(-A\cdot y) + (-A\cdot y)\cdot\frac{\partial}{\partial y}(-A\cdot y) \qquad a_y = A^2\cdot y$$

Hence at (0.7,2)

$$a_x = \left(\frac{1}{s}\right)^2 \times 0.7\cdot m \qquad a_x = 0.7\frac{m}{s^2}$$

$$a_y = \left(\frac{1}{s}\right)^2 \times 2\cdot m \qquad a_y = 2\frac{m}{s^2}$$

$$a = \sqrt{a_x^{\,2} + a_y^{\,2}} \qquad \theta = \operatorname{atan}\left(\frac{a_y}{a_x}\right) \qquad a = 2.12\frac{m}{s^2} \qquad \theta = 70.7\cdot deg$$

For the pressure gradient

$$\frac{\partial}{\partial x}p = \rho\cdot g_x - \rho\cdot a_x = -1000\cdot\frac{kg}{m^3} \times 0.7\cdot\frac{m}{s^2} \times \frac{N\cdot s^2}{kg\cdot m} \qquad \frac{\partial}{\partial x}p = -700\cdot\frac{Pa}{m} = -0.7\cdot\frac{kPa}{m}$$

$$\frac{\partial}{\partial y}p = \rho\cdot g_y - \rho\cdot a_y = 1000\cdot\frac{kg}{m^3} \times (-9.81 - 2)\cdot\frac{m}{s^2} \times \frac{N\cdot s^2}{kg\cdot m} \qquad \frac{\partial}{\partial y}p = -11800\cdot\frac{Pa}{m} = -11.8\cdot\frac{kPa}{m}$$

Problem 6.3

[2]

Given: Horizontal flow of water described by the velocity field

$$\vec{V} = (Ax+Bt)\hat{i} + (-Ay+Bt)\hat{j}$$

where: $A = 5\,s^{-1}$, $B = 10\,ft\cdot s^{-2}$, coordinates x, y in ft, t in s.

Find: (a) Expressions for (i) local, (ii) convective, (iii) total, acceleration

(b) Evaluate at point (2,2) for $t = 5\,s$

(c) Evaluate ∇p at same point and time

Solution:

Basic equations: $$\frac{D\vec{V}}{Dt} = \vec{a}_p = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{local} + \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\cancel{\frac{\partial \vec{V}}{\partial z}}^{0}}_{convective} \quad ; \quad \rho\vec{g} - \nabla p = \rho\frac{D\vec{V}}{Dt}$$

Assumptions: (1) frictionless flow

(2) $\rho = \text{constant} = 1.94\ slug/ft^3$

$$\frac{\partial \vec{V}}{\partial t} = \frac{\partial}{\partial t}\left[(Ax+Bt)\hat{i} + (-Ay+Bt)\hat{j}\right] = B\hat{i} + B\hat{j} = 10(\hat{i}+\hat{j})\ ft/s^2 \leftarrow \vec{a}_{local}$$

$$u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} = (Ax+Bt)\frac{\partial}{\partial x}\left[(Ax+Bt)\hat{i} + (-Ay+Bt)\hat{j}\right] + (-Ay+Bt)\frac{\partial}{\partial y}\left[(Ax+Bt)\hat{i} + (-Ay+Bt)\hat{j}\right]$$

$$= (Ax+Bt)\left[A\hat{i}\right] + (-Ay+Bt)\left[-A\hat{j}\right]$$

$$u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} = A(Ax+Bt)\hat{i} - A(-Ay+Bt)\hat{j} \longleftarrow$$

$$= \frac{5}{s}\left(\frac{5}{s}\times 2ft + \frac{10ft}{s^2}\times 5s\right)\hat{i} - \frac{5}{s}\left(-\frac{5}{s}\times 2ft + \frac{10ft}{s^2}\times 5s\right)\hat{j} = 300\hat{i} - 200\hat{j}\ \frac{ft}{s^2} \leftarrow \vec{a}_{conv}$$

$$\vec{a} = \vec{a}_{local} + \vec{a}_{conv} = \left[B + A(Ax+Bt)\right]\hat{i} + \left[B - A(-Ay+Bt)\right]\hat{j} = 310\hat{i} - 190\hat{j}\ \frac{ft}{s^2} \leftarrow \vec{a}$$

From Euler's equation,

$$\nabla p = \rho\vec{g} - \rho\frac{D\vec{V}}{Dt} = 1.94\frac{slug}{ft^3}\left[-32.2\hat{k} - (310\hat{i} - 190\hat{j})\right]\frac{ft}{s^2}\times\frac{lbf\cdot s^2}{ft\cdot slug}$$

$$\nabla p = -601\hat{i} + 367\hat{j} - 62\hat{k}\ \frac{lbf/ft^2}{ft} = -4.17\hat{i} + 2.56\hat{j} - 0.43\hat{k}\ psi/ft$$

Note: $\nabla\cdot\vec{V} = 0$ as required for incompressible flow

Problem 6.4

Given: Velocity field, $\vec{V} = (Ax - By)t\hat{i} - (Ay + Bx)t\hat{j}$

where $A = 1\ s^{-2}$

$B = 2\ s^{-2}$

coordinates x, y are in meters

Fluid density is $\rho = 1500\ kg/m^3$. Body forces are negligible

Find: ∇P at location $(1,2)$ at $t = 1\ s$.

Solution:

Basic equations: $\cancelto{0}{\rho\vec{g}} - \nabla P = \rho \frac{D\vec{V}}{Dt}$

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Assumptions: (1) frictionless flow

Substituting for the velocity field in the equation for $\frac{D\vec{V}}{Dt}$,

$$\frac{D\vec{V}}{Dt} = \frac{\partial}{\partial t}\left[(Ax-By)t\hat{i} - (Ay+Bx)t\hat{j}\right] + (Ax-By)t\frac{\partial}{\partial x}\left[(Ax-By)t\hat{i} - (Ay+Bx)t\hat{j}\right]$$
$$-(Ay+Bx)t\frac{\partial}{\partial y}\left[(Ax-By)t\hat{i} - (Ay+Bx)t\hat{j}\right]$$
$$= \left[(Ax-By)\hat{i} - (Ay+Bx)\hat{j}\right] + (Ax-By)t\left[At\hat{i} - Bt\hat{j}\right] - (Ay+Bx)t\left[-Bt\hat{i} - At\hat{j}\right]$$
$$= \hat{i}\left\{Ax - By + A^2xt^2 - \cancel{AByt^2} + \cancel{AByt^2} + B^2xt^2\right\} + \hat{j}\left\{-Ay - Bx - \cancel{ABxt^2} + B^2yt^2 + A^2yt^2 + \cancel{ABxt^2}\right\}$$
$$\frac{D\vec{V}}{Dt} = \hat{i}\left\{Ax - By + xt^2(A^2+B^2)\right\} + \hat{j}\left\{-Ay - Bx + yt^2(A^2+B^2)\right\}$$

Then,

$$\nabla P = -\rho\frac{D\vec{V}}{Dt} = -\rho\left[\hat{i}\left\{Ax - By + xt^2(A^2+B^2)\right\} + \hat{j}\left\{-Ay - Bx + yt^2(A^2+B^2)\right\}\right]$$

At location $(1,2)$ at $t = 1\ s$

$$\nabla P = -1500\frac{kg}{m^3}\left[\hat{i}\left\{\frac{1}{s^2}\times 1m - \frac{2}{s^2}\times 2 + 1m \times 1\ s^2\left(\frac{(1)^2+(2)^2}{s^4}\right)\right\}\right.$$
$$\left. + \hat{j}\left\{-\frac{1}{s^2}\times 2m - \frac{2}{s^2}\times 1m + 2m\times 1s^2\times\left(\frac{(1)^2+(2)^2}{s^4}\right)\right\}\right]\frac{N\cdot s^2}{kg\cdot m}$$

$$\nabla P = -(3.0\hat{i} + 9.0\hat{j})\ \frac{kN/m^2}{m} \longleftarrow$$ ∇P

Note: $\nabla\cdot\vec{V} = 0$ as required for incompressible flow

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 6.5

[2]

6.5 Consider the flow field with velocity given by $\vec{V} = [A(x^2 - y^2) - 3Bx]\hat{i} - [2Axy - 3By]\hat{j}$, where $A = 1\ \mathrm{ft}^{-1} \cdot \mathrm{s}^{-1}$, $B = 1\ \mathrm{s}^{-1}$, and the coordinates are measured in feet. The density is 2 slug/ft^3 and gravity acts in the negative y direction. Determine the acceleration of a fluid particle and the pressure gradient at point $(x, y) = (1, 1)$.

Given: Velocity field

Find: Acceleration of particle and pressure gradient at (1,1)

Solution:

Basic equations

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}} \qquad \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

For this flow

$$u(x,y) = A\cdot\left(x^2 - y^2\right) - 3\cdot B\cdot x \qquad v(x,y) = -2\cdot A\cdot x\cdot y + 3\cdot B\cdot y$$

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = \left[A\cdot\left(x^2 - y^2\right) - 3\cdot B\cdot x\right]\cdot\frac{\partial}{\partial x}\left[A\cdot\left(x^2 - y^2\right) - 3\cdot B\cdot x\right] \ldots$$

$$+ (-2\cdot A\cdot x\cdot y + 3\cdot B\cdot y)\cdot\frac{\partial}{\partial y}\left[A\cdot\left(x^2 - y^2\right) - 3\cdot B\cdot x\right]$$

$$a_x = (2\cdot A\cdot x - 3\cdot B)\cdot\left(A\cdot x^2 - 3\cdot B\cdot x + A\cdot y^2\right)$$

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = \left[A\cdot\left(x^2 - y^2\right) - 3\cdot B\cdot x\right]\cdot\frac{\partial}{\partial x}(-2\cdot A\cdot x\cdot y + 3\cdot B\cdot y) + (-2\cdot A\cdot x\cdot y + 3\cdot B\cdot y)\cdot\frac{\partial}{\partial y}(-2\cdot A\cdot x\cdot y + 3\cdot B\cdot y)$$

$$a_y = (3\cdot B\cdot y - 2\cdot A\cdot x\cdot y)\cdot(3\cdot B - 2\cdot A\cdot x) - 2\cdot A\cdot y\cdot\left[A\cdot\left(x^2 - y^2\right) - 3\cdot B\cdot x\right]$$

Hence at (1,1)

$$a_x = (2\cdot 1\cdot 1 - 3\cdot 1)\cdot\frac{1}{s} \times \left(1\cdot 1^2 - 3\cdot 1\cdot 1 + 1\cdot 1^2\right)\cdot\frac{ft}{s} \qquad a_x = 1\cdot\frac{ft}{s^2}$$

$$a_y = (3\cdot 1\cdot 1 - 2\cdot 1\cdot 1\cdot 1)\cdot\frac{1}{s} \times (3\cdot 1 - 2\cdot 1\cdot 1)\cdot\frac{ft}{s} - 2\cdot 1\cdot 1\cdot\frac{1}{s} \times \left[1\cdot\left(1^2 - 1^2\right) - 3\cdot 1\cdot 1\right]\cdot\frac{ft}{s} \qquad a_y = 7\cdot\frac{ft}{s^2}$$

$$a = \sqrt{a_x^2 + a_y^2} \qquad \theta = \operatorname{atan}\left(\frac{a_y}{a_x}\right) \qquad a = 7.1\cdot\frac{ft}{s^2} \qquad \theta = 81.9\cdot deg$$

For the pressure gradient

$$\frac{\partial}{\partial x}p = \rho\cdot g_x - \rho\cdot a_x = -2\cdot\frac{slug}{ft^3} \times 1\cdot\frac{ft}{s^2} \times \frac{lbf\cdot s^2}{slug\cdot ft} \qquad \frac{\partial}{\partial x}p = -2\cdot\frac{\frac{lbf}{ft^2}}{ft} = -0.0139\cdot\frac{psi}{ft}$$

$$\frac{\partial}{\partial y}p = \rho\cdot g_y - \rho\cdot a_y = 2\cdot\frac{slug}{ft^3} \times (-32.2 - 7)\cdot\frac{ft}{s^2} \times \frac{lbf\cdot s^2}{slug\cdot ft} \qquad \frac{\partial}{\partial y}p = -78.4\cdot\frac{\frac{lbf}{ft^2}}{ft} = -0.544\cdot\frac{psi}{ft}$$

Problem 6.6

[3]

6.6 Consider the flow field with velocity given by $\vec{V} = Ax\sin(2\pi\omega t)\hat{i} - Ay\sin(2\pi\omega t)\hat{j}$, where $A = 2\ \mathrm{s}^{-1}$ and $\omega = 1\ \mathrm{s}^{-1}$. The fluid density is 2 kg/m^3. Find expressions for the local acceleration, the convective acceleration, and the total acceleration. Evaluate these at point (1, 1) at $t = 0$, 0.5 and 1 seconds. Evaluate ∇p at the same point and times.

Given: Velocity field

Find: Expressions for local, convective and total acceleration; evaluate at several points; evaluate pressure gradient

Solution:

The given data is $A = 2\cdot\frac{1}{s}$ $\quad\omega = 1\cdot\frac{1}{s}$ $\quad\rho = 2\cdot\frac{kg}{m^3}$ $\quad u = A\cdot x\cdot \sin(2\cdot\pi\cdot\omega\cdot t)$ $\quad v = -A\cdot y\cdot \sin(2\cdot\pi\cdot\omega\cdot t)$

Check for incompressible flow $\quad\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$

Hence $\quad\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = A\cdot\sin(2\cdot\pi\cdot\omega\cdot t) - A\cdot\sin(2\cdot\pi\cdot\omega\cdot t) = 0$ $\quad$ Incompressible flow

The governing equation for acceleration is

$$\vec{a}_p = \underset{\substack{\text{total}\\ \text{acceleration}\\ \text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}}_{\substack{\text{convective}\\ \text{acceleration}}} + \underset{\substack{\text{local}\\ \text{acceleration}}}{\frac{\partial\vec{V}}{\partial t}}$$

The local acceleration is then $\quad$ *x* - component $\quad\frac{\partial}{\partial t}u = 2\cdot\pi\cdot A\cdot\omega\cdot x\cdot\cos(2\cdot\pi\cdot\omega\cdot t)$

y - component $\quad\frac{\partial}{\partial t}v = -2\cdot\pi\cdot A\cdot\omega\cdot y\cdot\cos(2\cdot\pi\cdot\omega\cdot t)$

For the present steady, 2D flow, the convective acceleration is

x - component $\quad u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = A\cdot x\cdot\sin(2\cdot\pi\cdot\omega\cdot t)\cdot(A\cdot\sin(2\cdot\pi\cdot\omega\cdot t)) + (-A\cdot y\cdot\sin(2\cdot\pi\cdot\omega\cdot t))\cdot 0 = A^2\cdot x\cdot\sin(2\cdot\pi\cdot\omega\cdot t)^2$

y - component $\quad u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = A\cdot x\cdot\sin(2\cdot\pi\cdot\omega\cdot t)\cdot 0 + (-A\cdot y\cdot\sin(2\cdot\pi\cdot\omega\cdot t))\cdot(-A\cdot\sin(2\cdot\pi\cdot\omega\cdot t)) = A^2\cdot y\cdot\sin(2\cdot\pi\cdot\omega\cdot t)^2$

The total acceleration is then $\quad$ *x* - component $\quad\frac{\partial}{\partial t}u + u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = 2\cdot\pi\cdot A\cdot\omega\cdot x\cdot\cos(2\cdot\pi\cdot\omega\cdot t) + A^2\cdot x\cdot\sin(2\cdot\pi\cdot\omega\cdot t)^2$

y - component $\quad\frac{\partial}{\partial t}v + u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = -2\cdot\pi\cdot A\cdot\omega\cdot y\cdot\cos(2\cdot\pi\cdot\omega\cdot t) + A^2\cdot y\cdot\sin(2\cdot\pi\cdot\omega\cdot t)^2$

Evaluating at point (1,1) at

$t = 0 \cdot s$ Local $12.6 \cdot \frac{m}{s^2}$ and $-12.6 \cdot \frac{m}{s^2}$ Convective $0 \cdot \frac{m}{s^2}$ and $0 \cdot \frac{m}{s^2}$

Total $12.6 \cdot \frac{m}{s^2}$ and $-12.6 \cdot \frac{m}{s^2}$

$t = 0.5 \cdot s$ Local $-12.6 \cdot \frac{m}{s^2}$ and $12.6 \cdot \frac{m}{s^2}$ Convective $0 \cdot \frac{m}{s^2}$ and $0 \cdot \frac{m}{s^2}$

Total $-12.6 \cdot \frac{m}{s^2}$ and $12.6 \cdot \frac{m}{s^2}$

$t = 1 \cdot s$ Local $12.6 \cdot \frac{m}{s^2}$ and $-12.6 \cdot \frac{m}{s^2}$ Convective $0 \cdot \frac{m}{s^2}$ and $0 \cdot \frac{m}{s^2}$

Total $12.6 \cdot \frac{m}{s^2}$ and $-12.6 \cdot \frac{m}{s^2}$

The governing equation (assuming inviscid flow) for computing the pressure gradient is $\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p$ (6.1)

Hence, the components of pressure gradient (neglecting gravity) are

$$\frac{\partial}{\partial x} p = -\rho \cdot \frac{Du}{Dt} \qquad \frac{\partial}{\partial x} p = -\rho \cdot \left(2 \cdot \pi \cdot A \cdot \omega \cdot x \cdot \cos(2 \cdot \pi \cdot \omega \cdot t) + A^2 \cdot x \cdot \sin(2 \cdot \pi \cdot \omega \cdot t)^2\right)$$

$$\frac{\partial}{\partial y} p = -\rho \cdot \frac{Dv}{Dt} \qquad \frac{\partial}{\partial x} p = -\rho \cdot \left(-2 \cdot \pi \cdot A \cdot \omega \cdot y \cdot \cos(2 \cdot \pi \cdot \omega \cdot t) + A^2 \cdot y \cdot \sin(2 \cdot \pi \cdot \omega \cdot t)^2\right)$$

Evaluated at (1,1) and time

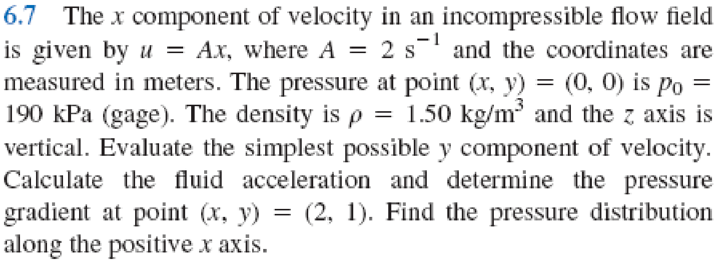
$t = 0 \cdot s$ *x* comp. $-25.1 \cdot \frac{Pa}{m}$ *y* comp. $25.1 \cdot \frac{Pa}{m}$

$t = 0.5 \cdot s$ *x* comp. $25.1 \cdot \frac{Pa}{m}$ *y* comp. $-25.1 \cdot \frac{Pa}{m}$

$t = 1 \cdot s$ *x* comp. $-25.1 \cdot \frac{Pa}{m}$ *y* comp. $25.1 \cdot \frac{Pa}{m}$

Problem 6.7

[2]



6.7 The x component of velocity in an incompressible flow field is given by $u = Ax$, where $A = 2\ \mathrm{s}^{-1}$ and the coordinates are measured in meters. The pressure at point $(x, y) = (0, 0)$ is $p_0 =$ 190 kPa (gage). The density is $\rho = 1.50\ \mathrm{kg/m^3}$ and the z axis is vertical. Evaluate the simplest possible y component of velocity. Calculate the fluid acceleration and determine the pressure gradient at point $(x, y) = (2, 1)$. Find the pressure distribution along the positive x axis.

Given: Velocity field

Find: Simplest y component of velocity; Acceleration of particle and pressure gradient at (2,1); pressure on x axis

Solution:

Basic equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \vec{a}_p = \underbrace{\frac{D\vec{V}}{Dt}}_{\text{total acceleration of a particle}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{convective acceleration}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local acceleration}} \qquad \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

For this flow $u(x,y) = A\cdot x$ $\quad \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ so $v(x,y) = -\int \frac{\partial}{\partial x}u\,dy = -\int A\,dy = -A\cdot y + c$

Hence $v(x,y) = -A\cdot y$ is the simplest y component of velocity

For acceleration

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = A\cdot x\cdot\frac{\partial}{\partial x}(A\cdot x) + (-A\cdot y)\cdot\frac{\partial}{\partial y}(A\cdot x) = A^2\cdot x \qquad a_x = A^2\cdot x$$

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = A\cdot x\cdot\frac{\partial}{\partial x}(-A\cdot y) + (-A\cdot y)\cdot\frac{\partial}{\partial y}(-A\cdot y) \qquad a_y = A^2\cdot y$$

Hence at (2,1)

$$a_x = \left(\frac{2}{s}\right)^2 \times 2\cdot m \qquad a_y = \left(\frac{2}{s}\right)^2 \times 1\cdot m \qquad a_x = 8\frac{m}{s^2} \qquad a_y = 4\frac{m}{s^2}$$

$$a = \sqrt{a_x{}^2 + a_y{}^2} \qquad \theta = \mathrm{atan}\left(\frac{a_y}{a_x}\right) \qquad a = 8.94\frac{m}{s^2} \qquad \theta = 26.6\cdot deg$$

For the pressure gradient

$$\frac{\partial}{\partial x}p = \rho\cdot g_x - \rho\cdot a_x = -1.50\cdot\frac{kg}{m^3}\times 8\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad \frac{\partial}{\partial x}p = -12\cdot\frac{Pa}{m}$$

$$\frac{\partial}{\partial y}p = \rho\cdot g_y - \rho\cdot a_y = -1.50\cdot\frac{kg}{m^3}\times 4\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad \frac{\partial}{\partial y}p = -6\cdot\frac{Pa}{m}$$

$$\frac{\partial}{\partial z}p = \rho\cdot g_z - \rho\cdot a_z = 1.50\times\frac{kg}{m^3}\times(-9.81)\cdot\frac{m}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad \frac{\partial}{\partial y}p = -14.7\cdot\frac{Pa}{m}$$

For the pressure on the x axis $\quad dp = \frac{\partial}{\partial x}p \qquad p - p_0 = \int_0^x \left(\rho\cdot g_x - \rho\cdot a_x\right)dx = \int_0^x \left(-\rho\cdot A^2\cdot x\right)dx = -\frac{1}{2}\cdot\rho\cdot A^2\cdot x^2$

$$p(x) = p_0 - \frac{1}{2}\cdot\rho\cdot A^2\cdot x^2 \qquad p(x) = 190\cdot kPa - \frac{1}{2}\cdot 1.5\cdot\frac{kg}{m^3}\times\left(\frac{2}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m}\times x^2 \qquad p(x) = 190 - \frac{3}{1000}\cdot x^2$$

(p in kPa, x in m)

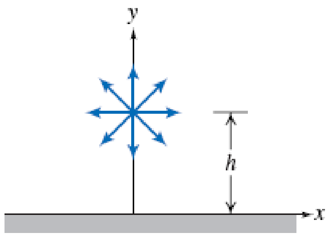
Problem 6.8

[3]

6.8 The velocity field for a plane source located distance $h = 1$ m above an infinite wall aligned along the x axis is given by

$$\vec{V} = \frac{q}{2\pi[x^2 + (y-h)^2]}[x\hat{\imath} + (y-h)\hat{\jmath}] + \frac{q}{2\pi[x^2 + (y+h)^2]}[x\hat{\imath} + (y+h)\hat{\jmath}]$$

where $q = 2\ \mathrm{m^3/s/m}$. The fluid density is 1000 kg/m^3 and body forces are negligible. Derive expressions for the velocity and acceleration of a fluid particle that moves along the wall, and plot from $x = 0$ to $x = +10h$. Verify that the velocity and acceleration normal to the wall are zero. Plot the pressure gradient $\partial p/\partial x$ along the wall. Is the pressure gradient along the wall adverse (does it oppose fluid motion) or not?



Given: Velocity field

Find: Expressions for velocity and acceleration along wall; plot; verify vertical components are zero; plot pressure gradient

Solution:

The given data is $q = 2\cdot\frac{\frac{\mathrm{m}^3}{\mathrm{s}}}{\mathrm{m}}$ $\quad h = 1\cdot\mathrm{m}$ $\quad \rho = 1000\cdot\frac{\mathrm{kg}}{\mathrm{m}^3}$

$$u = \frac{q\cdot x}{2\cdot\pi\left[x^2 + (y-h)^2\right]} + \frac{q\cdot x}{2\cdot\pi\left[x^2 + (y+h)^2\right]} \qquad v = \frac{q\cdot(y-h)}{2\cdot\pi\left[x^2 + (y-h)^2\right]} + \frac{q\cdot(y+h)}{2\cdot\pi\left[x^2 + (y+h)^2\right]}$$

The governing equation for acceleration is

$$\underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\vec{a}_p = \frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial\vec{V}}{\partial t}}$$

For steady, 2D flow this reduces to (after considerable math!)

x - component
$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = -\frac{q^2\cdot x\cdot\left[\left(x^2+y^2\right)^2 - h^2\cdot\left(h^2 - 4\cdot y^2\right)\right]}{\left[x^2 + (y+h)^2\right]^2\cdot\left[x^2 + (y-h)^2\right]^2\cdot\pi^2}$$

y - component
$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = -\frac{q^2\cdot y\cdot\left[\left(x^2+y^2\right)^2 - h^2\cdot\left(h^2 + 4\cdot x^2\right)\right]}{\pi^2\cdot\left[x^2 + (y+h)^2\right]^2\cdot\left[x^2 + (y-h)^2\right]^2}$$

For motion along the wall $\quad y = 0\cdot\mathrm{m}$

$u = \frac{q\cdot x}{\pi\cdot\left(x^2 + h^2\right)}$ $\qquad v = 0$ $\qquad$ (No normal velocity) $\qquad a_x = -\frac{q^2\cdot x\cdot\left(x^2 - h^2\right)}{\pi^2\cdot\left(x^2 + h^2\right)^3}$ $\qquad a_y = 0$ $\qquad$ (No normal acceleration)

The governing equation (assuming inviscid flow) for computing the pressure gradient is $\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$ (6.1)

Hence, the component of pressure gradient (neglecting gravity) along the wall is

$$\frac{\partial}{\partial x}p = -\rho\cdot\frac{Du}{Dt} \qquad \frac{\partial}{\partial x}p = \frac{\rho\cdot q^2\cdot x\cdot\left(x^2 - h^2\right)}{\pi^2\cdot\left(x^2 + h^2\right)^3}$$

The plots of velocity, acceleration, and pressure gradient are shown in the associated *Excel* workbook. From the plots it is clear that the fluid experiences an adverse pressure gradient from the origin to x = 1 m, then a negative one promoting fluid acceleration. If flow separates, it will likely be in the region x = 0 to x = h.

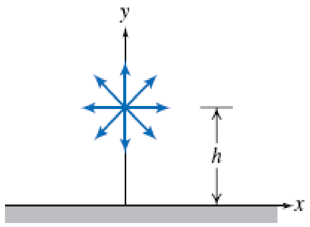
Problem 6.8

[3]

6.8 The velocity field for a plane source located distance $h = 1$ m above an infinite wall aligned along the x axis is given by

$$\vec{V} = \frac{q}{2\pi[x^2+(y-h)^2]}[x\hat{i}+(y-h)\hat{j}] + \frac{q}{2\pi[x^2+(y+h)^2]}[x\hat{i}+(y+h)\hat{j}]$$

where $q = 2$ m^3/s/m. The fluid density is 1000 kg/m^3 and body forces are negligible. Derive expressions for the velocity and acceleration of a fluid particle that moves along the wall, and plot from $x = 0$ to $x = +10h$. Verify that the velocity and acceleration normal to the wall are zero. Plot the pressure gradient $\partial p/\partial x$ along the wall. Is the pressure gradient along the wall adverse (does it oppose fluid motion) or not?



Given: Velocity field

Find: Expressions for velocity and acceleration along wall; plot; verify vertical components are zero; plot pressure gradient

Solution:

The velocity, acceleration and pressure gradient are given by

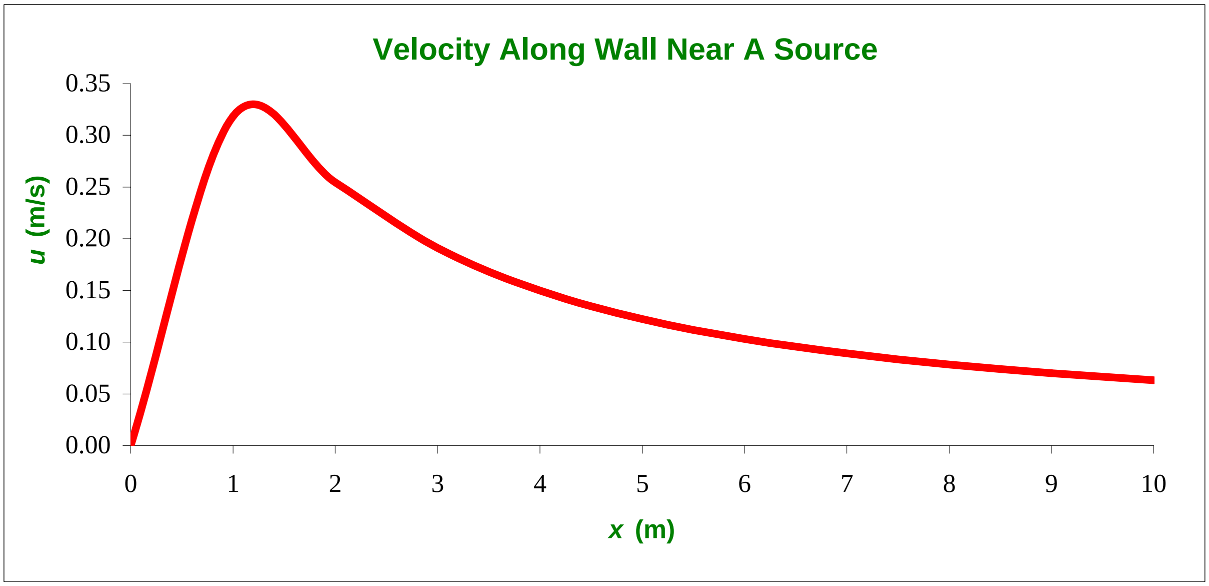
$$u = \frac{q\cdot x}{\pi\cdot\left(x^2+h^2\right)}$$

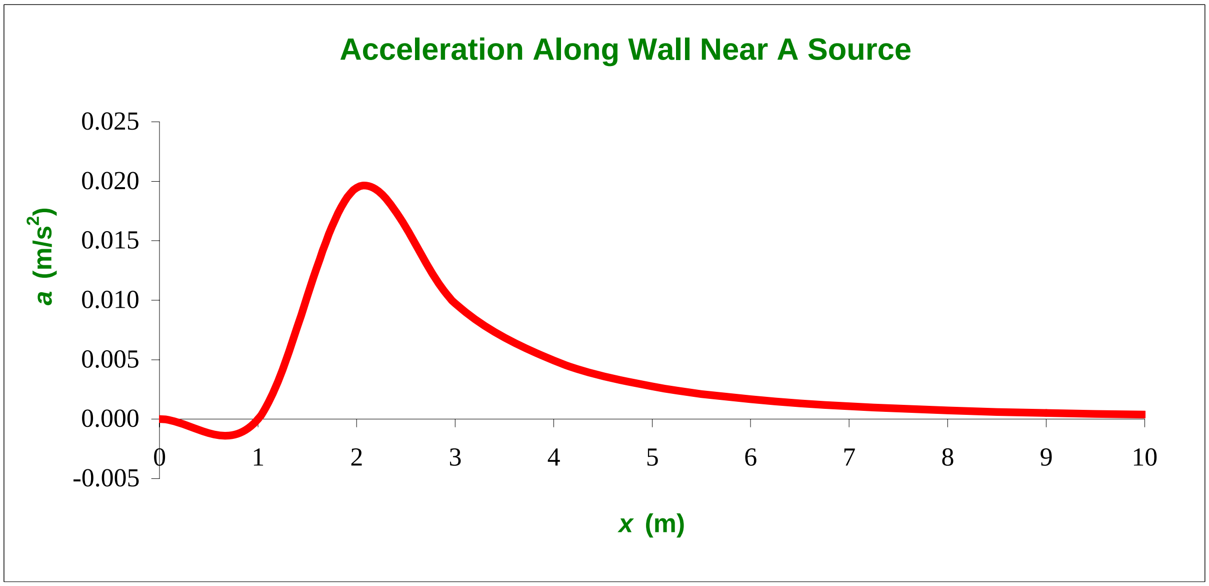
$$a_x = -\frac{q^2\cdot x\cdot\left(x^2-h^2\right)}{\pi^2\cdot\left(x^2+h^2\right)^3}$$

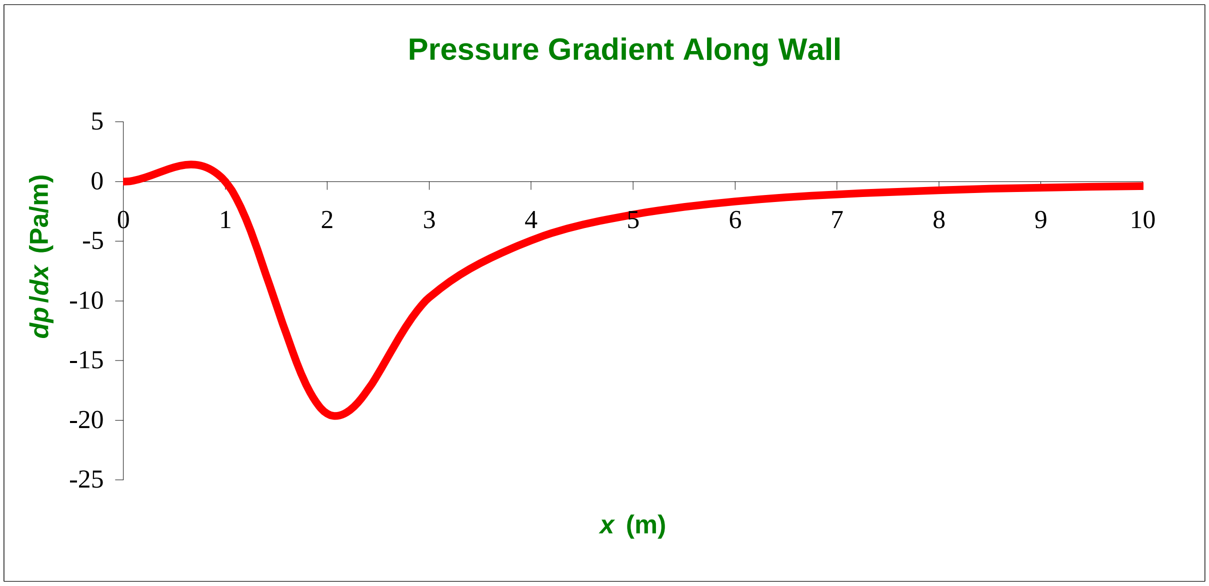
$$\frac{\partial}{\partial x}p = \frac{\rho\cdot q^2\cdot x\cdot\left(x^2-h^2\right)}{\pi^2\cdot\left(x^2+h^2\right)^3}$$

q = 2 m^3/s/m
h = 1 m
ρ = 1000 kg/m^3

x (m)	*u* (m/s)	*a* (m/s^2)	*dp* /*dx* (Pa/m)
0.0	0.00	0.00000	0.00
1.0	0.32	0.00000	0.00
2.0	0.25	0.01945	-19.45
3.0	0.19	0.00973	-9.73
4.0	0.15	0.00495	-4.95
5.0	0.12	0.00277	-2.77
6.0	0.10	0.00168	-1.68
7.0	0.09	0.00109	-1.09
8.0	0.08	0.00074	-0.74
9.0	0.07	0.00053	-0.53
10.0	0.06	0.00039	-0.39


Velocity Along Wall Near A Source
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
u (m/s)
0
1
2
3
4
5
6
7
8
9
10
x (m)


Acceleration Along Wall Near A Source
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
a (m/s2)
0
1
2
3
4
5
6
7
8
9
10
x (m)


Pressure Gradient Along Wall
5
0
-5
-10
-15
-20
-25
dp/dx (Pa/m)
0
1
2
3
4
5
6
7
8
9
10
x (m)

Problem 6.9

Given: The velocity distribution in a steady, 2-D flow field in the xy plane is given by $\vec{V} = (Ax-B)\hat{i} + (C-Ay)\hat{j}$, where $A = 2\ s^{-1}$, $B = 5\ m\cdot s^{-1}$, $C = 3\ m\cdot s^{-1}$, and the body force distribution is $\vec{g} = -g\hat{k}$

Find: (a) Does the velocity field represent the flow of an incompressible fluid?
(b) Find the stagnation point of the flow field.
(c) Obtain an expression for the pressure gradient.
(d) Evaluate Δp between origin and point (1,3) if $\rho = 1.2\ kg/m^3$

Solution:

(a) Apply the continuity equation, $\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\vec{V} = 0$, for the given conditions. If $\rho =$ constant, then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 = \frac{\partial}{\partial x}(2x-5) + \frac{\partial}{\partial y}(3-2y) = 2-2 = 0 \ \checkmark$$

∴ velocity field represents an incompressible flow ⟵

(b) At the stagnation point, $\vec{V} = 0$. For $\vec{V} = 0$, then

$$u = 2x-5 = 0 \quad \text{and} \quad v = (3-2y) = 0$$

Thus stagnation point is at $(x,y) = \left(\frac{5}{2}, \frac{3}{2}\right)$ ⟵

(c) Euler's equation, $\rho\vec{g} - \nabla p = \rho\frac{D\vec{V}}{Dt}$, can be used to obtain an expression for the pressure gradient

$$\nabla p = \rho\vec{g} - \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \rho\left[\underset{\text{steady}}{\cancel{\frac{\partial\vec{V}}{\partial t}}} + u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + \underset{w=0}{\cancel{w\frac{\partial\vec{V}}{\partial z}}}\right].$$

$$\nabla p = \rho\left[\vec{g} - u\frac{\partial\vec{V}}{\partial x} - v\frac{\partial\vec{V}}{\partial y}\right] = \rho\left[-g\hat{k} - (2x-5)2\hat{i} - (3-2y)(-2\hat{j})\right]$$

$$\nabla p = -\rho\left[(4x-10)\hat{i} + (4y-6)\hat{j} + g\hat{k}\right] \longleftarrow \qquad \nabla p$$

(d) Since $p = p(x,y,z)$ we can write

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz = -\rho(4x-10)dx - \rho(4y-6)dy - \rho g\,dz$$

We can integrate to obtain Δp between any two points in the field if, and only if, the integral of the right hand side is independent of the path of integration. This is true for the present case.

$$\therefore p_{1,3} - p_{0,0} = -\rho\left\{\int_0^1 (4x-10)dx + \int_0^3 (4y-6)dy\right\} = -\rho\left\{\left[2x^2-10x\right]_0^1 + \left[2y^2-6y\right]_0^3\right\}$$

$$= -\rho\{-8-0\} = 8\rho$$

$$p_{1,3} - p_{0,0} = 8\frac{m^2}{s^2} \times 1.2\frac{kg}{m^3}\cdot\frac{N\cdot s^2}{kg\cdot m} = 9.6\ N/m^2 \longleftarrow \qquad \Delta p$$

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Problem 6.10

[2]

Given: Frictionless, incompressible flow field with

$$\vec{V} = Ax\hat{i} - Ay\hat{j}$$
$$\vec{g} = -g\hat{k}$$

At $(0,0,0)$ $p = p_0$

Find: Expression for the pressure field $p(x,y,z)$

Solution:

Basic equations: $\rho\vec{B} - \nabla p = \rho\frac{D\vec{V}}{Dt}$

$$\frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}$$

$$\nabla p = \rho\left(\vec{g} - \frac{D\vec{V}}{Dt}\right) = \rho\left(-g\hat{k} - u\frac{\partial\vec{V}}{\partial x} - v\frac{\partial\vec{V}}{\partial y}\right)$$

$$= -\rho\left[g\hat{k} + Ax(A\hat{i}) - Ay(-A\hat{j})\right]$$

$$\nabla p = -\rho\left[A^2x\hat{i} + A^2y\hat{j} + g\hat{k}\right]$$

$$\hat{i}\frac{\partial p}{\partial x} + \hat{j}\frac{\partial p}{\partial y} + \hat{k}\frac{\partial p}{\partial z} = -\rho\left[A^2x\hat{i} + A^2y\hat{j} + g\hat{k}\right]$$

$$\frac{\partial p}{\partial x} = -\rho A^2 x \qquad \frac{\partial p}{\partial y} = -\rho A^2 y \qquad \frac{\partial p}{\partial z} = -\rho g$$

$$p = p(x,y,z)$$

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz = -\rho A^2x\,dx - \rho A^2y\,dy - \rho g\,dz$$

$$* \quad p - p_0 = \int_{p_0}^{p} dp = -\int_0^x \rho A^2x\,dx - \int_0^y \rho A^2y\,dy - \int_0^z \rho g\,dz$$

$$p - p_0 = -\rho\left[\frac{A^2x^2}{2} + \frac{A^2y^2}{2} + gz\right]$$

$$p = p_0 - \rho\left[\frac{A^2x^2}{2} + \frac{A^2y^2}{2} + gz\right] \longleftarrow$$

p

* We can integrate to obtain Δp between any two points in the flow field if, and only if, the integral of the right hand side is independent of the path of integration. This is true for the present case

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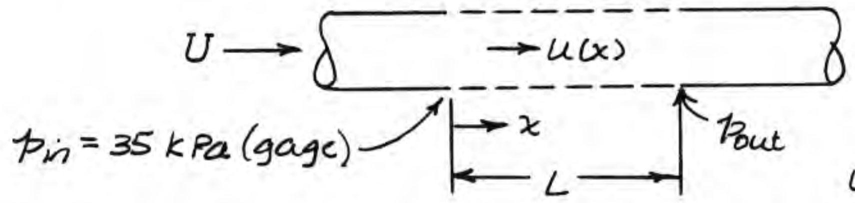
Problem 6.11

[2]

Given: Porous pipe with liquid ($\mu = 0$, $\rho = 900\ \mathrm{kg/m^3}$)



$U = 5$ m/s

$L = 0.3$ m

$u(x) = U(1 - x/2L)$

Find: (a) Expression for acceleration along ℄.

(b) Expression for pressure gradient along ℄.

(c) Evaluate p_{out}

Solution: Computing equations (acceleration and Euler in x-direction)

$$a_{p_x} = u\frac{\partial u}{\partial x} + \overset{=0(1)}{\cancel{v}}\frac{\partial u}{\partial y} + \overset{=0(1)}{\cancel{w}}\frac{\partial u}{\partial z} + \overset{=0(2)}{\cancel{\frac{\partial u}{\partial t}}}\ ; \quad \overset{=0(3)}{\cancel{\rho g_x}} - \frac{\partial p}{\partial x} = \rho a_{p_x}$$

Assumptions: (1) $v = w = 0$ along ℄

(2) steady flow

(3) $g_x = 0$

Then

$$a_{p_x} = u\frac{\partial u}{\partial x} = U\left(1 - \frac{x}{2L}\right)U\left(-\frac{1}{2L}\right) = -\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)$$

a_{p_x}

From Euler

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -\rho a_{p_x} = \rho\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)$$

$\frac{dp}{dx}$

Integrating,

$$p_{out} - p_{in} = \int_0^L \frac{dp}{dx}\,dx = \rho\frac{U^2}{2L}\int_0^L\left(1 - \frac{x}{2L}\right)dx = \rho\frac{U^2}{2L}\left(x - \frac{x^2}{4L}\right)\Big]_0^L$$

or

$$p_{out} = p_{in} + \frac{\rho U^2}{2L}\left(\frac{3}{4}L\right) = p_{in} + \frac{3}{8}\rho U^2$$

$$= 35\ \mathrm{kPa} + \frac{3}{8}\times 900\,\frac{\mathrm{kg}}{\mathrm{m^3}}\times(5)^2\,\frac{\mathrm{m^2}}{\mathrm{s^2}}\times\frac{\mathrm{N\cdot s^2}}{\mathrm{kg\cdot m}}$$

$p_{out} = 43.4$ kPa (gage)

p_{out}

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Problem 6.12

[2]

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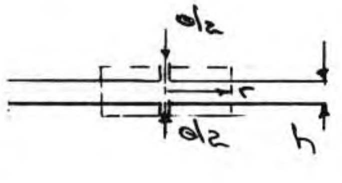
Given: Liquid, ρ = constant and negligible viscosity, is pumped at total volume flow rate, Q, through two small holes into the narrow gap between closely space parallel plates. The liquid flowing away from the holes has only radial motion. Flow may be assumed uniform at any section.

(a) Show that $V_r = Q/2\pi r h$, where h is the spacing between the plates.

(b) Obtain an expression for a_r and $\partial p/\partial r$

Solution:

Apply the conservation of mass to a CV with outer edge at r.



Basic equation: $$0 = \overset{0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) uniform flow at each section

Then

$$0 = \int_{CS} \vec{V}\cdot d\vec{A} = -2 \times \frac{Q}{2} + V_r\, 2\pi r h$$

and $$V_r = \frac{Q}{2\pi r h} \longleftarrow V_r$$

From Eq. 6.4a

$$g_r - \frac{1}{\rho}\frac{\partial p}{\partial r} = a_r = \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}$$

Since $V_r = V_r(r)$ and $V_\theta = 0$, then

$$a_r = V_r \frac{\partial V_r}{\partial r} = \frac{Q}{2\pi r h}\left[\frac{Q}{2\pi h}\left(-\frac{1}{r^2}\right)\right] = -\left(\frac{Q}{2\pi r h}\right)^2 \frac{1}{r}$$

$$a_r = -\frac{V_r^2}{r} \longleftarrow a_r$$

Since $g_r = 0$, then

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = a_r$$

$$\frac{\partial p}{\partial r} = -\rho a_r = \rho \frac{V_r^2}{r} \longleftarrow \frac{\partial p}{\partial r}$$

Problem 6.13

[3]

6.13 The velocity field for a plane vortex sink is given by $\vec{V} = (-q/2\pi r)\hat{e}_r + (K/2\pi r)\hat{e}_\theta$, where $q = 2\ \mathrm{m^3/s/m}$ and $K = 1\ \mathrm{m^3/s/m}$. The fluid density is $1000\ \mathrm{kg/m^3}$. Find the acceleration at $(1, 0)$, $(1, \pi/2)$, and $(2, 0)$. Evaluate ∇p under the same conditions.

Given: Velocity field

Find: The acceleration at several points; evaluate pressure gradient

Solution:

The given data is $q = 2 \cdot \dfrac{\frac{m^3}{s}}{m}$ $\quad K = 1 \cdot \dfrac{\frac{m^3}{s}}{m}$ $\quad \rho = 1000 \cdot \dfrac{kg}{m^3}$ $\quad V_r = -\dfrac{q}{2 \cdot \pi \cdot r}$ $\quad V_\theta = \dfrac{K}{2 \cdot \pi \cdot r}$

The governing equations for this 2D flow are

$$\rho a_r = \rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}\right) = \rho g_r - \frac{\partial p}{\partial r} \qquad (6.3a)$$

$$\rho a_\theta = \rho\left(\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} \qquad (6.3b)$$

The total acceleration for this steady flow is then

r - component $\quad a_r = V_r \cdot \dfrac{\partial}{\partial r}V_r + \dfrac{V_\theta}{r} \cdot \dfrac{\partial}{\partial \theta}V_r \qquad a_r = -\dfrac{q^2}{4 \cdot \pi^2 \cdot r^3}$

θ - component $\quad a_\theta = V_r \cdot \dfrac{\partial}{\partial r}V_\theta + \dfrac{V_\theta}{r} \cdot \dfrac{\partial}{\partial \theta}V_\theta \qquad a_\theta = \dfrac{q \cdot K}{4 \cdot \pi^2 \cdot r^3}$

Evaluating at point (1,0) $\quad a_r = -0.101 \dfrac{m}{s^2} \qquad a_\theta = 0.0507 \dfrac{m}{s^2}$

Evaluating at point (1,π/2) $\quad a_r = -0.101 \dfrac{m}{s^2} \qquad a_\theta = 0.0507 \dfrac{m}{s^2}$

Evaluating at point (2,0) $\quad a_r = -0.0127 \dfrac{m}{s^2} \qquad a_\theta = 0.00633 \dfrac{m}{s^2}$

From Eq. 6.3, pressure gradient is $\quad \dfrac{\partial}{\partial r}p = -\rho \cdot a_r \qquad \dfrac{\partial}{\partial r}p = \dfrac{\rho \cdot q^2}{4 \cdot \pi^2 \cdot r^3}$

$$\frac{1}{r} \cdot \frac{\partial}{\partial \theta}p = -\rho \cdot a_\theta \qquad \frac{1}{r} \cdot \frac{\partial}{\partial \theta}p = -\frac{\rho \cdot q \cdot K}{4 \cdot \pi^2 \cdot r^3}$$

Evaluating at point (1,0) $\quad \dfrac{\partial}{\partial r}p = 101 \cdot \dfrac{Pa}{m} \qquad \dfrac{1}{r} \cdot \dfrac{\partial}{\partial \theta}p = -50.5 \cdot \dfrac{Pa}{m}$

Evaluating at point (1,π/2) $\quad \dfrac{\partial}{\partial r}p = 101 \cdot \dfrac{Pa}{m} \qquad \dfrac{1}{r} \cdot \dfrac{\partial}{\partial \theta}p = -50.5 \cdot \dfrac{Pa}{m}$

Evaluating at point (2,0) $\quad \dfrac{\partial}{\partial r}p = 12.7 \cdot \dfrac{Pa}{m} \qquad \dfrac{1}{r} \cdot \dfrac{\partial}{\partial \theta}p = -6.33 \cdot \dfrac{Pa}{m}$

Problem 6.14

[3]

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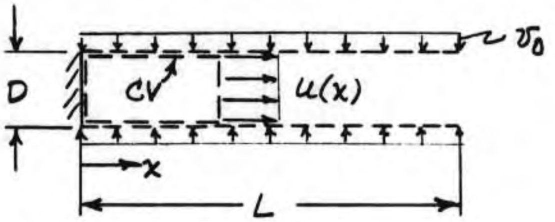
Given: Circular tube with porous wall; incompressible flow, uniform in x direction.



Find: (a) Algebraic expression for a_{p_x} at x.
(b) Pressure gradient at x.
(c) Integrate to obtain p at $x = 0$.

Solution: Apply conservation of mass using the CV shown.

Basic equations:
$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$a_{p_x} = u\frac{\partial u}{\partial x} + \overset{\approx 0(5)}{\cancel{v}}\frac{\partial u}{\partial y} + \overset{\approx 0(5)}{\cancel{w}}\frac{\partial u}{\partial z} + \overset{=0(1)}{\cancel{\frac{\partial u}{\partial t}}} \quad ; \quad -\frac{\partial p}{\partial x} + \rho g_x = \rho a_{p_x}$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each cross-section
(4) Horizontal; $g_x = 0$
(5) $v \approx 0$ in channel ($w \approx 0$ too)
(6) Inviscid flow

Then
$$\int \vec{V}\cdot d\vec{A} = \left\{-\left|v_0 \pi D x\right|\right\} + \left\{+\left|u\,\frac{\pi D^2}{4}\right|\right\} = 0 \quad \text{or} \quad u(x) = 4v_0\frac{x}{D}$$

and
$$a_{p_x} = 4v_0\frac{x}{D}\left(4v_0\frac{1}{D}\right) = 16v_0^2\frac{x}{D^2} \longleftarrow a_{p_x}$$

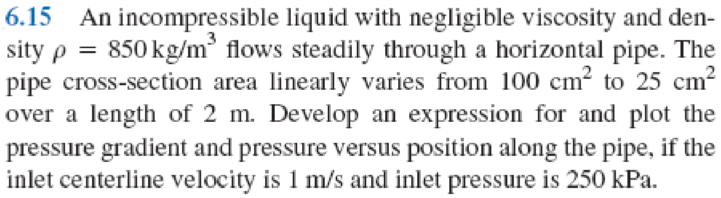
From the Euler equation,
$$-\frac{\partial p}{\partial x} = \rho a_{p_x} \quad \text{so} \quad \frac{\partial p}{\partial x} = -\rho a_{p_x} = -16\rho v_0^2\frac{x}{D^2} \longleftarrow \frac{\partial p}{\partial x}$$

Since $v \approx w \approx 0$, then $p(x)$ and $dp = \frac{\partial p}{\partial x}dx$. Integrating
$$\int_0^L dp = p_L - p(0) = \int_0^L -16\rho v_0^2\frac{x}{D^2}\,dx = -\frac{16\rho v_0^2}{D^2}\left.\frac{x^2}{2}\right]_0^L = -\frac{8\rho v_0^2 L^2}{D^2}$$

Thus, since $p_L = p_{atm}$, the gage pressure at $x = 0$ is
$$p(0) = 8\rho v_0^2\left(\frac{L}{D}\right)^2 \longleftarrow p(0)$$

Problem 6.15

[4]



6.15 An incompressible liquid with negligible viscosity and density $\rho = 850\,\text{kg/m}^3$ flows steadily through a horizontal pipe. The pipe cross-section area linearly varies from 100 cm^2 to 25 cm^2 over a length of 2 m. Develop an expression for and plot the pressure gradient and pressure versus position along the pipe, if the inlet centerline velocity is 1 m/s and inlet pressure is 250 kPa.

Given: Flow in a pipe with variable area

Find: Expression for pressure gradient and pressure; Plot them

Solution:

Assumptions: 1) Incompressible flow 2) Flow profile remains unchanged so centerline velocity can represent average velocity

Basic equations $Q = V\cdot A$

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}} \qquad \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

For this 1D flow $Q = u_i\cdot A_i = u\cdot A$ $\quad A = A_i - \frac{\left(A_i - A_e\right)}{L}\cdot x$ $\quad$ so $\quad u(x) = u_i\cdot\frac{A_i}{A} = u_i\cdot\frac{A_i}{A_i - \left[\frac{\left(A_i - A_e\right)}{L}\cdot x\right]}$

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = u_i\cdot\frac{A_i}{A_i - \left[\frac{\left(A_i - A_e\right)}{L}\cdot x\right]}\cdot\frac{\partial}{\partial x}\left[u_i\cdot\frac{A_i}{A_i - \left[\frac{\left(A_i - A_e\right)}{L}\cdot x\right]}\right] = \frac{A_i^{\,2}\cdot L^2\cdot u_i^{\,2}\cdot\left(A_e - A_i\right)}{\left(A_i\cdot L + A_e\cdot x - A_i\cdot x\right)^3}$$

For the pressure $\quad \frac{\partial}{\partial x}p = -\rho\cdot a_x - \rho\cdot g_x = -\frac{\rho\cdot A_i^{\,2}\cdot L^2\cdot u_i^{\,2}\cdot\left(A_e - A_i\right)}{\left(A_i\cdot L + A_e\cdot x - A_i\cdot x\right)^3}$

and $\quad dp = \frac{\partial}{\partial x}p\cdot dx \qquad p - p_i = \int_0^x \frac{\partial}{\partial x}p\,dx = \int_0^x -\frac{\rho\cdot A_i^{\,2}\cdot L^2\cdot u_i^{\,2}\cdot\left(A_e - A_i\right)}{\left(A_i\cdot L + A_e\cdot x - A_i\cdot x\right)^3}\,dx$

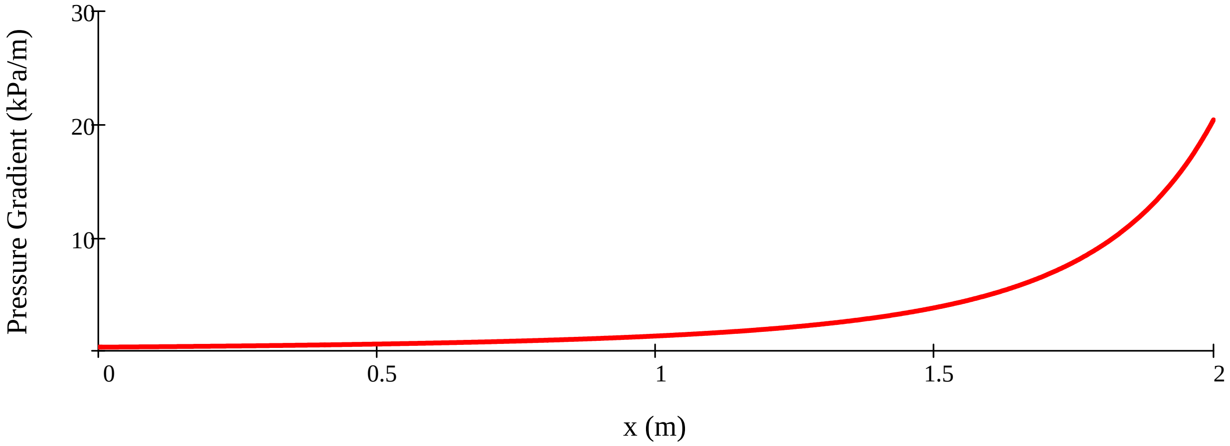
This is a tricky integral, so instead consider the following: $\quad \frac{\partial}{\partial x}p = -\rho\cdot a_x = -\rho\cdot u\cdot\frac{\partial}{\partial x}u = -\frac{1}{2}\cdot\rho\cdot\frac{\partial}{\partial x}\left(u^2\right)$

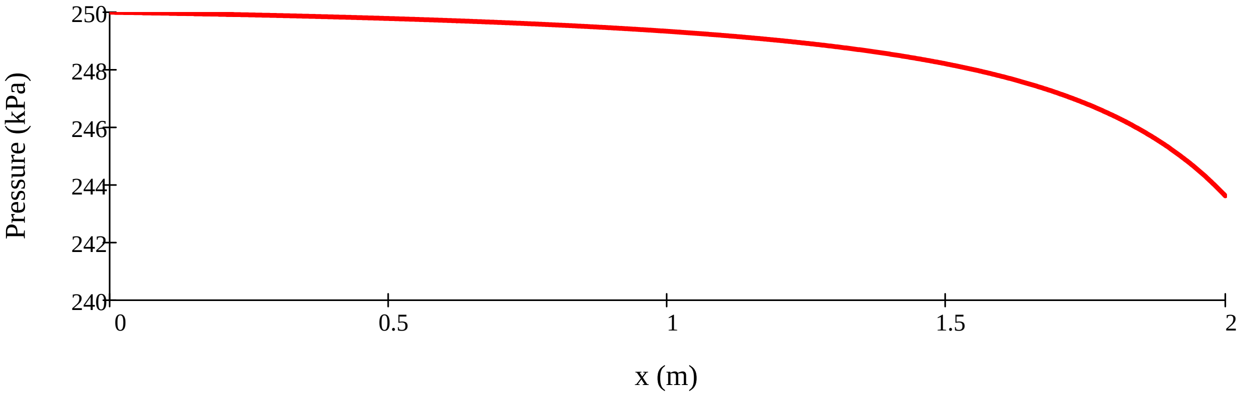
Hence $\quad p - p_i = \int_0^x \frac{\partial}{\partial x}p\,dx = -\frac{\rho}{2}\cdot\int_0^x \frac{\partial}{\partial x}\left(u^2\right)dx = \frac{\rho}{2}\cdot\left(u(x=0)^2 - u(x)^2\right)$

$p(x) = p_i + \frac{\rho}{2}\cdot\left(u_i^{\,2} - u(x)^2\right)$ $\qquad$ which we recognise as the Bernoulli equation!

$$p(x) = p_i + \frac{\rho\cdot u_i^{\,2}}{2}\cdot\left[1 - \left[\frac{A_i}{A_i - \left[\frac{\left(A_i - A_e\right)}{L}\cdot x\right]}\right]^2\right]$$

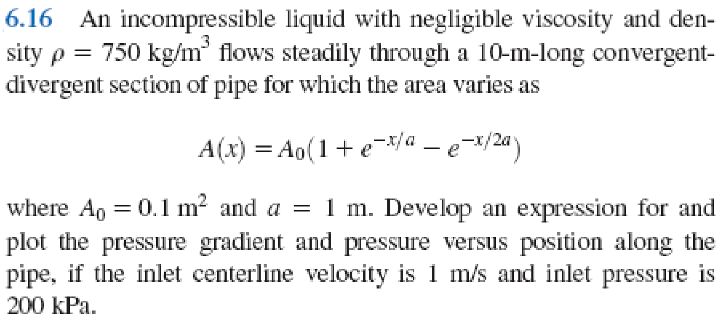
The following plots can be done in *Excel*





Problem 6.16

[4]



6.16 An incompressible liquid with negligible viscosity and density $\rho = 750\ \mathrm{kg/m^3}$ flows steadily through a 10-m-long convergent-divergent section of pipe for which the area varies as

$$A(x) = A_0(1 + e^{-x/a} - e^{-x/2a})$$

where $A_0 = 0.1\ \mathrm{m}^2$ and $a = 1$ m. Develop an expression for and plot the pressure gradient and pressure versus position along the pipe, if the inlet centerline velocity is 1 m/s and inlet pressure is 200 kPa.

Given:

Flow in a pipe with variable area

Find:

Expression for pressure gradient and pressure; Plot them

Solution:

Assumptions: 1) Incompressible flow 2) Flow profile remains unchanged so centerline velocity can represent average velocity

Basic equations

$$Q = V\cdot A \qquad \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\vec{a}_p = \frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}} \qquad \rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

For this 1D flow

$$Q = u_0\cdot A_0 = u\cdot A \qquad A(x) = A_0\cdot\left(1 + e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}}\right)$$

so

$$u(x) = u_0\cdot\frac{A_0}{A} = \frac{u_0}{\left(1 + e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}}\right)}$$

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = \frac{u_0}{\left(1 + e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}}\right)}\cdot\frac{\partial}{\partial x}\left[\frac{u_0}{\left(1 + e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}}\right)}\right] = \frac{u_0^{\,2}\cdot e^{-\frac{x}{2\cdot a}}\cdot\left(2\cdot e^{-\frac{x}{2\cdot a}} - 1\right)}{2\cdot a\cdot\left(e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}} + 1\right)^3}$$

For the pressure

$$\frac{\partial}{\partial x}p = -\rho\cdot a_x - \rho\cdot g_x = -\frac{\rho\cdot u_0^{\,2}\cdot e^{-\frac{x}{2\cdot a}}\cdot\left(2\cdot e^{-\frac{x}{2\cdot a}} - 1\right)}{2\cdot a\cdot\left(e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}} + 1\right)^3}$$

and

$$dp = \frac{\partial}{\partial x}p\cdot dx \qquad p - p_i = \int_0^x \frac{\partial}{\partial x}p\,dx = \int_0^x -\frac{\rho\cdot u_0^{\,2}\cdot e^{-\frac{x}{2\cdot a}}\cdot\left(2\cdot e^{-\frac{x}{2\cdot a}} - 1\right)}{2\cdot a\cdot\left(e^{-\frac{x}{a}} - e^{-\frac{x}{2\cdot a}} + 1\right)^3}\,dx$$

This is a tricky integral, so instead consider the following:

$$\frac{\partial}{\partial x}p = -\rho\cdot a_x = -\rho\cdot u\cdot\frac{\partial}{\partial x}u = -\frac{1}{2}\cdot\rho\cdot\frac{\partial}{\partial x}\left(u^2\right)$$

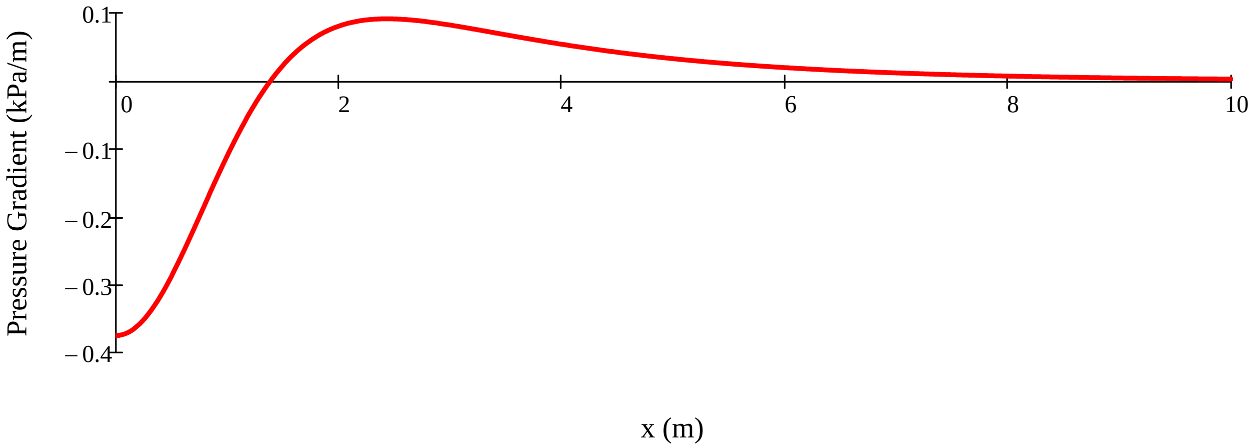
Hence

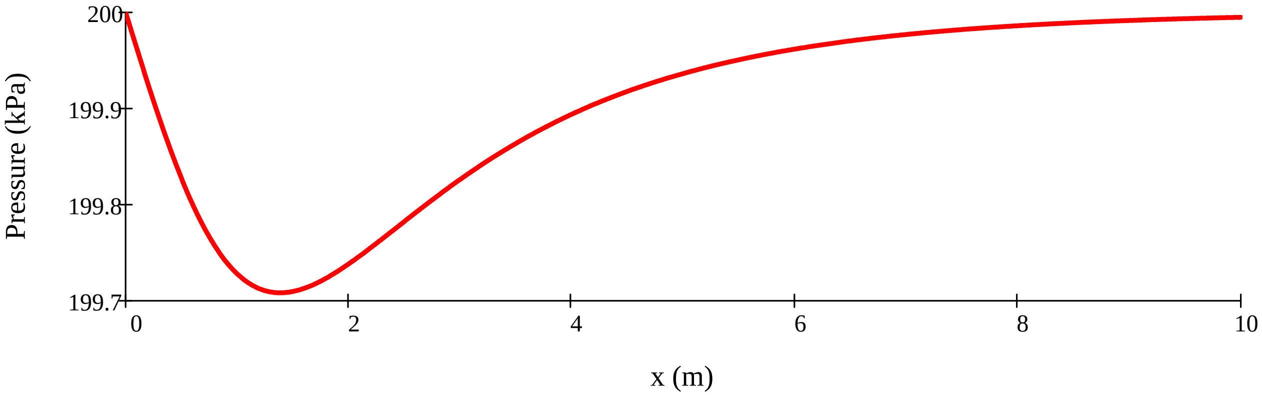
$$p - p_i = \int_0^x \frac{\partial}{\partial x} p \, dx = -\frac{\rho}{2} \cdot \int_0^x \frac{\partial}{\partial x}\left(u^2\right) dx = \frac{\rho}{2} \cdot \left(u(x = 0)^2 - u(x)^2\right)$$

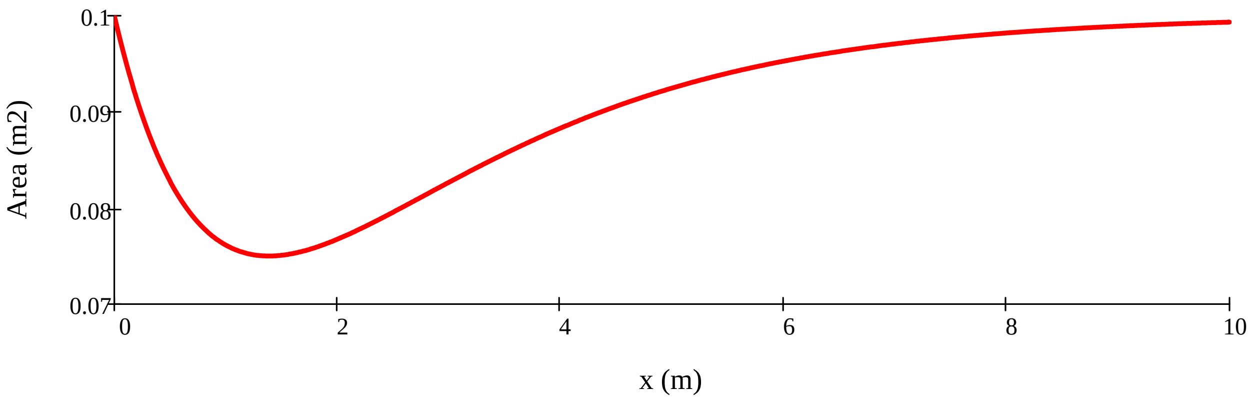
$$p(x) = p_0 + \frac{\rho}{2} \cdot \left(u_0^{\ 2} - u(x)^2\right)$$ which we recognise as the Bernoulli equation!

$$p(x) = p_0 + \frac{\rho \cdot u_0^{\ 2}}{2} \cdot \left[1 - \left[\frac{1}{\left(1 + e^{-\frac{x}{a}} - e^{-\frac{x}{2 \cdot a}}\right)}\right]^2\right]$$

The following plots can be done in *Excel*

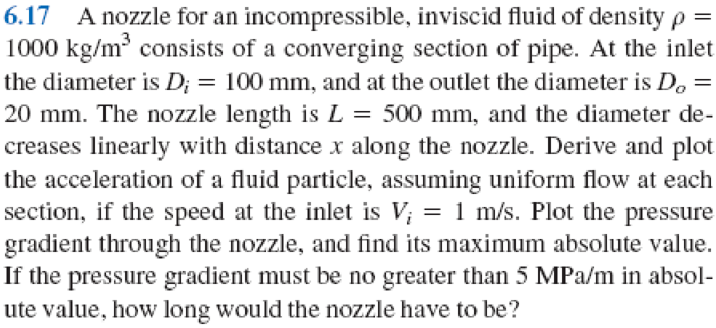






Problem 6.17

[3]



6.17 A nozzle for an incompressible, inviscid fluid of density $\rho = 1000\ \text{kg/m}^3$ consists of a converging section of pipe. At the inlet the diameter is $D_i = 100$ mm, and at the outlet the diameter is $D_o = 20$ mm. The nozzle length is $L = 500$ mm, and the diameter decreases linearly with distance x along the nozzle. Derive and plot the acceleration of a fluid particle, assuming uniform flow at each section, if the speed at the inlet is $V_i = 1$ m/s. Plot the pressure gradient through the nozzle, and find its maximum absolute value. If the pressure gradient must be no greater than 5 MPa/m in absolute value, how long would the nozzle have to be?

Given: Nozzle geometry

Find: Acceleration of fluid particle; Plot; Plot pressure gradient; find *L* such that pressure gradient < 5 MPa/m in absolute value

Solution:

The given data is $D_i = 0.1\cdot\text{m}$ $\quad D_o = 0.02\cdot\text{m}$ $\quad L = 0.5\cdot\text{m}$ $\quad V_i = 1\cdot\frac{\text{m}}{\text{s}}$ $\quad \rho = 1000\cdot\frac{\text{kg}}{\text{m}^3}$

For a linear decrease in diameter $\quad D(x) = D_i + \frac{D_o - D_i}{L}\cdot x$

From continuity $\quad Q = V\cdot A = V\cdot\frac{\pi}{4}\cdot D^2 = V_i\cdot\frac{\pi}{4}\cdot D_i^{\,2}$ $\quad Q = 0.00785\frac{\text{m}^3}{\text{s}}$

Hence $\quad V(x)\cdot\frac{\pi}{4}\cdot D(x)^2 = Q$ $\quad V(x) = \frac{4\cdot Q}{\pi\cdot\left(D_i + \frac{D_o - D_i}{L}\cdot x\right)^2}$

or $\quad V(x) = \frac{V_i}{\left(1 + \frac{D_o - D_i}{L\cdot D_i}\cdot x\right)^2}$

The governing equation for this flow is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} \qquad (6.2\text{a})$$

or, for steady 1D flow, in the notation of the problem

$$a_x = V\cdot\frac{d}{dx}V = \frac{V_i}{\left(1 + \frac{D_o - D_i}{L\cdot D_i}\cdot x\right)^2}\cdot\frac{d}{dx}\frac{V_i}{\left(1 + \frac{D_o - D_i}{L\cdot D_i}\cdot x\right)^2} \qquad a_x(x) = -\frac{2\cdot V_i^{\,2}\cdot\left(D_o - D_i\right)}{D_i\cdot L\cdot\left[1 + \frac{\left(D_o - D_i\right)}{D_i\cdot L}\cdot x\right]^5}$$

This is plotted in the associated *Excel* workbook

From Eq. 6.2a, pressure gradient is

$$\frac{\partial}{\partial x}p = -\rho\cdot a_x \qquad \frac{\partial}{\partial x}p = \frac{2\cdot\rho\cdot V_i^{\,2}\cdot\left(D_o - D_i\right)}{D_i\cdot L\cdot\left[1 + \frac{\left(D_o - D_i\right)}{D_i\cdot L}\cdot x\right]^5}$$

This is also plotted in the associated *Excel* workbook. Note that the pressure gradient is always negative: separation is unlikely to occur in the nozzle

At the inlet $\frac{\partial}{\partial x}p = -3.2 \cdot \frac{kPa}{m}$ At the exit $\frac{\partial}{\partial x}p = -10 \cdot \frac{MPa}{m}$

To find the length *L* for which the absolute pressure gradient is no more than 5 MPa/m, we need to solve

$$\left|\frac{\partial}{\partial x}p\right| \le 5 \cdot \frac{MPa}{m} = \frac{2 \cdot \rho \cdot V_i^2 \cdot \left(D_o - D_i\right)}{D_i \cdot L \cdot \left[1 + \frac{\left(D_o - D_i\right)}{D_i \cdot L} \cdot x\right]^5}$$

with *x* = *L* m (the largest pressure gradient is at the outlet)

Hence $$L \ge \frac{2 \cdot \rho \cdot V_i^2 \cdot \left(D_o - D_i\right)}{D_i \cdot \left(\frac{D_o}{D_i}\right)^5 \cdot \left|\frac{\partial}{\partial x}p\right|}$$ $L \ge 1 \cdot m$

This result is also obtained using *Goal Seek* in the *Excel* workbook

Problem 6.17

[3]

6.17 A nozzle for an incompressible, inviscid fluid of density $\rho = 1000\ \text{kg/m}^3$ consists of a converging section of pipe. At the inlet the diameter is $D_i = 100$ mm, and at the outlet the diameter is $D_o = 20$ mm. The nozzle length is $L = 500$ mm, and the diameter decreases linearly with distance x along the nozzle. Derive and plot the acceleration of a fluid particle, assuming uniform flow at each section, if the speed at the inlet is $V_i = 1$ m/s. Plot the pressure gradient through the nozzle, and find its maximum absolute value. If the pressure gradient must be no greater than 5 MPa/m in absolute value, how long would the nozzle have to be?

Given: Nozzle geometry

Find: Acceleration of fluid particle; Plot; Plot pressure gradient; find L such that pressure gradient < 5 MPa/m in absolute value

Solution:

The acceleration and pressure gradient are given by

$$a_x(x) = -\frac{2\cdot V_i^2\cdot(D_o - D_i)}{D_i\cdot L\cdot\left[1+\frac{(D_o - D_i)}{D_i\cdot L}\cdot x\right]^5}$$

$$\frac{\partial}{\partial x}p = \frac{2\cdot\rho\cdot V_i^2\cdot(D_o - D_i)}{D_i\cdot L\cdot\left[1+\frac{(D_o - D_i)}{D_i\cdot L}\cdot x\right]^5}$$

$D_i =$ 0.1 m

$D_o =$ 0.02 m

$L =$ 0.5 m

$V_i =$ 1 m/s

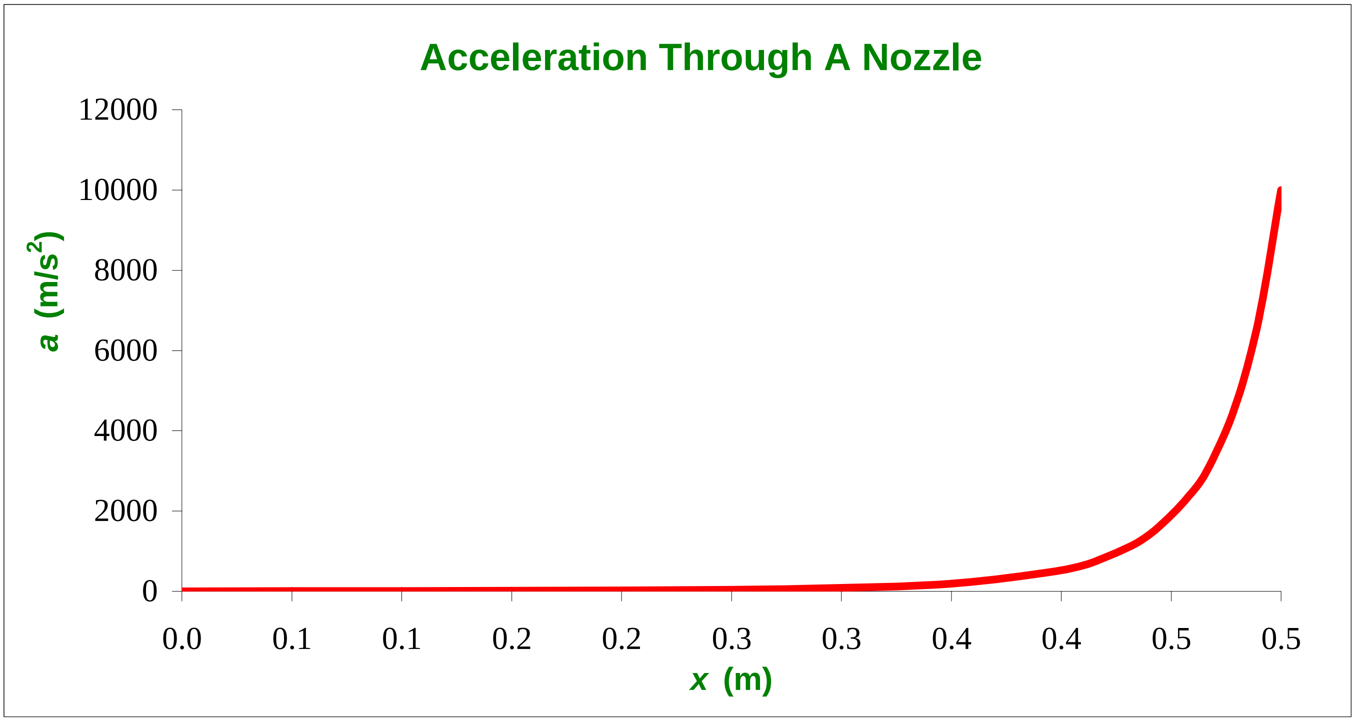
$\rho =$ 1000 kg/m^3

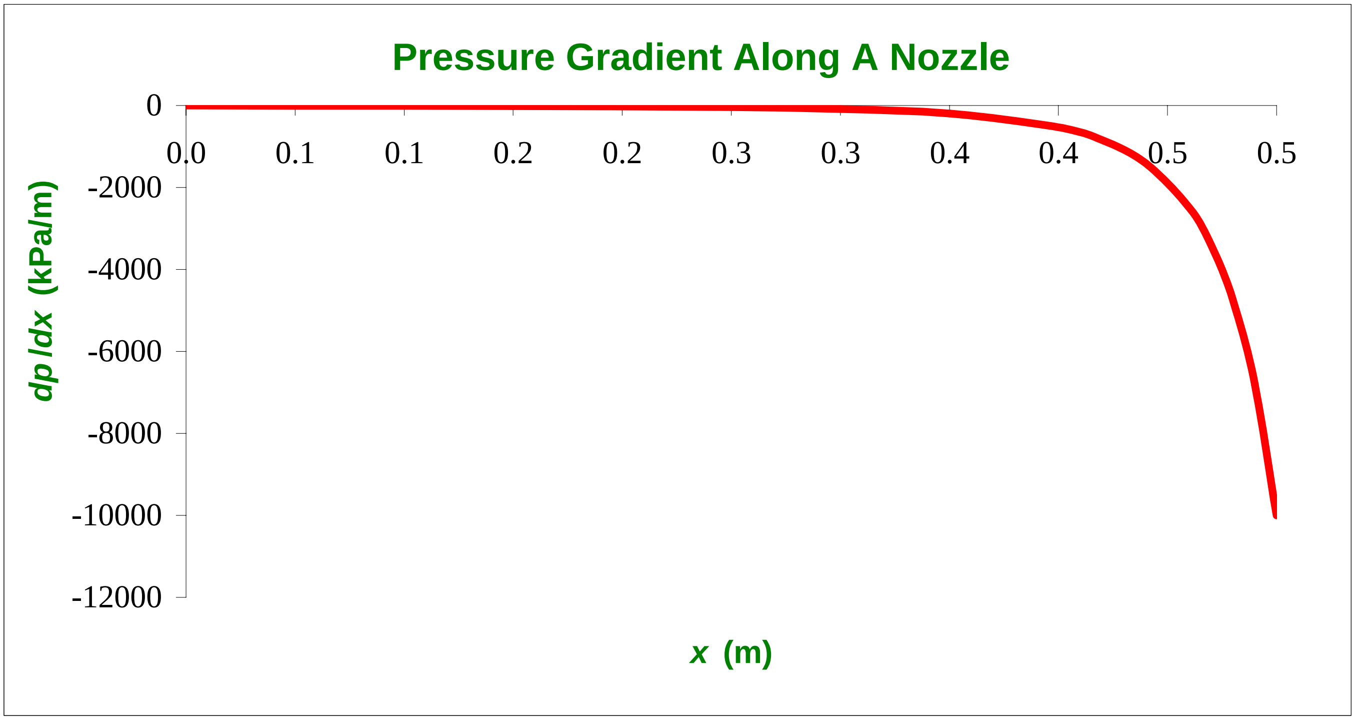
x (m)	*a* (m/s^2)	*dp*/*dx* (kPa/m)
0.000	3.20	-3.20
0.050	4.86	-4.86
0.100	7.65	-7.65
0.150	12.6	-12.6
0.200	22.0	-22.0
0.250	41.2	-41.2
0.300	84.2	-84.2
0.350	194	-194
0.400	529	-529
0.420	843	-843
0.440	1408	-1408
0.460	2495	-2495
0.470	3411	-3411
0.480	4761	-4761
0.490	6806	-6806
0.500	10000	-10000

For the length L required for the pressure gradient to be less than 5 MPa/m (abs) use *Goal Seek*

$L =$ 1.00 m

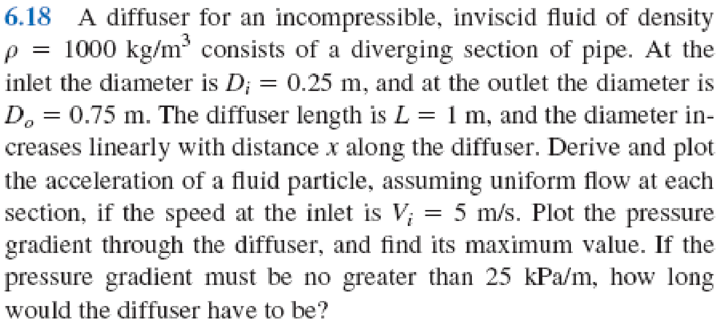
x (m)	*dp*/*dx* (kPa/m)
1.00	-5000


Acceleration Through A Nozzle
12000
10000
8000
6000
4000
2000
0
a (m/s2)
0.0
0.1
0.1
0.2
0.2
0.3
0.3
0.4
0.4
0.5
0.5
x (m)


Pressure Gradient Along A Nozzle
0
-2000
-4000
-6000
-8000
-10000
-12000
dp/dx (kPa/m)
0.0
0.1
0.1
0.2
0.2
0.3
0.3
0.4
0.4
0.5
0.5
x (m)

Problem 6.18

[3]



6.18 A diffuser for an incompressible, inviscid fluid of density $\rho = 1000\ \text{kg/m}^3$ consists of a diverging section of pipe. At the inlet the diameter is $D_i = 0.25$ m, and at the outlet the diameter is $D_o = 0.75$ m. The diffuser length is $L = 1$ m, and the diameter increases linearly with distance x along the diffuser. Derive and plot the acceleration of a fluid particle, assuming uniform flow at each section, if the speed at the inlet is $V_i = 5$ m/s. Plot the pressure gradient through the diffuser, and find its maximum value. If the pressure gradient must be no greater than 25 kPa/m, how long would the diffuser have to be?

Given: Diffuser geometry

Find: Acceleration of a fluid particle; plot it; plot pressure gradient; find *L* such that pressure gradient is less than 25 kPa/m

Solution:

The given data is $D_i = 0.25\cdot\text{m}$ $D_o = 0.75\cdot\text{m}$ $L = 1\cdot\text{m}$ $V_i = 5\cdot\frac{\text{m}}{\text{s}}$ $\rho = 1000\cdot\frac{\text{kg}}{\text{m}^3}$

For a linear increase in diameter $D(x) = D_i + \frac{D_o - D_i}{L}\cdot x$

From continuity $Q = V\cdot A = V\cdot\frac{\pi}{4}\cdot D^2 = V_i\cdot\frac{\pi}{4}\cdot D_i^{\,2}$ $Q = 0.245\,\frac{\text{m}^3}{\text{s}}$

Hence $V(x)\cdot\frac{\pi}{4}\cdot D(x)^2 = Q$ $V(x) = \dfrac{4\cdot Q}{\pi\cdot\left(D_i + \frac{D_o - D_i}{L}\cdot x\right)^2}$ or $V(x) = \dfrac{V_i}{\left(1 + \frac{D_o - D_i}{L\cdot D_i}\cdot x\right)^2}$

The governing equation for this flow is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} \qquad (6.2a)$$

or, for steady 1D flow, in the notation of the problem $a_x = V\cdot\frac{d}{dx}V = \dfrac{V_i}{\left(1 + \frac{D_o - D_i}{L\cdot D_i}\cdot x\right)^2}\cdot\dfrac{d}{dx}\dfrac{V_i}{\left(1 + \frac{D_o - D_i}{L\cdot D_i}\cdot x\right)^2}$

Hence $a_x(x) = -\dfrac{2\cdot V_i^{\,2}\cdot\left(D_o - D_i\right)}{D_i\cdot L\cdot\left[1 + \frac{\left(D_o - D_i\right)}{D_i\cdot L}\cdot x\right]^5}$

This is plotted in the associated *Excel* workbook

From Eq. 6.2a, pressure gradient is $\frac{\partial}{\partial x}p = -\rho\cdot a_x$ $\frac{\partial}{\partial x}p = \dfrac{2\cdot\rho\cdot V_i^{\,2}\cdot\left(D_o - D_i\right)}{D_i\cdot L\cdot\left[1 + \frac{\left(D_o - D_i\right)}{D_i\cdot L}\cdot x\right]^5}$

This is also plotted in the associated *Excel* workbook. Note that the pressure gradient is adverse: separation is likely to occur in the diffuser, and occur near the entrance

At the inlet $\frac{\partial}{\partial x}p = 100 \cdot \frac{kPa}{m}$ At the exit $\frac{\partial}{\partial x}p = 412 \cdot \frac{Pa}{m}$

To find the length *L* for which the pressure gradient is no more than 25 kPa/m, we need to solve

$$\frac{\partial}{\partial x}p \le 25 \cdot \frac{kPa}{m} = \frac{2 \cdot \rho \cdot V_i^{\,2} \cdot \left(D_o - D_i\right)}{D_i \cdot L \cdot \left[1 + \frac{\left(D_o - D_i\right)}{D_i \cdot L} \cdot x\right]^5}$$

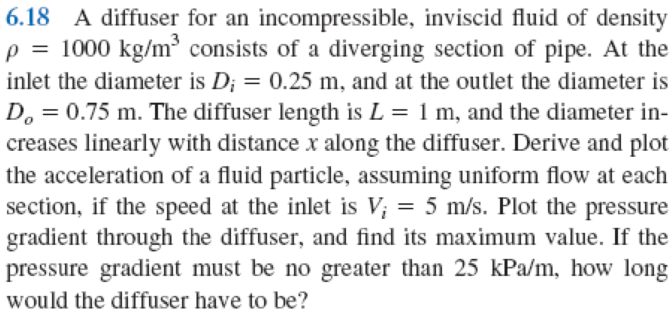
with *x* = 0 m (the largest pressure gradient is at the inlet)

Hence $L \ge \frac{2 \cdot \rho \cdot V_i^{\,2} \cdot \left(D_o - D_i\right)}{D_i \cdot \frac{\partial}{\partial x}p}$ $L \ge 4 \cdot m$

This result is also obtained using *Goal Seek* in the *Excel* workbook

Problem 6.18

[3]



6.18 A diffuser for an incompressible, inviscid fluid of density $\rho = 1000$ kg/m^3 consists of a diverging section of pipe. At the inlet the diameter is $D_i = 0.25$ m, and at the outlet the diameter is $D_o = 0.75$ m. The diffuser length is $L = 1$ m, and the diameter increases linearly with distance x along the diffuser. Derive and plot the acceleration of a fluid particle, assuming uniform flow at each section, if the speed at the inlet is $V_i = 5$ m/s. Plot the pressure gradient through the diffuser, and find its maximum value. If the pressure gradient must be no greater than 25 kPa/m, how long would the diffuser have to be?

Given: Diffuser geometry

Find: Acceleration of a fluid particle; plot it; plot pressure gradient; find L such that pressure gradient is less than 25 kPa/m

Solution:

The acceleration and pressure gradient are given by

$D_i =$ 0.25 m

$D_o =$ 0.75 m

$L =$ 1 m

$V_i =$ 5 m/s

$\rho =$ 1000 kg/m^3

$$a_x(x) = -\frac{2\cdot V_i^2\cdot(D_o - D_i)}{D_i\cdot L\cdot\left[1 + \frac{(D_o - D_i)}{D_i\cdot L}\cdot x\right]^5}$$

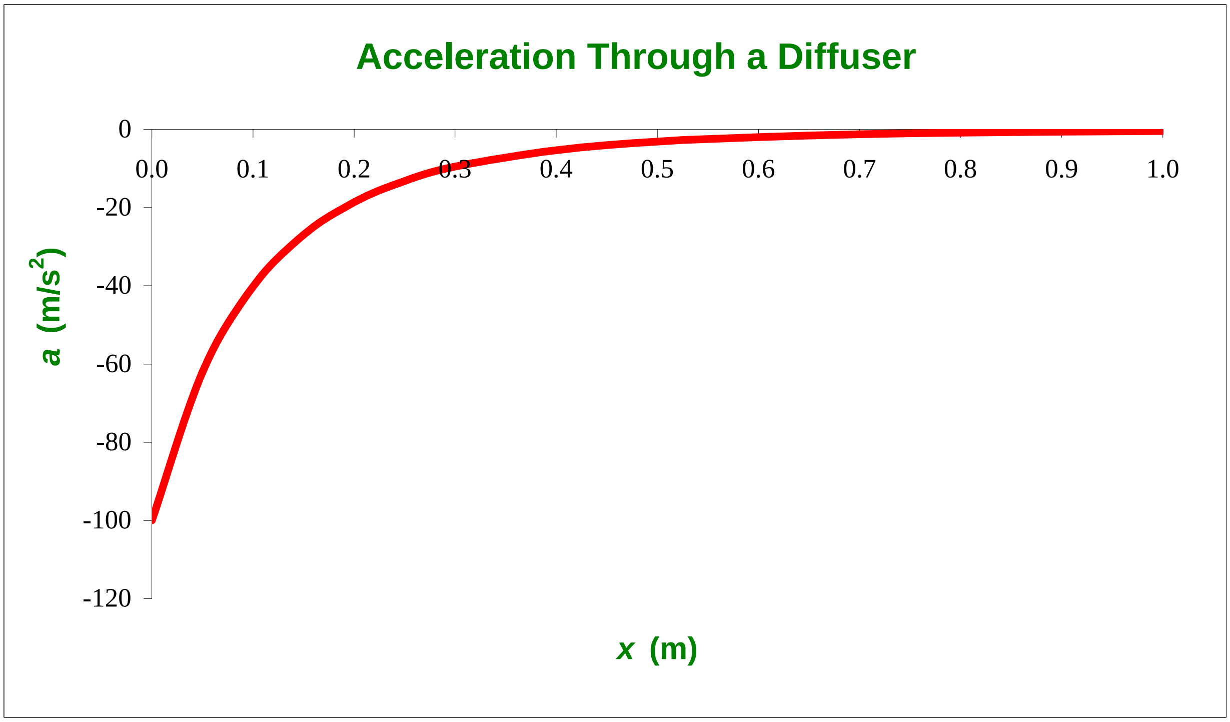
$$\frac{\partial}{\partial x}p = \frac{2\cdot\rho\cdot V_i^2\cdot(D_o - D_i)}{D_i\cdot L\cdot\left[1 + \frac{(D_o - D_i)}{D_i\cdot L}\cdot x\right]^5}$$

x (m)	a (m/s^2)	dp/dx (kPa/m)
0.00	-100	100
0.05	-62.1	62.1
0.10	-40.2	40.2
0.15	-26.9	26.93
0.20	-18.59	18.59
0.25	-13.17	13.17
0.30	-9.54	9.54
0.40	-5.29	5.29
0.50	-3.125	3.125
0.60	-1.940	1.940
0.70	-1.256	1.256
0.80	-0.842	0.842
0.90	-0.581	0.581
1.00	-0.412	0.412

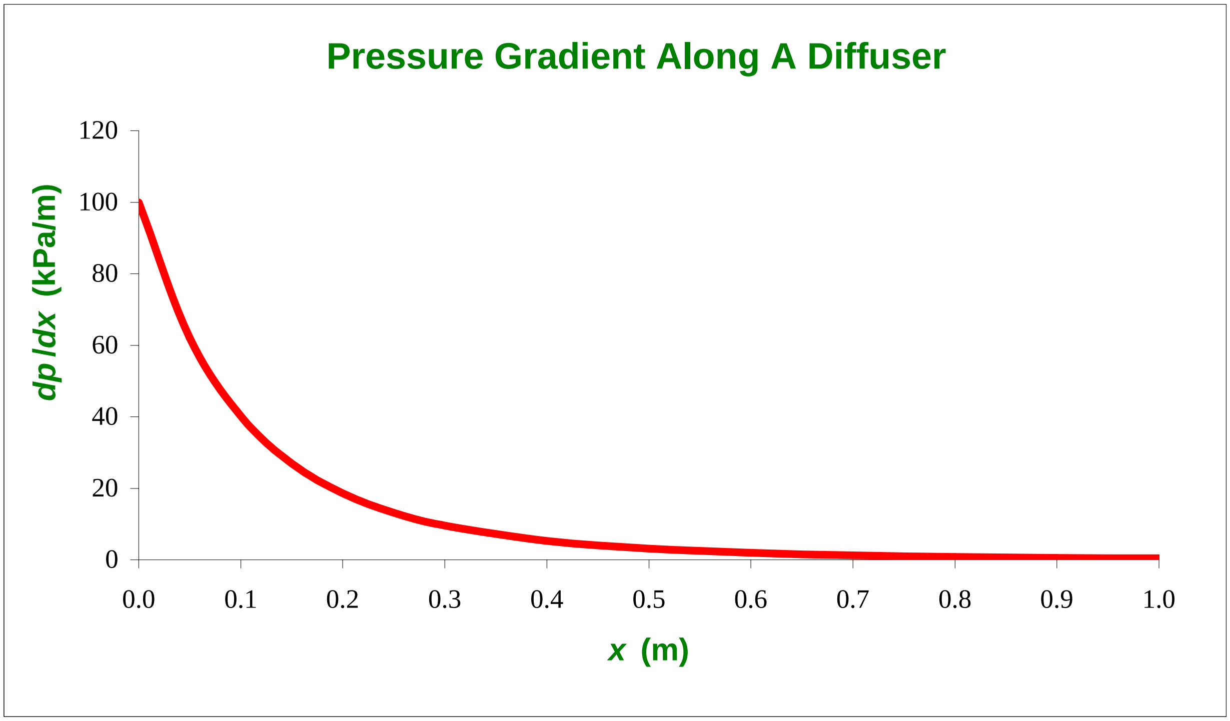
For the length L required for the pressure gradient to be less than 25 kPa/m use *Goal Seek*

$L =$ 4.00 m

x (m)	dp/dx (kPa/m)
0.0	25.0



Acceleration Through a Diffuser



Pressure Gradient Along A Diffuser

Problem 6.19

[4]

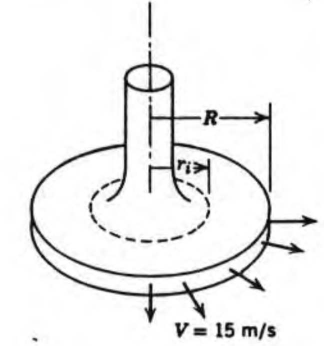
42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Steady, incompressible flow of air between parallel discs as shown

$\vec{V} = V \frac{R}{r} \hat{e}_r$ for $r_i \le r \le R$

where $V = 15$ m/s, $R = 75$ mm, $r_i = R/2$

Find: magnitude and direction of the net pressure force that acts on the upper plate between r_i and R.

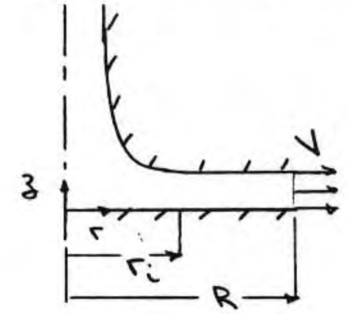


Solution:

Basic equations: $\rho\vec{g} - \nabla p = \rho \frac{D\vec{V}}{Dt}$ $\qquad \vec{F} = -\int p\, d\vec{A}$

Assumptions:
(1) incompressible flow
(2) steady flow
(3) frictionless flow
(4) uniform flow at each section.

To determine the pressure distribution $p(r)$, apply Eulers equation in the r direction



$$-\frac{\partial p}{\partial r} + \cancel{\rho g_r} = \rho a_r = \rho V_r \frac{\partial V_r}{\partial r}$$

$$\frac{\partial p}{\partial r} = -\rho V_r \frac{\partial V_r}{\partial r} = -\rho V \frac{R}{r} \frac{\partial}{\partial r}\left(V\frac{R}{r}\right) = \rho V \frac{R}{r} \frac{VR}{r^2}$$

$$\frac{dp}{dr} = \rho V^2 \frac{R^2}{r^3}$$

$$dp = \rho V^2 \frac{R^2}{r^3}\, dr$$

Integrating we obtain

$$p - p_{atm} = \int_{p_{atm}}^{p} dp = \rho V^2 R^2 \int_R^r r^{-3}\, dr = \rho V^2 R^2 \left[-\frac{1}{2r^2}\right]_R^r = \frac{1}{2}\rho V^2 R^2 \left[\frac{1}{R^2} - \frac{1}{r^2}\right]$$

Then

$$F_z = \int (p - p_{atm})\, dA = \int_{R/2}^{R} \frac{1}{2}\rho V^2 R^2 \left[\frac{1}{R^2} - \frac{1}{r^2}\right] 2\pi r\, dr = \rho V^2 R^2 \pi \left[\frac{r^2}{2R^2} - \ln r\right]_{R/2}^{R}$$

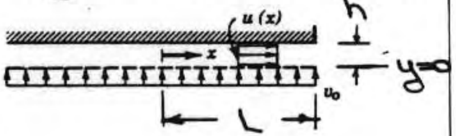
$$= \rho V^2 R^2 \pi \left[\frac{1}{2R^2}\left(R^2 - \frac{R^2}{4}\right) - \ln \frac{R}{R/2}\right] = \rho V^2 R^2 \pi \left[0.375 - \ln 2\right] = -0.318\pi \rho V^2 R^2$$

$$= -0.318\pi \times 1.23 \frac{kg}{m^3} \times (15)^2 \frac{m^2}{s^2} \times (0.075)^2\, m^2 \times \frac{N \cdot s^2}{kg \cdot m}$$

$F_z = -1.56$ N $\quad$ ($F_z < 0$, so force acts down) $\qquad F_z$

Problem 6.20

Given: Air flows into the narrow gap between closely spaced parallel plates through a porous surface as shown. The uniform velocity in the x direction is $u = v_0 x/h$. Assume the flow ~~to~~ incompressible with $\rho = 1.23\ \text{kg/m}^3$ and that friction is negligible.



$v_0 = 15\ \text{mm/s}$, $L = 22\ \text{mm}$, $h = 1.2\ \text{mm}$

Find: (a) the pressure gradient at the point (L, h)

(b) an equation for the flow streamlines in the cavity

Solution:

Eulers equation, $\rho\vec{g} - \nabla p = \rho\frac{D\vec{V}}{Dt}$, can be used to determine the pressure gradient for incompressible frictionless flow.

We need first to determine the velocity field. With $u = v_0 x/h$, for 2-D, incompressible flow we can use the continuity equation to determine v.

Since $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, then $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{v_0 x}{h}\right) = -\frac{v_0}{h}$

Then
$$v = \int\frac{\partial v}{\partial y}dy + f(x) = -\frac{v_0}{h}y + f(x)$$

But $v = v_0$ at $y = 0$ and hence $f(x) = v_0$ and $v = v_0\left(1 - \frac{y}{h}\right)$

Then
$$\nabla p = \rho\vec{g} - \rho\frac{D\vec{V}}{Dt} = \rho\left[\vec{g} - u\frac{\partial\vec{V}}{\partial x} - v\frac{\partial\vec{V}}{\partial y}\right] = \rho\left[-g\hat{j} - \frac{v_0 x}{h}\left(\frac{v_0}{h}\hat{i}\right) - v_0\left(1 - \frac{y}{h}\right)\left(-\frac{v_0}{h}\hat{j}\right)\right]$$

$$\nabla p = \rho\left[-g\hat{j} - \frac{v_0^2 x}{h^2}\hat{i} - \frac{v_0^2}{h}\left(1 - \frac{y}{h}\right)\hat{j}\right]$$

At the point $(x, y) = (L, h)$
$$\nabla p = \rho\left[-\frac{v_0^2 L}{h^2}\hat{i} - g\hat{j}\right]$$

$$= 1.23\frac{\text{kg}}{\text{m}^3}\left[-(15)^2\frac{\cancel{\text{mm}^2}}{\text{s}^2} \times 0.022\text{m} \times \frac{1}{(1.2)^2\cancel{\text{mm}^2}}\hat{i} - 9.81\frac{\text{m}}{\text{s}^2}\hat{j}\right] \cdot \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$\nabla p\Big]_{L,h} = -4.23\hat{i} - 12.1\hat{j}\ \text{N/m}^3 \longleftarrow \nabla p$$

(b) The slope of the streamlines is given by $\frac{dy}{dx} = \frac{v}{u}$

$\therefore \frac{dy}{dx} = \frac{v_0(1 - y/h)}{\frac{v_0 x}{h}}$ and separating variables, we can write

$\frac{d\left(\frac{y}{h}\right)}{(1 - y/h)} = \frac{d\left(\frac{x}{h}\right)}{x/h}$ Then integrating we obtain

$$-\ln(1 - y/h) = \ln\frac{x}{h} - \ln c$$

or
$$\frac{x}{h}\left(1 - \frac{y}{h}\right) = \text{constant} \longleftarrow$$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Given: Upper plane surface moving downward at constant speed V causes incompressible liquid layer to be squeezed between surfaces as shown. Depth w in z direction and $w \gg L$.

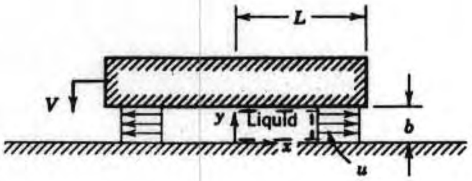
Find: (a) Show that $u = Vx/b$ within the gap $(b = b_0 - Vt)$

(b) expression for a_x

(c) $\partial p/\partial x$

(d) $p(x)$

(e) net pressure force on upper surface



Solution:

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

$-\nabla p + \rho\vec{g} = \rho\frac{D\vec{V}}{Dt}$ $\quad\quad \vec{F} = -\int p\,d\vec{A}$

(a) For the deformable CV shown

$$0 = \frac{\partial}{\partial t}\int_0^y \rho w x\,dy + \rho u w y = \rho w x\frac{dy}{dt} + \rho u w y$$

But $dy/dt = -V$ and hence $u = \frac{Vx}{y}$

If $y = b_0$ at $t=0$, then $y = b = b_0 - Vt$ at any time t

$$\therefore\ u = \frac{Vx}{b} \longleftarrow u(t)$$

(b) $$a_x = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + \overset{0(1)}{\cancel{v\frac{\partial u}{\partial y}}} + \overset{0(1)}{\cancel{w\frac{\partial u}{\partial z}}} + \frac{\partial u}{\partial t}$$

Assumptions: (1) $u \neq u(y)$, $w = 0$

$$a_x = \frac{Vx}{b}\left(\frac{V}{b}\right) + \frac{\partial u}{\partial b}\frac{\partial b}{\partial t} = \frac{V^2x}{b^2} + \left(-\frac{Vx}{b^2}\right)(-V) = \frac{2V^2x}{b^2} \longleftarrow a_x$$

(c) From Euler's equation in the x direction with $g_x = 0$

$$\frac{\partial p}{\partial x} = -\rho a_x = -\frac{2\rho V^2 x}{b^2} \longleftarrow \frac{\partial p}{\partial x}$$

(d) $$p - p_{atm} = \int_L^x \frac{\partial p}{\partial x}dx = \int_L^x -\frac{2\rho V^2}{b^2}x\,dx = -\frac{\rho V^2x^2}{b^2}\Big]_L^x = \frac{\rho V^2}{b^2}L^2\left[1 - \left(\frac{x}{L}\right)^2\right] \longleftarrow p(x)$$

(e) $$F_y = \int_A (p - p_{atm})\,dA = 2\int_0^L \frac{\rho V^2L^2}{b^2}\left[1 - \left(\frac{x}{L}\right)^2\right]w\,dx$$

$$= 2\int_0^1 \frac{\rho V^2L^3}{b^2}\left[1 - \left(\frac{x}{L}\right)^2\right]w\,d\left(\frac{x}{L}\right) = \frac{2\rho V^2L^3w}{b^2}\left[\left(\frac{x}{L}\right) - \frac{1}{3}\left(\frac{x}{L}\right)^3\right]_0^1$$

$$F_y = \frac{4\rho V^2L^3w}{3b^2} \quad \text{(upward, since } F_y > 0) \longleftarrow F_y$$

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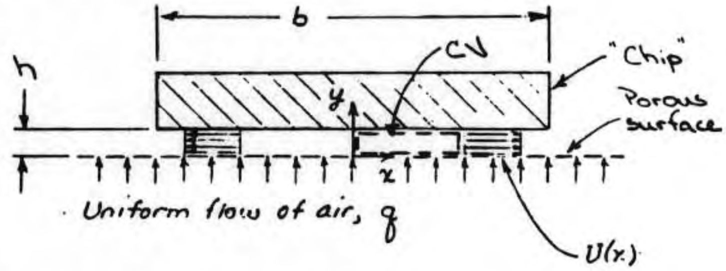
Problem 6.22

Given: Rectangular "chip" floats on thin layer of air of thickness, $h = 0.5$ mm above a porous surface as shown. Chip width $b = 20$ mm; length L (perpendicular to diagram) $>> b$; no flow in z direction. Flow in x direction under chip may be assumed uniform; $\rho =$ constant; neglect frictional effects

Find:
(a) Use a suitably chosen CV to show $U(x) = qx/h$ in the gap
(b) Find an expression for $\vec{a}_p$ in the gap
(c) Estimate the maximum value of $\vec{a}_p$
(d) Obtain an expression for $\partial p/\partial x$
(e) Sketch the pressure distribution under the chip
(f) Is the net pressure force on the chip directed up or down?
(g) Estimate the mass per unit length of the chip if $q = 0.06\ \text{m}^3/\text{sec}/\text{m}^2$

Solution:



Assumptions:
(1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) uniform flow at porous surface and in the gap at x.

(a) Apply continuity equation to CV, $\quad 0 = \cancelto{0(1)}{\frac{\partial}{\partial t}}\int \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

Then

$$0 = \{-|\rho q x L|\} + \{+|\rho U h L|\} \qquad \text{or} \qquad U = q\frac{x}{h} \longleftarrow \boxed{U(x)}$$

(b) Apply the substantial derivative definition

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \cancelto{\text{given}}{w}\frac{\partial \vec{V}}{\partial z} + \cancelto{0(1)}{\frac{\partial \vec{V}}{\partial t}}$$

Obtain v from differential continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\therefore \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{q}{h} \quad \text{and} \quad v - v_0 = \int_0^y -\frac{q}{h}\,dy + f(x) = -\frac{q}{h}y + f(x)$$

$$\text{or} \quad v = q\left(1 - \frac{y}{h}\right) \qquad [f(x) = 0 \text{ since } v = v_0 = q = \text{const. along } y = 0]$$

$$a_{p_x} = u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}} = q\frac{x}{h}\left(\frac{q}{h}\right) = \frac{q^2 x}{h^2}$$

$$a_{p_y} = \cancel{u\frac{\partial v}{\partial x}} + v\frac{\partial v}{\partial y} = q\left(1 - \frac{y}{h}\right)\left(-\frac{q}{h}\right) = \frac{q^2}{h}\left(\frac{y}{h} - 1\right)$$

$$\vec{a}_p = \frac{q^2 x}{h^2}\hat{i} + \frac{q^2}{h}\left(\frac{y}{h} - 1\right)\hat{j} = \frac{q^2}{h}\left[\frac{x}{h}\hat{i} + \left(\frac{y}{h} - 1\right)\hat{j}\right]$$

(c) The magnitude of $|\vec{a}_p| = \frac{q^2}{h}\left[\left(\frac{x}{h}\right)^2 + \left(\frac{y}{h} - 1\right)^2\right]^{1/2}$ is a maximum at $x = \frac{b}{2}$, $y = 0$

$$|\vec{a}_p|_{max} = \frac{q^2}{h}\left[\left(\frac{b}{2h}\right)^2 + 1\right]^{1/2} = 144\ \text{m/s}^2 \longleftarrow \boxed{|\vec{a}_p|_{m}}$$

(d) To obtain $\partial p/\partial x$ write the x component of the Euler equation

$$-\frac{\partial p}{\partial x} + \rho g_x = \rho a_{p_x} \quad \therefore \quad \frac{\partial p}{\partial x} = -\rho a_{p_x} = -\frac{\rho q_0^2 x}{h^2} \longleftarrow \frac{\partial p}{\partial x}$$

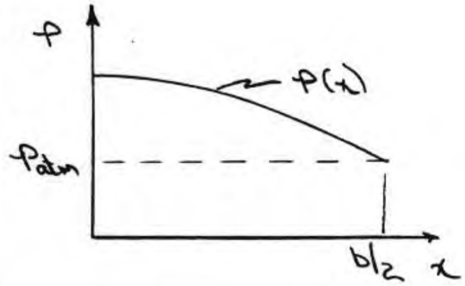
(e) To obtain an expression for the pressure distribution, $p(x)$ we need to separate variables and integrate noting that $p = p_{atm}$ at $x = b/2$. Thus.

$$p - p_{atm} = \int_{b/2}^{x} \frac{\partial p}{\partial x}\,dx = -\int_{b/2}^{x} \frac{\rho q_0^2}{h^2} x = -\frac{\rho q_0^2 x^2}{2h^2}\Bigg]_{b/2}^{x}$$

$$p - p_{atm} = \frac{\rho q_0^2}{2h^2}\left[\left(\frac{b}{2}\right)^2 - x^2\right] = \frac{\rho q_0^2 b^2}{8h^2}\left[1 - \left(\frac{2x}{b}\right)^2\right]$$

$$p = p_{atm} + \frac{\rho q_0^2 b^2}{8h^2}\left[1 - \left(\frac{2x}{b}\right)^2\right] \longleftarrow p(x)$$

(f) The net pressure force on the chip is up. Note that the pressure on the chip is greater than p_{atm} over the entire chip surface



(g) To estimate the mass per unit weight of the chip we must determine the net pressure force on the chip

$$F_{net} = \int_A (p - p_{atm})\,dA = 2\int_0^{b/2} \frac{\rho q_0^2 b^2}{8h^2}\left[1 - \left(\frac{2x}{b}\right)^2\right] L\,dx$$

$$= \frac{\rho q_0^2 b^2 L}{4h^2}\left[x - \frac{4}{3}\frac{x^3}{b^2}\right]_0^{b/2} = \frac{\rho q_0^2 b^2 L}{4h^2}\left[\frac{b}{2} - \frac{1}{3}\frac{b}{2}\right]$$

$$F_{net} = \frac{\rho q_0^2 b^3 L}{12h^2}$$

The weight of the chip, $W = Mg$, must be balanced by the net pressure force. Hence

$$Mg = F_{net} = \frac{\rho q_0^2 b^3 L}{12h^2}$$

$$\frac{M}{L} = \frac{\rho q_0^2 b^3}{12h^2 g}$$

$$= \frac{1}{12} \times 1.23\,\frac{kg}{m^3} \times (0.06)^2\,\frac{m^6}{s^2 \cdot m^4} \times (0.02)^3\,m^3 \times \frac{1}{(0.0005)^2\,m^2} \times \frac{s^2}{9.81\,m}$$

$$\frac{M}{L} = 1.20 \times 10^{-3}\,kg/m \longleftarrow \text{mass/length}$$

Problem 6.23

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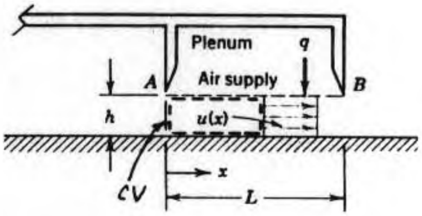


Given: Load pallet supported by air:

Flow is incompressible, uniform, and frictionless; $h \ll L$.

No flow across plane at $x = 0$.

Find: (a) Use a suitable CV to show $u(x) = qx/h$ in the gap.
(b) Calculate the acceleration of a fluid particle in the gap.
(c) Evaluate the pressure gradient, $\partial p/\partial x$.
(d) Sketch the pressure distribution; indicate pressure at $x = L$.

Solution: Choose a CV in the gap, from 0 to x, as shown.

Basic equations: $0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho \vec{V}\cdot d\vec{A}$

$$a_x = u\frac{\partial u}{\partial x} + v\overset{=0(3)}{\cancel{\frac{\partial u}{\partial y}}} + w\overset{=0(4)}{\cancel{\frac{\partial u}{\partial z}}} + \overset{=0(1)}{\cancel{\frac{\partial u}{\partial t}}} \qquad -\frac{\partial p}{\partial x} + \rho \overset{=0(5)}{\cancel{g_x}} = \rho a_{p_x}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) No variation with z
(5) Horizontal, so $g_x = 0$

From continuity,

$$0 = \{-|\cancel{\rho} q w x|\} + \{+|\cancel{\rho} u(x) w h|\} \quad \text{so} \quad u(x) = q\frac{x}{h}$$

$u(x)$

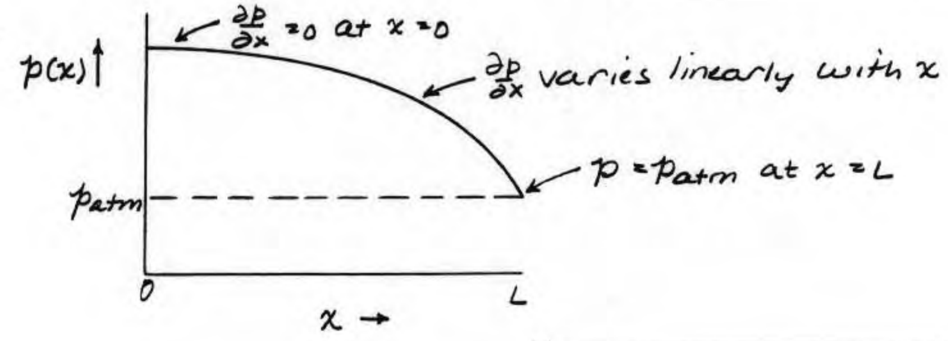
The acceleration is $a_{p_x} = \left(q\frac{x}{h}\right)\left(q\frac{1}{h}\right) = \frac{q^2 x}{h^2}$

a_{p_x}

The pressure gradient is $\frac{\partial p}{\partial x} = -\rho a_{p_x} = -\frac{\rho q^2 x}{h^2}$

$\frac{\partial p}{\partial x}$

Sketching:

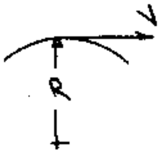


Sketc

Problem 6.24

[2]

Given: Air at 20 psia, 100°F flows around a smooth corner
Velocity = 150 ft/s
Radius of curvature of streamline is 3 in.



Find: (a) magnitude of centripetal acceleration in G's
(b) pressure gradient, $\frac{\partial p}{\partial r}$

Solution:

Basic equations: $\rho\vec{g} - \nabla p = \rho\frac{D\vec{V}}{Dt}$ ......(1)

$\frac{D\vec{V}}{Dt} = \vec{a}_p$ ...(2) $\qquad p = \rho RT$ ....(3)

Assumptions: (1) ρ = constant
(2) frictionless flow
(3) $\vec{g} = -g\hat{k}$

Writing the r component of equation (1)

$$\cancelto{0}{g_r} - \frac{1}{\rho}\frac{\partial p}{\partial r} = a_r = \cancelto{0}{\frac{\partial V_r}{\partial t}} + \cancelto{0}{V_r}\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\cancelto{0}{\frac{\partial V_r}{\partial \theta}} + \cancelto{0}{V_z}\underbrace{\frac{\partial V_r}{\partial z}}_{0} - \frac{V_\theta^2}{r}$$

$$a_r = -\frac{V_\theta^2}{r} \qquad \frac{a_r}{g} = -\frac{V_\theta^2}{rg} = -(150)^2\frac{ft^2}{s^2}\times\frac{1}{3\,in}\times\frac{12\,in}{ft}\times\frac{s^2}{32.2\,ft}$$

$$\frac{a_r}{g} = -2800 \text{ G's}$$

$\frac{a_r}{g}$

Also

$$\frac{\partial p}{\partial r} = \rho\frac{V_\theta^2}{r}$$

where $\rho = \frac{p}{RT} = 20\frac{lbf}{in^2}\times\frac{lbm\,°R}{53.3\,ft\text{-}lbf}\times\frac{1}{560°R}\times 144\frac{in^2}{ft^2}\times\frac{slug}{32.2\,lbm}$

$$\rho = 0.003\ slug/ft^3$$

$$\frac{\partial p}{\partial r} = \rho\frac{V_\theta^2}{r} = 0.003\frac{slug}{ft^3}\times(150)^2\frac{ft^2}{s^2}\times\frac{1}{3\,in}\times\frac{12\,in}{ft}\times\frac{lbf\text{-}s^2}{ft\text{-}slug}$$

$$\frac{\partial p}{\partial r} = 270\ \frac{lbf/ft^2}{ft}$$

$\frac{\partial p}{\partial r}$

Problem 6.25

[2]

6.25 The velocity field for a plane doublet is given in Table 6.2. Find an expression for the pressure gradient at any point (r, θ).

Given: Velocity field for doublet

Find: Expression for pressure gradient

Solution:

Basic equations

$$\rho a_r = \rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}\right) = \rho g_r - \frac{\partial p}{\partial r}$$

$$\rho a_\theta = \rho\left(\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta}$$

For this flow $\quad V_r(r,\theta) = -\frac{\Lambda}{r^2}\cdot\cos(\theta) \qquad V_\theta(r,\theta) = -\frac{\Lambda}{r^2}\cdot\sin(\theta) \qquad V_z = 0$

Hence for r momentum $\quad \rho\cdot g_r - \frac{\partial}{\partial r}p = \rho\cdot\left(V_r\cdot\frac{\partial}{\partial r}V_r + \frac{V_\theta}{r}\cdot\frac{\partial}{\partial \theta}V_r - \frac{V_\theta^{\,2}}{r}\right)$

Ignoring gravity

$$\frac{\partial}{\partial r}p = -\rho\cdot\left[\left(-\frac{\Lambda}{r^2}\cdot\cos(\theta)\right)\cdot\frac{\partial}{\partial r}\left(-\frac{\Lambda}{r^2}\cdot\cos(\theta)\right) + \frac{\left(-\frac{\Lambda}{r^2}\cdot\sin(\theta)\right)}{r}\cdot\frac{\partial}{\partial \theta}\left(-\frac{\Lambda}{r^2}\cdot\cos(\theta)\right) - \frac{\left(-\frac{\Lambda}{r^2}\cdot\sin(\theta)\right)^2}{r}\right] \qquad \frac{\partial}{\partial r}p = \frac{2\cdot\Lambda^2\cdot\rho}{r^5}$$

For θ momentum $\quad \rho\cdot g_\theta - \frac{1}{r}\cdot\frac{\partial}{\partial \theta}p = \rho\cdot\left(V_r\cdot\frac{\partial}{\partial r}V_\theta + \frac{V_\theta}{r}\cdot\frac{\partial}{\partial \theta}V_\theta + \frac{V_r\cdot V_\theta}{r}\right)$

Ignoring gravity

$$\frac{\partial}{\partial \theta}p = -r\cdot\rho\cdot\left[\left(-\frac{\Lambda}{r^2}\cdot\cos(\theta)\right)\cdot\frac{\partial}{\partial r}\left(-\frac{\Lambda}{r^2}\cdot\sin(\theta)\right) + \frac{\left(-\frac{\Lambda}{r^2}\cdot\sin(\theta)\right)}{r}\cdot\frac{\partial}{\partial \theta}\left(-\frac{\Lambda}{r^2}\cdot\sin(\theta)\right) + \frac{\left(-\frac{\Lambda}{r^2}\cdot\sin(\theta)\right)\cdot\left(-\frac{\Lambda}{r^2}\cdot\cos(\theta)\right)}{r}\right] \qquad \frac{\partial}{\partial \theta}p = 0$$

The pressure gradient is purely radial

Problem 6.26

[2]

Given: The velocity field for steady, frictionless, incompressible flow (from right to left) over a stationary circular cylinder of radius, a, is given by

$$\vec{V} = U\left[\left(\frac{a}{r}\right)^2 - 1\right]\cos\theta\,\hat{e}_r + U\left[\left(\frac{a}{r}\right)^2 + 1\right]\sin\theta\,\hat{e}_\theta$$

Consider flow along the streamline forming the cylinder surface, i.e. $r = a$

Find: The pressure gradient along cylinder surface

Plot $V(r)$ along $\theta = \pi/2$ for $r > a$

Solution:

Basic equation: $\cancel{\rho\vec{g}}^{\,0} - \nabla p = \rho\dfrac{D\vec{V}}{Dt}$

Assumptions: (1) neglect body force

Along the surface, $r = a$, $\vec{V} = 2U\sin\theta\,\hat{i}_\theta$.

Computing equations:

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = \cancel{\frac{\partial V_r}{\partial t}}^{0} + \cancel{V_r\frac{\partial V_r}{\partial r}}^{0} + \cancel{\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}}^{0} + \cancel{V_z\frac{\partial V_r}{\partial z}}^{0} - \frac{V_\theta^2}{r}$$

$$-\frac{1}{\rho r}\frac{\partial p}{\partial \theta} = \cancel{\frac{\partial V_\theta}{\partial t}}^{0} + \cancel{V_r\frac{\partial V_\theta}{\partial r}}^{0} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + \cancel{V_z\frac{\partial V_\theta}{\partial z}}^{0} + \cancel{\frac{V_r V_\theta}{r}}^{0}$$

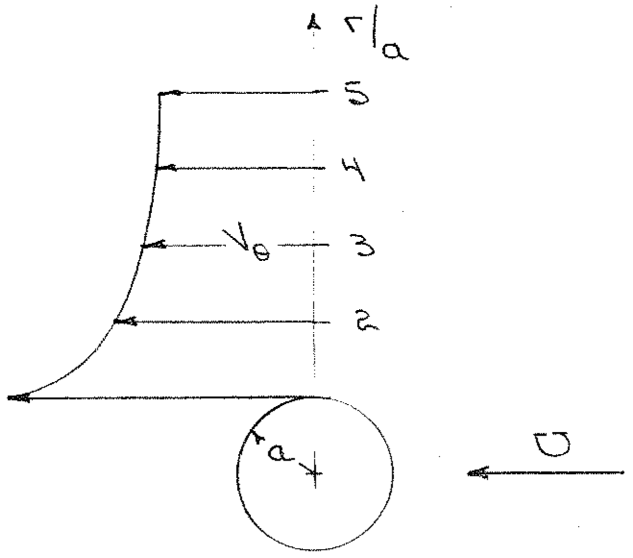
$$\frac{\partial p}{\partial r} = \rho\frac{V_\theta^2}{r} = \rho\frac{[2U\sin\theta]^2}{a} = \frac{4U^2\rho}{a}\sin^2\theta$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = -\rho\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} = -\rho\left[\frac{2U\sin\theta}{a}\right](2U\cos\theta) = -\frac{4U^2\rho}{a}\sin\theta\cos\theta$$

$$\nabla p = \hat{i}_r\frac{\partial p}{\partial r} + \hat{i}_\theta\frac{1}{r}\frac{\partial p}{\partial \theta} = \frac{4\rho U^2}{a}\sin\theta\left(\hat{i}_r\sin\theta - \hat{i}_\theta\cos\theta\right) \longleftarrow \nabla p$$

Along $\theta = \frac{\pi}{2}$, $\vec{V} = U\left[\left(\frac{a}{r}\right)^2 + 1\right]\hat{e}_\theta$

$\frac{r}{a}$	V_θ
1	$2U$
2	$1.25U$
3	$1.111U$
4	$1.063U$
5	$1.04U$



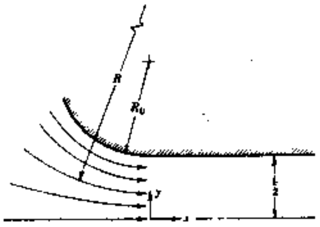
Problem 6.27

[2]

Given: Radius of curvature of streamlines at wind tunnel inlet is modeled as

$$R = \frac{L/2}{y} R_0$$

Speed along each streamline assumed constant at $V = 20$ m/s ; $L = 0.15$ m, $R_0 = 0.6$ m



Find: Δp between $y=0$ and tunnel wall $(y = L/2)$

Solution:

Basic equation: $\frac{\partial p}{\partial n} = \rho \frac{V^2}{R}$

Assumptions: (1) steady flow (2) frictionless flow
(3) neglect body forces
(4) constant speed along each streamline

At the inlet section, $p = p(y)$

$$\therefore \quad \frac{dp}{dn} = -\frac{dp}{dy} = \rho\frac{V^2}{R} = \rho V^2 \frac{2y}{R_0 L}$$

$$\therefore dp = -\frac{\rho V^2}{R_0 L} 2y\,dy$$

$$p_{L/2} - p_0 = \int_0^{L/2} dp = -\frac{2\rho V^2}{R_0 L}\int_0^{L/2} y\,dy = -\frac{2\rho V^2}{R_0 L}\left.\frac{y^2}{2}\right]_0^{L/2}$$

$$p_{L/2} - p_0 = -\frac{\rho V^2}{R_0 L}\frac{L^2}{4} = -\frac{\rho V L}{4R_0}$$

$$p_{L/2} - p_0 = -1.225\,\frac{kg}{m^3} \times \left(20\,\frac{m}{s}\right)^2 \times 0.15\,m \times \frac{1}{4} \times \frac{1}{0.6\,m} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$p_{L/2} - p_0 = -30.6 \text{ N/m}^2 \longleftarrow p_{L/2} - p_0$$

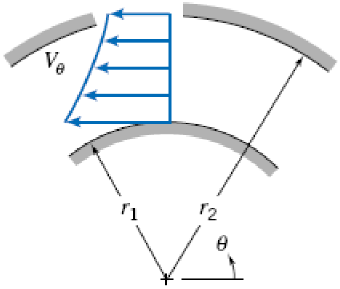
Problem 6.28

[3]

6.28 Repeat Example 6.1, but with the somewhat more realistic assumption that the flow is similar to a free vortex (irrotational) profile, $V_\theta = c/r$ (where c is a constant), as shown in Fig. P6.28. In doing so, prove that the flow rate is given by $Q = k\sqrt{\Delta p}$, where k is

$$k = w \ln\left(\frac{r_2}{r_1}\right)\sqrt{\frac{2r_2^2 r_1^2}{\rho(r_2^2 - r_1^2)}}$$

and w is the depth of the bend.



Given: Velocity field for free vortex flow in elbow

Find: Similar solution to Example 6.1; find k (above)

Solution:

Basic equation $\quad \dfrac{\partial}{\partial r}p = \dfrac{\rho \cdot V^2}{r} \quad$ with $\quad V = V_\theta = \dfrac{c}{r}$

Assumptions: 1) Frictionless 2) Incompressible 3) free vortex

For this flow $\quad p \neq p(\theta) \quad$ so $\quad \dfrac{\partial}{\partial r}p = \dfrac{d}{dr}p = \dfrac{\rho \cdot V^2}{r} = \dfrac{\rho \cdot c^2}{r^3}$

Hence

$$\Delta p = p_2 - p_1 = \int_{r_1}^{r_2} \frac{\rho \cdot c^2}{r^3}\, dr = \frac{\rho \cdot c^2}{2} \cdot \left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) = \frac{\rho \cdot c^2 \cdot \left(r_2^2 - r_1^2\right)}{2 \cdot r_1^2 \cdot r_2^2} \qquad (1)$$

Next we obtain c in terms of Q

$$Q = \int \vec{V}\, d\vec{A} = \int_{r_1}^{r_2} V \cdot w\, dr = \int_{r_1}^{r_2} \frac{w \cdot c}{r}\, dr = w \cdot c \cdot \ln\left(\frac{r_2}{r_1}\right)$$

Hence

$$c = \frac{Q}{w \cdot \ln\left(\frac{r_2}{r_1}\right)}$$

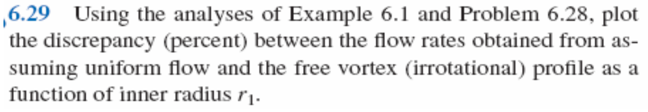
Using this in Eq 1

$$\Delta p = p_2 - p_1 = \frac{\rho \cdot c^2 \cdot \left(r_2^2 - r_1^2\right)}{2 \cdot r_1^2 \cdot r_2^2} = \frac{\rho \cdot Q^2 \cdot \left(r_2^2 - r_1^2\right)}{2 \cdot w^2 \cdot \ln\left(\frac{r_2}{r_1}\right)^2 \cdot r_1^2 \cdot r_2^2}$$

Solving for Q

$$Q = w \cdot \ln\left(\frac{r_2}{r_1}\right) \cdot \sqrt{\frac{2 \cdot r_1^2 \cdot r_2^2}{\rho \cdot \left(r_2^2 - r_1^2\right)}} \cdot \sqrt{\Delta p} \qquad k = w \cdot \ln\left(\frac{r_2}{r_1}\right) \cdot \sqrt{\frac{2 \cdot r_1^2 \cdot r_2^2}{\rho \cdot \left(r_2^2 - r_1^2\right)}}$$

Problem 6.29



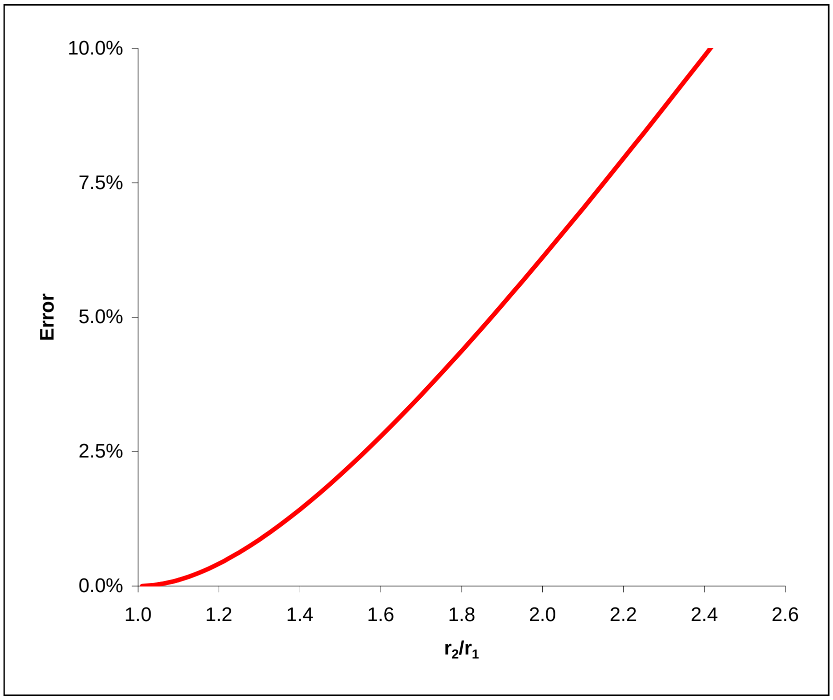
6.29 Using the analyses of Example 6.1 and Problem 6.28, plot the discrepancy (percent) between the flow rates obtained from assuming uniform flow and the free vortex (irrotational) profile as a function of inner radius r_1.

From Example 6.1: $Q_{Uniform} = V \cdot A = w \cdot (r_2 - r_1) \cdot \sqrt{\dfrac{1}{\rho \cdot \ln\left(\dfrac{r_2}{r_1}\right)}} \cdot \sqrt{\Delta p}$ or $\dfrac{Q_{Uniform} \cdot \sqrt{\rho}}{w \cdot r_1 \cdot \sqrt{\Delta p}} = \dfrac{\left(\dfrac{r_2}{r_1} - 1\right)}{\sqrt{\ln\left(\dfrac{r_2}{r_1}\right)}}$ Eq. 1

From Problem 6.28: $\dfrac{Q \cdot \sqrt{\rho}}{w \cdot r_1 \cdot \sqrt{\Delta p}} = \left(\dfrac{r_2}{r_1}\right) \cdot \ln\left(\dfrac{r_2}{r_1}\right) \cdot \sqrt{\dfrac{2}{\left[\left(\dfrac{r_2}{r_1}\right)^2 - 1\right]}}$ Eq. 2

Instead of plotting as a function of inner radius we plot as a function of r_2/r_1

r_2/r_1	Eq. 1	Eq. 2	Error
1.01	0.100	0.100	0.0%
1.05	0.226	0.226	0.0%
1.10	0.324	0.324	0.1%
1.15	0.401	0.400	0.2%
1.20	0.468	0.466	0.4%
1.25	0.529	0.526	0.6%
1.30	0.586	0.581	0.9%
1.35	0.639	0.632	1.1%
1.40	0.690	0.680	1.4%
1.45	0.738	0.726	1.7%
1.50	0.785	0.769	2.1%
1.55	0.831	0.811	2.4%
1.60	0.875	0.851	2.8%
1.65	0.919	0.890	3.2%
1.70	0.961	0.928	3.6%
1.75	1.003	0.964	4.0%
1.80	1.043	1.000	4.4%
1.85	1.084	1.034	4.8%
1.90	1.123	1.068	5.2%
1.95	1.162	1.100	5.7%
2.00	1.201	1.132	6.1%
2.05	1.239	1.163	6.6%
2.10	1.277	1.193	7.0%
2.15	1.314	1.223	7.5%
2.20	1.351	1.252	8.0%
2.25	1.388	1.280	8.4%
2.30	1.424	1.308	8.9%
2.35	1.460	1.335	9.4%
2.40	1.496	1.362	9.9%
2.45	1.532	1.388	10.3%
2.50	1.567	1.414	10.8%



Problem 6.30

Given: Velocity field $\vec{V} = (Ax + B)\hat{i} - Ay\hat{j}$ where $A = 1\,s^{-1}$, $B = 2\,m/s$ and coordinates are measured in meters.

Show: that streamlines are given by $(x + B/A)y = \text{constant}$.
Plot: streamlines through points $(x, y) = (1,1), (1,2), (2,2)$.

Find: (a) velocity vector & acceleration vector at (1,2); show these on streamline plot
(b) component of $\vec{a}_p$ along the streamline at (1,2); express as a vector.
(c) pressure gradient along streamline at (1,2) for air
(d) relative value of pressure at points (1,1) & (2,2)

Solution:

The slope of a streamline is $\left.\frac{dy}{dx}\right)_{s.l} = \frac{v}{u} = \frac{-Ay}{Ax+B} = \frac{-y}{x+B/A}$

Then $\frac{dy}{y} + \frac{dx}{x + B/A} = 0$ and $\ln y + \ln(x + B/A) = \ln c$.

and $(x + B/A)y = \text{constant}$ ← Streamlines

For
$(1,1)$ $(x+2)y = 3$
$(1,2)$ $(x+2)y = 6$
$(2,2)$. $(x+2)y = 8$
} These streamlines are shown in the plot at the end of the problem solution

The particle acceleration $\vec{a}_p = \frac{D\vec{V}}{Dt} = \cancelto{0(1)}{\frac{\partial \vec{V}}{\partial t}} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \cancelto{0(2)}{w\frac{\partial \vec{V}}{\partial z}}$.

Assumptions: (1) steady flow (given)
(2) 2-D (given) $\vec{V} \neq \vec{V}(z)$.

$\vec{a}_p = (Ax+B)\frac{\partial}{\partial x}\left[(Ax+B)\hat{i} - Ay\hat{j}\right] - Ay\frac{\partial}{\partial y}\left[(Ax+B)\hat{i} - Ay\hat{j}\right]$

$\vec{a}_p = (Ax+B)A\hat{i} - Ay(-A\hat{j}) = A(Ax+B)\hat{i} + A^2y\hat{j}$

At point (1,2).

$\vec{a}_p = \frac{1}{s}\left(\frac{1}{s}\times 1m + 2\frac{m}{s}\right)\hat{i} + \frac{1}{s^2}\times 2m\hat{j} = 3\hat{i} + 2\hat{j}\ m/s^2$ ← $\vec{a}_{(1,2)}$

$\vec{V} = \left(\frac{1}{s}\times 1m + 2\frac{m}{s}\right)\hat{i} - \frac{1}{s}2m\hat{j} = 3\hat{i} - 2\hat{j}\ m/s$ ← $\vec{V}_{(1,2)}$

$\vec{V}$ and $\vec{a}$ are shown on the streamline plot.

(b) The component of $\vec{a}_p$ along (tangent to) the streamline is given by $a_t = \vec{a}_p \cdot \hat{e}_t$ where $\hat{e}_t = \frac{\vec{V}}{|\vec{V}|}$

Thus $\hat{e}_t = \frac{3\hat{i} - 2\hat{j}}{\left[3^2 + (-2)^2\right]^{1/2}} = 0.832\hat{i} - 0.555\hat{j}$

and

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Problem 6.30

$$a_t = \vec{a}_p \cdot \hat{e}_t = (3\hat{i} + 2\hat{j})\,\text{m/s}^2 \cdot (0.832\hat{i} - 0.555\hat{j}) = 1.39\ \text{m/s}^2$$

$$\vec{a}_t = 1.39\,\hat{e}_t = 1.16\hat{i} - 0.771\hat{j}\ \text{m/s}^2 \qquad \vec{a}_{t(1,2)}$$

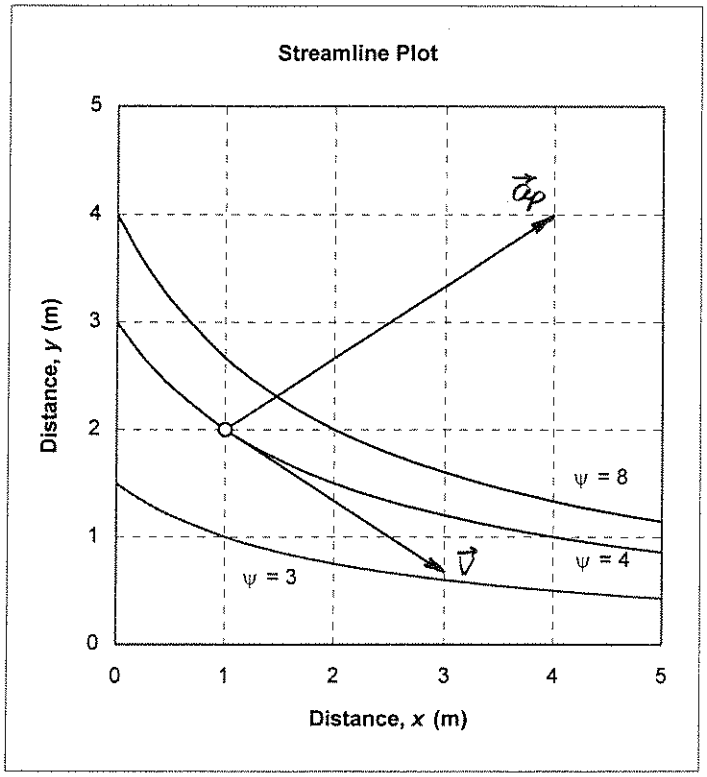
For frictionless flow, Euler's equation along a streamline (neglecting gravity, i.e. assuming flow in horizontal plain is

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s} = -\rho a_t = -1.23\,\frac{\text{kg}}{\text{m}^3} \times 1.39\,\frac{\text{m}}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$\frac{\partial p}{\partial s} = -1.71\ \text{N/m}^2/\text{m} \qquad \frac{\partial p}{\partial s}_{(1,2)}$$

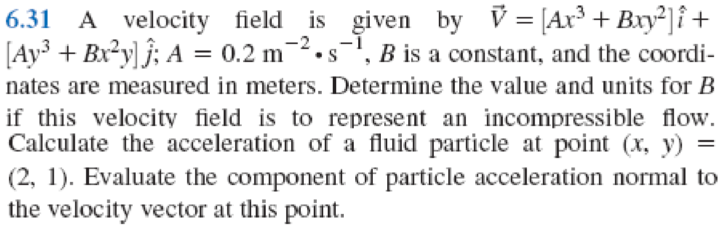
Looking at the streamline we would expect $p(2,2)$ to be less than $p(1,1)$ due to streamline curvature; Euler's equation normal to a streamline says

$$\partial p/\partial n = \frac{\rho V^2}{R}$$



Problem 6.31

[4]



6.31 A velocity field is given by $\vec{V} = [Ax^3 + Bxy^2]\,\hat{\imath} + [Ay^3 + Bx^2y]\,\hat{\jmath}$; $A = 0.2\ \mathrm{m^{-2} \cdot s^{-1}}$, B is a constant, and the coordinates are measured in meters. Determine the value and units for B if this velocity field is to represent an incompressible flow. Calculate the acceleration of a fluid particle at point $(x, y) = (2, 1)$. Evaluate the component of particle acceleration normal to the velocity vector at this point.

Given: Velocity field

Find: Constant B for incompressible flow; Acceleration of particle at (2,1); acceleration normal to velocity at (2,1)

Solution:

Basic equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

For this flow $u(x,y) = A\cdot x^3 + B\cdot x\cdot y^2$ $\qquad v(x,y) = A\cdot y^3 + B\cdot x^2\cdot y$

$$\frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}v(x,y) = \frac{\partial}{\partial x}\left(A\cdot x^3 + B\cdot x\cdot y^2\right) + \frac{\partial}{\partial y}\left(A\cdot y^3 + B\cdot x^2\cdot y\right) = 0$$

$$\frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}v(x,y) = (3\cdot A + B)\cdot\left(x^2 + y^2\right) = 0$$

Hence $\qquad B = -3\cdot A \qquad B = -0.6\frac{1}{m^2\cdot s}$

We can write $u(x,y) = A\cdot x^3 - 3\cdot A\cdot x\cdot y^2$ $\qquad v(x,y) = A\cdot y^3 - 3\cdot A\cdot x^2\cdot y$

Hence for a_x

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = \left(A\cdot x^3 - 3\cdot A\cdot x\cdot y^2\right)\cdot\frac{\partial}{\partial x}\left(A\cdot x^3 - 3\cdot A\cdot x\cdot y^2\right) + \left(A\cdot y^3 - 3\cdot A\cdot x^2\cdot y\right)\cdot\frac{\partial}{\partial y}\left(A\cdot x^3 - 3\cdot A\cdot x\cdot y^2\right)$$

$$a_x = 3\cdot A^2\cdot x\cdot\left(x^2 + y^2\right)^2$$

For a_y

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial v}v = \left(A\cdot x^3 - 3\cdot A\cdot x\cdot y^2\right)\cdot\frac{\partial}{\partial x}\left(A\cdot y^3 - 3\cdot A\cdot x^2\cdot y\right) + \left(A\cdot y^3 - 3\cdot A\cdot x^2\cdot y\right)\cdot\frac{\partial}{\partial y}\left(A\cdot y^3 - 3\cdot A\cdot x^2\cdot y\right)$$

$$a_y = 3\cdot A^2\cdot y\cdot\left(x^2 + y^2\right)^2$$

Hence at (2,1)

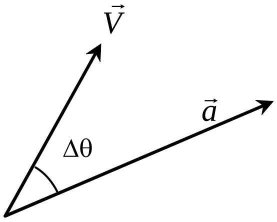
$$a_x = 3\cdot\left(\frac{0.2}{m^2\cdot s}\right)^2 \times 2\cdot m \times \left[(2\cdot m)^2 + (1\cdot m)^2\right]^2 \qquad a_x = 6.00\cdot\frac{m}{s^2}$$

$$a_y = 3\cdot\left(\frac{0.2}{m^2\cdot s}\right)^2 \times 1\cdot m \times \left[(2\cdot m)^2 + (1\cdot m)^2\right]^2 \qquad a_y = 3.00\cdot\frac{m}{s^2}$$

$$a = \sqrt{{a_x}^2 + {a_y}^2} \qquad a = 6.71\frac{m}{s^2}$$

We need to find the component of acceleration normal to the velocity vector

At (2,1) the velocity vector is at angle $\theta_{vel} = \operatorname{atan}\left(\frac{v}{u}\right) = \operatorname{atan}\left(\frac{A\cdot y^3 - 3\cdot A\cdot x^2\cdot y}{A\cdot x^3 - 3\cdot A\cdot x\cdot y^2}\right)$



$$\theta_{vel} = \operatorname{atan}\left(\frac{1^3 - 3\cdot 2^2\cdot 1}{2^3 - 3\cdot 2\cdot 1^2}\right) \qquad \theta_{vel} = -79.7\cdot\text{deg}$$

At (1,2) the acceleration vector is at angle $\theta_{accel} = \operatorname{atan}\left(\frac{a_y}{a_x}\right)$ $\quad \theta_{accel} = \operatorname{atan}\left(\frac{1}{2}\right)$ $\quad \theta_{accel} = 26.6\cdot\text{deg}$

Hence the angle between the acceleration and velocity vectors is $\Delta\theta = \theta_{accel} - \theta_{vel}$ $\quad \Delta\theta = 106\cdot\text{deg}$

The component of acceleration normal to the velocity is then $a_n = a\cdot\sin(\Delta\theta) = 6.71\cdot\frac{m}{s^2}\cdot\sin(106\cdot\text{deg})$ $\quad a_n = 6.45\cdot\frac{m}{s^2}$

Problem 6.32

[4] Part 1/2

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Given: The x component of velocity in a 2-D, incompressible flow field is
$u = Ax^2$ where $A = 1\ \text{ft}^{-1}\cdot\text{s}^{-1}$ and coordinates are in ft; $w = 0$ and $\partial/\partial z = 0$.

Find: (a) acceleration of fluid particle at $(x,y) = (1,2)$
(b) radius of curvature of streamline at $(1,2)$

Plot: streamline through $(1,2)$; show velocity and acceleration vectors on the plot.

Solution:

For 2-D incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, so $\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$

$$v = \int \frac{\partial v}{\partial y}\,dy + f(x) = \int -\frac{\partial u}{\partial x}\,dy + f(x) = -\int 2Ax\,dy + f(x) = -2Axy + f(x).$$

Choose the simplest solution, $f(x) = 0$, so $v = -2Axy$. Hence

$$\vec{V} = Ax^2\hat{i} - 2Axy\hat{j} = A\left[x^2\hat{i} - 2xy\hat{j}\right]$$

The acceleration of a fluid particle is

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} = Ax^2\left[A(2x\hat{i} - 2y\hat{j})\right] - 2Axy\left[-2Ax\hat{j}\right]$$

$$\vec{a}_p = 2A^2x^3\hat{i} + 2A^2x^2y\hat{j} = 2A^2x^2\left[x\hat{i} + y\hat{j}\right]$$

At the point $(1,2)$

$$\vec{a}_p = 2\times\frac{(1)^2}{\text{ft}^2\text{s}^2}\times(1)^2\text{m}^2\left[1\text{m}\,\hat{i} + 2\text{m}\,\hat{j}\right] = 2\hat{i} + 4\hat{j}\ \text{ft/s}^2 \longleftarrow \vec{a}_{(1,2)}$$

$$\vec{V} = \frac{1}{\text{ft}\cdot\text{s}}\left[(1)^2\text{m}^2\,\hat{i} - 2(1\text{m})(2\text{m})\hat{j}\right] = \hat{i} - 4\hat{j}\ \text{ft/s}$$

The unit vector tangent to the streamline is

$$\hat{e}_t = \frac{\vec{V}}{|\vec{V}|} = \frac{\hat{i} - 4\hat{j}}{\left[(1)^2 + (-4)^2\right]^{1/2}} = 0.243\hat{i} - 0.970\hat{j}$$

The unit vector normal to the streamline is

$$\hat{e}_n = \hat{e}_t \times \hat{k} = (0.243\hat{i} - 0.970\hat{j}) \times \hat{k} = -0.970\hat{i} - 0.243\hat{j}$$

The normal component of acceleration is

$$a_n = -\frac{V^2}{R} = \vec{a}\cdot\hat{e}_n = (2\hat{i} + 4\hat{j})\cdot(-0.970\hat{i} - 0.243\hat{j})$$

$$-\frac{V^2}{R} = -2.91\ \text{ft/s}^2$$

$$R = \frac{V^2}{2.91} = \frac{17\ \text{ft}^2/\text{s}^2}{2.91\ \text{ft/s}^2} = 5.84\ \text{ft} \longleftarrow R$$

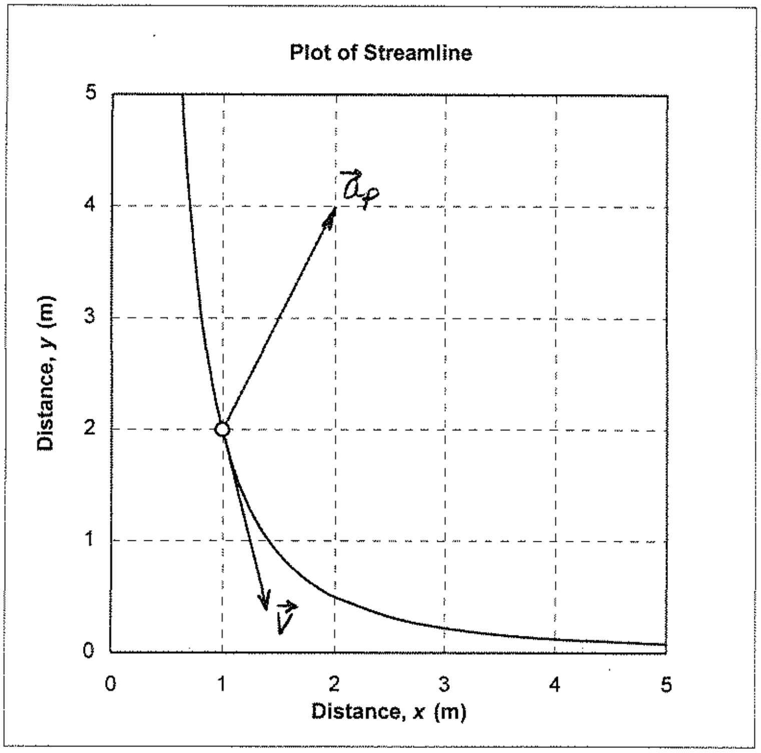
The slope of the streamline is given by

$$\left.\frac{dy}{dx}\right)_{s.l.} = \frac{v}{u} = \frac{-2Axy}{Ax^2} = \frac{-2y}{x}$$

Thus $\frac{dy}{y} + \frac{2dx}{x} = 0$ and $\ln y + \ln x^2 = \ln c$ or

$$x^2 y = c$$

The equation of the streamline through (1,2) is $x^2y = 2$.



Problem 6.33

Given: Incompressible, 2-D flow with $u = Axy$, $w = 0$; $A = 2\ ft^{-1} s^{-1}$

Find: (a) Acceleration of particle at $(x, y) = (2, 1)$.

(b) Radius of curvature of streamline at that point.

(c) Plot streamline, show velocity vector and acceleration vector.

Solution: For two-d. incompressible flow, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, so

$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -Ay$; Integrating, $v = -\frac{1}{2}Ay^2$; $\vec{V} = Axy\hat{i} - \frac{1}{2}Ay^2\hat{j}$.

The acceleration is

$$a_{p_x} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (Axy)(Ay) + (-\tfrac{1}{2}Ay^2)(Ax) = \tfrac{1}{2}A^2xy^2$$

$$a_{p_y} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (Axy)(0) + (-\tfrac{1}{2}Ay^2)(-Ay) = \tfrac{1}{2}A^2y^3$$

$$\vec{a}_p = \tfrac{1}{2}A^2xy^2\hat{i} + \tfrac{1}{2}A^2y^3\hat{j}\text{; at } (2,1)\quad \vec{a}_p = 4\hat{i} + 2\hat{j}\ (ft/s^2)$$

$\vec{a}_p$

Note $a_n = \frac{V^2}{R}$, so $R = \frac{V^2}{a_n}$, where a_n is acceleration normal to $\vec{V}$

At $(2,1)$, $\vec{V} = 4\hat{i} - 1\hat{j}$ ft/s, so $V^2 = (4)^2 + (1)^2 = 17\ ft^2/s^2$

To find a_n, dot $\vec{a}_p$ with $\hat{e}_n$, the unit normal vector. To find $\hat{e}_n$, set

$$\hat{e}_n = -\frac{v}{V}\hat{i} + \frac{u}{V}\hat{j} = \frac{1}{\sqrt{17}}\hat{i} + \frac{4}{\sqrt{17}}\hat{j}$$

$$a_n = \hat{e}_n \cdot \hat{a}_p = \frac{4}{\sqrt{17}} + \frac{8}{\sqrt{17}} = \frac{12}{\sqrt{17}} = 2.91\ ft/s^2$$

Substituting

$$R = \frac{V^2}{a_n} = 17\frac{ft^2}{s^2} \times \frac{s^2}{2.91\ ft} = 5.84\ ft$$

R

The streamline is $\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{Axy} = \frac{dy}{-\frac{1}{2}Ay^2}$ or $\frac{dx}{x} + 2\frac{dy}{y} = 0$

Integrating, $\ln x + 2\ln y = \ln c$ or $xy^2 = c$

For $(x, y) = (2, 1)$, then $c = 2\ ft^3$.

The plot and streamlines are on the following page.

Problem 6.33

Components of Velocity and Acceleration:

Input Parameters:

$A =$ 2 $\text{ft}^{-1}\text{s}^{-1}$

Calculated Values:

$C =$ 2 ft^3

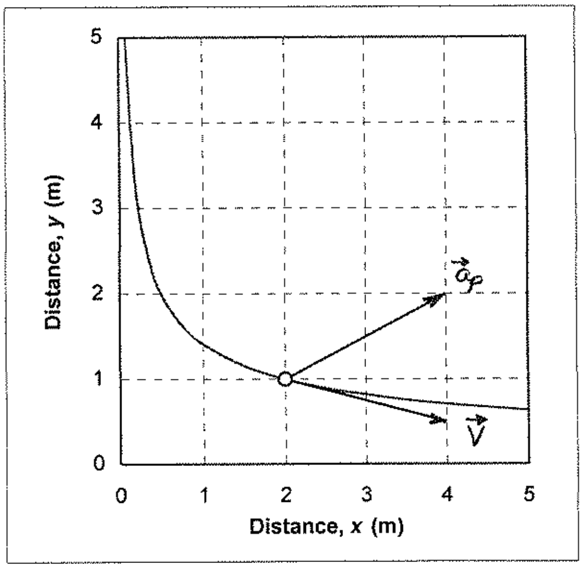
Coord. x	Coord. y	Velocity, V_x	Velocity, V_y	Velocity, V	Accel., a_x	Accel., a_y	Accel., a	Normal Accel., a_n
0.08	5.00							
0.2	3.16							
0.4	2.24							
0.5	2.00	2.00	-4.00	4.47	2.00	16.0	16.1	8.94
0.6	1.83							
0.8	1.58							
1.0	1.41	2.83	-2.00	3.46	2.83	5.66	6.32	6.25
1.5	1.15	3.46	-1.33	3.71	3.46	3.08	4.63	4.12
2.0	1.00	4.00	-1.00	4.12	4.00	2.00	4.47	2.91
2.5	0.89	4.47	-0.80	4.54	4.47	1.43	4.70	2.20
3.0	0.82	4.90	-0.67	4.94	4.90	1.09	5.02	1.74
3.5	0.76	5.29	-0.57	5.32	5.29	0.86	5.36	1.43
4.0	0.71	5.66	-0.50	5.68	5.66	0.71	5.70	1.20
4.5	0.67	6.00	-0.44	6.02	6.00	0.59	6.03	1.03
5.0	0.63	6.32	-0.40	6.34	6.32	0.51	6.34	0.90

Acceleration:

2	1
4	2

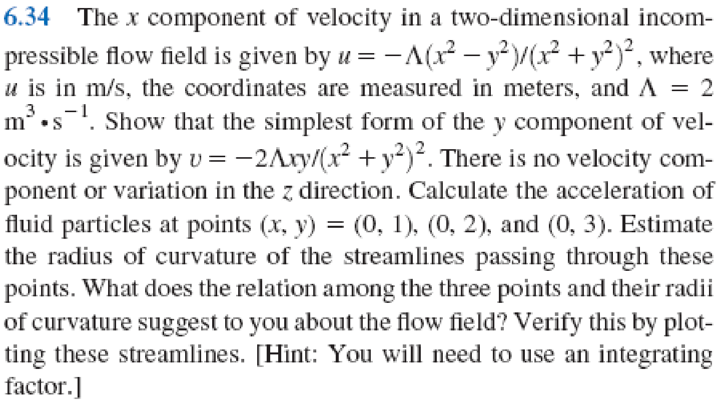
Velocity:

2	1
4	0.5



Problem 6.34

[4]



6.34 The x component of velocity in a two-dimensional incompressible flow field is given by $u = -\Lambda(x^2 - y^2)/(x^2 + y^2)^2$, where u is in m/s, the coordinates are measured in meters, and $\Lambda = 2$ $\text{m}^3 \cdot \text{s}^{-1}$. Show that the simplest form of the y component of velocity is given by $v = -2\Lambda xy/(x^2 + y^2)^2$. There is no velocity component or variation in the z direction. Calculate the acceleration of fluid particles at points $(x, y) = (0, 1)$, $(0, 2)$, and $(0, 3)$. Estimate the radius of curvature of the streamlines passing through these points. What does the relation among the three points and their radii of curvature suggest to you about the flow field? Verify this by plotting these streamlines. [Hint: You will need to use an integrating factor.]

Given: x component of velocity field

Find: y component of velocity field; acceleration at several points; estimate radius of curvature; plot streamlines

Solution:

The given data is $\Lambda = 2\cdot\dfrac{\text{m}^3}{\text{s}}$ $\qquad u = -\dfrac{\Lambda\cdot\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}$

The governing equation (continuity) is $\dfrac{\partial}{\partial x}u + \dfrac{\partial}{\partial y}v = 0$

Hence $$v = -\int \frac{du}{dx}\,dy = -\int \frac{2\cdot\Lambda\cdot x\cdot\left(x^2 - 3\cdot y^2\right)}{\left(x^2 + y^2\right)^3}\,dy$$

Integrating (using an integrating factor) $$v = -\frac{2\cdot\Lambda\cdot x\cdot y}{\left(x^2 + y^2\right)^2}$$

Alternatively, we could check that the given velocities u and v satisfy continuity

$$u = -\frac{\Lambda\cdot\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} \qquad \frac{\partial}{\partial x}u = \frac{2\cdot\Lambda\cdot x\cdot\left(x^2 - 3\cdot y^2\right)}{\left(x^2 + y^2\right)^3} \qquad v = -\frac{2\cdot\Lambda\cdot x\cdot y}{\left(x^2 + y^2\right)^2} \qquad \frac{\partial}{\partial y}v = -\frac{2\cdot\Lambda\cdot x\cdot\left(x^2 - 3\cdot y^2\right)}{\left(x^2 + y^2\right)^3}$$

so $\dfrac{\partial}{\partial x}u + \dfrac{\partial}{\partial y}v = 0$

The governing equation for acceleration is
$$\vec{a}_p = \underbrace{\frac{D\vec{V}}{Dt}}_{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}} = \underbrace{u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underbrace{\frac{\partial\vec{V}}{\partial t}}_{\substack{\text{local}\\\text{acceleration}}}$$

For steady, 2D flow this reduces to (after considerable math!)

x - component $\qquad a_x = u\cdot\dfrac{\partial}{\partial x}u + v\cdot\dfrac{\partial}{\partial y}u$

$$a_x = \left[-\frac{\Lambda\cdot\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}\right]\cdot\left[\frac{2\cdot\Lambda\cdot x\cdot\left(x^2-3\cdot y^2\right)}{\left(x^2+y^2\right)^3}\right]+\left[-\frac{2\cdot\Lambda\cdot x\cdot y}{\left(x^2+y^2\right)^2}\right]\cdot\left[\frac{2\cdot\Lambda\cdot y\cdot\left(3\cdot x^2-y^2\right)}{\left(x^2+y^2\right)^3}\right] \qquad a_x = -\frac{2\cdot\Lambda^2\cdot x}{\left(x^2+y^2\right)^3}$$

y - component $\quad a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v$

$$a_y = \left[-\frac{\Lambda\cdot\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}\right]\cdot\left[\frac{2\cdot\Lambda\cdot y\cdot\left(3\cdot x^2-y^2\right)}{\left(x^2+y^2\right)^3}\right]+\left[-\frac{2\cdot\Lambda\cdot x\cdot y}{\left(x^2+y^2\right)^2}\right]\cdot\left[\frac{2\cdot\Lambda\cdot y\cdot\left(3\cdot y^2-x^2\right)}{\left(x^2+y^2\right)^3}\right] \qquad a_y = -\frac{2\cdot\Lambda^2\cdot y}{\left(x^2+y^2\right)^3}$$

Evaluating at point (0,1) $\quad u = 2\cdot\frac{m}{s} \quad v = 0\cdot\frac{m}{s} \quad a_x = 0\cdot\frac{m}{s^2} \quad a_y = -8\cdot\frac{m}{s^2}$

Evaluating at point (0,2) $\quad u = 0.5\cdot\frac{m}{s} \quad v = 0\cdot\frac{m}{s} \quad a_x = 0\cdot\frac{m}{s^2} \quad a_y = -0.25\cdot\frac{m}{s^2}$

Evaluating at point (0,3) $\quad u = 0.222\cdot\frac{m}{s} \quad v = 0\cdot\frac{m}{s} \quad a_x = 0\cdot\frac{m}{s^2} \quad a_y = -0.0333\cdot\frac{m}{s^2}$

The instantaneous radius of curvature is obtained from $\quad a_{radial} = -a_y = -\frac{u^2}{r}$ $\quad$ or $\quad r = -\frac{u^2}{a_y}$

For the three points $\quad y = 1\,m \quad r = \frac{\left(2\cdot\frac{m}{s}\right)^2}{8\cdot\frac{m}{s^2}} \quad r = 0.5\,m$

$y = 2\,m \quad r = \frac{\left(0.5\cdot\frac{m}{s}\right)^2}{0.25\cdot\frac{m}{s^2}} \quad r = 1\,m$

$y = 3\,m \quad r = \frac{\left(0.2222\cdot\frac{m}{s}\right)^2}{0.03333\cdot\frac{m}{s^2}} \quad r = 1.5\cdot m$

The radius of curvature in each case is 1/2 of the vertical distance from the origin. The streamlines form circles tangent to the *x* axis

The streamlines are given by $\quad \frac{dy}{dx} = \frac{v}{u} = \frac{-\frac{2\cdot\Lambda\cdot x\cdot y}{\left(x^2+y^2\right)^2}}{-\frac{\Lambda\cdot\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}} = \frac{2\cdot x\cdot y}{\left(x^2-y^2\right)}$

so $\quad -2\cdot x\cdot y\cdot dx + \left(x^2-y^2\right)\cdot dy = 0$

This is an inexact integral, so an integrating factor is needed

First we try $$R = \frac{1}{-2\cdot x\cdot y}\cdot\left[\frac{d}{dx}\left(x^2 - y^2\right) - \frac{d}{dy}(-2\cdot x\cdot y)\right] = -\frac{2}{y}$$

Then the integrating factor is $$F = e^{\int -\frac{2}{y}\,dy} = \frac{1}{y^2}$$

The equation becomes an exact integral $$-2\cdot\frac{x}{y}\cdot dx + \frac{\left(x^2 - y^2\right)}{y^2}\cdot dy = 0$$

So $$u = \int -2\cdot\frac{x}{y}\,dx = -\frac{x^2}{y} + f(y)$$ and $$u = \int \frac{\left(x^2 - y^2\right)}{y^2}\,dy = -\frac{x^2}{y} - y + g(x)$$

Comparing solutions $$\psi = \frac{x^2}{y} + y$$ or $$x^2 + y^2 = \psi\cdot y = \text{const}\cdot y$$

These form circles that are tangential to the *x* axis, as shown in the associated *Excel* workbook

Problem 6.34

[4]

6.34 The x component of velocity in a two-dimensional incompressible flow field is given by $u = -\Lambda(x^2 - y^2)/(x^2 + y^2)^2$, where u is in m/s, the coordinates are measured in meters, and $\Lambda = 2\ \mathrm{m^3 \cdot s^{-1}}$. Show that the simplest form of the y component of velocity is given by $v = -2\Lambda xy/(x^2 + y^2)^2$. There is no velocity component or variation in the z direction. Calculate the acceleration of fluid particles at points $(x, y) = (0, 1)$, $(0, 2)$, and $(0, 3)$. Estimate the radius of curvature of the streamlines passing through these points. What does the relation among the three points and their radii of curvature suggest to you about the flow field? Verify this by plotting these streamlines. [Hint: You will need to use an integrating factor.]

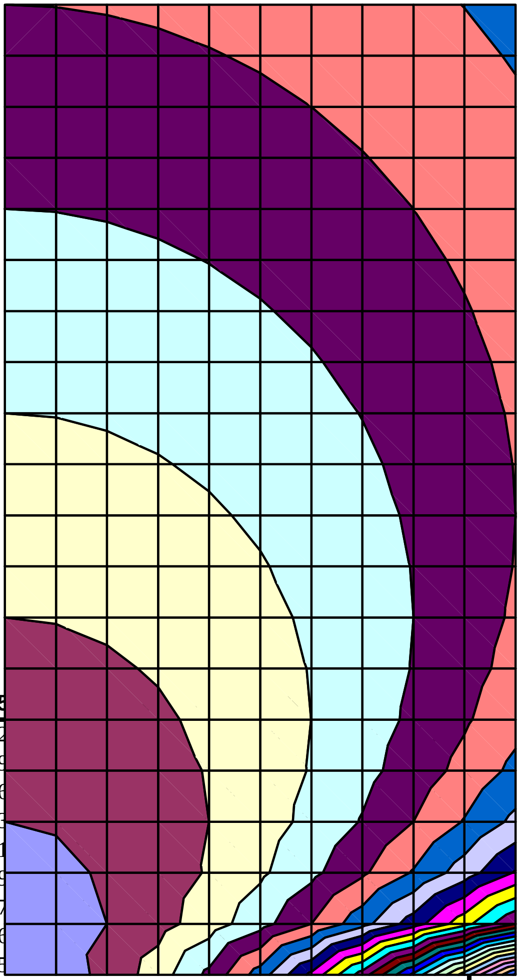
Given: x component of velocity field

Find: y component of velocity field; acceleration at several points; estimate radius of curvature; plot streamlines

Solution:

$$\psi = \frac{x^2}{y} + y$$

This function is computed and plotted below



y values

x values	0.10	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.5						
2.50	62.6	25.3	13.0	9.08	7.25	6.25	5.67	5.32	5.13	5.03	5.00	5.02	5.08	5.17	5.2						
2.25	50.7	20.5	10.6	7.50	6.06	5.30	4.88	4.64	4.53	4.50	4.53	4.59	4.69	4.81	4.9						
2.00	40.1	16.3	8.50	6.08	5.00	4.45	4.17	4.04	4.00	4.03	4.10	4.20	4.33	4.48	4.6						
1.75	30.7	12.5	6.63	4.83	4.06	3.70	3.54	3.50	3.53	3.61	3.73	3.86	4.02	4.19	4.3						
1.50	22.6	9.25	5.00	3.75	3.25	3.05	3.00	3.04	3.13	3.25	3.40	3.57	3.75	3.94	4.1						
1.25	15.7	6.50	3.63	2.83	2.56	2.50	2.54	2.64	2.78	2.94	3.13	3.32	3.52	3.73	3.9						
1.00	10.1	4.25	2.50	2.08	2.00	2.05	2.17	2.32	2.50	2.69	2.90	3.11	3.33	3.56	3.7						
0.75	5.73	2.50	1.63	1.50	1.56	1.70	1.88	2.07	2.28	2.50	2.73	2.95	3.19	3.42	3.6						
0.50	2.60	1.25	1.00	1.08	1.25	1.45	1.67	1.89	2.13	2.36	2.60	2.84	3.08	3.33	3.5						
0.25	0.73	0.50	0.63	0.83	1.06	1.30	1.54	1.79	2.03	2.28	2.53	2.77	3.02	3.27	3.52	3.77	4.02	4.26	4.51	4.76	5.01
0.00	0.10	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00

Given: The y component of velocity in a 2-D, incompressible flow field is
$v = -Axy$ where $A = 1\ m^{-1} \cdot s^{-1}$ and coordinates are in meters; $w = 0$ and $\partial/\partial z = 0$.

Find: (a) acceleration of fluid particle at $(x,y) = (1,2)$
(b) radius of curvature of streamline at $(1,2)$.

Plot: streamline through $(1,2)$; show velocity and acceleration vectors on the plot.

Solution:

For 2-D incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, so $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$.

$$u = \int \frac{\partial u}{\partial x} dx + f(y) = \int -\frac{\partial v}{\partial y} dx + f(y) = -\int (-Ax)\, dx + f(y) = \frac{Ax^2}{2} + f(y)$$

Choose the simplest solution, $f(y) = 0$, so $u = \frac{Ax^2}{2}$. Hence

$$\vec{V} = \frac{Ax^2}{2}\hat{i} - Axy\hat{j} = A\left(\frac{x^2}{2}\hat{i} - xy\hat{j}\right)$$

The acceleration of a fluid particle is

$$\vec{a}_p = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} = \frac{Ax^2}{2}(Ax\hat{i} - Ay\hat{j}) - Axy(-Ax\hat{j}).$$

$$\vec{a}_p = \frac{A^2x^3}{2}\hat{i} + \frac{A^2x^2y}{2}\hat{j} = \frac{A^2}{2}(x^3\hat{i} + x^2y\hat{j})$$

At the point $(1,2)$

$$\vec{a}_p = \frac{1}{2} \times (1)\frac{1}{m^2 s^2}\left[(1)^3 m^3\hat{i} + (1)^2(2)\,m^3\hat{j}\right] = 0.5\hat{i} + \hat{j}\ m/s^2 \longleftarrow \vec{a}(1,2)$$

$$\vec{V} = \frac{1}{m \cdot s}\left[\frac{1}{2}(1)^2 m^2\hat{i} - (1)(2)\,m^2\hat{j}\right] = 0.5\hat{i} - 2\hat{j}\ m/s$$

The unit vector tangent to the streamline is

$$\hat{e}_t = \frac{\vec{V}}{|\vec{V}|} = \frac{0.5\hat{i} - 2\hat{j}}{[(0.5)^2 + (-2)^2]^{1/2}} = 0.243\hat{i} - 0.970\hat{j}$$

The unit vector normal to the streamline is

$$\vec{e}_n = \hat{e}_t \times \hat{k} = (0.243\hat{i} - 0.970\hat{j}) \times \hat{k} = -0.970\hat{i} - 0.243\hat{j}$$

The normal component of acceleration is

$$a_n = -\frac{V^2}{R} = \vec{a} \cdot \hat{e}_n = (0.5\hat{i} + \hat{j}) \cdot (-0.970\hat{i} - 0.243\hat{j})$$

$$-\frac{V^2}{R} = -0.728\ m/s^2$$

$$R = \frac{V^2}{0.728} = \frac{4.25}{0.728}\frac{m^2/s^2}{m/s^2} = 5.84\ m \longleftarrow R(1,2)$$

The slope of the streamlines is given by

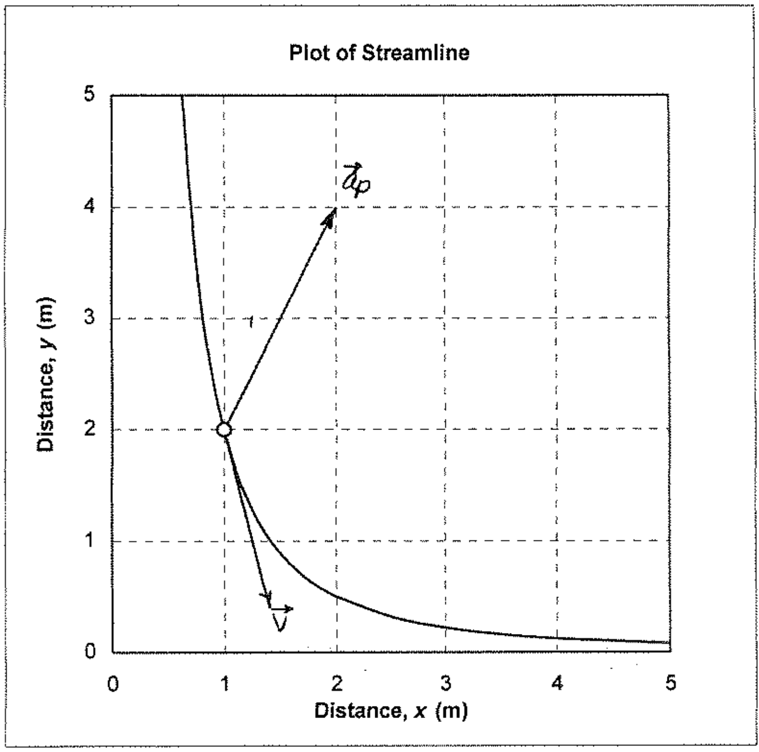
$$\left.\frac{dy}{dx}\right)_{sl} = \frac{v}{u} = \frac{-Axy}{Ax^2/2} = -\frac{2y}{x}$$

Problem 6.35

Thus $\frac{dy}{y} + \frac{2dx}{x} = 0$ and $\ln y + \ln x^2 = \ln c$ or

$$x^2 y = c$$

The equation of the streamline through (1,2) is $x^2 y = 2$



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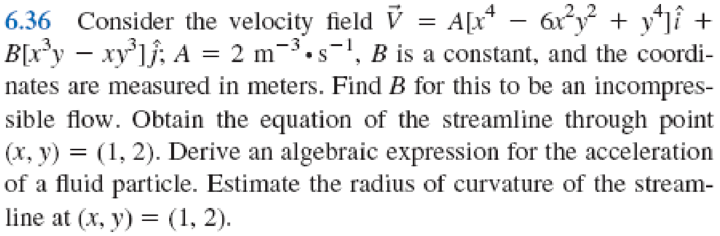
Problem 6.36

[5]



6.36 Consider the velocity field $\vec{V} = A[x^4 - 6x^2y^2 + y^4]\hat{i} + B[x^3y - xy^3]\hat{j}$; $A = 2\ \mathrm{m^{-3}\cdot s^{-1}}$, B is a constant, and the coordinates are measured in meters. Find B for this to be an incompressible flow. Obtain the equation of the streamline through point $(x, y) = (1, 2)$. Derive an algebraic expression for the acceleration of a fluid particle. Estimate the radius of curvature of the streamline at $(x, y) = (1, 2)$.

Given: Velocity field

Find: Constant B for incompressible flow; Equation for streamline through (1,2); Acceleration of particle; streamline curvatur

Solution:

Basic equations $\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

$$\vec{a}_p = \underset{\substack{\text{total}\\\text{acceleration}\\\text{of a particle}}}{\frac{D\vec{V}}{Dt}} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\substack{\text{convective}\\\text{acceleration}}} + \underset{\substack{\text{local}\\\text{acceleration}}}{\frac{\partial \vec{V}}{\partial t}}$$

For this flow $\quad u(x,y) = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right) \qquad v(x,y) = B\cdot\left(x^3\cdot y - x\cdot y^3\right)$

$$\frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}v(x,y) = \frac{\partial}{\partial x}\left[A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\right] + \frac{\partial}{\partial y}\left[B\cdot\left(x^3\cdot y - x\cdot y^3\right)\right] = 0$$

$$\frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}v(x,y) = B\cdot\left(x^3 - 3\cdot x\cdot y^2\right) + A\cdot\left(4\cdot x^3 - 12\cdot x\cdot y^2\right) = (4\cdot A + B)\cdot x\cdot\left(x^2 - 3\cdot y^2\right) = 0$$

Hence $\quad B = -4\cdot A \qquad\qquad B = -8\dfrac{1}{m^3\cdot s}$

Hence for a_x

$$a_x = u\cdot\frac{\partial}{\partial x}u + v\cdot\frac{\partial}{\partial y}u = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\cdot\frac{\partial}{\partial x}\left[A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\right] + \left[-4\cdot A\cdot\left(x^3\cdot y - x\cdot y^3\right)\right]\cdot\frac{\partial}{\partial y}\left[A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\right]$$

$$a_x = 4\cdot A^2\cdot x\cdot\left(x^2 + y^2\right)^3$$

For a_y

$$a_y = u\cdot\frac{\partial}{\partial x}v + v\cdot\frac{\partial}{\partial y}v = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)\cdot\frac{\partial}{\partial x}\left[-4\cdot A\cdot\left(x^3\cdot y - x\cdot y^3\right)\right] + \left[-4\cdot A\cdot\left(x^3\cdot y - x\cdot y^3\right)\right]\cdot\frac{\partial}{\partial y}\left[-4\cdot A\cdot\left(x^3\cdot y - x\cdot y^3\right)\right]$$

$$a_y = 4\cdot A^2\cdot y\cdot\left(x^2 + y^2\right)^3$$

For a streamline $\quad \dfrac{dy}{dx} = \dfrac{v}{u} \quad$ so $\quad \dfrac{dy}{dx} = \dfrac{-4\cdot A\cdot\left(x^3\cdot y - x\cdot y^3\right)}{A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)} = -\dfrac{4\cdot\left(x^3\cdot y - x\cdot y^3\right)}{\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)}$

Let $\quad u = \dfrac{y}{x} \qquad \dfrac{du}{dx} = \dfrac{d\left(\frac{y}{x}\right)}{dx} = \dfrac{1}{x}\cdot\dfrac{dy}{dx} + y\cdot\dfrac{d\left(\frac{1}{x}\right)}{dx} = \dfrac{1}{x}\cdot\dfrac{dy}{dx} - \dfrac{y}{x^2}$ so $\quad \dfrac{dy}{dx} = x\cdot\dfrac{du}{dx} + u$

Hence $$\frac{dy}{dx} = x\cdot\frac{du}{dx} + u = -\frac{4\cdot\left(x^3\cdot y - x\cdot y^3\right)}{\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)} = -\frac{4\cdot\left(1 - u^2\right)}{\left(\frac{1}{u} - 6\cdot u + u^3\right)} \quad u + \frac{4\cdot\left(1 - u^2\right)}{\left(\frac{1}{u} - 6\cdot u + u^3\right)}$$

$$x\cdot\frac{du}{dx} = -\left[u + \frac{4\cdot\left(1 - u^2\right)}{\left(\frac{1}{u} - 6\cdot u + u^3\right)}\right] = -\frac{u\cdot\left(u^4 - 10\cdot u^2 + 5\right)}{u^4 - 6\cdot u^2 + 1}$$

Separating variables $$\frac{dx}{x} = -\frac{u^4 - 6\cdot u^2 + 1}{u\cdot\left(u^4 - 10\cdot u^2 + 5\right)}\cdot du \qquad \ln(x) = -\frac{1}{5}\cdot\ln\left(u^5 - 10\cdot u^3 + 5\cdot u\right) + C$$

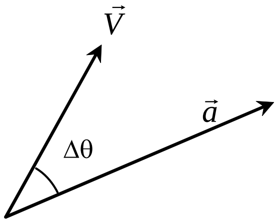
$$\left(u^5 - 10\cdot u^3 + 5\cdot u\right)\cdot x^5 = c \qquad y^5 - 10\cdot y^3\cdot x^2 + 5\cdot y\cdot x^4 = \text{const}$$

For the streamline through (1,2) $y^5 - 10\cdot y^3\cdot x^2 + 5\cdot y\cdot x^4 = -38$

Note that it would be MUCH easier to use the stream function method here!

To find the radius of curvature we use $a_n = -\frac{V^2}{R}$ or $|R| = \frac{V^2}{a_n}$

We need to find the component of acceleration normal to the velocity vector



At (1,2) the velocity vector is at angle $$\theta_{vel} = \operatorname{atan}\left(\frac{v}{u}\right) = \operatorname{atan}\left[-\frac{4\cdot\left(x^3\cdot y - x\cdot y^3\right)}{\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right)}\right]$$

$$\theta_{vel} = \operatorname{atan}\left[-\frac{4\cdot(2-8)}{1 - 24 + 16}\right] \qquad \theta_{vel} = -73.7\cdot\text{deg}$$

At (1,2) the acceleration vector is at angle $$\theta_{accel} = \operatorname{atan}\left(\frac{a_y}{a_x}\right) = \operatorname{atan}\left[\frac{4\cdot A^2\cdot y\cdot\left(x^2 + y^2\right)^3}{4\cdot A^2\cdot x\cdot\left(x^2 + y^2\right)^3}\right] = \operatorname{atan}\left(\frac{y}{x}\right)$$

$$\theta_{accel} = \operatorname{atan}\left(\frac{2}{1}\right) \qquad \theta_{accel} = 63.4\cdot\text{deg}$$

Hence the angle between the acceleration and velocity vectors is $\Delta\theta = \theta_{accel} - \theta_{vel}$ $\Delta\theta = 137\cdot\text{deg}$

The component of acceleration normal to the velocity is then $a_n = a\cdot\sin(\Delta\theta)$ where $a = \sqrt{a_x^2 + a_y^2}$

At (1,2) $$a_x = 4\cdot A^2\cdot x\cdot\left(x^2 + y^2\right)^3 = 500\cdot m^7 \times A^2 = 500\cdot m^7 \times \left(\frac{2}{m^3\cdot s}\right)^2 = 2000\cdot\frac{m}{s^2}$$

$$a_y = 4\cdot A^2\cdot y\cdot\left(x^2 + y^2\right)^3 = 1000\cdot m^7 \times A^2 = 1000\cdot m^7 \times \left(\frac{2}{m^3\cdot s}\right)^2 = 4000\cdot\frac{m}{s^2}$$

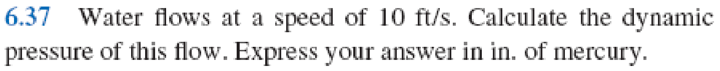
$$a = \sqrt{2000^2 + 4000^2}\cdot\frac{m}{s^2} \qquad a = 4472\frac{m}{s^2} \qquad a_n = a\cdot\sin(\Delta\theta) \qquad a_n = 3040\frac{m}{s^2}$$

$$u = A\cdot\left(x^4 - 6\cdot x^2\cdot y^2 + y^4\right) = -14\cdot\frac{m}{s} \qquad v = B\cdot\left(x^3\cdot y - x\cdot y^3\right) = 48\cdot\frac{m}{s} \qquad V = \sqrt{u^2 + v^2} = 50\cdot\frac{m}{s}$$

Then $$|R| = \frac{V^2}{a_n} \qquad R = \left(50\cdot\frac{m}{s}\right)^2 \times \frac{1}{3040}\cdot\frac{s^2}{m} \qquad R = 0.822\,m$$

Problem 6.37

[1]



6.37 Water flows at a speed of 10 ft/s. Calculate the dynamic pressure of this flow. Express your answer in in. of mercury.

Given: Water at speed 10 ft/s

Find: Dynamic pressure in in. Hg

Solution:

Basic equation $p_{dynamic} = \frac{1}{2} \cdot \rho \cdot V^2$ $\qquad p = \rho_{Hg} \cdot g \cdot \Delta h = SG_{Hg} \cdot \rho \cdot g \cdot \Delta h$

Hence $\Delta h = \frac{\rho \cdot V^2}{2 \cdot SG_{Hg} \cdot \rho \cdot g} = \frac{V^2}{2 \cdot SG_{Hg} \cdot g}$

$$\Delta h = \frac{1}{2} \times \left(10 \cdot \frac{ft}{s}\right)^2 \times \frac{1}{13.6} \times \frac{s^2}{32.2 \cdot ft} \times \frac{12 \cdot in}{1 \cdot ft} \qquad \Delta h = 1.37 \cdot in$$

Problem 6.38

[1]

Given: Standard air

Find: Dynamic pressure that corresponds to $V = 100\ km/hr$

Solution: Dynamic pressure is $p_{dyn} = \frac{1}{2}\rho V^2$

For standard air, $\rho = 1.23\ kg/m^3$

Then $$p_{dyn} = \frac{1}{2} \times 1.23 \frac{kg}{m^3} \times (100)^2 \frac{(km)^2}{(hr)^2} \times (1000)^2 \frac{m^2}{(km)^2} \times \frac{(hr)^2}{(3600)^2 s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$p_{dyn} = 475\ N/m^2 \longleftarrow$$ p_{dyn}

This may be expressed conveniently as a water column height.

$$p_{dyn} = \rho_{water}\, g\, h_{dyn}$$

$$h_{dyn} = \frac{p_{dyn}}{\rho_w g} = 475 \frac{N}{m^2} \times \frac{m^3}{999\ kg} \times \frac{s^2}{9.81\ m} \times \frac{kg \cdot m}{N \cdot s^2}$$

$$h_{dyn} = 0.0484\ m \quad \text{or} \quad 48.4\ mm \longleftarrow$$ h_{dyn}

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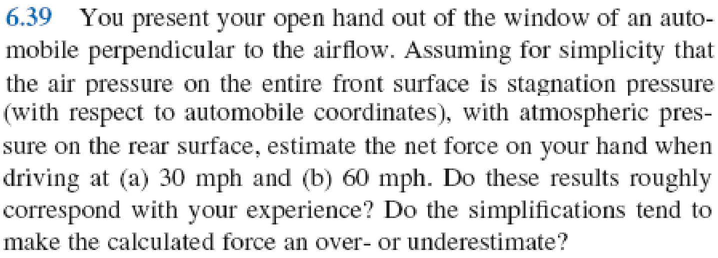
Problem 6.39

[1]



6.39 You present your open hand out of the window of an automobile perpendicular to the airflow. Assuming for simplicity that the air pressure on the entire front surface is stagnation pressure (with respect to automobile coordinates), with atmospheric pressure on the rear surface, estimate the net force on your hand when driving at (a) 30 mph and (b) 60 mph. Do these results roughly correspond with your experience? Do the simplifications tend to make the calculated force an over- or underestimate?

Given: Velocity of automobile

Find: Estimates of aerodynamic force on hand

Solution:

For air $\rho = 0.00238 \cdot \frac{\text{slug}}{\text{ft}^3}$

We need an estimate of the area of a typical hand. Personal inspection indicates that a good approximation is a square of sides 9 cm and 17 cm

$$A = 9 \cdot \text{cm} \times 17 \cdot \text{cm} \qquad A = 153\,\text{cm}^2$$

The governing equation is the Bernoulli equation (in coordinates attached to the vehicle)

$$p_{atm} + \frac{1}{2} \cdot \rho \cdot V^2 = p_{stag}$$

where *V* is the free stream velocity

Hence, for p_{stag} on the front side of the hand, and p_{atm} on the rear, by assumption,

$$F = \left(p_{stag} - p_{atm}\right) \cdot A = \frac{1}{2} \cdot \rho \cdot V^2 \cdot A$$

(a) $V = 30 \cdot \text{mph}$

$$F = \frac{1}{2} \cdot \rho \cdot V^2 \cdot A = \frac{1}{2} \times 0.00238 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(30 \cdot \text{mph} \cdot \frac{22 \cdot \frac{\text{ft}}{\text{s}}}{15 \cdot \text{mph}}\right)^2 \times 153 \cdot \text{cm}^2 \times \left(\frac{\frac{1}{12} \cdot \text{ft}}{2.54 \cdot \text{cm}}\right)^2 \qquad F = 0.379\,\text{lbf}$$

(b) $V = 60 \cdot \text{mph}$

$$F = \frac{1}{2} \cdot \rho \cdot V^2 \cdot A = \frac{1}{2} \times 0.00238 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(60 \cdot \text{mph} \cdot \frac{22 \cdot \frac{\text{ft}}{\text{s}}}{15 \cdot \text{mph}}\right)^2 \times 153 \cdot \text{cm}^2 \times \left(\frac{\frac{1}{12} \cdot \text{ft}}{2.54 \cdot \text{cm}}\right)^2 \qquad F = 1.52\,\text{lbf}$$

Problem 6.40

6.40 A jet of air from a nozzle is blown at right angles against a wall in which two pressure taps are located. A manometer connected to the tap directly in front of the jet shows a head of 0.15 in. of mercury above atmospheric. Determine the approximate speed of the air leaving the nozzle if it is at 50°F and 14.7 psia. At the second tap a manometer indicates a head of 0.10 in. of mercury above atmospheric; what is the approximate speed of the air there?

Given:
Air jet hitting wall generating pressures

Find:
Speed of air at two locations

Solution:

Basic equation $\frac{p}{\rho_{air}} + \frac{V^2}{2} + g \cdot z = \text{const}$ $\qquad \rho_{air} = \frac{p}{R_{air} \cdot T}$ $\qquad \Delta p = \rho_{Hg} \cdot g \cdot \Delta h = SG_{Hg} \cdot \rho \cdot g \cdot \Delta h$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the jet and where it hits the wall directly

$$\frac{p_{atm}}{\rho_{air}} + \frac{V_j^2}{2} = \frac{p_{wall}}{\rho_{air}} \qquad p_{wall} = \frac{\rho_{air} \cdot V_j^2}{2} \qquad \text{(working in gage pressures)}$$

For air $\quad \rho_{air} = 14.7 \cdot \frac{lbf}{in^2} \times \frac{144 \cdot in^2}{1 \cdot ft^2} \times \frac{lbm \cdot R}{53.33 \cdot ft \cdot lbf} \times \frac{1 \cdot slug}{32.2 \cdot lbm} \times \frac{1}{(50 + 460) \cdot R} \qquad \rho_{air} = 2.42 \times 10^{-3} \frac{slug}{ft^3}$

Hence $\quad p_{wall} = SG_{Hg} \cdot \rho \cdot g \cdot \Delta h = \frac{\rho_{air} \cdot V_j^2}{2} \qquad$ so $\qquad V_j = \sqrt{\frac{2 \cdot SG_{Hg} \cdot \rho \cdot g \cdot \Delta h}{\rho_{air}}}$

Hence $\quad V_j = \sqrt{2 \times 13.6 \times 1.94 \cdot \frac{slug}{ft^3} \times \frac{1}{2.42 \times 10^{-3}} \cdot \frac{ft^3}{slug} \times 32.2 \cdot \frac{ft}{s^2} \times 0.15 \cdot in \times \frac{1ft}{12 \cdot in}} \qquad V_j = 93.7 \frac{ft}{s}$

Repeating the analysis for the second point

$$\frac{p_{atm}}{\rho_{air}} + \frac{V_j^2}{2} = \frac{p_{wall}}{\rho_{air}} + \frac{V^2}{2} \qquad V = \sqrt{V_j^2 - \frac{2 \cdot p_{wall}}{\rho_{air}}} = \sqrt{V_j^2 - \frac{2 \cdot SG_{Hg} \cdot \rho \cdot g \cdot \Delta h}{\rho_{air}}}$$

Hence $\quad V = \sqrt{\left(93.7 \cdot \frac{ft}{s}\right)^2 - 2 \times 13.6 \times 1.94 \cdot \frac{slug}{ft^3} \times \frac{1}{2.42 \times 10^{-3}} \cdot \frac{ft^3}{slug} \times 32.2 \cdot \frac{ft}{s^2} \times 0.1 \cdot in \times \frac{1ft}{12 \cdot in}} \qquad V = 54.1 \frac{ft}{s}$

Problem 6.41

[2]

Given: Pitot static probe is used to measure speed in standard air.

$V = 100\ \text{m/s}$

Find: Manometer deflection in mm H_2O, corresponding to given conditions.

Solution:

Manometer reads $p_0 - p$ in mm of H_2O.

Basic equations: $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$ for flow

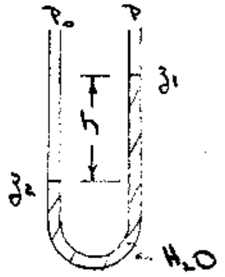
$\frac{dp}{dz} = -\rho g$ for manometer

Assumptions:
(1) steady flow
(2) incompressible flow
(3) flow along a streamline
(4) frictionless deceleration to p_0
(5) ρ = constant for manometer

From the Bernoulli equation

$$\frac{p_0}{\rho} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$p_0 - p = \rho\frac{V^2}{2}$$



For the manometer, $dp = -\rho g dz$

$$p_0 - p = \int_p^{p_0} dp = -\rho g(z_2 - z_1) = \rho g h$$

Then,

$$\rho_{H_2O}\, g h = \rho_{air}\frac{V^2}{2}$$

and

$$h = \frac{\rho_{air}}{\rho_{H_2O}}\frac{V^2}{2g} = \frac{1.23}{999} \times (100)^2\frac{m^2}{s^2} \times \frac{1}{2} \times \frac{s^2}{9.81\,m} \times \frac{10^3\,mm}{m} = 628\ mm \longleftarrow \boxed{h}$$

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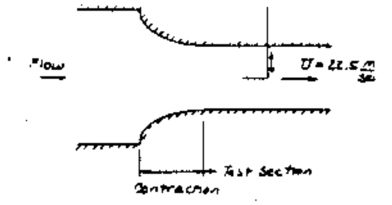
Problem 6.42

[2]

Given: Wind tunnel with inlet and test section as shown.

$\bar{U} = 22.5$ m/s , $p_{0s} = -6.0$ mm H$_2$O gage

$p_a = 99.1$ kPa (abs) , $T_a = 23°C$



Find: (a) $p_{dynamic}$ on tunnel centerline
(b) p_{static} " " "
(c) compare p_{static} at tunnel wall with that measured at centerline

Solution:

(a) By definition $p_{dyn} = \frac{1}{2}\rho \bar{U}^2$

Assume: (1) air behaves as an ideal gas, and (2) incompressible flow

Then $\rho = \frac{p}{RT} = 99.1\times10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287 N\cdot m} \times \frac{1}{(273+23)K} = 1.17 \, kg/m^3$

and $p_{dyn} = \frac{1}{2}\rho\bar{U}^2 = \frac{1}{2}\times 1.17\frac{kg}{m^3}\times(22.5)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = 296 \, N/m^2$ ⟵ p_{dyn}

(b) By definition $p_0 = p_s + p_{dyn}$

$\therefore p_s = p_0 - p_{dyn}$ where $p_0 = -6$ mm H$_2$O gage

then $p_0 - p_a = \rho g \Delta h = 999\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times -6\times10^{-3} m \times \frac{N\cdot s^2}{kg\cdot m}$

$p_{0gage} = -58.8 \, N/m^2$

$\therefore p_s = p_0 - p_{dyn} = -58.8 - 296 = -355 \, N/m^2$ gage ⟵ p_{static}

$\{$ or $p_s = -36.2$ mm H$_2$O (gage) $\}$

(c) Streamlines in the test section should be straight. Then in the test section the variation of static pressure is given by $\frac{\partial p}{\partial n} = 0$ and $p_{wall} = p_{centerline}$ ⟵

In the contraction section the streamlines are curved. The variation of static pressure normal to the streamlines is given by $\frac{\partial p}{\partial n} = \rho\frac{V^2}{R}$

and consequently the static pressure increases toward the centerline, i.e. $p_{wall} < p_{centerline}$

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Problem 6.43

[2]

Given: High-pressure hydraulic system subject to small leak.

Plot: Jet speed of a leak vs system-pressure for system pressures up to 40 MPa gage; explain how a high-speed jet of hydraulic fluid can cause injury

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Solution:

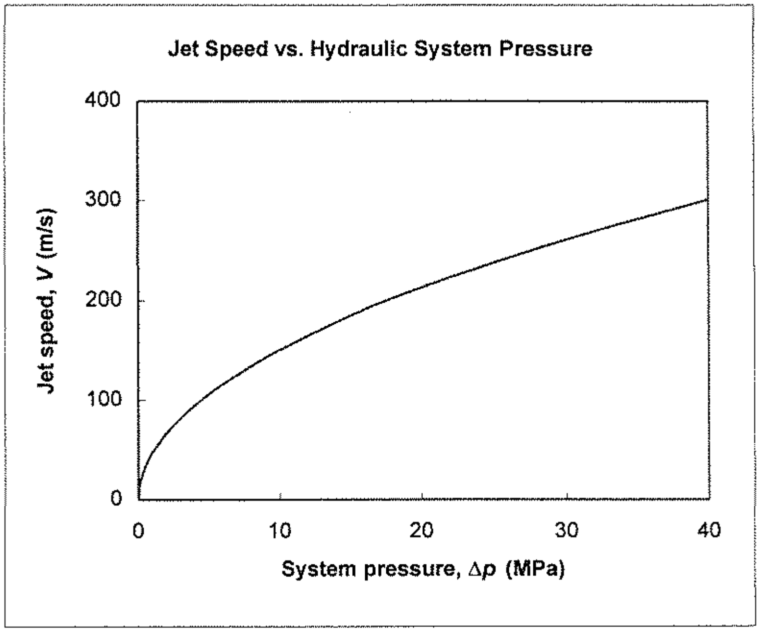
Basic equation: $\frac{p}{\rho}+\frac{V^2}{2}+gz=\text{constant}$

Assumptions: (1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline

The Bernoulli equation gives

$$V=\left[\frac{2(p_0-p_{atm})}{\rho}\right]^{1/2}$$

From Table A.2 (Appendix A) for lubricating oil SG = 0.88



The high stagnation pressure ruptures the skin causing the jet to penetrate the tissue.

Problem 6.44

[2]

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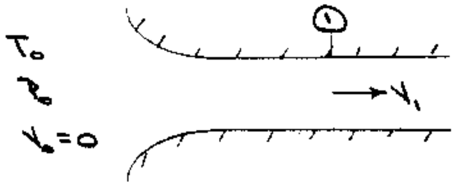
Given: Air flow in open circuit wind tunnel as shown.

$p_{atm} - p_1 = 45 \text{ mm } H_2O$

$T_0 = 25C$

$p_0 = p_{atm}$

Consider air to be incompressible.



Find: Air speed in tunnel at section ①

Solution:

Basic equations: $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$

Assumptions: (1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline
(5) air behaves as an ideal gas
(6) stagnation pressure = p_{atm}

From the Bernoulli equation, $\frac{p_0}{\rho} = \frac{p_1}{\rho} + \frac{V_1^2}{2}$.

$$p_0 - p_1 = p_{atm} - p_1 = \frac{1}{2}\rho V_1^2$$

$$V_1 = \left[\frac{2(p_{atm} - p_1)}{\rho}\right]^{1/2}$$

From the manometer reading. $p_{atm} - p_1 = \rho_{H_2O}\, g\, h$ Then

$$V_1 = \left[\frac{2\rho_{H_2O}\, g\, h}{\rho}\right]^{1/2}$$

From the ideal gas equation of state

$$\rho = \frac{p}{RT} = 100 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\, N \cdot m} \times \frac{1}{298\, K} = 1.17\, kg/m^3$$

$$V_1 = \left[\frac{2\rho_{H_2O}}{\rho} gh\right]^{1/2} = \left[2 \times \frac{999}{1.17} \times 9.81 \frac{m}{s^2} \times 0.045\, m\right]^{1/2} = 27.5\, m/s \longleftarrow V_1$$

Problem 6.45

[4]_____

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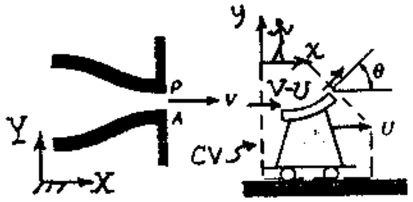
Given: Wheeled cart of Problem 4.123;

$V = 40\ m/s$
$A = 25\ mm^2$
Water, no friction on vane, $\theta = 120°$
Vane accelerates to the right



Find: At instant when $U = 15\ m/s$,

(a) stagnation pressure leaving nozzle, relative to fixed observer.
(b) Stagnation pressure leaving nozzle, relative to observer on vane.
(c) Absolute velocity of jet leaving vane.
(d) Stagnation pressure of jet leaving vane, relative to fixed observer.
(e) How would viscous forces increase, decrease, or leave unchanged the stagnation pressure in (d). How can you justify this?

Solution: Stagnation pressure is $p_0 = p + \frac{1}{2}\rho V^2$ or $p_0 - p = \frac{1}{2}\rho V^2$

At jet, $p_{0_j} = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 999 \frac{kg}{m^3} \times (40)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 799\ kPa\ (gage)$ ← p_{0j}

At cart, $p_{0,rel} = \frac{1}{2}\rho (V-U)^2 = \frac{1}{2} \times 999 \frac{kg}{m^3} \times (40-15)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 312\ kPa\ (gage)$ ← $p_{0,rel}$

Leaving vane, $\vec{V}_{abs} = U\hat{i} + (V-U)(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$$\vec{V}_{abs} = [U + (V-U)\cos\theta]\hat{i} + (V-U)\sin\theta\,\hat{j}$$

$$= \left[15\frac{m}{s} + (40-15)\frac{m}{s} \times \left(-\frac{1}{2}\right)\right]\hat{i} + (40-15)\frac{m}{s} \times 0.866\,\hat{j}$$

$$\vec{V}_{abs} = 2.5\hat{i} + 21.7\hat{j}\ m/s$$ ← $\vec{V}_{abs}$

The magnitude $|\vec{V}_{abs}| = [(2.5)^2 + (21.7)^2]^{1/2}\ m/s = 21.8\ m/s$

Leaving vane, $p_0 = \frac{1}{2}\rho |\vec{V}_{abs}|^2$, relative to a fixed observer. Thus

$$p_{0,fixed} = \frac{1}{2} \times 999 \frac{kg}{m^3} \times (21.8)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 237\ kPa\ (gage)$$ ← $p_{0,fixe}$

{The corresponding absolute pressures are 900, 413, and 338 kPa (abs).}

Discussion: Viscous forces would slow the jet speed relative to the vane. The jet would enter the vane with relative speed (V - U); it would leave the vane with speed α (V - U), where $\alpha < 1$.

Friction would reduce both components of relative velocity leaving the vane. The absolute velocity of the jet leaving the vane, as seen by a fixed observer, would decrease. Thus the stagnation pressure of the flow leaving the vane, relative to a fixed observer, would decrease.

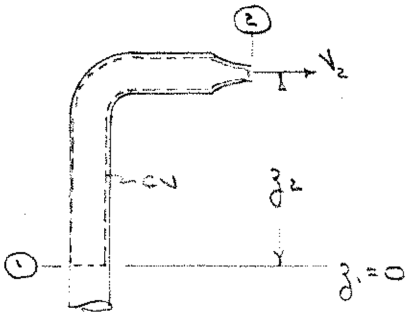
Problem 6.46

Given: Steady flow of water through elbow and nozzle as shown

$D_1 = 0.1\,m \qquad D_2 = 0.05\,m$

$p_2 = p_{atm} \qquad V_2 = 20\,m/s$

$z_1 = 0 \qquad z_2 = 4\,m$



Find: Gage pressure, p_1; p_1 if device were inverted

Solution: Apply continuity to cv shown to determine V_1; the Bernoulli equation is then applied along a streamline from ① to ② to determine p_1.

Basic equations:

$$0 = \overset{\approx 0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall}} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \overset{\approx 0(6)}{\cancel{gz_1}} = \overset{\approx 0(5)}{\cancel{\frac{p_2}{\rho}}} + \frac{V_2^2}{2} + gz_2$$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a steamline
(5) p_2 gage $= 0$
(6) $z_1 = 0$

From the continuity equation, $\quad 0 = -|\rho V_1 A_1| + |\rho V_2 A_2|$

then, $\quad V_1^2 = \left(\frac{A_2}{A_1}\right)^2 V_2^2 = \left(\frac{D_2}{D_1}\right)^4 V_2^2$

From the Bernoulli equation

$$p_1 = \rho\left[\frac{V_2^2 - V_1^2}{2} + gz_2\right] = \rho\left[\frac{V_2^2}{2}\left(1 - \frac{V_1^2}{V_2^2}\right) + gz_2\right] = \rho\left[\frac{V_2^2}{2}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) + gz_2\right]$$

$$p_1 = 999\,\frac{kg}{m^3}\left[\frac{1}{2} \times (20)^2\,\frac{m^2}{s^2} \times \left(1 - \left(\frac{1}{2}\right)^4\right) + 9.81\,\frac{m}{s^2} \times 4\,m\right] \times \frac{N \cdot s^2}{kg \cdot m}$$

$p_1 = 227\,kN/m^2 = 227\,kPa$ (gage) $\longleftarrow$ p_1

If device is inverted, $z_2 = -4\,m$ with $z_1 = 0$

$$p_1 = \rho\left[\frac{V_2^2}{2}\left\{1 - \left(\frac{D_2}{D_1}\right)^4\right\} + gz_2\right]$$

$$= 999\,\frac{kg}{m^3}\left[\frac{1}{2} \times (20)^2\,\frac{m^2}{s^2}\left\{1 - \left(\frac{1}{2}\right)^4\right\} + 9.81\,\frac{m}{s^2} \times (-4\,m)\right]\frac{N \cdot s^2}{kg \cdot m}$$

$p_1 = 148\,kN/m^2 = 148\,kPa$ (gage) $\longleftarrow$ p_1

Problem 6.47

[2]

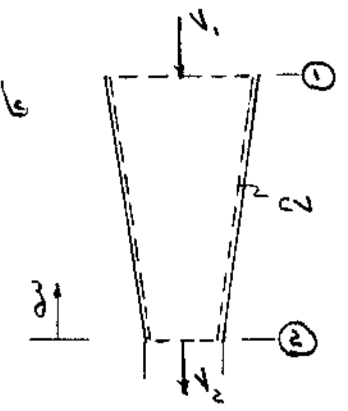
Given: Water flow in a circular duct

$D_1 = 0.3\,m \quad P_1 = 260\ kPa\,(gage) \quad \vec{V}_1 = -3\hat{k}\ m/s$

$z_1 = 10\,m$

$z_2 = 0 \quad D_2 = 0.15\,m$

Frictional effects may be neglected.



Find: Pressure, P_2

Solution: Apply continuity to cv shown to determine V_2; the Bernoulli equation is then applied along a streamline from ① to ② to determine P_2

Basic equations: $$0 = \overset{0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall}} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline
(5) uniform flow at sections ① and ②

From the continuity equation

$$0 = -|\rho V_1 A_1| + |\rho V_2 A_2|$$

Then, $$V_2 = \frac{A_1}{A_2} V_1 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{0.3}{0.15}\right)^2 \times 3\,\frac{m}{s} = 12\ m/s$$

From the Bernoulli equation,

$$P_2 = P_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) + \rho g (z_1 - z_2)$$

$$= 260\,\frac{kN}{m^2} + \frac{1}{2} \times 999\,\frac{kg}{m^3} \times \left[(3)^2 - (12)^2\right]\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} + 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 10\,m \times \frac{N \cdot s^2}{kg \cdot m}$$

$P_2 = 291\ kN/m^2 = 291\ kPa$ (gage).

P_2

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Problem 6.48

[2]

6.48 You are on a date. Your date runs out of gas unexpectedly. You come to your own rescue by siphoning gas from another car. The height difference for the siphon is about 6 in. The hose diameter is 1 in. What is your gasoline flow rate?

Given: Siphoning of gasoline

Find: Flow rate

Solution:

Basic equation $$\frac{p}{\rho_{gas}} + \frac{V^2}{2} + g \cdot z = \text{const}$$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the gas tank free surface and the siphon exit

$$\frac{p_{atm}}{\rho_{gas}} = \frac{p_{atm}}{\rho_{gas}} + \frac{V^2}{2} - g \cdot h$$

where we assume the tank free surface is slowly changing so V_{tank} <<, and h is the difference in levels

Hence $$V = \sqrt{2 \cdot g \cdot h}$$

The flow rate is then $$Q = V \cdot A = \frac{\pi \cdot D^2}{4} \cdot \sqrt{2 \cdot g \cdot h}$$

$$Q = \frac{\pi}{4} \times (1 \cdot \text{in})^2 \times \frac{1 \cdot \text{ft}^2}{144 \cdot \text{in}^2} \times \sqrt{2 \times 32.2 \frac{\text{ft}}{\text{s}^2} \times \frac{1}{2} \cdot \text{ft}} \qquad Q = 0.0309 \frac{\text{ft}^3}{\text{s}} \qquad Q = 13.9 \frac{\text{gal}}{\text{min}}$$

Problem 6.49

[2]

6.49 A pipe ruptures and benzene shoots 25 ft into the air. What is the pressure inside the pipe?

Given: Ruptured pipe

Find: Pressure in tank

Solution:

Basic equation $$\frac{p}{\rho_{ben}} + \frac{V^2}{2} + g\cdot z = \text{const}$$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the pipe and the rise height of the benzene

$$\frac{p_{pipe}}{\rho_{ben}} = \frac{p_{atm}}{\rho_{ben}} + g\cdot h$$ where we assume V_{pipe} <<, and h is the rise height

Hence $$p_{pipe} = \rho_{ben}\cdot g\cdot h = SG_{ben}\cdot\rho\cdot g\cdot h$$ where p_{pipe} is now the gage pressure

From Table A.2 $$SG_{ben} = 0.879$$

Hence $$p_{ben} = 0.879 \times 1.94\cdot\frac{\text{slug}}{\text{ft}^3} \times 32.2\cdot\frac{\text{ft}}{\text{s}^2} \times 25\cdot\text{ft} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\,\text{ft}} \qquad p_{ben} = 1373\,\frac{\text{lbf}}{\text{ft}^2} \qquad p_{ben} = 9.53\,\text{psi}$$ (gage)

Problem 6.50

6.50 A can of Coke has a small pinhole leak in it. The Coke is being sprayed vertically in the air to a height of 20 in. What is the pressure inside the can of Coke?

Given:

Ruptured Coke can

Find:

Pressure in can

Solution:

Basic equation $$\frac{p}{\rho_{Coke}} + \frac{V^2}{2} + g\cdot z = \text{const}$$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the coke can and the rise height of the coke

$$\frac{p_{can}}{\rho_{Coke}} = \frac{p_{atm}}{\rho_{Coke}} + g\cdot h$$ where we assume V_{Coke} <<, and h is the rise height

Hence $p_{Coke} = \rho_{Coke}\cdot g\cdot h = SG_{Coke}\cdot\rho\cdot g\cdot h$ where p_{pipe} is now the gage pressure

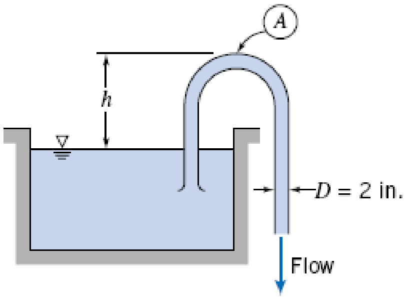
From a web search $SG_{DietCoke} = 1$ $SG_{RegularCoke} = 1.11$

Hence $$p_{Diet} = 1\times 1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times 20\cdot in\times\frac{1\cdot ft}{12\cdot in}\times\frac{lbf\cdot s^2}{slug\,ft}$$ $$p_{Diet} = 104\cdot\frac{lbf}{ft^2}$$ $$p_{Diet} = 0.723\cdot psi$$ (gage)

Hence $$p_{Regular} = 1.11\times 1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times 20\cdot in\times\frac{1\cdot ft}{12\cdot in}\times\frac{lbf\cdot s^2}{slug\,ft}$$ $$p_{Regular} = 116\cdot\frac{lbf}{ft^2}$$ $$p_{Regular} = 0.803\cdot psi$$ (gage)

Problem 6.51

6.51 The water flow rate through the siphon is 0.7 ft³/s, its temperature is 70°F, and the pipe diameter is 2 in. Compute the maximum allowable height, *h*, so that the pressure at point *A* is above the vapor pressure of the water. (Assume the flow is frictionless.)



Given: Flow rate through siphon

Find: Maximum height h to avoid cavitation

Solution:

Basic equation $\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$ $\qquad Q = V\cdot A$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

From continuity $V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$ $\qquad V = \frac{4}{\pi} \times 0.7\cdot\frac{ft^3}{s} \times \left(\frac{1}{2\cdot in}\right)^2 \times \left(\frac{12\cdot in}{1\cdot ft}\right)^2$ $\qquad V = 32.1\frac{ft}{s}$

Hence, applying Bernoulli between the free surface and point A

$$\frac{p_{atm}}{\rho} = \frac{p_A}{\rho} + g\cdot h + \frac{V^2}{2}$$

where we assume $V_{Surface}$ <<

Hence $p_A = p_{atm} - \rho\cdot g\cdot h - \rho\cdot\frac{V^2}{2}$

From the steam tables, at 70ºF the vapor pressure is $p_v = 0.363\cdot psi$

This is the lowest permissible value of p_A

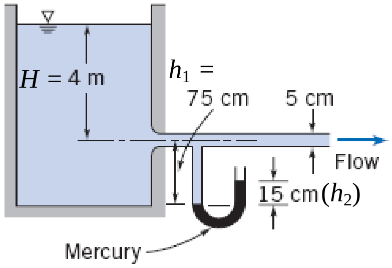
Hence $p_A = p_v = p_{atm} - \rho\cdot g\cdot h - \rho\cdot\frac{V^2}{2}$ or $h = \frac{p_{atm} - p_v}{\rho\cdot g} - \frac{V^2}{2\cdot g}$

Hence $h = (14.7 - 0.363)\cdot\frac{lbf}{in^2} \times \left(\frac{12\cdot in}{1\cdot ft}\right)^2 \times \frac{1}{1.94}\cdot\frac{ft^3}{slug} \times \frac{s^2}{32.2\cdot ft} \times \frac{slug\cdot ft}{lbf\cdot s^2} - \frac{1}{2} \times \left(32.18\frac{ft}{s}\right)^2 \times \frac{s^2}{32.2\cdot ft}$ $\qquad h = 17.0\,ft$

Problem 6.52

[2]

6.52 Water flows from a very large tank through a 5-cm-diameter tube. The dark liquid in the manometer is mercury. Estimate the velocity in the pipe and the rate of discharge from the tank. (Assume the flow is frictionless.)



Given: Flow through tank-pipe system

Find: Velocity in pipe; Rate of discharge

Solution:

Basic equation $\quad \dfrac{p}{\rho} + \dfrac{V^2}{2} + g \cdot z = \text{const} \qquad \Delta p = \rho \cdot g \cdot \Delta h \qquad Q = V \cdot A$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the free surface and the manometer location

$$\frac{p_{atm}}{\rho} = \frac{p}{\rho} - g \cdot H + \frac{V^2}{2}$$

where we assume $V_{Surface}$ <<, and H = 4 m

Hence $\quad p = p_{atm} + \rho \cdot g \cdot H - \rho \cdot \dfrac{V^2}{2}$

For the manometer $\quad p - p_{atm} = SG_{Hg} \cdot \rho \cdot g \cdot h_2 - \rho \cdot g \cdot h_1$

Note that we have water on one side and mercury on the other of the manometer

Combining equations $\quad \rho \cdot g \cdot H - \rho \cdot \dfrac{V^2}{2} = SG_{Hg} \cdot \rho \cdot g \cdot h_2 - \rho \cdot g \cdot h_1$ $\quad$ or $\quad V = \sqrt{2 \cdot g \cdot \left(H - SG_{Hg} \cdot h_2 + h_2\right)}$

Hence $\quad V = \sqrt{2 \times 9.81 \cdot \dfrac{m}{s^2} \times (4 - 13.6 \times 0.15 + 0.75) \cdot m} \qquad V = 7.29 \dfrac{m}{s}$

The flow rate is $\quad Q = V \cdot \dfrac{\pi \cdot D^2}{4} \qquad Q = \dfrac{\pi}{4} \times 7.29 \cdot \dfrac{m}{s} \times (0.05 \cdot m)^2 \qquad Q = 0.0143 \dfrac{m^3}{s}$

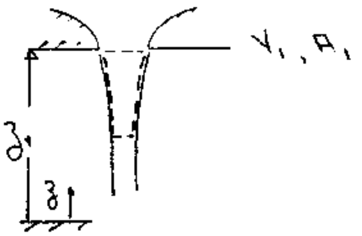
Problem 6.53

[2]

Given: Liquid stream leaving a nozzle pointing downward as shown

Assume uniform flow

Neglect friction



Find: Variation in jet area for $z < z_0$

Solution:

Basic equations: $$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{p}{\rho}+\frac{V^2}{2}+gz$$

$$0=\frac{\partial}{\partial t}\int_{CV}\rho\,d\forall+\int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline
(5) $p = p_1 = p_{atm}$
(6) uniform flow at a section

From the Bernoulli equation

$$V^2 = V_1^2 + 2g(z_1 - z)$$

From the continuity equation

$$0=\int_{CS}\rho\vec{V}\cdot d\vec{A} = -\{|\rho V_1 A_1|\}+\{|\rho V A|\}$$

and

$$V_1A_1 = VA \qquad \text{or} \qquad V = V_1\frac{A_1}{A}$$

Thus

$$V_1^2\left(\frac{A_1}{A}\right)^2 = V_1^2 + 2g(z_1 - z)$$

Solving for A,

$$A = A_1\sqrt{\frac{1}{1+\dfrac{2g(z_1-z)}{V_1^2}}} \longleftarrow \boxed{A(z)}$$

{Note: jet area decreases as z decreases, owing to the higher velocity}

Problem 6.54

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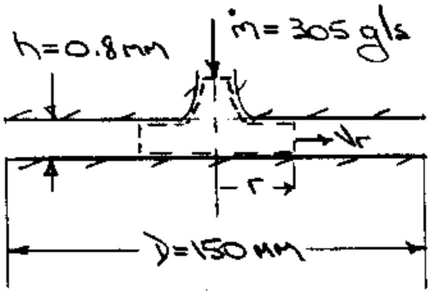


Given: Water flow between parallel disks discharging to atmosphere as shown.

Find: (a) theoretical static pressure between the disks at r = 50 mm.
(b) in actual laboratory situation, would the pressure be above or below the theoretical value?

Solution:

Basic equations: $0 = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V} \cdot d\vec{A}$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Assumptions: (1) steady flow
(2) incompressible flow
(3) flow along a streamline
(4) neglect friction
(5) uniform flow at each section

Apply continuity to the CV shown

$$0 = \{-\dot{m}\} + \{\rho V_r 2\pi r h\} \quad \text{so} \quad V = \frac{\dot{m}}{2\pi \rho r h}$$

$$V_1 = V_{r=50mm} = \frac{1}{2\pi} \times 0.305\frac{kg}{s} \times \frac{m^3}{999\,kg} \times \frac{1}{0.050\,m} \times \frac{1}{8\times10^{-4}\,m} = 1.21\,m/s$$

$$V_2 = V_{r=R} = \frac{1}{2\pi} \times 0.305\frac{kg}{s} \times \frac{m^3}{999\,kg} \times \frac{1}{0.075\,m} \times \frac{1}{8\times10^{-4}\,m} = 0.810\,m/s$$

From the Bernoulli equation

$$p_1 - p_2 = p_{r=50m} - p_{atm} = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

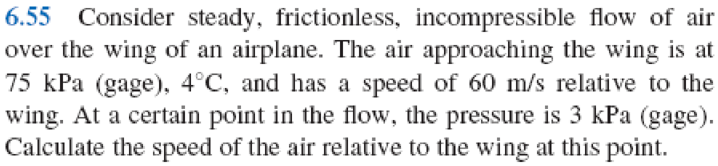
$$p_{r=50mm} = \frac{1}{2} \times 999\frac{kg}{m^3}\left[(0.810)^2 - (1.21)^2\right]\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$p_{r=50mm} = -404\,N/m^2\ (gage) \leftarrow p_{r=50mm}$$

Friction would cause a pressure drop in the flow direction. Since the discharge pressure is fixed at p_{atm}, the measured pressure would be greater than the theoretical value.

Problem 6.55

[2]



6.55 Consider steady, frictionless, incompressible flow of air over the wing of an airplane. The air approaching the wing is at 75 kPa (gage), 4°C, and has a speed of 60 m/s relative to the wing. At a certain point in the flow, the pressure is 3 kPa (gage). Calculate the speed of the air relative to the wing at this point.

Given: Air flow over a wing

Find: Air speed relative to wing at a point

Solution:

Basic equation $\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{const} \qquad p = \rho \cdot R \cdot T$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the upstream point (1) and the point on the wing (2)

$$\frac{p_1}{\rho} + \frac{{V_1}^2}{2} = \frac{p_2}{\rho} + \frac{{V_2}^2}{2} \qquad \text{where we ignore gravity effects}$$

Hence $$V_2 = \sqrt{{V_1}^2 + 2 \cdot \frac{\left(p_1 - p_2\right)}{\rho}}$$

For air $$\rho = \frac{p}{R \cdot T} \qquad \rho = (75 + 101) \times 10^3 \cdot \frac{N}{m^2} \times \frac{kg \cdot K}{286.9 \cdot N \cdot m} \times \frac{1}{(4 + 273) \cdot K} \qquad \rho = 2.21 \frac{kg}{m^3}$$

Then $$V = \sqrt{\left(60 \cdot \frac{m}{s}\right)^2 + 2 \times \frac{m^3}{2.21 \cdot kg} \times (75 - 3) \times 10^3 \cdot \frac{N}{m^2} \times \frac{kg \cdot m}{N \cdot s^2}} \qquad V = 262 \frac{m}{s}$$

NOTE: At this speed, significant density changes will occur, so this result is not very realistic

Problem 6.56

[2]

Given: Mercury barometer carried in car on windless day.

Outside: $T = 20°C$, $h_{bar} = 761$ mm Hg (corrected)

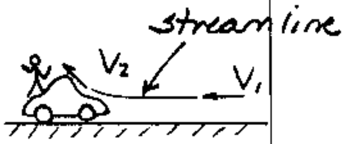
Inside: $V = 105$ km/hr, window open, $h_{bar} = 756$ mm Hg

Find: (a) Explain what is happening.
(b) Local speed of air flow past window, relative to car.

Solution: (a) Air speed relative to car is higher than in the freestream, thus lowering the pressure at window.

(b) Apply the Bernoulli equation in frame seen by an observer on the car:

Basic equation: $$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$



Assumptions: (1) Steady flow (seen by observer on car)
(2) Incompressible flow
(3) Neglect friction
(4) Flow along a streamline
(5) Neglect Δz

Then $$V_2^2 = \left[V_1^2 + 2\left(\frac{p_1 - p_2}{\rho}\right)\right] \quad \text{or} \quad V_2 = \left[V_1^2 + \frac{2(p_1 - p_2)}{\rho}\right]^{1/2} \qquad (1)$$

From fluid statics

$$p_1 - p_2 = \rho g(h_1 - h_2) = SG\,\rho_{H_2O}\,g\,\Delta h$$

$$= 13.6 \times 1000\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 0.005\,m \times \frac{N \cdot s^2}{kg \cdot m}$$

$$p_1 - p_2 = 667\ N/m^2$$

and from ideal gas

$$\rho = \frac{p}{RT} = 13.6 \times 1000\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 0.761\,m \times \frac{kg \cdot K}{287\,N \cdot m} \times \frac{1}{(273+20)\,K} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$\rho = 1.21\ kg/m^3$$

Substituting into Eq. 1

$$V_2 = \left[\left(105\,\frac{km}{hr} \times 1000\,\frac{m}{km} \times \frac{hr}{3600\,s}\right)^2 + 2 \times 667\,\frac{N}{m^2} \times \frac{m^2}{1.21\,kg} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2}$$

$V_2 = 44.2$ m/s (159 km/hr) relative to car

V_2

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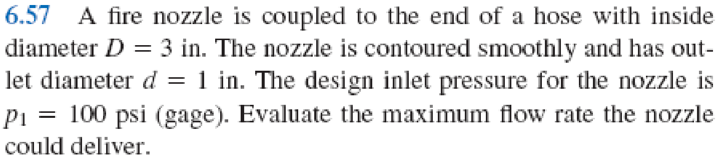
Problem 6.57

[2]



6.57 A fire nozzle is coupled to the end of a hose with inside diameter $D = 3$ in. The nozzle is contoured smoothly and has outlet diameter $d = 1$ in. The design inlet pressure for the nozzle is $p_1 = 100$ psi (gage). Evaluate the maximum flow rate the nozzle could deliver.

Given: Flow through fire nozzle

Find: Maximum flow rate

Solution:

Basic equation $\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$ $Q = V\cdot A$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the inlet (1) and exit (2)

$$\frac{p_1}{\rho} + \frac{V_1^{\,2}}{2} = \frac{p_2}{\rho} + \frac{V_2^{\,2}}{2}$$ where we ignore gravity effects

But we have $Q = V_1\cdot A_1 = V_1\cdot\frac{\pi\cdot D^2}{4} = V_2\cdot A_2 = \frac{\pi\cdot d^2}{4}$ so $V_1 = V_2\cdot\left(\frac{d}{D}\right)^2$

$$V_2^{\,2} - V_2^{\,2}\cdot\left(\frac{d}{D}\right)^4 = \frac{2\cdot\left(p_2 - p_1\right)}{\rho}$$

Hence $V_2 = \sqrt{\dfrac{2\cdot\left(p_1 - p_2\right)}{\rho\cdot\left[1 - \left(\frac{d}{D}\right)^4\right]}}$

Then $V_2 = \sqrt{2 \times \dfrac{\text{ft}^3}{1.94\cdot\text{slug}} \times (100 - 0)\cdot\dfrac{\text{lbf}}{\text{in}^2} \times \left(\dfrac{12\cdot\text{in}}{1\cdot\text{ft}}\right)^2 \times \dfrac{1}{1 - \left(\frac{1}{3}\right)^3} \times \dfrac{\text{slug}\,\text{ft}}{\text{lbf}\cdot\text{s}^2}}$ $V_2 = 124\cdot\frac{\text{ft}}{\text{s}}$

$Q = V_2\cdot\frac{\pi\cdot d^2}{4}$ $Q = \frac{\pi}{4} \times 124\cdot\frac{\text{ft}}{\text{s}} \times \left(\frac{1}{12}\cdot\text{ft}\right)^2$ $Q = 0.676\cdot\frac{\text{ft}^3}{\text{s}}$ $Q = 304\cdot\frac{\text{gal}}{\text{min}}$

Problem 6.58

[2]

Given: Indianapolis race car, $V_0 = 98.3$ m/s, on a straightaway.

Air inlet at location where $V = 25.5$ m/s along body surface.

Find: (a) Static pressure at inlet location.
(b) Express pressure rise as a fraction of the dynamic pressure.

Solution: Apply the Bernoulli equation, relative to the auto.

Basic equation:

$$\frac{p_\infty}{\rho} + \frac{V_\infty^2}{2} + g\cancel{z_\infty} = \frac{p}{\rho} + \frac{V^2}{2} + g\cancel{z}$$

Assumptions: (1) Steady flow (as seen by observer on auto)
(2) Incompressible flow ($V_\infty < 100$ m/sec)
(3) No friction
(4) Flow along a stream line
(5) Neglect changes in z
(6) Standard air: $\rho = 1.23$ kg/m³

Then

$$p - p_\infty = \frac{1}{2}\rho V_\infty^2 - \frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_\infty^2\left[1-\left(\frac{V}{V_\infty}\right)^2\right] = q\left[1-\left(\frac{V}{V_\infty}\right)^2\right]$$

$$q = \frac{1}{2}\rho V_\infty^2 = \frac{1}{2} \times 1.23\,\frac{\text{kg}}{\text{m}^3} \times (98.3)^2\,\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 5.94 \text{ kPa}$$

$$\frac{\Delta p}{q} = 1-\left(\frac{V}{V_\infty}\right)^2 = 1-\left(\frac{25.5}{98.3}\right)^2 = 0.933 \longleftarrow \qquad \Delta p/q$$

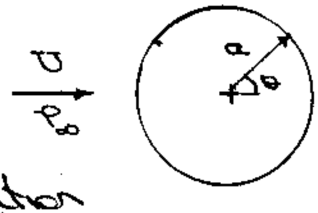
$$\text{and} \quad \Delta p = 0.933\,q = 0.933 \times 5.94 \text{ kPa} = 5.54 \text{ kPa} \longleftarrow \qquad p-p$$

Problem 6.59

[2]

Given: Steady, frictionless, incompressible flow over a stationary cylinder of radius, a.

$\vec{V} = U\left[1-\left(\frac{a}{r}\right)^2\right]\cos\theta\,\hat{e}_r - U\left[1+\left(\frac{a}{r}\right)^2\right]\sin\theta\,\hat{e}_\theta$



Find: (a) expression for pressure distribution along streamline forming cylinder, r = a.
(b) locations on cylinder where $p = p_\infty$.

Solution:

Basic equation: $\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}$

Assumptions: (1) steady flow (given).
(2) incompressible flow (given)
(3) frictionless flow (given)
(4) flow along a streamline.

Along the cylinder surface $r = a$ and $\vec{V} = -2U\sin\theta\,\hat{e}_\theta$

Applying the Bernoulli equation along the streamline r=a,

$$\frac{p}{\rho} + \frac{V^2}{2} = \frac{p_\infty}{\rho} + \frac{U^2}{2}$$

$$p = p_\infty + \frac{1}{2}\rho(U^2 - V^2) = p_\infty + \frac{1}{2}\rho(U^2 - 4U^2\sin^2\theta)$$

$$p = p_\infty + \frac{1}{2}\rho U^2(1 - 4\sin^2\theta) \qquad \underline{\hspace{6cm}} \; p$$

For $p = p_\infty$, $1 - 4\sin^2\theta = 0$ and $\sin\theta = \pm 0.5$

$$\therefore \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ \qquad \underline{\hspace{6cm}} \; \theta$$

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42-382 100 SHEETS EYE-EASE 5 SQUARE
42-389 200 SHEETS EYE-EASE 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
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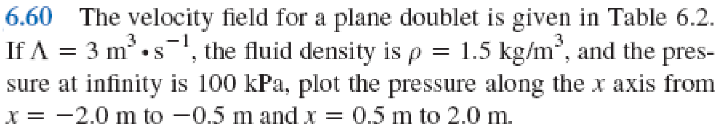
Problem 6.60

[3]



6.60 The velocity field for a plane doublet is given in Table 6.2. If $\Lambda = 3\ \mathrm{m}^3 \cdot \mathrm{s}^{-1}$, the fluid density is $\rho = 1.5\ \mathrm{kg/m}^3$, and the pressure at infinity is 100 kPa, plot the pressure along the x axis from $x = -2.0$ m to -0.5 m and $x = 0.5$ m to 2.0 m.

Given: Velocity field for plane doublet

Find: Pressure distribution along *x* axis; plot distribution

Solution:

The given data is $\Lambda = 3 \cdot \dfrac{\mathrm{m}^3}{\mathrm{s}}$ $\quad \rho = 1000 \cdot \dfrac{\mathrm{kg}}{\mathrm{m}^3}$ $\quad p_0 = 100 \cdot \mathrm{kPa}$

From Table 6.1 $\quad V_r = -\dfrac{\Lambda}{r^2} \cdot \cos(\theta) \qquad V_\theta = -\dfrac{\Lambda}{r^2} \cdot \sin(\theta)$

where V_r and V_θ are the velocity components in cylindrical coordinates (r,θ). For points along the *x* axis, $r = x$, $\theta = 0$, $V_r = u$ and $V_\theta = v = 0$

$$u = -\frac{\Lambda}{x^2} \qquad v = 0$$

The governing equation is the Bernoulli equation

$$\frac{p}{\rho} + \frac{1}{2} \cdot V^2 + g \cdot z = \text{const} \qquad \text{where} \qquad V = \sqrt{u^2 + v^2}$$

so (neglecting gravity) $\quad \dfrac{p}{\rho} + \dfrac{1}{2} \cdot u^2 = \text{const}$

Apply this to point arbitrary point (*x*,0) on the *x* axis and at infinity

At $\quad |x| \rightarrow \quad u \rightarrow 0 \qquad p \rightarrow p_0$

At point (*x*,0) $\quad u = -\dfrac{\Lambda}{x^2}$

Hence the Bernoulli equation becomes

$$\frac{p_0}{\rho} = \frac{p}{\rho} + \frac{\Lambda^2}{2 \cdot x^4} \qquad \text{or} \qquad p(x) = p_0 - \frac{\rho \cdot \Lambda^2}{2 \cdot x^4}$$

The plot of pressure is shown in the associated *Excel* workbook

Problem 6.60

[3]

6.60 The velocity field for a plane doublet is given in Table 6.2. If $\Lambda = 3\ \mathrm{m}^3 \cdot \mathrm{s}^{-1}$, the fluid density is $\rho = 1.5\ \mathrm{kg/m}^3$, and the pressure at infinity is 100 kPa, plot the pressure along the x axis from $x = -2.0$ m to -0.5 m and $x = 0.5$ m to 2.0 m.

Given: Velocity field for plane doublet

Find: Pressure distribution along x axis; plot distribution

Solution:

$$p(x) = p_0 - \frac{\rho \cdot \Lambda^2}{2 \cdot x^4}$$

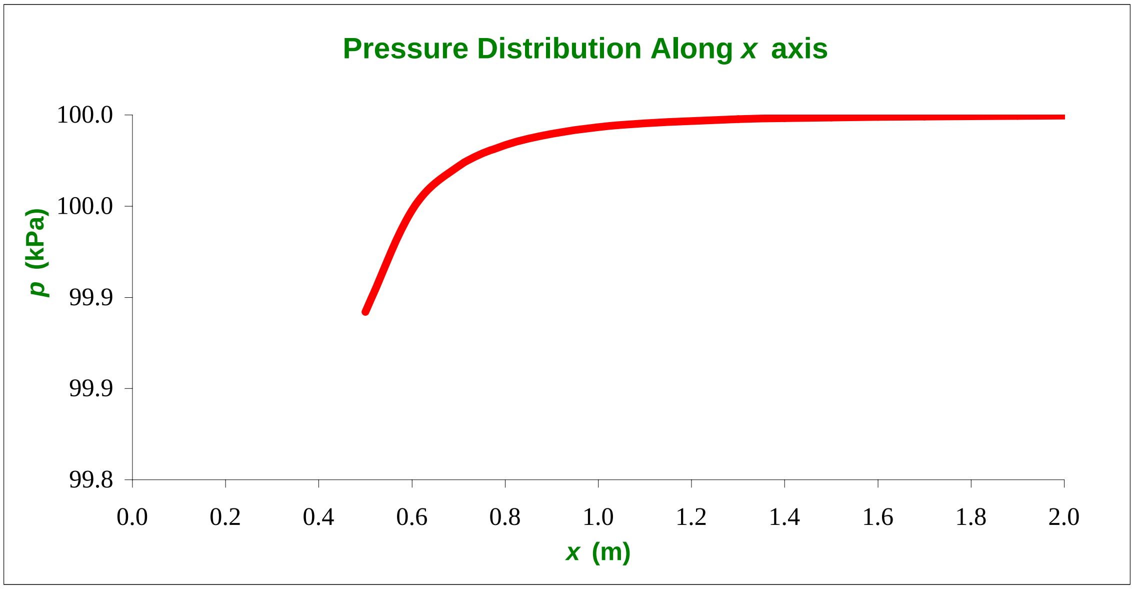
The given data is

$\Lambda =$ 3 $\mathrm{m^3/s}$

$\rho =$ 1.5 $\mathrm{kg/m^3}$

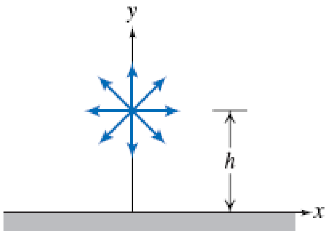
$p_0 =$ 100 kPa

x (m)	p (Pa)
0.5	99.892
0.6	99.948
0.7	99.972
0.8	99.984
0.9	99.990
1.0	99.993
1.1	99.995
1.2	99.997
1.3	99.998
1.4	99.998
1.5	99.999
1.6	99.999
1.7	99.999
1.8	99.999
1.9	99.999
2.0	100.000



Problem 6.61

6.61 The velocity field for a plane source at a distance h above an infinite wall aligned along the x axis was given in Problem 6.8. Using the data from that problem, plot the pressure distribution along the wall from $x = -10h$ to $x = +10h$ (assume the pressure at infinity is atmospheric). Find the net force on the wall if the pressure on the lower surface is atmospheric. Does the force tend to pull the wall towards the source, or push it away?



Given: Velocity field

Find: Pressure distribution along wall; plot distribution; net force on wall

Solution:

The given data is $q = 2\cdot\frac{\frac{m^3}{s}}{m}$ $\quad h = 1\cdot m \quad$ $\rho = 1000\cdot\frac{kg}{m^3}$

$$u = \frac{q\cdot x}{2\cdot\pi\left[x^2 + (y-h)^2\right]} + \frac{q\cdot x}{2\cdot\pi\left[x^2 + (y+h)^2\right]} \qquad v = \frac{q\cdot(y-h)}{2\cdot\pi\left[x^2 + (y-h)^2\right]} + \frac{q\cdot(y+h)}{2\cdot\pi\left[x^2 + (y+h)^2\right]}$$

The governing equation is the Bernoulli equation

$$\frac{p}{\rho} + \frac{1}{2}\cdot V^2 + g\cdot z = \text{const} \qquad \text{where} \qquad V = \sqrt{u^2 + v^2}$$

Apply this to point arbitrary point (x,0) on the wall and at infinity (neglecting gravity)

At $\quad |x| \to 0 \quad u \to 0 \quad v \to 0 \quad V \to 0$

At point (x,0) $\quad u = \frac{q\cdot x}{\pi\cdot\left(x^2 + h^2\right)} \quad v = 0 \quad V = \frac{q\cdot x}{\pi\cdot\left(x^2 + h^2\right)}$

Hence the Bernoulli equation becomes $\quad \frac{p_{atm}}{\rho} = \frac{p}{\rho} + \frac{1}{2}\cdot\left[\frac{q\cdot x}{\pi\cdot\left(x^2 + h^2\right)}\right]^2$

or (with pressure expressed as gage pressure) $\quad p(x) = -\frac{\rho}{2}\cdot\left[\frac{q\cdot x}{\pi\cdot\left(x^2 + h^2\right)}\right]^2$

(Alternatively, the pressure distribution could have been obtained from Problem 6.8, where the momentum equation was used to find the pressure gradient $\frac{\partial}{\partial x}p = \frac{\rho\cdot q^2\cdot x\cdot\left(x^2 - h^2\right)}{\pi^2\cdot\left(x^2 + h^2\right)^3}$ along the wall. Integration of this with respect to x leads to the same result for $p(x)$)

The plot of pressure is shown in the associated *Excel* workbook. From the plot it is clear that the wall experiences a negative gage pressure on the upper surface (and zero gage pressure on the lower), so the net force on the wall is upwards, towards the source

The force per width on the wall is given by $\quad F = \int_{-10\cdot h}^{10\cdot h}\left(p_{upper} - p_{lower}\right)dx \qquad F = -\frac{\rho\cdot q^2}{2\cdot\pi^2}\cdot\int_{-10\cdot h}^{10\cdot h}\frac{x^2}{\left(x^2 + h^2\right)^2}dx$

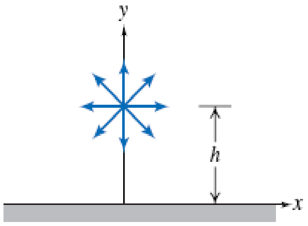
The integral is $$\int \frac{x^2}{\left(x^2+h^2\right)^2}\,dx \rightarrow \frac{\operatorname{atan}\left(\frac{x}{h}\right)}{2\cdot h} - \frac{x}{2\cdot h^2 + 2\cdot x^2}$$

so $$F = -\frac{\rho\cdot q^2}{2\cdot\pi^2\cdot h}\cdot\left(-\frac{10}{101} + \operatorname{atan}(10)\right)$$

$$F = -\frac{1}{2\cdot\pi^2} \times 1000\cdot\frac{kg}{m^3} \times \left(2\cdot\frac{m^2}{s}\right)^2 \times \frac{1}{1\cdot m} \times \left(-\frac{10}{101} + \operatorname{atan}(10)\right) \times \frac{N\cdot s^2}{kg\cdot m} \qquad F = -278\,\frac{N}{m}$$

Problem 6.61 [3]

6.61 The velocity field for a plane source at a distance h above an infinite wall aligned along the x axis was given in Problem 6.8. Using the data from that problem, plot the pressure distribution along the wall from $x = -10h$ to $x = +10h$ (assume the pressure at infinity is atmospheric). Find the net force on the wall if the pressure on the lower surface is atmospheric. Does the force tend to pull the wall towards the source, or push it away?



Given: Velocity field

Find: Pressure distribution along wall; plot distribution; net force on wall

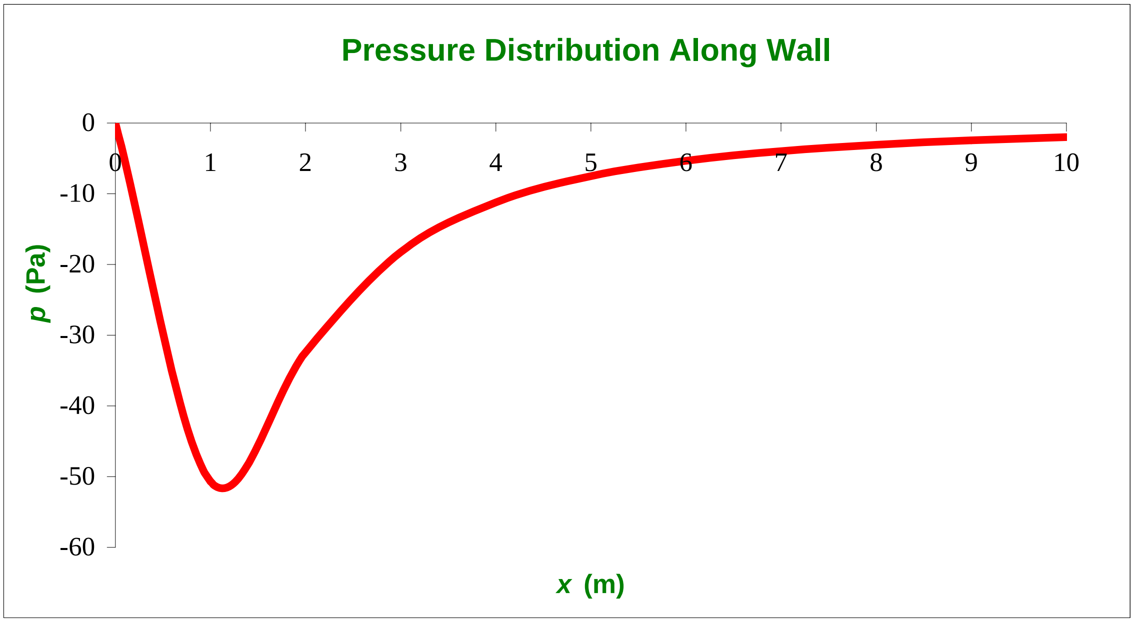
Solution:

$$p(x) = -\frac{\rho}{2}\cdot\left[\frac{q\cdot x}{\pi\cdot\left(x^2 + h^2\right)}\right]^2$$

The given data is

q = 2 m^3/s/m
h = 1 m
ρ = 1000 kg/m^3

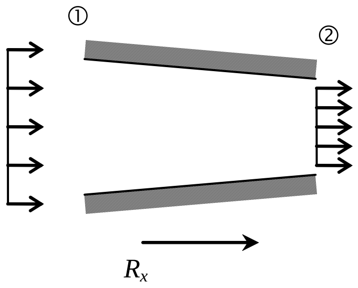
x (m)	p (Pa)
0.0	0.00
1.0	-50.66
2.0	-32.42
3.0	-18.24
4.0	-11.22
5.0	-7.49
6.0	-5.33
7.0	-3.97
8.0	-3.07
9.0	-2.44
10.0	-1.99



Problem 6.62

[3]

6.62 A fire nozzle is coupled to the end of a hose with inside diameter $D = 75$ mm. The nozzle is smoothly contoured and its outlet diameter is $d = 25$ mm. The nozzle is designed to operate at an inlet water pressure of 700 kPa (gage). Determine the design flow rate of the nozzle. (Express your answer in L/s.) Evaluate the axial force required to hold the nozzle in place. Indicate whether the hose coupling is in tension or compression.



Given:
Flow through fire nozzle

Find:
Maximum flow rate

Solution:

Basic equation $\quad \frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const} \qquad Q = V\cdot A \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!- + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the inlet (1) and exit (2)

$$\frac{p_1}{\rho} + \frac{V_1^{\,2}}{2} = \frac{p_2}{\rho} + \frac{V_2^{\,2}}{2} \qquad \text{where we ignore gravity effects}$$

But we have $\quad Q = V_1\cdot A_1 = V_1\cdot\frac{\pi\cdot D^2}{4} = V_2\cdot\frac{\pi\cdot d^2}{4} \qquad$ so $\qquad V_1 = V_2\cdot\left(\frac{d}{D}\right)^2$

$$V_2^{\,2} - V_2^{\,2}\cdot\left(\frac{d}{D}\right)^4 = \frac{2\cdot\left(p_2 - p_1\right)}{\rho}$$

Hence $\quad V_2 = \sqrt{\frac{2\cdot\left(p_1 - p_2\right)}{\rho\cdot\left[1-\left(\frac{d}{D}\right)^4\right]}}$

$$V_2 = \sqrt{2\times\frac{m^3}{1000\cdot kg}\times(700-0)\times10^3\cdot\frac{N}{m^2}\times\frac{1}{1-\left(\frac{25}{75}\right)^4}\times\frac{kg\cdot m}{N\cdot s^2}} \qquad V_2 = 37.6\frac{m}{s}$$

Then $\quad Q = V_2\cdot\frac{\pi\cdot d^2}{4} \qquad Q = \frac{\pi}{4}\times37.6\cdot\frac{m}{s}\times(0.025\cdot m)^2 \qquad Q = 0.0185\cdot\frac{m^3}{s} \qquad Q = 18.5\cdot\frac{L}{s}$

From x momentum $\quad R_x + p_1\cdot A_1 = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right) \qquad$ using gage pressures

Hence $\quad R_x = -p_1\cdot\frac{\pi\cdot D^2}{4} + \rho\cdot Q\cdot\left(V_2 - V_1\right) = -p_1\cdot\frac{\pi\cdot D^2}{4} + \rho\cdot Q\cdot V_2\cdot\left[1-\left(\frac{d}{D}\right)^2\right]$

$$R_x = -700\times10^3\cdot\frac{N}{m^2}\times\frac{\pi}{4}\cdot(0.075\cdot m)^2 + 1000\cdot\frac{kg}{m^3}\times0.0185\cdot\frac{m^3}{s}\times37.6\cdot\frac{m}{s}\times\left[1-\left(\frac{25}{75}\right)^3\right]\times\frac{N\cdot s^2}{kg\cdot m} \qquad R_x = -2423\,N$$

This is the force of the nozzle on the fluid; hence the force of the fluid on the nozzle is 2400 N to the right; the nozzle is in tension

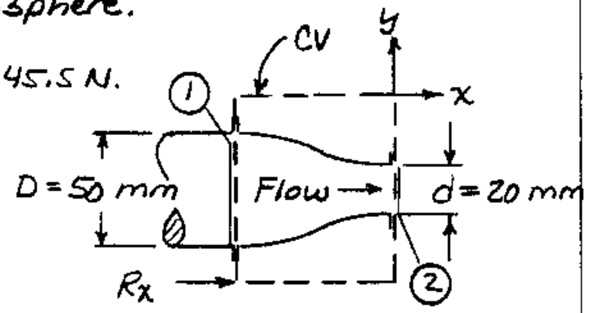
Problem 6.63

[3]

Given: Nozzle coupled to straight pipe by flanges, bolts. Water flow discharges to atmosphere.

For steady, inviscid flow, $R_x = -45.5\ \text{N}$.

Find: Volume flow rate.



Solution: Apply continuity, x momentum, and Bernoulli.

Basic equation:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \overset{=0(7)}{\cancel{\frac{p_2}{\rho}}} + \frac{V_2^2}{2} + g\cancel{z_2}$$

$$F_{S_x} + \overset{=0(6)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Uniform flow at each section
(3) Flow along a streamline
(4) Incompressible flow
(5) No friction
(6) Horizontal, $F_{B_x} = 0$, $z_1 = z_2$
(7) Use gage pressures

Then

$$0 = \{-V_1A_1\} + \{+V_2A_2\}\ ;\ V_2 = V_1\frac{A_1}{A_2} = V_1\left(\frac{D}{d}\right)^2\ ;\ Q = V_1A_1 = V_2A_2$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{V_2^2}{2}\ ;\ p_1 = \rho\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) = \rho\frac{V_1^2}{2}\left[\left(\frac{V_2}{V_1}\right)^2 - 1\right] = \rho\frac{V_1^2}{2}\left[\left(\frac{D}{d}\right)^4 - 1\right]$$

$$R_x + p_1A_1 - \overset{=0(7)}{\cancel{p_2}}A_2 = u_1\{-|\rho V_1A_1|\} + u_2\{+|\rho V_2A_2|\} = \rho V_1A_1(V_2 - V_1)$$

$$u_1 = V_1 \qquad u_2 = V_2$$

$$R_x + A_1\rho\frac{V_1^2}{2}\left[\left(\frac{D}{d}\right)^4 - 1\right] = \rho V_1^2A_1\left(\frac{V_2}{V_1} - 1\right) = \rho V_1^2A\left[\left(\frac{D}{d}\right)^2 - 1\right]$$

Thus

$$V_1^2 = \frac{-2R_x}{\rho A_1}\frac{1}{\left(\frac{D}{d}\right)^4 - 2\left(\frac{D}{d}\right)^2 + 1} \quad \text{so} \quad V_1 = \sqrt{\frac{-2R_x}{\rho A_1}}\frac{1}{\left(\frac{D}{d}\right)^2 - 1}$$

$$V_1 = \left[-2 \times -45.5\ \text{N} \times \frac{\text{m}^3}{999\ \text{kg}} \times \frac{4}{\pi(0.050)^2\text{m}^2} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{\frac{1}{2}}\frac{1}{\left(\frac{50}{20}\right)^2 - 1} = 1.30\ \text{m/s}$$

Finally,

$$Q = V_1A_1 = 1.30\ \frac{\text{m}}{\text{s}} \times \frac{\pi}{4}(0.050)^2\text{m}^2 = 2.55 \times 10^{-3}\ \text{m}^3/\text{s}$$

Q

{ Note: It is necessary to recognize that $R_x < 0$ for a nozzle, see Example Problem 4.7. }

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 6.64

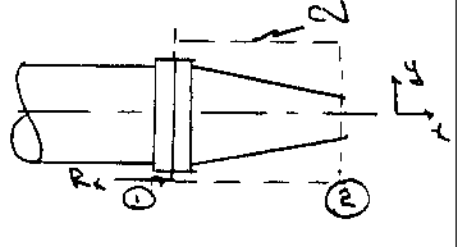
[3]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Water flows steadily through a pipe with diameter $D = 3.25$ in. and discharges through a nozzle ($d = 1.25$ in) to atmosphere. The flow rate is $Q = 24.5$ gal/min.

Find: (a) the minimum static pressure required in the pipe to produce this flowrate
(b) the horizontal force of the nozzle assembly on the pipe flange.

Solution:



Apply the Bernoulli equation along the central streamline between sections ① and ②

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}$$

Assumptions: (1) steady flow (2) incompressible flow
(3) frictionless flow (4) flow along a streamline
(5) $\Delta z = 0$ (6) uniform flow at each section

Then $p_1 = p_2 + \frac{\rho}{2}(V_2^2 - V_1^2) = p_2 + \frac{\rho V_2^2}{2}\left[1 - \left(\frac{V_1}{V_2}\right)^2\right]$

$p_2 = p_{atm}$ and from continuity, $A_2 V_2 = A_1 V_1$

$$\therefore\ p_{1g} = \frac{\rho}{2} V_2^2 \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]$$

$$V_2 = \frac{Q}{A} = \frac{4Q}{\pi d^2} = \frac{4}{\pi} \times 24.5 \frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{min}}{60\,\text{s}} \times \frac{1}{(1.25)^2\,\text{in}^2} \times \frac{144\,\text{in}^2}{\text{ft}^2}$$

$V_2 = 6.41$ ft/s and

$$p_{1g} = \frac{1}{2} \times 1.94 \frac{\text{slug}}{\text{ft}^3} \times (6.41)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}}\left[1 - \left(\frac{1.25}{3.25}\right)^4\right] = 39.0 \text{ psfgage}$$ p_1

(b) Apply the x momentum equation to the cv

$$F_{S_x} + F_{B_x} = \cancelto{0}{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

$$R_x + p_{1g} A_1 = u_1\{-\dot{m}\} + u_2\{\dot{m}\} = -V_1 \dot{m} + V_2 \dot{m}$$

$$R_x = -p_{1g}A_1 + \dot{m}(V_2 - V_1) = -p_{1g}A_1 + \rho Q V_2\left(1 - \frac{V_1}{V_2}\right)$$

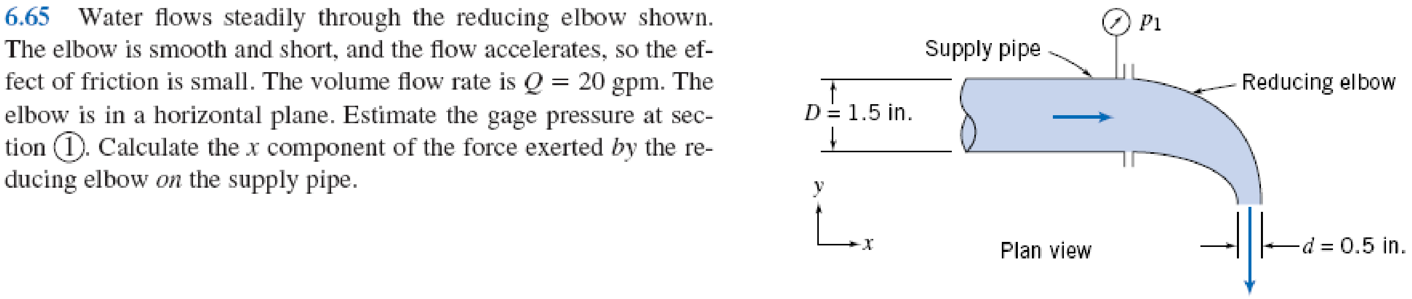
$$= -39 \frac{\text{lbf}}{\text{ft}^2} \times \frac{\pi}{4}\left(\frac{3.25}{12}\right)^2 \text{ft}^2 + 1.94 \frac{\text{slug}}{\text{ft}^3} \times 24.5 \frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{min}}{60\,\text{s}} \times 6.41 \frac{\text{ft}}{\text{s}}\left[1 - \left(\frac{1.25}{3.25}\right)^2\right]$$

$R_x = -2.25 + 0.58 = -1.67$ lbf

Force of nozzle on flange $K_x = -R_x = 1.67$ lbf K_x

Problem 6.65

[3]



6.65 Water flows steadily through the reducing elbow shown. The elbow is smooth and short, and the flow accelerates, so the effect of friction is small. The volume flow rate is $Q = 20$ gpm. The elbow is in a horizontal plane. Estimate the gage pressure at section ①. Calculate the x component of the force exerted *by* the reducing elbow *on* the supply pipe.

Given: Flow through reducing elbow

Find: Mass flow rate in terms of Δp, T_1 and D_1 and D_2

Solution:

Basic equations: $\frac{p}{\rho}+\frac{V^2}{2}+g\cdot z = \text{const}$ $\qquad F_x = F_{S_x}+F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$ $\qquad Q = V\cdot A$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline 5) Ignore elevation change 6) $p_2 = p_{atm}$

Available data: $Q = 20\cdot\text{gpm}$ $\quad Q = 0.0446\frac{ft^3}{s}$ $\quad D = 1.5\cdot\text{in}$ $\quad d = 0.5\cdot\text{in}$ $\quad \rho = 1.94\cdot\frac{slug}{ft^3}$

From contnuity $\quad V_1 = \frac{Q}{\left(\frac{\pi\cdot D^2}{4}\right)}$ $\quad V_1 = 3.63\frac{ft}{s}$ $\quad V_2 = \frac{Q}{\left(\frac{\pi\cdot d^2}{4}\right)}$ $\quad V_2 = 32.7\frac{ft}{s}$

Hence, applying Bernoulli between the inlet (1) and exit (2) $\quad \frac{p_1}{\rho}+\frac{V_1^2}{2} = \frac{p_2}{\rho}+\frac{V_2^2}{2}$

or, in gage pressures $\quad p_{1g} = \frac{\rho}{2}\cdot\left(V_2^2 - V_1^2\right)$ $\quad p_{1g} = 7.11\,\text{psi}$

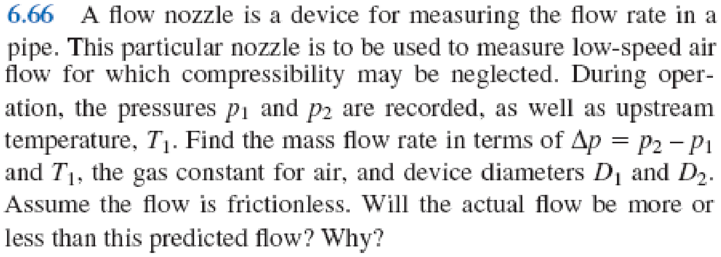
From x-momentum $\quad R_x + p_{1g}\cdot A_1 = u_1\cdot\left(-m_{rate}\right) + u_2\cdot\left(m_{rate}\right) = -m_{rate}\cdot V_1 = -\rho\cdot Q\cdot V_1$ $\quad$ because $\quad u_1 = V_1 \quad u_2 = 0$

$$R_x = -p_{1g}\cdot\frac{\pi\cdot D^2}{4} - \rho\cdot Q\cdot V_1 \qquad R_x = -12.9\,\text{lbf}$$

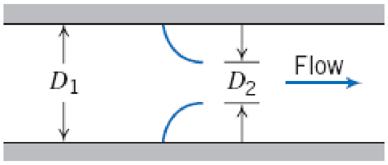
The force on the supply pipe is then $\quad K_x = -R_x \quad K_x = 12.9\,\text{lbf}$ $\quad$ on the pipe to the right

Problem 6.66

[2]



6.66 A flow nozzle is a device for measuring the flow rate in a pipe. This particular nozzle is to be used to measure low-speed air flow for which compressibility may be neglected. During operation, the pressures p_1 and p_2 are recorded, as well as upstream temperature, T_1. Find the mass flow rate in terms of $\Delta p = p_2 - p_1$ and T_1, the gas constant for air, and device diameters D_1 and D_2. Assume the flow is frictionless. Will the actual flow be more or less than this predicted flow? Why?



Given: Flow nozzle

Find: Mass flow rate in terms of Δp, T_1 and D_1 and D_2

Solution:

Basic equation $\qquad \frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const} \qquad Q = V\cdot A$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Hence, applying Bernoulli between the inlet (1) and exit (2)

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_2}{\rho} + \frac{V_2^2}{2} \qquad \text{where we ignore gravity effects}$$

But we have $\qquad Q = V_1\cdot A_1 = V_1\cdot\frac{\pi\cdot D_1^2}{4} = V_2\cdot\frac{\pi\cdot D_2^2}{4} \qquad$ so $\qquad V_1 = V_2\cdot\left(\frac{D_2}{D_1}\right)^2$

Note that we assume the flow at D_2 is at the same pressure as the entire section 2; this will be true if there is turbulent mixing

Hence $\qquad V_2^2 - V_2^2\cdot\left(\frac{D_2}{D_1}\right)^4 = \frac{2\cdot(p_2 - p_1)}{\rho}$

$$V_2 = \sqrt{\frac{2\cdot(p_1 - p_2)}{\rho\cdot\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}}$$

Then the mass flow rate is $m_{flow} = \rho\cdot V_2\cdot A_2 = \rho\cdot\frac{\pi\cdot D_2^2}{4}\cdot\sqrt{\frac{2\cdot(p_1 - p_2)}{\rho\cdot\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}} = \frac{\pi\cdot D_2^2}{2\cdot\sqrt{2}}\cdot\sqrt{\frac{\Delta p\cdot\rho}{\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}}$

Using $\qquad p = \rho\cdot R\cdot T \qquad m_{flow} = \frac{\pi\cdot D_2^2}{2\cdot\sqrt{2}}\cdot\sqrt{\frac{\Delta p\cdot p_1}{R\cdot T_1\cdot\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}}$

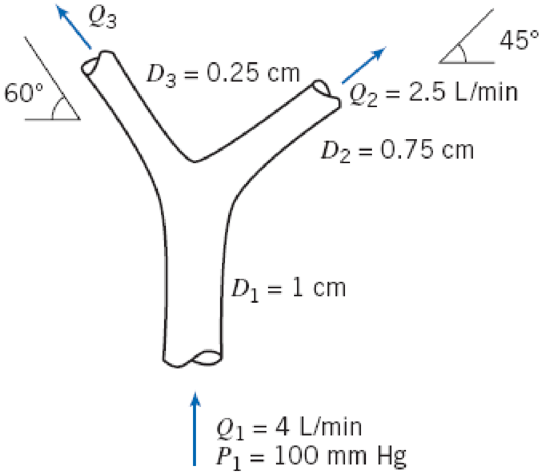
For a flow nozzle $\qquad m_{flow} = k\cdot\sqrt{\Delta p} \qquad$ where $\qquad k = \frac{\pi\cdot D_2^2}{2\cdot\sqrt{2}}\cdot\sqrt{\frac{p_1}{R\cdot T_1\cdot\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}}$

We can expect the actual flow will be less because there is actually significant loss in the device. Also the flow will experience a vena co that the minimum diameter is actually smaller than D_2. We will discuss this device in Chapter 8.

Problem 6.67

[4]

6.67 The branching of a blood vessel is shown. Blood at a pressure of 100 mm Hg flows in the main vessel at 4 L/min. Estimate the blood pressure in each branch, assuming that blood vessels behave as rigid tubes, that we have frictionless flow, and that the vessel lies in the horizontal plane. What is the force generated at the branch by the blood? You may approximate blood to have the same density as water.



Given: Flow through branching blood vessel

Find: Blood pressure in each branch; force at branch

Solution:

Basic equation $\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$ $\qquad \sum_{CV} Q = 0 \qquad Q = V\cdot A \qquad \Delta p = \rho\cdot g\cdot\Delta h$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

$$F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,d\forall + \int_{CS} v\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

For Q_3 we have $\sum_{CV} Q = -Q_1 + Q_2 + Q_3 = 0$ $\quad$ so $\quad Q_3 = Q_1 - Q_2 \qquad Q_3 = 1.5\cdot\frac{L}{\min}$

We will need each velocity

$$V_1 = \frac{Q_1}{A_1} = \frac{4\cdot Q_1}{\pi\cdot D_1^{\,2}} \qquad V_1 = \frac{4}{\pi}\times 4\cdot\frac{L}{\min}\times\frac{0.001\cdot m^3}{1\cdot L}\times\frac{1\cdot\min}{60\cdot s}\times\left(\frac{1}{0.01\cdot m}\right)^2 \qquad V_1 = 0.849\,\frac{m}{s}$$

Similarly $\quad V_2 = \frac{4\cdot Q_2}{\pi\cdot D_2^{\,2}} \qquad V_2 = 0.943\,\frac{m}{s} \qquad V_3 = \frac{4\cdot Q_3}{\pi\cdot D_3^{\,2}} \qquad V_3 = 5.09\,\frac{m}{s}$

Hence, applying Bernoulli between the inlet (1) and exit (2)

$$\frac{p_1}{\rho} + \frac{V_1^{\,2}}{2} = \frac{p_2}{\rho} + \frac{V_2^{\,2}}{2}$$ where we ignore gravity effects

$$p_2 = p_1 + \frac{\rho}{2}\cdot\left(V_1^{\,2} - V_2^{\,2}\right)$$

$$p_1 = SG_{Hg}\cdot\rho\cdot g\cdot h_1$$ where h_1 = 100 mm Hg

$$p_1 = 13.6\times 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 0.1\cdot m\times\frac{N\cdot s^2}{kg\cdot m} \qquad p_1 = 13.3\cdot kPa$$

Hence $p_2 = 13300\cdot\frac{N}{m^2} + \frac{1}{2}\cdot 1000\cdot\frac{kg}{m^3}\times\left(0.849^2 - 0.943^2\right)\cdot\left(\frac{m}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m}$ $\qquad p_2 = 13.2\cdot kPa$

In mm Hg $h_2 = \frac{p_2}{SG_{Hg}\cdot\rho\cdot g}$ $\qquad h_2 = \frac{1}{13.6}\times\frac{1}{1000}\cdot\frac{m^3}{kg}\times\frac{s^2}{9.81\cdot m}\times 13200\cdot\frac{N}{m^2}\times\frac{kg\cdot m}{s^2\cdot N}$ $\qquad h_2 = 98.9\cdot mm$

Similarly for exit (3) $p_3 = p_1 + \frac{\rho}{2}\cdot\left(V_1^{\ 2} - V_3^{\ 2}\right)$

$$p_3 = 13300\cdot\frac{N}{m^2} + \frac{1}{2}\cdot 1000\cdot\frac{kg}{m^3}\times\left(0.849^2 - 5.09^2\right)\cdot\left(\frac{m}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m} \qquad p_3 = 706\cdot Pa$$

In mm Hg $h_3 = \frac{p_3}{SG_{Hg}\cdot\rho\cdot g}$ $\qquad h_3 = \frac{1}{13.6}\times\frac{1}{1000}\cdot\frac{m^3}{kg}\times\frac{s^2}{9.81\cdot m}\times 706\cdot\frac{N}{m^2}\times\frac{kg\cdot m}{s^2\cdot N}$ $\qquad h_3 = 5.29\cdot mm$

Note that all pressures are gage.

For x momentum $R_x + p_3\cdot A_3\cdot\cos(60\cdot deg) - p_2\cdot A_2\cdot\cos(45\cdot deg) = u_3\cdot\left(\rho\cdot Q_3\right) + u_2\cdot\left(\rho\cdot Q_2\right)$

$$R_x = p_2\cdot A_2\cdot\cos(45\cdot deg) - p_3\cdot A_3\cdot\cos(60\cdot deg) + \rho\cdot\left(Q_2\cdot V_2\cdot\cos(45\cdot deg) - Q_3\cdot V_3\cdot\cos(60\cdot deg)\right)$$

$$R_x = 13200\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.0075\cdot m)^2}{4}\times\cos(45\cdot deg) - 706\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.0025\cdot m)^2}{4}\times\cos(60\cdot deg)\ ...$$

$$+\,1000\cdot\frac{kg}{m^3}\cdot\left(2.5\cdot\frac{L}{min}\cdot 0.943\cdot\frac{m}{s}\cdot\cos(45\cdot deg) - 1.5\cdot\frac{L}{min}\cdot 5.09\cdot\frac{m}{s}\cdot\cos(60\cdot deg)\right)\times\frac{10^{-3}\cdot m^3}{1\cdot L}\times\frac{1\cdot min}{60\cdot s}\times\frac{N\cdot s^2}{kg\times m} \qquad R_x = 0.375\,N$$

For y momentum $R_y - p_3\cdot A_3\cdot\sin(60\cdot deg) - p_2\cdot A_2\cdot\sin(45\cdot deg) = v_3\cdot\left(\rho\cdot Q_3\right) + v_2\cdot\left(\rho\cdot Q_2\right)$

$$R_y = p_2\cdot A_2\cdot\sin(45\cdot deg) + p_3\cdot A_3\cdot\sin(60\cdot deg) + \rho\cdot\left(Q_2\cdot V_2\cdot\sin(45\cdot deg) + Q_3\cdot V_3\cdot\sin(60\cdot deg)\right)$$

$$R_y = 13200\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.0075\cdot m)^2}{4}\times\sin(45\cdot deg) + 706\cdot\frac{N}{m^2}\times\frac{\pi\cdot(0.0025\cdot m)^2}{4}\cdot\sin(60\cdot deg)\ ...$$

$$+\,1000\cdot\frac{kg}{m^3}\cdot\left(2.5\cdot\frac{L}{min}\cdot 0.943\cdot\frac{m}{s}\cdot\sin(45\cdot deg) + 1.5\cdot\frac{L}{min}\cdot 5.09\cdot\frac{m}{s}\cdot\sin(60\cdot deg)\right)\times\frac{10^{-3}\cdot m^3}{1\cdot L}\times\frac{1\cdot min}{60\cdot s}\times\frac{N\cdot s^2}{kg\times m} \qquad R_y = 0.553\,N$$

Problem 6.68

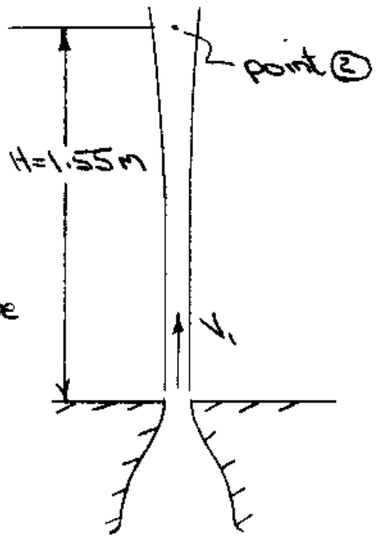
[3]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: A water jet is directed upward from a well-designed nozzle of area $A_1 = 600\ mm^2$; $V_1 = 6.3\ m/s$
The flow is steady and liquid stream does not break up. Point ② is $H = 1.55\ m$ above nozzle exit

Find: (a) V_2 (b) p_{0_2}
(c) force on flat plate placed normal to the flow at ②
(d) Sketch pressure distribution on the plate

Solution: Apply Bernoulli and then y-momentum equation



Basic eq.: $\cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + gz_1 = \cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2} + gz_2$

Assumptions: (1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline
(5) $p_1 = p_2 = p_{atm}$

Then

$$V_2 = \left[V_1^2 + 2g(z_1 - z_2)\right]^{1/2}$$

$$V_2 = \left[(6.3)^2 \frac{m^2}{s^2} + 2 \times 9.81 \frac{m}{s^2}(-1.55\ m)\right]^{1/2}$$

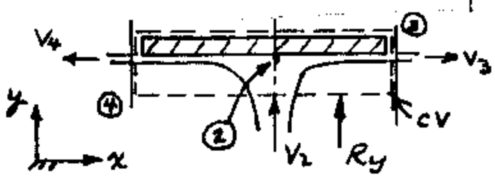
$V_2 = 3.05\ m/s$ ← V_2

By definition, $p_{0_2} = p_2 + \frac{1}{2}\rho V_2^2 = p_{atm} + \frac{1}{2}\rho V_2^2$, so

$$p_{0_2\,gage} = \frac{1}{2} \times 999 \frac{kg}{m^3} \times (3.05)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 4.65\ kPa\ (g)$$ ← p_{0_2}

Apply y-momentum equation to CV surrounding plate

Basic eq.: $F_{S_y} + \cancel{F_{B_y}}^{\approx 0(6)} = \cancel{\frac{\partial}{\partial t}}^{=0(1)} \int_{CV} v \rho\, d\forall + \int_{CS} v \rho \vec{V} \cdot d\vec{A}$



Assumptions: (6) neglect mass in CV
(7) V_2 enters CV uniformly
(8) $v_3 = v_4 = 0$

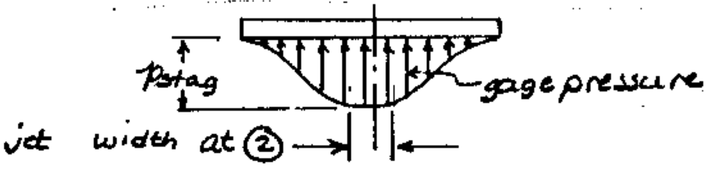
Then

$R_y = v_2\{-\rho V_1 A_1\} + \cancel{v_3}^{=0(8)}\{\dot m_3\} + \cancel{v_4}^{=0(6)}\{\dot m_4\} = -\rho V_1 A_1 V_2$ and

$$K_y = -R_y = \rho V_1 A_1 V_2 = 999 \frac{kg}{m^3} \times 6.3 \frac{m}{s} \times 600\ mm^2 \times 3.05 \frac{m}{s} \times \frac{m^2}{10^6\ mm^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$K_y = 11.5\ N$ (force up) ← K_y

The pressure distribution on the plate is as shown.

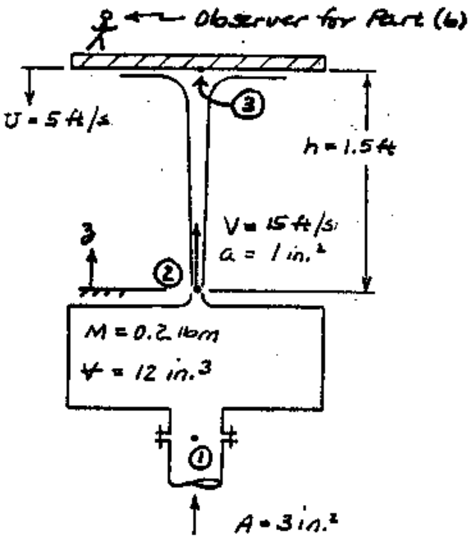


42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Given: A flat object moves downward, at speed $U = 5$ ft/sec, into the water jet of the spray system shown. The spray system, of mass $M = 0.200$ lbm and internal volume $\forall = 12$ in^3, operates under steady conditions

Find: (a) the minimum supply pressure required to produce the jet of the spray system.
(b) the maximum pressure exerted by the jet on the object when the object is at $z = 1.5$ ft.

Solution:



(a)
The minimum pressure occurs when friction is neglected, and so we apply the Bernoulli equation

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1}^{(5)} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}^{(5)}$$

Assume: (1) steady flow
(2) incompressible flow
(3) no friction
(4) flow along a streamline
(5) neglect $z_2 - z_1$
(6) $p_2 = p_{atm}$
(7) uniform flow at ① + ②

Then

$$p_{1g} = p_1 - p_{atm} = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{V_1}{V_2}\right)^2\right]$$

From continuity, $A_1V_1 = A_2V_2$, and $\frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{a}{A}$. Then,

$$p_{1g} = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{a}{A}\right)^2\right] = \frac{1}{2} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times (15)^2\,\frac{\text{ft}^2}{\text{s}^2}\left[1 - \left(\frac{1}{3}\right)^2\right] \times \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}} \times \frac{\text{ft}^2}{144\,\text{in}^2} = 1.35\ \text{psig} \qquad p_{1g}$$

Frictional effects would cause this value to be higher.

(b) The maximum pressure of the jet on the object is the stagnation pressure

$$p_0 = p + \frac{1}{2}\rho V_r^2$$

where V is the velocity of the impinging jet relative to the object

At $z = 1.5$ ft, the jet velocity, V_4, in the absence of the object can be calculated from

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \cancel{\frac{p_4}{\rho}} + \frac{V_4^2}{2} + gz_4$$

$$V_4 = \left[V_2^2 - 2g(z_4 - z_2)\right]^{1/2} = \left[(15)^2\,\frac{\text{ft}^2}{\text{s}^2} - 2 \times 32.2\,\frac{\text{ft}}{\text{s}^2}\,(1.5)\,\text{ft}\right]^{1/2} = 11.3\ \text{ft/s}$$

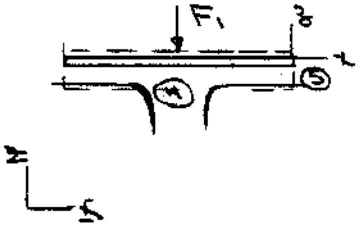
Then

$$V_{rel} = V_4 - (-U) = (11.3 + 5)\ \text{ft/s} = 16.3\ \text{ft/s}$$

and

$$p_0 - p_{atm} = p_{0g} = \frac{1}{2}\rho V_r^2 = \frac{1}{2} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times (16.3)^2\,\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}} \times \frac{\text{ft}^2}{144\,\text{in}^2} = 1.79\ \text{psig} \qquad p_{0g}$$

(c) To determine the force of the water on the object we apply the z component of the momentum equation to the CV shown



$$F_{S_z} + \overset{=0(9)}{\cancel{F_{B_z}}} = \overset{=0(8)}{\cancel{\frac{\partial}{\partial t}\int_{CV} w_{xyz}\,\rho\, dV}} + \int_{CS} w_{xyz}\left(\rho \vec{V}_{xyz}\cdot d\vec{A}\right)$$

Assumptions: (8) neglect $\frac{\partial}{\partial t}\int_{CV}$
(9) neglect body forces
(10) uniform radial flow at ⑤
(11) uniform vertical flow at ④ with $z_4 = 1.5$ ft

Then $-F_1 = -w_{4xyz}\left|\rho V_{4xyz} A_4\right|$

where F_1 is applied force necessary to maintain motion of plate at constant speed U

$$V_{4xyz} = V_4 - (-U) = V_4 + U$$

$$w_{4xyz} = V_{4xyz} = V_4 + U$$

$$\therefore\ F_1 = \rho (V_4 + U)^2 A_4$$

From continuity $A_2V_2 = A_4V_4$

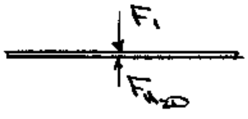
and $A_4 = \frac{V_2}{V_4} A_2 = \frac{15}{11.3} \times 1\,in^2 = 1.33\ in^2$

Then

$$F_1 = \rho (V_4 + U)^2 A_4 = 1.94\,\frac{slug}{ft^3}\,(11.3+5)^2\,\frac{ft^2}{s^2} \times 1.33\,in^2 \times \frac{ft^2}{144\,in^2} \times \frac{lbf\cdot s^2}{ft\cdot slug}$$

$F_1 = 4.76$ lbf (in the direction shown)

Since the plate is moving at constant speed, then



$\sum \vec{F}_{plate} = m\bar{a} = 0$ and

neglecting the weight of the plate then

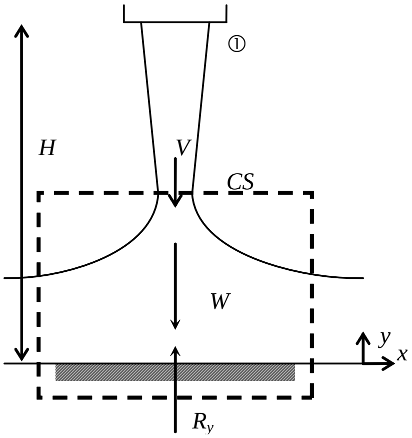
$$F_{H_2O} = F_1 = 4.76\ lbf$$

$$\vec{F}_{H_2O} = 4.76\,\hat{k}\ lbf$$

Problem 6.70

[4]

6.70 Water flows out of a kitchen faucet of 1.25 cm diameter at the rate of 0.1 L/s. The bottom of the sink is 45 cm below the faucet outlet. Will the cross-sectional area of the fluid stream increase, decrease, or remain constant between the faucet outlet and the bottom of the sink? Explain briefly. Obtain an expression for the stream cross section as a function of distance y above the sink bottom. If a plate is held directly under the faucet, how will the force required to hold the plate in a horizontal position vary with height above the sink? Explain briefly.



Given:

Flow through kitchen faucet

Find:

Area variation with height; force to hold plate as function of height

Solution:

Basic equation $\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{const}$ $\quad Q = V \cdot A$ $\quad F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v\,\rho\,dV + \int_{CS} v\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

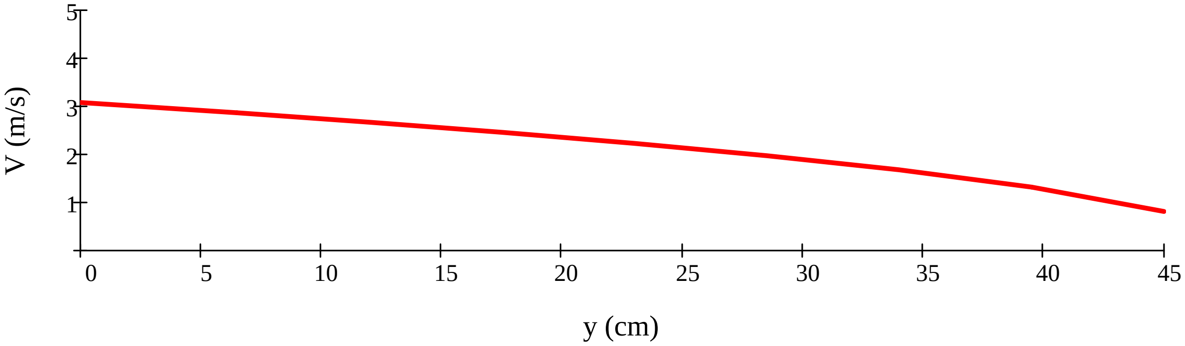
Hence, applying Bernoulli between the faucet (1) and any height y

$$\frac{V_1^2}{2} + g \cdot H = \frac{V^2}{2} + g \cdot y$$

where we assume the water is at p_{atm}

Hence $$V(y) = \sqrt{V_1^2 + 2 \cdot g \cdot (H - y)}$$

The problem doesn't require a plot, but it looks like $\quad V_1 = 0.815\,\frac{m}{s} \quad V(0 \cdot m) = 3.08\,\frac{m}{s}$



The speed increases as y decreases because the fluid particles "trade" potential energy for kinetic, just as a falling solid particle does!

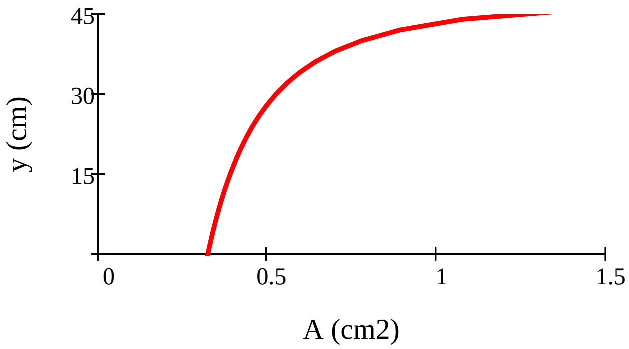
But we have $$Q = V_1 \cdot A_1 = V_1 \cdot \frac{\pi \cdot D^2}{4} = V \cdot A$$

Hence $$A = \frac{V_1 \cdot A_1}{V} \qquad A(y) = \frac{\pi \cdot D_1^2 \cdot V_1}{4 \cdot \sqrt{V_1^2 + 2 \cdot g \cdot (H - y)}}$$

The problem doesn't require a plot, but it looks like

$$A(H) = 1.23\,\mathrm{cm}^2$$

$$A(0) = 0.325\,\mathrm{cm}^2$$



The area decreases as the speed increases. If the stream falls far enough the flow will change to turbulent.

For the CV above $R_y - W = u_{in}\cdot\left(-\rho\cdot V_{in}\cdot A_{in}\right) = -V\cdot(-\rho\cdot Q)$

$$R_y = W + \rho\cdot V^2\cdot A = W + \rho\cdot Q\cdot\sqrt{V_1^{\,2} + 2\cdot g\cdot(H - y)}$$

Hence R_y increases in the same way as V as the height y varies; the maximum force is when y = $R_{ymax} = W + \rho\cdot Q\cdot\sqrt{V_1^{\,2} + 2\cdot g\cdot H}$

Problem 6.71

An old magic trick uses an empty thread spool and a playing card. The playing card is placed against the bottom of the spool. Contrary to intuition, when one blows downward through the central hole in the spool, the card is not blown away. Instead it is ‘‘sucked’’ up against the spool. Explain.

Open-Ended Problem Statement: An old magic trick uses an empty thread spool and a playing card. The playing card is placed against the bottom of the spool. Contrary to intuition, when one blows downward through the central hole in the spool, the card is not blown away. Instead it is ‘‘sucked’’ up against the spool. Explain.

Discussion: The secret to this “parlor trick” lies in the velocity distribution, and hence the pressure distribution, that exists between the spool and the playing cards.

Neglect viscous effects for the purposes of discussion. Consider the space between the end of the spool and the playing card as a pair of parallel disks. Air from the hole in the spool enters the annular space surrounding the hole, and then flows radially outward between the parallel disks. For a given flow rate of air the edge of the hole is the cross-section of minimum flow area and therefore the location of maximum air speed.

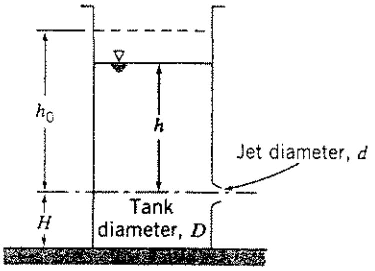
After entering the space between the parallel disks, air flows radially outward. The flow area becomes larger as the radius increases. Thus the air slows and its pressure increases. The largest flow area, slowest air speed, and highest pressure between the disks occur at the outer periphery of the spool where the air is discharged from an annular area.

The air leaving the annular space between the disk and card must be at atmospheric pressure. This is the location of the highest pressure in the space between the parallel disks. Therefore pressure at smaller radii between the disks must be lower, and hence the pressure between the disks is sub-atmospheric. Pressure above the card is less than atmospheric pressure; pressure beneath the card is atmospheric. Each portion of the card experiences a pressure difference acting upward. This causes a net pressure force to act upward on the whole card. The upward pressure force acting on the card tends to keep it from blowing off the spool when air is introduced through the central hole in the spool.

Viscous effects are present in the narrow space between the disk and card. However, they only reduce the pressure rise as the air flows outward, they do not dominate the flow behavior.

Problem 6.72

Given: Tank shown has well-rounded nozzle.
At time $t=0$, water level is h_0



Find: expression for h/h_0 as a function of time.

Plot: (a) h/h_0 vs t for $D/d = 10$, with h_0 as a parameter for $0.1 \le h/h_0 \le 1$ m

(b) h/h_0 vs t for $h_0 = 1$ m, with D/d as a parameter for $2 \le D/d \le 10$.

Solution:

Apply the Bernoulli equation along a streamline between the surface and the jet

Basic equation: $\overset{(5)}{\cancel{\frac{p_s}{\rho}}} + \frac{V_t^2}{2} + gz_s = \overset{(5)}{\cancel{\frac{p_j}{\rho}}} + \frac{V_j^2}{2} + gz_j$

Assumptions: (1) quasi-steady flow, ie neglect acceleration in tank

(2) incompressible flow

(3) neglect frictional effects

(4) flow along a streamline

(5) $p_t = p_j = p_{atm}$.

From continuity, $V_t A_t = V_j A_j$ or $V_j = V_t \frac{A_t}{A_j} = V_t \left(\frac{D}{d}\right)^2$

Solving,

$$\frac{V_t^2}{2} - \frac{V_j^2}{2} = \frac{V_t^2}{2}\left[1 - \left(\frac{V_j}{V_t}\right)^2\right] = g(z_j - z_s) = g[H - (H+h)] = -gh$$

Then

$$V_t = \left[\frac{2gh}{(V_j/V_t)^2 - 1}\right]^{1/2} = \left[\frac{2gh}{(A_t/A_j)^2 - 1}\right]^{1/2} = \left[\frac{2gh}{(D/d)^4 - 1}\right]^{1/2} = -\frac{dh}{dt}$$

Separating variables,

$$\frac{dh}{h^{1/2}} = -\left[\frac{2g}{(D/d)^4 - 1}\right]^{1/2} dt$$

Integrating,

$$2h^{1/2} = -\left[\frac{2g}{(D/d)^4 - 1}\right]^{1/2} t + c$$

At $t=0$, $h = h_0$, so $c = 2h_0^{1/2}$ and

$$h = \left\{ h_0^{1/2} - \frac{1}{2}\left[\frac{2g}{(D/d)^4 - 1}\right]^{1/2} t \right\}^2$$

Problem 6.72

Nondimensionalize (divide by h_0) to obtain

$$\frac{h}{h_0} = \left\{ 1 - \sqrt{\frac{g}{2h_0\{(D/d)^4 - 1\}}}\, t \right\}^2 \qquad h/h_0$$

Draining of a cylindrical liquid tank:

Plot of h/h_0 vs. t for $0.1 < h_0 < 1$ m

Input Data: D = 50 mm

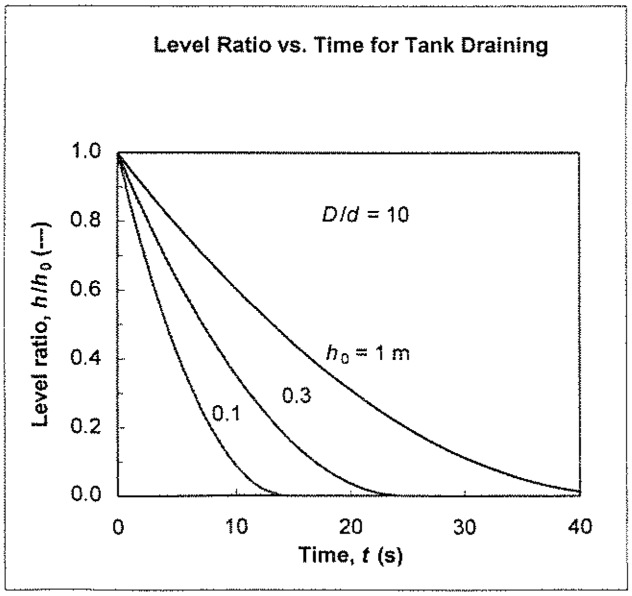
d = 5 mm

h_0 (m) =	0.1	0.3	1
Time, t (s)	**h/h_0 (---)**	**h/h_0 (---)**	**h/h_0 (---)**
0	1.00	1.00	1.00
2	0.739	0.845	0.913
4	0.518	0.703	0.831
6	0.336	0.574	0.752
8	0.193	0.458	0.677
10	0.090	0.355	0.606
12	0.025	0.265	0.539
14	0.000	0.188	0.476
16		0.125	0.417
18		0.074	0.362
20		0.037	0.310
22		0.012	0.263
24		0.001	0.219
26			0.180
28			0.144
30			0.113
32			0.085
34			0.061
36			0.041
38			0.025
40			0.013
45			0.000

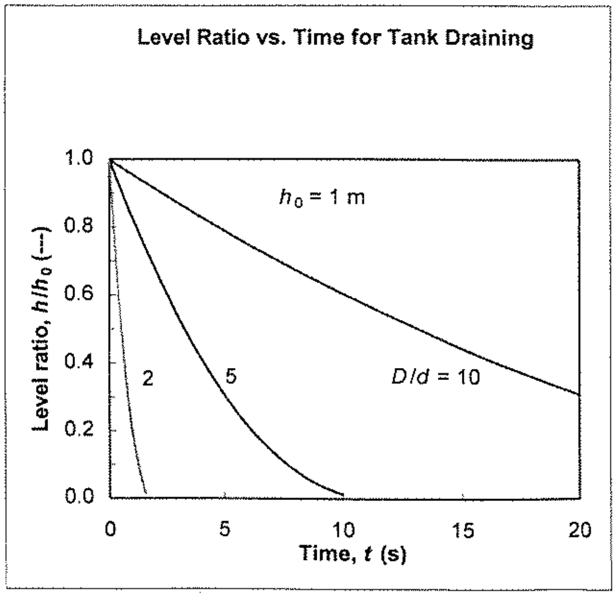
Plot of h/h_0 vs. t for $10 < D/d < 2$

h_0 = 1 m

D/d (---) =	2	5	10
Time, t (s)	**h/h_0 (---)**	**h/h_0 (---)**	**h/h_0 (---)**
0	1.00	1.00	1.00
0.5	0.523	0.913	0.978
1	0.199	0.831	0.956
1.5	0.029	0.752	0.935
1.6	0.013	0.737	0.930
3		0.539	0.872
4		0.417	0.831
5		0.310	0.791
6		0.219	0.752
7		0.144	0.714
8		0.085	0.677
9		0.041	0.641
10		0.013	0.606
12			0.539
14			0.476
16			0.417
18			0.362
20			0.310



Level Ratio vs. Time for Tank Draining

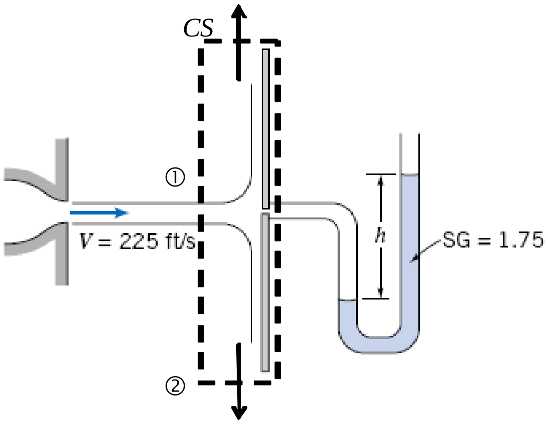


Level Ratio vs. Time for Tank Draining

Problem 6.73

[4]

6.73 A horizontal axisymmetric jet of air with 0.4 in. diameter strikes a stationary vertical disk of 7.5 in. diameter. The jet speed is 225 ft/s at the nozzle exit. A manometer is connected to the center of the disk. Calculate (a) the deflection, if the manometer liquid has SG = 1.75, (b) the force exerted by the jet on the disk, and (c) the force exerted on the disk if it is assumed that the stagnation pressure acts on the entire forward surface of the disk. Sketch the streamline pattern and plot the distribution of pressure on the face of the disk.



Given: Air jet striking disk

Find: Manometer deflection; Force to hold disk; Force assuming p_0 on entire disk; plot pressure distribution

Solution:

Basic equations: Hydrostatic pressure, Bernoulli, and momentum flux in x direction

$$\Delta p = SG\cdot\rho\cdot g\cdot\Delta h \qquad \frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction 4) Flow along streamline 5) Uniform flow 6) Horizontal flow ($g_x = 0$)

Applying Bernoulli between jet exit and stagnation point

$$\frac{p_{atm}}{\rho_{air}} + \frac{V^2}{2} = \frac{p_0}{\rho_{air}} + 0 \qquad p_0 - p_{atm} = \frac{1}{2}\cdot\rho_{air}\cdot V^2$$

But from hydrostatics $\quad p_0 - p_{atm} = SG\cdot\rho\cdot g\cdot\Delta h \quad$ so $\quad \Delta h = \dfrac{\frac{1}{2}\cdot\rho_{air}\cdot V^2}{SG\cdot\rho\cdot g} = \dfrac{\rho_{air}\cdot V^2}{2\cdot SG\cdot\rho\cdot g}$

$$\Delta h = 0.002377\cdot\frac{slug}{ft^3}\times\left(225\cdot\frac{ft}{s}\right)^2\times\frac{1}{2\cdot 1.75}\times\frac{ft^3}{1.94\cdot slug}\times\frac{s^2}{32.2\cdot ft} \qquad \Delta h = 0.55\cdot ft \qquad \Delta h = 6.60\cdot in$$

For x momentum $\quad R_x = V\cdot\left(-\rho_{air}\cdot A\cdot V\right) = -\rho_{air}\cdot V^2\cdot\dfrac{\pi\cdot d^2}{4}$

$$R_x = -0.002377\cdot\frac{slug}{ft^3}\times\left(225\cdot\frac{ft}{s}\right)^2\times\frac{\pi\cdot\left(\frac{0.4}{12}\cdot ft\right)^2}{4}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad R_x = -0.105\cdot lbf$$

The force of the jet on the plate is then $\quad F = -R_x \qquad F = 0.105\cdot lbf$

The stagnation pressure is $\quad p_0 = p_{atm} + \dfrac{1}{2}\cdot\rho_{air}\cdot V^2$

The force on the plate, assuming stagnation pressure on the front face, is

$$F = \left(p_0 - p\right)\cdot A = \frac{1}{2}\cdot\rho_{air}\cdot V^2\cdot\frac{\pi\cdot D^2}{4}$$

$$F = \frac{\pi}{8} \times 0.002377 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(225 \cdot \frac{\text{ft}}{\text{s}}\right)^2 \times \left(\frac{7.5}{12} \cdot \text{ft}\right)^2 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \qquad F = 18.5\,\text{lbf}$$

Obviously this is a huge overestimate!

For the pressure distribution on the disk, we use Bernoulli between the disk outside edge any radius r for radial flow

$$\frac{p_{atm}}{\rho_{air}} + \frac{1}{2} \cdot v_{edge}^{\ 2} = \frac{p}{\rho_{air}} + \frac{1}{2} \cdot v^2$$

We need to obtain the speed v as a function of radius. If we assume the flow remains constant thickness h, then

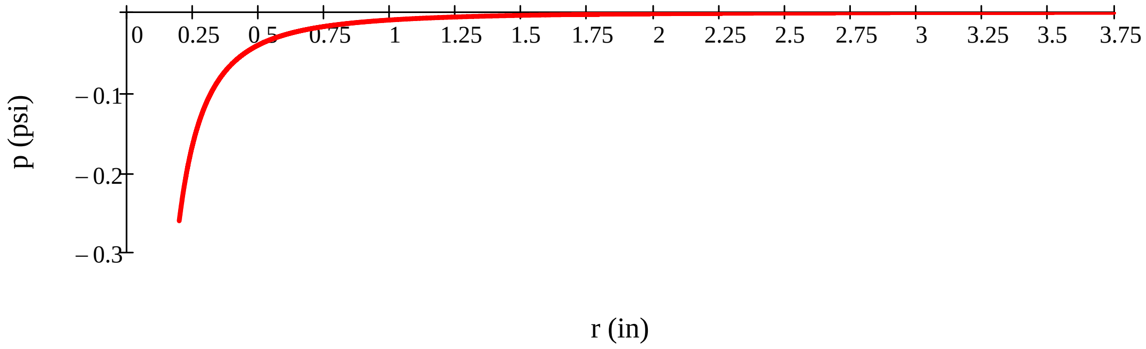
$$Q = v \cdot 2 \cdot \pi \cdot r \cdot h = V \cdot \frac{\pi \cdot d^2}{4} \qquad v(r) = V \cdot \frac{d^2}{8 \cdot h \cdot r}$$

We need an estimate for h. As an approximation, we assume that h = d (this assumption will change the scale of p(r) but not the basic shap

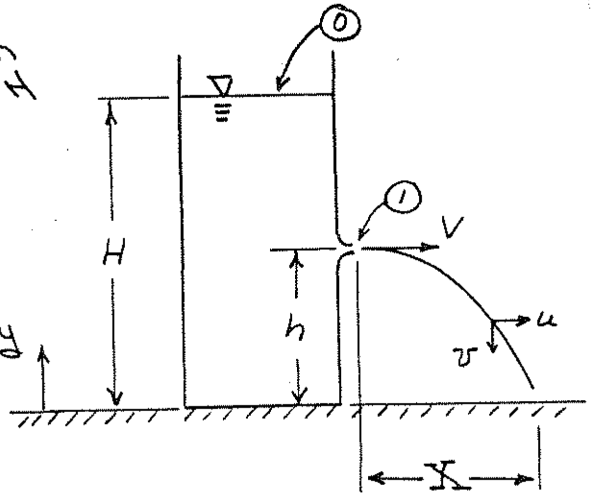
Hence $\quad v(r) = V \cdot \dfrac{d}{8 \cdot r}$

Using this in Bernoulli $\quad p(r) = p_{atm} + \dfrac{1}{2} \cdot \rho_{air} \cdot \left(v_{edge}^{\ 2} - v(r)^2\right) = p_{atm} + \dfrac{\rho_{air} \cdot V^2 \cdot d^2}{128} \cdot \left(\dfrac{4}{D^2} - \dfrac{1}{r^2}\right)$

Expressed as a gage pressure $\quad p(r) = \dfrac{\rho_{air} \cdot V^2 \cdot d^2}{128} \cdot \left(\dfrac{4}{D^2} - \dfrac{1}{r^2}\right)$



Problem 6.74



Given: Water level in tank shown is maintained at height H

Find: Elevation h to maximize range, X, of jet.

Plot: Jet speed, V, & distance, X as function of h, for $0 < h < H$.

Solution:

Apply Bernoulli equation between tank surface and jet.

Basic equation: $\cancel{\frac{p_0}{\rho}} + \cancel{\frac{V_0^2}{2}}^{\approx 0} + gy_0 = \cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + gy_1$

Assumptions: (1) steady flow (2) incompressible flow (3) flow along streamline (4) no friction

Then

$$gH = \frac{V^2}{2} + gh \quad \text{or} \quad V = \sqrt{2g(H-h)} \qquad (1)$$

Assume no air resistance in the stream. Then $u = \text{constant}$, and $X = ut = \sqrt{2g(H-h)}\, t$ — — — — — — — (2)

The only force acting on the stream is gravity

$$\Sigma F_y = -mg = ma_y = m\frac{dv}{dt} \; ; \; \text{thus} \; \frac{dv}{dt} = -g$$

Integrating we obtain $v = \cancel{v_0}^{0} - gt$ and

$$y = y_0 + \cancel{v_0 t}^{0} - \frac{1}{2}gt^2$$

Solving for t, $t = \left[\frac{2(y_0 - y)}{g}\right]^{1/2}$

The time of flight is then $t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2h}{g}}$

Substituting into Eq. 2

$$X = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}} = 2\sqrt{h(H-h)} \qquad (3)$$

X will be maximized when $h(H-h)$ is maximized, or when

$$\frac{d}{dh}\left[h(H-h)\right] = 0 = (H-h) + h(-1) = H - 2h \quad \text{or} \quad h = H/2 \longleftarrow h$$

The corresponding range is

$$X = 2\sqrt{\frac{H}{2}\cdot\frac{H}{2}} = H$$

See the next page for plots

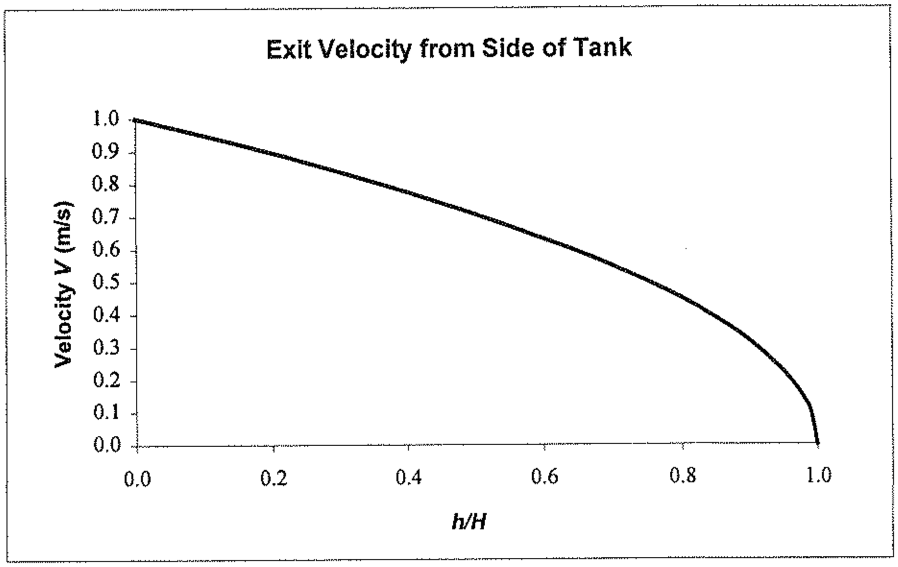
Problem 6.74

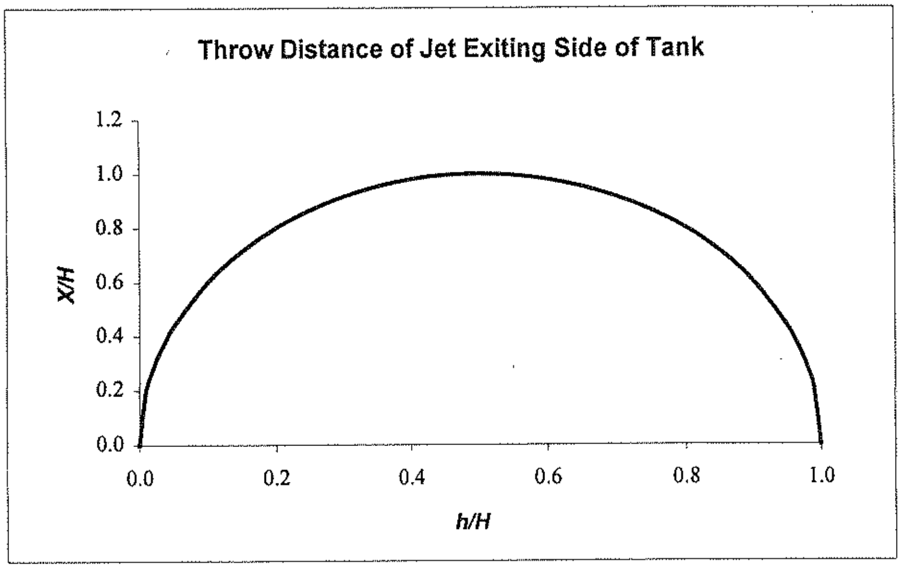
From Eq.1, $\frac{V}{\sqrt{2gH}} = \sqrt{1-\frac{h}{H}}$

From Eq.2, $\frac{X}{H} = 2\sqrt{\frac{h}{H}\left(1-\frac{h}{H}\right)}$

Exit velocity and throw distance from orifice in side of tank, versus height *h/H*

h/H	$V/(2gH)^{1/2}$	*X/H*
0.00	1.00	0.000
0.01	0.995	0.199
0.02	0.990	0.280
0.03	0.985	0.341
0.04	0.980	0.392
0.05	0.975	0.436
0.10	0.949	0.600
0.15	0.922	0.714
0.20	0.894	0.800
0.25	0.866	0.866
0.30	0.837	0.917
0.35	0.806	0.954
0.40	0.775	0.980
0.45	0.742	0.995
0.50	0.707	1.000
0.55	0.671	0.995
0.60	0.632	0.980
0.65	0.592	0.954
0.70	0.548	0.917
0.75	0.500	0.866
0.80	0.447	0.800
0.85	0.387	0.714
0.90	0.316	0.600
0.95	0.224	0.436
0.96	0.200	0.392
0.97	0.173	0.341
0.98	0.141	0.280
0.99	0.100	0.199
1.00	0.00	0.00



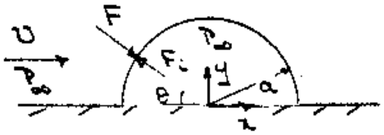


Problem 6.75

Given: Flow over a Quonset hut may be approximated by the velocity field

$$\vec{V} = U\left[1-\left(\frac{a}{r}\right)^2\right]\cos\theta\,\hat{e}_r - U\left[1+\left(\frac{a}{r}\right)^2\right]\sin\theta\,\hat{e}_\theta$$

with $0 \le \theta \le 2\pi$



The hut has a diameter, $D = 6m$, and a length, $L = 18m$

During a storm, $U = 100$ km/hr, $p_\infty = 720$ mm Hg, $T_\infty = 5C$

Find: The net force tending to lift the hut off its foundation.

Solution:

Basic equations: $\frac{p}{\rho}+\frac{V^2}{2}+gz=\text{const}$ $\quad F=\int p\,dA$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) frictionless flow
(4) flow along a streamline

Along the top half of the cylinder, $r=a$ and $\vec{V}=-2U\sin\theta\,\hat{e}_\theta$, $0\le\theta\le 2\pi$

Applying the Bernoulli equation along the streamline $(r=a)$

$$\frac{p}{\rho}+\frac{V^2}{2}=\frac{p_\infty}{\rho}+\frac{V_\infty^2}{2}$$

$$p-p_\infty=\frac{\rho}{2}\left(V_\infty^2-V^2\right)=\frac{\rho}{2}\left(U^2-4U^2\sin^2\theta\right)=\rho\frac{U^2}{2}\left(1-4\sin^2\theta\right)$$

$$F_{R_y}=\int_A (p_\infty-p)\,dA\sin\theta=\int_0^\pi (p_\infty-p)\sin\theta\,La\,d\theta$$

$$=\int_0^\pi \rho\frac{U^2}{2}\left(4\sin^2\theta-1\right)\sin\theta\,La\,d\theta=\rho\frac{U^2}{2}aL\left\{4\left[\frac{\cos^3\theta}{3}-\cos\theta\right]_0^\pi+\cos\theta\Big]_0^\pi\right\}$$

$$=\rho\frac{U^2}{2}aL\left\{4\left[\left(-\frac{1}{3}+1\right)-\left(\frac{1}{3}-1\right)\right]+(-1-1)\right\}$$

$$F_{R_y}=\rho\frac{U^2}{2}aL\left(\frac{10}{3}\right)=\frac{5}{3}\rho U^2aL$$

From the ideal gas equation of state

$$\rho=\frac{p}{RT}=720\,\text{mm Hg}\times\frac{\text{atm}}{760\,\text{mm}}\times1.01\times10^5\frac{N}{m^2\cdot\text{atm}}\times\frac{kg\cdot K}{287\,N\cdot m}\times\frac{1}{278\,K}=1.20\,kg/m^3$$

$$F_{R_y}=\frac{5}{3}\rho U^2aL=\frac{5}{3}\times1.20\frac{kg}{m^3}\times\left(10^5\right)^2\frac{m^2}{hr^2}\times\frac{hr^2}{(3600)^2s^2}\times3m\times18m\times\frac{N\cdot s^2}{kg\cdot m}$$

$F_{R_y}=83.3$ kN ——— F_{R_y}

Comment: The actual pressure distribution over the rear portion of the hut is not modelled well by ideal flow. The force calculated here is lower that the actual force.

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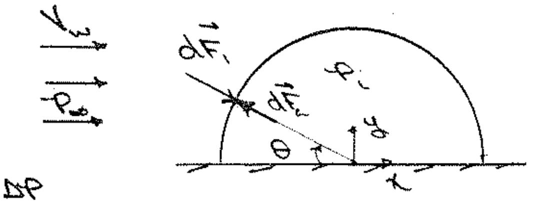
Problem 6.76

[4]

Given: Inflatable "bubble" structure modelled as circular semi-cylinder.
diameter, $D = 30\ \text{m}$
length $L = 70\ \text{m}$
Pressure inside is $p_i = p_\infty + \Delta p$
where $\Delta p = \rho_{H_2O}\, g\, \Delta h$ and $\Delta h = 10\ \text{mm}$.
Pressure distribution over outer surface is given by

$$\frac{p - p_\infty}{\frac{1}{2}\rho V_w^2} = 1 - 4\sin^2\theta \qquad V_w = 60\ \text{km/hr}$$



Find: net vertical force exerted on the structure.

Solution:

The force due to pressure is $F = \int p\, dA$.

The vertical component of dF_1 is $dF_{1v} = -p\, dA \sin\theta = -p\, R L\, d\theta \sin\theta$

The vertical component of dF_2 is $dF_{2v} = p_i\, dA \sin\theta = p_i R L\, d\theta \sin\theta$

Then, neglecting end effects

$$dF_{v\,net} = (p_i - p) RL \sin\theta\, d\theta = (p_\infty + \Delta p - p) RL \sin\theta\, d\theta$$

$$F_v = \int dF_v = \int_0^\pi \left[\Delta p - (p - p_\infty)\right] RL \sin\theta\, d\theta$$

$$= \int_0^\pi \left[\Delta p - \tfrac{1}{2}\rho V_w^2 (1 - 4\sin^2\theta)\right] RL \sin\theta\, d\theta$$

$$= RL\left\{\Delta p\left[-\cos\theta\right]_0^\pi - \tfrac{1}{2}\rho V_w^2 \left[-\cos\theta + 4\left(\cos\theta - \frac{\cos^3\theta}{3}\right)\right]_0^\pi\right.$$

$$= RL\left\{2\Delta p - \tfrac{1}{2}\rho V_w^2\left[2 + 4\left(-2 + \frac{2}{3}\right)\right]\right.$$

$$F_v = RL\left\{2\Delta p + \frac{5}{3}\rho V_w^2\right\} = RL\left\{2\rho_{H_2O}\, g\,\Delta h + \frac{5}{3}\rho V_w^2\right\}$$

$$F_v = 15\,\text{m} \times 70\,\text{m}\left\{2 \times 999\,\frac{\text{kg}}{\text{m}^3} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 0.01\,\text{m} + \frac{5}{3} \times 1.23\,\frac{\text{kg}}{\text{m}^3} \times (60)^2\,\frac{\text{km}^2}{\text{hr}^2} \times 10^6\,\frac{\text{m}^2}{\text{km}^2} \times \frac{\text{hr}^2}{(3600\,\text{s})^2}\right\} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$F_{v\,net} = 804\ \text{kN}$ $\longleftarrow$ $F_{v\,net}$

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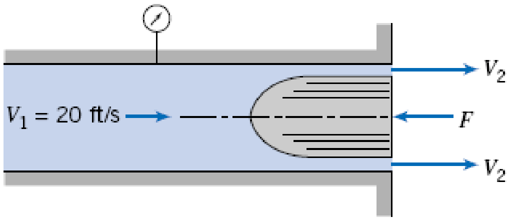
Problem 6.77

6.77 Water flows at low speed through a circular tube with inside diameter of 2 in. A smoothly contoured body of 1.5 in. diameter is held in the end of the tube where the water discharges to atmosphere. Neglect frictional effects and assume uniform velocity profiles at each section. Determine the pressure measured by the gage and the force required to hold the body.



Given: Water flow out of tube

Find: Pressure indicated by gage; force to hold body in place

Solution:

Basic equations: Bernoulli, and momentum flux in x direction

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad Q = V\cdot A \qquad F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!\!/ + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction 4) Flow along streamline 5) Uniform flow 6) Horizontal flow ($g_x = 0$)

Applying Bernoulli between jet exit and stagnation point

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_2}{\rho} + \frac{V_2^2}{2} = \frac{V_2^2}{2} \qquad \text{where we work in gage pressure}$$

$$p_1 = \frac{\rho}{2}\cdot\left(V_2^2 - V_1^2\right)$$

But from continuity $Q = V_1\cdot A_1 = V_2\cdot A_2$ $\qquad V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot\frac{D^2}{D^2 - d^2}$ $\qquad$ where D = 2 in and d = 1.5 in

$$V_2 = 20\cdot\frac{\text{ft}}{\text{s}}\cdot\left(\frac{2^2}{2^2 - 1.5^2}\right) \qquad V_2 = 45.7\frac{\text{ft}}{\text{s}}$$

Hence $\quad p_1 = \frac{1}{2}\times 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left(45.7^2 - 20^2\right)\cdot\left(\frac{\text{ft}}{\text{s}}\right)^2\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\,\text{ft}} \qquad p_1 = 1638\frac{\text{lbf}}{\text{ft}^2} \qquad p_1 = 11.4\,\text{psi}$ $\qquad$ (gage)

The x mometum is $\quad -F + p_1\cdot A_1 - p_2\cdot A_2 = u_1\cdot\left(-\rho\cdot V_1\cdot A_1\right) + u_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$

$$F = p_1\cdot A_1 + \rho\cdot\left(V_1^2\cdot A_1 - V_2^2\cdot A_2\right) \qquad \text{using gage pressures}$$

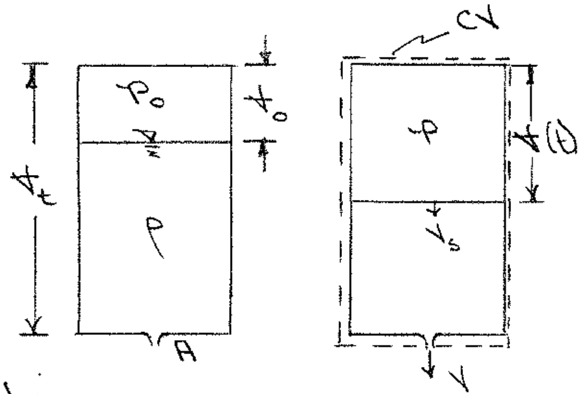
$$F = 11.4\cdot\frac{\text{lbf}}{\text{in}^2}\times\frac{\pi\cdot(2\cdot\text{in})^2}{4} + 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left[\left(20\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\frac{\pi\cdot(2\cdot\text{in})^2}{4} - \left(45.7\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\frac{\pi\cdot\left[(2\cdot\text{in})^2 - (1.5\cdot\text{in})^2\right]}{4}\right]\times\left(\frac{1\cdot\text{ft}}{12\cdot\text{in}}\right)^2\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\,\text{ft}}$$

$F = 14.1\,\text{lbf}$ $\qquad$ in the direction shown

Given: High-pressure air forces a stream of water from a tiny, rounded orifice, of area A, in a tank. The air expands slowly so the expansion may be considered isothermal.

Find: (a) algebraic expression for $\dot m$ leaving the tank
(b) " " " dM/dt in tank.
(c) expression for $M_w(t)$
(d) plot $M_w(t)$ for $0<t<40\,\text{min}$ if $\forall_0 = 5\,m^3$, $\forall_t = 10\,m^3$, $A = 25\,mm^2$, $p_0 = 1\,MPa$

Solution:



Basic equations: $\frac{p}{\rho}+\frac{V^2}{2}+gz=\text{const}$

$$0=\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall+\int_{CS}\rho\vec V\cdot d\vec A$$

Assumptions: (1) quasi steady flow $V_s << V$
(2) frictionless
(3) incompressible
(4) flow along a streamline
(5) uniform flow at outlet.
(6) neglect gravity
(7) $p >> p_{atm}$ $\therefore p_{abs} \approx p_{gage}$

Apply Bernoulli equation between liquid surface and orifice

$$V_j=\left[\frac{2(p-p_{atm})}{\rho}\right]^{1/2}\approx\sqrt{\frac{2p}{\rho}}$$

$$\dot m=\rho A V_j=\rho A\sqrt{\frac{2p}{\rho}}=\sqrt{2p\rho}\,A \longleftarrow \dot m$$

Rate of change of mass in tank is $\frac{dM}{dt}=\frac{\partial}{\partial t}\int \rho\, d\forall$

$$\frac{dM}{dt}=\rho_w\frac{d\forall_w}{dt}=-\rho_w\frac{d\forall_{air}}{dt}\quad(\forall_t=\forall_{air}+\forall_w) \longleftarrow \frac{dM}{dt}$$

For isothermal flow, $\frac{p}{\rho}=RT=\text{constant}=\frac{p_0}{\rho_0}$

where ρ is the air density and $\rho = M_{air}/\forall_{air}$

thus

$$p\forall=p_0\forall_0 \quad\text{or}\quad p=p_0\frac{\forall_0}{\forall}$$

From continuity

$$0=\rho_w\frac{d\forall_w}{dt}+\dot m$$

and

$$0=-\rho_w\frac{d\forall_{air}}{dt}+\sqrt{2p\rho_w}\,A$$

$$\frac{d\forall}{dt}=\sqrt{\frac{2p}{\rho_w}}=\sqrt{\frac{2p_0\forall_0}{\rho_w\forall}}$$

Problem 6.78

Separating variables, $\forall^{1/2}\,d\forall = \sqrt{\frac{2p_0\forall_0}{\rho}}\,A\,dt$

Integrating $\frac{2}{3}\forall^{3/2}\Big]_{\forall_0}^{\forall} = \sqrt{\frac{2p_0\forall_0}{\rho_w}}\,At$

$$\frac{2}{3}\left(\forall^{3/2}-\forall_0^{3/2}\right) = \frac{2\forall_0^{3/2}}{3}\left[\left(\frac{\forall}{\forall_0}\right)^{3/2}-1\right] = \sqrt{\frac{2p_0\forall_0}{\rho_w}}\,At$$

Then $\left(\frac{\forall}{\forall_0}\right)^{3/2} = \left[1+\frac{3}{2\forall_0^{3/2}}\sqrt{\frac{2p_0\forall_0}{\rho_w}}\,At\right]$

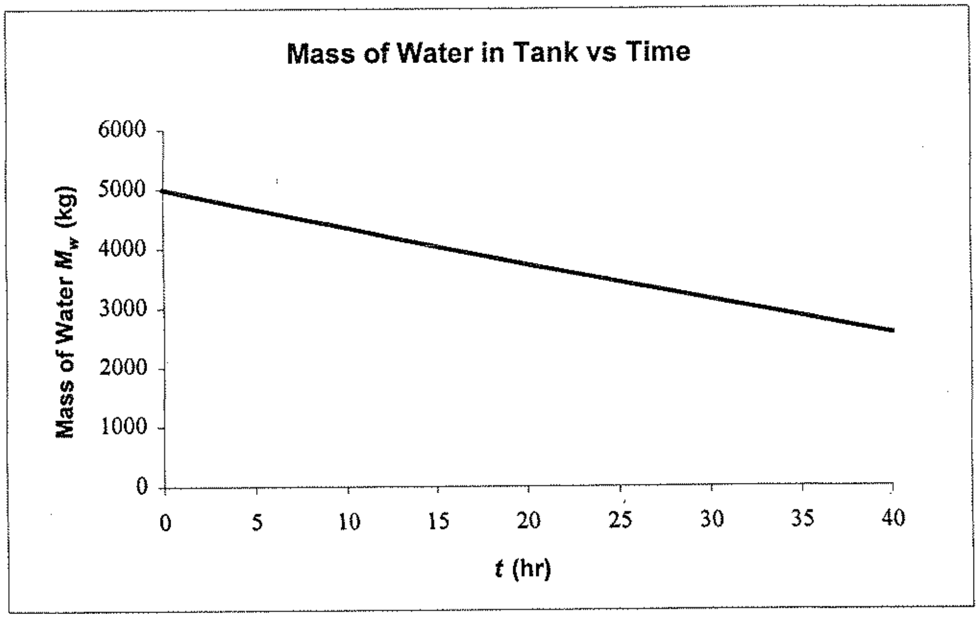
$$\frac{\forall}{\forall_0} = \left[1+1.5\sqrt{\frac{2p_0}{\rho_w}}\,\frac{At}{\forall_0}\right]^{2/3}$$

But $M_w = \rho_w(\forall_t-\forall) = \rho\forall_0\left\{\frac{\forall_t}{\forall_0}-\frac{\forall}{\forall_0}\right\}$

$\therefore M_w = \rho_w\forall_0\left\{\frac{\forall_t}{\forall_0}-\left[1+1.5\sqrt{\frac{2p_0}{\rho_w}}\,\frac{At}{\forall_0}\right]^{2/3}\right\}$

M_w

t (s)	M_w (kg)
0	4995
2	4862
4	4730
6	4600
8	4472
10	4345
12	4220
14	4096
16	3973
18	3851
20	3731
22	3612
24	3494
26	3377
28	3260
30	3145
32	3031
34	2918
36	2806
38	2695
40	2584



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Problem 6.79

[4] Part 1/2

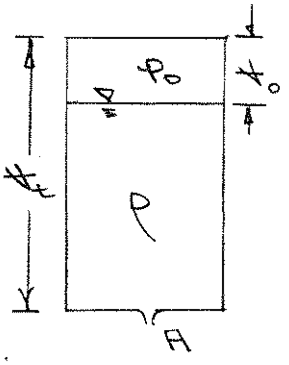
Given: High-pressure air forces a stream of water from a tiny rounded orifice, of area A, in a tank. The air expands rapidly so the expansion may be treated as adiabatic.

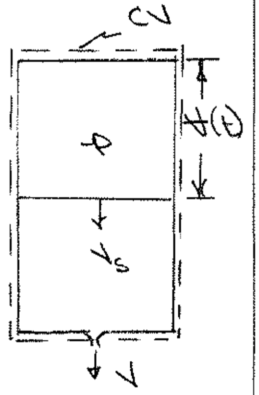
Find: (a) algebraic expression for $\dot{m}$ leaving the tank
(b) " " " dM/dt in the tank
(c) expression for $M_w(t)$; plot $M_w(t)$ for $0<t<40$ min if $\mathcal{V}_o = 5\,m^3$, $\mathcal{V}_t = 10\,m^3$, $A = 25\,mm^2$, $p_o = 1$ MPa

Solution:

Basic equations: $\dfrac{p}{\rho} + \dfrac{V^2}{2} + gz = \text{const}$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\mathcal{V} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$





Assumptions: (1) quasi steady flow
$V_s \ll V$
(2) frictionless
(3) incompressible
(4) flow along a streamline
(5) uniform flow at outlet
(6) neglect gravity
(7) $p \gg p_{atm} \therefore p_{abs} = p_{gage}$

Apply Bernoulli equation between liquid surface and orifice

$$V_j = \left[\frac{2(p - p_{atm})}{\rho}\right]^{1/2} \approx \sqrt{\frac{2p}{\rho}}$$

$$\dot{m} = \rho A V_j = \rho A \sqrt{\frac{2p}{\rho}} = \sqrt{2p\rho}\, A \longleftarrow \dot{m}$$

Rate of change of mass in tank is $\dfrac{dM}{dt} = \dfrac{\partial}{\partial t}\int \rho\, d\mathcal{V}$

$$\frac{dM}{dt} = \rho_w \frac{d\mathcal{V}_w}{dt} = -\rho_w \frac{d\mathcal{V}_{air}}{dt} \quad (\mathcal{V}_t = \mathcal{V}_{air} + \mathcal{V}_w) \longleftarrow \frac{dM}{dt}$$

For adiabatic expansion of air $p/\rho^k = \text{constant}$

Since mass of air is constant, $p_o \mathcal{V}_o^k = p \mathcal{V}^k$

From continuity, $-\rho_w \dfrac{d\mathcal{V}_{air}}{dt} + \sqrt{2p\rho_w}\, A = 0$

$$\frac{d\mathcal{V}_{air}}{dt} = \frac{A\sqrt{2}}{\sqrt{\rho_w}}\, p^{1/2} = A\sqrt{\frac{2}{\rho_w}}\left[\frac{p_o \mathcal{V}_o^k}{\mathcal{V}^k}\right]^{1/2} = A\sqrt{\frac{2p_o \mathcal{V}_o^k}{\rho_w}}\, V^{-k/2}$$

$$\mathcal{V}^{k/2}\, d\mathcal{V} = A\sqrt{\frac{2p_o \mathcal{V}_o^k}{\rho_w}}\, dt = c\, dt \quad \text{where } c = A\sqrt{\frac{2p_o \mathcal{V}_o^k}{\rho_w}}$$

Integrating

$$\left.\frac{2}{(k+2)} V^{\frac{k}{2}+1}\right]_{\mathcal{V}_o}^{\mathcal{V}} = ct$$

Problem 6.79

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$$\forall^{\left(\frac{k+2}{2}\right)} - \forall_0^{\left(\frac{k+2}{2}\right)} = \frac{k+2}{2} c t$$

$$\left(\frac{\forall}{\forall_0}\right)^{\left(\frac{k+2}{2}\right)} = 1 + \left(\frac{k+2}{2}\right) c t \cdot \frac{1}{\forall_0^{\frac{k+2}{2}}}$$

$$= 1 + \left(\frac{k+2}{2}\right) A \left[\frac{2 p_0 \forall_0^{k}}{\rho_w}\right]^{1/2} \cdot \left[\frac{1}{\forall_0^{(k+2)}}\right]^{1/2} t$$

$$\left(\frac{\forall}{\forall_0}\right)^{\frac{k+2}{2}} = 1 + \left(\frac{k+2}{2}\right) A \left[\frac{2 p_0}{\rho_w \forall_0^{2}}\right]^{1/2} t = 1 + \left(\frac{k+2}{2}\right) \frac{A}{\forall_0} \left[\frac{2 p_0}{\rho_w}\right]^{1/2} t$$

$$\frac{\forall}{\forall_0} = \left[1 + \frac{A}{\forall_0} \sqrt{\frac{2 p_0}{\rho_w}} \left(\frac{k+2}{2}\right) t\right]^{2/k+2}$$

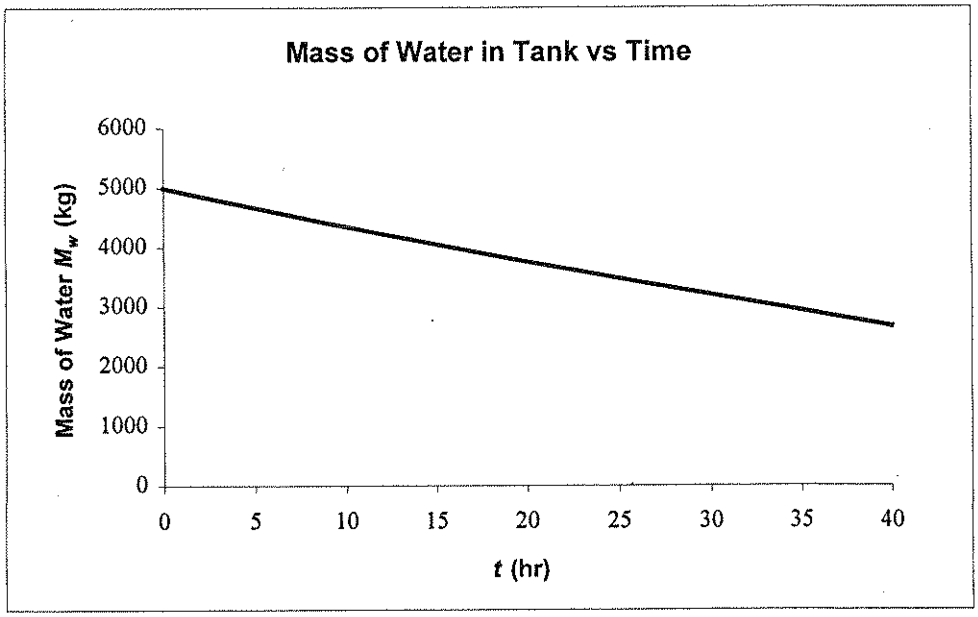
$$M_w = \rho_w (\forall_t - \forall) = \rho \forall_0 \left\{\frac{\forall_t}{\forall_0} - \frac{\forall}{\forall_0}\right\}.$$

$$M_w = \rho_w \forall_0 \left\{\frac{\forall_t}{\forall_0} - \left[1 + \frac{A}{\forall_0} \sqrt{\frac{2 p_0}{\rho_w}} \left(\frac{k+2}{2}\right) t\right]^{2/k+2}\right\}$$

$$M_w = \rho_w \forall_0 \left\{\frac{\forall_t}{\forall_0} - \left[1 + 1.70 \sqrt{\frac{2 p_0}{\rho_w}} \frac{A t}{\forall_0}\right]^{0.588}\right\}$$

M_w

t (s)	M_w (kg)
0	4995
2	4862
4	4732
6	4603
8	4477
10	4353
12	4231
14	4110
16	3991
18	3874
20	3759
22	3645
24	3532
26	3420
28	3310
30	3202
32	3094
34	2988
36	2882
38	2778
40	2675



Problem 6.80

[5]

Describe the pressure distribution on the exterior of a multistory building in a steady wind. Identify the locations of the maximum and minimum pressures on the outside of the building. Discuss the effect of these pressures on infiltration of outside air into the building.

Open-Ended Problem Statement: Describe the pressure distribution on the exterior of a multistory building in a steady wind. Identify the locations of the maximum and minimum pressures on the outside of the building. Discuss the effect of these pressures on infiltration of outside air into the building.

Discussion: A multi-story building acts as a bluff-body obstruction in a thick atmospheric boundary layer. The boundary-layer velocity profile causes the air speed near the top of the building to be highest and that toward the ground to be lower.

Obstruction of air flow by the building causes regions of stagnation pressure on upwind surfaces. The stagnation pressure is highest where the air speed is highest. Therefore the maximum surface pressure occurs near the roof on the upwind side of the building. Minimum pressure on the upwind surface of the building occurs near the ground where the air speed is lowest.

The minimum pressure on the entire building will likely be in the low-speed, low-pressure wake region on the downwind side of the building.

Static pressure inside the building will tend to be an average of all the surface pressures that act on the outside of the building. It is never possible to seal all openings completely. Therefore air will tend to infiltrate into the building in regions where the outside surface pressure is above the interior pressure, and will tend to pass out of the building in regions where the outside surface pressure is below the interior pressure. Thus generally air will tend to move through the building from the upper floors toward the lower floors, and from the upwind side to the downwind side.

Problem 6.81

Imagine a garden hose with a stream of water flowing out through a nozzle. Explain why the end of the hose may be unstable when held a half meter or so from the nozzle end.

Open-Ended Problem Statement: Imagine a garden hose with a stream of water flowing out through a nozzle. Explain why the end of the hose may be unstable when held a half meter or so from the nozzle end.

Discussion: Water flowing out of the nozzle tends to exert a thrust force on the end of the hose. The thrust force is aligned with the flow from the nozzle and is directed toward the hose.

Any misalignment of the hose will lead to a tendency for the thrust force to bend the hose further. This will quickly become unstable, with the result that the free end of the hose will "flail" about, spraying water from the nozzle in all directions.

This instability phenomenon can be demonstrated easily in the backyard. However, it will tend to do least damage when the person demonstrating it is wearing a bathing suit!

Problem 6.82

[5]

An aspirator provides suction by using a stream of water flowing through a venturi. Analyze the shape and dimensions of such a device. Comment on any limitations on its use.

Open-Ended Problem Statement: An aspirator provides suction by using a stream of water flowing through a venturi. Analyze the shape and dimensions of such a device. Comment on any limitations on its use.

Discussion: The basic shape of the aspirator channel should be a converging nozzle section to reduce pressure followed by a diverging diffuser section to promote pressure recovery. The basic shape is that of a venturi flow meter.

If the diffuser exhausts to atmosphere, the exit pressure will be atmospheric. The pressure rise in the diffuser will cause the pressure at the diffuser inlet (venturi throat) to be below atmospheric.

A small tube can be brought in from the side of the throat to aspirate another liquid or gas into the throat as a result of the reduced pressure there.

The following comments can be made about limitations on the aspirator:

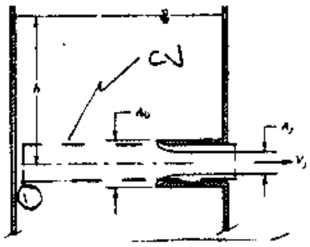
1. It is desirable to minimize the area of the aspirator tube compared to the flow area of the venturi throat. This minimizes the disturbance of the main flow through the venturi and promotes the best possible pressure recovery in the diffuser.
2. It is desirable to avoid cavitation in the throat of the venturi. Cavitation alters the effective shape of the flow channel and destroys the pressure recovery in the diffuser. To avoid cavitation, the reduced pressure must always be above the vapor pressure of the driver liquid.
3. It is desirable to limit the flow rate of gas into the venturi throat. A large amount of gas can alter the flow pattern and adversely affect pressure recovery in the diffuser.

The best combination of specific dimensions could be determined experimentally by a systematic study of aspirator performance. A good starting point probably would be to use dimensions similar to those of a commercially available venturi flow meter.

Problem 6.83

[5]

Given: Reentrant orifice in the side of a large tank. Pressure along the tank walls is essentially hydrostatic.



Find: the contraction coefficient,

$C_c = A_j/A_0$

Solution:

Apply the x-component of the momentum equation to the CV shown

$$F_{S_x} + \cancel{F_{B_x}} = \cancelto{0(1)}{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) steady flow
(2) uniform flow at jet exit.
(3) hydrostatic pressure variation across CS①, $V_1 \approx 0$
(4) x momentum flux across horizontal portion of CS is negligible.
(5) ρ = constant

then

$$\int_{A_0} p\,dA_1 = \dot{m}V_j = \rho V_j A_j V_j = \rho A_j V_j^2$$

$$\bar{p}_1 A_0 = \rho g h A_0 = \rho A_j V_j^2$$

$$\therefore \frac{A_0}{A_j} = \frac{V_j^2}{gh}$$

Apply the Bernoulli equation along the central streamline from ① to the jet exit.

$$\frac{p_1}{\rho} + \cancelto{0(3)}{\frac{V_1^2}{2}} + \cancel{gz_1} = \cancelto{0(gage)}{\frac{p_j}{\rho}} + \frac{V_j^2}{2} + \cancel{gz_j}$$

Assumptions: (6) frictionless flow

$$p_1 = \rho g h = \rho\frac{V_j^2}{2}$$

$$\therefore \frac{V_j^2}{2} = gh$$

and

$$\frac{A_0}{A_j} = \frac{V_j^2}{gh} = 2$$

$$\therefore C_c = \frac{A_j}{A_0} = \frac{1}{2} \longleftarrow$$

C_c

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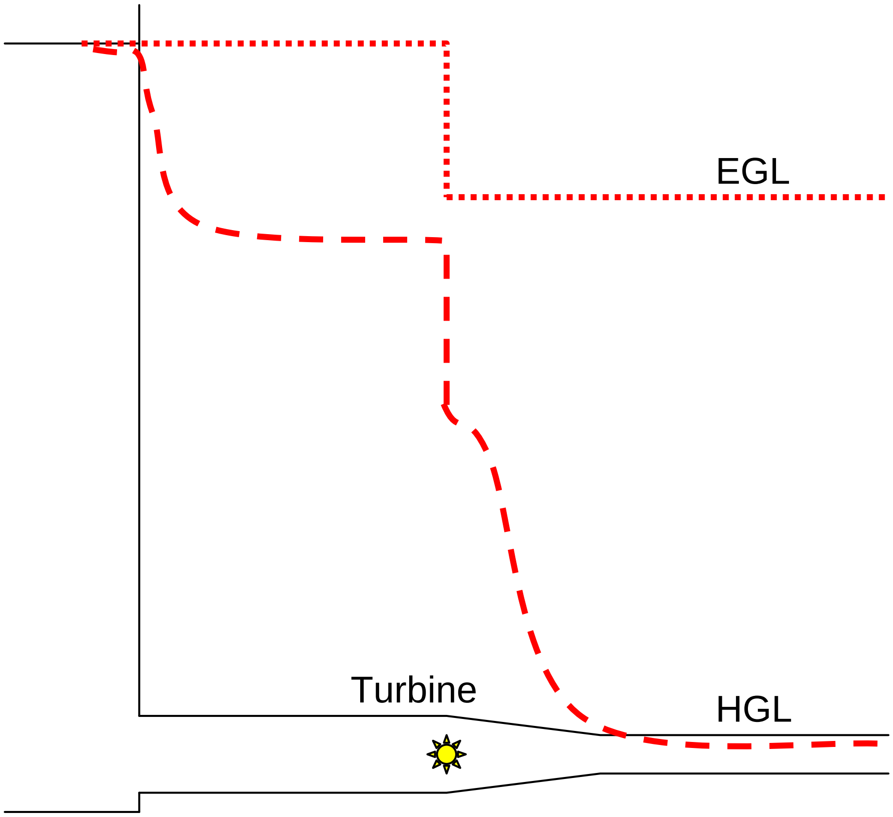
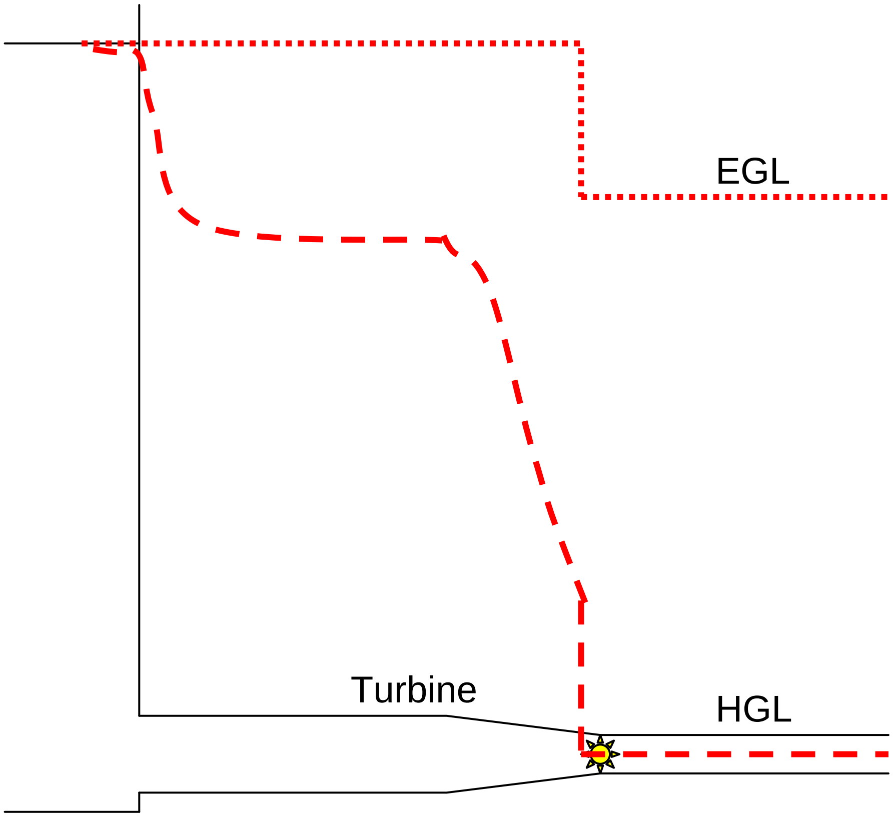
Problem 6.84

Carefully sketch the energy grade lines (EGL) and hydraulic grade lines (HGL) for the system shown in Fig. 6.6 if the pipe is horizontal (i.e., the outlet is at the base of the reservoir), and a water turbine (extracting energy) is located at (a) point ②, or (b) at point ③. In Chapter 8 we will investigate the effects of friction on internal flows. Can you anticipate and sketch the effect of friction on the EGL and HGL for cases (a) and (b)?

(a) Note that the effect of friction would be that the EGL would tend to drop: suddenly at the contraction, gradually in the large pipe, more steeply in the small pipe. The HGL would then "hang" below the HGL in a manner similar to that shown.



(b) Note that the effect of friction would be that the EGL would tend to drop: suddenly at the contraction, gradually in the large pipe, more steeply in the small pipe. The HGL would then "hang" below the HGL in a manner similar to that shown.

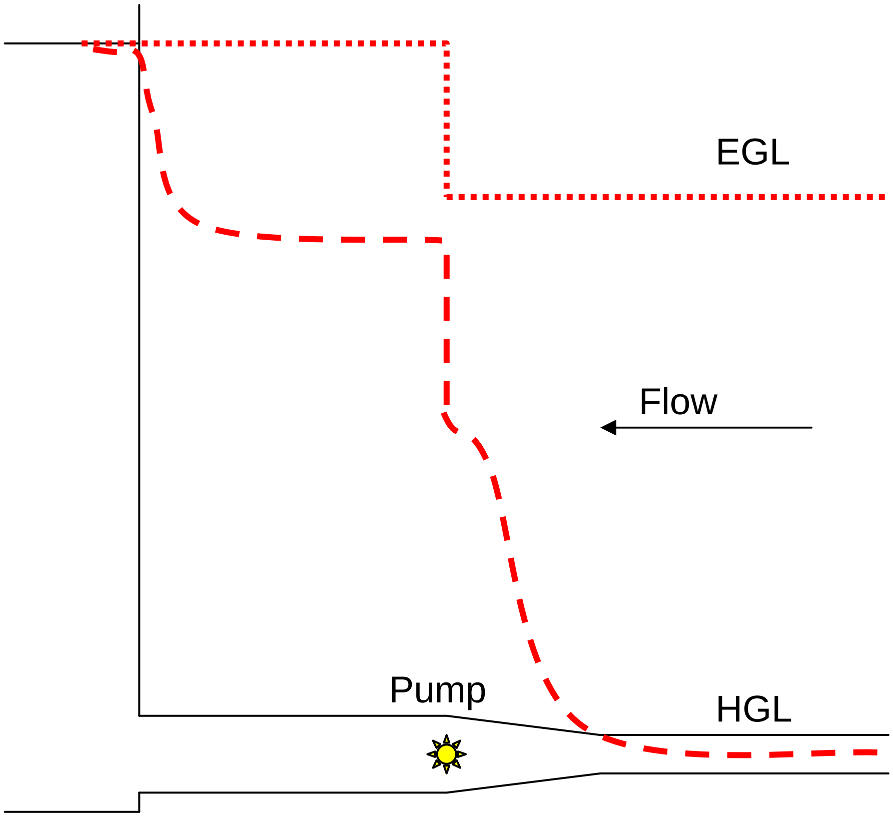


Problem 6.85

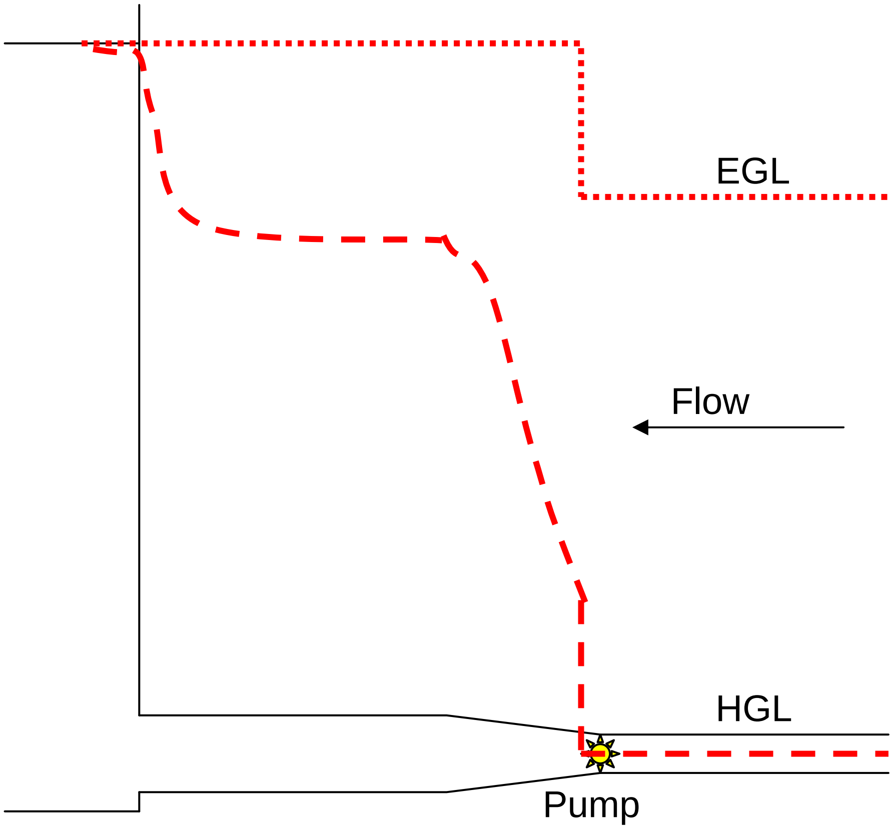
[2]

Carefully sketch the energy grade lines (EGL) and hydraulic grade lines (HGL) for the system shown in Fig. 6.6 if a pump (adding energy to the fluid) is located at (a) point ②, or (b) at point ③, such that flow is into the reservoir. In Chapter 8 we will investigate the effects of friction on internal flows. Can you anticipate and sketch the effect of friction on the EGL and HGL for cases (a) and (b)?

(a) Note that the effect of friction would be that the EGL would tend to drop from right to left: steeply in the small pipe, gradually in the large pipe, and suddenly at the expansion. The HGL would then "hang" below the HGL in a manner similar to that shown.



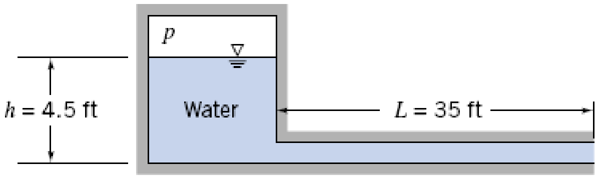
(b) Note that the effect of friction would be that the EGL would tend to drop from right to left: steeply in the small pipe, gradually in the large pipe, and suddenly at the expansion. The HGL would then "hang" below the HGL in a manner similar to that shown.



Problem *6.86

[2]

***6.86** Compressed air is used to accelerate water from a tube. Neglect the velocity in the reservoir and assume the flow in the tube is uniform at any section. At a particular instant, it is known that $V = 6$ ft/s and $dV/dt = 7.5$ ft/s^2. The cross-sectional area of the tube is $A = 32$ in.^{2}. Determine the pressure in the tank at this instant.



Given: Unsteady water flow out of tube

Find: Pressure in the tank

Solution:

Basic equation: Unsteady Bernoulli

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \int_1^2 \frac{\partial V}{\partial t}\,ds$$

Assumptions: 1) Unsteady flow 2) Incompressible 3) No friction 4) Flow along streamline 5) Uniform flow 6) Horizontal flow ($g_x = 0$)

Applying unsteady Bernoulli between reservoir and tube exit

$$\frac{p}{\rho} + g \cdot h = \frac{V^2}{2} + \int_1^2 \frac{\partial}{\partial t} V\,ds = \frac{V^2}{2} + \frac{dV}{dt} \cdot \int_1^2 1\,ds$$

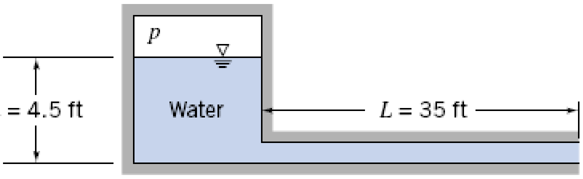
where we work in gage pressure

Hence $$p = \rho \cdot \left(\frac{V^2}{2} - g \cdot h + \frac{dV}{dt} \cdot L \right)$$

Hence $$p = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(\frac{6^2}{2} - 32.2 \times 4.5 + 7.5 \times 35 \right) \cdot \left(\frac{\text{ft}}{\text{s}} \right)^2 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug ft}}$$ $$p = 263 \cdot \frac{\text{lbf}}{\text{ft}^2} \qquad p = 1.83 \cdot \text{psi}$$ (gage)

Problem *6.87

*6.87 If the water in the pipe in Problem 6.86 is initially at rest and the air pressure is 3 psig, what will be the initial acceleration of the water in the pipe?



Given: Unsteady water flow out of tube

Find: Initial acceleration

Solution:

Basic equation: Unsteady Bernoulli

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 + \int_1^2 \frac{\partial V}{\partial t} ds$$

Assumptions: 1) Unsteady flow 2) Incompressible 3) No friction 4) Flow along streamline 5) Uniform flow 6) Horizontal flow ($g_x = 0$)

Applying unsteady Bernoulli between reservoir and tube exit

$$\frac{p}{\rho} + g \cdot h = \int_1^2 \frac{\partial}{\partial t} V \, ds = \frac{dV}{dt} \cdot \int_1^2 1 \, ds = a_x \cdot L$$

where we work in gage pressure

Hence $$a_x = \frac{1}{L} \cdot \left(\frac{p}{\rho} + g \cdot h \right)$$

Hence $$a_x = \frac{1}{35 \cdot \text{ft}} \times \left[3 \cdot \frac{\text{lbf}}{\text{in}^2} \times \left(\frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \right)^2 \times \frac{\text{ft}^3}{1.94 \cdot \text{slug}} \times \frac{\text{slug} \, \text{ft}}{\text{s}^2 \cdot \text{lbf}} + 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times 4.5 \cdot \text{ft} \right]$$

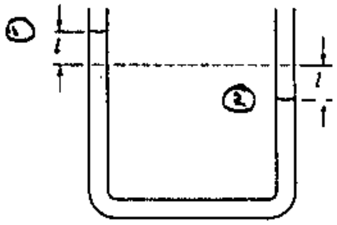
$$a_x = 10.5 \cdot \frac{\text{ft}}{\text{s}^2}$$

Note that we obtain the same result if we treat the water in the pipe as a single body at rest with gage pressure p + ρgh at the left end!

Problem *6.88

[5]______

Given: U-tube manometer of constant area as shown
Manometer fluid is initially deflected and then released



Find: a differential equation for l as a function of time

Solution

Basic equation: $\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \int_1^2 \frac{\partial V_s}{\partial t}\, ds$

Assumptions: (1) incompressible flow
(2) frictionless flow
(3) flow along a streamline

Since $p_1 = p_2 = p_{atm}$ and $V_1^2 = V_2^2$, then

$$g(z_1 - z_2) = \int_1^2 \frac{\partial V_s}{\partial t}\, ds$$

Let L = total length of column
l = deflection

Then $ds = dL$
$V_s = V = \frac{dl}{dt}$

$$\therefore \quad 2gl = \int_1^2 \frac{\partial V}{\partial t}\, dL = \frac{\partial V}{\partial t}\int_1^2 dL = L\frac{\partial V}{\partial t}$$

Since $V = -\frac{dl}{dt}$

$$2gl = L\frac{\partial V}{\partial t} = -L\frac{d^2 l}{dt^2}$$

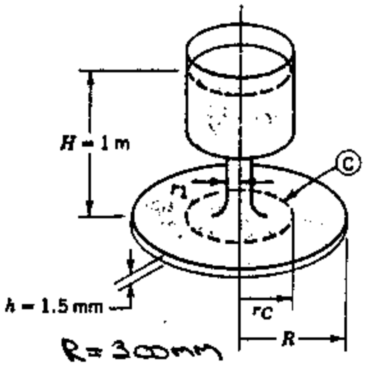
Finally $\frac{d^2 l}{dt^2} + \frac{2g}{L} l = 0$ ←

Problem *6.89

[4]

Given: Flow between parallel disks shown is started from rest at $t=0$. The reservoir level is maintained constant; $r_1 = 50\text{mm}$.

Find: Rate of change of volume flow, dQ/dt, at $t=0$



R=300mm

Solution:

Apply the unsteady Bernoulli equation from the surface to the exit.

$$\cancel{\frac{p_s}{\rho}} + \cancel{\frac{V_s^2}{2}}^{\approx 0} + g z_s = \cancel{\frac{p_e}{\rho}} + \frac{V_e^2}{2} + \cancel{g z_e}^{0} + \int_1^2 \frac{\partial V_s}{\partial t}\, ds$$

$$gH = \frac{V_e^2}{2} + \int_1^2 \frac{\partial V_s}{\partial t}\, ds.$$

Assumptions: (1) frictionless flow
(2) incompressible flow
(3) flow along a streamline.

For uniform flow at any section between the plates, for $r \geq r_1$, the volume flow rate is given by

$$Q = \int \vec{V} \cdot d\vec{A} = V_r 2\pi r h \quad \text{and} \quad V_r = \frac{Q}{2\pi r h}$$

At the exit $V_e = Q/2\pi R h$

Assume that the rate of change of fluid velocity in the reservoir (out to $r = r_1$) is negligible. Then

$$\int_1^2 \frac{\partial V_s}{\partial t}\, ds = \frac{\partial}{\partial t}\int_1^2 V_r\, dr = \frac{\partial}{\partial t}\int_1^2 \frac{Q}{2\pi h}\frac{dr}{r} = \frac{\ln R/r_1}{2\pi h}\frac{dQ}{dt}$$

Then substituting into the unsteady Bernoulli equation, we obtain

$$gH = \frac{Q^2}{8\pi^2 R^2 h^2} + \frac{\ln R/r_1}{2\pi h}\frac{dQ}{dt}$$

At $t=0$, $Q=0$ and

$$\frac{dQ}{dt} = \frac{2\pi h g H}{\ln R/r_1}$$

$$= 2\pi \times 0.0015\,\text{m} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 1\,\text{m} \times \frac{1}{\ln \frac{300}{50}}$$

$$\frac{dQ}{dt} = 0.0516\ \text{m}^3/\text{s}/\text{s} \longleftarrow \left.\frac{dQ}{dt}\right)_{t=0}$$

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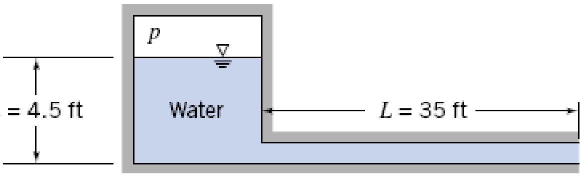
Problem *6.90

[4]

*6.90 If the water in the pipe of Problem 6.86 is initially at rest, and the air pressure is maintained at 1.5 psig, derive a differential equation for the velocity V in the pipe as a function of time, integrate, and plot V versus t for $t = 0$ to 5 s.



Given: Unsteady water flow out of tube

Find: Differential equation for velocity; Integrate; Plot v versus time

Solution:

Basic equation: Unsteady Bernoulli $\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 + \int_1^2 \frac{\partial V}{\partial t} ds$

Assumptions: 1) Unsteady flow 2) Incompressible 3) No friction 4) Flow along streamline 5) Uniform flow 6) Horizontal flow ($g_x = 0$)

Applying unsteady Bernoulli between reservoir and tube exit

$$\frac{p}{\rho} + g \cdot h = \frac{V^2}{2} + \int_1^2 \frac{\partial}{\partial t} V\, ds = \frac{V^2}{2} + \frac{dV}{dt} \cdot \int_1^2 1\, ds = \frac{V^2}{2} + \frac{dV}{dt} \cdot L$$

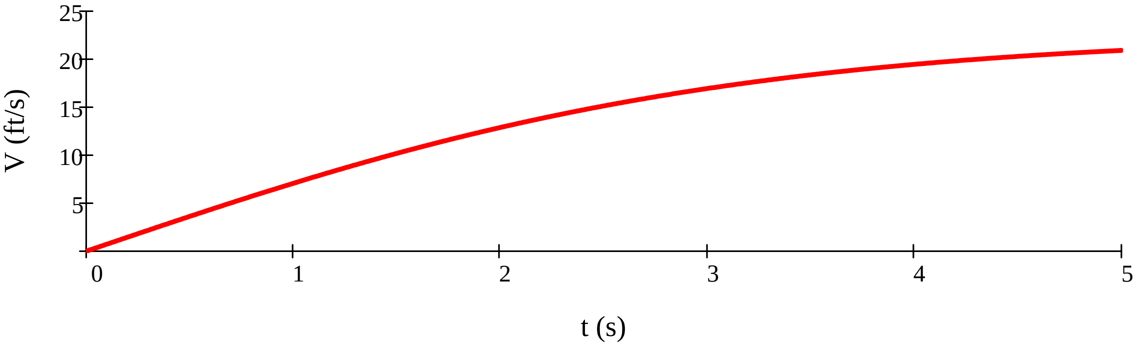
where we work in gage pressure

Hence $$\frac{dV}{dt} + \frac{V^2}{2 \cdot L} = \frac{1}{L} \cdot \left(\frac{p}{\rho} + g \cdot h\right)$$ is the differential equation for the flow

Separating variables $$\frac{L \cdot dV}{\frac{p}{\rho} + g \cdot h - \frac{V^2}{2}} = dt$$

Integrating and using limits V(0) = 0 and V(t) = V

$$V(t) = \sqrt{2 \cdot \left(\frac{p}{\rho} + g \cdot h\right)} \cdot \tanh\left(\sqrt{\frac{\frac{p}{\rho} + g \cdot h}{2 \cdot L^2}} \cdot t\right)$$



This graph is suitable for plotting in *Excel*

For large times $$V = \sqrt{2 \cdot \left(\frac{p}{\rho} + g \cdot h\right)} \qquad V = 22.6 \frac{ft}{s}$$

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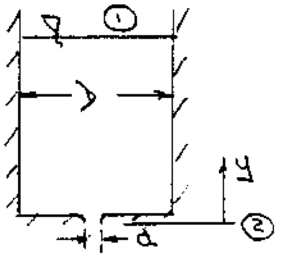
Given: A cylindrical tank of diameter, $D = 50\,mm$, drains through an opening, $d = 5\,mm$, in the body of the tank. If the flow is assumed to be quasi-steady, the speed of the liquid leaving the tank may be approximated by $V = \sqrt{2gy}$, where y is the height from the tank bottom to the free surface.

Find: Using the Bernoulli equation for unsteady flow along a streamline, evaluate the minimum diameter ratio, D/d, required to justify the assumption that flow from the tank is quasi-steady.

Solution:

For incompressible, frictionless flow along a streamline, the unsteady Bernoulli equation is



$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gy_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gy_2 + \int_1^2 \frac{\partial V_s}{\partial t}\,dy$$

$p_1 = p_2 = p_{atm}$, $y_2 = 0$

From continuity $V_1 A_1 = V_2 A_2 = V_j A_j$

$$\therefore \quad \frac{1}{2} V_j^2 \left(\frac{A_j}{A_1}\right) + gy_1 = \frac{1}{2} V_j^2 + \int_1^2 \frac{\partial V_s}{\partial t}\,dy$$

or,
$$gy_1 = \frac{1}{2} V_j^2 \left[1 - \left(\frac{A_j}{A_1}\right)^2\right] + \int_1^2 \frac{\partial V_s}{\partial t}\,dy$$

If we assume quasi-steady flow, we say that $\int_1^2 \frac{\partial V_s}{\partial t}\,dy$ is negligible and hence $\frac{2gy}{V_j^2[1 - AR^2]} = 1$ where $AR = \frac{A_j}{A_1}$

Now, $\int_1^2 \frac{\partial V_s}{\partial t} \approx y \frac{dV_s}{dt} = y \frac{dV_1}{dt} = y \frac{d}{dt}\left(V_j \frac{A_j}{A_1}\right) = y \frac{A_j}{A_1} \frac{dV_j}{dt}$

Thus for the assumption to be reasonable we must have

$$\left| y \frac{A_j}{A_1} \frac{dV_j}{dt} \right| \ll gy \qquad \text{or} \qquad \left| \frac{A_j}{A_1} \frac{dV_j}{dt} \right| \ll g$$

Under the assumption of quasi-steady flow

$$V_j = \left[2gy \frac{1}{(1 - AR^2)}\right]^{1/2} \quad \text{where } AR = A_j/A_1$$

then,
$$\frac{dV_j}{dt} = \sqrt{\frac{2g}{(1 - AR^2)}} \; \frac{1}{2\sqrt{y}} \; \frac{dy}{dt} = \frac{dy}{dt} \sqrt{\frac{g}{2y(1 - AR^2)}}$$

Since
$$\frac{dy}{dt} = -V_1 = -V_j \frac{A_j}{A_1} \quad \text{, then}$$

$$\frac{dV_j}{dt} = -V_j \frac{A_j}{A_1} \sqrt{\frac{g}{2y(1 - AR^2)}} = -\frac{A_j}{A_1} \sqrt{\frac{V_j^2 (1 - AR^2)}{2gy}} \; \frac{g}{(1 - AR^2)}$$

and
$$\frac{dV_j}{dt} = -\frac{A_j}{A_1} \frac{g}{(1 - AR^2)}$$

For $\left|\frac{A_j}{A_1}\frac{dV_j}{dt}\right| << g$, then $\left(\frac{A_j}{A_1}\right)^2\frac{1}{(1-AR^2)} << 1$

If we take $\left(\frac{A_j}{A_1}\right)^2\frac{1}{(1-AR^2)} \approx 0.01$

then,

$$\left(\frac{A_j}{A_1}\right)^2 = 0.01(1-AR^2) = 0.01\left[1-\left(\frac{A_j}{A_1}\right)^2\right]$$

and

$$1.01\left(\frac{A_j}{A_1}\right)^2 = 0.01$$

$$\frac{A_j}{A_1} = 0.0995$$

or

$$\frac{D_j}{D_1} = \left(\frac{A_j}{A_1}\right)^{1/2} = 0.32 \longleftarrow$$

In problem 4.44, $D_j/D_1 = d/D = 0.1$ and hence the assumption of quasi-steady flow is valid.

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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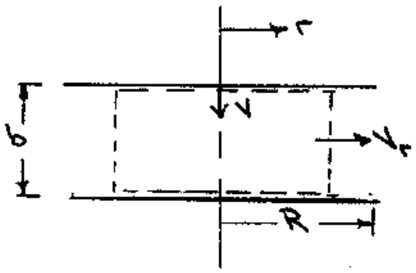
Problem *6.92

[5]

Given: Two circular discs of radius, R, are separated by a distance, b.
Upper disc moves toward the lower one at speed, V.
Fluid between discs is incompressible and is squeezed out radially.
Assume frictionless flow and uniform radial flow and any radial section.
Pressure surrounding disc is at p_{atm}



Find: gage pressure at $r=0$

Solution:

Basic equation:
$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2+\int_1^2\frac{\partial V_s}{\partial t}\,ds$$
$$0=\frac{\partial}{\partial t}\int_{CV}\rho\,d\forall+\int_{CS}\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) incompressible flow
(2) frictionless flow
(3) flow along a streamline
(4) uniform radial flow at any r
(5) neglect elevation changes.

Then,
$$0=\frac{\partial}{\partial t}\int_{CV}\rho\,d\forall+\int_{CS}\rho\vec{V}\cdot d\vec{A}=\frac{\partial}{\partial t}(\rho\pi r^2 b)+\rho V_r 2\pi r b$$
$$=\rho\pi r^2\frac{\partial b}{\partial t}+\rho V_r 2\pi r b. \quad \text{But } \frac{\partial b}{\partial t}=-V$$
$$\therefore 0=-\rho\pi r^2 V+\rho V_r 2\pi r b \quad \text{and } V_r=V\frac{r}{2b}$$

Applying the Bernoulli equation between point ① ($r=r$) and point ② ($r=R$)
$$p_1-p_2=\frac{\rho}{2}\left[V_2^2-V_1^2\right]+\int_r^R\rho\frac{\partial V_r}{\partial t}\,dr \quad \text{Now, } \frac{\partial V_r}{\partial t}=\frac{\partial}{\partial t}\left(V\frac{r}{2b}\right)=\frac{rV}{2}\left(-\frac{1}{b^2}\frac{db}{dt}\right)=\frac{V^2 r}{2b^2}$$
$$=\frac{\rho}{2}\left[\left(\frac{VR}{2b}\right)^2-\left(\frac{Vr}{2b}\right)^2\right]+\int_r^R\rho\frac{V^2 r}{2b^2}\,dr$$
$$=\frac{\rho V^2}{8b^2}\left[R^2-r^2\right]+\frac{\rho V^2}{4b^2}r^2\Big]_r^R=\frac{\rho V^2}{8b^2}\left[R^2-r^2\right]+\frac{\rho V^2}{4b^2}\left[R^2-r^2\right]$$
$$p_1-p_{atm}=\frac{3}{8}\frac{\rho V^2}{b^2}\left[R^2-r^2\right]=\frac{3}{8}\rho\frac{V^2R^2}{b^2}\left[1-\left(\frac{r}{R}\right)^2\right]$$

When $r=0$ $\quad p_1=p_0$
$$\therefore p_0-p_{atm}=\frac{3}{8}\rho\frac{V^2R^2}{b^2}\longleftarrow$$

Problem *6.93

[2]

Given: Two vortex flows with velocity fields

$$\vec{V}_1 = \omega r \hat{e}_\theta \qquad \vec{V}_2 = \frac{K}{2\pi r}\hat{e}_\theta$$

Determine: if the Bernoulli equation can be applied between different radii for each flow.

Solution: Since $V_r = 0$, the streamlines are concentric circles.

In order for it to be possible to apply the Bernoulli equation between different radii, it is necessary that the flow be irrotational.

Basic equation: $\vec{\omega} = \frac{1}{2}\nabla\times\vec{V}$

Flow (1)

$$\nabla\times\vec{V}_1 = \left(\hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z}\right)\times\omega r\hat{e}_\theta$$

$$= \hat{e}_r\times\hat{e}_\theta\frac{\partial}{\partial r}(\omega r) + \hat{e}_r\times\omega r\cancelto{0}{\frac{\partial\hat{e}_\theta}{\partial r}} + \cancelto{0}{\hat{e}_\theta\times\hat{e}_\theta}\frac{1}{r}\frac{\partial(\omega r)}{\partial\theta} + \hat{e}_\theta\times\frac{\omega r}{r}\frac{\partial\hat{e}_\theta}{\partial\theta}$$

$$= \hat{k}\omega + \hat{e}_\theta\times\omega(-\hat{e}_r)$$

$$\nabla\times\vec{V}_1 = 2\omega\hat{k}$$

$\therefore$ Flow (1) is rotational and Bernoulli equation cannot be applied between different radii.

Flow (2)

$$\nabla\times\vec{V}_2 = \left(\hat{e}_r\frac{\partial}{\partial r} + \hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{k}\frac{\partial}{\partial z}\right)\times\frac{K}{2\pi r}\hat{e}_\theta$$

$$= \hat{e}_r\times\hat{e}_\theta\frac{\partial}{\partial r}\left(\frac{K}{2\pi r}\right) + \hat{e}_r\times\left(\frac{K}{2\pi r}\right)\cancelto{0}{\frac{\partial\hat{e}_\theta}{\partial r}} + \cancelto{0}{\hat{e}_\theta\times\hat{e}_\theta}\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{K}{2\pi r}\right) + \hat{e}_\theta\times\frac{1}{r}\left(\frac{K}{2\pi r}\right)\frac{\partial\hat{e}_\theta}{\partial\theta}$$

$$= -\hat{k}\frac{K}{2\pi r^2} + \hat{e}_\theta\frac{K}{2\pi r^2}\times(-\hat{e}_r)$$

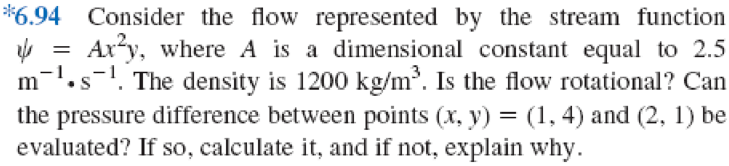
$$= -\hat{k}\frac{K}{2\pi r^2} + \hat{k}\frac{K}{2\pi r^2}$$

$$\nabla\times\vec{V}_2 = 0$$

Since the flow field is irrotational, Bernoulli equation can be applied between different radii if the flow is also incompressible and frictionless.

Problem *6.94

[2]



*6.94 Consider the flow represented by the stream function $\psi = Ax^2y$, where A is a dimensional constant equal to 2.5 $\mathrm{m^{-1}\cdot s^{-1}}$. The density is 1200 kg/m^3. Is the flow rotational? Can the pressure difference between points $(x, y) = (1, 4)$ and $(2, 1)$ be evaluated? If so, calculate it, and if not, explain why.

Given: Stream function

Find: If the flow is irrotational; Pressure difference between points (1,4) and (2,1)

Solution:

Basic equations: Incompressibility because ψ exists $\quad u = \frac{\partial}{\partial y}\psi \quad v = -\frac{\partial}{\partial x}\psi \quad$ Irrotationality $\quad \frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

$$\psi(x,y) = A\cdot x^2\cdot y$$

$$u(x,y) = \frac{\partial}{\partial y}\psi(x,y) = \frac{\partial}{\partial y}\left(A\cdot x^2\cdot y\right) \qquad u(x,y) = A\cdot x^2$$

$$v(x,y) = -\frac{\partial}{\partial x}\psi(x,y) = -\frac{\partial}{\partial x}\left(A\cdot x^2\cdot y\right) \qquad v(x,y) = -2\cdot A\cdot x\cdot y$$

Hence $\quad \frac{\partial}{\partial x}v(x,y) - \frac{\partial}{\partial y}u(x,y) \rightarrow -2\cdot A\cdot y \qquad \frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u \neq 0 \quad$ so flow is NOT IRROTATIONAL

Since flow is rotational, we must be on same streamline to be able to use Bernoulli

At point (1,4) $\quad \psi(1,4) = 4A \qquad$ and at point (2,1) $\quad \psi(2,1) = 4A$

Hence these points are on same streamline so Bernoulli can be used. The velocity at a point is $\quad V(x,y) = \sqrt{u(x,y)^2 + v(x,y)^2}$

Hence at (1,4) $\quad V_1 = \sqrt{\left[\frac{2.5}{\mathrm{m\cdot s}}\times(1\cdot\mathrm{m})^2\right]^2 + \left(-2\times\frac{2.5}{\mathrm{m\cdot s}}\times 1\cdot\mathrm{m}\times 4\cdot\mathrm{m}\right)^2} \qquad V_1 = 20.2\,\frac{\mathrm{m}}{\mathrm{s}}$

Hence at (2,1) $\quad V_2 = \sqrt{\left[\frac{2.5}{\mathrm{m\cdot s}}\times(2\cdot\mathrm{m})^2\right]^2 + \left(-2\times\frac{2.5}{\mathrm{m\cdot s}}\times 2\cdot\mathrm{m}\times 1\cdot\mathrm{m}\right)^2} \qquad V_2 = 14.1\,\frac{\mathrm{m}}{\mathrm{s}}$

Using Bernoulli $\quad \frac{p_1}{\rho} + \frac{1}{2}\cdot V_1^2 = \frac{p_2}{\rho} + \frac{1}{2}\cdot V_2^2 \qquad \Delta p = \frac{\rho}{2}\cdot\left(V_2^2 - V_1^2\right)$

$$\Delta p = \frac{1}{2}\times 1200\cdot\frac{\mathrm{kg}}{\mathrm{m}^3}\times\left(14.1^2 - 20.2^2\right)\cdot\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2\times\frac{\mathrm{N\cdot s^2}}{\mathrm{kg\cdot m}} \qquad \Delta p = -126\cdot\mathrm{kPa}$$

Problem *6.95

[2]

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Given: Two-dimensional flow represented by the velocity field $\vec{V} = (Ax - By)t\hat{i} - (Bx + Ay)t\hat{j}$, where $A = 1\,s^{-2}$, $B = 2\,s^{-2}$, t is in s, and coordinates are in meters.

Find: (a) Is this a possible incompressible flow?
(b) Is the flow steady or unsteady?
(c) Show that the flow is irrotational
(d) Derive an expression for the velocity potential

Solution: For incompressible flow, $\nabla \cdot \vec{V} = 0$

For given flow $\nabla \cdot \vec{V} = \frac{\partial}{\partial x}(Ax - By)t - \frac{\partial}{\partial y}(Bx + Ay)t = At - At = 0$

∴ velocity field represents a possible incompressible flow

The flow is unsteady since $\vec{V} = \vec{V}(x, y, t)$

The rotation is given by $\vec{\omega} = \frac{1}{2}\nabla \times \vec{V} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$

$$\vec{\omega} = \frac{1}{2}\left[\frac{\partial}{\partial x} - (Bx + Ay)t - \frac{\partial}{\partial y}(Ax - By)t\right] = -Bt + Bt = 0$$

$\vec{\omega} = 0$, so flow is irrotational

From the definition of the velocity potential, $\vec{V} = -\nabla\phi$

$u = -\frac{\partial\phi}{\partial x}$ and $\phi = \int -u\,dx + f(y,t) = \int -(Ax - By)t\,dx + f(y,t)$

$$\phi = \left(-A\frac{x^2}{2} + Bxy\right)t + f(y,t)$$

$v = -\frac{\partial\phi}{\partial y}$ and $\phi = \int -v\,dy + g(x,t) = \int (Bx + Ay)t\,dy + g(x,t)$

$$\phi = \left(Bxy + A\frac{y^2}{2}\right)t + g(x,t)$$

Comparing the two expressions for ϕ we conclude

$$f(y,t) = \frac{A}{2}y^2 t \quad \text{and} \quad g(x,t) = -\frac{A}{2}x^2 t$$

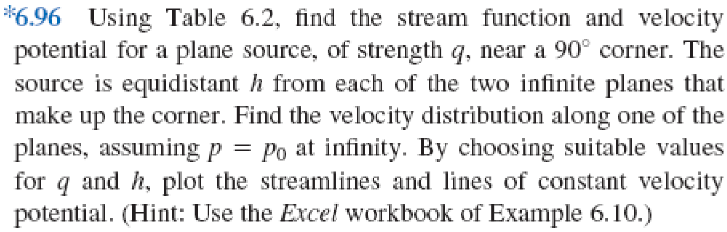
Hence,

$$\phi = \left\{\frac{A}{2}(y^2 - x^2) + Bxy\right\}t$$

ϕ

Problem *6.96

[3]



*6.96 Using Table 6.2, find the stream function and velocity potential for a plane source, of strength q, near a 90° corner. The source is equidistant h from each of the two infinite planes that make up the corner. Find the velocity distribution along one of the planes, assuming $p = p_0$ at infinity. By choosing suitable values for q and h, plot the streamlines and lines of constant velocity potential. (Hint: Use the *Excel* workbook of Example 6.10.)

Given: Data from Table 6.2

Find: Stream function and velocity potential for a source in a corner; plot; velocity along one plane

Solution:

From Table 6.2, for a source at the origin $\psi(r,\theta) = \frac{q}{2\cdot\pi}\cdot\theta$ $\phi(r,\theta) = -\frac{q}{2\cdot\pi}\cdot\ln(r)$

Expressed in Cartesian coordinates $\psi(x,y) = \frac{q}{2\cdot\pi}\cdot\operatorname{atan}\left(\frac{y}{x}\right)$ $\phi(x,y) = -\frac{q}{4\cdot\pi}\cdot\ln\left(x^2 + y^2\right)$

To build flow in a corner, we need image sources at three locations so that there is symmetry about both axes. We need sources at (*h*,*h*), (*h*,- *h*), (- *h*,*h*), and (- *h*,- *h*)

Hence the composite stream function and velocity potential are

$$\psi(x,y) = \frac{q}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x-h}\right) + \operatorname{atan}\left(\frac{y+h}{x-h}\right) + \operatorname{atan}\left(\frac{y+h}{x+h}\right) + \operatorname{atan}\left(\frac{y-h}{x+h}\right)\right)$$

$$\phi(x,y) = -\frac{q}{4\cdot\pi}\cdot\ln\left[\left[(x-h)^2 + (y-h)^2\right]\cdot\left[(x-h)^2 + (y+h)^2\right]\right] - \frac{q}{4\cdot\pi}\cdot\left[(x+h)^2 + (y+h)^2\right]\cdot\left[(x+h)^2 + (y-h)^2\right]$$

By a similar reasoning the horizontal velocity is given by

$$u = \frac{q\cdot(x-h)}{2\cdot\pi\left[(x-h)^2 + (y-h)^2\right]} + \frac{q\cdot(x-h)}{2\cdot\pi\left[(x-h)^2 + (y+h)^2\right]} + \frac{q\cdot(x+h)}{2\cdot\pi\left[(x+h)^2 + (y+h)^2\right]} + \frac{q\cdot(x+h)}{2\cdot\pi\left[(x+h)^2 + (y+h)^2\right]}$$

Along the horizontal wall ($y = 0$)

$$u = \frac{q\cdot(x-h)}{2\cdot\pi\left[(x-h)^2 + h^2\right]} + \frac{q\cdot(x-h)}{2\cdot\pi\left[(x-h)^2 + h^2\right]} + \frac{q\cdot(x+h)}{2\cdot\pi\left[(x+h)^2 + h^2\right]} + \frac{q\cdot(x+h)}{2\cdot\pi\left[(x+h)^2 + h^2\right]}$$

or $$u(x) = \frac{q}{\pi}\cdot\left[\frac{x-h}{(x-h)^2 + h^2} + \frac{x+h}{(x+h)^2 + h^2}\right]$$

Problem *6.96

[3]

*6.96 Using Table 6.2, find the stream function and velocity potential for a plane source, of strength q, near a 90° corner. The source is equidistant h from each of the two infinite planes that make up the corner. Find the velocity distribution along one of the planes, assuming $p = p_0$ at infinity. By choosing suitable values for q and h, plot the streamlines and lines of constant velocity potential. (Hint: Use the *Excel* workbook of Example 6.10.)

Given: Data from Table 6.2

Find: Stream function and velocity potential for a source in a corner; plot; velocity along one plane

Solution:

$$\psi(x,y) = \frac{q}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x-h}\right) + \operatorname{atan}\left(\frac{y+h}{x-h}\right) + \operatorname{atan}\left(\frac{y+h}{x+h}\right) + \operatorname{atan}\left(\frac{y-h}{x+h}\right)\right)$$

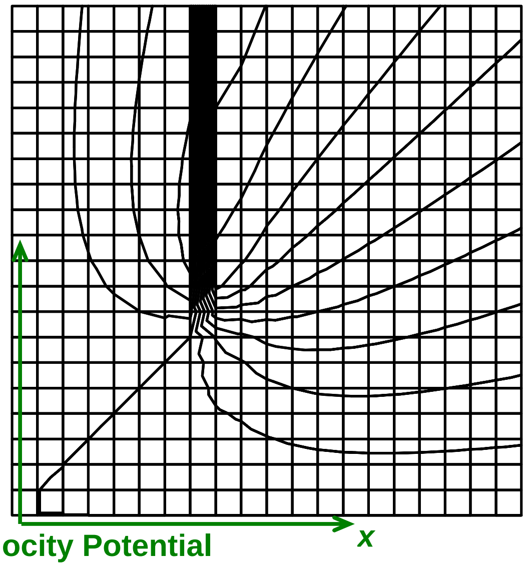
$$\phi(x,y) = -\frac{q}{4\cdot\pi}\cdot\ln\left[\left[(x-h)^2+(y-h)^2\right]\cdot\left[(x-h)^2+(y+h)^2\right]\right] - \frac{q}{4\cdot\pi}\cdot\left[(x+h)^2+(y+h)^2\right]\cdot\left[(x+h)^2+(y-h)^2\right]$$

#NAME?	**Stream Function**

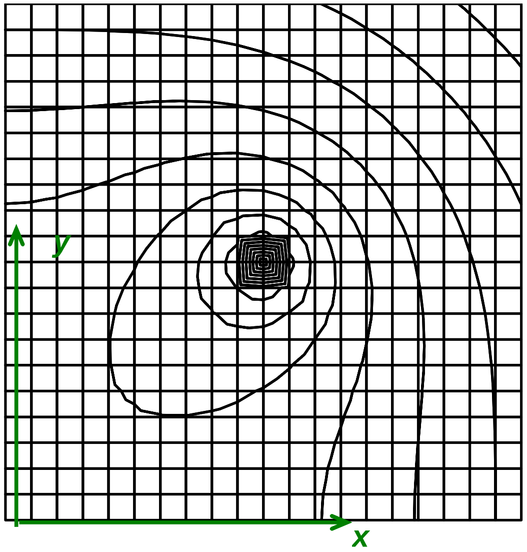
#NAME?	**Velocity Potential**

Note that the plot is
from x = 0 to 5 and y = 0 to 5

Stream Function



Velocity Potential



Problem *6.97

***6.97** The flow field for a plane source at a distance h above an infinite wall aligned along the x axis is given by

$$\vec{V} = \frac{q}{2\pi[x^2 + (y-h)^2]}[x\hat{i} + (y-h)\hat{j}] + \frac{q}{2\pi[x^2 + (y+h)^2]}[x\hat{i} + (y+h)\hat{j}]$$

where q is the strength of the source. The flow is irrotational and incompressible. Derive the stream function and velocity potential. By choosing suitable values for q and h, plot the streamlines and lines of constant velocity potential. (Hint: Use the *Excel* workbook of Example 6.10.)

Given: Velocity field of irrotational and incompressible flow

Find: Stream function and velocity potential; plot

Solution:

The velocity field is
$$u = \frac{q\cdot x}{2\cdot\pi\left[x^2 + (y-h)^2\right]} + \frac{q\cdot x}{2\cdot\pi\left[x^2 + (y+h)^2\right]} \qquad v = \frac{q\cdot(y-h)}{2\cdot\pi\left[x^2 + (y-h)^2\right]} + \frac{q\cdot(y+h)}{2\cdot\pi\left[x^2 + (y+h)^2\right]}$$

The governing equations are
$$u = \frac{\partial}{\partial y}\psi \qquad v = -\frac{\partial}{\partial x}\psi \qquad u = -\frac{\partial}{\partial x}\phi \qquad v = -\frac{\partial}{\partial y}\phi$$

Hence for the stream function
$$\psi = \int u(x,y)\,dy = \frac{q}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x}\right) + \operatorname{atan}\left(\frac{y+h}{x}\right)\right) + f(x)$$

$$\psi = -\int v(x,y)\,dx = \frac{q}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x}\right) + \operatorname{atan}\left(\frac{y+h}{x}\right)\right) + g(y)$$

The simplest expression for ψ is
$$\psi(x,y) = \frac{q}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x}\right) + \operatorname{atan}\left(\frac{y+h}{x}\right)\right)$$

For the stream function
$$\phi = -\int u(x,y)\,dx = -\frac{q}{4\cdot\pi}\cdot\ln\left[\left[x^2 + (y-h)^2\right]\cdot\left[x^2 + (y+h)^2\right]\right] + f(y)$$

$$\phi = -\int v(x,y)\,dy = -\frac{q}{4\cdot\pi}\cdot\ln\left[\left[x^2 + (y-h)^2\right]\cdot\left[x^2 + (y+h)^2\right]\right] + g(x)$$

The simplest expression for φ is
$$\phi(x,y) = -\frac{q}{4\cdot\pi}\cdot\ln\left[\left[x^2 + (y-h)^2\right]\cdot\left[x^2 + (y+h)^2\right]\right]$$

Problem *6.97

[3]

***6.97** The flow field for a plane source at a distance h above an infinite wall aligned along the x axis is given by

$$\vec{V} = \frac{q}{2\pi[x^2 + (y-h)^2]}[x\hat{i} + (y-h)\hat{j}] + \frac{q}{2\pi[x^2 + (y+h)^2]}[x\hat{i} + (y+h)\hat{j}]$$

where q is the strength of the source. The flow is irrotational and incompressible. Derive the stream function and velocity potential. By choosing suitable values for q and h, plot the streamlines and lines of constant velocity potential. (Hint: Use the *Excel* workbook of Example 6.10.)

Given: Velocity field of irrotational and incompressible flow

Find: Stream function and velocity potential; plot

Solution:

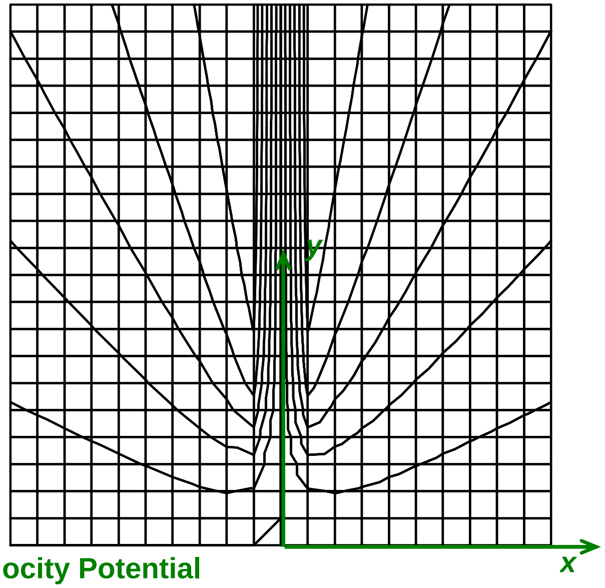
$$\psi(x,y) = \frac{q}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x}\right) + \operatorname{atan}\left(\frac{y+h}{x}\right)\right)$$

$$\phi(x,y) = -\frac{q}{4\cdot\pi}\cdot\ln\left[\left[x^2 + (y-h)^2\right]\cdot\left[x^2 + (y+h)^2\right]\right]$$

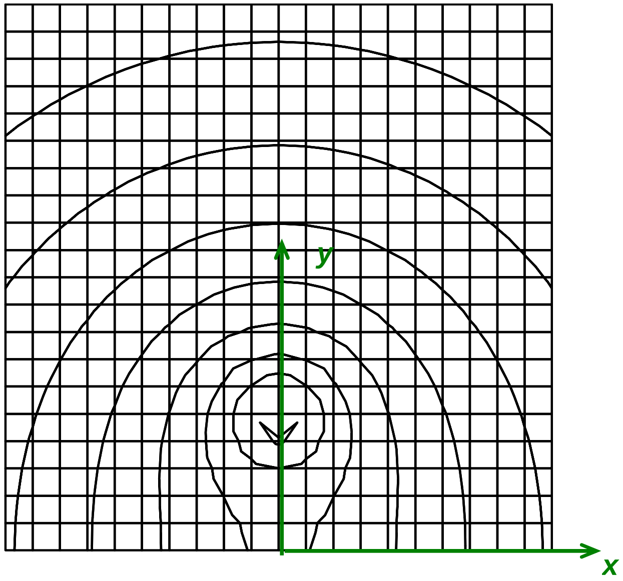
#NAME?	**Stream Function**
#NAME?	**Velocity Potential**

Note that the plot is
from x = -2.5 to 2.5 and y = 0 to 5

Stream Function

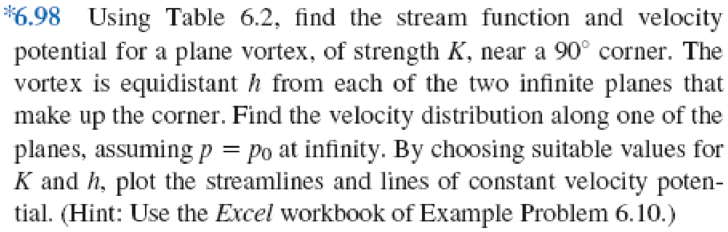


Velocity Potential



Problem *6.98

[3]



*6.98 Using Table 6.2, find the stream function and velocity potential for a plane vortex, of strength K, near a 90° corner. The vortex is equidistant h from each of the two infinite planes that make up the corner. Find the velocity distribution along one of the planes, assuming $p = p_0$ at infinity. By choosing suitable values for K and h, plot the streamlines and lines of constant velocity potential. (Hint: Use the *Excel* workbook of Example Problem 6.10.)

Given: Data from Table 6.2

Find: Stream function and velocity potential for a vortex in a corner; plot; velocity along one plane

Solution:

From Table 6.2, for a vortex at the origin $\phi(r,\theta) = \frac{K}{2\cdot\pi}\cdot\theta$ $\psi(r,\theta) = -\frac{K}{2\cdot\pi}\cdot\ln(r)$

Expressed in Cartesian coordinates $\phi(x,y) = \frac{q}{2\cdot\pi}\cdot\operatorname{atan}\left(\frac{y}{x}\right)$ $\psi(x,y) = -\frac{q}{4\cdot\pi}\cdot\ln\left(x^2+y^2\right)$

To build flow in a corner, we need image vortices at three locations so that there is symmetry about both axes. We need vortices at (h,h), (h,- h), (- h,h), and (- h,- h). Note that some of them must have strengths of - K!

Hence the composite velocity potential and stream function are

$$\phi(x,y) = \frac{K}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x-h}\right) - \operatorname{atan}\left(\frac{y+h}{x-h}\right) + \operatorname{atan}\left(\frac{y+h}{x+h}\right) - \operatorname{atan}\left(\frac{y-h}{x+h}\right)\right)$$

$$\psi(x,y) = -\frac{K}{4\cdot\pi}\cdot\ln\left[\frac{(x-h)^2+(y-h)^2}{(x-h)^2+(y+h)^2}\cdot\frac{(x+h)^2+(y+h)^2}{(x+h)^2+(y-h)^2}\right]$$

By a similar reasoning the horizontal velocity is given by

$$u = -\frac{K\cdot(y-h)}{2\cdot\pi\left[(x-h)^2+(y-h)^2\right]} + \frac{K\cdot(y+h)}{2\cdot\pi\left[(x-h)^2+(y+h)^2\right]} - \frac{K\cdot(y+h)}{2\cdot\pi\left[(x+h)^2+(y+h)^2\right]} + \frac{K\cdot(y-h)}{2\cdot\pi\left[(x+h)^2+(y-h)^2\right]}$$

Along the horizontal wall (y = 0)

$$u = \frac{K\cdot h}{2\cdot\pi\left[(x-h)^2+h^2\right]} + \frac{K\cdot h}{2\cdot\pi\left[(x-h)^2+h^2\right]} - \frac{K\cdot h}{2\cdot\pi\left[(x+h)^2+h^2\right]} - \frac{K\cdot h}{2\cdot\pi\left[(x+h)^2+h^2\right]}$$

or $$u(x) = \frac{K\cdot h}{\pi}\cdot\left[\frac{1}{(x-h)^2+h^2} - \frac{1}{(x+h)^2+h^2}\right]$$

Problem *6.98

[3]

***6.98** Using Table 6.2, find the stream function and velocity potential for a plane vortex, of strength K, near a 90° corner. The vortex is equidistant h from each of the two infinite planes that make up the corner. Find the velocity distribution along one of the planes, assuming $p = p_0$ at infinity. By choosing suitable values for K and h, plot the streamlines and lines of constant velocity potential. (Hint: Use the *Excel* workbook of Example Problem 6.10.)

Given: Data from Table 6.2

Find: Stream function and velocity potential for a vortex in a corner; plot; velocity along one plane

Solution:

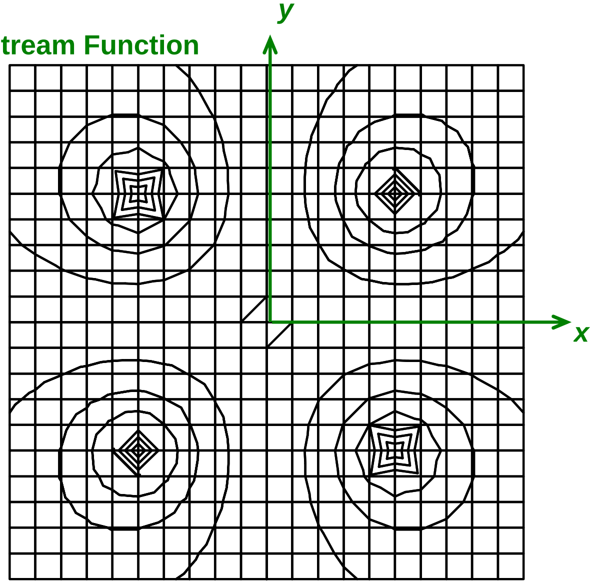
$$\phi(x,y) = \frac{K}{2\cdot\pi}\cdot\left(\operatorname{atan}\left(\frac{y-h}{x-h}\right) - \operatorname{atan}\left(\frac{y+h}{x-h}\right) + \operatorname{atan}\left(\frac{y+h}{x+h}\right) - \operatorname{atan}\left(\frac{y-h}{x+h}\right)\right)$$

$$\psi(x,y) = -\frac{K}{4\cdot\pi}\cdot\ln\left[\frac{(x-h)^2+(y-h)^2}{(x-h)^2+(y+h)^2}\cdot\frac{(x+h)^2+(y+h)^2}{(x+h)^2+(y-h)^2}\right]$$

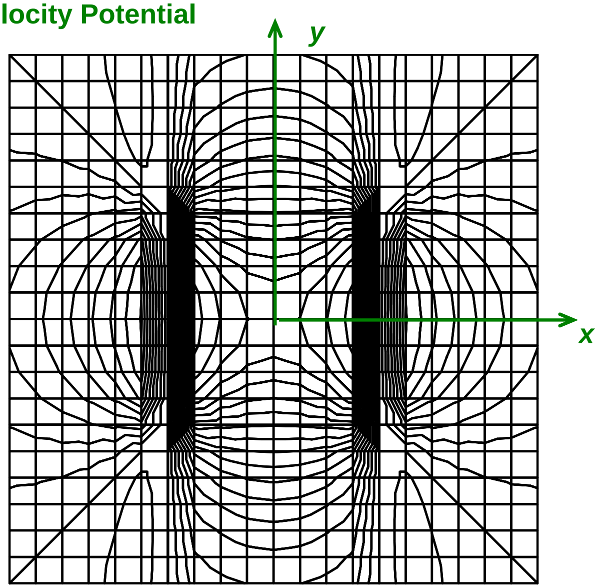
#NAME?	**Stream Function**
	#NAME?
#NAME?	**Velocity Potential**

Note that the plot is
from x = -5 to 5 and y = -5 to 5

Stream Function



Velocity Potential



Problem *6.99 [NOTE: Typographical Error - Wrong Function!]

[2]

Given: Flow field represented by $\psi = Ax^2y - By^3$, where $A = 1\ m^{-1}\cdot s^{-1}$, $B = \frac{1}{3}\ m^{-1}\cdot s^{-1}$, and coordinates are in meters.

Find: an expression for the velocity potential, ϕ

Solution:

The velocity field is determined from the stream function

$$\left.\begin{aligned} u &= \partial\psi/\partial y = Ax^2 - 3By^2 \\ v &= -\partial\psi/\partial x = -2Axy \end{aligned}\right\} \therefore \vec{V} = (Ax^2 - 3By^2)\hat{i} - 2Axy\hat{j}$$

The rotation is given by $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$$\omega_z = \frac{1}{2}(-2Ay + 6By) = \frac{1}{2}\left(-2\times 1\,y + 6\times\frac{1}{3}y\right) = 0$$

Since $\omega_z = 0$, the flow is irrotational and $\vec{V} = -\nabla\phi$

Then

$$u = -\frac{\partial\phi}{\partial x} \quad \text{and} \quad \phi = \int -u\,dx + f(y) = \int(-Ax^2 + 3By^2)\,dx + f(y)$$

$$\phi = -\frac{A}{3}x^3 + 3Bxy^2 + f(y)$$

$$v = -\frac{\partial\phi}{\partial y} \quad \text{and} \quad \phi = \int -v\,dy + g(x) = \int 2Axy\,dy + g(x)$$

$$\phi = Axy^2 + g(x)$$

Comparing the two expressions for ϕ we

- note that $Axy^2 = 3Bxy^2$ $\quad (A=1, B=\frac{1}{3})$
- conclude that $g(x) = -\frac{A}{3}x^3$, $f(y) = 0$

Hence $\phi = Axy^2 - \frac{A}{3}x^3$ or $\phi = 3Bxy^2 - \frac{A}{3}x^3$ $\longleftarrow$ ϕ

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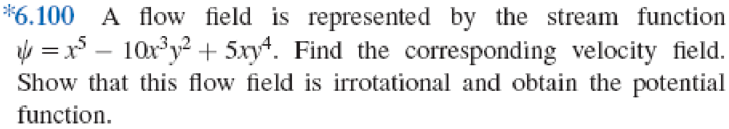
Problem *6.100

[2]



***6.100** A flow field is represented by the stream function $\psi = x^5 - 10x^3y^2 + 5xy^4$. Find the corresponding velocity field. Show that this flow field is irrotational and obtain the potential function.

Given: Stream function

Find: Velocity field; Show flow is irrotational; Velocity potential

Solution:

Basic equations: Incompressibility because ψ exists $u = \frac{\partial}{\partial y}\psi \qquad v = -\frac{\partial}{\partial x}\psi \qquad u = -\frac{\partial}{\partial x}\varphi \qquad v = -\frac{\partial}{\partial y}\varphi$

Irrotationality $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

$$\psi(x,y) = x^5 - 10\cdot x^3\cdot y^2 + 5\cdot x\cdot y^4$$

$$u(x,y) = \frac{\partial}{\partial y}\psi(x,y) \qquad u(x,y) \rightarrow 20\cdot x\cdot y^3 - 20\cdot x^3\cdot y$$

$$v(x,y) = -\frac{\partial}{\partial x}\psi(x,y) \qquad v(x,y) \rightarrow 30\cdot x^2\cdot y^2 - 5\cdot x^4 - 5\cdot y^4$$

$$\frac{\partial}{\partial x}v(x,y) - \frac{\partial}{\partial y}u(x,y) \rightarrow 0$$ Hence flow is IRROTATIONAL

Hence $u = -\frac{\partial}{\partial x}\varphi$ so $\varphi(x,y) = -\int u(x,y)\,dx + f(y) = 5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + f(y)$

$v = -\frac{\partial}{\partial y}\varphi$ so $\varphi(x,y) = -\int v(x,y)\,dy + g(x) = 5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5 + g(x)$

Comparing, the simplest velocity potential is then $\varphi(x,y) = 5\cdot x^4\cdot y - 10\cdot x^2\cdot y^3 + y^5$

Problem *6.101

Given: Flow field represented by the potential function,

$$\phi = Ax^2 + Bxy - Ay^2$$

Find: (a) Verify that the flow is incompressible
(b) Determine the corresponding stream function, ψ

Solution:

The velocity field is given by $\vec{V} = -\nabla\phi$

$$\vec{V} = -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(Ax^2 + Bxy - Ay^2) = -\hat{i}(2Ax + By) - \hat{j}(Bx - 2Ay)$$

If the flow is incompressible, then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}(-)(2Ax + By) + \frac{\partial}{\partial y}(-)(Bx - 2Ay) = -2A + 2A = 0$$

$\therefore$ flow is incompressible $\longleftarrow$

From the definition of ψ, $\quad u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

Thus,

$$u = -2Ax - By = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \psi = -\int (2Ax + By)\,dy + f(x)$$

$$\psi = -2Axy - B\frac{y^2}{2} + f(x)$$

Then,

$$v = -Bx + 2Ay = -\frac{\partial \psi}{\partial x} = 2Ay - \frac{df}{dx}$$

and $\quad -\frac{df}{dx} = -Bx \quad$ or $\quad f = \frac{1}{2}Bx^2 + \text{constant}$

$$\therefore \psi = -2Axy - B\frac{y^2}{2} + B\frac{x^2}{2} + \text{constant}$$

Setting the constant equal to zero, we obtain

$$\psi = \frac{B}{2}(x^2 - y^2) - 2Axy \longleftarrow \psi$$

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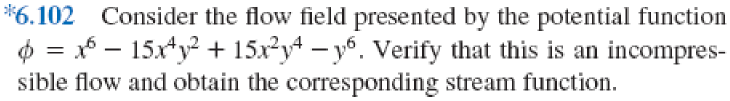
Problem *6.102

[2]



*6.102 Consider the flow field presented by the potential function $\phi = x^6 - 15x^4y^2 + 15x^2y^4 - y^6$. Verify that this is an incompressible flow and obtain the corresponding stream function.

Given: Velocity potential

Find: Show flow is incompressible; Stream function

Solution:

Basic equations: Irrotationality because φ exists $u = \frac{\partial}{\partial y}\psi$ $\quad v = -\frac{\partial}{\partial x}\psi$ $\quad u = -\frac{\partial}{\partial x}\varphi$ $\quad v = -\frac{\partial}{\partial y}\varphi$

Incompressibility $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$

$$\varphi(x,y) = x^6 - 15\cdot x^4\cdot y^2 + 15\cdot x^2\cdot y^4 - y^6$$

$$u(x,y) = -\frac{\partial}{\partial x}\varphi(x,y) \qquad u(x,y) \rightarrow 60\cdot x^3\cdot y^2 - 6\cdot x^5 - 30\cdot x\cdot y^4$$

$$v(x,y) = -\frac{\partial}{\partial y}\varphi(x,y) \qquad v(x,y) \rightarrow 30\cdot x^4\cdot y - 60\cdot x^2\cdot y^3 + 6\cdot y^5$$

Hence $\frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}v(x,y) \rightarrow 0$ Hence flow is INCOMPRESSIBLE

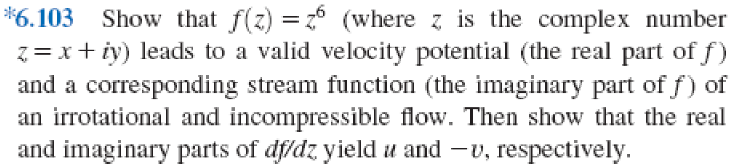
Hence $u = \frac{\partial}{\partial y}\psi$ so $\psi(x,y) = \int u(x,y)\,dy + f(x) = 20\cdot x^3\cdot y^3 - 6\cdot x^5\cdot y - 6\cdot x\cdot y^5 + f(x)$

$v = -\frac{\partial}{\partial x}\psi$ so $\psi(x,y) = -\int v(x,y)\,dx + g(y) = 20\cdot x^3\cdot y^3 - 6\cdot x^5\cdot y - 6\cdot x\cdot y^5 + g(y)$

Comparing, the simplest stream function is then $\psi(x,y) = 20\cdot x^3\cdot y^3 - 6\cdot x^5\cdot y - 6\cdot x\cdot y^5$

Problem *6.103

[4]



***6.103** Show that $f(z) = z^6$ (where z is the complex number $z = x + iy$) leads to a valid velocity potential (the real part of f) and a corresponding stream function (the imaginary part of f) of an irrotational and incompressible flow. Then show that the real and imaginary parts of df/dz yield u and $-v$, respectively.

Given: Complex function

Find: Show it leads to velocity potential and stream function of irrotational incompressible flow; Show that df/dz leads to u and v

Solution:

Basic equations: Irrotationality because φ exists $\quad u = \frac{\partial}{\partial y}\psi \quad v = -\frac{\partial}{\partial x}\psi \quad u = -\frac{\partial}{\partial x}\varphi \quad v = -\frac{\partial}{\partial y}\varphi$

Incompressibility $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ Irrotationality $\frac{\partial}{\partial x}v - \frac{\partial}{\partial y}u = 0$

$$f(z) = z^6 = (x + i\cdot y)^6$$

Expanding $\quad f(z) = x^6 - 15\cdot x^4\cdot y^2 + 15\cdot x^2\cdot y^4 - y^6 + i\cdot\left(6\cdot x\cdot y^5 + 6\cdot x^5\cdot y - 20\cdot x^3\cdot y^3\right)$

We are thus to check the following

$$\varphi(x,y) = x^6 - 15\cdot x^4\cdot y^2 + 15\cdot x^2\cdot y^4 - y^6 \qquad \psi(x,y) = 6\cdot x\cdot y^5 + 6\cdot x^5\cdot y - 20\cdot x^3\cdot y^3$$

$$u(x,y) = -\frac{\partial}{\partial x}\varphi(x,y) \qquad u(x,y) \rightarrow 60\cdot x^3\cdot y^2 - 6\cdot x^5 - 30\cdot x\cdot y^4$$

$$v(x,y) = -\frac{\partial}{\partial y}\varphi(x,y) \qquad v(x,y) \rightarrow 30\cdot x^4\cdot y - 60\cdot x^2\cdot y^3 + 6\cdot y^5$$

An alternative derivation of u and v is

$$u(x,y) = \frac{\partial}{\partial y}\psi(x,y) \qquad u(x,y) \rightarrow 6\cdot x^5 - 60\cdot x^3\cdot y^2 + 30\cdot x\cdot y^4$$

$$v(x,y) = -\frac{\partial}{\partial x}\psi(x,y) \qquad v(x,y) \rightarrow 60\cdot x^2\cdot y^3 - 30\cdot x^4\cdot y - 6\cdot y^5$$

Note that the values of u and v are of opposite sign using ψ and φ!different which is the same result using φ! To resolve this we could either let f = -φ+iψ; altenatively we could use a different definition of φ that many authors use:

$$u = \frac{\partial}{\partial x}\varphi \qquad v = \frac{\partial}{\partial y}\varphi$$

Hence $\quad \frac{\partial}{\partial x}v(x,y) - \frac{\partial}{\partial y}u(x,y) \rightarrow 0 \quad$ Hence flow is IRROTATIONAL

Hence $\quad \frac{\partial}{\partial x}u(x,y) + \frac{\partial}{\partial y}v(x,y) \rightarrow 0 \quad$ Hence flow is INCOMPRESSIBLE

Next we find $\quad \frac{df}{dz} = \frac{d\left(z^6\right)}{dz} = 6\cdot z^5 = 6\cdot(x + i\cdot y)^5 = \left(6\cdot x^5 - 60\cdot x^3\cdot y^2 + 30\cdot x\cdot y^4\right) + i\cdot\left(30\cdot x^4\cdot y + 6\cdot y^5 - 60\cdot x^2\cdot y^3\right)$

Hence we see $\quad \frac{df}{dz} = u - i\cdot v \quad$ Hence the results are verified; $\quad u = \mathrm{Re}\left(\frac{df}{dz}\right)$ and $v = -\mathrm{Im}\left(\frac{df}{dz}\right)$

These interesting results are explained in Problem 6.104!

Problem *6.104

[4]

***6.104** Show that *any* differentiable function $f(z)$ of the complex number $z = x + iy$ leads to a valid potential (the real part of f) and a corresponding stream function (the imaginary part of f) of an incompressible, irrotational flow. To do so, prove using the chain rule that $f(z)$ automatically satisfies the Laplace equation. Then show that $df/dz = u - iv$.

Given: Complex function

Find: Show it leads to velocity potential and stream function of irrotational incompressible flow;
Show that df/dz leads to u and v

Solution:

Basic equations: $u = \frac{\partial}{\partial y}\psi \qquad v = -\frac{\partial}{\partial x}\psi \qquad u = -\frac{\partial}{\partial x}\varphi \qquad v = -\frac{\partial}{\partial y}\varphi$

First consider $\frac{\partial}{\partial x}f = \frac{\partial}{\partial x}z\cdot\frac{d}{dz}f = 1\cdot\frac{d}{dz}f = \frac{d}{dz}f \qquad (1)$ and also $\frac{\partial}{\partial y}f = \frac{\partial}{\partial y}z\cdot\frac{d}{dz}f = i\cdot\frac{d}{dz}f = i\cdot\frac{d}{dz}f \qquad (2)$

Hence $\frac{\partial^2}{\partial x^2}f = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}f\right) = \frac{d}{dz}\left(\frac{d}{dz}f\right) = \frac{d^2}{dz^2}f$ and $\frac{\partial^2}{\partial y^2}f = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}f\right) = i\cdot\frac{d}{dz}\left(i\cdot\frac{d}{dz}f\right) = -\frac{d^2}{dz^2}f$

Combining $\frac{\partial^2}{\partial x^2}f + \frac{\partial^2}{\partial y^2}f = \frac{d^2}{dz^2}f - \frac{d^2}{dz^2}f = 0$

Any differentiable function f(z) automatically satisfies the Laplace Equation; so do its real and imaginary parts!

We demonstrate derivation of velocities u and v

From Eq 1 $\frac{d}{dz}f = \frac{d}{dz}(\varphi + i\cdot\psi) = \frac{\partial}{\partial x}(\varphi + i\cdot\psi) = \frac{\partial}{\partial x}\varphi + i\cdot\frac{\partial}{\partial x}\psi = -u - i\cdot v$

From Eq 2 $\frac{d}{dz}f = \frac{d}{dz}(\varphi + i\cdot\psi) = \frac{1}{i}\cdot\frac{\partial}{\partial y}(\varphi + i\cdot\psi) = -i\cdot\frac{\partial}{\partial y}\varphi + \frac{\partial}{\partial y}\psi = i\cdot v + u$

There appears to be an incompatibilty here,
but many authors define φ as $u = \frac{\partial}{\partial x}\varphi \qquad v = \frac{\partial}{\partial y}\varphi$ or in other words, as the negative of our definition

Alternatively, we can use out φ but set $f = -\varphi + i\cdot\psi$

Then

From Eq 1 $\frac{d}{dz}f = \frac{d}{dz}(\varphi + i\cdot\psi) = \frac{\partial}{\partial x}(\varphi + i\cdot\psi) = \frac{\partial}{\partial x}\varphi + i\cdot\frac{\partial}{\partial x}\psi = u - i\cdot v$

From Eq 2 $\frac{d}{dz}f = \frac{d}{dz}(\varphi + i\cdot\psi) = \frac{1}{i}\cdot\frac{\partial}{\partial y}(\varphi + i\cdot\psi) = -i\cdot\frac{\partial}{\partial y}\varphi + \frac{\partial}{\partial y}\psi = -i\cdot v + u$

Hence we have demonstrated that $\frac{df}{dz} = u - i\cdot v$ if we set $u = \frac{\partial}{\partial x}\varphi \qquad v = \frac{\partial}{\partial y}\varphi$

Problem *6.105

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Given: Flow field represented by the velocity potential $\phi = Ax + Bx^2 - By^2$, where $A = 1\ \text{m}\cdot\text{s}^{-1}$, $B = 1\ \text{s}^{-1}$, and coordinates are measured in meters.

Find: (a) expression for the velocity field
(b) stream function
(c) pressure difference between points $(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (1,2)$

Solution

The velocity field is determined from the velocity potential

$$\left.\begin{aligned} u &= -\partial\phi/\partial x = -A - 2Bx \\ v &= -\partial\phi/\partial y = 2By \end{aligned}\right\} \quad \vec{V} = -(A+2Bx)\hat{i} + 2By\hat{j} \longleftarrow \vec{V}$$

From the definition of the stream function, $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$

Then

$$\psi = \int u\,dy + f(x) = \int -(A+2Bx)\,dy + f(x)$$

$$\psi = -Ay - 2Bxy + f(x)$$

Also,

$$\psi = \int -v\,dx + g(y) = \int -2By\,dx + g(y)$$

$$\psi = -2Bxy + g(y)$$

Comparing the two expressions for ψ we conclude

$$f(x) = 0 \quad , \quad g(y) = -Ay$$

$$\therefore \psi = -(Ay + 2Bxy) \longleftarrow \psi$$

Since $\nabla^2\phi = 2B - 2B = 0$, the flow is irrotational and the Bernoulli equation can be applied between any two points in the flow field

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2} \qquad \left\{ \begin{aligned} &\text{Assume: } \rho = \text{constant} \\ &z_1 = z_2 \end{aligned} \right\}$$

$$\vec{V}(0,0) = -A\hat{i} = -\hat{i}\ \text{m/s} \qquad V_{0,0} = 1\ \text{m/s}$$

$$\vec{V}(1,2) = -(A+2B)\hat{i} + 4B\hat{j} = -3\hat{i} + 4\hat{j}\ \text{m/s} \qquad \therefore V_{1,2} = 5\ \text{m/s}$$

$$\therefore p_1 - p_2 = \rho\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) = \frac{\rho}{2}\left(V_2^2 - V_1^2\right)$$

Assume fluid is water

$$p_1 - p_2 = \frac{1}{2} \times 999\,\frac{\text{kg}}{\text{m}^3}\,(25-1)\,\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 12\ \text{kN/m}^2 \longleftarrow$$

Problem *6.106

Given: Incompressible flow field represented by $\psi = 3Ax^2y - Ay^3$ where $A = 1\ m^{-1}\cdot s^{-1}$

Show: that this flow field is irrotational

Find: the velocity potential ϕ

Plot: streamlines and potential lines, and visually verify that they are orthogonal

Solution:

For a 2-D incompressible, irrotational flow $\nabla^2\psi = 0$ (6.30)

For the flow field.

$$\nabla^2\psi = \frac{\partial^2}{\partial x^2}(3Ax^2y - Ay^3) + \frac{\partial^2}{\partial y^2}(3Ax^2y - Ay^3) = 6Ay - 6Ay = 0 \longleftarrow \text{irrotational}$$

The velocity field is given by $\vec{V} = u\hat{i} + v\hat{j}$

$$\left.\begin{array}{l} u = \partial\psi/\partial y = 3Ax^2 - 3Ay^2 = 3A(x^2-y^2). \\ v = -\partial\psi/\partial x = -6Axy \end{array}\right\} \vec{V} = 3A(x^2-y^2)\hat{i} - 6Axy\hat{j}$$

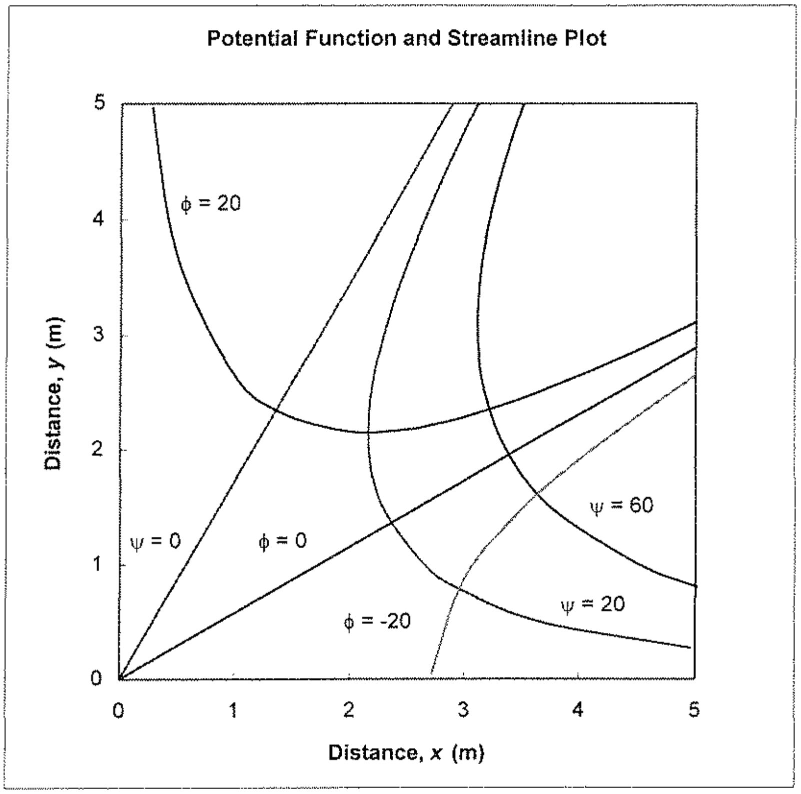
The velocity potential is defined such that $u = -\frac{\partial\phi}{\partial x}$ & $v = -\frac{\partial\phi}{\partial y}$

Then, $\phi = -\int u\,dx + f(y) = -\int 3A(x^2-y^2)\,dx + f(y) = -Ax^3 + 3Axy^2 + f(y)$ ---(1)

Also, $\phi = -\int v\,dy + g(x) = \int 6Axy\,dy + g(x) = 3Axy^2 + g(x)$ ......(2)

Equating expressions for ϕ (Eqs 1 and 2) we see that

$g(x) = -Ax^3$ and $f(y) = 0$ $\quad\therefore\ \phi = 3Axy^2 - Ax^3 \longleftarrow \phi$



Problem *6.107

Given: Flow field represented by the velocity potential $\phi = Ay^3 - Bx^2y$, where $A = \frac{1}{3}\,m^{-1}\cdot s^{-1}$, $B = 1\,m^{-1}\cdot s^{-1}$, and the coordinates are measured in meters.

Find: (a) expression for the magnitude of the velocity vector
(b) the stream function.

Plot: streamlines and potential lines, and visually verify that they are orthogonal.

Solution:

The velocity field is determined from the velocity potential

$$\left.\begin{aligned} u &= -\partial\phi/\partial x = 2Bxy = 2xy \\ v &= -\partial\phi/\partial y = -3Ay^2 + Bx^2 = x^2 - y^2 \end{aligned}\right\} \vec{V} = 2xy\,\hat{i} + (x^2 - y^2)\,\hat{j}$$

$$V = \left[u^2 + v^2\right]^{1/2} = \left[4x^2y^2 + (x^2-y^2)^2\right]^{1/2} = \left[4x^2y^2 + x^4 - 2x^2y^2 + y^4\right]^{1/2}$$

$$V = \left[x^4 + 2x^2y^2 + y^4\right]^{1/2} = \left[(x^2+y^2)^2\right]^{1/2} = x^2 + y^2 \longleftarrow V$$

The stream function is defined such that $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$

Then, $\psi = \int u\,dy + f(x) = \int 2Bxy\,dy + f(x) = Bxy^2 + f(x)$ ----(1)

Also, $\psi = \int -v\,dx + g(y) = \int(3Ay^2 - Bx^2)\,dx + g(y) = 3Axy^2 - \frac{B}{3}x^3 + g(y)$ ---(2)

Comparing the two expressions for ψ, we

- note that $Bxy^2 = 3Ax^2y$ $(B = 1,\ A = \frac{1}{3})$, and
- conclude that $f(x) = -\frac{B}{3}x^3$ and $g(y) = 0$

$\therefore\ \psi = Bxy^2 - \frac{B}{3}x^3$ or $\psi = 3Ax^2y - \frac{B}{3}x^3 \longleftarrow \psi$

with $A = \frac{1}{3}$, $B = 1$

$$\psi = x^2y - \frac{x^3}{3} = x\left(y^2 - \frac{x^2}{3}\right)$$

For $\psi = 0$, $x = 0$ or $y = 0.577x$

For $\psi = -4$, $y^2 = \frac{x^2}{3} - \frac{4}{x}$

For $\psi = 4$, $y^2 = \frac{x^2}{3} + \frac{4}{x}$

See the next page for plots

Problem *6.107

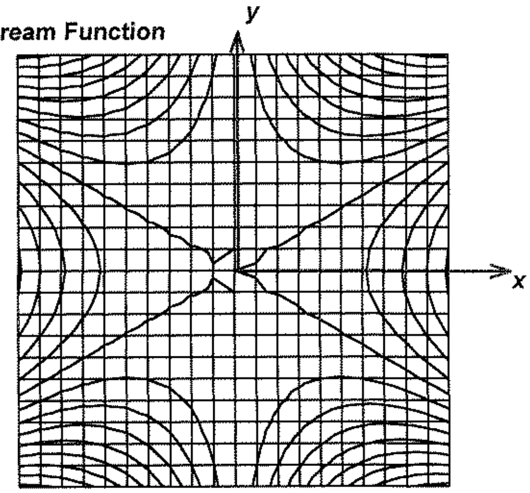
[2] Part 2/2

Using *Excel*, the stream function and velocity potential can be plotted.
The data below was obtained using the workbook for Example Problem 6.10.
Note the orthogonality of ψ and ϕ!

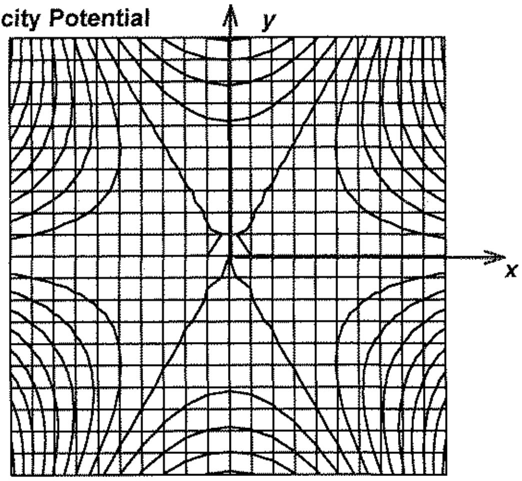
#NAME?	**Stream Function**
#NAME?	**Velocity Potential**

Note that the plot is
from x = -5 to 5 and y = -5 to 5

Stream Function



Velocity Potential



Problem *6.108

Given: Irrotational flow represented by $\psi = Bxy$, where $B = 0.25\ s^{-1}$ and the coordinates are measured in meters.

Find: (a) the rate of flow between points $(x_1, y_1) = (2,2)$ and $(x_2, y_2) = (3,3)$

(b) the velocity potential for this flow

Plot: streamlines and potential lines, and visually verify that they are orthogonal.

Solution:

The volume flow rate (per unit depth) between points ① and ② is given by

$$Q_{12} = \psi_2 - \psi_1 = B[x_2y_2 - x_1y_1] = 0.25\,s^{-1}[3m \times 3m - 2m \times 2m]$$

$$Q_{12} = 1.25\ m^3/s/m \longleftarrow Q_{12}$$

The velocity field is determined from the stream function

$$u = \partial\psi/\partial y = Bx \qquad v = -\partial\psi/\partial x = -By \qquad \therefore \vec{V} = Bx\hat{i} - By\hat{j}$$

For irrotational flow $\vec{V} = -\nabla\phi$ and $u = -\partial\phi/\partial x$, $v = -\partial\phi/\partial y$

and

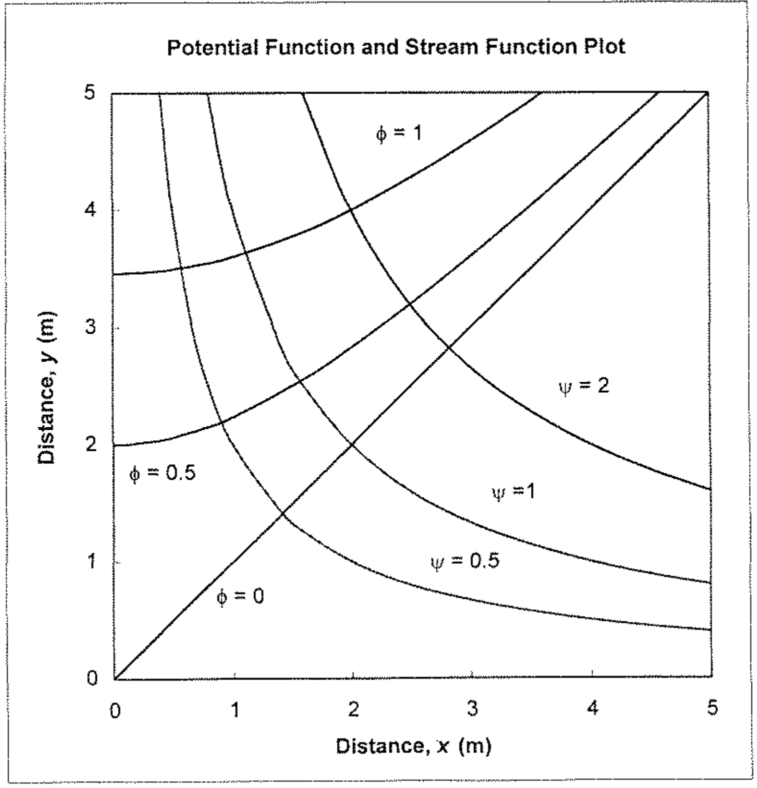
$$\phi = -\int u\,dx + f(y) = -\int Bx\,dx + f(y) = -\frac{B}{2}x^2 + f(y) \quad \text{.....(1)}$$

Also

$$\phi = -\int v\,dy + g(x) = \int By\,dy + g(x) = \frac{B}{2}y^2 + g(x) \quad \text{......(2)}$$

Equating expressions for ϕ (Eqs 1 and 2) we conclude that

$$f(y) = \frac{B}{2}y^2, \quad g(x) = -\frac{B}{2}x^2 \quad \text{and} \quad \phi = \frac{B}{2}(y^2 - x^2) \longleftarrow \phi$$



Problem *6.109

Given: Two-dimensional, inviscid flow with velocity field $\vec{V} = (Ax+B)\hat{i} + (C-Ay)\hat{j}$, where $A = 3\,s^{-1}$, $B = 6\,m/s$, $C = 4\,m/s$ and the coordinates are measured in meters. The body force distribution is $\vec{B} = -g\hat{k}$; $\rho = 825\,kg/m^3$.

Find: (a) if this is a possible incompressible flow
(b) stagnation point(s) of the flow field
(c) if the flow is irrotational
(d) the velocity potential (if one exists)
(e) pressure difference between origin and point $(x,y,z) = (2,2,2)$

Plot: a few streamlines in the upper half plane.

Solution:

For incompressible flow $\nabla \cdot \vec{V} = 0$. For this flow

$$\nabla \cdot \vec{V} = \frac{\partial}{\partial x}(Ax+B) + \frac{\partial}{\partial y}(C-Ay) = A - A = 0$$

$\therefore$ velocity field represents possible incompressible flow. ←

At the stagnation point $u = v = 0$. $(\vec{V} = 0)$

$$u = 0 = (Ax+B) \quad \therefore \; x = -B/A = \frac{-6\,m\cdot s^{-1}}{3\,s^{-1}} = -2\,m$$

$$v = 0 = (C-Ay) \quad \therefore \; y = C/A = \frac{4\,m\cdot s^{-1}}{3\,s^{-1}} = 4/3\,m$$

Stagnation point is at $(x,y) = (-2, 4/3)\,m$. ←

The fluid rotation (for a 2-D flow) is given by $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

For this flow $$\omega_z = \frac{1}{2}\left[\frac{\partial(C-Ay)}{\partial x} - \frac{\partial(Ax+B)}{\partial y}\right] = 0$$

$\therefore$ flow is irrotational ←

Then, $\vec{V} = -\nabla\phi$ and $u = -\partial\phi/\partial x$ and $v = -\partial\phi/\partial y$.

and $$\phi = \int -u\,dx + f(y) = -\int (Ax+B)\,dx + f(y) = -\frac{Ax^2}{2} - Bx + f(y) \quad \text{...(1)}$$

Also $$\phi = -\int v\,dy + g(x) = -\int (C-Ay)\,dy + g(x) = \frac{Ay^2}{2} - Cy + g(x) \quad \text{---(2)}$$

Equating the two expressions for ϕ (Eqs. 1 and 2) we note that

$$g(x) = -\left(\frac{Ax^2}{2} + Bx\right) \quad \text{and} \quad f(y) = \frac{Ay^2}{2} - Cy$$

$$\therefore \; \phi = \frac{A}{2}(y^2 - x^2) - Bx - Cy \quad \longleftarrow \phi$$

Since the flow is irrotational we can apply the Bernoulli equation between any two points in the flow field.

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

At point 1 $(0,0,0)$, $\vec{V} = B\hat{i} + C\hat{j} = 6\hat{i} + 4\hat{j}\,m/s$, $V_1^2 = 52\,m^2/s^2$

At point 2 (2,2,2) $\vec{V}_2 = [3s^{-1}\times 2m + 6m/s]\hat{i} + [4m/s - 3s^{-1}\times 2m]\hat{j}$

$\vec{V}_2 = 12\hat{i} - 2\hat{j}\ m/s \quad , \quad V_2^2 = 148\ m^2/s^2$

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2) + \rho g(z_2 - z_1) = \rho\left[\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$= 825\ \frac{kg}{m^3}\times\left[\frac{1}{2}\times(148-52)\frac{m^2}{s^2} + 9.81\frac{m}{s^2}\times(2m)\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_1 - p_2 = 55.8\ kPa \longleftarrow \Delta p$

The stream function is defined such that $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$

Then, $\psi = \int u\,dy + f(x) = \int (Ax+B)\,dy + f(x) = Axy + By + f(x)$ ....(1)

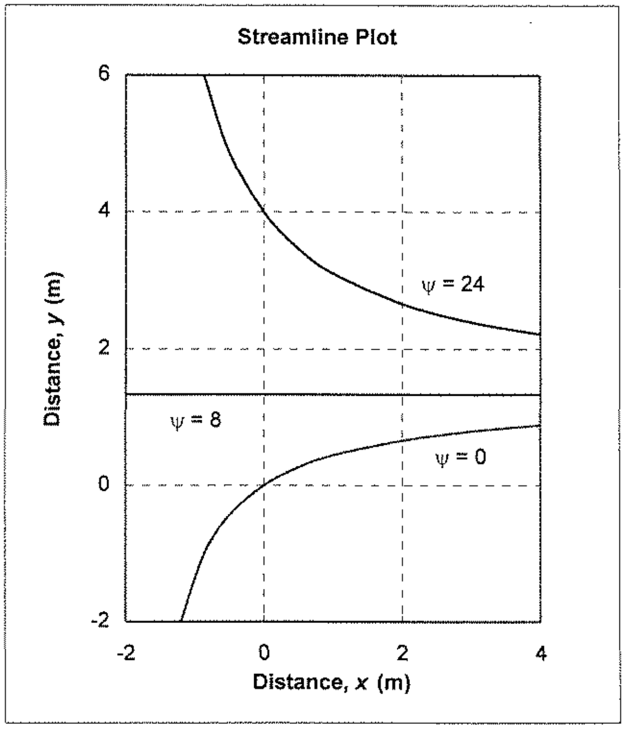
Also, $\psi = -\int v\,dx + g(y) = \int(-C+Ay)\,dx + g(y) = -Cx + Axy + g(y)$ ....(2)

Equating the two expressions for ψ (Eqs 1 and 2) we note that

$f(x) = -Cx$, $g(y) = By$ and $\therefore\ \psi = Axy + By - Cx \longleftarrow \psi$

The stagnation streamline goes through the stagnation point $(-2, \frac{4}{3})$

$\psi_{stag} = 3s^{-1}\times(-2m)\times\frac{4}{3}m + 6m\cdot s^{-1}\times\frac{4}{3}m - 4m\cdot s^{-1}\times(-2m) = 8\ m^2/s \longleftarrow \psi_{stag}$



Problem *6.110

Given: Flow past a circular cylinder of Example Problem 6.11.

Find: (a) Show that $V_r = 0$ along the lines $(r,\theta) = (r, \pm\pi/2)$
(b) Plot V_θ/U versus r for $r \geq a$. along line $(r, \pi/2)$
(c) Find distance beyond which the influence of the cylinder on the velocity is less that 1% of U.

Solution.

From Example Problem 6.11

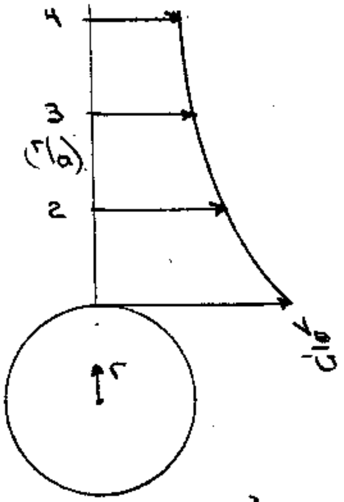
$$\vec{V} = \left(-\frac{\Lambda\cos\theta}{r^2} + U\cos\theta\right)\hat{i}_r + \left(-\frac{\Lambda\sin\theta}{r^2} - U\sin\theta\right)\hat{i}_\theta \quad \ldots\ldots (1)$$

Then $V_r = \left(-\frac{\Lambda}{r^2} + U\right)\cos\theta$. For $\theta = \pm\frac{\pi}{2}$, $\cos\theta = 0$ and $V_r = 0$

$V_\theta = -\left(\frac{\Lambda}{r^2} + U\right)\sin\theta$, but $\frac{\Lambda}{U} = a^2$

$\therefore V_\theta = -\left(\frac{a^2}{r^2} + 1\right)U\sin\theta$ For $\theta = \pi/2$.

$$\frac{V_\theta}{U} = -\left(1 + \frac{a^2}{r^2}\right)$$



$$\vec{V} = U\cos\theta\left(1 - \frac{a^2}{r^2}\right)\hat{i} - U\sin\theta\left(1 + \frac{a^2}{r^2}\right)\hat{j}$$

For $\theta = \pi/2$

$\frac{V}{U} = 1 + \frac{a^2}{r^2}$ If $\frac{V}{U} = 1.01$ then $\frac{a^2}{r^2} = 0.01$ or $\frac{a}{r} = 0.1$

$\therefore \frac{V}{U} < 1\%$ for $r > 10a$

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42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem *6.111

[3]

Consider flow around a circular cylinder with freestream velocity from right to left and a counterclockwise free vortex. Show that the lift force on the cylinder can be expressed as $F_L = -\rho U\Gamma$, as illustrated in Example 6.12.

Open-Ended Problem Statement: Consider flow around a circular cylinder with freestream velocity from right to left and a counterclockwise free vortex. Show that the lift force on the cylinder can be expressed as $F_L = -\rho U\Gamma$, as illustrated in Example 6.12.

Discussion: The only change in this flow from the flow of Example 6.12 is that the directions of the freestream velocity and the vortex are changed. This changes the sign of the freestream velocity from U to $-U$ and the sign of the vortex strength from K to $-K$. Consequently the signs of both terms in the equation for lift are changed. Therefore the direction of the lift force remains unchanged.

The analysis of Example 6.12 shows that only the term involving the vortex strength contributes to the lift force. Therefore the expression for lift obtained with the changed freestream velocity and vortex strength is identical to that derived in Example 6.12. Thus the general solution of Example 6.12 holds for any orientation of the freestream and vortex velocities. For the present case, $F_L = -\rho U\Gamma$, as shown for the general case in Example 6.12.

Problem *6.112

[3]

Given: A tornado is modelled by the superposition of a sink (strength, $q = 2800\ m^2/sec$) and a free vortex (strength, $K = 5600\ m^2/sec$)

Find: (a) Expressions for ψ and ϕ
(b) Estimate the radius beyond which the flow may be treated as incompressible.
(c) Find the gage pressure at that radius.

Solution:

$$\psi = \psi_{si} + \psi_{vo} = -\frac{q}{2\pi}\theta - \frac{K}{2\pi}\ln r \longleftarrow \psi$$

$$\phi = \phi_{si} + \phi_{vo} = \frac{q}{2\pi}\ln r - \frac{K}{2\pi}\theta \longleftarrow \phi$$

$$\vec{V} = V_r\hat{i}_r + V_\theta\hat{i}_\theta \qquad V_{r\,si} = -\frac{q}{2\pi r},\ V_{r\,vo} = 0;\ V_{\theta\,si} = 0,\ V_{\theta\,vo} = \frac{K}{2\pi r}$$

$$\therefore \vec{V} = -\frac{q}{2\pi r}\hat{i}_r + \frac{K}{2\pi r}\hat{i}_\theta$$

Then

$$V = (V_r^2 + V_\theta^2)^{1/2} = \left[\left(-\frac{q}{2\pi}\right)^2 + \left(\frac{K}{2\pi}\right)^2\right]^{1/2}\frac{1}{r}$$

For incompressible flow $M \leq 0.3$. For standard air this corresponds to $V < 102\ m/sec$

Then, for incompressible flow

$$V = 102\ m/sec < \left[q^2 + K^2\right]^{1/2}\frac{1}{2\pi r}$$

or

$$r > \left[q^2 + K^2\right]^{1/2}\frac{1}{2\pi}\times\frac{s}{102\ m} = \left[(2800)^2 + (5600)^2\right]^{1/2}\frac{m^2}{s^2}\times\frac{1}{2\pi}\times\frac{s}{102\ m}$$

$$r > 9.77\ m \longleftarrow r$$

To determine the gage pressure at this radius, apply the Bernoulli equation for irrotational flow

$$\frac{p_\infty}{\rho} + \cancel{\frac{V_\infty^2}{2}}^{\approx 0} + \cancel{gz_\infty} = \frac{p}{\rho} + \frac{V^2}{2} + \cancel{gz} \qquad \text{assume } \Delta z = 0$$

Then

$$p_{gage} = p - p_\infty = -\frac{1}{2}\rho V^2 = -\frac{1}{2}\times 1.225\frac{kg}{m^3}\times(102)^2\frac{m^2}{s^2}\times\frac{N\,s^2}{kg\cdot m}$$

$$p_{gage} = -6.37\ kPa \quad (\text{for standard air}) \longleftarrow p_{gage}$$

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42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem *6.113

[3]

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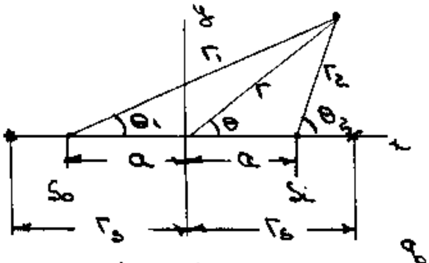
NATIONAL

Given: Flow past a Rankine body is formed from the superposition of a uniform flow ($\bar{U} = 20\,m/s$) in the +x direction and a source and a sink of equal strengths ($q = 3\pi\ m^2/s$) located on the x axis at $x=-a$ and $x=a$, respectively.

Find: (a) expressions for ψ, ϕ, and $\vec{V}$
(b) the value of ψ = constant on the stagnation streamline.
(c) the stagnation points if $a = 0.3\,m$.

Solution:



$$\psi = \psi_{so} + \psi_{si} + \psi_{uf} = \frac{q}{2\pi}\theta_1 - \frac{q}{2\pi}\theta_2 + \bar{U}y$$

$$\psi = \frac{q}{2\pi}(\theta_1 - \theta_2) + \bar{U} r \sin\theta \longleftarrow \psi$$

$$\phi = \phi_{so} + \phi_{si} + \phi_{uf} = -\frac{q}{2\pi}\ln r_1 + \frac{q}{2\pi}\ln r_2 - \bar{U}x$$

$$\phi = \frac{q}{2\pi}\ln\frac{r_2}{r_1} - \bar{U} r \cos\theta \longleftarrow \phi$$

$$u = u_{so} + u_{si} + u_{uf} = \frac{q}{2\pi r_1}\cos\theta_1 - \frac{q}{2\pi r_2}\cos\theta_2 + U$$

$$v = v_{so} + v_{si} + v_{uf} = \frac{q}{2\pi r_1}\sin\theta_1 - \frac{q}{2\pi r_2}\sin\theta_2$$

$$\vec{V} = u\hat{i} + v\hat{j} = \left\{\frac{q}{2\pi}\left(\frac{\cos\theta_1}{r_1} - \frac{\cos\theta_2}{r_2}\right) + U\right\}\hat{i} + \frac{q}{2\pi}\left(\frac{\sin\theta_1}{r_1} - \frac{\sin\theta_2}{r_2}\right)\hat{j} \longleftarrow \vec{V}$$

At stagnation point $\vec{V} = 0$ $\quad y = 0 \quad \theta_1 = \theta_2 = 0$

$r_2 = r_s - a \quad , r_1 = r_s + a$

$$\therefore u = 0 = \frac{q}{2\pi}\left(\frac{1}{r_s + a} - \frac{1}{r_s - a}\right) + U = \frac{q}{2\pi}\left[\frac{(r_s - a) - (r_s + a)}{(r_s^2 - a^2)}\right] + U$$

$$0 = \frac{-qa}{\pi(r_s^2 - a^2)} + \bar{U} \qquad \text{or} \qquad (r_s^2 - a^2) = \frac{qa}{\pi \bar{U}}$$

$$r_s = \left(a^2 + \frac{qa}{\pi\bar{U}}\right)^{1/2} = a\left(1 + \frac{q}{\pi\bar{U}a}\right)^{1/2}$$

For $a = 0.3\,m$

$$r = 0.3\,m\left[1 + \frac{3\pi}{\pi}\frac{m^2}{s}\cdot\frac{s}{20\,m}\cdot\frac{1}{0.3\,m}\right]^{1/2} = 0.367\,m$$

Stagnation points located at $\theta = 0, \pi \quad r = 0.367\,m$ $\longleftarrow$

Since $\psi = \frac{q}{2\pi}(\theta_1 - \theta_2) + \bar{U}y$ and $\theta_1 = \theta_2$ & $y = 0$ at stagnation

$$\psi_{stag} = 0 \longleftarrow$$

Problem *6.114

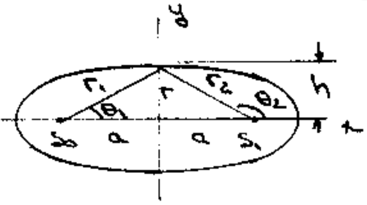
42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Given: Flow past a Rankine body is formed from the superposition of a uniform flow ($\bar{U} = 20\text{ m/s}$) in the $+x$ direction, and a source and a sink of equal strengths ($q = 3\pi\ \text{m}^2/\text{s}$) located on the x axis at $x = -a$ and $x = a$, respectively.

Find: (a) the half width of the body
(b) $\vec{V}$ and p at the points $(0, \pm h)$

Solution:

$$\psi = \psi_{so} + \psi_{si} + \psi_{uf} = \frac{q}{2\pi}(\theta_1 - \theta_2) + \bar{U} r \sin\theta$$



At stagnation point $\theta_1 = \theta_2$ and $\theta = 0, \pi$

$\therefore \psi_{stag} = 0$ and equation of stag. streamline is

$$0 = \frac{q}{2\pi}(\theta_1 - \theta_2) + \bar{U} r \sin\theta$$

or $$r = \frac{q}{2\pi}\frac{(\theta_2 - \theta_1)}{\bar{U}\sin\theta}$$

At half width, $\theta = \frac{\pi}{2}$, $\theta_2 = \pi - \theta_1$, and $r = h = \frac{q}{2\pi}\frac{[(\pi - \theta_1) - \theta_1]}{\bar{U}}$

$\therefore h\bar{U} = \frac{q}{2\pi}[\pi - 2\theta_1] = \frac{q}{2} - \frac{q\theta_1}{\pi}$ or $\theta_1 = \frac{\pi}{2} - \frac{\bar{U}h\pi}{q}$

Since $h = a\tan\theta_1$

$$\frac{h}{a} = \tan\left(\frac{\pi}{2} - \frac{\bar{U}h\pi}{q}\right) = \cot\left(\frac{\bar{U}h\pi}{q}\right)$$

Substituting values, $\frac{h}{0.3} = \cot\left(\frac{20}{3}h\right)$. Trial and error solution gives

$h = 0.1615\text{ m}$ ——— h

The velocity field is given by $\vec{V} = \hat{i}u + \hat{j}v$

$$\vec{V} = \left\{\frac{q}{2\pi}\left(\frac{\cos\theta_1}{r_1} - \frac{\cos\theta_2}{r_2}\right) + \bar{U}\right\}\hat{i} + \frac{q}{2\pi}\left(\frac{\sin\theta_1}{r_1} - \frac{\sin\theta_2}{r_2}\right)\hat{j}$$

At $(0, h)$, $r_1 = r_2$, $\theta_2 = \pi - \theta_1$ $\therefore \sin\theta_2 = \sin\theta_1$, $\cos\theta_2 = -\cos\theta_1$

and $\vec{V} = \left(\frac{q\cos\theta_1}{r_1} + \bar{U}\right)\hat{i}$

$\theta_1 = \tan^{-1}\frac{h}{a} = \tan^{-1}\frac{0.1615}{0.3} = 28.3°$ $\qquad r_1 = [a^2 + h^2]^{1/2} = [0.3^2 + 0.1615^2]^{1/2} = 0.341\text{ m}$

$$\vec{V} = \left(\frac{q\cos\theta_1}{r_1} + \bar{U}\right)\hat{i} = \left(3\pi\frac{\text{m}^2}{\text{sec}} \times \frac{\cos 28.3°}{0.341\text{ m}} + 20\frac{\text{m}}{\text{s}}\right)\hat{i} = 44.3\hat{i}\ \text{m/s}$$ ——— V

To find the gage pressure apply the Bernoulli equation between the point at conditions at ∞

$$\frac{p_\infty}{\rho} + \frac{\bar{U}^2}{2} = \frac{p}{\rho} + \frac{V^2}{2}$$

$$p_{gage} = p - p_\infty = \frac{1}{2}\rho(\bar{U}^2 - V^2) = \frac{1}{2} \times 1.225\frac{\text{kg}}{\text{m}^3}\left[(20)^2 - (44.3)^2\right]\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$p_{gag} = -957\ \text{N/m}^2$ ——— p_{gag}

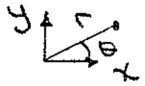
Problem *6.115

Given: Flow field formed by superposition of a uniform flow in the + x direction ($U = 10$ m/s) and a counterclockwise vortex, with strength $K = 16\pi$ m^2/s, located at the origin

Find: (a) ψ, ϕ, and $\vec{V}$ for the flow field
(b) stagnation point(s)

Plot: streamlines and lines of constant potential

Solution:



$$\psi = \psi_{uf} + \psi_v = Uy - \frac{K}{2\pi}\ln r = Ur\sin\theta - \frac{K}{2\pi}\ln r \longleftarrow \psi$$

$$\phi = \phi_{uf} + \phi_v = -Ux - \frac{K}{2\pi}\theta = -Ur\cos\theta - \frac{K}{2\pi}\theta \longleftarrow \phi$$

$$V_r = -\frac{\partial\phi}{\partial r} = U\cos\theta \quad , \quad V_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U\sin\theta + \frac{K}{2\pi r}$$

$$\vec{V} = U\cos\theta\,\hat{e}_r + \left(\frac{K}{2\pi r} - U\sin\theta\right)\hat{e}_\theta \longleftarrow \vec{V}$$

At stagnation point, $\vec{V} = 0$

$V_r = 0$ at $\theta = \pm\frac{\pi}{2}$; $V_\theta = 0$ on $r = \frac{K}{2\pi U\sin\theta}$

$$\therefore \vec{V} = 0 \text{ at } (r,\theta) = \frac{K}{2\pi U}, \pi/2 \longleftarrow \text{Stagnation}$$

See the next page for plots

Problem *6.115

[3] Part 2/2

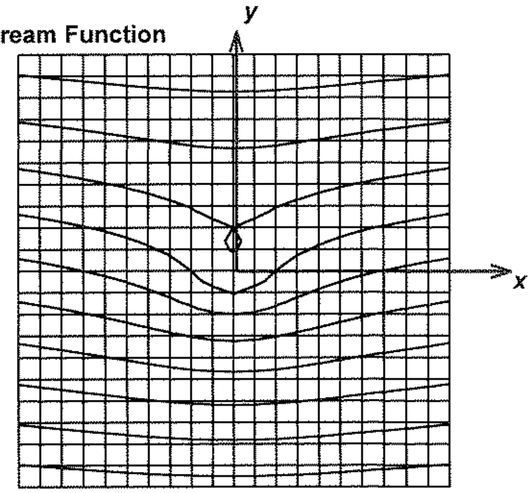
Using *Excel*, the stream function and velocity potential can be plotted.
The data below was obtained using the workbook for Example Problem 6.10.
Note the orthogonality of ψ and ϕ!

#NAME?	**Stream Function**

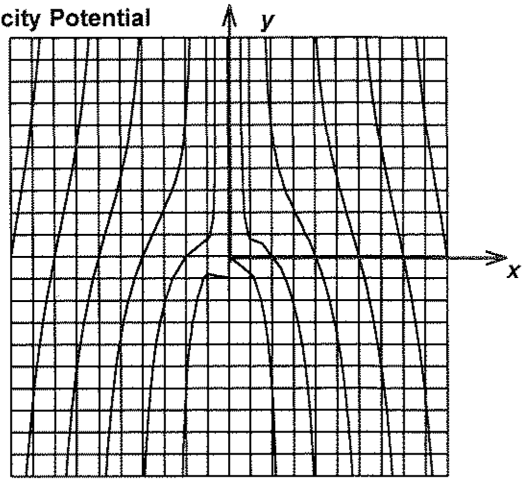
#NAME?	**Velocity Potential**

Note that the plot is
from $x = -5$ to 5 and $y = -5$ to 5

Stream Function



Velocity Potential



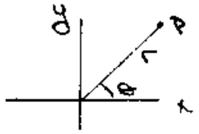
Problem *6.116

[4]

Given: Flow field formed by combining a uniform flow in the +x direction ($U = 50\ \text{m/s}$) and a sink (of strength, $q = 90\ \text{m}^2/\text{s}$) at the origin.

Find: the net force per unit depth needed to hold in place (in standard air) the surface shape formed by the stagnation streamline

Solution:



$\psi = \psi_{uf} + \psi_{si} = Uy - \frac{q}{2\pi}\theta = Ur\sin\theta - \frac{q}{2\pi}\theta$ ----(1)

$u = u_{uf} + u_{si}$; $u_{uf} = U$, $u_{si} = -V_r\cos\theta = -\frac{q}{2\pi r}\frac{x}{r}$ $\therefore u = U - \frac{q}{2\pi}\frac{x}{r^2}$

$v = v_{uf} + v_{si}$; $v_{uf} = 0$, $v_{si} = -V_r\sin\theta = -\frac{q}{2\pi r}\frac{y}{r}$ $\therefore v = -\frac{q}{2\pi}\frac{y}{r^2}$

$\therefore \vec{V} = u\hat{i} + v\hat{j} = \left(U - \frac{q}{2\pi}\frac{x}{r^2}\right)\hat{i} - \frac{q}{2\pi}\frac{y}{r^2}\hat{j}$

At the stagnation point, $\vec{V} = 0$

$\therefore -\frac{q}{2\pi}\frac{y}{r^2} = 0$, i.e. $y = 0$. Also $U - \frac{q}{2\pi}\frac{x}{r^2} = 0$ $\therefore U = \frac{q}{2\pi}\frac{x}{(x^2+y^2)}$

and $x_{stag} = \frac{q}{2\pi U} = \frac{90\,\text{m}^2}{\text{s}} \times \frac{1}{2\pi} \times \frac{\text{s}}{50\,\text{m}} = 0.286\,\text{m}$

At stagnation point, $y = 0$ and $\theta = 0$. From eq (1), then $\psi_{stag} = 0$

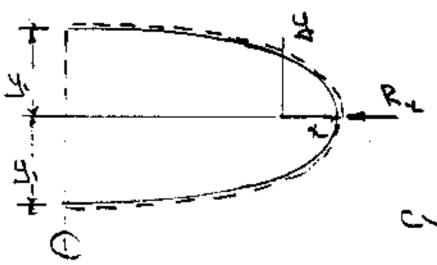
The equation of the stagnation streamline is then,

$\psi = 0 = Ur\sin\theta - \frac{q}{2\pi}\theta$ or $r_{stag} = \frac{q\theta}{2\pi U\sin\theta}$

Since $y = r\sin\theta$, then along the stagnation streamline $y = \frac{q\theta}{2\pi U}$.

For upstream, $\theta \to \pi$ and $y = y_1 \to \frac{q}{2U}$.

The surface shape formed by the stagnation streamline is then as follows:



There is no flow across this streamline. The flow in through the left face must be equal to the flow (q) which leaves through the sink at the origin.

Applying the x momentum equation to the CV shown. R_x is force required to hold shape in place

$-R_x = \int u\,\rho\vec{V}\cdot d\vec{A} = -U\dot{m}_1 = -U\rho q b$

$\therefore \frac{R_x}{b} = \rho q U$

For standard air $\rho = 1.225\ \text{kg/m}^3$ and

$\frac{R_x}{b} = 1.225\frac{\text{kg}}{\text{m}^3} \times 90\frac{\text{m}^2}{\text{s}} \times 50\frac{\text{m}}{\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 5.51\ \text{kN/m}$

$\vec{R}_x/b = -5.51\,\hat{i}\ \text{kN/m}$ $\xleftarrow{R_x/b}$

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Problem *6.117

[4] Part 1/2

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Given: Flow field obtained by combining a uniform flow in the +x direction ($U = 30$ m/s) and a source (of strength $q = 150$ m²/s) located at the origin.

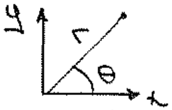
Plot: the ratio of the local velocity V to the free stream velocity U as a function of θ along the stagnation streamline.

Find: (a) points on the stagnation streamline where the velocity reaches its maximum value

(b) gage pressure at this location if $\rho = 1.2$ kg/m³

Solution:

Superposition of a uniform flow and source gives flow around a half body.



$$\psi = \psi_{uf} + \psi_{so} = Uy + \frac{q}{2\pi}\theta = U r \sin\theta + \frac{q}{2\pi}\theta \tag{1}$$

$$u = u_{uf} + u_{so}\ ;\ u_{uf} = U\ ;\ u_{so} = v_r\cos\theta = \frac{q}{2\pi r}\frac{x}{r} \qquad \therefore u = U + \frac{q}{2\pi}\frac{x}{r^2}$$

$$v = v_{uf} + v_{so}\ ;\ v_{uf} = 0\ ;\ v_{so} = v_r\sin\theta = \frac{q}{2\pi r}\frac{y}{r} \qquad \therefore v = \frac{q}{2\pi}\frac{y}{r^2}$$

$$\therefore \vec{V} = u\hat{i} + v\hat{j} = \left(U + \frac{q}{2\pi}\frac{x}{r^2}\right)\hat{i} + \frac{q}{2\pi}\frac{y}{r^2}\hat{j} \tag{2}$$

Then,

$$V^2 = u^2 + v^2 = \left(U + \frac{q}{2\pi r}\cos\theta\right)^2 + \left(\frac{q}{2\pi r}\sin\theta\right)^2$$

$$= U^2 + \left(\frac{q}{2\pi r}\right)^2\cos^2\theta + \frac{Uq}{\pi r}\cos\theta + \left(\frac{q}{2\pi r}\right)^2\sin^2\theta$$

$$V^2 = U^2 + \left(\frac{q}{2\pi r}\right)^2 + \frac{Uq}{\pi r}\cos\theta \tag{3}$$

To determine the equation of the stagnation streamline, we locate the stagnation point ($\vec{V} = 0$). From Eq. 2 $y = 0$ and

$$U + \frac{q}{2\pi}\frac{x}{r^2} = 0 = U + \frac{q}{2\pi}\frac{x}{(x^2 + \cancel{y^2})} = U + \frac{q}{2\pi x} \quad \text{and} \quad x_{stag} = -\frac{q}{2\pi U}$$

$$x_{stag} = -\frac{q}{2\pi U} = -\frac{1}{2\pi} \times 150\frac{m^2}{s} \times \frac{s}{30\,m} = -0.796\,m$$

At the stagnation point $y = 0$ and $\theta = \pi$. From Eq. 1 $\psi_{stag} = \frac{q}{2}$

the equation of the stagnation streamline is then:

$$\psi_{stag} = \frac{q}{2} = U r \sin\theta + \frac{q}{2\pi}\theta$$. Solving for r, we obtain

$$r = \frac{1}{U\sin\theta}\left(\frac{q}{2} - \frac{q}{2\pi}\theta\right) = \frac{q(\pi - \theta)}{2\pi U\sin\theta} \tag{4}$$

Substituting this value of r into the expression for V^2 [Eq. 3] we obtain

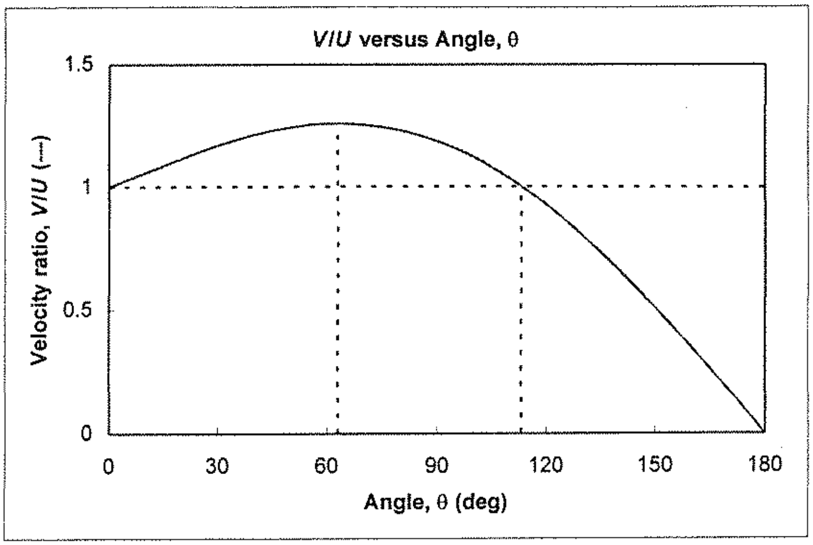
$$V^2 = U^2 + \left[\frac{\cancel{q}}{\cancel{2\pi}} \times \frac{2\pi U\sin\theta}{\cancel{q}(\pi - \theta)}\right]^2 + \frac{U\cancel{q}\cos\theta}{\cancel{\pi}} \times \frac{2\pi U\sin\theta}{\cancel{q}(\pi - \theta)}$$

$$V^2 = U^2 + \frac{U^2\sin^2\theta}{(\pi - \theta)^2} + \frac{2U^2\sin\theta\cos\theta}{(\pi - \theta)} = U^2\left[1 + \frac{\sin^2\theta}{(\pi - \theta)^2} + \frac{2\sin\theta\cos\theta}{(\pi - \theta)}\right]$$

Along the stagnation streamline

$$\frac{V}{U} = \left[1 + \frac{\sin^2\theta}{(\pi - \theta)^2} + \frac{2\sin\theta\cos\theta}{(\pi - \theta)}\right]^{1/2} \tag{5}$$

V/U is plotted as a function of θ



From the plot we see that V/U is a maximum at $\theta = 63°$ (also at $\theta = 297°$ from symmetry with respect to the x axis.

At $\theta = 63°$, Eq 5 gives $V/U_{max} = 1.26$

, Eq. 4 gives $r = 150\,\frac{m^2}{s} \times \frac{(\pi - 0.35\pi)}{2\pi \sin 63°} \times \frac{s}{30\,m} = 1.82\,m$.

Thus $V = V_{max}$ at $r = 1.82\,m$ and $\theta = 63°, 297°$ ← $(r,\theta)_{V_{max}}$

To determine the gage pressure at this point, write the Bernoulli equation between a point upstream and the point of maximum velocity

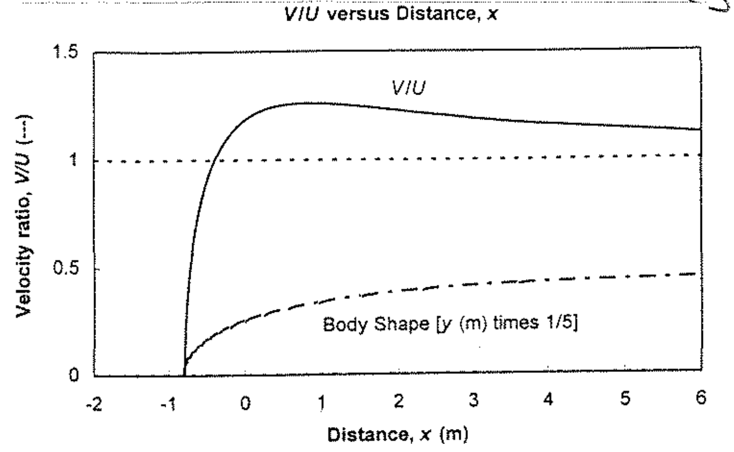
$$\frac{p_\infty}{\rho} + \frac{U^2}{2} = \frac{p}{\rho} + \frac{V_{max}^2}{2}$$

$$\therefore\ p - p_\infty = \frac{\rho}{2}\left[U^2 - V_{max}^2\right] = \frac{1}{2}\rho U^2\left[1 - \left(\frac{V_{max}}{U}\right)^2\right]$$

$$= \frac{1}{2} \times 1.2\,\frac{kg}{m^3} \times (30)^2\,\frac{m^2}{s^2}\left[1 - (1.26)^2\right] \times \frac{N \cdot s^2}{kg \cdot m}.$$

$p - p_\infty = 317\,N/m^2$ ← p_{gage}

Note: From the plot we see that $V/U = 1.0$, and hence $p = p_\infty$, at $\theta = 113°$. The corresponding r is 1.01 m.



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Given: Flow field obtained by superposing a uniform flow in the +x direction ($U = 25\ \text{m/s}$) and a source (of strength q) at the origin. Stagnation point is at $x = -1.0\ \text{m}$.

Find: (a) expressions for ψ, ϕ, $\vec{V}$
(b) source strength, q.

Plot: streamlines and potential lines.

Solution:

$$\psi = \psi_{u.f} + \psi_{so} = Uy + \frac{q}{2\pi}\theta = Ur\sin\theta + \frac{q}{2\pi}\theta \qquad \psi$$

$$\phi = \phi_{u.f} + \phi_{so} = -Ux - \frac{q}{2\pi}\ln r = -Ur\cos\theta - \frac{q}{2\pi}\ln r \qquad \phi$$

$$u = u_{u.f} + u_{so}\ ;\ u_{u.f} = U\ ;\ u_{so} = V_r\cos\theta = \frac{q}{2\pi r}\frac{x}{r} \qquad \therefore u = U + \frac{q}{2\pi}\frac{x}{r^2}$$

$$v = v_{u.f} + v_{so}\ ;\ v_{u.f} = 0\ ;\ v_{so} = V_r\sin\theta = \frac{q}{2\pi r}\frac{y}{r} \qquad \therefore v = \frac{q}{2\pi}\frac{y}{r^2}$$

$$\vec{V} = u\hat{i} + v\hat{j} = \left\{U + \frac{q}{2\pi}\frac{x}{(x^2+y^2)}\right\}\hat{i} + \frac{q}{2\pi}\frac{y}{(x^2+y^2)}\hat{j} \qquad \vec{V}$$

At the stagnation point $\vec{V} = 0$ $\quad x = -1.0\ \text{m}$ $\quad y = 0\ (v = 0)$.

For $u = 0 = U + \frac{q}{2\pi}\frac{x}{(x^2+\cancel{y^2})}$ $\quad \therefore q = -2\pi U x_{stag}$

$$q = -2\pi \times 25\frac{\text{m}}{\text{s}} \times (-1.0\ \text{m}) = 50\pi\ \text{m}^2/\text{s} \qquad q$$

At the stagnation point, $\theta = \pi$ $\quad \therefore \psi_{stag} = \frac{q}{2\pi}\theta = \frac{q}{2}$

The equation of the stagnation streamline is then

$$q/2 = Ur\sin\theta + \frac{q}{2\pi}\theta \qquad \text{and} \qquad r = \frac{q(\pi-\theta)}{2\pi U\sin\theta}$$

At $\theta = \pi/2$, $\quad r = \frac{q}{4U} = 50\pi\frac{\text{m}^2}{\text{s}} \times \frac{1}{4} \times \frac{\text{s}}{25\ \text{m}} = \pi/2$

Far downstream $\theta \to 0$ and the y coordinate of the body

$$y = r\sin\theta = \frac{q(\pi-\theta)}{2\pi U} \text{ approaches } \frac{q}{2U} = \frac{50\pi}{2 \times 25} = \pm\pi\ \text{m} \qquad y_{\theta\to 0}$$

See the next page for plots

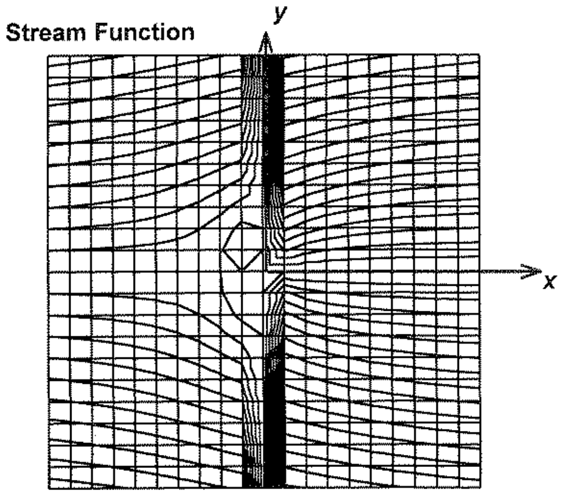
Problem *6.118

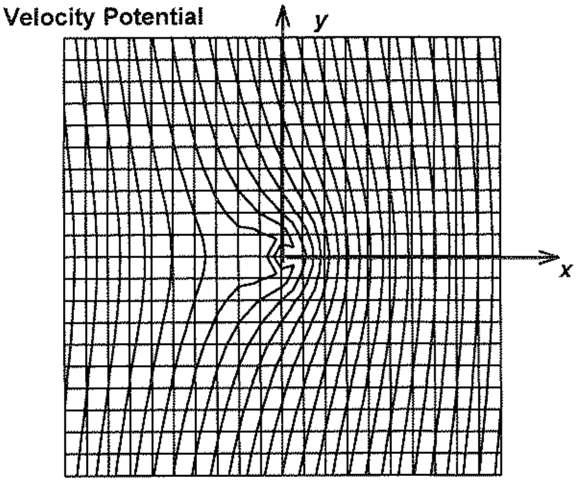
Using *Excel*, the stream function and velocity potential can be plotted.
The data below was obtained using the workbook for Example Problem 6.10.
Note the orthogonality of ψ and ϕ!

#NAME? **Stream Function**

#NAME? **Velocity Potential**

Note that the plot is
from $x = -5$ to 5 and $y = -5$ to 5





Problem 7.1

[2]

Given: The slope of the free surface of a steady wave in one-dimensional flow in a shallow liquid layer is described by the equation

$$\frac{\partial h}{\partial x} = -\frac{u}{g}\frac{\partial u}{\partial x}$$

Find: Nondimensionalize the equation (using length scale, L, and velocity scale, V_0)
Obtain the dimensionless groups that characterize this flow.

Solution:

To nondimensionalize the equation, all lengths are divided by the reference length, L, and all vecolities are divided by the reference velocity, V_0

Denoting the nondimensional quantities by an asterisk,

$$h^* = \frac{h}{L}\quad , \quad x^* = \frac{x}{L}\quad , \quad u^* = \frac{u}{V_0}$$

Substituting into the governing equation

$$\frac{\partial(h^* L)}{\partial(x^* L)} = -\frac{V_0 u^*}{g}\,\frac{\partial(V_0 u^*)}{\partial(L x^*)}$$

$$\frac{\partial h^*}{\partial x^*} = -\frac{V_0^2}{gL}\,\frac{\partial u^*}{\partial x^*}$$

The dimensionless group is $\frac{V_0^2}{gL}$ This is the square of the Froude number.

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Problem 7.2

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Given: The propagation speed of small amplitude waves in a region of uniform depth is given by

$$c^2 = \left(\frac{\sigma}{\rho}\frac{2\pi}{\lambda} + \frac{g\lambda}{2\pi}\right)\tanh\frac{2\pi h}{\lambda}$$

where h is the depth of the undisturbed liquid

λ is the wavelength.

Find: Obtain the dimensionless groups that characterize the equation. (Use L as a characteristic length and V_0 as a characteristic velocity)

Solution:

$$c^2 = \left(\frac{\sigma}{\rho}\frac{2\pi}{\lambda} + \frac{g\lambda}{2\pi}\right)\tanh\frac{2\pi h}{\lambda}$$

To nondimensionalize the equation, all lengths are divided by L and all velocities are divided by V_0.

Denoting nondimensional quantities by an asterisk, then

$$\lambda^* = \frac{\lambda}{L} \qquad h^* = \frac{h}{L} \qquad c^* = \frac{c}{V_0}$$

Then

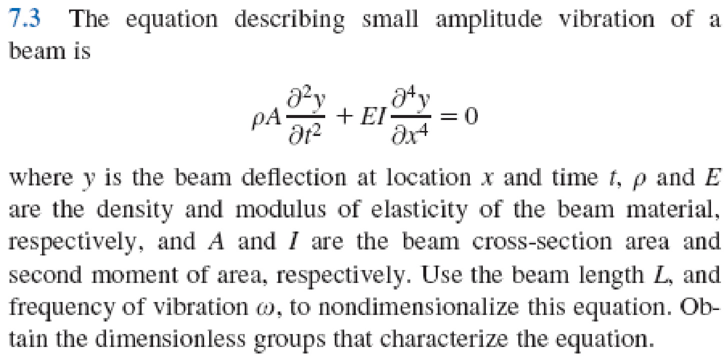
$$c^{*2} V_0^2 = \left(\frac{\sigma}{\rho}\frac{2\pi}{L\lambda^*} + \frac{g\lambda^* L}{2\pi}\right)\tanh\frac{2\pi h^* L}{\lambda^* L}$$

$$c^{*2} = \left(\frac{\sigma}{\rho L V_0^2}\frac{2\pi}{\lambda^*} + \frac{gL}{V_0^2}\frac{\lambda^*}{2\pi}\right)\tanh\frac{2\pi h^*}{\lambda^*}$$

$\therefore$ Dimensionless groups are $\frac{\sigma}{\rho L V_0^2}$, $\frac{gL}{V_0^2}$

Problem 7.3

[2]



7.3 The equation describing small amplitude vibration of a beam is

$$\rho A\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = 0$$

where y is the beam deflection at location x and time t, ρ and E are the density and modulus of elasticity of the beam material, respectively, and A and I are the beam cross-section area and second moment of area, respectively. Use the beam length L, and frequency of vibration ω, to nondimensionalize this equation. Obtain the dimensionless groups that characterize the equation.

Given: Equation for beam

Find: Dimensionless groups

Solution:

Denoting nondimensional quantities by an asterisk

$$A^* = \frac{A}{L^2} \qquad y^* = \frac{y}{L} \qquad t^* = t\omega \qquad I^* = \frac{I}{L^4} \qquad x^* = \frac{x}{L}$$

Hence $$A = L^2 A^* \qquad y = L\,y^* \qquad t = \frac{t^*}{\omega} \qquad I = L^4 I^* \qquad x = L\,x^*$$

Substituting into the governing equation $$\rho L^2 L\omega^2 A^* \frac{\partial^2 y^*}{\partial t^{*2}} + EL^4\frac{1}{L^4}LI^*\frac{\partial^4 y^*}{\partial x^{*4}} = 0$$

The final dimensionless equation is $$A^*\frac{\partial^2 y^*}{\partial t^{*2}} + \left(\frac{E}{\rho L^2\omega^2}\right)I^*\frac{\partial^4 y^*}{\partial x^{*4}} = 0$$

The dimensionless group is $$\left(\frac{E}{\rho L^2\omega^2}\right)$$

Problem 7.4

Given: One-dimensional, unsteady flow in a thin liquid layer is described by the equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x}$$

Find: Nondimensionalize the equation (using length scale, L, and velocity scale, V_0)
Obtain the dimensionless groups that characterize this flow.

Solution:

To nondimensionalize the equation, all lengths are divided by the reference length, L, velocity is divided by the reference velocity, V_0, and time is divided by the ratio, L/V_0.

Denoting the nondimensional quantities by an asterisk,

$$x^* = \frac{x}{L}, \quad h^* = \frac{h}{L}, \quad u^* = \frac{u}{V_0}, \quad t^* = \frac{t}{L/V_0}$$

Substituting into the governing equation

$$\frac{\partial(V_0 u^*)}{\partial(L t^*/V_0)} + u^* V_0 \frac{\partial(V_0 u^*)}{\partial(x^* L)} = -g\frac{\partial(h^* L)}{\partial(x^* L)}$$

$$\frac{V_0^2}{L}\frac{\partial u^*}{\partial t^*} + \frac{V_0^2}{L}u^*\frac{\partial u^*}{\partial x^*} = -g\frac{\partial h^*}{\partial x^*}$$

Multiplying through by L/V_0^2,

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} = -\frac{gL}{V_0^2}\frac{\partial h^*}{\partial x^*}$$

The dimensionless group is $\frac{gL}{V_0^2}$. This is one over the square of the Froude number.

Problem 7.5

Given: For steady, incompressible, two-dimensional flow, the Prandtl boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{-------- (1)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \quad \text{---(2)}$$

Find: Nondimensionalize these equations (using L and V_0 as characteristic length and velocity, respectively) and identify the resulting similarity parameters.

Solution:

Denoting nondimensional quantities by an asterisk.

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L} \quad u^* = \frac{u}{V_0} \quad v^* = \frac{v}{V_0}$$

Substituting into Eq. 1, we obtain

$$\frac{\partial(u^* V_0)}{\partial(x^* L)} + \frac{\partial(v^* V_0)}{\partial(y^* L)} = 0 = \frac{V_0}{L}\frac{\partial u^*}{\partial x^*} + \frac{V_0}{L}\frac{\partial v^*}{\partial y^*}$$

or

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Consider each term in Eq. 2.

$$u\frac{\partial u}{\partial x} = u^* V_0 \frac{\partial(u^* V_0)}{\partial(x^* L)} = \frac{V_0^2}{L} u^* \frac{\partial u^*}{\partial x^*}$$

$$v\frac{\partial u}{\partial y} = v^* V_0 \frac{\partial(u^* V_0)}{\partial(y^* L)} = \frac{V_0^2}{L} v^* \frac{\partial u^*}{\partial x^*}$$

Leave 3rd term as is for the moment

$$\nu\frac{\partial^2 u}{\partial y^2} = \nu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \nu\frac{\partial}{\partial y}\left(\frac{\partial u^* V_0}{\partial y^* L}\right) = \nu\frac{V_0}{L}\frac{\partial}{\partial y}\left(\frac{\partial u^*}{\partial y^*}\right) = \nu\frac{V_0}{L}\frac{\partial}{\partial(y^* L)}\frac{\partial u^*}{\partial y^*} = \nu\frac{V_0}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Substituting into Eq. 2.

$$\frac{V_0^2}{L} u^* \frac{\partial u^*}{\partial x^*} + \frac{V_0^2}{L} v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{V_0}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Multiplying through by $\frac{L}{V_0^2}$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{L}{\rho V_0^2}\frac{\partial p}{\partial x} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}} = -L\frac{\partial\left(\frac{p}{\rho V_0^2}\right)}{\partial(x^* L)} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

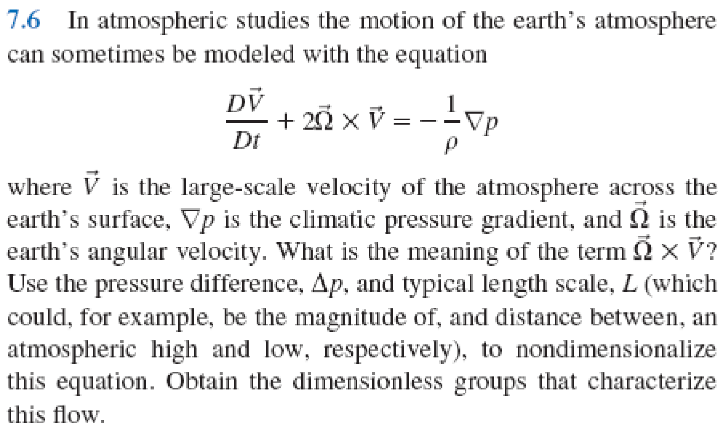
Define the non-dimensional pressure $p^* = \frac{p}{\rho V_0^2}$, then

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y} = -\frac{\partial p^*}{\partial x^*} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The similarity parameter is $\frac{\nu}{V_0 L} = \frac{1}{Re}$

Problem 7.6

[2]



7.6 In atmospheric studies the motion of the earth's atmosphere can sometimes be modeled with the equation

$$\frac{D\vec{V}}{Dt} + 2\vec{\Omega} \times \vec{V} = -\frac{1}{\rho}\nabla p$$

where $\vec{V}$ is the large-scale velocity of the atmosphere across the earth's surface, ∇p is the climatic pressure gradient, and $\vec{\Omega}$ is the earth's angular velocity. What is the meaning of the term $\vec{\Omega} \times \vec{V}$? Use the pressure difference, Δp, and typical length scale, L (which could, for example, be the magnitude of, and distance between, an atmospheric high and low, respectively), to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given:

Equations for modeling atmospheric motion

Find:

Non-dimensionalized equation; Dimensionless groups

Solution:

Recall that the total acceleration is

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$$

Nondimensionalizing the velocity vector, pressure, angular velocity, spatial measure, and time, (using a typical velocity magnitude V and angular velocity magnitude Ω):

$$\vec{V}* = \frac{\vec{V}}{V} \qquad p* = \frac{p}{\Delta p} \qquad \vec{\Omega}* = \frac{\vec{\Omega}}{\Omega} \qquad x* = \frac{x}{L} \qquad t* = t\frac{V}{L}$$

Hence

$$\vec{V} = V\vec{V}* \qquad p = \Delta p\, p* \qquad \vec{\Omega} = \Omega\vec{\Omega}* \qquad x = L\,x* \qquad t = \frac{L}{V}t*$$

Substituting into the governing equation

$$V\frac{V}{L}\frac{\partial \vec{V}*}{\partial t*} + V\frac{V}{L}\vec{V}*\cdot\nabla*\vec{V}* + 2\Omega V\,\vec{\Omega}*\times\vec{V}* = -\frac{1}{\rho}\frac{\Delta p}{L}\nabla p*$$

The final dimensionless equation is

$$\frac{\partial \vec{V}*}{\partial t*} + \vec{V}*\cdot\nabla*\vec{V}* + 2\left(\frac{\Omega L}{V}\right)\vec{\Omega}*\times\vec{V} = -\frac{\Delta p}{\rho V^2}\nabla p*$$

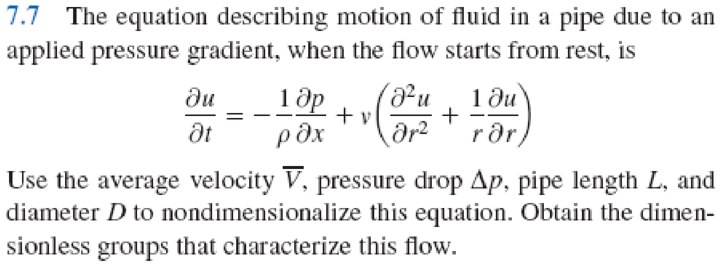
The dimensionless groups are

$$\frac{\Delta p}{\rho \vec{V}^2} \qquad \frac{\Omega L}{V}$$

The second term on the left of the governing equation is the Coriolis force due to a rotating coordinate system. This is a very significant term in atmospheric studies, leading to such phenomena as geostrophic flow.

Problem 7.7

[2]



7.7 The equation describing motion of fluid in a pipe due to an applied pressure gradient, when the flow starts from rest, is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

Use the average velocity $\overline{V}$, pressure drop Δp, pipe length L, and diameter D to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given: Equations Describing pipe flow

Find: Non-dimensionalized equation; Dimensionless groups

Solution:

Nondimensionalizing the velocity, pressure, spatial measures, and time:

$$u^* = \frac{u}{\overline{V}} \qquad p^* = \frac{p}{\Delta p} \qquad x^* = \frac{x}{L} \qquad r^* = \frac{r}{L} \qquad t^* = t\frac{\overline{V}}{L}$$

Hence

$$u = \overline{V}\,u^* \qquad p = \Delta p\,p^* \qquad x = L\,x^* \qquad r = D\,r^* \qquad t = \frac{L}{\overline{V}}t^*$$

Substituting into the governing equation

$$\frac{\partial u}{\partial t} = \overline{V}\frac{\overline{V}}{L}\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\Delta p\frac{1}{L}\frac{\partial p^*}{\partial x^*} + \nu\overline{V}\frac{1}{D^2}\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

The final dimensionless equation is

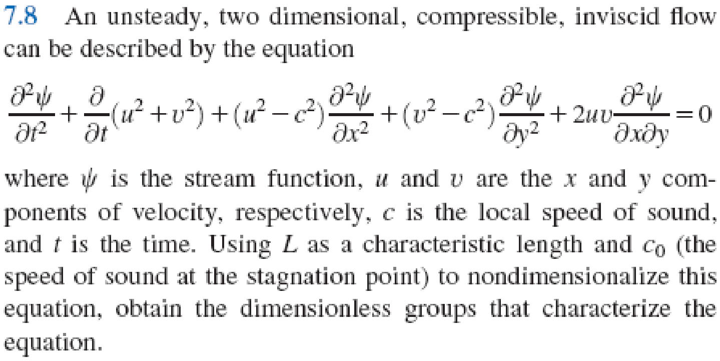
$$\frac{\partial u^*}{\partial t^*} = -\frac{\Delta p}{\rho\overline{V}^2}\frac{\partial p^*}{\partial x^*} + \left(\frac{\nu}{D\overline{V}}\right)\left(\frac{L}{D}\right)\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

The dimensionless groups are

$$\frac{\Delta p}{\rho\overline{V}^2} \qquad \frac{\nu}{D\overline{V}} \qquad \frac{L}{D}$$

Problem 7.8

[2]



7.8 An unsteady, two dimensional, compressible, inviscid flow can be described by the equation

$$\frac{\partial^2\psi}{\partial t^2}+\frac{\partial}{\partial t}(u^2+v^2)+(u^2-c^2)\frac{\partial^2\psi}{\partial x^2}+(v^2-c^2)\frac{\partial^2\psi}{\partial y^2}+2uv\frac{\partial^2\psi}{\partial x\partial y}=0$$

where ψ is the stream function, u and v are the x and y components of velocity, respectively, c is the local speed of sound, and t is the time. Using L as a characteristic length and c_0 (the speed of sound at the stagnation point) to nondimensionalize this equation, obtain the dimensionless groups that characterize the equation.

Given: Equation for unsteady, 2D compressible, inviscid flow

Find: Dimensionless groups

Solution:

Denoting nondimensional quantities by an asterisk

$$x^*=\frac{x}{L} \qquad y^*=\frac{y}{L} \qquad u^*=\frac{u}{c_0} \qquad v^*=\frac{v}{c_0} \qquad c^*=\frac{c}{c_0} \qquad t^*=\frac{t\,c_0}{L} \qquad \psi^*=\frac{\psi}{L\,c_0}$$

Note that the stream function indicates volume flow rate/unit depth!

Hence

$$x=L\,x^* \qquad y=L\,y^* \qquad u=c_0\,u^* \qquad v=c_0\,v^* \qquad c=c_0\,c^* \qquad t=\frac{L\,t^*}{c_0} \qquad \psi=L\,c_0\,\psi^*$$

Substituting into the governing equation

$$\left(\frac{c_0^3}{L}\right)\frac{\partial^2\psi^*}{\partial t^{*2}}+\left(\frac{c_0^3}{L}\right)\frac{\partial\left(u^{*2}+v^{*2}\right)}{\partial t}+\left(\frac{c_0^3}{L}\right)\left(u^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial x^{*2}}+\left(\frac{c_0^3}{L}\right)\left(v^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial y^{*2}}+\left(\frac{c_0^3}{L}\right)2u^*v^*\frac{\partial^2\psi^*}{\partial x^*\partial y^*}=0$$

The final dimensionless equation is

$$\frac{\partial^2\psi^*}{\partial t^{*2}}+\frac{\partial\left(u^{*2}+v^{*2}\right)}{\partial t}+\left(u^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial x^{*2}}+\left(v^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial y^{*2}}+2u^*v^*\frac{\partial^2\psi^*}{\partial x^*\partial y^*}=0$$

No dimensionless group is needed for this equation!

Problem 7.9

Given: At low speeds, drag is independent of fluid density.

$F = F(\mu, V, D)$

Find: Appropriate dimensionless parameters.

Solution: Apply Buckingham Π procedure.

① $F \quad \mu \quad V \quad D$ $\qquad n = 4$ parameters

② Select primary dimensions M, L, t.

③
$$\begin{array}{cccc} F & \mu & V & D \\ \frac{ML}{t^2} & \frac{M}{Lt} & \frac{L}{t} & L \end{array}$$
$r = 3$ primary dimensions

④ μ, V, D $\quad m = r = 3$ repeating parameters

⑤ Then $n - m = 1$ dimensionless group will result. Setting up a dimensional equation,

$$\Pi_1 = \mu^a V^b D^c F$$
$$= \left(\frac{M}{Lt}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & a + 1 = 0 & a = -1 \\ L: & -a + b + c + 1 = 0 & c = -1 \\ t: & -a - b - 2 = 0 & b = -1 \end{array} \qquad \therefore \Pi_1 = \frac{F}{\mu V D}$$

Π_1

⑥ Check, using F, L, t primary dimensions.

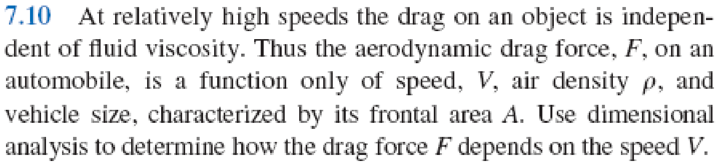
$$\Pi_1 = F \frac{L^2}{Ft} \frac{t}{L} \frac{1}{L} = [1] \quad \checkmark$$

Since the procedure produces only one dimensionless group, it must be a constant. Thus

$$\Pi_1 = \frac{F}{\mu V D} \quad \text{or} \quad F \propto \mu V D$$

Problem 7.10

[2]



7.10 At relatively high speeds the drag on an object is independent of fluid viscosity. Thus the aerodynamic drag force, F, on an automobile, is a function only of speed, V, air density ρ, and vehicle size, characterized by its frontal area A. Use dimensional analysis to determine how the drag force F depends on the speed V.

Given: That drag depends on speed, air density and frontal area

Find: How drag force depend on speed

Solution:

Apply the Buckingham Π procedure

① $F \quad V \quad \rho \quad A$ $\qquad n = 4$ parameters

② Select primary dimensions M, L, t

③
$$\begin{array}{cccc} F & V & \rho & A \\ \dfrac{ML}{t^2} & \dfrac{L}{t} & \dfrac{M}{L^3} & L^2 \end{array}$$
$r = 3$ primary dimensions

④ $V \quad \rho \quad A$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 1$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = V^a \rho^b A^c F$$

$$= \left(\frac{L}{t}\right)^a \left(\frac{M}{L^3}\right)^b \left(L^2\right)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rc|l} M: & b + 1 = 0 & b = -1 \\ L: & a - 3b + 2c + 1 = 0 & c = -1 \\ t: & -a - 2 = 0 & a = -2 \end{array}$$

Hence

$$\Pi_1 = \frac{F}{\rho V^2 A}$$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{F}{\dfrac{Ft^2}{L^4}\dfrac{L^2}{t^2}L^2} = [1]$$

The relation between drag force F and speed V must then be

$$F \propto \rho V^2 A \propto V^2$$

The drag is proportional to the *square* of the speed.

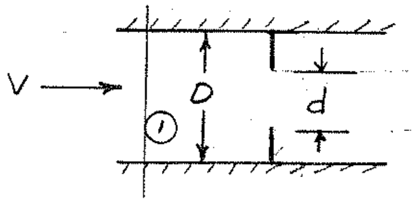
Problem 7.11

[2]

Given: Flow through an orifice plate

$$\Delta p = p_1 - p_2 = f(\rho, \mu, V, D, d)$$

Find: Dimensionless parameters.



Solution: Choose ρ, V, and D as repeating variables.

① $\Delta p \quad \rho \quad \mu \quad V \quad D \quad d \qquad n = 6$ parameters

② select primary dimensions M, L, t

③ $\Delta p \quad \rho \quad \mu \quad V \quad D \quad d$

$\frac{M}{Lt^2} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad \frac{L}{t} \quad L \quad L \qquad r = 3$ primary dimensions

④ $\rho, V, D \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result. Setting up dimensional equations,

$$\Pi_1 = \rho^a V^b D^c \Delta p = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{M}{Lt^2}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + b + c - 1 = 0$
$t: -b - 2 = 0 \qquad b = -2$

$c = 1 - b + 3a = 0$

$$\therefore \Pi_1 = \frac{\Delta p}{\rho V^2}$$

$$\Pi_2 = \rho^a V^b D^c \mu = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{M}{Lt}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + b + c - 1 = 0$
$t: -b - 1 = 0 \qquad b = -1$

$c = 1 - b + 3a = -1$

$$\therefore \Pi_2 = \frac{\mu}{\rho V D}$$

$$\Pi_3 = \rho^a V^b D^c d = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c L = M^0 L^0 t^0$$

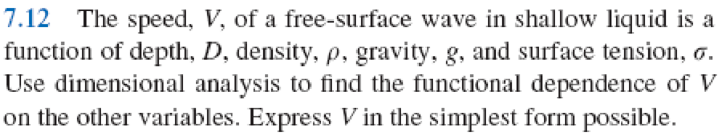
$$\left.\begin{array}{ll} M: a + 0 = 0 & a = 0 \\ L: -3a + b + c + 1 = 0 & \\ t: -b + 0 = 0 & b = 0 \end{array}\right\} c = -1 \; ; \; \Pi_3 = \frac{d}{D}$$

Thus $\Pi_1 = f(\Pi_2, \Pi_3)$ or $\frac{\Delta p}{\rho V^2} = f\left(\frac{\mu}{\rho V D}, \frac{d}{D}\right) \longleftarrow \frac{\Delta p}{\rho V^2}$

⑥ Check, using F, L, t $\quad \Pi_1 = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{t^2}{L^2} = [1]$ ✓, $\Pi_2 = Re = [1]$ ✓, $\Pi_3 = \frac{L}{L} = [1]$ ✓

Problem 7.12

[2]



7.12 The speed, V, of a free-surface wave in shallow liquid is a function of depth, D, density, ρ, gravity, g, and surface tension, σ. Use dimensional analysis to find the functional dependence of V on the other variables. Express V in the simplest form possible.

Given: That speed of shallow waves depends on depth, density, gravity and surface tension

Find: Dimensionless groups; Simplest form of V

Solution:

Apply the Buckingham Π procedure

① $V \quad D \quad \rho \quad g \quad \sigma$ $\qquad n = 5$ parameters

② Select primary dimensions M, L, t

③
$$\left\{ \begin{array}{ccccc} V & D & \rho & g & \sigma \\ \dfrac{L}{t} & L & \dfrac{M}{L^3} & \dfrac{L}{t^2} & \dfrac{M}{t^2} \end{array} \right\}$$
$r = 3$ primary dimensions

④ $g \quad \rho \quad D$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = g^a \rho^b D^c V = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{L}{t} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rl|l} M: & b = 0 & b = 0 \\ L: & a - 3b + c + 1 = 0 & c = -\dfrac{1}{2} \\ t: & -2a - 1 = 0 & a = -\dfrac{1}{2} \end{array}$$
Hence $\quad \Pi_1 = \dfrac{V}{\sqrt{gD}}$

$$\Pi_2 = g^a \rho^b D^c \sigma = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{M}{t^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rl|l} M: & b + 1 = 0 & b = -1 \\ L: & a - 3b + c = 0 & c = -2 \\ t: & -2a - 2 = 0 & a = -1 \end{array}$$
Hence $\quad \Pi_2 = \dfrac{\sigma}{g\rho D^2}$

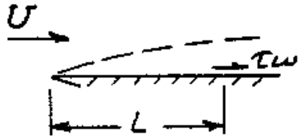
⑥ Check using F, L, t as primary dimensions
$$\Pi_1 = \frac{\dfrac{L}{t}}{\left(\dfrac{L}{t^2} L\right)^{\frac{1}{2}}} = [1] \qquad \Pi_2 = \frac{\dfrac{F}{L}}{\dfrac{L}{t^2} \dfrac{Ft^2}{L^4} L^2} = [1]$$

The relation between drag force speed V is
$$\Pi_1 = f(\Pi_2) \qquad \frac{V}{\sqrt{gD}} = f\left(\frac{\sigma}{g\rho D^2}\right) \qquad V = \sqrt{gD}\, f\left(\frac{\sigma}{g\rho D^2}\right)$$

Problem 7.13

[2]

Given: Wall shear stress, τ_w, in a boundary layer, depends on ρ, μ, L, and U.



Find: (a) Dimensionless groups.
(b) Express the functional relationship.

Solution: Step ① $\tau_w \quad \rho \quad \mu \quad L \quad U \qquad n=5$

Step ② Choose M, L, t. $\tau_w = \frac{F}{L^2} \times \frac{ML}{Ft^2} = \frac{M}{Lt^2}$

Step ③ $\frac{M}{Lt^2} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad L \quad \frac{L}{t} \qquad r=3$

Step ④ Select ρ, L, U

Step ⑤ $\Pi_1 = \tau_w \rho^a L^b U^c = \frac{M}{Lt^2}\left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{L}{t}\right)^c = M^0 L^0 t^0$

$$\left.\begin{aligned} M&: 0 = 1+a & a&=-1 \\ L&: 0 = -1-3a+b+c & b&=3a-c+1=0 \\ t&: 0=-2-c & c&=-2 \end{aligned}\right\} \Pi_1 = \frac{\tau_w}{\rho U^2}$$

Π_1

$\Pi_2 = \mu \rho^a L^b U^c = \frac{M}{Lt}\left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{L}{t}\right)^c = M^0 L^0 t^0$

$$\left.\begin{aligned} M&: 0 = 1+a & a&=-1 \\ L&: 0 = -1-3a+b+c & b&=3a-c+1=-1 \\ t&: 0=-1-c & c&=-1 \end{aligned}\right\} \Pi_2 = \frac{\mu}{\rho U L}$$

Π_2

Step ⑥: Check using F, L, t: $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{\tau_w}{\rho U^2} = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{t^2}{L^2} = \frac{FL^4t^2}{FL^4t^2} = 1$ ✓✓

$\Pi_2 = \frac{\mu}{\rho U L} = \frac{Ft}{L^2}\frac{L^4}{Ft^2}\frac{t}{L}\frac{1}{L} = \frac{FL^4t^2}{FL^4t^2} = 1$ ✓✓

The functional relationship is

$\Pi_1 = f(\Pi_2)$

f

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Problem 7.14

[2]______

Given: The boundary layer thickness, δ, on a smooth flat plate in incompressible flow without pressure gradient is a function of U (free stream velocity), ρ, μ, and x (distance)

Find: suitable dimensionless parameters

Solution: Apply Buckingham π-theorem

① $\delta \quad U \quad \rho \quad \mu \quad x$ $\qquad$ $n = 5$ parameters

② Select M, L, t as primary dimensions

③ $\delta \quad U \quad \rho \quad \mu \quad x$

$L \quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad L$ $\qquad$ $r = 3$ primary dimensions

④ ρ, U, x $\qquad$ $m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result.

Setting up dimensional equations.

$$\pi_1 = \rho^a U^b x^c \delta$$

$$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c L$$

Equating exponents,

M: $0 = a$ $\qquad \therefore a = 0$

L: $0 = -3a + b + c + 1$ $\qquad c = -1$

t: $0 = -b$ $\qquad \therefore b = 0$

$$\therefore \pi_1 = \frac{\delta}{x}$$

$$\pi_2 = \rho^a U^b x^c \mu$$

$$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$$

Equating exponents,

M: $0 = a + 1$ $\qquad \therefore a = -1$

L: $0 = -3a + b + c - 1$ $\qquad c = -1$

t: $0 = -b - 1$ $\qquad \therefore b = -1$

$$\pi_2 = \frac{\mu}{\rho U x}$$

and $\dfrac{\delta}{x} = f\left(\dfrac{\rho U x}{\mu}\right)$ $\longleftarrow$ $\dfrac{\delta}{x}$

⑥ Check using F, L, t dimensions

$$\pi_1 = \frac{L}{L} = [1] \checkmark$$

$$\pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \frac{t}{L} \frac{1}{L} = [1] \checkmark$$

Problem 7.15

[2]

7.15 If an object is light enough it can be supported on the surface of a fluid by surface tension. Tests are to be done to investigate this phenomenon. The weight, W, supportable in this way depends on the object's perimeter, p, and the fluid's density, ρ, surface tension σ, and gravity, g. Determine the dimensionless parameters that characterize this problem.

Given: That light objects can be supported by surface tension

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $W \quad p \quad \rho \quad g \quad \sigma$ $\qquad$ $n = 5$ parameters

② Select primary dimensions M, L, t

③ $$\left\{\begin{array}{ccccc} W & p & \rho & g & \sigma \\ \dfrac{ML}{t^2} & L & \dfrac{M}{L^3} & \dfrac{L}{t^2} & \dfrac{M}{t^2} \end{array}\right\}$$ $r = 3$ primary dimensions

④ $g \quad \rho \quad p$ $\qquad$ $m = r = 3$ repeat parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = g^a \rho^b p^c W = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rr|l} M: & b+1=0 & b=-1 \\ L: & a-3b+c+1=0 & c=-3 \\ t: & -2a-2=0 & a=-1 \end{array}$$
Hence $\Pi_1 = \dfrac{W}{g\rho p^3}$

$$\Pi_2 = g^a \rho^b p^c \sigma = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{M}{t^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rr|l} M: & b+1=0 & b=-1 \\ L: & a-3b+c=0 & c=-2 \\ t: & -2a-2=0 & a=-1 \end{array}$$
Hence $\Pi_2 = \dfrac{\sigma}{g\rho p^2}$

⑥ Check using F, L, t as primary dimensions $\quad \Pi_1 = \dfrac{F}{\dfrac{L}{t^2}\dfrac{Ft^2}{L^4}L^3} = [1] \qquad \Pi_2 = \dfrac{\dfrac{F}{L}}{\dfrac{L}{t^2}\dfrac{Ft^2}{L^4}L^2} = [1]$

Note: Any combination of Π_1 and Π_2 is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{Wp}{\sigma}$, so Π_1 and Π_2 are not unique!

Problem 7.16

[2]

Given: The mean velocity, $\bar{u}$, for turbulent pipe or boundary layer flow, may be correlated in terms of the wall shear stress, τ_w, distance from the wall, y, and fluid properties, ρ and μ.

Find: (a) dimensionless parameter containing $\bar{u}$ and one containing y that are suitable for organizing experimental data

(b) show that the result may be written as

$$\frac{\bar{u}}{u_*} = f\left(\frac{y u_*}{\nu}\right) \quad \text{where } u_* = (\tau_w/\rho)^{1/2}$$

Solution: Apply the Buckingham π-Theorem

① $\bar{u} \quad \tau_w \quad y \quad \rho \quad \mu \qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③ $\frac{L}{t} \quad \frac{M}{Lt^2} \quad L \quad \frac{M}{L^3} \quad \frac{M}{Lt}$

④ $\tau_w, y, \rho \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$$\pi_1 = \tau_w^a y^b \rho^c \bar{u} \qquad\qquad \pi_2 = \tau_w^a y^b \rho^c \mu$$

$$M^0L^0t^0 = \left(\frac{M}{Lt^2}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{L}{t} \qquad\qquad M^0L^0t^0 = \left(\frac{M}{Lt^2}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt}$$

Summing exponents

M: $a + c = 0 \quad \therefore a = -c$

L: $-a + b - 3c + 1 = 0$

t: $-2a - 1 = 0 \quad \therefore a = -1/2$

$a = -1/2, \; c = 1/2, \; b = 0$

$$\pi_1 = \bar{u}\frac{\rho^{1/2}}{\tau_w^{1/2}} = \frac{\bar{u}}{\sqrt{\tau_w/\rho}}$$

Summing exponents

M: $a + c + 1 = 0 \quad \therefore c = -a - 1$

L: $-a + b - 3c - 1 = 0$

t: $-2a - 1 = 0 \quad \therefore a = -1/2$

$a = -1/2, \; c = -1/2, \; b = -1$

$$\pi_2 = \frac{\mu}{\tau_w^{1/2}\rho^{1/2} y} = \frac{\mu}{\rho y \sqrt{\frac{\tau_w}{\rho}}}$$

$$\pi_1 = f(\pi_2) \quad \text{or} \quad \frac{\bar{u}}{\sqrt{\tau_w/\rho}} = f\left(\frac{\mu}{\rho y \sqrt{\tau_w/\rho}}\right)$$

Since $\sqrt{\tau_w/\rho} = u_*$, then

$$\frac{\bar{u}}{u_*} = f\left(\frac{\mu}{\rho y u_*}\right) = f\left(\frac{\nu}{y u_*}\right) = g\left(\frac{y u_*}{\nu}\right) \longleftarrow \frac{\bar{u}}{u_*}$$

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Problem 7.17

[2]

Given: Velocity, V, of a free surface gravity wave in deep water is a function of λ (wavelength) D, ρ, and g

Find: Dependence of V on other variables.

Solution: Apply Buckingham Π-theorem

① $V \quad \lambda \quad D \quad \rho \quad g$ $\qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③
$$\begin{array}{ccccc} V & \lambda & D & \rho & g \\ \frac{L}{t} & L & L & \frac{M}{L^3} & \frac{L}{t^2} \end{array}$$
$r = 3$ primary dimensions

④ ρ, D, g $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$$\Pi_1 = \rho^a D^b g^c V$$

$$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c \frac{L}{t}$$

Summing exponents,

$M: \quad a = 0$
$L: \quad -3a + b + c + 1 = 0$
$t: \quad -2c - 1 = 0$

i.e. $a = 0$
$c = -\frac{1}{2}$
$b = 3a - c - 1 = -\frac{1}{2}$

$$\therefore \Pi_1 = \frac{V}{\sqrt{gD}}$$

$$\Pi_2 = \rho^a D^b g^c \lambda$$

$$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c L$$

Summing exponents,

$M: \quad a = 0$
$L: \quad -3a + b + c + 1 = 0$
$t: \quad -2c = 0$

i.e. $a = 0$
$c = 0$
$b = 3a - c - 1 = -1$

$$\therefore \Pi_2 = \frac{\lambda}{D}$$

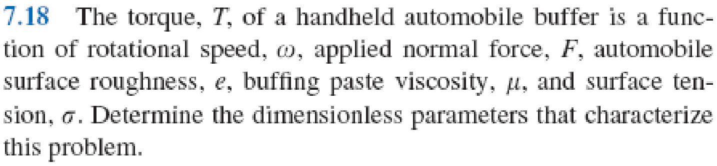
Thus $\dfrac{V}{\sqrt{gD}} = f\left(\dfrac{\lambda}{D}\right)$ or $V = \sqrt{gD}\, f\left(\dfrac{\lambda}{D}\right) \longleftarrow$ V

⑥ Check using F, L, t

$\Pi_1 = \dfrac{L}{t} \cdot \dfrac{1}{\left(\frac{L}{t^2} L\right)^{1/2}} = [1]$ ✓ $\qquad \Pi_2 = \dfrac{L}{L} = [1]$

Problem 7.18

[2]



7.18 The torque, T, of a handheld automobile buffer is a function of rotational speed, ω, applied normal force, F, automobile surface roughness, e, buffing paste viscosity, μ, and surface tension, σ. Determine the dimensionless parameters that characterize this problem.

Given: That automobile buffer depends on several parameters

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $T \quad \omega \quad F \quad e \quad \mu \quad \sigma$ $\qquad n = 6$ parameters

② Select primary dimensions M, L, t

③
$$\left\{\begin{array}{cccccc} T & \omega & F & e & \mu & \sigma \\ \dfrac{ML^2}{t^2} & \dfrac{1}{t} & \dfrac{ML}{t^2} & L & \dfrac{M}{Lt} & \dfrac{M}{t^2} \end{array}\right\}$$
$r = 3$ primary dimensions

④ $F \quad e \quad \omega$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 3$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = F^a e^b \omega^c T = \left(\frac{ML}{t^2}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rc|l} M: & a+1=0 & a=-1 \\ L: & a+b+2=0 & b=-1 \\ t: & -2a-c-2=0 & c=0 \end{array}$$

Hence $\qquad \Pi_1 = \dfrac{T}{Fe}$

$$\Pi_2 = F^a e^b \omega^c \mu = \left(\frac{ML}{t^2}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rc|l} M: & a+1=0 & a=-1 \\ L: & a+b-1=0 & b=2 \\ t: & -2a-c-1=0 & c=1 \end{array}$$

Hence $\qquad \Pi_2 = \dfrac{\mu e^2 \omega}{F}$

$$\Pi_3 = F^a e^b \omega^c \sigma = \left(\frac{ML}{t^2}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rc|l} M: & a+1=0 & a=-1 \\ L: & a+b=0 & b=1 \\ t: & -2a-c-2=0 & c=0 \end{array}$$

Hence $\qquad \Pi_3 = \dfrac{\sigma e}{F}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{FL}{FL} = [1] \qquad \Pi_2 = \frac{\frac{Ft}{L^2} L^2 \frac{1}{t}}{F} = [1] \qquad \Pi_3 = \frac{\frac{F}{L} L}{F} = [1]$$

Note: Any combination of Π_1, Π_2 and Π_3 is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{T}{\mu \omega e^3}$, so Π_1, Π_2 and Π_3 are not unique!

Problem 7.19

[2]

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Given: Volume flow rate, Q, over a weir is a function of: upstream height, h, gravity, g, and channel width, b.

Find: Expression for Q (using dimensional analysis)

Solution: Apply Buckingham π - theorem

① List $Q \quad h \quad g \quad b \qquad n = 4$ parameters

② Choose F, L, t as primary dimensions

③ Dimensions $\frac{L^3}{t} \quad L \quad \frac{L}{t^2} \quad L$

④ Repeating variables $g, h \qquad m = r = 2$

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$\pi_1 = g^a h^b Q$

$L^0 t^0 = \left(\frac{L}{t^2}\right)^a L^b \left(\frac{L^3}{t}\right)$

Equating exponents

$L: \; 0 = a + b + 3$

$t \quad 0 = -2a - 1$

$\therefore a = -\frac{1}{2}$

$b = -2\frac{1}{2}$

$\therefore \pi_1 = \frac{Q}{g^{1/2} h^{2.5}}$

$\pi_1 = \frac{Q}{h^2\sqrt{gh}}$

$\pi_2 = g^a h^c b$

$L^0 t^0 = \left(\frac{L}{t^2}\right)^a L^c L$

Equating exponents

$L: \; 0 = a + c + 1$

$t: \; 0 = -2a$

$\therefore a = 0$

$c = -1$

$\pi_2 = \frac{b}{h}$

(this is obvious by inspection)

Then $\frac{Q}{h^2\sqrt{gh}} = f\left(\frac{b}{h}\right)$

$Q = h^2\sqrt{gh}\; f\left(\frac{b}{h}\right) \longleftarrow$ Q

Problem 7.20

[2]

Given: Capillary waves form on a liquid free surface. The speed of the wave is a function of σ (surface tension), λ (the wave length) and ρ

Find: The wave speed as a function of the variables

Solution: Apply Buckingham Π - theorem

① $V \quad \sigma \quad \lambda \quad \rho$ $n = 4$ parameters

② Select M, L, t as primary dimensions

③ $V \quad \sigma \quad \lambda \quad \rho$

$\frac{L}{t} \quad \frac{M}{t^2} \quad L \quad \frac{M}{L^3}$ $r = 3$ primary dimensions

④ σ, λ, ρ $m = r = 3$ repeating parameters

⑤ Then $n - m = 1$ dimensionless group will result

Setting up dimensional equation

$$\Pi_1 = \sigma^a \lambda^b \rho^c V$$

$$M^0 L^0 t^0 = \left(\frac{M}{t^2}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{L}{t}$$

Summing exponents

M: $a + c = 0$ $c = -a = \frac{1}{2}$

L: $b - 3c + 1 = 0$ $b = 3c - 1 = \frac{1}{2}$

t: $-2a - 1 = 0$ $\therefore$ $a = -\frac{1}{2}$

$$\therefore \Pi_1 = \left(\frac{\rho \lambda}{\sigma}\right)^{\frac{1}{2}} V = \text{constant} \qquad \therefore V \propto \sqrt{\frac{\sigma}{\rho \lambda}}$$

V

⑥ Check using F, L, t

$$\Pi_1 = \left(\frac{Ft^2}{L^4} \cdot L \cdot \frac{L}{F}\right)^{1/2} \frac{L}{t} = [1] \checkmark$$

Problem 7.21

[2]

Given: Load-carrying capacity, W, (of a journal bearing) depends on: diameter, D; length, l; clearance, c; angular speed, ω; lubricant viscosity, μ

Find: Dimensionless parameters that characterize the problem.

Solution: Apply Buckingham π - theorem

① List $W \quad D \quad l \quad c \quad \omega \quad \mu$ $\quad n=6$ parameters

② Choose F, L, t as primary dimensions

③ Dimensions $F \quad L \quad L \quad L \quad \frac{1}{t} \quad \frac{Ft}{L^2}$

④ Repeating variables D, ω, μ $\quad m=r=3$

⑤ Then $n-m=3$ dimensionless groups will result

By inspection, $\pi_1 = \frac{l}{D} \qquad \pi_2 = \frac{c}{D}$

Set up dimensional equation to determine π_3

$$\pi_3 = D^a \omega^b \mu^e W$$

$$F^0 L^0 t^0 = L^a \left(\frac{1}{t}\right)^b \left(\frac{Ft}{L^2}\right)^e F$$

Equating exponents:

$F \qquad 0 = e+1 \qquad \therefore e=-1$

$L \qquad 0 = a-2e \qquad \therefore a=-2$

$t \qquad 0 = -b+e \qquad \therefore b=-1$

and

$$\pi_3 = \frac{W}{D^2 \omega \mu}$$

⑥ Check using M, L, t dimensions

$$\pi_3 = \frac{ML}{t^2} \times \frac{1}{L^2} \times t \times \frac{Lt}{M} = [1] \checkmark$$

$$\therefore \frac{W}{D^2 \omega \mu} = f\left(\frac{l}{D}, \frac{c}{D}\right)$$ W

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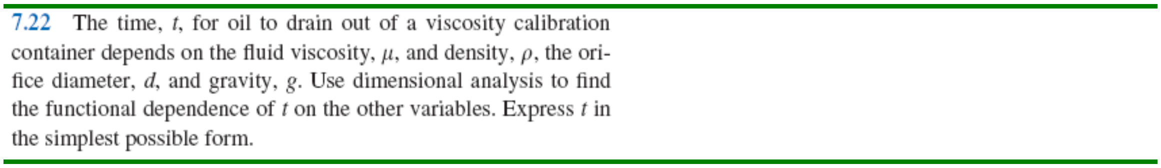
Problem 7.22 (In Excel)

[2]



7.22 The time, t, for oil to drain out of a viscosity calibration container depends on the fluid viscosity, μ, and density, ρ, the orifice diameter, d, and gravity, g. Use dimensional analysis to find the functional dependence of t on the other variables. Express t in the simplest possible form.

Given: That drain time depends on fluid viscosity and density, orifice diameter, and gravity

Find: Functional dependence of t on other variables

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 5$
The number of primary dimensions is: $r = 3$
The number of repeat parameters is: $m = r = 3$
The number of Π groups is: $n - m = 2$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: **Choose ρ, g, d**

	M	**L**	**t**
ρ	1	-3	
g		1	-2
d		1	

Π GROUPS:

	M	**L**	**t**
t	0	0	1

Π_1: $a =$ **0**, $b =$ **0.5**, $c =$ **-0.5**

	M	**L**	**t**
μ	1	-1	-1

Π_2: $a =$ **-1**, $b =$ **-0.5**, $c =$ **-1.5**

The following Π groups from Example 7.1 are not used:

M	**L**	**t**
0	0	0

Π_3: $a =$ **0**, $b =$ **0**, $c =$ **0**

M	**L**	**t**
0	0	0

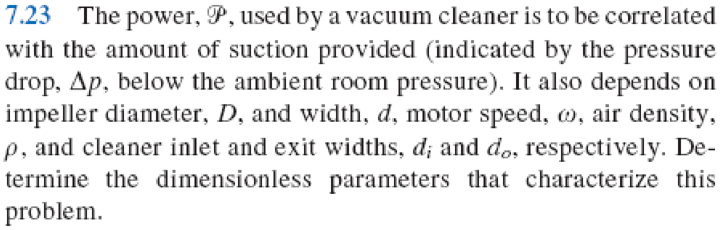
Π_4: $a =$ **0**, $b =$ **0**, $c =$ **0**

Hence $\Pi_1 = t\sqrt{\dfrac{g}{d}}$ and $\Pi_2 = \dfrac{\mu}{\rho g^{\frac{1}{2}} d^{\frac{3}{2}}} \rightarrow \dfrac{\mu^2}{\rho^2 g d^3}$ with $\Pi_1 = f(\Pi_2)$

The final result is $t = \sqrt{\dfrac{d}{g}}\, f\left(\dfrac{\mu^2}{\rho^2 g d^3}\right)$

Problem 7.23

[2]



7.23 The power, $\mathscr{P}$, used by a vacuum cleaner is to be correlated with the amount of suction provided (indicated by the pressure drop, Δp, below the ambient room pressure). It also depends on impeller diameter, D, and width, d, motor speed, ω, air density, ρ, and cleaner inlet and exit widths, d_i and d_o, respectively. Determine the dimensionless parameters that characterize this problem.

Given: That the power of a vacuum depends on various parameters

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $\mathscr{P}$ $\quad \Delta p \quad D \quad d \quad \omega \quad \rho \quad d_i \quad d_o$ $\qquad n = 8$ parameters

② Select primary dimensions M, L, t

③
$$\left\{\begin{array}{cccccccc} \mathscr{P} & \Delta p & D & d & \omega & \rho & d_i & d_o \\ \dfrac{ML^2}{t^3} & \dfrac{M}{Lt^2} & L & L & \dfrac{1}{t} & \dfrac{M}{L^3} & L & L \end{array}\right\}$$
$r = 3$ primary dimensions

④ $\rho \quad D \quad \omega$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 5$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega^c \mathscr{P} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^3} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rc|l} M: & a+1=0 & a=-1 \\ L: & -3a+b+2=0 & b=-5 \\ t: & -c-3=0 & c=-3 \end{array}$$
Hence $\Pi_1 = \dfrac{\mathscr{P}}{\rho D^5 \omega^3}$

$$\Pi_2 = \rho^a D^b \omega^c \Delta p = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rc|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-2=0 & c=-2 \end{array}$$
Hence $\Pi_2 = \dfrac{\Delta p}{\rho D^2 \omega^2}$

The other Π groups can be found by inspection: $\Pi_3 = \dfrac{d}{D} \qquad \Pi_4 = \dfrac{d_i}{D} \qquad \Pi_5 = \dfrac{d_o}{D}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{\dfrac{FL}{t}}{\dfrac{Ft^2}{L^4} L^5 \dfrac{1}{t^3}} = [1] \qquad \Pi_2 = \frac{\dfrac{F}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t^2}} = [1] \qquad \Pi_3 = \Pi_4 = \Pi_5 = \frac{L}{L} = [1]$$

Note: Any combination of Π_1, Π_2 and Π_3 is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{\mathscr{P}}{\Delta p D^3 \omega}$, so the Π's are not unique!

Problem 7.24

[2]

Given: Power per unit cross-sectional area, E, transmitted by a sound wave, depends on wave speed, V, amplitude, r, frequency, n, and medium density, ρ.

Find: General form of dependence of E on the other variables.

Solution: Step ① $E \quad V \quad r \quad n \quad \rho \qquad n = 5$

Step ② Choose M, L, t. $E = \frac{P}{L^2} = \frac{FL}{t} \times \frac{1}{L^2} = \frac{F}{Lt} \times \frac{ML}{Ft^2} = \frac{M}{t^3}$

Step ③ $\frac{M}{t^3} \quad \frac{L}{t} \quad L \quad \frac{1}{t} \quad \frac{M}{L^3} \qquad r = 3$

Step ④ Choose ρ, V, r

Step ⑤ $\Pi_1 = \rho^a V^b r^c E = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{M}{t^3} = M^0 L^0 t^0$

$$\left.\begin{array}{lll} M: a+1=0 & a=-1 & \\ L: -3a+b+c=0 & & c=-3a-b=3(-1)-(-3)=0 \\ t: -b-3=0 & b=-3 & \end{array}\right\} \quad \Pi_1 = \frac{E}{\rho V^3} \qquad \Pi_1$$

$\Pi_2 = \rho^a V^b r^c n = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{1}{t} = M^0 L^0 t^0$

$$\left.\begin{array}{lll} M: a+0=0 & a=0 & \\ L: -3a+b+c=0 & & c=3a-b=3(0)-(-1)=1 \\ t: -b-1=0 & b=-1 & \end{array}\right\} \quad \Pi_2 = \frac{nr}{V} \qquad \Pi_2$$

Step ⑥ Check using FLt: $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{E}{\rho V^3} = \frac{FL}{tL^2} \frac{L^4}{Ft^2} \frac{t^3}{L^3} = \frac{FL^5t^3}{FL^5t^3} = 1$ ✓✓

$\Pi_2 = \frac{nr}{V} = \frac{1}{t} L \times \frac{t}{L} = \frac{Lt}{Lt} = 1$ ✓✓

Problem 7.25

[2]

Given: Draining of a tank from initial level, h_0.

Time, τ, depends on tank diameter, D, orifice diameter, d, acceleration of gravity, g, density, ρ, and viscosity, μ.

Find: (a) Number of dimensionless parameters
(b) Number of repeating variables.
(c) Π-parameter containing viscosity.

Solution: Step① τ $\quad h_0$ $\quad D$ $\quad d$ $\quad g$ $\quad \rho$ $\quad \mu$

Step② Choose MLt system $\quad$ $(n = 7)$

Step③ t $\quad L$ $\quad L$ $\quad L$ $\quad \frac{L}{t^2}$ $\quad \frac{M}{L^3}$ $\quad \frac{M}{Lt}$

$(r=3)$

Then $n-r = 7-3 = 4$ parameters will result. $\quad$ Π's

Step④ $r=3$, so choose 3 variables: ρ, d, g

Step⑤ $\Pi_1 = \rho^a d^b g^c \mu = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$

$M: a+1=0 \qquad a=-1$

$L: -3a+b+c-1=0 \qquad b = 3a-c+1 = 3(-1)-(-\frac{1}{2})+1$

$t: -2c-1=0 \qquad c = -\frac{1}{2} \qquad b = -\frac{3}{2}$

$$\Pi_1 = \frac{\mu}{\rho\, d^{3/2} g^{1/2}}$$ Π_1

Step⑥ Check, using FLt system.

$\mu = \frac{Ft}{L^2}$; $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{Ft}{L^2}\,\frac{L^4}{Ft^2}\,\frac{1}{L^{3/2}}\,\frac{t}{L^{1/2}} = \frac{FL^4t^2}{FL^4t^2} = 1$ ✓✓

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Problem 7.26

[2]

Given: Power, $\mathcal{P}$, required to drive a fan depends on ρ, Q, D and ω.

Find: Dependence of $\mathcal{P}$ on other parameters.

Solution: Apply Buckingham π procedure.

① $\mathcal{P} \quad \rho \quad Q \quad D \quad \omega$ $\qquad n = 5$ parameters

② Choose primary dimensions M, L, t

③ $\mathcal{P} \quad \rho \quad Q \quad D \quad \omega$

$$\frac{ML^2}{t^3} \quad \frac{M}{L^3} \quad \frac{L^3}{t} \quad L \quad \frac{1}{t} \qquad r = 3 \text{ primary dimensions}$$

④ $\rho, D, \omega \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up dimensional equations,

$$\Pi_1 = \rho^a D^b \omega^c \mathcal{P}$$

$$= \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \left(\frac{ML^2}{t^3}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: \ a + 1 = 0 \qquad a = -1$
$L: \ -3a + b + 2 = 0 \qquad b = -5$
$t: \ -c - 3 = 0 \qquad c = -3$

$$\therefore \ \Pi_1 = \frac{\mathcal{P}}{\rho D^5 \omega^3}$$

$$\Pi_2 = \rho^d D^e \omega^f Q$$

$$= \left(\frac{M}{L^3}\right)^d (L)^e \left(\frac{1}{t}\right)^f \left(\frac{L^3}{t}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: \ d + 0 = 0 \qquad d = 0$
$L: \ -3d + e + 3 = 0 \qquad e = -3$
$t: \ -f - 1 = 0 \qquad f = -1$

$$\therefore \ \Pi_2 = \frac{Q}{D^3 \omega}$$

⑥ Check using primary dimensions F, L, t

$$\Pi_1 = \frac{FL}{t} \frac{L^4}{Ft^2} \frac{1}{L^5} t^3 = [1] \checkmark \qquad \Pi_2 = \frac{L^3}{t} \frac{1}{L^3} t = [1] \checkmark$$

Thus $\Pi_1 = f(\Pi_2)$, or $\dfrac{\mathcal{P}}{\rho D^5 \omega^3} = f\left(\dfrac{Q}{D^3 \omega}\right) \longleftarrow$ $\qquad \mathcal{P}$

Problem 7.27

[3]

Given: Continuous belt moving vertically through a viscous liquid bath

The volume rate of liquid loss, Q, is a function of μ, ρ, g, h (thickness of liquid layer), and V

Find: form of dependence of Q on other variables.

Solution: Apply Buckingham π-theorem.

① $Q \quad \mu \quad \rho \quad g \quad h \quad V \qquad n = 6$ parameters

② Select M, L, t as primary dimensions

③ $\begin{array}{cccccc} Q & \mu & \rho & g & h & V \\ \frac{L^3}{t} & \frac{M}{Lt} & \frac{M}{L^3} & \frac{L}{t^2} & L & \frac{L}{t} \end{array} \qquad r = 3$ primary dimensions

④ $\rho, V, h \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result.

Setting up dimensional equations.

$\pi_1 = \rho^a V^b h^c Q$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{L^3}{t}$

Equating exponents,

M: $0 = a$

L: $0 = -3a + b + c + 3$

t: $0 = -b - 1$

i.e. $a = 0$

$b = -1$

$c = -2$

$\therefore \pi_1 = \frac{Q}{Vh^2}$

$\pi_2 = \rho^a V^b h^c \mu$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$

Equating exponents,

M: $0 = a + 1$

L: $0 = -3a + b + c - 1$

t: $0 = -b - 1$

i.e. $a = -1$

$b = -1$

$c = -1$

$\therefore \pi_2 = \frac{\mu}{\rho V h}$

$\pi_3 = \rho^a V^b h^c g$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{L}{t^2}$

Equating exponents,

M: $0 = a$

L: $0 = -3a + b + c + 1$

t: $0 = -b - 2$

i.e. $a = 0$

$b = -2$

$c = 1$

$\therefore \pi_3 = \frac{gh}{V^2}$

Then

$$\frac{Q}{Vh^2} = f\left(\frac{\rho V h}{\mu}, \frac{V^2}{gh}\right) \longleftarrow \quad Q$$

⑥ Check using F, L, t dimensions

$\pi_1 = \frac{L^3}{t} \cdot \frac{t}{L} \cdot \frac{1}{L^2} = [1]✓ \qquad \pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = [1]✓ \qquad \pi_3 = \frac{L}{t^2} \cdot L \cdot \frac{t^2}{L^2} = [1]✓$

Problem 7.28

[2]

Given: Water is drained from a tank of diameter D, through a smoothly rounded drain hole of diameter, d. The initial mass flow rate, $\dot{m}$, from the tank is written in functional form as

$$\dot{m} = \dot{m}(h_o, D, d, g, \rho, \mu)$$

where h_o is the initial water depth in the tank
g is the acceleration of gravity
ρ and μ are fluid properties.

Find: (a) the number of dimensionless groups required to correlate the data
(b) the number of repeating variables that must be selected to determine the dimensionless parameters.
(c) the π parameter that contains the fluid viscosity, μ.

Solution: Apply the Buckingham π-theorem

① List: $\dot{m}$ h_o D d g ρ μ $n=7$ parameters

② Select M, L, t as primary dimensions

③ Dimensions $\frac{M}{t}$ L L L $\frac{L}{t^2}$ $\frac{M}{L^3}$ $\frac{M}{Lt}$ $r=3$ prim. dim

④ Choose repeating variables ρ, d, g $m=3$ repeating parameters ← m

$\therefore$ expect $n-m = 7-3 = 4$ dimensionless parameters ← n

⑤ $\pi_1 = \rho^a d^b g^c \mu$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c \frac{M}{Lt}$$

t: $0 = -2c-1$ $\therefore c = -\frac{1}{2}$
M $0 = a+1$ $\therefore a = -1$
L $0 = -3a+b+c-1$ $\therefore b = 3a-c+1 = -\frac{3}{2}$

$$\pi_1 = \frac{\mu}{\rho d^{3/2} g^{1/2}}$$ ← π

⑥ Check $\pi_1 = \frac{\cancel{F}t}{L^2} \times \frac{L^4}{\cancel{F}t^2} \times \frac{1}{L^{3/2}} \times \frac{t}{L^{1/2}} = [1]$ ✓

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Problem 7.29

[3]

Given: Diameter, d, of liquid droplets formed in fuel injection process is a function of ρ, μ, σ (surface tension), V, D.

Find: (a) number of dimensionless ratios required to characterize the process
(b) the dimensionless ratios.

Solution: Apply Buckingham π-theorem

① $d \quad \rho \quad \mu \quad \sigma \quad V \quad D$ $\qquad n = 6$ parameters

② Select M, L, t as primary dimensions

③ $d \quad \rho \quad \mu \quad \sigma \quad V \quad D$

$L \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad \frac{M}{t^2} \quad \frac{L}{t} \quad L$ $\qquad r = 3$ primary dimensions

④ ρ, D, V $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result.

Setting up dimensional equations

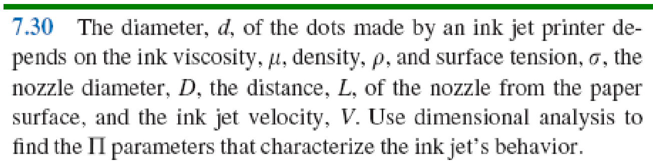
$\pi_1 = \rho^a D^b V^c d$	$\pi_2 = \rho^a D^b V^c \mu$	$\pi_3 = \rho^a D^b V^c \sigma$
$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t}\right)^c L$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t}\right)^c \frac{M}{Lt}$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t}\right)^c \frac{M}{t^2}$
Summing exponents,	Summing exponents	Summing exponents
M: $a = 0$	M: $a + 1 = 0$	M: $a + 1 = 0$
L: $-3a + b + c + 1 = 0$	L: $-3a + b + c - 1 = 0$	L: $-3a + b + c = 0$
t: $-c = 0$	t: $-c - 1 = 0$	t: $-c - 2 = 0$
i.e. $a = 0$	i.e. $a = -1$	i.e. $a = -1$
$c = 0$	$c = -1$	$c = -2$
$b = -1$	$b = 3a - c + 1 = -1$	$b = 3a - c = -1$
$\therefore \pi_1 = \frac{d}{D}$	$\therefore \pi_2 = \frac{\mu}{\rho V D}$	$\therefore \pi_3 = \frac{\sigma}{\rho D V^2}$

⑥ Check using F, L, t dimensions

$\pi_1 = \frac{L}{L} = [1]$ ✓ $\qquad \pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = [1]$ ✓ $\qquad \pi_3 = \frac{F}{L} \cdot \frac{L^4}{Ft^2} \cdot \frac{1}{L} \cdot \frac{t^2}{L^2} = [1]$ ✓

Problem 7.30 (In Excel)

[3]



7.30 The diameter, d, of the dots made by an ink jet printer depends on the ink viscosity, μ, density, ρ, and surface tension, σ, the nozzle diameter, D, the distance, L, of the nozzle from the paper surface, and the ink jet velocity, V. Use dimensional analysis to find the Π parameters that characterize the ink jet's behavior.

Given: That dot size depends on ink viscosity, density, and surface tension, and geometry

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 7$

The number of primary dimensions is: $r = 3$

The number of repeat parameters is: $m = r = 3$

The number of Π groups is: $n - m = 4$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents *a* , *b* , and *c* for each.

REPEATING PARAMETERS: **Choose ρ, V, D**

	M	**L**	**t**
ρ	1	-3	
V		1	-1
D		1	

Π GROUPS:

	M	**L**	**t**
d	0	1	0

Π_1:

$a =$ **0**
$b =$ **0**
$c =$ **-1**

	M	**L**	**t**
μ	1	-1	-1

Π_2:

$a =$ **-1**
$b =$ **-1**
$c =$ **-1**

	M	**L**	**t**
σ	1	0	-2

Π_3:

$a =$ **-1**
$b =$ **-2**
$c =$ **-1**

	M	**L**	**t**
L	0	1	0

Π_4:

$a =$ **0**
$b =$ **0**
$c =$ **-1**

Hence

$$\Pi_1 = \frac{d}{D} \qquad \Pi_2 = \frac{\mu}{\rho VD} \rightarrow \frac{\rho VD}{\mu} \qquad \Pi_3 = \frac{\sigma}{\rho V^2 D} \qquad \Pi_4 = \frac{L}{D}$$

Note that groups Π_1 and Π_4 can be obtained by inspection

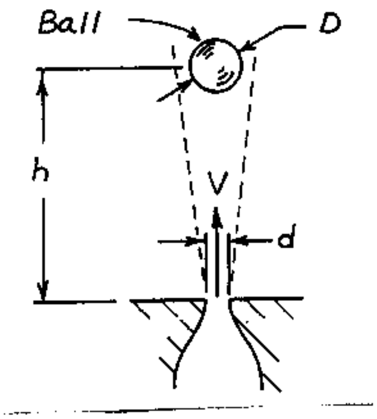
Problem 7.31

[3]

Given: Ball in jet

$$h = h(d, D, \rho, V, \mu, W)$$

Find: Pi parameters



Solution: Apply Buckingham procedure

① $h \quad d \quad D \quad \rho \quad V \quad \mu \quad W \qquad n = 7$

② M, L, t

③ $L \quad L \quad L \quad \frac{M}{L^3} \quad \frac{L}{t} \quad \frac{M}{Lt} \quad \frac{ML}{t^2} \qquad m = 3 \qquad n - m = 7 - 3 = 4$ parameters

④ Choose ρ, V, d as repeating parameters.

⑤ $\rho^a V^b d^c W = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$

$M: a + 1 = 0 \qquad a = -1$

$L: -3a + b + c + 1 = 0 \qquad c = -2$

$t: -b - 2 = 0 \qquad b = -2$

$$\Pi_1 = \frac{W}{\rho V^2 d^2} \longleftarrow \Pi_1$$

⑥ Check: $F \times \frac{L^4}{Ft^2} \frac{t^2}{L^2} \times \frac{1}{L^2} = 1$ ✓✓

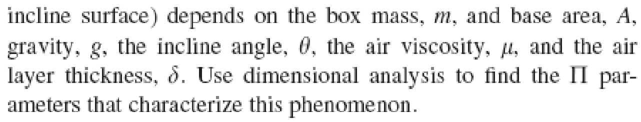
$$\Pi_2 = \rho^a V^b d^c \mu = \frac{\mu}{\rho V d}$$

$$\Pi_3 = \rho^a V^b d^c h = \frac{h}{d}$$

$$\Pi_4 = \rho^a V^b d^c D = \frac{D}{d}$$

Problem 7.32 (In Excel)

[3]



7.32 The terminal speed *V* of shipping boxes sliding down an incline on a layer of air (injected through numerous pinholes in the incline surface) depends on the box mass, *m*, and base area, *A*, gravity, *g*, the incline angle, *θ*, the air viscosity, *μ*, and the air layer thickness, *δ*. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Speed depends on mass, area, gravity, slope, and air viscosity and thickness

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 7$

The number of primary dimensions is: $r = 3$

The number of repeat parameters is: $m = r = 3$

The number of Π groups is: $n - m = 4$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents *a*, *b*, and *c* for each.

REPEATING PARAMETERS: Choose *g*, δ, *m*

	M	L	t
g		1	-2
δ		1	
m	1		

Π GROUPS:

	M	L	t
V	0	1	-1

Π$_1$:

a = **-0.5**
b = **-0.5**
c = **0**

	M	L	t
μ	1	-1	-1

Π$_2$:

a = **-0.5**
b = **1.5**
c = **-1**

	M	L	t
θ	0	0	0

Π$_3$:

a = **0**
b = **0**
c = **0**

	M	L	t
A	0	2	0

Π$_4$:

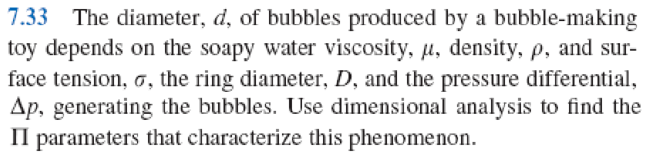
a = **0**
b = **-2**
c = **0**

Hence
$$\Pi_1 = \frac{V}{g^{\frac{1}{2}}\delta^{\frac{1}{2}}} \rightarrow \frac{V^2}{g\delta} \qquad \Pi_2 = \frac{\mu\delta^{\frac{3}{2}}}{g^{\frac{1}{2}}m} \rightarrow \frac{\mu^2\delta^3}{m^2 g} \qquad \Pi_3 = \theta \qquad \Pi_4 = \frac{A}{\delta^2}$$

Note that the Π_1 , Π_3 and Π_4 groups can be obtained by inspection

Problem 7.33 (In Excel)

[3]



7.33 The diameter, d, of bubbles produced by a bubble-making toy depends on the soapy water viscosity, μ, density, ρ, and surface tension, σ, the ring diameter, D, and the pressure differential, Δp, generating the bubbles. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Bubble size depends on viscosity, density, surface tension, geometry and pressure

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 6$
The number of primary dimensions is: $r = 3$
The number of repeat parameters is: $m = r = 3$
The number of Π groups is: $n - m = 3$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents *a* , *b* , and *c* for each.

REPEATING PARAMETERS: **Choose ρ, Δp, D**

	M	**L**	**t**
ρ	1	-3	
Δp	1	-1	-2
D		1	

Π GROUPS:

	M	**L**	**t**
d	0	1	0

Π_1: $a =$ **0**, $b =$ **0**, $c =$ **-1**

	M	**L**	**t**
μ	1	-1	-1

Π_2: $a =$ **-0.5**, $b =$ **-0.5**, $c =$ **-1**

	M	**L**	**t**
σ	1	0	-2

Π_3: $a =$ **0**, $b =$ **-1**, $c =$ **-1**

	M	**L**	**t**
	0	0	0

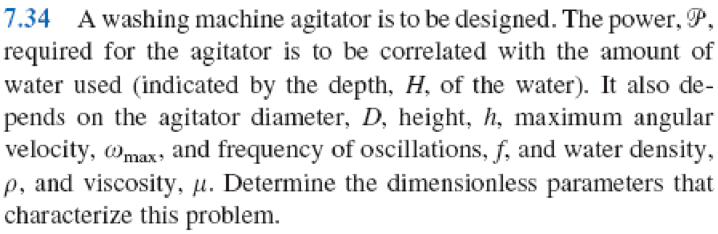
Π_4: $a =$ **0**, $b =$ **0**, $c =$ **0**

Hence $$\Pi_1 = \frac{d}{D} \qquad \Pi_2 = \frac{\mu}{\rho^{\frac{1}{2}}\Delta p^{\frac{1}{2}} D} \rightarrow \frac{\mu^2}{\rho \Delta p D^2} \qquad \Pi_3 = \frac{\sigma}{D \Delta p}$$

Note that the Π_1 group can be obtained by inspection

Problem 7.34

[2]



7.34 A washing machine agitator is to be designed. The power, $\mathscr{P}$, required for the agitator is to be correlated with the amount of water used (indicated by the depth, H, of the water). It also depends on the agitator diameter, D, height, h, maximum angular velocity, $\omega_{\max}$, and frequency of oscillations, f, and water density, ρ, and viscosity, μ. Determine the dimensionless parameters that characterize this problem.

Given: That the power of a washing machine agitator depends on various parameters

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $\mathscr{P}$ H D h $\omega_{\max}$ f ρ μ $n = 8$ parameters

② Select primary dimensions M, L, t

③
$$\left\{\begin{array}{cccccccc} \mathscr{P} & H & D & h & \omega_{\max} & f & \rho & \mu \\ \dfrac{ML^2}{t^3} & L & L & L & \dfrac{1}{t} & \dfrac{1}{t} & \dfrac{M}{L^3} & \dfrac{M}{Lt} \end{array}\right\}$$
$r = 3$ primary dimensions

④ ρ D $\omega_{\max}$ $m = r = 3$ repeat parameters

⑤ Then $n - m = 5$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega_{\max}^c \mathscr{P} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^3} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rl|l} M: & a+1=0 & a=-1 \\ L: & -3a+b+2=0 & b=-5 \\ t: & -c-3=0 & c=-3 \end{array}$$

Hence $\Pi_1 = \dfrac{\mathscr{P}}{\rho D^5 \omega_{\max}^3}$

$$\Pi_2 = \rho^a D^b \omega_{\max}^c \mu = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rl|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-1=0 & c=-1 \end{array}$$

Hence $\Pi_2 = \dfrac{\mu}{\rho D^2 \omega_{\max}}$

The other Π groups can be found by inspection: $\Pi_3 = \dfrac{H}{D}$ $\Pi_4 = \dfrac{h}{D}$ $\Pi_5 = \dfrac{f}{\omega_{\max}}$

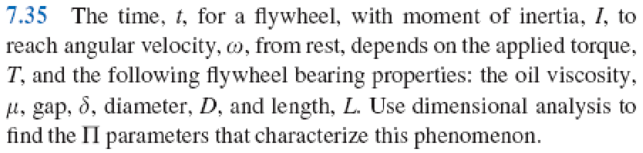
⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{\dfrac{FL}{t}}{\dfrac{Ft^2}{L^4} L^5 \dfrac{1}{t^3}} = [1] \qquad \Pi_2 = \frac{\dfrac{Ft}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t}} = [1] \qquad \Pi_3 = \Pi_4 = \Pi_5 = [1]$$

Note: Any combination of Π's is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{\mathscr{P}}{D^3 \omega_{\max}^2 \mu}$, so the Π's are not unique!

Problem 7.35 (In Excel)

[3]



7.35 The time, t, for a flywheel, with moment of inertia, I, to reach angular velocity, ω, from rest, depends on the applied torque, T, and the following flywheel bearing properties: the oil viscosity, μ, gap, δ, diameter, D, and length, L. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Time to speed up depends on inertia, speed, torque, oil viscosity and geometry

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 8$

The number of primary dimensions is: $r = 3$

The number of repeat parameters is: $m = r = 3$

The number of Π groups is: $n - m = 5$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: **Choose ω, D, T**

	M	**L**	**t**
ω			-1
D		1	
T	1	2	-2

Π GROUPS:

Two Π groups can be obtained by inspection: δ/D and L/D. The others are obtained below

	M	**L**	**t**
t	0	0	1

Π_1:

$a =$ **1**
$b =$ **0**
$c =$ **0**

	M	**L**	**t**
μ	1	-1	-1

Π_2:

$a =$ **1**
$b =$ **3**
$c =$ **-1**

	M	**L**	**t**
I	1	2	0

Π_3:

$a =$ **2**
$b =$ **0**
$c =$ **-1**

	M	**L**	**t**
	0	0	0

Π_4:

$a =$ **0**
$b =$ **0**
$c =$ **0**

Hence the Π groups are

$$t\omega \qquad \frac{\delta}{D} \qquad \frac{L}{D} \qquad \frac{\mu\omega D^3}{T} \qquad \frac{I\omega^2}{T}$$

Note that the Π_1 group can also be easily obtained by inspection

Problem 7.36

Given: Pressurized tank drained through a smooth nozzle, area A.

$$\dot{m} = \dot{m}(\Delta p, h, \rho, A, g)$$

Find: (a) Number of independent dimensionless parameters.
(b) Obtain the parameters.
(c) State the functional relationship for $\dot{m}$.

Solution: Apply the Buckingham Π-theorem.

① $\dot{m}$ $\quad \Delta p$ $\quad h$ $\quad \rho$ $\quad A$ $\quad g$ $\quad n = 6$ parameters

② Select M, L, t as primary dimensions

③ $\frac{M}{t}$ $\quad \frac{M}{Lt^2}$ $\quad L$ $\quad \frac{M}{L^3}$ $\quad L^2$ $\quad \frac{L}{t^2}$ $\quad r = 3$ primary dimensions
$m = r = 3$

④ Choose ρ, A, g as repeating parameters.

⑤ Then $n - m = 6 - 3 = 3$ dimensionless parameters result.

$n-m$

Set up dimensional equations:

$$\Pi_1 = \rho^a A^b g^c \dot{m}$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a (L^2)^b \left(\frac{L}{t^2}\right)^c \frac{M}{t}$$

Equating exponents:

$M: \; a + 1 = 0 \qquad a = -1$
$L: \; -3a + 2b + c = 0$
$t: \; -2c - 1 = 0 \qquad c = -\frac{1}{2}$

$$\therefore b = \frac{1}{2}(3a - c) = -\frac{5}{4}$$

$$\Pi_1 = \frac{\dot{m}}{\rho A^{5/4} g^{1/2}}$$

$$\Pi_2 = \rho^a A^b g^c \Delta p$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a (L^2)^b \left(\frac{L}{t^2}\right)^c \frac{M}{Lt^2}$$

Equating exponents:

$M: \; a + 1 = 0 \qquad a = -1$
$L: \; -3a + 2b + c - 1 = 0$
$t: \; -2c - 2 = 0 \qquad c = -1$

$$\therefore b = \frac{1}{2}(1 + 3a - c) = -\frac{1}{2}$$

$$\Pi_2 = \frac{\Delta p}{\rho A^{1/2} g}$$

$$\Pi_3 = \rho^a A^b g^c h$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a (L^2)^b \left(\frac{L}{t^2}\right)^c L$$

Equating exponents:

$M: \; a = 0 \qquad a = 0$
$L: \; -3a + 2b + c + 1 = 0$
$t: \; -2c + 0 = 0 \qquad c = 0$

$$\therefore b = \frac{1}{2}(-1 + 3a - c) = -\frac{1}{2}$$

$$\Pi_3 = \frac{h}{A^{1/2}}$$

Π's

⑥ Check using FLt dimensions: $\dot{m} = \frac{M}{t}\frac{Ft^2}{ML} = \frac{Ft}{L}$; $\rho = \frac{M}{L^3}\frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{Ft}{L}\frac{L^4}{Ft^2}\frac{1}{L^{5/2}}\frac{t}{L^{1/2}} = [1]$ ✓✓

$\Pi_2 = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{1}{L}\frac{t^2}{L} = [1]$ ✓✓

$\Pi_3 = \frac{L}{L} = [1]$ ✓✓

Thus

$$\Pi_1 = f(\Pi_2, \Pi_3) \qquad \frac{\dot{m}}{\rho A^{5/4} g^{1/2}} = f\left(\frac{\Delta p}{\rho A^{1/2} g}, \frac{h}{A^{1/2}}\right)$$

or

$$\dot{m} = \rho A^{5/4} g^{1/2} f\left(\frac{\Delta p}{\rho A^{1/2} g}, \frac{h}{A^{1/2}}\right)$$

$\dot{m}$

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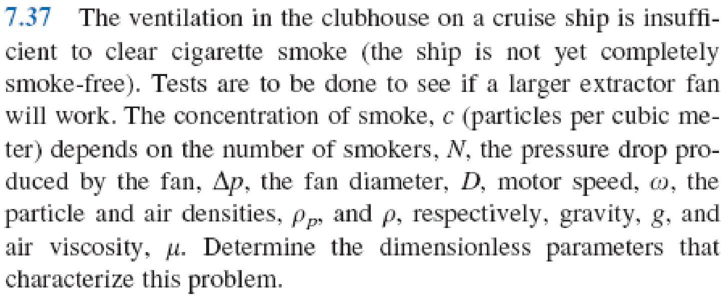
Problem 7.37

[2]



7.37 The ventilation in the clubhouse on a cruise ship is insufficient to clear cigarette smoke (the ship is not yet completely smoke-free). Tests are to be done to see if a larger extractor fan will work. The concentration of smoke, c (particles per cubic meter) depends on the number of smokers, N, the pressure drop produced by the fan, Δp, the fan diameter, D, motor speed, ω, the particle and air densities, ρ_p, and ρ, respectively, gravity, g, and air viscosity, μ. Determine the dimensionless parameters that characterize this problem.

Given: Ventilation system of cruise ship clubhouse

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $c \quad N \quad \Delta p \quad D \quad \omega \quad \rho_p \quad \rho \quad g \quad \mu \qquad n = 9$ parameters

② Select primary dimensions M, L, t

③
$$\left\{\begin{array}{ccccccccc} c & N & \Delta p & D & \omega & \rho_p & \rho & g & \mu \\ \dfrac{1}{L^3} & 1 & \dfrac{M}{Lt^2} & L & \dfrac{1}{t} & \dfrac{M}{L^3} & \dfrac{M}{L^3} & \dfrac{L}{t^2} & \dfrac{M}{Lt} \end{array}\right\} \qquad r = 3 \text{ primary dimensions}$$

④ $\rho \quad D \quad \omega \qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 6$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega^c \Delta p = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rr|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-2=0 & c=-2 \end{array} \qquad \text{Hence} \qquad \Pi_1 = \frac{\Delta p}{\rho D^2 \omega^2}$$

$$\Pi_2 = \rho^a D^b \omega^c \mu = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rr|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-1=0 & c=-1 \end{array} \qquad \text{Hence} \qquad \Pi_2 = \frac{\mu}{\rho D^2 \omega}$$

The other Π groups can be found by inspection: $\Pi_3 = cD^3 \qquad \Pi_4 = N \qquad \Pi_5 = \dfrac{\rho_p}{\rho} \qquad \Pi_6 = \dfrac{g}{D\omega^2}$

⑥ Check using F, L, t as primary dimensions

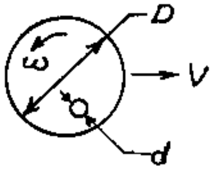
$$\Pi_1 = \frac{\dfrac{F}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t^2}} = [1] \qquad \Pi_2 = \frac{\dfrac{Ft}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t}} = [1] \qquad \Pi_3 = \Pi_4 = \Pi_5 = \Pi_6 = [1]$$

Note: Any combination of Π's is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{\Delta p}{\omega\mu}$, so the Π's are not unique!

Problem 7.38

Given: Aerodynamic torque on spinning ball,

$$T = f(V, \rho, \mu, D, \omega, d)$$



Find: Dimensionless parameters

Solution: Apply Buckingham procedure.

① List: $T \quad V \quad \rho \quad \mu \quad D \quad \omega \quad d \qquad n = 7$

② Choose M, L, t

③ $\frac{ML^2}{t^2} \quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad L \quad \frac{1}{t} \quad L \qquad m = 3$

④ Choose ρ, V, D $\qquad n - m = 4$ parameters

⑤ $\Pi_1 = \rho^a V^b D^c T = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + b + c + 2 = 0 \qquad c = -3 \qquad \Pi_1 = \frac{T}{\rho V^2 D^3}$
$t: -b - 2 = 0 \qquad b = -2$

Π_1

⑥ Check: $\Pi_1 = FL \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} \times \frac{1}{L^3} = 1$ ✓✓

$$\Pi_2 = \frac{\mu}{\rho V D}$$

$$\Pi_3 = \frac{\omega D}{V}$$

$$\Pi_4 = \frac{d}{D}$$

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$$

$$\frac{T}{\rho V^2 D^3} = f\left(\frac{\mu}{\rho V D}, \frac{\omega D}{V}, \frac{d}{D}\right)$$

T

Problem 7.39

[3]

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NATIONAL

Given: Power loss, $\mathcal{P}$, depends on: length, l, diameter, D; clearance, c; angular speed, ω; viscosity, μ; mean pressure, p.

Find: (a) Dimensionless parameters that characterize the problem
(b) Functional form of dependence of $\mathcal{P}$ on these parameters

Solution: Apply Buckingham π - theorem

① $\mathcal{P} \quad l \quad D \quad c \quad \omega \quad \mu \quad p \qquad n = 7$ parameters

② Select F, L, t as primary dimensions

③

$\mathcal{P}$	l	D	c	ω	μ	p
$\frac{FL}{t}$	L	L	L	$\frac{1}{t}$	$\frac{Ft}{L^2}$	$\frac{F}{L^2}$

④ $D, \omega, p \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 4$ dimensionless groups will result.

Setting up dimensional equations

$\pi_1 = D^a \omega^b p^e \mathcal{P}$	$\pi_2 = D^a \omega^b p^e l$	$\pi_3 = D^a \omega^b p^e c$	$\pi_4 = D^a \omega^b p^e \mu$
$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e \frac{FL}{t}$	$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e L$	$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e L$	$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e \frac{Ft}{L^2}$
Equating exponents,	Equating exponents,	Equating exponents,	Equating exponents,
$F: \; 0 = e+1$	$F: \; 0 = e$	$F: \; 0 = e$	$F: \; 0 = e+1$
$L: \; 0 = a - 2e + 1$	$L: \; 0 = a - 2e + 1$	$L: \; 0 = a - 2e + 1$	$L: \; 0 = a - 2e - 2$
$t: \; 0 = -b - 1$	$t: \; 0 = -b$	$t: \; 0 = -b$	$t: \; 0 = -b + 1$
$\therefore \; e = -1$	$\therefore \; e = 0$	$\therefore \; e = 0$	$\therefore \; e = -1$
$a = -3$	$a = -1$	$a = -1$	$a = 0$
$b = -1$	$b = 0$	$b = 0$	$b = 1$
$\pi_1 = \frac{\mathcal{P}}{p\omega D^3}$	$\pi_2 = \frac{l}{D}$	$\pi_3 = \frac{c}{D}$	$\pi_4 = \frac{\mu\omega}{p}$

Then, $\frac{\mathcal{P}}{p\omega D^3} = f\left(\frac{\mu\omega}{p}, \frac{c}{D}, \frac{l}{D}\right) \longleftarrow$ $\mathcal{P}$

⑥ Check using M, L, t dimensions

$\pi_1 = \frac{ML^2}{t^3} \times \frac{Lt^2}{M} \times t \times \frac{1}{L^3} = [1]$ ✓

$\pi_2 = \frac{L}{L} = [1]$ ✓ $\qquad \pi_3 = \frac{L}{L} = [1]$ ✓

$\pi_4 = \frac{M}{Lt} \times \frac{1}{t} \times \frac{Lt^2}{M} = [1]$ ✓

Problem 7.40

[3]

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Given: Thrust, F_t, of a marine propeller is thought to depend on: ρ (water density), D (diameter), V (speed of advance), g (acceleration of gravity), ω (angular speed of propeller), p (pressure in the liquid), and μ (liquid viscosity)

Find: Dimensionless parameters that characterize propeller performance.

Solution: Apply Buckingham π-theorem

① List: $F_t \quad \rho \quad D \quad V \quad g \quad \omega \quad p \quad \mu \quad (n=8)$

② Choose M, L, t as primary dimensions

③ Dimensions: $\frac{ML}{t^2} \quad \frac{M}{L^3} \quad L \quad \frac{L}{t} \quad \frac{L}{t^2} \quad \frac{1}{t} \quad \frac{M}{Lt^2} \quad \frac{M}{Lt}$

④ Repeating variables $\rho, V, D \qquad m=r=3$

⑤ Then $n-m=5$ dimensionless groups will result

Setting up dimensional equations

$$\pi_1 = \rho^a V^b D^c F_t$$
$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{ML}{t^2} \qquad \left\{\begin{array}{l} M: 0=a+1 \\ t: 0=-b-2 \\ L: 0=-3a+b+c+1 \end{array}\right\} \begin{array}{l} a=-1 \\ b=-2 \\ c=-2 \end{array} \qquad \therefore \pi_1 = \frac{F_t}{\rho V^2 D^2}$$

$$\pi_2 = \rho^a V^b D^c g$$
$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{L}{t^2}\right) \qquad \left\{\begin{array}{l} M: 0=a \\ t: 0=-b-2 \\ L: 0=-3a+b+c+1 \end{array}\right\} \begin{array}{l} a=0 \\ b=-2 \\ c=1 \end{array} \qquad \therefore \pi_2 = \frac{gD}{V^2}$$

$$\pi_3 = \rho^a V^b D^c \omega$$
$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{1}{t}\right) \qquad \left\{\begin{array}{l} M: 0=a \\ t: 0=-b-1 \\ L: 0=-3a+b+c \end{array}\right\} \begin{array}{l} a=0 \\ b=-1 \\ c=1 \end{array} \qquad \therefore \pi_3 = \frac{\omega D}{V}$$

$$\pi_4 = \rho^a V^b D^c p$$
$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt^2} \qquad \left\{\begin{array}{l} M: 0=a+1 \\ t: 0=-b-2 \\ L: 0=-3a+b+c-1 \end{array}\right\} \begin{array}{l} a=-1 \\ b=-2 \\ c=0 \end{array} \qquad \therefore \pi_4 = \frac{p}{\rho V^2}$$

$$\pi_5 = \rho^a V^b D^c \mu$$
$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt} \qquad \left\{\begin{array}{l} M: 0=a+1 \\ t: 0=-b-1 \\ L: 0=-3a+b+c-1 \end{array}\right\} \begin{array}{l} a=-1 \\ b=-1 \\ c=-1 \end{array} \qquad \therefore \pi_5 = \frac{\mu}{\rho V D}$$

Dimensionless parameters are $\frac{F_t}{\rho V^2 D^2}, \ \frac{gD}{V^2}, \ \frac{\omega D}{V}, \ \frac{p}{\rho V^2}, \ \frac{\mu}{\rho V D}$ ←

⑥ Check using F, L, t

$\pi_1 = F \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} \times \frac{1}{L^2} = [1] \checkmark \qquad \pi_2 = \frac{L}{t^2} \times L \times \frac{t^2}{L^2} = [1] \checkmark$

$\pi_3 = \frac{1}{t} \times L \times \frac{t}{L} = [1], \quad \pi_4 = \frac{F}{L^2} \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} = [1]$

$\pi_5 = \frac{1}{Re} = [1] \checkmark$

Problem 7.41

[4]

Given: Fan-assisted convection oven; $\dot{Q}$ = heat transfer rate (energy/time).

$$\dot{Q} = f(c_p, \theta, L, \rho, \mu, V)$$

Find: (a) Number of basic dimensions included in these variables.
(b) Number of Π-parameters.
(c) Obtain the parameters.

Solution: Apply the Buckingham Π-theorem.

① $\dot{Q}$ $\quad c_p$ $\quad \Theta$ $\quad L$ $\quad \rho$ $\quad \mu$ $\quad V$ $\qquad n = 7$ parameters

② Select F, L, t, T (temperature) as primary dimensions.

③ $\frac{FL}{t}$ $\quad \frac{L^2}{t^2 T}$ $\quad T$ $\quad L$ $\quad \frac{Ft^2}{L^4}$ $\quad \frac{Ft}{L^2}$ $\quad \frac{L}{t}$ $\qquad r = 4$ primary dimensions

④ Choose ρ, V, L, Θ as repeating parameters. $\qquad m = r = 4$

⑤ Then $n - m = 7 - 4 = 3$ dimensionless parameters result. $\qquad n-m$

Set up dimensional equations:

$\Pi_1 = \rho^a V^b L^c \theta^d \dot{Q}$

$F^0 L^0 t^0 T^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L}{t}\right)^b (L)^c (T)^d \frac{FL}{t}$

Equating exponents:

$F: a + 1 = 0 \qquad a = -1$
$L: -4a + b + c + 1 = 0$
$t: 2a - b - 1 = 0 \qquad b = -3$
$T: d = 0$

$\therefore c = -1 + 4a - b = -2$

$$\Pi_1 = \frac{\dot{Q}}{\rho V^3 L^2}$$

$\Pi_2 = \rho^a V^b L^c \theta^d c_p$

$F^0 L^0 t^0 T^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L}{t}\right)^b (L)^c (T)^d \frac{L^2}{t^2 T}$

Equating exponents:

$F: a = 0 \qquad a = 0$
$L: -4a + b + c + 2 = 0$
$t: 2a - b - 2 = 0 \qquad b = -2$
$T: d - 1 = 0 \qquad d = 1$

$\therefore c = -2 + 4a - b = 0$

$$\Pi_2 = \frac{c_p \theta}{V^2}$$

$\Pi_3 = \rho^a V^b L^c \theta^d \mu$

$F^0 L^0 t^0 T^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L}{t}\right)^b (L)^c (T)^d \frac{Ft}{L^2}$

Equating exponents:

$F: a + 1 = 0 \qquad a = -1$
$L: -4a + b + c - 2 = 0$
$t: 2a - b + 1 = 0 \qquad b = -1$
$T: d = 0 \qquad d = 0$

$\therefore c = 2 + 4a - b = -1$

$$\Pi_3 = \frac{\mu}{\rho V L}$$

⑥ Check, using MLtT dimensions: $\dot{Q} = ML^2/t^3$; $\mu = M/Lt$

$\Pi_1 = \frac{ML^2}{t^3} \frac{L^3}{M} \frac{t^3}{L^3} \frac{1}{L^2} = [1]$ ✓✓

$\Pi_2 = \frac{L^2}{t^2 T} T \frac{t^2}{L^2} = [1]$ ✓✓

$\Pi_3 = \frac{M}{Lt} \frac{L^3}{M} \frac{t}{L} \frac{1}{L} = [1]$ ✓✓

Thus

$$\Pi_1 = f(\Pi_2, \Pi_3) \qquad \frac{\dot{Q}}{\rho V^3 L^2} = f\left(\frac{c_p \Theta}{V^2}, \frac{\mu}{\rho V L}\right)$$

or

$$\dot{Q} = \rho V^3 L^2 f\left(\frac{c_p \Theta}{V^2}, \frac{\mu}{\rho V L}\right)$$

$\dot{Q}$

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Problem 7.42

[3]

Given: Power, $\mathscr{P}$, required to drive a propeller is a function of V, D, ω (angular velocity), μ, ρ, and c (speed of sound)

Find: (a) number of dimensionless groups required to characterize situation
(b) the dimensionless groups

Solution: Apply Buckingham π-theorem

① $\mathscr{P} \quad V \quad D \quad \omega \quad \mu \quad \rho \quad c$ $\qquad n = 7$ parameters

② Select M, L, t as primary dimensions

③ $\mathscr{P} \quad V \quad D \quad \omega \quad \mu \quad \rho \quad c$

$\frac{ML^2}{t^3} \quad \frac{L}{t} \quad L \quad \frac{1}{t} \quad \frac{M}{Lt} \quad \frac{M}{L^3} \quad \frac{L}{t}$ $\qquad r = 3$ primary dimensions

④ V, D, ρ $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 4$ dimensionless groups will result.

Setting up dimensional equations:

$\pi_1 = V^a D^b \rho^c \mathscr{P}$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{ML^2}{t^3}$

Summing exponents,

M: $c + 1 = 0 \quad \therefore c = -1$

L: $a + b - 3c + 2 = 0$

t: $-a - 3 = 0 \quad \therefore a = -3$

$b = 3c - 2 - a = -2$

$\therefore \pi_1 = \frac{\mathscr{P}}{\rho D^2 V^3}$

$\pi_2 = V^a D^b \rho^c \omega$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{1}{t}$

Summing exponents,

M: $c = 0$

L: $a + b - 3c = 0$

t: $-a - 1 = 0 \quad \therefore a = -1$

$b = 3c - a = 1$

$\therefore \pi_2 = \frac{\omega D}{V}$

$\pi_3 = V^a D^b \rho^c \mu$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt}$

Summing exponents,

M: $c + 1 = 0 \quad \therefore c = -1$

L: $a + b - 3c - 1 = 0$

t $-a - 1 = 0 \quad \therefore a = -1$

$b = 3c + 1 - a = -1$

$\therefore \pi_3 = \frac{\mu}{\rho D V}$

$\pi_4 = V^a D^b \rho^c c$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{L}{t}$

Summing exponents,

M: $c = 0$

L: $a + b - 3c + 1 = 0$

t: $-a - 1 = 0 \quad \therefore a = -1$

$b = 3c - a - 1 = 0$

$\therefore \pi_4 = \frac{c}{V}$

Dimensionless groups are: $\frac{\mathscr{P}}{\rho D^2 V^3}, \frac{\omega D}{V}, \frac{\mu}{\rho D V}, \frac{c}{V}$

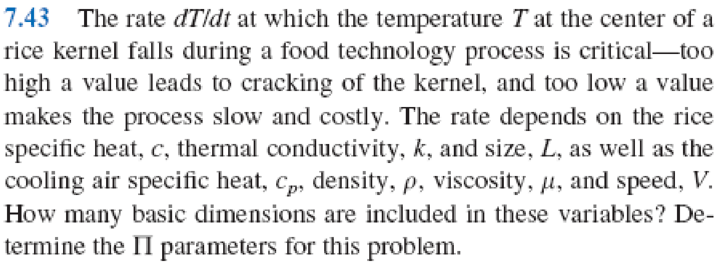
⑥ Check using F, L, t

$\pi_1 = \frac{FL}{t} \cdot \frac{L^4}{Ft^2} \frac{1}{L^2} \frac{t^3}{L^3} = [1]$✓ $\qquad \pi_2 = \frac{1}{t} L \cdot \frac{t}{L} = [1]$✓

$\pi_3 = \frac{1}{Re} = [1]$✓ $\qquad \pi_4 = \frac{L}{t} \cdot \frac{t}{L} = [1]$✓

Problem 7.43

[3]



7.43 The rate dT/dt at which the temperature T at the center of a rice kernel falls during a food technology process is critical—too high a value leads to cracking of the kernel, and too low a value makes the process slow and costly. The rate depends on the rice specific heat, c, thermal conductivity, k, and size, L, as well as the cooling air specific heat, c_p, density, ρ, viscosity, μ, and speed, V. How many basic dimensions are included in these variables? Determine the Π parameters for this problem.

Given: That the cooling rate depends on rice properties and air properties

Find: The Π groups

Solution:

Apply the Buckingham Π procedure

① $dT/dt \quad c \quad k \quad L \quad c_p \quad \rho \quad \mu \quad V$ $\qquad$ $n = 8$ parameters

② Select primary dimensions M, L, t and T (temperature)

③

$$\begin{array}{cccccccc} dT/dt & c & k & L & c_p & \rho & \mu & V \\ \frac{T}{t} & \frac{L^2}{t^2T} & \frac{ML}{t^2T} & L & \frac{L^2}{t^2T} & \frac{M}{L^3} & \frac{M}{Lt} & \frac{L}{t} \end{array}$$

$r = 4$ primary dimensions

④ $V \quad \rho \quad L \quad c_p$ $\qquad$ $m = r = 4$ repeat parameters

Then $n - m = 4$ dimensionless groups will result. By inspection, one Π group is c/c_p. Setting up a dimensional equation,

$$\Pi_1 = V^a \rho^b L^c c_p^d \frac{dT}{dt} = \left(\frac{L}{t}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \left(\frac{L^2}{t^2T}\right)^d \frac{T}{t} = T^0 M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rc|c} T: & -d + 1 = 0 & d = 1 \\ M: & b = 0 & b = 0 \\ L: & a - 3b + c + 2d = 0 & a + c = -2 \rightarrow c = 1 \\ t: & -a - 2d - 1 = 0 & a = -3 \end{array}$$

Hence $\quad \Pi_1 = \dfrac{dT}{dt}\dfrac{Lc_p}{V^3}$

By a similar process, we find $\qquad \Pi_2 = \dfrac{k}{\rho L^2 c_p} \qquad$ and $\qquad \Pi_3 = \dfrac{\mu}{\rho L V}$

Hence

$$\frac{dT}{dt}\frac{Lc_p}{V^3} = f\left(\frac{c}{c_p}, \frac{k}{\rho L^2 c_p}, \frac{\mu}{\rho L V}\right)$$

Problem 7.44

[4]

Given: Water hammer caused by sudden closure of valve in pipeline.

$$p_{max} = f(\rho, V_0, E_v)$$

Find: (a) How many dimensionless groups needed to characterize?
(b) Functional relationship in terms of Π groups.

Solution: Step ①: List $\quad p_{max} \quad \rho \quad V_0 \quad E_v \qquad n = 4$

Step ②: Choose M, L, t

Step ③: $\quad \dfrac{M}{Lt^2} \quad \dfrac{M}{L^3} \quad \dfrac{L}{t} \quad \dfrac{M}{Lt^2}$

Check dimensional matrix:

$$\begin{array}{c|cccc} & p_{max} & \rho & V_0 & E_v \\ \hline M & 1 & 1 & 0 & 1 \\ L & -1 & -3 & 1 & -1 \\ t & -2 & 0 & -1 & -2 \end{array}$$

For this matrix, $r = 2$

Step ④: Choose ρ, V_0

Step ⑤: $\Pi_1 = \rho^a V_0^b p_{max} = \left(\dfrac{M}{L^3}\right)^a \left(\dfrac{L}{t}\right)^b \dfrac{M}{Lt^2} = M^0 L^0 t^0$

$$\left.\begin{array}{ll} M: a + 1 = 0 & a = -1 \\ L: -3a + b - 1 = 0 & \\ t: -b - 2 = 0 & b = -2 \end{array}\right\} \checkmark \qquad \Pi_1 = \frac{p_{max}}{\rho V_0^2}$$ Π_1

$\Pi_2 = \rho^a V_0^b E_v = \left(\dfrac{M}{L^3}\right)^a \left(\dfrac{L}{t}\right)^b \dfrac{M}{Lt^2} = M^0 L^0 t^0$

By inspection $\qquad \Pi_2 = \dfrac{E_v}{\rho V_0^2}$ Π_2

Step ⑥: Check using FLt: $\rho = \dfrac{M}{L^3} \times \dfrac{Ft^2}{ML} = \dfrac{Ft^2}{L^4}$

$$\Pi_1 = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{t^2}{L^2} = \frac{FL^4t^2}{FL^4t^2} = 1 \quad \checkmark\checkmark$$

The functional relationship is $\Pi_1 = f(\Pi_2)$. Thus

$$\frac{p_{max}}{\rho V_0^2} = f\left(\frac{E_v}{\rho V_0^2}\right)$$ f

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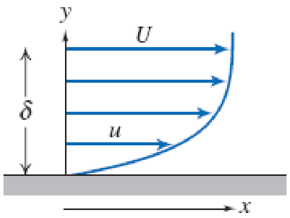
Problem 7.45

[2]

7.45 The fluid velocity u at any point in a boundary layer depends on the distance y of the point above the surface, the free-stream velocity U and free-stream velocity gradient dU/dx, the fluid kinematic viscosity ν, and the boundary layer thickness δ. How many dimensionless groups are required to describe this problem? Find: (a) two Π groups by inspection, (b) one Π that is a standard fluid mechanics group, and (c) any remaining Π groups using the Buckingham Pi theorem.



Given: Boundary layer profile

Find: Two Π groups by inspection; One Π that is a standard fluid mechanics group; Dimensionless groups

Solution:

Two obvious Π groups are u/U and y/δ. A dimensionless group common in fluid mechanics is $U\delta/\nu$ (Reynolds number)

Apply the Buckingham Π procedure

① $u \quad y \quad U \quad dU/dx \quad \nu \quad \delta$ $\qquad n = 6$ parameters

② Select primary dimensions M, L, t

③ $$\left\{\begin{array}{cccccc} u & y & U & dU/dx & \nu & \delta \\ \frac{L}{t} & L & \frac{L}{t} & \frac{1}{t} & \frac{L^2}{t} & L \end{array}\right\}$$ $m = r = 3$ primary dimensions

④ $U \quad \delta$ $\qquad m = r = 2$ repeat parameters

⑤ Then $n - m = 4$ dimensionless groups will result. We can easily do these by inspection

$$\Pi_1 = \frac{u}{U} \qquad \Pi_2 = \frac{y}{\delta} \qquad \Pi_3 = \frac{(dU/dy)\delta}{U} \qquad \Pi_4 = \frac{\nu}{\delta U}$$

⑥ Check using F, L, t as primary dimensions, is not really needed here

Note: Any combination of Π's can be used; they are not unique!

Problem 7.46

[3]

Given: Airship to operate at 20 m/sec in standard air.
Model built to 1/20 scale tested at same air temperature.
Model is tested at 75 m/sec

Find: (a) Criterion for dynamic similarity.
(b) Wind tunnel pressure.
(c) Prototype drag if drag force on model is 250 N.

Solution

Dimensional analysis predicts $\frac{F}{\rho V^2 L^2} = f\left(\frac{\rho V L}{\mu}\right)$

Consequently for similarity, $\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p \longleftarrow$

Since L is fixed, and $\mu_p = \mu_m$ (because T is the same)

$$\rho_m = \rho_p \frac{V_p}{V_m}\frac{L_p}{L_m}\frac{\mu_m}{\mu_p} = \rho_p \frac{20}{75}(20)(1) = 5.33\,\rho_p$$

From ideal gas law, $p = \rho R T$

$\therefore \frac{p_m}{p_p} = \frac{\rho_m}{\rho_p} = 5.33$ and $p_m = 5.33\,p_p = 5.33 \times 101$ kPa $= 5.39\times10^5$ Pa $\longleftarrow$ p_m

From the force ratios,

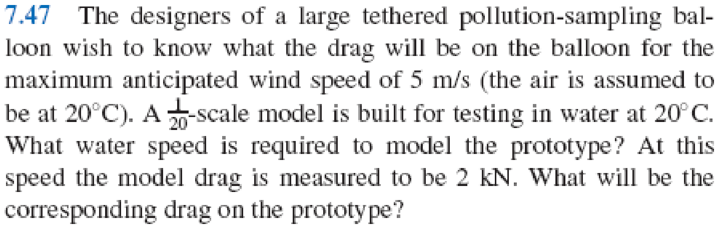
$$F_p = F_m \frac{\rho_p}{\rho_m}\frac{V_p^2}{V_m^2}\frac{L_p^2}{L_m^2} = F_m \frac{1}{5.33}\left(\frac{20}{75}\right)^2 (20)^2 = 5.34\,F_m$$

Thus

$F_p = 5.34\,F_m = 5.34 \times 250$ N $= 1.34$ kN $\longleftarrow$ F_p

Problem 7.47

[3]



7.47 The designers of a large tethered pollution-sampling balloon wish to know what the drag will be on the balloon for the maximum anticipated wind speed of 5 m/s (the air is assumed to be at 20°C). A $\frac{1}{20}$-scale model is built for testing in water at 20°C. What water speed is required to model the prototype? At this speed the model drag is measured to be 2 kN. What will be the corresponding drag on the prototype?

Given: Model scale for on balloon

Find: Required water model water speed; drag on protype based on model drag

Solution:

From Appendix A (inc. Fig. A.2) $\rho_{air} = 1.24 \cdot \frac{kg}{m^3}$ $\mu_{air} = 1.8 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}$ $\rho_w = 999 \cdot \frac{kg}{m^3}$ $\mu_w = 10^{-3} \cdot \frac{N \cdot s}{m^2}$

The given data is $V_{air} = 5 \cdot \frac{m}{s}$ $L_{ratio} = 20$ $F_w = 2 \cdot kN$

For dynamic similarity we assume $\frac{\rho_w \cdot V_w \cdot L_w}{\mu_w} = \frac{\rho_{air} \cdot V_{air} \cdot L_{air}}{\mu_{air}}$

Then $V_w = V_{air} \cdot \frac{\mu_w}{\mu_{air}} \cdot \frac{\rho_{air}}{\rho_w} \cdot \frac{L_{air}}{L_w} = V_{air} \cdot \frac{\mu_w}{\mu_{air}} \cdot \frac{\rho_{air}}{\rho_w} \cdot L_{ratio} = 5 \cdot \frac{m}{s} \times \left(\frac{10^{-3}}{1.8 \times 10^{-5}} \right) \times \left(\frac{1.24}{999} \right) \times 20$ $V_w = 6.90 \frac{m}{s}$

For the same Reynolds numbers, the drag coefficients will be the same so we have $\frac{F_{air}}{\frac{1}{2} \cdot \rho_{air} \cdot A_{air} \cdot V_{air}^2} = \frac{F_w}{\frac{1}{2} \cdot \rho_w \cdot A_w \cdot V_w^2}$

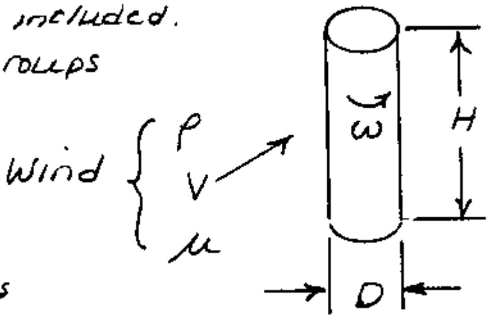
where $\frac{A_{air}}{A_w} = \left(\frac{L_{air}}{L_w} \right)^2 = L_{ratio}^2$

Hence the prototype drag is $F_{air} = F_w \cdot \frac{\rho_{air}}{\rho_w} \cdot L_{ratio}^2 \cdot \left(\frac{V_{air}}{V_w} \right)^2 = 2000 \cdot N \times \left(\frac{1.24}{999} \right) \times 20^2 \times \left(\frac{5}{6.9} \right)^2$ $F_{air} = 522\,N$

Problem 7.48

Given: Vessel to be powered by rotating cylinder. Model to be tested to estimate power neededed to rotate cylinder.

Find: (a) Parameters that should be included.
(b) Important dimensionless groups



Solution: $\mathcal{P} = f(\rho, \omega, D, \mu, H, V)$

① $\rho \quad \omega \quad D \quad \mu \quad H \quad V \quad \mathcal{P} \qquad n = 7$

② Choose M, L, t as primary dimensions

③ $\frac{M}{L^3} \quad \frac{1}{t} \quad L \quad \frac{M}{Lt} \quad L \quad \frac{L}{t} \quad \frac{ML^2}{t^3} \qquad r = 3$ primary dimensions

④ $\rho, \omega, D \qquad m = 3 \qquad m = r = 3$ repeating parameters

⑤ Then expect $n - m = 4$ dimensionless groups

$\Pi_1 = \rho^a \omega^b D^c \mathcal{P} = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{ML^2}{t^3}$

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + c + 2 = 0 \qquad c = -5$
$t: -b - 3 = 0 \qquad b = -3$

$$\Pi_1 = \frac{\mathcal{P}}{\rho \omega^3 D^5} \qquad \Pi_1$$

$\Pi_2 = \rho^a \omega^b D^c V = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{L}{t}$

$M: a + 0 = 0 \qquad a = 0$
$L: -3a + c + 1 = 0 \qquad c = -1$
$t: -b - 1 = 0 \qquad b = -1$

$$\Pi_2 = \frac{V}{\omega D} \qquad \Pi_2$$

$\Pi_3 = \rho^a \omega^b D^c H$

By inspection $\Pi_3 = \frac{H}{D} \qquad \Pi_3$

$\Pi_4 = \rho^a \omega^b D^c \mu = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{M}{Lt}$

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + c - 1 = 0 \qquad c = -2$
$t: -b - 1 = 0 \qquad b = -1$

$$\Pi_4 = \frac{\mu}{\rho \omega D^2}$$

Thus $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$ or $\frac{\mathcal{P}}{\rho \omega^3 D^5} = f\left(\frac{V}{\omega D}, \frac{H}{D}, \frac{\mu}{\rho \omega D^2}\right)$

$\frac{\mathcal{P}}{\rho \omega^3 D^5}$

⑥ Check, using F, L, t

$\Pi_1 = \frac{FL}{t} \frac{L^4}{Ft^2} \frac{t^3}{1} \frac{1}{L^5} = [1]$ ✓ $\qquad \Pi_2 = \frac{L}{t} \frac{t}{1} \frac{1}{L} = [1]$ ✓

$\Pi_3 = \frac{L}{L} = [1]$ ✓ $\qquad \Pi_4 = \frac{Ft}{L^2} \frac{L^4}{Ft^2} \frac{t}{1} \frac{1}{L^2} = [1]$ ✓

Problem 7.49

Given: Desire to match Reynolds number in two flows: one of air and one of water, using the same size model.

Find: Which flow must have the higher speed, and by how much.

Solution: Set $Re_w = \frac{\rho_w V_w L_w}{\mu_w} = Re_a = \frac{\rho_a V_a L_a}{\mu_a}$

Since $L_w = L_a$, then $\frac{V_a}{V_w} = \frac{\rho_w}{\rho_a}\frac{\mu_a}{\mu_w} = \frac{\nu_a}{\nu_w}$

From Tables A.8 and A.10, at 20°C, $\nu_w = 1.00 \times 10^{-6}\, m^2/s$ and $\nu_a = 1.51 \times 10^{-5}\, m^2/s$.

Thus $\frac{V_a}{V_w} = 1.51 \times 10^{-5} \frac{m^2}{s} \times \frac{s}{1.00 \times 10^{-6}\, m^2} = 15.1$

Therefore V_a must be larger than V_w. ← V_a

In fact, to match Re,

$V_a = 15.1\, V_w$ ← V_a

Problem 7.50

[3]

Given: Measurements of drag force are made on a model car in a towing tank filled with freshwater; $L_m/L_p = 1/5$. The dimensionless force ratio becomes constant at model test speeds above $V_m = 4\,m/s$. At this speed the drag force on the model is $F_{D_m} = 182\,N$.

Find: (a) State conditions required to assure dynamic similarity between model and prototype

(b) Determine required speed ratio V_m/V_p to assure dynamically similar conditions

(c) Calculate expected prototype drag when operating in air at speed, $V_p = 90\,km/hr$.

Solution:

(a) The flows must be geometrically and kinematically similar, and have equal Reynolds numbers to be dynamically similar.

- geometric similarity requires true model in all respects
- kinematic similarity requires same flow pattern, i.e. no free-surface effects or cavitation.
- the problem may be stated as $F_D = f(\rho, V, L, \mu)$.

Dimensional analysis gives

$$\frac{F_D}{\rho V^2 L^2} = f\left(\frac{\mu}{\rho V L}\right) = g(Re).$$

(b) Matching Reynolds numbers between model & prototype flows gives $\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$ Assume T = 20°C

$$\frac{V_m}{V_p} = \frac{\nu_m}{\nu_p}\times\frac{L_p}{L_m} = 1\times10^{-6}\,\frac{m^2}{s}\times 1.51\times10^{5}\,\frac{s}{m^2}\times 5 = 0.331 \longleftarrow \frac{V_m}{V_p}$$

(c) For dynamically similar conditions, $\left.\frac{F_D}{\rho V^2 L^2}\right)_m = \left.\frac{F_D}{\rho V^2 L^2}\right)_p$

$$\therefore F_{D_p} = F_{D_m}\,\frac{\rho_p}{\rho_m}\times\left(\frac{V_p}{V_m}\right)^2\times\left(\frac{L_p}{L_m}\right)^2$$

$$= 182\,N\times\frac{1.20}{999}\times\left(\frac{90\,km}{hr}\times\frac{1000\,m}{km}\times\frac{hr}{3600\,s}\times\frac{s}{4\,m}\right)^2(5)^2$$

$$F_{D_p} = 214\,N \longleftarrow F_{D_p}$$

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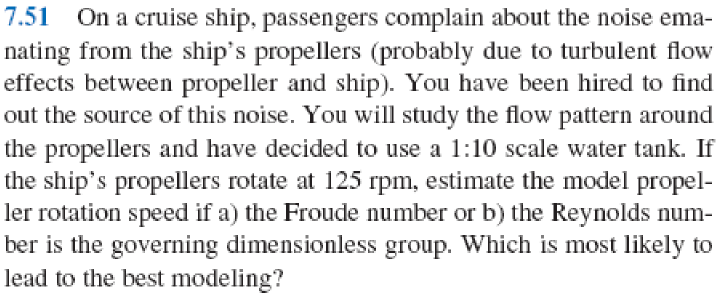
Problem 7.51

[2]



7.51 On a cruise ship, passengers complain about the noise emanating from the ship's propellers (probably due to turbulent flow effects between propeller and ship). You have been hired to find out the source of this noise. You will study the flow pattern around the propellers and have decided to use a 1:10 scale water tank. If the ship's propellers rotate at 125 rpm, estimate the model propeller rotation speed if a) the Froude number or b) the Reynolds number is the governing dimensionless group. Which is most likely to lead to the best modeling?

Given: Flow around ship's propeller

Find: Model propeller speed using Froude number and Reynolds number

Solution:

Basic equations $\quad Fr = \frac{V}{\sqrt{g \cdot L}} \qquad Re = \frac{V \cdot L}{\nu}$

Using the Froude number $\quad Fr_m = \frac{V_m}{\sqrt{g \cdot L_m}} = Fr_p = \frac{V_p}{\sqrt{g \cdot L_p}}$ or $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$ (1)

But the angular velocity is given by $\quad V = L \cdot \omega$ so $\frac{V_m}{V_p} = \frac{L_m}{L_p} \cdot \frac{\omega_m}{\omega_p}$ (2)

Comparing Eqs. 1 and 2 $\quad \frac{L_m}{L_p} \cdot \frac{\omega_m}{\omega_p} = \sqrt{\frac{L_m}{L_p}} \qquad \frac{\omega_m}{\omega_p} = \sqrt{\frac{L_p}{L_m}}$

The model rotation speed is then $\quad \omega_m = \omega_p \cdot \sqrt{\frac{L_p}{L_m}} \qquad \omega_m = 125 \cdot \mathrm{rpm} \times \sqrt{\frac{10}{1}} \qquad \omega_m = 395\,\mathrm{rpm}$

Using the Reynolds number $\quad Re_m = \frac{V_m \cdot L_m}{\nu_m} = Re_p = \frac{V_p \cdot L_p}{\nu_p}$ or $\frac{V_m}{V_p} = \frac{L_p}{L_m} \cdot \frac{\nu_m}{\nu_p} = \frac{L_p}{L_m}$ (3)

(We have assumed the viscosities of the sea water and model water are comparable)

Comparing Eqs. 2 and 3 $\quad \frac{L_m}{L_p} \cdot \frac{\omega_m}{\omega_p} = \frac{L_p}{L_m} \qquad \frac{\omega_m}{\omega_p} = \left(\frac{L_p}{L_m}\right)^2$

The model rotation speed is then $\quad \omega_m = \omega_p \cdot \left(\frac{L_p}{L_m}\right)^2 \qquad \omega_m = 125 \cdot \mathrm{rpm} \times \left(\frac{10}{1}\right)^2 \qquad \omega_m = 12500\,\mathrm{rpm}$

Of the two models, the Froude number appears most realistic; at 12,500 rpm serious cavitation will occur. Both flows will likely have high Reynolds numbers so that the flow becomes independent of Reynolds number; the Froude number is likely to be a good indicator of static pressure to dynamic pressure for this (although cavitation number would be better).

Problem 7.52

Given: Prototype torpedo, $D = 533$ mm, $l = 6.7$ m operates in water at a speed of 28 m/s. Model ($1/5$ scale) is to be tested in a wind tunnel. Maximum wind tunnel speed is 110 m/sec; $T = 20°C$; pressure is variable.
At dynamically similar test conditions, $F_{D\,model} = 618\,N$

Find: (a) required wind tunnel pressure for dynamically similar test
(b) expected drag force on prototype

Solution:

Assume $F_D = F_D(V, D, \rho, \mu)$. From the Buckingham π-theorem, for $n = 5$, with $m = r = 3$, we would expect two dimensionless groups.

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}\right)$$

To attain dynamically similar model test, $\left.\frac{\rho V D}{\mu}\right)_m = \left.\frac{\rho V D}{\mu}\right)_p$

$$\therefore \quad \rho_m = \rho_p \frac{V_p}{V_m} \frac{D_p}{D_m} \frac{\mu_m}{\mu_p}$$

For air at 20°C $\mu_m = 1.81 \times 10^{-5}\ N \cdot s/m^2$
water at 20°C $\mu_p = 1 \times 10^{-3}\ N \cdot s/m^2$

$$\rho_m = 998\,\frac{kg}{m^3} \times \frac{28}{110} \times 5 \times \frac{1.81 \times 10^{-5}}{1 \times 10^{-3}} = 23.0\ kg/m^3.$$

From the ideal gas equation of state,

$$p = \rho_m R T_m = 23.0\,\frac{kg}{m^3} \times 287\,\frac{N \cdot m}{kg \cdot K} \times 293\,K = 1.93\ MPa\,(abs) \longleftarrow$$

p

For dynamically similar flows,

$$\left.\frac{F_D}{\rho V^2 D^2}\right)_m = \left.\frac{F_D}{\rho V^2 D^2}\right)_p$$

$$\therefore \quad F_{Dp} = F_{Dm} \frac{\rho_p}{\rho_m} \left(\frac{V_p}{V_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2$$

$$= 618\,N \times \frac{998}{23.0} \left(\frac{28}{110}\right)^2 (5)^2$$

$$F_{Dp} = 43.4\ kN \longleftarrow$$

F_{Dp}

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Problem 7.53

[3]

Given: Drag force, F, of an airfoil at zero angle of attack is a function of ρ, μ, V, and L.

Model test conditions:

$$\frac{L_m}{L_p} = \frac{1}{10} \qquad Re_m = 5.5\times10^6 \text{ based on chord length}$$

$$T = 15^\circ C \quad , \; P = 10 \text{ atmospheres}$$

Prototype data: chord length, $L = 2\,m$

$$T = 15^\circ C \qquad P = 101 \text{ kPa}$$

Find: (a) velocity, V_m, of model test
(b) corresponding prototype velocity.

Solution

Dimensional analysis predicts $\quad \dfrac{F}{\rho V^2 L^2} = f\left(\dfrac{\rho V L}{\mu}\right)$

$$Re_m = \frac{\rho V L}{\mu}\Big)_m \quad \text{and hence} \quad V_m = \frac{Re_m\,\mu_m}{\rho_m L_m}$$

To determine ρ_m assume air behaves as an ideal gas.

$$\rho_m = \frac{p_m}{RT_m} = 10\times101\times10^3\,\frac{N}{m^2}\times\frac{kg\cdot K}{287\,N\cdot m}\times\frac{1}{288\,K} = 12.2\ kg/m^3$$

From Table A.10, Appendix A, $\mu_m = 1.79\times10^{-5}\ N\cdot s/m^2$

$$V_m = \frac{Re_m\,\mu_m}{\rho_m L_m} = 5.5\times10^6\times1.79\times10^{-5}\,\frac{N\cdot sec}{m^2}\times\frac{m^3}{12.2\,kg}\times\frac{1}{0.2\,m}\times\frac{kg\cdot m}{N\cdot sec^2}$$

$$V_m = 40.3\ m/s \longleftarrow$$

V_m

For dynamic similarity $\quad \dfrac{\rho V L}{\mu}\Big)_m = \dfrac{\rho V L}{\mu}\Big)_p$

$$V_p = V_m\,\frac{\mu_p}{\mu_m}\,\frac{\rho_m}{\rho_p}\,\frac{L_m}{L_p} = V_m\,\frac{\mu_p}{\mu_m}\,\frac{p_m}{p_p}\,\frac{T_p}{T_m}\,\frac{L_m}{L_p}$$

$$V_p = 40.3\,\frac{m}{s}\times(1)\times(10)\times(1)\times\left(\frac{1}{10}\right) = 40.3\ m/s$$

V_p

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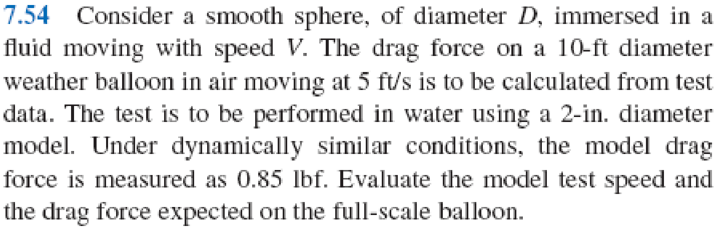
Problem 7.54

[2]



7.54 Consider a smooth sphere, of diameter *D*, immersed in a fluid moving with speed *V*. The drag force on a 10-ft diameter weather balloon in air moving at 5 ft/s is to be calculated from test data. The test is to be performed in water using a 2-in. diameter model. Under dynamically similar conditions, the model drag force is measured as 0.85 lbf. Evaluate the model test speed and the drag force expected on the full-scale balloon.

Given: Model of weather balloon

Find: Model test speed; drag force expected on full-scale balloon

Solution:

From Buckingham Π
$$\frac{F}{\rho \cdot V^2 \cdot D^2} = f\left(\frac{\nu}{V \cdot D}, \frac{V}{c}\right) = F(\mathrm{Re}, M)$$

For similarity $\mathrm{Re}_p = \mathrm{Re}_m$ and $M_p = M_m$ (Mach number criterion satisified because M<<)

Hence
$$\mathrm{Re}_p = \frac{V_p \cdot D_p}{\nu_p} = \mathrm{Re}_m = \frac{V_m \cdot D_m}{\nu_m}$$

$$V_m = V_p \cdot \frac{\nu_m}{\nu_p} \cdot \frac{D_p}{D_m}$$

From Table A.7 at 68ºF $\nu_m = 1.08 \times 10^{-5} \cdot \frac{ft^2}{s}$ From Table A.9 at 68ºF $\nu_p = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

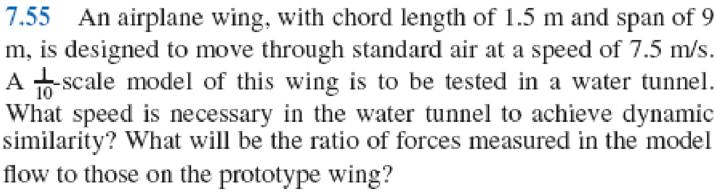
$$V_m = 5 \cdot \frac{ft}{s} \times \left(\frac{1.08 \times 10^{-5} \cdot \frac{ft^2}{s}}{1.62 \times 10^{-4} \cdot \frac{ft^2}{s}}\right) \times \left(\frac{10 \cdot ft}{\frac{1}{6} \cdot ft}\right) \qquad V_m = 20.0 \frac{ft}{s}$$

Then
$$\frac{F_m}{\rho_m \cdot V_m^2 \cdot D_m^2} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot D_p^2} \qquad F_p = F_m \cdot \frac{\rho_p}{\rho_{m.}} \cdot \frac{V_p^2}{V_m^2} \cdot \frac{D_p^2}{D_m^2}$$

$$F_p = 0.85 \cdot lbf \times \left(\frac{0.00234 \cdot \frac{slug}{ft^3}}{1.94 \cdot \frac{slug}{ft^3}}\right) \times \left(\frac{5 \frac{ft}{s}}{20 \cdot \frac{ft}{s}}\right)^2 \times \left(\frac{10 \cdot ft}{\frac{1}{6} \cdot ft}\right)^2 \qquad F_p = 0.231\,lbf$$

Problem 7.55

[2]



7.55 An airplane wing, with chord length of 1.5 m and span of 9 m, is designed to move through standard air at a speed of 7.5 m/s. A $\frac{1}{10}$-scale model of this wing is to be tested in a water tunnel. What speed is necessary in the water tunnel to achieve dynamic similarity? What will be the ratio of forces measured in the model flow to those on the prototype wing?

Given: Model of wing

Find: Model test speed for dynamic similarity; ratio of model to prototype forces

Solution:

We would expect $F = F(l, s, V, \rho, \mu)$ where F is the force (lift or drag), l is the chord and s the span

From Buckingham Π

$$\frac{F}{\rho \cdot V^2 \cdot l \cdot s} = f\left(\frac{\rho \cdot V \cdot l}{\mu}, \frac{l}{s}\right)$$

For dynamic similarity

$$\frac{\rho_m \cdot V_m \cdot l_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot l_p}{\mu_p}$$

Hence

$$V_m = V_p \cdot \frac{\rho_p}{\rho_m} \cdot \frac{l_p}{l_m} \cdot \frac{\mu_m}{\mu_p}$$

From Table A.8 at 20ºC $\mu_m = 1.01 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}$ From Table A.10 at 20ºC $\mu_p = 1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}$

$$V_m = 7.5 \cdot \frac{m}{s} \times \left(\frac{1.21 \cdot \frac{kg}{m^3}}{998 \cdot \frac{kg}{m^3}}\right) \times \left(\frac{10}{1}\right) \times \left(\frac{1.01 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}}{1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}}\right) \qquad V_m = 5.07 \frac{m}{s}$$

Then

$$\frac{F_m}{\rho_m \cdot V_m^2 \cdot l_m \cdot s_m} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot l_p \cdot s_p} \qquad \frac{F_m}{F_p} = \frac{\rho_m}{\rho_p} \cdot \frac{V_m^2}{V_p^2} \cdot \frac{l_m \cdot s_m}{l_p \cdot s_p} = \frac{998}{1.21} \times \left(\frac{5.07}{7.5}\right)^2 \times \frac{1}{10} \times \frac{1}{10} = 3.77$$

Problem 7.56

[3]

Given: Fluid dynamic characteristics of a golf ball are to be tested using a model in a wind tunnel.

dependent variables: F_D, F_L

independent variables should include ω & d (dimple depth)

Golf pro can hit prototype ($D = 1.68$ in) at $V = 240$ ft/s and $\omega = 9000$ rpm. Prototype is to be modeled in wind tunnel with $V = 80$ ft/s.

Find: (a) suitable dimensionless parameters

(b) required diameter of model

(c) required rotational speed of model

Solution: Assume the functional dependence to be given by

$$F_D = F_D(D, V, \omega, d, \rho, \mu) \quad \text{and} \quad F_L = F_L(D, V, \omega, d, \rho, \mu)$$

From the Buckingham π-theorem, for $n = 7$ and $m = r = 3$, we would expect four dimensionless groups

$$\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}, \frac{\omega D}{V}, \frac{d}{D}\right) \quad \text{and} \quad \frac{F_L}{\rho V^2 D^2} = g\left(\frac{\rho V D}{\mu}, \frac{\omega D}{V}, \frac{d}{D}\right) \longleftarrow \pi\text{'s}$$

To determine the required diameter of the model,

$$\left.\frac{\rho V D}{\mu}\right)_m = \left.\frac{\rho V D}{\mu}\right)_p \qquad \therefore D_m = \frac{\rho_p}{\rho_m}\frac{V_p}{V_m}\frac{\mu_m}{\mu_p} D_p = 1 \times \frac{240}{80} \times 1 \times D_p$$

$$D_m = 3 D_p = 3 \times 1.68 \text{ in} = 5.04 \text{ in.} \longleftarrow D_m$$

To determine the required rotational speed of the model,

$$\left.\frac{\omega D}{V}\right)_m = \left.\frac{\omega D}{V}\right)_p \qquad \therefore \omega_m = \omega_p \frac{D_p}{D_m}\frac{V_m}{V_p} = \omega_p \frac{1}{3} \times \frac{80}{240} = \frac{1}{9}\omega_p$$

$$\omega_m = \frac{1}{9}\omega_p = \frac{1}{9} \times 9000 \text{ rpm} = 1000 \text{ rpm} \longleftarrow \omega_m$$

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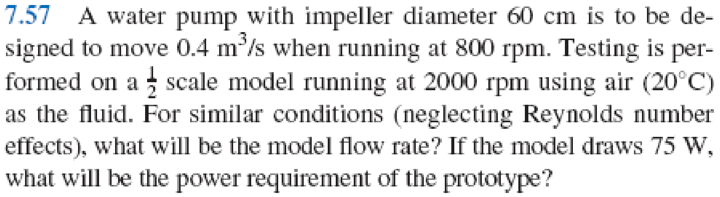
Problem 7.57

[3]



7.57 A water pump with impeller diameter 60 cm is to be designed to move 0.4 m^3/s when running at 800 rpm. Testing is performed on a $\frac{1}{2}$ scale model running at 2000 rpm using air (20°C) as the fluid. For similar conditions (neglecting Reynolds number effects), what will be the model flow rate? If the model draws 75 W, what will be the power requirement of the prototype?

Given: Model of water pump

Find: Model flow rate for dynamic similarity (ignoring Re); Power of prototype

Solution:

From Buckingham Π $\frac{Q}{\omega \cdot D^3}$ and $\frac{P}{\rho \cdot \omega^3 \cdot D^5}$ where Q is flow rate, ω is angular speed, d is diameter, and ρ is density (these Π groups will be discussed in Chapter 10

For dynamic similarity $$\frac{Q_m}{\omega_m \cdot D_m^3} = \frac{Q_p}{\omega_p \cdot D_p^3}$$

Hence $$Q_m = Q_p \cdot \frac{\omega_m}{\omega_p} \cdot \left(\frac{D_m}{D_p}\right)^3$$

$$Q_m = 0.4 \cdot \frac{m^3}{s} \times \left(\frac{2000}{800}\right) \times \left(\frac{1}{2}\right)^3 \qquad Q_m = 0.125 \frac{m^3}{s}$$

From Table A.8 at 20ºC $\rho_p = 998 \cdot \frac{kg}{m^3}$ From Table A.10 at 20ºC $\mu_m = 1.21 \cdot \frac{kg}{m^3}$

Then $$\frac{P_m}{\rho_m \cdot \omega_m^3 \cdot D_m^5} = \frac{P_p}{\rho_p \cdot \omega_p^3 \cdot D_p^5}$$

$$P_p = P_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3 \cdot \left(\frac{D_p}{D_m}\right)^5$$

$$P_p = 75 \cdot W \times \frac{998}{1.21} \times \left(\frac{800}{2000}\right)^3 \times \left(\frac{2}{1}\right)^5 \qquad P_p = 127 \, kW$$

Problem 7.58

[2]

7.58 A model test is performed to determine the flight characteristics of a Frisbee. Dependent parameters are drag force, F_D, and lift force, F_L. The independent parameters should include angular speed, ω, and roughness height, h. Determine suitable dimensionless parameters and express the functional dependence among them. The test (using air) on a $\frac{1}{4}$-scale model Frisbee is to be geometrically, kinematically, and dynamically similar to the prototype. The prototype values are $V_p = 5$ m/s and $\omega_p = 100$ rpm. What values of V_m and ω_m should be used?

Given: Model of Frisbee

Find: Dimensionless parameters; Model speed and angular speed

Solution:

The functional dependence is $F = F(D, V, \omega, h, \rho, \mu)$ where F represents lift or drag

From Buckingham Π $\dfrac{F}{\rho \cdot V^2 \cdot D^2} = f\left(\dfrac{\rho \cdot V \cdot D}{\mu}, \dfrac{\omega \cdot D}{V}, \dfrac{h}{D}\right)$

For dynamic similarity $\dfrac{\rho_m \cdot V_m \cdot D_m}{\mu_m} = \dfrac{\rho_p \cdot V_p \cdot D_p}{\mu_p}$ $V_m = V_p \cdot \dfrac{\rho_p}{\rho_m} \cdot \dfrac{D_p}{D_m} \cdot \dfrac{\mu_m}{\mu_p}$ $V_m = 5 \cdot \dfrac{m}{s} \times (1) \times \left(\dfrac{4}{1}\right) \times (1)$ $V_m = 20 \dfrac{m}{s}$

Also $\dfrac{\omega_m \cdot D_m}{V_m} = \dfrac{\omega_p \cdot D_p}{V_p}$ $\omega_m = \omega_p \cdot \dfrac{D_p}{D_m} \cdot \dfrac{V_m}{V_p}$ $\omega_m = 100 \cdot rpm \times \left(\dfrac{4}{1}\right) \times \left(\dfrac{20}{5}\right)$ $\omega_m = 1600\,rpm$

Given: Model of hydrofoil boat (1:20 scale) is to be tested in water at 130°F. Prototype operates at speed of 60 knots in water at 45°F.

To model cavitation correctly, cavitation number must be duplicated.

Find: ambient pressure at which model test must be run.

Solution:

To duplicate the Froude number between model and prototype requires

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \quad \text{or} \quad \frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2} = \frac{1}{\sqrt{20}}$$

$$\text{and } V_m = \frac{1}{\sqrt{20}} V_p = \frac{1}{\sqrt{20}} 60 \text{ knot} = 13.4 \text{ knot}$$

For $Ca_m = Ca_p$, then

$$\left.\frac{p - p_v}{\frac{1}{2}\rho V^2}\right)_m = \left.\frac{p - p_v}{\frac{1}{2}\rho V^2}\right)_p$$

or

$$p_m = p_{v_m} + (p - p_v)_p \frac{V_m^2}{V_p^2} \qquad (\text{assuming } \rho_m \approx \rho_p)$$

and

$$p_m = p_{v_m} + (p - p_v)_p \cdot \frac{1}{20}$$

From the Table A.7, at $T = 130°F$ $p_{v_m} = 2.23$ psia

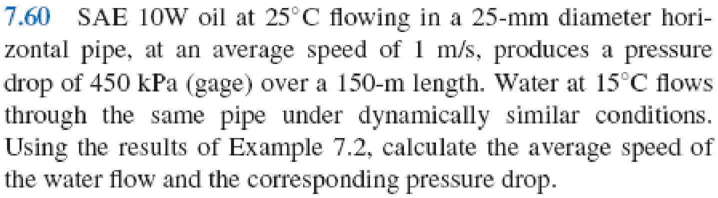
$T = 45$ F $p_{v_p} = 0.15$ psia

$$\therefore p_m = 2.23 \text{ psia} + (14.7 - 0.15) \text{ psia} \cdot \frac{1}{20}$$

$$p_m = 2.96 \text{ psia}$$ p_m

Problem 7.60

[2]



7.60 SAE 10W oil at 25°C flowing in a 25-mm diameter horizontal pipe, at an average speed of 1 m/s, produces a pressure drop of 450 kPa (gage) over a 150-m length. Water at 15°C flows through the same pipe under dynamically similar conditions. Using the results of Example 7.2, calculate the average speed of the water flow and the corresponding pressure drop.

Given: Oil flow in pipe and dynamically similar water flow

Find: Average water speed and pressure drop

Solution:

From Example 7.2 $\quad \frac{\Delta p}{\rho\cdot V^2} = f\left(\frac{\mu}{\rho\cdot V\cdot D}, \frac{l}{D}, \frac{e}{D}\right)$

For dynamic similarity $\quad \frac{\mu_{H2O}}{\rho_{H2O}\cdot V_{H2O}\cdot D_{H2O}} = \frac{\mu_{Oil}}{\rho_{Oil}\cdot V_{Oil}\cdot D_{Oil}}$ so $\quad V_{H2O} = \frac{\mu_{H2O}}{\rho_{H2O}}\cdot\frac{\rho_{Oil}}{\mu_{Oil}}\cdot V_{oil} = \frac{\nu_{H2O}}{\nu_{Oil}}\cdot V_{Oil}$

From Fig. A.3 at 25ºC $\quad \nu_{Oil} = 8\times 10^{-5}\cdot\frac{m^2}{s}$ $\quad$ From Table A.8 at 15ºC $\quad \nu_{H2O} = 1.14\times 10^{-6}\cdot\frac{m^2}{s}$

Hence $\quad V_{H2O} = \frac{1.14\times 10^{-6}\cdot\frac{m^2}{s}}{8\times 10^{-5}\cdot\frac{m^2}{s}}\times 1\cdot\frac{m}{s}$ $\quad V_{H2O} = 0.0142\frac{m}{s}$

Then $\quad \frac{\Delta p_{Oil}}{\rho_{Oil}\cdot V_{Oil}^2} = \frac{\Delta p_{H2O}}{\rho_{H2O}\cdot V_{H2O}^2}$ $\quad \Delta p_{H2O} = \frac{\rho_{H2O}\cdot V_{H2O}^2}{\rho_{Oil}\cdot V_{Oil}^2}\cdot\Delta p_{Oil}$

From Table A.2 $\quad SG_{Oil} = 0.92$

$$\Delta p_{H2O} = \frac{1}{0.92}\times\left(\frac{0.0142}{1}\right)^2\times 450\cdot kPa \qquad \Delta p_{H2O} = 98.6\cdot Pa$$

Problem 7.61

[3]_______

Given: The frequency, f, of vortex shedding from the rear of a bluff cylinder is a function of ρ, V, d, μ

Two cylinders is standard air, $\frac{d_1}{d_2} = 2$

Find: (a) functional relationship for f, using dimensional analysis
(b) V_1/V_2 for dynamic similarity
(c) f_1/f_2

Solution: Apply Buckingham π theorem.

① $f \quad \rho \quad V \quad d \quad \mu$ $\qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③ $$\begin{array}{ccccc} f & \rho & V & d & \mu \\ \frac{1}{t} & \frac{M}{L^3} & \frac{L}{t} & L & \frac{M}{Lt} \end{array}$$ $r = 3$ primary dimensions

④ ρ, V, d $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$\pi_1 = \rho^a V^b d^c f$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{1}{t}$

Equating exponents,

$M: \quad 0 = a$

$L: \quad 0 = -3a + b + c \qquad c = 1$

$t \quad 0 = -b - 1 \qquad \therefore b = -1$

$\therefore \pi_1 = \frac{fd}{V}$

$\pi_2 = \rho^a V^b d^c \mu$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$

Equating exponents,

$M: \quad 0 = a + 1 \qquad \therefore a = -1$

$L: \quad 0 = -3a + b + c - 1 \qquad c = -1$

$t: \quad 0 = -b - 1 \qquad \therefore b = -1$

$\therefore \pi_2 = \frac{\mu}{\rho V d}$

⑥ Check using F, L, t dimensions

$\pi_1 = \frac{1}{t} \cdot L \cdot \frac{t}{L} = [1]$ ✓ $\qquad \pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = [1]$ ✓

$$\therefore \quad \frac{fd}{V} = g\left(\frac{\rho V d}{\mu}\right)$$ $\frac{fd}{V}$

To achieve dynamic similarity between geometrically similar flows, we must duplicate all but one of the dimensionless groups

$$\left(\frac{\rho V d}{\mu}\right)_1 = \left(\frac{\rho V d}{\mu}\right)_2 \Rightarrow \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} \frac{\mu_1}{\mu_2} \frac{d_2}{d_1} = 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$$ V_1/V_2

If $\left(\frac{\rho V d}{\mu}\right)_1 = \left(\frac{\rho V d}{\mu}\right)_2$, then $\left(\frac{fd}{V}\right)_1 = \left(\frac{fd}{V}\right)_2$

$$\text{and} \quad \frac{f_1}{f_2} = \frac{V_1}{V_2} \frac{d_2}{d_1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$ f_1/f_2

Problem 7.62

[3]

Given: $\frac{1}{8}$-scale model of tractor-trailer rig tested in pressurized wind tunnel.

$W = 0.305\ m \qquad V = 75.0\ m/s$

$H = 0.476\ m \qquad F_D = 128\ N$

$L = 2.48\ m \qquad \rho = 3.23\ kg/m^3$

Find: (a) Aerodynamic drag coefficient of model.

(b) Compare Reynolds number for model with prototype at V = 55 mph.

(c) Aerodynamic drag on prototype at V = 55 mph, with headwind, V_w = 10 mph.

Solution: Defining equations: $F_D = C_D A \frac{1}{2}\rho V^2$; $Re = \frac{\rho V L}{\mu}$

Then $$C_{D_m} = \frac{F_{D_m}}{\frac{1}{2}\rho_m V_m^2 A_m}$$

Assume $A_m = W_m H_m = 0.305\ m \times 0.476\ m = 0.145\ m^2$

$$C_{D_m} = 2 \times 128\ N \times \frac{m^3}{3.23\ kg} \times \frac{s^2}{(75)^2 m^2} \times \frac{1}{0.145\ m^2} \times \frac{kg \cdot m}{N \cdot s^2} = 0.0972$$ $\longleftarrow$ C_{D_m}

$$\frac{Re_m}{Re_p} = \frac{\rho_m V_m L_m}{\mu_m} \times \frac{\mu_p}{\rho_p V_p L_p} = \frac{\rho_m}{\rho_p}\frac{V_m}{V_p}\frac{L_m}{L_p} \quad (\text{assume air: } \mu_m = \mu_p)$$

For the prototype, $V_p = 55 \frac{mi}{hr} \times 5280 \frac{ft}{mi} \times \frac{hr}{3600\ s} \times 0.305 \frac{m}{ft} = 24.6\ m/s$

$$\frac{Re_m}{Re_p} = \left(\frac{3.23}{1.23}\right)\left(\frac{75.0}{24.6}\right)\left(\frac{1}{8}\right) = 1.00 \quad \therefore Re_m = Re_p$$ $\longleftarrow$ Re

Since $Re_m = Re_p$, then $C_{D_p} = C_{D_m}$, assuming geometric and kinematic similarity, so

$$F_{D_p} = C_{D_p} A_p \frac{1}{2}\rho_p (V_p + V_w)^2$$

With V_w = 10 mph, $V_p + V_w = \frac{65}{55} \times 24.6\ m/s = 29.1\ m/s$

Thus

$$F_{D_p} = 0.0972 \times (8)^2 0.145\ m^2 \times \frac{1}{2} \times 1.23 \frac{kg}{m^3} (29.1)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$F_{D_p} = 470\ N$ $\longleftarrow$ F_{D_p}

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Problem 7.63

[2]

7.63 On a cruise ship, passengers complain about the amount of smoke that becomes entrained behind the cylindrical smoke stack. You have been hired to study the flow pattern around the stack, and have decided to use a 1:12.5 scale model of the 4.75 m smoke stack. What range of wind tunnel speeds could you use if the ship speed for which the problem occurs is 15 knots to 25 knots?

Given: Flow around cruise ship smoke stack

Find: Range of wind tunnel speeds

Solution:

For dynamic similarity $\dfrac{V_m \cdot D_m}{\nu_m} = \dfrac{V_p \cdot D_p}{\nu_m}$ or $V_m = \dfrac{D_p}{D_m} \cdot V_p = \dfrac{1}{12.5} \cdot V_p = 0.08 \cdot V_p$

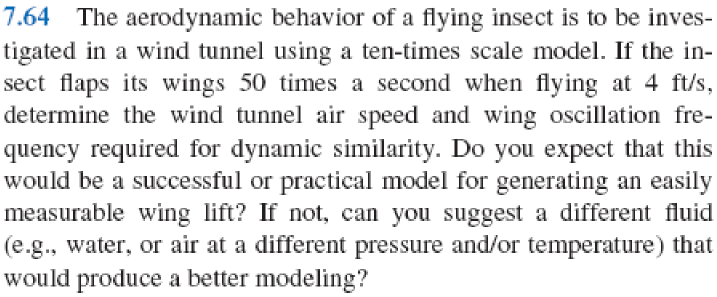
From Wikipedia $1 \cdot \text{knot} = 1.852 \dfrac{\text{km}}{\text{hr}} = 1.852 \cdot \dfrac{\text{km}}{\text{hr}} \times \dfrac{1000 \cdot \text{m}}{\text{km}} \times \dfrac{1 \cdot \text{hr}}{3600 \cdot \text{s}} = 0.514 \cdot \dfrac{\text{m}}{\text{s}}$

Hence for $V_p = 15 \cdot \text{knot} = 15 \cdot \text{knot} \times \dfrac{0.514 \cdot \frac{\text{m}}{\text{s}}}{1 \cdot \text{knot}}$ $\quad V_p = 7.72 \cdot \dfrac{\text{m}}{\text{s}}$ $\quad V_m = 0.08 \times 7.72 \cdot \dfrac{\text{m}}{\text{s}}$ $\quad V_m = 0.618 \dfrac{\text{m}}{\text{s}}$

$V_p = 25 \cdot \text{knot} = 25 \cdot \text{knot} \times \dfrac{0.514 \cdot \frac{\text{m}}{\text{s}}}{1 \cdot \text{knot}}$ $\quad V_p = 12.86 \cdot \dfrac{\text{m}}{\text{s}}$ $\quad V_m = 0.08 \times 12.86 \cdot \dfrac{\text{m}}{\text{s}}$ $\quad V_m = 1.03 \dfrac{\text{m}}{\text{s}}$

Problem 7.64

[2]



7.64 The aerodynamic behavior of a flying insect is to be investigated in a wind tunnel using a ten-times scale model. If the insect flaps its wings 50 times a second when flying at 4 ft/s, determine the wind tunnel air speed and wing oscillation frequency required for dynamic similarity. Do you expect that this would be a successful or practical model for generating an easily measurable wing lift? If not, can you suggest a different fluid (e.g., water, or air at a different pressure and/or temperature) that would produce a better modeling?

Given: Model of flying insect

Find: Wind tunnel speed and wing frequency; select a better model fluid

Solution:

For dynamic similarity the following dimensionless groups must be the same in the insect and model (these are Reynolds number and Strouhal number, and can be obtained from a Buckingham Π analysis)

$$\frac{V_{insect}\cdot L_{insect}}{\nu_{air}} = \frac{V_m\cdot L_m}{\nu_m} \qquad \frac{\omega_{insect}\cdot L_{insect}}{V_{insect}} = \frac{\omega_m\cdot L_m}{V_m}$$

From Table A.9 (68ºF) $\rho_{air} = 0.00234\cdot\frac{slug}{ft^3}$ $\qquad \nu_{air} = 1.62\times10^{-4}\cdot\frac{ft^2}{s}$

The given data is $\omega_{insect} = 50\cdot Hz$ $\qquad V_{insect} = 4\cdot\frac{ft}{s}$ $\qquad \frac{L_{insect}}{L_m} = \frac{1}{10}$

Hence in the wind tunnel $V_m = V_{insect}\cdot\frac{L_{insect}}{L_m}\cdot\frac{\nu_m}{\nu_{air}} = V_{insect}\cdot\frac{L_{insect}}{L_m} = 4\cdot\frac{ft}{s}\times\frac{1}{10}$ $\qquad V_m = 0.4\cdot\frac{ft}{s}$

Also $\omega_m = \omega_{insect}\cdot\frac{V_m}{V_{insect}}\cdot\frac{L_{insect}}{L_m} = 50\cdot Hz\times\frac{0.4}{4}\times\frac{1}{10}$ $\qquad \omega_m = 0.5\cdot Hz$

It is unlikely measurable wing lift can be measured at such a low wing frequency (unless the measured lift was averaged, using an integrator circuit). Maybe try hot air (200ºF) for the model

For hot air try $\nu_{hot} = 2.4\times10^{-4}\cdot\frac{ft^2}{s}$ instead of $\nu_{air} = 1.62\times10^{-4}\cdot\frac{ft^2}{s}$

Hence $\frac{V_{insect}\cdot L_{insect}}{\nu_{air}} = \frac{V_m\cdot L_m}{\nu_{hot}}$ $\qquad V_m = V_{insect}\cdot\frac{L_{insect}}{L_m}\cdot\frac{\nu_{hot}}{\nu_{air}} = 4\cdot\frac{ft}{s}\times\frac{1}{10}\times\frac{2.4\times10^{-4}}{1.62\times10^{-4}}$ $\qquad V_m = 0.593\cdot\frac{ft}{s}$

Also $\omega_m = \omega_{insect}\cdot\frac{V_m}{V_{insect}}\cdot\frac{L_{insect}}{L_m} = 50\cdot Hz\times\frac{0.593}{4}\times\frac{1}{10}$ $\qquad \omega_m = 0.741\cdot Hz$

Hot air does not improve things much. Try modeling in wa $\nu_w = 1.08\times10^{-5}\cdot\frac{ft^2}{s}$

Hence $\frac{V_{insect}\cdot L_{insect}}{\nu_{air}} = \frac{V_m\cdot L_m}{\nu_w}$ $\qquad V_m = V_{insect}\cdot\frac{L_{insect}}{L_m}\cdot\frac{\nu_w}{\nu_{air}} = 4\cdot\frac{ft}{s}\times\frac{1}{10}\times\frac{1.08\times10^{-5}}{1.62\times10^{-4}}$ $\qquad V_m = 0.0267\cdot\frac{ft}{s}$

Also $\omega_m = \omega_{insect}\cdot\frac{V_m}{V_{insect}}\cdot\frac{L_{insect}}{L_m} = \omega_{insect}\cdot\frac{V_m}{V_{insect}}\cdot L_{ratio} = 50\cdot Hz\times\frac{0.0267}{4}\times\frac{1}{10}$ $\qquad \omega_m = 0.033\cdot Hz$

This is even worse! It seems the best bet is hot (very hot) air for the wind tunnel. Alternatively, choose a much smaller wind tunnel model, e.g., a 2.5 X model would lead to V_m = 1.6 ft/s and ω_m = 8 Hz

Problem 7.65

[3]

Given: Model test of tractor-trailer rig in standard air.

$F_D = f(A, V, \rho, \mu)$; scale is 1:4 ; $A_m = 0.625\ m^2$

At $V_m = 89.6$ m/s, $F_D = 2.46$ kN

Find: (a) Dimensionless parameters.
(b) Conditions for dynamic similarity.
(c) Drag force on prototype at $V_p = 22.4$ m/s (no wind).
(d) Power to overcome aero drag.

Solution: ① $F_D \quad A \quad V \quad \rho \quad \mu$ | ② $M\ L\ t$

③ $\frac{ML}{t^2} \quad L^2 \quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{M}{Lt}$ | ④ $\rho V A$

⑤ $\Pi_1 = \rho^a V^b A^c F_D = M^0 L^0 t^0$ $\qquad$ $\Pi_2 = \rho^a V^b A^c \mu$

M: $a + 1 = 0$ | $a = -1$
L: $-3a + b + 2c + 1 = 0$ | $c = -1$
t: $-b - 2 = 0$ | $b = -2$

M: $a + 1 = 0$ | $a = -1$
L: $-3a + b + 2c - 1 = 0$ | $c = -\frac{1}{2}$
t: $-b - 1 = 0$ | $b = -1$

$$\Pi_1 = \frac{F_D}{\rho V^2 A} \qquad \Pi_2 = \frac{\mu}{\rho V A^{1/2}}$$

Π_1, Π_2

⑥ $\Pi_1 = F \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} \times \frac{1}{L^2} = 1$ ✓✓ $\qquad$ $\Pi_2 = \frac{Ft}{L^2} \times \frac{L^4}{Ft^2} \times \frac{t}{L} \times \frac{1}{L} = 1$ ✓✓

For dynamic similarity, must have geometric and kinematic similarity and $Re_m = Re_p$. Then $\left.\frac{F_D}{\rho V^2 A}\right)_m = \left.\frac{F_D}{\rho V^2 A}\right)_p$

For the prototype,

$$F_{D_p} = F_{D_m} \frac{\rho_p}{\rho_m} \left(\frac{V_p}{V_m}\right)^2 \frac{A_p}{A_m} = F_{D_m} \left(\frac{1.23}{1.23}\right)\left(\frac{22.4}{89.6}\right)^2 (4)^2 = F_{D_m} = 2.46\ kN$$

F_{D_p}

The power requirement is

$$P = F_{D_p} V_p = 2.46\ kN \times 22.4\ \frac{m}{s} \times \frac{W \cdot s}{N \cdot m} = 55.1\ kW \quad (73.9\ hp)$$

P

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Problem 7.66

[2]

7.66 Tests are performed on a 1:5 scale boat model. What must be the kinematic viscosity of the model fluid if friction and wave drag phenomena are to be correctly modeled? The full size boat will be used in a freshwater lake where the average water temperature is 10°C.

Given: Model of boat

Find: Model kinematic viscosity for dynamic similarity

Solution:

For dynamic similarity $$\frac{V_m \cdot L_m}{\nu_m} = \frac{V_p \cdot L_p}{\nu_p} \quad (1) \qquad \frac{V_m}{\sqrt{g \cdot L_m}} = \frac{V_p}{\sqrt{g \cdot L_p}} \quad (2)$$

(from Buckingham Π; the first is the Reynolds number, the second the Froude number)

Hence from Eq 2 $$\frac{V_m}{V_p} = \sqrt{\frac{g \cdot L_m}{g \cdot L_p}} = \sqrt{\frac{L_m}{L_p}}$$

Using this in Eq 1 $$\nu_m = \nu_p \cdot \frac{V_m}{V_p} \cdot \frac{L_m}{L_p} = \nu_p \cdot \sqrt{\frac{L_m}{L_p}} \cdot \frac{L_m}{L_p} = \nu_p \cdot \left(\frac{L_m}{L_p}\right)^{\frac{3}{2}}$$

From Table A.8 at 10ºC $$\nu_p = 1.3 \times 10^{-6} \cdot \frac{m^2}{s} \qquad \nu_m = 1.3 \times 10^{-6} \cdot \frac{m^2}{s} \times \left(\frac{1}{5}\right)^{\frac{3}{2}} \qquad \nu_m = 1.16 \times 10^{-7} \frac{m^2}{s}$$

Problem 7.67

[4]

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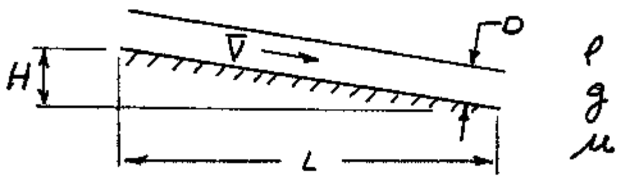
Given: Model glacier using glycerine. Assume ice is Newtonian and 10^6 x as viscous.

$D = 15\ m$
$H = 1.5\ m$
$L = 1850\ m$
} model



In lab test, model instructor reappears in $\tau = 9.6\ hr$.

Find: (a) Develop suitable dimensionless parameters.
(b) Estimate time when instructor will reappear.

Solution: ① $\bar{V}$ ρ g μ D H L $n = 7$

② MLt $\frac{L}{t}$ $\frac{M}{L^3}$ $\frac{L}{t^2}$ $\frac{M}{Lt}$ L L L $m = r = 3$

④ Choose ρ, g, D as repeating variables: $n - m = 7 - 3 = 4$ parameters

⑤ $\Pi_1 = \rho^a g^b D^c \bar{V} = M^0 L^0 t^0$

M: $a + 0 = 0$ | $a = 0$
L: $-3a + b + c + 1 = 0$ | $c = -b - 1 = -\frac{1}{2}$
t: $-2b - 1 = 0$ | $b = -\frac{1}{2}$

$\Pi_1 = \frac{\bar{V}}{\sqrt{gD}}$ (Froude no.)

$\Pi_2 = \rho^a g^b D^c \mu = M^0 L^0 t^0$

M: $a + 1 = 0$ $a = -1$
L: $-3a + b + c - 1 = 0$ $c = 3a - b + 1 = -\frac{3}{2}$
t: $-2b - 1 = 0$ $b = -\frac{1}{2}$

$\Pi_2 = \frac{\mu}{\rho g^{1/2} D^{3/2}} \sim \frac{\mu}{\rho \sqrt{gD}\, D}$ (Reynolds no)

$\Pi_3 = \frac{H}{D}$, $\Pi_4 = \frac{L}{D}$ (by inspection)

⑥ Check: obvious from forms above. $\Pi_1 = f(\Pi_2, \underbrace{\Pi_3, \Pi_4}_{\text{geometric similarity}})$

For dynamic similarity, $\Pi_{2m} = \Pi_{2p} = \frac{\mu_m}{\rho_m g_m^{1/2} D_m^{3/2}} = \frac{\mu_p}{\rho_p g_p^{1/2} D_p^{3/2}}$, so

$$\frac{D_m}{D_p} = \left(\frac{\mu_m}{\mu_p}\frac{\rho_p}{\rho_m}\right)^{2/3} = \left(\frac{\mu_m}{\mu_p}\frac{SG_p}{SG_m}\right)^{2/3} = \left(\frac{1}{10^6} \times \frac{0.92}{1.26}\right)^{2/3} = 8.11 \times 10^{-5}$$

$\left\{ \begin{array}{l} SG_{ice} = 0.92 \text{ (Table A.1} \\ SG_{glycerin} = 1.26 \text{ (A.2)} \end{array} \right.$

So $\frac{L_m}{L_p} = 8.11 \times 10^{-5}$; $L_m = 8.11 \times 10^{-5} L_p = 8.11 \times 10^{-5} \times 1850\ m = 0.150\ m$

From Π_1, $\frac{\bar{V}_m}{\bar{V}_p} = \sqrt{\frac{D_m}{D_p}} = 9.00 \times 10^{-3}$

The time to reappear is $\tau = L/\bar{V}$, so $\tau_p = L_p/\bar{V}_p$, $\tau_m = L_m/\bar{V}_m$

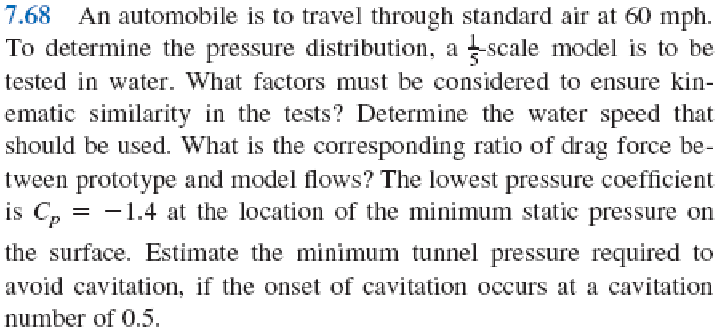
$$\frac{\tau_p}{\tau_m} = \frac{L_p}{L_m}\frac{\bar{V}_m}{\bar{V}_p} = \frac{D_p}{D_m}\sqrt{\frac{D_m}{D_p}} = \sqrt{\frac{D_p}{D_m}} = \frac{1}{9.00 \times 10^{-3}} = 111$$

Thus $\tau_p = 111\, \tau_m = 111 \times 9.6\ hr = 1070\ hr$ (~45 days) τ_p

{ The instructor will reappear before the semester ends! }

Problem 7.68

[3]



7.68 An automobile is to travel through standard air at 60 mph. To determine the pressure distribution, a $\frac{1}{5}$-scale model is to be tested in water. What factors must be considered to ensure kinematic similarity in the tests? Determine the water speed that should be used. What is the corresponding ratio of drag force between prototype and model flows? The lowest pressure coefficient is $C_p = -1.4$ at the location of the minimum static pressure on the surface. Estimate the minimum tunnel pressure required to avoid cavitation, if the onset of cavitation occurs at a cavitation number of 0.5.

Given: Model of automobile

Find: Factors for kinematic similarity; Model speed; ratio of protype and model drags; minimum pressure for no cavitation

Solution:

For dynamic similarity $\frac{\rho_m \cdot V_m \cdot L_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot L_p}{\mu_p}$ $\qquad V_m = V_p \cdot \frac{\rho_p}{\rho_m} \cdot \frac{L_p}{L_m} \cdot \frac{\mu_m}{\mu_p}$

For air (Table A.9) and water (Table A.7) at 68ºF

$$\rho_p = 0.00234 \cdot \frac{\text{slug}}{\text{ft}^3} \qquad \mu_p = 3.79 \times 10^{-7} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$\rho_m = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \qquad \mu_m = 2.10 \times 10^{-5} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$V_m = 60 \cdot \text{mph} \times \frac{88 \cdot \frac{\text{ft}}{\text{s}}}{60 \cdot \text{mph}} \times \left(\frac{0.00234}{1.94}\right) \times \left(\frac{5}{1}\right) \times \left(\frac{2.10 \times 10^{-5}}{3.79 \times 10^{-7}}\right) \qquad V_m = 29.4 \cdot \frac{\text{ft}}{\text{s}}$$

Then $\frac{F_m}{\rho_m \cdot V_m^2 \cdot L_m^2} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot L_p^2}$

Hence $\frac{F_p}{F_m} = \frac{\rho_p \cdot V_p^2 \cdot L_p^2}{\rho_m \cdot V_m^2 \cdot L_m^2} = \left(\frac{0.00234}{1.94}\right) \times \left(\frac{88}{29.4}\right)^2 \times \left(\frac{5}{1}\right)^2$ $\qquad \frac{F_p}{F_m} = 0.270$

For Ca = 0.5 $\frac{p_{min} - p_v}{\frac{1}{2} \cdot \rho \cdot V^2} = 0.5$ so we get $p_{min} = p_v + \frac{1}{4} \cdot \rho \cdot V^2$ for the water tank

From steam tables, for water at 68ºF $p_v = 0.339 \cdot \text{psi}$ so

$$p_{min} = 0.339 \cdot \text{psi} + \frac{1}{4} \times 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(29.4 \cdot \frac{\text{ft}}{\text{s}}\right)^2 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \left(\frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^2 \qquad p_{min} = 3.25\,\text{psi}$$

This is the minimum allowable pressure in the water tank; we can use it to find the required tank pressure

$$C_p = -1.4 = \frac{p_{min} - p_{tank}}{\frac{1}{2} \cdot \rho \cdot V^2} \qquad p_{tank} = p_{min} + \frac{1.4}{2} \cdot \rho \cdot V^2 = p_{min} + 0.7 \cdot \rho \cdot V^2$$

$$p_{tank} = 3.25 \cdot \text{psi} + 0.7 \times 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(29.4 \cdot \frac{\text{ft}}{\text{s}}\right)^2 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \left(\frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^2 \qquad p_{tank} = 11.4\,\text{psi}$$

Problem 7.69

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Given: Submarine model (1:30 scale) to be tested in fresh water under two conditions:

(1) on the surface at 20 kt (prototype)

(2) far below the surface at 0.5 kt (prototype)

Find: (a) Speed for model test on surface

(b) Speed for model test submerged

(c) Ratio of full-scale to model drag force.

Solution: On the surface, match the Froude number, $Fr = \frac{V}{\sqrt{gL}}$

Thus $Fr_m = \frac{V_m}{\sqrt{gL_m}} = Fr_p = \frac{V_p}{\sqrt{gL_p}}$ or $V_m = V_p\sqrt{\frac{L_m}{L_p}}$

For 1:30 scale,

$$V_m = 20\,\text{kt}\sqrt{\frac{1}{30}} = 3.65\,\text{kt}$$

$$V_m = 3.65\,\frac{\text{nm}}{\text{hr}} \times 1852\,\frac{\text{m}}{\text{nm}} \times \frac{\text{hr}}{3600\,\text{s}} = 1.88\,\text{m/s}$$ V_m

Submerged, match the Reynolds number, $Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$

Thus $Re_m = \frac{V_m L_m}{\nu_m} = Re_p = \frac{V_p L_p}{\nu_p}$ or $V_m = V_p \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}$

From Table A.2, for seawater, SG = 1.025 and $\mu = 1.08 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2$ at 20°C. Thus

$$\nu_p = \frac{\mu_p}{\rho_p} = \frac{\mu_p}{SG\,\rho_{H_2O}} = 1.08\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2} \times \frac{\text{m}^3}{(1.025)\,1000\,\text{kg}} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = 1.05\times10^{-6}\,\frac{\text{m}^2}{\text{s}}$$

From Table A.8, fresh water at 20°C has $\nu = 1.00\times10^{-6}\,\text{m}^2/\text{s}$.

For 1:30 scale

$$V_m = 0.5\,\text{kt} \times \frac{30}{1} \times 1.00\times10^{-6}\,\frac{\text{m}^2}{\text{s}} \times \frac{\text{s}}{1.05\times10^{-6}\,\text{m}^2} = 14.3\,\text{kt}$$

$$V_m = 14.3\,\frac{\text{nm}}{\text{hr}} \times 1852\,\frac{\text{m}}{\text{nm}} \times \frac{\text{hr}}{3600\,\text{s}} = 7.36\,\text{m/s}$$ V_m

Under dynamically similar conditions the drag coefficients, $C_D = \frac{F_D}{\rho V^2 A}$, will be identical. Thus

$$\frac{F_p}{\rho_p V_p^2 L_p^2} = \frac{F_m}{\rho_m V_m^2 L_m^2} \quad \text{or} \quad F_p = F_m \frac{\rho_p}{\rho_m}\frac{V_p^2}{V_m^2}\frac{L_p^2}{L_m^2}$$

$$F_p = F_m \frac{1.025}{0.999} \times \left(\frac{0.5}{14.3}\right)^2 \left(\frac{30}{1}\right)^2 = 1.13 \text{ (submerged)},\ 2.77\times10^4 \text{ (surface)}$$ F_p/F_m

Problem 7.70

[3]

Given: The drag force on a circular cylinder immersed in a water flow can be expressed as

$$F_D = f(D, l, V, \rho, \mu)$$

The static pressure distribution on a circular cylinder can be expressed in terms of the dimensionless pressure coefficient

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2}$$

At the location of minimum static pressure on the cylinder surface, $C_p = -2.4$. The onset of cavitation occurs at $Ca = 0.5$

Find: (a) expression for dimensionless drag force

(b) an estimate of maximum speed V at which cylinder could be towed in water (at p_{atm}) without causing cavitation

Solution:

$F_D = f(D, l, V, \rho, \mu)$. From the Buckingham π-theorem, for $n=6$, with $m=r=3$, we would expect three dimensionless groups.

$$\frac{F}{\rho V^2 D^2} = f_1\left(\frac{l}{D}, \frac{\rho V D}{\mu}\right)$$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2} \qquad Ca = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

For $C_{p_{min}} = -2.4$, $p_{min} - p_\infty = \frac{1}{2}\rho V_{max}^2 (C_{p_{min}})$ $\therefore p_{min} = p_\infty + \frac{1}{2}\rho V_{max}^2 C_{p_{min}}$

For $Ca = \frac{1}{2}$, $p_{min} - p_v = \frac{1}{2}\rho V_{max}^2 Ca$ $\therefore p_{min} = p_v + \frac{1}{2}\rho V_{max}^2 Ca$

Equation expressions for p_{min},

$$p_\infty + \frac{1}{2}\rho V_{max}^2 C_{p_{min}} = p_v + \frac{1}{2}\rho V_{max}^2 Ca$$

$$\frac{1}{2}\rho V_{max}^2 \left[Ca - C_{p_{min}}\right] = p_\infty - p_v$$

$$V_{max} = \left\{\frac{2(p_\infty - p_v)}{\rho\left[Ca - C_{p_{min}}\right]}\right\}^{1/2}$$

For water at 68°F (Table A.7), $p_v = 0.339$ psia

$$\therefore V_{max} = \left\{2 \times (14.7 - 0.339)\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{ft}^3}{1.94\,\text{slug}} \times \frac{1}{[0.5-(-2.4)]} \times 144\frac{\text{in}^2}{\text{ft}^2} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right\}^{1/2}$$

$$V_{max} = 27.1\ \text{ft/s} \quad (8.26\ \text{m/s}) \longleftarrow$$

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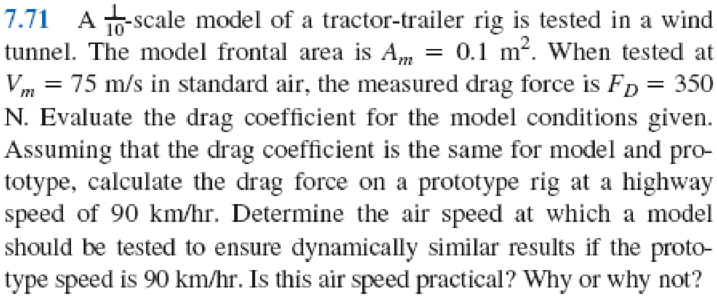
Problem 7.71

[3]



7.71 A $\frac{1}{10}$-scale model of a tractor-trailer rig is tested in a wind tunnel. The model frontal area is $A_m = 0.1\ \mathrm{m}^2$. When tested at $V_m = 75$ m/s in standard air, the measured drag force is $F_D = 350$ N. Evaluate the drag coefficient for the model conditions given. Assuming that the drag coefficient is the same for model and prototype, calculate the drag force on a prototype rig at a highway speed of 90 km/hr. Determine the air speed at which a model should be tested to ensure dynamically similar results if the prototype speed is 90 km/hr. Is this air speed practical? Why or why not?

Given: Model of tractor-trailer truck

Find: Drag coefficient; Drag on prototype; Model speed for dynamic similarity

Solution:

For kinematic similarity we need to ensure the geometries of model and prototype are similar, as is the incoming flow field

The drag coefficient is
$$C_D = \frac{F_m}{\frac{1}{2}\cdot\rho_m\cdot V_m^2\cdot A_m}$$

For air (Table A.10) at 20ºC
$$\rho_m = 1.21\cdot\frac{kg}{m^3} \qquad \mu_p = 1.81\times 10^{-5}\cdot\frac{N\cdot s}{m^2}$$

$$C_D = 2\times 350\cdot N\times\frac{m^3}{1.21\cdot kg}\times\left(\frac{s}{75\cdot m}\right)^2\times\frac{1}{0.1\cdot m^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad C_D = 1.028$$

This is the drag coefficient for model and prototype

For the rig
$$F_p = \frac{1}{2}\cdot\rho_p\cdot V_p^2\cdot A_p\cdot C_D \qquad \text{with} \qquad \frac{A_p}{A_m} = \left(\frac{L_p}{L_m}\right)^2 = 100 \qquad A_p = 10\cdot m^2$$

$$F_p = \frac{1}{2}\times 1.21\cdot\frac{kg}{m^3}\times\left(90\cdot\frac{km}{hr}\times\frac{1000\cdot m}{1\cdot km}\times\frac{1\cdot hr}{3600\cdot s}\right)^2\times 10\cdot m^2\times 1.028\times\frac{N\cdot s^2}{kg\cdot m} \qquad F_p = 3.89\,kN$$

For dynamic similarity
$$\frac{\rho_m\cdot V_m\cdot L_m}{\mu_m} = \frac{\rho_p\cdot V_p\cdot L_p}{\mu_p} \qquad V_m = V_p\cdot\frac{\rho_p}{\rho_m}\cdot\frac{L_p}{L_m}\cdot\frac{\mu_m}{\mu_p} = V_p\cdot\frac{L_p}{L_m}$$

$$V_m = 90\cdot\frac{km}{hr}\times\frac{1000\cdot m}{1\cdot km}\times\frac{1\cdot hr}{3600\cdot s}\times\frac{10}{1} \qquad V_m = 250\frac{m}{s}$$

For air at standard conditions, the speed of sound is $c = \sqrt{k\cdot R\cdot T}$

$$c = \sqrt{1.40\times 286.9\cdot\frac{N\cdot m}{kg\cdot K}\times(20+273)\cdot K\times\frac{kg\cdot m}{s^2\cdot N}} \qquad c = 343\frac{m}{s}$$

Hence we have
$$M = \frac{V_m}{c} = \frac{250}{343} = 0.729$$
which indicates compressibility is significant - this model speed is impractical (and unnecessary)

Problem 7.72

[4]

Given: Circular container partially filled with water is rotated about its axis at constant angular velocity, ω.
The velocity V_θ is a function of: location, r, time from start, τ, angular velocity, ω, density, ρ, and viscosity, μ.
Water is replaced with honey and cylinder is rotated at the same value of ω.

Find: (a) dimensionless parameters that characterize the problem.
(b) Determine whether honey will attain steady state motion as quickly as water.
(c) Explain why Re would not be an important parameter in scaling the steady state motion of the liquid.

Solution:

$$V_\theta = V_\theta(\omega, r, \tau, \rho, \mu)$$

From the Buckingham π-theorem, for n=6 and m=r=3, we would expect three dimensionless groups.

$$\frac{V_\theta}{\omega r} = f\left(\frac{\mu}{\rho \omega r^2}, \omega\tau\right)$$

$\frac{V_\theta}{\omega r}$

From the above results $\pi_2 = \frac{\mu}{\rho\omega r^2}$ contains the fluid properties ρ, μ.

$\pi_3 = \omega\tau$ contains the time τ

$$\pi_2\pi_3 = \frac{\mu}{\rho\omega r^2}\omega\tau = \frac{\mu\tau}{\rho r^2} = \frac{\nu\tau}{r^2} \quad \text{where } \nu = \mu/\rho$$

For steady flow at the same radius

$$\left.\frac{\nu\tau}{r^2}\right)_{Honey} = \left.\frac{\nu\tau}{r^2}\right)_{water}$$

$$\therefore \tau_H = \frac{\nu_{water}}{\nu_{Honey}}\tau_{water}$$

Since $\nu_{Honey} > \nu_{water}$ ($\mu_{Honey} > \mu_{water}$ and $\rho_H \approx \rho_w$)

$$\tau_H < \tau_{water}$$

τ_H

At steady state conditions, we have solid body rotation there are no viscous forces. Hence Re is not important.

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Problem 7.73

[3]

Given: Recommended procedures for wind tunnel tests of trucks & buses suggest:

$A_{model} / A_{test\ section} < 0.05$

$h_{model} / h_{test\ section} < 0.30$ (h = height)

$w_{model\ at\ max\ yaw\ (20°)} / w_{test\ sect} < 0.30$ (w = projected width)

$V_{max} < 300\ ft/s$

Wind tunnel test section is $h = 1.5\ ft$, $w = 2\ ft$.

Prototype has: $h = 13.5\ ft$, $w = 8\ ft$, Length $= 65\ ft$.

Find: (a) scale ratio of largest model that meets the recommended criteria.

(b) Use results of Ex. Prob 7.5 to assess whether an adequate value of Re can be achieved in the test facility.

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Solution:

Let s = scale ratio. Then $h_m = s h_p$, $w_m = s w_p$, $l_m = s l_p$

(1) height criteria

$$h_m = 0.30\, h_{test\ section} = 0.3(1.5\ ft) = 0.45\ ft$$

$$s = \frac{h_m}{h_p} = \frac{0.45\ ft}{13.5\ ft} = 0.0333 \qquad \left\{ \frac{1}{s} = 30 \right\}$$

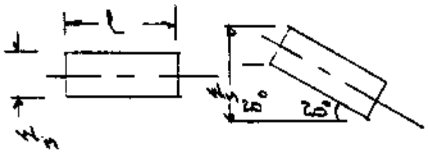
(2) frontal area criteria

$$A_{model} = 0.05\, A_{test\ sect} = 0.05 \times 1.5\ ft \times 2\ ft = 0.15\ ft^2$$

$$A_{model} = s^2 A_p = s^2 [13.5\ ft \times 8\ ft] = s^2 (108)\ ft^2 = 0.15$$

$$\therefore s = \left(\frac{0.15}{108} \right)^{1/2} = 0.0373 \qquad \left\{ \frac{1}{s} = 26.8 \right\}$$

(3) width criteria



$$w_{m_{20°}} = l_m \sin 20° + w_m \cos 20°$$

$$= s (l_p \sin 20° + w_p \cos 20°)$$

$$w_{m_{20°}} = s [65 \sin 20° + 8 \cos 20°]\ ft = 29.7\, s\ ft$$

From constraint, $w_{m_{20°}} = 0.30\, w_{test\ sect} = 0.30 (2\ ft) = 0.6\ ft$

$$\therefore 0.6\ ft = 29.7 s\ ft \quad \text{and} \quad s = 0.0202 \qquad \left\{ \frac{1}{s} = 49.5 \right\}$$

The width criteria is the most stringent $\therefore s = \frac{1}{50}$

Model $= \frac{1}{50}$ Prototype ←

From Ex. Prob 7.5, C_D = const for $Re > 4 \times 10^5$

with $Re = \frac{\rho V w}{\mu} = \frac{V w}{\nu}$ Standard air $\nu = 1.57 \times 10^{-4}\ ft^2/s$

For current model test, $Re = 300 \frac{ft}{sec} \times \left(\frac{1}{50} \times 8\ ft \right) \times \frac{s}{1.57 \times 10^{-4}\ ft^2} = 3.06 \times 10^5$

$\therefore$ Adequate Re cannot be achieved

Problem 7.74

[2]

Given: Power, P, to drive a fan depends on ρ, Q, D, and ω.

Condition	D(mm)	Q (m³/s)	ω(rpm)
1	200	0.4	2400
2	400	?	1850

Find: Volume flow rate at Condition 2, for dynamic similarity.

Solution: step ① $P \quad \rho \quad Q \quad D \quad \omega$

step ② MLt ③: $\frac{ML^2}{t^3} \quad \frac{M}{L^3} \quad \frac{L^3}{t} \quad L \quad \frac{1}{t}$ ④ ρ, ω, D

⑤ $\Pi_1 = \rho^a \omega^b D^c P = M^0 L^0 t^0$

$M: a+1=0 \quad | \; a=-1$

$L: -3a + c + 2 = 0 \quad | \; c = 3a-2 = -5$

$t: -b-3=0 \quad | \; b=-3$

$$\Pi_1 = \frac{P}{\rho \omega^3 D^5}$$

$\Pi_2 = \rho^a \omega^b D^c Q = M^0 L^0 t^0$

$M: a+0=0 \quad | \; a=0$

$L: -3a + c + 3 = 0 \quad | \; c=-3$

$t: -b-1=0 \quad | \; b=-1$

$$\Pi_2 = \frac{Q}{\omega D^3}$$

⑥ $\Pi_1 = \frac{FL}{t} \times \frac{L^4}{Ft^2} \times t^3 \times \frac{1}{L^5} = \frac{FL^5t^3}{FL^5t^3} = 1$ ✓✓ $\quad \Pi_2 = \frac{L^3}{t} \times t \times \frac{1}{L^3} = \frac{L^3 t}{L^3 t} = 1$ ✓✓

Thus $\Pi_1 = f(\Pi_2)$ or $\frac{P}{\rho \omega^3 D^5} = f\left(\frac{Q}{\omega D^3}\right)$

For dynamic similarity, need geometric and kinematic similarity and

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

Thus

$$Q_2 = Q_1 \frac{\omega_2}{\omega_1}\left(\frac{D_2}{D_1}\right)^3 = 0.4\,\text{m}^3/\text{s}\;\frac{1850\,\text{rpm}}{2400\,\text{rpm}}\left(\frac{200\,\text{mm}}{400\,\text{mm}}\right)^3 = 2.47\,\text{m}^3/\text{s}$$

Q_2

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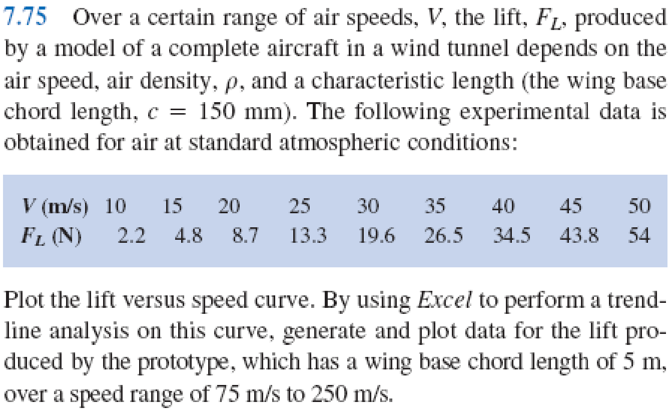
Problem 7.75 (In Excel)

[3]



7.75 Over a certain range of air speeds, V, the lift, F_L, produced by a model of a complete aircraft in a wind tunnel depends on the air speed, air density, ρ, and a characteristic length (the wing base chord length, $c = 150$ mm). The following experimental data is obtained for air at standard atmospheric conditions:

V (m/s)	10	15	20	25	30	35	40	45	50
F_L (N)	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54

Plot the lift versus speed curve. By using *Excel* to perform a trendline analysis on this curve, generate and plot data for the lift produced by the prototype, which has a wing base chord length of 5 m, over a speed range of 75 m/s to 250 m/s.

Given: Data on model of aircraft

Find: Plot of lift vs speed of model; also of prototype

Solution:

V_m (m/s)	10	15	20	25	30	35	40	45	50
F_m (N)	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54.0

This data can be fit to

$$F_m = \frac{1}{2}\cdot\rho\cdot A_m\cdot C_D\cdot V_m^{\,2} \qquad \text{or} \qquad F_m = k_m\cdot V_m^{\,2}$$

From the trendline, we see that

$k_m = 0.0219$ N/(m/s)^2

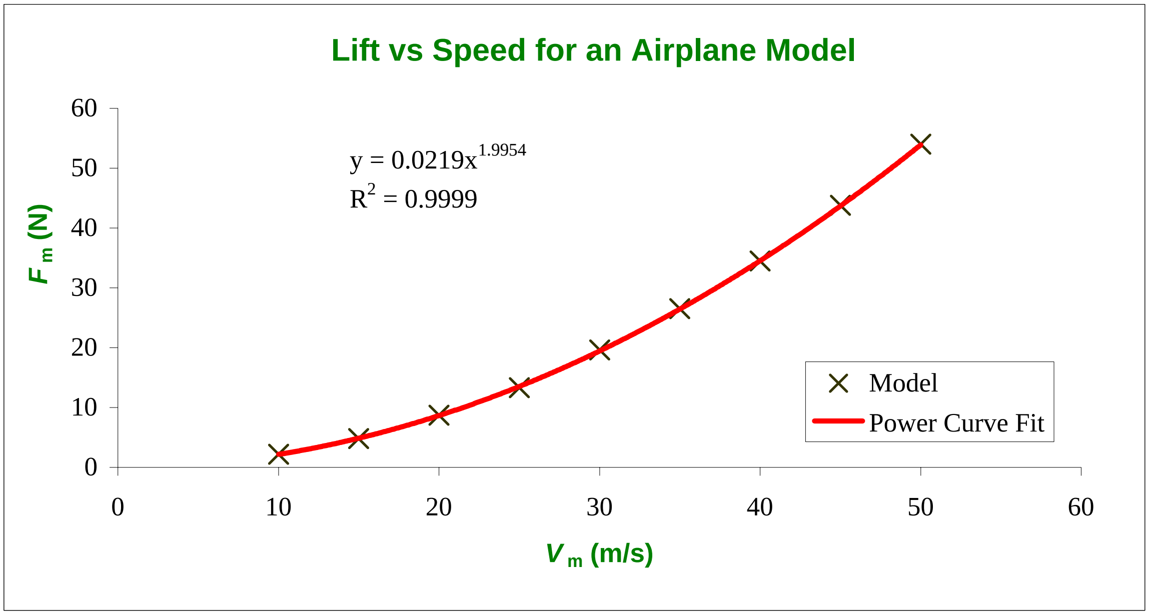
(And note that the power is 1.9954 or 2.00 to three signifcant figures, confirming the relation is quadratic)

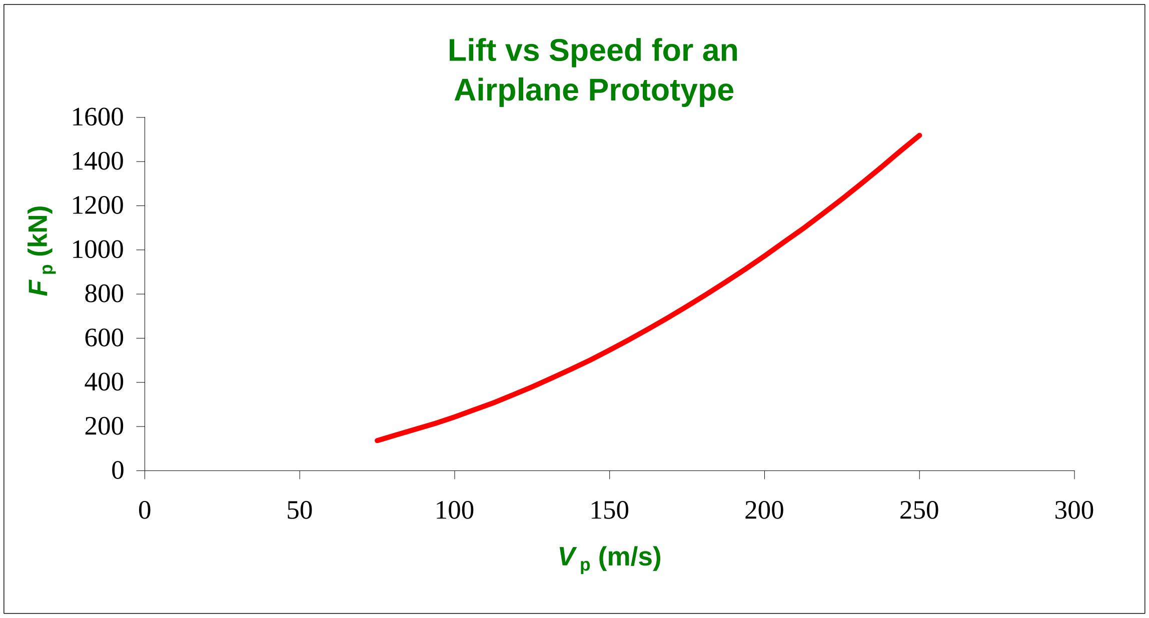
Also, $k_p = 1110\ k_m$

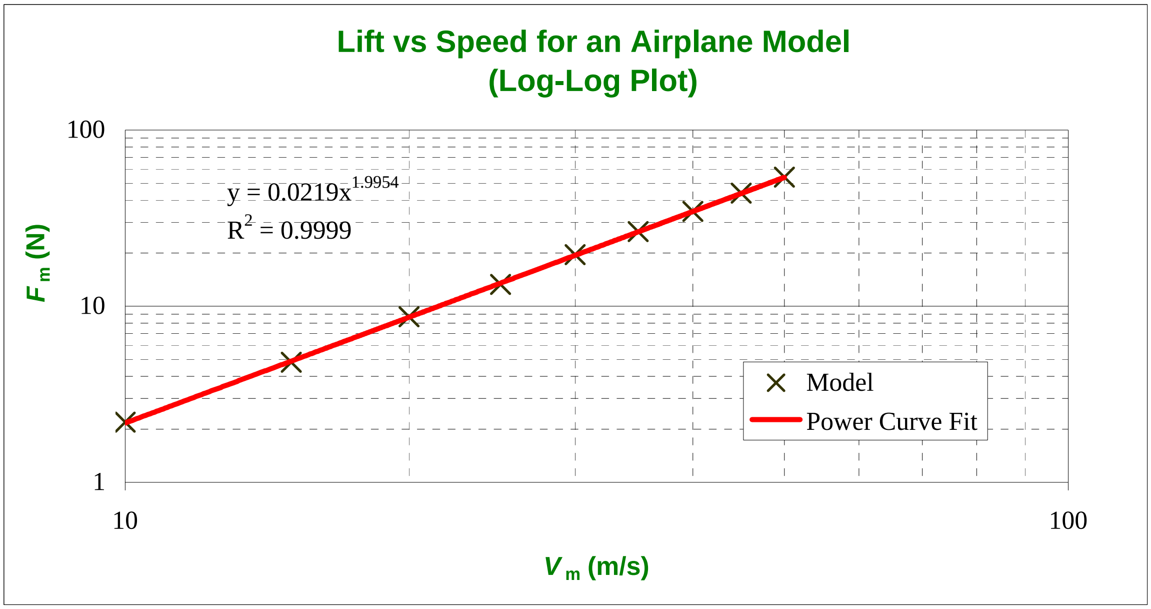
Hence,

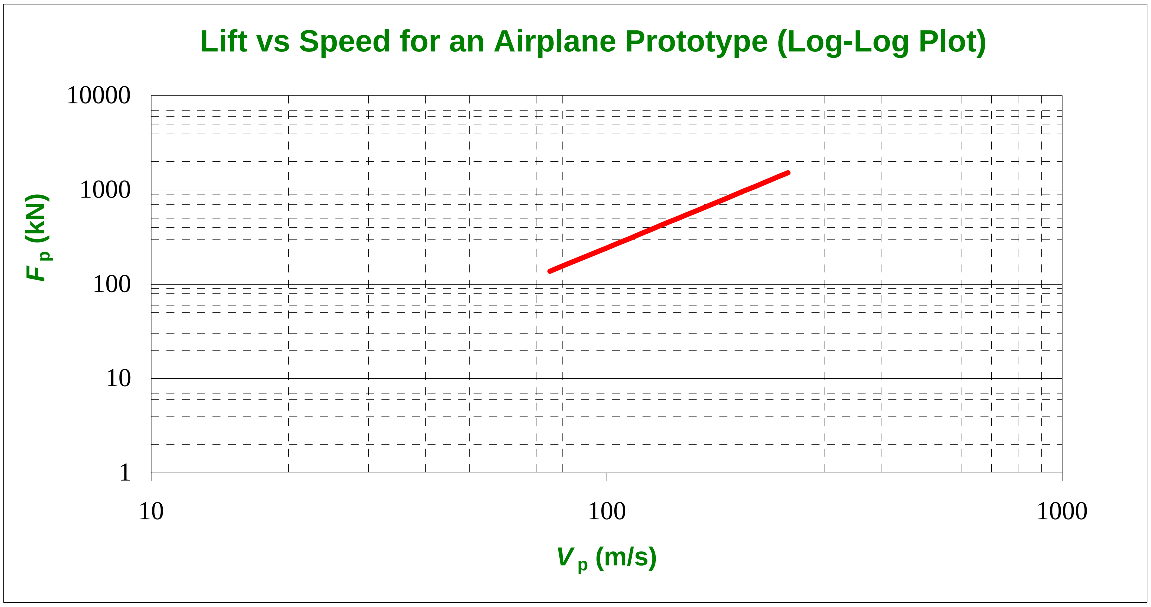
$k_p = 24.3$ N/(m/s)^2 $\qquad F_p = k_p V_m^{\,2}$

V_p (m/s)	75	100	125	150	175	200	225	250
F_p (kN) (Trendline)	137	243	380	547	744	972	1231	1519


Lift vs Speed for an Airplane Model
y = 0.0219x^1.9954
R^2 = 0.9999
60
50
40
30
20
10
0
F m (N)
0
10
20
30
40
50
60
V m (m/s)
Model
Power Curve Fit


Lift vs Speed for an Airplane Prototype
1600
1400
1200
1000
800
600
400
200
0
F p (kN)
0
50
100
150
200
250
300
V p (m/s)


Lift vs Speed for an Airplane Model
(Log-Log Plot)
y = 0.0219x^1.9954
R^2 = 0.9999
Model
Power Curve Fit
F m (N)
V m (m/s)
100
10
1
10
100


Lift vs Speed for an Airplane Prototype (Log-Log Plot)
F p (kN)
V p (m/s)
10000
1000
100
10
1
10
100
1000

Problem 7.76

[2]

Given: Information relating to geometrically similar model test of centrifugal pump:

	Variable	Prototype	Model
Pressure Rise	Δp		29.3 kPa
Volume Flow Rate	Q	1.25 m³/min	
Density	ρ	800 kg/m³	999 kg/m³
Angular Speed	ω	183 rad/s	367 rad/s
Diameter	D	150 mm	50 mm

Find: Missing values for dynamically similar conditions.

Solution: Apply Buckingham Π-theorem. Assume $\Delta p = f(Q, \rho, \omega, D)$

① $\Delta p \quad Q \quad \rho \quad \omega \quad D \qquad n = 5$ parameters

② Choose M, L, t as fundamental dimensions.

③ $\frac{M}{Lt^2} \quad \frac{L^3}{t} \quad \frac{M}{L^3} \quad \frac{L}{t} \quad L \qquad r = 3$ primary dimensions

④ Let ρ, ω, and D be repeating variables. $\quad m = r = 3$

⑤ Then $n - m = 5 - 3 = 2$ dimensionless parameters result. ⑥ Check:

$$\Pi_1 = \rho^a \omega^b D^c \Delta p = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{M}{Lt^2} = M^0 L^0 t^0$$

$$\left.\begin{array}{ll} M: a + 1 = 0 & a = -1 \\ L: -3a + c - 1 = 0 & c = -2 \\ t: -b - 2 = 0 & b = -2 \end{array}\right\} \Pi_1 = \frac{\Delta p}{\rho \omega^2 D^2} \qquad \Pi_1 = \frac{F}{L^2} \frac{L^4}{Ft^2} \frac{t^2}{1} \frac{1}{L^2} = [1] \checkmark\checkmark$$

$$\Pi_2 = \rho^a \omega^b D^c Q = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{L^3}{t} = M^0 L^0 t^0$$

$$\left.\begin{array}{ll} M: a = 0 & a = 0 \\ L: -3a + c + 3 = 0 & c = -3 \\ t: -b - 1 = 0 & b = -1 \end{array}\right\} \Pi_2 = \frac{Q}{\omega D^3} \qquad \Pi_2 = \frac{L^3}{t} \frac{t}{1} \frac{1}{L^3} = [1] \checkmark\checkmark$$

Thus $\Pi_1 = f(\Pi_2)$ for this situation. Flows are geometrically similar. Assume kinematic similarity. Then for dynamic similarity, if $\Pi_{2m} = \Pi_{2p}$ then $\Pi_{1m} = \Pi_{1p}$.

$$\Pi_{2m} = \frac{Q_m}{\omega_m D_m^3} = \Pi_{2p} = \frac{Q_p}{\omega_p D_p^3}\,; \quad Q_m = Q_p \left(\frac{\omega_m}{\omega_p}\right)\left(\frac{D_m}{D_p}\right)^3 = Q_p \left(\frac{367}{183}\right)\left(\frac{50}{150}\right)^3 = 0.0743\, Q_p$$

$$Q_m = 0.0743 \times 1.25 \frac{m^3}{min} = 0.0928\ m^3/min \longleftarrow$$

Q_m

$$\Pi_{1m} = \frac{\Delta p_m}{\rho_m \omega_m^2 D_m^2} = \Pi_{1p} = \frac{\Delta p_p}{\rho_p \omega_p^2 D_p^2}\,; \quad \Delta p_p = \Delta p_m \frac{\rho_p}{\rho_m} \left(\frac{\omega_p}{\omega_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2$$

$$\Delta p_p = \Delta p_m \left(\frac{800}{999}\right)\left(\frac{183}{367}\right)^2\left(\frac{150}{50}\right)^2 = 1.79 \times 29.3\ kPa = 52.5\ kPa \longleftarrow$$

Δp_p

{This result neglects any effect of viscosity.}

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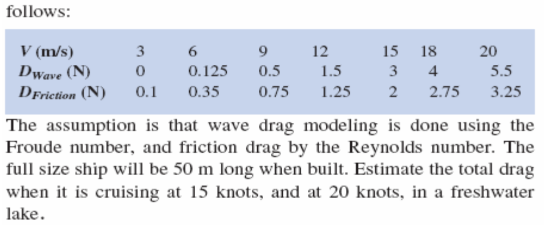
Problem 7.77

[3]



7.77 Tests are performed on a 1-m long ship model in a water tank. Results obtained (after doing some data analysis) are as follows:

V (m/s)	3	6	9	12	15	18	20
D_{Wave} (N)	0	0.125	0.5	1.5	3	4	5.5
$D_{Friction}$ (N)	0.1	0.35	0.75	1.25	2	2.75	3.25

The assumption is that wave drag modeling is done using the Froude number, and friction drag by the Reynolds number. The full size ship will be 50 m long when built. Estimate the total drag when it is cruising at 15 knots, and at 20 knots, in a freshwater lake.

For drag we can use $C_D = \dfrac{D}{\frac{1}{2}\rho V^2 A}$ As a suitable scaling area for A we use L^2 $C_D = \dfrac{D}{\frac{1}{2}\rho V^2 L^2}$

Model: $L =$ 1 m

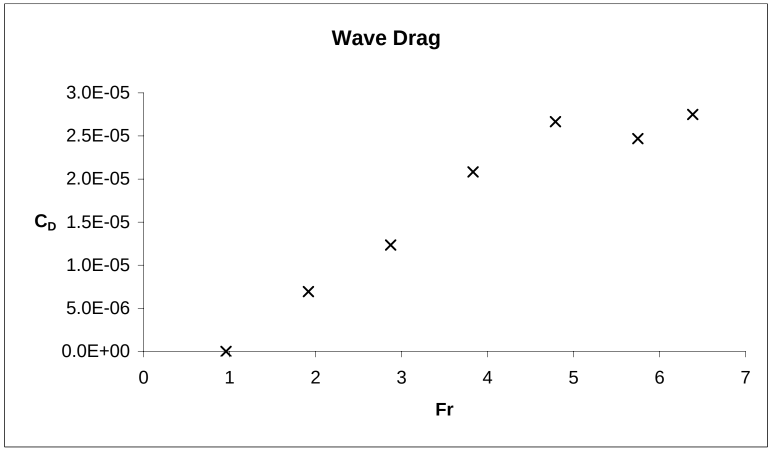
For water $\rho =$ 1000 kg/m^3

$\mu =$ 1.01E-03 N·s/m^2

The data is:

V (m/s)	3	6	9	12	15	18	20
D_{Wave} (N)	0	0.125	0.5	1.5	3	4	5.5
$D_{Friction}$ (N)	0.1	0.35	0.75	1.25	2	2.75	3.25

Fr	0.958	1.916	2.873	3.831	4.789	5.747	6.386
Re	2.97E+06	5.94E+06	8.91E+06	1.19E+07	1.49E+07	1.78E+07	1.98E+07
$C_{D(Wave)}$	0.00E+00	6.94E-06	1.23E-05	2.08E-05	2.67E-05	2.47E-05	2.75E-05
$C_{D(Friction)}$	2.22E-05	1.94E-05	1.85E-05	1.74E-05	1.78E-05	1.70E-05	1.63E-05



Wave Drag

The friction drag coefficient becomes a constant, as expected, at high Re.
The wave drag coefficient appears to be linear with Fr, over most values

Ship: $L =$ 50 m

V (knot)	15	20
V (m/s)	7.72	10.29
Fr	0.348	0.465
Re	3.82E+08	5.09E+08

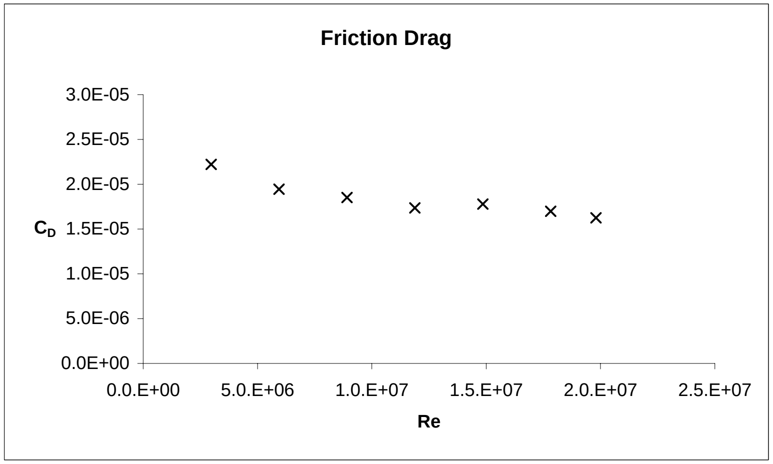
$$D = \frac{1}{2}\rho V^2 L^2 C_D$$

Hence for the ship we have very high Re, and low Fr.
From the graph we see the friction C_D levels out at about 1.75 x 10^{-5}
From the graph we see the wave C_D is negligibly small

$C_{D(Wave)}$	0	0
$C_{D(Friction)}$	1.75E-05	1.75E-05

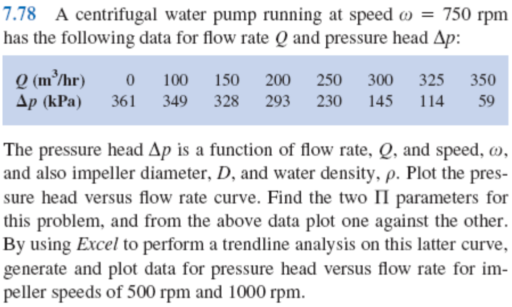
D_{Wave} (N)	0	0
$D_{Friction}$ (N)	1303	2316

D_{Total} (N)	1303	2316



Friction Drag

Problem 7.78 (In Excel)



7.78 A centrifugal water pump running at speed $\omega = 750$ rpm has the following data for flow rate Q and pressure head Δp:

Q (m^3/hr)	0	100	150	200	250	300	325	350
Δp (kPa)	361	349	328	293	230	145	114	59

The pressure head Δp is a function of flow rate, Q, and speed, ω, and also impeller diameter, D, and water density, ρ. Plot the pressure head versus flow rate curve. Find the two Π parameters for this problem, and from the above data plot one against the other. By using *Excel* to perform a trendline analysis on this latter curve, generate and plot data for pressure head versus flow rate for impeller speeds of 500 rpm and 1000 rpm.

Given: Data on centrifugal water pump

Find: Π groups; plot pressure head vs flow rate for range of speeds

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 5$
The number of primary dimensions is: $r = 3$
The number of repeat parameters is: $m = r = 3$
The number of Π groups is: $n - m = 2$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents *a* , *b* , and *c* for each.

REPEATING PARAMETERS: **Choose ρ, *g* , *d***

	M	**L**	**t**
ρ	1	-3	
ω			-1
D		1	

Π GROUPS:

	M	**L**	**t**
Δp	1	-1	-2

Π_1: $a = -1$, $b = -2$, $c = -2$

	M	**L**	**t**
Q	0	3	-1

Π_2: $a = 0$, $b = -1$, $c = -3$

The following Π groups from Example 7.1 are not used:

M	**L**	**t**
0	0	0

Π_3: $a = 0$, $b = 0$, $c = 0$

M	**L**	**t**
0	0	0

Π_4: $a = 0$, $b = 0$, $c = 0$

Hence $\Pi_1 = \dfrac{\Delta p}{\rho\omega^2 D^2}$ and $\Pi_2 = \dfrac{Q}{\omega D^3}$ with $\Pi_1 = f(\Pi_2)$.

Based on the plotted data, it looks like the relation between Π_1 and Π_2 may be parabolic

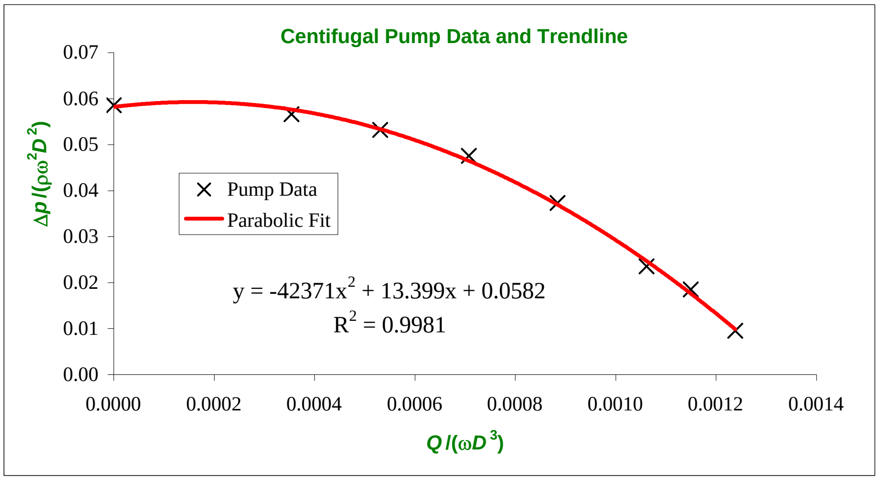
Hence
$$\frac{\Delta p}{\rho\omega^2 D^2} = a + b\left(\frac{Q}{\omega D^3}\right) + c\left(\frac{Q}{\omega D^3}\right)^2$$

The data is

Q (m^3/hr)	0	100	150	200	250	300	325	350
Δp (kPa)	361	349	328	293	230	145	114	59

ρ = 999 kg/m^3
ω = 750 rpm
D = 1 m (*D* is not given; use *D* = 1 m as a scale)

$Q/(\omega D^3)$	0.00000	0.000354	0.000531	0.000707	0.000884	0.00106	0.00115	0.00124
$\Delta p/(\rho\omega^2 D^2)$	0.0586	0.0566	0.0532	0.0475	0.0373	0.0235	0.0185	0.00957



From the *Trendline* analysis

a = 0.0582
b = 13.4
c = -42371

and $$\Delta p = \rho\omega^2 D^2\left[a + b\left(\frac{Q}{\omega D^3}\right) + c\left(\frac{Q}{\omega D^3}\right)^2\right]$$

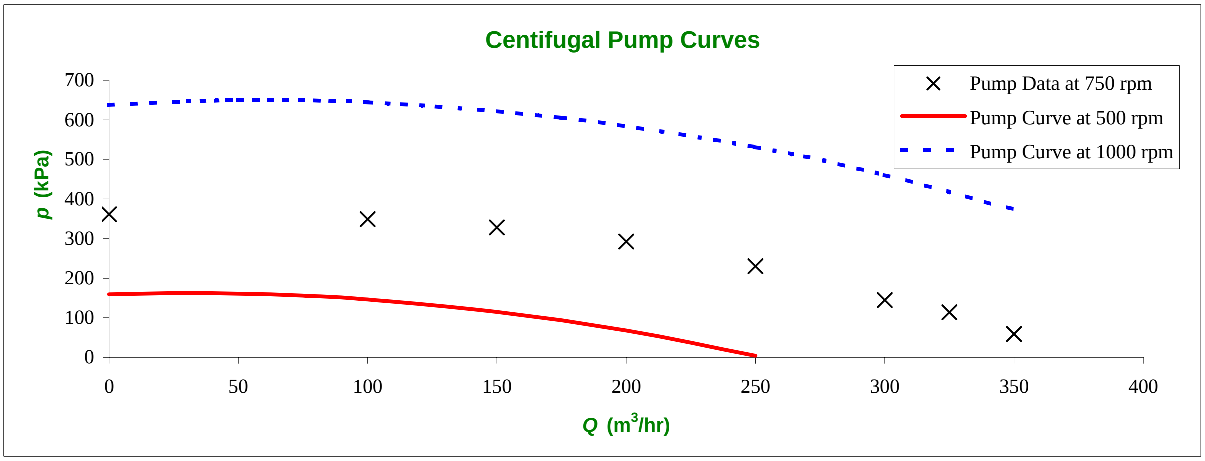
Finally, data at 500 and 1000 rpm can be calculated and plotted

ω = 500 rpm

Q (m^3/hr)	0	25	50	75	100	150	200	250
Δp (kPa)	159	162	161	156	146	115	68	4

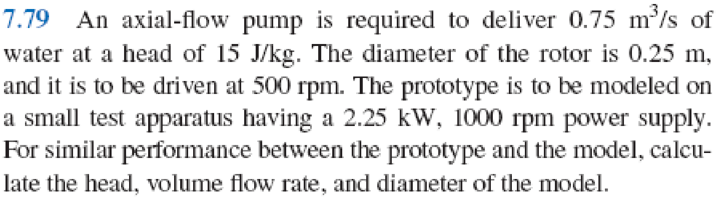
ω = 1000 rpm

Q (m^3/hr)	0	25	50	100	175	250	300	350
Δp (kPa)	638	645	649	644	606	531	460	374



Problem 7.79

[3]



7.79 An axial-flow pump is required to deliver 0.75 m^3/s of water at a head of 15 J/kg. The diameter of the rotor is 0.25 m, and it is to be driven at 500 rpm. The prototype is to be modeled on a small test apparatus having a 2.25 kW, 1000 rpm power supply. For similar performance between the prototype and the model, calculate the head, volume flow rate, and diameter of the model.

Given: Model of water pump

Find: Model head, flow rate and diameter

Solution:

From Buckingham Π $\quad \dfrac{h}{\omega^2 \cdot D^2} = f\left(\dfrac{Q}{\omega \cdot D^3}, \dfrac{\rho \cdot \omega \cdot D^2}{\mu}\right) \quad$ and $\quad \dfrac{P}{\omega^3 \cdot D^5} = f\left(\dfrac{Q}{\omega \cdot D^3}, \dfrac{\rho \cdot \omega \cdot D^2}{\mu}\right)$

Neglecting viscous effects $\quad \dfrac{Q_m}{\omega_m \cdot D_m^3} = \dfrac{Q_p}{\omega_p \cdot D_p^3} \quad$ then $\quad \dfrac{h_m}{\omega_m^2 \cdot D_m^2} = \dfrac{h_p}{\omega_p^2 \cdot D_p^2} \quad$ and $\quad \dfrac{P_m}{\omega_m^3 \cdot D_m^5} = \dfrac{P_p}{\omega_p^3 \cdot D_p^5}$

Hence if $\quad \dfrac{Q_m}{Q_p} = \dfrac{\omega_m}{\omega_p} \cdot \left(\dfrac{D_m}{D_p}\right)^3 = \dfrac{1000}{500} \cdot \left(\dfrac{D_m}{D_p}\right)^3 = 2 \cdot \left(\dfrac{D_m}{D_p}\right)^3 \quad (1)$

then $\quad \dfrac{h_m}{h_p} = \dfrac{\omega_m^2}{\omega_p^2} \cdot \dfrac{D_m^2}{D_p^2} = \left(\dfrac{1000}{500}\right)^2 \cdot \dfrac{D_m^2}{D_p^2} = 4 \cdot \dfrac{D_m^2}{D_p^2} \quad (2)$

and $\quad \dfrac{P_m}{P_p} = \dfrac{\omega_m^3}{\omega_p^3} \cdot \dfrac{D_m^5}{D_p^5} = \left(\dfrac{1000}{500}\right)^3 \cdot \dfrac{D_m^5}{D_p^5} = 8 \cdot \dfrac{D_m^5}{D_p^5} \quad (3)$

We can find P_p from $\quad P_p = \rho \cdot Q \cdot h = 1000 \cdot \dfrac{kg}{m^3} \times 0.75 \cdot \dfrac{m^3}{s} \times 15 \cdot \dfrac{J}{kg} = 11.25 \cdot kW$

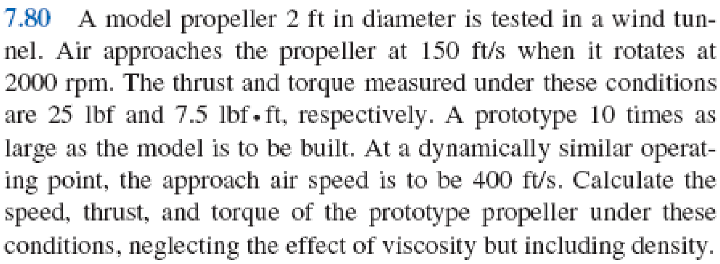
From Eq 3 $\quad \dfrac{P_m}{P_p} = 8 \cdot \dfrac{D_m^5}{D_p^5} \quad$ so $\quad D_m = D_p \cdot \left(\dfrac{1}{8} \cdot \dfrac{P_m}{P_p}\right)^{\frac{1}{5}} \quad D_m = 0.25 \cdot m \times \left(\dfrac{1}{8} \times \dfrac{2.25}{11.25}\right)^{\frac{1}{5}} \quad D_m = 0.120\,m$

From Eq 1 $\quad \dfrac{Q_m}{Q_p} = 2 \cdot \left(\dfrac{D_m}{D_p}\right)^3 \quad$ so $\quad Q_m = Q_p \cdot 2 \cdot \left(\dfrac{D_m}{D_p}\right)^3 \quad Q_m = 0.75 \cdot \dfrac{m^3}{s} \times 2 \times \left(\dfrac{0.12}{0.25}\right)^3 \quad Q_m = 0.166\,\dfrac{m^3}{s}$

From Eq 2 $\quad \dfrac{h_m}{h_p} = 4 \cdot \left(\dfrac{D_m}{D_p}\right)^2 \quad$ so $\quad h_m = h_p \cdot 4 \cdot \left(\dfrac{D_m}{D_p}\right)^2 \quad h_m = 15 \cdot \dfrac{J}{kg} \times 4 \times \left(\dfrac{0.12}{0.25}\right)^2 \quad h_m = 13.8\,\dfrac{J}{kg}$

Problem 7.80

[3]



7.80 A model propeller 2 ft in diameter is tested in a wind tunnel. Air approaches the propeller at 150 ft/s when it rotates at 2000 rpm. The thrust and torque measured under these conditions are 25 lbf and 7.5 lbf•ft, respectively. A prototype 10 times as large as the model is to be built. At a dynamically similar operating point, the approach air speed is to be 400 ft/s. Calculate the speed, thrust, and torque of the prototype propeller under these conditions, neglecting the effect of viscosity but including density.

Given: Data on model propeller

Find: Speed, thrust and torque on prototype

Solution:

There are two problems here: Determine $F_t = f_1(D, \omega, V, \mu, \rho)$ and also $T = f_2(D, \omega, V, \mu, \rho)$. Since μ is to be ignored, do not select it as a repeat parameter; instead select D, ω, ρ as repeats.

Apply the Buckingham Π procedure

① $F_t \quad D \quad \omega \quad V \quad \mu \quad \rho$ $\qquad n = 6$ parameters

② Select primary dimensions M, L, t

③ $$\left\{ \begin{array}{cccccc} F_t & D & \omega & V & \mu & \rho \\ \frac{M\,L}{t^2} & L & \frac{1}{t} & \frac{L}{t} & \frac{M}{Lt} & \frac{M}{L^3} \end{array} \right\}$$ $r = 3$ primary dimensions

④ $\rho \quad D \quad \omega$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 5$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega^c F_t = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rc|l} M: & a + 1 = 0 & a = -1 \\ L: & -3a + b + 1 = 0 & b = -4 \\ t: & -c - 2 = 0 & c = -2 \end{array}$$
Hence $\Pi_1 = \dfrac{F_t}{\rho D^4 \omega^2}$

$$\Pi_2 = \rho^a D^b \omega^c V = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{L}{t} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rc|l} M: & a = 0 & a = 0 \\ L: & -3a + b + 1 = 0 & b = -1 \\ t: & -c - 1 = 0 & c = -1 \end{array}$$
Hence $\Pi_2 = \dfrac{V}{D\omega}$

$$\Pi_3 = \rho^a D^b \omega^c \mu = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{ll|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-1=0 & c=-1 \end{array}$$
Hence $\Pi_3 = \dfrac{\mu}{\rho D^2 \omega}$

⑥ Check using *F*, *L*, *t* as primary dimensions

$$\Pi_1 = \frac{F}{\frac{Ft^2}{L^4} L^4 \frac{1}{t^2}} = [1] \qquad \Pi_2 = \frac{\frac{L}{t}}{L\frac{1}{t}} = [1] \qquad \Pi_3 = \frac{\frac{Ft}{L^2}}{\frac{Ft^2}{L^4} L^2 \frac{1}{t}} = [1]$$

Then $\Pi_1 = f_1(\Pi_2, \Pi_3)$ $\qquad \dfrac{F_t}{\rho D^4 \omega^2} = f_1\left(\dfrac{V}{D\omega}, \dfrac{\mu}{\rho D^2 \omega}\right)$

If viscous effects are neglected $\dfrac{F_t}{\rho D^4 \omega^2} = g_1\left(\dfrac{V}{D\omega}\right)$

For dynamic similarity $\dfrac{V_m}{D_m \omega_m} = \dfrac{V_p}{D_p \omega_p}$

so $\omega_p = \dfrac{D_m}{D_p}\dfrac{V_p}{V_m}\omega_m = \left(\dfrac{1}{10}\right) \times \left(\dfrac{400}{150}\right) \times 2000\,\text{rpm} = 533\,\text{rpm}$

Under these conditions $\dfrac{F_{t_m}}{\rho D_m^4 \omega_m^2} = \dfrac{F_{t_p}}{\rho D_p^4 \omega_p^2}$ (assuming $\rho_m = \rho_p$)

or $F_{t_p} = \dfrac{D_p^4}{D_m^4}\dfrac{\omega_p^2}{\omega_m^2} F_{t_m} = \left(\dfrac{10}{1}\right)^4 \times \left(\dfrac{533}{2000}\right)^2 \times 25\,\text{lbf} = 1.78 \times 10^4\,\text{lbf}$

For the torque we can avoid repeating a lot of the work

$$\Pi_4 = \rho^a D^b \omega^c T = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{ll|l} M: & a+1=0 & a=-1 \\ L: & -3a+b+2=0 & b=-5 \\ t: & -c-2=0 & c=-2 \end{array}$$
Hence $\Pi_4 = \dfrac{T}{\rho D^5 \omega^2}$

Then $\Pi_4 = f_2(\Pi_2, \Pi_3)$ $\qquad \dfrac{T}{\rho D^5 \omega^2} = f_2\left(\dfrac{V}{D\omega}, \dfrac{\mu}{\rho D^2 \omega}\right)$

If viscous effects are neglected $\dfrac{T}{\rho D^5 \omega^2} = g_2\left(\dfrac{V}{D\omega}\right)$

For dynamic similarity $\dfrac{T_m}{\rho D_m^5 \omega_m^2} = \dfrac{T_p}{\rho D_p^5 \omega_p^2}$

or $T_p = \dfrac{D_p^5}{D_m^5}\dfrac{\omega_p^2}{\omega_m^2} T_m = \left(\dfrac{10}{1}\right)^5 \times \left(\dfrac{533}{2000}\right)^2 \times 7.5\,\text{lbf}\cdot\text{ft} = 5.33 \times 10^4\,\text{lbf}\cdot\text{ft}$

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Given: For a marine propeller (see Problem 7.40) the thrust force, F_t, is

$$F_t = F_t(\rho, D, V, g, \omega, p, \mu)$$

Neglecting viscous effects, and pressure, then

$$F_t = F_t(\rho, D, V, g, \omega)$$

Assume that Torque, T, and power, $\mathcal{P}$, depend on same parameters

$$T = T(\rho, D, V, g, \omega)$$

$$\mathcal{P} = \mathcal{P}(\rho, D, V, g, \omega)$$

Find: Derive scaling "laws" for propellers that relate F_t, T, and $\mathcal{P}$ to other variables.

Solution: Apply Buckingham π-theorem

① $\rho \quad D \quad V \quad g \quad \omega \quad F_t \quad T \quad \mathcal{P}$

② Choose F, L, t as primary dimensions

③ $\frac{Ft^2}{L^4} \quad L \quad \frac{L}{t} \quad \frac{L}{t^2} \quad \frac{1}{t} \quad F \quad FL \quad \frac{FL}{t}$

④ Repeating variables ρ, ω, D

⑤ Then $n-m = 5$ dimensionless groups (2 independent, 3 dependent)

Setting up dimensional equations

$$\pi_1 = \rho^a \omega^b D^c V \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{L}{t} \qquad \begin{Bmatrix} F: & 0 = a \\ t: & 0 = 2a - b - 1 \\ L: & 0 = -4a + c + 1 \end{Bmatrix} \quad \begin{matrix} a = 0 \\ b = -1 \\ c = -1 \end{matrix} \qquad \therefore \pi_1 = \frac{V}{\omega D}$$

$$\pi_2 = \rho^a \omega^b D^c g \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{L}{t^2} \qquad \begin{Bmatrix} F: & 0 = a \\ t: & 0 = 2a - b - 2 \\ L: & 0 = -4a + c + 1 \end{Bmatrix} \quad \begin{matrix} a = 0 \\ b = -2 \\ c = -1 \end{matrix} \qquad \therefore \pi_2 = \frac{g}{\omega^2 D}$$

$$\pi_3 = \rho^a \omega^b D^c F_t \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c F \qquad \begin{Bmatrix} F: & 0 = a + 1 \\ t: & 0 = 2a - b \\ L: & 0 = -4a + c \end{Bmatrix} \quad \begin{matrix} a = -1 \\ b = -2 \\ c = -4 \end{matrix} \qquad \therefore \pi_3 = \frac{F_t}{\rho \omega^2 D^4}$$

$$\pi_4 = \rho^a \omega^b D^c T \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c FL \qquad \begin{Bmatrix} F: & 0 = a + 1 \\ t: & 0 = 2a - b \\ L: & 0 = -4a + c + 1 \end{Bmatrix} \quad \begin{matrix} a = -1 \\ b = -2 \\ c = -5 \end{matrix} \qquad \therefore \pi_4 = \frac{T}{\rho \omega^2 D^5}$$

$$\pi_5 = \rho^a \omega^b D^c \mathcal{P} \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{FL}{t} \qquad \begin{Bmatrix} F: & 0 = a + 1 \\ t: & 0 = 2a - b - 1 \\ L: & 0 = -4a + c + 1 \end{Bmatrix} \quad \begin{matrix} a = -1 \\ b = -3 \\ c = -5 \end{matrix} \qquad \therefore \pi_5 = \frac{\mathcal{P}}{\rho \omega^3 D^5}$$

Then scaling "laws" are

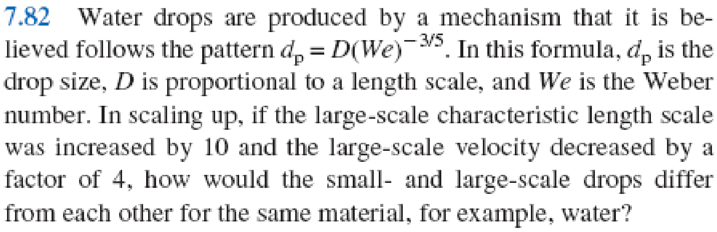
$$\frac{F_t}{\rho \omega^2 D^4} = f_1\left(\frac{V}{\omega D}, \frac{g}{\omega^2 D}\right)$$

$$\frac{T}{\rho \omega^2 D^5} = f_2\left(\frac{V}{\omega D}, \frac{g}{\omega^2 D}\right)$$

$$\frac{\mathcal{P}}{\rho \omega^3 D^5} = f_3\left(\frac{V}{\omega D}, \frac{g}{\omega^2 D}\right)$$

Problem 7.82

[2]



7.82 Water drops are produced by a mechanism that it is believed follows the pattern $d_p = D(We)^{-3/5}$. In this formula, d_p is the drop size, D is proportional to a length scale, and We is the Weber number. In scaling up, if the large-scale characteristic length scale was increased by 10 and the large-scale velocity decreased by a factor of 4, how would the small- and large-scale drops differ from each other for the same material, for example, water?

Given: Water drop mechanism

Find: Difference between small and large scale drops

Solution:

Given relation $$d = D\cdot(We)^{-\frac{3}{5}} = D\cdot\left(\frac{\rho\cdot V^2\cdot D}{\sigma}\right)^{-\frac{3}{5}}$$

For dynamic similarity $$\frac{d_m}{d_p} = \frac{D_m\cdot\left(\frac{\rho\cdot V_m^{\,2}\cdot D_m}{\sigma}\right)^{-\frac{3}{5}}}{D_p\cdot\left(\frac{\rho\cdot V_p^{\,2}\cdot D_p}{\sigma}\right)^{-\frac{3}{5}}} = \left(\frac{D_m}{D_p}\right)^{\frac{2}{5}}\cdot\left(\frac{V_m}{V_p}\right)^{-\frac{6}{5}}$$ where d_p stands for $d_{prototype}$ not the original d_p!

Hence $$\frac{d_m}{d_p} = \left(\frac{1}{10}\right)^{\frac{2}{5}}\times\left(\frac{4}{1}\right)^{-\frac{6}{5}}$$ $$\frac{d_m}{d_p} = 0.075$$

The small scale droplets are 7.5% of the size of the large scale

Problem 7.83

[2]

Given: The kinetic energy ratio is a figure of merit defined as the ratio of kinetic energy flux in a wind tunnel test section to the drive power.

Find: an estimate of the kinetic energy ratio for the 40×80 wind tunnel at NASA-Ames.

Solution:

From text (p. 319). for NASA-Ames tunnel:

$A = 40\,ft \times 80\,ft = 3200\,ft^2$, $\mathcal{P} = 125{,}000\,hp$

$V_{max} = 300 \frac{kmi}{hr} \times \frac{6080\,ft}{k\text{-}mi} \times \frac{hr}{3600\,s} = 507\ ft/s$

$\text{K.E ratio} = \frac{\text{K.E flux}}{\text{Power in}} = \frac{\dot{m}\frac{V^2}{2}}{\mathcal{P}} = \frac{\rho V A\, V^2}{2\mathcal{P}} = \frac{\rho V^3 A}{2\mathcal{P}}$

Assuming standard air,

$\text{K.E. ratio} = \frac{1}{2} \times 0.00238 \frac{slug}{ft^3} \times (507)^3 \frac{ft^3}{s^3} \times 3200\,ft^2 \times \frac{1}{125{,}000\,hp} \times \frac{hp \cdot s}{550\,ft \cdot lbf} \times \frac{lbf \cdot s^2}{slug \cdot ft}$

$\text{K.E ratio} = 7.22$

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Problem 7.84

[3]

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Given: A 1:16 scale model of a 20 m long truck is tested in a wind tunnel at speed $V_m = 80\,m/s$. The axial pressure gradient at this speed is $dh/dx = -1.2\,mm\,H_2O/m$. The frontal area of the prototype is $A_p = 10\,m^2$. $C_D = 0.85$

Find: (a) Estimate the horizontal buoyancy correction
(b) Express the correction as a fraction of the measured C_D.

Solution:

The horizontal buoyancy force, F_B, is the difference in the pressure force between the front and back of the model due to the pressure gradient in the tunnel

$$F_B = (p_f - p_b)A = \rho_w g \frac{dh}{dx} L_m A_m \qquad (\Delta p = \rho_w g \Delta h)$$

$$L_m = \frac{L_p}{16} \qquad A_m = \frac{A_p}{(16)^2}$$

$$\therefore F_B = 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (-1.2)\times 10^{-3}\,\frac{m}{m} \times \frac{20\,m}{16} \times \frac{10\,m^2}{(16)^2} \times \frac{N\cdot s^2}{kg\cdot m}$$

$$F_B = -0.574\,N$$ F_B

The horizontal buoyancy correction should be added to the measured drag force on the model

The measured drag force on the model is given by

$$F_{D_m} = \frac{1}{2}\rho V^2 A_m C_D = \frac{1}{2}\rho V^2 \frac{A_p}{(16)^2} C_D$$

Assume air at standard conditions, $\rho = 1.23\,kg/m^3$

$$F_{D_m} = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (80)^2\,\frac{m^2}{s^2} \times \frac{10\,m^2}{(16)^2} \times 0.85 \times \frac{N\cdot s^2}{kg\cdot m}$$

$$F_{D_m} = 131\,N$$

$$\frac{F_B}{F_{D_m}} = \frac{-0.574}{131} = -4.38\times 10^{-3} = -0.44\,\%$$ $\frac{F_B}{F_{D_m}}$

Problem 7.85

[3]

Given: Wind tunnel test of 1:16 model bus in standard air.

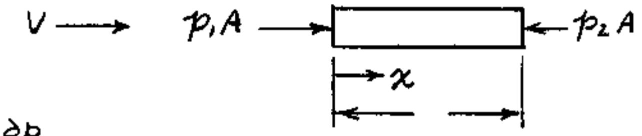
$W = 152\text{ mm}$ $\quad V = 26.5\text{ m/s}$ $\quad$ Pressure gradient:
$H = 200\text{ mm}$ $\quad F_D = 6.09\text{ N}$ (measured) $\quad \frac{dp}{dx} = -11.8\text{ N/m}^2\text{/m}$
$L = 762\text{ mm}$

Find: (a) Estimate the horizontal buoyancy correction.
(b) Calculate the corrected model drag coefficient.
(c) Evaluate the drag force on the prototype at 100 km/hr on a calm day.

Solution: Apply definitions

Computing equations: $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$ $\quad$ Assume $A = WH$

The buoyancy force will be



$$F_B = p_1 A - p_2 A = (p_1 - p_2)A$$

But $p_2 = p_1 + \frac{\partial p}{\partial x}\Delta x + \cdots \approx p_1 + \frac{\partial p}{\partial x} L$

Therefore $p_1 - p_2 = -\frac{\partial p}{\partial x} L$, and $F_B \approx -\frac{\partial p}{\partial x} LA = -\frac{\partial p}{\partial x} LWH$

$$F_B \approx -(-11.8)\frac{N}{m^3} \times 0.762\, m \times 0.152\, m \times 0.200\, m = 0.273\, N \text{ (to right)}$$ F_B

The corrected drag force is

$$F_{D_c} = F_{D_m} - F_B = (6.09 - 0.273)\, N = 5.82\, N$$

The corrected model drag coefficient is

$$C_{D_m} = \frac{F_{D_c}}{\frac{1}{2}\rho V^2 A} = 2 \times 5.82\, N \times \frac{m^3}{1.23\, kg} \times \frac{s^2}{(26.5)^2 m^2} \times \frac{1}{(0.200)(0.152) m^2} \times \frac{kg \cdot m}{N \cdot s^2} = 0.443$$ C_D

Assume the test was conducted at high enough Reynolds number so $C_{D_p} = C_{D_m}$. Then

$$F_{D_p} = C_{D_p} A_p \tfrac{1}{2}\rho V_p^2$$

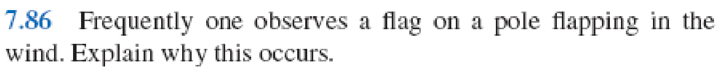
$$= \frac{1}{2} \times 0.443 \times 0.200(16)\, m \times 0.152(16)\, m \times 1.23\frac{kg}{m^3} \times \left[100\frac{km}{hr} \times 1000\frac{m}{km} \times \frac{hr}{3600\, s}\right]^2 \frac{N \cdot s^2}{kg \cdot m}$$

$F_{D_p} = 1.64\text{ kN}$ (prototype at 100 km/hr) $\quad F_{D_p}$

{Rolling resistance must be included to obtain the total tractive effort needed to propel the full-scale vehicle.}

Problem 7.86

[4]



7.86 Frequently one observes a flag on a pole flapping in the wind. Explain why this occurs.

Given:

Flapping flag on a flagpole

Find:

Explanation of the flappinh

Solution:

Open-Ended Problem Statement: Frequently one observes a flag on a pole "flapping" in the wind. Explain why this occurs. What dimensionless parameters might characterize the phenomenon? Why?

Discussion: The natural wind contains significant fluctuations in air speed and direction. These fluctuations tend to disturb the flag from an initially plane position.

When the flag is bent or curved from the plane position, the flow nearby must follow its contour. Flow over a convex surface tends to be faster, and have lower pressure, than flow over a concave curved surface. The resulting pressure forces tend to exaggerate the curvature of the flag. The result is a seemingly random "flapping" motion of the flag.

The rope or chain used to raise the flag may also flap in the wind. It is much more likely to exhibit a periodic motion than the flag itself. The rope is quite close to the flag pole, where it is influenced by any vortices shed from the pole. If the Reynolds number is such that periodic vortices are shed from the pole, they will tend to make the rope move with the same frequency. This accounts for the periodic thump of a rope or clank of a chain against the pole.

The vortex shedding phenomenon is characterized by the Strouhal number, $St = fD/V_\infty$, where f is the vortex shedding frequency, D is the pole diameter, and D is the wind speed. The Strouhal number is constant at approximately 0.2 over a broad range of Reynolds numbers.

Problem 7.87

Open-Ended Problem Statement: Explore the variation in wave propagation speed given by the equation of Problem 7.2 for a free-surface flow of water. Find the operating depth to minimize the speed of capillary waves (waves with small wavelength, also called *ripples*). First assume wavelength is much smaller than water depth. Then explore the effect of depth. What depth do you recommend for a water table used to visualize compressible-flow wave phenomena? What is the effect of reducing surface tension by adding a surfactant?

Discussion: The equation given in Problem 7.2 contains three terms. The first term contains surface tension and gives a speed inversely proportional to wavelength. This term will be important when small wavelengths are considered.

The second term contains gravity and gives a speed proportional to wavelength. This term will be important when long wavelengths are considered.

The argument of the hyperbolic tangent is proportional to water depth and inversely proportional to wavelength. For small wavelengths, this term should approach unity since the hyperbolic tangent of a large number approaches one.

See the spreadsheet for numerical values and a plot.

Problem 7.87

Input Parameters:

g = 9.81 m/s^2 Acceleration of gravity

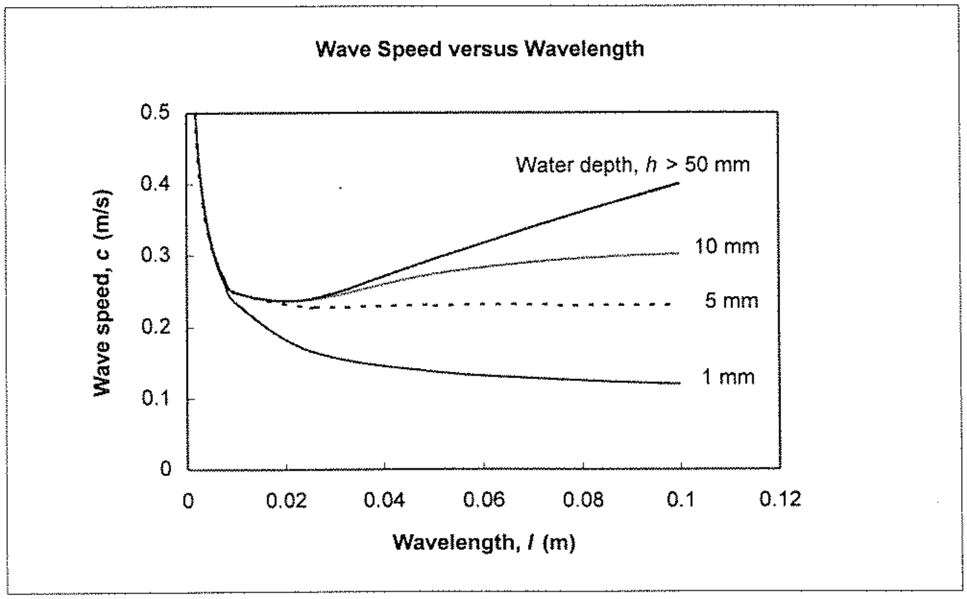
h = 0.01 m Liquid depth (for hyperbolic tangent calculation)

ρ = 999 kg/m^3 Liquid density

σ = 0.0728 N/m Surface tension

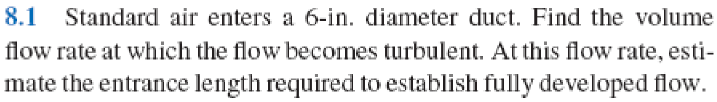
Calculated Values:

	h (m) =	0.001	0.005	0.01	0.05	0.1	0.5
Wavelength, λ (m)	**tanh (---)** (h = 10 mm)	**Wave Speed, c (m/s)**					
0.00185	1.00	0.500	0.500	0.500	0.500	0.500	0.500
0.003	1.00	0.396	0.397	0.397	0.397	0.397	0.397
0.005	1.00	0.313	0.315	0.315	0.315	0.315	0.315
0.0075	1.00	0.263	0.270	0.270	0.270	0.270	0.270
0.01	1.00	0.233	0.248	0.248	0.248	0.248	0.248
0.025	0.987	0.167	0.227	0.238	0.239	0.239	0.239
0.05	0.850	0.138	0.229	0.275	0.295	0.295	0.295
0.075	0.685	0.126	0.229	0.294	0.351	0.351	0.351
0.1	0.557	0.120	0.228	0.303	0.400	0.401	0.401
0.2	0.304	0.110	0.226	0.312	0.537	0.560	0.561
0.5	0.125	0.104	0.223	0.314	0.660	0.815	0.884
0.75	0.0836	0.102	0.223	0.314	0.681	0.896	1.08
1	0.0627	0.101	0.222	0.314	0.690	0.933	1.25
2	0.0314	0.100	0.222	0.314	0.698	0.975	1.69
5	0.0126	0.100	0.222	0.313	0.700	0.988	2.09
7.5	0.00838	0.0994	0.222	0.313	0.700	0.989	2.15
10	0.00628	0.0993	0.222	0.313	0.700	0.990	2.18
Froude Speed, $(gh)^{1/2}$ (m/s)		0.0990	0.221	0.313	0.700	0.990	2.21



Problem 8.1

[1]



8.1 Standard air enters a 6-in. diameter duct. Find the volume flow rate at which the flow becomes turbulent. At this flow rate, estimate the entrance length required to establish fully developed flow.

Given:

Air entering duct

Find:

Flow rate for turbulence; Entrance length

Solution:

The governing equations are $Re = \frac{V \cdot D}{\nu}$ $\quad Re_{crit} = 2300$ $\quad Q = \frac{\pi}{4} \cdot D^2 \cdot V$

The given data is $D = 6 \cdot in$ $\quad$ From Table A.9 $\quad \nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

$L_{laminar} = 0.06 \cdot Re_{crit} \cdot D$ $\quad$ or, for turbulent, L_{turb} = 25D to 40D

Hence

$$Re_{crit} = \frac{\frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot D}{\nu} \quad \text{or} \quad Q = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D}{4}$$

$$Q = 2300 \times \frac{\pi}{4} \times 1.62 \times 10^{-4} \cdot \frac{ft^2}{s} \times \frac{1}{2} \cdot ft \qquad Q = 0.146 \cdot \frac{ft^3}{s}$$

For laminar flow $\quad L_{laminar} = 0.06 \cdot Re_{crit} \cdot D$ $\quad L_{laminar} = 0.06 \times 2300 \times 6 \cdot in$ $\quad L_{laminar} = 69.0 \cdot ft$

For turbulent flow $\quad L_{min} = 25 \cdot D$ $\quad L_{min} = 12.5 \cdot ft$ $\quad L_{max} = 40 \cdot D$ $\quad L_{max} = 20 \cdot ft$

Problem 8.2

[2]

Given: Incompressible flow in a circular channel.
Re = 1800 in a section where the channel diameter is $D = 10\text{ mm}$.

Find: (i) general expression for Re in terms of
(a) volume flow rate, Q, and channel diameter, D
(b) mass flow rate, $\dot{m}$, and channel diameter, D.
(ii) Re for same flow rate and $D = 6\text{ mm}$.

Solution:

Assume steady, incompressible flow

Definitions: $Re = \frac{\rho D \bar{V}}{\mu}$, $Q = A\bar{V}$, $\dot{m} = \rho A \bar{V}$ and $A = \frac{\pi D^2}{4}$

Then,

$$Re = \frac{\rho D \bar{V}}{\mu} = \frac{\rho D}{\mu}\frac{Q}{A} = \frac{\rho D}{\mu}\frac{Q4}{\pi D^2} = \frac{4Q}{\pi D}\frac{\rho}{\mu} = \frac{4Q}{\pi D \nu} \longleftarrow \quad Re$$

Also

$$Re = \frac{\rho D \bar{V}}{\mu} = \frac{D}{\mu}\frac{\rho \bar{V} A}{A} = \frac{D}{\mu}\frac{\dot{m}4}{\pi D^2} = \frac{4\dot{m}}{\pi D \mu} \longleftarrow \quad Re$$

From Eq (i) a

$$Q = \frac{\pi D \nu Re}{4}$$

Then for same flow rate in sections with different channel diameter,

$$D_1 Re_1 = D_2 Re_2$$

$$Re_2 = \frac{D_1}{D_2} Re_1 = \frac{10\text{ mm}}{6\text{ mm}} \times 1800 = 3000 \longleftarrow \quad Re_2$$

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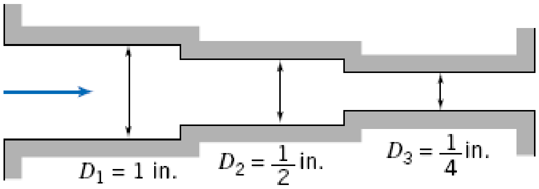
Problem 8.3

[3]

8.3 Standard air flows in a pipe system in which diameter is decreased in two stages from 1 in., to $\frac{1}{2}$ in., to $\frac{1}{4}$ in. Each section is 5 ft long. As the flow rate is increased, which section will become turbulent first? Determine the flow rates at which one, two, then all three sections first become turbulent. At each of these flow rates, determine which sections, if any, attain fully developed flow.



Given: Air entering pipe system

Find: Flow rate for turbulence in each section; Which become fully developed

Solution:

From Table A.9 $\quad \nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

The given data is $\quad L = 5 \cdot ft \quad D_1 = 1 \cdot in \quad D_2 = \frac{1}{2} \cdot in \quad D_3 = \frac{1}{4} \cdot in$

The critical Reynolds number is $\quad Re_{crit} = 2300$

Writing the Reynolds number as a function of flow rate

$$Re = \frac{V \cdot D}{\nu} = \frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot \frac{D}{\nu} \qquad \text{or} \qquad Q = \frac{Re \cdot \pi \cdot \nu \cdot D}{4}$$

Then the flow rates for turbulence to begin in each section of pipe are

$$Q_1 = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D_1}{4} \qquad Q_1 = 2300 \times \frac{\pi}{4} \times 1.62 \times 10^{-4} \cdot \frac{ft^2}{s} \times \frac{1}{12} \cdot ft \qquad Q_1 = 0.0244 \frac{ft^3}{s}$$

$$Q_2 = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D_2}{4} \qquad Q_2 = 0.0122 \frac{ft^3}{s} \qquad Q_3 = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D_3}{4} \qquad Q_3 = 0.00610 \frac{ft^3}{s}$$

Hence, **smallest** pipe becomes turbulent first, then second, then the largest.

For the smallest pipe transitioning to turbulence (Q_3)

For pipe 3 $\quad Re_3 = 2300 \quad L_{laminar} = 0.06 \cdot Re_3 \cdot D_3 \quad L_{laminar} = 2.87\,ft \quad$ $L_{laminar} < L$: Not fully developed

or, for turbulent, $\quad L_{min} = 25 \cdot D_3 \quad L_{min} = 0.521\,ft \quad L_{max} = 40 \cdot D_3 \quad L_{max} = 0.833\,ft \quad$ $L_{max/min} < L$: Not fully developed

For pipes 1 and 2 $\quad L_{laminar} = 0.06 \cdot \left(\frac{4 \cdot Q_3}{\pi \cdot \nu \cdot D_1}\right) \cdot D_1 \quad L_{laminar} = 2.87\,ft \quad$ $L_{laminar} < L$: Not fully developed

$L_{laminar} = 0.06 \cdot \left(\frac{4 \cdot Q_3}{\pi \cdot \nu \cdot D_2}\right) \cdot D_2 \quad L_{laminar} = 2.87\,ft \quad$ $L_{laminar} < L$: Not fully developed

For the middle pipe transitioning to turbulence (Q_2)

For pipe 2 $Re_2 = 2300$ $L_{laminar} = 0.06 \cdot Re_2 \cdot D_2$ $L_{laminar} = 5.75\,ft$

$L_{laminar} >$ L: Fully developed

or, for turbulent, $L_{min} = 25 \cdot D_2$ $L_{min} = 1.04\,ft$ $L_{max} = 40 \cdot D_2$ $L_{max} = 1.67\,ft$

$L_{max/min} <$ L: Not fully developed

For pipes 1 and 3 $L_1 = 0.06 \cdot \left(\frac{4 \cdot Q_2}{\pi \cdot \nu \cdot D_1}\right) \cdot D_1$ $L_1 = 5.75\,ft$

$L_{3min} = 25 \cdot D_3$ $L_{3min} = 0.521\,ft$ $L_{3max} = 40 \cdot D_3$ $L_{3max} = 0.833\,ft$

$L_{max/min} <$ L: Not fully developed

For the large pipe transitioning to turbulence (Q_1)

For pipe 1 $Re_1 = 2300$ $L_{laminar} = 0.06 \cdot Re_1 \cdot D_1$ $L_{laminar} = 11.5\,ft$

$L_{laminar} >$ L: Fully developed

or, for turbulent, $L_{min} = 25 \cdot D_1$ $L_{min} = 2.08\,ft$ $L_{max} = 40 \cdot D_1$ $L_{max} = 3.33\,ft$

$L_{max/min} <$ L: Not fully developed

For pipes 2 and 3 $L_{2min} = 25 \cdot D_2$ $L_{2min} = 1.04\,ft$ $L_{2max} = 40 \cdot D_2$ $L_{2max} = 1.67\,ft$

$L_{max/min} <$ L: Not fully developed

$L_{3min} = 25 \cdot D_3$ $L_{3min} = 0.521\,ft$ $L_{3max} = 40 \cdot D_3$ $L_{3max} = 0.833\,ft$

$L_{max/min} <$ L: Not fully developed

Problem 8.4

[2]

8.4 For flow in circular tubes, transition to turbulence usually occurs around $Re \approx 2300$. Investigate the circumstances under which the flows of (a) standard air and (b) water at 15°C become turbulent. On log-log graphs, plot: the average velocity, the volume flow rate, and the mass flow rate, at which turbulence first occurs, as functions of tube diameter.

Given: That transition to turbulence occurs at about *Re* = 2300

Find: Plots of average velocity and volume and mass flow rates for turbulence for air and water

Solution:

From Tables A.8 and A.10 $\rho_{air} = 1.23\cdot\frac{kg}{m^3}$ $\nu_{air} = 1.45\times10^{-5}\cdot\frac{m^2}{s}$ $\rho_w = 999\cdot\frac{kg}{m^3}$ $\nu_w = 1.14\times10^{-6}\cdot\frac{m^2}{s}$

The governing equations are $Re = \frac{V\cdot D}{\nu}$ $Re_{crit} = 2300$

For the average velocity $V = \frac{Re_{crit}\cdot\nu}{D}$

Hence for air $V_{air} = \frac{2300\times1.45\times10^{-5}\cdot\frac{m^2}{s}}{D}$ $V_{air} = \frac{0.0334\cdot\frac{m^2}{s}}{D}$

For water $V_w = \frac{2300\times1.14\times10^{-6}\cdot\frac{m^2}{s}}{D}$ $V_w = \frac{0.00262\cdot\frac{m^2}{s}}{D}$

For the volume flow rates $Q = A\cdot V = \frac{\pi}{4}\cdot D^2\cdot V = \frac{\pi}{4}\cdot D^2\cdot\frac{Re_{crit}\cdot\nu}{D} = \frac{\pi\cdot Re_{crit}\cdot\nu}{4}\cdot D$

Hence for air $Q_{air} = \frac{\pi}{4}\times2300\times1.45\cdot10^{-5}\cdot\frac{m^2}{s}\cdot D$ $Q_{air} = 0.0262\cdot\frac{m^2}{s}\times D$

For water $Q_w = \frac{\pi}{4}\times2300\times1.14\cdot10^{-6}\cdot\frac{m^2}{s}\cdot D$ $Q_w = 0.00206\cdot\frac{m^2}{s}\times D$

Finally, the mass flow rates are obtained from volume flow rates

$m_{air} = \rho_{air}\cdot Q_{air}$ $m_{air} = 0.0322\cdot\frac{kg}{m\cdot s}\times D$

$m_w = \rho_w\cdot Q_w$ $m_w = 2.06\cdot\frac{kg}{m\cdot s}\times D$

These results are plotted in the associated *Excel* workbook

Problem 8.4 [2]

8.4 For flow in circular tubes, transition to turbulence usually occurs around *Re* ≈ 2300. Investigate the circumstances under which the flows of (a) standard air and (b) water at 15°C become turbulent. On log-log graphs, plot: the average velocity, the volume flow rate, and the mass flow rate, at which turbulence first occurs, as functions of tube diameter.

Given: That transition to turbulence occurs at about *Re* = 2300

Find: Plots of average velocity and volume and mass flow rates for turbulence for air and water

Solution:

The relations needed are

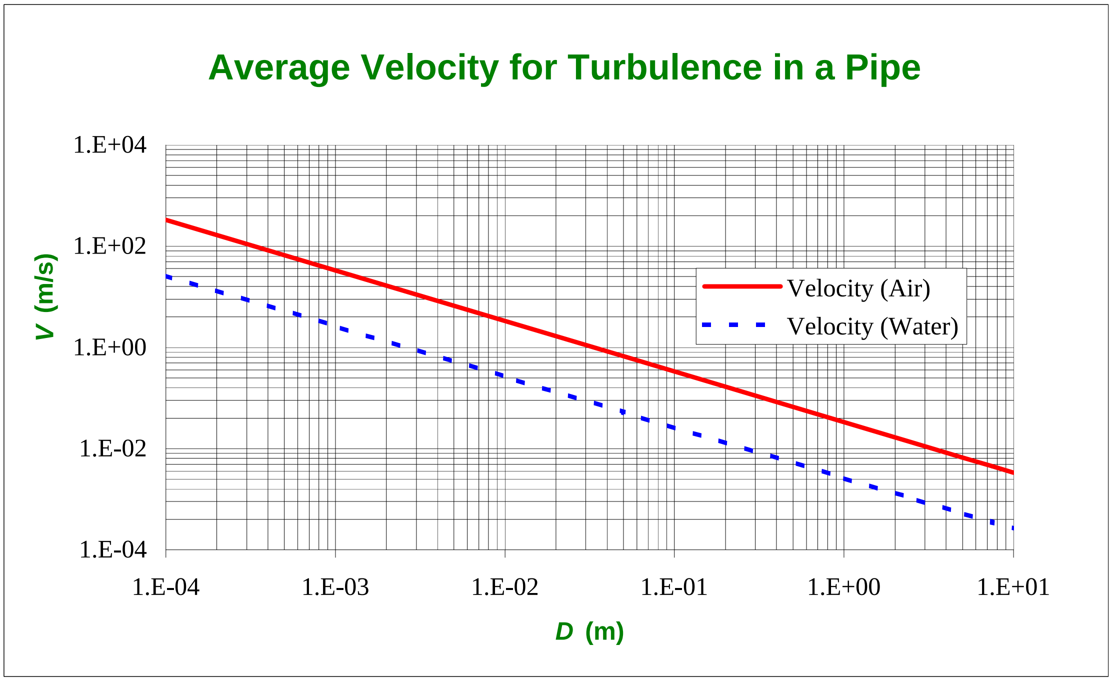
$$Re_{crit} = 2300 \qquad V = \frac{Re_{crit}\cdot\nu}{D} \qquad Q = \frac{\pi\cdot Re_{crit}\cdot\nu}{4}\cdot D \qquad m_{rate} = \rho\cdot Q$$

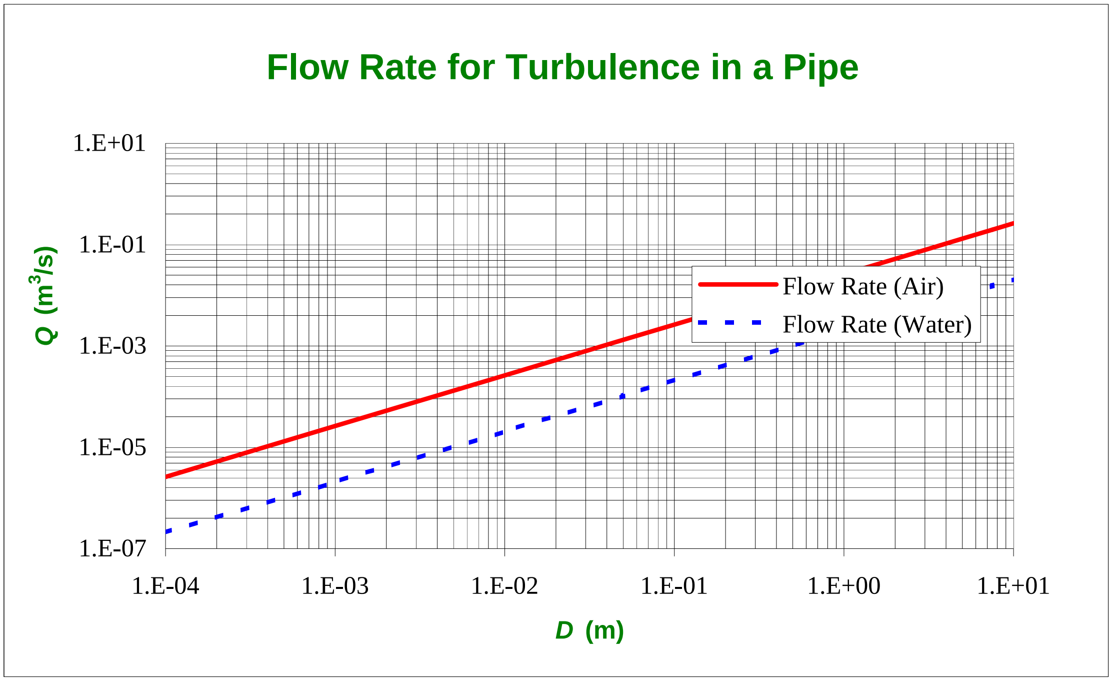
From Tables A.8 and A.10 the data required is

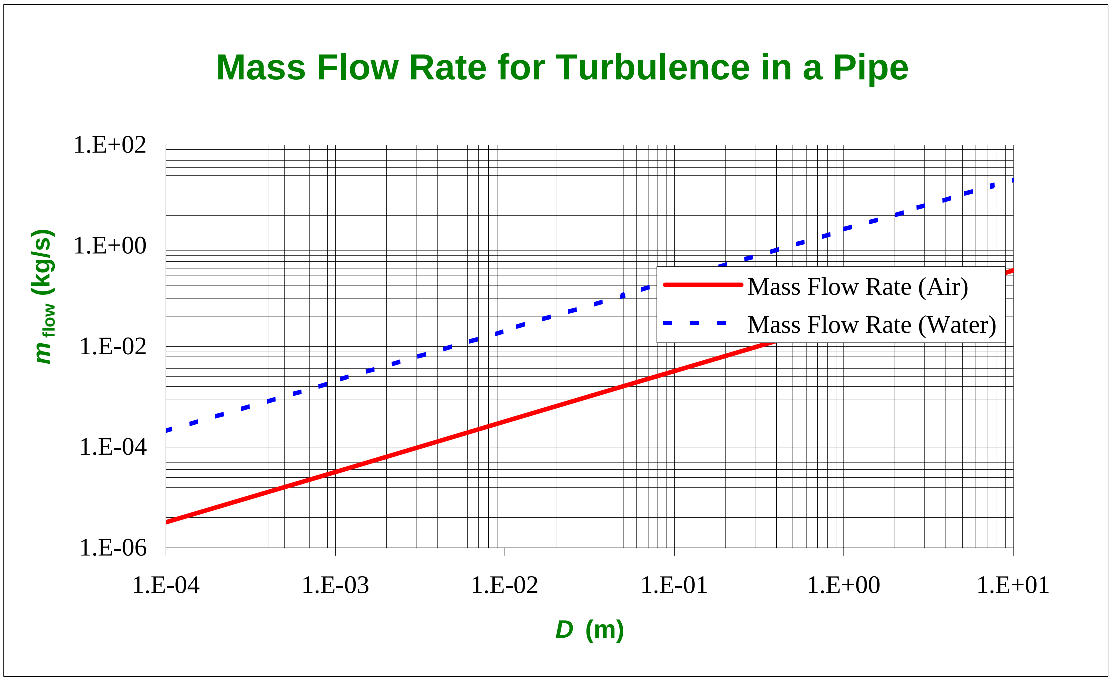
$\rho_{air} =$ 1.23 kg/m^3 $\qquad$ $\nu_{air} =$ 1.45E-05 m^2/s

$\rho_w =$ 999 kg/m^3 $\qquad$ $\nu_w =$ 1.14E-06 m^2/s

D **(m)**	0.0001	0.001	0.01	0.05	1.0	2.5	5.0	7.5	10.0
V $_{air}$ **(m/s)**	333.500	33.350	3.335	0.667	3.34E-02	1.33E-02	6.67E-03	4.45E-03	3.34E-03
V $_w$ **(m/s)**	26.2	2.62	0.262	5.24E-02	2.62E-03	1.05E-03	5.24E-04	3.50E-04	2.62E-04
Q $_{air}$ **(m^3/s)**	2.62E-06	2.62E-05	2.62E-04	1.31E-03	2.62E-02	6.55E-02	1.31E-01	1.96E-01	2.62E-01
Q $_w$ **(m^3/s)**	2.06E-07	2.06E-06	2.06E-05	1.03E-04	2.06E-03	5.15E-03	1.03E-02	1.54E-02	2.06E-02
m $_{air}$ **(kg/s)**	3.22E-06	3.22E-05	3.22E-04	1.61E-03	3.22E-02	8.05E-02	1.61E-01	2.42E-01	3.22E-01
m $_w$ **(kg/s)**	2.06E-04	2.06E-03	2.06E-02	1.03E-01	2.06E+00	5.14E+00	1.03E+01	1.54E+01	2.06E+01




Flow Rate for Turbulence in a Pipe
Q (m3/s)
1.E+01
1.E-01
1.E-03
1.E-05
1.E-07
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
1.E+01
D (m)
Flow Rate (Air)
Flow Rate (Water)


Mass Flow Rate for Turbulence in a Pipe
m flow (kg/s)
1.E+02
1.E+00
1.E-02
1.E-04
1.E-06
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
1.E+01
D (m)
Mass Flow Rate (Air)
Mass Flow Rate (Water)

Problem 8.5

Given: Laminar flow in the entrance section of a pipe shown schematically in Fig. 8.1.

Find: Sketch centerline velocity, static pressure, and wall shear stress as functions of distance along the pipe. Explain significant features of the plots, comparing them with fully developed flow. Can the Bernoulli equation be applied anywhere in the flow field? If so, where? Explain briefly.

Discussion: The centerline velocity, static pressure, and wall shear stress variations are sketched on the next page. Each variation sketch is aligned vertically with the corresponding sections of the developing pipe flow in Fig. 8.1.

Boundary layers grow on the tube wall, reducing the velocity near the wall. The velocity reduction becomes more pronounced farther downstream. Consequently the centerline velocity must increase in the streamwise direction to carry the same mass flow rate across each section of the tube. (When laminar flow becomes fully developed, the centerline velocity becomes twice the average velocity at any cross-section.)

Frictional effects are concentrated within the boundary layers. The boundary layers do not join at the tube centerline for some distance along the tube. Therefore in the center region outside the boundary layers flow may still be considered to behave as though it were inviscid.

Flow outside the boundary layers is steady, frictionless, incompressible, and along a streamline. These are the restrictions required to apply the Bernoulli equation. Therefore the Bernoulli equation may be applied as a reasonable model for the actual flow outside the boundary layers. The Bernoulli equation predicts that pressure decreases as flow speed increases.

After the boundary layers merge at the centerline of the channel the entire flow is affected by friction. Therefore it is no longer possible to apply the Bernoulli equation.

When flow becomes fully developed the rate of change of pressure with distance becomes constant. In the entrance region the pressure falls more rapidly; the increased pressure gradient is caused by increased shear stress at the wall (larger than for fully developed flow) and by the developing velocity profile, which causes momentum flux to increase.

In fully developed flow the pressure curve becomes linear; the pressure drops the same amount for each length along the tube. The pressure distribution curve at the end of the entrance length becomes asymptotic to the linear variation for fully developed flow.

The wall shear stress initially is large, because the boundary layers are thin. The shear stress decreases as the boundary layers become thicker. At the end of the entrance length the shear stress asymptotically approaches the constant value for fully developed flow.

Problem 8.5

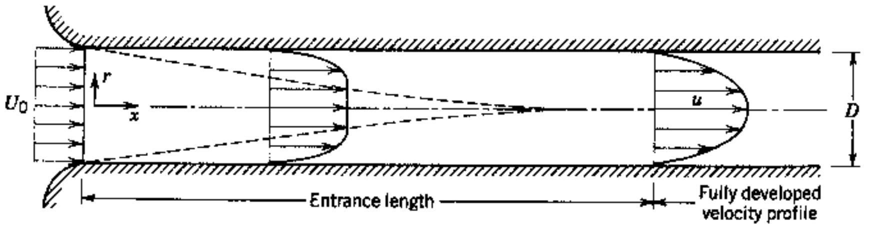
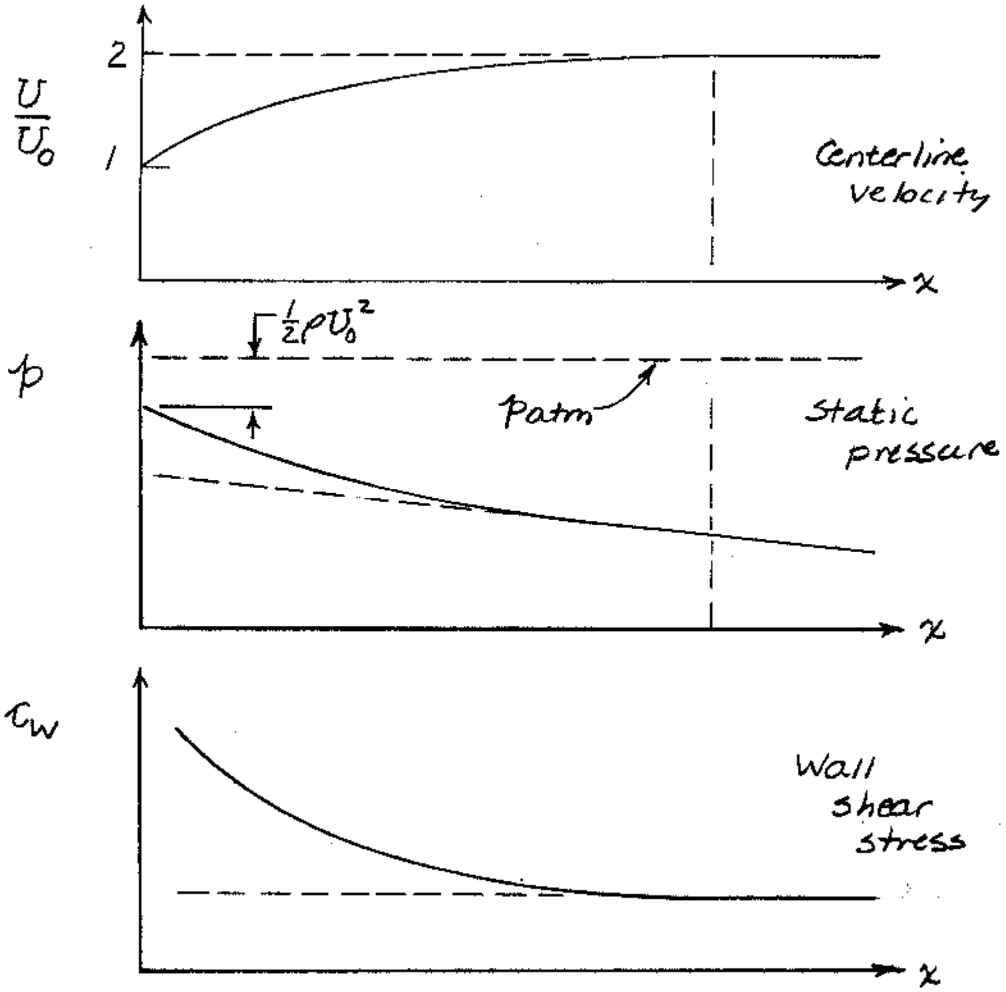


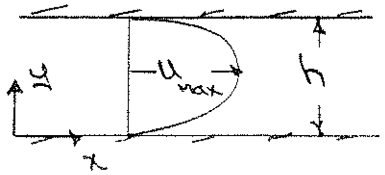
Fig. 8.1 Flow in the entrance region of a pipe.



Problem 8.6

Given: Incompressible flow between parallel plates with

$$u = u_{max}(Ay^2 + By + C)$$



Find: (a) constants A, B, C using appropriate boundary conditions

(b) Q per unit depth b.

(c) $\bar{V}/u_{max}$

Solution:

(a) Available boundary conditions: (1) $y=0,\ u=0$

(2) $y=h,\ u=0$

(3) $y=h/2,\ u=u_{max}$

From B.C (1) $u(0) = 0 = u_{max}C \quad \therefore \quad C = 0 \leftarrow$ C

From B.C (2) $u(h) = 0 = u_{max}(Ah^2 + Bh) \quad \text{---(i)}$

From B.C (3) $u(h/2) = u_{max} = u_{max}\left(A\frac{h^2}{4} + B\frac{h}{2}\right) \quad \text{---(ii)}$

From Eq (i), $B = -Ah$. Substituting into Eq (ii) gives

$$\cancel{u_{max}} = \cancel{u_{max}}\left(A\frac{h^2}{4} - A\frac{h^2}{2}\right) \quad \therefore \quad A = -\frac{4}{h^2} \leftarrow$$ A

and $B = -Ah = \frac{4}{h} \leftarrow$ B

Then

$$u = u_{max}(Ay^2 + By + C) = u_{max}\left(-4\frac{y^2}{h^2} + 4\frac{y}{h}\right) = 4u_{max}\left[\frac{y}{h} - \left(\frac{y}{h}\right)^2\right]$$

(b) $$Q = \int_0^h u\,b\,dy = \int_0^h 4u_{max}\left[\frac{y}{h} - \frac{y^2}{h^2}\right]b\,dy = 4u_{max}b\left[\frac{y^2}{2h} - \frac{y^3}{3h^2}\right]_0^h$$

$$Q = 4b\,u_{max}\left[\frac{h}{2} - \frac{h}{3}\right] = \frac{2}{3}u_{max}bh$$

$$Q/b = \frac{2}{3}u_{max}h \leftarrow$$ Q/b

(c) Since $Q = \bar{V}A = \bar{V}bh$

$$\frac{Q}{b} = \bar{V}h = \frac{2}{3}u_{max}h$$

and

$$\frac{\bar{V}}{u_{max}} = \frac{2}{3} \leftarrow$$ $\frac{\bar{V}}{u_{max}}$

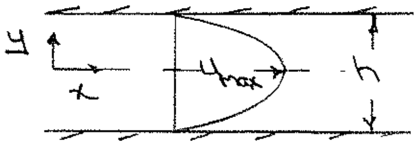
Problem 8.7

[2]

Given: Velocity profile for flow between stationary parallel plates.

$$u = a(h^2/4 - y^2)$$

where a = constant



Find: Ratio $\bar{V}/u_{max}$

Solution: First find u_{max}, by setting $\frac{du}{dy} = 0$

$$\frac{du}{dy} = -2ay \quad ; \quad \frac{du}{dy} = 0 \text{ at } y = 0$$

$$u_{max} = u(0) = a\frac{h^2}{4}$$

From the definition of $\bar{V}$,

$$\bar{V} = \frac{Q}{A} = \frac{1}{A}\int u\,dA = \frac{1}{h}\int_{-h/2}^{h/2} u\,dy$$

$$= \frac{1}{h}\int_{-h/2}^{h/2} a\left(\frac{h^2}{4} - y^2\right) = \frac{a}{h}\left[\frac{h^2 y}{4} - \frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$\bar{V} = \frac{a}{h}\left[\left(\frac{h^3}{8} - \frac{h^3}{24}\right) - \left(-\frac{h^3}{8} + \frac{h^3}{24}\right)\right] = \frac{a}{h}\left[\frac{h^3}{4} - \frac{h^3}{12}\right]$$

$$\bar{V} = \frac{1}{6}ah^2$$

and

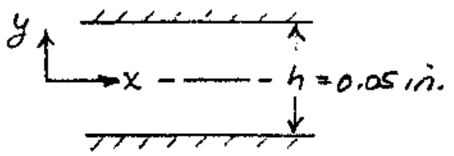
$$\frac{\bar{V}}{u_{max}} = \frac{ah^2}{6}\,\frac{4}{ah^2} = \frac{2}{3} \longleftarrow \frac{\bar{V}}{u_{max}}$$

Problem 8.8

[3]

Given: Fully developed laminar flow between parallel plates.

$\mu = 2.40 \times 10^{-5} \frac{lbf \cdot s}{ft^2}$; $\frac{\partial p}{\partial x} = -4 \frac{lbf}{ft^3}$



Find: (a) Derive and plot equation for shear stress versus y.
(b) Maximum shear stress.

Solution: From Eq. 8.7, with a = h, $u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{2y}{h}\right)^2\right]$

By symmetry, the origin for y must be located at the channel centerline. Apply Newton's law of viscosity.

$$\tau_{yx} = \mu \frac{du}{dy}$$

Assumption: Newtonian fluid

Then

$$\tau_{yx} = \mu \frac{d}{dy}\left\{-\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{2y}{h}\right)^2\right]\right\} = y\frac{\partial p}{\partial x}$$

τ_{yx}

For u > 0, $\partial p/\partial x < 0$. Thus $\tau_{yx} < 0$ for $y > 0$ and $\tau_{yx} > 0$ for $y < 0$.

On the upper plate (a minus y surface), $\tau_{yx} < 0$, so shear stress acts to the right.

On the lower plate (a plus y surface), $\tau_{yx} > 0$, so shear stress acts to the right.

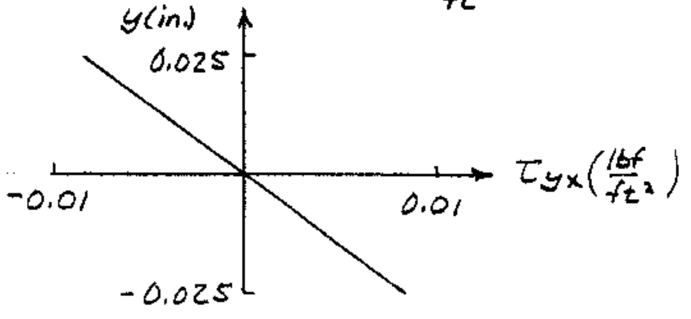
The maximum stress occurs when $y = \pm h/2$. Thus

$$\tau_{max} = \tau_{yx}\left(\frac{h}{2}\right) = \frac{h}{2}\frac{\partial p}{\partial x} = \frac{1}{2} \times 0.05\,in. \times \frac{ft.}{12\,in.} \times \left(-4.0\frac{lbf}{ft^3}\right) = -0.00835\frac{lbf}{ft^2}$$

τ_{ma}

or $\tau_{max} = \tau_{yx}\left(-\frac{h}{2}\right) = 0.00835\frac{lbf}{ft^2}$

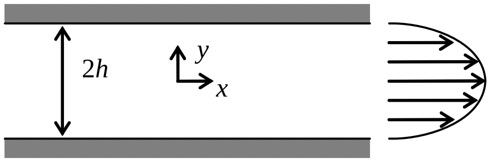
Plot:



Problem 8.9

[2]

8.9 Viscous oil flows steadily between parallel plates. The flow is fully developed and laminar. The pressure gradient is 1.25 kPa/m and the channel half-width is $h = 1.5$ mm. Calculate the magnitude and direction of the wall shear stress at the upper plate surface. Find the volume flow rate through the channel ($\mu = 0.50\ \mathrm{N \cdot s/m^2}$).



Given: Laminar flow between flat plates

Find: Shear stress on upper plate; Volume flow rate per width

Solution:

Basic equation $\tau_{yx} = \mu \cdot \frac{du}{dy}$ $\quad u(y) = -\frac{h^2}{2 \cdot \mu} \cdot \frac{dp}{dx} \cdot \left[1 - \left(\frac{y}{h}\right)^2\right]$ (from Eq. 8.7)

Then $\tau_{yx} = \frac{-h^2}{2} \cdot \frac{dp}{dx} \cdot \left(-\frac{2 \cdot y}{h^2}\right) = -y \cdot \frac{dp}{dx}$

At the upper surface $y = h$ $\quad \tau_{yx} = -1.5 \cdot \mathrm{mm} \times \frac{1 \cdot \mathrm{m}}{1000 \cdot \mathrm{mm}} \times 1.25 \times 10^3 \cdot \frac{\mathrm{N}}{\mathrm{m}^2 \cdot \mathrm{m}}$ $\quad \tau_{yx} = -1.88\,\mathrm{Pa}$

The volume flow rate is $Q = \int u\,dA = \int_{-h}^{h} u \cdot b\,dy = -\frac{h^2 \cdot b}{2 \cdot \mu} \cdot \frac{dp}{dx} \cdot \int_{-h}^{h} \left[1 - \left(\frac{y}{h}\right)^2\right] dy$ $\quad Q = -\frac{2 \cdot h^3 \cdot b}{3 \cdot \mu} \cdot \frac{dp}{dx}$

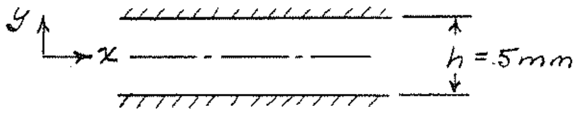
$$\frac{Q}{b} = -\frac{2}{3} \times \left(1.5 \cdot \mathrm{mm} \times \frac{1 \cdot \mathrm{m}}{1000 \cdot \mathrm{mm}}\right)^3 \times 1.25 \times 10^3 \cdot \frac{\mathrm{N}}{\mathrm{m}^2 \cdot \mathrm{m}} \times \frac{\mathrm{m}^2}{0.5 \cdot \mathrm{N} \cdot \mathrm{s}} \qquad \frac{Q}{b} = -5.63 \times 10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

Problem 8.10

[2]

Given: Laminar, fully developed flow between parallel plates

$$\mu = 0.5\,\frac{N\cdot s}{m^2}\ ;\ \frac{\partial p}{\partial x} = -1000\,\frac{N}{m^3}$$



Find: (a) Shear stress on upper plate.
(b) Volume flow rate per unit width.

width = b

Solution: From Eq. 8.7 with $a = h$,

$$u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{2y}{h}\right)^2\right]$$

Then

$$\tau_{yx} = \mu\frac{du}{dy} = -\frac{h^2}{8}\frac{\partial p}{\partial x}\left(-\frac{8y}{h^2}\right) = y\frac{\partial p}{\partial x}$$

At upper surface, $y = h/2$, and

$$\tau_{yx} = \frac{0.005\,m}{2}\times -1000\,\frac{N}{m^3} = -2.5\,N/m^2$$

The upper plate is a negative y surface. Thus since $\tau_{yx} < 0$, stress acts to right, in $+x$ direction. ← τ_{yx}

The volume flow rate is

$$Q = \int_A u\,dA = \int_{-h/2}^{h/2} u\,b\,dy = 2\int_0^{h/2} u\,b\,dy = 2\left(\frac{h}{2}\right)b\int_0^1 u\,d\left(\frac{2y}{h}\right)$$

or

$$\frac{Q}{b} = h\int_0^1 u\,d\eta \quad \text{where } \eta = \frac{2y}{h} \quad \text{and} \quad u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}(1-\eta^2)$$

Thus

$$\frac{Q}{b} = h\int_0^1 -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}(1-\eta^2)\,d\eta = -\frac{h^3}{8\mu}\frac{\partial p}{\partial x}\left(\eta - \frac{1}{3}\eta^3\right)\Big|_0^1 = -\frac{h^3}{12\mu}\frac{\partial p}{\partial x}$$

$$\frac{Q}{b} = -\frac{1}{12}\times(0.005)^3\,m^3\times\frac{m^2}{0.5\,N\cdot s}\times -1000\,\frac{N}{m^3} = 20.8\times10^{-6}\,m^2/s$$ ← Q/b

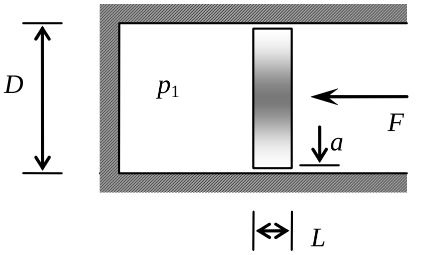
Note $u > 0$, so flow is from left to right.

NATIONAL 42 381 50 SHEETS 5 SQUARE 42 382 100 SHEETS 5 SQUARE 42 389 200 SHEETS 5 SQUARE

Problem 8.11

[3]

8.11 Oil is confined in a 4-in. diameter cylinder by a piston having a radial clearance of 0.001 in. and a length of 2 in. A steady force of 4500 lbf is applied to the piston. Assume the properties of SAE 30 oil at 120°F. Estimate the rate at which oil leaks past the piston.



Given:

Piston cylinder assembly

Find:

Rate of oil leak

Solution:

Basic equation $\frac{Q}{l} = \frac{a^3 \cdot \Delta p}{12 \cdot \mu \cdot L}$ $\qquad Q = \frac{\pi \cdot D \cdot a^3 \cdot \Delta p}{12 \cdot \mu \cdot L}$ (from Eq. 8.6c; we assume laminar flow and verify this is correct after solving)

For the system $\Delta p = p_1 - p_{atm} = \frac{F}{A} = \frac{4 \cdot F}{\pi \cdot D^2}$

$$\Delta p = \frac{4}{\pi} \times 4500 \cdot lbf \times \left(\frac{1}{4 \cdot in} \times \frac{12 \cdot in}{1 \cdot ft}\right)^2 \qquad \Delta p = 358 \cdot psi$$

At 120ºF (about 50ºC), from Fig. A.2 $\quad \mu = 0.06 \times 0.0209 \cdot \frac{lbf \cdot s}{ft^2} \qquad \mu = 1.25 \times 10^{-3} \cdot \frac{lbf \cdot s}{ft^2}$

$$Q = \frac{\pi}{12} \times 4 \cdot in \times \left(0.001 \cdot in \times \frac{1 \cdot ft}{12 \cdot in}\right)^3 \times 358 \cdot \frac{lbf}{in^2} \times \frac{144 \cdot in^2}{1 \cdot ft^2} \times \frac{ft^2}{1.25 \times 10^{-3} lbf \cdot s} \times \frac{1}{2 \cdot in} \qquad Q = 1.25 \times 10^{-5} \cdot \frac{ft^3}{s} \qquad Q = 0.0216 \cdot \frac{in^3}{s}$$

Check Re: $\quad V = \frac{Q}{A} = \frac{Q}{a \cdot \pi \cdot D} \qquad V = \frac{1}{\pi} \times 1.25 \times 10^{-5} \frac{ft^3}{s} \times \frac{1}{.001 \cdot in} \times \frac{1}{4 \cdot in} \times \left(\frac{12 \cdot in}{1 \cdot ft}\right)^2 \qquad V = 0.143 \cdot \frac{ft}{s}$

$$Re = \frac{V \cdot a}{\nu} \qquad \nu = 6 \times 10^{-5} \times 10.8 \frac{ft^2}{s} \qquad \nu = 6.48 \times 10^{-4} \cdot \frac{ft^2}{s}$$ (at 120ºF, from Fig. A.3)

$$Re = 0.143 \cdot \frac{ft}{s} \times 0.001 \cdot in \times \frac{1 \cdot ft}{12 \cdot in} \times \frac{s}{6.48 \times 10^{-4}\ ft^2} \qquad Re = 0.0184$$ so flow is very much laminar

The speed of the piston is approximately

$$V_p = \frac{Q}{\left(\frac{\pi \cdot D^2}{4}\right)} \qquad V_p = \frac{4}{\pi} \times 1.25 \times 10^{-5} \frac{ft^3}{s} \times \left(\frac{1}{4 \cdot in} \times \frac{12 \cdot in}{1 \cdot ft}\right)^2 \qquad V_p = 1.432 \times 10^{-4} \cdot \frac{ft}{s}$$

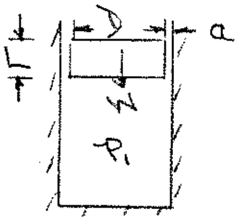
The piston motion is negligible so our assumption of flow between parallel plates is reasonable

Problem 8.12

Given: Hydraulic jack supports supports a load of 9000 kg
piston diameter $D = 100$ mm
radial clearance $a = 0.05$ mm
piston length $L = 120$ mm
Fluid has viscosity of SAE 30 oil at 30°C

Find: Leakage rate of fluid past the piston

Solution:



Model the flow as steady, fully developed laminar flow between stationary parallel plates, i.e., neglect motion of the piston.

Then, the leakage flow rate can be evaluated from Eq. 8.6c (in the text)

$$\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L} \quad \text{where } l = \pi D$$

From Fig. A.2 at T = 30°C, $\mu = 3.0 \times 10^{-1}$ N.s/m²

$$\Delta p = p_1 - p_{atm} \quad \text{and} \quad p_1 = \frac{W}{A} = \frac{Mg}{A} = \frac{4Mg}{\pi D^2}$$

$$p_1 = \frac{4}{\pi} \times 9000\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times \frac{1}{(0.1\,\text{m})^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 11.2\,\text{MPa}$$

$$Q = \frac{\pi D a^3 \Delta p}{12 \mu L} = \frac{\pi}{12} \times (0.1\,\text{m}) \times (5 \times 10^{-5}\,\text{m})^3 \times 11.2 \times 10^6\,\frac{\text{N}}{\text{m}^2} \times \frac{1}{0.3}\,\frac{\text{m}^2}{\text{N}\cdot\text{s}} \times \frac{1}{0.12\,\text{m}}$$

$$Q = 1.01 \times 10^{-6}\,\text{m}^3/\text{s} = 1.01 \times 10^{-3}\,\text{L/s} \longleftarrow Q$$

Check $Re = \frac{\rho a \bar{V}}{\mu} = \frac{a \bar{V}}{\nu}$ where $\nu = 2.8 \times 10^{-4}$ m²/s (Fig. A.3)

$$\bar{V} = \frac{Q}{A} = \frac{Q}{al} = \frac{Q}{a \pi D} = \frac{1}{\pi} \times 1.01 \times 10^{-6}\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{5 \times 10^{-5}\,\text{m}} \times \frac{1}{0.1\,\text{m}} = 0.0643\,\text{m/s}$$

$$Re = \frac{a \bar{V}}{\nu} = 5 \times 10^{-5}\,\text{m} \times 0.0643\,\frac{\text{m}}{\text{s}} \times \frac{1}{2.8 \times 10^{-4}}\,\frac{\text{s}}{\text{m}^2} = 0.011$$

∴ flow is definitely laminar

Piston moving down at speed v displaces liquid at rate Q where

$$Q = \frac{\pi D^2}{4} v$$

Then

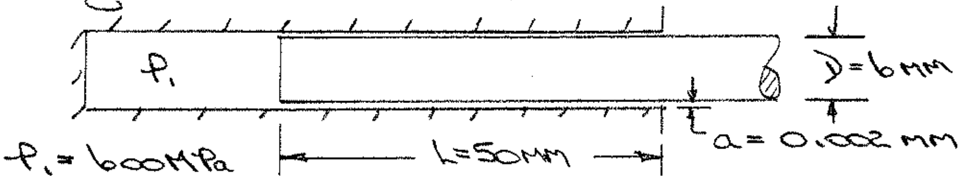
$$v = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 1.01 \times 10^{-6}\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{(0.1\,\text{m})^2} = 1.29 \times 10^{-4}\,\text{m/s}$$

Since $\frac{v}{\bar{V}} = \frac{1.29 \times 10^{-4}\,\text{m/s}}{0.0643\,\text{m/s}} = 2.0 \times 10^{-3}$, motion of piston can be neglected.

Problem 8.13

[3]

Given: Piston-cylinder device with SAE 10W oil at 35°



Find: Leakage flow rate

Solution: Computing equation: $\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L}$ (8.6c)

Assumptions: (1) Laminar flow
(2) Fully developed flow (L >> a)

For SAE 10W oil at 35°C, $\mu = 3.8 \times 10^{-2}\ N \cdot s/m^2$ (Fig. A.2)

For this configuration, $l = \pi D$, since $a << D$. Then

$$Q = \frac{a^3 \Delta p\, l}{12 \mu L} = \frac{\pi a^3 \Delta p D}{12 \mu L}$$

$$Q = \frac{\pi}{12} \times (2 \times 10^{-6}\ m)^3 \times 6 \times 10^8\ \frac{N}{m^2} \times 0.006\ m \times 3.8 \times 10^{-2}\ \frac{m^2}{N \cdot s} \times \frac{1}{0.05\ m}$$

$Q = 3.97 \times 10^{-9}\ m^3/s = 3.97 \times 10^{-6}\ L/s$ ← Q

Check Re to assure laminar flow

$$\bar{V} = \frac{Q}{A} = \frac{Q}{\pi D a} = \frac{1}{\pi} \times 3.97 \times 10^{-9}\ \frac{m^3}{s} \times \frac{1}{0.006\ m} \times \frac{1}{2 \times 10^{-6}\ m} = 0.105\ m/s$$

SG = 0.88 (Table A.2) ; $\rho = SG\, \rho_{H_2O}$

$$Re = \frac{\rho \bar{V} a}{\mu} = \frac{SG\, \rho_{H_2O} \bar{V} a}{\mu}$$

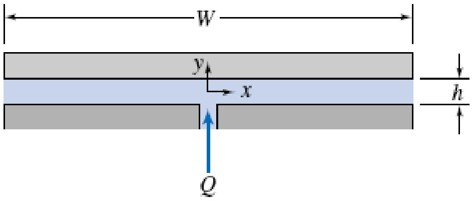
$$= 0.88 \times 999\ \frac{kg}{m^3} \times 0.105\ \frac{m}{s} \times 2 \times 10^{-6}\ m \times 3.8 \times 10^{-2}\ \frac{m^2}{N \cdot s}$$

Re = 0.005 << 2300 so flow is definitely laminar!

Problem 8.14

[3]

8.14 A hydrostatic bearing is to support a load of 50,000 N per meter of length perpendicular to the diagram. The bearing is supplied with SAE 30 oil at 35°C and 700 kPa (gage) through the central slit. Since the oil is viscous and the gap is small, the flow may be considered fully developed. Calculate (a) the required width of the bearing pad, (b) the resulting pressure gradient, *dp/dx*, and (c) the gap height, if $Q = 1$ mL/min per meter of length.



Given: Hydrostatic bearing

Find: Required pad width; Pressure gradient; Gap height

Solution:

For a laminar flow (we will verify this assumption later), the pressure gradient is consta $p(x) = p_i \cdot \left(1 - \frac{2 \cdot x}{W}\right)$

where p_i = 700 kPa is the inlet pressure (gage)

Hence the total force in the y direction due to pressure is $F = b \cdot \int p\, dx$ where b is the pad width into the paper

$$F = b \cdot \int_{-\frac{W}{2}}^{\frac{W}{2}} p_i \cdot \left(1 - \frac{2 \cdot x}{W}\right) dx \qquad F = p_i \cdot \frac{b \cdot W}{2}$$

This must be equal to the applied load F. Hence $W = \frac{2}{p_i} \cdot \frac{F}{b}$ $\quad W = 2 \times \frac{m^2}{700 \times 10^3 \cdot N} \times \frac{50000 \cdot N}{m}$ $\quad W = 0.143\,m$

The pressure gradient is then $\frac{dp}{dx} = -\frac{\Delta p}{\frac{W}{2}} = -\frac{2 \cdot \Delta p}{W} = -2 \times \frac{700 \times 10^3 \cdot N}{m^2} \times \frac{1}{0.143 \cdot m} = -9.79 \cdot \frac{MPa}{m}$

The flow rate is given $\frac{Q}{l} = -\frac{h^3}{12 \cdot \mu} \cdot \left(\frac{dp}{dx}\right)$ (Eq. 8.6c)

Hence, for h we have $h = \left(-\frac{12 \cdot \mu \cdot \frac{Q}{l}}{\frac{dp}{dx}}\right)^{\frac{1}{3}}$ At 35ºC, from Fig. A.2 $\mu = 0.15 \cdot \frac{N \cdot s}{m^2}$

$$h = \left[-12 \times \left(-\frac{m^3}{9.79 \times 10^6 \cdot N}\right) \times 0.15 \cdot \frac{N \cdot s}{m^2} \times \frac{1 \cdot mL}{min \cdot m} \times \frac{10^{-6} \cdot m^3}{1 \cdot mL} \times \frac{1 \cdot min}{60 \cdot s}\right]^{\frac{1}{3}} \qquad h = 1.452 \times 10^{-5}\,m$$

Check Re: $Re = \frac{V \cdot D}{\nu} = \frac{D}{\nu} \cdot \frac{Q}{A} = \frac{h}{\nu} \cdot \frac{Q}{b \cdot h} = \frac{1}{\nu} \cdot \frac{Q}{l}$ $\quad \nu = 1.6 \times 10^{-4} \frac{m^2}{s}$ (at 35ºC, from Fig. A.3)

$Re = \frac{s}{1.6 \times 10^{-4} \cdot m^2} \times \frac{1 \cdot mL}{min \cdot m} \times \frac{10^{-6} \cdot m^3}{1 \cdot mL} \times \frac{1 \cdot min}{60 \cdot s}$ $\quad Re = 1.04 \times 10^{-4}$ so flow is very much laminar

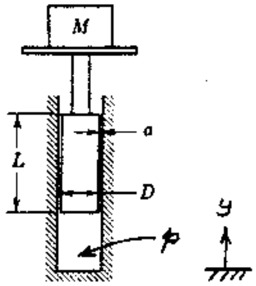
Problem 8.15

[4]

Given: Piston-cylinder device, as shown.

$D = 6\ mm \quad L = 25\ mm$

Liquid is SAE 30 oil at 20°C.



Find: (a) M to develop $p = 1.5$ MPa (gage)

(b) Leakage flow rate in terms of a

(c) Maximum a to provide < 1 mm/min movement.

Solution: The mass may be found from a force balance on the piston.

$$\Sigma F_y = \frac{\pi D^2}{4}(p - p_{atm}) - Mg = 0 \qquad \text{so} \quad M = \frac{\pi D^2}{4g} p_{gage}$$

$$M = \frac{\pi}{4} \times (0.006)^2 m^2 \times 1.5 \times 10^6 \frac{N}{m^2} \times \frac{s^2}{9.81\ m} \times \frac{kg \cdot m}{N \cdot s^2} = 4.32\ kg$$

M

The leakage flow rate may be evaluated for flow between flat plates. From Eq. 8.6c, neglecting motion of the piston,

$$\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L} \qquad \text{or, since } l = \pi D, \quad Q = \frac{\pi}{12} \frac{a^3 \Delta p D}{\mu L} \sim a^3$$

Q

The piston, moving downward at speed, v, displaces liquid at rate

$$Q = \frac{\pi D^2}{4} v = \frac{\pi}{4} (0.006)^2 m^2 \times 0.001 \frac{m}{min} \times \frac{min}{60\ s} = 4.71 \times 10^{-10}\ m^3/s$$

Then, with $\mu = 0.42\ N \cdot sec/m^2$ (at 20°C, Fig. A.2),

$$a = \left[\frac{12 \mu Q L}{\pi D \Delta p}\right]^{1/3} = \left[\frac{12}{\pi} \times 0.42 \frac{N \cdot s}{m^2} \times 4.71 \times 10^{-10} \frac{m^3}{s} \times 0.025\ m \times \frac{1}{0.006\ m} \times \frac{m^2}{1.5 \times 10^6\ N}\right]^{1/3}$$

$$a = 1.28 \times 10^{-5}\ m \quad (12.8\ \mu m)$$

a

Check assumptions: $\bar{V} = \frac{Q}{A} = \frac{Q}{\pi D a} = \frac{1}{\pi} \times 4.71 \times 10^{-10} \frac{m^3}{s} \times \frac{1}{0.006\ m} \times \frac{1}{1.28 \times 10^{-5}\ m} = 1.95 \frac{mm}{s}$

Thus $\frac{v}{\bar{V}} = 1 \frac{mm}{min} \times \frac{sec}{1.95\ mm} \times \frac{min}{60\ s} = 0.00855 < 0.01$

Therefore piston motion is negligible.

Also $Re = \frac{\bar{V} a}{\nu}$; $\nu = \frac{\mu}{\rho} = \frac{\mu}{SG\, \rho_{H_2O}}$. From Table A.2 (Appendix A), SG = 0.92

$$\nu = 0.42 \frac{N \cdot s}{m^2} \times \frac{m^3}{(0.92)1000\ kg} \times \frac{kg \cdot m}{N \cdot s^2} = 4.57 \times 10^{-4}\ m^2/s$$

$$Re = 1.95 \times 10^{-3} \frac{m}{s} \times 1.28 \times 10^{-5} m \times \frac{s}{4.57 \times 10^{-4}\ m^2} = 5.46 \times 10^{-5} \lll 1$$

Therefore flow is surely laminar!

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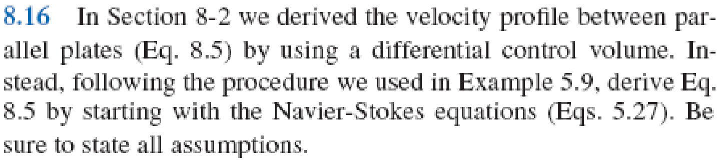
Problem 8.16

[2]



8.16 In Section 8-2 we derived the velocity profile between parallel plates (Eq. 8.5) by using a differential control volume. Instead, following the procedure we used in Example 5.9, derive Eq. 8.5 by starting with the Navier-Stokes equations (Eqs. 5.27). Be sure to state all assumptions.

Given: Navier-Stokes Equations

Find: Derivation of Eq. 8.5

Solution:

The Navier-Stokes equations are

$$\cancelto{4}{\frac{\partial u}{\partial x}}+\frac{\partial v}{\partial y}+\cancelto{3}{\frac{\partial w}{\partial z}}=0 \qquad (5.1c)$$

$$\rho\left(\cancelto{1}{\frac{\partial u}{\partial t}}+u\cancelto{4}{\frac{\partial u}{\partial x}}+v\cancelto{5}{\frac{\partial u}{\partial y}}+w\cancelto{3}{\frac{\partial u}{\partial z}}\right)=\cancelto{6}{\rho g_x}-\frac{\partial p}{\partial x}+\mu\left(\cancelto{4}{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}+\cancelto{3}{\frac{\partial^2 u}{\partial z^2}}\right) \qquad (5.27a)$$

$$\rho\left(\cancelto{1}{\frac{\partial v}{\partial t}}+u\cancelto{4}{\frac{\partial v}{\partial x}}+v\cancelto{5}{\frac{\partial v}{\partial y}}+w\cancelto{3}{\frac{\partial v}{\partial z}}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\cancelto{4}{\frac{\partial^2 v}{\partial x^2}}+\cancelto{5}{\frac{\partial^2 v}{\partial y^2}}+\cancelto{3}{\frac{\partial^2 v}{\partial z^2}}\right) \qquad (5.27b)$$

$$\rho\left(\cancelto{1}{\frac{\partial w}{\partial t}}+u\cancelto{3}{\frac{\partial w}{\partial x}}+v\cancelto{3}{\frac{\partial w}{\partial y}}+w\cancelto{3}{\frac{\partial w}{\partial z}}\right)=\cancelto{3}{\rho g_z}-\cancelto{3}{\frac{\partial p}{\partial z}}+\mu\left(\cancelto{3}{\frac{\partial^2 w}{\partial x^2}}+\cancelto{3}{\frac{\partial^2 w}{\partial y^2}}+\cancelto{3}{\frac{\partial^2 w}{\partial z^2}}\right) \qquad (5.27c)$$

The following assumptions have been applied:

(1) Steady flow (given).
(2) Incompressible flow; ρ = constant.
(3) No flow or variation of properties in the z direction; $w = 0$ and $\partial/\partial z = 0$.
(4) Fully developed flow, so no properties except pressure p vary in the x direction; $\partial/\partial x = 0$.
(5) See analysis below.
(6) No body force in the x direction; $g_x = 0$

Assumption (1) eliminates time variations in any fluid property. Assumption (2) eliminates space variations in density. Assumption (3) states that there is no z component of velocity and no property variations in the z direction. All terms in the z component of the Navier–Stokes equation cancel. After assumption (4) is applied, the continuity equation reduces to $\partial v/\partial y = 0$. Assumptions (3) and (4) also indicate that $\partial v/\partial z = 0$ and $\partial v/\partial x = 0$. Therefore v must be constant. Since v is zero at the solid surface, then v must be zero everywhere. The fact that $v = 0$ reduces the Navier–Stokes equations further, as indicated by (5). Hence for the y direction

$$\frac{\partial p}{\partial y}=\rho g$$

which indicates a hydrostatic variation of pressure. In the x direction, after assumption (6) we obtain

$$\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial x}=0$$

Integrating twice

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + \frac{c_1}{\mu}y + c_2$$

To evaluate the constants, c_1 and c_2, we must apply the boundary conditions. At $y = 0$, $u = 0$. Consequently, $c_2 = 0$. At $y = a$, $u = 0$. Hence

$$0 = \frac{1}{2\mu}\frac{\partial p}{\partial x}a^2 + \frac{c_1}{\mu}a$$

which gives

$$c_1 = -\frac{1}{2\mu}\frac{\partial p}{\partial x}a$$

and finally

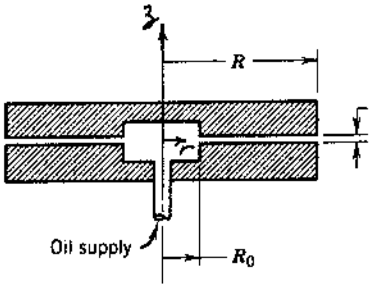
$$u = \frac{a^2}{2\mu}\frac{\partial p}{\partial x}\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

Problem 8.17

[5]

Given: Viscous flow in narrow gap between parallel disks, as shown.

Flow rate is Q, accelerations are small.
Velocity profile same as fully developed.



Find: (a) Expression for $\bar{V}(r)$, (b) dp/dr in gap
(c) Expression for $p(r)$.
(d) Show net force to hold upper plate is

$$F = \frac{3\mu Q R^2}{h^3}\left[1-\left(\frac{R_0}{R}\right)^2\right]$$

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Solution: From the definition of mean velocity, $Q = \bar{V}2\pi r h$ so $\bar{V} = \dfrac{Q}{2\pi r h}$ $\bar{V}(r)$

The pressure change with radius can be evaluated by analogy to Eq. 8.6b

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)h^3 \quad \text{with } l = 2\pi r \quad \text{so } \frac{Q}{2\pi r} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial r}\right)h^3$$

Thus

$$\frac{dp}{dr} = -\frac{6\mu Q}{\pi h^3 r}$$

$\dfrac{dp}{dr}$

Integrating to find $p(r)$,

$$\int_p^{p_{atm}} dp = p_{atm} - p = \int_r^R -\frac{6\mu Q}{\pi h^3 r}\,dr = -\frac{6\mu Q}{\pi h^3}\ln r\Big]_r^R = \frac{6\mu Q}{\pi h^3}\ln(r/R)$$

Thus $p(r) = p_{atm} - \dfrac{6\mu Q}{\pi h^3}\ln(r/R)$ $(R_0 < r < R)$; $p = p_0$ $r < R_0$ $p(r)$

The force on the upper plate is $dF_z = (p(r) - p_{atm})2\pi r\,dr$

Integrating and using gage pressures (note $p_{0g} = -\dfrac{6\mu Q}{\pi h^3}\ln\left(\dfrac{R_0}{R}\right)$)

$$F_z = p_0\pi R_0^2 + \int_{R_0}^R p(r)\,2\pi r\,dr = p_0\pi R_0^2 + 2\pi R^2\int_{R_0/R}^1 p(r)\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)$$

$$= p_0\pi R_0^2 + 2\pi R^2\int_{R_0/R}^1 -\frac{6\mu Q}{\pi h^3}\ln\left(\frac{r}{R}\right)\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) = p_0\pi R_0^2 - \frac{12\mu Q R^2}{h^3}\left(\frac{r}{R}\right)^2\left[\frac{1}{2}\ln\left(\frac{r}{R}\right) - \frac{1}{4}\right]_{R_0/R}^1$$

$$= p_0\pi R_0^2 - \frac{12\mu Q R^2}{h^3}\left\{(1)\left[\frac{1}{2}(0) - \frac{1}{4}\right] - \left(\frac{R_0}{R}\right)^2\left[\frac{1}{2}\ln\left(\frac{R_0}{R}\right) - \frac{1}{4}\right]\right\}$$

$$= -\frac{6\mu Q R^2}{h^3}\left(\frac{R_0}{R}\right)^2\ln\left(\frac{R_0}{R}\right) - \frac{6\mu Q R^2}{h^3}\left[-\frac{1}{2} - \left(\frac{R_0}{R}\right)^2\ln\left(\frac{R_0}{R}\right) + \frac{1}{2}\left(\frac{R_0}{R}\right)^2\right]$$

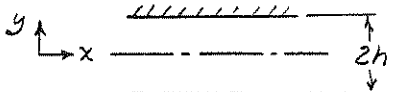
$$F_z = \frac{3\mu Q R^2}{h^3}\left[1 - \left(\frac{R_0}{R}\right)^2\right]$$

F_z

Problem 8.18

Given: Power-law model for non-Newtonian liquid, $\tau_{yx} = k\left(\frac{du}{dy}\right)^n$

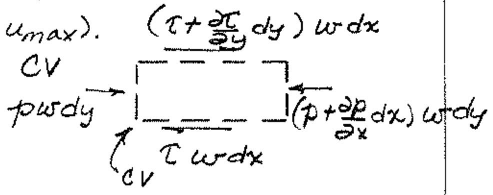
Find: Show $u = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n}\frac{nh}{n+1}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$



for fully developed laminar flow between plates.

Plot: Profiles u/U vs. y/h for $n = 0.7$, 1.0, and 1.3 $(U = u_{max})$.

Solution: Apply momentum equation to differential CV



$(\tau + \frac{\partial \tau}{\partial y}dy)\,w\,dx$
$p\,w\,dy$
$(p+\frac{\partial p}{\partial x}dx)\,w\,dy$
$\tau\,w\,dx$
CV

Basic equation:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}} + \overset{=0(3)}{\cancel{\int_{CS} u\rho\, \vec{V}\cdot d\vec{A}}}$$

Assumptions: (1) Horizontal flow
(2) Steady flow
(3) Fully developed flow

Then

$$p\,w\,dy + \left(\tau + \frac{\partial \tau}{\partial y}dy\right)w\,dx - \left(p + \frac{\partial p}{\partial x}dx\right)w\,dy - \tau\,w\,dx = 0 \quad \text{or} \quad \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

Since $\tau = \tau(y)$ and $p = p(x)$, then $\frac{d\tau}{dy} = \frac{\partial p}{\partial x} = \text{constant}$ and $\tau = y\frac{\partial p}{\partial x}$ or

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n = y\frac{\partial p}{\partial x} = -y\frac{\Delta p}{L}$$

Thus $$\frac{du}{dy} = -\left(\frac{1}{k}\frac{\Delta p}{L}\right)^{1/n} y^{1/n}$$

Integrating

$$u = -\left(\frac{1}{k}\frac{\Delta p}{L}\right)^{1/n}\frac{1}{1/n+1}\,y^{1/n+1} + c = -\left(\frac{1}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{n}{n+1}\,y^{\frac{n+1}{n}} + c$$

But $u = 0$ at $y = h$, so $$c = \left(\frac{1}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{n}{n+1}\,h^{\frac{n+1}{n}}$$

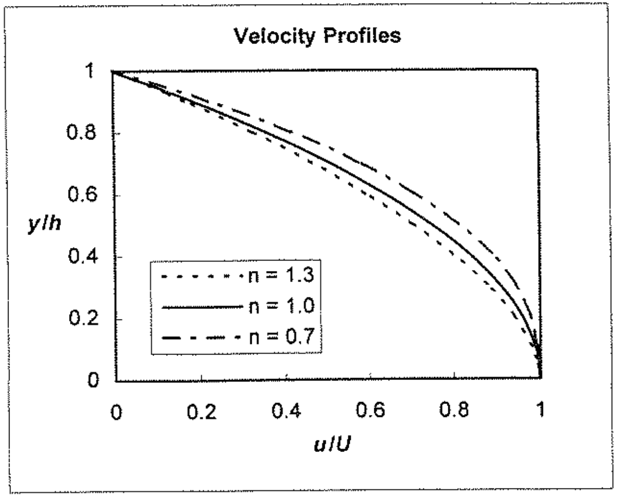
and

$$u = \left(\frac{1}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{n}{n+1}\,h^{\frac{n+1}{n}}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

or

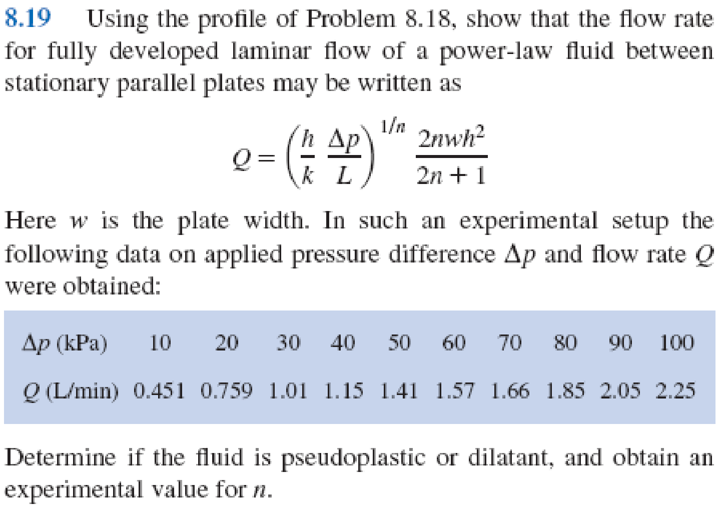
$$u = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{nh}{n+1}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

	n = 0.7	n = 1.0	n = 1.3
y/h	u/U	u/U	u/U
0	1	1	1
0.03	1.000	0.999	0.998
0.06	0.999	0.996	0.993
0.1	0.996	0.990	0.983
0.2	0.980	0.960	0.942
0.3	0.946	0.910	0.881
0.4	0.892	0.840	0.802
0.5	0.814	0.750	0.707
0.6	0.711	0.640	0.595
0.7	0.580	0.510	0.468
0.8	0.418	0.360	0.326
0.9	0.226	0.190	0.170
1	0	0	0



Problem 8.19

[3]



8.19 Using the profile of Problem 8.18, show that the flow rate for fully developed laminar flow of a power-law fluid between stationary parallel plates may be written as

$$Q = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n} \frac{2nwh^2}{2n+1}$$

Here w is the plate width. In such an experimental setup the following data on applied pressure difference Δp and flow rate Q were obtained:

Δp (kPa)	10	20	30	40	50	60	70	80	90	100
Q (L/min)	0.451	0.759	1.01	1.15	1.41	1.57	1.66	1.85	2.05	2.25

Determine if the fluid is pseudoplastic or dilatant, and obtain an experimental value for n.

Given: Laminar velocity profile of power-law fluid flow between parallel plates

Find: Expression for flow rate; from data determine the type of fluid

Solution:

The velocity profile is

$$u = \left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}}\cdot\frac{n\cdot h}{n+1}\cdot\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

The flow rate is then

$$Q = w\cdot\int_{-h}^{h} u\,dy$$

or, because the flow is symmetric

$$Q = 2\cdot w\cdot\int_{0}^{h} u\,dy$$

The integral is computed as

$$\int 1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}dy = y\cdot\left[1-\frac{n}{2\cdot n+1}\cdot\left(\frac{y}{h}\right)^{\frac{2\cdot n+1}{n}}\right]$$

Using this with the limits

$$Q = 2\cdot w\cdot\left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}}\cdot\frac{n\cdot h}{n+1}\cdot h\cdot\left[1-\frac{n}{2\cdot n+1}\cdot(1)^{\frac{2\cdot n+1}{n}}\right]$$

$$Q = \left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}}\cdot\frac{2\cdot n\cdot w\cdot h^2}{2\cdot n+1}$$

The associated *Excel* spreadsheet shows computation of *n*.

Problem 8.19

8.19 Using the profile of Problem 8.18, show that the flow rate for fully developed laminar flow of a power-law fluid between stationary parallel plates may be written as

$$Q = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n} \frac{2nwh^2}{2n+1}$$

Here w is the plate width. In such an experimental setup the following data on applied pressure difference Δp and flow rate Q were obtained:

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Determine if the fluid is pseudoplastic or dilatant, and obtain an experimental value for n.

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Find: Expression for flow rate; from data determine the type of fluid

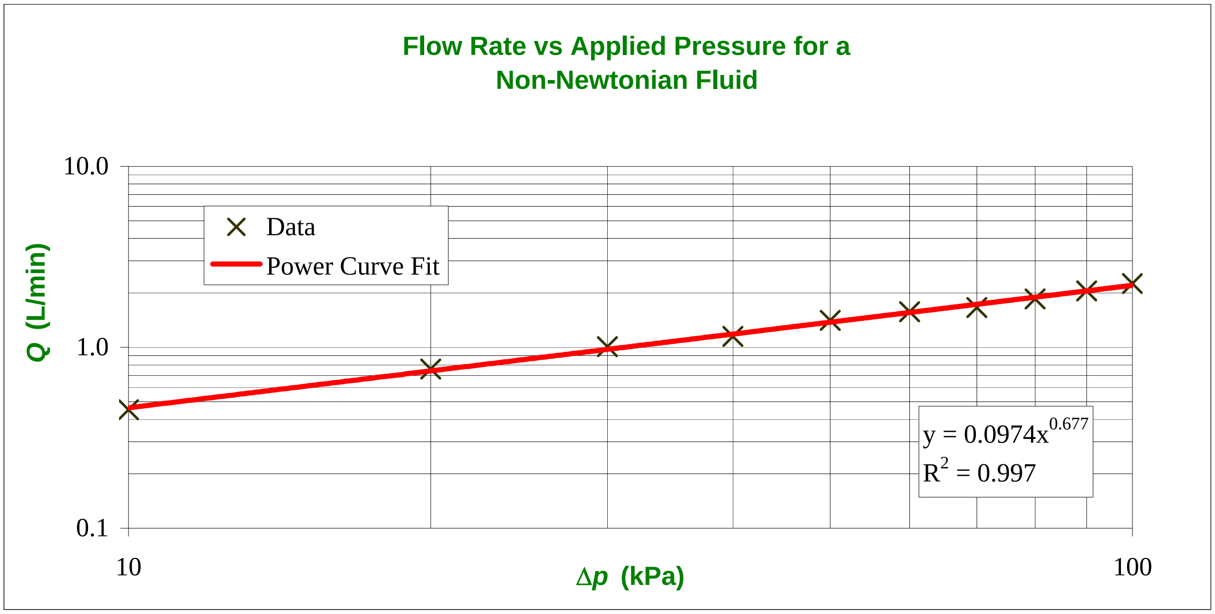
Solution:

The data is

Δp (kPa)	10	20	30	40	50	60	70	80	90	100
Q (L/min)	0.451	0.759	1.01	1.15	1.41	1.57	1.66	1.85	2.05	2.25

This must be fitted to $Q = \left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}} \cdot \frac{2\cdot n\cdot w\cdot h^2}{2\cdot n + 1}$ or $Q = k\cdot \Delta p^{\frac{1}{n}}$

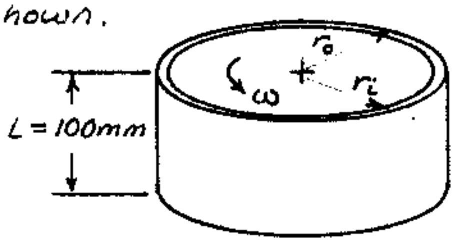
We can fit a power curve to the data



Hence $1/n =$ 0.677 $n =$ 1.48

Problem 8.20

Given: Sealed journal bearing rotating as shown.



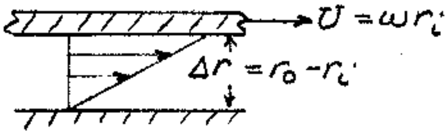
$r_o = 26\,mm$, $r_i = 25\,mm$

Gap contains oil in laminar motion with linear velocity profile.

$\omega = 2800\,rpm$ and Torque, $T = 0.2\,N\cdot m$

Find: (a) Viscosity of oil
(b) Will torque increase or decrease with time? Why?

Solution: "Unfold" bearing since gap is small, and consider as flow between parallel plates. Apply Newton's law of viscosity.



Basic equation: $\tau_{yx} = \mu \frac{du}{dy}$

Assumption: Linear velocity profile

Then $\tau_{yx} = \mu \frac{U}{\Delta r} = \frac{\mu \omega r_i}{\Delta r}$

and

$$T = r_i\,(2\pi r_i L \tau_{yx}) = 2\pi r_i^2 L \tau_{yx} = \frac{2\pi \mu \omega r_i^3 L}{\Delta r}$$

Solving, $\mu = \frac{\Delta r\, T}{2\pi \omega r_i^3 L}$

$$\mu = \frac{1}{2\pi} \times 0.001\,m \times 0.2\,N\cdot m \times \frac{min}{2800\,rev} \times \frac{1}{(0.025)^3 m^3} \times \frac{1}{0.1\,m} \times \frac{rev}{2\pi\,rad} \times 60\frac{s}{min}$$

$$\mu = 0.0695\,N\cdot s / m^2$$

μ

Bearing is sealed, so oil temperature will increase as energy is dissipated by friction. For liquids, μ decreases as T increases. Thus torque will decrease, since it is proportional to μ.

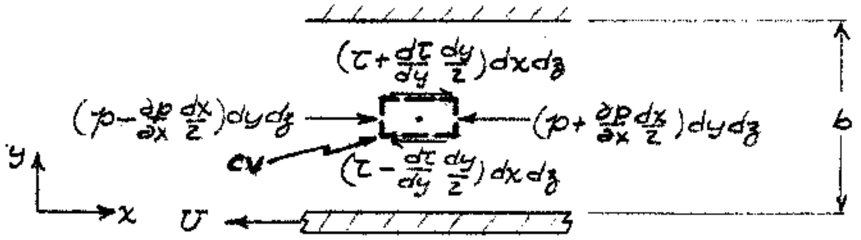
Problem 8.21

[2]

Given: Water at 60°C flows between large flat plates.

$U = 0.3\,m/s$

$b = 3\,mm$



$(\tau + \frac{d\tau}{dy}\frac{dy}{2})dx\,dz$
$(p - \frac{\partial p}{\partial x}\frac{dx}{2})dy\,dz$
$(p + \frac{\partial p}{\partial x}\frac{dx}{2})dy\,dz$
$(\tau - \frac{d\tau}{dy}\frac{dy}{2})dx\,dz$
CV
y
x
U
b

Find: Pressure gradient required for zero net flow at a section.

Solution: Apply momentum equation using CV and coordinates shown.

Basic equations:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \overset{=0(3)}{\cancel{\int_{CS}}} u\rho\vec{V}\cdot d\vec{A}\,, \quad \tau = \tau_{yx} = \mu\frac{du}{dy}$$

Assumptions: (1) $F_{B_x} = 0$

(2) Steady flow

(3) Fully-developed flow

(4) Newtonian fluid

Then $F_{S_x} = 0$. Substituting the force terms (see page 315 for details) gives

$$\frac{\partial p}{\partial x} = \frac{d\tau_{yx}}{dy} = \frac{d}{dy}\left(\mu\frac{du}{dy}\right) = \mu\frac{d^2u}{dy^2} \quad \text{or} \quad \frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating twice,

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + c_1 y + c_2$$

To evaluate the constants c_1 and c_2, we must use the boundary conditions. At $y = 0$, $u = -U$, so $c_2 = -U$. At $y = b$, $u = 0$, so

$$0 = \frac{1}{2\mu}\frac{\partial p}{\partial x}b^2 + c_1 b - U \quad \text{or} \quad c_1 = \frac{U}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x}b$$

Thus

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - by) + U\left(\frac{y}{b} - 1\right)$$

To find the flowrate, we integrate

$$\frac{Q}{w} = \int_0^b u\,dy = \int_0^b\left[\frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - by) + U\left(\frac{y}{b} - 1\right)\right]dy = -\frac{1}{12\mu}\frac{\partial p}{\partial x}b^3 - \frac{Ub}{2}$$

For $Q = 0$, with $\mu = 4.63 \times 10^{-4}\,\frac{N \cdot s}{m^2}$ from Table A.8,

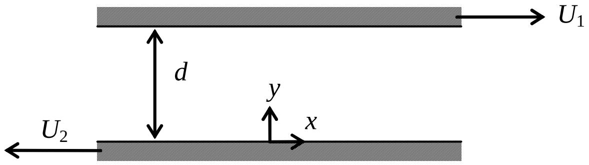
$$\frac{\partial p}{\partial x} = -\frac{6U\mu}{b^2} = -6 \times 0.3\frac{m}{s} \times 4.63 \times 10^{-4}\frac{N \cdot s}{m^2} \times \frac{1}{(0.003)^2 m^2} = -92.6\,N/m^2 \cdot m \qquad \frac{\partial p}{\partial x}$$

Thus pressure must decrease in x direction for zero net flowrate. ←

Problem 8.22

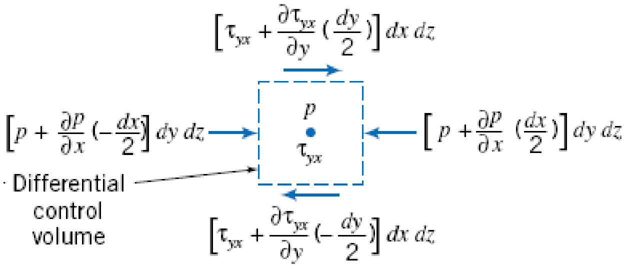
[2]

8.22 Consider fully developed laminar flow between infinite parallel plates separated by gap width $d = 10$ mm. The upper plate moves to the right with speed $U_2 = 0.5$ m/s; the lower plate moves to the left with speed $U_1 = 0.25$ m/s. The pressure gradient in the direction of flow is zero. Develop an expression for the velocity distribution in the gap. Find the volume flow rate per unit depth passing a given cross-section.



Given: Laminar flow between moving plates

Find: Expression for velocity; Volume flow rate per depth



$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\left(\frac{dy}{2}\right)\right] dx\,dz$
$\left[p + \frac{\partial p}{\partial x}\left(-\frac{dx}{2}\right)\right] dy\,dz$
p
τ_{yx}
$\left[p + \frac{\partial p}{\partial x}\left(\frac{dx}{2}\right)\right] dy\,dz$
Differential control volume
$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\left(-\frac{dy}{2}\right)\right] dx\,dz$

Solution:

Using the analysis of Section 8-2, the sum of forces in the x direction is

$$\left[\tau + \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2} - \left(\tau - \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2}\right)\right]\cdot b\cdot dx + \left(p - \frac{\partial}{\partial x}p\cdot\frac{dx}{2} - p + \frac{\partial}{\partial x}p\cdot\frac{dx}{2}\right)\cdot b\cdot dy = 0$$

Simplifying $\frac{d\tau}{dy} = \frac{dp}{dx} = 0$ or $\mu\cdot\frac{d^2u}{dy^2} = 0$

Integrating twice $u = c_1\cdot y + c_2$

Boundary conditions: $u(0) = -U_1$ $\quad c_2 = -U_1$ $\quad u(y = d) = U_2$ $\quad c_1 = \frac{U_1 + U_2}{d}$

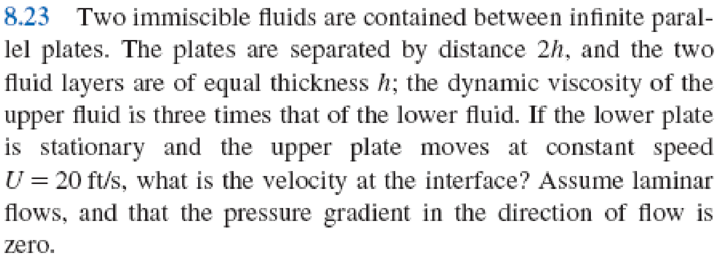
Hence $u(y) = \left(U_1 + U_2\right)\cdot\frac{y}{d} - U_1$ $\quad u(y) = 75\cdot y - 0.25$ (u in m/s, y in m)

The volume flow rate is $Q = \int u\,dA = b\cdot\int u\,dy$ $\quad Q = b\cdot\int_0^d \left[\left(U_1 + U_2\right)\cdot\frac{y}{d} - U_1\right] dx$

$$Q = b\cdot d\cdot\frac{\left(U_2 - U_1\right)}{2} \qquad \frac{Q}{b} = 10\cdot mm \times \frac{1\cdot m}{1000\cdot mm} \times \frac{1}{2} \times (0.5 - 0.25) \times \frac{m}{s} \qquad Q = 0.00125\,\frac{\frac{m^3}{s}}{m}$$

Problem 8.23

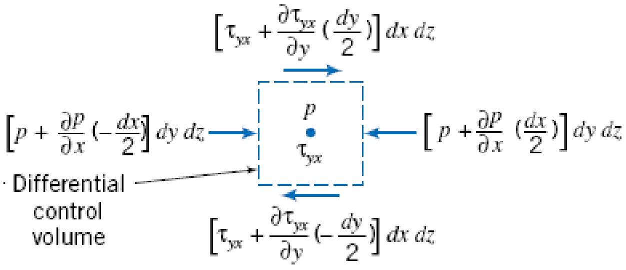
[3]



8.23 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness h; the dynamic viscosity of the upper fluid is three times that of the lower fluid. If the lower plate is stationary and the upper plate moves at constant speed $U = 20$ ft/s, what is the velocity at the interface? Assume laminar flows, and that the pressure gradient in the direction of flow is zero.

Given: Laminar flow of two fluids between plates

Find: Velocity at the interface



$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\left(\frac{dy}{2}\right)\right] dx\, dz$
$\left[p + \frac{\partial p}{\partial x}\left(-\frac{dx}{2}\right)\right] dy\, dz$
p
τ_{yx}
$\left[p + \frac{\partial p}{\partial x}\left(\frac{dx}{2}\right)\right] dy\, dz$
Differential control volume
$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\left(-\frac{dy}{2}\right)\right] dx\, dz$

Solution:

Using the analysis of Section 8-2, the sum of forces in the x direction is

$$\left[\tau + \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2} - \left(\tau - \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2}\right)\right]\cdot b\cdot dx + \left(p - \frac{\partial}{\partial x}p\cdot\frac{dx}{2} - p + \frac{\partial}{\partial x}p\cdot\frac{dx}{2}\right)\cdot b\cdot dy = 0$$

Simplifying $\quad \frac{d\tau}{dy} = \frac{dp}{dx} = 0 \quad$ or $\quad \mu\cdot\frac{d^2u}{dy^2} = 0$

Applying this to fluid 1 (lower fluid) and fluid 2 (upper fluid), integrating twice yields $\quad u_1 = c_1\cdot y + c_2 \quad u_2 = c_3\cdot y + c_4$

We need four BCs. Three are obvious $\quad y = 0 \quad u_1 = 0 \quad y = h \quad u_1 = u_2 \quad y = 2\cdot h \quad u_2 = U$

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$$y = h \qquad \mu_1\cdot\frac{du_1}{dy} = \mu_2\cdot\frac{du_2}{dy}$$

Using these four BCs $\quad 0 = c_2 \quad c_1\cdot h + c_2 = c_3\cdot h + c_4 \quad U = c_3\cdot 2\cdot h + c_4 \quad \mu_1\cdot c_1 = \mu_2\cdot c_3$

Hence $\quad c_2 = 0$

From the 2nd and 3rd equations $\quad c_1\cdot h - U = -c_3\cdot h \quad$ and $\quad \mu_1\cdot c_1 = \mu_2\cdot c_3$

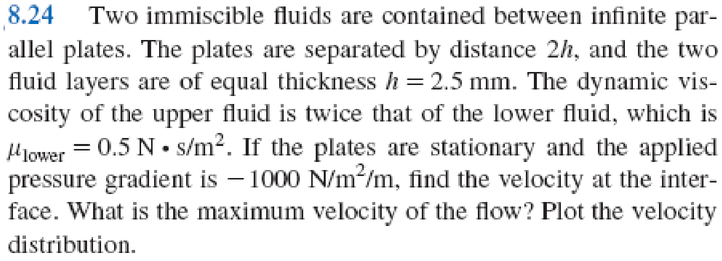
Hence $\quad c_1\cdot h - U = -c_3\cdot h = -\frac{\mu_1}{\mu_2}\cdot h\cdot c_1 \qquad c_1 = \frac{U}{h\cdot\left(1 + \frac{\mu_1}{\mu_2}\right)}$

Hence for fluid 1 (we do not need to complete the analysis for fluid 2) $\quad u_1 = \frac{U}{h\cdot\left(1 + \frac{\mu_1}{\mu_2}\right)}\cdot y$

Evaluating this at $y = h$, where $u_1 = u_{\text{interface}}$ $\quad u_{\text{interface}} = \frac{20\cdot\frac{ft}{s}}{\left(1 + \frac{1}{3}\right)} \qquad u_{\text{interface}} = 15\cdot\frac{ft}{s}$

Problem 8.24

[3]



8.24 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness $h = 2.5$ mm. The dynamic viscosity of the upper fluid is twice that of the lower fluid, which is $\mu_{\text{lower}} = 0.5\ \text{N} \cdot \text{s/m}^2$. If the plates are stationary and the applied pressure gradient is $-1000\ \text{N/m}^2\text{/m}$, find the velocity at the interface. What is the maximum velocity of the flow? Plot the velocity distribution.

Given: Properties of two fluids flowing between parallel plates; applied pressure gradient

Find: Velocity at the interface; maximum velocity; plot velocity distribution

Solution:

Given data $\quad k = \dfrac{dp}{dx} = -1000 \cdot \dfrac{Pa}{m} \qquad h = 2.5 \cdot mm$

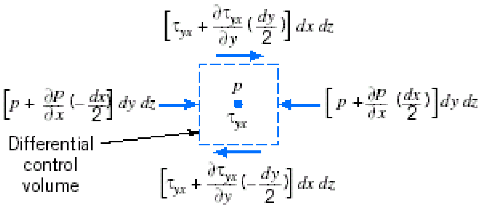
$$\mu_1 = 0.5 \cdot \frac{N \cdot s}{m^2} \qquad \mu_2 = 2 \cdot \mu_1 \qquad \mu_2 = 1 \cdot \frac{N \cdot s}{m^2}$$

(Lower fluid is fluid 1; upper is fluid 2)

Following the analysis of Section 8-2, analyse the forces on a differential CV of either fluid

The net force is zero for steady flow, so

$$\left[\tau + \frac{d\tau}{dy} \cdot \frac{dy}{2} - \left(\tau - \frac{d\tau}{dy} \cdot \frac{dy}{2}\right)\right] \cdot dx \cdot dz + \left[p - \frac{dp}{dx} \cdot \frac{dx}{2} - \left(p + \frac{dp}{dx} \cdot \frac{dx}{2}\right)\right] \cdot dy \cdot dz = 0$$


$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} \left(\frac{dy}{2}\right)\right] dx\, dz$
$\left[p + \frac{\partial p}{\partial x} \left(-\frac{dx}{2}\right)\right] dy\, dz$
p
τ_{yx}
$\left[p + \frac{\partial p}{\partial x} \left(\frac{dx}{2}\right)\right] dy\, dz$
Differential control volume
$\left[\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} \left(-\frac{dy}{2}\right)\right] dx\, dz$

Simplifying $\quad \dfrac{d\tau}{dy} = \dfrac{dp}{dx} = k \qquad$ so for each fluid $\quad \mu \cdot \dfrac{d^2}{dy^2} u = k$

Applying this to fluid 1 (lower fluid) and fluid 2 (upper fluid), integrating twice yields

$$u_1 = \frac{k}{2 \cdot \mu_1} \cdot y^2 + c_1 \cdot y + c_2 \qquad u_2 = \frac{k}{2 \cdot \mu_2} \cdot y^2 + c_3 \cdot y + c_4$$

For convenience the origin of coordinates is placed at the centerline

We need four BCs. Three are obvious $\quad y = -h \qquad u_1 = 0 \qquad (1)$

$y = 0 \qquad u_1 = u_2 \qquad (2)$

$y = h \qquad u_2 = 0 \qquad (3)$

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$$y = 0 \qquad \mu_1 \cdot \frac{du_1}{dy} = \mu_2 \cdot \frac{du_2}{dy} \qquad (4)$$

Using these four BCs

$$0 = \frac{k}{2\cdot\mu_1}\cdot h^2 - c_1\cdot h + c_2$$

$$c_2 = c_4$$

$$0 = \frac{k}{2\cdot\mu_2}\cdot h^2 + c_3\cdot h + c_4$$

$$\mu_1\cdot c_1 = \mu_2\cdot c_3$$

Hence, after some algebra

$$c_1 = \frac{k\cdot h}{2\cdot\mu_1}\cdot\frac{\left(\mu_2 - \mu_1\right)}{\left(\mu_2 + \mu_1\right)} \qquad c_2 = c_4 = -\frac{k\cdot h^2}{\mu_2 + \mu_1} \qquad c_3 = \frac{k\cdot h}{2\cdot\mu_2}\cdot\frac{\left(\mu_2 - \mu_1\right)}{\left(\mu_2 + \mu_1\right)}$$

The velocity distributions are then

$$u_1 = \frac{k}{2\cdot\mu_1}\cdot\left[y^2 + y\cdot h\cdot\frac{\left(\mu_2 - \mu_1\right)}{\left(\mu_2 + \mu_1\right)}\right] - \frac{k\cdot h^2}{\mu_2 + \mu_1} \qquad u_2 = \frac{k}{2\cdot\mu_2}\cdot\left[y^2 + y\cdot h\cdot\frac{\left(\mu_2 - \mu_1\right)}{\left(\mu_2 + \mu_1\right)}\right] - \frac{k\cdot h^2}{\mu_2 + \mu_1}$$

Evaluating either velocity at *y* = 0, gives the velocity at the interface

$$u_{interface} = -\frac{k\cdot h^2}{\mu_2 + \mu_1} \qquad u_{interface} = 4.17\times 10^{-3}\frac{m}{s}$$

The plots of these velocity distributions are shown in the associated *Excel* workbook, as is the determination of the maximum velocity.

From *Excel* $$u_{max} = 4.34\times 10^{-3}\cdot\frac{m}{s}$$

Problem 8.24

[3]

8.24 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness $h = 2.5$ mm. The dynamic viscosity of the upper fluid is twice that of the lower fluid, which is $\mu_{\text{lower}} = 0.5\ \text{N}\cdot\text{s/m}^2$. If the plates are stationary and the applied pressure gradient is $-1000\ \text{N/m}^2/\text{m}$, find the velocity at the interface. What is the maximum velocity of the flow? Plot the velocity distribution.

Given: Properties of two fluids flowing between parallel plates; applied pressure gradient

Find: Velocity at the interface; maximum velocity; plot velocity distribution

Solution:

The data is

$k =$ -1000 Pa/m
$h =$ 2.5 mm
$\mu_1 =$ 0.5 N.s/m^2
$\mu_2 =$ 1.0 N.s/m^2

The velocity distribution is

$$u_1 = \frac{k}{2\cdot\mu_1}\cdot\left[y^2 + y\cdot h\cdot\frac{(\mu_2-\mu_1)}{(\mu_2+\mu_1)}\right] - \frac{k\cdot h^2}{\mu_2+\mu_1} \qquad u_2 = \frac{k}{2\cdot\mu_2}\cdot\left[y^2 + y\cdot h\cdot\frac{(\mu_2-\mu_1)}{(\mu_2+\mu_1)}\right] - \frac{k\cdot h^2}{\mu_2+\mu_1}$$

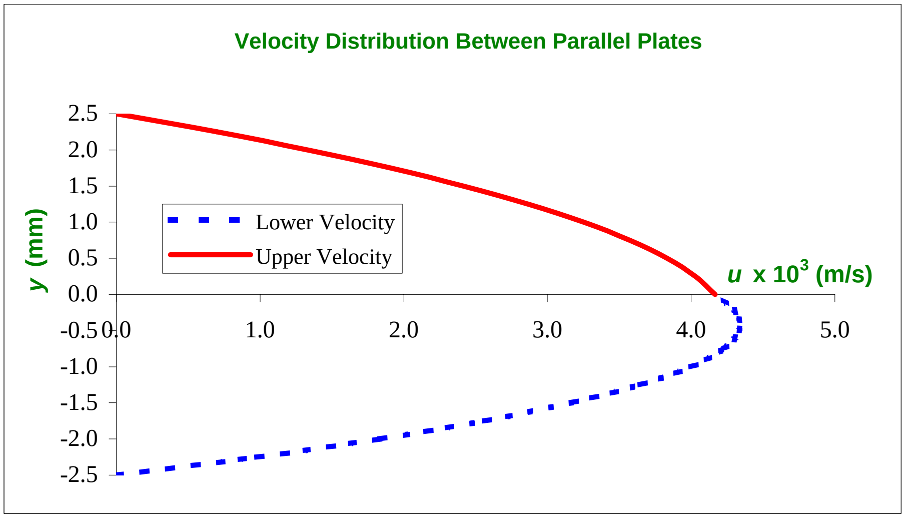
y (mm)	$u_1 \times 10^3$ (m/s)	$u_2 \times 10^3$ (m/s)
-2.50	0.000	NA
-2.25	0.979	NA
-2.00	1.83	NA
-1.75	2.56	NA
-1.50	3.17	NA
-1.25	3.65	NA
-1.00	4.00	NA
-0.75	4.23	NA
-0.50	4.33	NA
-0.25	4.31	NA
0.00	4.17	4.17
0.25	NA	4.03
0.50	NA	3.83
0.75	NA	3.57
1.00	NA	3.25
1.25	NA	2.86
1.50	NA	2.42
1.75	NA	1.91
2.00	NA	1.33
2.25	NA	0.698
2.50	NA	0.000

The lower fluid has the highest velocity

We can use *Solver* to find the maximum

(Or we could differentiate to find the maximum)

y (mm)	$u_{\max} \times 10^3$ (m/s)
-0.417	4.34



Problem 8.25

[2]

8.25 The dimensionless velocity profile for fully developed laminar flow between infinite parallel plates with the upper plate moving at constant speed U is shown in Fig. 8.6. Find the pressure gradient $\partial p/\partial x$ at which (a) the upper plate and (b) the lower plate experience zero shear stress, in terms of U, a, and μ. Plot the dimensionless velocity profiles for these cases.

Given: Velocity profile between parallel plates

Find: Pressure gradients for zero stress at upper/lower plates; plot

Solution:

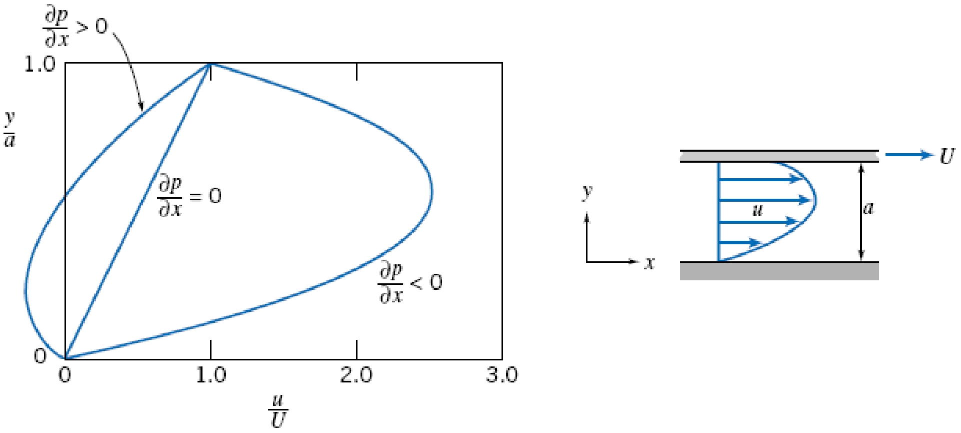


Fig. 8.6 Dimensionless velocity profile for fully developed laminar flow between infinite parallel plates: upper plate moving with constant speed, U.

From Eq. 8.8, the velocity distribution is

$$u = \frac{U\cdot y}{a} + \frac{a^2}{2\cdot\mu}\cdot\left(\frac{\partial}{\partial x}p\right)\cdot\left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right]$$

The shear stress is

$$\tau_{yx} = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{U}{a} + \frac{a^2}{2}\cdot\left(\frac{\partial}{\partial x}p\right)\cdot\left(2\cdot\frac{y}{a^2} - \frac{1}{a}\right)$$

(a) For $\tau_{yx} = 0$ at $y = a$

$$0 = \mu\cdot\frac{U}{a} + \frac{a}{2}\cdot\frac{\partial}{\partial x}p \qquad \frac{\partial}{\partial x}p = -\frac{2\cdot U\cdot\mu}{a^2}$$

The velocity distribution is then

$$u = \frac{U\cdot y}{a} - \frac{a^2}{2\cdot\mu}\cdot\frac{2\cdot U\cdot\mu}{a^2}\cdot\left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right] \qquad \frac{u}{U} = 2\cdot\frac{y}{a} - \left(\frac{y}{a}\right)^2$$

(b) For $\tau_{yx} = 0$ at $y = 0$

$$0 = \mu\cdot\frac{U}{a} - \frac{a}{2}\cdot\frac{\partial}{\partial x}p \qquad \frac{\partial}{\partial x}p = \frac{2\cdot U\cdot\mu}{a^2}$$

The velocity distribution is then

$$u = \frac{U\cdot y}{a} + \frac{a^2}{2\cdot\mu}\cdot\frac{2\cdot U\cdot\mu}{a^2}\cdot\left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right] \qquad \frac{u}{U} = \left(\frac{y}{a}\right)^2$$

The velocity distributions are plotted in the associated *Excel* workbook

Problem 8.25

[2]

8.25 The dimensionless velocity profile for fully developed laminar flow between infinite parallel plates with the upper plate moving at constant speed U is shown in Fig. 8.6. Find the pressure gradient $\partial p/\partial x$ at which (a) the upper plate and (b) the lower plate experience zero shear stress, in terms of U, a, and μ. Plot the dimensionless velocity profiles for these cases.

Given: Velocity profile between parallel plates

Find: Pressure gradients for zero stress at upper/lower plates; plot

Solution:

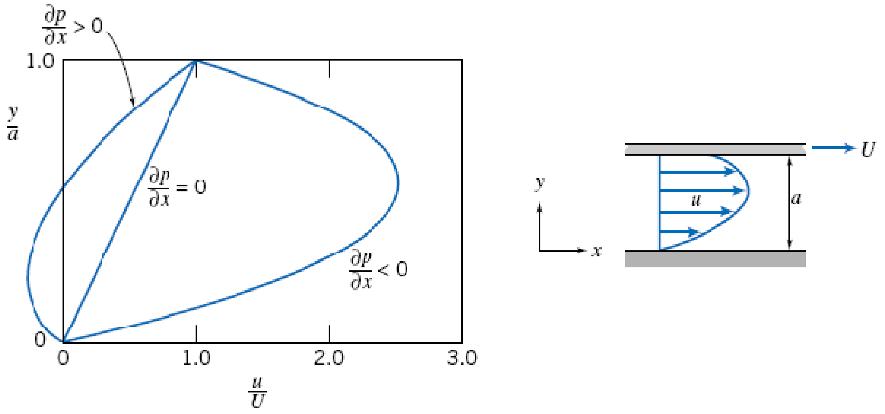
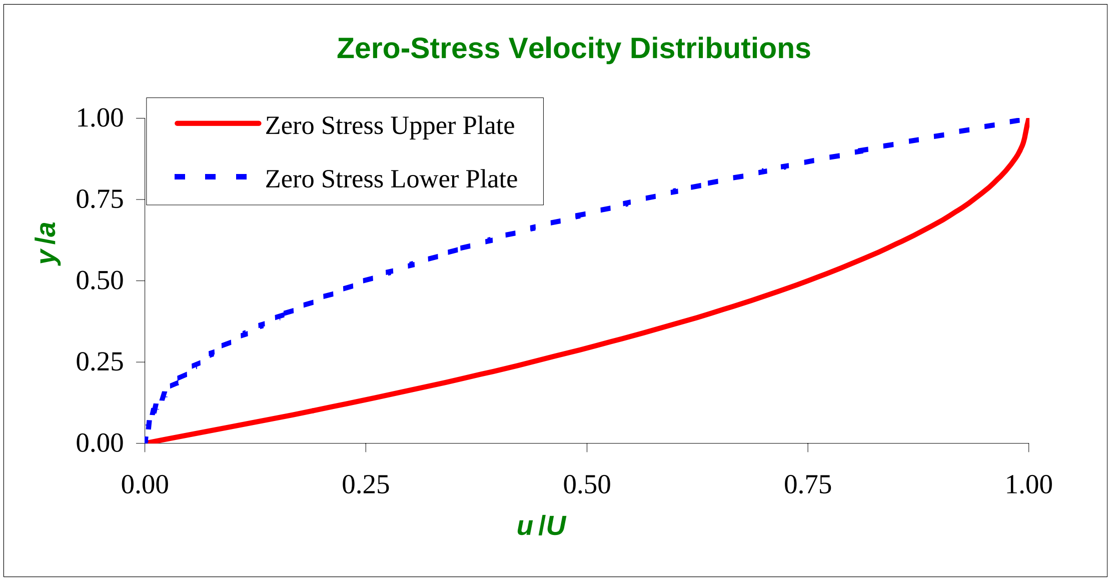

$\frac{\partial p}{\partial x} > 0$
1.0
$\frac{y}{a}$
$\frac{\partial p}{\partial x} = 0$
$\frac{\partial p}{\partial x} < 0$
0
0
1.0
2.0
3.0
$\frac{u}{U}$
y
x
u
a
U

Fig. 8.6 Dimensionless velocity profile for fully developed laminar flow between infinite parallel plates: upper plate moving with constant speed, U.

(a) For zero shear stress at upper plate $\frac{u}{U} = 2 \cdot \frac{y}{a} - \left(\frac{y}{a}\right)^2$

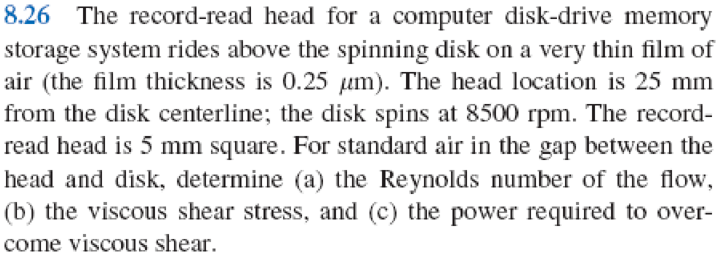
(b) For zero shear stress at lower plate $\frac{u}{U} = \left(\frac{y}{a}\right)^2$

y/a	(a) u/U	(b) u/U
0.0	0.000	0.000
0.1	0.190	0.010
0.2	0.360	0.040
0.3	0.510	0.090
0.4	0.640	0.160
0.5	0.750	0.250
0.6	0.840	0.360
0.7	0.910	0.490
0.8	0.960	0.640
0.9	0.990	0.810
1.0	1.00	1.000



Problem 8.26

[2]



8.26 The record-read head for a computer disk-drive memory storage system rides above the spinning disk on a very thin film of air (the film thickness is 0.25 μm). The head location is 25 mm from the disk centerline; the disk spins at 8500 rpm. The record-read head is 5 mm square. For standard air in the gap between the head and disk, determine (a) the Reynolds number of the flow, (b) the viscous shear stress, and (c) the power required to overcome viscous shear.

Given: Computer disk drive

Find: Flow Reynolds number; Shear stress; Power required

Solution:

For a distance R from the center of a disk spinning at speed ω

$$V = R\cdot\omega \qquad V = 25\cdot\text{mm}\times\frac{1\cdot\text{m}}{1000\cdot\text{mm}}\times 8500\cdot\text{rpm}\times\frac{2\cdot\pi\cdot\text{rad}}{\text{rev}}\times\frac{1\cdot\text{min}}{60\cdot\text{s}} \qquad V = 22.3\cdot\frac{\text{m}}{\text{s}}$$

The gap Reynolds number is $Re = \frac{\rho\cdot V\cdot a}{\mu} = \frac{V\cdot a}{\nu}$ $\qquad \nu = 1.45\times 10^{-5}\cdot\frac{\text{m}^2}{\text{s}}$ from Table A.10 at 15ºC

$$Re = 22.3\cdot\frac{\text{m}}{\text{s}}\times 0.25\times 10^{-6}\cdot\text{m}\times\frac{\text{s}}{1.45\times 10^{-5}\cdot\text{m}^2} \qquad Re = 0.384$$

The flow is definitely laminar

The shear stress is then $\tau = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{V}{a}$ $\qquad \mu = 1.79\times 10^{-5}\cdot\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ from Table A.10 at 15ºC

$$\tau = 1.79\times 10^{-5}\cdot\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times 22.3\cdot\frac{\text{m}}{\text{s}}\times\frac{1}{0.25\times 10^{-6}\cdot\text{m}} \qquad \tau = 1.60\cdot\text{kPa}$$

The power required is $P = T\cdot\omega$ where torque T is given by $T = \tau\cdot A\cdot R$ with $A = (5\cdot\text{mm})^2$ $\qquad A = 2.5\times 10^{-5}\,\text{m}^2$

$$P = \tau\cdot A\cdot R\cdot\omega \qquad P = 1600\cdot\frac{\text{N}}{\text{m}^2}\times 2.5\times 10^{-5}\cdot\text{m}^2\times 25\cdot\text{mm}\times\frac{1\cdot\text{m}}{1000\cdot\text{mm}}\times 8500\cdot\text{rpm}\times\frac{2\cdot\pi\cdot\text{rad}}{\text{rev}}\times\frac{1\cdot\text{min}}{60\cdot\text{s}} \qquad P = 0.890\,\text{W}$$

Problem 8.27

[2]

Given: Steady, incompressible, fully developed laminar flow down an incline (of angle θ).
Velocity profile (Example Problem 5.9) is

$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - y^2/2\right)$$

Find: Kinematic viscosity ν of liquid for $h = 0.8\text{ mm}$, $\theta = 30°$ and $u_{max} = 15.7\text{ mm/s}$

Plot: the velocity profile

Solution:

$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - y^2/2\right) = \frac{g \sin\theta}{\nu}\left(hy - y^2/2\right)$$

$u = u_{max}$ at $y = h$

$$\therefore\ u_{max} = \frac{g\sin\theta}{\nu}\left(h^2 - h^2/2\right) = \frac{g \sin\theta h^2}{2\nu}$$

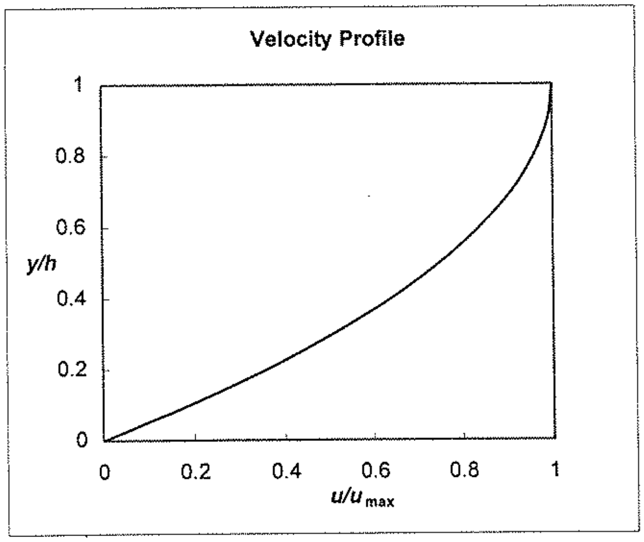
and

$$\nu = \frac{g \sin\theta h^2}{2 u_{max}} = \frac{\sin 30°}{2} \times 9.81\frac{\text{m}}{\text{s}^2} \times \left(0.8\times10^{-3}\text{ m}\right)^2 \times \frac{\text{s}}{15.7\times10^{-3}\text{ m}}$$

$$\nu = 1.00\times10^{-4}\ \text{m}^2/\text{s} \longleftarrow \nu$$

Plot $$\frac{u}{u_{max}} = \frac{g \cancel{\sin\theta}}{\cancel{\nu}}\left(hy - y^2/2\right) \times \frac{2\cancel{\nu}}{g\cancel{\sin\theta} h^2} = 2\frac{y}{h} - \left(\frac{y}{h}\right)^2$$

u/u_{max}	y/h
0	0
0.0396	0.02
0.098	0.05
0.190	0.1
0.360	0.2
0.510	0.3
0.640	0.4
0.750	0.5
0.840	0.6
0.910	0.7
0.960	0.8
0.990	0.9
1.00	1.0

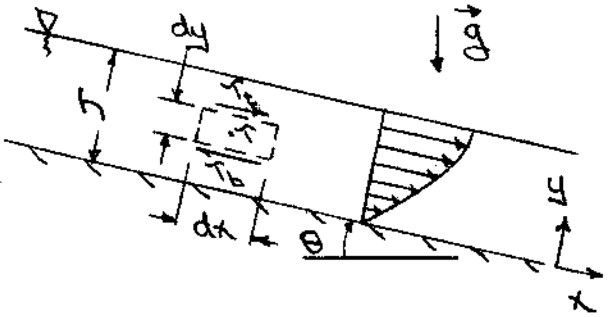


Problem 8.28

Given: Fully developed, laminar flow of an incompressible liquid down an inclined surface. The thickness, h, of the liquid layer is constant.

Find: (a) the velocity profile by use of a suitably chosen differential control volume. (b) volume flow rate, Q/w

Solution: Flow is fully developed, so $u = u(y)$ and $\tau = \tau(y)$.



Expand τ in a Taylor series about the center of the differential CV

$$\tau_t = \tau + \frac{d\tau}{dy}\frac{dy}{2}$$

$$\tau_b = \tau + \frac{d\tau}{dy}\left(-\frac{dy}{2}\right)$$

The boundary conditions on the velocity profile are:

@ $y=0$, $u=0$ (no slip).

@ $y=h$, $\frac{du}{dy}=0$ (no shear stress).

Apply the x component of the momentum equation to the differential CV shown

$$F_{S_x} + F_{B_x} = \overset{0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \overset{0(2)}{\cancel{\int_{CS} u\rho\vec{V}\cdot d\vec{A}}}$$

Assumptions: (1) steady flow

(2) fully developed flow, so u and τ are functions of y only

(3) no variation of pressure in the x direction

Then

$$F_{S_x} + F_{B_x} = 0 = \left(\tau + \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz - \left(\tau - \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz + \rho g\sin\theta\, dx\,dy\,dz$$

or

$$\frac{d\tau}{dy} = -\rho g\sin\theta$$

Integrating, $\tau = -\rho g\sin\theta\, y + c_1$

But $\tau = 0$ @ $y=h$, $\therefore$ $c_1 = \rho g\sin\theta\, h$, and

$$\frac{du}{dy} = \frac{\rho g\sin\theta}{\mu}(h-y)$$

Integrating again,

$$u = \frac{\rho g\sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) + c_2$$

At $y=0$, $u=0$, so $c_2 = 0$ and hence

$$u = \frac{\rho g\sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) \longleftarrow u$$

$$Q/w = \int_0^h u\,dy = \frac{\rho g\sin\theta}{\mu}\int_0^h\left(hy - \frac{y^2}{2}\right)dy = \frac{\rho g\sin\theta}{\mu}\left[\frac{hy^2}{2} - \frac{y^3}{6}\right]_0^h$$

$$Q/w = \rho g\sin\theta\, h^3/3\mu \longleftarrow Q/w$$

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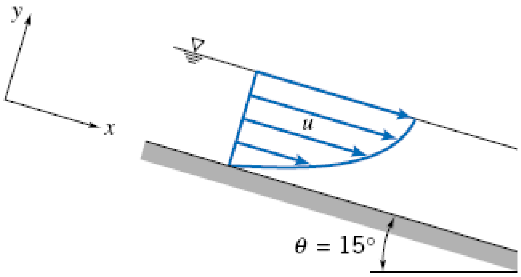
Problem 8.29

[2]

8.29 The velocity distribution for flow of a thin viscous film down an inclined plane surface was developed in Example 5.9. Consider a film 7 mm thick, of liquid with SG = 1.2 and dynamic viscosity of 1.60 N • s/m^2. Derive an expression for the shear stress distribution within the film. Calculate the maximum shear stress within the film and indicate its direction. Evaluate the volume flow rate in the film, in mm^3/s per millimeter of surface width. Calculate the film Reynolds number based on average velocity.



Given: Velocity distribution on incline

Find: Expression for shear stress; Maximum shear; volume flow rate/mm width; Reynolds number

Solution:

From Example 5.9 $$u(y) = \frac{\rho \cdot g \cdot \sin(\theta)}{\mu} \cdot \left(h \cdot y - \frac{y^2}{2} \right)$$

For the shear stress $$\tau = \mu \cdot \frac{du}{dy} = \rho \cdot g \cdot \sin(\theta) \cdot (h - y)$$

τ is a maximum at y = 0 $$\tau_{max} = \rho \cdot g \cdot \sin(\theta) \cdot h = SG \cdot \rho_{H2O} \cdot g \cdot \sin(\theta) \cdot h$$

$$\tau_{max} = 1.2 \times 1000 \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \sin(15 \cdot deg) \times 0.007 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad \tau_{max} = 21.3\,Pa$$

This stress is in the x direction on the wall

The flow rate is $$Q = \int u\,dA = w \cdot \int_0^h u(y)\,dy = w \cdot \int_0^h \frac{\rho \cdot g \cdot \sin(\theta)}{\mu} \cdot \left(h \cdot y - \frac{y^2}{2} \right) dy \qquad Q = \frac{\rho \cdot g \cdot \sin(\theta) \cdot w \cdot h^3}{3 \cdot \mu}$$

$$\frac{Q}{w} = \frac{1}{3} \times 1.2 \times 1000 \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \sin(15 \cdot deg) \times (0.007 \cdot m)^3 \times \frac{m^2}{1.60 \cdot N \cdot s} \cdot \frac{N \cdot s^2}{kg \cdot m} = 2.18 \times 10^{-4} \frac{\frac{m^3}{s}}{m} \qquad \frac{Q}{w} = 217 \frac{\frac{mm^3}{s}}{mm}$$

The average velocity is $$V = \frac{Q}{A} = \frac{Q}{w \cdot h} \qquad V = 217 \cdot \frac{\frac{mm^3}{s}}{mm} \times \frac{1}{7 \cdot mm} \qquad V = 31.0 \frac{mm}{s}$$

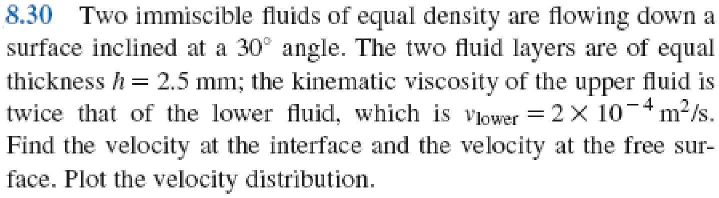
The gap Reynolds number is $$Re = \frac{\rho \cdot V \cdot h}{\mu}$$

$$Re = 1.2 \times 1000 \frac{kg}{m^3} \times 31 \cdot \frac{mm}{s} \times 7 \cdot mm \times \frac{m^2}{1.60 \cdot N \cdot s} \times \left(\frac{1 \cdot m}{1000 \cdot mm} \right)^2 \qquad Re = 0.163$$

The flow is definitely laminar

Problem 8.30

[3]



8.30 Two immiscible fluids of equal density are flowing down a surface inclined at a 30° angle. The two fluid layers are of equal thickness $h = 2.5$ mm; the kinematic viscosity of the upper fluid is twice that of the lower fluid, which is $\nu_{\text{lower}} = 2 \times 10^{-4}\ \text{m}^2/\text{s}$. Find the velocity at the interface and the velocity at the free surface. Plot the velocity distribution.

Given: Data on flow of liquids down an incline

Find: Velocity at interface; velocity at free surface; plot

Solution:

Given data $h = 2.5 \cdot \text{mm}$ $\theta = 30 \cdot \text{deg}$ $\nu_1 = 2 \times 10^{-4} \cdot \frac{\text{m}^2}{\text{s}}$ $\nu_2 = 2 \cdot \nu_1$

(The lower fluid is designated fluid 1, the upper fluid 2)

From Example 5.9 (or Exanple 8.3 with *g* replaced with *g*sinθ), a free body analysis leads to (for either fluid)

$$\frac{d^2}{dy^2}u = -\frac{\rho \cdot g \cdot \sin(\theta)}{\mu}$$

Applying this to fluid 1 (lower fluid) and fluid 2 (upper fluid), integrating twice yields

$$u_1 = -\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_1} \cdot y^2 + c_1 \cdot y + c_2 \qquad u_2 = -\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_2} \cdot y^2 + c_3 \cdot y + c_4$$

We need four BCs. Two are obvious $y = 0$ $u_1 = 0$ (1)

$y = h$ $u_1 = u_2$ (2)

The third BC comes from the fact that there is no shear stress at the free surface

$y = 2 \cdot h$ $\mu_2 \cdot \frac{du_2}{dy} = 0$ (3)

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$y = h$ $\mu_1 \cdot \frac{du_1}{dy} = \mu_2 \cdot \frac{du_2}{dy}$ (4)

Using these four BCs $c_2 = 0$

$$-\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_1} \cdot h^2 + c_1 \cdot h + c_2 = -\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_2} \cdot h^2 + c_3 \cdot h + c_4$$

$$-\rho \cdot g \cdot \sin(\theta) \cdot 2 \cdot h + \mu_2 \cdot c_3 = 0$$

$$-\rho \cdot g \cdot \sin(\theta) \cdot h + \mu_1 \cdot c_1 = -\rho \cdot g \cdot \sin(\theta) \cdot h + \mu_2 \cdot c_3$$

Hence, after some algebra $c_1 = \frac{2\cdot\rho\cdot g\cdot\sin(\theta)\cdot h}{\mu_1}$ $c_2 = 0$

$$c_3 = \frac{2\cdot\rho\cdot g\cdot\sin(\theta)\cdot h}{\mu_2} \qquad c_4 = 3\cdot\rho\cdot g\cdot\sin(\theta)\cdot h^2\cdot\frac{(\mu_2-\mu_1)}{2\cdot\mu_1\cdot\mu_2}$$

The velocity distributions are then $u_1 = \frac{\rho\cdot g\cdot\sin(\theta)}{2\cdot\mu_1}\cdot\left(4\cdot y\cdot h - y^2\right)$ $u_2 = \frac{\rho\cdot g\cdot\sin(\theta)}{2\cdot\mu_2}\cdot\left[3\cdot h^2\cdot\frac{(\mu_2-\mu_1)}{\mu_1} + 4\cdot y\cdot h - y^2\right]$

Rewriting in terms of ν_1 and ν_2 (ρ is constant and equal for both fluids)

$$u_1 = \frac{g\cdot\sin(\theta)}{2\cdot\nu_1}\cdot\left(4\cdot y\cdot h - y^2\right) \qquad u_2 = \frac{g\cdot\sin(\theta)}{2\cdot\nu_2}\cdot\left[3\cdot h^2\cdot\frac{(\nu_2-\nu_1)}{\nu_1} + 4\cdot y\cdot h - y^2\right]$$

(Note that these result in the same expression if $\nu_1 = \nu_2$, i.e., if we have one fluid)

Evaluating either velocity at $y = h$, gives the velocity at the interface

$$u_{interface} = \frac{3\cdot g\cdot h^2\cdot\sin(\theta)}{2\cdot\nu_1} \qquad u_{interface} = 0.23\,\frac{m}{s}$$

Evaluating u_2 at $y = 2h$ gives the velocity at the free surface

$$u_{freesurface} = g\cdot h^2\cdot\sin(\theta)\cdot\frac{(3\cdot\nu_2+\nu_1)}{2\cdot\nu_1\cdot\nu_2} \qquad u_{freesurface} = 0.268\,\frac{m}{s}$$

The velocity distributions are plotted in the associated *Excel* workbook

Problem 8.30

[3]

8.30 Two immiscible fluids of equal density are flowing down a surface inclined at a 30° angle. The two fluid layers are of equal thickness $h = 2.5$ mm; the kinematic viscosity of the upper fluid is twice that of the lower fluid, which is $\nu_{\text{lower}} = 2 \times 10^{-4}$ m²/s. Find the velocity at the interface and the velocity at the free surface. Plot the velocity distribution.

Given: Data on flow of liquids down an incline

Find: Velocity at interface; velocity at free surface; plot

Solution:

h = 2.5 mm

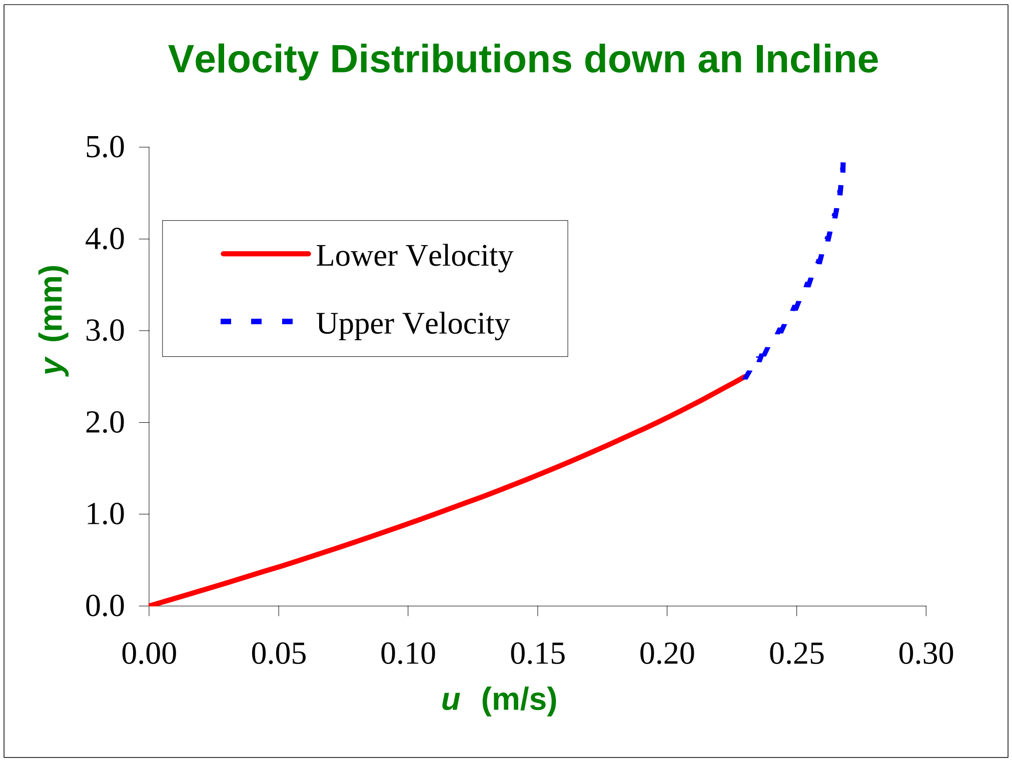
θ = 30 deg

ν_1 = 2.00E-04 m^2/s

ν_2 = 4.00E-04 m^2/s

$$u_1 = \frac{g \cdot \sin(\theta)}{2 \cdot \nu_1} \cdot \left(4 \cdot y \cdot h - y^2\right) \qquad u_2 = \frac{g \cdot \sin(\theta)}{2 \cdot \nu_2} \cdot \left[3 \cdot h^2 \cdot \frac{(\nu_2 - \nu_1)}{\nu_1} + 4 \cdot y \cdot h - y^2\right]$$

y (mm)	u_1 (m/s)	u_2 (m/s)
0.000	0.000	
0.250	0.0299	
0.500	0.0582	
0.750	0.0851	
1.000	0.110	
1.250	0.134	
1.500	0.156	
1.750	0.177	
2.000	0.196	
2.250	0.214	
2.500	0.230	0.230
2.750		0.237
3.000		0.244
3.250		0.249
3.500		0.254
3.750		0.259
4.000		0.262
4.250		0.265
4.500		0.267
4.750		0.268
5.000		0.268



Problem 8.31

[3]

8.31 Consider fully developed flow between parallel plates with the upper plate moving at $U = 5$ ft/s; the spacing between the plates is $a = 0.1$ in. Determine the flow rate per unit depth for the case of zero pressure gradient. If the fluid is air, evaluate the shear stress on the lower plate and plot the shear stress distribution across the channel for the zero pressure gradient case. Will the flow rate increase or decrease if the pressure gradient is adverse? Determine the pressure gradient that will give zero shear stress at $y = 0.25a$. Plot the shear stress distribution across the channel for the latter case.

Given: Flow between parallel plates

Find: Shear stress on lower plate; Plot shear stress; Flow rate for pressure gradient; Pressure gradient for zero shear; Plot

Solution:

From Section 8-2 $$u(y) = \frac{U \cdot y}{a} + \frac{a^2}{2 \cdot \mu} \cdot \frac{dp}{dx} \cdot \left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right]$$

For dp/dx = 0 $$u = U \cdot \frac{y}{a} \qquad \frac{Q}{l} = \int_0^a u(y)\,dy = w \cdot \int_0^a U \cdot \frac{y}{a}\,dy = \frac{U \cdot a}{2} \qquad Q = \frac{1}{2} \times 5 \cdot \frac{ft}{s} \times \frac{0.1}{12} \cdot ft \qquad Q = 0.0208 \frac{\frac{ft^3}{s}}{ft}$$

For the shear stress $$\tau = \mu \cdot \frac{du}{dy} = \frac{\mu \cdot U}{a} \quad \text{when } dp/dx = 0 \qquad \mu = 3.79 \times 10^{-7} \cdot \frac{lbf \cdot s}{ft^2} \quad \text{(Table A.9)}$$

The shear stress is constant - no need to plot!

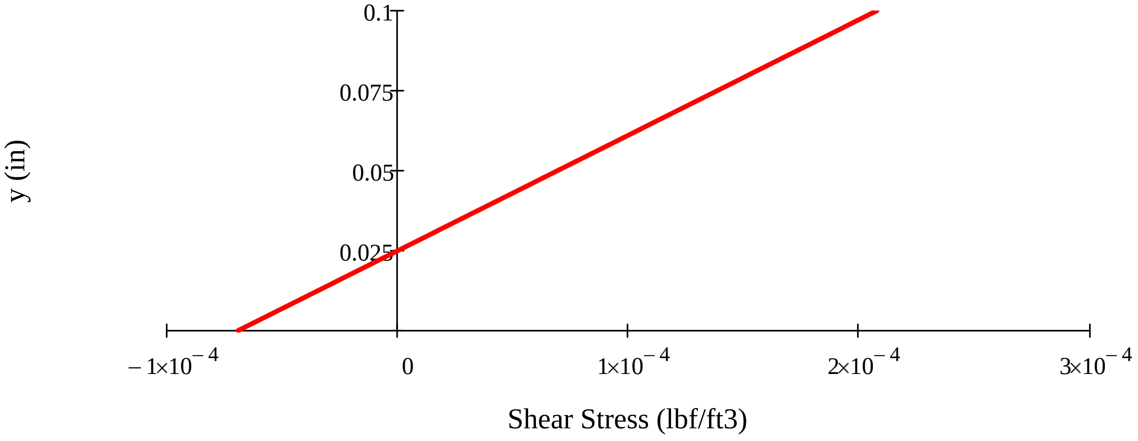
$$\tau = 3.79 \times 10^{-7} \cdot \frac{lbf \cdot s}{ft^2} \times 5 \cdot \frac{ft}{s} \times \frac{12}{0.1 \cdot ft} \times \left(\frac{1 \cdot ft}{12 \cdot in}\right)^2 \qquad \tau = 1.58 \times 10^{-6} psi$$

Q will decrease if dp/dx > 0; it will increase if dp/dx < 0.

For non- zero dp/dx: $$\tau = \mu \cdot \frac{du}{dy} = \frac{\mu \cdot U}{a} + a \cdot \frac{dp}{dx} \cdot \left(\frac{y}{a} - \frac{1}{2}\right)$$

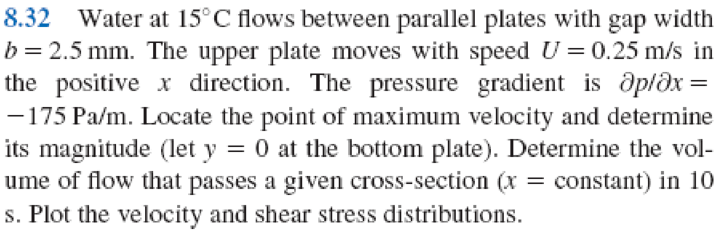
At y = 0.25a, we get $$\tau(y = 0.25 \cdot a) = \mu \cdot \frac{U}{a} + a \cdot \frac{dp}{dx} \cdot \left(\frac{1}{4} - \frac{1}{2}\right) = \mu \cdot \frac{U}{a} - \frac{a}{4} \cdot \frac{dp}{dx}$$

Hence this stress is zero when $$\frac{dp}{dx} = \frac{4 \cdot \mu \cdot U}{a^2} = 4 \times 3.79 \times 10^{-7} \cdot \frac{lbf \cdot s}{ft^2} \times 5 \cdot \frac{ft}{s} \times \left(\frac{12}{0.1 \cdot ft}\right)^2 = 0.109 \cdot \frac{\frac{lbf}{ft^2}}{ft} = 7.58 \times 10^{-4} \frac{psi}{ft}$$



Problem 8.32

[3]



8.32 Water at 15°C flows between parallel plates with gap width $b = 2.5$ mm. The upper plate moves with speed $U = 0.25$ m/s in the positive x direction. The pressure gradient is $\partial p/\partial x = -175$ Pa/m. Locate the point of maximum velocity and determine its magnitude (let $y = 0$ at the bottom plate). Determine the volume of flow that passes a given cross-section (x = constant) in 10 s. Plot the velocity and shear stress distributions.

Given: Flow between parallel plates

Find: Location and magnitude of maximum velocity; Volume flow in 10 s; Plot velocity and shear stress

Solution:

From Section 8-2 $$u(y) = \frac{U\cdot y}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]$$

For u_{max} set du/dx = 0 $$\frac{du}{dy} = 0 = \frac{U}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left(\frac{2\cdot y}{b^2} - \frac{1}{a}\right) = \frac{U}{b} + \frac{1}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot(2\cdot y - b)$$

Hence $u = u_{max}$ at $y = \dfrac{b}{2} - \dfrac{\mu\cdot U}{b\cdot\dfrac{dp}{dx}}$ From Table A.8 at 15ºC $\mu = 1.14\times 10^{-3}\cdot\dfrac{N\cdot s}{m^2}$

$$y = \frac{0.0025\cdot m}{2} - 1.14\times 10^{-3}\cdot\frac{N\cdot s}{m^2}\times 0.25\cdot\frac{m}{s}\times\frac{1}{0.0025\cdot m}\times\left(-\frac{m^3}{175\cdot N}\right)$$

$y = 1.90\times 10^{-3}\cdot m$ $y = 1.90\cdot mm$

Hence $$u_{max} = \frac{U\cdot y}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]$$ with y = 1.90 mm

$$u_{max} = 0.25\cdot\frac{m}{s}\times\left(\frac{1.90}{2.5}\right) + \frac{1}{2}\times(0.0025\cdot m)^2\times\frac{m^2}{1.14\times 10^{-3}\cdot N\cdot s}\times\left(-\frac{175\cdot N}{m^3}\right)\times\left[\left(\frac{1.90}{2.5}\right)^2 - \left(\frac{1.90}{2.5}\right)\right]$$

$u_{max} = 0.278\dfrac{m}{s}$

$$\frac{Q}{w} = \int_0^b u(y)\,dy = w\cdot\int_0^b\left[\frac{U\cdot y}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]\right]dy = \frac{U\cdot b}{2} - \frac{b^3}{12\cdot\mu}\cdot\frac{dp}{dx}$$

$$\frac{Q}{w} = \frac{1}{2}\times 0.25\cdot\frac{m}{s}\times 0.0025\cdot m - \frac{1}{12}\times(0.0025\cdot m)^3\times\frac{m^2}{1.14\times 10^{-3}\cdot N\cdot s}\times\left(-\frac{175\cdot N}{m^3}\right)$$

$\dfrac{Q}{w} = 5.12\times 10^{-4}\,\dfrac{m^2}{s}$

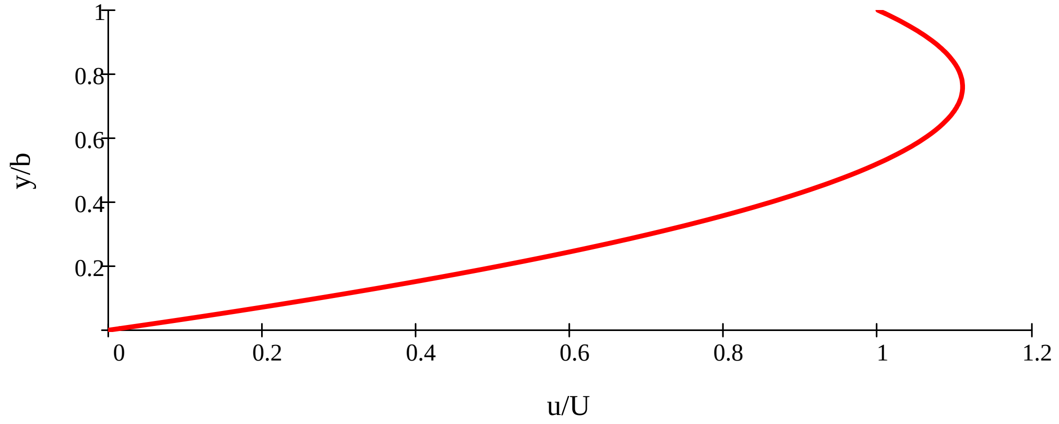
$$\text{Flow} = \frac{Q}{w}\cdot\Delta t = 5.12\times 10^{-4}\,\frac{m^2}{s}\times 10\cdot s$$

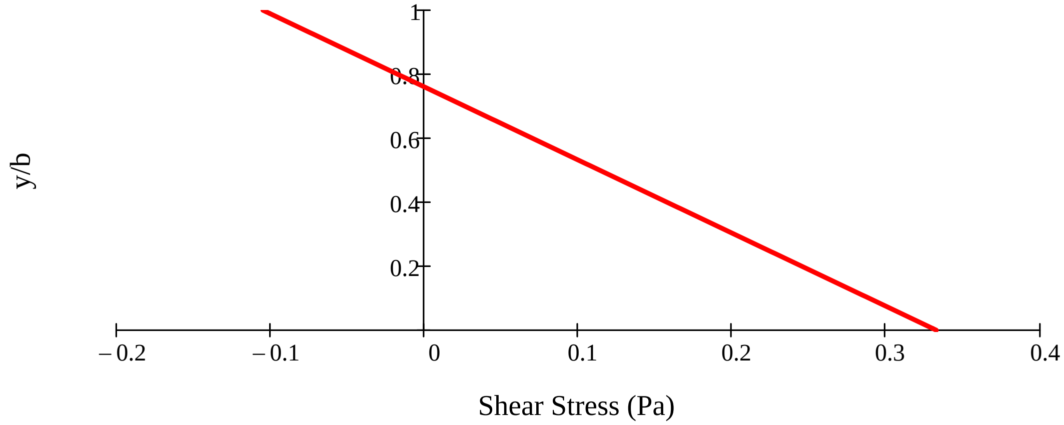
$$\text{Flow} = 5.12\times 10^{-3}\,m^2 = 5.12\times 10^{-3}\,\frac{m^3}{m}$$

The velocity profile is $$\frac{u}{U} = \frac{y}{b} + \frac{b^2}{2\cdot\mu\cdot U}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]$$

For the shear stress $$\tau = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{U}{b} + \frac{b}{2}\cdot\frac{dp}{dx}\cdot\left[2\cdot\left(\frac{y}{b}\right) - 1\right]$$

The graphs on the next page can be plotted in *Excel*


y/b
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
u/U

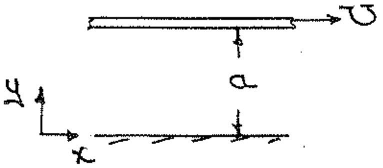

y/b
1
0.8
0.6
0.4
0.2
− 0.2
− 0.1
0
0.1
0.2
0.3
0.4
Shear Stress (Pa)

Problem 8.33

Given: Velocity profile for fully developed laminar flow of air between parallel plates

$$u = \frac{Uy}{a} + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

$U = 2\ \text{m/s}$ $\quad a = 2.5\ \text{mm}$



Find: (a) pressure gradient for which net flow is zero; plot expected $u(y)$ and $\tau_{yx}(y)$

(b) expected $u(y)$ and $\tau_{yx}(y)$ for case where $u = 2U$ at $y/a = 0.5$

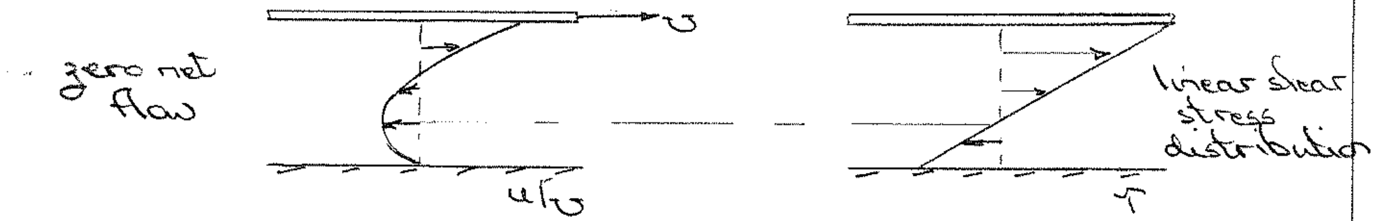
Solution:

Computing equations:
$$Q/l = \frac{Ua}{2} - \frac{a^3}{12\mu}\left(\frac{\partial p}{\partial x}\right) \qquad (8.9b)$$
$$\tau_{yx} = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right] \qquad (8.9a).$$

For $Q = 0$, from Eq. 8.9b (assuming $T = 15°C$)

$$\frac{\partial p}{\partial x} = \frac{6\mu U}{a^2} = 6 \times 1.79\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2\ \frac{\text{m}}{\text{s}} \times \frac{1}{(2.5\times10^{-3}\ \text{m})^2} = 34.4\ \text{N/m}^2/\text{m} \quad \xleftarrow{} \frac{\partial p}{\partial x}$$

For this adverse pressure gradient



(b) For $u = 2U$ at $y/a = 0.5$

$$2U = 0.5U + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\frac{1}{4} - \frac{1}{2}\right] \quad \text{and} \quad \frac{3}{2}U = -\frac{a^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial p}{\partial x} = -\frac{12U\mu}{a^2} = -12 \times 2\frac{\text{m}}{\text{s}} \times 1.79\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times \frac{1}{(2.5\times10^{-3}\ \text{m})^2} = -68.7\ \text{N/m}^2/\text{m}$$

$$\tau = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right] \qquad \{\text{shear stress is linear}\}$$

$y = 0$ $\quad \tau = \mu\frac{U}{a} - \frac{a}{2}\left(\frac{\partial p}{\partial x}\right) = 1.79\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times \frac{2\ \text{m}}{\text{s}} \times \frac{1}{2.5\times10^{-3}\ \text{m}} - \frac{2.5\times10^{-3}\ \text{m}}{2}\left(-68.7\frac{\text{N}}{\text{m}^3}\right) = 0.10\frac{\text{N}}{\text{m}^2}$

$y = a$ $\quad \tau = \mu\frac{U}{a} + \frac{a}{2}\left(\frac{\partial p}{\partial x}\right) = -0.0716\ \text{N/m}^2$

Note that the point of zero shear stress is not at $y/a = 0.5$ and hence $y/a = 0.5$ is not the location of maximum velocity. Maximum velocity occurs at $y/a > 0.5$.

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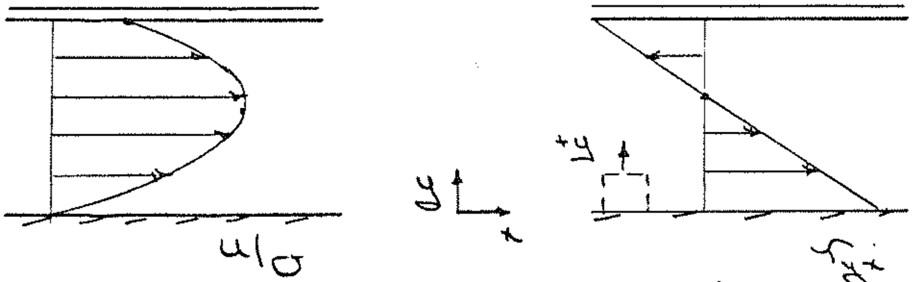
Problem 8.33

To find the location of zero shear set $\tau_{yx} = 0$, then

$$0 = \frac{\mu U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left(\frac{y}{a} - \frac{1}{2}\right) \quad \text{and} \quad \frac{y}{a} = \frac{1}{2} - \frac{\mu U}{a^2\left(\frac{\partial p}{\partial x}\right)}.$$

$$\frac{y}{a} = 0.5 - 1.79\times10^{-5}\,\frac{N\cdot s}{m^2} \times \frac{2\,m}{s} \times \frac{1}{(2.5\times10^{-3}\,m)^2} \times \frac{1}{(-68.7)}\,\frac{m^3}{N} = 0.583$$

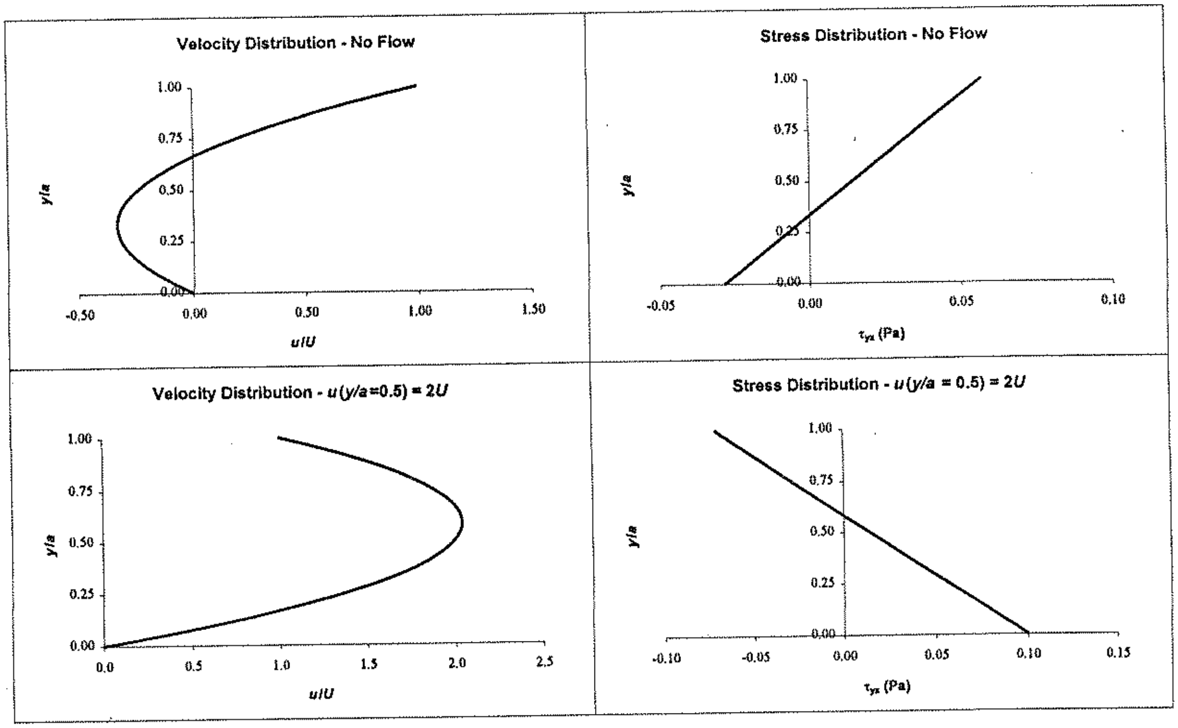
For this case ($u = 2U$ at $y/a = 0.5$) the velocity and shear stress distributions would be as follows.



The shear stress is positive ($du/dy > 0$) below $y/a = 0.583$; positive stress acts in positive x direction on a positive y surface.

The shear stress is negative ($du/dy < 0$) above $y/a = 0.583$; negative stress acts in the negative x direction on a positive y surface.

From *Excel*, the plots are


Velocity Distribution - No Flow

Stress Distribution - No Flow

Velocity Distribution - $u(y/a=0.5) = 2U$

Stress Distribution - $u(y/a = 0.5) = 2U$

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Problem 8.34

[4]

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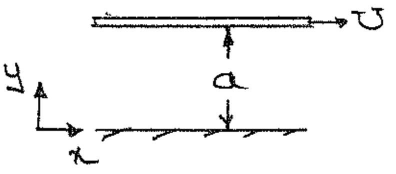
Given: Velocity profile for fully developed laminar flow of water between parallel plates

$$u = \frac{Uy}{a} + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right]$$

$U = 2\ \text{m/s} \qquad a = 2.5\ \text{mm}$



Find: (a) Volum flow rate for zero pressure gradient.
(b) shear stress on lower plate; sketch $\tau(y)$
(c) effect of mild adverse pressure gradient on Q
(d) pressure gradient for zero shear at $y/a = 0.25$; sketch $\tau(y)$.

Solution:

Computing equations: $\tau_{yx} = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$ (8.9a)

$Q/l = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$ (8.9b)

For $\partial p/\partial x = 0$, $Q/l = \frac{Ua}{2} = \frac{1}{2} \times 2\,\frac{\text{m}}{\text{s}} \times 2.5\times10^{-3}\,\text{m} = 2.50\times10^{-3}\ \text{m}^3/\text{s}/\text{m}$ Q

The shear stress is $\tau_{yx} = \mu\frac{U}{a}$ {At 15°C, $\mu = 1.14\times10^{-3}\ \text{N}\cdot\text{s}/\text{m}^2$ (Table A.8}

$\tau_{yx} = 1.14\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2\,\frac{\text{m}}{\text{s}} \times \frac{1}{2.5\times10^{-3}\,\text{m}} = 0.912\ \text{N}/\text{m}^2$ $\longleftarrow$ τ_{yx}

The shear stress is constant across the channel (curve 1 below)

For $\partial p/\partial x > 0$, Eq. 8.9b indicates that Q will decrease.

For $\tau = 0$ at $y/a = 0.25$

$$\tau_{yx} = 0 = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{1}{4} - \frac{1}{2}\right] = \mu\frac{U}{a} - \frac{a}{4}\left(\frac{\partial p}{\partial x}\right)$$

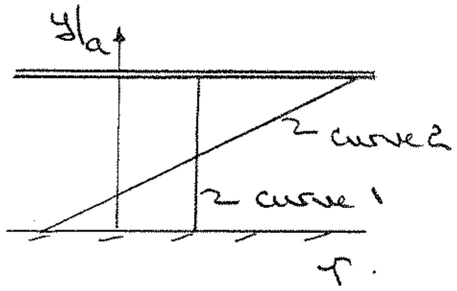
$$\frac{\partial p}{\partial x} = \frac{4\mu U}{a^2} = 4\times1.14\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2\,\frac{\text{m}}{\text{s}} \times \frac{1}{(2.5\times10^{-3}\,\text{m})^2} = 1.46\ \text{kN}/\text{m}^2/\text{m}$$ $\longleftarrow$ $\frac{\partial p}{\partial x}$

For this pressure gradient

$$\tau_{yx} = 1.14\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2\,\frac{\text{m}}{\text{s}} \times \frac{1}{2.5\times10^{-3}\,\text{m}} + 2.5\times10^{-3}\,\text{m} \times 1.46\times10^{3}\,\frac{\text{N}}{\text{m}^3}\left[\frac{y}{a} - 0.5\right]$$

$$\tau_{yx} = 0.912\ \text{N}/\text{m}^2 + 3.65\,\frac{\text{N}}{\text{m}^2}\left(\frac{y}{a} - 0.5\right)$$

$$\left.\begin{aligned} \tau_{yx})_{y=0} &= -0.913\ \text{N}/\text{m}^2 \\ \tau_{yx})_{y=a} &= 2.74\ \text{N}/\text{m}^2. \end{aligned}\right\} \text{curve 2}$$



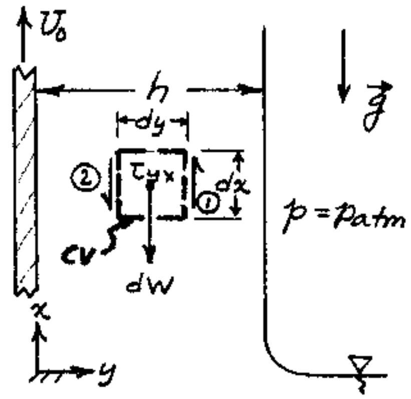
Problem 8.35

[3]

Given: Belt moving steadily through bath as shown.

Assume zero shear at film/air surface, and no pressure forces.

Find: (a) Boundary conditions for velocity at $y=0$, $y=h$.
(b) Velocity profile.



Bath: ρ, μ

Solution: Choose CV $dx\,dy\,dz$ as shown.
Apply x component of momentum equation.

Basic equations:

$$F_{S_x} + F_{B_x} = \overset{=0(2)}{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \overset{=0(3)}{\int_{CS} u\rho \vec{V}\cdot d\vec{A}} \quad ; \quad \tau_{yx} = \mu \frac{du}{dy} = \tau$$

Assumptions: (1) F_{S_x} due to shear forces only
(2) Steady flow
(3) Fully-developed flow

Then

$$F_{S_x} + F_{B_x} = F_{①} - F_{②} + F_{B_x} = \left(\tau + \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz - \left(\tau - \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz - \rho g\, dx\,dy\,dz = 0$$

or $\dfrac{d\tau}{dy} = \rho g$. Integrating

$$\tau = \rho g y + c_1 = \mu \frac{du}{dy} \quad \text{or} \quad \frac{du}{dy} = \frac{\rho g y}{\mu} + \frac{c_1}{\mu}. \text{ Integrating again,}$$

$$u = \frac{\rho g y^2}{2\mu} + \frac{c_1}{\mu} y + c_2$$

To evaluate the constants c_1 and c_2, apply the boundary conditions:

At $y=0$, $u = U_0$, so $c_2 = U_0$

At $y=h$, $\tau = 0$, so $\dfrac{du}{dy} = 0$, and $c_1 = -\rho g h$ ← BC

Substituting,

$$u = \frac{\rho g y^2}{2\mu} - \frac{\rho g h y}{\mu} + U_0 = \frac{\rho g}{\mu}\left(\frac{y^2}{2} - hy\right) + U_0$$ ← u

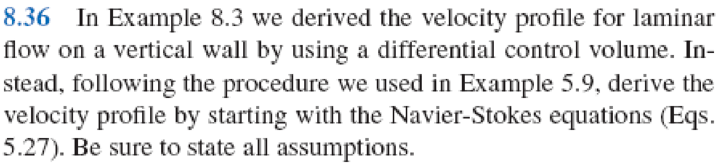
{Note that at $y=h$,

$$u = \frac{\rho g}{\mu}\left(-\frac{h^2}{2}\right) + U_0 \neq 0$$

Thus the solution is determined only when U_0 and h are known.}

Problem 8.36

[2]



8.36 In Example 8.3 we derived the velocity profile for laminar flow on a vertical wall by using a differential control volume. Instead, following the procedure we used in Example 5.9, derive the velocity profile by starting with the Navier-Stokes equations (Eqs. 5.27). Be sure to state all assumptions.

Given: Navier-Stokes Equations

Find: Derivation of Eq. 8.5

Solution:

The Navier-Stokes equations are (using the coordinates of Example 8.3, so that *x* is vertical, *y* is horizontal)

$$\cancelto{4}{\frac{\partial u}{\partial x}}+\frac{\partial v}{\partial y}+\cancelto{3}{\frac{\partial w}{\partial z}}=0 \qquad (5.1c)$$

$$\rho\left(\cancelto{1}{\frac{\partial u}{\partial t}}+u\cancelto{4}{\frac{\partial u}{\partial x}}+v\cancelto{5}{\frac{\partial u}{\partial y}}+w\cancelto{3}{\frac{\partial u}{\partial z}}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\cancelto{4}{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}+\cancelto{3}{\frac{\partial^2 u}{\partial z^2}}\right) \qquad (5.27a)$$

$$\rho\left(\cancelto{1}{\frac{\partial v}{\partial t}}+u\cancelto{4}{\frac{\partial v}{\partial x}}+v\cancelto{5}{\frac{\partial v}{\partial y}}+w\cancelto{3}{\frac{\partial v}{\partial z}}\right)=\cancelto{6}{\rho g_y}-\frac{\partial p}{\partial y}+\mu\left(\cancelto{4}{\frac{\partial^2 v}{\partial x^2}}+\cancelto{5}{\frac{\partial^2 v}{\partial y^2}}+\cancelto{3}{\frac{\partial^2 v}{\partial z^2}}\right) \qquad (5.27b)$$

$$\rho\left(\cancelto{1}{\frac{\partial w}{\partial t}}+u\cancelto{3}{\frac{\partial w}{\partial x}}+v\cancelto{3}{\frac{\partial w}{\partial y}}+w\cancelto{3}{\frac{\partial w}{\partial z}}\right)=\cancelto{3}{\rho g_z}-\cancelto{3}{\frac{\partial p}{\partial z}}+\mu\left(\cancelto{3}{\frac{\partial^2 w}{\partial x^2}}+\cancelto{3}{\frac{\partial^2 w}{\partial y^2}}+\cancelto{3}{\frac{\partial^2 w}{\partial z^2}}\right) \qquad (5.27c)$$

The following assumptions have been applied:

(1) Steady flow (given).
(2) Incompressible flow; ρ = constant.
(3) No flow or variation of properties in the *z* direction; $w = 0$ and $\partial/\partial z = 0$.
(4) Fully developed flow, so no properties except possibly pressure *p* vary in the *x* direction; $\partial/\partial x = 0$.
(5) See analysis below.
(6) No body force in the *y* direction; $g_y = 0$

Assumption (1) eliminates time variations in any fluid property. Assumption (2) eliminates space variations in density. Assumption (3) states that there is no *z* component of velocity and no property variations in the *z* direction. All terms in the *z* component of the Navier–Stokes equation cancel. After assumption (4) is applied, the continuity equation reduces to $\partial v/\partial y = 0$. Assumptions (3) and (4) also indicate that $\partial v/\partial z = 0$ and $\partial v/\partial x = 0$. Therefore *v* must be constant. Since *v* is zero at the solid surface, then *v* must be zero everywhere. The fact that $v = 0$ reduces the Navier–Stokes equations further, as indicated by (5). Hence for the *y* direction

$$\frac{\partial p}{\partial y}=0$$

which indicates the pressure is a constant across the layer. However, at the free surface $p = p_{atm}$ = constant. Hence we conclude that *p* = constant throughout the fluid, and so

$$\frac{\partial p}{\partial x}=0$$

In the x direction, we obtain

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho g = 0$$

Integrating twice

$$u = -\frac{1}{2\mu}\rho g y^2 + \frac{c_1}{\mu} y + c_2$$

To evaluate the constants, c_1 and c_2, we must apply the boundary conditions. At $y = 0$, $u = 0$. Consequently, $c_2 = 0$. At $y = a$, $du/dy = 0$ (we assume air friction is negligible). Hence

$$\tau(y = \delta) = \mu \left.\frac{du}{dy}\right|_{y=\delta} = -\frac{1}{\mu}\rho g \delta + \frac{c_1}{\mu} = 0$$

which gives

$$c_1 = \rho g \delta$$

and finally

$$u = -\frac{1}{2\mu}\rho g y^2 + \frac{\rho g}{\mu} y = \frac{\rho g}{\mu}\delta^2 \left[\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

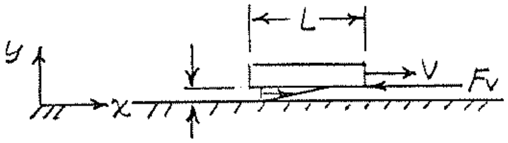
Problem 8.37

[4]

Given: Microchip supported on air film, on a horizontal surface. Chips are $L = 11.7$ mm long, $w = 9.35$ mm wide, and have mass $m = 0.325$ g. The air film is $h = 0.125$ mm thick. The initial speed of the chips is $V_0 = 1.75$ mm/s; they slow from viscous shear.

Find: (a) Differential equation for chip motion during deceleration.
(b) Time required for chip to lose 5 percent of V_0.
(c) Plot of chip speed vs. time, with labels and comments.

Solution: Apply Newton's law of viscosity



Basic equations: $\tau_{yx} = \mu \dfrac{du}{dy}$

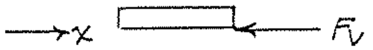
$$F_V = \tau A \qquad \Sigma F = m a_x$$

Assume: (1) Newtonian fluid
(2) Linear velocity profile in narrow gap
(3) Air at STP

Then

$$\tau_{yx} = \mu \frac{du}{dy} = \mu \frac{V}{h}; \quad F_V = \tau A = \mu \frac{V}{h} wL = \frac{\mu V w L}{h}$$

The free-body diagram for the chip is



$$\Sigma F_x = -F_V = -\frac{\mu V w L}{h} = m\frac{dV}{dt}; \quad \frac{dV}{V} = -\frac{\mu w L}{mh}\,dt$$

D.E.

Integrating, $\displaystyle\int_{V_0}^{V} \frac{dV}{V} = \ln \frac{V}{V_0} = -\frac{\mu w L}{mh} t$

Thus

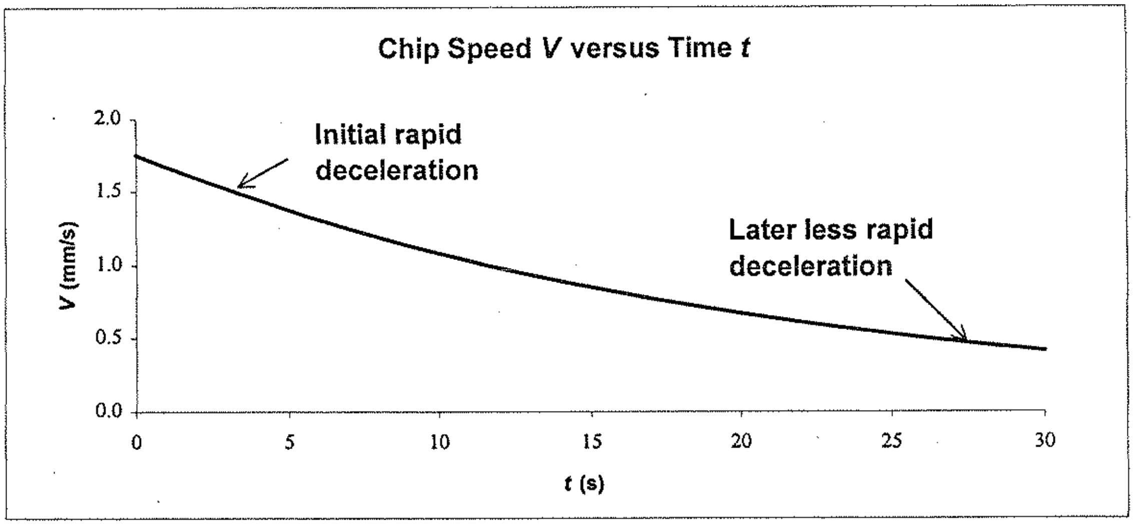
$$t = -\frac{mh}{\mu w L} \ln \frac{V}{V_0}$$

$$t = -0.325\,\text{g} \times 0.125\,\text{mm} \times \frac{\text{m}\cdot\text{s}}{1.79\times10^{-5}\,\text{kg}} \times \frac{1}{9.35\,\text{mm}} \times \frac{1}{11.7\,\text{mm}} \times \ln 0.95 \times \frac{\text{kg}}{1000\,\text{g}} \times \frac{1000\,\text{mm}}{\text{m}}$$

$t = 1.06$ s

t

From *Excel*, the plot of speed vs. time is:



Problem 8.38

[4]

Given: Free-surface waves begin to form on a laminar liquid film flowing down an inclined surface whenever the Reynolds number, based on mass flow per unit width of film, is larger than about 33.

Find: Estimate of the maximum thickness of a laminar film of water that remains free from waves while flowing down a vertical surface.

Solution: The mass flow rate is $\dot{m} = \rho \bar{V} A = \rho \bar{V} w \delta$, so $\dot{m}/w = \rho \bar{V} \delta$.

Thus

$$Re = \frac{\rho \bar{V} \delta}{\mu} = \frac{\bar{V}\delta}{\nu} = 33 \text{ (maximum)}$$

Using the result for average velocity from Example 8.3

$$\bar{V} = \frac{\rho g \delta^2}{3\mu}$$

Thus

$$\frac{\rho \bar{V} \delta}{\mu} = \frac{\rho^2 g \delta^3}{3\mu^2} = 33$$

Solving for δ,

$$\delta = \left[\frac{99\mu^2}{\rho^2 g}\right]^{1/3}$$

At $T = 20°\text{C}$, $\mu = 1.00\times10^{-3}\ \text{kg/m}\cdot\text{s}$ and $\rho = 998\ \text{kg/m}^3$ (Table A.8). Substituting,

$$\delta = \left[99 \times (1.00\times10^{-3})^2 \frac{\text{kg}^2}{\text{m}^2\cdot\text{s}^2} \times \frac{\text{m}^6}{(998)^2\ \text{kg}^2} \times \frac{\text{s}^2}{9.81\ \text{m}}\right]^{1/3}$$

$$\delta = 2.16\times10^{-4}\ \text{m} \quad \text{or} \quad 0.216\ \text{mm} \longleftarrow \delta_{max}$$

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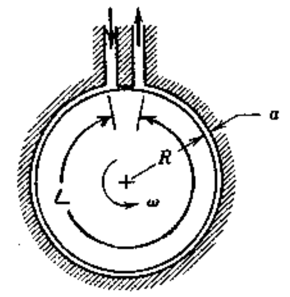
Problem 8.39

[5]

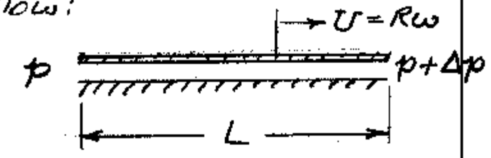
Given: Viscous-shear pump, as shown.

b = width normal to diagram; $a \ll R$



Find: Performance characteristics
(a) Pressure differential
(b) Input power
(c) Efficiency
as functions of volume flow rate.

Solution: Since $a \ll R$, unwrap to form flow between parallel plates. Apply Eqs. 8.9 to fully developed flow:



Volume flow rate is $\frac{Q}{b} = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$

Substituting $U = R\omega$ and $\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$, then

$$\Delta p = \frac{12\mu L}{a^3}\left(\frac{\omega R a}{2} - \frac{Q}{b}\right) = \frac{6\mu L R\omega}{a^2}\left(1 - \frac{2Q}{abR\omega}\right)$$

Δp

Torque is $T = \tau R(bL) = RLb\tau$. Power is $P = T\omega$. From Eq. 8.9a, at $y = a$,

$$P = RLb\omega\left[\mu\frac{R\omega}{a} + \frac{\Delta p}{L}\frac{a}{2}\right] = RLb\omega\left[\mu\frac{R\omega}{a} + \frac{6\mu LR\omega}{a^2}\left(1 - \frac{2Q}{abR\omega}\right)\frac{a}{2L}\right]$$

$$P = RLb\omega\left[\frac{\mu R\omega}{a}\left(4 - \frac{6Q}{abR\omega}\right)\right] = \frac{\mu Lb(R\omega)^2}{a}\left(4 - \frac{6Q}{abR\omega}\right)$$

P

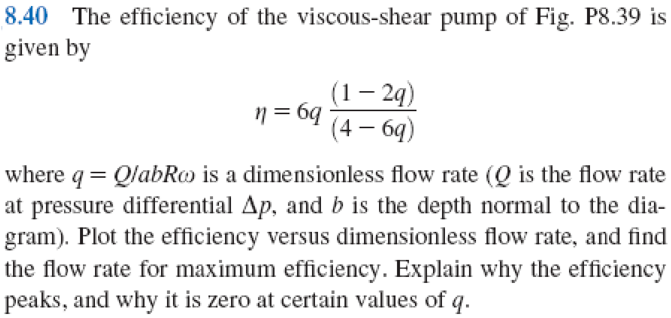
Output power is $Q\Delta p$, so efficiency is

$$\eta = \frac{Q\Delta p}{P} = \frac{6\mu QLR\omega}{a^2}\left(1 - \frac{2Q}{abR\omega}\right)\frac{a}{\mu Lb(R\omega)^2}\frac{1}{\left(4 - \frac{6Q}{abR\omega}\right)}$$

$$\eta = \frac{6Q}{abR\omega}\frac{\left(1 - \frac{2Q}{abR\omega}\right)}{\left(4 - \frac{6Q}{abR\omega}\right)}$$

η

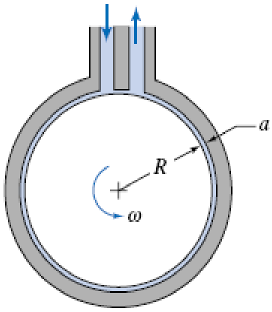
Problem 8.40 (In Excel)



8.40 The efficiency of the viscous-shear pump of Fig. P8.39 is given by

$$\eta = 6q\,\frac{(1-2q)}{(4-6q)}$$

where $q = Q/abR\omega$ is a dimensionless flow rate (Q is the flow rate at pressure differential Δp, and b is the depth normal to the diagram). Plot the efficiency versus dimensionless flow rate, and find the flow rate for maximum efficiency. Explain why the efficiency peaks, and why it is zero at certain values of q.



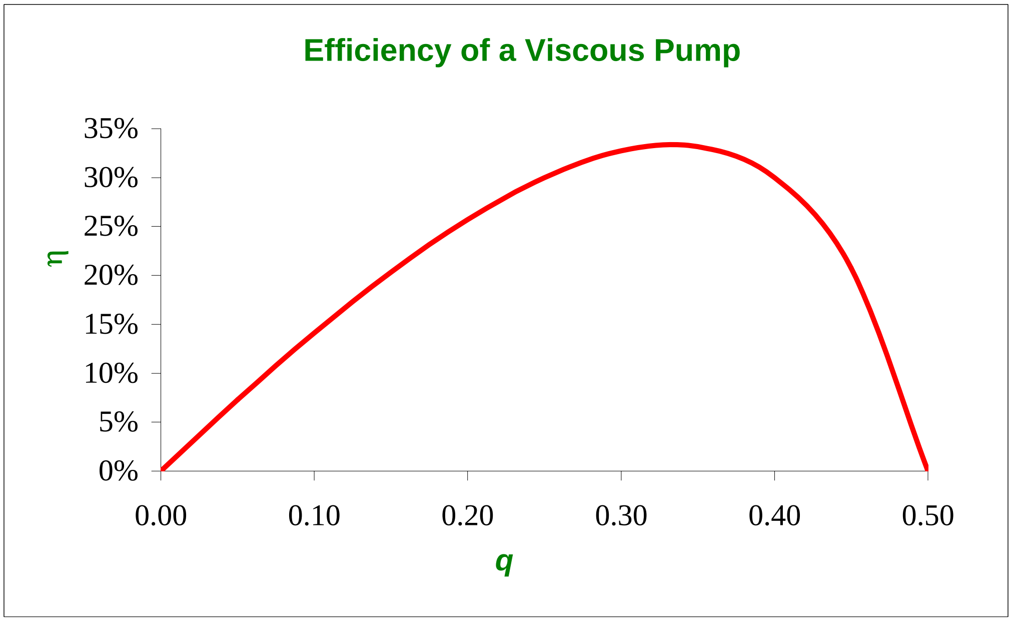
P8.39, P8.40

Given: Expression for efficiency

Find: Plot; find flow rate for maximum efficiency; explain curve

Solution:

q	η
0.00	0.0%
0.05	7.30%
0.10	14.1%
0.15	20.3%
0.20	25.7%
0.25	30.0%
0.30	32.7%
0.35	33.2%
0.40	30.0%
0.45	20.8%
0.50	0.0%



For the maximum efficiency point we can use *Solver* (or alternatively differentiate)

q	η
0.333	33.3%

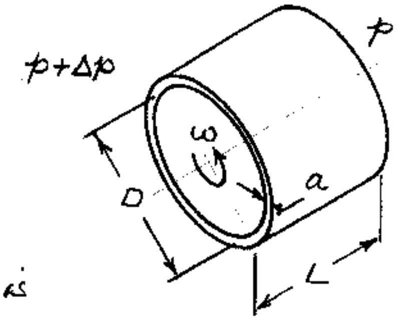
The efficiency is zero at zero flow rate because there is no output at all
The efficiency is zero at maximum flow rate $\Delta p = 0$ so there is no output
The efficiency must therefore peak somewhere between these extremes

Problem 8.41

Given: Annular gap seal as shown.

Power required to pump oil, P_p.

Power to overcome viscous dissipation, P_V.



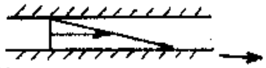
Find: (a) Expressions for P_p, P_V

(b) Show total power minimized when a is chosen so that $P_V = 3P_p$.

Solution: Apply Eqs. 8.6 and 8.9 for flow between parallel plates.

Assumptions: (1) $a \ll D$, so unfold to flat plates

(2) No pressure gradient circumferentially


$\rightarrow V = \omega R = \frac{\omega D}{2}$

The viscous power is the product of viscous torque times ω:

$$P_V = T\omega = \tau(2\pi RL)R\omega = \mu\frac{V}{a}\left(2\pi\frac{D}{2}L\right)\frac{D}{2}\omega = \mu\frac{\omega D}{2a}\pi DL\frac{D}{2}\omega = \frac{\pi\mu\omega^2D^3L}{4a}$$

P_V

The pump power is the product of flow rate times pressure drop.

$$P_p = Q\Delta p$$

From Eq. 8.6c, $Q = \frac{la^3\Delta p}{12\mu L} = \frac{\pi Da^3\Delta p}{12\mu L}$, so $P_p = \frac{\pi Da^3\Delta p^2}{12\mu L}$

P_p

The total power required is $P_T = P_V + P_p = \frac{\pi\mu\omega^2D^3L}{4a} + \frac{\pi Da^3\Delta p^2}{12\mu L}$

It may be minimized by setting $\frac{dP_T}{da} = 0$. Thus

$$\frac{dP_T}{da} = -\frac{\pi\mu\omega^2D^3L}{4a^2} + \frac{\pi Da^2\Delta p^2}{4\mu L} = 0 \qquad (1)$$

This can be written

$$\frac{dP_T}{da} = -\frac{1}{a}P_V + \frac{3}{a}P_p = 0$$

which is satisfied when $3P_p - P_V = 0$ or $P_V = 3P_p$

Optimu

Equation 1 also can be solved for a at optimum conditions:

$$a^4 = \frac{\mu^2\omega^2D^2L^2}{\Delta p^2} \quad \text{or} \quad a^2 = \frac{\mu\omega DL}{\Delta p} \quad \text{or} \quad \frac{a}{D} = \sqrt{\frac{\mu\omega L}{D\Delta p}} \text{ (optimum)}$$

Problem 8.42

Given: "Viscous timer," consisting of a cylindrical mass inside a circular tube filled with viscous liquid, creating a narrow annular gap.

Find: (a) The flow field created when the mass falls under gravity.
(b) Whether this would make a satisfactory timer, and if so, for what range of time intervals.
(c) Effect of temperature change on measured time interval.

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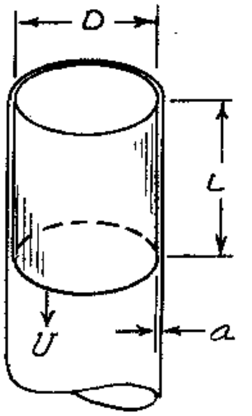
Solution: Apply conservation of mass to a CV enclosing the cylinder and the moving mass:



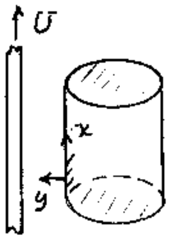
Then: $Q = U\frac{\pi D^2}{4} = \bar{V}\pi D a = \bar{V} l a$ (1)

Assume: (1) Gap is narrow, $a \ll D$
(2) Unroll gap so flat, $l = \pi D$
(3) Steady flow
(4) Fully developed laminar flow

Under these assumptions, the flow field in the gap is that for flow between parallel plates with one plate moving.

Flow Field

Place coordinates on the moving mass:

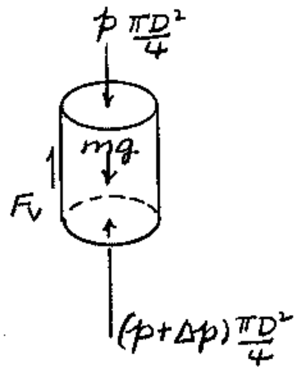


Then the volume flow rate (Eq. 8.9b) is

$$\frac{Q}{l} = \frac{Q}{\pi D} = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$$

But $\frac{\partial p}{\partial x} = -\frac{\Delta p_v}{L}$, where Δp_v is the pressure drop driving viscous flow, so

$$\frac{Q}{l} = \frac{Ua}{2} - \frac{1}{12\mu}\left(-\frac{\Delta p_v}{L}\right)a^3 = \frac{Ua}{2} + \frac{\Delta p_v a^3}{12\mu L} \qquad (2)$$



The pressure change across the moving mass is

$$\Delta p = \rho_l g L + \Delta p_v \qquad (3)$$

Summing forces on the moving mass gives

$$\Sigma F_x = \Delta p\frac{\pi D^2}{4} - mg + F_v = m\cancel{\frac{dU}{dt}}^{=0(3)}$$

But $mg = \rho_m\frac{\pi D^2}{4}L$ and $F_v = \tau_s \pi D L$

From Eq. 8.9a, $\tau_s = \mu\frac{U}{a} - \frac{a}{2}\left(\frac{\partial p}{\partial x}\right) = \mu\frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}$

Substituting, $\Delta p\frac{\pi D^2}{4} - \rho_m\frac{\pi D^2}{4}Lg + \left[\mu\frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}\right]\pi D L = 0$

or $\Delta p = \rho_m g L - \left[\mu\frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}\right]\frac{4L}{D}$ (4)

Problem 8.42

Combining Eqs. 1 and 2 gives $\frac{UD}{4} = \frac{Ua}{2} + \frac{\Delta p_v a^3}{12\mu L}$

Thus

$$\Delta p_v = \frac{12\mu L}{a^3}\left[\frac{UD}{4} - \overset{\ll D}{\cancel{\frac{Ua}{2}}}\right] = \frac{3\mu ULD}{a^3} \qquad (5)$$

Δp_v

Combining Eqs. 3 and 4 gives $\Delta p = \rho_\ell g L + \Delta p_v = \rho_m g L - \left[\mu \frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}\right]\frac{4L}{D}$

Using Eq. 5,

$$\rho_\ell g L + \frac{3\mu ULD}{a^3} = \rho_m g L - \mu \frac{U}{a}\frac{4L}{D} - \frac{a}{2}\,\frac{3\mu ULD}{La^3}\,\frac{4L}{D}$$

Simplifying and re-arranging,

$$(\rho_m - \rho_\ell) g L = \frac{3\mu ULD}{a^3} + \frac{4\mu UL}{aD} + \frac{6\mu UL}{a^2} \approx \frac{3\mu ULD}{a^3}$$

Finally, using $\rho = SG\,\rho_{H_2O}$,

$$U = \frac{(SG_m - SG_\ell)\,\rho_{H_2O}\, g\, a^3}{3\mu D}$$

U

The time interval for the mass to move distance H is

$$\Delta t = \frac{H}{U} = \frac{3\mu D}{(SG_m - SG_\ell)\,\rho_{H_2O}\, g\, a^3} \qquad (6)$$

Δt

Equation 6 shows that the time interval for the mass to fall any distance *H* is proportional to liquid viscosity μ and inversely proportional to gap width *a* cubed. A temperature change would affect the diameter of the measuring tube and the diameter of the falling mass. A temperature change also would affect the viscosity of the liquid in the tube.

Speed of the falling mass is proportional to the cube of gap width. If the coefficient of thermal expansion of the falling mass were greater than that of the glass measuring tube (which seems likely), then the width of the annular gap would decrease with increasing temperature. This would tend to slow the falling mass. The total amount of thermal expansion would depend on the diameter of the mass and tube. The effect on gap width would be greater, the larger the tube diameter compared to the initial gap width.

It might be possible to "tailor" the thermal expansion coefficient of the cylinder, by using a suitable material, to closely match that of the falling mass. Then there would be no differential thermal expansion between the mass and tube, and changes in temperature would not affect the gap width.

Speed of the falling mass is inversely proportional to liquid viscosity. Liquid viscosity decreases sharply as temperature increases (the viscosity of SAE 30 oil drops more than 10 percent as its temperature increases from 20°C to 25°C, see Fig. A.2). This would tend to increase the speed of the falling mass.

The entire device could be maintained at constant temperature.

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Problem 8.43

Open-Ended Design Problem: Automotive design is tending toward all-wheel drive to improve vehicle performance and safety when traction is poor. An all-wheel drive vehicle must have an interaxle differential to allow operation on dry roads. Numerous vehicles are being built using multiplate viscous drives for interaxle differentials. Perform the analysis and design needed to define the torque transmitted by the differential for a given speed difference, in terms of the design parameters. Identify suitable dimensions for a viscous differential to transmit a torque of 150 N · m at a speed loss of 125 rpm, using lubricant with the properties of SAE 30 oil. Discuss how to find the minimum material cost for the differential, if the plate cost per square meter is constant.

Solution: From Problem 2.59, $dT = r\,dF = r\tau\,dA$

But $\tau = \mu\frac{du}{dy} = \mu\frac{u}{h} = \mu\frac{r\Delta\omega}{h}$; $dA = 2\pi r\,dr$

Thus $dT = r\mu\frac{r\Delta\omega}{h}2\pi r\,dr = \frac{2\pi\mu\Delta\omega}{h}r^3\,dr$; $T = \frac{\pi\mu\Delta\omega}{2h}\left[R_o^4 - R_i^4\right]$

or $T = \frac{\pi\mu\Delta\omega}{2h}R^4(1-\alpha^4)$ where $\alpha = R_i/R$ $\longleftarrow$ T

This value is per gap. Each rotor has 2 gaps to a housing. For n gaps

$$T_n = \frac{n\pi\mu\Delta\omega}{2h}R^4(1-\alpha^4) \qquad (1)$$

$\longleftarrow$ T_n

From Eq. 1, assuming $\mu = 0.18$ kg/m·s (Fig. A.2) and $\alpha = \frac{1}{2}$, so $1-\alpha^4 = 1-\frac{1}{16} \approx 1$, then

$$\frac{nR^4}{h} = \frac{2T_n}{\pi\mu\Delta\omega} = \frac{2}{\pi}\times 150\,\text{N}\cdot\text{m}\times\frac{\text{m}^2}{0.18\,\text{N}\cdot\text{s}}\times\frac{\text{min}}{125\,\text{rev}}\times\frac{\text{rev}}{2\pi\,\text{rad}}\times 60\frac{\text{s}}{\text{min}} = 40.5\ \text{m}^3 = C$$

or

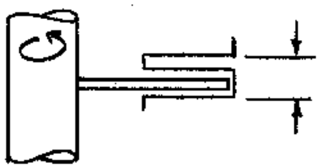
$$R^4 = C\frac{h}{n}$$

For $n = 100$ and $h = 0.2$ mm, $R^4 = 40.5\,\text{m}^3 \times 0.0002\,\text{m} \times \frac{1}{100} = 8.11\times10^{-5}\,\text{m}^4$

$$R = \left[8.11\times10^{-5}\right]^{1/4}\text{m} = 0.0949\,\text{m} \quad (\text{or } D \approx 190\ \text{mm})$$

$\longleftarrow$ R

The stack length might be



≈ 2.5 mm for $n = 2$, or 125 mm for $n = 100$ $\longleftarrow$

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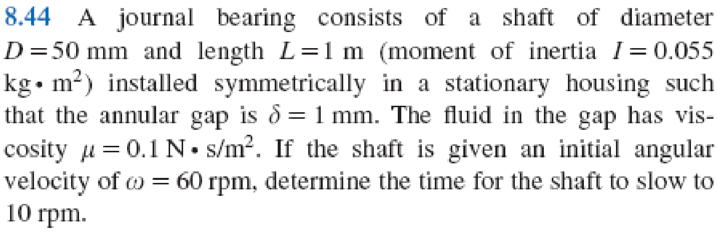
Problem 8.44

[3]



8.44 A journal bearing consists of a shaft of diameter $D = 50$ mm and length $L = 1$ m (moment of inertia $I = 0.055$ kg • m^2) installed symmetrically in a stationary housing such that the annular gap is $\delta = 1$ mm. The fluid in the gap has viscosity $\mu = 0.1$ N • s/m^2. If the shaft is given an initial angular velocity of $\omega = 60$ rpm, determine the time for the shaft to slow to 10 rpm.

Given: Data on a journal bearing

Find: Time for the bearing to slow to 10 rpm

Solution:

The given data is $D = 50 \cdot mm$ $L = 1 \cdot m$ $I = 0.055 \cdot kg \cdot m^2$ $\delta = 1 \cdot mm$

$$\mu = 0.1 \cdot \frac{N \cdot s}{m^2} \qquad \omega_i = 60 \cdot rpm \qquad \omega_f = 10 \cdot rpm$$

The equation of motion for the slowing bearing is $I \cdot \alpha = \text{Torque} = -\tau \cdot A \cdot \frac{D}{2}$

where α is the angular acceleration and τ is the viscous stress, and $A = \pi \cdot D \cdot L$ is the surface area of the bearing

As in Example 8.2 the stress is given by $\tau = \mu \cdot \frac{U}{\delta} = \frac{\mu \cdot D \cdot \omega}{2 \cdot \delta}$

where *U* and ω are the instantaneous linear and angular velocities.

Hence
$$I \cdot \alpha = I \cdot \frac{d\omega}{dt} = -\frac{\mu \cdot D \cdot \omega}{2 \cdot \delta} \cdot \pi \cdot D \cdot L \cdot \frac{D}{2} = -\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta} \cdot \omega$$

Separating variables
$$\frac{d\omega}{\omega} = -\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta \cdot I} \cdot dt$$

Integrating and using IC $\omega = \omega_0$
$$\omega(t) = \omega_i \cdot e^{-\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta \cdot I} \cdot t}$$

The time to slow down to ω_f = 10 rpm is obtained from solving
$$\omega_f = \omega_i \cdot e^{-\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta \cdot I} \cdot t}$$

so
$$t = -\frac{4 \cdot \delta \cdot I}{\mu \cdot \pi \cdot D^3 \cdot L} \cdot \ln\left(\frac{\omega_f}{\omega_i}\right) \qquad t = 10\,s$$

Problem 8.45

Given: Fully developed laminar flow in a pipe, with

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Find: Radius from pipe axis at which u equals the average velocity, $\bar{V}$.

Solution: First determine $\bar{V}$.

$$\bar{V} = \frac{Q}{A} = \frac{1}{\pi R^2}\int_A u\,dA = \frac{1}{\pi R^2}\int_0^R \left\{-\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right]\right\} 2\pi r\,dr$$

$$= -\frac{R^2}{2\mu}\frac{\partial p}{\partial x}\int_0^1 \left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = -\frac{R^2}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^1$$

$$\bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}$$

Then $u = \bar{V}$ when

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right] = \bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}$$

or $$1-\left(\frac{r}{R}\right)^2 = \frac{1}{2}$$

or $$\left(\frac{r}{R}\right)^2 = \frac{1}{2}$$

$$r = \frac{R}{\sqrt{2}} = 0.707R \longleftarrow$$

r

Problem 8.46

[2]

Given: Water and SAE 10W oil flowing at 40°C through a 6 mm tube.

Find, for each fluid:

(a) The maximum flowrate for laminar flow.
(b) The corresponding pressure gradient.

Solution: Laminar flow is expected for $Re \leq 2300$. Expressing this in terms of flowrate,

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{Q D}{A \nu} = \frac{4}{\pi D^2} \frac{Q D}{\nu} = \frac{4Q}{\pi \nu D} \quad \text{or} \quad Q = \frac{\pi \nu D Re}{4}$$

Thus

$$Q_{max} = \frac{\pi \nu D Re_{max}}{4} = \frac{\pi}{4} \times 2300 \times 0.006\,m \times \nu \frac{m^2}{s} = 10.8\,\nu \left(\frac{m^3}{s}\right)$$

Also, $Q = -\frac{\pi R^4}{8\mu} \frac{\partial p}{\partial x}$ for laminar flow, according to Eq. 8.13b. Then

$$\frac{\partial p}{\partial x} = -\frac{8 \mu Q}{\pi R^4} = -\frac{128\,\mu Q}{\pi D^4}$$

so

$$\frac{\partial p}{\partial x} = -\frac{128}{\pi} \times \mu \frac{N \cdot s}{m^2} \times Q \frac{m^3}{s} \times \frac{1}{(0.006)^4\,m^4} = -3.14 \times 10^{10} \mu Q \left(\frac{N}{m^3}\right)$$

Using data from Appendix A, at 40°C,

Fluid	$\nu\left(\frac{m^2}{s}\right)$	$Q\left(\frac{m^3}{s}\right)$	$\mu\left(\frac{N \cdot s}{m^2}\right)$	$\mu Q\,(N \cdot m)$	$\frac{\partial p}{\partial x}\left(\frac{N}{m^3}\right)$
Water	6.57×10^{-7}	7.10×10^{-6}	6.51×10^{-4}	4.62×10^{-9}	-145
SAE 10W oil	3.8×10^{-5}	4.10×10^{-4}	3.4×10^{-2}	1.39×10^{-5}	-4.36×10^{5}

$\frac{\partial p}{\partial x}$

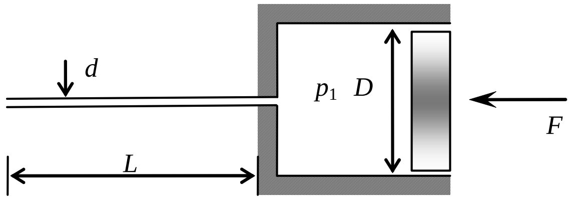
Q_{ma}

$$\left\{ \text{Note } Q \sim \nu = \frac{\mu}{\rho} \text{ and } \frac{\partial p}{\partial x} \sim \mu Q \sim \frac{\mu^2}{\rho}. \right\}$$

Problem 8.47

[2]

8.47 A hypodermic needle, with inside diameter $d = 0.005$ in. and length $L = 1$ in., is used to inject saline solution with viscosity five times that of water. The plunger diameter is $D = 0.375$ in.; the maximum force that can be exerted by a thumb on the plunger is $F = 7.5$ lbf. Estimate the volume flow rate of saline that can be produced.



Given: Hyperdermic needle

Find: Volume flow rate of saline

Solution:

Basic equation $$Q = \frac{\pi \cdot \Delta p \cdot d^4}{128 \cdot \mu \cdot L}$$ (Eq. 8.13c; we assume laminar flow and verify this is correct after solving)

For the system $$\Delta p = p_1 - p_{atm} = \frac{F}{A} = \frac{4 \cdot F}{\pi \cdot D^2}$$

$$\Delta p = \frac{4}{\pi} \times 7.5 \cdot \text{lbf} \times \left(\frac{1}{0.375 \cdot \text{in}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \right)^2 \qquad \Delta p = 67.9 \cdot \text{psi}$$

At 68°F, from Table A.7 $$\mu_{H2O} = 2.1 \times 10^{-5} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2} \qquad \mu = 5 \cdot \mu_{H2O} \qquad \mu = 1.05 \times 10^{-4} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$Q = \frac{\pi}{128} \times 67.9 \cdot \frac{\text{lbf}}{\text{in}^2} \times \frac{144 \cdot \text{in}^2}{1 \cdot \text{ft}^2} \times \left(0.005 \cdot \text{in} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \right)^4 \times \frac{\text{ft}^2}{1.05 \times 10^{-4} \text{lbf} \cdot \text{s}} \times \frac{1}{1 \cdot \text{in}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}}$$

$$Q = 8.27 \times 10^{-7} \cdot \frac{\text{ft}^3}{\text{s}} \qquad Q = 1.43 \times 10^{-3} \cdot \frac{\text{in}^3}{\text{s}} \qquad Q = 0.0857 \cdot \frac{\text{in}^3}{\text{min}}$$

Check Re: $$V = \frac{Q}{A} = \frac{Q}{\frac{\pi \cdot d^2}{4}} \qquad V = \frac{4}{\pi} \times 8.27 \times 10^{-7} \frac{\text{ft}^3}{\text{s}} \times \left(\frac{1}{.005 \cdot \text{in}} \right)^2 \times \left(\frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \right)^2 \qquad V = 6.07 \cdot \frac{\text{ft}}{\text{s}}$$

$$Re = \frac{\rho \cdot V \cdot d}{\mu} \qquad \rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$$ (assuming saline is close to water)

$$Re = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times 6.07 \cdot \frac{\text{ft}}{\text{s}} \times 0.005 \cdot \text{in} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \times \frac{\text{ft}^2}{1.05 \times 10^{-4} \cdot \text{lbf} \cdot \text{s}} \times \frac{\text{slug} \cdot \text{ft}}{\text{s}^2 \cdot \text{lbf}} \qquad Re = 46.7$$

Flow is laminar

Problem 8.48

[3]

8.48 In engineering science there are often analogies to be made between disparate phenomena. For example, the applied pressure difference Δp and corresponding volume flow rate Q in a tube can be compared to the applied DC voltage V across and current I through an electrical resistor, respectively. By analogy, find a formula for the "resistance" of laminar flow of fluid of viscosity μ in a tube length of L and diameter D, corresponding to electrical resistance R. For a tube 100 mm long with inside diameter 0.3 mm, find the maximum flow rate and pressure difference for which this analogy will hold for (a) kerosine and (b) castor oil (both at 40°C). When the flow exceeds this maximum, why does the analogy fail?

Given: Data on a tube

Find: "Resistance" of tube; maximum flow rate and pressure difference for which electrical analogy holds for (a) kerosine and (b) castor oil

Solution:

The given data is $L = 100 \cdot \text{mm}$ $\quad D = 0.3 \cdot \text{mm}$

From Fig. A.2 and Table A.2

Kerosene: $\mu = 1.1 \times 10^{-3} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$ $\quad \rho = 0.82 \times 990 \cdot \frac{\text{kg}}{\text{m}^3} = 812 \cdot \frac{\text{kg}}{\text{m}^3}$

Castor oil: $\mu = 0.25 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$ $\quad \rho = 2.11 \times 990 \cdot \frac{\text{kg}}{\text{m}^3} = 2090 \cdot \frac{\text{kg}}{\text{m}^3}$

For an electrical resistor $V = R \cdot I$ (1)

The governing equation for the flow rate for laminar flow in a tube is Eq. 8.13c

$$Q = \frac{\pi \cdot \Delta p \cdot D^4}{128 \cdot \mu \cdot L}$$

or

$$\Delta p = \frac{128 \cdot \mu \cdot L}{\pi \cdot D^4} \cdot Q \qquad (2)$$

By analogy, current *I* is represented by flow rate *Q*, and voltage *V* by pressure drop Δp.
Comparing Eqs. (1) and (2), the "resistance" of the tube is

$$R = \frac{128 \cdot \mu \cdot L}{\pi \cdot D^4}$$

The "resistance" of a tube is directly proportional to fluid viscosity and pipe length, and strongly dependent on the inverse of diameter

The analogy is only valid for $\text{Re} < 2300$ or $\frac{\rho \cdot V \cdot D}{\mu} < 2300$

Writing this constraint in terms of flow rate $\frac{\rho \cdot \frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot D}{\mu} < 2300$ or $Q_{max} = \frac{2300 \cdot \mu \cdot \pi \cdot D}{4 \cdot \rho}$

The corresponding maximum pressure gradient is then obtained from Eq. (2)

$$\Delta p_{max} = \frac{128 \cdot \mu \cdot L}{\pi \cdot D^4} \cdot Q_{max} = \frac{32 \cdot 2300 \cdot \mu^2 \cdot L}{\rho \cdot D^3}$$

(a) For kerosine $Q_{max} = 7.34 \times 10^{-7} \frac{m^3}{s}$ $\Delta p_{max} = 406\,kPa$

(b) For castor oil $Q_{max} = 6.49 \times 10^{-5} \frac{m^3}{s}$ $\Delta p_{max} = 8156\,MPa$

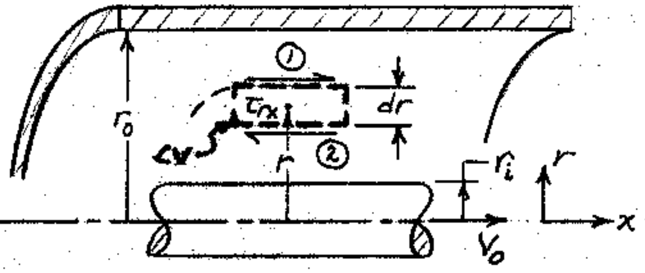
The analogy fails when Re > 2300 because the flow becomes turbulent, and "resistance" to flow is then no longer linear with flow rate

Problem 8.49

[4]______

Given: Fully-developed laminar flow in an annulus as shown. The inner section is stationary; the outer moves at V_0.

Assume $\frac{\partial p}{\partial x} = 0$.



Find: (a) $\tau(r)$ in terms of c_1.

(b) $V(r)$ in terms of c_1, c_2.

(c) Evaluate c_1, c_2.

Solution: Apply x component of momentum equation, using annular CV shown.

Basic Equations: $F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \overset{=0(3)}{\cancel{\int_{CS}}} u\rho\vec{V}\cdot d\vec{A}$; $\tau_{rx} = \mu\frac{du}{dr} = \tau$

Assumptions: (1) $F_{B_x} = 0$

(2) steady flow

(3) Fully-developed flow

Then

$$F_{S_x} = F_{①} - F_{②} = \left(\tau + \frac{d\tau}{dr}\frac{dr}{2}\right)2\pi\left(r+\frac{dr}{2}\right)dx - \left(\tau - \frac{d\tau}{dr}\frac{dr}{2}\right)2\pi\left(r-\frac{dr}{2}\right)dx = 0$$

Neglecting products of differentials, this reduces to

$$\tau + r\frac{d\tau}{dr} = 0 \quad \text{or} \quad \frac{d}{dr}(r\tau) = 0$$

Thus $r\tau = c_1$ or $\tau = \frac{c_1}{r}$ $\tau(r)$

But $\tau = \mu\frac{du}{dr}$, so $\frac{du}{dr} = \frac{c_1}{\mu r}$

and $u = \frac{c_1}{\mu}\ln r + c_2$ $u(r)$

To evaluate constants c_1 and c_2, use boundary conditions.

At $r = r_i$, $u = V_0$, so $V_0 = \frac{c_1}{\mu}\ln r_i + c_2$

At $r = r_0$, $u = 0$, so $0 = \frac{c_1}{\mu}\ln r_0 + c_2$ and $c_2 = -\frac{c_1}{\mu}\ln r_0$

Thus, subtracting, $V_0 = \frac{c_1}{\mu}\ln\left(\frac{r_i}{r_0}\right)$ or $c_1 = \frac{\mu V_0}{\ln(r_i/r_0)}$ so $c_2 = -\frac{V_0 \ln r_0}{\ln(r_i/r_0)}$

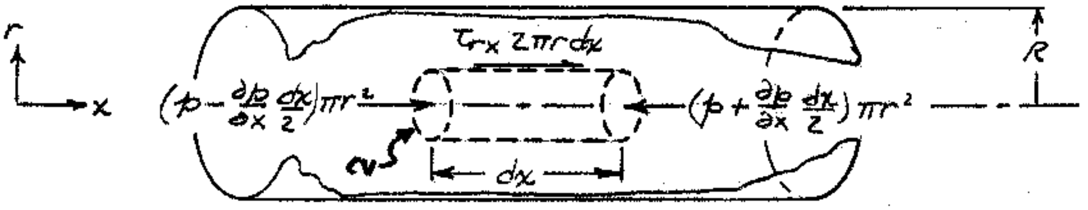
Finally

$$u = \frac{V_0}{\ln(r_i/r_0)}(\ln r - \ln r_i) = V_0\frac{\ln(r/r_0)}{\ln(r_i/r_0)}$$ $u(r)$

Problem 8.50

[4]

Given: Fully-developed laminar flow in a circular pipe, with cylindrical control volume as shown.


r
x
$\tau_{rx}\, 2\pi r\, dx$
$(p - \frac{\partial p}{\partial x}\frac{dx}{2})\pi r^2$
$(p + \frac{\partial p}{\partial x}\frac{dx}{2})\pi r^2$
CV
dx
R

Find: (a) Forces acting on CV.
(b) Expression for velocity distribution.

Solution: The forces on a CV of radius r are shown above.

Apply the x component of momentum to CV shown.

Basic equations:

$$F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \overset{=0(3)}{\cancel{\int_{CS}}} u\rho\vec{V}\cdot d\vec{A}\,, \qquad \tau_{rx} = \mu\frac{du}{dr}$$

Assumptions: (1) $F_{B_x} = 0$
(2) Steady flow
(3) Fully-developed flow

Then

$$F_{S_x} = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)\pi r^2 + \tau_{rx}\, 2\pi r\, dx - \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)\pi r^2 = 0$$

Cancelling and combining terms,

$$-r\frac{\partial p}{\partial x} + 2\tau_{rx} = 0 \qquad \text{or} \qquad \tau_{rx} = \mu\frac{du}{dr} = \frac{r}{2}\frac{\partial p}{\partial x}$$

Thus $\quad \dfrac{du}{dr} = \dfrac{r}{2\mu}\dfrac{\partial p}{\partial x}$

and

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + c_1$$

To evaluate c_1, apply the boundary condition $u = 0$ at $r = R$. Thus

$$c_1 = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}$$

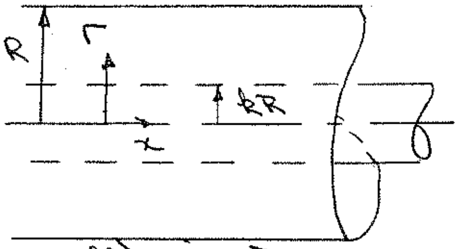
and

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}(r^2 - R^2) = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1 - \left(\frac{r}{R}\right)^2\right] \longleftarrow u$$

which is identical to Eq. 8.12.

Problem 8.51

[4] Part 1/2



Given: Fully developed laminar flow with pressure gradient, $\partial p/\partial x$, in the annulus shown

(a) Show that the velocity profile is given by $u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1-\left(\frac{r}{R}\right)^2 + \frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right]$

(b) Obtain an expression for the location ($\alpha = r/R$) of maximum u as a function of k.

(c) Plot α vs k.

(d) Compare limiting case, $k \to 0$, with flow in circular pipe.

Solution: We may use the results of the differential control ~~volume~~ analysis of Section 8-3 to write

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + \frac{c_1}{\mu}\ln r + c_2 \quad \text{----(1)}$$

The boundary conditions are $u=0$ at $r=R$
$u=0$ at $r=kR$.

Substituting the boundary conditions

$$0 = \frac{R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{c_1}{\mu}\ln R + c_2 \quad \text{----(2)}$$

$$0 = \frac{k^2R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{c_1}{\mu}\ln kR + c_2 \quad \text{----(3)}$$

Subtracting, $0 = \frac{R^2}{4\mu}\frac{\partial p}{\partial x}(1-k^2) + \frac{c_1}{\mu}(\ln R - \ln kR)$

$$\therefore \quad c_1 = -\frac{R^2}{4}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}$$

From Eq. 2

$$c_2 = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{R^2}{4\mu}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}\ln R$$

Substituting for c_1 and c_2 into Eq. 1 gives

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} - \frac{R^2}{4\mu}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}\ln r - \frac{R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{R^2}{4\mu}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}\ln R$$

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\left[r^2 - R^2 - \frac{R^2(1-k^2)}{\ln(1/k)}(\ln r - \ln R)\right]$$

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2 + \frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right] \longleftarrow u$$

To locate max u, set $\tau_{rx} = \mu\frac{du}{dr} = 0$

$$\tau_{rx} = \mu\frac{du}{dr} = -\frac{R^2}{4}\frac{\partial p}{\partial x}\left[-\frac{2r}{R^2} + \frac{(1-k^2)}{\ln(1/k)}\frac{1}{r}\right]$$

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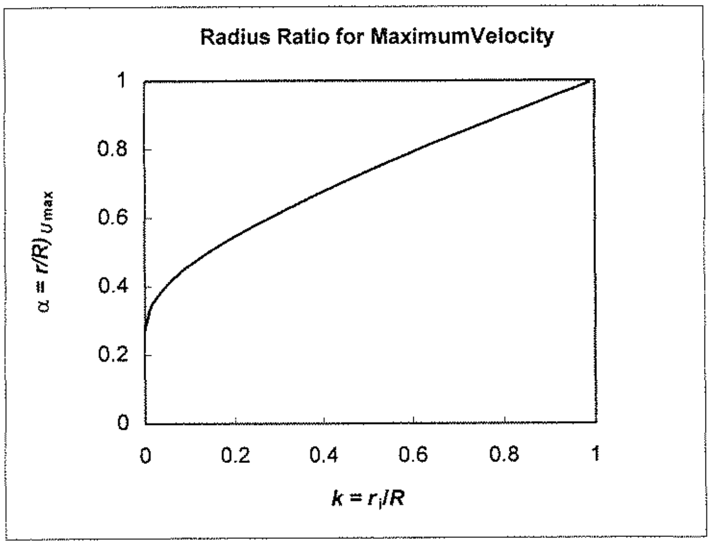
Set $\tau_{rx} = 0$ at $r = \alpha R$. Then

$$0 = -\frac{2\alpha R}{R^2} + \frac{(1-k^2)}{\ln(1/k)}\frac{1}{\alpha R}$$

and

$$\alpha = \left[\frac{1}{2}\frac{(1-k^2)}{\ln(1/k)}\right]^{1/2} \longleftarrow \alpha = \alpha(k)$$

$k = r_i/R$	$\alpha = r/R)_{u\,max}$
0.001	0.269
0.01	0.329
0.02	0.357
0.05	0.408
0.08	0.444
0.1	0.464
0.2	0.546
0.4	0.677
0.6	0.791
0.8	0.898
0.95	0.975
0.99	0.995



For $k \to 0$, $\alpha \to 0$ and $u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$

This agrees with the results for flow in a circular pipe

As $k \to 1.0$, $\alpha \to 1.0$ and the flow behaves like flow between stationary infinite parallel plates.

Problem 8.52

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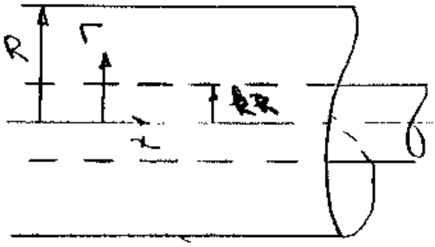

Given: Fully developed laminar flow in the annulus shown, with pressure gradient $\partial p/\partial x$.
The velocity profile is given by

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2+\frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right]$$

(a) Show that the volume flow rate is given by

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}\left[(1-k^4)-\frac{(1-k^2)^2}{\ln(1/k)}\right]$$

(b) Obtain an expression for the average velocity

(c) Compare limiting case, $k \to 0$, with flow in a circular pipe.

Solution: The volume flow rate is given by

$$Q = \int u\,dA = \int_{kR}^{R} u\,2\pi r\,dr = 2\pi\int_{kR}^{R} u\,r\,dr$$

$$= 2\pi\left(-\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\right)\int_{kR}^{R}\left[r-\frac{r^3}{R^2}+\frac{(1-k^2)}{\ln(1/k)}\,r\ln\frac{r}{R}\right]dr$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\int_{k}^{1}\left[\frac{r}{R}-\left(\frac{r}{R}\right)^3+\frac{(1-k^2)}{\ln(1/k)}\,\frac{r}{R}\ln\frac{r}{R}\right]d\left(\frac{r}{R}\right)$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2-\frac{1}{4}\left(\frac{r}{R}\right)^4+\frac{(1-k^2)}{\ln(1/k)}\left\{\left(\frac{r}{R}\right)^2\left[\frac{1}{2}\ln\left(\frac{r}{R}\right)-\frac{1}{4}\right]\right\}\right]_k^1$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{2}-\frac{k^2}{2}-\frac{1}{4}+\frac{k^4}{4}+\frac{(1-k^2)}{\ln(1/k)}\left\{-\frac{1}{4}-k^2\left[\frac{1}{2}\ln k-\frac{1}{4}\right]\right\}\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{4}-\frac{k^2}{2}+\frac{k^4}{4}+\frac{(1-k^2)}{\ln(1/k)}\left\{-\frac{1}{4}+\frac{k^2}{4}-k^2\,\frac{1}{2}\ln k\right\}\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1-2k^2+k^4}{4}+\frac{(1-k^2)}{\ln(1/k)}\,\frac{(k^2-1)}{4}-\frac{(1-k^2)}{\ln(1/k)}\,k^2\,\frac{1}{2}\ln k\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1-2k^2+k^4}{4}-\frac{(1-k^2)^2}{4\ln(1/k)}+\frac{k^2-k^4}{2}\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1-\cancel{2k^2}+k^4+\cancel{2k^2}-2k^4}{4}-\frac{(1-k^2)^2}{4\ln(1/k)}\right]$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}\left[(1-k^4)-\frac{(1-k^2)^2}{\ln(1/k)}\right] \longleftarrow Q$$

The average velocity, $\bar{V} = \frac{Q}{A}$

The area is given by

$$A = \int dA = \int 2\pi r\,dr = 2\pi R^2 \int_k^1 \frac{r}{R}\, d\left(\frac{r}{R}\right)$$

$$A = 2\pi R^2 \left[\frac{1}{2}\left(\frac{r}{R}\right)^2 \right]_k^1 = 2\pi R^2 \times \frac{1}{2}(1-k^2) = \pi R^2 (1-k^2)$$

Thus

$$\bar{V} = \frac{Q}{A} = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x} \times \frac{1}{\pi R^2}\left[\frac{(1-k^4)}{(1-k^2)} - \frac{(1-k^2)}{\ln(1/k)} \right]$$

$$\bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}\left[\frac{(1-k^4)}{(1-k^2)} - \frac{(1-k^2)}{\ln(1/k)} \right] \longleftarrow \bar{V}$$

For $k \to 0$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x} \quad \text{and} \quad \bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}$$

These agree with the results for flow in a circular pipe.

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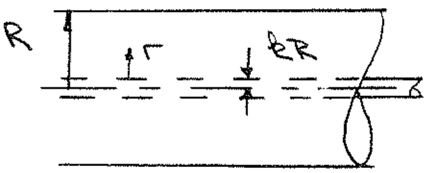
Problem 8.53

[4]

Given: Fully developed laminar flow in a circular pipe is converted to flow in an annulus by insertion of a thin wire along the centerline



(a) Use results of Problem 8.52 to obtain an expression for the percent change in pressure drop as a function of radius ratio k.

(b) Plot percent change in Δp vs k for $0.001 \le k \le 0.10$

Solution: The results of problem 8.48 give

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]$$

Thus

$$\frac{\Delta p}{L} = -\frac{\partial p}{\partial x} = \frac{8\mu Q}{\pi R^4} \times \frac{1}{\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]}$$

For $k=0$, $\frac{\Delta p}{L} = \frac{8\mu Q}{\pi R^4}$

$$\text{Percent change} = \frac{\Delta p/L - \Delta p/L)_{k=0}}{\Delta p/L)_{k=0}} = \frac{1}{\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]} - 1$$

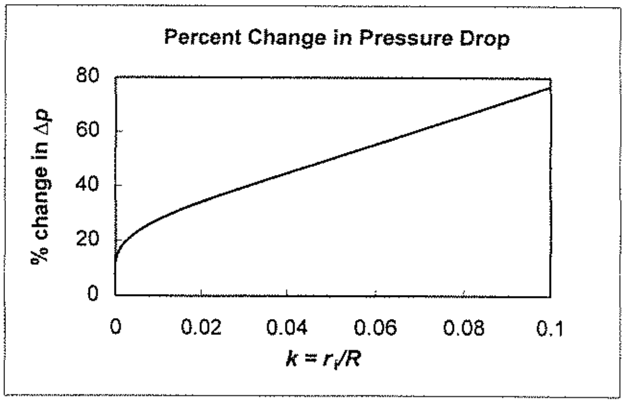
$$\%\text{ change} = \frac{1 - \left[(1-k^4) - \frac{(1-k^2)}{\ln(1/k)}\right]}{\left[(1-k^4) - \frac{(1-k^2)}{\ln(1/k)}\right]}$$

For small k,

$$\%\text{ change} = \frac{1 - \left[1 - \frac{1}{\ln(1/k)}\right]}{\left[1 - \frac{1}{\ln(1/k)}\right]} = \frac{1 - \left[1 + \frac{1}{\ln k}\right]}{\left[1 + \frac{1}{\ln k}\right]} = \frac{-\frac{1}{\ln k}}{\left[1 + \frac{1}{\ln k}\right]}$$

$$\%\text{ change} = -\frac{1}{\ln k\left(1 + \frac{1}{\ln k}\right)} \times 100 \longleftarrow \%\text{ change}$$

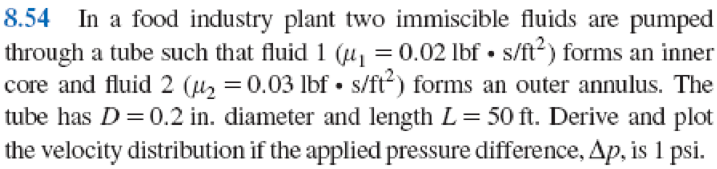
$k = r_i/R$	% change in Δp
0.0001	12.2
0.0002	13.3
0.0005	15.1
0.001	16.9
0.002	19.2
0.005	23.3
0.01	27.7
0.02	34.3
0.05	50.1
0.1	76.8



The plot shows that even the smallest of wires causes a significant increase in pressure drop for a given flow rate.

Problem 8.54

[3]



8.54 In a food industry plant two immiscible fluids are pumped through a tube such that fluid 1 ($\mu_1 = 0.02 \text{ lbf} \cdot \text{s/ft}^2$) forms an inner core and fluid 2 ($\mu_2 = 0.03 \text{ lbf} \cdot \text{s/ft}^2$) forms an outer annulus. The tube has $D = 0.2$ in. diameter and length $L = 50$ ft. Derive and plot the velocity distribution if the applied pressure difference, Δp, is 1 psi.

Given: Two-fluid flow in tube

Find: Velocity distribution; Plot

Solution:

Given data $D = 0.2 \cdot \text{in}$ $L = 50 \cdot \text{ft}$ $\Delta p = -1 \cdot \text{psi}$ $\mu_1 = 0.02 \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$ $\mu_2 = 0.03 \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$

From Section 8-3 for flow in a pipe, Eq. 8.11 can be applied to either fluid

$$u = \frac{r^2}{4 \cdot \mu} \cdot \left(\frac{\partial}{\partial x} p \right) + \frac{c_1}{\mu} \cdot \ln(r) + c_2$$

Applying this to fluid 1 (inner fluid) and fluid 2 (outer fluid)

$$u_1 = \frac{r^2}{4 \cdot \mu_1} \cdot \frac{\Delta p}{L} + \frac{c_1}{\mu_1} \cdot \ln(r) + c_2 \qquad u_2 = \frac{r^2}{4 \cdot \mu_2} \cdot \frac{\Delta p}{L} + \frac{c_3}{\mu_2} \cdot \ln(r) + c_4$$

We need four BCs. Two are obvious $r = \frac{D}{2}$ $u_2 = 0$ (1) $r = \frac{D}{4}$ $u_1 = u_2$ (2)

The third BC comes from the fact that the axis is a line of symmetry

$$r = 0 \qquad \frac{du_1}{dr} = 0 \qquad (3)$$

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$$r = \frac{D}{4} \qquad \mu_1 \cdot \frac{du_1}{dr} = \mu_2 \cdot \frac{du_2}{dr} \qquad (4)$$

Using these four BCs

$$\frac{\left(\frac{D}{2}\right)^2}{4 \cdot \mu_2} \cdot \frac{\Delta p}{L} + \frac{c_3}{\mu_2} \cdot \ln\left(\frac{D}{2}\right) + c_4 = 0 \qquad \frac{\left(\frac{D}{4}\right)^2}{4 \cdot \mu_1} \cdot \frac{\Delta p}{L} + \frac{c_1}{\mu_1} \cdot \ln\left(\frac{D}{4}\right) + c_2 = \frac{\left(\frac{D}{4}\right)^2}{4 \cdot \mu_2} \cdot \frac{\Delta p}{L} + \frac{c_3}{\mu_2} \cdot \ln\left(\frac{D}{4}\right) + c_4$$

$$\lim_{r \to 0} \frac{c_1}{\mu_1 \cdot r} = 0 \qquad \frac{D}{8} \cdot \frac{\Delta p}{L} + \frac{4 \cdot c_1}{D} = \frac{D}{8} \cdot \frac{\Delta p}{L} + \frac{4 \cdot c_3}{D}$$

Hence, after some algebra

$c_1 = 0$ (To avoid singularity) $c_2 = -\frac{D^2 \cdot \Delta p}{64 \cdot L} \frac{\left(\mu_2 + 3 \cdot \mu_1\right)}{\mu_1 \cdot \mu_2}$ $c_3 = 0$ $c_4 = -\frac{D^2 \cdot \Delta p}{16 \cdot L \cdot \mu_2}$

The velocity distributions are then

$$u_1(r) = \frac{\Delta p}{4 \cdot \mu_1 \cdot L} \cdot \left[r^2 - \left(\frac{D}{2}\right)^2 \cdot \frac{\left(\mu_2 + 3 \cdot \mu_1\right)}{4 \cdot \mu_2} \right] \qquad u_2(r) = \frac{\Delta p}{4 \cdot \mu_2 \cdot L} \cdot \left[r^2 - \left(\frac{D}{2}\right)^2 \right]$$

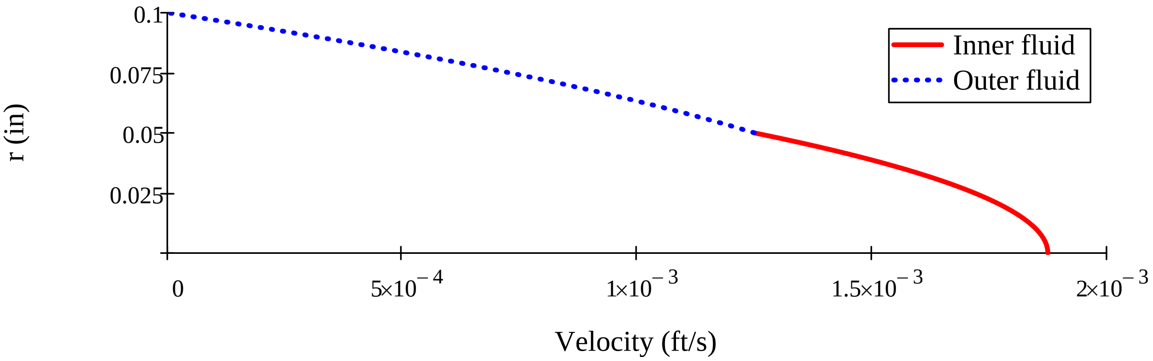
(Note that these result in the same expression if $\mu_1 = \mu_2$, i.e., if we have one fluid)

Evaluating either velocity at $r = D/4$ gives the velocity at the interface

$$u_{interface} = -\frac{3 \cdot D^2 \cdot \Delta p}{64 \cdot \mu_2 \cdot L} \qquad u_{interface} = -\frac{3}{64} \times \left(\frac{0.2}{12} \cdot ft\right)^2 \times \left(-1 \cdot \frac{lbf}{in^2}\right) \times \frac{144 \cdot in^2}{1 \cdot ft^2} \times \frac{ft^2}{0.03 \cdot lbf \cdot s} \times \frac{1}{50 \cdot ft} \qquad u_{interface} = 1.25 \times 10^{-3} \frac{ft}{s}$$

Evaluating u_1 at $r = 0$ gives the maximum velocity

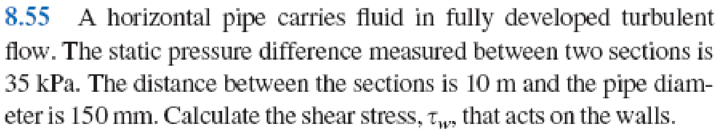
$$u_{max} = -\frac{D^2 \cdot \Delta p \cdot \left(\mu_2 + 3 \cdot \mu_1\right)}{64 \cdot \mu_1 \cdot \mu_2 \cdot L} \qquad u_{max} = -\frac{1}{64} \times \left(\frac{0.2}{12} \cdot ft\right)^2 \times \left(-1 \cdot \frac{lbf}{in^2}\right) \times \frac{0.03 + 3 \times 0.02}{0.02 \times 0.03} \cdot \frac{ft^2}{lbf \cdot s} \times \frac{1}{50 \cdot ft} \qquad u_{max} = 1.88 \times 10^{-3} \frac{ft}{s}$$



The velocity distributions can be plotted in *Excel*

Problem 8.55

[2]



8.55 A horizontal pipe carries fluid in fully developed turbulent flow. The static pressure difference measured between two sections is 35 kPa. The distance between the sections is 10 m and the pipe diameter is 150 mm. Calculate the shear stress, τ_w, that acts on the walls.

Given: Turbulent pipe flow

Find: Wall shear stress

Solution:

Basic equation $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV\!\!\!\!- + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A} \qquad \text{(Eq. 4.18a)}$$

Assumptions 1) Horizontal pipe 2) Steady flow 3) Fully developed flow

With these assumptions the x momentum equation becomes

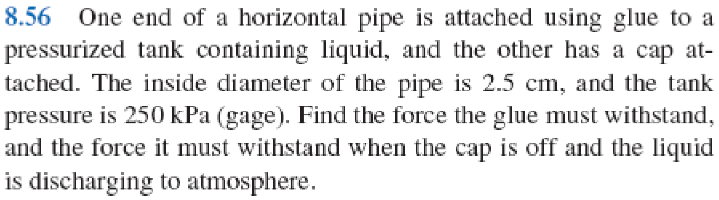
$$p_1\cdot\frac{\pi\cdot D^2}{4} + \tau_w\cdot\pi\cdot D\cdot L - p_2\cdot\frac{\pi\cdot D^2}{4} = 0 \qquad \text{or} \qquad \tau_w = \frac{(p_2 - p_1)\cdot D}{4\cdot L} = -\frac{\Delta p\cdot D}{4\cdot L}$$

$$\tau_w = -\frac{1}{4}\times 35\times 10^3\cdot\frac{N}{m^2}\times 150\cdot mm\times\frac{1\cdot m}{1000\cdot mm}\times\frac{1}{10\cdot m} \qquad \tau_w = -131\,Pa$$

Since τ_w is negative it acts to the left on the fluid, to the right on the pipe wall

Problem 8.56

[3]



8.56 One end of a horizontal pipe is attached using glue to a pressurized tank containing liquid, and the other has a cap attached. The inside diameter of the pipe is 2.5 cm, and the tank pressure is 250 kPa (gage). Find the force the glue must withstand, and the force it must withstand when the cap is off and the liquid is discharging to atmosphere.

Given: Pipe glued to tank

Find: Force glue must hold when cap is on and off

Solution:

Basic equation
$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!/ + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A} \qquad \text{(Eq. 4.18a)}$$

First solve when the cap is on. In this static case

$$F_{glue} = \frac{\pi \cdot D^2}{4}\cdot p_1 \qquad \text{where } p_1 \text{ is the tank pressure}$$

Second, solve for when flow is occuring:

Assumptions 1) Horizontal pipe 2) Steady flow 3) Fully developed flow

With these assumptions the x momentum equation becomes

$$p_1\cdot\frac{\pi\cdot D^2}{4} + \tau_w\cdot\pi\cdot D\cdot L - p_2\cdot\frac{\pi\cdot D^2}{4} = 0$$

Here p_1 is again the tank pressure and p_2 is the pressure at the pipe exit; the pipe exit pressure is p_{atm} = 0 kPa gage. Hence

$$F_{pipe} = F_{glue} = -\tau_w\cdot\pi\cdot D\cdot L = \frac{\pi\cdot D^2}{4}\cdot p_1$$

We conclude that in each case the force on the glue is the same! When the cap is on the glue has to withstand the tank pressure; when the cap is off, the glue has to hold the pipe in place against the friction of the fluid on the pipe, which is equal in magnitude to the pressure drop.

$$F_{glue} = \frac{\pi}{4}\times\left(2.5\cdot cm\times\frac{1\cdot m}{100\cdot cm}\right)^2\times 250\times 10^3\cdot\frac{N}{m^2} \qquad F_{glue} = 123\,N$$

Problem 8.57

[2]

8.57 The pressure drop between two taps separated in the streamwise direction by 30 ft in a horizontal, fully developed channel flow of water is 1 psi. The cross-section of the channel is a 1 in. × $9\frac{1}{2}$ in. rectangle. Calculate the average wall shear stress.

Given: Flow through channel

Find: Average wall stress

Solution:

Basic equation $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\mathcal{V} + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad \text{(Eq. 4.18a)}$$

Assumptions 1) Horizontal pipe 2) Steady flow 3) Fully developed flow

With these assumptions the x momentum equation becomes

$$p_1\cdot W\cdot H + \tau_w\cdot 2\cdot L\cdot(W+H) - p_2\cdot W\cdot H = 0 \qquad \text{or} \qquad \tau_w = \left(p_2 - p_1\right)\cdot\frac{W\cdot H}{2\cdot(W+H)\cdot L} \qquad \tau_w = -\Delta p\cdot\frac{\frac{H}{L}}{2\cdot\left(1+\frac{H}{W}\right)}$$

$$\tau_w = -\frac{1}{2}\times 1\cdot\frac{lbf}{in^2}\times\frac{144\cdot in^2}{ft^2}\times\frac{1\cdot in\times\frac{1\cdot ft}{12\cdot in}}{30\cdot ft}\times\left(\frac{1}{1+\frac{9.5\cdot in\times\frac{1\cdot ft}{12\cdot in}}{30\cdot ft}}\right) \qquad \tau_w = -0.195\frac{lbf}{ft^2} \qquad \tau_w = -1.35\times 10^{-3}\,psi$$

Since $\tau_w < 0$, it acts to the left on the fluid, to the right on the channel wall

Problem 8.58

[2]

8.58 Kerosine is pumped through a smooth tube with inside diameter $D = 30$ mm at close to the critical Reynolds number. The flow is unstable and fluctuates between laminar and turbulent states, causing the pressure gradient to intermittently change from approximately −4.5 kPa/m to −11 kPa/m. Which pressure gradient corresponds to laminar, and which to turbulent, flow? For each flow, compute the shear stress at the tube wall, and sketch the shear stress distributions.

Given: Data on pressure drops in flow in a tube

Find: Which pressure drop is laminar flow, which turbulent

Solution:

Given data $\quad \frac{\partial}{\partial x} p_1 = -4.5 \cdot \frac{kPa}{m} \qquad \frac{\partial}{\partial x} p_2 = -11 \cdot \frac{kPa}{m} \qquad D = 30 \cdot mm$

From Section 8-4, a force balance on a section of fluid leads to

$$\tau_w = -\frac{R}{2} \cdot \frac{\partial}{\partial x} p = -\frac{D}{4} \cdot \frac{\partial}{\partial x} p$$

Hence for the two cases

$$\tau_{w1} = -\frac{D}{4} \cdot \frac{\partial}{\partial x} p_1 \qquad \tau_{w1} = 33.8\,Pa$$

$$\tau_{w2} = -\frac{D}{4} \cdot \frac{\partial}{\partial x} p_2 \qquad \tau_{w2} = 82.5\,Pa$$

Because both flows are at the same nominal flow rate, the higher pressure drop must correspond to the turbulent flow, because, as indicated in Section 8-4, turbulent flows experience additional stresses. Also indicated in Section 8-4 is that for both flows the shear stress varies from zero at the centerline to the maximums computed above at the walls.

The stress distributions are linear in both cases: Maximum at the walls and zero at the centerline.

Problem 8.59

Given: Liquid with viscosity and density of water in laminar flow in a smooth capillary tube. $D = 0.25\ mm$, $L = 50\ mm$.

Find: (a) Maximum volume flow rate.
(b) Pressure drop to produce this flow rate.
(c) Corresponding wall shear stress.

Solution: Flow will be laminar for $Re < 2300$.

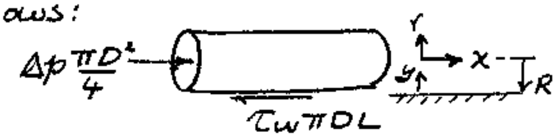
$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{Q}{A}\frac{D}{\nu} = \frac{4Q}{\pi D^2}\frac{D}{\nu} = \frac{4Q}{\pi \nu D} < 2300$$

Thus (at $T = 20°C$)

$$Q < \frac{2300\pi\nu D}{4} = \frac{2300\pi}{4} \times 1.0\times10^{-6}\frac{m^2}{s} \times 0.00025\,m = 4.52\times10^{-7}\,m^3/s \qquad Q$$

(This flow rate corresponds to 27.1 mL/min.)

A force balance on a fluid element shows:



$$\sum F_x = \Delta p \frac{\pi D^2}{4} - \tau_w \pi D L = 0$$

or

$$\Delta p = \tau_w \frac{4L}{D}$$

For laminar pipe flow, $u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$, from Eq. 8.14. Thus

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = -\mu \left.\frac{\partial u}{\partial r}\right)_{r=R} = -\mu u_{max}\left(-\frac{2r}{R^2}\right)_{r=R} = \frac{2\mu u_{max}}{R}$$

But $u_{max} = 2\bar{V}$, so $\tau_w = \frac{2\mu 2\bar{V}}{D/2} = \frac{8\mu\bar{V}}{D} = \frac{8\rho\nu\bar{V}}{D}$

Also

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 4.52\times10^{-7}\frac{m^3}{s} \times \frac{1}{(0.00025)^2 m^2} = 9.21\ m/s$$

Thus

$$\tau_w = 8 \times 999\frac{kg}{m^3} \times 1.0\times10^{-6}\frac{m^2}{s} \times 9.21\frac{m}{s} \times \frac{1}{0.00025\,m} \times \frac{N\cdot s^2}{kg\cdot m} = 294\ N/m^2\ (294\ Pa) \qquad \tau_w$$

and

$$\Delta p = 4 \times 0.05\,m \times \frac{1}{0.00025\,m} \times 294\frac{N}{m^2} = 235\ kPa \qquad \Delta p$$

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Problem 8.60

[3]

Given: Velocity profiles for pipe flow

$$\frac{u}{U} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \text{ (turbulent)}; \quad \frac{u}{U} = 1 - \left(\frac{r}{R}\right)^2 \text{ (laminar)}$$

Find: (a) value of r/R at which $u = \bar{V}$ for each profile.

Plot: r/R vs n for $6 \le n \le 10$

Solution:

Definition: $\bar{V} = \frac{Q}{A} = \frac{1}{A}\int u\,dA$

For laminar flow, $\bar{V} = \frac{1}{\pi R^2}\int_0^R U\left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r\,dr = 2U\int_0^1 \left[1 - \left(\frac{r}{R}\right)^2\right]\frac{r}{R}\,d\left(\frac{r}{R}\right)$

$$\bar{V} = 2U\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^1 = \frac{U}{2}$$

Thus $u = \bar{V}$ when $1 - \left(\frac{r}{R}\right)^2 = \frac{\bar{V}}{U} = \frac{1}{2}$ or $\frac{r}{R} = 0.707$ ← laminar

For turbulent flow, $\bar{V} = \frac{1}{\pi R^2}\int_0^R U\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} 2\pi r\,dr$

$$\bar{V} = 2U\int_0^1 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \frac{r}{R}\,d\left(\frac{r}{R}\right)$$

To integrate let $m = 1 - \frac{r}{R}$. Then $\frac{r}{R} = 1 - m$, $d\left(\frac{r}{R}\right) = -dm$

and

$$\bar{V} = 2U\int_1^0 m^{\frac{1}{n}}(1-m)(-dm) = 2U\int_0^1 \left(m^{\frac{1}{n}} - m^{1+\frac{1}{n}}\right)dm$$

$$= 2U\left[\frac{n}{(n+1)} m^{\frac{1}{n}+1} - \frac{n}{2n+1} m^{2+\frac{1}{n}}\right]_0^1 = 2U\left[\frac{n}{n+1} - \frac{n}{2n+1}\right]$$

$$\bar{V} = 2U\left[\frac{n(2n+1) - n(n+1)}{(n+1)(2n+1)}\right] = U\frac{2n^2}{(n+1)(2n+1)} \quad \text{..........(8.24)}$$

For $n = 7$, $\bar{V} = U\frac{2(7)^2}{8 \times 15} = 0.817U$

Thus $u = \bar{V}$ when $\left(1 - \frac{r}{R}\right)^{1/7} = 0.817$ or $\frac{r}{R} = 1 - (0.817)^7 = 0.758$ ← turb

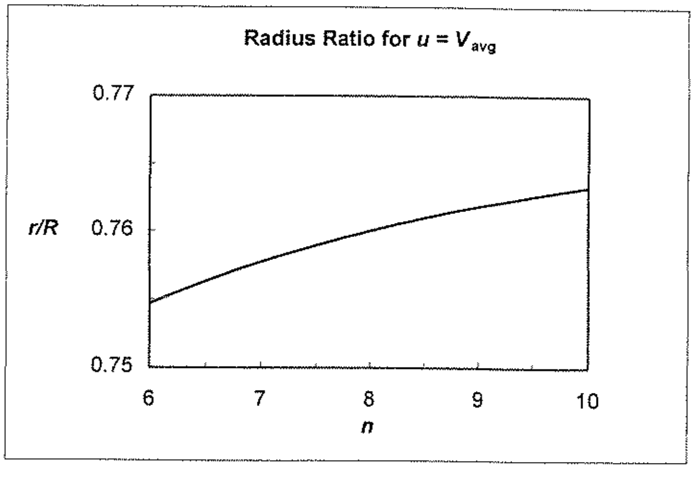
From Eq 8.24, $u = \bar{V}$ when

$$\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = \frac{2n^2}{(n+1)(2n+1)}$$

or

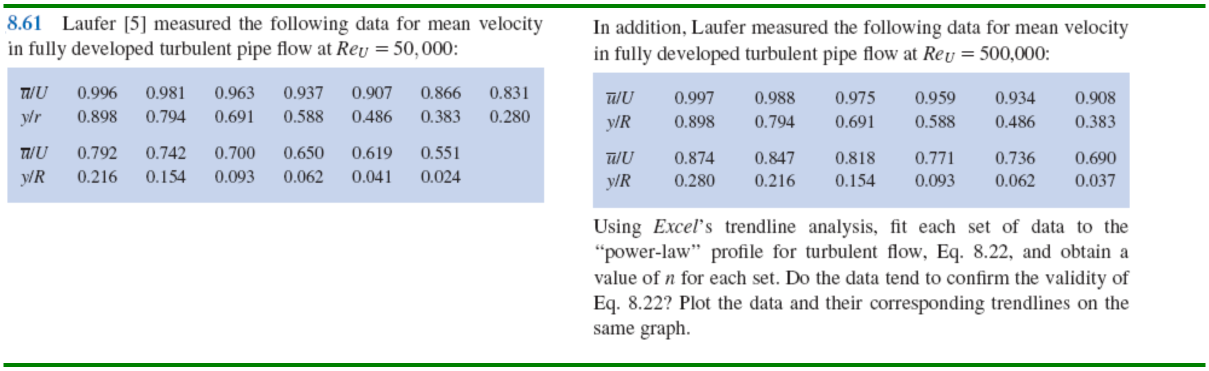
$$\frac{r}{R} = 1 - \left[\frac{2n^2}{(n+1)(2n+1)}\right]^n$$

r/R is plotted vs n.



Problem 8.61

[3]



8.61 Laufer [5] measured the following data for mean velocity in fully developed turbulent pipe flow at $Re_U = 50,000$:

$\overline{u}/U$	0.996	0.981	0.963	0.937	0.907	0.866	0.831
y/r	0.898	0.794	0.691	0.588	0.486	0.383	0.280

$\overline{u}/U$	0.792	0.742	0.700	0.650	0.619	0.551
y/R	0.216	0.154	0.093	0.062	0.041	0.024

In addition, Laufer measured the following data for mean velocity in fully developed turbulent pipe flow at $Re_U = 500,000$:

$\overline{u}/U$	0.997	0.988	0.975	0.959	0.934	0.908
y/R	0.898	0.794	0.691	0.588	0.486	0.383

$\overline{u}/U$	0.874	0.847	0.818	0.771	0.736	0.690
y/R	0.280	0.216	0.154	0.093	0.062	0.037

Using *Excel*'s trendline analysis, fit each set of data to the "power-law" profile for turbulent flow, Eq. 8.22, and obtain a value of n for each set. Do the data tend to confirm the validity of Eq. 8.22? Plot the data and their corresponding trendlines on the same graph.

Given: Data on mean velocity in fully developed turbulent flow

Find: Trendlines for each set; values of n for each set; plot

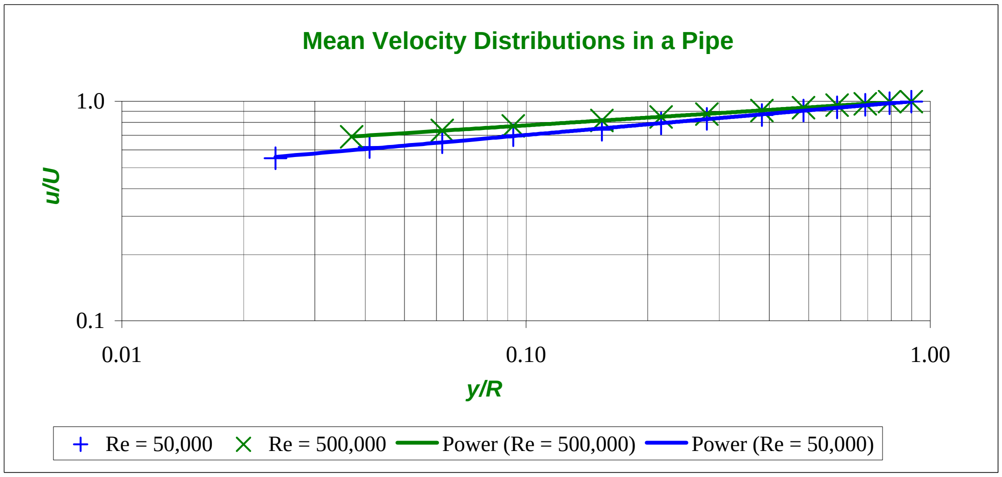
Solution:

y/R	*u/U*
0.898	0.996
0.794	0.981
0.691	0.963
0.588	0.937
0.486	0.907
0.383	0.866
0.280	0.831
0.216	0.792
0.154	0.742
0.093	0.700
0.062	0.650
0.041	0.619
0.024	0.551

y/R	*u/U*
0.898	0.997
0.794	0.998
0.691	0.975
0.588	0.959
0.486	0.934
0.383	0.908
0.280	0.874
0.216	0.847
0.154	0.818
0.093	0.771
0.062	0.736
0.037	0.690

Equation 8.22 is

$$\frac{\overline{u}}{U} = \left(\frac{y}{R}\right)^{1/n} = \left(1 - \frac{r}{R}\right)^{1/n}$$



Applying the *Trendline* analysis to each set of data:

At Re = 50,000

$u/U = 1.017(y/R)^{0.161}$

with $R^2 = 0.998$ (high confidence)

Hence $1/n = 0.161$

$n = 6.21$

At Re = 500,000

$u/U = 1.017(y/R)^{0.117}$

with $R^2 = 0.999$ (high confidence)

Hence $1/n = 0.117$

$n = 8.55$

Both sets of data tend to confirm the validity of Eq. 8.22

Problem 8.62

[3]

Given: Power-law exponent n as a function of Re_U and ratio $\bar{V}/U$ as a function of n.

$$n = -1.7 + 1.8 \log Re_U \qquad (8.23)$$

$$\bar{V}/U = \frac{2n^2}{(n+1)(2n+1)} \qquad (8.24)$$

Plot: $\bar{V}/U$ vs $Re_{\bar{V}}$

Solution:

Prepare a Table of values

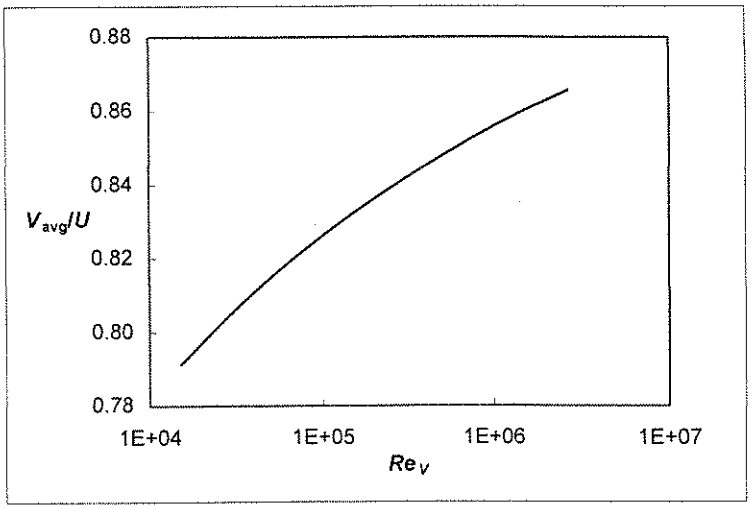
Re_U

n from Eq 8.23

$\bar{V}/U$ from Eq 8.24

$$Re_{\bar{V}} = \frac{\bar{V}}{U} \times Re_U$$

Re_U	n	Re_V	V_{avg}/U
1.90E+04	6.00	1.50E+04	0.791
3.60E+04	6.50	2.90E+04	0.805
6.85E+04	7.00	5.59E+04	0.817
1.29E+05	7.50	1.07E+05	0.827
2.45E+05	8.00	2.05E+05	0.837
4.65E+05	8.50	3.93E+05	0.845
8.80E+05	9.00	7.50E+05	0.853
1.67E+06	9.50	1.44E+06	0.860
3.16E+06	10.0	2.74E+06	0.866



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Problem 8.63

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Given: Velocity profiles for pipe flow:

$\frac{u}{U} = 1-\left(\frac{r}{R}\right)^2$ (laminar); $\frac{u}{U} = \left(1-\frac{r}{R}\right)^{\frac{1}{n}}$ (turbulent)

Momentum coefficient, β, where $\beta \dot{m} \bar{V} = \int_A u \rho u \, dA$

Find: (a) β for laminar profile.

(b) β for turbulent profile with $n=7$

Plot: β vs n for turbulent profile over range $6 \le n \le 10$, and compare with laminar profile

Solution:

$$\beta = \frac{1}{\dot{m}\bar{V}}\int_A u\rho u \, dA = \frac{1}{\rho \bar{V} \pi R^2 \bar{V}}\int_0^R u \rho u \, 2\pi r \, dr$$

Noting that $\frac{u}{U} = f(r/R)$

$$\beta = \frac{1}{\rho \pi R^2}\left[\frac{U}{\bar{V}}\right]^2 \int_0^R \left(\frac{u}{U}\right)^2 2\rho\pi r \, dr = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left(\frac{u}{U}\right)^2 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \text{ --- } \beta$$

For laminar flow, $\frac{u}{U} = 1-\left(\frac{r}{R}\right)^2$, so $\left(\frac{u}{U}\right)^2 = 1 - 2\left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^4$, and

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left[\left(\frac{r}{R}\right) - 2\left(\frac{r}{R}\right)^3 + \left(\frac{r}{R}\right)^5\right] d\left(\frac{r}{R}\right) = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right]$$

$\beta = \frac{1}{3}\left[\frac{U}{\bar{V}}\right]^2$. For this case $U = 2\bar{V}$ so

$$\beta = \frac{1}{3}[2]^2 = \frac{4}{3} \longleftarrow \beta_{\text{laminar}}$$

For turbulent flow, $\frac{u}{U} = \left(1-\frac{r}{R}\right)^{\frac{1}{n}}$, so $\left(\frac{u}{U}\right)^2 = \left(1-\frac{r}{R}\right)^{\frac{2}{n}}$, and

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left(1-\frac{r}{R}\right)^{\frac{2}{n}} \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

To integrate, let $m = 1 - \frac{r}{R}$. Then $\frac{r}{R} = 1-m$, $d\left(\frac{r}{R}\right) = -dm$, so

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \int_1^0 m^{\frac{2}{n}}(1-m)(-dm) = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left(m^{\frac{2}{n}} - m^{1+\frac{2}{n}}\right) dm$$

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{n}{(n+2)} m^{\frac{2}{n}+1} - \frac{n}{(2n+2)} m^{2+\frac{2}{n}}\right]_0^1 = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{n}{(n+2)} - \frac{n}{(2n+2)}\right]$$

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{(2n+2)n - (n+2)n}{(n+2)(2n+2)}\right] = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{n^2}{(n+2)(2n+2)}\right] \text{ -- } (1)$$

From Eq. 8.24, $\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$

For $n=7$, $\frac{\bar{V}}{U} = 0.817$, so

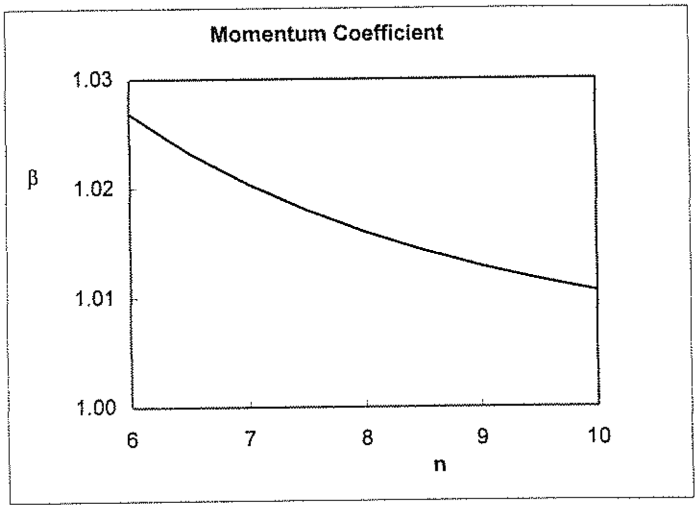
$$\beta = \left[\frac{1}{0.817}\right]^2 \frac{2(7)^2}{(9)(16)} = 1.02 \longleftarrow \beta_{\text{turb}}$$

Problem 8.63

To plot β vs n

- $\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$ ....(8.24)
- $\beta = \left[\frac{U}{\bar{V}}\right]^2 \frac{n^2}{(n+2)(n+1)}$
- $\beta = \frac{(n+1)(2n+1)^2}{4n^2(n+2)}$

n	β
6.0	1.027
6.5	1.023
7.0	1.020
7.5	1.018
8.0	1.016
8.5	1.014
9.0	1.013
9.5	1.012
10.0	1.011



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Problem 8.64

[3]

8.64 Consider fully developed laminar flow of water between stationary parallel plates. The maximum flow speed, plate spacing, and width are 20 ft/s, 0.075 in. and 1.25 in. respectively. Find the kinetic energy coefficient, α.

Given: Laminar flow between parallel plates

Find: Kinetic energy coefficient, α

Solution:

Basic Equation: The kinetic energy coefficient, α is given by

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} \qquad (8.26b)$$

From Section 8-2, for flow between parallel plates

$$u = u_{\max}\left[1-\left(\frac{y}{a/2}\right)^2\right] = \frac{3}{2}\bar{V}\left[1-\left(\frac{y}{a/2}\right)^2\right]$$

since $u_{\max} = \frac{3}{2}\bar{V}$.

Substituting

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} = \frac{\int_A \rho u^3 dA}{\rho \bar{V} A \bar{V}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{V}}\right)^3 dA = \frac{1}{wa}\int_{-\frac{a}{2}}^{\frac{a}{2}} \left(\frac{u}{\bar{V}}\right)^3 w\,dy = \frac{2}{a}\int_0^{\frac{a}{2}} \left(\frac{u}{\bar{V}}\right)^3 dy$$

Then

$$\alpha = \frac{2}{a}\frac{a}{2}\int_0^1 \left(\frac{u}{u_{\max}}\right)^3 \left(\frac{u_{\max}}{\bar{V}}\right)^3 d\left(\frac{y}{a/2}\right) = \left(\frac{3}{2}\right)^3 \int_0^1 \left(1-\eta^2\right)^3 d\eta$$

where $\eta = \frac{y}{a/2}$

Evaluating,

$$\left(1-\eta^2\right)^3 = 1 - 3\eta^2 + 3\eta^4 - \eta^6$$

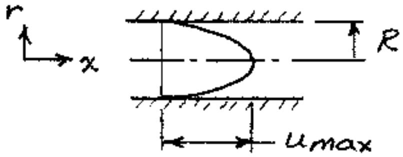
The integral is then

$$\alpha = \left(\frac{3}{2}\right)^3 \int_0^1 \left(1-3\eta^2+3\eta^4-\eta^6\right) d\eta = \left(\frac{3}{2}\right)^3 \left[\eta - \eta^3 + \frac{3}{5}\eta^5 - \frac{1}{7}\eta^7\right]_0^1 = \frac{27}{8}\frac{16}{35} = 1.54$$

Problem 8.65

[3]

Given: Fully developed laminar flow in a circular tube.



Find: Kinetic energy coefficient, α.

Solution: Apply definition of kinetic energy coefficient,

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2}, \quad \dot{m} = \rho \bar{V} A \qquad (8.26b)$$

From the analysis of Section 8-3, for flow in a circular tube,

$$u = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 2\bar{V}\left[1-\left(\frac{r}{R}\right)^2\right] \quad \text{since} \quad u_{max} = 2\bar{V}$$

Substituting into Eq. 8.25b,

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} = \frac{\int_A \rho u^3 dA}{\rho \bar{V} A\, \bar{V}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{V}}\right)^3 dA = \frac{1}{\pi R^2}\int_0^R \left(\frac{u}{\bar{V}}\right)^3 2\pi r\, dr = 2\int_0^1 \left(\frac{u}{\bar{V}}\right)^3 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

Then

$$\alpha = 2\int_0^1 \left(\frac{u}{u_{max}}\right)^3 \left(\frac{u_{max}}{\bar{V}}\right)^3 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2(2)^3 \int_0^1 (1-\eta^2)^3 \eta\, d\eta \quad \text{where } \eta = \frac{r}{R}$$

Evaluating,

$$(1-\eta^2)^3 \eta = \eta - 3\eta^3 + 3\eta^5 - \eta^7$$

The integral is

$$\int_0^1 (1-\eta^2)^3 \eta\, d\eta = \left[\frac{\eta^2}{2} - \frac{3}{4}\eta^4 + \frac{3}{6}\eta^6 - \frac{1}{8}\eta^8\right]_0^1 = \frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8} = \frac{1}{8}$$

Substituting,

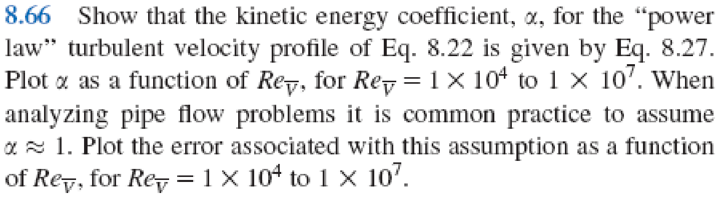
$$\alpha = 16\int_0^1 (1-\eta^2)^3 \eta\, d\eta = 16 \times \frac{1}{8} = 2 \longleftarrow$$

α

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 8.66

[3]



8.66 Show that the kinetic energy coefficient, α, for the "power law" turbulent velocity profile of Eq. 8.22 is given by Eq. 8.27. Plot α as a function of $Re_{\overline{V}}$, for $Re_{\overline{V}} = 1 \times 10^4$ to 1×10^7. When analyzing pipe flow problems it is common practice to assume $\alpha \approx 1$. Plot the error associated with this assumption as a function of $Re_{\overline{V}}$, for $Re_{\overline{V}} = 1 \times 10^4$ to 1×10^7.

Given: Definition of kinetic energy correction coefficient α

Find: α for the power-law velocity profile; plot

Solution:

Equation 8.26b is

$$\alpha = \frac{\int \rho \cdot V^3\, dA}{m_{rate} \cdot V_{av}^2}$$

where *V* is the velocity, m_{rate} is the mass flow rate and V_{av} is the average velocity

For the power-law profile (Eq. 8.22)

$$V = U \cdot \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

For the mass flow rate

$$m_{rate} = \rho \cdot \pi \cdot R^2 \cdot V_{av}$$

Hence the denominator of Eq. 8.26b is

$$m_{rate} \cdot V_{av}^2 = \rho \cdot \pi \cdot R^2 \cdot V_{av}^3$$

We next must evaluate the numerator of Eq. 8.26b

$$\int \rho \cdot V^3\, dA = \int \rho \cdot 2 \cdot \pi \cdot r \cdot U^3 \cdot \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} dr$$

$$\int_0^R \rho \cdot 2 \cdot \pi \cdot r \cdot U^3 \cdot \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} dr = \frac{2 \cdot \pi \cdot \rho \cdot R^2 \cdot n^2 \cdot U^3}{(3+n) \cdot (3 + 2 \cdot n)}$$

To integrate substitute

$$m = 1 - \frac{r}{R} \qquad dm = -\frac{dr}{R}$$

Then

$$r = R \cdot (1 - m) \qquad dr = -R \cdot dm$$

$$\int_0^R \rho \cdot 2 \cdot \pi \cdot r \cdot U^3 \cdot \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} dr = -\int_1^0 \rho \cdot 2 \cdot \pi \cdot R \cdot (1 - m) \cdot m^{\frac{3}{n}} \cdot R\, dm$$

Hence
$$\int \rho\cdot V^3\,dA = \int_0^1 \rho\cdot 2\cdot\pi\cdot R\cdot\left(m^{\frac{3}{n}} - m^{\frac{3}{n}+1}\right)\cdot R\,dm$$

$$\int \rho\cdot V^3\,dA = \frac{2\cdot R^2\cdot n^2\cdot\rho\cdot\pi\cdot U^3}{(3+n)\cdot(3+2\cdot n)}$$

Putting all these results together
$$\alpha = \frac{\int \rho\cdot V^3\,dA}{m_{rate}\cdot V_{av}^{\ 2}} = \frac{\dfrac{2\cdot R^2\cdot n^2\cdot\rho\cdot\pi\cdot U^3}{(3+n)\cdot(3+2\cdot n)}}{\rho\cdot\pi\cdot R^2\cdot V_{av}^{\ 3}}$$

$$\alpha = \left(\frac{U}{V_{av}}\right)^3\cdot\frac{2\cdot n^2}{(3+n)\cdot(3+2\cdot n)}$$

To plot α versus Re_{Vav} we use the following parametric relations

$$n = -1.7 + 1.8\cdot\log\left(Re_u\right) \qquad \text{(Eq. 8.23)}$$

$$\frac{V_{av}}{U} = \frac{2\cdot n^2}{(n+1)\cdot(2\cdot n+1)} \qquad \text{(Eq. 8.24)}$$

$$Re_{Vav} = \frac{V_{av}}{U}\cdot Re_U$$

$$\alpha = \left(\frac{U}{V_{av}}\right)^3\cdot\frac{2\cdot n^2}{(3+n)\cdot(3+2\cdot n)} \qquad \text{(Eq. 8.27)}$$

A value of Re_U leads to a value for *n*; this leads to a value for V_{av}/U; these lead to a value for Re_{Vav} and α

The plots of α, and the error in assuming α = 1, versus Re_{Vav} are shown in the associated *Excel* workbook

Problem 8.66

[3]

8.66 Show that the kinetic energy coefficient, α, for the "power law" turbulent velocity profile of Eq. 8.22 is given by Eq. 8.27. Plot α as a function of $Re_{\bar{V}}$, for $Re_{\bar{V}} = 1 \times 10^4$ to 1×10^7. When analyzing pipe flow problems it is common practice to assume α ≈ 1. Plot the error associated with this assumption as a function of $Re_{\bar{V}}$, for $Re_{\bar{V}} = 1 \times 10^4$ to 1×10^7.

Given: Definition of kinetic energy correction coefficient α

Find: α for the power-law velocity profile; plot

Solution:

$$n = -1.7 + 1.8 \cdot \log\left(Re_u\right) \qquad \text{(Eq. 8.23)}$$

$$\frac{V_{av}}{U} = \frac{2 \cdot n^2}{(n+1) \cdot (2 \cdot n + 1)} \qquad \text{(Eq. 8.24)}$$

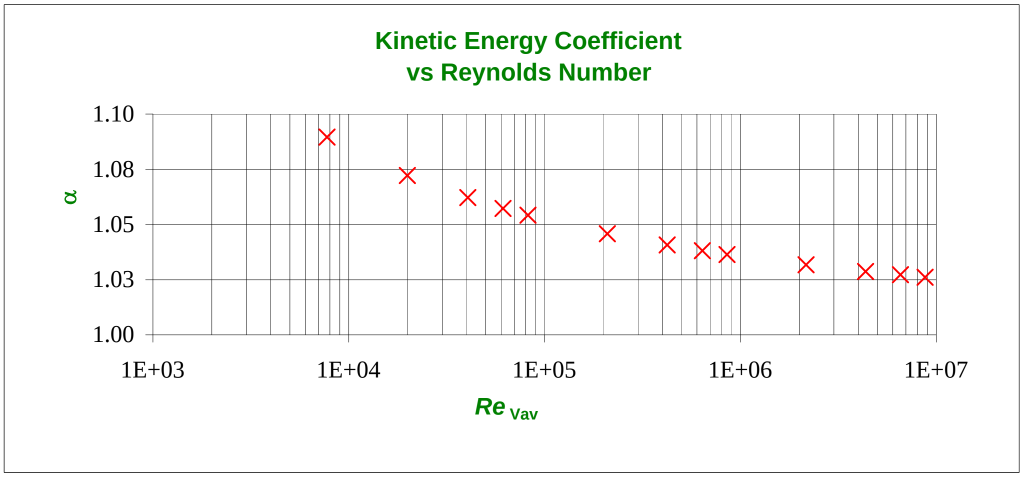
$$Re_{V_{av}} = \frac{V_{av}}{U} \cdot Re_U$$

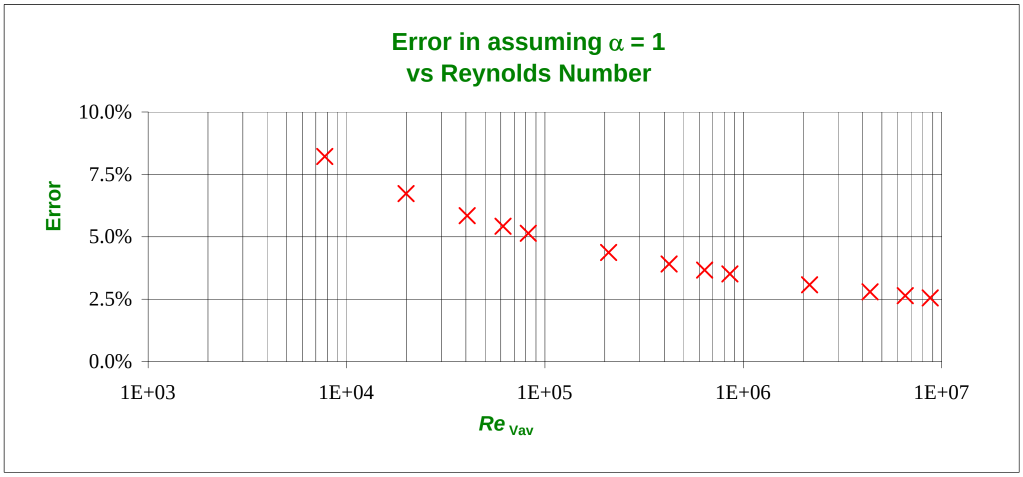
$$\alpha = \left(\frac{U}{V_{av}}\right)^3 \cdot \frac{2 \cdot n^2}{(3+n) \cdot (3 + 2 \cdot n)} \qquad \text{(Eq. 8.27)}$$

this leads to a value for V_{av}/U;

these lead to a value for Re_{Vav} and α

Re_U	n	V_{av}/U	Re_{Vav}	α	α Error
1.00E+04	5.50	0.776	7.76E+03	1.09	8.2%
2.50E+04	6.22	0.797	1.99E+04	1.07	6.7%
5.00E+04	6.76	0.811	4.06E+04	1.06	5.9%
7.50E+04	7.08	0.818	6.14E+04	1.06	5.4%
1.00E+05	7.30	0.823	8.23E+04	1.05	5.1%
2.50E+05	8.02	0.837	2.09E+05	1.05	4.4%
5.00E+05	8.56	0.846	4.23E+05	1.04	3.9%
7.50E+05	8.88	0.851	6.38E+05	1.04	3.7%
1.00E+06	9.10	0.854	8.54E+05	1.04	3.5%
2.50E+06	9.82	0.864	2.16E+06	1.03	3.1%
5.00E+06	10.4	0.870	4.35E+06	1.03	2.8%
7.50E+06	10.7	0.873	6.55E+06	1.03	2.6%
1.00E+07	10.9	0.876	8.76E+06	1.03	2.5%

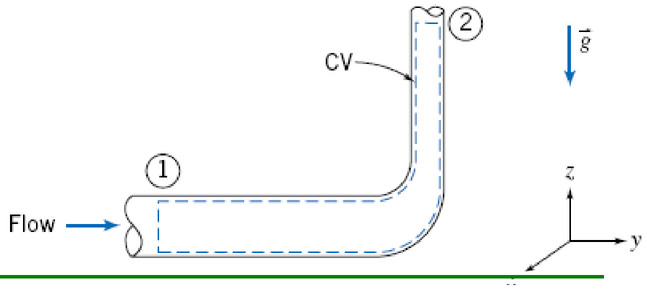




Problem 8.67

[2]

8.67 Measurements are made for the flow configuration shown in Fig. 8.12. At the inlet, section ①, the pressure is 70 kPa (gage), the average velocity is 1.75 m/s, and the elevation is 2.25 m. At the outlet, section ②, the pressure, average velocity, and elevation are 45 kPa (gage), 3.5 m/s, and 3 m, respectively. Calculate the head loss in meters. Convert to units of energy per unit mass.



Given: Data on flow through elbow

Find: Head loss

Solution:

Basic equation $$\left(\frac{p_1}{\rho\cdot g}+\alpha\cdot\frac{V_1^{\,2}}{2\cdot g}+z_1\right)-\left(\frac{p_2}{\rho\cdot g}+\alpha\cdot\frac{V_2^{\,2}}{2\cdot g}+z_2\right)=\frac{h_{lT}}{g}=H_{lT}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1

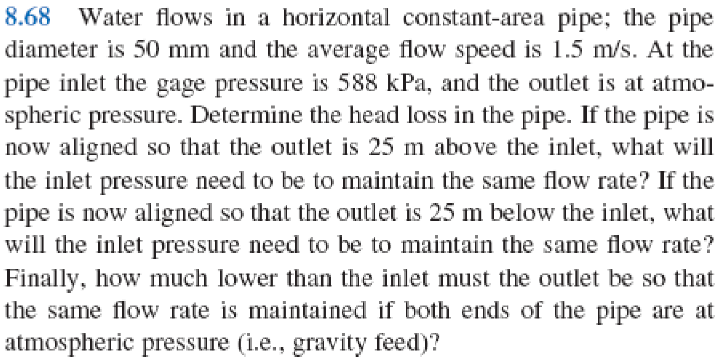
Then $$H_{lT}=\frac{p_1-p_2}{\rho\cdot g}+\frac{V_1^{\,2}-V_2^{\,2}}{2\cdot g}+z_1-z_2$$

$$H_{lT} = (70-45)\times 10^3\cdot\frac{N}{m^2}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}\times\frac{s^2}{9.81\cdot m}+\frac{1}{2}\times\left(1.75^2-3.5^2\right)\cdot\left(\frac{m}{s}\right)^2\times\frac{s^2}{9.81\cdot m}+(2.25-3)\cdot m \qquad H_{lT} = 1.33\,m$$

In terms of energy/mass $$h_{lT} = g\cdot H_{lT} \qquad h_{lT} = 9.81\cdot\frac{m}{s^2}\times 1.33\cdot m\times\frac{N\cdot s^2}{kg\cdot m} \qquad h_{lT} = 13.0\cdot\frac{N\cdot m}{kg}$$

Problem 8.68

[2]



8.68 Water flows in a horizontal constant-area pipe; the pipe diameter is 50 mm and the average flow speed is 1.5 m/s. At the pipe inlet the gage pressure is 588 kPa, and the outlet is at atmospheric pressure. Determine the head loss in the pipe. If the pipe is now aligned so that the outlet is 25 m above the inlet, what will the inlet pressure need to be to maintain the same flow rate? If the pipe is now aligned so that the outlet is 25 m below the inlet, what will the inlet pressure need to be to maintain the same flow rate? Finally, how much lower than the inlet must the outlet be so that the same flow rate is maintained if both ends of the pipe are at atmospheric pressure (i.e., gravity feed)?

Given: Data on flow in a pipe

Find: Head loss for horizontal pipe; inlet pressure for different alignments; slope for gravity feed

Solution:

Given or available data $D = 50\cdot\text{mm}$ $\rho = 1000\cdot\frac{\text{kg}}{\text{m}^3}$

The governing equation between inlet (1) and exit (2) is

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} \qquad (8.29)$$

Horizontal pipe data $p_1 = 588\cdot\text{kPa}$ $p_2 = 0\cdot\text{kPa}$ (Gage pressures)

$z_1 = z_2$ $V_1 = V_2$

Equation 8.29 becomes $h_{lT} = \frac{p_1 - p_2}{\rho}$ $h_{lT} = 588\cdot\frac{\text{J}}{\text{kg}}$

For an inclined pipe with the same flow rate, the head loss will be the same as above; in addition we have the following new data

$z_1 = 0\cdot\text{m}$ $z_2 = 25\cdot\text{m}$

Equation 8.29 becomes $p_1 = p_2 + \rho\cdot g\cdot(z_2 - z_1) + \rho\cdot h_{lT}$ $p_1 = 833\cdot\text{kPa}$

For a declining pipe with the same flow rate, the head loss will be the same as above; in addition we have the following new data

$z_1 = 0\cdot\text{m}$ $z_2 = -25\cdot\text{m}$

Equation 8.29 becomes $p_1 = p_2 + \rho\cdot g\cdot(z_2 - z_1) + \rho\cdot h_{lT}$ $p_1 = 343\cdot\text{kPa}$

For a gravity feed with the same flow rate, the head loss will be the same as above; in addition we have the following new data

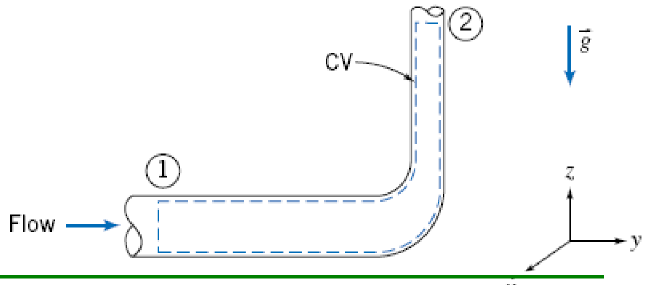
$p_1 = 0\cdot\text{kPa}$ (Gage)

Equation 8.29 becomes $z_2 = z_1 - \frac{h_{lT}}{g}$ $z_2 = -60\,\text{m}$

Problem 8.69

[2]

8.69 For the flow configuration of Fig. 8.12, it is known that the head loss is 1 m. The pressure drop from inlet to outlet is 50 kPa, the velocity doubles from inlet to outlet, and the elevation increase is 2 m. Compute the inlet water velocity.



Given: Data on flow through elbow

Find: Inlet velocity

Solution:

Basic equation
$$\left(\frac{p_1}{\rho\cdot g}+\alpha\cdot\frac{V_1^2}{2\cdot g}+z_1\right)-\left(\frac{p_2}{\rho\cdot g}+\alpha\cdot\frac{V_2^2}{2\cdot g}+z_2\right)=\frac{h_{lT}}{g}=H_{lT}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1

Then
$$V_2^2-V_1^2=\left(2\cdot V_1\right)^2-V_1^2=3\cdot V_1^2=\frac{2\cdot\left(p_1-p_2\right)}{\rho}+2\cdot g\cdot\left(z_1-z_2\right)-2\cdot g\cdot H_{lT}$$

$$V_1=\sqrt{\frac{2}{3}\cdot\left[\frac{\left(p_1-p_2\right)}{\rho}+g\cdot\left(z_1-z_2\right)-g\cdot H_{lT}\right]}$$

$$V_1=\sqrt{\frac{2}{3}\times\left[50\times10^3\cdot\frac{N}{m^2}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}+\frac{9.81\cdot m}{s^2}\times(-2)\cdot m-9.81\cdot\frac{m}{s^2}\times1\cdot m\right]}$$

$$V_1=3.70\,\frac{m}{s}$$

Problem 8.70

[2]

8.70 Consider the pipe flow from the water tower of Example 8.7. After another 10 years the pipe roughness has increased such that the flow is fully turbulent and $f = 0.04$. Find by how much the flow rate is decreased.

Given: Increased friction factor for water tower flow

Find: How much flow is decreased

Solution:

Basic equation from Example 8.7

$$V_2 = \sqrt{\frac{2\cdot g\cdot\left(z_1 - z_2\right)}{f\cdot\left(\frac{L}{D} + 8\right) + 1}}$$

where $L = 680\cdot ft$ $D = 4\cdot in$ $z_1 - z_2 = 80\cdot ft$

With f = 0.0308, we obtain $V_2 = 8.97\cdot\frac{ft}{s}$ and Q = 351 gpm

We need to recompute with f = 0.04

$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 80\cdot ft\times\frac{1}{0.04\cdot\left(\frac{680}{\frac{4}{12}} + 8\right) + 1}} \qquad V_2 = 7.88\frac{ft}{s}$$

Hence

$$Q = V_2\cdot A = V_2\cdot\frac{\pi\cdot D^2}{4}$$

$$Q = 7.88\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{4}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 309\,gpm$$

(From Table G.2 1 ft^3 = 7.48 gal)

Hence the flow is decreased by $(351 - 309)\cdot gpm = 42\,gpm$

Problem 8.71

[2]

8.71 Consider the pipe flow from the water tower of Problem 8.70. To increase delivery, the pipe length is reduced from 600 ft to 300 ft (the flow is still fully turbulent and $f \approx 0.04$). What is the flow rate?

Given: Increased friction factor for water tower flow, and reduced length

Find: How much flow is decreased

Solution:

Basic equation from Example 8.7

$$V_2 = \sqrt{\frac{2\cdot g\cdot\left(z_1 - z_2\right)}{f\cdot\left(\frac{L}{D} + 8\right) + 1}}$$

where now we have $L = 380\cdot ft$ $\quad D = 4\cdot in$ $\quad z_1 - z_2 = 80\cdot ft$

We need to recompute with f = 0.04

$$V_2 = \sqrt{2 \times 32.2\cdot\frac{ft}{s^2} \times 80\cdot ft \times \frac{1}{0.04\cdot\left(\frac{380}{\frac{4}{12}} + 8\right) + 1}} \qquad V_2 = 10.5\frac{ft}{s}$$

Hence

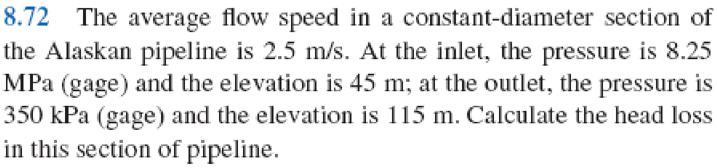
$$Q = V_2\cdot A = V_2\cdot\frac{\pi\cdot D^2}{4}$$

$$Q = 10.5\cdot\frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{4}{12}\cdot ft\right)^2 \times \frac{7.48\cdot gal}{1\cdot ft^3} \times \frac{60\cdot s}{1\cdot min} \qquad Q = 411\,gpm$$

(From Table G.2 1 ft^3 = 7.48 gal)

Problem 8.72

[2]



8.72 The average flow speed in a constant-diameter section of the Alaskan pipeline is 2.5 m/s. At the inlet, the pressure is 8.25 MPa (gage) and the elevation is 45 m; at the outlet, the pressure is 350 kPa (gage) and the elevation is 115 m. Calculate the head loss in this section of pipeline.

Given: Data on flow through Alaskan pipeline

Find: Head loss

Solution:

Basic equation
$$\left(\frac{p_1}{\rho_{oil}\cdot g}+\alpha\cdot\frac{V_1^{\,2}}{2\cdot g}+z_1\right)-\left(\frac{p_2}{\rho_{oil}\cdot g}+\alpha\cdot\frac{V_2^{\,2}}{2\cdot g}+z_2\right)=\frac{h_{lT}}{g}=H_{lT}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) SG = 0.9 (Table A.2)

Then
$$H_{lT}=\frac{p_1-p_2}{SG_{oil}\cdot\rho_{H2O}\cdot g}+z_1-z_2$$

$$H_{lT} = (8250-350)\times 10^3\cdot\frac{N}{m^2}\times\frac{1}{0.9}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}\times\frac{s^2}{9.81\cdot m}+(45-115)\cdot m \qquad H_{lT}=825\,m$$

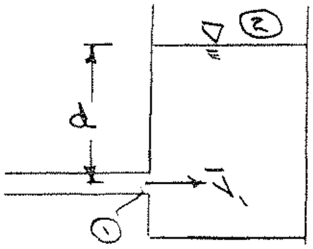
In terms of energy/mass $h_{lT}=g\cdot H_{lT}$
$$h_{lT}=9.81\cdot\frac{m}{s^2}\times 825\cdot m\times\frac{N\cdot s^2}{kg\cdot m} \qquad h_{lT}=8.09\cdot\frac{kN\cdot m}{kg}$$

Problem 8.73

Given: Water flows from a horizontal tube into a very large tank as shown.

$d = 2.5\,m$, $h_\ell = 2\,J/kg$

Find: Average flow speed in tube



Solution:

Apply definition of head loss, Eq 8.29,

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{\ell_T}$$

At free surface, $V_2 = 0$, $p_2 = p_{atm}$

At tube discharge $p_1 = \rho g d$, $z_1 = 0$. Assume $\alpha_1 \simeq 1$

Then

$$g d + \frac{\bar{V}_1^2}{2} - g d = h_{\ell_T}$$

$$\bar{V}_1^2 = 2 h_{\ell_T} = 2 \times 2 \frac{N \cdot m}{kg} \times \frac{kg \cdot m}{N \cdot s^2} = 4\, m^2/s^2$$

$$\bar{V}_1 = 2\, m/s \xleftarrow{\hspace{6em}} \bar{V}_1$$

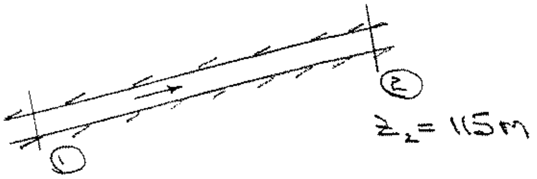
Problem 8.74

[2]

Given: Section of Alaskan pipeline with conditions shown

$h_{l_{1\to 2}} = 6.9$ kJ/kg

Find: outlet pressure, p_2



$z_2 = 115$ m

$p_1 = 8.5$ MPa

$z_1 = 45$ m

Solution:

Computing equation: $\left(\frac{p_1}{\rho} + \alpha_1 \cancel{\frac{\bar{V}_1^2}{2}} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cancel{\frac{\bar{V}_2^2}{2}} + g z_2\right) = h_{l_T}$ (8.29)

Assumptions: (1) incompressible flow, so $\bar{V}_1 = \bar{V}_2$

(2) fully developed so $\alpha_1 = \alpha_2$

(3) SG crude oil = 0.90 (Table A.2)

Then

$$p_2 = p_1 + \rho g (z_1 - z_2) - \rho h_{l_T}$$

$$= 8.5\times10^6 \, \mathrm{N/m^2} + 0.9\times999 \, \frac{\mathrm{kg}}{\mathrm{m^3}} \times 9.81 \, \frac{\mathrm{m}}{\mathrm{s^2}} \times (-70\,\mathrm{m}) \times \frac{\mathrm{N\cdot s^2}}{\mathrm{kg\cdot m}}$$

$$- 0.9\times999 \, \frac{\mathrm{kg}}{\mathrm{m^3}} \times 6.9\times10^3 \, \frac{\mathrm{N\cdot m}}{\mathrm{kg}}$$

$p_2 = 1.68$ MPa ← p_2

Problem 8.75

[2]

Given: Water flow at $Q = 3\,\text{gpm}$ through a horizontal 5/8 in. diameter garden hose. Pressure drop in $L = 50\,\text{ft}$ is 12.3 psi.

Find: Head loss

Solution: Computing equation is

$$h_{l_T} = \left(\frac{p_1}{\rho} + \cancelto{(1)}{\alpha_1 \frac{\bar{V}_1^2}{2}} + \cancelto{(3)}{g z_1}\right) - \left(\frac{p_2}{\rho} + \cancelto{(1)}{\alpha_2 \frac{\bar{V}_2^2}{2}} + \cancelto{(3)}{g z_2}\right)$$

Assumptions: (1) Incompressible flow, so $\bar{V}_1 = \bar{V}_2$

(2) Fully developed so $\alpha_1 = \alpha_2$

(3) Horizontal, so $z_1 = z_2$

Then $$h_{l_T} = \frac{p_1 - p_2}{\rho} = 12.3\,\frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{1.94\,\text{slug}} \times 144\,\frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}$$

$$h_{l_T} = 913\ \text{ft}^2/\text{s}^2 \longleftarrow h_{l_T}$$

Also $$H_{l_T} = \frac{h_{l_T}}{g} = 913\,\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{s}^2}{32.2\,\text{ft}} = 28.4\ \text{ft} \longleftarrow H_{l_T}$$

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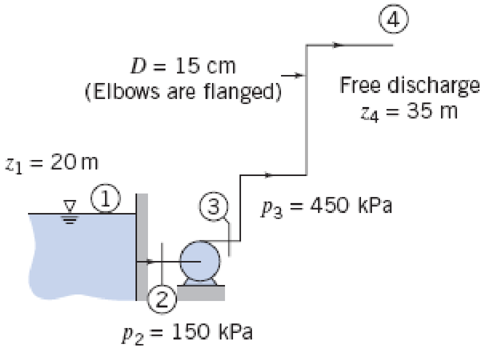
Problem 8.76

[3]

8.76 Water is pumped at the rate of 0.075 m^3/s from a reservoir 20 m above a pump to a free discharge 35 m above the pump. The pressure on the intake side of the pump is 150 kPa and the pressure on the discharge side is 450 kPa. All pipes are commercial steel of 15 cm diameter. Determine (a) the head supplied by the pump and (b) the total head loss between the pump and point of free discharge.



Given: Data on flow from reservoir

Find: Head from pump; head loss

Solution:

Basic equations

$$\left(\frac{p_3}{\rho \cdot g} + \alpha \cdot \frac{V_3^2}{2 \cdot g} + z_3\right) - \left(\frac{p_4}{\rho \cdot g} + \alpha \cdot \frac{V_4^2}{2 \cdot g} + z_4\right) = \frac{h_{lT}}{g} = H_{lT}$$ for flow from 3 to 4

$$\left(\frac{p_3}{\rho \cdot g} + \alpha \cdot \frac{V_3^2}{2 \cdot g} + z_3\right) - \left(\frac{p_2}{\rho \cdot g} + \alpha \cdot \frac{V_2^2}{2 \cdot g} + z_2\right) = \frac{\Delta h_{pump}}{g} = H_{pump}$$ for flow from 2 to 3

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) $V_2 = V_3 = V_4$ (constant area pipe)

Then for the pump

$$H_{pump} = \frac{p_3 - p_2}{\rho \cdot g}$$

$$H_{pump} = (450 - 150) \times 10^3 \cdot \frac{N}{m^2} \times \frac{m^3}{1000 \cdot kg} \times \frac{kg \cdot m}{s^2 \cdot N} \times \frac{s^2}{9.81 \cdot m} \qquad H_{pump} = 30.6\,m$$

In terms of energy/mass

$$h_{pump} = g \cdot H_{pump} \qquad h_{pump} = 9.81 \cdot \frac{m}{s^2} \times 30.6 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad h_{pump} = 300 \cdot \frac{N \cdot m}{kg}$$

For the head loss from 3 to 4

$$H_{lT} = \frac{p_3 - p_4}{\rho \cdot g} + z_3 - z_4$$

$$H_{lT} = (450 - 0) \times 10^3 \cdot \frac{N}{m^2} \times \frac{m^3}{1000 \cdot kg} \times \frac{kg \cdot m}{s^2 \cdot N} \times \frac{s^2}{9.81 \cdot m} + (0 - 35) \cdot m \qquad H_{lT} = 10.9\,m$$

In terms of energy/mass

$$h_{lT} = g \cdot H_{lT} \qquad h_{lT} = 9.81 \cdot \frac{m}{s^2} \times 10.9 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad h_{lT} = 107 \cdot \frac{N \cdot m}{kg}$$

Problem 8.77

[2]

Given: Data measured in fully developed turbulent pipe flow at $Re_U = 50{,}000$ in air:

$\frac{\bar{u}}{U}$	0.343	0.318	0.300	0.264	0.228	0.221	0.179	0.152	0.140
$\frac{y}{R}$	0.0082	0.0075	0.0071	0.0061	0.0055	0.0051	0.0041	0.0034	0.0030

$U = 9.8$ ft/s and $R = 4.86$ in.

Find: (a) Evaluate best-fit value of $d\bar{u}/dy$ from plot.

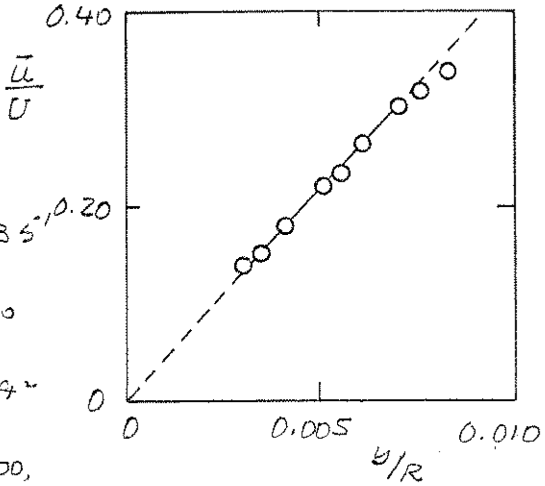
(b) $\tau_w = \mu\, d\bar{u}/dy$

(c) τ_w calculated from friction factor.

Solution: "Best-fit" slope is $\left\{\begin{matrix}\text{from analysis}\\ \text{in Excel file}\end{matrix}\right\}$

$$\frac{d(\bar{u}/U)}{d(y/R)} \approx \frac{\Delta(\bar{u}/U)}{\Delta(y/R)} = 39.8$$

$$\frac{d\bar{u}}{dy} = \frac{U\,d(\bar{u}/U)}{R\,d(y/R)} = 39.8 \times 9.8\,\frac{\text{ft}}{\text{s}} \times \frac{1}{4.86\,\text{in.}} \times 12\,\frac{\text{in.}}{\text{ft}} = 963\ \text{s}^{-1}$$



$\frac{d\bar{u}}{dy}$ ←

For standard air, $\mu = 3.72 \times 10^{-7}\ \text{lbf}\cdot\text{s}/\text{ft}^2$, so

$$\tau_w = \mu\frac{d\bar{u}}{dy} = 3.72\times10^{-7}\,\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2} \times \frac{963}{\text{s}} = 3.58\times10^{-4}\ \text{lbf}/\text{ft}^2$$

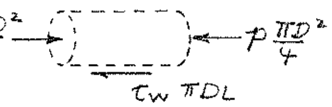
τ_w ←

Friction factor is $f = f(Re, e/D)$. For $Re_U = 50{,}000$, $n = 6.8$ from Eq. 8.23. Then from Eq. 8.24,

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} = 0.812 \quad \text{and} \quad Re_{\bar{V}} = 0.812\,Re_U = 0.812 \times 50{,}000 = 40{,}600$$

Assuming smooth pipe, $f = 0.0219$ from Eq. 8.37

Balancing forces on a fluid element: $(p+\Delta p)\frac{\pi D^2}{4}$



Then $(p+\Delta p)\frac{\pi D^2}{4} - \tau_w \pi D L - p\frac{\pi D^2}{4} = 0$

$$\tau_w = \frac{R}{2}\frac{\Delta p}{L} = \frac{D}{4L} f \frac{L}{D} \rho\frac{\bar{V}^2}{2} = \frac{f}{8}\rho\bar{V}^2\ ; \quad \bar{V} = 0.812\,U = 0.812 \times 9.8\,\frac{\text{ft}}{\text{sec}} = 7.96\ \text{ft/sec}$$

Substituting,

$$\tau_w = \frac{0.0219}{8} \times 0.00238\,\frac{\text{slug}}{\text{ft}^3} \times (7.96)^2\,\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 4.13\times10^{-4}\ \text{lbf}/\text{ft}^2$$

← τ_w

The result calculated from the friction factor is 15 % higher than that evaluated graphically!

Problem 8.78

Given: Small-diameter (i.d. = 0.5 mm) capillary tube made from drawn aluminum is used in place of an expansion valve in a home refrigerator

Find: corresponding relative roughness; with regard to fluid flow, can tube be considered "smooth"?

Solution:

For drawn tubing, from Table 8.1, $e = 0.0015$ mm

Then with $D = 0.5$ mm, $\frac{e}{D} = \frac{0.0015}{0.5} = 0.003$

Looking at the Moody diagram (Fig. 8.13), it is clear that this tube cannot be considered smooth for turbulent flow through the tube.

For laminar flow ($Re < 2300$) the relative roughness has no effect on the flow.

Problem 8.79

[2]

8.79 A smooth, 75-mm diameter pipe carries water (65°C) horizontally. When the mass flow rate is 0.075 kg/s, the pressure drop is measured to be 7.5 Pa per 100 m of pipe. Based on these measurements, what is the friction factor? What is the Reynolds number? Does this Reynolds number generally indicate laminar or turbulent flow? Is the flow actually laminar or turbulent?

Given:

Data on flow in a pipe

Find:

Friction factor; Reynolds number; if flow is laminar or turbulent

Solution:

Given data $D = 75 \cdot \mathrm{mm}$ $\quad \dfrac{\Delta p}{L} = 0.075 \cdot \dfrac{\mathrm{Pa}}{\mathrm{m}}$ $\quad m_{rate} = 0.075 \cdot \dfrac{\mathrm{kg}}{\mathrm{s}}$

From Appendix A $\rho = 1000 \cdot \dfrac{\mathrm{kg}}{\mathrm{m}^3}$ $\quad \mu = 4 \cdot 10^{-4} \cdot \dfrac{\mathrm{N \cdot s}}{\mathrm{m}^2}$

The governing equations between inlet (1) and exit (2) are

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) = h_l \qquad (8.29)$$

$$h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (8.34)$$

For a constant area pipe $V_1 = V_2 = V$

Hence Eqs. 8.29 and 8.34 become $f = \dfrac{2 \cdot D}{L \cdot V^2} \cdot \dfrac{(p_1 - p_2)}{\rho} = \dfrac{2 \cdot D}{\rho \cdot V^2} \cdot \dfrac{\Delta p}{L}$

For the velocity $V = \dfrac{m_{rate}}{\rho \cdot \dfrac{\pi}{4} \cdot D^2}$ $\quad V = 0.017 \dfrac{\mathrm{m}}{\mathrm{s}}$

Hence $f = \dfrac{2 \cdot D}{\rho \cdot V^2} \cdot \dfrac{\Delta p}{L}$ $\quad f = 0.0390$

The Reynolds number is $Re = \dfrac{\rho \cdot V \cdot D}{\mu}$ $\quad Re = 3183$

This Reynolds number indicates the flow is turbulent.

(From Eq. 8.37, at this Reynolds number the friction factor for a smooth pipe is *f* = 0.043; the friction factor computed above thus indicates that, within experimental error, the flow corresponds to turbulent flow in a smooth pipe)

Problem 8.80

[3]

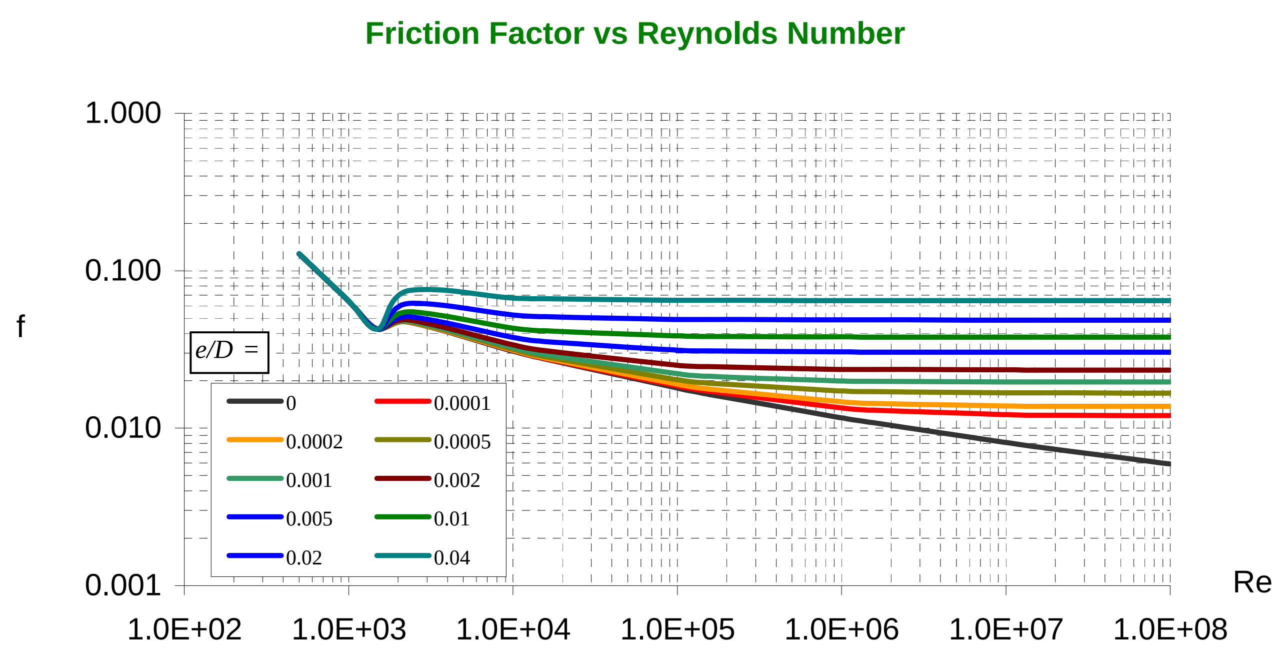
8.80 Using Eqs. 8.36 and 8.37, generate the Moody chart of Fig. 8.13.

Solution:

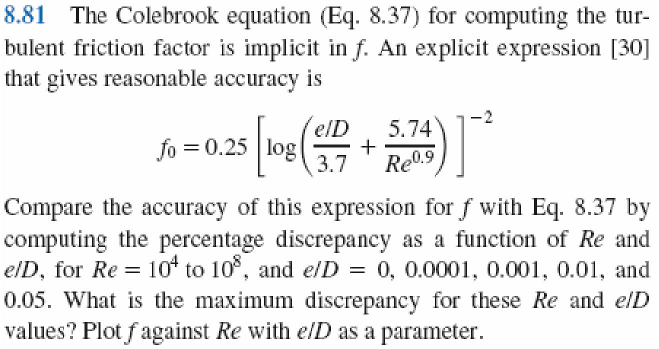
Using the add-in function *Friction factor* from the web site

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.04

Re	*f*									
500	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280
1.00E+03	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640
1.50E+03	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427
2.30E+03	0.0473	0.0474	0.0474	0.0477	0.0481	0.0489	0.0512	0.0549	0.0619	0.0747
1.00E+04	0.0309	0.0310	0.0312	0.0316	0.0324	0.0338	0.0376	0.0431	0.0523	0.0672
1.50E+04	0.0278	0.0280	0.0282	0.0287	0.0296	0.0313	0.0356	0.0415	0.0511	0.0664
1.00E+05	0.0180	0.0185	0.0190	0.0203	0.0222	0.0251	0.0313	0.0385	0.0490	0.0649
1.50E+05	0.0166	0.0172	0.0178	0.0194	0.0214	0.0246	0.0310	0.0383	0.0489	0.0648
1.00E+06	0.0116	0.0134	0.0147	0.0172	0.0199	0.0236	0.0305	0.0380	0.0487	0.0647
1.50E+06	0.0109	0.0130	0.0144	0.0170	0.0198	0.0235	0.0304	0.0379	0.0487	0.0647
1.00E+07	0.0081	0.0122	0.0138	0.0168	0.0197	0.0234	0.0304	0.0379	0.0486	0.0647
1.50E+07	0.0076	0.0121	0.0138	0.0167	0.0197	0.0234	0.0304	0.0379	0.0486	0.0647
1.00E+08	0.0059	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0647



Problem 8.81



8.81 The Colebrook equation (Eq. 8.37) for computing the turbulent friction factor is implicit in f. An explicit expression [30] that gives reasonable accuracy is

$$f_0 = 0.25\left[\log\left(\frac{e/D}{3.7}+\frac{5.74}{Re^{0.9}}\right)\right]^{-2}$$

Compare the accuracy of this expression for f with Eq. 8.37 by computing the percentage discrepancy as a function of Re and e/D, for $Re = 10^4$ to 10^8, and $e/D = 0$, 0.0001, 0.001, 0.01, and 0.05. What is the maximum discrepancy for these Re and e/D values? Plot f against Re with e/D as a parameter.

Using the above formula for f_0, and Eq. 8.37 for f_1

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

Re	f_0									
1.00E+04	0.0310	0.0311	0.0313	0.0318	0.0327	0.0342	0.0383	0.0440	0.0534	0.0750
2.50E+04	0.0244	0.0247	0.0250	0.0258	0.0270	0.0291	0.0342	0.0407	0.0508	0.0731
5.00E+04	0.0208	0.0212	0.0216	0.0226	0.0242	0.0268	0.0325	0.0395	0.0498	0.0724
7.50E+04	0.0190	0.0195	0.0200	0.0212	0.0230	0.0258	0.0319	0.0390	0.0494	0.0721
1.00E+05	0.0179	0.0185	0.0190	0.0204	0.0223	0.0253	0.0316	0.0388	0.0493	0.0720
2.50E+05	0.0149	0.0158	0.0167	0.0186	0.0209	0.0243	0.0309	0.0383	0.0489	0.0717
5.00E+05	0.0131	0.0145	0.0155	0.0178	0.0204	0.0239	0.0307	0.0381	0.0488	0.0717
7.50E+05	0.0122	0.0139	0.0150	0.0175	0.0201	0.0238	0.0306	0.0380	0.0487	0.0716
1.00E+06	0.0116	0.0135	0.0148	0.0173	0.0200	0.0237	0.0305	0.0380	0.0487	0.0716
5.00E+06	0.0090	0.0124	0.0140	0.0168	0.0197	0.0235	0.0304	0.0379	0.0487	0.0716
1.00E+07	0.0081	0.0122	0.0139	0.0168	0.0197	0.0235	0.0304	0.0379	0.0486	0.0716
5.00E+07	0.0066	0.0120	0.0138	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716
1.00E+08	0.0060	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716

Using the add-in function *Friction factor* from the Web

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

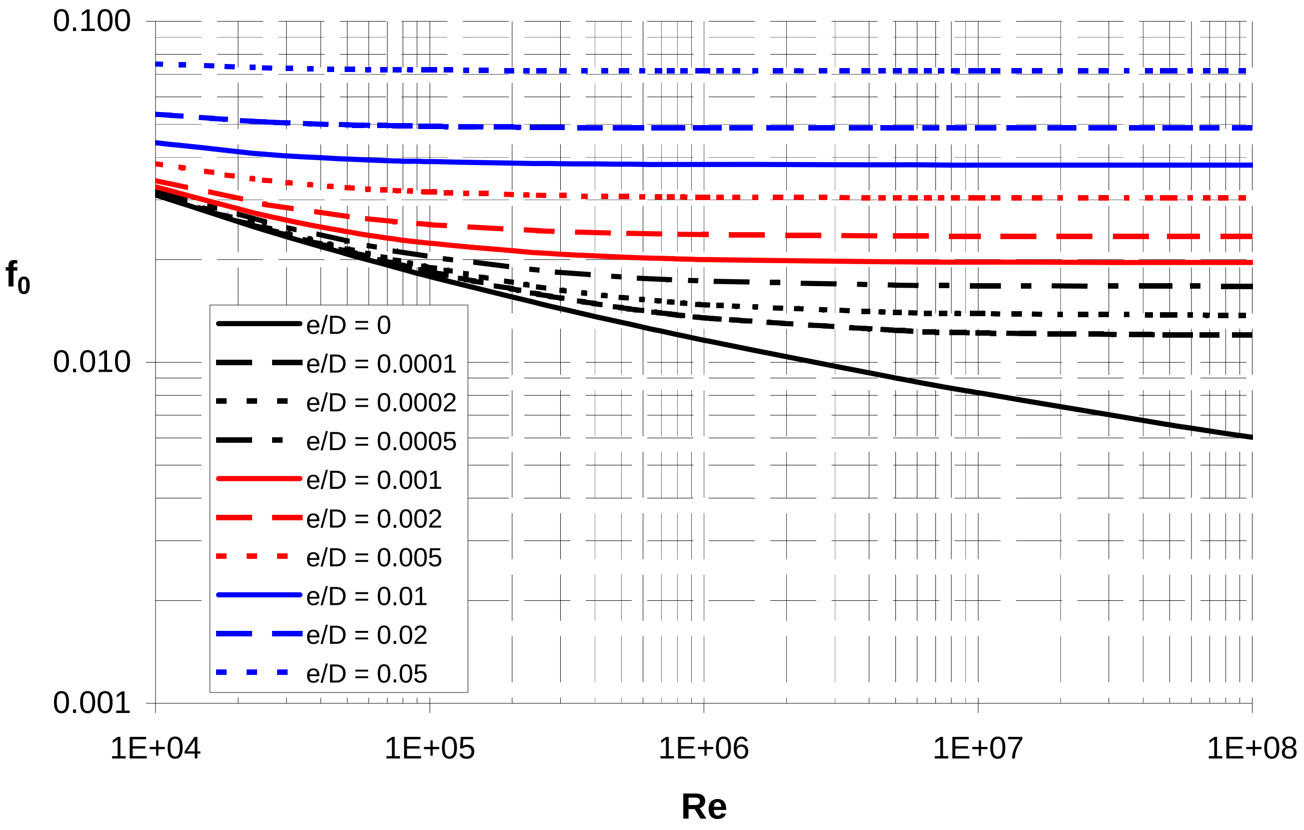
Re	f									
1.00E+04	0.0309	0.0310	0.0312	0.0316	0.0324	0.0338	0.0376	0.0431	0.0523	0.0738
2.50E+04	0.0245	0.0248	0.0250	0.0257	0.0268	0.0288	0.0337	0.0402	0.0502	0.0725
5.00E+04	0.0209	0.0212	0.0216	0.0226	0.0240	0.0265	0.0322	0.0391	0.0494	0.0720
7.50E+04	0.0191	0.0196	0.0200	0.0212	0.0228	0.0256	0.0316	0.0387	0.0492	0.0719
1.00E+05	0.0180	0.0185	0.0190	0.0203	0.0222	0.0251	0.0313	0.0385	0.0490	0.0718
2.50E+05	0.0150	0.0158	0.0166	0.0185	0.0208	0.0241	0.0308	0.0381	0.0488	0.0716
5.00E+05	0.0132	0.0144	0.0154	0.0177	0.0202	0.0238	0.0306	0.0380	0.0487	0.0716
7.50E+05	0.0122	0.0138	0.0150	0.0174	0.0200	0.0237	0.0305	0.0380	0.0487	0.0716
1.00E+06	0.0116	0.0134	0.0147	0.0172	0.0199	0.0236	0.0305	0.0380	0.0487	0.0716
5.00E+06	0.0090	0.0123	0.0139	0.0168	0.0197	0.0235	0.0304	0.0379	0.0486	0.0716
1.00E+07	0.0081	0.0122	0.0138	0.0168	0.0197	0.0234	0.0304	0.0379	0.0486	0.0716
5.00E+07	0.0065	0.0120	0.0138	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716
1.00E+08	0.0059	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716

The error can now be computed

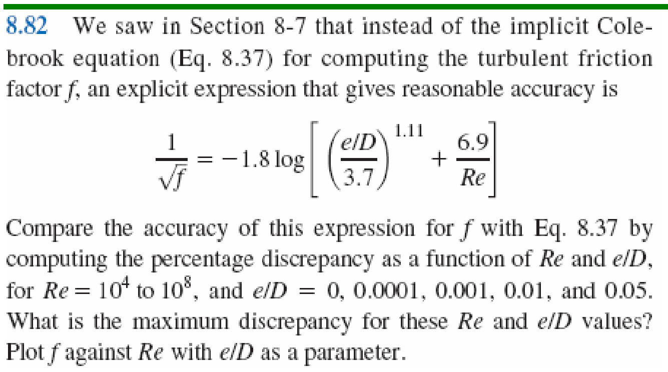
e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

Re	Error (%)									
1.00E+04	0.29%	0.36%	0.43%	0.61%	0.88%	1.27%	1.86%	**2.12%**	2.08%	1.68%
2.50E+04	0.39%	0.24%	0.11%	0.21%	0.60%	1.04%	1.42%	1.41%	1.21%	0.87%
5.00E+04	0.63%	0.39%	0.19%	0.25%	0.67%	1.00%	1.11%	0.98%	0.77%	0.52%
7.50E+04	0.69%	0.38%	0.13%	0.35%	0.73%	0.95%	0.93%	0.77%	0.58%	0.38%
1.00E+05	0.71%	0.33%	0.06%	0.43%	0.76%	0.90%	0.81%	0.64%	0.47%	0.30%
2.50E+05	0.65%	0.04%	0.28%	0.64%	0.72%	0.66%	0.48%	0.35%	0.24%	0.14%
5.00E+05	0.52%	0.26%	0.51%	0.64%	0.59%	0.47%	0.31%	0.21%	0.14%	0.08%
7.50E+05	0.41%	0.41%	0.58%	0.59%	0.50%	0.37%	0.23%	0.15%	0.10%	0.06%
1.00E+06	0.33%	0.49%	0.60%	0.54%	0.43%	0.31%	0.19%	0.12%	0.08%	0.05%
5.00E+06	0.22%	0.51%	0.39%	0.24%	0.16%	0.10%	0.06%	0.03%	0.02%	0.01%
1.00E+07	0.49%	0.39%	0.27%	0.15%	0.10%	0.06%	0.03%	0.02%	0.01%	0.01%
5.00E+07	1.15%	0.15%	0.09%	0.05%	0.03%	0.02%	0.01%	0.01%	0.00%	0.00%
1.00E+08	1.44%	0.09%	0.06%	0.03%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%

The maximum discrepancy is 2.12% at *Re* = 10,000 and *e/D* = 0.01



Problem 8.82



8.82 We saw in Section 8-7 that instead of the implicit Colebrook equation (Eq. 8.37) for computing the turbulent friction factor *f*, an explicit expression that gives reasonable accuracy is

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{e/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

Compare the accuracy of this expression for *f* with Eq. 8.37 by computing the percentage discrepancy as a function of *Re* and *e/D*, for $Re = 10^4$ to 10^8, and $e/D = 0$, 0.0001, 0.001, 0.01, and 0.05. What is the maximum discrepancy for these *Re* and *e/D* values? Plot *f* against *Re* with *e/D* as a parameter.

Using the above formula for f_0, and Eq. 8.37 for f_1

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

Re	f_0									
1.00E+04	0.0309	0.0310	0.0311	0.0315	0.0322	0.0335	0.0374	0.0430	0.0524	0.0741
2.50E+04	0.0244	0.0245	0.0248	0.0254	0.0265	0.0285	0.0336	0.0401	0.0502	0.0727
5.00E+04	0.0207	0.0210	0.0213	0.0223	0.0237	0.0263	0.0321	0.0391	0.0495	0.0722
7.50E+04	0.0189	0.0193	0.0197	0.0209	0.0226	0.0254	0.0316	0.0387	0.0492	0.0720
1.00E+05	0.0178	0.0183	0.0187	0.0201	0.0220	0.0250	0.0313	0.0385	0.0491	0.0719
2.50E+05	0.0148	0.0156	0.0164	0.0183	0.0207	0.0241	0.0308	0.0382	0.0489	0.0718
5.00E+05	0.0131	0.0143	0.0153	0.0176	0.0202	0.0238	0.0306	0.0381	0.0488	0.0717
7.50E+05	0.0122	0.0137	0.0148	0.0173	0.0200	0.0237	0.0305	0.0381	0.0488	0.0717
1.00E+06	0.0116	0.0133	0.0146	0.0172	0.0199	0.0236	0.0305	0.0380	0.0488	0.0717
5.00E+06	0.0090	0.0123	0.0139	0.0168	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717
1.00E+07	0.0081	0.0122	0.0139	0.0168	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717
5.00E+07	0.0066	0.0120	0.0138	0.0167	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717
1.00E+08	0.0060	0.0120	0.0138	0.0167	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717

Using the add-in function *Friction factor* from the Web

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

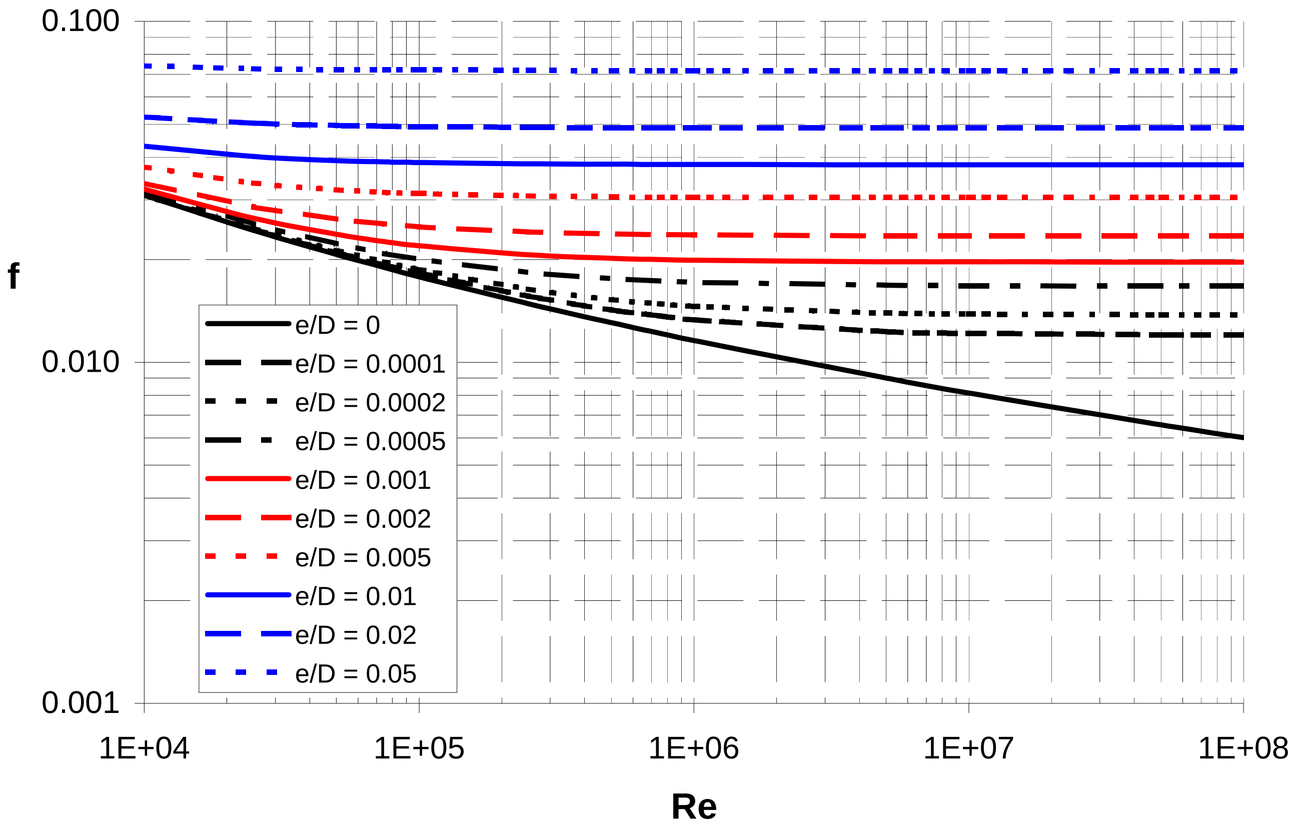
Re	*f*									
1.00E+04	0.0309	0.0310	0.0312	0.0316	0.0324	0.0338	0.0376	0.0431	0.0523	0.0738
2.50E+04	0.0245	0.0248	0.0250	0.0257	0.0268	0.0288	0.0337	0.0402	0.0502	0.0725
5.00E+04	0.0209	0.0212	0.0216	0.0226	0.0240	0.0265	0.0322	0.0391	0.0494	0.0720
7.50E+04	0.0191	0.0196	0.0200	0.0212	0.0228	0.0256	0.0316	0.0387	0.0492	0.0719
1.00E+05	0.0180	0.0185	0.0190	0.0203	0.0222	0.0251	0.0313	0.0385	0.0490	0.0718
2.50E+05	0.0150	0.0158	0.0166	0.0185	0.0208	0.0241	0.0308	0.0381	0.0488	0.0716
5.00E+05	0.0132	0.0144	0.0154	0.0177	0.0202	0.0238	0.0306	0.0380	0.0487	0.0716
7.50E+05	0.0122	0.0138	0.0150	0.0174	0.0200	0.0237	0.0305	0.0380	0.0487	0.0716
1.00E+06	0.0116	0.0134	0.0147	0.0172	0.0199	0.0236	0.0305	0.0380	0.0487	0.0716
5.00E+06	0.0090	0.0123	0.0139	0.0168	0.0197	0.0235	0.0304	0.0379	0.0486	0.0716
1.00E+07	0.0081	0.0122	0.0138	0.0168	0.0197	0.0234	0.0304	0.0379	0.0486	0.0716
5.00E+07	0.0065	0.0120	0.0138	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716
1.00E+08	0.0059	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716

The error can now be computed

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

Re	Error (%)									
1.00E+04	0.01%	0.15%	0.26%	0.46%	0.64%	0.73%	0.55%	0.19%	0.17%	0.43%
2.50E+04	0.63%	0.88%	1.02%	1.20%	1.22%	1.03%	0.51%	0.11%	0.14%	0.29%
5.00E+04	0.85%	1.19%	1.32%	1.38%	1.21%	0.84%	0.28%	0.00%	0.16%	0.24%
7.50E+04	0.90%	1.30%	1.40%	1.35%	1.07%	0.65%	0.16%	0.06%	0.17%	0.23%
1.00E+05	0.92%	1.34%	**1.42%**	1.28%	0.94%	0.52%	0.09%	0.09%	0.18%	0.22%
2.50E+05	0.84%	1.33%	1.25%	0.85%	0.47%	0.16%	0.07%	0.15%	0.19%	0.21%
5.00E+05	0.70%	1.16%	0.93%	0.48%	0.19%	0.00%	0.13%	0.18%	0.20%	0.20%
7.50E+05	0.59%	0.99%	0.72%	0.30%	0.07%	0.07%	0.16%	0.18%	0.20%	0.20%
1.00E+06	0.50%	0.86%	0.57%	0.20%	0.01%	0.10%	0.17%	0.19%	0.20%	0.20%
5.00E+06	0.07%	0.17%	0.01%	0.11%	0.15%	0.18%	0.19%	0.20%	0.20%	0.20%
1.00E+07	0.35%	0.00%	0.09%	0.15%	0.18%	0.19%	0.20%	0.20%	0.20%	0.20%
5.00E+07	1.02%	0.16%	0.18%	0.19%	0.20%	0.20%	0.20%	0.20%	0.20%	0.20%
1.00E+08	1.31%	0.18%	0.19%	0.20%	0.20%	0.20%	0.20%	0.20%	0.20%	0.20%

The maximum discrepancy is 1.42% at *Re* = 100,000 and *e/D* = 0.0002



Problem 8.83

[2]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Moody diagram gives Darcy friction factor, f.

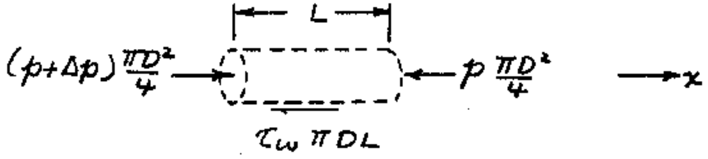
Fanning friction factor is $f_F \equiv \dfrac{\tau_w}{\frac{1}{2}\rho\bar{V}^2}$

Find: Relate Darcy and Fanning friction factors for fully developed pipe flow. Show $f = 4f_F$.

Solution: Consider cylindrical CV containing fluid in pipe; apply force balance, definition of f.

Basic equations: $\Sigma F_x = 0$

$$\Delta p = f\frac{L}{D}\rho\frac{\bar{V}^2}{2}$$



From the force balance,

$$(p+\Delta p)\frac{\pi D^2}{4} - \tau_w \pi DL - p\frac{\pi D^2}{4} = 0 \qquad \text{or} \qquad \tau_w = \frac{D}{4}\frac{\Delta p}{L}$$

Substituting,

$$\tau_w = \frac{D}{4L}\, f\,\frac{L}{D}\,\rho\frac{\bar{V}^2}{2} = f\,\rho\frac{\bar{V}^2}{8}$$

But

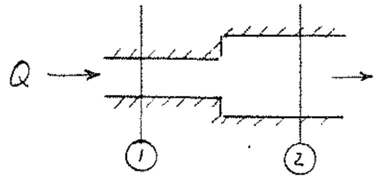
$$f_F \equiv \frac{\tau_w}{\frac{1}{2}\rho\bar{V}^2} = \frac{f\rho\bar{V}^2}{8}\,\frac{2}{\rho\bar{V}^2} = \frac{f}{4}$$

f_F

Problem 8.84

Given: Water flow through sudden enlargement from 25 mm to 50 mm diameter. $Q = 1.25$ liters per minute.

Find: Pressure rise across enlargement.
Comparison with value for frictionless flow.



Solution: Apply energy equation for pipe flow.

$D_1 = 25\text{ mm}$ $D_2 = 50\text{ mm}$

Computing equation: $\dfrac{p_1}{\rho} + \alpha_1 \dfrac{\bar{V}_1^2}{2} + \cancel{gz_1} = \dfrac{p_2}{\rho} + \alpha_2 \dfrac{\bar{V}_2^2}{2} + \cancel{gz_2} + h_{l_T}$

$h_{l_T} = K \dfrac{\bar{V}_1^2}{2}$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section: $\alpha_1 = \alpha_2 = 1$
(4) Horizontal section

Then

$$p_2 - p_1 = \frac{\rho}{2}(\bar{V}_1^2 - \bar{V}_2^2) - \rho h_{l_{T_{12}}}$$

From continuity, $\bar{V}_1 A_1 = \bar{V}_2 A_2$, so $\bar{V}_2 = \bar{V}_1 \dfrac{A_1}{A_2} = \bar{V}_1 \left(\dfrac{D_1}{D_2}\right)^2$; $\bar{V}_2^2 = \bar{V}_1^2 \left(\dfrac{D_1}{D_2}\right)^4$

From Fig. 8.14, at $AR = \left(\dfrac{D_1}{D_2}\right)^2 = \dfrac{1}{4}$, $K = 0.56$.

$$\bar{V}_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times 1.25 \frac{\text{L}}{\text{s}} \times 10^{-3} \frac{\text{m}^3}{\text{L}} \times \frac{1}{(25 \times 10^{-3})^2 \text{m}^2} = 2.55 \text{ m/s}$$

Substituting,

$$p_2 - p_1 = \frac{\rho \bar{V}_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] - K\rho\frac{\bar{V}_1^2}{2} = \frac{1}{2}\rho \bar{V}_1^2 \left[1 - \left(\frac{D_1}{D_2}\right)^4 - K\right]$$

$$= \frac{1}{2} \times 999 \frac{\text{kg}}{\text{m}^3} \times (2.55)^2 \frac{\text{m}^2}{\text{s}^2} \left[1 - \left(\frac{1}{2}\right)^4 - 0.56\right] \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$$

$p_2 - p_1 = 1.22 \text{ kPa}$ Δp_a

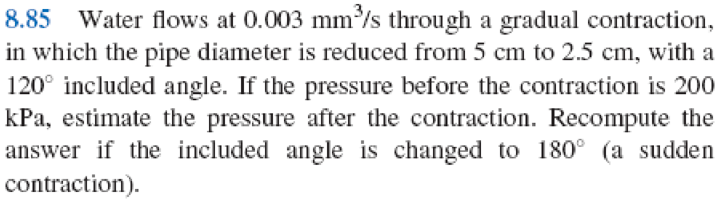
For frictionless flow, $K = 0$, and

$p_2 - p_1 = \frac{1}{2}\rho \bar{V}_1^2 \left[1 - \left(\frac{D_1}{D_2}\right)^4\right] = 3.04 \text{ kPa}$ Δp_f

Thus $\dfrac{\Delta p_{\text{actual}}}{\Delta p_{\text{frictionless}}} = \dfrac{1.22}{3.04} = 0.403$ or 40.3% Ratio

Problem 8.85

[2]



8.85 Water flows at 0.003 mm³/s through a gradual contraction, in which the pipe diameter is reduced from 5 cm to 2.5 cm, with a 120° included angle. If the pressure before the contraction is 200 kPa, estimate the pressure after the contraction. Recompute the answer if the included angle is changed to 180° (a sudden contraction).

Given: Flow through gradual contraction

Find: Pressure after contraction; compare to sudden contraction

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lm} \qquad h_{lm}=K\cdot\frac{V_2^2}{2} \qquad Q=V\cdot A$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Horizontal

For an included angle of 120^{o} and an area ratio $\frac{A_2}{A_1}=\left(\frac{D_2}{D_1}\right)^2=\left(\frac{2.5}{5}\right)^2=0.25$ we find from Table 8. $K = 0.27$

Hence the energy equation becomes $\left(\frac{p_1}{\rho}+\frac{V_1^2}{2}\right)-\left(\frac{p_2}{\rho}+\frac{V_2^2}{2}\right)=K\cdot\frac{V_2^2}{2}$ with $V_1=\frac{4\cdot Q}{\pi\cdot D_1^2}$ $V_2=\frac{4\cdot Q}{\pi\cdot D_2^2}$

$$p_2=p_1-\frac{\rho}{2}\cdot\left[(1+K)\cdot V_2^2-V_1^2\right]=p_2-\frac{8\cdot\rho\cdot Q^2}{\pi^2}\cdot\left[\frac{(1+K)}{D_2^4}-\frac{1}{D_1^4}\right]$$

$$p_2=200\times10^3\cdot\frac{N}{m^2}-\frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left[\frac{0.003\cdot mm^3}{s}\cdot\left(\frac{1\cdot m}{1000\cdot mm}\right)^3\right]^2\times\left[(1+0.27)\times\frac{1}{(0.025\cdot m)^4}-\frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_2 = 200\cdot kPa$ No change because the flow rate is miniscule!

Repeating the above analysis for an included angle of 180º (sudden contraction) $K = 0.41$

$$p_2=200\times10^3\cdot\frac{N}{m^2}-\frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left[\frac{0.003\cdot mm^3}{s}\cdot\left(\frac{1\cdot m}{1000\cdot mm}\right)^3\right]^2\times\left[(1+0.41)\times\frac{1}{(0.025\cdot m)^4}-\frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_2 = 200\cdot kPa$ No change because the flow rate is miniscule!

The flow rate has a typo: it is much too small, and should be $Q=0.003\cdot\frac{m^3}{s}$ not $Q=0.003\cdot\frac{mm^3}{s}$

$$p_2=200\times10^3\cdot\frac{N}{m^2}-\frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left(\frac{0.003\cdot m^3}{s}\right)^2\times\left[(1+0.27)\times\frac{1}{(0.025\cdot m)^4}-\frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m} \qquad p_2=177\cdot kPa$$

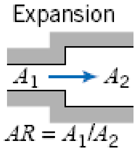
Repeating the above analysis for an included angle of 180º (sudden contraction) $K = 0.41$

$$p_2=200\times10^3\cdot\frac{N}{m^2}-\frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left(\frac{0.003\cdot m^3}{s}\right)^2\times\left[(1+0.41)\times\frac{1}{(0.025\cdot m)^4}-\frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m} \qquad p_2=175\cdot kPa$$

There is slightly more loss in the sudden contraction

Problem 8.86

8.86 Air at standard conditions flows through a sudden expansion in a circular duct. The upstream and downstream duct diameters are 75 mm and 225 mm, respectively. The pressure downstream is 5 mm of water higher than that upstream. Determine the average speed of the air approaching the expansion and the volume flow rate.



Given: Flow through sudden expansion

Find: Inlet speed; Volume flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lm}$ $\quad h_{lm} = K\cdot\frac{V_1^2}{2}$ $\quad Q = V\cdot A$ $\quad \Delta p = \rho_{H2O}\cdot g\cdot\Delta h$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Horizontal

Hence the energy equation becomes

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) = K\cdot\frac{V_1^2}{2}$$

From continuity $V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot AR$

Hence $\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_1^2\cdot AR^2}{2}\right) = K\cdot\frac{V_1^2}{2}$

Solving for V_1 $\quad V_1 = \sqrt{\frac{2\cdot(p_2 - p_1)}{\rho\cdot(1 - AR^2 - K)}}$ $\quad AR = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{75}{225}\right)^2 = 0.111$ $\quad$ so from Fig. 8.14 $\quad K = 0.8$

Also $p_2 - p_1 = \rho_{H2O}\cdot g\cdot\Delta h = 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times\frac{5}{1000}\cdot m\times\frac{N\cdot s^2}{kg\cdot m} = 49.1\cdot Pa$

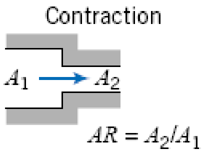
Hence $V_1 = \sqrt{2\times 49.1\cdot\frac{N}{m^2}\times\frac{m^3}{1.23\cdot kg}\times\frac{1}{(1 - 0.111^2 - 0.8)}\times\frac{kg\cdot m}{N\cdot s^2}}$ $\quad V_1 = 20.6\frac{m}{s}$

$Q = V_1\cdot A_1 = \frac{\pi\cdot D_1^2}{4}\cdot V_1$ $\quad Q = \frac{\pi}{4}\times\left(\frac{75}{1000}\cdot m\right)^2\times 20.6\cdot\frac{m}{s}$ $\quad Q = 0.0910\cdot\frac{m^3}{s}$ $\quad Q = 5.46\cdot\frac{m^3}{min}$

Problem 8.87

[2]

8.87 Water flows through a 2-in. diameter tube that suddenly contracts to 1 in. diameter. The pressure drop across the contraction is 0.5 psi. Determine the volume flow rate.



Given: Flow through sudden contraction

Find: Volume flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lm}$ $\quad h_{lm}=K\cdot\frac{V_2^2}{2}$ $\quad Q=V\cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Horizontal

Hence the energy equation becomes

$$\left(\frac{p_1}{\rho}+\frac{V_1^2}{2}\right)-\left(\frac{p_2}{\rho}+\frac{V_2^2}{2}\right)=K\cdot\frac{V_2^2}{2}$$

From continuity $V_1=V_2\cdot\frac{A_2}{A_1}=V_2\cdot AR$

Hence $\left(\frac{p_1}{\rho}+\frac{V_2^2\cdot AR^2}{2}\right)-\left(\frac{p_2}{\rho}+\frac{V_2^2}{2}\right)=K\cdot\frac{V_2^2}{2}$

Solving for V_2 $\quad V_2=\sqrt{\frac{2\cdot(p_1-p_2)}{\rho\cdot\left(1-AR^2+K\right)}}$ $\quad AR=\left(\frac{D_2}{D_1}\right)^2=\left(\frac{1}{2}\right)^2=0.25$ so from Fig. 8.14 $\quad K=0.4$

Hence $V_2=\sqrt{2\times 0.5\cdot\frac{lbf}{in^2}\times\left(\frac{12\cdot in}{1\cdot ft}\right)^2\times\frac{ft^3}{1.94\cdot slug}\times\frac{1}{\left(1-0.25^2+0.4\right)}\times\frac{slug\cdot ft}{lbf\cdot s^2}}$ $\quad V_2=7.45\cdot\frac{ft}{s}$

$Q=V_2\cdot A_2=\frac{\pi\cdot D_2^2}{4}\cdot V_2$ $\quad Q=\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2\times 7.45\cdot\frac{ft}{s}$ $\quad Q=0.0406\cdot\frac{ft^3}{s}$ $\quad Q=2.44\cdot\frac{ft^3}{min}$ $\quad Q=18.2\,gpm$

Problem 8.88

[2]

8.88 In an undergraduate laboratory you have been assigned the task of developing a crude flow meter for measuring the flow in a 400-mm diameter water pipe system. You are to install a 200-mm diameter section of pipe, and a water manometer to measure the pressure drop at the sudden contraction. Derive an expression for the theoretical calibration constant k in $Q = k\sqrt{\Delta h}$, where Q is the volume flow rate in L/min, and Δh is the manometer deflection in mm. Plot the theoretical calibration curve for a flow rate range of 0 to 200 L/min. Would you expect this to be a practical device for measuring flow rate?

Given: Data on a pipe sudden contraction

Find: Theoretical calibration constant; plot

Solution:

Given data $D_1 = 400\cdot\text{mm}$ $D_2 = 200\cdot\text{mm}$

The governing equations between inlet (1) and exit (2) are

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l \qquad (8.29)$$

where

$$h_l = K\cdot\frac{V_2^2}{2} \qquad (8.40a)$$

Hence the pressure drop is (assuming $\alpha = 1$)

$$\Delta p = p_1 - p_2 = \rho\cdot\left(\frac{V_2^2}{2} - \frac{V_1^2}{2} + K\cdot\frac{V_2^2}{2}\right)$$

For the sudden contraction

$$V_1\cdot\frac{\pi}{4}\cdot D_1^2 = V_2\cdot\frac{\pi}{4}\cdot D_2^2 = Q$$

or

$$V_2 = V_1\cdot\left(\frac{D_1}{D_2}\right)^2$$

so

$$\Delta p = \frac{\rho\cdot V_1^2}{2}\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K) - 1\right]$$

For the pressure drop we can use the manometer equation

$$\Delta p = \rho\cdot g\cdot\Delta h$$

Hence

$$\rho\cdot g\cdot\Delta h = \frac{\rho\cdot V_1^2}{2}\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K) - 1\right]$$

In terms of flow rate *Q*

$$\rho\cdot g\cdot\Delta h = \frac{\rho}{2}\cdot\frac{Q^2}{\left(\frac{\pi}{4}\cdot D_1^{\,2}\right)^2}\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K)-1\right]$$

or

$$g\cdot\Delta h = \frac{8\cdot Q^2}{\pi^2\cdot D_1^{\,4}}\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K)-1\right]$$

Hence for flow rate *Q* we find

$$Q = k\cdot\sqrt{\Delta h}$$

where

$$k = \sqrt{\frac{g\cdot\pi^2\cdot D_1^{\,4}}{8\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K)-1\right]}}$$

For *K*, we need the aspect ratio *AR*

$$AR = \left(\frac{D_2}{D_1}\right)^2 \qquad AR = 0.25$$

From Fig. 8.15

$$K = 0.4$$

Using this in the expression for *k*, with the other given values

$$k = \sqrt{\frac{g\cdot\pi^2\cdot D_1^{\,4}}{8\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K)-1\right]}} = 0.12\cdot\frac{m^{\frac{5}{2}}}{s}$$

For Δ*h* in mm and *Q* in L/min

$$k = 228\,\frac{\frac{L}{min}}{mm^{\frac{1}{2}}}$$

The plot of theoretical *Q* versus flow rate Δ*h* is shown in the associated *Excel* workbook

Problem 8.88

[2]

8.88 In an undergraduate laboratory you have been assigned the task of developing a crude flow meter for measuring the flow in a 400-mm diameter water pipe system. You are to install a 200-mm diameter section of pipe, and a water manometer to measure the pressure drop at the sudden contraction. Derive an expression for the theoretical calibration constant k in $Q = k\sqrt{\Delta h}$, where Q is the volume flow rate in L/min, and Δh is the manometer deflection in mm. Plot the theoretical calibration curve for a flow rate range of 0 to 200 L/min. Would you expect this to be a practical device for measuring flow rate?

Given: Data on a pipe sudden contraction

Find: Theoretical calibration constant; plot

Solution:

$D_1 =$ 400 mm

$D_1 =$ 200 mm

$K =$ 0.4

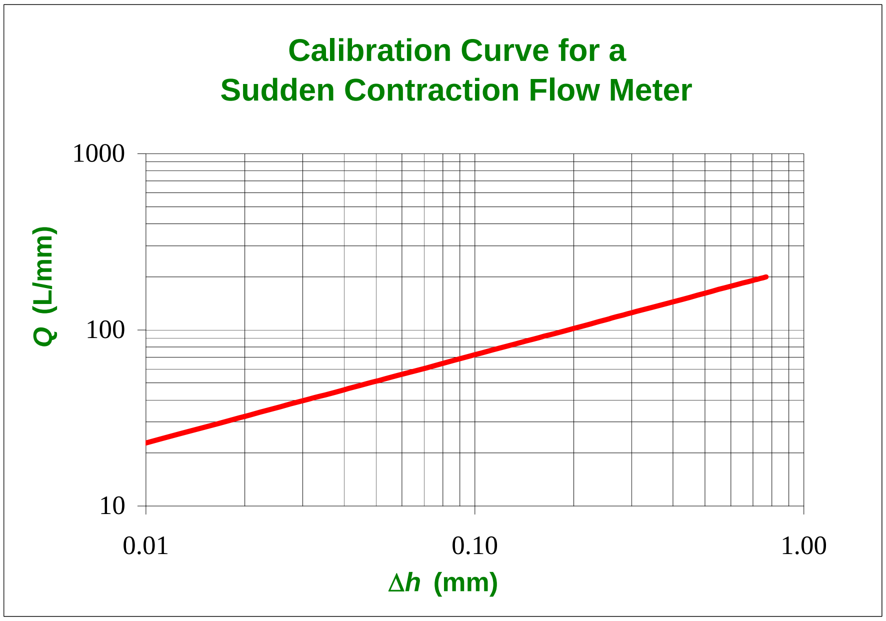
$k =$ 228 L/min/mm$^{1/2}$

$$Q = k\cdot\sqrt{\Delta h}$$

$$k = \sqrt{\frac{g\cdot\pi^2\cdot D_1^{\,4}}{8\cdot\left[\left(\frac{D_1}{D_2}\right)^4(1+K)-1\right]}}$$

The values for Δh are quite low; this would not be a good meter - it is not sensitive enough.
In addition, it is non-linear.

Δh (mm)	Q (L/min)
0.010	23
0.020	32
0.030	40
0.040	46
0.050	51
0.075	63
0.100	72
0.150	88
0.200	102
0.250	114
0.300	125
0.400	144
0.500	161
0.600	177
0.700	191
0.767	200



Calibration Curve for a Sudden Contraction Flow Meter

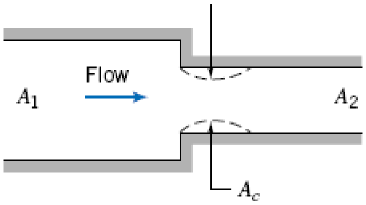
Problem 8.89

[3]

8.89 Flow through a sudden contraction is shown. The minimum flow area at the vena contracta is given in terms of the area ratio by the contraction coefficient [31],

$$C_c = \frac{A_c}{A_2} = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$$

The loss in a sudden contraction is mostly a result of the vena contracta: The fluid accelerates into the contraction, there is flow separation (as shown by the dashed lines), and the vena contracta acts as a miniature sudden expansion with significant secondary flow losses. Use these assumptions to obtain and plot estimates of the minor loss coefficient for a sudden contraction, and compare with the data presented in Fig. 8.15.



Given: Contraction coefficient for sudden contraction

Find: Expression for minor head loss; compare with Fig. 8.15; plot

Solution:

We analyse the loss at the "sudden expansion" at the vena contracta

The governing CV equations (mass, momentum, and energy) are

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad (4.12)$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \, \rho \, d\forall + \int_{CS} u \, \rho \vec{V} \cdot d\vec{A} \qquad (4.18a)$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t}\int_{CV} e \, \rho \, d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A} \qquad (4.56)$$

Assume: 1) Steady flow 2) Incompressible flow 3) Uniform flow at each section 4) Horizontal: no body force 5) No shaft work 6) Neglect viscous friction 7) Neglect gravity

The mass equation becomes

$$V_c \cdot A_c = V_2 \cdot A_2 \qquad (1)$$

The momentum equation becomes

$$p_c \cdot A_2 - p_2 \cdot A_2 = V_c \cdot \left(-\rho \cdot V_c \cdot A_c\right) + V_2 \cdot \left(\rho \cdot V_2 \cdot A_2\right)$$

or (using Eq. 1)

$$p_c - p_2 = \rho \cdot V_c \cdot \frac{A_c}{A_2} \cdot \left(V_2 - V_c\right) \qquad (2)$$

The energy equation becomes

$$Q_{rate} = \left(u_c + \frac{p_c}{\rho} + V_c^{\,2}\right) \cdot \left(-\rho \cdot V_c \cdot A_c\right) + \left(u_2 + \frac{p_2}{\rho} + V_2^{\,2}\right) \cdot \left(\rho \cdot V_2 \cdot A_2\right)$$

or (using Eq. 1)

$$h_{lm} = u_2 - u_c - \frac{Q_{rate}}{m_{rate}} = \frac{V_c^{\,2} - V_2^{\,2}}{2} + \frac{p_c - p_2}{\rho} \qquad (3)$$

Combining Eqs. 2 and 3 $$h_{lm} = \frac{V_c^2 - V_2^2}{2} + V_c \cdot \frac{A_c}{A_2} \cdot \left(V_2 - V_c\right)$$

$$h_{lm} = \frac{V_c^2}{2} \cdot \left[1 - \left(\frac{V_2}{V_c}\right)^2\right] + V_c^2 \cdot \frac{A_c}{A_2} \cdot \left[\left(\frac{V_2}{V_c}\right) - 1\right]$$

From Eq. 1 $$C_c = \frac{A_c}{A_2} = \frac{V_2}{V_c}$$

Hence $$h_{lm} = \frac{V_c^2}{2} \cdot \left(1 - C_c^2\right) + V_c^2 \cdot C_c \cdot \left(C_c - 1\right)$$

$$h_{lm} = \frac{V_c^2}{2} \cdot \left(1 - C_c^2 + 2 \cdot C_c^2 - 2 \cdot C_c\right)$$

$$h_{lm} = \frac{V_c^2}{2} \cdot \left(1 - C_c\right)^2 \qquad (4)$$

But we have $$h_{lm} = K \cdot \frac{V_2^2}{2} = K \cdot \frac{V_c^2}{2} \cdot \left(\frac{V_2}{V_c}\right)^2 = K \cdot \frac{V_c^2}{2} \cdot C_c^2 \qquad (5)$$

Hence, comparing Eqs. 4 and 5 $$K = \frac{\left(1 - C_c\right)^2}{C_c^2}$$

So, finally $$K = \left(\frac{1}{C_c} - 1\right)^2$$

where $$C_c = 0.62 + 0.38 \cdot \left(\frac{A_2}{A_1}\right)^3$$

This result, and the curve of Fig. 8.15, are shown in the associated *Excel* workbook. The agreement is reasonable.

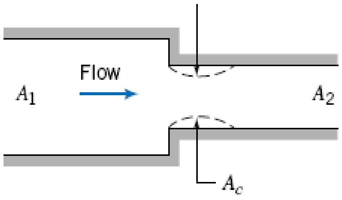
Problem 8.89

[3]

8.89 Flow through a sudden contraction is shown. The minimum flow area at the vena contracta is given in terms of the area ratio by the contraction coefficient [31],

$$C_c = \frac{A_c}{A_2} = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$$

The loss in a sudden contraction is mostly a result of the vena contracta: The fluid accelerates into the contraction, there is flow separation (as shown by the dashed lines), and the vena contracta acts as a miniature sudden expansion with significant secondary flow losses. Use these assumptions to obtain and plot estimates of the minor loss coefficient for a sudden contraction, and compare with the data presented in Fig. 8.15.



Given: Contraction coefficient for sudden contraction

Find: Expression for minor head loss; compare with Fig. 8.15; plot

Solution:

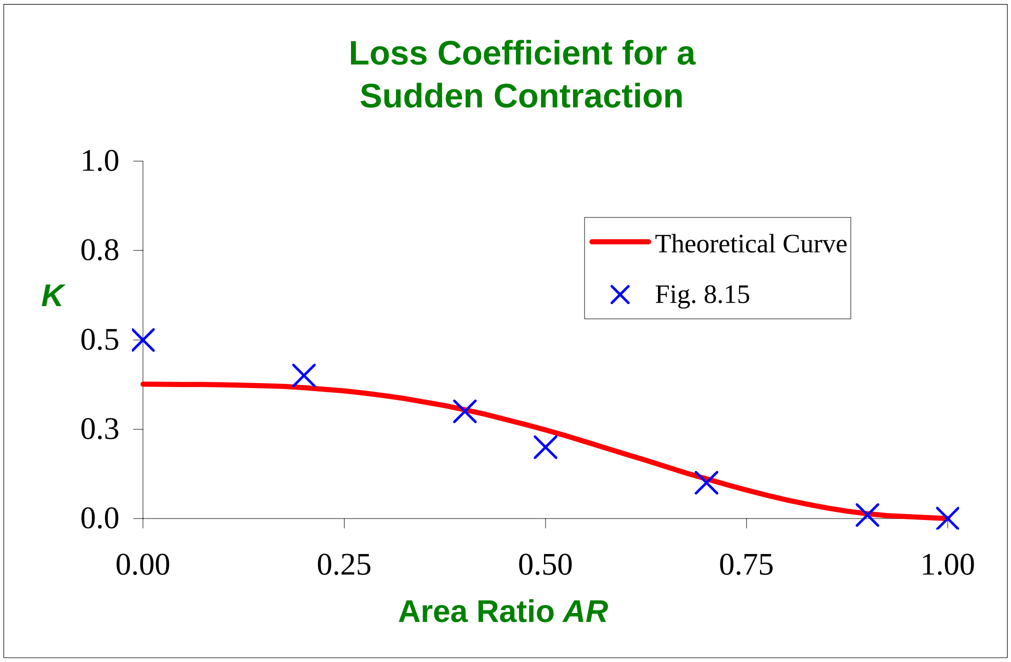
The CV analysis le

$$K = \left(\frac{1}{C_c} - 1\right)^2$$

$$C_c = 0.62 + 0.38 \cdot \left(\frac{A_2}{A_1}\right)^3$$

A_2/A_1	K_{CV}	$K_{\text{Fig. 8.15}}$
0.0	0.376	0.50
0.1	0.374	
0.2	0.366	0.40
0.3	0.344	
0.4	0.305	0.30
0.5	0.248	0.20
0.6	0.180	
0.7	0.111	0.10
0.8	0.052	
0.9	0.013	0.01
1.0	0.000	0.00

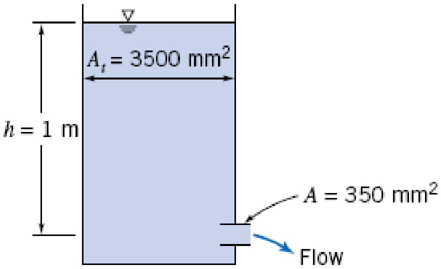
(Data from Fig. 8.15
is "eyeballed")
Agreement is reasonable



Problem 8.90

[2]

8.90 Water flows from the tank shown through a very short pipe. Assume the flow is quasi-steady. Estimate the flow rate at the instant shown. How could you improve the flow system if a larger flow rate were desired?



Given: Flow through short pipe

Find: Volume flow rate; How to improve flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT}$ $\quad h_{lT} = h_l + h_{lm} = f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2} + K\cdot\frac{V_2^2}{2}$ $\quad Q = V\cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) L << so ignore h_l 5) Reentrant

Hence between the free surface (Point 1) and the exit (2) the energy equation becomes

$$\frac{V_1^2}{2} + g\cdot z_1 - \frac{V_2^2}{2} = K\cdot\frac{V_2^2}{2}$$

From continuity $V_1 = V_2\cdot\frac{A_2}{A_1}$

Hence $\frac{V_2^2}{2}\cdot\left(\frac{A_2}{A_1}\right)^2 + g\cdot h - \frac{V_2^2}{2} = K\cdot\frac{V_2^2}{2}$

Solving for V_2 $\quad V_2 = \sqrt{\frac{2\cdot g\cdot h}{\left[1 + K - \left(\frac{A_2}{A_1}\right)^2\right]}}$ $\quad$ and from Table 8.2 $\quad K = 0.78$

Hence $\quad V_2 = \sqrt{2 \times 9.81\cdot\frac{m}{s^2} \times 1\cdot m \times \frac{1}{\left[1 + 0.78 - \left(\frac{350}{3500}\right)^2\right]}}$ $\quad V_2 = 3.33\frac{m}{s}$

$Q = V_2\cdot A_2$ $\quad Q = 3.33\cdot\frac{m}{s} \times 350\cdot mm^2 \times \left(\frac{1\cdot m}{1000\cdot mm}\right)^2$ $\quad Q = 1.17 \times 10^{-3}\frac{m^3}{s}$ $\quad Q = 0.070\frac{m^3}{min}$

The flow rate could be increased by (1) rounding the entrance and/or (2) adding a diffuser (both somewhat expensive)

Problem 8.91

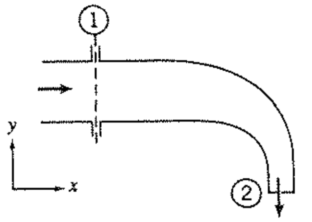
[3]

Given: Consider again flow through the elbow analyzed in Example Problem 4.6

$p_1 = 221\text{ kPa}$ $\quad A_1 = 0.01\text{ m}^2$

$V_2 = 16\text{ m/s}$ $\quad A_2 = 0.0025\text{ m}^2$

$p_2 = p_{atm}$



Find: Minor head loss coefficient for the elbow

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+\cancel{gz_1}\right)-\left(\overset{=0(4)}{\cancel{\frac{p_2}{\rho}}}+\alpha_2\frac{\bar{V}_2^2}{2}+\cancel{gz_2}\right)=h_{l_T}=\overset{=0}{\cancel{h_l}}+h_{l_m}$$

Assumptions: (1) $\alpha_1 = \alpha_2 = 1$

(2) neglect Δz

(3) uniform, incompressible flow so $\bar{V}_1A_1 = \bar{V}_2A_2$

(4) use gage pressures

From continuity $$\bar{V}_1 = \bar{V}_2\frac{A_2}{A_1} = 16\,\frac{\text{m}}{\text{s}}\times\frac{0.0025\text{ m}^2}{0.01\text{ m}^2} = 4\text{ m/s}$$

Then

$$h_{l_m} = \frac{p_{1g}}{\rho}+\frac{\bar{V}_1^2}{2}-\frac{\bar{V}^2}{2} = (221-101)10^3\,\frac{\text{N}}{\text{m}^2}\times\frac{\text{m}^3}{999\text{ kg}}\times\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}$$
$$+\frac{1}{2}\left[(4)^2-(16)^2\right]\frac{\text{m}^2}{\text{s}^2}$$

$$h_{l_m} = 0.120\text{ m}^2/\text{s}^2$$

But $h_{l_m} = K\dfrac{\bar{V}_2^2}{2}$; $K = \dfrac{2h_{l_m}}{\bar{V}_2^2} = 2\times0.120\,\dfrac{\text{m}^2}{\text{s}^2}\times\dfrac{\text{s}^2}{(16)^2\text{ m}^2} = 9.38\times10^{-4}$ $\quad K$

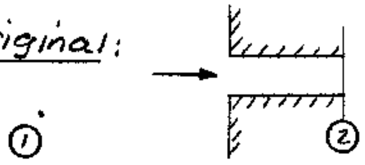
Problem 8.92

[2]

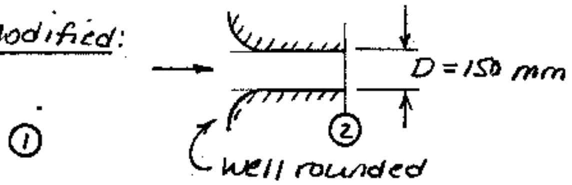
42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Air flow from a clean room through a duct of 150 mm diameter.

Original:



Modified:



$h_1 - h_2 = 2.5 \text{ mm } H_2O$

Friction losses negligible, compared to inlet and exit losses.

Find: Increase in volume flow rate for modified duct.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equations:

$$\frac{p_1}{\rho} + \alpha_1 \overset{\approx 0(1)}{\cancel{\frac{\bar{V}_1^2}{2}}} + \cancel{g z_1} = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + \cancel{g z_2} + h_{l_T}$$

$$h_{l_T} = \overset{\approx 0(4)}{\cancel{h_l}} + h_{l_m} \; ; \; h_{l_m} = K_{ent} \frac{\bar{V}_2^2}{2} \; ; \; \Delta p = \rho_{H_2O} g \Delta h$$

Assumptions: (1) $\bar{V}_1 \approx 0$

(2) Neglect elevation changes

(3) Uniform flow at exit

(4) Neglect frictional losses

Then

$$\frac{\Delta p}{\rho} = \frac{p_1 - p_2}{\rho} = \frac{\bar{V}_2^2}{2} + K_{ent} \frac{\bar{V}_2^2}{2} = \frac{\bar{V}_2^2}{2}(1 + K_{ent}) = \frac{\rho_{H_2O} g \Delta h}{\rho}$$

or

$$\bar{V}_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho(1 + K_{ent})}} = \sqrt{\frac{2 \rho_{H_2O} g \Delta h}{\rho(1 + K_{ent})}}$$

From Table 8.2, $K_{ent} = 0.5$ for square-edged, $K_{ent} = 0.04$ for rounded entrance.

$$\bar{V}_2 = \sqrt{\frac{2}{1.50} \times 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.0025 \, m \times \frac{m^3}{1.23 \, kg}} = 5.15 \, m/s$$

$$\bar{V}_2 (\text{modified}) = \sqrt{\frac{2}{1.04} \times 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.0025 \, m \times \frac{m^3}{1.23 \, kg}} = 6.19 \, m/s$$

Since $Q = \bar{V} A$, then

$$\Delta Q = (\bar{V}_{2m} - \bar{V}_2) A = (6.19 - 5.15) \frac{m}{s} \times \frac{\pi}{4} (0.150)^2 m^2 = 0.0184 \, m^3/s$$

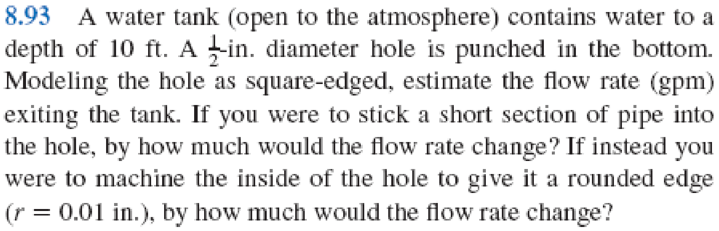
ΔQ

The percentage improvement is

$$\% = \frac{\Delta Q}{Q} \times 100 = \frac{\bar{V}_{2m} - \bar{V}_2}{\bar{V}_2} \times 100 = \frac{6.19 - 5.15}{5.15} \times 100 = 20.2 \text{ percent}$$

Problem 8.93

[3]



8.93 A water tank (open to the atmosphere) contains water to a depth of 10 ft. A $\frac{1}{2}$-in. diameter hole is punched in the bottom. Modeling the hole as square-edged, estimate the flow rate (gpm) exiting the tank. If you were to stick a short section of pipe into the hole, by how much would the flow rate change? If instead you were to machine the inside of the hole to give it a rounded edge ($r = 0.01$ in.), by how much would the flow rate change?

Given: Flow out of water tank

Find: Volume flow rate using hole; Using short pipe section; Using rounded edge

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_{lT} \qquad h_{lT} = h_l + h_{lm} = f\cdot\frac{L}{D}\cdot\frac{V_2^{\,2}}{2} + K\cdot\frac{V_2^{\,2}}{2} \qquad Q = V\cdot A$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_l << 5) L << so $h_l = 0$

Hence for all three cases, between the free surface (Point 1) and the exit (2) the energy equation becomes

$$g\cdot z_1 - \frac{V_2^{\,2}}{2} = K\cdot\frac{V_2^{\,2}}{2}$$

Solving for V_2
$$V_2 = \sqrt{\frac{2\cdot g\cdot h}{(1+K)}}$$

From Table 8.2 $K_{hole} = 0.5$ for a hole (assumed to be square-edged) $K_{pipe} = 0.78$ for a short pipe (rentrant)

Also, for a rounded edge $\frac{r}{D} = \frac{0.01\cdot in}{0.5\cdot in} = 0.02$ so from Table 8.2 $K_{round} = 0.28$

Hence for the hole
$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 10\cdot ft\times\frac{1}{(1+0.5)}} \qquad V_2 = 20.7\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 20.7\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 12.7\cdot gpm$$

Hence for the pipe
$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 10\cdot ft\times\frac{1}{(1+0.78)}} \qquad V_2 = 19.0\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 19.0\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 11.6\cdot gpm$$

Hence the change in flow rate is $11.6 - 12.7 = -1.1\cdot gpm$ The pipe leads to a LOWER flow rate

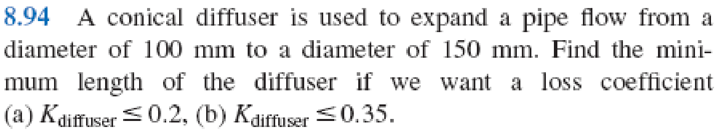
Hence for the rounded
$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 10\cdot ft\times\frac{1}{(1+0.28)}} \qquad V_2 = 22.4\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 22.4\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 13.7\cdot gpm$$

Hence the change in flow rate is $13.7 - 12.7 = 1.0\cdot gpm$ The rounded edge leads to a HIGHER flow rate

Problem 8.94

[2]



8.94 A conical diffuser is used to expand a pipe flow from a diameter of 100 mm to a diameter of 150 mm. Find the minimum length of the diffuser if we want a loss coefficient (a) $K_{\text{diffuser}} \leq 0.2$, (b) $K_{\text{diffuser}} \leq 0.35$.

Given: Data on inlet and exit diameters of diffuser

Find: Minimum lengths to satisfy requirements

Solution:

Given data $D_1 = 100 \cdot mm$ $D_2 = 150 \cdot mm$

The governing equations for the diffuser are

$$h_{lm} = K \cdot \frac{V_1^2}{2} = \left(C_{pi} - C_p\right) \cdot \frac{V_1^2}{2} \qquad (8.44)$$

and

$$C_{pi} = 1 - \frac{1}{AR^2} \qquad (8.42)$$

Combining these we obtain an expression for the loss coefficient *K*

$$K = 1 - \frac{1}{AR^2} - C_p \qquad (1)$$

The area ratio *AR* is $AR = \left(\frac{D_2}{D_1}\right)^2$ $AR = 2.25$

The pressure recovery coefficient C_p is obtained from Eq. 1 above once we select *K*; then, with C_p and *AR* specified, the minimum value o N/R_1 (where *N* is the length and R_1 is the inlet radius) can be read from Fig. 8.15

(a) $K = 0.2$ $C_p = 1 - \frac{1}{AR^2} - K$ $C_p = 0.602$

From Fig. 8.15 $\frac{N}{R_1} = 5.5$ $R_1 = \frac{D_1}{2}$ $R_1 = 50 \cdot mm$

$N = 5.5 \cdot R_1$ $N = 275 \cdot mm$

(b) $K = 0.35$ $C_p = 1 - \frac{1}{AR^2} - K$ $C_p = 0.452$

From Fig. 8.15 $\frac{N}{R_1} = 3$

$N = 3 \cdot R_1$ $N = 150 \cdot mm$

Problem 8.95

8.95 A conical diffuser of length 6 in. is used to expand a pipe flow from a diameter of 2 in. to a diameter of 3.5 in. For a water flow rate of 750 gal/min, estimate the static pressure rise. What is the approximate value of the loss coefficient?

Given: Data on geometry of conical diffuser; flow rate

Find: Static pressure rise; loss coefficient

Solution:

Basic equations $$C_p = \frac{p_2 - p_1}{\frac{1}{2}\cdot\rho\cdot V_1^2} \quad (8.41) \qquad h_{lm} = K\cdot\frac{V_1^2}{2} = \left(C_{pi} - C_p\right)\cdot\frac{V_1^2}{2} \quad (8.44) \qquad C_{pi} = 1 - \frac{1}{AR^2} \quad (8.42)$$

Given data $D_1 = 2\cdot in$ $\quad D_2 = 3.5\cdot in$ $\quad N = 6\cdot in$ $\quad$ (N = length) $\quad Q = 750\cdot gpm$

From Eq. 8.41 $$\Delta p = p_2 - p_1 = \frac{1}{2}\cdot\rho\cdot V_1^2\cdot C_p \quad (1)$$

Combining Eqs. 8.44 and 8.42 we obtain an expression for the loss coefficient *K* $$K = 1 - \frac{1}{AR^2} - C_p \quad (2)$$

The pressure recovery coefficient C_p for use in Eqs. 1 and 2 above is obtained from Fig. 8.15 once compute *AR* and the dimensionless length N/R_1 (where R_1 is the inlet radius)

The aspect ratio *AR* is $$AR = \left(\frac{D_2}{D_1}\right)^2 \qquad AR = \left(\frac{3.5}{2}\right)^2 \qquad AR = 3.06$$

$$R_1 = \frac{D_1}{2} \qquad R_1 = 1\cdot in \quad \text{Hence} \quad \frac{N}{R_1} = 6$$

From Fig. 8.15, with *AR* = 3.06 and the dimensionless length N/R_1 = 6, we find $C_p = 0.6$

To complete the calculations we need V_1 $$V_1 = \frac{Q}{\frac{\pi}{4}\cdot D_1^2} \qquad V_1 = \frac{4}{\pi}\times 750\cdot\frac{gal}{min}\times\frac{1\cdot ft^3}{7.48\cdot gal}\times\frac{1\cdot min}{60\cdot s}\times\left(\frac{1}{\frac{2}{12}\cdot ft}\right)^2 \qquad V_1 = 76.6\cdot\frac{ft}{s}$$

We can now compute the pressure rise and loss coefficient from Eqs. 1 and 2 $\quad \Delta p = \frac{1}{2}\cdot\rho\cdot V_1^2\cdot C_p$

$$\Delta p = \frac{1}{2}\times 1.94\cdot\frac{slug}{ft^3}\times\left(76.6\cdot\frac{ft}{s}\right)^2\times 0.6\times\frac{lbf\cdot s^2}{slug\cdot ft}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2 \qquad \Delta p = 23.7\cdot psi$$

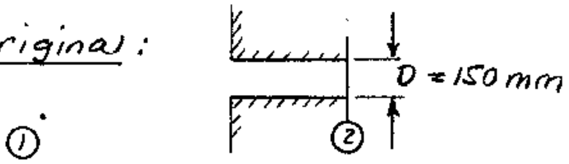
$$K = 1 - \frac{1}{AR^2} - C_p \qquad K = 1 - \frac{1}{3.06^2} - 0.6 \qquad K = 0.293$$

Problem 8.96

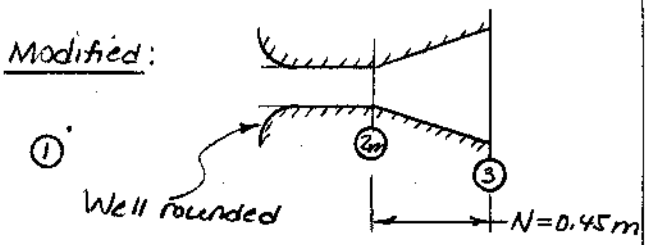
[4]

Given: Air flow from a clean room through a duct of 150 mm diameter.

Original:



Modified:



$h_1 - h_2 = 2.5\,mm\ H_2O$

$h_1 - h_3 = 2.5\,mm\ H_2O$

Neglect friction losses compared to "minor" losses.

Find: (a) Area ratio and angle for optimum conical diffuser.
(b) Flow rate for modified system.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equations:

$$\frac{p_1}{\rho} + \cancelto{\approx 0(1)}{\alpha_1 \frac{\bar{V}_1^2}{2}} + \cancel{g z_1} = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + \cancel{g z_2} + h_{l_T} \quad \text{(or to section 3)}$$

$$h_{l_T} = \cancelto{\approx 0(4)}{h_l} + h_{l_m}\,; \quad h_{l_m} = K_{ent} + h_{l_{diffuser}}\,; \quad \Delta p = \rho_{H_2O}\, g\, \Delta h$$

$$\text{From Eq. 8.42, } h_{l_{diffuser}} = \frac{\bar{V}_2^2}{2}\left[1 - \frac{1}{AR^2} - C_p\right]$$

Assumptions: (1) $\bar{V}_1 \approx 0$ (3) Uniform flow at each section
(2) Neglect Δz (4) Neglect frictional losses

For the original system, $\frac{p_1 - p_2}{\rho} = \frac{\bar{V}_2^2}{2} + K_{ent}\frac{\bar{V}_2^2}{2} = 1.5\,\frac{\bar{V}_2^2}{2} = \frac{\rho_{H_2O}\, g\, \Delta h}{\rho}$ $(K_{ent} = 0.5)$

Thus

$$\bar{V}_2 = \sqrt{\frac{2}{1.5}\,\frac{\rho_{H_2O}\, g\, \Delta h}{\rho}} = \sqrt{\frac{2}{1.5} \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 0.0025\,m \times \frac{m^3}{1.23\,kg}} = 5.15\ m/s$$

For the modified system, $\frac{p_1 - p_3}{\rho} = \frac{\bar{V}_3^2}{2} + K_{ent}\frac{\bar{V}_2^2}{2} + \frac{\bar{V}_2^2}{2}\left[1 - \frac{1}{AR^2} - C_p\right] = \frac{\bar{V}_2^2}{2}\left[1 + K_{ent} - C_p\right]$

since $\bar{V}_3^2 = \bar{V}_2^2 \frac{1}{AR^2}$. Thus the best diffuser has the highest C_p.

From Fig. 8.16, $C_p = f(N/R_1, AR)$. $N/R_1 = 2N/D_1 = 2 \times \frac{0.45\,m}{0.15\,m} = 6$. From the figure, the best diffuser is

$$C_p \approx 0.62 \text{ at } AR \approx 2.7 \text{ and } 2\phi \approx 12 \text{ deg}$$

AR, 2

For the modified system,

$$\bar{V}_2 = \sqrt{\frac{2}{1 + K_{ent} - C_p}\,\frac{\rho_{H_2O}\, g\, \Delta h}{\rho}} = \sqrt{\frac{2}{1 + 0.04 - 0.62} \times 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times 0.0025\,m \times \frac{m^3}{1.23\,kg}} = 9.74\ m/s$$

and

$$Q = \bar{V}_2 A_2 = 9.74\,\frac{m}{s} \times \frac{\pi}{4}(0.15)^2\,m^2 = 0.172\ m^3/s$$

Q

$$\left\{ \text{The improvement is } \frac{Q_m - Q}{Q} \times 100 = \frac{\bar{V}_m - \bar{V}}{\bar{V}} \times 100 = \frac{9.74 - 5.15}{5.15} \times 100 = 89.1 \text{ percent more} \right\}$$

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Problem 8.97

[3]

8.97 By applying the basic equations to a control volume starting at the expansion and ending downstream, analyze flow through a sudden expansion (assume the inlet pressure p_1 acts on the area A_2 at the expansion). Develop an expression for and plot the minor head loss across the expansion as a function of area ratio, and compare with the data of Fig. 8.15.

Given: Sudden expansion

Find: Expression for minor head loss; compare with Fig. 8.15; plot

Solution:

The governing CV equations (mass, momentum, and energy) are

$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV\!\!\!\!- + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad (4.12)$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \, \rho \, dV\!\!\!\!- + \int_{CS} u \, \rho \vec{V} \cdot d\vec{A} \qquad (4.18a)$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e \, \rho \, dV\!\!\!\!- + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho \vec{V} \cdot d\vec{A} \qquad (4.56)$$

Assume: 1) Steady flow 2) Incompressible flow 3) Uniform flow at each section 4) Horizontal: no body force 5) No shaft work 6) Neglect viscous friction 7) Neglect gravity

The mass equation becomes

$$V_1 \cdot A_1 = V_2 \cdot A_2 \qquad (1)$$

The momentum equation becomes

$$p_1 \cdot A_2 - p_2 \cdot A_2 = V_1 \cdot \left(-\rho \cdot V_1 \cdot A_1\right) + V_2 \cdot \left(\rho \cdot V_2 \cdot A_2\right)$$

or (using Eq. 1)

$$p_1 - p_2 = \rho \cdot V_1 \cdot \frac{A_1}{A_2} \cdot \left(V_2 - V_1\right) \qquad (2)$$

The energy equation becomes

$$Q_{rate} = \left(u_1 + \frac{p_1}{\rho} + V_1^{\,2}\right) \cdot \left(-\rho \cdot V_1 \cdot A_1\right) + \left(u_2 + \frac{p_2}{\rho} + V_2^{\,2}\right) \cdot \left(\rho \cdot V_2 \cdot A_2\right)$$

or (using Eq. 1)

$$h_{lm} = u_2 - u_1 - \frac{Q_{rate}}{m_{rate}} = \frac{V_1^{\,2} - V_2^{\,2}}{2} + \frac{p_1 - p_2}{\rho} \qquad (3)$$

Combining Eqs. 2 and 3

$$h_{lm} = \frac{V_1^{\,2} - V_2^{\,2}}{2} + V_1 \cdot \frac{A_1}{A_2} \cdot \left(V_2 - V_1\right)$$

$$h_{lm} = \frac{V_1^{\,2}}{2} \cdot \left[1 - \left(\frac{V_2}{V_1}\right)^2\right] + V_1^{\,2} \cdot \frac{A_1}{A_2} \cdot \left[\left(\frac{V_2}{V_1}\right) - 1\right]$$

From Eq. 1
$$AR = \frac{A_1}{A_2} = \frac{V_2}{V_1}$$

Hence
$$h_{lm} = \frac{V_1^2}{2}\cdot\left(1 - AR^2\right) + V_1^2\cdot AR\cdot(AR - 1)$$

$$h_{lm} = \frac{V_1^2}{2}\cdot\left(1 - AR^2 + 2\cdot AR^2 - 2\cdot AR\right)$$

$$h_{lm} = K\cdot\frac{V_1^2}{2} = (1 - AR)^2\cdot\frac{V_1^2}{2}$$

Finally
$$K = (1 - AR)^2$$

This result, and the curve of Fig. 8.15, are shown in the associated *Excel* workbook. The agreement is excellent

Problem 8.97

[3]

8.97 By applying the basic equations to a control volume starting at the expansion and ending downstream, analyze flow through a sudden expansion (assume the inlet pressure p_1 acts on the area A_2 at the expansion). Develop an expression for and plot the minor head loss across the expansion as a function of area ratio, and compare with the data of Fig. 8.15.

Given: Sudden expansion

Find: Expression for minor head loss; compare with Fig. 8.15; plot

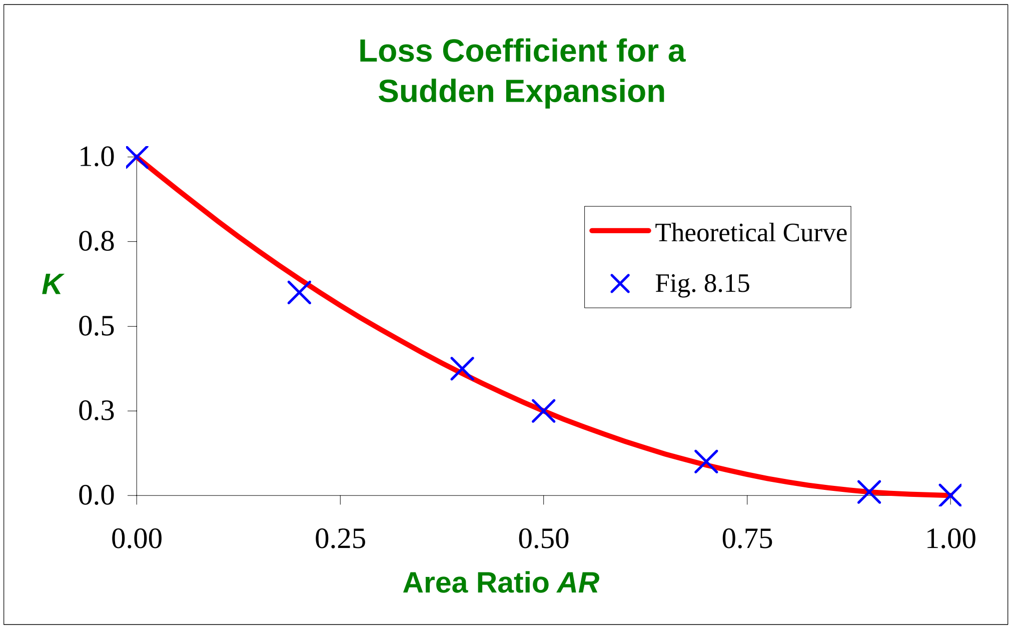
Solution:

From the CV analysis

$$K = (1 - AR)^2$$

AR	K_{CV}	$K_{\text{Fig. 8.15}}$
0.0	1.00	1.00
0.1	0.81	
0.2	0.64	0.60
0.3	0.49	
0.4	0.36	0.38
0.5	0.25	0.25
0.6	0.16	
0.7	0.09	0.10
0.8	0.04	
0.9	0.01	0.01
1.0	0.00	0.00

(Data from Fig. 8.15
is "eyeballed")
Agreement is excellent



Problem 8.98

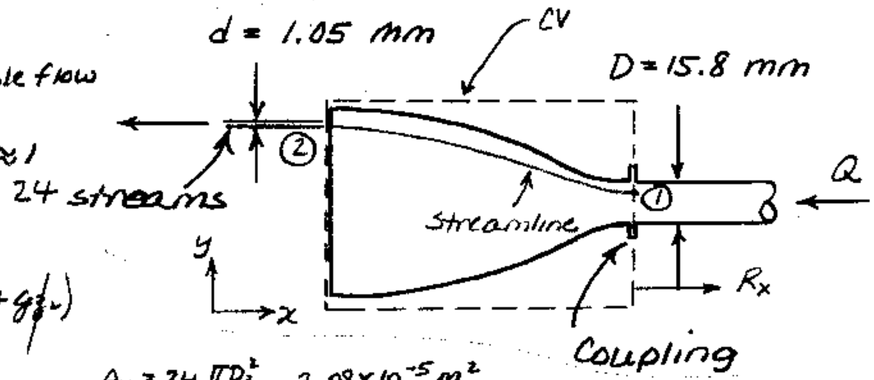
[3]

Given: Water at 45°C enters a shower head through a circular tube with 15.8 mm inside diameter. The water leaves in 24 streams, each of 1.05 mm diameter. The volume flow rate is 5.67 L/min.

Find:
(a) Estimate of the minimum water pressure needed at the inlet to the shower head.
(b) Force needed to hold the shower head onto the end of the circular tube, indicating clearly whether this is a compression or a tension force.

Solution: Apply the energy equation for steady, incompressible pipe flow, and the x component of momentum, using the CV shown.

Assume:
(1) Steady flow
(2) Incompressible flow
(3) Neglect changes in z
(4) Uniform flow: $\alpha_1 = \alpha_2 \approx 1$
(5) Use gage pressures



Then

$$\frac{p_1}{\rho}+\alpha_1\frac{\bar V_1^2}{2}+g\cancel{z_1}-\left(\overset{=0\,(gage)}{\cancel{\frac{p_2}{\rho}}}+\alpha_2\frac{\bar V_2^2}{2}+g\cancel{z_2}\right)$$

$$= h_{l_T} = \overset{=0}{\cancel{h_l}} + h_{l_m}$$

$$A_2 = 24\,\frac{\pi D_2^2}{4} = 2.08\times10^{-5}\,m^2$$

$$A_1 = \frac{\pi D_1^2}{4} = 1.96\times10^{-4}\,m^2$$

$$\bar V_1 = \frac{Q}{A_1} = 5.67\,\frac{L}{min}\times\frac{1}{1.96\times10^{-4}\,m^2}\times\frac{m^3}{1000\,L}\times\frac{min}{60\,s} = 0.487\,m/s$$

$$\bar V_2 = \bar V_1\frac{A_1}{A_2} = 0.487\,\frac{m}{s}\times\frac{1.96\times10^{-4}\,m^2}{2.08\times10^{-5}\,m^2} = 4.59\,m/s$$

Use $K = 0.5$, for a square-edged orifice, $\rho = 990\,kg/m^3$ (Table A.8). Then

$$p_1 = \frac{\rho}{2}\left(\bar V_2^2 + K\bar V_2^2 - \bar V_1^2\right) = \frac{\rho}{2}\left[(1+K)\bar V_2^2 - \bar V_1^2\right]$$

$$p_1 = \frac{1}{2}\times 990\,\frac{kg}{m^3}\left[(1+0.5)(4.59)^2-(0.487)^2\right]\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = 15.5\,kPa\ (gage)$$ p_1

Use momentum to find force:

Basic equation: $$F_{S_x} + \overset{=0(6)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec V\cdot d\vec A$$

Assume: (6) $F_{B_x} = 0$

Then $$R_x - p_{1g}A_1 = u_1\{\overset{Step ①}{-\rho Q}\} + u_2\{+\rho Q\} = -V_1\{-\rho Q\} + (-V_2)\{+\rho Q\} = \rho Q(V_1 - V_2)$$

Step ②: $u_1 = -V_1$ $\quad u_2 = -V_2$

$$R_x = p_{1g}A_1 + \rho Q(V_1 - V_2) = 15.5\times10^3\,\frac{N}{m^2}\times1.96\times10^{-4}\,m^2 + 990\,\frac{kg}{m^3}\times5.67\,\frac{L}{min}\times(0.487-4.59)\,\frac{m}{s}\times\frac{m^3}{1000\,L}\times\frac{min}{60\,s}$$

$R_x = 2.65\ N$ (in direction shown, i.e., tension) R_x

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Problem 8.99

[2]

8.99 Analyze flow through a sudden expansion to obtain an expression for the upstream average velocity $\overline{V}_1$ in terms of the pressure change $\Delta p = p_2 - p_1$, area ratio AR, fluid density ρ, and loss coefficient K. If the flow were frictionless, would the flow rate indicated by a measured pressure change be higher or lower than a real flow, and why? Conversely, if the flow were frictionless, would a given flow generate a larger or smaller pressure change, and why?

Given: Sudden expansion

Find: Expression for upstream average velocity

Solution:

The governing equation is

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} \qquad (8.29)$$

$$h_{lT} = h_l + K \cdot \frac{V^2}{2}$$

Assume: 1) Steady flow 2) Incompressible flow 3) $h_l = 0$ 4) $\alpha_1 = \alpha_2 = 1$ 5) Neglect gravity

The mass equation is $V_1 \cdot A_1 = V_2 \cdot A_2$ so $V_2 = V_1 \cdot \frac{A_1}{A_2}$

$$V_2 = AR \cdot V_1 \qquad (1)$$

Equation 8.29 becomes

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_1}{\rho} + \frac{V_1^2}{2} + K \cdot \frac{V_1^2}{2}$$

or (using Eq. 1)

$$\frac{\Delta p}{\rho} = \frac{p_2 - p_1}{\rho} = \frac{V_1^2}{2} \cdot \left(1 - AR^2 - K\right)$$

Solving for V_1

$$V_1 = \sqrt{\frac{2 \cdot \Delta p}{\rho \cdot \left(1 - AR^2 - K\right)}}$$

If the flow were frictionless, $K = 0$, so

$$V_{inviscid} = \sqrt{\frac{2 \cdot \Delta p}{\rho \cdot \left(1 - AR^2\right)}} < V_1$$

Hence the flow rate indicated by a given Δp would be lower

If the flow were frictionless, $K = 0$, so

$$\Delta p_{invscid} = \frac{V_1^2}{2} \cdot \left(1 - AR^2\right)$$

compared to

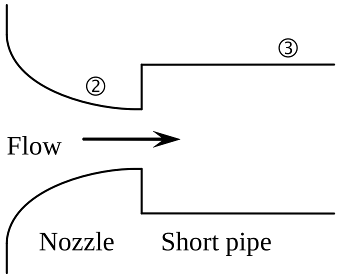
$$\Delta p = \frac{V_1^2}{2} \cdot \left(1 - AR^2 - K\right)$$

Hence a given flow rate would generate a larger Δp for inviscid flow

Problem 8.100

[4]

8.100 Water discharges to atmosphere from a large reservoir through a moderately rounded horizontal nozzle of 1 in. diameter. The free surface is 5 ft above the nozzle exit plane. Calculate the change in flow rate when a short section of 2-in. diameter pipe is attached to the end of the nozzle to form a sudden expansion. Determine the location and estimate the magnitude of the minimum pressure with the sudden expansion in place. If the flow were frictionless (with the sudden expansion in place), would the minimum pressure be higher, lower, or the same? Would the flow rate be higher, lower, or the same?



Given: Flow out of water tank through a nozzle

Find: Change in flow rate when short pipe section is added; Minimum pressure; Effect of frictionless flow

Solution:

Basic equations $$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}\qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_2^{\,2}}{2}+K\cdot\frac{V_2^{\,2}}{2}\qquad Q=V\cdot A$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_l << 5) L << so h_l = 0

Hence for the nozzle case, between the free surface (Point 1) and the exit (2) the energy equation becomes

$$g\cdot z_1-\frac{V_2^{\,2}}{2}=K_{nozzle}\cdot\frac{V_2^{\,2}}{2}$$

Solving for V_2 $$V_2=\sqrt{\frac{2\cdot g\cdot z_1}{\left(1+K_{nozzle}\right)}}$$

For a rounded edge, we choose the first value from Table 8.2 $$K_{nozzle}=0.28$$

Hence $$V_2=\sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 5\cdot ft\times\frac{1}{(1+0.28)}}\qquad V_2=15.9\cdot\frac{ft}{s}$$

$$Q=V_2\cdot A_2\qquad Q=15.9\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min}\qquad Q=9.73\cdot gpm\qquad Q=0.0217\frac{ft^3}{s}$$

When a small piece of pipe is added the energy equation between the free surface (Point 1) and the exit (3) becomes

$$g\cdot z_1-\frac{V_3^{\,2}}{2}=K_{nozzle}\cdot\frac{V_2^{\,2}}{2}+K_e\cdot\frac{V_2^{\,2}}{2}$$

From continuity $$V_3=V_2\cdot\frac{A_2}{A_3}=V_2\cdot AR$$

Solving for V_2 $$V_2=\sqrt{\frac{2\cdot g\cdot z_1}{\left(AR^2+K_{nozzle}+K_e\right)}}$$

We need the AR for the sudden expansion $$AR=\frac{A_2}{A_3}=\left(\frac{D_2}{D_3}\right)^2=\left(\frac{1}{2}\right)^2=0.25$$

From Fig. 8.14 for AR = 0.25 $$K_e=0.6$$

Hence $$V_2 = \sqrt{2 \times 32.2\cdot\frac{ft}{s^2} \times 5\cdot ft \times \frac{1}{\left(0.25^2 + 0.28 + 0.6\right)}}$$ $$V_2 = 18.5\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 18.5\cdot\frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{0.5}{12}\cdot ft\right)^2 \times \frac{7.48\cdot gal}{1\cdot ft^3} \times \frac{60\cdot s}{1\cdot min} \qquad Q = 11.32\cdot gpm \qquad Q = 0.0252\frac{ft^3}{s}$$

Comparing results we see the flow increases from 0.0217 ft^3/s to 0.0252 ft^3/s $$\frac{\Delta Q}{Q} = \frac{0.0252 - 0.0217}{0.0217} = 16.1\cdot\%$$

The flow increases because the effect of the pipe is to allow an exit pressure at the nozzle LESS than atmospheric!

The minimum pressure point will now be at Point 2 (it was atmospheric before adding the small pipe). The energy equation between 1 and 2 is

$$g\cdot z_1 - \left(\frac{p_2}{\rho} + \frac{V_2^{\,2}}{2}\right) = K_{nozzle}\cdot\frac{V_2^{\,2}}{2}$$

Solving for p_2 $$p_2 = \rho\cdot\left[g\cdot z_1 - \frac{V_2^{\,2}}{2}\cdot\left(K_{nozzle} + 1\right)\right]$$

Hence $$p_2 = 1.94\cdot\frac{slug}{ft^3} \times \left[32.2\cdot\frac{ft}{s^2} \times 5\cdot ft - \frac{1}{2} \times \left(18.5\cdot\frac{ft}{s}\right)^2 \times (0.28 + 1)\right] \times \frac{lbf\cdot s^2}{slug\cdot ft} \qquad p_2 = -113\frac{lbf}{ft^2} \qquad p_2 = -0.782\,psi$$

If the flow were frictionless the the two loss coeffcients would be zero. Instead of $$V_2 = \sqrt{\frac{2\cdot g\cdot z_1}{\left(AR^2 + K_{nozzle} + K_e\right)}}$$

We'd have $$V_2 = \sqrt{\frac{2\cdot g\cdot z_1}{AR^2}}$$ which is larger

If V_2 is larger, then p_2, through Bernoulli, would be lower (more negative)

Problem 8.101

[2]

Given: Steady flow of water from a large tank through a length of smooth plastic tubing, with $D = 3.18$ mm and $L = 15.3$ m.

Find: (a) Maximum volume flow rate for laminar flow.
(b) Estimate maximum water level in tank for laminar flow ($\alpha = 2$ and $K_{ent} = 1.4$)

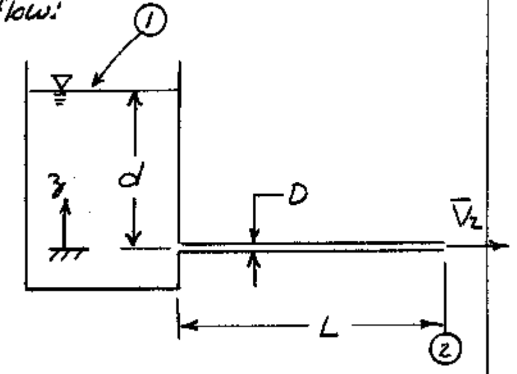
Solution: Assume water at 20°C. From Table A.8, $\rho = 998\ \mathrm{kg/m^3}$, $\nu = 1.00\times10^{-6}\ \mathrm{m^2/s}$.

$$Re = \frac{\rho \bar V D}{\mu} = \frac{\bar V D}{\nu} \le 2300\ ;\ \bar V_{max} = \frac{2300\,\nu}{D} = 2300 \times 1.00\times10^{-6}\frac{m^2}{s}\times\frac{1}{0.00318\ m} = 0.723\ m/s$$ $\vec{V}$

$$Q = \bar V A\ ;\ A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.00318)^2 m^2 = 7.94\times10^{-6}\ m^2$$

$$Q = 0.723\frac{m}{s}\times 7.94\times10^{-6}\ m^2 = 5.74\times10^{-6}\frac{m^3}{s}\times10^3\frac{L}{m^3}\times 60\frac{s}{min} = 0.345\ L/min$$ Q

Apply energy equation for steady, ρ = constant pipe flow:



Computing Equation:
$$\left(\frac{\cancel{p_1}}{\rho}+\alpha_1\frac{\overset{\approx 0}{\cancel{\bar V_1^2}}}{2}+gz_1\right)-\left(\frac{\cancel{p_2}}{\rho}+\alpha_2\frac{\bar V_2^2}{2}+g\overset{=0}{\cancel{z_2}}\right)=h_{l_T}$$

$$h_{l_T} = h_{l_m} + h_l$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$
(2) $\bar V_1 \approx 0$
(3) $K_{ent} = 1.4$ (given)

Then $$g d = \alpha_2\frac{\bar V_2^2}{2} + K_{ent}\frac{\bar V_2^2}{2} + f\frac{L}{D}\frac{\bar V_2^2}{2} \quad \text{or} \quad d = \frac{\bar V_2^2}{2g}\left(\alpha_2 + K_{ent} + f\frac{L}{D}\right)$$

For laminar flow, $f = \frac{64}{Re} = \frac{64}{2300} = 0.0278$. Substituting

$$d = \frac{1}{2}\times(0.723)^2\frac{m^2}{s^2}\times\frac{s^2}{9.81\ m}\left(2.0 + 1.4 + 0.0278\ \frac{15.3\ m}{0.00318\ m}\right)$$

$$d = 3.65\ m$$ d

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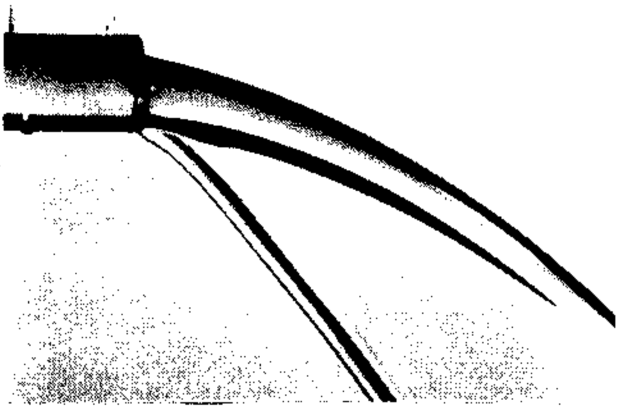
Problem 8.102

Open-Ended Problem Statement: You are asked to compare the behavior of fully developed laminar flow and fully developed turbulent flow in a horizontal pipe under different conditions. For the same flow rate, which will have the larger centerline velocity? Why? If the pipe discharges to atmosphere what would you expect the trajectory of the discharge stream to look like (for the same flow rate)? Sketch your expectations for each case. For the same flow rate, which flow would give the larger wall shear stress? Why? Sketch the shear stress distribution τ/τ_w as a function of radius for each flow. For the same Reynolds number, which flow would have the larger pressure drop per unit length? Why? For a given imposed pressure differential, which flow would have the larger flow rate? Why?

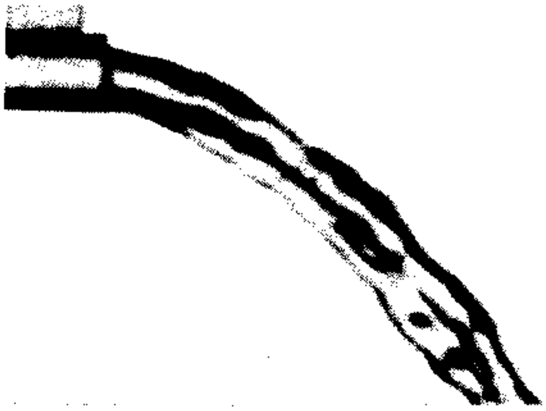
Discussion: In the following fully developed laminar flow and fully developed turbulent flow in a pipe are compared:

(a) For the same flow rate, laminar flow has the higher maximum velocity, because the turbulent velocity profile is more blunt.

(b) The trajectory of the discharge stream spreads out for laminar flow because of the large variation in velocity across the pipe exit. For turbulent flow the exit profile is more nearly uniform (except for the region adjacent to the wall) and hence the trajectory is more uniform. Since centerline velocity is larger for laminar flow, liquid travels the greatest horizontal distance. Trajectories for the two flow cases are shown below:



(i) Laminar flow



(ii) Turbulent flow

(c) For the same flow rate (same mean velocity), turbulent flow has larger wall shear stress because of the larger velocity gradient at the pipe wall. For fully developed flow the pressure force driving the flow is balanced by the shear force at the wall.

(d) Shear stress varies linearly with radius for both flow cases, from its maximum value at the wall to zero at the pipe centerline.

(e) For the same Reynolds number, turbulent flow has a larger pressure drop per unit length because the friction factor is larger.

(f) For a given pressure drop (per unit length), laminar flow has the larger flow rate (larger mean velocity), because it has the smaller friction factor.

The two flow cases are compared in the NCFMF video *Turbulence*, in which R. W. Stewart uses a clever experimental setup to contrast the two flow regimes at constant volume flow rate by varying the liquid viscosity. The trajectories of the liquid streams leaving the end of the pipe are particularly well shown.

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Problem 8.103

[3]

8.103 Estimate the minimum level in the water tank of Problem 8.101 such that the flow will be turbulent.

Given: Data on water flow from a tank/tubing system

Find: Minimum tank level for turbulent flow

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}=\sum_{\text{major}} h_l+\sum_{\text{minor}} h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes $g\cdot d-\alpha\cdot\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$

This can be solved expicitly for height *d*, or solved using *Solver*

Given data:

L = 15.3 m

D = 3.18 mm

K_{ent} = 1.4

α = 2

Tabulated or graphical data:

ν = 1.00E-06 m^2/s

ρ = 998 kg/m^3

(Appendix A)

Computed results:

Re = 2300 (Transition *Re*)

V = 0.723 m/s

α = 1 (Turbulent)

f = 0.0473 (Turbulent)

d = 6.13 m (Vary *d* to minimize error in energy equation)

Energy equation:
(Using *Solver*)

Left (m^2/s)	Right (m^2/s)	Error
59.9	59.9	0.00%

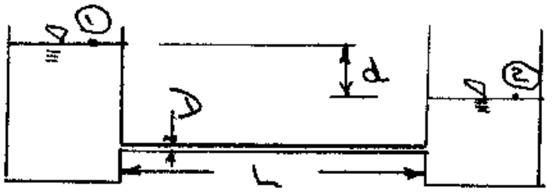
Note that we used α = 1 (turbulent); using α = 2 (laminar) gives *d* = 6.16 m

Problem 8.104

Given: System for measuring pressure drop for water flow in smooth tube as shown

$D = 15.9\ \text{mm} \quad L = 3.56\ \text{m}$

square-edged entrance to pipe



Find: (a) volume flow rate needed for turbulent flow in pipe
(b) reservoir height differential needed for turbulent pipe flow

Solution:

Flow will be turbulent for $Re_D > 2300$

$$Re_D = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{Q D}{A \nu} = \frac{Q D}{\nu}\frac{4}{\pi D^2} = \frac{4Q}{\pi \nu D} \quad \text{so} \quad Q = \frac{\pi \nu D\, Re}{4}$$

Assume $T = 20°C$, $\nu = 1.00 \times 10^{-6}\ \text{m}^2/\text{s}$ (Table A.8)

For $Re = 2300$,

$$Q = \frac{\pi}{4} \times 1.0 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}} \times 15.9 \times 10^{-3}\ \text{m} \times 2300 = 2.87 \times 10^{-5}\ \text{m}^3/\text{s}$$ ______ Q

Basic equations: $$\left(\cancel{\frac{p_1}{\rho}} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}} + g z_1\right) - \left(\cancel{\frac{p_2}{\rho}} + \cancel{\alpha_2 \frac{\bar{V}_2^2}{2}} + g z_2\right) = h_{l_T} \quad (8.29)$$

$$h_{l_T} = h_l + h_{l_m} \qquad h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}, \quad h_{l_m} = K\frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2) $\bar{V}_1 = \bar{V}_2 = 0$
(3) $K_{ent} = 0.5$ (Table 8.2), $K_{exit} = 1.0$

Then,

$$z_1 - z_2 = \frac{\bar{V}^2}{2g}\left[f\frac{L}{D} + K_{ent} + K_{exit}\right] \quad \text{------ (1)}$$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 2.87 \times 10^{-5}\ \frac{\text{m}}{\text{s}} \times \frac{1}{(15.9 \times 10^{-3}\ \text{m})^2} = 0.145\ \text{m/s}$$

For turbulent flow in a smooth pipe at $Re = 2300$,

$f = 0.05$ (Fig 8.13)

From Eq. 1

$$d = z_1 - z_2 = \frac{(0.145)^2}{2}\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{s}^2}{9.81\ \text{m}}\left[0.05 \times \frac{3.56 \times 10^3}{15.9} + 0.5 + 1.0\right]$$

$d = 0.0136\ \text{m}$ or $13.6\ \text{mm}$ ______ d

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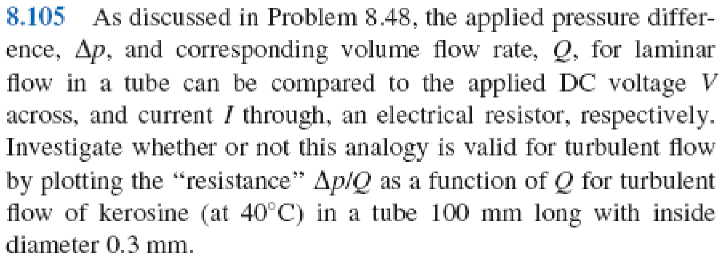
Problem 8.105

[3]



8.105 As discussed in Problem 8.48, the applied pressure difference, Δp, and corresponding volume flow rate, Q, for laminar flow in a tube can be compared to the applied DC voltage V across, and current I through, an electrical resistor, respectively. Investigate whether or not this analogy is valid for turbulent flow by plotting the "resistance" $\Delta p/Q$ as a function of Q for turbulent flow of kerosine (at 40°C) in a tube 100 mm long with inside diameter 0.3 mm.

Given:

Data on a tube

Find:

"Resistance" of tube for flow of kerosine; plot

Solution:

The given data is $L = 100 \cdot mm$ $D = 0.3 \cdot mm$

From Fig. A.2 and Table A.2 $\mu = 1.1 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}$ $\rho = 0.82 \times 990 \cdot \frac{kg}{m^3} = 812 \cdot \frac{kg}{m^3}$ (Kerosene)

For an electrical resistor $V = R \cdot I$ (1)

The governing equations for turbulent flow are

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_l \qquad (8.29)$$

$$h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad (8.37)$$

Simplifying Eqs. 8.29 and 8.34 for a horizontal, constant-area pipe

$$\frac{p_1 - p_2}{\rho} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} = f \cdot \frac{L}{D} \cdot \frac{\left(\frac{Q}{\frac{\pi}{4} \cdot D^2}\right)^2}{2} \quad \text{or} \quad \Delta p = \frac{8 \cdot \rho \cdot f \cdot L}{\pi^2 \cdot D^5} \cdot Q^2 \qquad (2)$$

By analogy, current *I* is represented by flow rate *Q*, and voltage *V* by pressure drop Δ*p***.**
Comparing Eqs. (1) and (2), the "resistance" of the tube is

$$R = \frac{\Delta p}{Q} = \frac{8 \cdot \rho \cdot f \cdot L \cdot Q}{\pi^2 \cdot D^5}$$

The "resistance" of a tube is not constant, but is proportional to the "current" *Q*! Actually, the dependence is not quite linear, because *f* decreases slightly (and nonlinearly) with *Q*. The analogy fails!

The analogy is hence invalid for $Re > 2300$ or $\frac{\rho \cdot V \cdot D}{\mu} > 2300$

Writing this constraint in terms of flow rate $\frac{\rho \cdot \frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot D}{\mu} > 2300$ or $Q > \frac{2300 \cdot \mu \cdot \pi \cdot D}{4 \cdot \rho}$

Flow rate above which analogy fails $Q = 7.34 \times 10^{-7} \frac{m^3}{s}$

The plot of "resistance" versus flow rate is shown in the associated *Excel* workbook

Problem 8.105

[3]

8.105 As discussed in Problem 8.48, the applied pressure difference, Δp, and corresponding volume flow rate, Q, for laminar flow in a tube can be compared to the applied DC voltage V across, and current I through, an electrical resistor, respectively. Investigate whether or not this analogy is valid for turbulent flow by plotting the "resistance" $\Delta p/Q$ as a function of Q for turbulent flow of kerosine (at 40°C) in a tube 100 mm long with inside diameter 0.3 mm.

Given: Data on a tube

Find: "Resistance" of tube for flow of kerosine; plot

Solution:

The "resistance" is $$R = \frac{\Delta p}{Q} = \frac{8 \cdot \rho \cdot f \cdot L \cdot Q}{\pi^2 \cdot D^5}$$

The "resistance" of a tube is not constant, but is proportional to the "current" *Q*! Actually, the dependence is not quite linear, because *f* decreases slightly (and nonlinearly) with *Q*. The analogy fails!

Given data:

L = 100 mm
D = 0.3 mm

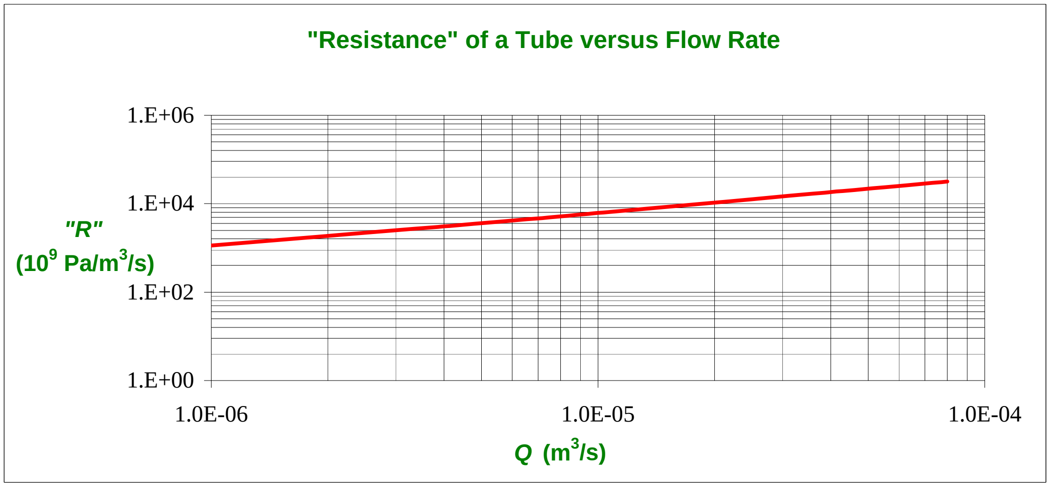
Tabulated or graphical data:

μ = 1.01E-03 $\mathrm{N.s/m^2}$
SG_{ker} = 0.82
ρ_w = 990 $\mathrm{kg/m^3}$
ρ = 812 $\mathrm{kg/m^3}$
(Appendix A)

Computed results:

Q ($\mathrm{m^3/s}$)	V (m/s)	*Re*	*f*	*"R"* (10^9 $\mathrm{Pa/m^3/s}$)
1.0E-06	14.1	3.4E+03	0.0419	1133
2.0E-06	28.3	6.8E+03	0.0343	1855
4.0E-06	56.6	1.4E+04	0.0285	3085
6.0E-06	84.9	2.0E+04	0.0257	4182
8.0E-06	113.2	2.7E+04	0.0240	5202
1.0E-05	141.5	3.4E+04	0.0228	6171
2.0E-05	282.9	6.8E+04	0.0195	10568
4.0E-05	565.9	1.4E+05	0.0169	18279
6.0E-05	848.8	2.0E+05	0.0156	25292
8.0E-05	1131.8	2.7E+05	0.0147	31900

The "resistance" is not constant; the analogy is invalid for turbulent flow



Problem 8.106

[3]

8.106 Plot the required reservoir depth of water to create flow in a smooth tube of diameter 10 mm and length 100 m, for a flow rate range of 1 L/min through 10 L/min.

Given: Data on tube geometry

Find: Plot of reservoir depth as a function of flow rate

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}=\sum_{\text{major}} h_l+\sum_{\text{minor}} h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes $g\cdot d-\alpha\cdot\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$

This can be solved expicitly for height *d*, or solved using *Solver*

$$d=\frac{V^2}{2\cdot g}\cdot\left(\alpha+f\cdot\frac{L}{D}+K\right)$$

Given data:

L = 100 m

D = 10 mm

α = 1 (All flows turbulent)

Tabulated or graphical data:

μ = 1.01E-03 N.s/m^2

ρ = 998 kg/m^3

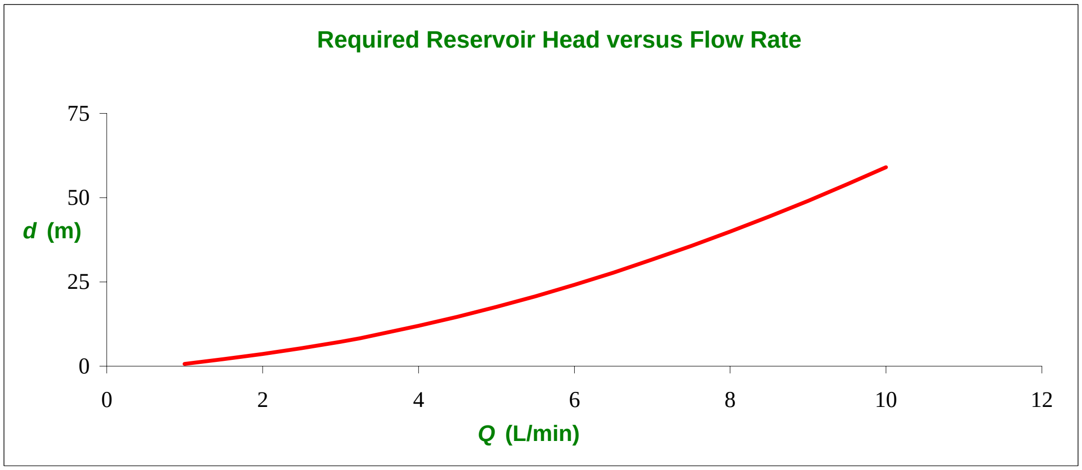
(Table A.8)

K_{ent} = 0.5 (Square-edged)

(Table 8.2)

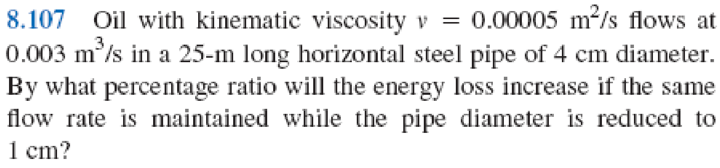
Computed results:

Q (L/min)	*V* (m/s)	*Re*	*f*	*d* (m)
1	0.2	2.1E+03	0.0305	0.704
2	0.4	4.2E+03	0.0394	3.63
3	0.6	6.3E+03	0.0350	7.27
4	0.8	8.4E+03	0.0324	11.9
5	1.1	1.0E+04	0.0305	17.6
6	1.3	1.3E+04	0.0291	24.2
7	1.5	1.5E+04	0.0280	31.6
8	1.7	1.7E+04	0.0270	39.9
9	1.9	1.9E+04	0.0263	49.1
10	2.1	2.1E+04	0.0256	59.1



Problem 8.107

[3]



8.107 Oil with kinematic viscosity $\nu = 0.00005\ \mathrm{m^2/s}$ flows at $0.003\ \mathrm{m^3/s}$ in a 25-m long horizontal steel pipe of 4 cm diameter. By what percentage ratio will the energy loss increase if the same flow rate is maintained while the pipe diameter is reduced to 1 cm?

Given: Flow of oil in a pipe

Find: Percentage change in loss if diameter is reduced

Solution:

Basic equations

$$h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad f = \frac{64}{Re} \quad \text{Laminar} \qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \quad \text{Turbulent}$$

Here

$$V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = \frac{4}{\pi}\times 0.003\cdot\frac{m^3}{s}\times\left(\frac{1}{0.04\cdot m}\right)^2 \qquad V = 2.39\frac{m}{s}$$

Then

$$Re = \frac{V\cdot D}{\nu} \qquad Re = 2.39\cdot\frac{m}{s}\times 0.04\cdot m\times\frac{s}{0.00005\cdot m^2} \qquad Re = 1912$$

The flow is LAMINAR

$$h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h_l = \frac{64}{Re}\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h_l = \frac{64}{1912}\times\frac{25\cdot m}{0.04\cdot m}\times\frac{\left(2.39\cdot\frac{m}{s}\right)^2}{2} \qquad h_l = 643\cdot\frac{ft^2}{s^2}$$

When the diameter is reduced

$$V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = \frac{4}{\pi}\times 0.003\cdot\frac{m^3}{s}\times\left(\frac{1}{0.01\cdot m}\right)^2 \qquad V = 38.2\frac{m}{s}$$

$$Re = \frac{V\cdot D}{\nu} \qquad Re = 38.2\cdot\frac{m}{s}\times 0.01\cdot m\times\frac{s}{0.00005\cdot m^2} \qquad Re = 7640$$

The flow is TURBULENT For a steel pipe, from table 8.1 $\quad e = 0.046\cdot mm$

Given

$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0389$$

$$h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h_l = 0.0389\times\frac{25\cdot m}{0.01\cdot m}\times\frac{\left(38.2\cdot\frac{m}{s}\right)^2}{2} \qquad h_l = 7.64\times 10^5\cdot\frac{ft^2}{s^2}$$

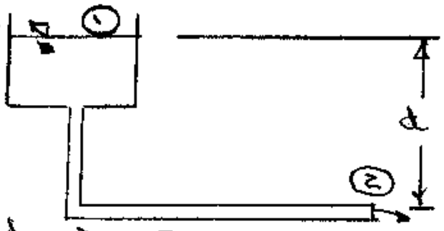
The increase in loss is

$$\frac{7.64\times 10^5\ \frac{ft^2}{s^2}}{643\ \frac{ft^2}{s^2}} = 1188$$

This is a HUGH increase! As a percentage increase of 118800%. Hence choice of diameter is very important! The increase is because the diameter reduces by a factor of four and the velocity therefore increases by a factor of 16, and is squared!

Problem 8.108

Given: System for measuring pressure drop for water flow in smooth pipe supplies water from an overhead constant-head tank.



System includes:
- square-edged entrance
- two 45° standard elbows
- two 90° standard elbows.
- fully open gate valve
- pipe length $L = 9.8\,m$, diameter $D = 15.9\,mm$

Find: elevation of water surface in supply tank above pipe discharge needed to achieve $Re_D = 10^5$.

Solution:

$Re = \frac{\rho D \bar{V}}{\mu} = \frac{D\bar{V}}{\nu}$ Assume $T = 20°c$, $\nu = 1.00\times10^{-6}\,m^2/s$ (Table A.8).

For $Re = 10^5$, $\bar{V} = \frac{Re\,\nu}{D} = 10^5 \times 1.0\times10^{-6}\,\frac{m^2}{s} \times \frac{1}{15.9\times10^{-3}\,m} = 6.29\,m/s$

Basic equations: $\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{lT}$ (8.29)

$h_{lT} = h_l + h_{lm}$, $h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$, $h_{lm} = f\frac{\bar{V}^2}{2}\sum\frac{L_e}{D} + K_{ent}\frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2) $\bar{V}_1 = 0$ (3) $\alpha_2 = 1.0$

Then,

$$g(z_1 - z_2) = \frac{\bar{V}^2}{2} + f\frac{L}{D}\frac{\bar{V}^2}{2} + f\frac{\bar{V}^2}{2}\sum\left(\frac{L_e}{D}\right) + K_{ent}\frac{\bar{V}^2}{2}$$

$$d = (z_1 - z_2) = \frac{\bar{V}^2}{2g}\left[1 + f\frac{L}{D} + 2f\left(\frac{L_e}{D}\right)_{45°el} + 2f\left(\frac{L_e}{D}\right)_{90°el} + f\left(\frac{L_e}{D}\right)_{gv} + K_{ent}\right]$$

From Table 8.2 $K_{ent} = 0.5$

From Table 8.4 $(L_e/D)_{45°el} = 16$, $(L_e/D)_{90°el} = 30$, $(L_e/D)_{gv} = 8$

For $Re = 10^5$ in smooth pipe, $f = 0.018$ (Fig. 8.13)

Thus

$$d = \frac{1}{2}\times(6.29)^2\,\frac{m^2}{s^2}\times\frac{s^2}{9.81\,m}\left[1 + 0.018\times\frac{9.8\times10^3}{15.9} + 2(0.018)16 + 2(0.018)30 + 0.018(8) + 0.5\right]$$

$d = 29.0\,m$ ← d

This value of d indicates that it will not be possible to obtain a value of $Re = 10^5$ in the flow system. The maximum value of Re will be considerably less than 10^5.

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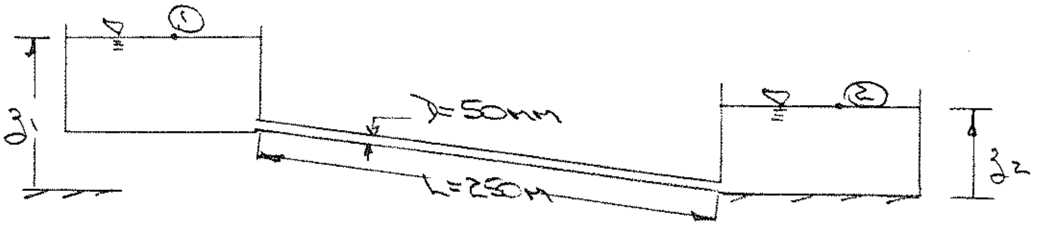
Problem 8.109

Given: Water flow by gravity between two reservoirs through straight galvanized iron pipe. Required flow rate is Q



Plot: required elevation difference Δz vs Q for $0 \le Q \le 0.01\,\text{m}^3/\text{s}$
Estimate: fraction of Δz due to minor losses

Plot: (a) Δz and (b) minor loss / total loss versus Q

Solution:

Apply the energy equation for steady incompressible flow between sections ① and ②.

Basic equations: $\left(\cancel{\frac{p_1}{\rho}}^{(1)} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}}^{=0(2)} + g z_1\right) - \left(\cancel{\frac{p_2}{\rho}}^{(1)} + \cancel{\alpha_2 \frac{\bar{V}_2^2}{2}}^{=0(2)} + g z_2\right) = h_{l_T}$ (8.29)

$h_{l_T} = h_l + h_{lm}$; $h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$; $h_{lm} = K_{ent} \frac{\bar{V}^2}{2} + K_{exit} \frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (given)
(2) $\bar{V}_1 = \bar{V}_2 \approx 0$
(3) square edged entrance

For square edged entrance (Table 8.2) $K_{ent} = 0.5$; also $K_{exit} = 1.0$

For water at 20°C, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\text{s}$ (Table A.8)

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{Q}{A}\frac{D}{\nu} = \frac{QD}{\nu} \cdot \frac{4}{\pi D^2} = \frac{4Q}{\pi \nu D} \quad \text{------} \quad (1)$$

To plot Δz vs Q

$$\bar{V} = \frac{4Q}{\pi D^2} = 509\, Q\,(\text{m}^3/\text{s})$$

$$\Delta z = \frac{\bar{V}^2}{2g}\left[K_{ent} + K_{exit} + f\frac{L}{D}\right] = \frac{\bar{V}^2}{2g}\left[1.5 + 5000 f\right]$$

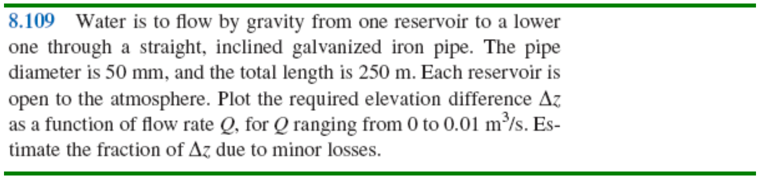
where $f = f(Re,\ e/D = 0.003)$

$$\frac{h_{lm}}{h_{l_T}} = \frac{K_{ent} + K_{exit}}{K_{ent} + K_{exit} + f\frac{L}{D}} = \frac{1.5}{1.5 + 5000 f}$$

The ratio h_{lm}/h_{l_T} increases with increasing Re because f decreases with increasing Re.

Problem 8.109

[3]



8.109 Water is to flow by gravity from one reservoir to a lower one through a straight, inclined galvanized iron pipe. The pipe diameter is 50 mm, and the total length is 250 m. Each reservoir is open to the atmosphere. Plot the required elevation difference Δz as a function of flow rate Q, for Q ranging from 0 to 0.01 m^3/s. Estimate the fraction of Δz due to minor losses.

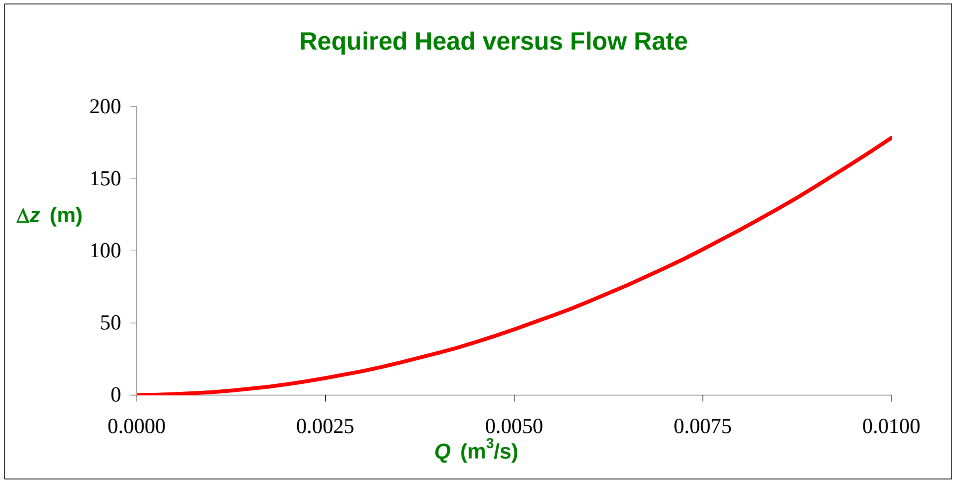
Given: Data on reservoir/pipe system

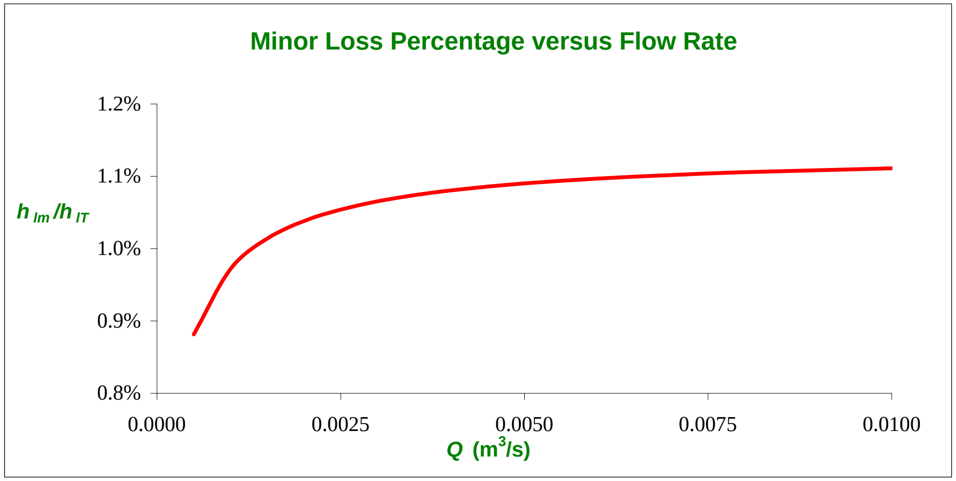
Find: Plot elevation as a function of flow rate; fraction due to minor losses

Solution:

L = 250 m
D = 50 mm
e/D = 0.003
K_{ent} = 0.5
K_{exit} = 1.0
ν = 1.01E-06 m^2/s

Q (m^3/s)	V (m/s)	Re	f	Δz (m)	h_{lm}/h_{lT}
0.0000	0.000	0.00E+00		0.000	
0.0005	0.255	1.26E+04	0.0337	0.562	0.882%
0.0010	0.509	2.52E+04	0.0306	2.04	0.972%
0.0015	0.764	3.78E+04	0.0293	4.40	1.01%
0.0020	1.02	5.04E+04	0.0286	7.64	1.04%
0.0025	1.27	6.30E+04	0.0282	11.8	1.05%
0.0030	1.53	7.56E+04	0.0279	16.7	1.07%
0.0035	1.78	8.82E+04	0.0276	22.6	1.07%
0.0040	2.04	1.01E+05	0.0275	29.4	1.08%
0.0045	2.29	1.13E+05	0.0273	37.0	1.09%
0.0050	2.55	1.26E+05	0.0272	45.5	1.09%
0.0055	2.80	1.39E+05	0.0271	54.8	1.09%
0.0060	3.06	1.51E+05	0.0270	65.1	1.10%
0.0065	3.31	1.64E+05	0.0270	76.2	1.10%
0.0070	3.57	1.76E+05	0.0269	88.2	1.10%
0.0075	3.82	1.89E+05	0.0269	101	1.10%
0.0080	4.07	2.02E+05	0.0268	115	1.11%
0.0085	4.33	2.14E+05	0.0268	129	1.11%
0.0090	4.58	2.27E+05	0.0268	145	1.11%
0.0095	4.84	2.40E+05	0.0267	161	1.11%
0.0100	5.09	2.52E+05	0.0267	179	1.11%

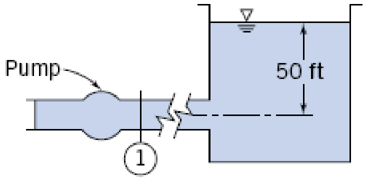




Problem 8.110

[2]

8.110 Water from a pump flows through a 9-in. diameter commercial steel pipe for a distance of 4 miles from the pump discharge to a reservoir open to the atmosphere. The level of the water in the reservoir is 50 ft above the pump discharge, and the average speed of the water in the pipe is 10 ft/s. Calculate the pressure at the pump discharge.



Given: Flow from pump to reservoir

Find: Pressure at pump discharge

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{{V_1}^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{{V_2}^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{{V_1}^2}{2}+K_{exit}\cdot\frac{{V_1}^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) $V_2 <<$

Hence the energy equation between Point 1 and the free surface (Point 2) becomes

$$\left(\frac{p_1}{\rho}+\frac{V^2}{2}\right)-\left(g\cdot z_2\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{exit}\cdot\frac{V^2}{2}$$

Solving for p_1
$$p_1=\rho\cdot\left(g\cdot z_2-\frac{V^2}{2}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{exit}\cdot\frac{V^2}{2}\right)$$

From Table A.7 (68ºF) $\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}$ $\qquad \nu = 1.08\times10^{-5}\cdot\frac{\text{ft}^2}{\text{s}}$

$$Re=\frac{V\cdot D}{\nu} \qquad Re=10\cdot\frac{\text{ft}}{\text{s}}\times\frac{9}{12}\cdot\text{ft}\times\frac{\text{s}}{1.08\times10^{-5}\cdot\text{ft}^2} \qquad Re=6.94\times10^5$$

Turbulent

For commercial steel pipe $e = 0.00015\cdot\text{ft}$ (Table 8.1) so $\frac{e}{D}=0.0002$

Flow is turbulent: Given
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0150$$

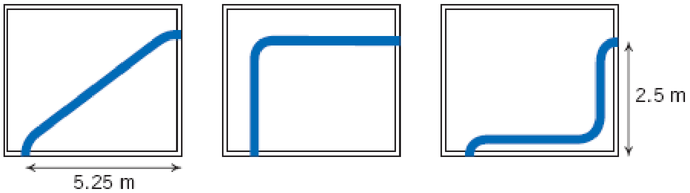
For the exit $K_{exit}=1.0$ so we find $p_1=\rho\cdot\left(g\cdot z_2+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\right)$

$$p_1=1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left[32.2\cdot\frac{\text{ft}}{\text{s}^2}\times50\cdot\text{ft}+.0150\times\frac{4\cdot\text{mile}}{0.75\cdot\text{ft}}\times\frac{5280\cdot\text{ft}}{1\text{mile}}\times\frac{1}{2}\times\left(10\cdot\frac{\text{ft}}{\text{s}}\right)^2\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \qquad p_1=4.41\times10^4\cdot\frac{\text{lbf}}{\text{ft}^2} \qquad p_1=306\cdot\text{psi}$$

Problem 8.111

[3]

8.111 A 5-cm diameter potable water line is to be run through a maintenance room in a commercial building. Three possible layouts for the water line are proposed, as shown. Which is the best option, based on minimizing losses? Assume galvanized iron, and a flow rate of 350 L/min.



(*a*) Two miter bends (*b*) A standard elbow (*c*) Three standard elbows

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Given: Flow through three different layouts

Find: Which has minimum loss

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}\qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+\sum_{\text{Minor}}\left(f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}\right)$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore additional length of elbows

For a flow rate of $Q=350\cdot\frac{L}{min}$ $\quad V=\frac{Q}{A}=\frac{4\cdot Q}{\pi\cdot D^2}$ $\quad V=\frac{4}{\pi}\times350\cdot\frac{L}{min}\times\frac{0.001\cdot m^3}{1\cdot L}\times\frac{1\cdot min}{60\cdot s}\times\left(\frac{1}{0.05\cdot m}\right)^2$ $\quad V=2.97\frac{m}{s}$

For water at 20ºC $\quad \nu=1.01\times10^{-6}\cdot\frac{m^2}{s}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re=2.97\cdot\frac{m}{s}\times0.05\cdot m\times\frac{s}{1.01\times10^{-6}\cdot m^2}$ $\quad Re=1.47\times10^5$

Flow is turbulent. From Table 8.1 $\quad e=0.15\cdot mm$ $\quad \frac{e}{D}=6.56\times10^{-4}$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0201$

For Case (a) $\quad L=\sqrt{5.25^2+2.5^2}\cdot m$ $\quad L=5.81\,m$ $\quad$ Two 45º miter bends (Fig. 8.16), for each $\frac{L_e}{D}=13$

Hence the energy equation is $\quad \frac{p_1}{\rho}-\frac{p_2}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+2\cdot f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$

Solving for Δp $\quad \Delta p=p_1-p_2=\rho\cdot f\cdot\frac{V^2}{2}\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D}\right)$

$$\Delta p=1000\cdot\frac{kg}{m^3}\times.0201\times\left(2.97\cdot\frac{m}{s}\right)^2\times\left(\frac{5.81}{0.05}+2\cdot13\right)\times\frac{N\cdot s^2}{kg\cdot m}\qquad \Delta p=25.2\,kPa$$

For Case (b) $\quad L=(5.25+2.5)\cdot m$ $\quad L=7.75\,m$ $\quad$ One standard 90º elbow (Table 8.4) $\frac{L_e}{D}=30$

Hence the energy equation is $\quad \frac{p_1}{\rho}-\frac{p_2}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$

Solving for Δp $\quad \Delta p=p_1-p_2=\rho\cdot f\cdot\frac{V^2}{2}\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)$

$$\Delta p = 1000 \cdot \frac{kg}{m^3} \times .0201 \times \left(2.97 \cdot \frac{m}{s}\right)^2 \times \left(\frac{7.75}{0.05} + 30\right) \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 32.8\,kPa$$

For Case (c) $L = (5.25 + 2.5) \cdot m$ $L = 7.75\,m$ Three standard 90º elbows, for each $\frac{L_e}{D} = 30$

Hence the energy equation is $\frac{p_1}{\rho} - \frac{p_2}{\rho} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + 3 \cdot f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2}$

Solving for Δp $\Delta p = p_1 - p_2 = \rho \cdot f \cdot \frac{V^2}{2} \cdot \left(\frac{L}{D} + 3 \cdot \frac{L_e}{D}\right)$

$$\Delta p = 1000 \cdot \frac{kg}{m^3} \times .0201 \times \left(2.97 \cdot \frac{m}{s}\right)^2 \times \left(\frac{7.75}{0.05} + 3 \times 30\right) \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 43.4\,kPa$$

Hence we conclude Case (a) is the best and Case (c) is the worst

Problem 8.112

[2]

Given: Air at a flowrate of 35 m^3/min at standard conditions in a smooth duct 0.3 m square.

Find: Pressure drop in mm H_2O per 30 m of horizontal duct.

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section. Use hydraulic diameter.

Basic equation: $$\frac{p_1}{\rho}+\cancelto{(1)}{\frac{\bar V_1^2}{2}}+\cancelto{(2)}{g z_1}=\frac{p_2}{\rho}+\cancelto{(1)}{\frac{\bar V_2^2}{2}}+\cancelto{(2)}{g z_2}+f\frac{L}{D_h}\frac{\bar V^2}{2}+\cancelto{=0(3)}{h_{\ell m}}\,;\quad D_h=\frac{4A}{P_w}$$

Assumptions: (1) $\bar V_1=\bar V_2$

(2) Horizontal

(3) $h_{\ell m}=0$

Then

$$\Delta p = p_1 - p_2 = f\frac{L}{D_h}\,\rho\frac{\bar V^2}{2}$$

From continuity, $\bar V=\frac{Q}{A}=35\,\frac{m^3}{min}\times\frac{1}{(0.3)^2 m^2}\times\frac{min}{60\,sec}=6.48\ m/s$

$$D_h=\frac{4A}{P_w}=4\times(0.3)^2 m^2\times\frac{1}{4(0.3)m}=0.3\,m\,;\quad \nu=1.45\times10^{-5}\,m^2/s\ \text{(Table A.10)}$$

$$Re=\frac{\bar V D_h}{\nu}=6.48\,\frac{m}{s}\times0.3\,m\times\frac{s}{1.46\times10^{-5}m^2}=1.33\times10^5$$

$f = 0.017$ (Fig. 8.13)

Then $$\Delta p=\frac{0.017}{2}\times\frac{30m}{0.3m}\times1.23\,\frac{kg}{m^3}\times(6.48)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}=43.9\ N/m^2$$ Δp

For a manometer, $\Delta p=\rho_{H_2O}\,g\,\Delta h$

$$\Delta h=\frac{\Delta p}{\rho_{H_2O}\,g}=43.9\,\frac{N}{m^2}\times\frac{m^3}{999\,kg}\times\frac{s^2}{9.81\,m}\times\frac{kg\cdot m}{N\cdot s^2}=0.00448\ m$$

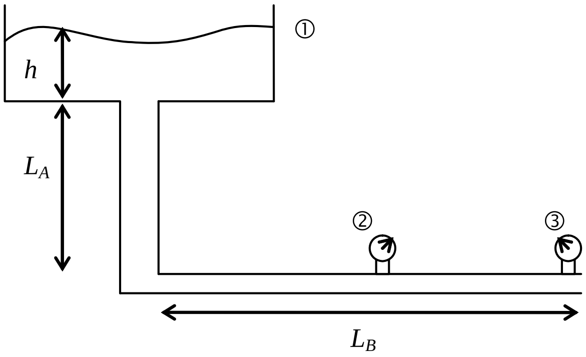
Thus

$\Delta h = 4.48$ mm H_2O (per 30 m of duct) Δh

(This is Δp expressed in mm of water.)

Problem 8.113

8.113 A pipe friction experiment is to be designed, using water, to reach a Reynolds number of 100,000. The system will use 5 cm smooth PVC pipe from a constant-head tank to the flow bench and 20 m of smooth 2.5 cm PVC line mounted horizontally for the test section. The water level in the constant-head tank is 0.5 m above the entrance to the 5 cm PVC line. Determine the required average speed of water in the 2.5 cm pipe. Estimate the feasibility of using a constant-head tank. Calculate the pressure difference expected between taps 5 m apart in the horizontal test section.



Given: Pipe friction experiment

Find: Required average speed; Estimate feasibility of constant head tank; Pressure drop over 5 m

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}\qquad h_{lT}=h_A+h_B=f_A\cdot\frac{L_A}{D_A}\cdot\frac{V_A^2}{2}+f_B\cdot\frac{L_B}{D_B}\cdot\frac{V_B^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore minor losses

We wish to have $Re_B = 10^5$

Hence, from $Re_B=\frac{V_B\cdot D_B}{\nu}$ $\quad V_B=\frac{Re_B\cdot\nu}{D_B}$ $\quad$ and for water at 20ºC $\quad \nu = 1.01\times10^{-6}\cdot\frac{m^2}{s}$

$$V_B = 10^5\times1.01\times10^{-6}\cdot\frac{m^2}{s}\times\frac{1}{0.025\cdot m}\qquad V_B=4.04\frac{m}{s}$$

We will also need $\quad V_A=V_B\cdot\left(\frac{D_B}{D_A}\right)^2\quad V_A=4.04\cdot\frac{m}{s}\times\left(\frac{2.5}{5}\right)^2\quad V_A=1.01\frac{m}{s}$

$$Re_A=\frac{V_A\cdot D_A}{\nu}\qquad Re_A=1.01\cdot\frac{m}{s}\times0.05\cdot m\times\frac{s}{1.01\times10^{-6}\cdot m^2}\qquad Re_A=5\times10^4$$

Both tubes have turbulent flow

For PVC pipe (from Googling!) $\quad e = 0.0015\cdot mm$

For tube A $\quad$ Given $\quad \frac{1}{\sqrt{f_A}}=-2.0\cdot\log\left(\frac{\frac{e}{D_A}}{3.7}+\frac{2.51}{Re_A\cdot\sqrt{f_A}}\right)\qquad f_A=0.0210$

For tube B $\quad$ Given $\quad \frac{1}{\sqrt{f_B}}=-2.0\cdot\log\left(\frac{\frac{e}{D_B}}{3.7}+\frac{2.51}{Re_B\cdot\sqrt{f_B}}\right)\qquad f_B=0.0183$

Applying the energy equation between Points 1 and 3

$$g\cdot\left(L_A+h\right)-\frac{V_B^2}{2}=f_A\cdot\frac{L_A}{D_A}\cdot\frac{V_A^2}{2}+f_B\cdot\frac{L_B}{D_B}\cdot\frac{V_B^2}{2}$$

Solving for L_A $$L_A = \frac{\frac{V_B^2}{2}\cdot\left(1+f_B\cdot\frac{L_B}{D_B}\right)-g\cdot h}{\left(g-\frac{f_A}{D_A}\cdot\frac{V_A^2}{2}\right)}$$

$$L_A = \frac{\frac{1}{2}\times\left(4.04\cdot\frac{m}{s}\right)^2\times\left(1+0.0183\times\frac{20}{0.025}\right)-9.81\cdot\frac{m}{s^2}\times 0.5\cdot m}{9.81\cdot\frac{m}{s^2}-\frac{0.0210}{2}\times\frac{1}{0.05\cdot m}\times\left(1.01\cdot\frac{m}{s}\right)^2}$$ $$L_A = 12.8\,m$$

Most ceilings are about 3.5 m or 4 m, so this height is IMPRACTICAL

Applying the energy equation between Points 2 and 3

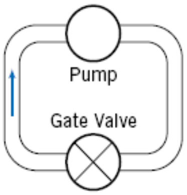
$$\left(\frac{p_2}{\rho}+\frac{V_B^2}{2}\right)-\left(\frac{p_3}{\rho}+\frac{V_B^2}{2}\right) = f_B\cdot\frac{L}{D_B}\cdot\frac{V_B^2}{2}$$ or $$\Delta p = \rho\cdot f_B\cdot\frac{L}{D_B}\cdot\frac{V_B^2}{2}$$

$$\Delta p = 1000\cdot\frac{kg}{m^3}\times\frac{0.0183}{2}\times\frac{5\cdot m}{0.025\cdot m}\times\left(4.04\cdot\frac{m}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m}$$ $$\Delta p = 29.9\cdot kPa$$

Problem 8.114

[3]

8.114 A system for testing variable-output pumps consists of the pump, four standard elbows, and an open gate valve forming a closed circuit as shown. The circuit is to absorb the energy added by the pump. The tubing is 75-mm diameter cast iron, and the total length of the circuit is 20 m. Plot the pressure difference required from the pump for water flow rates Q ranging from 0.01 m^3/s to 0.06 m^3/s.



Given: Data on circuit

Find: Plot pressure difference for a range of flow rates

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\ 2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\ 2}}{2}+g\cdot z_2\right)=h_{lT}=\sum_{major}h_l+\sum_{minor}h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for the circuit (1 = pump inlet, 2 = pump outlet)

$$\frac{p_1-p_2}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+4\cdot f\cdot L_{elbow}\cdot\frac{V^2}{2}+f\cdot L_{valve}\cdot\frac{V^2}{2} \qquad \text{or} \qquad \Delta p=\rho\cdot f\cdot\frac{V^2}{2}\cdot\left(\frac{L}{D}+4\cdot\frac{L_{elbow}}{D}+\frac{L_{valve}}{D}\right)$$

Given data:

L = 20 m

D = 75 mm

Tabulated or graphical data:

e = 0.26 mm

(Table 8.1)

μ = 1.00E-03 $N.s/m^2$

ρ = 999 kg/m^3

(Appendix A)

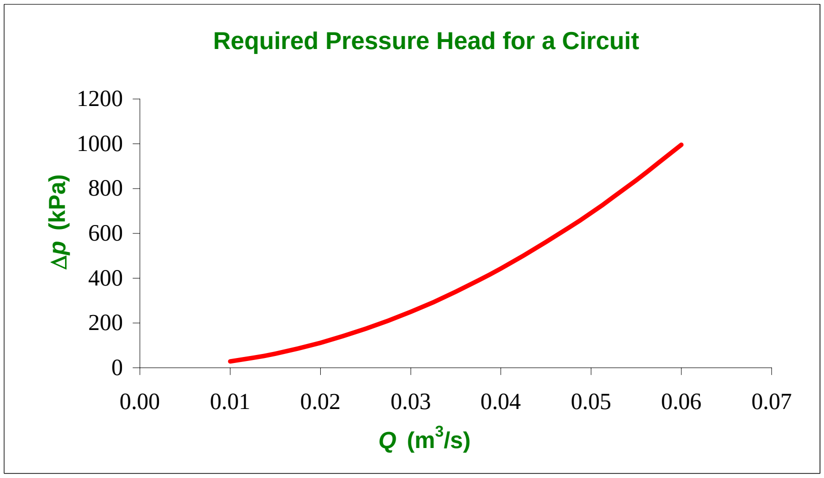
Gate valve L_e/D = 8

Elbow L_e/D = 30

(Table 8.4)

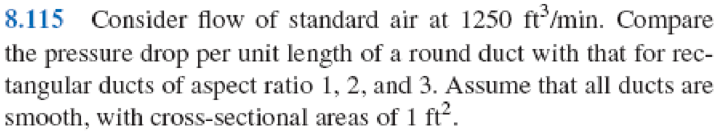
Computed results:

Q (m^3/s)	V (m/s)	Re	f	Δp (kPa)
0.010	2.26	1.70E+05	0.0280	28.3
0.015	3.40	2.54E+05	0.0277	63.1
0.020	4.53	3.39E+05	0.0276	112
0.025	5.66	4.24E+05	0.0276	174
0.030	6.79	5.09E+05	0.0275	250
0.035	7.92	5.94E+05	0.0275	340
0.040	9.05	6.78E+05	0.0274	444
0.045	10.2	7.63E+05	0.0274	561
0.050	11.3	8.48E+05	0.0274	692
0.055	12.4	9.33E+05	0.0274	837
0.060	13.6	1.02E+06	0.0274	996



Problem 8.115

[3]



8.115 Consider flow of standard air at 1250 ft^3/min. Compare the pressure drop per unit length of a round duct with that for rectangular ducts of aspect ratio 1, 2, and 3. Assume that all ducts are smooth, with cross-sectional areas of 1 ft^2.

Given: Same flow rate in various ducts

Find: Pressure drops of each compared to round duct

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_l$ $\quad D_h = \frac{4\cdot A}{P_w}$ $\quad e = 0$ (Smooth)

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore minor losses

The energy equation simplifies to

$$\Delta p = p_1 - p_2 = \rho\cdot f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad \text{or} \qquad \frac{\Delta p}{L} = \rho\cdot\frac{f}{D_h}\cdot\frac{V^2}{2}$$

But we have $\quad V = \frac{Q}{A} \quad V = 1250\cdot\frac{ft^3}{min}\times\frac{1\cdot min}{60\cdot s}\times\frac{1}{1\cdot ft^2} \quad V = 20.8\frac{ft}{s}$

From Table A.9 $\quad \nu = 1.62\times 10^{-4}\cdot\frac{ft^2}{s} \quad \rho = 0.00234\cdot\frac{slug}{ft^3}$ $\quad$ at 68ºF

Hence $\quad Re = \frac{V\cdot D_h}{\nu} \quad Re = 20.8\cdot\frac{ft}{s}\times\frac{s}{1.62\times 10^{-4}\cdot ft^2}\times D_h = 1.284\times 10^5\cdot D_h$ $\quad$ (D_h in ft)

For a round duct $\quad D_h = D = \sqrt{\frac{4\cdot A}{\pi}} \quad D_h = \sqrt{\frac{4}{\pi}\times 1\cdot ft^2} \quad D_h = 1.13\,ft$

For a rectangular duct $\quad D_h = \frac{4\cdot A}{P_w} = \frac{4\cdot b\cdot h}{2\cdot(b+h)} = \frac{2\cdot h\cdot ar}{1+ar}$ $\quad$ where $\quad ar = \frac{b}{h}$

But $\quad h = \frac{b}{ar}$ $\quad$ so $\quad h^2 = \frac{b\cdot h}{ar} = \frac{A}{ar}$ $\quad$ or $\quad h = \sqrt{\frac{A}{ar}}$ $\quad$ and $\quad D_h = \frac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$

The results are:

Round $\quad D_h = 1.13\cdot ft \quad Re = 1.284\times 10^5\cdot\frac{1}{ft}\cdot D_h \quad Re = 1.45\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \quad f = 0.0167 \quad \frac{\Delta p}{L} = \rho\cdot\frac{f}{D_h}\cdot\frac{V^2}{2} \quad \frac{\Delta p}{L} = 7.51\times 10^{-3}\cdot\frac{lbf}{ft^3}$

$ar = 1 \quad D_h = \frac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A} \quad D_h = 1\,ft \quad Re = 1.284\times 10^5\cdot\frac{1}{ft}\cdot D_h \quad Re = 1.28\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \quad f = 0.0171 \quad \frac{\Delta p}{L} = \rho\cdot\frac{f}{D_h}\cdot\frac{V^2}{2} \quad \frac{\Delta p}{L} = 8.68\times 10^{-3}\cdot\frac{lbf}{ft^3}$

Hence the square duct experiences a percentage increase in pressure drop of $\dfrac{8.68\times10^{-3}-7.51\times10^{-3}}{7.51\times10^{-3}}=15.6\%$

$ar = 2$ $\quad D_h=\dfrac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$ $\quad D_h=0.943\,ft$ $\quad Re=1.284\times10^{5}\cdot\dfrac{1}{ft}\cdot D_h$ $\quad Re=1.21\times10^{5}$

Given $\quad \dfrac{1}{\sqrt{f}}=-2.0\cdot\log\left(\dfrac{\dfrac{e}{D_h}}{3.7}+\dfrac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0173$ $\quad \dfrac{\Delta p}{L}=\rho\cdot\dfrac{f}{D_h}\cdot\dfrac{V^2}{2}$ $\quad \dfrac{\Delta p}{L}=9.32\times10^{-3}\cdot\dfrac{lbf}{ft^3}$

Hence the 2 x 1 duct experiences a percentage increase in pressure drop of $\dfrac{9.32\times10^{-3}-7.51\times10^{-3}}{7.51\times10^{-3}}=24.1\%$

$ar = 3$ $\quad D_h=\dfrac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$ $\quad D_h=0.866\,ft$ $\quad Re=1.284\times10^{5}\cdot\dfrac{1}{ft}\cdot D_h$ $\quad Re=1.11\times10^{5}$

Given $\quad \dfrac{1}{\sqrt{f}}=-2.0\cdot\log\left(\dfrac{\dfrac{e}{D_h}}{3.7}+\dfrac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0176$ $\quad \dfrac{\Delta p}{L}=\rho\cdot\dfrac{f}{D_h}\cdot\dfrac{V^2}{2}$ $\quad \dfrac{\Delta p}{L}=0.01\cdot\dfrac{lbf}{ft^3}$

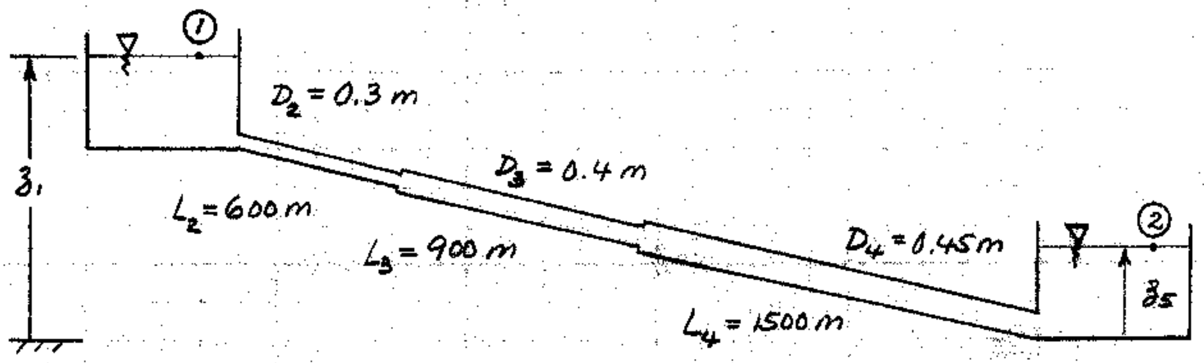
Hence the 3 x 1 duct experiences a percentage increase in pressure drop of $\dfrac{0.01-7.51\times10^{-3}}{7.51\times10^{-3}}=33.2\%$

Note that f varies only about 7%; the large change in $\Delta p/L$ is primarily due to the $1/D_h$ factor

Problem 8.116

[3] Part 1/2

Given: Reservoirs connected by three clean, cast iron pipes in series. The flow is water at $0.11\ m^3/s$ and $15^\circ C$.



Find: Elevation difference, $z_1 - z_5$

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section.

Basic equations: $$\cancel{\frac{p_1}{\rho}}^{(1)} + \cancel{\frac{\bar{V}_1^2}{2}}^{(2)} + g z_1 = \cancel{\frac{p_5}{\rho}}^{(1)} + \cancel{\frac{\bar{V}_5^2}{2}}^{(2)} + g z_5 + h_{lT}$$

$$h_{lT} = \sum f \frac{L}{D}\frac{\bar{V}^2}{2} + h_{lm}\ ;\ h_{lm} = K_{ent}\frac{\bar{V}_2^2}{2} + \overset{\approx 0 (3)}{\cancel{\sum h_{l\,exp}}} + K_{exit}\frac{\bar{V}_4^2}{2}$$

Assumptions: (1) $p_1 = p_5 = p_{atm}$
(2) $\bar{V}_1 = \bar{V}_5 \simeq 0$
(3) Neglect $h_{l\,exp}$ at pipe joints (note all minor losses are probably small due to long lengths of straight pipe sections, but we will check)

For non-smooth pipe, $f = f(Re, e/D)$. $\mu = 1.1 \times 10^{-3}\ N\cdot s/m^2$ from Table A.8.

Section ②: $e/D_2 = 0.26\ mm / 300\ mm = 0.00087$ (for cast iron, $e = 0.26\ mm$, Table 8.1)

$$\bar{V}_2 = \frac{Q}{A_2} = 0.11\frac{m^3}{s} \times \frac{4}{\pi}\frac{1}{(0.3)^2 m^2} = 1.56\ m/s$$

$$Re_2 = \frac{\rho \bar{V}_2 D_2}{\mu} = 999\frac{kg}{m^3} \times 1.56\frac{m}{s} \times 0.3\ m \times \frac{m^2}{1.14 \times 10^{-3}\ N\cdot s} \times \frac{N\cdot s^2}{kg\cdot m} = 4.10 \times 10^5$$

From Fig 8.13, $f_2 = 0.020$

Section ③: $e/D_3 = 0.00065$

$$\bar{V}_3 = \frac{Q}{A_3} = 0.11\frac{m^3}{s} \times \frac{4}{\pi}\frac{1}{(0.4)^2 m^2} = 0.875\ m/s$$

$$Re_3 = \frac{\rho \bar{V}_3 D_3}{\mu} = 999\frac{kg}{m^3} \times 0.875\frac{m}{s} \times 0.4\ m \times \frac{m^2}{1.14 \times 10^{-3}\ N\cdot s} \times \frac{N\cdot s^2}{kg\cdot m} = 3.07 \times 10^5$$

From Fig. 8.13, $f_3 = 0.019$

Section ④: $e/D_4 = 0.00058$

$$\bar{V}_4 = \frac{Q}{A_4} = 0.11\,\frac{m^3}{s} \times \frac{4}{\pi}\,\frac{1}{(0.45)^2 m^2} = 0.692\ m/s$$

$$Re_4 = \frac{\rho \bar{V}_4 D_4}{\mu} = 999\,\frac{kg}{m^3} \times 0.692\,\frac{m}{s} \times 0.45\,m \times \frac{m^2}{1.14\times10^{-3}\,N\cdot s} \times \frac{N\cdot s^2}{kg\cdot m} = 2.73\times10^5$$

From Fig. 8.13, $f_4 = 0.0185$

Then $$\sum f \frac{L}{D}\frac{\bar{V}^2}{2} = 0.020 \times \frac{600\,m}{0.3\,m} \times \frac{1}{2}(1.56)^2\frac{m^2}{s^2} + 0.019 \times \frac{900\,m}{0.4\,m} \times \frac{1}{2}(0.875)^2\frac{m^2}{s^2}$$

$$+ 0.0185 \times \frac{1500\,m}{0.45\,m} \times \frac{1}{2}(0.692)^2\frac{m^2}{s^2} = 79.8\ m^2/s^2$$

The minor loss coefficients are $K_{ent} = 0.5$ (Table 8.2) and $K_{exit} = 1.0$. Thus,

$$h_{\ell m} = K_{ent}\frac{\bar{V}_2^2}{2} + K_{exit}\frac{\bar{V}_4^2}{2}$$

$$h_{\ell m} = 0.5 \times \frac{1}{2} \times (1.56)^2\frac{m^2}{s^2} + 1.0 \times \frac{1}{2} \times (0.692)^2\frac{m^2}{s^2} = 0.848\ m^2/s^2$$

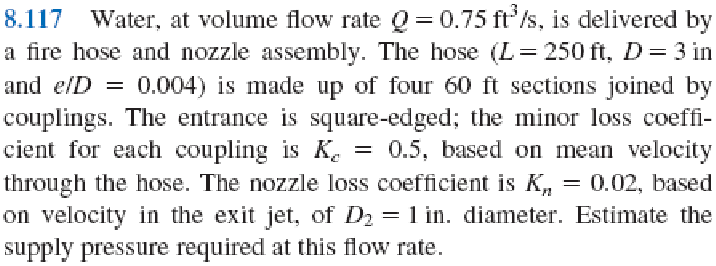
Therefore minor losses are roughly 1 percent of the frictional losses, so they may be neglected. Thus from the energy equation

$$z_1 - z_5 = \sum f \frac{L}{D}\frac{\bar{V}^2}{2g} = 79.8\,\frac{m^2}{s^2} \times \frac{s^2}{9.81\,m} = 8.13\ m \longleftarrow$$

$z_1 - z_5$

Problem 8.117

[3]



8.117 Water, at volume flow rate $Q = 0.75\ \text{ft}^3/\text{s}$, is delivered by a fire hose and nozzle assembly. The hose ($L = 250$ ft, $D = 3$ in and $e/D = 0.004$) is made up of four 60 ft sections joined by couplings. The entrance is square-edged; the minor loss coefficient for each coupling is $K_c = 0.5$, based on mean velocity through the hose. The nozzle loss coefficient is $K_n = 0.02$, based on velocity in the exit jet, of $D_2 = 1$ in. diameter. Estimate the supply pressure required at this flow rate.

Given: Flow through fire hose and nozzle

Find: Supply pressure

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+\sum_{\text{Minor}}\left(K\cdot\frac{V^2}{2}\right)$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) $p_2 = p_{atm}$ so $p_2 = 0$ gage

Hence the energy equation between Point 1 at the supply and the nozzle exit (Point n); let the velocity in the hose be V

$$\frac{p_1}{\rho}-\frac{V_n^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+\left(K_e+4\cdot K_c\right)\cdot\frac{V^2}{2}+K_n\cdot\frac{V_n^2}{2}$$

From continuity $\quad V_n=\left(\frac{D}{D_2}\right)^2\cdot V \quad$ and $\quad V=\frac{Q}{A}=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=\frac{4}{\pi}\times 0.75\cdot\frac{\text{ft}^3}{\text{s}}\times\frac{1}{\left(\frac{1}{4}\cdot\text{ft}\right)^2} \qquad V=15.3\,\frac{\text{ft}}{\text{s}}$

Solving for p_1
$$p_1=\frac{\rho\cdot V^2}{2}\cdot\left[f\cdot\frac{L}{D}+K_e+4\cdot K_c+\left(\frac{D}{D_2}\right)^4\cdot\left(1+K_n\right)\right]$$

From Table A.7 (68ºF) $\quad \rho=1.94\cdot\frac{\text{slug}}{\text{ft}^3} \qquad \nu=1.08\times10^{-5}\cdot\frac{\text{ft}^2}{\text{s}}$

$$Re=\frac{V\cdot D}{\nu} \qquad Re=15.3\cdot\frac{\text{ft}}{\text{s}}\times\frac{3}{12}\cdot\text{ft}\times\frac{\text{s}}{1.08\times10^{-5}\cdot\text{ft}^2} \qquad Re=3.54\times10^{5}$$ Turbulent

For the hose $\quad \frac{e}{D}=0.004$

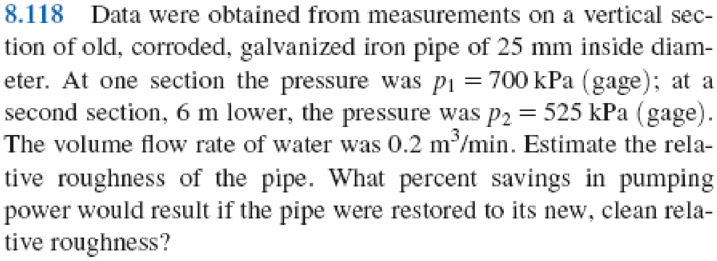
Flow is turbulent: Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0287$

$$p_1=\frac{1}{2}\times1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left(15.3\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\left[0.0287\times\frac{250}{\frac{1}{4}}+0.5+4\times0.5+\left(\frac{3}{1}\right)^4\times(1+0.02)\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$p_1=2.58\times10^4\cdot\frac{\text{lbf}}{\text{ft}^2} \qquad p_1=179\cdot\text{psi}$$

Problem 8.118

[3]



8.118 Data were obtained from measurements on a vertical section of old, corroded, galvanized iron pipe of 25 mm inside diameter. At one section the pressure was $p_1 = 700$ kPa (gage); at a second section, 6 m lower, the pressure was $p_2 = 525$ kPa (gage). The volume flow rate of water was 0.2 m^3/min. Estimate the relative roughness of the pipe. What percent savings in pumping power would result if the pipe were restored to its new, clean relative roughness?

Given: Flow down corroded iron pipe

Find: Pipe roughness; Power savings with new pipe

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{2}}{2}+g\cdot z_2\right)=h_l \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes

$$\left(\frac{p_1}{\rho}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+g\cdot z_2\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

and
$$V=\frac{Q}{A}=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=\frac{4}{\pi}\times 0.2\cdot\frac{m^3}{min}\times\frac{1\cdot min}{60\cdot s}\times\frac{1}{(0.025\cdot m)^2} \qquad V=6.79\frac{m}{s}$$

In this problem we can compute directly f and Re, and hnece obtain e/D

Solving for f
$$f=\frac{2\cdot D}{L\cdot V^2}\cdot\left(\frac{p_1-p_2}{\rho}+g\left(z_1-z_2\right)\right)$$

$$f=2\times\frac{0.025}{6}\times\left(\frac{s}{6.79\cdot m}\right)^2\times\left[(700-525)\times 10^3\cdot\frac{N}{m^2}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}+9.81\cdot\frac{m}{s^2}\times 6\cdot m\right] \qquad f=0.0423$$

From Table A.8 (20ºF) $\nu=1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\qquad Re=\frac{V\cdot D}{\nu}$ $\qquad Re=6.79\cdot\frac{m}{s}\times 0.025\cdot m\times\frac{s}{1.01\times 10^{-6}\cdot m^2}$ $\qquad Re=1.68\times 10^5$

Flow is turbulent: Given $\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad \frac{e}{D}=0.0134$

New pipe (Table 8.1) $e=0.15\cdot mm$ $\qquad \frac{e}{D}=0.006$

Given $\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad f=0.0326$

In this problem
$$\Delta p=p_1-p_2=\rho\cdot\left[g\cdot\left(z_2-z_1\right)+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\right]$$

Hence
$$\Delta p_{new}=1000\cdot\frac{kg}{m^3}\times\left[9.81\cdot\frac{m}{s^2}\times(-6\cdot m)+\frac{0.0326}{2}\times\frac{6}{0.025}\times\left(6.79\cdot\frac{m}{s}\right)^2\right]\times\frac{N\cdot s^2}{kg\cdot m} \qquad \Delta p_{new}=121\cdot kPa$$

Compared to $\Delta p_{old}=175\cdot kPa$ we find $\qquad \frac{\Delta p_{old}-\Delta p_{new}}{\Delta p_{old}}=30.6\cdot\%$

Problem 8.119

[3]

8.119 Flow in a tube may alternate between laminar and turbulent states for Reynolds numbers in the transition zone. Design a bench-top experiment consisting of a constant-head cylindrical transparent plastic tank with depth graduations, and a length of plastic tubing (assumed smooth) attached at the base of the tank through which the water flows to a measuring container. Select tank and tubing dimensions so that the system is compact, but will operate in the transition zone range. Design the experiment so that you can easily increase the tank head from a low range (laminar flow) through transition to turbulent flow, and vice versa. (Write instructions for students on recognizing when the flow is laminar or turbulent.) Generate plots (on the same graph) of tank depth against Reynolds number, assuming laminar or turbulent flow.

Given: Proposal for bench top experiment

Find: Design it; Plot tank depth versus Re

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}=\sum_{\text{major}}h_l+\sum_{\text{minor}}h_{lm}\qquad(8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu}\qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\quad(8.34)\qquad h_{lm}=K\cdot\frac{V^2}{2}\quad(8.40a)\qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}\quad(8.40b)$$

$$f=\frac{64}{Re}\quad(8.36)\quad(\text{Laminar})\qquad\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)\quad(8.37)\quad(\text{Turbulent})$$

The energy equation (Eq. 8.29) becomes

$$g\cdot H-\alpha\cdot\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

This can be solved explicity for reservoir height *H*

$$H=\frac{V^2}{2\cdot g}\cdot\left(\alpha+f\cdot\frac{L}{D}+K\right)$$

Choose data:

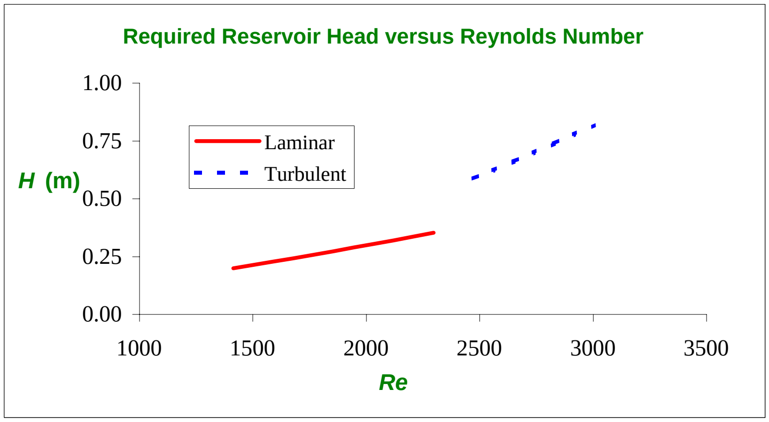
L = 1.0 m
D = 3.0 mm
e = 0.0 mm
α = 2 (Laminar)
= 1 (Turbulent)

Tabulated or graphical data:

μ = 1.00E-03 N.s/m^2
ρ = 999 kg/m^3
(Appendix A)
K_{ent} = 0.5 (Square-edged)
(Table 8.2)

Computed results:

Q (L/min)	*V* (m/s)	*Re*	Regime	*f*	*H* (m)
0.200	0.472	1413	Laminar	0.0453	0.199
0.225	0.531	1590	Laminar	0.0403	0.228
0.250	0.589	1767	Laminar	0.0362	0.258
0.275	0.648	1943	Laminar	0.0329	0.289
0.300	0.707	2120	Laminar	0.0302	0.320
0.325	0.766	2297	Laminar	0.0279	0.353
0.350	0.825	2473	Turbulent	0.0462	0.587
0.375	0.884	2650	Turbulent	0.0452	0.660
0.400	0.943	2827	Turbulent	0.0443	0.738
0.425	1.002	3003	Turbulent	0.0435	0.819
0.450	1.061	3180	Turbulent	0.0428	0.904



The flow rates are realistic, and could easily be measured using a tank/timer system
The head required is also realistic for a small-scale laboratory experiment
Around *Re* = 2300 the flow may oscillate between laminar and turbulent:
Once turbulence is triggered (when *H* > 0.353 m), the resistance to flow increases
requiring *H* >0.587 m to maintain; hence the flow reverts to laminar, only to trip over
again to turbulent! This behavior will be visible: the exit flow will switch back and
forth between smooth (laminar) and chaotic (turbulent)

Problem 8.120

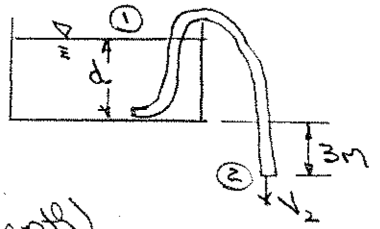
[4]

Given: Small swimming pool is drained using a garden hose.

Hose: $D = 20\,mm$, $L = 30\,m$

$e = 0.2\,mm$

$\bar{V}_2 = 1.2\,m/s$



Find: Water depth at instant shown.
If the flow were inviscid (at this depth) what would be the velocity

Solution:

Apply the energy equation for steady incompressible flow between sections ① and ②

Basic equations: $$\left(\cancel{\frac{p_1}{\rho}}+\overset{\approx 0\,(2)}{\cancel{\alpha_1\frac{\bar{V}_1^2}{2}}}+gz_1\right)-\left(\cancel{\frac{p_2}{\rho}}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T} \qquad (8.29)$$

$$h_{l_T}=h_l+h_{l_m}\ ;\quad h_l=f\frac{L}{D}\frac{\bar{V}^2}{2}\ ;\quad h_{l_m}=K_{ent}\frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$

(2) $\bar{V}_1 = 0$, $\alpha_2 = 1.0$

(3) square edged entrance

Then

$$z_1 - z_2 = d + 3m = f\frac{L}{D}\frac{\bar{V}^2}{2g}+K_{ent}\frac{\bar{V}^2}{2g}+\frac{\bar{V}_2^2}{2g}=\frac{\bar{V}^2}{2g}\left[f\frac{L}{D}+K_{ent}+1\right]$$

$$\therefore\ d=\frac{\bar{V}^2}{2g}\left[f\frac{L}{D}+K_{ent}+1\right]-3m \qquad (1)$$

For square edged entrance (Table 8.2) $K_{ent} = 0.5$

$$Re=\frac{D\bar{V}}{\nu}=0.020\,m\times1.2\frac{m}{s}\times\frac{1}{1.00\times10^{-6}}\frac{s}{m^2}=2.4\times10^4$$ {assume $T = 20°C$ Table A.8}

$e/D = 0.2/20 = 0.01$. From Fig. 8.13, $f = 0.04$

Then from Eq. 1

$$d=\frac{(1.2)^2}{2}\frac{m^2}{s^2}\times\frac{s^2}{9.81\,m}\left[0.04\times\frac{30}{0.02}+0.5+1\right]-3m = 1.51\,m \longleftarrow d$$

For frictionless flow, $h_{l_T}=f\frac{L}{D}\frac{\bar{V}^2}{2}+K_{ent}\frac{\bar{V}^2}{2}=0$ and Eq. 1 gives $$d=\frac{\bar{V}^2}{2g}-3m$$

and $$\bar{V}=\left[2g(d+3m)\right]^{1/2}=\left[2\times9.81\frac{m}{s^2}(1.51+3)m\right]^{1/2}$$

$$\bar{V} = 9.41\,m/s \longleftarrow \bar{V}_{inviscid}$$

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Problem 8.121

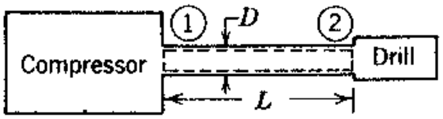
[2]

Given: Air flow through a line, of length L and diameter $D = 40$ mm

$p_1 = 670$ kPa(g) $p_2 = 650$ kPa(g)

$T_1 = 40°\text{C}$ $\dot{m} = 0.25$ kg/s

$\rho \approx$ constant



Find: Allowable length of hose

Solution:

Computing equation: $$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}=h_l+h_{l_m} \quad (8.29)$$

where $h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$ $\quad h_{l_m} = K\frac{\bar{V}^2}{2}$

For $\rho = c$, then $\bar{V}_1 = \bar{V}_2$, since $A_1 = A_2$. Since p_1 and p_2 are given, neglect minor losses. Assume $\alpha_1 = \alpha_2$ and neglect elevation changes. Then Eq. 8.29 can be written as

$$\frac{p_1 - p_2}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2} \quad \text{or} \quad L = \frac{(p_1-p_2)}{\rho}\frac{2D}{f\bar{V}^2}$$

The density is

$$\rho = \rho_1 = \frac{p_1}{RT_1} = 7.91\times10^5\,\frac{\text{N}}{\text{m}^2}\times\frac{\text{kg}\cdot\text{K}}{287\,\text{N}\cdot\text{m}}\times\frac{1}{313\,\text{K}} = 8.81\ \text{kg/m}^3$$

From continuity

$$\bar{V} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\pi\rho D^2} = \frac{4}{\pi}\times0.25\,\frac{\text{kg}}{\text{sec}}\times\frac{\text{m}^3}{8.81\,\text{kg}}\times\frac{1}{(0.04)^2\,\text{m}^2} = 22.6\ \text{m/sec}$$

For air at 40°C, $\mu = 1.91\times10^{-5}$ kg/m·s (Table A.10), so

$$Re = \frac{\rho\bar{V}D}{\mu} = 8.81\,\frac{\text{kg}}{\text{m}^3}\times22.6\,\frac{\text{m}}{\text{sec}}\times0.04\,\text{m}\times\frac{\text{m}\cdot\text{sec}}{1.91\times10^{-5}\,\text{kg}} = 4.17\times10^5$$

Assume smooth pipe; then from Fig. 8.13, $f = 0.0134$

Substituting gives

$$L = \frac{(p_1-p_2)}{\rho}\frac{2D}{f\bar{V}^2}$$

$$= 20\times10^3\,\frac{\text{N}}{\text{m}^2}\times2\times0.04\,\text{m}\times\frac{\text{m}^3}{8.81\,\text{kg}}\times\frac{1}{0.0134}\times\frac{\text{sec}^2}{(22.6)^2\,\text{m}^2}\times\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{sec}^2}$$

$L = 26.5$ m ← L

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42-389 200 SHEETS 5 SQUARE
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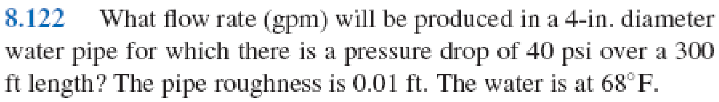
Problem 8.122

[3]



8.122 What flow rate (gpm) will be produced in a 4-in. diameter water pipe for which there is a pressure drop of 40 psi over a 300 ft length? The pipe roughness is 0.01 ft. The water is at 68°F.

Given: Flow in horizontal pipe

Find: Flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_l$ $\qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes

$$\frac{p_1}{\rho}-\frac{p_2}{\rho}=\frac{\Delta p}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Solving for V $\quad V=\sqrt{\frac{2\cdot D\cdot\Delta p}{L\cdot\rho\cdot f}}\qquad V=\frac{k}{\sqrt{f}}\qquad (1)$

$k=\sqrt{\frac{2\cdot D\cdot\Delta p}{L\cdot\rho}}\qquad k=\sqrt{2\times\frac{\frac{1}{3}}{300}\times 40\cdot\frac{\text{lbf}}{\text{in}^2}\times\left(\frac{12\cdot\text{in}}{1\cdot\text{ft}}\right)^2\times\frac{\text{ft}^3}{1.94\cdot\text{slug}}\times\frac{\text{slug ft}}{\text{s}^2\cdot\text{lbf}}}\qquad k=2.57\cdot\frac{\text{ft}}{\text{s}}$

We also have $\quad Re=\frac{V\cdot D}{\nu}$ or $\quad Re=c\cdot V\quad (2)\quad$ where $\quad c=\frac{D}{\nu}$

From Table A.7 (68ºF) $\quad \nu=1.08\times10^{-5}\cdot\frac{\text{ft}^2}{\text{s}}\qquad c=\frac{1}{3}\cdot\text{ft}\times\frac{\text{s}}{1.08\times10^{-5}\cdot\text{ft}^2}\qquad c=3.09\times10^4\cdot\frac{\text{s}}{\text{ft}}$

In addition $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)\qquad (3)$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f=0.1\quad$ then $\quad V=\frac{k}{\sqrt{f}}\qquad V=8.12\cdot\frac{\text{ft}}{\text{s}}\qquad Re=c\cdot V\qquad Re=2.51\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)\qquad f=0.0573\qquad V=\frac{k}{\sqrt{f}}\qquad V=10.7\cdot\frac{\text{ft}}{\text{s}}\qquad Re=c\cdot V\qquad Re=3.31\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)\qquad f=0.0573\qquad V=\frac{k}{\sqrt{f}}\qquad V=10.7\cdot\frac{\text{ft}}{\text{s}}\qquad Re=c\cdot V\qquad Re=3.31\times10^5$

The flow rate is then $\quad Q=V\cdot\frac{\pi\cdot D^2}{4}\qquad Q=10.7\cdot\frac{\text{ft}}{\text{s}}\times\frac{\pi}{4}\times\left(\frac{1}{3}\cdot\text{ft}\right)^2\times\frac{7.48\cdot\text{gal}}{1\cdot\text{ft}^3}\times\frac{60\cdot\text{s}}{1\cdot\text{min}}\qquad Q=419\cdot\text{gpm}$

Note that we could use *Excel*'s *Solver* for this problem

Problem 8.123

[3]

8.123 When you drink you beverage with a straw, you need to overcome both gravity and friction in the straw. Estimate the fraction of the total effort you put into quenching your thirst of each factor, making suitable assumptions about the liquid and straw properties, and your drinking rate (for example, how long it would take you to drink a 12 oz drink if you drank it all in one go (quite a feat with a straw). Is the flow laminar or turbulent? (Ignore minor losses.)

Given: Drinking of a beverage

Find: Fraction of effort of drinking of friction and gravity

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{{V_1}^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{{V_2}^2}{2}+g\cdot z_2\right)=h_l \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes, between the bottom of the straw (Point 1) and top (Point 2)

$$g\cdot z_1-\left(\frac{p_2}{\rho}+g\cdot z_2\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

where p_2 is the gage pressure in the mouth

The negative gage pressure the mouth must create is therefore due to two parts

$$p_{grav}=-\rho\cdot g\cdot\left(z_2-z_1\right) \qquad p_{fric}=-\rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assuming a person can drink 12 fluid ounces in 5 s
$$Q=\frac{\frac{12}{128}\cdot gal}{5\cdot s}\times\frac{1\cdot ft^3}{7.48\cdot gal} \qquad Q=2.51\times10^{-3}\frac{ft^3}{s}$$

Assuming a straw is 6 in long diameter 0.2 in, with roughness $e=5\times10^{-5}in$ (from Googling!)

$$V=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=\frac{4}{\pi}\times2.51\times10^{-3}\frac{ft^3}{s}\times\left(\frac{1}{0.2\cdot in}\times\frac{12\cdot in}{1\cdot ft}\right)^2 \qquad V=11.5\frac{ft}{s}$$

From Table A.7 (68ºF) $\nu=1.08\times10^{-5}\cdot\frac{ft^2}{s}$ (for water, but close enough)

$$Re=\frac{V\cdot D}{\nu} \qquad Re=11.5\cdot\frac{ft}{s}\times\frac{0.2}{12}\cdot ft\times\frac{s}{1.08\times10^{-5}ft^2} \qquad Re=1.775\times10^4$$

Given
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0272$$

Then
$$p_{grav}=-1.94\cdot\frac{slug}{ft^3}\times32.2\cdot\frac{ft}{s^2}\times\frac{1}{2}\cdot ft\times\frac{lbf\cdot s^2}{slug\,ft} \qquad p_{grav}=-31.2\frac{lbf}{ft^2} \qquad p_{grav}=-0.217\,psi$$

and
$$p_{fric}=-1.94\cdot\frac{slug}{ft^3}\times0.0272\times\frac{6}{0.2}\times\frac{1}{2}\times\left(11.5\cdot\frac{ft}{s}\right)^2\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad p_{fric}=-105\frac{lbf}{ft^2} \qquad p_{fric}=-0.727\,psi$$

Hence the fraction due to friction is $\frac{p_{fric}}{p_{fric}+p_{grav}}=77\%$ and gravity is $\frac{p_{grav}}{p_{fric}+p_{grav}}=23\%$

These results will vary depending on assumptions, but it seems friction is significant!

Problem 8.124

[2]

Given: Gasoline flow in a horizontal pipeline at 15°C. The distance and pressure drop between pumping stations are 13 km and 1.4 MPa, respectively. The pipe is 0.6 m in diameter. Its roughness corresponds to galvanized iron.

Find: Volume flow rate.

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section.

Basic equation: $$\frac{p_1}{\rho} + \overset{=0(1)}{\cancel{\frac{\bar{V}_1^2}{2}}} + \overset{=0(2)}{\cancel{g z_1}} = \frac{p_2}{\rho} + \overset{=0(1)}{\cancel{\frac{\bar{V}_2^2}{2}}} + \overset{=0(2)}{\cancel{g z_2}} + h_{l_T}\ ; \quad h_{l_T} = f\frac{L}{D}\frac{\bar{V}^2}{2} + \overset{=0(1)}{\cancel{h_{l_m}}}$$

Assumptions: (1) Constant area pipe, so $\bar{V}_1 = \bar{V}_2$, $h_{l_m} = 0$

(2) Level, so $z_1 = z_2$

Thus

$$\frac{p_1 - p_2}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad \text{or} \qquad \bar{V} = \left[\frac{2D(p_1 - p_2)}{\rho f L}\right]^{\frac{1}{2}}$$

But $f = f(Re, e/D)$, and the Reynolds number is not known. Therefore iteration is required. Choose f in the fully-rough zone. From Table 8.1, $e = 0.15\,mm$; $e/D = 0.00025$. Then from Fig. 8.13, $f \simeq 0.014$. {From Eq. 8.37, using Excel's solver, $f = 0.014$}. Then,

$$\bar{V} = \left[2 \times 0.6\,m \times 1.4 \times 10^6 \frac{N}{m^2} \times \frac{m^3}{(0.72)1000\,kg} \times \frac{1}{0.014} \times \frac{1}{13 \times 10^3\,m} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}}$$

{SG = 0.72, Table A.2}

$$\bar{V} = 3.58\ m/s$$

Now compute Re and check on guess for f. Choose $\mu \simeq 5 \times 10^{-4}\,N \cdot s/m^2$ (Fig. A.2).*

$$Re = \frac{\rho \bar{V} D}{\mu} = (0.72)1000\frac{kg}{m^3} \times 3.58\frac{m}{s} \times 0.6\,m \times \frac{m^2}{5 \times 10^{-4}\,N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} = 3.09 \times 10^6$$

Checking on Fig. 8.13, flow is essentially in the fully-rough zone, and initial guess for f was okay. Thus

$$Q = \bar{V}A = 3.58\frac{m}{s} \times \frac{\pi}{4}(0.6)^2 m^2 = 1.01\ m^3/s$$

Q

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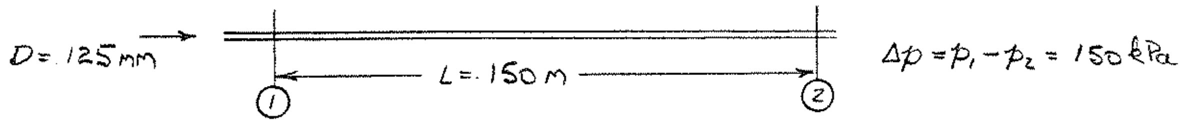
* Note gasoline is between heptane and octane.

Problem 8.125

[2]

Given: Steady flow of water in 5 in. diameter, horizontal, cast-iron pipe.



Find: Volume flow rate.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation:
$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)+h_{l_T}$$
$$h_{l_T}=h_l+h_{lm}=f\frac{L}{D}\frac{\bar{V}^2}{2}+K\frac{\bar{V}^2}{2}$$

Assumptions: (1) Fully developed flow: $\alpha_1\bar{V}_1^2=\alpha_2\bar{V}_2^2$
(2) Horizontal: $z_1=z_2$
(3) Constant area, so $K=0$

Then
$$\frac{\Delta p}{\rho}=h_{l_T}=f\frac{L}{D}\frac{\bar{V}^2}{2} \quad \text{so} \quad \bar{V}=\sqrt{\frac{2\Delta p D}{\rho f L}}$$

Since flow rate (hence Re and f) are unknown, must iterate. Guess a trial value of f in the fully rough zone. From Table 8.1, e = 0.26 mm

Then $e/D = \frac{0.26}{125} = 0.0021$. Then from Eq. 8.37*, $f = 0.0237$ for $Re \geq 6\times10^5$

$$\bar{V}=\left[2\times150\times10^3\,\frac{N}{m^2}\times0.125\,m\times\frac{m^3}{999\,kg}\times\frac{1}{0.0237}\times\frac{1}{150\,m}\times\frac{kg\cdot m}{N\cdot s^2}\right]^{1/2}=3.25\,m/s$$

and, checking Re, with $\nu = 1.14\times10^{-6}\,m^2/s$ at $T = 15°C$ (Table A.8),

$$Re=\frac{\bar{V}D}{\nu}=3.25\,\frac{m}{s}\times0.125\,m\times\frac{s}{1.14\times10^{-6}\,m^2}=3.56\times10^5$$

The friction factor at this Re is still $f = 0.0242$ (2% error), so convergence is ok.

$$Q=\bar{V}A=3.25\,\frac{m}{s}\times\frac{\pi}{4}\times(0.125\,m)^2=0.0399\,m^3/s$$ Q

Using $f = 0.0242$, $\bar{V} = 3.22$ m/s and $Q = 0.0395\,m^3/s$

* Value of f = 0.237 obtained using Excel's Solver (or Goal Seek)

Problem 8.126

Given: Steady flow of water through a cast iron pipe of diameter $D = 125$ mm. The pressure drop over a length of pipe, $L = 150$ m is $p_1 - p_2 = 150$ kPa. Section 2 is located 15 m above section 1.

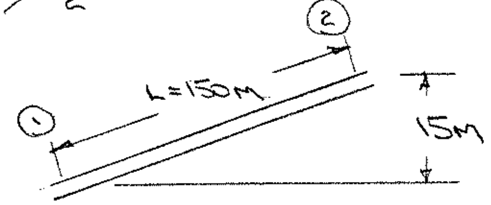
Find: the volume flow rate, Q.

Solution: Apply the energy equation for steady, incompressible pipe flow

Computing equation:

$$\left(\frac{p_1}{\rho} + \cancel{\alpha_1\frac{\bar V_1^2}{2}} + gz_1\right) - \left(\frac{p_2}{\rho} + \cancel{\alpha_2\frac{\bar V_2^2}{2}} + gz_2\right) = h_{l_T} \qquad (1)$$

$$h_{l_T} = h_l + h_{l_m} = f\frac{L}{D}\frac{\bar V^2}{2} + \cancel{K\frac{\bar V^2}{2}} \rightarrow 0\,(4) \qquad (2)$$



Assumptions:
(1) $\bar V_1 = \bar V_2$ from continuity
(2) $\alpha_1 = \alpha_2$
(3) $z_2 - z_1 = 15$ m
(4) neglect minor losses

For cast iron pipe with $D = 125$ mm $\frac{e}{D} = 0.0021$ ($e = 0.26$ mm, Table 8.1)

Since $f = f(Re)$ and $\bar V$ is unknown, iteration will be required

From Eqs (1) and (2)

$$\left(\frac{p_1}{\rho} + gz_1\right) - \left(\frac{p_2}{\rho} + gz_2\right) = f\frac{L}{D}\frac{\bar V^2}{2}$$

Then

$$f\bar V^2 = \frac{2D}{L}\left[\frac{(p_1 - p_2)}{\rho} + g(z_1 - z_2)\right]$$

$$f\bar V^2 = 2\times\frac{0.125\,m}{150\,m}\left[150\times10^3\,\frac{N}{m^2}\times\frac{m^3}{999\,kg}\times\frac{kg\cdot m}{N\cdot s^2}\right] + 9.81\,\frac{m}{s^2}\times(-15\,m)$$

$$f\bar V^2 = 0.005\ m^2/s^2$$

Assume flow in fully rough region, $f = 0.0237$, then $\bar V = 0.46$ m/s

Check Re. Assume $T = 15°C$, $\nu = 1.14\times10^{-6}$ m²/s (Table A.8)

$$\text{Then } Re = \frac{D\bar V}{\nu} = 0.125\,m\times0.46\,\frac{m}{s}\times 1.14\times10^{-6}\,\frac{s}{m^2} = 50{,}400$$

From Eq. 8.37 with $Re = 50{,}400$, $e/D = 0.0021$, then using Excel's Solver (or Goal Seek)

$$f = 0.0267 \text{ and } \bar V = 0.433 \text{ m/s}$$

With this value of $\bar V$, $Re = 47{,}500$, $f = 0.0268$, $\bar V = 0.432$ m/s

Then

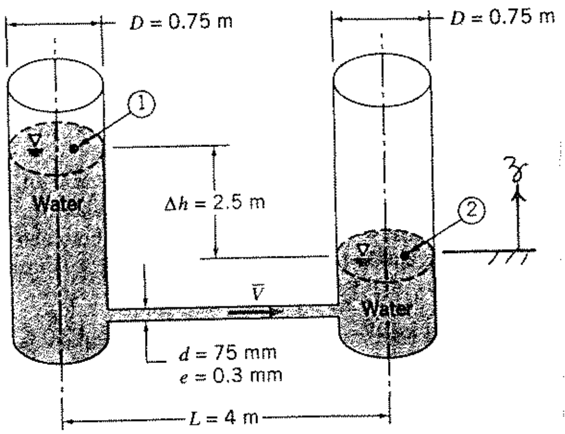
$$Q = A\bar V = \frac{\pi D^2}{4}\bar V = \frac{\pi}{4}(0.125\,m)^2\times0.432\,\frac{m}{s} = 0.0053\ m^3/s \leftarrow Q$$

Problem 8.127

[2]

Given: Two open standpipes shown.
Water flows by gravity.

Find: Estimate of rate of change of water level in left standpipe.



Solution: Apply the energy equation for quasi-steady, incompressible pipe flow.

Computing equation:

$$\left(\cancel{\frac{p_1}{\rho}}+\alpha_1\cancel{\frac{\bar{V}_1^2}{2}}+gz_1\right)-\left(\cancel{\frac{p_2}{\rho}}+\alpha_2\cancel{\frac{\bar{V}_2^2}{2}}+g\cancel{z_2}^{=0}\right)=h_{lT};\; h_{lT}=h_l+h_{lm}=\left[f\frac{(L-D)}{d}+K_{ent}+K_{exit}\right]\frac{\bar{V}^2}{2}$$

From continuity, $A_1V_1 = A_p\bar{V}$

Assumptions: (1) Neglect unsteady effects
(2) Incompressible flow
(3) $p_1 = p_2 = p_{atm}$
(4) $\bar{V}_1 = \bar{V}_2$ since diameters are equal

Then $g\Delta h = h_{lT} = \left[f\frac{(L-D)}{d}+K_{ent}+K_{exit}\right]\frac{\bar{V}^2}{2}$

Flow rate (hence speed) is unknown, so assume flow is in fully rough zone.

$\frac{e}{D} = \frac{0.3}{75} = 0.004$, so $f \approx 0.0285$ from Eq. 8.37 (using Excel's Solver or Goal Seek)

From Table 8.2, $K_{ent} = 0.5$; from Fig. 8.15, $K_{exit} = 1$. Then

$$\bar{V} = \left[\frac{2g\Delta h}{f\frac{(L-D)}{d}+K_{ent}+K_{exit}}\right]^{\frac{1}{2}} = \left[\frac{2\times 9.81\frac{m}{s^2}\times 2.5\,m}{0.028\left(\frac{4-0.75}{0.075}\right)+0.5+1.0}\right]^{\frac{1}{2}} = 4.23\ m/s$$

Check Re and f. For water at 20°C, $\nu = 1.00\times10^{-6}\,m^2/s$ (Table A.8)

$$Re = \frac{\bar{V}d}{\nu} = 4.23\frac{m}{s}\times 0.075\,m\times\frac{s}{1.00\times10^{-6}\,m^2} = 3.18\times10^5$$

From Equation 8.37, $f \approx 0.0288$, so this is satisfactory agreement. (~1%)

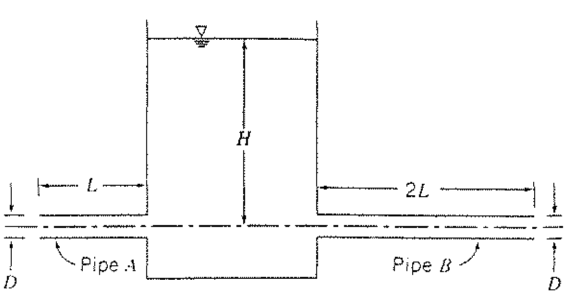
$$V_1 = \frac{A_p}{A_1}V_p = \left(\frac{d}{D}\right)^2 V_p = \left(\frac{0.075}{0.75}\right)^2\times 4.23\frac{m}{s} = 0.0423\ m/s\ (down)$$

The water level in the left tank falls at about 42.3 mm/s

Problem 8.128

[2]

Given: Two galvanized iron pipes connected to large water reservoir as shown.



Determine: (a) which pipe will pass the larger flow rate (without calculations); justify

(b) the larger flow rate if $H = 10\,m$, $D = 50\,mm$, $L = 50\,m$.

Solution:

Flow through each pipe is governed by the energy equation for steady incompressible flow

Basic equations: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T}$ (8.29)

$$h_{l_T} = h_l + h_{l_m} = f \frac{L}{D}\frac{\bar{V}^2}{2} + K_{ent}\frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_3 = p_{atm}$

(2) $\bar{V}_1 = 0$, $\alpha_2 = \alpha_3 = 1.0$

Then

$$g(z_1 - z_2) = h_{l_T} + \frac{\bar{V}_2^2}{2} = \frac{\bar{V}_2^2}{2}\left[f\frac{L}{D} + K_{ent} + 1\right] \quad \text{(pipe A)}$$

$$g(z_1 - z_3) = h_{l_T} + \frac{\bar{V}_3^2}{2} = \frac{\bar{V}_3^2}{2}\left[f\frac{2L}{D} + K_{ent} + 1\right] \quad \text{(pipe B)}$$

Since $z_1 - z_2 = z_1 - z_3$, then $\bar{V}_2 > \bar{V}_3$ and $Q_A > Q_B$ ⟵

Through pipe A $\quad gH = \frac{\bar{V}_2^2}{2}\left[f\frac{L}{D} + K_{ent} + 1\right]$

From Table 8.1 $e = 0.15\,m$ $\therefore$ $e/D = 0.15/50 = 0.003$

Assume water at 20°C, $\nu = 1.00 \times 10^{-6}\,m^2/s$ (Table A.8)

Choose friction factor $f = 0.0263^*$ (in fully rough region)

Then

$$\bar{V}_2 = \left\{\frac{2gH}{\left[f\frac{L}{D} + K_{ent} + 1\right]}\right\}^{1/2} = \left\{2 \times 9.81\frac{m}{s^2} \times 10\,m \times \frac{1}{\left[0.0263 \times \frac{50}{0.05} + 0.5 + 1.0\right]}\right\}$$

$$\bar{V}_2 = 2.66\,m/s$$

Check $Re = \frac{D\bar{V}}{\nu} = 0.05\,m \times 2.66\,\frac{m}{s} \times \frac{s}{1.00 \times 10^{-6}\,m^2} = 1.33 \times 10^5$

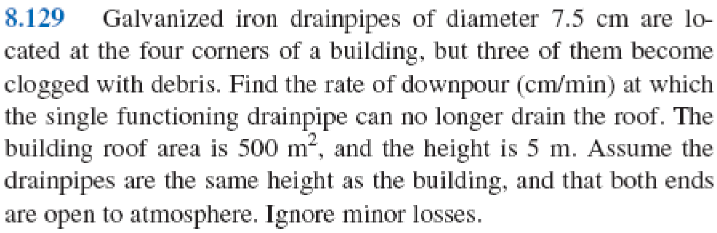
At this Re, $f = 0.0272^*$ and $\bar{V}_2 = 2.62\,m/s$

$$Q = A\bar{V} = \frac{\pi D^2}{4}\bar{V} = \frac{\pi}{4} \times (0.05\,m)^2 \times 2.62\,\frac{m}{s} = 5.14 \times 10^{-3}\,m^3/s \longleftarrow Q_A$$

* Value obtained from Eq. 8.37, using Excel Solver (or Goal Seek)

Problem 8.129

[3]



8.129 Galvanized iron drainpipes of diameter 7.5 cm are located at the four corners of a building, but three of them become clogged with debris. Find the rate of downpour (cm/min) at which the single functioning drainpipe can no longer drain the roof. The building roof area is 500 m^2, and the height is 5 m. Assume the drainpipes are the same height as the building, and that both ends are open to atmosphere. Ignore minor losses.

Given:

Galvanized drainpipe

Find:

Maximum downpour it can handle

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_l$ $\qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes $g\cdot z_1 - g\cdot z_2 = g\cdot\left(z_1 - z_2\right) = g\cdot h = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$ $\qquad h = L$

Solving for V $\quad V = \sqrt{\frac{2\cdot D\cdot g\cdot h}{L\cdot f}} = \sqrt{\frac{2\cdot D\cdot g}{f}}$ $\qquad V = \frac{k}{\sqrt{f}}$ $\qquad$ (1)

$k = \sqrt{2\cdot D\cdot g}$ $\qquad k = \sqrt{2\times 0.075\cdot m\times 9.81\cdot\frac{m}{s^2}}$ $\qquad k = 1.21\frac{m}{s}$

We also have $\quad Re = \frac{V\cdot D}{\nu}$ $\quad$ or $\quad Re = c\cdot V$ $\quad$ (2) $\qquad$ where $\quad c = \frac{D}{\nu}$

From Table A.7 (20ºC) $\quad \nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\qquad c = 0.075\cdot m\times\frac{s}{1.01\times 10^{-6}\cdot m^2}$ $\qquad c = 7.43\times 10^4\cdot\frac{s}{m}$

In addition $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad$ (3) $\qquad e = 0.15\,mm$ $\quad$ (Table 8.1)

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f = 0.01$ $\quad$ then $\quad V = \frac{k}{\sqrt{f}}$ $\quad V = 12.13\frac{m}{s}$ $\quad Re = c\cdot V$ $\quad Re = 9.01\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0236$ $\quad V = \frac{k}{\sqrt{f}}$ $\quad V = 7.90\frac{m}{s}$ $\quad Re = c\cdot V$ $\quad Re = 5.86\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0237$ $\quad V = \frac{k}{\sqrt{f}}$ $\quad V = 7.88\frac{m}{s}$ $\quad Re = c\cdot V$ $\quad Re = 5.85\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0237$ $\quad V = \frac{k}{\sqrt{f}}$ $\quad V = 7.88\frac{m}{s}$ $\quad Re = c\cdot V$ $\quad Re = 5.85\times 10^5$

The flow rate is then $Q = V \cdot \frac{\pi \cdot D^2}{4}$ $Q = 7.88 \cdot \frac{m}{s} \times \frac{\pi}{4} \times (0.075 \cdot m)^2$ $Q = 0.0348 \cdot \frac{m^3}{s}$

The downpour rate is then $\frac{Q}{A_{roof}} = \frac{0.0348 \cdot \frac{m^3}{s}}{500 \cdot m^2} \times \frac{100 \cdot cm}{1 \cdot m} \times \frac{60 \cdot s}{1 \cdot min} = 0.418 \cdot \frac{cm}{min}$ The drain can handle 0.418 cm/min

Note that we could use *Excel*'s *Solver* for this problem

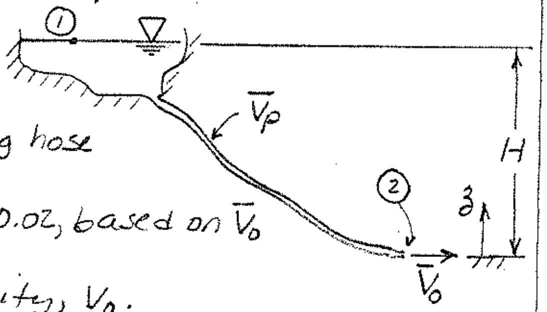
Problem 8.130

Given: Site for hydraulic mining, $H = 300\,m$, $L = 900\,m$.

Hose with $D = 75\,mm$, $e/D = 0.01$.

Couplings, $\frac{L_e}{D} = 20$, every 10 m along hose

Nozzle diameter, $d = 25\,mm$; $K = 0.02$, based on $\bar{V}_0$



Find: (a) Estimate maximum outlet velocity, V_0.
(b) Determine maximum force of jet on rock face.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\left(\overset{=0(1)}{\cancel{\frac{p_1}{\rho}}} + \alpha_1 \overset{\approx 0(2)}{\cancel{\frac{\bar{V}_1^2}{2}}} + g z_1\right) - \left(\overset{=0(3)}{\cancel{\frac{p_2}{\rho}}} + \alpha_2 \frac{\bar{V}_2^2}{2} + g \cancel{z_2}\right) = h_{\ell T}$$

Assume: (1) $p_1 = 0$; (2) $\bar{V}_1 = 0$; (3) $p_2 = 0$; (4) $\alpha_2 = 1$; (5) $z_2 = 0$; (6) Fully-rough zone

Then $$gH = h_{\ell T} + \frac{\bar{V}_2^2}{2} = f\frac{L}{D}\frac{\bar{V}_p^2}{2} + f \times 90 \frac{L_e}{D}\frac{\bar{V}_p^2}{2} + K\frac{\bar{V}_0^2}{2} + \frac{\bar{V}_2^2}{2}$$

From continuity $\bar{V}_p A_p = \bar{V}_0 A_0$; $\bar{V}_2 = \bar{V}_0 \frac{A_0}{A_2}$; $\bar{V}_2^2 = \bar{V}_0^2 \left(\frac{A_0}{A_2}\right)^2 = \bar{V}_0^2 \left(\frac{d}{D}\right)^4$

Substituting, $$gH = \left[f\left(\frac{L}{D} + 90\frac{L_e}{D}\right)\left(\frac{d}{D}\right)^4 + 1 + K\right]\frac{\bar{V}_0^2}{2}$$

$$\bar{V}_0 = \left[\frac{2gH}{f\left(\frac{L}{D} + 90\frac{L_e}{D}\right)\left(\frac{d}{D}\right)^4 + 1 + K}\right]^{1/2}$$; in fully-rough zone ($\frac{e}{D} = 0.01$), $f = 0.038^*$ (Eq. 8.37)

$$\bar{V}_0 = \left[2 \times 9.81\frac{m}{s^2} \times 300\,m \times \frac{1}{0.038\left(\frac{900\,m}{0.075\,m} + 90\,(20)\right)\left(\frac{0.025}{0.075}\right)^4 + 1 + 0.02}\right]^{1/2} = 28.0\,m/s\ (est.)$$

Check for fully-rough flow zone:

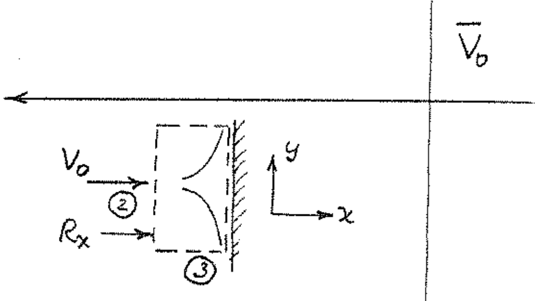
$Re = \frac{\bar{V}_p D}{\nu}$; $\bar{V}_p = \bar{V}_0\left(\frac{d}{D}\right)^4 = 28.0\frac{m}{s}\left(\frac{1}{3}\right)^4 = 0.346\,m/s$ {Assume $T = 20°C$}

$Re = 0.346\frac{m}{sec} \times 0.075\,m \times \frac{1}{1 \times 10^{-6}\,m^2} = 2.60 \times 10^4$; at $\frac{e}{D} = 0.01$, $f = 0.040$ (Eq. 8.37)

The new estimate is

$$\bar{V}_0 = \sqrt{\frac{0.038}{0.040}}\,\bar{V}_0(est) = \sqrt{\frac{0.038}{0.040}}\,28.0\frac{m}{s} = 27.3\,m/s$$

Apply momentum to find force: CV is shown.



$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) No pressure forces
(2) $F_{B_x} = 0$
(3) Steady flow

(4) Uniform flow at each cross-section

Then

$$R_x = u_2\{-\rho \bar{V}_0 A_0\} + u_3\{+\rho \bar{V}_0 A_0\}$$

$$u_2 = \bar{V}_0 \qquad u_3 = 0$$

$$R_x = -\rho \bar{V}_0^2 A_0$$

The force on the rock face is

$$K_x = -R_x = \rho \bar{V}_0^2 A_0$$

$$= 999 \frac{kg}{m^3} \times (27.3)^2 \frac{m^2}{s^2} \times \frac{\pi}{4}(0.025)^2 m^2 \times \frac{N \cdot s^2}{kg \cdot m}$$

$$K_x = 365\ N \text{ (to right)}$$

K_x

* Values of f obtained from Eq. 8.37 using Excel's Solver (or Goal Seek)

Problem 8.131

[3]

8.131 Investigate the effect of tube roughness on flow rate by computing the flow generated by a pressure difference $\Delta p = 100$ kPa applied to a length $L = 100$ m of tubing, with diameter $D = 25$ mm. Plot the flow rate against tube relative roughness e/D for e/D ranging from 0 to 0.05 (this could be replicated experimentally by progressively roughening the tube surface). Is it possible that this tubing could be roughened so much that the flow could be slowed to a laminar flow rate?

Given: Flow in a tube

Find: Effect of tube roughness on flow rate; Plot

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}=\sum_{\text{major}} h_l+\sum_{\text{minor}} h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for flow in a tube

$$p_1-p_2=\Delta p=\rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

This cannot be solved explicitly for velocity V, (and hence flow rate Q) because f depends on V; solution for a given relative roughness e/D requires iteration (or use of *Solver*)

Fluid is not specified: use water

Given data:

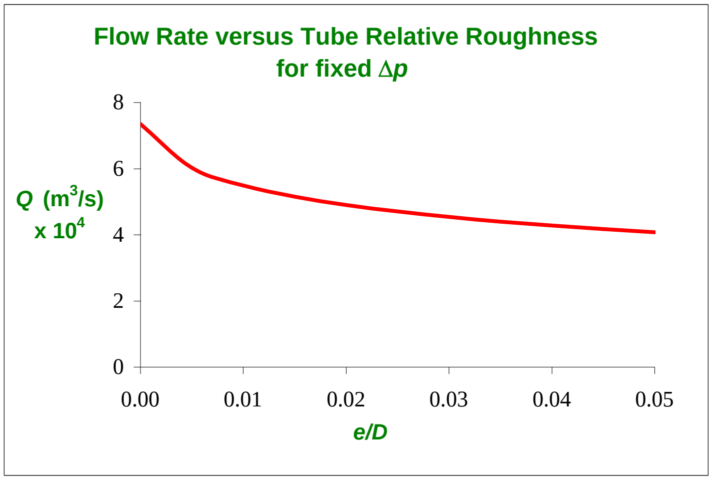
Δp = 100 kPa
D = 25 mm
L = 100 m

Tabulated or graphical data:

μ = 1.00E-03 N.s/m^2
ρ = 999 kg/m^3
(Water - Appendix A)

Computed results:

e/D	*V* (m/s)	*Q* (m^3/s) x 10^4	*Re*	Regime	*f*	Δp (kPa)	Error
0.000	1.50	7.35	37408	Turbulent	0.0223	100	0.0%
0.005	1.23	6.03	30670	Turbulent	0.0332	100	0.0%
0.010	1.12	5.49	27953	Turbulent	0.0400	100	0.0%
0.015	1.05	5.15	26221	Turbulent	0.0454	100	0.0%
0.020	0.999	4.90	24947	Turbulent	0.0502	100	0.0%
0.025	0.959	4.71	23939	Turbulent	0.0545	100	0.0%
0.030	0.925	4.54	23105	Turbulent	0.0585	100	0.0%
0.035	0.897	4.40	22396	Turbulent	0.0623	100	0.0%
0.040	0.872	4.28	21774	Turbulent	0.0659	100	0.0%
0.045	0.850	4.17	21224	Turbulent	0.0693	100	0.0%
0.050	0.830	4.07	20730	Turbulent	0.0727	100	0.0%



It is not possible to roughen the tube sufficiently to slow the flow down to a laminar flow for this Δp. Even a relative roughness of 0.5 (a physical impossibility!) would not work.

Problem 8.132 [3]

8.132 Investigate the effect of tube length on water flow rate by computing the flow generated by a pressure difference $\Delta p = 100$ kPa applied to a length L of smooth tubing, of diameter $D = 25$ mm. Plot the flow rate against tube length for flow ranging from low speed laminar to fully turbulent.

Given: Flow in a tube

Find: Effect of tube length on flow rate; Plot

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) = h_{lT} = \sum_{\text{major}} h_l + \sum_{\text{minor}} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K \cdot \frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for flow in a tube

$$p_1 - p_2 = \Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

This cannot be solved explicitly for velocity V, (and hence flow rate Q) because f depends on V; solution for a given L requires iteration (or use of *Solver*)

Fluid is not specified: use water

Given data:

$\Delta p =$ 100 m

$D =$ 25 mm

Tabulated or graphical data:

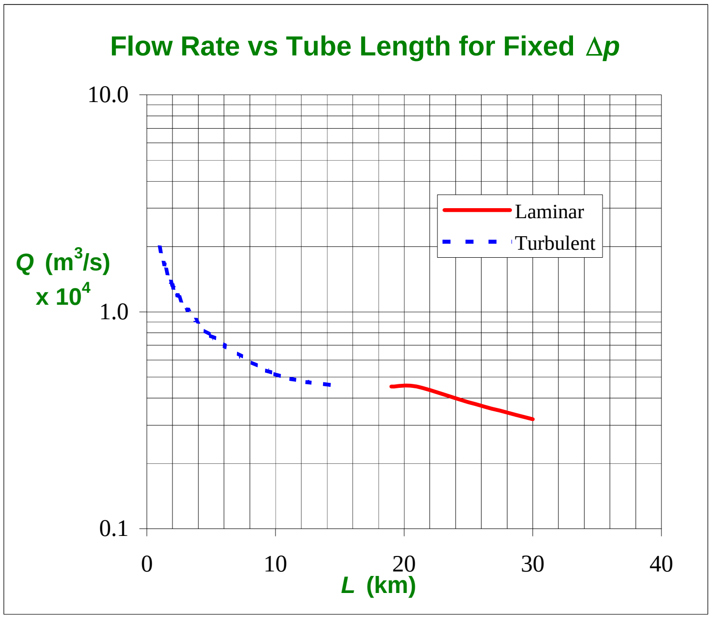
$\mu =$ 1.00E-03 $\mathrm{N.s/m^2}$

$\rho =$ 999 $\mathrm{kg/m^3}$

(Water - Appendix A)

Computed results:

L (km)	V (m/s)	Q $(\mathrm{m^3/s}) \times 10^4$	*Re*	Regime	*f*	Δp (kPa)	Error
1.0	0.40	1.98	10063	Turbulent	0.0308	100	0.0%
1.5	0.319	1.56	7962	Turbulent	0.0328	100	0.0%
2.0	0.270	1.32	6739	Turbulent	0.0344	100	0.0%
2.5	0.237	1.16	5919	Turbulent	0.0356	100	0.0%
5.0	0.158	0.776	3948	Turbulent	0.0401	100	0.0%
10	0.105	0.516	2623	Turbulent	0.0454	100	0.0%
15	0.092	0.452	2300	Turbulent	0.0473	120	20.2%
19	0.092	0.452	2300	Laminar	0.0278	90	10.4%
21	0.092	0.452	2300	Laminar	0.0278	99	1.0%
25	0.078	0.383	1951	Laminar	0.0328	100	0.0%
30	0.065	0.320	1626	Laminar	0.0394	100	0.0%



The "critical" length of tube is between 15 and 20 km.
For this range, the fluid is making a transition between laminar and turbulent flow, and is quite unstable. In this range the flow oscillates between laminar and turbulent; no consistent solution is found (i.e., an *Re* corresponding to turbulent flow needs an *f* assuming laminar to produce the Δp required, and vice versa!)

More realistic numbers (e.g., tube length) are obtained for a fluid such as SAE 10W oil
(The graph will remain the same except for scale)

Problem 8.133

[5]

8.133 For the pipe flow into a reservoir of Example 8.5 consider the effect of pipe roughness on flow rate, assuming the pressure of the pump is maintained at 153 kPa. Plot the flow rate against pipe roughness ranging from smooth ($e = 0$) to very rough ($e = 3.75$ mm). Also consider the effect of pipe length (again assuming the pump always produces 153 kPa) for smooth pipe. Plot the flow rate against pipe length for $L = 100$ m through $L = 1000$ m.

Given: Flow from a reservoir

Find: Effect of pipe roughness and pipe length on flow rate; Plot

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}=\sum_{\text{major}} h_l+\sum_{\text{minor}} h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for this flow (see Example 8.5)

$$p_{pump}=\Delta p=\rho\cdot\left(g\cdot d+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\right)$$

We need to solve this for velocity *V*, (and hence flow rate *Q*) as a function of roughness *e*, then length *L*. This cannot be solved explicitly for velocity *V*, (and hence flow rate *Q*) because *f* depends on *V*; solution for a given relative roughness *e/D* or length *L* requires iteration (or use of *Solver*)

Given data:

Δp = 153 kPa
D = 75 mm
L = 100 m

Tabulated or graphical data:

μ = 1.00E-03 N.s/m^2
ρ = 999 kg/m^3
(Water - Appendix A)

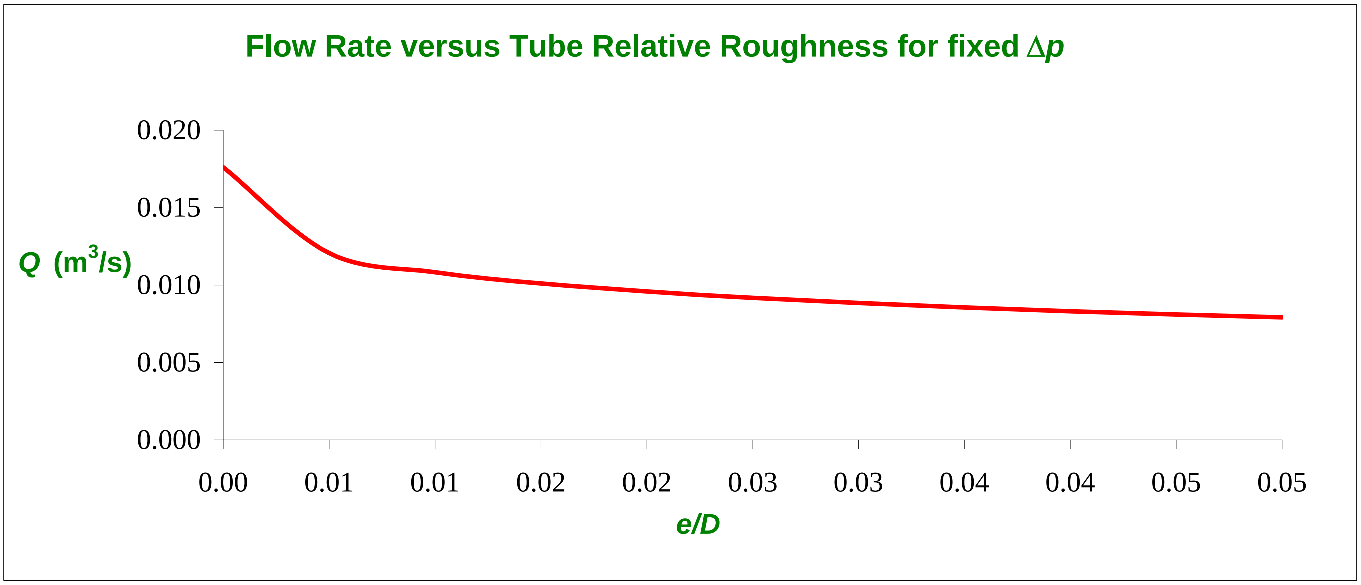
Computed results:

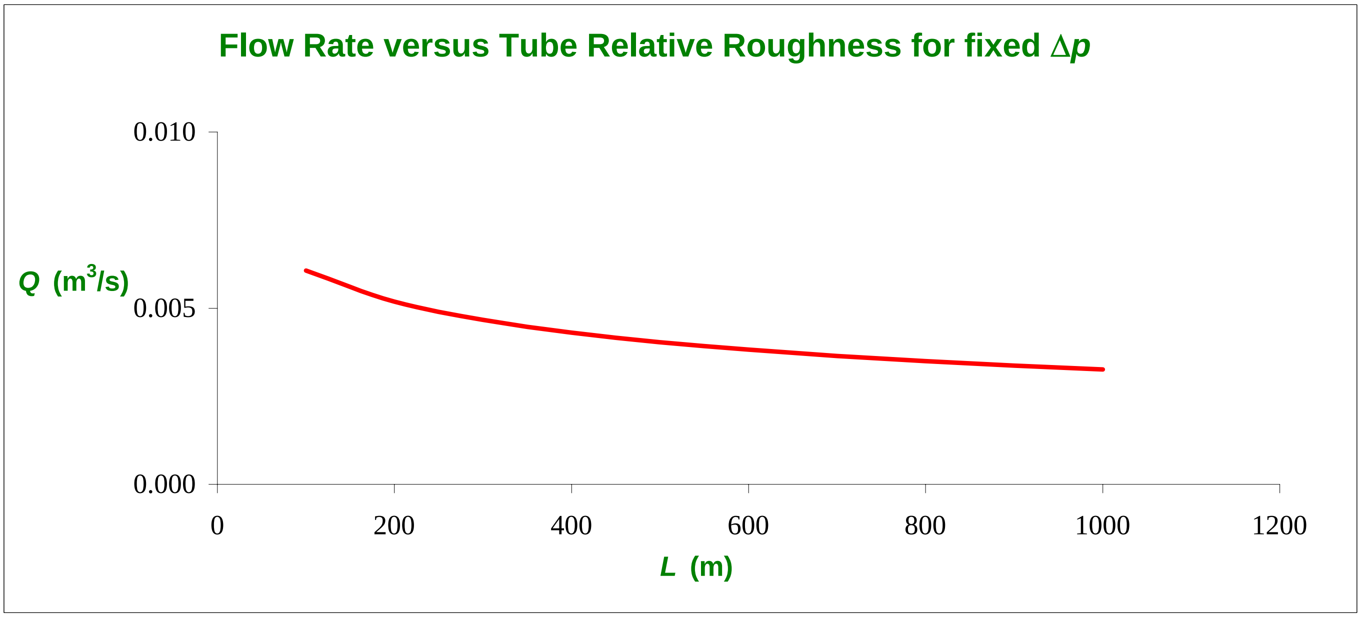
e/D	*V* (m/s)	*Q* (m^3/s)	*Re*	Regime	*f*	Δp (kPa)	Error
0.000	3.98	0.0176	2.98E+05	Turbulent	0.0145	153	0.0%
0.005	2.73	0.0121	2.05E+05	Turbulent	0.0308	153	0.0%
0.010	2.45	0.0108	1.84E+05	Turbulent	0.0382	153	0.0%
0.015	2.29	0.0101	1.71E+05	Turbulent	0.0440	153	0.0%
0.020	2.168	0.00958	1.62E+05	Turbulent	0.0489	153	0.0%
0.025	2.076	0.00917	1.56E+05	Turbulent	0.0533	153	0.0%
0.030	2.001	0.00884	1.50E+05	Turbulent	0.0574	153	0.0%
0.035	1.937	0.00856	1.45E+05	Turbulent	0.0612	153	0.0%
0.040	1.882	0.00832	1.41E+05	Turbulent	0.0649	153	0.0%
0.045	1.833	0.00810	1.37E+05	Turbulent	0.0683	153	0.0%
0.050	1.790	0.00791	1.34E+05	Turbulent	0.0717	153	0.0%

It is not possible to roughen the tube sufficiently to slow the flow down to a laminar flow for this Δp.

Computed results:

L (m)	*V* (m/s)	*Q* (m^3/s)	*Re*	Regime	*f*	Δ*p* (kPa)	Error
100	1.37	0.00606	1.03E+05	Turbulent	0.1219	153	0.0%
200	1.175	0.00519	8.80E+04	Turbulent	0.0833	153	0.0%
300	1.056	0.00467	7.92E+04	Turbulent	0.0686	153	0.0%
400	0.975	0.00431	7.30E+04	Turbulent	0.0604	153	0.0%
500	0.913	0.004036	6.84E+04	Turbulent	0.0551	153	0.0%
600	0.865	0.003821	6.48E+04	Turbulent	0.0512	153	0.0%
700	0.825	0.003645	6.18E+04	Turbulent	0.0482	153	0.0%
800	0.791	0.003496	5.93E+04	Turbulent	0.0459	153	0.0%
900	0.762	0.003368	5.71E+04	Turbulent	0.0439	153	0.0%
1000	0.737	0.003257	5.52E+04	Turbulent	0.0423	153	0.0%





Problem 8.134

[3]

8.134 Water for a fire protection system is supplied from a water tower through a 150 mm cast-iron pipe. A pressure gage at a fire hydrant indicates 600 kPa when no water is flowing. The total pipe length between the elevated tank and the hydrant is 200 m. Determine the height of the water tower above the hydrant. Calculate the maximum volume flow rate that can be achieved when the system is flushed by opening the hydrant wide (assume minor losses are 10 percent of major losses at this condition). When a fire hose is attached to the hydrant, the volume flow rate is 0.75 m^3/min. Determine the reading of the pressure gage at this flow condition.

Given: System for fire protection

Find: Height of water tower; Maximum flow rate; Pressure gage reading

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}=\sum_{\text{major}}h_l+\sum_{\text{minor}}h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=0.1\cdot h_l \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad (\text{Laminar}) \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad (\text{Turbulent})$$

For no flow the energy equation (Eq. 8.29) applied between the water tower free surface (state 1; height *H*) and the pressure gage is

$$g\cdot H=\frac{p_2}{\rho} \qquad \text{or} \qquad H=\frac{p_2}{\rho\cdot g} \qquad (1)$$

The energy equation (Eq. 8.29) becomes, for maximum flow (and α = 1)

$$g\cdot H-\frac{V^2}{2}=h_{lT}=(1+0.1)\cdot h_l \qquad \text{or} \qquad g\cdot H=\frac{V^2}{2}\cdot\left(1+1.1\cdot f\cdot\frac{L}{D}\right) \qquad (2)$$

This can be solved for *V* (and hence *Q*) by iterating, or by using *Solver*

The energy equation (Eq. 8.29) becomes, for restricted flow

$$g\cdot H-\frac{p_2}{\rho}+\frac{V^2}{2}=h_{lT}=(1+0.1)\cdot h_l \qquad p_2=\rho\cdot g\cdot H-\rho\cdot\frac{V^2}{2}\cdot\left(1+1.1\cdot\rho\cdot f\cdot\frac{L}{D}\right) \qquad (3)$$

Given data:

p_2 = 600 kPa

(Closed)

D = 150 mm

L = 200 m

Q = 0.75 m^3/min

(Open)

Tabulated or graphical data:

e = 0.26 mm

(Table 8.1)

μ = 1.00E-03 N.s/m^2

ρ = 999 kg/m^3

(Water - Appendix A)

Computed results:

Closed:

H = 61.2 m

(Eq. 1)

Fully open:

V = 5.91 m/s

Re = 8.85E+05

f = 0.0228

Eq. 2, solved by varying *V* using *Solver* :

Left (m^2/s)	Right (m^2/s)	Error
601	601	0%

Q = 0.104 m^3/s

Partially open:

Q = 0.75 m^3/min

V = 0.71 m/s

Re = 1.06E+05

f = 0.0243

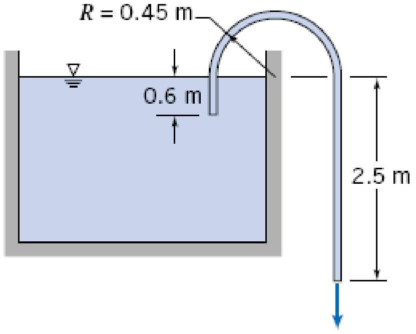
p_2 = 591 kPa

(Eq. 3)

Problem 8.135

[3]

8.135 The siphon shown is fabricated from 50 mm i.d. drawn aluminum tubing. The liquid is water at 15°C. Compute the volume flow rate through the siphon. Estimate the minimum pressure inside the tube.



Given: Syphon system

Find: Flow rate; Minimum pressure

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+h_{lm}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1

Hence the energy equation applied between the tank free surface (Point 1) and the tube exit (Point 2, z = 0) becomes

$$g\cdot z_1-\frac{V_2^2}{2}=g\cdot z_1-\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{ent}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$$

From Table 8.2 for reentrant entrance $K_{ent} = 0.78$

For the bend $\frac{R}{D}=9$ so from Fig. 8.16 $\frac{L_e}{D}=28$ for a 90° bend so for a 180° bend $\frac{L_e}{D}=56$

Solving for V
$$V=\sqrt{\frac{2\cdot g\cdot h}{\left[1+K_{ent}+f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)\right]}} \qquad (1) \qquad \text{and} \qquad h = 2.5\cdot m$$

The two lengths are $L_e = 56\cdot D$ $\quad L_e = 2.8\,m$ $\quad L = (0.6+\pi\cdot 0.45+2.5)\cdot m$ $\quad L = 4.51\,m$

We also have $Re=\frac{V\cdot D}{\nu}$ or $Re = c\cdot V$ (2) where $c=\frac{D}{\nu}$

From Table A.7 (15°C) $\nu = 1.14\times 10^{-6}\cdot\frac{m^2}{s}$ $\quad c = 0.05\cdot m\times\frac{s}{1.14\times 10^{-6}\cdot m^2}$ $\quad c = 4.39\times 10^4\cdot\frac{s}{m}$

In addition
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad (3) \qquad e = 0.0015\,mm \quad \text{(Table 8.1)}$$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f = 0.01$ then

$$V=\sqrt{\frac{2\cdot g\cdot h}{\left[1+K_{ent}+f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)\right]}} \qquad V = 3.89\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.71\times 10^5$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0164$$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left[1 + K_{ent} + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}} \qquad V = 3.43\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.50\times 10^5$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0168$$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left[1 + K_{ent} + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}} \qquad V = 3.40\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.49\times 10^5$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0168$$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left[1 + K_{ent} + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}} \qquad V = 3.40\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.49\times 10^5$$

Note that we could use *Excel*'s *Solver* for this problem

The minimum pressure occurs at the top of the curve (Point 3). Applying the energy equation between Points 1 and 3

$$g\cdot z_1 - \left(\frac{p_3}{\rho} + \frac{{V_3}^2}{2} + g\cdot z_3\right) = g\cdot z_1 - \left(\frac{p_3}{\rho} + \frac{V^2}{2} + g\cdot z_3\right) = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + K_{ent}\cdot\frac{V^2}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$$

where we have $\frac{L_e}{D} = 28$ for the first 90^o of the bend, and $L = \left(0.6 + \frac{\pi\times 0.45}{2}\right)\cdot m$ $\qquad L = 1.31\,m$

$$p_3 = \rho\cdot\left[g\cdot\left(z_1 - z_3\right) - \frac{V^2}{2}\cdot\left[1 + K_{ent} + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]\right]$$

$$p_3 = 1000\cdot\frac{kg}{m^3}\times\left[9.81\cdot\frac{m}{s^2}\times(-0.45\cdot m) - \left(3.4\cdot\frac{m}{s}\right)^2\cdot\left[1 + 0.78 + 0.0168\cdot\left(\frac{1.31}{0.05} + 28\right)\right]\right]\times\frac{N\cdot s^2}{kg\cdot m} \qquad p_3 = -35.5\,kPa$$

Problem 8.136

[4]

8.136 A large open water tank has a horizontal 2.5 cm diameter cast iron drainpipe of length 1.5 m attached at its base, If the depth of water is 3.5 m, find the flow rate (m^3/hr) if the pipe entrance is a) reentrant, b) square-edged and c) rounded (r = 3.75 mm).

Given: Tank with drainpipe

Find: Flow rate for rentrant, square-edged, and rounded entrances

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\ 2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\ 2}}{2}+g\cdot z_2\right)=h_{lT}$ $\qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{ent}\cdot\frac{V^2}{2}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1

Hence the energy equation applied between the tank free surface (Point 1) and the pipe exit (Point 2, z = 0) becomes

$$g\cdot z_1-\frac{V_2^{\ 2}}{2}=g\cdot z_1-\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{ent}\cdot\frac{V^2}{2}$$

Solving for V $\quad V=\sqrt{\frac{2\cdot g\cdot h}{\left(1+K_{ent}+f\cdot\frac{L}{D}\right)}}$ $\quad$ (1) $\quad$ and $\quad h=(1.5+3.5)\cdot m \quad h=5\,m$

We also have $\quad Re=\frac{V\cdot D}{\nu}$ $\quad$ or $\quad Re=c\cdot V$ $\quad$ (2) $\quad$ where $\quad c=\frac{D}{\nu}$

From Table A.7 (20ºC) $\quad \nu=1.01\times10^{-6}\cdot\frac{m^2}{s}$ $\quad c=0.025\cdot m\times\frac{s}{1.01\times10^{-6}\cdot m^2}$ $\quad c=2.48\times10^4\cdot\frac{s}{m}$

In addition $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad$ (3) $\quad e=0.26\cdot mm$ $\quad$ (Table 8.1)

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

For a reentrant entrance, from Table 8.2 $\quad K_{ent}=0.78$

Make a guess for f $\quad f=0.01$ $\quad$ then

$V=\sqrt{\frac{2\cdot g\cdot h}{\left(1+K_{ent}+f\cdot\frac{L}{D}\right)}}$ $\quad V=6.42\frac{m}{s}$ $\quad Re=c\cdot V$ $\quad Re=1.59\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0388$

$V=\sqrt{\frac{2\cdot g\cdot h}{\left(1+K_{ent}+f\cdot\frac{L}{D}\right)}}$ $\quad V=4.89\frac{m}{s}$ $\quad Re=c\cdot V$ $\quad Re=1.21\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0389$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 4.88\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.21\times 10^{5}$$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad f = 0.0389$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 4.88\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.21\times 10^{5}$$

Note that we could use *Excel*'s *Solver* for this problem

The flow rate is then $\quad Q = V\cdot\frac{\pi\cdot D^{2}}{4} \qquad Q = 4.88\cdot\frac{m}{s}\times\frac{\pi}{4}\times(0.025\cdot m)^{2} \qquad Q = 2.4\times 10^{-3}\cdot\frac{m^{3}}{s} \qquad Q = 8.62\cdot\frac{m^{3}}{hr}$

For a square-edged entrance, from Table 8.2 $\quad K_{ent} = 0.5$

Make a guess for f $\qquad f = 0.01 \quad$ then

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 6.83\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.69\times 10^{5}$$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad f = 0.0388$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 5.06\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.25\times 10^{5}$$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad f = 0.0389$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 5.06\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.25\times 10^{5}$$

The flow rate is then $\quad Q = V\cdot\frac{\pi\cdot D^{2}}{4} \qquad Q = 5.06\cdot\frac{m}{s}\times\frac{\pi}{4}\times(0.025\cdot m)^{2} \qquad Q = 2.48\times 10^{-3}\cdot\frac{m^{3}}{s} \qquad Q = 8.94\cdot\frac{m^{3}}{hr}$

For a rounded entrance, from Table 8.2 $\quad \frac{r}{D} = \frac{3.75}{25} = 0.15 \qquad K_{ent} = 0.04$

Make a guess for f $\qquad f = 0.01 \quad$ then

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 7.73\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.91\times 10^{5}$$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $f = 0.0387$

$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}}$ $V = 5.40 \frac{m}{s}$ $Re = c \cdot V$ $Re = 1.34 \times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $f = 0.0389$

$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}}$ $V = 5.39 \frac{m}{s}$ $Re = c \cdot V$ $Re = 1.34 \times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $f = 0.0389$

$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}}$ $V = 5.39 \frac{m}{s}$ $Re = c \cdot V$ $Re = 1.34 \times 10^5$

Note that we could use *Excel*'s *Solver* for this problem

The flow rate is then $Q = V \cdot \frac{\pi \cdot D^2}{4}$ $Q = 5.39 \cdot \frac{m}{s} \times \frac{\pi}{4} \times (0.025 \cdot m)^2$ $Q = 2.65 \times 10^{-3} \cdot \frac{m^3}{s}$ $Q = 9.52 \cdot \frac{m^3}{hr}$

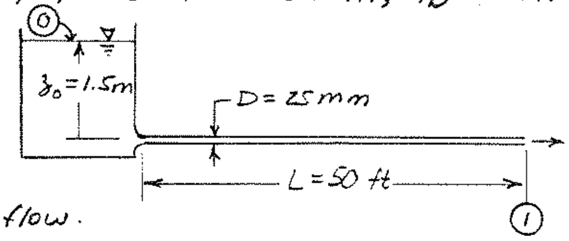
In summary: Renentrant: $Q = 8.62 \cdot \frac{m^3}{hr}$ Square-edged: $Q = 8.94 \cdot \frac{m^3}{hr}$ Rounded: $Q = 9.52 \cdot \frac{m^3}{hr}$

Problem 8.137

[4]

Given: Roman water supply system from Example Problem 8.10, but with 50 foot length of straight pipe with $D = 25\text{ mm}$, $e/D = 0.01$.

Find: (a) Flow rate delivered.
(b) Effect of adding a diffuser.



Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\cancel{\frac{p_0}{\rho}} + \cancel{\alpha_0\frac{\bar{V}_0^2}{2}} + gz_0 = \cancel{\frac{p_1}{\rho}} + \alpha_1\frac{\bar{V}_1^2}{2} + g\cancel{z_1} + h_{lT}\ ;\quad h_{lT} = \left(f\frac{L}{D} + K_{ent}\right)\frac{\bar{V}_1^2}{2}$$

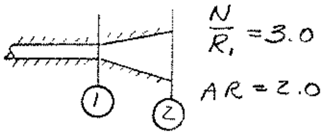
Assumptions: (1) $p_0 = p_1 = p_{atm}$ (3) $\alpha_1 \approx 1$
(2) $\bar{V}_1 \approx 0$ (4) $K_{ent} = 0.04$

Then
$$gz_0 = \frac{\bar{V}_1^2}{2} + \left(f\frac{L}{D} + K_{ent}\right)\frac{\bar{V}_1^2}{2} \quad \text{or} \quad \bar{V}_1 = \sqrt{\frac{2gz_0}{1 + f\frac{L}{D} + K}}$$

For $e/D = 0.01$, $f = 0.038$ from Eq. 8.37*, so
$$\bar{V}_1 = \left[2 \times 9.81\frac{\text{m}}{\text{s}^2} \times 1.5\text{m} \times \frac{1}{1 + 0.038 \times 50\text{ft} \times \frac{1}{25\text{mm}} \times 304.8\frac{\text{mm}}{\text{ft}} + 0.04}\right]^{1/2} = 1.10\text{ m/s}$$

Checking, assuming $T = 20°\text{C}$,
$$Re = \frac{\bar{V}D}{\nu} = 1.10\frac{\text{m}}{\text{sec}} \times 0.025\text{ m} \times \frac{\text{sec}}{1.0 \times 10^{-6}\text{m}^2} = 2.75 \times 10^4\ ;\ \text{from Eq. 8.37*},\ f \approx 0.040,\ \text{so}$$
$$\bar{V}_1 = \left[2 \times 9.81\frac{\text{m}}{\text{s}^2} \times 1.5\text{m} \times \frac{1}{1 + 0.040 \times 50\text{ft} \times \frac{1}{25\text{mm}} \times 304.8\frac{\text{mm}}{\text{ft}} + 0.04}\right]^{1/2} = 1.08\text{ m/s}$$
$$Q = \bar{V}_1 A = 1.08\frac{\text{m}}{\text{s}} \times \frac{\pi}{4}(0.025)^2\text{m}^2 = 5.30 \times 10^{-4}\text{ m}^3/\text{s} \quad (\text{no diffuser}) \qquad Q$$

The diffuser would increase head loss by $K_{diffuser} = 0.3$ (see Example 8.10), but would reduce $\bar{V}_2$ to $\frac{1}{2}\bar{V}_1$. The energy equation would be
$$gz_0 = \frac{\bar{V}_2^2}{2} + \left(f\frac{L}{D} + K_{ent} + K_{diff}\right)\frac{\bar{V}_1^2}{2} = \left(\frac{1}{4} + f\frac{L}{D} + K_{ent} + K_d\right)\frac{\bar{V}_1^2}{2}$$



Thus
$$\bar{V}_1 = \sqrt{\frac{2gz_0}{0.25 + f\frac{L}{D} + K_{ent} + K_{diff}}}$$
$$\bar{V}_1 = \left[2 \times 9.81\frac{\text{m}}{\text{s}^2} \times 1.5\text{m} \times \frac{1}{0.25 + 0.040 \times 50\text{ft} \times \frac{1}{25\text{mm}} \times 304.8\frac{\text{mm}}{\text{ft}} + 0.04 + 0.3}\right]^{1/2} = 1.09\text{ m/s}$$

and
$$Q = \bar{V}_1 A = 1.09\frac{\text{m}}{\text{s}} \times \frac{\pi}{4}(0.025)^2\text{m}^2 = 5.35 \times 10^{-4}\text{ m}^3/\text{s} \quad (\text{with diffuser}) \qquad Q$$

{The diffuser increases flow rate only slightly (~1 percent), because loss is dominated by fL/D.}

* Values of f obtained using Excel's Solver (or Goal Seek)

Problem 8.138

[5]

8.138 A 7500 gallon tank of kerosine is to be emptied by a gravity feed using a drain hose of diameter 1 in., roughness 0.01 in., and length 50 ft. The top of the tank is open to the atmosphere and the hose exits to an open chamber. If the kerosense level is initially 10 ft above the drain exit, estimate (by assuming steady flow) the initial drainage rate. Estimate the flow rate when the kerosene level is down to 5 ft, and down to 1 ft. Based on these three estimates, make a rough estimate of the time it took to drain to the 1 ft level.

Given: Tank with drain hose

Find: Flow rate at different instants; Estimate of drain time

Solution:

Basic equations $$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_l \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore minor loss at entrance (L >>; verify later)

Hence the energy equation applied between the tank free surface (Point 1) and the hose exit (Point 2, z = 0) becomes

$$g\cdot z_1-\frac{V_2^{\,2}}{2}=g\cdot z_1-\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Solving for V $$V=\sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}} \qquad (1) \qquad \text{and} \qquad h = 10\cdot ft \qquad \text{initially}$$

We also have $$Re=\frac{V\cdot D}{\nu} \qquad \text{or} \qquad Re=c\cdot V \quad (2) \qquad \text{where} \qquad c=\frac{D}{\nu}$$

From Fig. A.2 (20ºC) $$\nu = 1.8\times10^{-6}\cdot\frac{m^2}{s}\times\frac{10.8\cdot\frac{ft^2}{s}}{1\cdot\frac{m^2}{s}} \qquad \nu = 1.94\times10^{-5}\frac{ft^2}{s}$$

$$c=\frac{1}{12}\cdot ft\times\frac{s}{1.94\times10^{-5}\cdot ft^2} \qquad c = 4.30\times10^{3}\cdot\frac{s}{ft}$$

In addition $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad (3) \qquad \text{with} \qquad e = 0.01\cdot in \qquad D = 1\,in$$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f = 0.01 \quad$ then

$$V=\sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}} \qquad V = 9.59\cdot\frac{ft}{s} \qquad Re = c\cdot V \qquad Re = 4.12\times10^{4}$$

Given $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad f = 0.0393 \quad V=\sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}} \quad V = 5.12\cdot\frac{ft}{s} \quad Re = c\cdot V \quad Re = 2.20\times10^{4}$$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0405$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 5.04\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 2.17\times 10^{4}$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0405$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 5.04\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 2.17\times 10^{4}$

Note that we could use *Excel*'s *Solver* for this problem Note: $f\cdot\frac{L}{D} = 24.3$ $K_e = 0.5$ $h_{lm} < h_l$

The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 5.04\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.0275\cdot\frac{ft^3}{s}$ $Q = 12.3\cdot gpm$

Next we recompute everything for $h = 5\cdot ft$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0405$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 3.57\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 1.53\times 10^{4}$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0415$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 3.52\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 1.51\times 10^{4}$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0415$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 3.52\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 1.51\times 10^{4}$

The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 3.52\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.0192\cdot\frac{ft^3}{s}$ $Q = 8.62\cdot gpm$

Next we recompute everything for $h = 1\cdot ft$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0415$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 1.58\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 6.77\times 10^{3}$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0452$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 1.51\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 6.50\times 10^{3}$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0454$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1+f\cdot\frac{L}{D}\right)}}$ $V = 1.51\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 6.48\times 10^{3}$

The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 1.51\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.00824\cdot\frac{ft^3}{s}$ $Q = 3.70\cdot gpm$

Initially we have dQ/dt = -12.3 gpm, then -8.62 gpm, then -3.70 gpm. These occur at h = 10 ft, 5 ft and 1 ft. The corresponding volumes in the tank are then Q = 7500 gal, 3750 gal, and 750 gal. Using *Excel* we can fit a power trendline to the dQ/dt versus Q data to find, approximately

$\frac{dQ}{dt} = -0.12\cdot Q^{\frac{1}{2}}$ where dQ/dt is in gpm and t is min. Solving this with initial condition Q = 7500 gpm when t = 0 gives

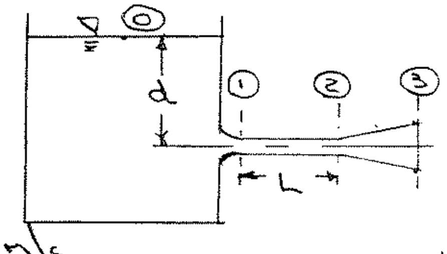
$t = \frac{1}{0.06}\cdot\left(\sqrt{7500}-\sqrt{Q}\right)$ Hence, when Q = 750 gal (h = 1 ft) $t = \frac{1}{0.06}\cdot\left(\sqrt{7500}-\sqrt{750}\right)\cdot min$ $t = 987\,min$ $t = 16.4\,hr$

Given: Pipe of length L inserted between the nozzle (attached to the water main) and diffuser of Example Problem 8.10.

$D_1 = 25\text{ mm}$, $K_{ent} = 0.04$, $d = 1.5\text{ m}$

Diffuser: $N/R_2 = 3.0$, $AR = 2.0$

$K_{diff} = 0.3$



Flow with nozzle alone:

$Q_i = 2.61\times10^{-3}\text{ m}^3/\text{s}$, $\bar V_1 = 5.32\text{ m/s}$

Flow with nozzle and diffuser $(L=0)$ $Q_d = 3.47\times10^{-3}\text{ m}^3/\text{s}$

Find: Length (L) of pipe with $e/D = 0.01$ required to give flow rate Q_i with diffuser in place; compare with commissioner's requirement of $L = 50$ ft (15.2 m)

Plot: Q/Q_i vs L/D

Solution:

Apply the energy equation for steady, incompressible flow between the water surface and the diffuser discharge.

Basic equations: $$\left(\cancel{\frac{p_0}{\rho}}+\cancel{\alpha_0\frac{\bar V_0^2}{2}}^{\approx 0(2)}+gz_0\right)-\left(\cancel{\frac{p_3}{\rho}}+\alpha_3\frac{\bar V_3^2}{2}+gz_3\right)=h_{l_T} \quad (8.29)$$

$$h_{l_T}=h_l+h_{l_m}\ ;\ h_l=f\frac{L}{D}\frac{\bar V^2}{2}\ ,\ h_{l_m}=(K_{ent}+K_{diff})\frac{\bar V_2^2}{2}$$

Assumptions: (1) $p_0 = p_3 = p_{atm}$

(2) $\bar V_0 = 0$, $\alpha_3 = 1.0$

(3) water @ 20°C, $\nu = 1.00\times10^{-6}\text{ m}^2/\text{s}$

Then,
$$g(z_0-z_3) = gd = f\frac{L}{D}\frac{\bar V^2}{2}+(K_{ent}+K_{diff})\frac{\bar V_2^2}{2}+\frac{\bar V_3^2}{2}$$

From continuity $A_2\bar V_2 = A_3\bar V_3$ $\therefore V_3 = \frac{\bar V_2}{AR}$

$$gd = f\frac{L}{D}\frac{\bar V_2^2}{2}+\left(K_{ent}+K_{diff}+\frac{1}{AR^2}\right)\frac{\bar V_2^2}{2} = f\frac{L}{D}\frac{\bar V_2^2}{2}+0.59\frac{\bar V_2^2}{2} \quad (1)$$

and
$$L = \frac{D}{f}\left[\frac{2gd}{\bar V_2^2}-0.590\right] \quad (2)$$

For $\bar V_1 = 5.32\text{ m/s}$, $Re = \frac{D\bar V}{\nu} = 0.025\text{ m}\times 5.32\frac{\text{m}}{\text{s}}\times\frac{\text{s}}{1.00\times10^{-6}\text{ m}^2} = 1.33\times10^5$

with $e/D = 0.01$, $f = 0.038$ (Fig. 8.13) and

$$L = \frac{0.025\text{ m}}{0.038}\left[2\times9.81\frac{\text{m}}{\text{s}^2}\times1.5\text{ m}\times\frac{\text{s}^2}{(5.32)^2\text{ m}^2}-0.590\right] = 0.296\text{ m} \longleftarrow L$$

(0.971 ft, $L/D = 11.8$)

This is significantly less than the 50 ft required by the water commissioner. He was extremely conservative.

Note that $Q/Q_i = \bar V/\bar V_i$ where $V_i = 5.32\text{ m/s}$

Increasing L reduces $\bar V$

National® Brand

Problem 8.139

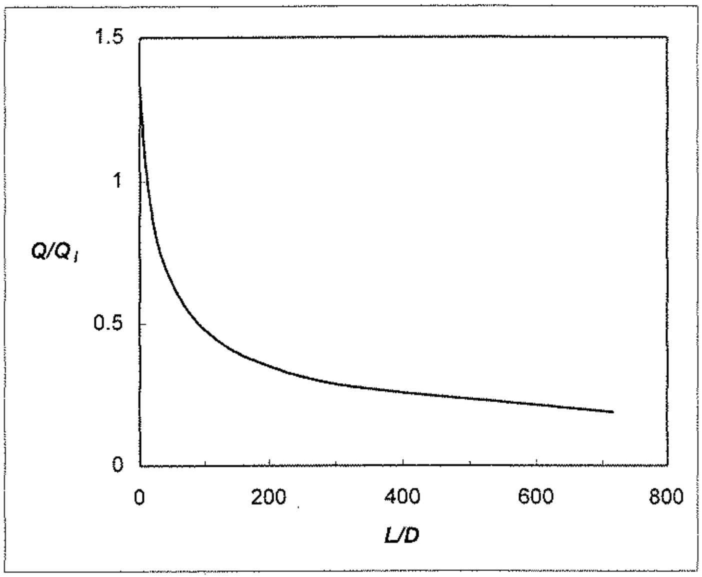
With $L = 0$ $\quad Q/Q_i = 1.33$

$L = 0.226$ m $\quad (L/D = 11.8) \quad Q/Q_i = 1.00$

As L is increased $\bar{V}_2$ (and hence Re) will decrease; the friction factor will increase slightly from 0.038.

The plot of Q/Q_i $(\bar{V}/\bar{V}_i)$ is best done by assuming values of $\bar{V}_1$ and solving Eq. 2 for L.

V_{avg} (m/s)	*Re* (---)	*f* (---)	*L/D* (---)	V/V_i (---)
7.06	1.77E+05	0.0382	0.0	1.33
5.32	1.33E+05	0.0384	11.7	1.00
5.0	1.25E+05	0.0384	15.3	0.940
4.5	1.13E+05	0.0384	22.5	0.846
4.0	1.00E+05	0.0385	32.4	0.752
3.5	8.75E+04	0.0386	47.0	0.658
3.0	7.50E+04	0.0387	69.3	0.564
2.5	6.25E+04	0.0388	106	0.470
2.0	5.00E+04	0.0391	173	0.376
1.5	3.75E+04	0.0394	317	0.282
1.0	2.50E+04	0.0402	718	0.188

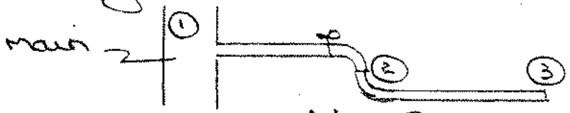


Problem 8.140

Given: Water flow from spigot (at 60°F) through an old hose with $D = 0.75$ in and $e = 0.022$ in. Pressure at main remains constant at 50 psig; pressure at spigot varies with flow rate.
One 50 ft. length of hose delivers 15 gpm

Find: (a) pressure at spigot (psig) for this case.
(b) delivery with two 50-ft lengths of hose connected.

Solution:



Apply the energy equation for steady, incompressible flow between the spigot ② and the hose discharge ③

Basic equations: $\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar V_2^2}{2}+gz_2\right)-\left(\frac{p_3}{\rho}+\alpha_3\frac{\bar V_3^2}{2}+gz_3\right)=h_{l_T}$ (8.29)

$$h_{l_T}=h_l+h_{l_m}\quad,\quad h_l=f\frac{L}{D}\frac{\bar V^2}{2}$$

Assumptions: (1) $p_3 = p_{atm}$ (4) Turbulent flow so $\Delta p_{1\to2}\propto Q^2$
(2) $\bar V_2=\bar V_3$, $\alpha_2=\alpha_3=1.0$
(3) $z_2=z_3$

Then
$$p_2=\rho f\frac{L}{D}\frac{\bar V^2}{2} \qquad (1)$$

$$\bar V=\frac{Q}{A}=\frac{4Q}{\pi D^2}=\frac{4}{\pi}\times 15\frac{gal}{min}\times\frac{min}{60\,s}\times\frac{ft^3}{7.48\,gal}\times\left(\frac{12}{0.75}\right)^2 ft^2 = 10.9\ ft/s$$

$$Re=\frac{D\bar V}{\nu}=\frac{0.75}{12}ft\times 10.9\frac{ft}{s}\times\frac{s}{1.21\times10^{-5}\,ft^2}=5.63\times10^4$$ {ν from Table A.7}

$$e/D = 0.022/0.75 = 0.0293$$

From Eq. 8.37 $f = 0.056$*. From Eq. 1,

$$p_2 = 1.94\frac{slug}{ft^3}\times 0.056\times\frac{50\,ft}{0.75\,in}\times\frac{12\,in}{ft}\times\frac{1}{2}\times(10.9)^2\frac{ft^2}{s^2}\times\frac{lbf\cdot s^2}{slug\cdot ft}\times\frac{ft^2}{144\,in^2}$$

p_2 = 35.9 psi gage ← p_2

The pressure drop from the main ① to the spigot ② is proportional to the square of the flow rate. Obtain the loss coefficient using the energy equation between ① and ②.

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar V_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar V_2^2}{2}+gz_2\right)=K\frac{\bar V_2^2}{2}$$

Assumptions: (4) $\bar V_1 = 0$ (5) $z_1 = z_2$

$$p_1-p_2=\rho\left[K\frac{\bar V_2^2}{2}+\frac{\bar V_2^2}{2}\right]=\rho\frac{\bar V_2^2}{2}\left[K+1\right]$$

$$K=\frac{\Delta p}{\frac{1}{2}\rho\bar V^2}-1=2(50-35.9)144\frac{lbf}{ft^2}\times\frac{ft^3}{1.94\,slug\times(10.9)^2\,ft^2}\,s^2\times\frac{slug\cdot ft}{lbf\cdot s^2}-1$$

$K = 16.6$

* Value of f obtained using Excel's Solver (or Goal Seek).

Problem 8.140

To find the delivery with two hoses, again apply the energy equation from the main ① to the end ④ of the second hose

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_4}{\rho} + \alpha_4\frac{\bar{V}_4^2}{2} + gz_4\right) = f\frac{L_{24}}{D}\frac{\bar{V}_4^2}{2} + K\frac{\bar{V}_4^2}{2}$$

$p_4 = p_{atm}$, $z_1 = z_4$, $\bar{V}_1 = 0$, $\alpha_4 = 1$.

$$\frac{p_1 - p_{atm}}{\rho} = \frac{p_{1g}}{\rho} = \frac{\bar{V}_4^2}{2}\left(f\frac{L_{24}}{D} + K + 1\right)$$ and.

$$\bar{V}_4 = \left[\frac{2\,p_{1g}}{\rho\left(2f\frac{L}{D} + K + 1\right)}\right]^{1/2}$$

Delivery will be reduced somewhat with two lengths of hose, but f will not change much. Assume $f \approx 0.056$ and check

$$\bar{V}_4 = \left[2 \times 50\,\frac{lbf}{in^2} \times 144\,\frac{in^2}{ft^2} \times \frac{ft^3}{1.94\,slug} \times \frac{1}{\left(0.056 \times \frac{100\,ft}{0.75\,in} \times \frac{12\,in}{ft} + 16.6 + 1\right)} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2}$$

$\bar{V}_4 = 8.32$ ft/s.

Checking,

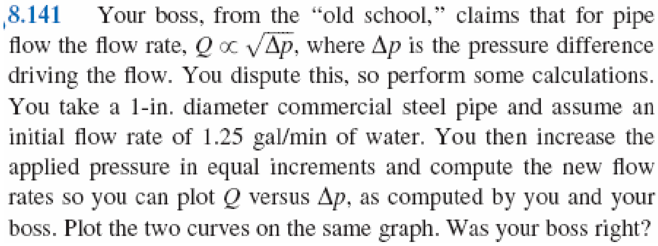
$$Re = \frac{D\bar{V}}{\nu} = \frac{0.75}{12}\,ft \times 8.32\,\frac{ft}{s} \times 1.21\times10^{-5}\,\frac{s}{ft^2} = 4.30\times10^4$$, so $f \approx 0.056$

Thus with two hoses,

$$Q = \bar{V}A = 8.32\,\frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{0.75}{12}\right)^2 ft^2 \times 7.48\,\frac{gal}{ft^3} \times 60\,\frac{s}{min} = 11.5\ gpm \longleftarrow Q$$

{Similar calculations could be performed using any desired number of hose lengths.}

Problem 8.141



8.141 Your boss, from the "old school," claims that for pipe flow the flow rate, $Q \propto \sqrt{\Delta p}$, where Δp is the pressure difference driving the flow. You dispute this, so perform some calculations. You take a 1-in. diameter commercial steel pipe and assume an initial flow rate of 1.25 gal/min of water. You then increase the applied pressure in equal increments and compute the new flow rates so you can plot Q versus Δp, as computed by you and your boss. Plot the two curves on the same graph. Was your boss right?

Applying the energy equation between inlet and exit:

$$\frac{\Delta p}{\rho} = f\frac{L}{D}\frac{V^2}{2} \quad \text{or} \quad \frac{\Delta p}{L} = \frac{\rho f}{D}\frac{V^2}{2}$$

"Old school":

$$\frac{\Delta p}{L} = \left(\frac{\Delta p}{L}\right)_0 \left(\frac{Q_0}{Q}\right)^2$$

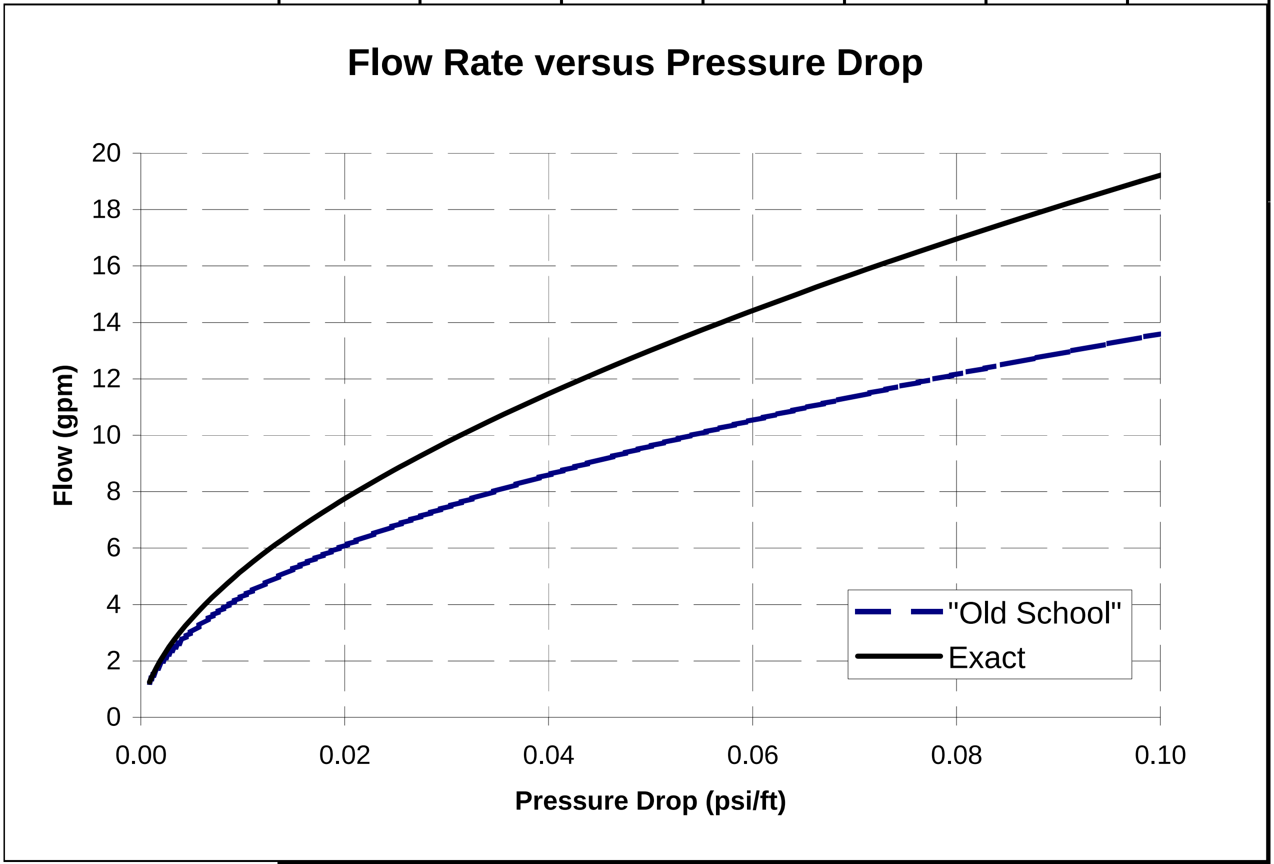
D = 1 in

e = 0.00015 ft

ν = 1.08E-05 ft^2/s

ρ = 1.94 slug/ft^3

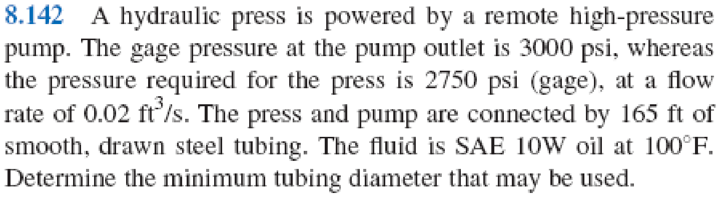
Q (gpm)	Q (ft^3/s)	V (ft/s)	Re	f	Δp (old school) (psi)	Δp (psi/ft)
1.25	0.00279	0.511	3940	0.0401	0.00085	0.00085
1.50	0.00334	0.613	4728	0.0380	0.00122	0.00115
1.75	0.00390	0.715	5516	0.0364	0.00166	0.00150
2.00	0.00446	0.817	6304	0.0350	0.00216	0.00189
8.75	0.01950	3.575	27582	0.0240	0.04142	0.02477
9.00	0.02005	3.677	28370	0.0238	0.04382	0.02604



Your boss was wrong!

Problem 8.142

[3]



8.142 A hydraulic press is powered by a remote high-pressure pump. The gage pressure at the pump outlet is 3000 psi, whereas the pressure required for the press is 2750 psi (gage), at a flow rate of 0.02 ft^3/s. The press and pump are connected by 165 ft of smooth, drawn steel tubing. The fluid is SAE 10W oil at 100°F. Determine the minimum tubing diameter that may be used.

Given: Hydraulic press system

Find: Minimum required diameter of tubing

Solution:

Basic equations

$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_l \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

The flow rate is low and it's oil, so try assuming laminar flow. Then, from Eq. 8.13c

$$\Delta p=\frac{128\cdot\mu\cdot Q\cdot L}{\pi\cdot D^4} \qquad \text{or} \qquad D=\left(\frac{128\cdot\mu\cdot Q\cdot L}{\pi\cdot\Delta p}\right)^{\frac{1}{4}}$$

For SAE 10W oil at 100ºF (Fig. A.2, 38ºC)

$$\mu=3.5\times10^{-2}\cdot\frac{N\cdot s}{m^2}\times\frac{0.0209\cdot\frac{lbf\cdot s}{ft^2}}{1\cdot\frac{N\cdot s}{m^2}} \qquad \mu=7.32\times10^{-4}\,\frac{lbf\cdot s}{ft^2}$$

Hence

$$D=\left[\frac{128}{\pi}\times7.32\times10^{-4}\,\frac{lbf\cdot s}{ft^2}\times0.02\cdot\frac{ft^3}{s}\times165\cdot ft\times\frac{in^2}{(3000-2750)\cdot lbf}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2\right]^{\frac{1}{4}} \qquad D=0.0407\,ft \qquad D=0.488\,in$$

Check Re to assure flow is laminar

$$V=\frac{Q}{A}=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=\frac{4}{\pi}\times0.02\cdot\frac{ft^3}{s}\times\left(\frac{12}{0.488}\cdot\frac{1}{ft}\right)^2 \qquad V=15.4\,\frac{ft}{s}$$

From Table A.2 $SG_{oil}=0.92$ so

$$Re=\frac{SG_{oil}\cdot\rho_{H2O}\cdot V\cdot D}{\mu}$$

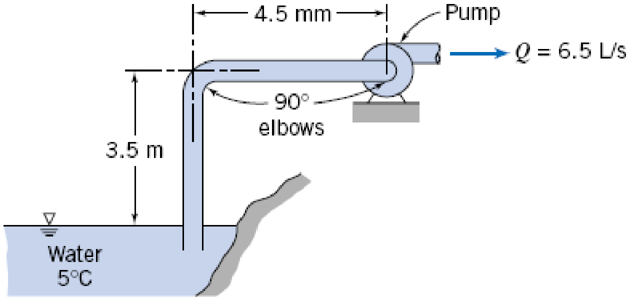
$$Re=0.92\times1.94\cdot\frac{slug}{ft^3}\times15.4\cdot\frac{ft}{s}\times\frac{0.488}{12}\cdot ft\times\frac{ft^2}{7.32\times10^{-4}\,lbf\cdot s}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad Re=1527$$

Hence the flow is laminar, Re < 2300. The minimum diameter is 0.488 in, so 0.5 in ID tube should be chosen

Problem 8.143

[4]

8.143 A pump is located 4.5 m to one side of, and 3.5 m above a reservoir. The pump is designed for a flow rate of 6 L/s. For satisfactory operation, the static pressure at the pump inlet must not be lower than −6 m of water gage. Determine the smallest standard commercial steel pipe that will give the required performance.



Given: Flow out of reservoir by pump

Find: Smallest pipe needed

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_2^{\,2}}{2}+K_{ent}\cdot\frac{V_2^{\,2}}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V_2^{\,2}}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_l <<

Hence for flow between the free surface (Point 1) and the pump inlet (2) the energy equation becomes

$$-\frac{p_2}{\rho}-g\cdot z_2-\frac{V_2^{\,2}}{2}=-\frac{p_2}{\rho}-g\cdot z_2-\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{ent}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad \text{and} \quad p=\rho\cdot g\cdot h$$

Solving for $h_2 = p_2/\rho g$
$$h_2=-z_2-\frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K_{ent}\right] \qquad (1)$$

From Table 8.2 $K_{ent}=0.78$ for rentrant, and from Table 8.4 two standard elbows lead to $\frac{L_e}{D}=2\times 30=60$

We also have $e=0.046\cdot mm$ (Table 8.1) $\nu=1.51\times 10^{-6}\cdot\frac{m^2}{s}$ (Table A.8)

and we are given $Q=6\cdot\frac{L}{s}$ $Q=6\times 10^{-3}\frac{m^3}{s}$ $z_2=3.5\cdot m$ $L=(3.5+4.5)\cdot m$ $L=8m$ $h_2=-6\cdot m$

Equation 1 is tricky because D is unknown, so V is unknown (even though Q is known), L/D and L_e/D are unknown, and Re and hence f are unknown! We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simultaneously by varying D, but here we try guesses:

$$D=2.5\cdot cm \quad V=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=12.2\frac{m}{s} \qquad Re=\frac{V\cdot D}{\nu} \qquad Re=2.02\times 10^5$$

Given
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0238$$

$$h_2=-z_2-\frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K_{ent}\right] \qquad h_2=-78.45\,m$$ but we need -6 m!

$$D=5\cdot cm \quad V=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=3.06\frac{m}{s} \qquad Re=\frac{V\cdot D}{\nu} \qquad Re=1.01\times 10^5$$

Given
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0219$$

$h_2 = -z_2 - \frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right) + K_{ent}\right]$ $h_2 = -6.16\,m$ but we need -6 m!

$D = 5.1\cdot cm$ $V = \frac{4\cdot Q}{\pi\cdot D^2}$ $V = 2.94\,\frac{m}{s}$ $Re = \frac{V\cdot D}{\nu}$ $Re = 9.92\times 10^4$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0219$

$h_2 = -z_2 - \frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right) + K_{ent}\right]$ $h_2 = -5.93\,m$

To within 1%, we can use 5-5.1 cm tubing; this corresponds to standard 2 in pipe.

Problem 8.144

[4]

8.144 Determine the minimum size smooth rectangular duct with an aspect ratio of 2 that will pass 2850 cfm of standard air with a head loss of 1.25 in. of water per 100 ft of duct.

Given: Flow of air in rectangular duct

Find: Minimum required size

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_l \qquad h_l=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad D_h=\frac{4\cdot A}{P_w}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

Hence for flow between the inlet (Point 1) and the exit (2) the energy equation becomes

$$\frac{p_1}{\rho}-\frac{p_2}{\rho}=\frac{\Delta p}{\rho}=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad \text{and} \qquad \Delta p=\rho_{H2O}\cdot g\cdot\Delta h$$

For a rectangular duct $D_h=\dfrac{4\cdot b\cdot h}{2\cdot(b+h)}=\dfrac{2\cdot h^2\cdot ar}{h\cdot(1+ar)}=\dfrac{2\cdot h\cdot ar}{1+ar}$ and also $A=b\cdot h=h^2\cdot\dfrac{b}{h}=h^2\cdot ar$

Hence $\Delta p=\rho\cdot f\cdot L\cdot\dfrac{V^2}{2}\cdot\dfrac{(1+ar)}{2\cdot h\cdot ar}=\rho\cdot f\cdot L\cdot\dfrac{Q^2}{2\cdot A^2}\cdot\dfrac{(1+ar)}{2\cdot h\cdot ar}=\dfrac{\rho\cdot f\cdot L\cdot Q^2}{4}\cdot\dfrac{(1+ar)}{ar^3}\cdot\dfrac{1}{h^5}$

Solving for h
$$h=\left[\frac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\frac{(1+ar)}{ar^3}\right]^{\frac{1}{5}} \qquad (1)$$

We are given $Q=2850\cdot\dfrac{ft^3}{min}$ $\quad L=100\cdot ft$ $\quad e=0\cdot ft$ $\quad ar=2$

and also $\Delta p=\rho_{H2O}\cdot g\cdot\Delta h$ $\quad \Delta p=1.94\cdot\dfrac{slug}{ft^3}\times 32.2\dfrac{ft}{s^2}\times\dfrac{1.25}{12}\cdot ft\times\dfrac{lbf\cdot s^2}{slug\cdot ft}$ $\quad \Delta p=6.51\cdot\dfrac{lbf}{ft^2}$

$\rho=0.00234\cdot\dfrac{slug}{ft^3}$ $\quad \nu=1.62\times 10^{-4}\cdot\dfrac{ft^2}{s}$ $\quad$ (Table A.9)

Equation 1 is tricky because h is unknown, so D_h is unknown, hence V is unknown (even though Q is known), and Re and hence f are unknown! We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simmultaneously by varying h, but here we try guesses:

$f=0.01$ $\quad h=\left[\dfrac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\dfrac{(1+ar)}{ar^3}\right]^{\frac{1}{5}}$ $\quad h=0.597\cdot ft$ $\quad V=\dfrac{Q}{h^2\cdot ar}$ $\quad V=66.6\cdot\dfrac{ft}{s}$

$D_h=\dfrac{2\cdot h\cdot ar}{1+ar}$ $\quad D_h=0.796\cdot ft$ $\quad Re=\dfrac{V\cdot D_h}{\nu}$ $\quad Re=3.27\times 10^5$

Given $\quad \dfrac{1}{\sqrt{f}}=-2.0\cdot\log\left(\dfrac{\frac{e}{D_h}}{3.7}+\dfrac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0142$

$$h = \left[\frac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\frac{(1+ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h = 0.641\cdot ft \qquad V = \frac{Q}{h^2\cdot ar} \qquad V = 57.8\cdot\frac{ft}{s}$$

$$D_h = \frac{2\cdot h\cdot ar}{1+ar} \qquad D_h = 0.855\cdot ft \qquad Re = \frac{V\cdot D_h}{\nu} \qquad Re = 3.05\times 10^5$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0144$$

$$h = \left[\frac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\frac{(1+ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h = 0.643\cdot ft \qquad V = \frac{Q}{h^2\cdot ar} \qquad V = 57.5\cdot\frac{ft}{s}$$

$$D_h = \frac{2\cdot h\cdot ar}{1+ar} \qquad D_h = 0.857\cdot ft \qquad Re = \frac{V\cdot D_h}{\nu} \qquad Re = 3.04\times 10^5$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0144$$

$$h = \left[\frac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\frac{(1+ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h = 0.643\cdot ft \qquad V = \frac{Q}{h^2\cdot ar} \qquad V = 57.5\cdot\frac{ft}{s}$$

$$D_h = \frac{2\cdot h\cdot ar}{1+ar} \qquad D_h = 0.857\cdot ft \qquad Re = \frac{V\cdot D_h}{\nu} \qquad Re = 3.04\times 10^5$$

In this process h and f have converged to a solution. The minimum dimensions are 0.642 ft by 1.28 ft, or 7.71 in by 15.4 in

Problem 8.145

[3]

Given: New industrial plant requires water supply of 5.7 m^3/min. The gage pressure at the main, 50 m from the plant, is 800 kPa. The supply line will have 4 elbows in a total length of 65 m. Pressure in the plant must be at least 500 kPa (gage).

Find: Minimum line size of galvanized iron to install.

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section ($\alpha \simeq 1$).

Basic equation: $$\frac{p_1}{\rho} + \cancelto{(2)}{\frac{\bar{V}_1^2}{2}} + \cancelto{(3)}{g z_1} = \frac{p_2}{\rho} + \cancelto{(2)}{\frac{\bar{V}_2^2}{2}} + \cancelto{(3)}{g z_2} + f \frac{L}{D} \frac{\bar{V}^2}{2} + h_{\ell m}$$

Assumptions: (1) $p_1 - p_2 \leq 300 \text{ kPa} = \Delta p$

(2) Fully developed flow in constant-area pipe, $\bar{V}_1 = \bar{V}_2 = \bar{V}$

(3) $z_1 = z_2$

(4) $h_{\ell m} = 4 \left(\frac{L_e}{D}\right)_{elbow} \frac{\bar{V}^2}{2} = 120 \frac{\bar{V}^2}{2}$ $\left(\frac{L_e}{D} = 30, \text{ from Table 8.5}\right)$

Then

$$\frac{\Delta p}{\rho} = f\left(\frac{L}{D} + 120\right)\frac{\bar{V}^2}{2} \quad \text{or} \quad \Delta p = \rho f\left(\frac{L}{D} + 120\right)\frac{\bar{V}^2}{2}$$

Since D is unknown, iteration is required. The calculating equations are:

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 5.7 \frac{m^3}{min} \times \frac{1}{D^2 m^2} \times \frac{min}{60 s} = \frac{0.121}{D^2} (m/s)$$

$$Re = \frac{\bar{V}D}{\nu} = \frac{4Q}{\pi \nu D} = \frac{4}{\pi} \times 5.7 \frac{m^3}{min} \times \frac{s}{1.14 \times 10^{-6} m^2} \times \frac{1}{D m} \times \frac{min}{60 s} = \frac{1.06 \times 10^5}{D} \quad (T = 15°C)$$

$e = 0.15$ mm (Table 8.1), f from Eq. 8.37*, $L = 65$ m. D from Table 8.5.

D (nom.)	D (m)	$\bar{V}$ (m/s)	Re (–)	e/D (–)	f (–)	L/D (–)	Δp (kPa)
3	0.0779	19.9	1.36×10^6	0.0019	0.024	834	4530
5	0.128	7.39	8.29×10^5	0.0012	0.021	508	360
6	0.154	5.10	6.89×10^5	0.001	0.020	422	141

Pipe friction calculations are accurate only within about ± 10 percent. Line resistance (and consequently Δp) will increase with age.

Recommend installation of 6 in. (nominal) line. D

* Values of f obtained using Excel's Solver (or Goal Seek)

Problem 8.146

[4]

8.146 Air at 20°C flows in a horizontal square cross-section duct made from commercial steel. The duct is 25 m long. What size (length of a side) duct is required to convey 2 m^3/s of air with a pressure drop of 1.5 cm H_2O?

Given: Flow of air in square duct

Find: Minimum required size

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_l$ $\quad h_l=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2}$ $\quad D_h=\frac{4\cdot A}{P_w}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

Hence for flow between the inlet (Point 1) and the exit (2) the energy equation becomes

$$\frac{p_1}{\rho}-\frac{p_2}{\rho}=\frac{\Delta p}{\rho}=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad \text{and} \qquad \Delta p=\rho_{H2O}\cdot g\cdot\Delta h$$

For a square duct $D_h=\frac{4\cdot h\cdot h}{2\cdot(h+h)}=h$ and also $A=h\cdot h=h^2$

Hence $\Delta p=\rho\cdot f\cdot L\cdot\frac{V^2}{2\cdot h}=\rho\cdot f\cdot L\cdot\frac{Q^2}{2\cdot h\cdot A^2}=\frac{\rho\cdot f\cdot L\cdot Q^2}{2\cdot h^5}$

Solving for h $\quad h=\left(\frac{\rho\cdot f\cdot L\cdot Q^2}{2\cdot\Delta p}\right)^{\frac{1}{5}}$ (1)

We are given $Q=2\cdot\frac{m^3}{s}$ $\quad L=25\cdot m$ $\quad e=0.046\cdot mm$ (Table 8.1)

and also $\Delta p=\rho_{H2O}\cdot g\cdot\Delta h$ $\quad \Delta p=1000\cdot\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times 0.015\cdot m\times\frac{N\cdot s^2}{kg\cdot m}$ $\quad \Delta p=147\,Pa$

$\rho=1.21\cdot\frac{kg}{m^3}$ $\quad \nu=1.50\times 10^{-5}\cdot\frac{m^2}{s}$ (Table A.10)

Equation 1 is tricky because h is unknown, so D_h is unknown, hence V is unknown (even though Q is known), and Re and hence f are unknown! We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simmultaneously by varying h, but here we try guesses:

$f=0.01$ $\quad h=\left(\frac{\rho\cdot f\cdot L\cdot Q^2}{2\cdot\Delta p}\right)^{\frac{1}{5}}$ $\quad h=0.333\,m$ $\quad V=\frac{Q}{h^2}$ $\quad V=18.0\cdot\frac{m}{s}$

$D_h=h$ $\quad D_h=0.333\,m$ $\quad Re=\frac{V\cdot D_h}{\nu}$ $\quad Re=4.00\times 10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0152$

$h = \left(\frac{\rho \cdot f \cdot L \cdot Q^2}{2 \cdot \Delta p}\right)^{\frac{1}{5}}$ $h = 0.362\,m$ $V = \frac{Q}{h^2}$ $V = 15.2\,\frac{m}{s}$

$D_h = h$ $D_h = 0.362 \cdot m$ $Re = \frac{V \cdot D_h}{\nu}$ $Re = 3.68 \times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $f = 0.0153$

$h = \left(\frac{\rho \cdot f \cdot L \cdot Q^2}{2 \cdot \Delta p}\right)^{\frac{1}{5}}$ $h = 0.363\,m$ $V = \frac{Q}{h^2}$ $V = 15.2\,\frac{m}{s}$

In this process h and f have converged to a solution. The minimum dimensions are 0.363 m by 0.363 m, or 36.3 cm by 36.3 cm

Problem 8.147

[3]

8.147 Investigate the effect of tube diameter on water flow rate by computing the flow generated by a pressure difference, $\Delta p = 100\,\text{kPa}$, applied to a length $L = 100\,\text{m}$ of smooth tubing. Plot the flow rate against tube diameter for a range that includes laminar and turbulent flow.

Given: Flow in a tube

Find: Effect of diameter; Plot flow rate versus diameter

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_l \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (8.34)$$

$$f = \frac{64}{Re} \qquad (8.36) \qquad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad (8.37) \qquad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for flow in a tube

$$p_1 - p_2 = \Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

This cannot be solved explicitly for velocity *V* (and hence flow rate *Q*), because *f* depends on *V*; solution for a given diameter *D* requires iteration (or use of *Solver*)

Fluid is not specified: use water (basic trends in plot apply to any fluid)

Given data:

Δp = 100 kPa

L = 100 m

Tabulated or graphical data:

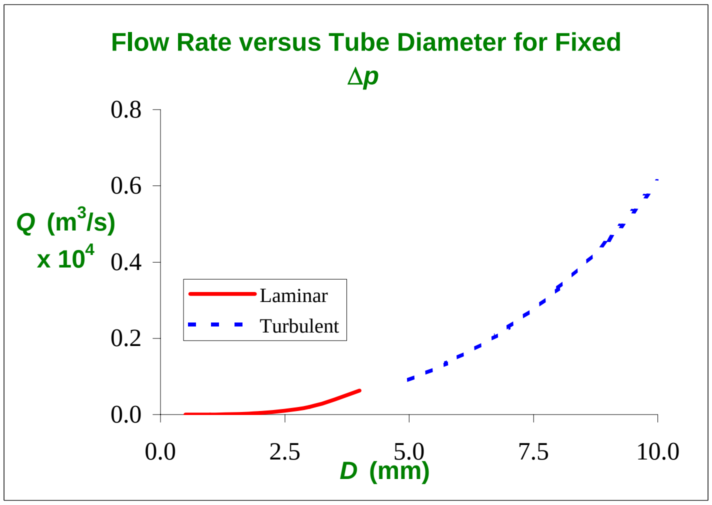
μ = 1.00E-03 N.s/m^2

ρ = 999 kg/m^3

(Water - Appendix A)

Computed results:

D (mm)	*V* (m/s)	*Q* (m^3/s) x 10^4	*Re*	Regime	*f*	Δp (kPa)	Error
0.5	0.00781	0.0000153	4	Laminar	16.4	100	0.0%
1.0	0.0312	0.000245	31	Laminar	2.05	100	0.0%
2.0	0.125	0.00393	250	Laminar	0.256	100	0.0%
3.0	0.281	0.0199	843	Laminar	0.0759	100	0.0%
4.0	0.500	0.0628	1998	Laminar	0.0320	100	0.0%
5.0	0.460	0.0904	2300	Turbulent	0.0473	100	0.2%
6.0	0.530	0.150	3177	Turbulent	0.0428	100	0.0%
7.0	0.596	0.229	4169	Turbulent	0.0394	100	0.0%
8.0	0.659	0.331	5270	Turbulent	0.0368	100	0.0%
9.0	0.720	0.458	6474	Turbulent	0.0348	100	0.0%
10.0	0.778	0.611	7776	Turbulent	0.0330	100	0.0%


Flow Rate versus Tube Diameter for Fixed Δp

Problem 8.148 [4]

8.148 What diameter water pipe is required to handle 1200 gpm and a 50 psi pressure drop? The pipe length is 500 ft, and roughness is 0.01 ft. The water is at 68°F.

Given: Flow of water in circular pipe

Find: Minimum required diameter

Solution:

Basic equations $\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1{}^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2{}^2}{2}+g\cdot z_2\right)=h_l$ $\quad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$ and also $A=\frac{\pi\cdot D^2}{4}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

Hence for flow between the inlet (Point 1) and the exit (2) the energy equation becomes

$$\frac{p_1}{\rho}-\frac{p_2}{\rho}=\frac{\Delta p}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Hence $\Delta p=\rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2}=\rho\cdot f\cdot\frac{L}{D}\cdot\frac{Q^2}{2\cdot A^2}=\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot D^5}$

Solving for D $\quad D=\left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}\qquad(1)$

We are given $\quad Q=1200\cdot gpm\qquad L=500\cdot ft\qquad e=0.01\cdot ft\qquad \Delta p=50\cdot psi$

and also $\quad \rho=1.94\cdot\frac{slug}{ft^3}\qquad \nu=1.08\times10^{-5}\cdot\frac{ft^2}{s}\qquad$ (Table A.7)

Equation 1 is tricky because D is unknown, hence V is unknown (even though Q is known), and Re and hence f are unknown!
We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simultaneously by varying D, but here we try guesses:

$f=0.01\qquad D=\left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}\qquad D=0.379\cdot ft\qquad V=\frac{4\cdot Q}{\pi\cdot D^2}\qquad V=23.7\cdot\frac{ft}{s}\qquad Re=\frac{V\cdot D}{\nu}\qquad Re=8.32\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)f\quad=0.0543$

$D=\left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}\qquad D=0.531\cdot ft\qquad V=\frac{4\cdot Q}{\pi\cdot D^2}\qquad V=12.1\cdot\frac{ft}{s}\qquad Re=\frac{V\cdot D}{\nu}\qquad Re=5.93\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)\qquad f=0.0476$

$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}$ $D = 0.518\cdot ft$ $V = \frac{4\cdot Q}{\pi\cdot D^2}$ $V = 12.7\cdot\frac{ft}{s}$ $Re = \frac{V\cdot D}{\nu}$ $Re = 6.09\times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0481$

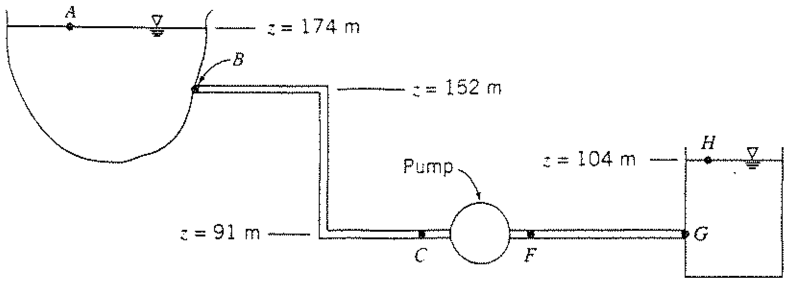
$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}$ $D = 0.519\cdot ft$ $V = \frac{4\cdot Q}{\pi\cdot D^2}$ $V = 12.7\cdot\frac{ft}{s}$ $Re = \frac{V\cdot D}{\nu}$ $Re = 6.08\times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0480$

$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}$ $D = 0.519\cdot ft$ $V = \frac{4\cdot Q}{\pi\cdot D^2}$ $V = 12.7\cdot\frac{ft}{s}$ $Re = \frac{V\cdot D}{\nu}$ $Re = 6.08\times 10^5$

In this process D and f have converged to a solution. The minimum diameter is 0.519 ft or 6.22 in

Given: Portion of water supply system designed to provide $Q = 1310\ L/s$ at $T = 20°C$.



System B → C
- square edged entrance
- 3 gate valves
- 4 45° elbows
- 2 90° elbows
- 760 m pipe
- $p_C = 197$ kPa gage

System F → G
- 760 m pipe
- 2 gate valves
- 4 90° elbows.

All pipe is cast iron, $D = 508$ mm

Find: (a) average velocity in pipeline
(b) gage pressure p_F
(c) shear stress on pipe centerline at C
(d) power input to pump if efficiency $\eta = 80\%$
(e) wall shear stress at G.

Solution:

Since $Q = A\bar{V}$, $\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 1310 \frac{L}{s} \times \frac{1}{(0.508 m)^2} \times 10^{-3} \frac{m^3}{L} = 6.46\ m/s$ $\bar{V}$

To determine the pressure at point F, apply the energy equation for steady, incompressible flow between F and G.

Basic equation: $\left(\frac{p_F}{\rho} + \alpha \frac{\bar{V}_F^2}{2} + g z_F\right) - \left(\overset{(2)}{\cancel{\frac{p_H}{\rho}}} + \alpha \overset{\approx 0 (1)}{\cancel{\frac{\bar{V}_H^2}{2}}} + g z_H\right) = h_{l_T}$ (8.29).

$h_{l_T} = h_l + h_{l_m}$, $h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$, $h_{l_m} = \frac{\bar{V}^2}{2} \Sigma f \frac{L_e}{D} + \frac{\bar{V}^2}{2} K_{exit}$

Assume: (1) $\bar{V}_H = 0$ (large storage tank) (2) $p_H = p_{atm}$
(3) $\alpha_F \approx 1.0$.

Then $\frac{p_F}{\rho} = h_{l_T} + g(z_H - z_F) - \frac{\bar{V}_F^2}{2}$

$\frac{p_F}{\rho} = h_{l_{F-G}} + 2 h_{l_{m_{gv}}} + 4 h_{l_{m_{90°el}}} + h_{l_{m_{exit}}} + g(z_H - z_F) - \frac{\bar{V}_F^2}{2}$

$\frac{p_F}{\rho} = f \frac{L}{D} \frac{\bar{V}^2}{2} + 2 f \left(\frac{L_e}{D}\right)_{gv} \frac{\bar{V}^2}{2} + 4 f \left(\frac{L_e}{D}\right)_{90°el} \frac{\bar{V}^2}{2} + K_{exit} \frac{\bar{V}^2}{2} + g(z_H - z_F) - \frac{\bar{V}_F^2}{2}$ ----(1)

From Table 8.4 $\left(\frac{L_e}{D}\right)_{gv} = 8$, $\left(\frac{L_e}{D}\right)_{90°el} = 30$; also $K_{exit} = 1$

$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = 0.508 m \times 6.46 \frac{m}{s} \times \frac{s}{1.00 \times 10^{-6} m^2} = 3.28 \times 10^6$ (ν from Table A.8)

From Table 8.1, $e = 0.26$ mm $\therefore e/D = 0.00051$

From Eq. 8.37, $f = 0.017$ (using Excel's Solver [or Goal Seek])

From Eq (1)

$\frac{p_F}{\rho} = f \frac{\bar{V}^2}{2} \left[\frac{L}{D} + 2\left(\frac{L_e}{D}\right)_{gv} + 4\left(\frac{L_e}{D}\right)_{90°el} \right] + g(z_H - z_F)$

Problem 8.149

$$\frac{p_F}{\rho} = f\frac{\bar{V}^2}{2}\left[\frac{760}{0.508} + 2(8) + 4(30)\right] + g(z_H - z_F) = f\frac{\bar{V}^2}{2}(1630) + g(z_H - z_F)$$

$$p_F = \rho\left[1630\, f\frac{\bar{V}^2}{2} + g(z_H - z_F)\right]$$

$$= 999\,\frac{kg}{m^3}\left[\frac{1630}{2}\times 0.017\times(6.46)^2\,\frac{m^2}{s^2} + 9.81\,\frac{m}{s^2}(104-91)\,m\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_F = 705$ kPa (gage) ← p_F

For fully developed flow in a pipe $\tau = \frac{r}{2}\frac{\partial p}{\partial x}$ (8.15)

At the pipe centerline, $\tau = 0$ ← $\tau_{℄}$

To determine the power input to the fluid apply the energy equation across the pump. Assuming 100% efficiency

$$\frac{\dot{W}_{pump}}{\dot{m}} = \left(\frac{p}{\rho} + \cancel{\frac{\bar{V}^2}{2}} + \cancel{gz}\right)_{discharge} - \left(\frac{p}{\rho} + \cancel{\frac{\bar{V}^2}{2}} + \cancel{gz}\right)_{suction} \qquad (8.47)$$

$$\dot{W}_{pump} = \left(\frac{p_F}{\rho} - \frac{p_C}{\rho}\right)\rho A\bar{V} = (p_F - p_C)Q$$

$$\dot{W}_{pump} = (705-197)\times 10^3\,\frac{N}{m^2}\times 1310\,\frac{L}{s}\times 10^{-3}\,\frac{m^3}{L} = 6.65\times 10^5\,\frac{N\cdot m}{s}$$

The actual pump input, $\dot{W}_{pump})_{act} = \dot{W}_{pump})_{ideal}/\eta$

$\dot{W}_{pump})_{actual} = 8.32\times 10^5$ N·m/s = 832 kW ← $\dot{W}_{actual}$

From Eq. 8.15 $\tau_w = \frac{R}{2}\frac{\partial p}{\partial x}$

Along the pipe from F to G $\frac{\Delta p}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2}$

$$\therefore \frac{\partial p}{\partial x} = \frac{\Delta p}{L} = \rho\frac{f}{D}\frac{\bar{V}^2}{2} = 999\,\frac{\cancel{kg}}{m^3}\times\frac{0.017}{0.508\,m}\times\frac{1}{2}\times(6.46)^2\,\frac{m^2}{\cancel{s^2}}\times\frac{N\cdot \cancel{s^2}}{\cancel{kg}\cdot m}$$

$\frac{\partial p}{\partial x} = 698$ N/m²/m

$\therefore \tau_w = \frac{R}{2}\frac{\partial p}{\partial x} = \frac{0.254\,m}{2}\times 698\,\frac{N}{m^3} = 88.6$ N/m² ← τ_w

Problem 8.150

Given: An air-pipe friction experiment utilizes smooth brass tube, $D = 63.5$ mm, $L = 1.52$ m.
At one flow condition $\Delta p = 12.3$ mm Merian red oil, $U_c = 23.1$ m/s.

Find: (a) $Re_{\bar{V}}$
(b) friction factor f; compare with value for Fig. 8.13.

Solution:

Apply the energy equation for steady, incompressible flow along the pipe

Basic equation: $$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} \qquad (8.29)$$

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

Computing equation: $$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} \qquad (8.24)$$

Assumptions: (1) power law profile, $n = 7$
(2) $\alpha_1 = \alpha_2$, $z_1 = z_2$
(3) air at $T = 15^\circ C$ $\nu = 1.46\times10^{-5}$ m²/s (Table A.10).

From Eq. 8.24 with $n = 7$

$$\frac{\bar{V}}{U} = \frac{2(7)^2}{(8)(15)} = 0.817$$

$$Re_{\bar{V}} = \frac{D\bar{V}}{\nu} = 0.0635\,\text{m} \times 0.817 \times 23.1\frac{\text{m}}{\text{s}} \times \frac{1.45\times10^{-5}}{}\frac{\text{s}}{\text{m}^2} = 8.26\times10^4 \quad \underline{\qquad Re_{\bar{V}}}$$

From Eq. 8.29 $$\Delta p/\rho = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

and

$$f = \frac{\Delta p}{\rho\frac{L}{D}\frac{\bar{V}^2}{2}} = \frac{\rho_{oil}\,g\,\Delta h}{\rho_{air}\frac{L}{D}\frac{\bar{V}^2}{2}} = \frac{2\rho_{H_2O}SG_{oil}\,g\,\Delta h}{\rho_{air}\frac{L}{D}\bar{V}^2} \qquad \begin{Bmatrix} SG = 0.827 \\ \text{Table A.1} \end{Bmatrix}.$$

$$f = 2\times\frac{10^3}{1.23}\times 0.827 \times 9.81\frac{\text{m}}{\text{s}^2}\times 0.0123\,\text{m}\times\frac{0.0635\,\text{m}}{1.52\,\text{m}}\times\frac{}{(0.817\times23.1)^2}\frac{\text{s}^2}{\text{m}^2}$$

$$f = 0.0190 \quad \underline{\qquad\qquad\qquad f}$$

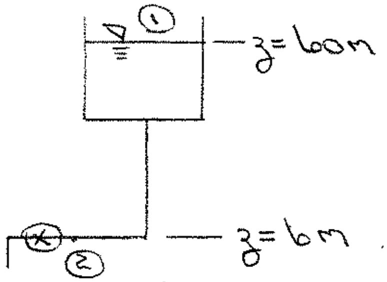
From Eq. 8.37 at $Re = 8.26\times10^4$ for smooth tube, $f = 0.0187$

The value of f is obtained using Excel's Solver (or Goal Seek)

Problem *8.151

Given: Oil flowing from a large tank on a hill to a tanker at the wharf. In stopping the flow, valve on wharf at such a rate that $p_2 = 1\text{ MPa}$ is maintained in the line immediately upstream of the valve. Assume:

Length of line from tank to valve	3 km
Inside diameter of line	200 mm
Elevation of oil surface in tank	60 m
Elevation of valve on wharf	6 m
Instantaneous flow rate	2.5 m³/min
Head loss in line (exclusive of valve being closed) at this rate of flow	23 m of oil
Specific gravity of oil	0.88



Find: the initial instantaneous rate of change of volume flow rate.

Solution: For unsteady flow with friction, we modify the unsteady Bernoulli equation (Eq. 6.21) to include a head loss term.

Computing equation: $$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \int_1^2 \frac{\partial V_s}{\partial t}\,ds + h_\ell$$

Assume: (1) $V_1 \approx 0$ (3) ρ = constant
(2) $p_1 = p_{atm}$

Then
$$\int_1^2 \frac{\partial V_s}{\partial t}\,ds = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) - h_\ell - \frac{V_2^2}{2}$$

If we neglect velocity in the tank except for small region near the inlet to the pipe, then

$$\int_1^2 \frac{\partial V_s}{\partial t}\,ds \approx \int_0^L \frac{\partial V_s}{\partial t}\,ds$$. Since $V_s = V_2$ everywhere, then

$$\int_0^L \frac{\partial V_s}{\partial t}\,ds = L\frac{dV_2}{dt}$$ and

$$\frac{dV_2}{dt} = \frac{1}{L}\left[\frac{p_1 - p_2}{\rho} + g(z_1 - z_2) - h_\ell - \frac{V_2^2}{2}\right], \quad V_2 = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

Note $h_\ell = h_\ell(V)$ and hence this result can only be used to obtain the initial instantaneous rate of change of flow velocity

$$\therefore \left.\frac{dV_2}{dt}\right)_{initial} = \frac{1}{3\times10^3\,m}\left[10^6\,\frac{N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{1}{0.88}\,\frac{kg\cdot m}{N\cdot s^2} + 9.81\frac{m}{s^2}\times 54\,m\right.$$
$$\left. - 23\,m \times 9.81\frac{m}{s^2} - \frac{1}{2}\left\{\frac{4}{\pi}\times 2.5\frac{m^3}{min}\times\frac{1}{(0.2\,m)^2}\times\frac{1\,min}{60\,s}\right\}^2\right]$$

$$\left.\frac{dV_2}{dt}\right)_{initial} = -0.278\ m/s/s$$

The instantaneous rate of change of volume flow rate is

$$dQ/dt = \frac{d}{dt}(AV) = A\frac{dV}{dt} = \frac{\pi D^2}{4}\frac{dV}{dt}$$

$$dQ/dt = \frac{\pi}{4}(0.2\,m)^2 \times \left(-0.278\,\frac{m/s}{s} \times \frac{60\,s}{min}\right) = -0.524\ m^3/s/min \leftarrow dQ/dt$$

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42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
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Problem *8.152

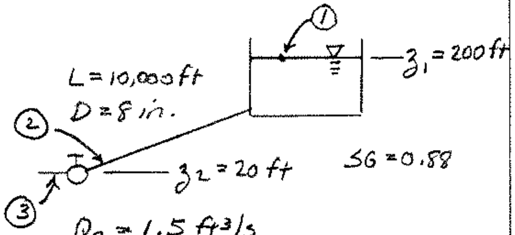
Given: Problem 8.151 describes a situation in which flow in a long pipeline from a hilltop tank is slowed gradually to avoid a large pressure rise.

Find: Expansion of this analysis to predict and plot the closing schedule (valve loss coefficient versus time) needed to maintain the maximum pressure at the valve at or below a given value throughout the process of stopping the flow from the tank.

Solution: Apply the unsteady Bernoulli equation with a head loss term added.

Computing equation:

$$\overset{\approx 0}{\cancel{\frac{p_1}{\rho}}} + \overset{\approx 0(1)}{\cancel{\frac{V_1^2}{2}}} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \int_1^2 \frac{\partial V}{\partial t}\,ds + h_{\ell T}$$



$Q_0 = 1.5$ ft³/s

Line loss = 75 ft of oil

Assume: (1) $V_1 \approx 0$ (3) ρ = constant

(2) $p_1 = p_{atm}$

At the initial condition, $V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times \left(\frac{12}{8}\right)^2 \frac{1}{ft^2} \times 1.5 \frac{ft^3}{s} = 4.30$ ft/s

$$H_{\ell T} = 75\,ft = \frac{h_{\ell T}}{g} = f\frac{L}{D}\frac{V^2}{2g}\,;\quad f\frac{L}{D} = H_{\ell T}\frac{2g}{V^2} = 2 \times 75\,ft \times 32.2\frac{ft}{s^2} \times \frac{s^2}{(4.30)^2 ft^2} = 261$$

Neglecting velocity in tank, $\int_1^2 \frac{\partial V}{\partial t}\,ds \approx \int_0^L \frac{\partial V}{\partial t}\,ds = \frac{dV}{dt}L$

Thus $\frac{dV}{dt} = \frac{1}{L}\left[-\frac{p_2}{\rho} + g(z_1 - z_2) - f\frac{L}{D}\frac{V^2}{2} - \frac{V^2}{2}\right]$

Substituting values,

$$\frac{dV}{dt} = \frac{1}{10{,}000\,ft}\left[-150\frac{lbf}{in.^2} \times \frac{ft^3}{(0.88)\,1.94\,slug} \times 144\frac{in.^2}{ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2} + 32.2\frac{ft}{s^2}(200-20)\,ft - (261+1)\frac{V^2}{2}\right]$$

$$\frac{dV}{dt} = -0.686\frac{ft}{s^2} - 0.0131\,V^2 = -(a^2 + b^2V^2)\,;$$

$a = \sqrt{0.686} = 0.828$; V in ft/s

$b = \sqrt{0.0131} = 0.114$

Separating variables and integrating

$$\int_{V_0}^{V} \frac{dV}{a^2 + b^2 t} = \frac{1}{ab}\tan^{-1}\frac{bV}{a}\Big]_{V_0}^{V} = \frac{1}{ab}\left[\tan^{-1}\frac{bV}{a} - \tan^{-1}\frac{bV_0}{a}\right] = -\int_0^t dt = -t$$

Thus

$$\tan^{-1}\frac{bV}{a} = -abt + \tan^{-1}\frac{bV_0}{a} \quad \text{or} \quad V = \frac{a}{b}\tan\left[\tan^{-1}\frac{bV_0}{a} - abt\right] \longleftarrow$$

V(t)

The pressure must drop across the valve:

$$\frac{p_2}{\rho} + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2} - \left(\frac{p_3}{\rho} + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right) = h_{\ell T} = K_V\frac{V^2}{2} \quad \text{or} \quad K_V = \frac{2(p_2 - p_3)}{\rho V^2} \approx \frac{2p_2}{\rho V^2}$$

$$\text{At } t=0,\; K_V = 2 \times 150\frac{lbf}{in.^2} \times \frac{ft^3}{(0.88)1.94\,slug} \times \frac{s^2}{(4.3)^2 ft^2} \times 144\frac{in.^2}{ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 1{,}370 \quad (t=0) \longleftarrow$$

$K_V(0)$

Calculations and plots are shown on the spreadsheet, next page.

Problem *8.152

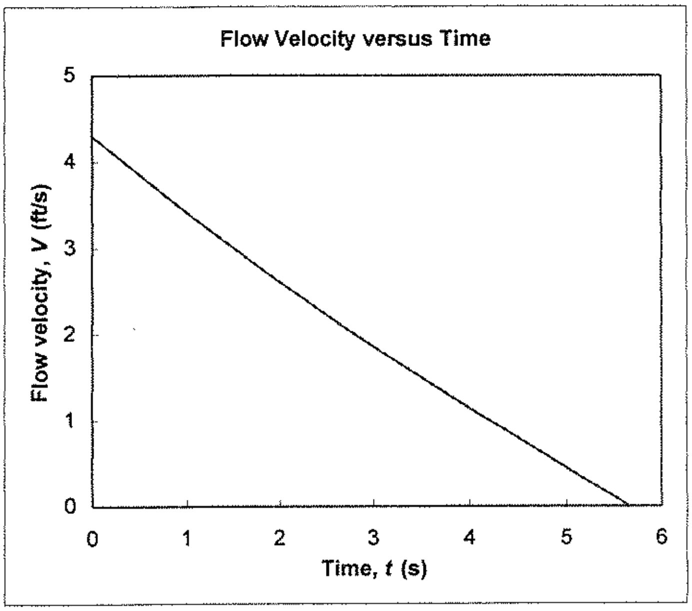
[5] Part 2/2

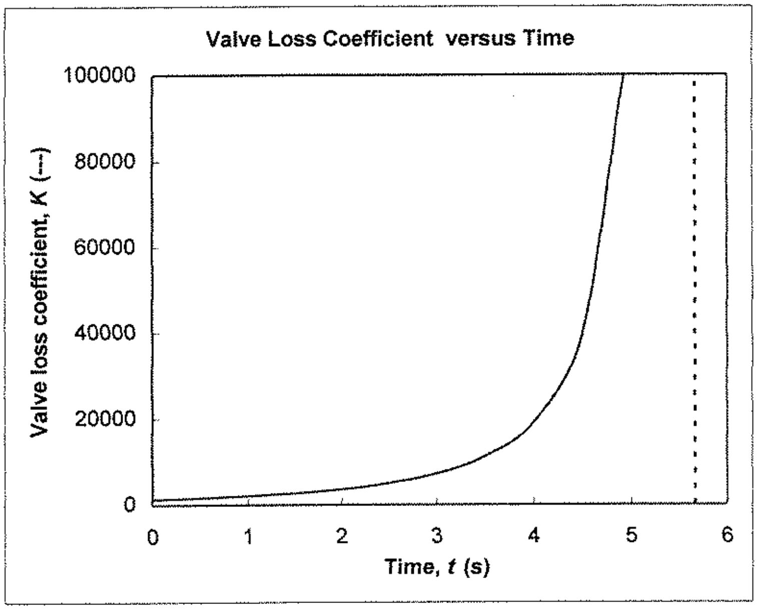
a = 0.828 (---)

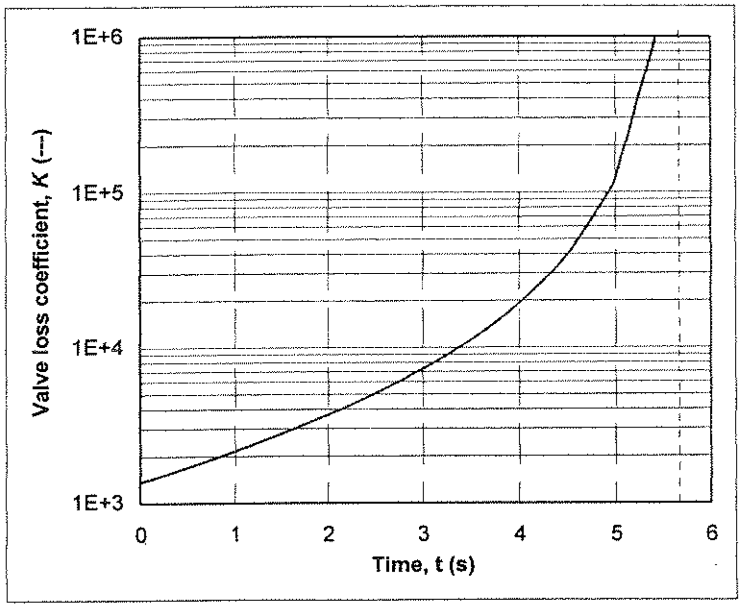
b = 0.114 (---)

V_0 = 4.3 (ft/s)

t (s)	***V*** (ft/s)	***K*** (---)
0.0	4.300	1.37E+3
0.5	3.849	1.71E+3
1.0	3.421	2.16E+3
1.5	3.011	2.79E+3
2.0	2.616	3.70E+3
2.5	2.235	5.07E+3
3.0	1.865	7.28E+3
3.5	1.504	1.12E+4
4.0	1.150	1.92E+4
4.5	0.801	3.95E+4
4.93	0.503	1.00E+5
5.0	0.455	1.22E+5
5.431	0.159	1.00E+6
5.5	0.112	2.03E+6
5.663	0.000	2.92E+13

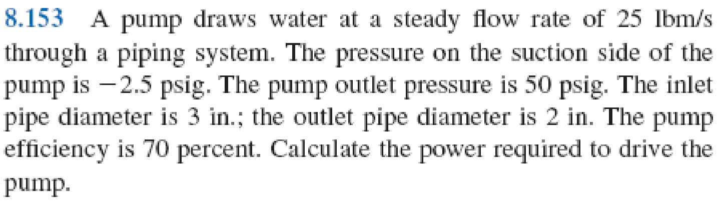






Problem 8.153

[2]



8.153 A pump draws water at a steady flow rate of 25 lbm/s through a piping system. The pressure on the suction side of the pump is −2.5 psig. The pump outlet pressure is 50 psig. The inlet pipe diameter is 3 in.; the outlet pipe diameter is 2 in. The pump efficiency is 70 percent. Calculate the power required to drive the pump.

Given: Flow through water pump

Find: Power required

Solution:

Basic equations

$$h_{pump} = \left(\frac{p_d}{\rho} + \frac{V_d^2}{2} + g\cdot z_d\right) - \left(\frac{p_s}{\rho} + \frac{V_s^2}{2} + g\cdot z_s\right) \qquad V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence for the inlet

$$V_s = \frac{4}{\pi} \times 25\cdot\frac{lbm}{s} \times \frac{1\cdot slug}{32.2\cdot lbm} \times \frac{ft^3}{1.94\cdot slug} \times \left(\frac{12}{3}\cdot\frac{1}{ft}\right)^2 \qquad V_s = 8.15\frac{ft}{s} \qquad p_s = -2.5\cdot psi$$

For the outlet

$$V_d = \frac{4}{\pi} \times 25\cdot\frac{lbm}{s} \times \frac{1\cdot slug}{32.2\cdot lbm} \times \frac{ft^3}{1.94\cdot slug} \times \left(\frac{12}{2}\cdot\frac{1}{ft}\right)^2 \qquad V_d = 18.3\frac{ft}{s} \qquad p_d = 50\cdot psi$$

Then

$$h_{pump} = \frac{p_d - p_s}{\rho} + \frac{V_d^2 - V_s^2}{2} \qquad \text{and} \qquad W_{pump} = m_{pump}\cdot h_{pump}$$

$$W_{pump} = m_{pump}\cdot\left(\frac{p_d - p_s}{\rho} + \frac{V_d^2 - V_s^2}{2}\right)$$

Note that the software cannot render a dot, so the power is W_{pump} and mass flow rate is m_{pump}!

$$W_{pump} = 25\cdot\frac{lbm}{s} \times \frac{1\cdot slug}{32.2\cdot lbm} \times \left[(50 - -2.5)\cdot\frac{lbf}{in^2} \times \left(\frac{12\cdot in}{1\cdot ft}\right)^2 \times \frac{ft^3}{1.94\cdot slug} + \frac{1}{2} \times \left(18.3^2 - 8.15^2\right)\cdot\left(\frac{ft}{s}\right)^2 \times \frac{lbf\cdot s^2}{slug\cdot ft}\right] \times \frac{1\cdot hp}{550\cdot\frac{ft\cdot lbf}{s}}$$

$W_{pump} = 5.69\,hp$ For an efficiency of $\eta = 70\cdot\%$ $\qquad W_{required} = \frac{W_{pump}}{\eta} \qquad W_{required} = 8.13\,hp$

Problem 8.154

8.154 The pressure rise across a water pump is 75 kPa when the volume flow rate is 25 L/s. If the pump efficiency is 80 percent, determine the power input to the pump.

Given: Flow through water pump

Find: Power required

Solution:

Basic equations $$h_{pump} = \left(\frac{p_d}{\rho} + \frac{V_d^2}{2} + g\cdot z_d\right) - \left(\frac{p_s}{\rho} + \frac{V_s^2}{2} + g\cdot z_s\right) \qquad V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

In this case we assume $D_s = D_d$ so $V_s = V_d$

Then $$h_{pump} = \frac{p_d - p_s}{\rho} = \frac{\Delta p}{\rho} \qquad \text{and} \qquad W_{pump} = m_{pump}\cdot h_{pump}$$

$$W_{pump} = m_{pump}\cdot\frac{\Delta p}{\rho} = \rho\cdot Q\cdot\frac{\Delta p}{\rho} = Q\cdot\Delta p$$

Note that the software cannot render a dot, so the power is W_{pump} and mass flow rate is m_{pump}!

$$W_{pump} = 25\cdot\frac{L}{s} \times \frac{0.001\cdot m^3}{1\cdot L} \times 75 \times 10^3\cdot\frac{N}{m^2} \qquad W_{pump} = 1.88\,kW$$

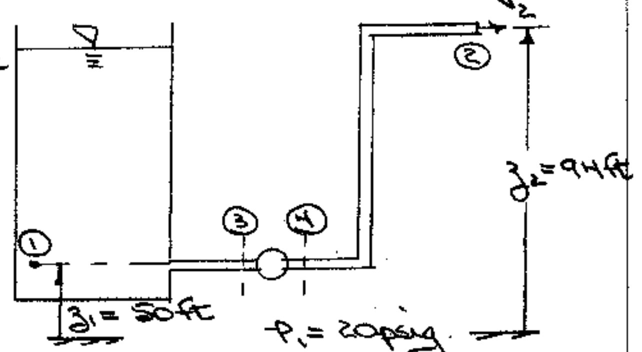
For an efficiency of $\eta = 80\cdot\%$ $$W_{required} = \frac{W_{pump}}{\eta} \qquad W_{required} = 2.34\cdot kW$$

Problem 8.155

[3]

Given: Pump in piping system shown moves $Q = 0.439\ ft^3/s$ of water.

System includes:
- $L = 290\ ft$ galvanized pipe $D = 2.5\ in.$ (nominal)
- 2 gate valves (open)
- 1 angle valve (open)
- 7 standard 90° elbows
- 1 square edge entrance
- 1 free discharge



Find: pressure rise, $p_4 - p_3$, across pump.

Solution:

Computing equation:
$$\left(\frac{p_1}{\rho} + \cancelto{0(1)}{\alpha_1\frac{\bar{V}_1^2}{2}} + gz_1\right) - \left(\cancelto{0(2)}{\frac{p_2}{\rho}} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump} = h_{l_T} \qquad (8.48)$$

$$h_{l_T} = h_l + h_{l_m}, \quad h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}, \quad h_{l_m} = \frac{\bar{V}^2}{2}\left\{\sum f\left(\frac{L_e}{D}\right) + \sum K\right\}$$

Assumptions: (1) $\bar{V}_1 = 0$ (2) $p_2 = p_{atm}$ (3) $\alpha_2 = 1.0$ (4) $T = 60°F$

Then,
$$\Delta h_{pump} = h_{l_T} + g(z_2 - z_1) + \frac{\bar{V}_2^2}{2} - \frac{p_1}{\rho} \quad \text{..............(1)}$$

$$h_{l_T} = \frac{\bar{V}_2^2}{2}\left[f\frac{L}{D} + 2f\left(\frac{L_e}{D}\right)_{gv} + f\left(\frac{L_e}{D}\right)_{av} + 7f\left(\frac{L_e}{D}\right)_{90°el} + K_{ent}\right] \quad \text{......(2)}$$

From Table 8.4 $L_e/D)_{gv} = 8$, $L_e/D)_{av} = 150$, $L_e/D)_{90°el} = 30$

From Table 8.2 $K_e = 0.5$. From Table 8.5 $D = 2.47\ in$

From Table 8.1 $e = 0.0005\ ft$ $\therefore\ e/D = 0.0005 \times \frac{12}{2.47} = 0.0024$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.439\frac{ft^3}{s} \times \left(\frac{12}{2.47\ ft}\right)^2 = 13.2\ ft/s$$

$$Re = \frac{D\bar{V}}{\nu} = \frac{2.47\ ft}{12} \times 13.2\frac{ft}{s} \times \frac{s}{1.21\times10^{-5}\ ft^2} = 2.25\times10^5 \qquad \{\nu \text{ from Table A.7}\}$$

From Fig. 8.13, $f = 0.025$.

From Eq. 2
$$h_{l_T} = \frac{1}{2}\times(13.2)^2\frac{ft^2}{s^2}\left[0.025\times\frac{290\times12}{2.47} + 2(0.025)(8) + (0.025)(150) + 7(0.025)(30) + 0.5\right]$$

$h_{l_T} = 3930\ ft^2/s^2$. Then from Eq. 1

$$\Delta h_{pump} = 3930\frac{ft^2}{s^2} + 32.2\frac{ft}{s^2}(44\ ft) + \frac{1}{2}(13.2)^2\frac{ft^2}{s^2} - 20\frac{lbf}{in^2}\times\frac{ft^3}{1.94\ slug}\times\frac{144\ in^2}{ft^2}\times\frac{slug\cdot ft}{lbf\cdot s^2}$$

$$\Delta h_{pump} = 3950\ ft^2/s^2$$

Apply the energy equation across the pump
$$\Delta h_{pump} = \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + \cancel{gz}\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + \cancel{gz}\right)_{suction} \qquad (8.47)$$

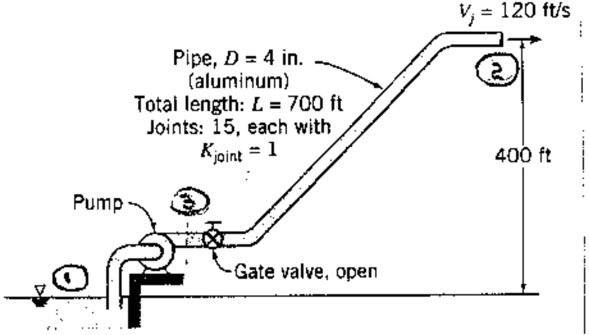
$$\Delta p = \rho\,\Delta h_{pump} = 1.94\frac{slug}{ft^3}\times 3950\frac{ft^2}{s^2}\times\frac{lbf\cdot s^2}{ft\cdot slug}\times\frac{ft^2}{144\ in^2} = 53.2\ lbf/in^2 \ \underline{(p_4 - p_3)}$$

Problem 8.156

Given: Cooling water supply system

$Q = 600$ gpm

$\eta_{pump} = 0.70$



Find: (a) minimum pressure needed at pump outlet
(b) power requirement

Solution:

Computing equations: $\left(\cancel{\frac{p_1}{\rho}} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}} + \cancel{g z_1}\right) - \left(\cancel{\frac{p_2}{\rho}} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) + \Delta h_{pump} = h_{l_T}$ (8.48)

$$h_{l_T} = h_l + h_{l_m}, \quad h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}, \quad h_{l_m} = \frac{\bar{V}^2}{2}\left(\sum K + \sum f\left(\frac{L_e}{D}\right)\right)$$

Assumptions: (1) $\bar{V}_1 = 0$ (2) $\alpha_2 = \alpha_3 = 1$ (3) $p_1 = p_2 = p_{atm}$

Then

$$\Delta h_{pump} = g z_2 + \frac{\bar{V}_2^2}{2} + f \frac{L}{D} \frac{\bar{V}^2}{2} + \frac{\bar{V}^2}{2}\left[K_{ent} + f\left(\frac{L_e}{D}\right)_{90^\circ el} + 2f\left(\frac{L_e}{D}\right)_{45^\circ el} + 15 K_J\right] \quad (1)$$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 600 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,sec} \times \frac{1}{(0.333\,ft)^2} = 15.3\ ft/s$$

$$Re = \frac{D\bar{V}}{\nu} = \frac{1}{3} ft \times 15.3 \frac{ft}{s} \times 1.24 \times 10^{5} \frac{s}{ft^2} = 4.11 \times 10^5 \qquad \{\nu \text{ at } T = 60^\circ F, \text{ Table A.7}\}$$

Table 8.1, $e = 5 \times 10^{-6}$ ft (drawn tubing) $\therefore e/D = 1.5 \times 10^{-5}$

From Fig. 8.13, $f = 0.0135$

From Table 8.1, $K_{ent} = 0.78$

From Table 8.4, $L_e/D)_{g.v} = 8$, $L_e/D)_{90^\circ el} = 30$, $L_e/D)_{45^\circ el} = 16$

Then from Eq. (1)

$$\Delta h_{pump} = 32.2 \frac{ft}{s^2} \times 400\,ft + \frac{1}{2}(120)^2 \frac{ft^2}{s^2} + 0.0135 \times \frac{700}{0.333} \frac{1}{2} \times (15.3)^2 \frac{ft^2}{s^2}$$
$$+ \frac{1}{2}(15.3)^2 \frac{ft^2}{s^2}\left[0.78 + 0.0135(30) + 2 \times 0.0135(16) + 15(1)\right]$$

$\Delta h_{pump} = 2.53 \times 10^4\ ft^2/s^2$

The theoretical power input to the pump is given by $\dot{W}_{pump} = \dot{m}\,\Delta h_{pump}$

From the definition of efficiency, $\eta = \dot{W}_{theor}/\dot{W}_{act}$, then

$$\dot{W}_{act} = \frac{\dot{m}\,\Delta h_{pump}}{\eta} = \frac{\rho Q\,\Delta h_{pump}}{\eta}$$

$$\dot{W}_{act} = \frac{1.94}{0.7} \frac{slug}{ft^3} \times 600 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times 2.53 \times 10^4 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{ft \cdot slug} \times \frac{hp \cdot s}{550\,ft \cdot lbf} = \underline{170\,hp} \quad \dot{W}_{in}$$

The discharge pressure from the pump is obtained by applying Eq 8.48 between sections ① and ③ neglecting losses in the inlet section, elevation change, and kinetic energy at ③

$$p_3 - p_A = \rho\,\Delta h_{pump} = 1.94 \frac{slug}{ft^3} \times 2.53 \times 10^4 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{ft \cdot s} \times \frac{ft^2}{144\,in^2} = \underline{341\,psi} \quad p_3$$

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42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.

Problem 8.157

Given: Water supply system requires $Q = 100$ gpm pumped to reservoir at elevation of 340 m. Water pressure at pump inlet (street level) is 400 kPa (gage). Piping is to be commercial steel; $\bar{V}_{max} = 3.5$ m/s

Find:
(a) Minimum pipe diameter
(b) pressure rise across the pump
(c) minimum power needed to drive the pump

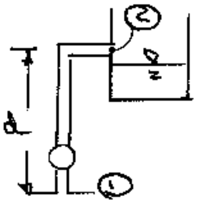
Solution:

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} \quad \therefore D = \left[\frac{4Q}{\pi \bar{V}}\right]^{1/2} = \left[\frac{4}{\pi} \times \frac{100\,\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{min}}{60\,\text{s}} \times \left(\frac{0.305\,\text{m}}{\text{ft}}\right)^3 \times \frac{\text{s}}{3.5\,\text{m}}\right]^{1/2}$$

$$D = 0.048\,\text{m} = 48\,\text{mm} \longleftarrow D$$

Computing equations:
$$\left(\frac{p_1}{\rho} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}} + gz_1\right) - \left(\overset{0(2)}{\cancel{\frac{p_2}{\rho}}} + \underset{(1)}{\cancel{\alpha_2 \frac{\bar{V}_2^2}{2}}} + gz_2\right) + \Delta h_{pump} = h_{l_T} \quad (8.48)$$

$$h_{l_T} = h_l + \overset{(3)}{\cancel{h_{l_m}}}, \quad h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$



Assume:
(1) $\alpha_1 = \alpha_2$
(2) $p_2 = p_{atm}$
(3) minor losses are negligible.
(4) water at 20°C ($\nu = 1\times10^{-6}$ m²/s, Table A.8).

Then
$$\Delta h_{pump} = h_l - \frac{p_1}{\rho} + g(z_2 - z_1) = f \frac{L}{D} \frac{\bar{V}^2}{2} - \frac{p_1}{\rho} + gd \quad \ldots\ldots\ldots (1)$$

From Table 8.1, $e = 0.046$ mm $\therefore e/D = 0.046/48 = 0.00096$

$$Re = \frac{D\bar{V}}{\nu} = 0.048\,\text{m} \times 3.5\,\frac{\text{m}}{\text{s}} \times \frac{\text{s}}{1\times10^{-6}\,\text{m}^2} = 1.68\times10^{5}$$

From Fig. 8.13, $f = 0.021$. Then from Eq. 1

$$\Delta h_{pump} = 0.021 \times \frac{340}{0.048} \times \frac{1}{2}(3.5)^2\frac{\text{m}^2}{\text{s}^2} - 400\times10^3\,\frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{998\,\text{kg}} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} + 9.81\,\frac{\text{m}}{\text{s}^2} \times 340\,\text{m}$$

$$\Delta h_{pump} = 3{,}850\ \text{m}^2/\text{s}^2 \quad \text{(This is head added to fluid).}$$

$$\Delta h_{pump} = \frac{\dot{W}_{pump}}{\dot{m}} = \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{suction} \quad (8.47)$$

Assume: (5) $\bar{V}_{disch} = \bar{V}_{suct}$; $z_{discharge} \approx z_{suction}$

$$\Delta p = \rho\,\Delta h = 998\,\frac{\text{kg}}{\text{m}^3} \times 3850\,\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 3840\,\text{kPa} \longleftarrow \Delta p$$

Also from Eq. 8.47

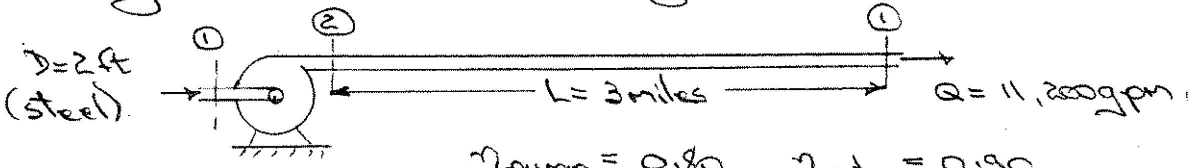
$$\dot{W}_{pump} = \dot{m}\,\Delta h_{pump} = \rho Q\,\Delta h_{pump}$$

$$\dot{W}_{pump} = 998\,\frac{\text{kg}}{\text{m}^3} \times 100\,\frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \left(\frac{0.305\,\text{m}}{\text{ft}}\right)^3 \times \frac{\text{min}}{60\,\text{s}} \times 3850\,\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$\dot{W}_{pump} = 24.3\,\text{kW}\ (32.6\,\text{hp}) \longleftarrow \dot{W}_{pump}$$

Problem 8.158

Given: Chilled-water pipe system for campus air conditioning makes a loop of length $L = 3$ miles.



$\eta_{pump} = 0.80$, $\eta_{motor} = 0.90$

$C = \$0.12/(kW\cdot hr)$

Find: (a) the pressure drop, $p_2 - p_1$
(b) rate of energy addition to the water
(c) daily cost of electrical energy for pumping

Solution:

Apply energy equation for steady, incompressible pipe flow from pump discharge around loop to pump inlet

Computing equations: $\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar V_2^2}{2}+gz_2\right)-\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar V_1^2}{2}+gz_1\right)=h_{l_T}$ (8.29)

$h_{l_T} = h_l + \overset{(3)}{\cancel{h_{l_m}}}$ $\quad h_l = f\frac{L}{D}\frac{\bar V^2}{2}$

Assumptions: (1) $\alpha_1=\alpha_2$, (2) $z_1=z_2$ (3) neglect minor losses

Then

$p_2 - p_1 = f\frac{L}{D}\rho\frac{\bar V^2}{2}$, $\bar V = \frac{Q}{A} = \frac{11{,}200\,gal}{min}\times\frac{4}{\pi(2)^2 ft^2}\times\frac{ft^3}{7.48\,gal}\times\frac{min}{60\,s} = 7.94\,ft/s$

Assume $T = 50°F$, so $\nu = 1.40\times10^{-5}\,ft^2/s$.

$Re = \frac{D\bar V}{\nu} = 2\,ft\times 7.94\,\frac{ft}{s}\times\frac{s}{1.40\times10^{-5}\,ft^2} = 1.13\times10^6$

From Table 8.1, $e = 0.00015\,ft$; $\therefore e/D = 0.000075$. Then, from Fig. 8.13, $f = 0.013$, and

$\Delta p = p_2-p_1 = 0.013\times\frac{3\,mi}{2\,ft}\times\frac{5280\,ft}{mi}\times 1.94\,\frac{slug}{ft^3}\times\frac{1}{2}\times(7.94)^2\frac{ft^2}{s^2}\times\frac{lbf\cdot s^2}{ft\cdot slug}\times\frac{ft^2}{144\,in^2}$

$\Delta p = 43.7\,psi. \xleftarrow{\hspace{6cm}} \Delta p$

To determine the energy per unit mass applied by the pump

$\frac{\dot W_{pump}}{\dot m} = \left(\frac{p}{\rho}+\cancel{\frac{\bar V^2}{2}}+\cancel{gz}\right)_{discharge}-\left(\frac{p}{\rho}+\cancel{\frac{\bar V^2}{2}}+\cancel{gz}\right)_{suction}$ (8.45)

$\dot W_{pump} = \dot m\,\frac{\Delta p}{\rho} = Q\,\Delta p$

$\dot W_{pump} = 11{,}200\,\frac{gal}{min}\times\frac{ft^3}{7.48\,gal}\times\frac{min}{60\,s}\times\frac{43.7\,lbf}{in^2}\times\frac{144\,in^2}{ft^2}\times\frac{hp\cdot s}{550\,ft\cdot lbf} = 286\,hp \xleftarrow{} \dot W_{pump}$

The actual energy required to run the pump is

$P = \frac{\dot W_{pump}}{\eta_{pump}\,\eta_{motor}} = 286\,hp\times\frac{1}{0.80}\times\frac{1}{0.90} = 397\,hp.$

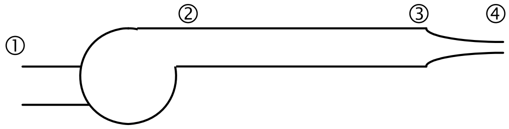
The daily cost is

$C = \frac{\$0.12}{kW\cdot hr}\times 397\,hp\times 0.746\,\frac{kW}{hp}\times\frac{24\,hr}{day} = \$853$

Problem 8.159

[4]

8.159 A fire nozzle is supplied through 100 m of 3.5-cm diameter, smooth, rubber-lined hose. Water from a hydrant is supplied to a booster pump on board the pumper truck at 350 kPa (gage). At design conditions, the pressure at the nozzle inlet is 700 kPa (gage), and the pressure drop along the hose is 750 kPa per 100 m of length. Determine (a) the design flow rate, (b) the nozzle exit velocity, assuming no losses in the nozzle, and (c) the power required to drive the booster pump, if its efficiency is 70 percent.



Given: Fire nozzle/pump system

Find: Design flow rate; nozzle exit velocity; pump power needed

Solution:

Basic equations $\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)-\left(\frac{p_3}{\rho}+\alpha\cdot\frac{V_3^2}{2}+g\cdot z_3\right)=h_l$ $\quad h_l=f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}$ for the hose

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 2 and 3 is approximately 1 4) No minor loss

$$\frac{p_3}{\rho}+\frac{V_3^2}{2}+g\cdot z_3=\frac{p_4}{\rho}+\frac{V_4^2}{2}+g\cdot z_4$$ for the nozzle (assuming Bernoulli applies)

$$\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)-\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)=h_{pump}$$ for the pump

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) No minor loss

Hence for the hose $\frac{\Delta p}{\rho}=\frac{p_2-p_3}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$ or $V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$

We need to iterate to solve this for V because f is unknown until Re is known. This can be done using *Excel*'s Solver, but here:

$\Delta p=750\cdot\text{kPa}$ $\quad L=100\cdot\text{m}$ $\quad e=0$ $\quad D=3.5\cdot\text{cm}$ $\quad \rho=1000\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad \nu=1.01\times10^{-6}\cdot\frac{\text{m}^2}{\text{s}}$

Make a guess for f $\quad f=0.01$ $\quad V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V=7.25\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re=2.51\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0150$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V=5.92\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re=2.05\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0156$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V=5.81\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re=2.01\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f=0.0156$

$V = \sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\qquad V = 5.80\,\frac{m}{s}$ $\qquad Re = \frac{V\cdot D}{\nu}$ $\qquad Re = 2.01\times 10^{5}$

$Q = V\cdot A = \frac{\pi\cdot D^{2}}{4}\cdot V$ $\qquad Q = \frac{\pi}{4}\times(0.035\cdot m)^{2}\times 5.80\cdot\frac{m}{s}$ $\qquad Q = 5.58\times 10^{-3}\,\frac{m^{3}}{s}$ $\qquad Q = 0.335\,\frac{m^{3}}{min}$

For the nozzle $\quad \frac{p_3}{\rho}+\frac{V_3^{2}}{2}+g\cdot z_3 = \frac{p_4}{\rho}+\frac{V_4^{2}}{2}+g\cdot z_4$ $\quad$ so $\quad V_4 = \sqrt{\frac{2\cdot\left(p_3-p_4\right)}{\rho}+V_3^{2}}$

$V_4 = \sqrt{2\times 700\times 10^{3}\cdot\frac{N}{m^{2}}\times\frac{m^{3}}{1000\cdot kg}\times\frac{kg\cdot m}{s^{2}\cdot N}+\left(5.80\cdot\frac{m}{s}\right)^{2}}$ $\qquad V_4 = 37.9\,\frac{m}{s}$

For the pump $\quad \left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{2}}{2}+g\cdot z_2\right)-\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{2}}{2}+g\cdot z_1\right) = h_{pump}$ $\quad$ so $\quad h_{pump} = \frac{p_2-p_1}{\rho}$

$p_1 = 350\cdot kPa$ $\qquad p_2 = 700\cdot kPa + 750\cdot kPa$ $\qquad p_2 = 1450\,kPa$

The pump power is $\quad P_{pump} = m_{pump}\cdot h_{pump}$ $\quad$ where P_{pump} and m_{pump} are the pump power and mass flow rate (software cannot render a dot!)

$P_{pump} = \rho\cdot Q\cdot\frac{\left(p_2-p_1\right)}{\rho} = Q\cdot\left(p_2-p_1\right)$ $\qquad P_{pump} = 5.58\times 10^{-3}\cdot\frac{m^{3}}{s}\times(1450-350)\times 10^{3}\cdot\frac{N}{m^{2}}$ $\qquad P_{pump} = 6.14\,kW$

$P_{required} = \frac{P_{pump}}{\eta}$ $\qquad P_{required} = \frac{6.14\cdot kW}{70\cdot\%}$ $\qquad P_{required} = 8.77\,kW$

Problem 8.160

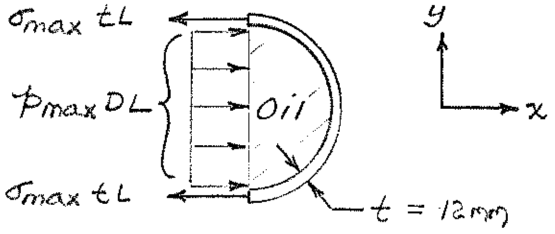
[4] Part 1/2

Given: Heavy crude oil ($SG = 0.925$) pumped through a level pipeline at a rate of 400,000 barrels per day (1 bbl = 42 gal). Pipe is 600 mm in diameter with 12 mm wall thickness. Maximum allowable stress in pipe wall is 275 MPa. Minimum pressure in oil is 500 kPa ($\nu = 1.0\times10^{-4}\ m^2/s$) Pipeline is steel.

Find: (a) Maximum allowable spacing between pumping stations.
(b) Power added to oil at each pumping station.

Solution: First find the maximum pressure allowable in pipe.
Consider a free body diagram of a segment of length, L:

Basic equation: $\sum F_x = 0$



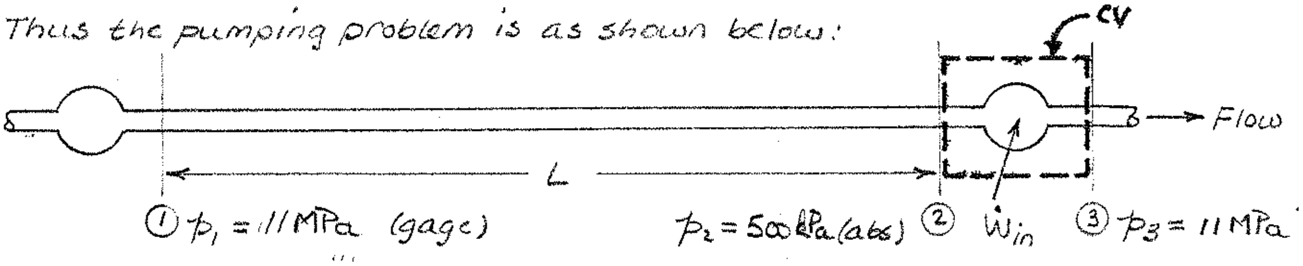
Assumption: Neglect hydrostatic pressure variation, and atmospheric pressure

Then

$$\sum F_x = p_{max} DL - 2\sigma_{max} tL = 0$$

$$p_{max} = 2\sigma_{max}\frac{t}{D} = 2\times 275\ MPa \times \frac{12mm}{600mm} = 11\ MPa\ (gage)$$

Thus the pumping problem is as shown below:



To find L, apply the energy equation for steady, incompressible flow that is uniform at each section.

Basic equation: $\frac{p_1}{\rho} + \cancelto{(1)}{\frac{\bar{V}_1^2}{2}} + \cancelto{(2)}{gz_1} = \frac{p_2}{\rho} + \cancelto{(1)}{\frac{\bar{V}_2^2}{2}} + \cancelto{(2)}{gz_2} + h_{l_T}\ ;\ h_{l_T} = f\frac{L}{D}\frac{\bar{V}^2}{2} + \cancelto{=0(3)}{h_{l_m}}$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$
(2) $z_1 = z_2$ (level)
(3) $h_{l_m} = 0$, since straight, constant area pipe

Then

$$f\frac{L}{D}\frac{\bar{V}^2}{2} = \frac{p_1 - p_2}{\rho} \quad or \quad L = \frac{D}{f}\left(\frac{p_1-p_2}{\rho}\right)\frac{2}{\bar{V}^2} \quad \text{- - - - - - - - - -}(1)$$

$$\bar{V} = \frac{Q}{A} = 4\times10^5\frac{bbl}{day}\times\frac{day}{24hr}\times\frac{hr}{3600s}\times\frac{42gal}{bbl}\times\frac{4qt}{gal}\times\frac{9.46\times10^{-4}m^3}{qt}\times\frac{4}{\pi}\frac{1}{(0.6m)^2} = 2.60 m/s$$

$f = f(Re, e/D)$. From Table 8.1, $e = 0.046mm$, so $e/D = 7.7\times10^{-5}$ Reynolds number is

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{\bar{V}D}{\nu} = 2.6\frac{m}{s}\times 0.6m\times\frac{s}{1.0\times10^{-4}m^2} = 1.56\times10^4$$

From Eq 8.37, $f = 0.0277$ (Using Excel's Solver or Goal Seek)

Thus, substituting into Eq. 1

$$L = \frac{0.6\,\mathrm{m}}{0.0277}\left[11\times10^6\,\frac{\mathrm{N}}{\mathrm{m}^2} - (500-101)\times10^3\,\frac{\mathrm{N}}{\mathrm{m}^2}\right]\times\frac{\mathrm{m}^3}{(0.925)\,999\,\mathrm{kg}}\times 2\times\frac{\mathrm{s}^2}{(2.16)^2\,\mathrm{m}^2}\times\frac{\mathrm{kg\cdot m}}{\mathrm{N\cdot s}^2}$$

$$L = 72.8\ \mathrm{km}$$

L

To find pump power delivered to the oil, apply the energy equation to the CV shown, between sections ② and ③

$$\left(\frac{p}{\rho}+\alpha\frac{\bar{V}^2}{2}+gz\right)_{discharge}-\left(\frac{p}{\rho}+\alpha\frac{\bar{V}^2}{2}+gz\right)_{suction}=\frac{\dot{W}_{pump}}{\dot{m}}=\Delta h_{pump} \qquad (8.45)$$

Since $\bar{V}$ = constant and elevation change is small, this reduces to

$$\Delta h_{pump} = \frac{p_3-p_2}{\rho}$$

$$= \left[11\times10^6-(500-101)\times10^3\right]\frac{\mathrm{N}}{\mathrm{m}^2}\times\frac{\mathrm{m}^3}{(0.925)\,999\,\mathrm{kg}}\times\frac{\mathrm{kg\cdot m}}{\mathrm{N\cdot s}^2}$$

$$\Delta h_{pump} = 1.15\times10^4\ \mathrm{m^2/s^2}$$

The mass flow rate is

$$\dot{m} = \rho Q = (0.925)\,999\,\frac{\mathrm{kg}}{\mathrm{m}^3}\times 400{,}000\,\frac{\mathrm{bbl}}{\mathrm{day}}\times 42\,\frac{\mathrm{gal}}{\mathrm{bbl}}\times 9.46\times10^{-4}\,\frac{\mathrm{m}^3}{\mathrm{qt}}\times\frac{4\,\mathrm{qt}}{\mathrm{gal}}\times\frac{\mathrm{day}}{24\,\mathrm{hr}}\times\frac{\mathrm{hr}}{3600\,\mathrm{s}}$$

$$\dot{m} = 680\ \mathrm{kg/s}$$

The power added to the oil is

$$\dot{W}_{pump} = \dot{m}\,\Delta h_{pump}$$

$$= 680\,\frac{\mathrm{kg}}{\mathrm{s}}\times 1.15\times10^4\,\frac{\mathrm{m}^2}{\mathrm{s}^2}\times\frac{\mathrm{N\cdot s}^2}{\mathrm{kg\cdot m}}$$

$$\dot{W}_{pump} = 7{,}730\ \mathrm{kW}$$

$\dot{W}_{pum}$

Note pump efficiency does not affect the power that must be added to the oil.

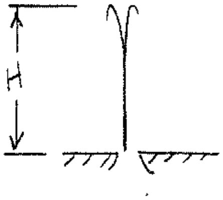
Problem 8.161

[4]

Given: Fountain on Purdue's Engineering Mall has $Q = 550$ gpm and $H = 10$ m (32.8 ft)

Find: Estimate of annual cost to operate the fountain.

Solution: Model fountain as a vertical jet (this will give maximum cost).



Computing equations:

$$C(\$/yr) = C_e\left(\frac{\$}{kW\cdot hr}\right) \times \mathcal{P}_{motor}(kW) \times N(hr/yr)$$

Assume $C_e = \$0.12/kW\cdot hr$

$$\mathcal{P}_{motor} = \frac{\mathcal{P}_{hydraulic}}{\eta_{pump}\eta_{motor}} \; ; \; \eta_{motor} = 0.9, \; \eta_{pump} = 0.8$$

$$\mathcal{P}_{hydraulic} = Q\Delta p$$

$$N = 365 \frac{days}{yr} \times 24 \frac{hr}{day} = 8{,}760 \; hr/yr$$

The minimum required Δp is $\rho g H$, so

$$\Delta p = 1.94 \frac{slug}{ft^3} \times 32.2 \frac{ft}{s^2} \times 32.8 \; ft \times \frac{lbf \cdot s^2}{slug \cdot ft} = 2.05 \times 10^3 \; lbf/ft^2$$

Combining,

$$C = \frac{\$0.12}{kW\cdot hr} \times \frac{1}{0.8(0.9)} \times 550 \frac{gal}{min} \times 2.05 \times 10^3 \frac{lbf}{ft^2} \times 8{,}760 \frac{hr}{yr}$$
$$\times \frac{ft^3}{7.48 \; gal} \times \frac{hp\cdot min}{33{,}000 \; ft\cdot lbf} \times 0.746 \frac{kW}{hp}$$

$$C = \$4980/yr \longleftarrow$$

The fountain does not operate year-round. It might be more fair to say $C \approx \$13$ per day of operation.

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem 8.162

[5]

13-782 500 SHEETS, FILLER 5 SQUARE
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42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
National Brand

Given: Petroleum products transported long distances by pipeline, e.g., the Alaskan pipeline (see Example Problem 8.6).

Find: (a) Estimate of energy needed to pump typical petroleum product, expressed as a fraction of throughput energy carried by pipeline.

(b) Statement and critical assessment of assumptions.

Solution: From Example Problem 8.6, for the Alaskan pipeline, $Q = 1.6\times10^6$ bpd.

Thus $Q = 1.6\times10^6 \frac{\text{bbl}}{\text{day}} \times 42 \frac{\text{gal}}{\text{bbl}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{day}}{24\,\text{hr}} \times \frac{\text{hr}}{3600\,\text{s}} = 104\,\text{ft}^3/\text{s}$

and

$$\dot m = \rho Q = SG\,\rho_{H_2O}Q = 0.93 \times 1.94 \frac{\text{slug}}{\text{ft}^3} \times 104 \frac{\text{ft}^3}{\text{s}} = 188\,\text{slug/s}$$

The energy content of a typical petroleum product is about 18,000 Btu/lbm, so the throughput energy is

$$\dot E = e\dot m = 18{,}000 \frac{\text{Btu}}{\text{lbm}} \times 188 \frac{\text{slug}}{\text{s}} \times 32.2 \frac{\text{lbm}}{\text{slug}} = 1.09\times10^8\,\text{Btu/s}$$

$\dot E$

From Example Problem 8.6, each pumping station requires 36,800 hp, and they are located L = 120 mi apart.

The entire pipeline is about 750 mi long. Thus there must be $N = 750/120$ or about $N = 7$ pumping stations. Thus the total energy required to pump must be

$$\mathcal{P} = N\dot W = 7\,\text{stations} \times 36{,}800 \frac{\text{hp}}{\text{station}} = 258{,}000\,\text{hp}$$

$\mathcal{P}$

Expressed as a fraction of throughput energy

$$\frac{\mathcal{P}}{\dot E} = 258{,}000\,\text{hp} \times \frac{\text{s}}{1.09\times10^8\,\text{Btu}} \times 2545 \frac{\text{Btu}}{\text{hp}\cdot\text{hr}} \times \frac{\text{hr}}{3600\,\text{s}} = 1.67\times10^{-3} \text{ or } 0.00167$$

Thus about 0.167% of energy is used for transporting petroleum.

The assumptions outlined above appear reasonable. The computed result is probably accurate within ±10%.

A more universal metric would be energy per unit mass and distance, e.g., energy per ton-mile of transport.

$$\frac{E}{M\cdot L} = \frac{E/t}{M/t\cdot L} = \frac{\mathcal{P}}{\dot m L} = 36{,}800\,\text{hp} \times \frac{\text{s}}{188\,\text{slug}} \times \frac{1}{120\,\text{mi}} \times 2545 \frac{\text{Btu}}{\text{hp}\cdot\text{hr}} \times \frac{\text{slug}}{32.2\,\text{lbm}} \times 2000 \frac{\text{lbm}}{\text{ton}} \times \frac{\text{hr}}{3600\,\text{s}}$$

Thus

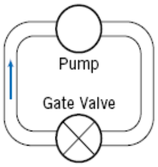
$$e = \frac{\mathcal{P}}{\dot m L} = 71.6\,\text{Btu/ton}\cdot\text{mi}$$

e

This specific metric allows direct comparison with other modes of transport.

Problem 8.163

8.163 The pump testing system of Problem 8.114 is run with a pump that generates a pressure difference given by $\Delta p = 750 - 15 \times 10^4 Q^2$ where Δp is in kPa, and the generated flow rate is Q m^3/s. Find the water flow rate, pressure difference, and power supplied to the pump if it is 70 percent efficient.



Given: Flow in a pump testing system

Find: Flow rate; Pressure difference; Power

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) = h_{lT} = \sum_{\text{major}} h_l + \sum_{\text{minor}} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for the circuit (1 = pump outlet, 2 = pump inlet)

$$\frac{p_1 - p_2}{\rho} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + 4 \cdot f \cdot L_{elbow} \cdot \frac{V^2}{2} + f \cdot L_{valve} \cdot \frac{V^2}{2}$$

or

$$\Delta p = \rho \cdot f \cdot \frac{V^2}{2} \cdot \left(\frac{L}{D} + 4 \cdot \frac{L_{elbow}}{D} + \frac{L_{valve}}{D}\right) \qquad (1)$$

This must be matched to the pump characteristic equation; at steady state, the pressure generated by the pump just equals that lost to friction in the circuit

$$\Delta p = 750 - 15 \times 10^4 \cdot Q^2 \qquad (2)$$

Finally, the power supplied to the pump, efficiency η, is

$$Power = \frac{Q \cdot \Delta p}{\eta} \qquad (3)$$

Given data:

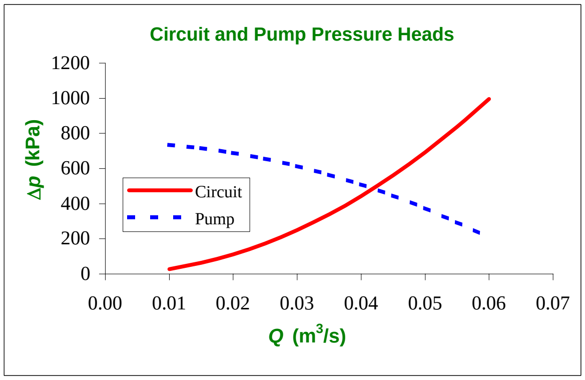
L = 20 m
D = 75 mm
η_{pump} = 70%

Tabulated or graphical data:

e = 0.26 mm
(Table 8.1)
μ = 1.00E-03 N.s/m^2
ρ = 999 kg/m^3
(Appendix A)
Gate valve L_e/D = 8
Elbow L_e/D = 30
(Table 8.4)

Computed results:

Q (m^3/s)	V (m/s)	Re	f	Δp (kPa) (Eq 1)	Δp (kPa) (Eq 2)
0.010	2.26	1.70E+05	0.0280	28.3	735
0.015	3.40	2.54E+05	0.0277	63.1	716
0.020	4.53	3.39E+05	0.0276	112	690
0.025	5.66	4.24E+05	0.0276	174	656
0.030	6.79	5.09E+05	0.0275	250	615
0.035	7.92	5.94E+05	0.0275	340	566
0.040	9.05	6.78E+05	0.0274	444	510
0.045	10.2	7.63E+05	0.0274	561	446
0.050	11.3	8.48E+05	0.0274	692	375
0.055	12.4	9.33E+05	0.0274	837	296
0.060	13.6	1.02E+06	0.0274	996	210



						Error
0.0419	9.48	7.11E+05	0.0274	487	487	0

Using *Solver* !

Power = 29.1 kW (Eq. 3)

Problem 8.164 Equations

8.164 A water pump can generate a pressure difference Δp (psi) given by $\Delta p = 145 - 0.1\,Q^2$, where the flow rate is Q ft^3/s. It supplies a pipe of diameter 20 in., roughness 0.5 in., and length 2500 ft. Find the flow rate, pressure difference, and the power supplied to the pump if it is 70 percent efficient. If the pipe were replaced with one of roughness 0.25 in., how much would the flow increase, and what would the required power be?

Given: Pump/pipe system

Find: Flow rate, pressure drop, and power supplied; Effect of roughness

Solution:

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad \left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) = h_{lT} - \Delta h_{pump} \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad \text{(Turbulent)}$$

The energy equation becomes for the system (1 = pipe inlet, 2 = pipe outlet)

$$\Delta h_{pump} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad \text{or} \quad \Delta p_{pump} = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (1)$$

This must be matched to the pump characteristic equation; at steady state, the pressure generated by the pump just equals that lost to friction in the circuit

$$\Delta p_{pump} = 145 - 0.1 \cdot Q^2 \qquad (2)$$

Finally, the power supplied to the pump, efficiency η, is

$$Power = \frac{Q \cdot \Delta p}{\eta} \qquad (3)$$

Tabulated or graphical data:

$\mu =$ 2.10E-05 lbf·s/ft^2

$\rho =$ 1.94 slug/ft^3

(Appendix A)

Given data:

$L =$ 2500 ft

$D =$ 20 in

$\eta_{pump} =$ 70%

Computed results: $e =$ 0.5 in

Q (ft^3/s)	V (ft/s)	Re	f	Δp (psi) (Eq 1)	Δp (psi) (Eq 2)
10	4.58	7.06E+05	0.0531	11.3	135.0
12	5.50	8.47E+05	0.0531	16.2	130.6
14	6.42	9.88E+05	0.0531	22.1	125.4
16	7.33	1.13E+06	0.0531	28.9	119.4
18	8.25	1.27E+06	0.0531	36.5	112.6
20	9.17	1.41E+06	0.0531	45.1	105.0
22	10.08	1.55E+06	0.0531	54.6	96.6
24	11.00	1.69E+06	0.0531	64.9	87.4
26	11.92	1.83E+06	0.0531	76.2	77.4
28	12.83	1.98E+06	0.0531	88.4	66.6
30	13.75	2.12E+06	0.0531	101.4	55.0

						Error	
26.1	12.0	1.84E+06	0.0531	76.8	76.8	0.00	Using *Solver* !

Power = 750 hp (Eq. 3)

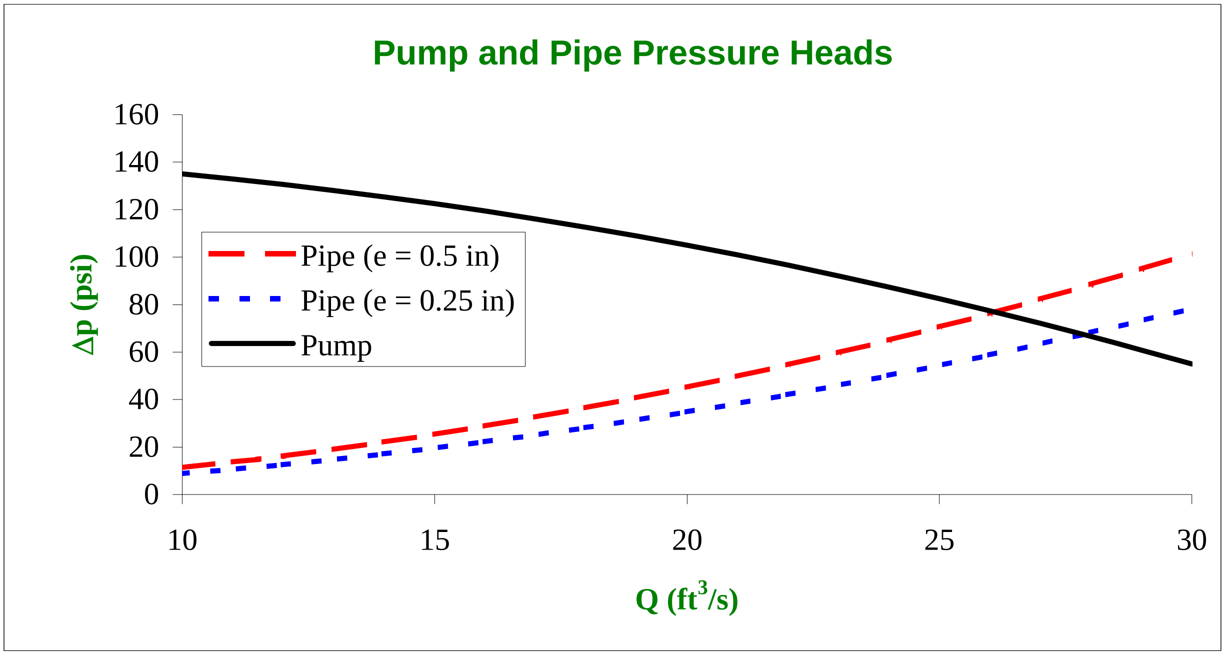
Repeating, with smoother pipe

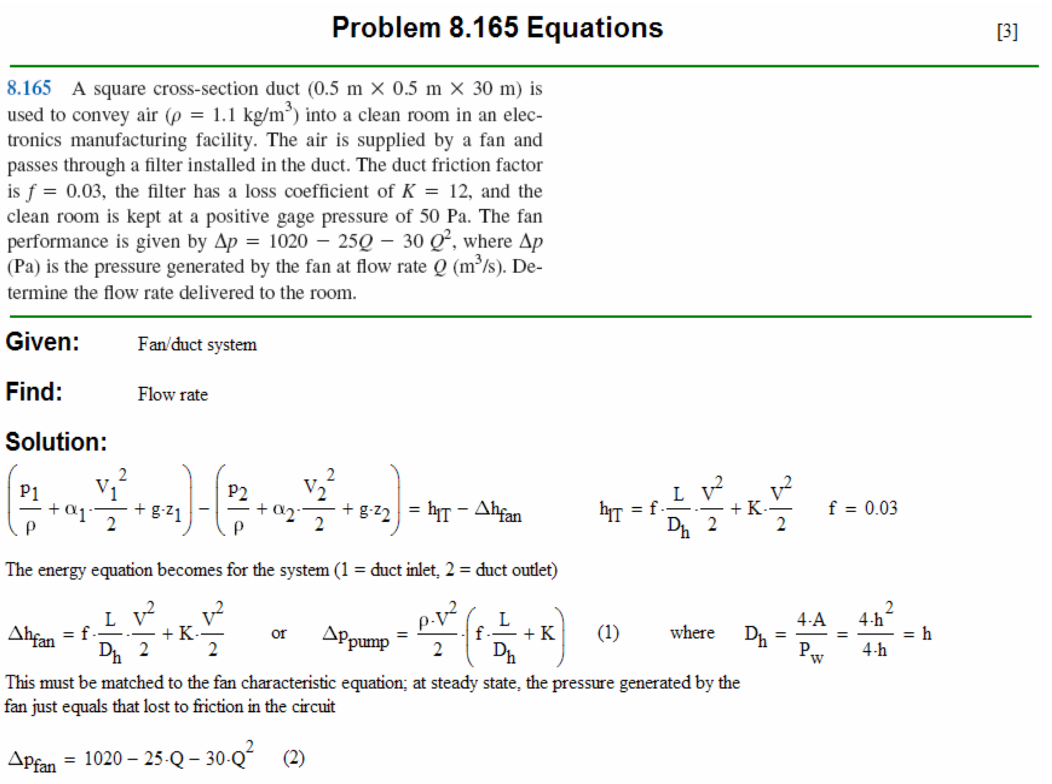
Computed results: *e* = 0.25 in

Q (ft^3/s)	*V* (ft/s)	*Re*	*f*	Δ*p* (psi) (Eq 1)	Δ*p* (psi) (Eq 2)
10	4.58	7.06E+05	0.0410	8.71	135.0
12	5.50	8.47E+05	0.0410	12.5	130.6
14	6.42	9.88E+05	0.0410	17.1	125.4
16	7.33	1.13E+06	0.0410	22.3	119.4
18	8.25	1.27E+06	0.0410	28.2	112.6
20	9.17	1.41E+06	0.0410	34.8	105.0
22	10.08	1.55E+06	0.0410	42.1	96.6
24	11.00	1.69E+06	0.0410	50.1	87.4
26	11.92	1.83E+06	0.0410	58.8	77.4
28	12.83	1.98E+06	0.0410	68.2	66.6
30	13.75	2.12E+06	0.0410	78.3	55.0

						Error	
27.8	12.8	1.97E+06	0.0410	67.4	67.4	0.00	Using *Solver* !

Power = 702 hp (Eq. 3)





Problem 8.165 Equations

[3]

8.165 A square cross-section duct (0.5 m × 0.5 m × 30 m) is used to convey air (ρ = 1.1 kg/m^3) into a clean room in an electronics manufacturing facility. The air is supplied by a fan and passes through a filter installed in the duct. The duct friction factor is f = 0.03, the filter has a loss coefficient of K = 12, and the clean room is kept at a positive gage pressure of 50 Pa. The fan performance is given by $\Delta p = 1020 - 25Q - 30\,Q^2$, where Δp (Pa) is the pressure generated by the fan at flow rate Q (m^3/s). Determine the flow rate delivered to the room.

Given: Fan/duct system

Find: Flow rate

Solution:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}-\Delta h_{fan} \qquad h_{lT}=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2} \qquad f=0.03$$

The energy equation becomes for the system (1 = duct inlet, 2 = duct outlet)

$$\Delta h_{fan}=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2} \quad \text{or} \quad \Delta p_{pump}=\frac{\rho\cdot V^2}{2}\cdot\left(f\cdot\frac{L}{D_h}+K\right) \quad (1) \qquad \text{where} \quad D_h=\frac{4\cdot A}{P_w}=\frac{4\cdot h^2}{4\cdot h}=h$$

This must be matched to the fan characteristic equation; at steady state, the pressure generated by the fan just equals that lost to friction in the circuit

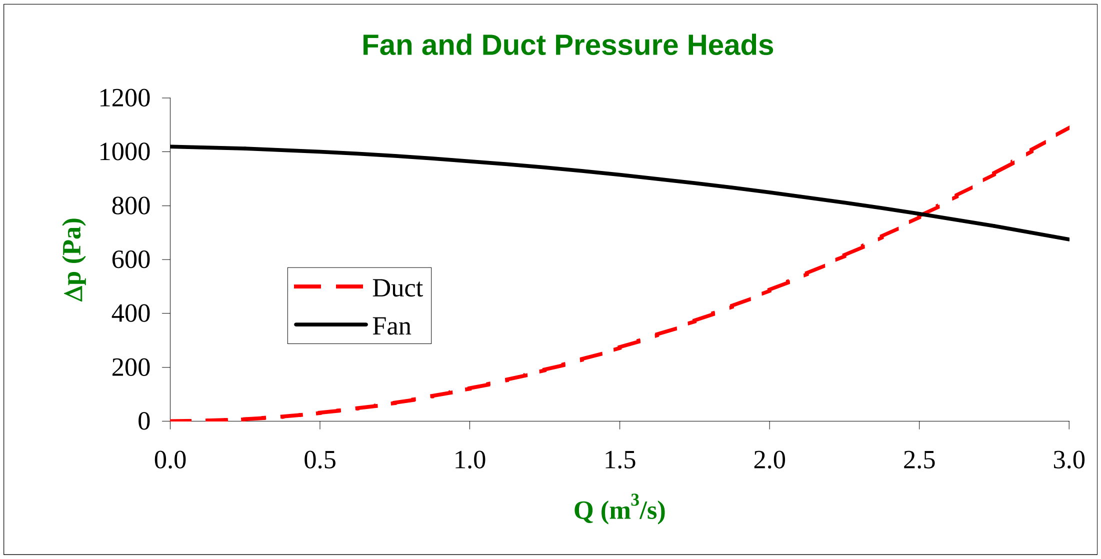
$$\Delta p_{fan}=1020-25\cdot Q-30\cdot Q^2 \quad (2)$$

Given data:

L =	30	m
D_h =	0.5	m
K =	12	
f =	0.03	
ρ =	1.1	kg/m^3

Computed results:

Q (m^3/s)	V (m/s)	Δp (Pa) (Eq 1)	Δp (Pa) (Eq 2)	Error	
0.00	0.00	0	1020		
0.25	1.00	8	1012		
0.50	2.00	30	1000		
0.75	3.00	68	984		
1.00	4.00	121	965		
1.25	5.00	190	942		
1.50	6.00	273	915		
1.75	7.00	372	884		
2.00	8.00	486	850		
2.25	9.00	615	812		
2.50	10.00	759	770		
2.75	11.00	918	724		
3.00	12.00	1093	675		
2.51	10.06	768	768	0.00	Using *Solver* !



Problem 8.166

Given: Fan with outlet dimensions of 8 × 16 in. Head vs Capacity curve is approximately

$$H\,(\text{in. } H_2O) = 30 - 10^{-7}\left[Q\,(\text{ft}^3/\text{min})\right]^2$$

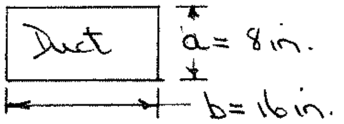
Find: Air flow rate delivered into a 200 ft. length of straight 8 × 16 in. duct.

Solution:

Basic equation: $\left(\frac{p_1}{\rho g} + \cancelto{(1)}{\alpha_1 \frac{\bar{V}_1^2}{2g}} + \cancelto{(2)}{z_1}\right) - \left(\frac{p_2}{\rho g} + \cancelto{(1)}{\alpha_2 \frac{\bar{V}_2^2}{2g}} + \cancelto{(2)}{z_2}\right) = H_{l_T}$ (8.30)

$$H_{l_T} = f\frac{L}{D_h}\frac{\bar{V}^2}{2g} + h_{l_m} \quad ; \quad D_h = \frac{4A}{P_w}$$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$, $\alpha_1 = \alpha_2 \simeq 1$
(2) $z_1 = z_2$
(3) $h_{l_m} = 0$



$A = ab = \frac{8}{12}\text{ ft} \times \frac{16}{12}\text{ ft} = 0.889\text{ ft}^2$

$D_h = \frac{4A}{P_w} = \frac{4A}{2(a+b)} = \frac{2 \times 0.889\text{ ft}^2}{(2/3 + 4/3)\text{ ft}} = 0.889\text{ ft}$

From Eq. 8.30 $\quad \Delta p = f\frac{L}{D}\rho_{air}\frac{\bar{V}^2}{2} = f\frac{L}{D}\frac{\rho}{2}\frac{Q^2}{A^2} = \gamma_{H_2O}H_{duct}$

where H_{duct} is the pressure drop in head of water

$$H_{duct} = \frac{fL\rho_{air}Q^2}{2\gamma_{H_2O}D_hA^2} = \frac{f}{2} \times \frac{200\text{ ft}}{0.889\text{ ft}} \times 0.00238\frac{\text{slug}}{\text{ft}^3} \times \frac{\text{ft}^3}{62.4\text{ lbf}} \times \frac{1}{(0.889)^2\text{ ft}^4} \times \left[Q\frac{\text{ft}^3}{\text{min}} \times \frac{\text{min}}{60\text{ s}}\right]^2 \times \frac{12\text{ in}}{\text{ft}}$$

$H_{duct} = 1.81 \times 10^{-5} fQ^2$ (where H is in in. H_2O) ………(1)

For a smooth duct, $f = f(Re)$

$Re = \frac{\bar{V}D_h}{\nu} = \frac{D_hQ}{\nu A}$. For $T = 68°F$, from Table A.9, $\nu = 1.62 \times 10^{-4}\text{ ft}^2/\text{s}$

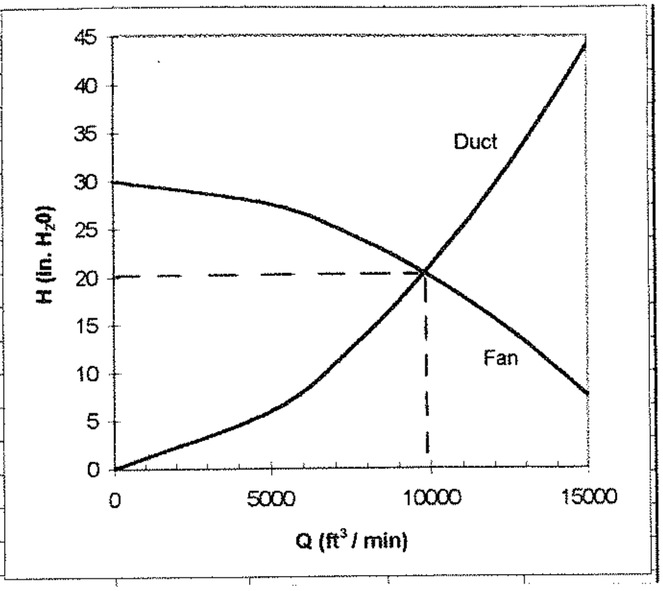
$$Re = \frac{0.889\text{ ft}}{0.889\text{ ft}^2} \times \frac{\text{s}}{1.62 \times 10^{-4}\text{ ft}^2} \times Q\frac{\text{ft}^3}{\text{min}} \times \frac{\text{min}}{60\text{ s}} = 103\,Q$$

To determine the air flow rate delivered we need to determine the operating point of the fan.
The operating point is at the intersection of the
- fan head capacity curve, and the
- system curve (the head loss in the duct)

This is shown on the plot below.
Note that the friction factor f is determined from the Colebrook equation (8.37a) using Eq. 8.37b for the initial estimate of f.

Problem 8.166

Q	Re	H (fan)	f_0	f	H (duct)
(ft^3/min)	(-)	(in.H_2O)	(-)	(-)	(in.H_2O)
0		30			0
5,000	5.15E+05	27.5	0.0130	0.0131	5.9
7,500	7.73E+05	24.4	0.0121	0.0122	12.4
10,000	1.03E+06	20.0	0.0115	0.0116	21.0
12,500	1.29E+06	14.4	0.0111	0.0112	31.6
15,000	1.55E+06	7.5	0.0108	0.0108	44.1



Operating point

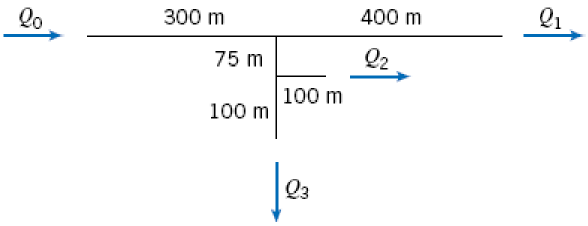
$Q \approx 10{,}000\ ft^3/min$

$H = 20\ in.\ H_2O$

Problem *8.167

[5]

*8.167 The water pipe system shown is constructed from 75 mm galvanized iron pipe. Minor losses may be neglected. The inlet is at 250 kPa (gage), and all exits are at atmospheric pressure. Find the flow rates Q_0, Q_1, Q_2, and Q_3. If the flow in the 400 m branch is closed off ($Q_1 = 0$), find the increase in flows Q_2, and Q_3.



Given: Pipe system

Find: Flow in each branch; Effect of shutting 400 m branch

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\ 2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\ 2}}{2} + g \cdot z_2\right) = h_l \qquad (8.29) \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (8.34)$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \quad (8.36) \qquad \frac{1}{f^{0.5}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot f^{0.5}}\right) \quad \text{(Turbulent)} \quad (8.37)$$

The energy equation (Eq. 8.29) can be simplified to $\Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$

This can be written for each pipe section

In addition we have the following contraints

$$Q_0 = Q_1 + Q_4 \qquad (1)$$

$$Q_4 = Q_2 + Q_3 \qquad (2)$$

$$\Delta p = \Delta p_0 + \Delta p_1 \qquad (3)$$

$$\Delta p = \Delta p_0 + \Delta p_4 + \Delta p_2 \qquad (4)$$

$$\Delta p_2 = \Delta p_3 \qquad (5)$$

(Pipe 4 is the 75 m unlabeled section)
We have 5 unknown flow rates (or, equivalently, velocities) and five equations

The workbook for Example 8.11 is modified for use in this problem

Pipe Data:

Pipe	*L* (m)	*D* (mm)	*e* (mm)
0	300	75	0.15
1	400	75	0.15
2	100	75	0.15
3	100	75	0.15
4	75	75	0.15

Fluid Properties:

ρ = 999 kg/m^3

μ = 0.001 N.s/m^2

Available Head:

Δp = 250 kPa

Flows:

Q_0 (m^3/s)	Q_1 (m^3/s)	Q_2 (m^3/s)	Q_3 (m^3/s)	Q_4 (m^3/s)
0.00928	0.00306	0.00311	0.00311	0.00623

V_0 (m/s)	V_1 (m/s)	V_2 (m/s)	V_3 (m/s)	V_4 (m/s)
2.10	0.692	0.705	0.705	1.41

Re_0	Re_1	Re_2	Re_3	Re_4
1.57E+05	5.18E+04	5.28E+04	5.28E+04	1.06E+05

f_0	f_1	f_2	f_3	f_4
0.0245	0.0264	0.0264	0.0264	0.0250

Heads:

Δp_0 (kPa)	Δp_1 (kPa)	Δp_2 (kPa)	Δp_3 (kPa)	Δp_4 (kPa)
216.4	33.7	8.7	8.7	24.8

Constraints:

(1) $Q_0 = Q_1 + Q_4$
0.00%

(2) $Q_4 = Q_2 + Q_3$
0.01%

(3) $\Delta p = \Delta p_0 + \Delta p_1$
0.03%

(4) $\Delta p = \Delta p_0 + \Delta p_4 + \Delta p_2$
0.01%

(5) $\Delta p_2 = \Delta p_3$
0.00%

Error: **0.05%** Vary Q_0, Q_1, Q_2, Q_3 and Q_4 using *Solver* to minimize total error

Problem 8.168

[3]

*8.168 A cast-iron pipe system consists of a 150 ft section of water pipe, after which the flow branches into two 150 ft sections, which then meet in a final 150 ft section. Minor losses may be neglected. All sections are 1.5 in. diameter, except one of the two branches, which is 1 in. diameter. If the applied pressure across the system is 50 psi, find the overall flow rate and the flow rates in each of the two branches.

Given: Water pipe system

Find: Flow rates

Solution:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_l \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad \text{(Turbulent)}$$

The energy equation can be simplified to $\Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$

This can be written for each pipe section

Pipe A (first section) $\Delta p_A = \rho \cdot f_A \cdot \frac{L_A}{D_A} \cdot \frac{V_A^2}{2}$ (1)

Pipe B (1.5 in branch) $\Delta p_B = \rho \cdot f_B \cdot \frac{L_B}{D_B} \cdot \frac{V_B^2}{2}$ (2)

Pipe C (1 in branch) $\Delta p_C = \rho \cdot f_C \cdot \frac{L_C}{D_C} \cdot \frac{V_C^2}{2}$ (3)

Pipe D (last section) $\Delta p_D = \rho \cdot f_D \cdot \frac{L_D}{D_D} \cdot \frac{V_D^2}{2}$ (4)

In addition we have the following contraints

$Q_A = Q_D$ (5)

$Q_A = Q_B + Q_C$ (6)

$\Delta p = \Delta p_A + \Delta p_B + \Delta p_D$ (7)

$\Delta p_B = \Delta p_C$ (8)

We have 4 unknown flow rates (or, equivalently, velocities) and four equations (5 - 8); Eqs 1 - 4 relate pressure drops to flow rates (velocities)

The workbook for Example Problem 8.11 is modified for use in this problem

Pipe Data:

Pipe	L (ft)	D (in)	e (ft)
A	150	1.5	0.00085
B	150	1.5	0.00085
C	150	1	0.00085
D	150	1.5	0.00085

Fluid Properties:

ρ = 1.94 slug/ft^3

μ = 2.10E-05 lbf·s/ft^2

Available Head:

Δp = 50 psi

Flows:

Q_A (ft^3/s)	Q_B (ft^3/s)	Q_C (ft^3/s)	Q_D (ft^3/s)
0.103	0.077	0.026	0.103

V_A (ft/s)	V_B (ft/s)	V_C (ft/s)	V_D (ft/s)
8.41	6.28	4.78	8.41

Re_A	Re_B	Re_C	Re_D
9.71E+04	7.25E+04	3.68E+04	9.71E+04

f_A	f_B	f_C	f_D
0.0342	0.0345	0.0397	0.0342

Heads:

Δp_A (psi)	Δp_B (psi)	Δp_C (psi)	Δp_D (psi)
19.5	11.0	11.0	19.5

Constraints:

(5) $Q_A = Q_D$ **0.00%**

(6) $Q_A = Q_B + Q_C$ **0.05%**

(7) $\Delta p = \Delta p_A + \Delta p_B + \Delta p_D$ **0.00%**

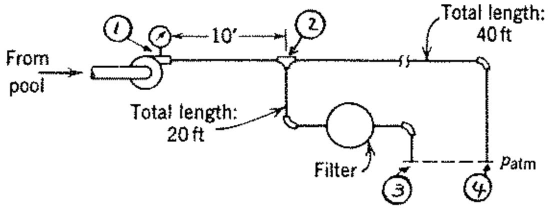
(8) $\Delta p_B = \Delta p_C$ **0.00%**

Error: **0.05%** Vary Q_A, Q_B, Q_C, and Q_D using *Solver* to minimize total error

Problem *8.169

[4]

Given: Partial-flow filtration system;



Pipes are 3/4 in. nominal PVC (smooth plastic) with $D = 0.824$ in.

Pump delivers 30 gpm at 75°F.

Filter pressure drop is $\Delta p(\text{psi}) = 0.6[Q(\text{gpm})]^2$.

Find: (a) Pressure at pump outlet.
(b) Flow rate through each branch of system.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2 + h_{lT}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) $\alpha_1\bar{V}_1^2 = \alpha_2\bar{V}_2^2$; (2) $z_1 = z_2$, (3) $h_{lm} = 0$ for $1 \to 2$, (4) Ignore "tee" at ②

The flow rate is $Q_{12} = 30$ gpm $(0.0668\ \text{ft}^3/\text{sec})$, so $\bar{V} = \frac{Q}{A} = 18.0$ ft/sec. Then

$$Re = \frac{\bar{V}D}{\nu} = 18.0\frac{\text{ft}}{\text{sec}} \times \left(\frac{0.824}{12}\right)\text{ft} \times \frac{\text{sec}}{1.0 \times 10^{-5}\ \text{ft}^2} = 1.24 \times 10^5, \text{ so } f = 0.017$$

$$\Delta p_{12} = f\frac{L}{D}\rho\frac{\bar{V}^2}{2} = 0.017 \times \frac{10\ \text{ft}}{0.824\ \text{in.}} \times \frac{1}{2} \times 1.94\frac{\text{slug}}{\text{ft}^3} \times (18.0)^2\frac{\text{ft}^2}{\text{sec}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{ft}}{12\ \text{in.}} = 5.40\ \text{psi}$$

Branch flow rates are unknown, but flow split must produce the same drop in each branch. Solve by iteration to obtain

$Q_{23} = 5.2$ gpm; $\bar{V}_{23} = 3.12$ ft/s; $Re = 2.15 \times 10^4$, and $f = 0.025^*$

$$\Delta p_{23} = f\left(\frac{L}{D} + 2\frac{L_e}{D}\right)\rho\frac{\bar{V}^2}{2} + 0.6Q^2$$

$$\Delta p_{23} = 0.025\left[\frac{240}{0.824} + 2(30)\right]\frac{1}{2} \times 1.94\frac{\text{slug}}{\text{ft}^3} \times (3.12)^2\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{ft}^2}{144\ \text{in.}^2} + 0.6(5.2)^2\frac{\text{lbf}}{\text{in.}^2} = 16.8\ \text{psi}$$

$Q_{24} = 24.8$ gpm; $\bar{V}_{24} = 14.9$ ft/s; $Re = 1.03 \times 10^5$, and $f = 0.018$

$$\Delta p_{24} = f\left(\frac{L}{D} + \frac{L_e}{D}\right)\rho\frac{\bar{V}^2}{2} = 0.018\left(\frac{480}{0.824} + 30\right)\frac{1}{2} \times 1.94\frac{\text{slug}}{\text{ft}^3} \times (14.9)^2\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{ft}^2}{144\ \text{in.}^2} = 16.5\ \text{psi}$$

The pump outlet pressure is

$$\Delta p_{pump} = \Delta p_{12} + \Delta p_{23} = (5.4 + 16.8)\ \text{psi} = 22.2\ \text{psi}$$

Δp

The branch flow rates are

$Q_{23} \approx 5.2$ gpm

Q_{23}

$Q_{24} \approx 24.8$ gpm

Q_{24}

* Value of f obtained from Eq 8.37 using Excel's solver (or Goal Seek)

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Problem 8.170

Open-Ended Problem Statement: Why does the shower temperature change when a toilet is flushed? Sketch pressure curves for the hot and cold water supply systems to explain what happens.

Discussion: Assume the pressure in the water main servicing the dwelling remains constant. The hot and cold water flow rates reaching the shower are controlled by valve(s) in the shower. Assuming a water heater temperature of 140°F, a cold water temperature of 60°F, and a shower water temperature of 100°F, the hot and cold flow rates must be equal. The two water streams mix before reaching the shower head, then spray out into the shower itself at 100°F.

Supply curves and system curves for the hot and cold water streams are shown below. Diagram *a* is the cold water system and diagram *b* is the hot water system. The numerical values are representative of an actual system.

In general the supply curves for the hot and cold streams are not the same. The difference is caused by the two systems having different pipe lengths and different fittings.

Each stream operates at the flow rate where the curves intersect. An equal flow split is accomplished by adjusting the shower valves to vary their resistances.

Flushing the toilet temporarily increases the flow rate of cold water to the bathroom. This reduces the cold water supply pressure reaching the shower. The system curves do not change because the valve settings stay the same. Therefore the flow rate of cold water must decrease to again match the supply and system curves (diagram *c*).

When the flow rate of cold water decreases the shower temperature increases, as experience testifies!

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Problem 8.170

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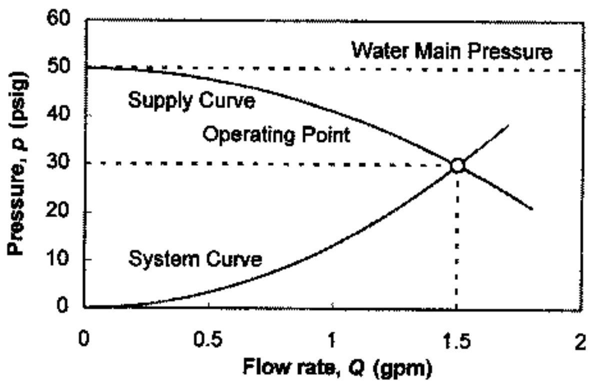
(*a*) **Cold water system:**

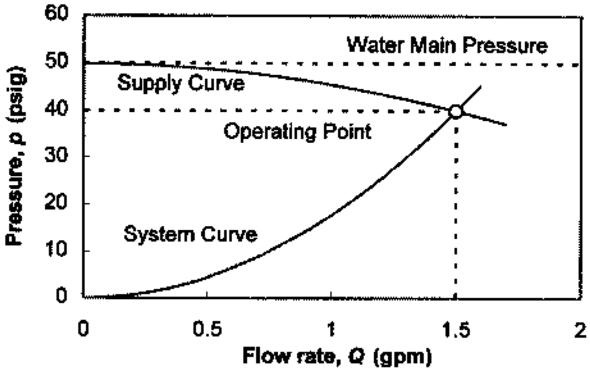
Q (gpm)	System Curve p (psig)	Supply Curve p (psig)
0	0.00	50.0
0.2	0.53	49.6
0.4	2.13	48.6
0.6	4.80	46.8
0.8	8.53	44.3
1.0	13.3	41.1
1.2	19.2	37.2
1.4	26.1	32.6
1.6	34.1	27.2
1.7	38.5	24.3
1.8		21.2



(*a*) **Cold water system curves**

(*b*) **Hot water system:**

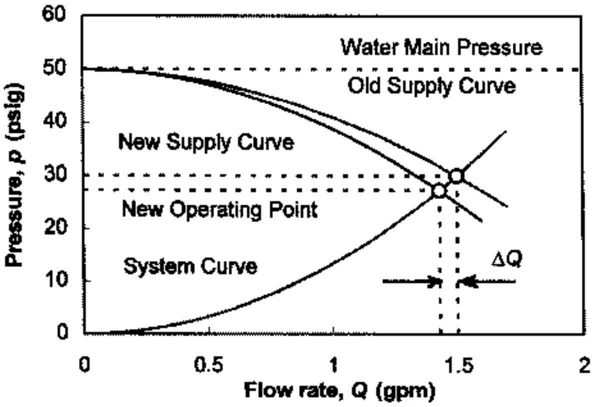
Q (gpm)	System Curve p (psig)	Supply Curve p (psig)
0	0.00	50.0
0.2	0.71	49.8
0.4	2.84	49.3
0.6	6.40	48.4
0.8	11.38	47.2
1.0	17.78	45.6
1.2	25.60	43.6
1.4	34.84	41.3
1.6	45.51	38.6
1.7		37.2
1.8		



(*b*) **Hot water system curves**

(*c*) **Cold water system: toilet flush**

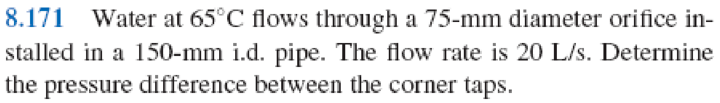
Q (gpm)	System Curve p (psig)	Old Supply Curve p (psig)	New Supply Curve p (psig)
0	0.00	50.0	50.0
0.2	0.53	49.6	49.6
0.4	2.13	48.6	48.2
0.6	4.80	46.8	46.0
0.8	8.53	44.3	42.9
1.0	13.3	41.1	38.9
1.2	19.2	37.2	34.0
1.4	26.1	32.6	28.2
1.430	27.27	31.8	27.28
1.6	34.1	27.2	21.6
1.7	38.5	24.3	
1.8			



(*c*) **Cold water system curves: toilet flush**

Problem 8.171

[2]



8.171 Water at 65°C flows through a 75-mm diameter orifice installed in a 150-mm i.d. pipe. The flow rate is 20 L/s. Determine the pressure difference between the corner taps.

Given: Flow through an orifice

Find: Pressure drop

Solution:

Basic equation $m_{actual} = K \cdot A_t \cdot \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = K \cdot A_t \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

For the flow coefficient $K = K\left(Re_{D1}, \frac{D_t}{D_1}\right)$

At 65ºC,(Table A.8) $\rho = 980 \cdot \frac{kg}{m^3}$ $\quad \nu = 4.40 \times 10^{-7} \cdot \frac{m^2}{s}$

$V = \frac{Q}{A}$ $\quad V = \frac{4}{\pi} \times \frac{1}{(0.15 \cdot m)^2} \times 20 \cdot \frac{L}{s} \times \frac{0.001 \cdot m^3}{1 \cdot L}$ $\quad V = 1.13 \frac{m}{s}$

$Re_{D1} = \frac{V \cdot D}{\nu}$ $\quad Re_{D1} = 1.13 \cdot \frac{m}{s} \times 0.15 \cdot m \times \frac{s}{4.40 \times 10^{-7} \cdot m^2}$ $\quad Re_{D1} = 3.85 \times 10^5$

$\beta = \frac{D_t}{D_1}$ $\quad \beta = \frac{75}{150}$ $\quad \beta = 0.5$

From Fig. 8.20 $K = 0.624$

Then $\Delta p = \left(\frac{m_{actual}}{K \cdot A_t}\right)^2 \cdot \frac{1}{2 \cdot \rho} = \left(\frac{\rho \cdot Q}{K \cdot A_t}\right)^2 \cdot \frac{1}{2 \cdot \rho} = \frac{\rho}{2} \cdot \left(\frac{Q}{K \cdot A_t}\right)^2$

$\Delta p = \frac{1}{2} \times 980 \cdot \frac{kg}{m^3} \times \left[20 \cdot \frac{L}{s} \times \frac{0.001 \cdot m^3}{1 \cdot L} \times \frac{1}{0.624} \times \frac{4}{\pi} \times \frac{1}{(0.075 \cdot m)^2}\right]^2$ $\quad \Delta p = 25.8\,kPa$

Problem 8.172

[2]

Given: Square-edged orifice, $d_t = 100\,mm$, used to meter air flow in a 150 mm i.d. line. The pressure upstream of the orifice is $p_1 = 600\,kPa$. The pressure drop across the orifice is $\Delta p = 750\,mm\,H_2O$. The air temperature is 25°C

Find: the volume flow rate of air in the line.

Solution: Apply analysis of section 8-10; data from Fig. 8.23 apply

Computing equation: $\dot{m}_{actual} = KA_t\sqrt{2\rho(p_1 - p_2)}$ (8.56)

Since $\dot{m} = \rho Q$, then

$$Q = \frac{\dot{m}}{\rho} = KA_t\sqrt{\frac{2(p_1-p_2)}{\rho}}$$

$$p_1 - p_2 = 750\,mm\,H_2O = \rho g \Delta h_{H_2O} = 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 0.75\,m \times \frac{N\cdot s^2}{kg\cdot m} = 7.35\,kPa$$

For this small Δp, the assumption of incompressible flow is certainly valid

$$\rho = \frac{p_1}{RT} = 701\times10^3\frac{N}{m^2} \times \frac{kg\cdot K}{287\,J} \times \frac{1}{298\,K} \times \frac{J}{N\cdot m} = 8.20\,kg/m^3$$

The flow coefficient $K = K\left(Re_{D_1}, \frac{D_t}{D_1}\right)$

Assume $Re > 2\times10^5$. For $\beta = \frac{D_t}{D_1} = \frac{2}{3}$, from Fig. 8.20, $K = 0.675$

$$Q = KA_t\sqrt{\frac{2(p_1-p_2)}{\rho}} = 0.675\,\frac{\pi}{4}(0.1\,m)^2\left[2\times 7350\frac{N}{m^2}\times\frac{m^3}{8.20\,kg}\times\frac{kg\cdot m}{N\cdot s^2}\right]^{1/2}$$

$Q = 0.224\,m^3/s$ $\longleftarrow$ Q

Check Re. At T = 25°C $\mu = 1.84\times10^{-5}\,N\cdot s/m^2$ (Table A.10)

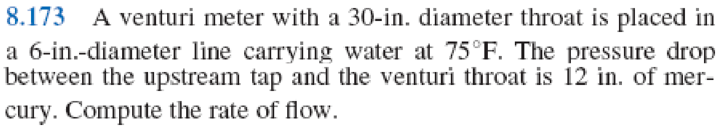
$$Re = \frac{\rho D V}{\mu} = \frac{\rho D Q}{\mu A} = \frac{\rho D Q}{\mu}\frac{4}{\pi D^2} = \frac{4\rho Q}{\pi\mu D}$$

$$Re = \frac{4}{\pi}\times 8.2\frac{kg}{m^3}\times 0.224\frac{m^3}{s}\times\frac{1}{1.84\times10^{-5}}\frac{m^2}{N\cdot s}\times\frac{1}{0.15\,m}\times\frac{N\cdot s^2}{kg\cdot m}$$

$Re = 8.47\times10^5$ ✓ assumption is valid

Problem 8.173

[2]



8.173 A venturi meter with a 30-in. diameter throat is placed in a 6-in.-diameter line carrying water at 75°F. The pressure drop between the upstream tap and the venturi throat is 12 in. of mercury. Compute the rate of flow.

Given: Flow through a venturi meter (NOTE: Throat is obviously 3 in not 30 in!)

Find: Flow rate

Solution:

Basic equation
$$m_{actual} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

For $Re_{D1} > 2 \times 10^5$, $0.980 < C < 0.995$. Assume $C = 0.99$, then check Re

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{3}{6} \qquad \beta = 0.5$$

Also
$$\Delta p = \rho_{Hg} \cdot g \cdot \Delta h = SG_{Hg} \cdot \rho \cdot g \cdot \Delta h$$

Then
$$Q = \frac{m_{actual}}{\rho} = \frac{C \cdot A_t}{\rho \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p} = \frac{\pi \cdot C \cdot D_t^2}{4 \cdot \rho \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot SG_{Hg} \cdot \rho \cdot g \cdot \Delta h} = \frac{\pi \cdot C \cdot D_t^2}{4 \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot SG_{Hg} \cdot g \cdot \Delta h}$$

$$Q = \frac{\pi}{4 \times \sqrt{1-0.5^4}} \times 0.99 \times \left(\frac{1}{4} \cdot ft\right)^2 \times \sqrt{2 \times 13.6 \times 32.2 \cdot \frac{ft}{s^2} \times 1 \cdot ft} \qquad Q = 1.49 \cdot \frac{ft^3}{s}$$

Hence
$$V = \frac{Q}{A} = \frac{4 \cdot Q}{\pi \cdot D_1^2} \qquad V = \frac{4}{\pi} \times \frac{1}{\left(\frac{1}{2} \cdot ft\right)^2} \times 1.49 \cdot \frac{ft^3}{s} \qquad V = 7.59 \cdot \frac{ft}{s}$$

At 75°F,(Table A.7) $\nu = 9.96 \times 10^{-6} \cdot \frac{ft^2}{s}$

$$Re_{D1} = \frac{V \cdot D_1}{\nu} \qquad Re_{D1} = 7.59 \cdot \frac{ft}{s} \times \frac{1}{2} \cdot ft \times \frac{s}{9.96 \times 10^{-6} \cdot ft^2} \qquad Re_{D1} = 3.81 \times 10^5$$

Thus $Re_{D1} > 2 \times 10^5$. The volume flow rate is $Q = 1.49 \cdot \frac{ft^3}{s}$

Problem 8.174

[2]

8.174 A smooth 200 m pipe, 100 mm diameter connects two reservoirs (the entrance and exit of the pipe are sharp-edged). At the midpoint of the pipe is an orifice plate with diameter 40 mm. If the water levels in the reservoirs differ by 30 m, estimate the pressure differential indicated by the orifice plate and the flow rate.

Given: Reservoir-pipe system

Find: Orifice plate pressure difference; Flow rate

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}=h_l+\Sigma h_{lm} \qquad (8.29)$$

$$h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad (8.34) \qquad\qquad h_{lm}=K\cdot\frac{V^2}{2} \qquad (8.40a)$$

$$f=\frac{64}{Re} \quad \text{(Laminar)} \quad (8.36) \qquad \frac{1}{f^{0.5}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot f^{0.5}}\right) \quad \text{(Turbulent)} \quad (8.37)$$

There are three minor losses: at the entrance; at the orifice plate; at the exit. For each $h_{lm}=K\cdot\frac{V^2}{2}$

The energy equation (Eq. 8.29) becomes ($\alpha = 1$)

$$g\cdot\Delta H=\frac{V^2}{2}\cdot\left(f\cdot\frac{L}{D}+K_{ent}+K_{orifice}+K_{exit}\right) \qquad (1)$$

(ΔH is the difference in reservoir heights)

This cannot be solved for V (and hence Q) because f depends on V; we can solve by manually iterating, or by using *Solver*

The tricky part to this problem is that the orifice loss coefficient $K_{orifice}$ is given in Fig. 8.23 as a percentage of pressure differential Δp across the orifice, which is unknown until V is known!

The mass flow rate is given by

$$m_{rate}=K\cdot A_t\cdot\sqrt{2\cdot\rho\cdot\Delta p} \qquad (2)$$

where K is the orifice flow coefficient, A_t is the orifice area, and Δp is the pressure drop across the orifice

Equations 1 and 2 form a set for solving for TWO unknowns: the pressure drop Δp across the orifice (leading to a value for $K_{orifice}$) and the velocity V. The easiest way to do this is by using *Solver*

Given data:

ΔH = 30 m

L = 200 m

D = 100 mm

D_t = 40 mm

β = 0.40

Tabulated or graphical data:

K_{ent} = 0.50 (Fig. 8.14)

K_{exit} = 1.00 (Fig. 8.14)

Loss at orifice = 80% (Fig. 8.23)

μ = 0.001 N.s/m^2

ρ = 999 kg/m^3

(Water - Appendix A)

Computed results:

Orifice loss coefficient:

K = 0.61

(Fig. 8.20
Assuming high *Re*)

Flow system:

V = 2.25 m/s

Q = 0.0176 m^3/s

Re = 2.24E+05

f = 0.0153

Orifice pressure drop

Δp = 265 kPa

Eq. 1, solved by varying V AND Δp, using *Solver*:

Left (m^2/s)	Right (m^2/s)	Error
294	293	0.5%

Eq. 2 and $m_{\text{rate}} = \rho Q$ compared, varying V AND Δp

	(From Q)	(From Eq. 2)	Error
m_{rate} (kg/s) =	17.6	17.6	0.0%

Total Error	0.5%

Procedure using *Solver*:
a) Guess at V and Δp
b) Compute error in Eq. 1
c) Compute error in mass flow rate
d) Minimize total error
e) Minimize total error by varying V and Δp

Problem 8.175

[2]

Given: Flow of gasoline through a venturi meter.

$SG = 0.73$, $D_1 = 2.0$ in., $D_t = 1.0$ in., $\Delta h = 380$ mm Hg

Find: Volume flow rate of gasoline.

Solution: Apply the analysis of Section 8-10.3.

Computing equations:

$$\dot{m}_{actual} = \frac{C A_t}{\sqrt{1-\beta^4}} \sqrt{2\rho(p_1 - p_2)} \qquad (8.52)$$

$$C = 0.99 \text{ for } Re_{D_1} > 2 \times 10^5$$

For the manometer, $\Delta p = \rho_{Hg} g \Delta h = SG_{Hg} \rho_{H_2O} g \Delta h$

Then

$$Q = \frac{\dot{m}}{\rho} = \frac{C A_t}{\sqrt{1-\beta^4}} \sqrt{\frac{2\Delta p}{\rho}} = \frac{C A_t}{\sqrt{1-\beta^4}} \sqrt{\frac{2 SG_{Hg} \rho_{H_2O} g \Delta h}{SG_{gas} \rho_{H_2O}}} = \frac{C A_t}{\sqrt{1-\beta^4}} \sqrt{\frac{2 SG_{Hg} g \Delta h}{SG_{gas}}}$$

$$Q = \frac{0.99}{\sqrt{1-(0.5)^4}} \frac{\pi}{4}(0.0254)^2 m^2 \sqrt{2 \times \frac{13.6}{0.73} \times 9.81 \frac{m}{s^2} \times 0.38\, m} = 0.00611 \, m^3/s$$

Now check Reynolds number:

$$\bar{V}_1 = \frac{Q}{A_1} = 0.00611 \frac{m^3}{s} \times \frac{4}{\pi (0.0508)^2 m^2} = 3.01 \, m/s$$

Assume viscosity midway between octane and heptane at 20°C. From Fig. A.1,

$\mu \approx 5.0 \times 10^{-4} \, N \cdot s / m^2$

$$Re_{D_1} = \frac{\rho \bar{V}_1 D_1}{\mu} = (0.73)\, 1000 \frac{kg}{m^3} \times 3.01 \frac{m}{s} \times 0.0508\, m \times \frac{m^2}{5.0 \times 10^{-4} N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} = 2.23 \times 10^5$$

Thus assumption that $C = 0.99$ is okay

$$Q = 0.00611 \, m^3/s \longleftarrow Q$$

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Problem 8.176

[2]

8.176 Consider a horizontal 50 × 25 mm venturi with water flow. For a differential pressure of 150 kPa, calculate the volume flow rate.

Given: Flow through an venturi meter

Find: Flow rate

Solution:

Basic equation $m_{actual} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \left(p_1 - p_2\right)} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

For $Re_{D1} > 2 \times 10^5$, $0.980 < C < 0.995$. Assume C = 0.99, then check Re

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{25}{50} \qquad \beta = 0.5$$

Then $Q = \frac{m_{actual}}{\rho} = \frac{C \cdot A_t}{\rho \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p} = \frac{\pi \cdot C \cdot D_t^{\,2}}{4 \cdot \sqrt{1-\beta^4}} \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$

$$Q = \frac{\pi}{4 \times \sqrt{1-0.5^4}} \times 0.99 \times (0.025 \cdot \mathrm{m})^2 \times \sqrt{2 \times 150 \times 10^3 \cdot \frac{\mathrm{N}}{\mathrm{m}^2} \times \frac{\mathrm{m}^3}{1000 \cdot \mathrm{kg}} \times \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2 \cdot \mathrm{N}}} \qquad Q = 8.69 \times 10^{-3} \frac{\mathrm{m}^3}{\mathrm{s}}$$

Hence $V = \frac{Q}{A} = \frac{4 \cdot Q}{\pi \cdot D_1^{\,2}} \qquad V = \frac{4}{\pi} \times \frac{1}{(0.05 \cdot \mathrm{m})^2} \times 8.69 \times 10^{-3} \frac{\mathrm{m}^3}{\mathrm{s}} \qquad V = 4.43 \frac{\mathrm{m}}{\mathrm{s}}$

At 20$^\mathrm{o}$C (Table A.8) $\nu = 1.01 \times 10^{-6} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$

$$Re_{D1} = \frac{V \cdot D}{\nu} \qquad Re_{D1} = 4.43 \cdot \frac{\mathrm{m}}{\mathrm{s}} \times 0.05 \cdot \mathrm{m} \times \frac{\mathrm{s}}{1.01 \times 10^{-6} \cdot \mathrm{m}^2} \qquad Re_{D1} = 2.19 \times 10^5$$

Thus $Re_{D1} > 2 \times 10^5$. The volume flow rate is $Q = 8.69 \times 10^{-3} \frac{\mathrm{m}^3}{\mathrm{s}}$ $\qquad Q = 0.522 \cdot \frac{\mathrm{m}^3}{\mathrm{min}}$

Problem 8.177

[3]

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Given: Test of 1.6 ℓ internal combustion engine at 6000 rpm. Meter air with flow nozzle, $\Delta h \leq 0.25$ m. Manometer reads to ± 0.5 mm of water.

Find: (a) Flow nozzle diameter required.
(b) Minimum rate of air flow that can be measured ± 2 percent.

Solution: Apply computing equation for flow nozzle.

Computing equation: $\dot{m} = KA_t\sqrt{2\rho(p_1 - p_2)}$ (8.54)

Assumptions: (1) $K = 0.97$ (Section 8-10.2b.)
(2) $\beta = 0$ (nozzle inlet is from atmosphere)
(3) Four-stroke cycle engine with 100 percent volumetric efficiency ($\forall$/rev = displacement/2)
(4) Standard air

Then

$$\dot{m} = \rho Q = 1.23\,\frac{kg}{m^3} \times \frac{1.6\,\ell}{2\,rev} \times 6000\,\frac{rev}{min} \times \frac{m^3}{1000\,\ell} \times \frac{min}{60\,s} = 0.0984\ kg/s$$

Solving for A_t,

$$A_t = \frac{\dot{m}}{K\sqrt{2\rho\Delta p}} = \frac{\dot{m}}{K\sqrt{2\rho\rho_{H_2O}\,g\,\Delta h}}$$

$$A_t = 0.0984\,\frac{kg}{s} \times \frac{1}{0.97}\left[\frac{1}{2} \times \frac{m^3}{1.23\,kg} \times \frac{m^3}{999\,kg} \times \frac{s^2}{9.81\,m} \times \frac{1}{0.25\,m}\right]^{\frac{1}{2}} = 1.31 \times 10^{-3}\,m^2$$

$$A_t = \frac{\pi D_t^2}{4}\ ;\ D_t = \sqrt{\frac{4A_t}{\pi}} = 40.8\ mm$$

D_t

The allowable error is ± 2 percent, or ± 0.02. As discussed in Appendix E, the square-root relationship halves the experimental uncertainty. Thus

$$e = \pm 0.02 \text{ when } e_{\Delta h} = \pm 0.04\ ;\ \Delta h_{min} = \frac{\pm 0.5\,mm}{\pm 0.04} = 12.5\ mm$$

$$\dot{m}_{min} \simeq \dot{m}\sqrt{\frac{\Delta h_{min}}{\Delta h}} = 0.0984\,\frac{kg}{s}\sqrt{\frac{12.5\,mm}{250\,mm}} = 0.0220\ kg/s$$

$\dot{m}_{min}$

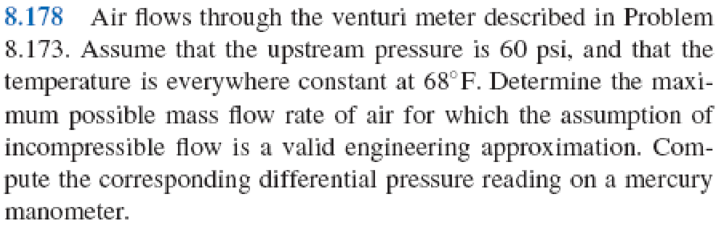
The air flow rate could be measured with ±2 percent accuracy down to about

$$\omega = 6000\,rpm\,\frac{0.0220}{0.0984} = 1340\ rpm$$

with this setup.

Problem 8.178

[4]



8.178 Air flows through the venturi meter described in Problem 8.173. Assume that the upstream pressure is 60 psi, and that the temperature is everywhere constant at 68°F. Determine the maximum possible mass flow rate of air for which the assumption of incompressible flow is a valid engineering approximation. Compute the corresponding differential pressure reading on a mercury manometer.

Given: Flow through a venturi meter (NOTE: Throat is obviously 3 in not 30 in!)

Find: Maximum flow rate for incompressible flow; Pressure reading

Solution:

Basic equation $$m_{actual} = \frac{C\cdot A_t}{\sqrt{1-\beta^4}}\cdot\sqrt{2\cdot\rho\cdot\left(p_1-p_2\right)} = \frac{C\cdot A_t}{\sqrt{1-\beta^4}}\cdot\sqrt{2\cdot\rho\cdot\Delta p}$$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

Assumptions: 1) Neglect density change 2) Use ideal gas equation for density

Then $$\rho = \frac{p}{R_{air}\cdot T} \qquad \rho = 60\cdot\frac{lbf}{in^2}\times\left(\frac{12\cdot in}{1\cdot ft}\right)^2\times\frac{lbm\cdot R}{53.33\cdot ft\cdot lbf}\times\frac{1\cdot slug}{32.2\cdot lbm}\cdot\frac{1}{(68+460)\cdot R} \qquad \rho = 9.53\times10^{-3}\cdot\frac{slug}{ft^3}$$

For incompressible flow V must be less than about 100 m/s or 330 ft/s at the throat. Hence

$$m_{actual} = \rho\cdot V_2\cdot A_2 \qquad m_{actual} = 9.53\times10^{-3}\,\frac{slug}{ft^3}\times330\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{4}\cdot ft\right)^2 \qquad m_{actual} = 0.154\cdot\frac{slug}{s}$$

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{3}{6} \qquad \beta = 0.5$$

Also $$\Delta p = \rho_{Hg}\cdot g\cdot\Delta h \qquad \Delta h = \frac{\Delta p}{\rho_{Hg}\cdot g}$$

and in addition $$\Delta p = \frac{1}{2\cdot\rho}\cdot\left(\frac{m_{actual}}{C\cdot A_t}\right)^2\cdot\left(1-\beta^4\right) \qquad \text{so} \qquad \Delta h = \frac{\left(1-\beta^4\right)}{2\cdot\rho\cdot\rho_{Hg}\cdot g}\cdot\left(\frac{m_{actual}}{C\cdot A_t}\right)^2$$

For $Re_{D1} > 2 \times 10^5$, $0.980 < C < 0.995$. Assume C = 0.99, then check Re

$$\Delta h = \frac{\left(1-0.5^4\right)}{2}\times\frac{ft^3}{9.53\times10^{-3}\,slug}\times\frac{ft^3}{13.6\cdot1.94\cdot slug}\times\frac{s^2}{32.2\cdot ft}\times\left[0.154\,\frac{slug}{s}\times\frac{1}{0.99}\times\frac{4}{\pi}\times\left(\frac{4}{1\cdot ft}\right)^2\right]^2 \qquad \Delta h = 0.581\cdot ft \qquad \Delta h = 6.98\cdot in$$

Hence $$V = \frac{Q}{A} = \frac{4\cdot m_{actual}}{\pi\cdot\rho\cdot D_1^{\,2}} \qquad V = \frac{4}{\pi}\times\frac{ft^3}{9.53\times10^{-3}\,slug}\times\frac{1}{\left(\frac{1}{2}\cdot ft\right)^2}\times0.154\,\frac{slug}{s} \qquad V = 82.3\cdot\frac{ft}{s}$$

At 68°F,(Table A.7) $$\nu = 1.08\times10^{-5}\cdot\frac{ft^2}{s}$$

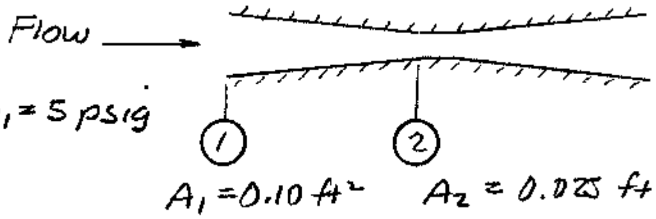
$$Re_{D1} = \frac{V\cdot D_1}{\nu} \qquad Re_{D1} = 82.3\cdot\frac{ft}{s}\times\frac{1}{2}\cdot ft\times\frac{s}{1.08\times10^{-5}\cdot ft^2} \qquad Re_{D1} = 3.81\times10^6$$

Thus $Re_{D1} > 2 \times 10^5$. The mass flow rate is $m_{actual} = 0.154\,\frac{slug}{s}$ and pressure $\Delta h = 6.98\,in$ Hg

Problem 8.179

[4]

Given: Water at 70°F flows through a Venturi.



Find: Estimate the maximum flow rate with no cavitation. (Express answer in cfs.)

Solution: Apply flowmeter equation.

Computing equation: $\dot m = \frac{C A_t}{\sqrt{1-\beta^4}}\sqrt{2\rho(p_1-p_2)}$; $\beta^2 = A_t/A_1$

Assume $C = 0.99$ for $Re_{D_1} \geq 2\times10^5$.

Cavitation occurs when $p_2 \leq p_v$. From Steam table, $p_v = 0.363$ psia at 70 F. Thus

$$p_1 - p_2 = (14.7+5.0) - 0.363 = 19.3\ psi$$

and

$$\dot m = 0.99 \times 0.025\ ft^2 \times \frac{1}{\sqrt{1-(0.025/0.1)^2}}\left[2 \times 1.94\ \frac{slug}{ft^3} \times 19.3\ \frac{lbf}{in.^2} \times 144\ \frac{in.^2}{ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2}$$

$$\dot m = 2.65\ slug/s$$

But $\dot m = \rho \bar V A = \rho Q$, so

$$Q = \frac{\dot m}{\rho} = 2.65\ \frac{slug}{sec} \times \frac{ft^3}{1.94\ slug} = 1.37\ ft^3/s \longleftarrow Q$$

{Note $Q = 1.37\ \frac{ft^3}{s} \times 7.48\ \frac{gal}{ft^3} \times 60\ \frac{s}{min} = 613\ gpm$.}

At 70 F, $\nu = 1.05\times10^{-5}\ ft^2/s$ (Table A.7). $Re_{D_1} = \frac{\bar V_1 D_1}{\nu}$ $A_1 = \frac{\pi D_1^2}{4}$; so

$$D_1 = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4}{\pi}\times 0.1\ ft^2} = 0.357\ ft\ (4.28\ in.)\ ;\ \bar V_1 = \frac{Q}{A_1} = 1.37\ \frac{ft^3}{s} \times \frac{1}{0.1\ ft^2} = 13.7\ ft/s$$

Then $Re_{D_1} = 13.7\ \frac{ft}{s} \times 0.357\ ft \times \frac{s}{1.05\times10^{-5}\ ft^2} = 4.66\times10^5$, so $C = 0.99$ is okay. ✓✓

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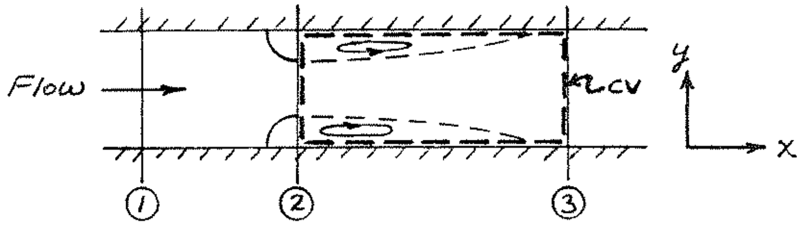
Problem 8.180

Given: Flow nozzle installation in pipe as shown.



Find: Head loss between sections ① and ③, expressed in coefficient form, $C_\ell = \frac{p_1 - p_3}{p_1 - p_2}$, Show $C_\ell = \frac{1 - A_2/A_1}{1 + A_2/A_1}$

Plot: C_ℓ vs. D_2/D_1

Solution: Apply the Bernoulli, continuity, momentum and energy equations, using the CV shown.

Basic equations: $$\frac{p_1}{\rho} + \frac{\bar V_1^2}{2} + \overset{(4)}{\cancel{gz_1}} = \frac{p_2}{\rho} + \frac{\bar V_2^2}{2} + \overset{(4)}{\cancel{gz_2}}$$

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec V \cdot d\vec A$$

$$F_{S_x} + \overset{=0(5)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho \, d\forall + \int_{CS} u\rho \vec V \cdot d\vec A$$

$$\dot Q + \overset{=0(6)}{\cancel{\dot W_s}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} e\rho \, d\forall + \int_{CS} \left(u + \frac{\bar V^2}{2} + \overset{(4)}{\cancel{gz}} + \frac{p}{\rho}\right) \rho \vec V \cdot d\vec A$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) No friction between ① and ②
(4) Neglect elevation terms
(5) $F_{B_x} = 0$
(6) $\dot W_s = 0$
(7) Uniform flow at each section

From continuity,
$$Q = \bar V_1 A_1 = \bar V_2 A_2 = \bar V_3 A_3$$

Apply Bernoulli along a streamline from ① to ②, noting $A_1 = A_3$,

$$\frac{p_1 - p_2}{\rho} = \frac{\bar V_2^2 - \bar V_1^2}{2} = \frac{\bar V_2^2}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{\bar V_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right]$$

From momentum, and using continuity,

$$F_{S_x} = p_2 A_1 - p_3 A_3 = \bar V_2\{-|\rho \bar V_2 A_2|\} + \bar V_3\{+|\rho \bar V_3 A_3|\} = (\bar V_3 - \bar V_2)\rho \bar V_3 A_3$$

or $$\frac{p_3 - p_2}{\rho} = \bar V_3(\bar V_2 - \bar V_3) = \bar V_2 \frac{A_2}{A_3}\left[\bar V_2 - \bar V_2 \frac{A_2}{A_3}\right] = \bar V_2^2 \frac{A_2}{A_3}\left(1 - \frac{A_2}{A_3}\right)$$

From energy,

$$\dot Q = \left(u_2 + \frac{\bar V_2^2}{2} + \frac{p_2}{\rho}\right)\{-|\rho \bar V_2 A_2|\} + \left(u_3 + \frac{\bar V_3^2}{2} + \frac{p_3}{\rho}\right)\{|\rho \bar V_3 A_3|\}$$

or $h_{\ell_{23}} = u_3 - u_2 - \frac{\dot{Q}}{\dot{m}} = \frac{\bar{V}_2^2 - \bar{V}_3^2}{2} - \frac{p_3 - p_2}{\rho} = \frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right] - \frac{p_3 - p_2}{\rho}$

But $h_{\ell_{12}} \simeq 0$ by assumption (3), so $h_{\ell_{13}} \simeq h_{\ell_{23}}$ and using momentum

$$h_{\ell_{13}} \simeq \frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right] - \bar{V}_2^2 \frac{A_2}{A_3}\left(1 - \frac{A_2}{A_3}\right)$$

After a little algebra, this may be written

$$h_{\ell_{13}} \simeq \frac{\bar{V}_2^2}{2}\left(1 - \frac{A_2}{A_3}\right)^2$$

Dividing by $(p_1 - p_2)/\rho$, a loss coefficient is derived as

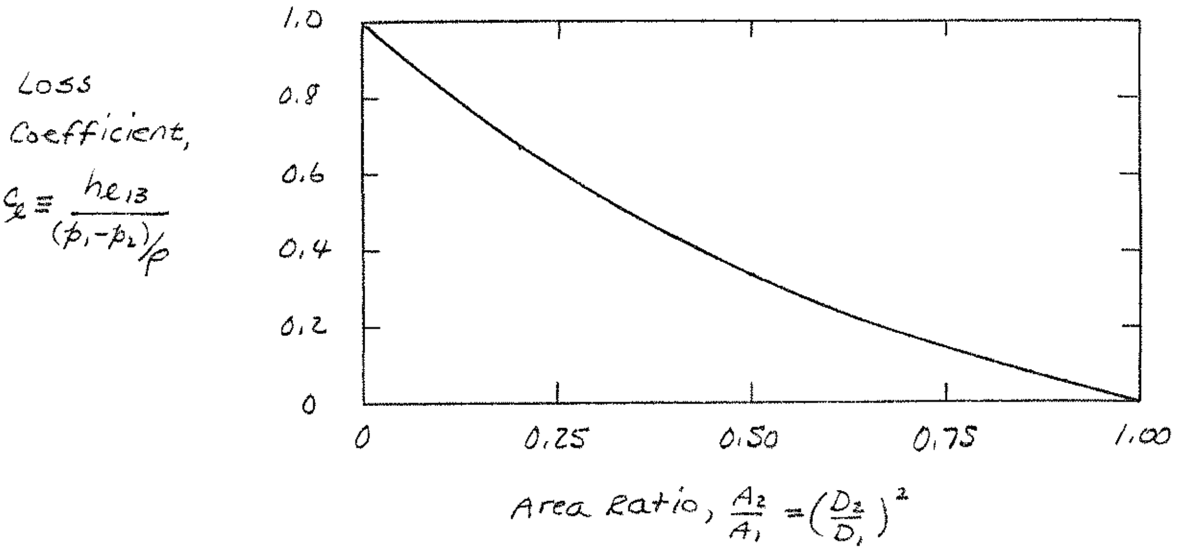
$$C_\ell = \frac{h_{\ell_{13}}}{(p_1 - p_2)/\rho} = \frac{\frac{\bar{V}_2^2}{2}\left(1 - \frac{A_2}{A_3}\right)^2}{\frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right]} = \frac{\left(1 - A_2/A_3\right)^2}{\left[1 - \left(A_2/A_3\right)^2\right]}$$

But $1 - \left(\frac{A_2}{A_3}\right)^2 = \left(1 + \frac{A_2}{A_3}\right)\left(1 - \frac{A_2}{A_3}\right)$, so

$$C_\ell = \frac{h_{\ell_{13}}}{(p_1 - p_2)/\rho} = \frac{1 - A_2/A_3}{1 + A_2/A_3} \longleftarrow$$

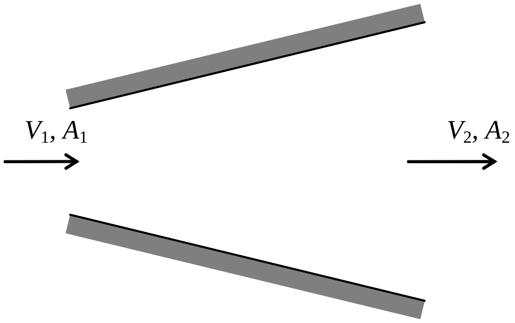
C_ℓ

Plotting:



Problem 8.181

8.181 Derive Eq. 8.42, the pressure loss coefficient for a diffuser assuming ideal (frictionless) flow.



Given: Flow through a diffuser

Find: Derivation of Eq. 8.42

Solution:

Basic equations $$C_p = \frac{p_2 - p_1}{\frac{1}{2}\cdot\rho\cdot V_1^2} \qquad \frac{p_1}{\rho} + \frac{V_1^2}{2} + g\cdot z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g\cdot z_2 \qquad Q = V\cdot A$$

Assumptions: 1) All the assumptions of the Bernoulli equation 2) Horizontal flow 3) No flow separation

From Bernoulli $$\frac{p_2 - p_1}{\rho} = \frac{V_1^2}{2} - \frac{V_2^2}{2} = \frac{V_1^2}{2} - \frac{V_1^2}{2}\cdot\left(\frac{A_1}{A_2}\right)^2$$ using continuity

Hence $$C_p = \frac{p_2 - p_1}{\frac{1}{2}\cdot\rho\cdot V_1^2} = \frac{1}{\frac{1}{2}\cdot V_1^2}\cdot\left[\frac{V_1^2}{2} - \frac{V_1^2}{2}\cdot\left(\frac{A_1}{A_2}\right)^2\right] = 1 - \left(\frac{A_1}{A_2}\right)^2$$

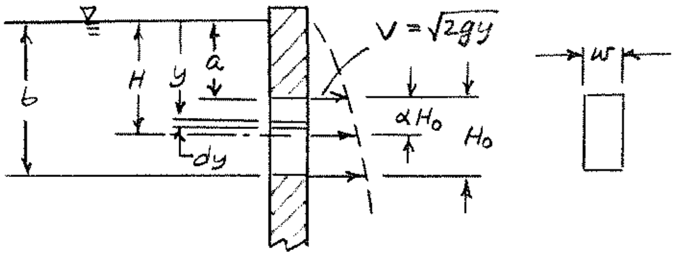
Finally $$C_p = 1 - \frac{1}{AR^2}$$ which is Eq. 8.42.

This result is not realistic as a real diffuser is very likely to have flow separation

Problem 8.182

Open-Ended Problem Statement: In some western states, water for mining and irrigation was sold by the "miner's inch," the rate at which water flows through an opening in a vertical plank of 1 in.^{2} area, up to 4 in. tall, under a head of 6 to 9 in. Develop an equation to predict the flow rate through such an orifice. Specify clearly the aspect ratio of the opening, thickness of the plank, and datum level for measurement of head (top, bottom, or middle of the opening). Show that the unit of measure varies from 38.4 (in Colorado) to 50 (in Arizona, Idaho, Nevada, and Utah) miner's inches equal to 1 ft^3/s.

Analysis: The geometry of the opening in a vertical plank is shown. The analysis includes the effect on flow speed of the variation in water depth vertically across the opening.



Aspect ratio, $ar \equiv \frac{H_0}{w}$; area, $A = wH_0 = 1\ in.^2$

$\alpha = 0, \frac{1}{2}, 1$

$$Q_{geom} = \int V\,dA = \int_a^b \sqrt{2gy}\,w\,dy = w\sqrt{2g}\left[\frac{2}{3}y^{3/2}\right]_a^b = \frac{2}{3}wa\sqrt{2ga}\left[\left(\frac{b}{a}\right)^{3/2} - 1\right]$$

For $ar = 1$, $\alpha = 0$, $a = H = 9\ in.$, $b = 10.0\ in.$, $w = 1.0\ in.$

$$Q_{geom} = \frac{2}{3} \times 1.0\ in. \times 9\ in. \left[2 \times 32.2\ \frac{ft}{s^2} \times 9\ in. \times \frac{ft}{12\ in.}\right]^{\frac{1}{2}}\left[\left(\frac{10}{9}\right)^{3/2} - 1\right]\frac{ft^2}{144\ in.^2} = 0.0496\ \frac{ft^3}{s}$$

$Q_{actual} = 0.6\,Q_{geom} = 0.0297\ ft^3/s$; thus $1/0.0297 = 33.6\ MI = 1\ cfs$

MI

Numerical results are presented in the spread sheet on the next page.

Discussion: All results assume a *vena contracta* in the liquid jet leaving the opening, reducing the effective flow area to 60 percent of the geometric area of the opening.

The calculated unit of measure varies from 31.3 to 52.4 miner's inch per cubic foot of water flow per second. This range encompasses the 38.4 and 50 values given in the problem statement.

Trends may be summarized as follows. The largest flow rate occurs when datum *H* is measured to the top of the opening in the vertical plank. This gives the deepest submergence and thus the highest flow speeds through the opening.

When *ar* = 1, the opening is square; when *ar* = 16, the opening is 4 inches tall and 1/4 inch wide. Increasing *ar* from 1 to 16 increases the flow rate through the opening when *H* is measured to the top of the opening, because it increases the submergence of the lower portion of the opening, thus increasing the flow speeds. When *H* is measured to the center of the opening *ar* has almost no effect on flow rate. When *H* is measured to the bottom of the opening, increasing *ar* reduces the flow rate. For this case, the depth of the opening decreases as *ar* becomes larger.

Plank thickness does not affect calculated flow rates since a *vena contracta* is assumed. In this flow model, water separates from the interior edges of the opening in the vertical plank. Only if the plank were several inches thick might the stream reattach and affect the flow rate.

The actual relationship between Q_{flow} and Q_{geom} might be a weak function of aspect ratio. The flow separates from all four edges of the opening in the vertical plank. At large *ar*, contraction on the narrow ends of the stream has a relatively small effect on flow area. As *ar* approaches 1 the effect becomes more pronounced, but would need to be measured experimentally. Assuming a constant 60 percent area fraction certainly gives reasonable trends.

Problem 8.182

Computation of "Miner's Inch" in Engineering Units:

a = depth to top of opening (in.)
ar = aspect ratio of opening (---)
A = area of opening 1 in.^{2}
b = depth to bottom of opening (in.)
H = nominal head (in.)
H_0 = height of opening (in.)
MI = "miner's inch" (mixed)
Q = volume flow rate (ft^3/s)
w = width of opening (in.)

Assume $Q_{flow} = 0.6 \times Q_{geometric}$ to account for contraction of the stream leaving the opening.

(*a*) Measure *H* to top of opening:

H	ar	H_0	a	b	w	Q_{geom}	Q_{flow}	MI/cfs
9	1	1.00	9.00	10.0	1.00	0.0496	0.0297	33.6
9	2	1.41	9.00	10.4	0.707	0.0501	0.0301	33.3
9	4	2.00	9.00	11.0	0.500	0.0509	0.0305	32.8
9	8	2.83	9.00	11.8	0.354	0.0519	0.0311	32.1
9	16	4.00	9.00	13.0	0.250	0.0533	0.0320	31.3
6	1	1.00	6.00	7.00	1.00	0.0410	0.0246	40.6
6	2	1.41	6.00	7.41	0.707	0.0416	0.0250	40.0
6	4	2.00	6.00	8.00	0.500	0.0425	0.0255	39.2
6	8	2.83	6.00	8.83	0.354	0.0437	0.0262	38.1
6	16	4.00	6.00	10.0	0.250	0.0454	0.0272	36.7

(*b*) Measure *H* to middle of opening:

H	ar	H_0	a	b	w	Q_{geom}	Q_{flow}	MI/cfs
9	1	1.00	8.50	9.50	1.00	0.0483	0.0290	34.5
9	2	1.41	8.29	9.71	0.707	0.0483	0.0290	34.5
9	4	2.00	8.00	10.0	0.500	0.0482	0.0289	34.6
9	8	2.83	7.59	10.4	0.354	0.0482	0.0289	34.6
9	16	4.00	7.00	11.0	0.250	0.0482	0.0289	34.6
6	1	1.00	5.50	6.50	1.00	0.0394	0.0236	42.3
6	2	1.41	5.29	6.71	0.707	0.0394	0.0236	42.3
6	4	2.00	5.00	7.00	0.500	0.0394	0.0236	42.3
6	8	2.83	4.59	7.41	0.354	0.0393	0.0236	42.4
6	16	4.00	4.00	8.00	0.250	0.0392	0.0235	42.5

(*c*) Measure *H* to bottom of opening:

H	ar	H_0	a	b	w	Q_{geom}	Q_{flow}	MI/cfs
9	1	1.00	8.00	9.00	1.00	0.0469	0.0281	35.5
9	2	1.41	7.59	9.00	0.707	0.0463	0.0278	36.0
9	4	2.00	7.00	9.00	0.500	0.0455	0.0273	36.7
9	8	2.83	6.17	9.00	0.354	0.0442	0.0265	37.7
9	16	4.00	5.00	9.00	0.250	0.0424	0.0254	39.3
6	1	1.00	5.00	6.00	1.00	0.0377	0.0226	44.2
6	2	1.41	4.59	6.00	0.707	0.0370	0.0222	45.1
6	4	2.00	4.00	6.00	0.500	0.0359	0.0215	46.4
6	8	2.83	3.17	6.00	0.354	0.0343	0.0206	48.6
6	16	4.00	2.00	6.00	0.250	0.0318	0.0191	52.4

National®Brand

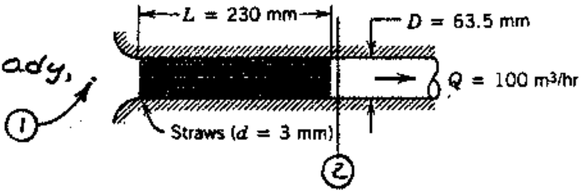
Problem 8.183

Given: Pipe-flow experiment with flow straightener made from straws.

Find: (a) Reynolds number for flow in each straw. $K_{ent} = 1.4$

(b) Friction factor for flow in each straw. $\alpha = 2.0$

(c) Gage pressure at exit from straws.



Solution: Apply energy equation for steady, incompressible pipe flow.

Computing equation:

$$\frac{p_1}{\rho} + \alpha_1 \overset{\approx 0(1)}{\cancel{\frac{\bar{V}_1^2}{2}}} + g\cancel{z_1} = \frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + g\cancel{z_2} + h_{lT}$$

$$h_{lT} = h_l + h_{lm} = f\frac{L}{D}\frac{\bar{V}^2}{2} + K_{ent}\frac{\bar{V}^2}{2} = \left(f\frac{L}{D} + K_{ent}\right)\frac{\bar{V}^2}{2}$$

Assumptions: (1) Flow from atmosphere: $p_1 = p_{atm}$, $\bar{V}_1 \approx 0$

(2) Horizontal

(3) Neglect thickness of straws

Then

$$\bar{V}_2 = \frac{Q}{A} = 100\,\frac{m^3}{hr} \times \frac{4}{\pi\,(0.0635)^2 m^2} \times \frac{hr}{3600\,s} = 8.77\ m/s$$

$$Re_d = \frac{\bar{V}_2 d}{\nu} = 8.77\,\frac{m}{sec} \times 0.003\,m \times \frac{sec}{1.46\times 10^{-5}\,m^2} = 1800$$ Re_d

For laminar flow,

$$f = \frac{64}{Re} = \frac{64}{1800} = 0.0356$$ f

The gage pressure at ② is

$$p_{2g} = -\rho\frac{\bar{V}_2^2}{2}\left(\alpha_2 + K_{ent} + f\frac{L}{D}\right)$$

$$= -\frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (8.77)^2\frac{m^2}{s^2}\left(2.0 + 1.4 + 0.0356 \times \frac{230\,mm}{3\,mm}\right)\frac{N\cdot s^2}{kg\cdot m}$$

$$p_{2g} = -290\ N/m^2\ (gage)$$ p_{2g}

This pressure drop is equivalent to

$$\Delta h = \frac{\Delta p}{\rho_{H_2O}\, g} = 290\,\frac{N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{s^2}{9.81\,m} \times \frac{kg\cdot m}{N\cdot s^2} = 29.6\ mm\,H_2O$$

Comments: (1) This pressure drop is large enough to measure readily. The straws could be used as a flowmeter.

(2) Straws would eliminate any swirl from the flow.

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 8.184

[4] Part 1/2

Given: Volume flow rate in a circular duct is to be measured using a "Pitot traverse," by measuring the velocity in each of several area segments across the duct, then summing.

Find: Comment on the way the traverse should be set up. Quantify and plot the expected error in measurement of flow rate as a function of the number of radial locations used in the traverse.

Solution: First divide the duct cross section into segments of equal area. Then measure velocity at the mean area of each segment.

Assume flow is turbulent, and that the velocity profile is well represented by the 1/7-power profile. From Eq. 8.24 the ratio of average flow velocity to centerline velocity is 0.817.

Distinguish two cases, depending on whether velocity is measured at the centerline.

Case 1: Measure velocity at the duct centerline, plus at (k - 1) other locations.

For $k = 1$, the sole measurement is at the duct centerline. This measures the centerline velocity U, which is 1/0.817 = 1.22 times the average flow velocity $\overline{\ }$. Thus the volume flow rate estimated by this 1-point measurement is 22 percent larger than the true value.

For $k = 2$, the duct is divided into two segments of equal area. The centerline velocity is measured and assigned the half of the duct area surrounding the centerline. The second measurement point is located at the midpoint of the remaining half of the duct area. Thus this point is located at the radius that encloses 3/4 of the duct area, or $r_2/R = (3/4)^{1/2} = 0.866$, as shown on the attached spreadsheet. The velocity ratio at this point is $\bar{u}/U = 0.92$. Averaging the segmental flow rates gives $(1.22 + 0.92)/2 = 1.07$. Thus the volume flow rate estimated by this 2-point measurement is 7 percent high.

For $k = 3$, the duct is divided into three portions of equal area. The centerline velocity is measured and assigned the one-third of the duct area surrounding the centerline. The second measurement point is located at the midpoint of the second one-third of the duct area. This point is located at the radius that encloses half the duct area, or at $r_2/R = (1/2)^{1/2} = 0.707$. The third measurement point is located at the midpoint of the third one-third of the duct area. This point is located at the radius enclosing 5/6 of the duct area, or at $r_3/R = (5/6)^{1/2} = 0.913$.

Results of calculations for $k = 4$ and 5 are also given on the spreadsheet.

Case 2: Measure velocity at k locations, not including the centerline.

For $k = 1$, the radius is chosen at half the duct area. Thus $r_1/R = (1/2)^{1/2} = 0.707$, $\bar{u}/U = 0.839$, and $\bar{u}/\overline{\ } = 1.03$, or about 3 percent too high, as shown on the spreadsheet.

For $k = 2$, the duct is divided into two equal areas. The first measurement is made at the midpoint of the inner area, where the radius includes one fourth of the total area. The second is made at the midpoint of the outer area, where the radius includes three fourths of the total duct area. The results are shown; the flow rate estimate is high by about 1.4 percent.

For $k = 3$, the duct is divided into three equal areas. The first measurement is made at the midpoint of the inner 1/3 of the duct area, where the radius includes 1/6 of the total area. The second is made at the midpoint of the second 1/3 of the duct area, where the radius includes 1/2 of the total duct area. The third is made at the midpoint of the third 1/3 of the duct area, where the radius includes 5/6 of the total duct area. The results are shown; the flow rate estimate is high by about 0.9 percent.

Results of calculations for $k = 4$ and 5 also are given on the spreadsheet.

Remarkably, Case 2 gives less than 2 percent error for any number of locations.

Problem 8.184

$V_{bar}/U =$ 0.817 $n =$ 7 $k =$ Number of measurement points

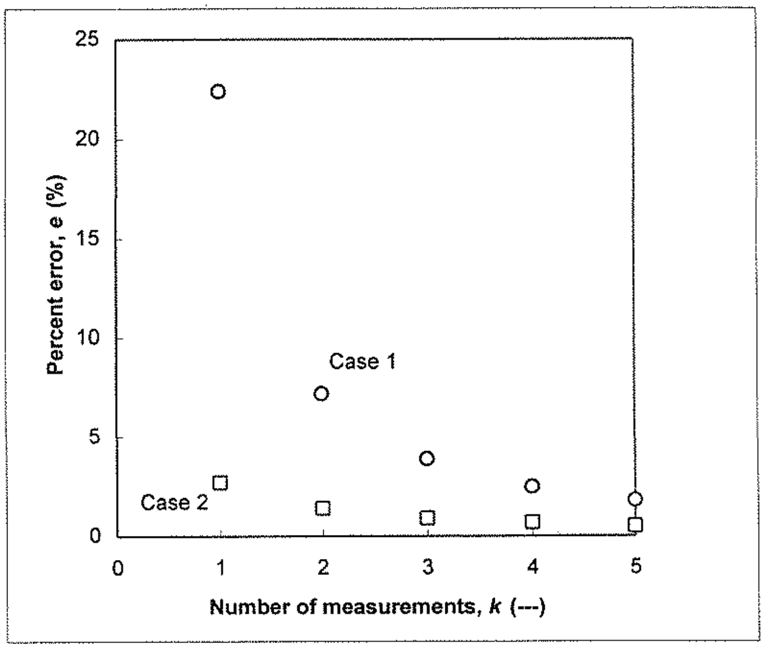
Case 1: Measure at centerline plus at (k - 1) other locations

k	i	r_i/R	u/U	u/V_{bar}	(%) Error
1	1	0.000	1.000	1.22	22.4
2	1	0.000	1.000	1.22	
	2	0.866	0.750	0.92	
				1.07	7.2
3	1	0.000	1.000	1.22	
	2	0.707	0.839	1.03	
	3	0.913	0.706	0.864	
				1.04	3.9
4	1	0.000	1.000	1.22	
	2	0.612	0.873	1.07	
	3	0.791	0.800	0.98	
	4	0.935	0.676	0.828	
				1.03	2.5
5	1	0.000	1.000	1.22	
	2	0.548	0.893	1.09	
	3	0.707	0.839	1.03	
	4	0.837	0.772	0.945	
	5	0.949	0.654	0.801	
				1.02	1.8

Case 2: Measure at k locations not including the centerline

k	i	r_i/R	u/U	u/V_{bar}	(%) Error
1	1	0.707	0.839	1.03	2.7
2	1	0.500	0.906	1.11	
	2	0.866	0.750	0.92	
				1.01	1.4
3	1	0.408	0.928	1.14	
	2	0.707	0.839	1.03	
	3	0.913	0.706	0.86	
				1.01	0.9
4	1	0.354	0.940	1.15	
	2	0.612	0.873	1.07	
	3	0.791	0.800	0.98	
	4	0.935	0.676	0.83	
				1.01	0.7
5	1	0.316	0.947	1.16	
	2	0.548	0.893	1.09	
	3	0.707	0.839	1.03	
	4	0.837	0.772	0.95	
	5	0.949	0.654	0.80	
				1.01	0.5

	Case 1	Case 2
k	e (%)	e (%)
1	22.4	2.7
2	7.2	1.4
3	3.9	0.9
4	2.5	0.7
5	1.8	0.5



Problem 8.185

Open-Ended Problem Statement: The chilled-water pipeline system that provides air conditioning for the Purdue University campus is described in Problem 8.158. The pipe diameter is selected to minimize total cost (capital cost plus operating cost). Annualized costs are compared, since capital cost occurs once and operating cost continues for the life of the system. The optimum diameter depends on both cost factors and operating conditions; the analysis must be repeated when these variables change. Perform a pipeline optimization analysis. Solve Problem 8.158 arranging your calculations to study the effect of pipe diameter on annual pumping cost. (Assume friction factor remains constant.) Obtain an expression for total annual cost per unit delivery (e.g., dollars per cubic meter), assuming construction cost varies as the square of pipe diameter. Obtain an analytic relation for the pipe diameter that yields minimum total cost per unit delivery. Assume the present chilled-water pipeline was optimized for a 20-year life with 5 percent annual interest. Repeat the optimization for a design to operate at 30 percent larger flow rate. Plot the annual cost for electrical energy for pumping and the capital cost, using the flow conditions of Problem 8.158, with pipe diameter varied from 300 to 900 mm. Show how the diameter may be chosen to minimize total cost. How sensitive are the results to interest rate?

(From Problem 8.158: The pipe makes a loop 3 miles in length. The pipe diameter is 2 ft and the material is steel. The maximum design volume flow rate is 11,200 gpm. The circulating pump is driven by an electric motor. The efficiencies of pump and motor are $\eta_p = 0.80$ and $\eta_m = 0.90$, respectively. Electricity cost is \$0.067/(kW·hr).)

Analysis: From Problem 8.158, the electrical energy for pumping costs \$174,000 per year for 11,200 gallons per minute circulation. The present line, with $D = 24$ in., is optimized for this flow rate. $\dot{W} = Q\Delta p$, so $\dot{W}/Q = \Delta p$.

The optimum pipe diameter minimizes total annualized cost, for construction and operation of the pipeline, $C_t = C_c + C_p$. Construction cost C_c is a one-time cost. Annualized pumping cost C_p is computed by summing the present worth of each annual pumping cost over the lifetime of the pipeline. For 20 years at 5 percent per year, spwf = 13.1 (see spreadsheet). Costs may be expressed in terms of diameter as

$$C_t = C_c + C_p = K_c D^2 + \frac{K_p}{D^5} \qquad (1)$$

For the optimum diameter, $dC_t/dD = 2K_c D - 5K_p D^{-6} = 0$, so

$$K_c = \frac{5K_p}{2D^7} = \frac{5C_p}{2D^2} = \frac{5}{2} \times (13.1)\$174{,}000 \times \frac{1}{(24)^2\,\text{in.}^2} = \$9890/\text{in.}^2 \longleftarrow K_c$$

From Eq. 1,

$$K_p = C_p D^5 = (13.1)\$174{,}000 \times (24)^5\,\text{in.}^5 = 1.81 \times 10^{13}\ \$\cdot\text{in.}^5 \longleftarrow K_p$$

Calculations with these values are shown on the spreadsheet.

To optimize at a new, larger flow rate, note $C_p \sim \Delta p \sim f\frac{L}{D}\frac{\rho\bar{V}^2}{2} = f\frac{L}{D}\frac{\rho}{2}\left(\frac{Q}{A}\right)^2 \sim f\frac{Q^2}{D^5}$

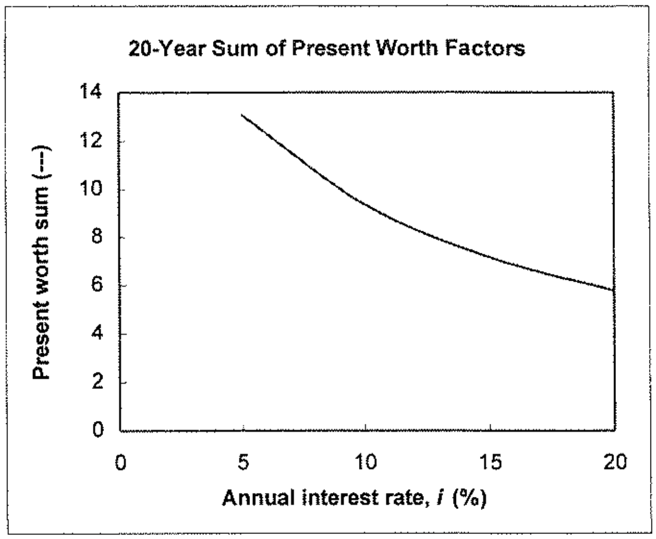
Thus

$K_p(\text{new}) = K_p(\text{old})\left(\frac{Q_{new}}{Q_{old}}\right)^2 = (1.3)^2 K_p(\text{old}) = 3.06 \times 10^{13}\ \$\cdot\text{in.}^5$ The new optimum is at $D \approx 25.9$ in., as shown on the second plot.

Results are not too sensitive to interest rate; only K_p varies. $D_{opt} \to 25$ in. for $i = 15\%$.

Problem 8.185

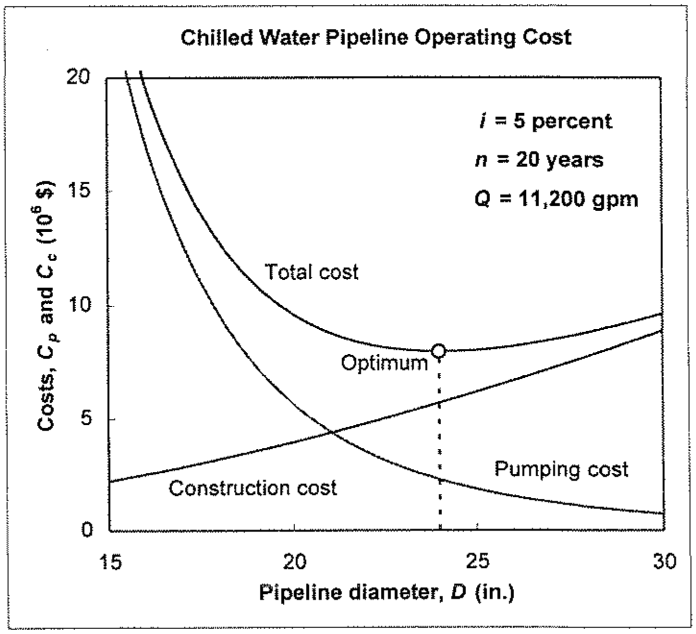
	Annual interest rate (%)			
i =	5	10	15	20
Year	*pwf*	*pwf*	*pwf*	*pwf*
1	1.00	1.00	1.00	1.00
2	0.952	0.909	0.870	0.833
3	0.907	0.826	0.756	0.694
4	0.864	0.751	0.658	0.579
5	0.823	0.683	0.572	0.482
6	0.784	0.621	0.497	0.402
7	0.746	0.564	0.432	0.335
8	0.711	0.513	0.376	0.279
9	0.677	0.467	0.327	0.233
10	0.645	0.424	0.284	0.194
11	0.614	0.386	0.247	0.162
12	0.585	0.350	0.215	0.135
13	0.557	0.319	0.187	0.112
14	0.530	0.290	0.163	0.0935
15	0.505	0.263	0.141	0.0779
16	0.481	0.239	0.123	0.0649
17	0.458	0.218	0.107	0.0541
18	0.436	0.198	0.0929	0.0451
19	0.416	0.180	0.0808	0.0376
20	0.396	0.164	0.0703	0.0313
Sum:	**13.1**	**9.4**	**7.2**	**5.8**



K_c = 9,890 \$/in.^{2} Cost of construction per diameter squared

K_p = 1.81E+13 \$*in.^{5} Present worth 20-yr cost of pumping 11,200 gpm

Pipe Diameter, *D* (in.)	**Cost of Pumping, C_p (10^6 \$)**	**Cost to Construct, C_c (10^6 \$)**	**Total Cost, C_t (10^6 \$)**
15	23.9	2.23	26.1
16	17.3	2.53	19.8
17	12.8	2.86	15.6
18	9.59	3.20	12.8
19	7.32	3.57	10.9
20	5.67	3.96	9.62
21	4.44	4.36	8.80
22	3.52	4.79	8.30
23	2.82	5.23	8.05
24	2.28	5.70	7.97
25	1.86	6.18	8.04
26	1.53	6.69	8.21
27	1.26	7.21	8.47
28	1.05	7.75	8.81
29	0.884	8.32	9.20
30	0.746	8.90	9.65

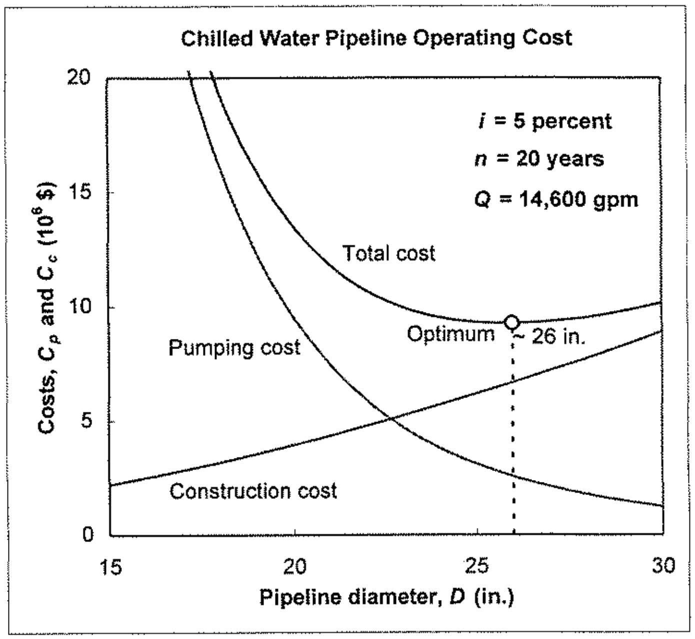


Problem 8.185

K_c = 9,890 \$/in.^{2} Cost of construction per diameter squared

K_p = 3.06E+13 \$*in.^{5} Present worth 20-yr cost of pumping 14,600 gpm

Pipe Diameter, D (in.)	**Cost of Pumping, C_p (10^6 \$)**	**Cost to Construct, C_c (10^6 \$)**	**Total Cost, C_t (10^6 \$)**
15	40.3	2.23	42.6
16	29.2	2.53	31.8
17	21.6	2.86	24.4
18	16.2	3.20	19.4
19	12.4	3.57	15.9
20	9.57	3.96	13.53
21	7.50	4.36	11.86
22	5.95	4.79	10.73
23	4.76	5.23	9.99
24	3.85	5.70	9.54
25	3.14	6.18	9.32
26	2.58	6.69	9.26
27	2.14	7.21	9.35
28	1.78	7.75	9.53
29	1.49	8.32	9.81
30	1.26	8.90	10.2



Problem 9.1

[2]

9.1 A model of a river towboat is to be tested at 1:18 scale. The boat is designed to travel at 3.5 m/s in fresh water at 10°C. Estimate the distance from the bow where transition occurs. Where should transition be stimulated on the model towboat?

Given: Model of riverboat

Find: Distance at which transition occurs

Solution:

Basic equation $Re_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{U \cdot x}{\nu}$ and transition occurs at about $Re_x = 5 \times 10^5$

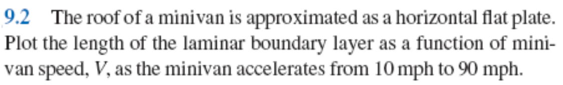
For water at 10ºC $\nu = 1.30 \times 10^{-6} \cdot \frac{m^2}{s}$ (Table A.8) and we are given $U = 3.5 \cdot \frac{m}{s}$

Hence $x_p = \frac{\nu \cdot Re_x}{U}$ $x_p = 0.186\,m$ $x_p = 18.6 \cdot cm$

For the model $x_m = \frac{x_p}{18}$ $x_m = 0.0103\,m$ $x_m = 10.3 \cdot mm$

Problem 9.2

[2]



9.2 The roof of a minivan is approximated as a horizontal flat plate. Plot the length of the laminar boundary layer as a function of minivan speed, *V*, as the minivan accelerates from 10 mph to 90 mph.

Given: Minivan traveling at various speeds

Find: Plot of boundary layer length as function of speed

Solution:

Governing equations:

The critical Reynolds number for transition to turbulence is

$$Re_{crit} = \rho V L_{crit}/\mu = 500000$$

The critical length is then

$$L_{crit} = 500000\mu/V\rho$$

Tabulated or graphical data:

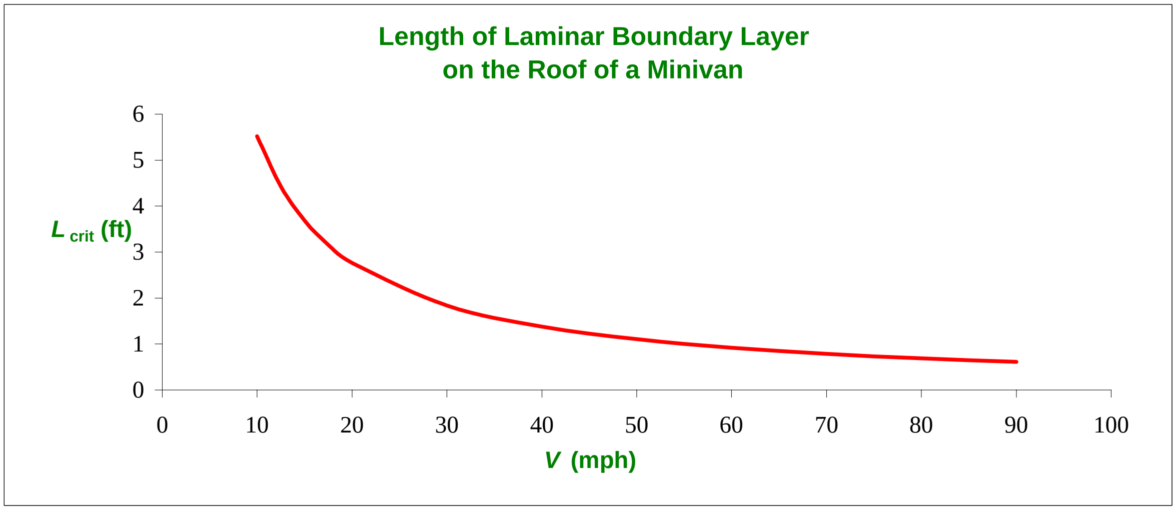
$\mu =$ 3.79E-07 lbf.s/ft^2

$\rho =$ 0.00234 slug/ft^3

(Table A.9, 68^{o}F)

Computed results:

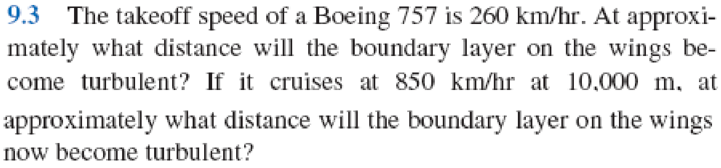
V (mph)	L_{crit} (ft)
10	5.52
13	4.42
15	3.68
18	3.16
20	2.76
30	1.84
40	1.38
50	1.10
60	0.920
70	0.789
80	0.690
90	0.614



Length of Laminar Boundary Layer on the Roof of a Minivan

Problem 9.3

[3]



9.3 The takeoff speed of a Boeing 757 is 260 km/hr. At approximately what distance will the boundary layer on the wings become turbulent? If it cruises at 850 km/hr at 10,000 m, at approximately what distance will the boundary layer on the wings now become turbulent?

Given: Boeing 757

Find: Point at which transition occurs; Same point at 10,000 m

Solution:

Basic equation $\mathrm{Re}_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{U \cdot x}{\nu}$ and transition occurs at about $\mathrm{Re}_x = 5 \times 10^5$

For air at 20ºC $\nu = 1.50 \times 10^{-5} \cdot \frac{m^2}{s}$ (Table A.10) and we are given $U = 260 \cdot \frac{km}{hr}$

Hence $x_p = \frac{\nu \cdot \mathrm{Re}_x}{U}$ $x_p = 0.104\,m$ $x_p = 10.4\,cm$

At 10,000 m $T = 223.3 \cdot K$ (Table A.3) $T = -49.8\,°C$

We need to estimate ν or μ at this temperature. From Appendix A-3

$$\mu = \frac{b \cdot \sqrt{T}}{1 + \frac{S}{T}} \qquad b = 1.458 \times 10^{-6} \cdot \frac{kg}{m \cdot s \cdot K^{\frac{1}{2}}} \qquad S = 110.4 \cdot K$$

Hence $\mu = \frac{b \cdot \sqrt{T}}{1 + \frac{S}{T}}$ $\mu = 1.458 \times 10^{-5} \frac{N \cdot s}{m^2}$

For air at 10,000 m (Table A.3)

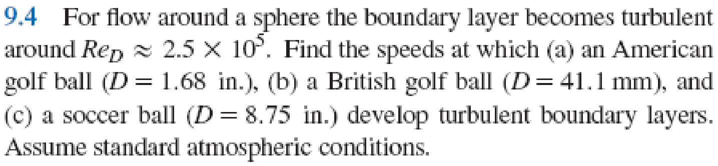
$$\frac{\rho}{\rho_{SL}} = 0.3376 \qquad \rho_{SL} = 1.225 \cdot \frac{kg}{m^3} \qquad \rho = 0.3376 \cdot \rho_{SL} \qquad \rho = 0.414 \frac{kg}{m^3}$$

$\nu = \frac{\mu}{\rho}$ $\nu = 3.53 \times 10^{-5} \frac{m^2}{s}$ and we are given $U = 850 \cdot \frac{km}{hr}$

Hence $x_p = \frac{\nu \cdot \mathrm{Re}_x}{U}$ $x_p = 0.0747\,m$ $x_p = 7.47\,cm$

Problem 9.4

[2]



9.4 For flow around a sphere the boundary layer becomes turbulent around $Re_D \approx 2.5 \times 10^5$. Find the speeds at which (a) an American golf ball ($D = 1.68$ in.), (b) a British golf ball ($D = 41.1$ mm), and (c) a soccer ball ($D = 8.75$ in.) develop turbulent boundary layers. Assume standard atmospheric conditions.

Given: Flow around American and British golf balls, and soccer ball

Find: Speed at which boundary layer becomes turbulent

Solution:

Basic equation $Re_D = \frac{\rho \cdot U \cdot D}{\mu} = \frac{U \cdot D}{\nu}$ and transition occurs at about $Re_D = 2.5 \times 10^5$

For air $\nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$ (Table A.9)

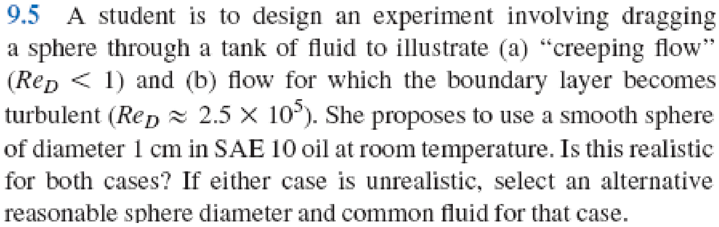
For the American golf ball $D = 1.68 \cdot in$ Hence $U = \frac{\nu \cdot Re_D}{D}$ $U = 289 \cdot \frac{ft}{s}$ $U = 197\,mph$ $U = 88.2\,\frac{m}{s}$

For the British golf ball $D = 41.1 \cdot mm$ Hence $U = \frac{\nu \cdot Re_D}{D}$ $U = 300 \cdot \frac{ft}{s}$ $U = 205\,mph$ $U = 91.5\,\frac{m}{s}$

For soccer ball $D = 8.75 \cdot in$ Hence $U = \frac{\nu \cdot Re_D}{D}$ $U = 55.5 \cdot \frac{ft}{s}$ $U = 37.9\,mph$ $U = 16.9\,\frac{m}{s}$

Problem 9.5

[2]



9.5 A student is to design an experiment involving dragging a sphere through a tank of fluid to illustrate (a) "creeping flow" ($Re_D < 1$) and (b) flow for which the boundary layer becomes turbulent ($Re_D \approx 2.5 \times 10^5$). She proposes to use a smooth sphere of diameter 1 cm in SAE 10 oil at room temperature. Is this realistic for both cases? If either case is unrealistic, select an alternative reasonable sphere diameter and common fluid for that case.

Given:

Experiment with 1 cm diameter sphere in SAE 10 oil

Find:

Reasonableness of two flow extremes

Solution:

Basic equation $\quad Re_D = \frac{\rho \cdot U \cdot D}{\mu} = \frac{U \cdot D}{\nu} \quad$ and transition occurs at about

For SAE 10 $\quad \nu = 1.1 \times 10^{-4} \cdot \frac{m^2}{s} \quad$ (Fig. A.3 at 20ºC) $\quad$ and $\quad D = 1 \cdot cm$

For $\quad Re_D = 1 \quad$ we find $\quad U = \frac{\nu \cdot Re_D}{D} \quad U = 0.011 \cdot \frac{m}{s} \quad U = 1.10 \cdot \frac{cm}{s} \quad$ which is reasonable

For $\quad Re_D = 2.5 \times 10^5 \quad U = \frac{\nu \cdot Re_D}{D} \quad U = 2750 \frac{m}{s} \quad$ which is much too high!

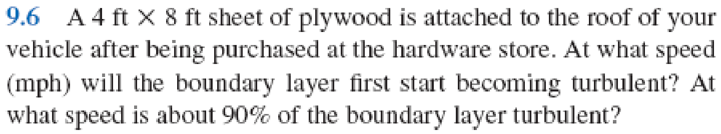
Note that for $\quad Re_D = 2.5 \times 10^5 \quad$ we need to increase the sphere diameter D by a factor of about 1000, or reduce the viscosity ν b
the same factor, or some combination of these. One possible solution is

For water $\quad \nu = 1.01 \times 10^{-6} \cdot \frac{m^2}{s} \quad$ (Table A.8 at 20ºC) $\quad$ and $\quad D = 10 \cdot cm$

For $\quad Re_D = 2.5 \times 10^5 \quad$ we find $\quad U = \frac{\nu \cdot Re_D}{D} \quad U = 2.52 \cdot \frac{m}{s} \quad$ which is reasonable

Hence one solution is to use a 10 cm diameter sphere in a water tank.

Problem 9.6



9.6 A 4 ft × 8 ft sheet of plywood is attached to the roof of your vehicle after being purchased at the hardware store. At what speed (mph) will the boundary layer first start becoming turbulent? At what speed is about 90% of the boundary layer turbulent?

Given: Sheet of plywood attached to the roof of a car

Find: Speed at which boundary layer becomes turbulent; Speed at which 90% is turbulent

Solution:

Basic equation $Re_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{U \cdot x}{\nu}$ and transition occurs at about $Re_x = 5 \times 10^5$

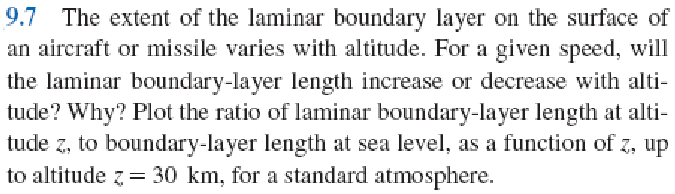
For air $\nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$ (Table A.9)

For the plywood $x = 8 \cdot ft$ Hence $U = \frac{\nu \cdot Re_x}{x}$ $U = 10.1 \cdot \frac{ft}{s}$ $U = 6.90 \cdot mph$

When 90% of the boundary layer is turbulent $x = 0.1 \times 8 \cdot ft$ Hence $U = \frac{\nu \cdot Re_x}{x}$ $U = 101 \cdot \frac{ft}{s}$ $U = 69.0 \cdot mph$

Problem 9.7

[2]



9.7 The extent of the laminar boundary layer on the surface of an aircraft or missile varies with altitude. For a given speed, will the laminar boundary-layer length increase or decrease with altitude? Why? Plot the ratio of laminar boundary-layer length at altitude z, to boundary-layer length at sea level, as a function of z, up to altitude $z = 30$ km, for a standard atmosphere.

Given: Aircraft or missile at various altitudes

Find: Plot of boundary layer length as function of altitude

Solution:

Governing equations:

The critical Reynolds number for transition to turbulence is

$$Re_{crit} = \rho U L_{crit}/\mu = 500000$$

The critical length is then

$$L_{crit} = 500000\mu/U\rho$$

Let L_0 be the length at sea level (density ρ_0 and viscosity μ_0). Then

$$L_{crit}/L_0 = (\mu/\mu_0)/(\rho/\rho_0)$$

The viscosity of air increases with temperature so generally decreases with elevation;
the density also decreases with elevation, but much more rapidly.
Hence we expect that the length ratio increases with elevation

For the density ρ, we use data from Table A.3.
For the viscosity μ, we use the Sutherland correlation (Eq. A.1)

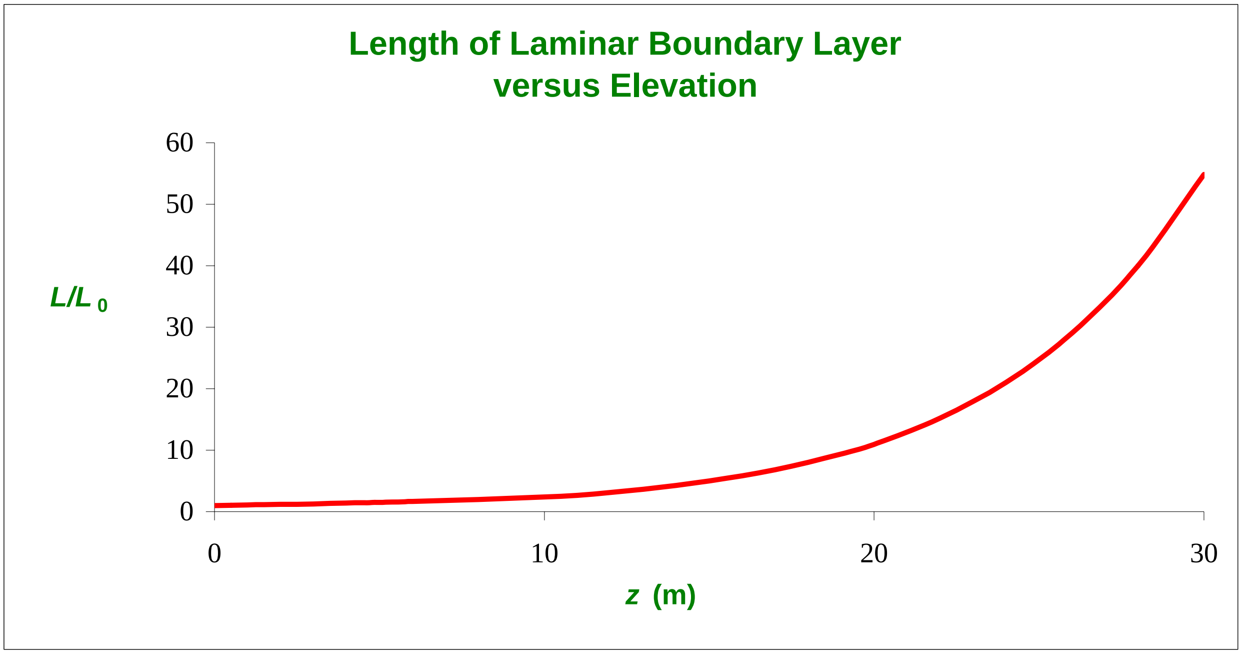
$$\mu = bT^{1/2}/(1+S/T)$$

$b =$ 1.46E-06 kg/m.s.K$^{1/2}$

$S =$ 110.4 K

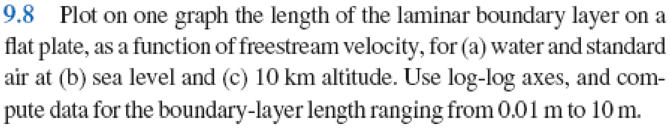
Computed results:

z (km)	T (K)	ρ/ρ_0	μ/μ_0	L_{crit}/L_0
0.0	288.2	1.0000	1.000	1.000
0.5	284.9	0.9529	0.991	1.04
1.0	281.7	0.9075	0.982	1.08
1.5	278.4	0.8638	0.973	1.13
2.0	275.2	0.8217	0.965	1.17
2.5	271.9	0.7812	0.955	1.22
3.0	268.7	0.7423	0.947	1.28
3.5	265.4	0.7048	0.937	1.33
4.0	262.2	0.6689	0.928	1.39
4.5	258.9	0.6343	0.919	1.45
5.0	255.7	0.6012	0.910	1.51
6.0	249.2	0.5389	0.891	1.65
7.0	242.7	0.4817	0.872	1.81
8.0	236.2	0.4292	0.853	1.99
9.0	229.7	0.3813	0.834	2.19
10.0	223.3	0.3376	0.815	2.41
11.0	216.8	0.2978	0.795	2.67
12.0	216.7	0.2546	0.795	3.12
13.0	216.7	0.2176	0.795	3.65
14.0	216.7	0.1860	0.795	4.27
15.0	216.7	0.1590	0.795	5.00
16.0	216.7	0.1359	0.795	5.85
17.0	216.7	0.1162	0.795	6.84
18.0	216.7	0.0993	0.795	8.00
19.0	216.7	0.0849	0.795	9.36
20.0	216.7	0.0726	0.795	10.9
22.0	218.6	0.0527	0.800	15.2
24.0	220.6	0.0383	0.806	21.0
26.0	222.5	0.0280	0.812	29.0
28.0	224.5	0.0205	0.818	40.0
30.0	226.5	0.0150	0.824	54.8



Problem 9.8

[2]



9.8 Plot on one graph the length of the laminar boundary layer on a flat plate, as a function of freestream velocity, for (a) water and standard air at (b) sea level and (c) 10 km altitude. Use log-log axes, and compute data for the boundary-layer length ranging from 0.01 m to 10 m.

Given: Laminar boundary layer (air & water)

Find: Plot of boundary layer length as function of speed (at various altitudes for air)

Solution:

Governing equations:

The critical Reynolds number for transition to turbulence is

$$Re_{\text{crit}} = UL_{\text{crit}}/\mu = 500000$$

The critical length is then

$$L_{\text{crit}} = 500000\mu/U\rho$$

For air at sea level and 10 km, we can use tabulated data for density ρ from Table A.3.
For the viscosity μ, use the Sutherland correlation (Eq. A.1)

$$\mu = bT^{1/2}/(1+S/T)$$

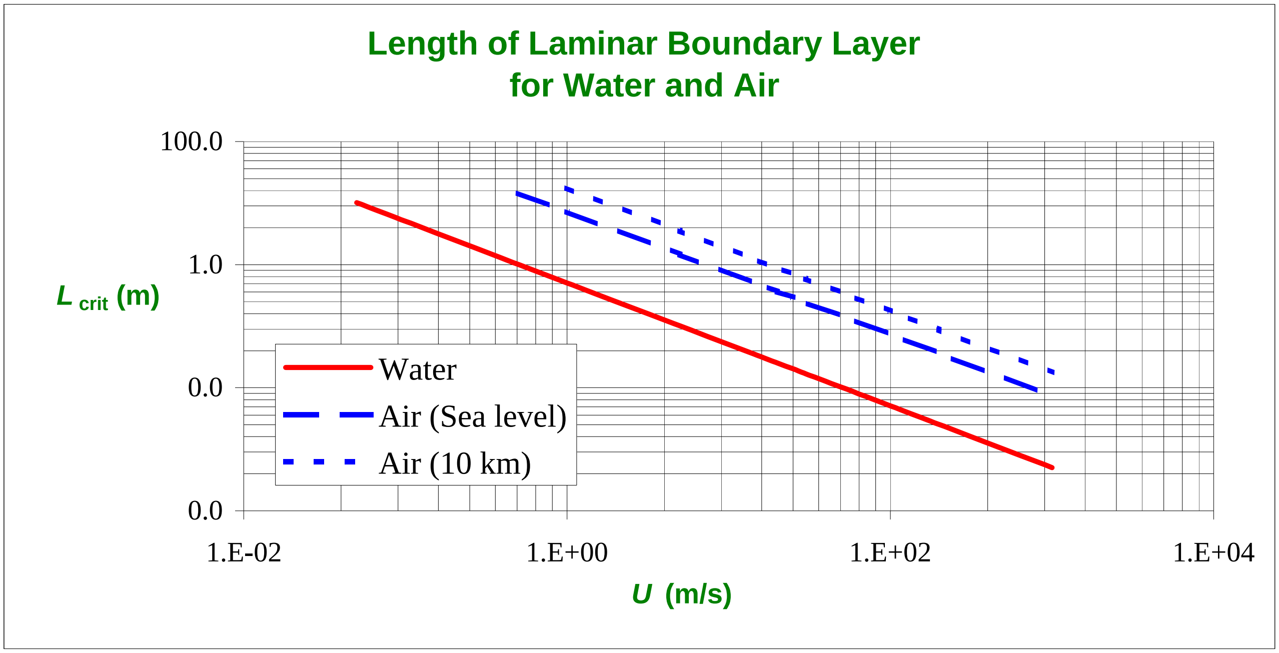
b = 1.46E-06 kg/m.s.K$^{1/2}$

S = 110.4 K

Air (sea level, T = 288.2 K):	Air (10 K, T = 223.3 K):	Water (20$^{\text{o}}$C):
ρ = 1.225 kg/m^3	ρ = 0.414 kg/m^3	ρ = 998 slug/ft^3
(Table A.3)	(Table A.3)	μ = 1.01E-03 N.s/m^2
μ = 1.79E-05 N.s/m^2	μ = 1.46E-05 N.s/m^2	(Table A.8)
(Sutherland)	(Sutherland)	

Computed results:

U **(m/s)**	**Water**	**Air (Sea level)**	**Air (10 km)**
	L_{crit} (m)	**L_{crit} (m)**	**L_{crit} (m)**
0.05	10.12	146.09	352.53
0.10	5.06	73.05	176.26
0.5	1.01	14.61	35.25
1.0	0.506	7.30	17.63
5.0	0.101	1.46	3.53
15	0.0337	0.487	1.18
20	0.0253	0.365	0.881
25	0.0202	0.292	0.705
30	0.0169	0.243	0.588
50	0.0101	0.146	0.353
100	0.00506	0.0730	0.176
200	0.00253	0.0365	0.0881
1000	0.00051	0.0073	0.0176



Problem 9.9

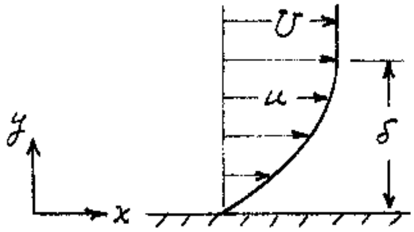
[2]

Given: Sinusoidal velocity profile for laminar boundary layer

$$u = A\sin(By) + C$$

Find: (a) State three applicable boundary conditions.
(b) Evaluate A, B and C.

Solution: For the boundary layer, at



(1) $y = 0, \quad u = 0$ (no slip)

(2) $y = \delta, \quad u = U$

(3) $\frac{\partial u}{\partial y} = 0$ (no shear stress)

B.C.

Applying these boundary conditions,

(1) $u(0) = A\sin(0) + C = 0 \qquad \therefore C = 0$

(2) $u(\delta) = A\sin(B\delta) = U$

$$\frac{\partial u}{\partial y} = AB\cos(By)$$

(3) $\left.\frac{\partial u}{\partial y}\right)_{y=\delta} = AB\cos(B\delta) = 0 \qquad \therefore B\delta = \frac{\pi}{2} \quad \text{or } B = \frac{\pi}{2\delta}$

Then from (2), $A\sin(B\delta) = A\sin(\frac{\pi}{2}) = A = U$, and then

$$u = U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$$A = U, \; B = \frac{\pi}{2\delta}, \; C = 0$$

A,B,C

Problem 9.10

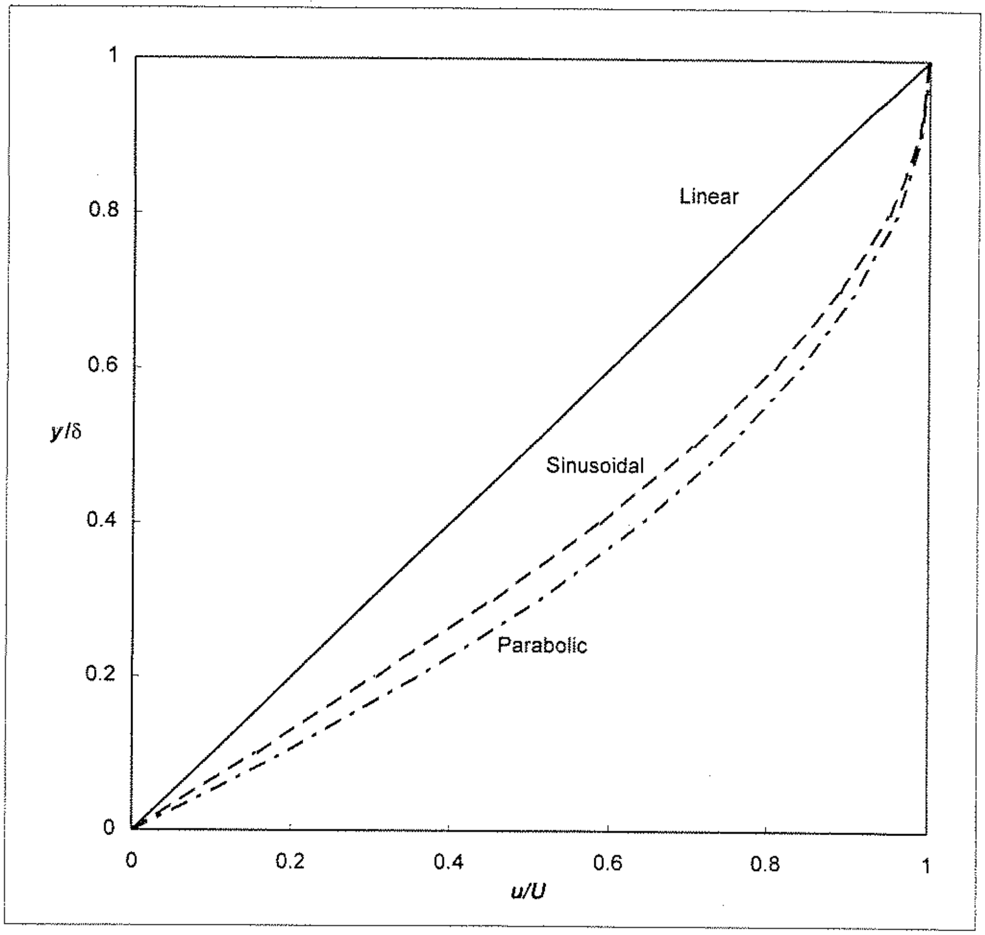
Given: Linear, parabolic, and sinusoidal velocity profiles for laminar boundary layer.

Linear $\frac{u}{U} = \frac{y}{\delta}$

Parabolic $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ Sinusoidal $\frac{u}{U} = \sin \frac{\pi}{2}\left(\frac{y}{\delta}\right)$

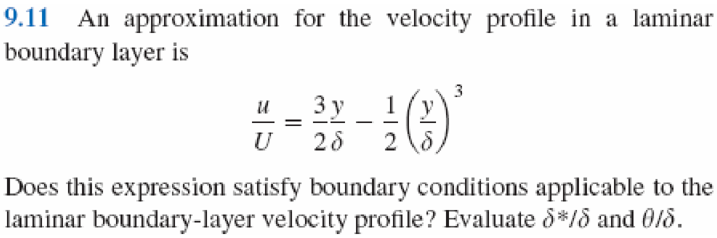
Find: Compare shapes by plotting $\frac{y}{\delta}$ vs. $\frac{u}{U}$.

Solution:



Problem 9.11

[2]



9.11 An approximation for the velocity profile in a laminar boundary layer is

$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Given: Laminar boundary layer profile

Find: If it satisfies BC's; Evaluate δ^*/δ and θ/δ

Solution:

The boundary layer equation is $\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ for which $u = U$ at $y = \delta$

The BC's are $u(0) = 0$ $\left.\frac{du}{dy}\right|_{y=\delta} = 0$

At $y = 0$ $\frac{u}{U} = \frac{3}{2}(0) - \frac{1}{2}(0)^3 = 0$

At $y = \delta$ $\frac{du}{dy} = U\left(\frac{3}{2}\frac{1}{\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right)\Bigg|_{y=\delta} = U\left(\frac{3}{2}\frac{1}{\delta} - \frac{3}{2}\frac{\delta^2}{\delta^3}\right) = 0$

For δ^*: $\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right)dy = \int_0^\delta \left(1 - \frac{u}{U}\right)dy$

Then $\frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^1 \left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta$

with $\frac{u}{U} = \frac{3}{2}\eta - \frac{1}{2}\eta^3$

Hence $\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 \left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)d\eta = \left[\eta - \frac{3}{4}\eta^2 + \frac{1}{8}\eta^4\right]_0^1 = \frac{3}{8} = 0.375$

For θ: $\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$

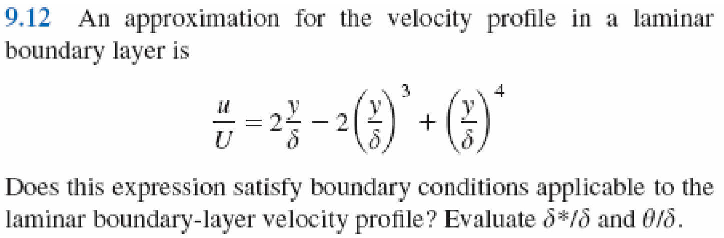
Then $\frac{\theta}{\delta} = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\eta$

Hence $\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\eta = \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)d\eta = \int_0^1 \left(\frac{3}{2}\eta - \frac{9}{4}\eta^2 - \frac{1}{2}\eta^3 + \frac{3}{2}\eta^4 - \frac{1}{4}\eta^6\right)d\eta$

$$\frac{\theta}{\delta} = \left[\frac{3}{4}\eta^2 - \frac{3}{4}\eta^3 - \frac{1}{8}\eta^4 + \frac{3}{10}\eta^5 - \frac{1}{28}\eta^7\right]_0^1 = \frac{39}{280} = 0.139$$

Problem 9.12

[2]



9.12 An approximation for the velocity profile in a laminar boundary layer is

$$\frac{u}{U}=2\frac{y}{\delta}-2\left(\frac{y}{\delta}\right)^3+\left(\frac{y}{\delta}\right)^4$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Given:

Laminar boundary layer profile

Find:

If it satisfies BC's; Evaluate δ^*/δ and θ/δ

Solution:

The boundary layer equation is $\frac{u}{U}=2\frac{y}{\delta}-2\left(\frac{y}{\delta}\right)^3+\left(\frac{y}{\delta}\right)^4$ for which $u = U$ at $y = \delta$

The BC's are $u(0)=0 \qquad \left.\frac{du}{dy}\right|_{y=\delta}=0$

At $y = 0$ $\quad \frac{u}{U}=2(0)-2(0)^3+(0)^4=0$

At $y = \delta$ $\quad \frac{du}{dy}=U\left(2\frac{1}{\delta}-6\frac{y^2}{\delta^3}+4\frac{y^3}{\delta^4}\right)\Bigg|_{y=\delta}=U\left(2\frac{1}{\delta}-6\frac{\delta^2}{\delta^3}+4\frac{\delta^3}{\delta^4}\right)=0$

For δ^*: $\quad \delta^*=\int_0^\infty\left(1-\frac{u}{U}\right)dy=\int_0^\delta\left(1-\frac{u}{U}\right)dy$

Then $\quad \frac{\delta^*}{\delta}=\frac{1}{\delta}\int_0^\delta\left(1-\frac{u}{U}\right)dy=\int_0^1\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right)=\int_0^1\left(1-\frac{u}{U}\right)d\eta$

with $\quad \frac{u}{U}=2\eta-2\eta^3+\eta^4$

Hence $\quad \frac{\delta^*}{\delta}=\int_0^1\left(1-\frac{u}{U}\right)d\eta=\int_0^1\left(1-2\eta+2\eta^3-\eta^4\right)d\eta=\left[\eta-\eta^2+\frac{1}{2}\eta^4-\frac{1}{5}\eta^5\right]_0^1=\frac{3}{10}=0.3$

For θ: $\quad \theta=\int_0^\infty\frac{u}{U}\left(1-\frac{u}{U}\right)dy=\int_0^\delta\frac{u}{U}\left(1-\frac{u}{U}\right)dy$

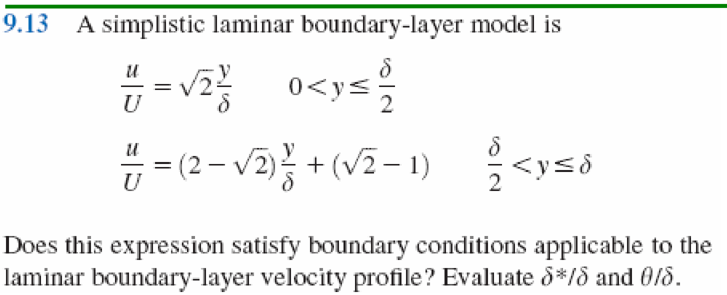
Then $\quad \frac{\theta}{\delta}=\frac{1}{\delta}\int_0^\delta\frac{u}{U}\left(1-\frac{u}{U}\right)dy=\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right)=\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d\eta$

Hence

$$\frac{\theta}{\delta}=\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d\eta=\int_0^1\left(2\eta-\eta^3+\eta^4\right)\left(1-2\eta+\eta^3-\eta^4\right)d\eta=\int_0^1\left(2\eta-4\eta^2-2\eta^3+9\eta^4-4\eta^5-4\eta^6+4\eta^7-\eta^8\right)d\eta$$

$$\frac{\theta}{\delta}=\left[\eta^{2}-\frac{4}{3}\eta^{3}-\frac{1}{2}\eta^{4}+\frac{9}{5}\eta^{5}-\frac{4}{7}\eta^{7}+\frac{1}{2}\eta^{8}-\frac{1}{9}\eta^{9}\right]_{0}^{1}=\frac{37}{315}=0.117$$

Problem 9.13



9.13 A simplistic laminar boundary-layer model is

$$\frac{u}{U} = \sqrt{2}\frac{y}{\delta} \qquad 0 < y \le \frac{\delta}{2}$$

$$\frac{u}{U} = (2-\sqrt{2})\frac{y}{\delta} + (\sqrt{2}-1) \qquad \frac{\delta}{2} < y \le \delta$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Given:

Laminar boundary layer profile

Find:

If it satisfies BC's; Evaluate δ^*/δ and θ/δ

Solution:

The boundary layer equation is

$$\frac{u}{U} = \sqrt{2}\frac{y}{\delta} \qquad 0 < y < \frac{\delta}{2}$$

$$\frac{u}{U} = \left(2-\sqrt{2}\right)\frac{y}{\delta} + \left(\sqrt{2}-1\right) \qquad \frac{\delta}{2} < y < \delta$$ for which $u = U$ at $y = \delta$

The BC's are

$$u(0) = 0 \qquad \left.\frac{du}{dy}\right|_{y=\delta} = 0$$

At $y = 0$

$$\frac{u}{U} = \sqrt{2}(0) = 0$$

At $y = \delta$

$$\frac{du}{dy} = U\left[\left(2-\sqrt{2}\right)\frac{1}{\delta}\right]_{y=\delta} \neq 0$$ so it fails the outer BC.

This simplistic distribution is a piecewise linear profile: The first half of the layer has velocity gradient $\sqrt{2}\frac{U}{\delta} = 1.414\frac{U}{\delta}$, and the second half has velocity gradient $\left(2-\sqrt{2}\right)\frac{U}{\delta} = 0.586\frac{U}{\delta}$. At $y = \delta$, we make another transition to zero velocity gradient.

For δ^*:

$$\delta^* = \int_0^\infty \left(1-\frac{u}{U}\right)dy = \int_0^\delta \left(1-\frac{u}{U}\right)dy$$

Then

$$\frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1-\frac{u}{U}\right)dy = \int_0^1 \left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1-\frac{u}{U}\right)d\eta$$

with

$$\frac{u}{U} = \sqrt{2}\eta \qquad 0 < \eta < \frac{1}{2}$$

$$\frac{u}{U} = \left(2-\sqrt{2}\right)\eta + \left(\sqrt{2}-1\right) \qquad \frac{1}{2} < \eta < 1$$

Hence

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1-\frac{u}{U}\right)d\eta = \int_0^{1/2}\left(1-\sqrt{2}\eta\right)d\eta + \int_{1/2}^1\left[1-\left(2-\sqrt{2}\right)\eta-\left(\sqrt{2}-1\right)\right]d\eta = \left[\frac{1}{2\sqrt{2}}\left(\sqrt{2}\eta-1\right)^2\right]_0^{1/2} + \left[\frac{1}{2}(\eta-1)^2\left(\sqrt{2}-2\right)\right]_{1/2}^1$$

$$\frac{\delta *}{\delta}=\left[\frac{1}{2}-\frac{\sqrt{2}}{8}\right]+\left[\frac{1}{4}-\frac{\sqrt{2}}{8}\right]=\frac{3}{4}-\frac{\sqrt{2}}{4}=0.396$$

For θ:

$$\theta=\int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy=\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

Then

$$\frac{\theta}{\delta}=\frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy=\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right)=\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta$$

Hence, after a LOT of work

$$\frac{\theta}{\delta}=\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta=\int_0^{1/2}\sqrt{2}\eta\left(1-\sqrt{2}\eta\right)d\eta+\int_{1/2}^1\left[\left(\left(2-\sqrt{2}\right)\eta+\left(\sqrt{2}-1\right)\right)\left(1-\left(2-\sqrt{2}\right)\eta-\left(\sqrt{2}-1\right)\right)\right]d\eta$$

$$\frac{\theta}{\delta}=\left[\sqrt{2}\eta^2\left(\frac{\sqrt{2}\eta}{3}-\frac{1}{2}\right)\right]_0^{1/2}+\left[\left(\frac{1}{3}\left(\sqrt{2}-2\right)\left(\eta-1\right)-\frac{1}{2}\right)\left(\sqrt{2}-2\right)\left(\eta-1\right)^2\right]_{1/2}^1=\frac{\sqrt{2}}{8}-\frac{1}{12}+\frac{\sqrt{2}}{24}=\frac{\sqrt{2}}{6}-\frac{1}{12}=0.152$$

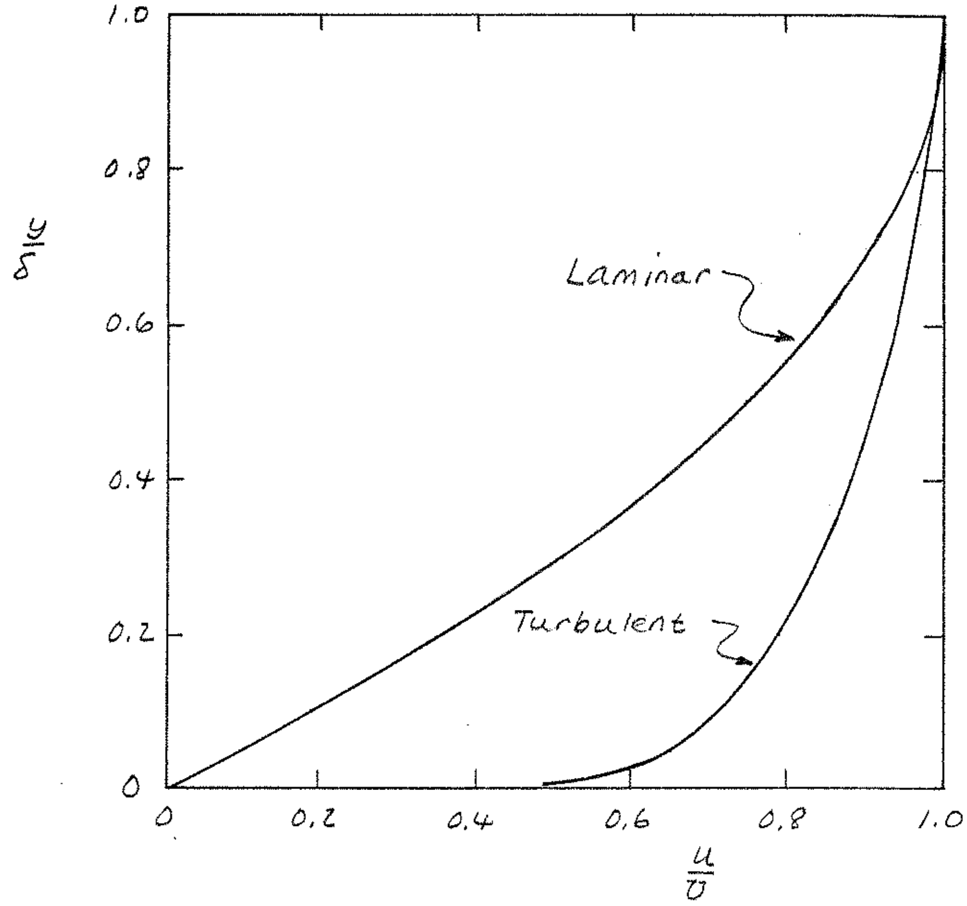
Problem 9.14

Given: "Power law" velocity profile for turbulent boundary layer and parabolic profile for laminar boundary layer,

"Power law" $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$ Parabolic $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Find: Compare shapes by plotting $\frac{y}{\delta}$ vs. $\frac{u}{U}$.

Solution:



Note that the "power law" profile gives an infinite value of du/dy at y=0, since

$$\frac{du}{dy} = \frac{d\left(\frac{u}{U}\right)}{d\left(\frac{y}{\delta}\right)} = \frac{1}{7}\frac{1}{\left(\frac{y}{\delta}\right)^{6/7}} \rightarrow \infty \text{ as } \frac{y}{\delta} \rightarrow 0.$$

Problem 9.15

[3]

National Brand

Given: Linear, parabolic, and sinusoidal profiles used to represent the laminar boundary layer velocity profile.

Evaluate: the ratio θ/δ for each profile.

Solution:

Definition: $\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy$ (9.2)

then,

$$\theta/\delta = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\eta$$

Linear profile $\frac{u}{U} = y/\delta = \eta$

$$\theta/\delta = \int_0^1 \eta(1-\eta)d\eta = \int_0^1 (\eta-\eta^2)d\eta = \left[\frac{1}{2}\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{6} = 0.167 \longleftarrow$$

Parabolic profile $\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$

$$\theta/\delta = \int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta = \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)d\eta$$

$$\theta/\delta = \left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1 = \left[1 - \frac{5}{3} + 1 - \frac{1}{5}\right] = \frac{2}{15} = 0.133 \longleftarrow$$

Sinusoidal profile $\frac{u}{U} = \sin\frac{\pi}{2}\frac{y}{\delta} = \sin\frac{\pi}{2}\eta$

$$\theta/\delta = \int_0^1 \sin\frac{\pi}{2}\eta\left(1-\sin\frac{\pi}{2}\eta\right)d\eta = \int_0^1\left(\sin\frac{\pi}{2}\eta - \sin^2\frac{\pi}{2}\eta\right)d\eta$$

$$\theta/\delta = \left[-\frac{2}{\pi}\cos\frac{\pi}{2}\eta - \frac{2}{\pi}\left\{\frac{\pi\eta}{4} - \frac{1}{4}\sin\pi\eta\right\}\right]_0^1 = -0-\left(-\frac{2}{\pi}\right) - \frac{2}{\pi}\left(\frac{\pi}{4}\right) - 0$$

$$\theta/\delta = \frac{2}{\pi} - \frac{1}{2} = 0.137 \longleftarrow$$

Summarizing:

Profile	Expression	θ/δ
Linear	$u/U = \eta$	0.167
Parabolic	$u/U = 2\eta - \eta^2$	0.133
Sinusoidal	$u/U = \sin\frac{\pi}{2}\eta$	0.137

Problem 9.16

[2]

Given: Linear, parabolic, and sinusoidal velocity profiles for laminar boundary layer,

(a) linear $u/U = y/\delta$

(b) parabolic $u/U = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$

(c) sinusoidal $u/U = \sin\frac{\pi}{2}\frac{y}{\delta}$

Find: ratio δ^*/δ for each profile.

Solution:

Definition: $\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right)dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right)dy$ (9.1)

Then, $\frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^1 \left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta$

(a) Linear profile $u/U = y/\delta = \eta$

$$\delta^*/\delta = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 (1-\eta)\,d\eta = \left[\eta - \frac{1}{2}\eta^2\right]_0^1 = \underline{\frac{1}{2}}$$

(b) Parabolic profile $u/U = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$

$$\delta^*/\delta = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 (1 - 2\eta + \eta^2)\,d\eta = \left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1 = \underline{\frac{1}{3}}$$

(c) Sinusoidal profile $u/U = \sin\frac{\pi}{2}\frac{y}{\delta} = \sin\frac{\pi}{2}\eta$

$$\delta^*/\delta = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 \left(1 - \sin\frac{\pi}{2}\eta\right)d\eta = \left[\eta + \frac{2}{\pi}\cos\frac{\pi}{2}\eta\right]_0^1 = 1 - \frac{2}{\pi} = \underline{0.363}$$

Problem 9.17

[2]

Given: "Power-law" velocity profile for turbulent boundary layer and parabolic profile for laminar boundary layer,

"Power-law" $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ Parabolic $\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$

Evaluate: (and compare) ratios δ^*/δ and θ/δ for each profile

Solution:

Definitions: $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$ (9.1)

$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$ (9.2)

Then $\delta^*/\delta = \frac{1}{\delta}\int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \frac{u}{U}\right) d\eta$

For the "power-law" profile

$$\delta^*/\delta = \int_0^1 \left(1 - \eta^{1/7}\right) d\eta = \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}$$

For the parabolic profile

$$\delta^*/\delta = \int_0^1 \left(1 - 2\eta + \eta^2\right) d\eta = \left[\eta - \eta^2 + \frac{\eta^3}{3}\right]_0^1 = \frac{1}{3}$$

thus δ^*/δ (turbulent) $= \frac{3}{8}\ \delta^*/\delta$ (laminar) ——— δ^*/δ

Also $\theta/\delta = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta$.

For the "power-law" profile

$$\theta/\delta = \int_0^1 \eta^{1/7}\left(1 - \eta^{1/7}\right) d\eta = \int_0^1 \left(\eta^{1/7} - \eta^{2/7}\right) d\eta = \left[\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}\right]_0^1 = \frac{7}{72}$$

For the parabolic profile

$$\theta/\delta = \int_0^1 \left(2\eta - \eta^2\right)\left(1 - 2\eta + \eta^2\right) d\eta = \int_0^1 \left(2\eta - 5\eta^2 + 4\eta^3 - \eta^4\right) d\eta$$

$$\theta/\delta = \left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{\eta^5}{5}\right]_0^1 = \left[1 - \frac{5}{3} + 1 - \frac{1}{5}\right] = \frac{2}{15}$$

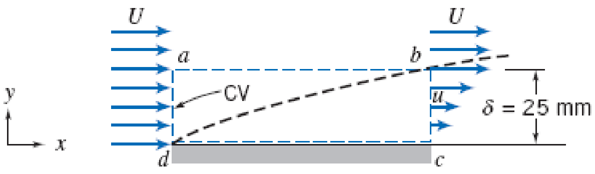
thus θ/δ (turbulent) $= 0.729\ \theta/\delta$ (laminar) ——— θ/δ

Profile	δ^*/δ	θ/δ
Power-law	0.125	0.0972
Parabolic	0.333	0.133

Problem 9.18

[3]

9.18 A fluid, with density $\rho = 800\ \text{kg/m}^3$, flows at $U = 3$ m/s over a flat plate 3 m long and 1 m wide. At the trailing edge the boundary-layer thickness is $\delta = 25$ mm. Assume the velocity profile is linear, as shown, and that the flow is two-dimensional (flow conditions are independent of z). Using control volume *abcd*, shown by dashed lines, compute the mass flow rate across surface *ab*. Determine the drag force on the upper surface of the plate. Explain how this (viscous) drag can be computed from the given data even though we do not know the fluid viscosity (see Problem 9.41).



Given: Data on fluid and boundary layer geometry

Find: Mass flow rate across *ab*; Drag

Solution:

The given data is $\rho = 800\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad U = 3\cdot\frac{\text{m}}{\text{s}}$ $\quad L = 3\cdot\text{m}$ $\quad \delta = 25\cdot\text{mm}$ $\quad b = 1\cdot\text{m}$

Governing equations:

Mass $$\frac{\partial}{\partial t}\int_{\text{CV}}\rho\, d\forall + \int_{\text{CS}}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Momentum $$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{\text{CV}}\vec{V}\,\rho\, d\forall + \int_{\text{CS}}\vec{V}\,\rho\vec{V}\cdot d\vec{A} \qquad (4.17\text{a})$$

Assumptions: (1) Steady flow (2) No pressure force (3) No body force in *x* direction (4) Uniform flow at *a*

Applying these to the CV *abcd*

Mass $$(-\rho\cdot U\cdot b\cdot\delta) + \int_0^{\delta}\rho\cdot u\cdot b\, dy + m_{ab} = 0$$

For the boundary layer $$\frac{u}{U} = \frac{y}{\delta} = \eta \qquad \frac{dy}{\delta} = d\eta$$

Hence $$m_{ab} = \rho\cdot U\cdot b\cdot\delta - \int_0^1 \rho\cdot U\cdot\eta\cdot\delta\, dy = \rho\cdot U\cdot b\cdot\delta - \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta$$

$$m_{ab} = \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta \qquad m_{ab} = 30\,\frac{\text{kg}}{\text{s}}$$

Momentum $$R_x = U\cdot(-\rho\cdot U\cdot\delta) + m_{ab}\cdot u_{ab} + \int_0^{\delta} u\cdot\rho\cdot u\cdot b\, dy$$

Note that $u_{ab} = U$ and $\int_0^{\delta} u\cdot\rho\cdot u\cdot b\, dy = \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^2\, d\eta$

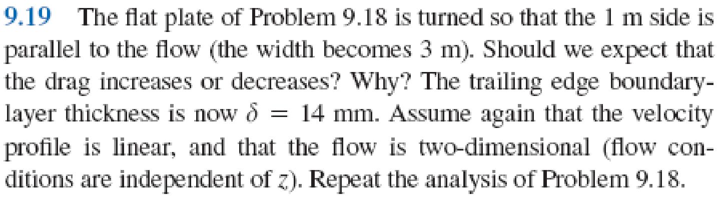
$$R_x = -\rho\cdot U^2\cdot b\cdot\delta + \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta\cdot U + \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^2\, dy$$

$$R_x = -\rho\cdot U^2\cdot b\cdot\delta + \frac{1}{2}\cdot\rho\cdot U^2\cdot\delta + \frac{1}{3}\cdot\rho\cdot U^2\cdot\delta \qquad R_x = -\frac{1}{6}\cdot\rho\cdot U^2\cdot b\cdot\delta \qquad R_x = -30\,\text{N}$$

We are able to compute the boundary layer drag even though we do not know the viscosity because it is the viscosity that creates the boundary layer in the first place

Problem 9.19

[3]



9.19 The flat plate of Problem 9.18 is turned so that the 1 m side is parallel to the flow (the width becomes 3 m). Should we expect that the drag increases or decreases? Why? The trailing edge boundary-layer thickness is now $\delta = 14$ mm. Assume again that the velocity profile is linear, and that the flow is two-dimensional (flow conditions are independent of z). Repeat the analysis of Problem 9.18.

Given: Data on fluid and boundary layer geometry

Find: Mass flow rate across *ab*; Drag; Compare to Problem 9.18

Solution:

The given data is $\rho = 800\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad U = 3\cdot\frac{\text{m}}{\text{s}}$ $\quad L = 1\cdot\text{m}$ $\quad \delta = 14\cdot\text{mm}$ $\quad b = 3\cdot\text{m}$

Governing equations:

Mass

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Momentum

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV}\vec{V}\,\rho\, d\forall + \int_{CS}\vec{V}\,\rho\vec{V}\cdot d\vec{A} \qquad (4.17a)$$

Assumptions: (1) Steady flow (2) No pressure force (3) No body force in *x* direction (4) Uniform flow at *a*

Applying these to the CV *abcd*

Mass

$$(-\rho\cdot U\cdot b\cdot\delta) + \int_0^\delta \rho\cdot u\cdot b\, dy + m_{ab} = 0$$

For the boundary layer

$$\frac{u}{U} = \frac{y}{\delta} = \eta \qquad\qquad \frac{dy}{\delta} = d\eta$$

Hence

$$m_{ab} = \rho\cdot U\cdot b\cdot\delta - \int_0^1 \rho\cdot U\cdot\eta\cdot\delta\, dy = \rho\cdot U\cdot b\cdot\delta - \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta$$

$$m_{ab} = \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta \qquad\qquad m_{ab} = 50.4\,\frac{\text{kg}}{\text{s}}$$

Momentum

$$R_x = U\cdot(-\rho\cdot U\cdot\delta) + m_{ab}\cdot u_{ab} + \int_0^\delta u\cdot\rho\cdot u\cdot b\, dy$$

Note that $u_{ab} = U$ and $\int_0^\delta u\cdot\rho\cdot u\cdot b\, dy = \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^2\, d\eta$

$$R_x = -\rho\cdot U^2\cdot b\cdot\delta + \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta\cdot U + \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^2\, dy$$

$$R_x = -\rho\cdot U^2\cdot b\cdot\delta + \frac{1}{2}\cdot\rho\cdot U^2\cdot\delta + \frac{1}{3}\cdot\rho\cdot U^2\cdot\delta$$

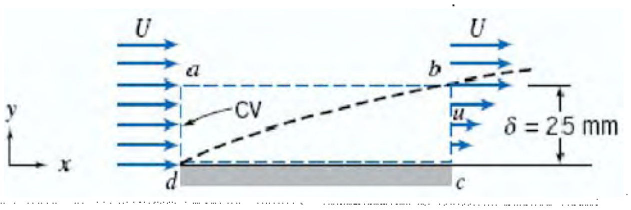
$$R_x = -\frac{1}{6}\cdot\rho\cdot U^2\cdot b\cdot\delta \qquad\qquad R_x = -50.4\,\text{N}$$

We should expect the drag to be larger than for Problem 9.18 because the viscous friction is mostly concentrated near the leading edge (which is only 1 m wide in Problem 9.18 but 3 m here). The reason viscous stress is highest at the front region is that the boundary layer is very small (δ <<) so $\tau = \mu du/dy \sim \mu U/\delta$ >>

Problem 9.20

[3]

Given: Fluid flow over a thin flat plate of width, $b = 1.0\ m$. Flow is two-dimensional. Assume that in the boundary layer the velocity profile is parabolic. (The plate is 3 m long.)



At bc, $\frac{u}{U} = 2\eta - \eta^2$; $\eta = \frac{y}{\delta}$

$U = 3.0\ m/s$

$\rho = 800\ kg/m^3$

Find: (a) Mass flowrate across ab.
(b) x component (and direction) of force needed to hold plate.

Solution: Apply the continuity and x component momentum equations.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall}} + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

$$F_{S_x} + \overset{=0(3)}{\cancel{F_{B_x}}} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}} + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) No pressure forces
(3) $F_{B_x} = 0$
(4) Uniform flow at da

Then

$$0 = \{-|\rho U b\delta|\} + \int_0^\delta \rho u b\, dy + \dot{m}_{ab}$$

But $\int_0^\delta \rho u b\, dy = \rho U b\delta \int_0^1 (2\eta - \eta^2)\, d\eta = \rho U b\delta \left[\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{2}{3}\rho U b\delta$

Thus $\dot{m}_{ab} = \rho U b\delta - \frac{2}{3}\rho U b\delta = \frac{1}{3}\rho U b\delta$

$$\dot{m}_{ab} = \frac{1}{3} \times 800\,\frac{kg}{m^3} \times 3\,\frac{m}{s} \times 1\ m \times 0.025\ m = 20\ kg/s$$

$\dot{m}_{ab}$

From momentum,

$$R_x = u_{da}\{-|\rho U b\delta|\} + u_{ab}\,\dot{m}_{ab} + \int_0^\delta u\rho u b\, dy\ ;\ u_{da} = u_{ab} = U$$

But $\int_0^\delta u\rho u b\, dy = \rho U^2 b\delta \int_0^1 (2\eta - \eta^2)\, d\eta = \rho U^2 b\delta\left[\frac{4}{3}\eta^3 - \eta^4 + \frac{1}{5}\eta^5\right]_0^1 = \frac{8}{15}\rho U^2 b\delta$

Thus $R_x = -\rho U^2 b\delta + \frac{1}{3}\rho U^2 b\delta + \frac{8}{15}\rho U^2 b\delta = -\frac{2}{15}\rho U^2 b\delta$

$$R_x = -\frac{2}{15} \times 800\,\frac{kg}{m^3} \times (3)^2\,\frac{m^2}{s^2} \times 1\ m \times 0.025\ m \times \frac{N\cdot s^2}{kg\cdot m} = -\ 24\ N$$

This force must be applied to the control volume by the plate. Thus to hold the plate,

$F_x = R_x = -\ 24$ (to the left)

F_x

Problem 9.21

[2]

9.21 The test section of a low speed wind tunnel is 1.5 meters long, preceded by a nozzle and with a diffuser at the outlet. The tunnel cross-section is 20 cm × 20 cm. The wind tunnel is to operate with 40°C air and have a design velocity of 50 m/s in the test section. A potential problem with such a wind tunnel is boundary-layer blockage. The boundary-layer displacement thickness reduces the effective cross-sectional area (the test area, in which we have uniform flow), and in addition the uniform flow will be accelerated. If these effects are pronounced, we end up with a smaller useful test cross section with a velocity somewhat higher than anticipated. If the boundary-layer thickness is 10 mm at the entrance and 25 mm at the exit, and the boundary-layer velocity profile is given by $u/U = (y/\delta)^{1/7}$, estimate the displacement thickness at the end of the test section and the percent change in the uniform velocity between the inlet and outlet.

Given: Data on wind tunnel and boundary layers

Find: Displacement thickness at exit; Percent change in uniform velocity through test section

Solution:

The solution involves using mass conservation in the inviscid core, allowing for the fact that as the boundary layer grows it reduces the size of the core. One approach would be to integrate the 1/7 law velocity profile to compute the mass flow in the boundary layer; an easier approach is to simply use the displacement thickness!

Basic equations $\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$ (4.12) $\quad \delta_{disp} = \int_0^{\delta}\left(1 - \frac{u}{U}\right)dy$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$ and $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

The design data is $U_{design} = 50\cdot\frac{m}{s}$ $\quad w = 20\cdot cm$ $\quad h = 20\cdot cm$ $\quad A_{design} = w\cdot h$ $\quad A_{design} = 0.04\, m^2$

The volume flow rate is $Q = U_{design}\cdot A_{design}$ $\quad Q = 2\frac{m^3}{s}$

We also have $\delta_{in} = 10\cdot mm$ $\quad \delta_{exit} = 25\cdot mm$

Hence $\delta_{disp} = \int_0^{\delta}\left(1 - \frac{u}{U}\right)dy = \int_0^{\delta}\left[1 - \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]dy = \delta\cdot\int_0^1\left(1 - \eta^{\frac{1}{7}}\right)d\eta$ where $\eta = \frac{y}{\delta}$ $\quad \delta_{disp} = \frac{\delta}{8}$

Hence at the inlet and exit

$\delta_{dispin} = \frac{\delta_{in}}{8}$ $\quad \delta_{dispin} = 1.25\cdot mm$ $\quad \delta_{dispexit} = \frac{\delta_{exit}}{8}$ $\quad \delta_{dispexit} = 3.125\cdot mm$

Hence the areas are $A_{in} = \left(w - 2\cdot\delta_{dispin}\right)\cdot\left(h - 2\cdot\delta_{dispin}\right)$ $A_{in} = 0.0390\cdot m^2$

$A_{exit} = \left(w - 2\cdot\delta_{dispexit}\right)\cdot\left(h - 2\cdot\delta_{dispexit}\right)$ $A_{exit} = 0.0375\cdot m^2$

Applying mass conservation between "design" conditions and the inlet

$$\left(-\rho\cdot U_{design}\cdot A_{design}\right) + \left(\rho\cdot U_{in}\cdot A_{in}\right) = 0$$

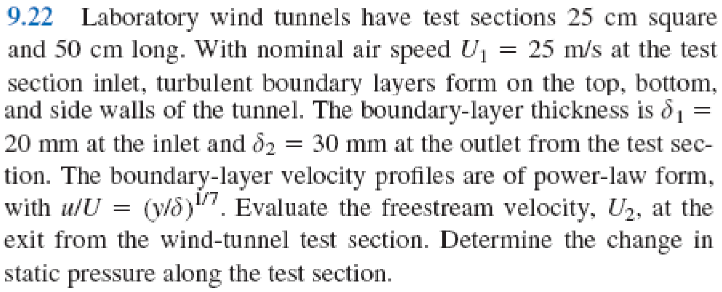
or $U_{in} = U_{design}\cdot\frac{A_{design}}{A_{in}}$ $U_{in} = 51.3\frac{m}{s}$

Also $U_{exit} = U_{design}\cdot\frac{A_{design}}{A_{exit}}$ $U_{exit} = 53.3\frac{m}{s}$

The percent change in uniform velocity is then $\frac{U_{exit} - U_{in}}{U_{in}} = 3.91\cdot\%$ The exit displacement thickness is $\delta_{dispexit} = 3.125\cdot mm$

Problem 9.22

[2]



9.22 Laboratory wind tunnels have test sections 25 cm square and 50 cm long. With nominal air speed $U_1 = 25$ m/s at the test section inlet, turbulent boundary layers form on the top, bottom, and side walls of the tunnel. The boundary-layer thickness is $\delta_1 = 20$ mm at the inlet and $\delta_2 = 30$ mm at the outlet from the test section. The boundary-layer velocity profiles are of power-law form, with $u/U = (y/\delta)^{1/7}$. Evaluate the freestream velocity, U_2, at the exit from the wind-tunnel test section. Determine the change in static pressure along the test section.

Given: Data on wind tunnel and boundary layers

Find: Uniform velocity at exit; Change in static pressure through the test section

Solution:

Basic equations $$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \quad (4.12) \qquad \delta_{disp} = \int_0^\delta \left(1-\frac{u}{U}\right)dy \qquad \frac{p}{\rho}+\frac{V^2}{2}+g\cdot z = \text{const}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$ and $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

The given data is $U_1 = 25\cdot\frac{m}{s}$ $h = 25\cdot cm$ $A = h^2$ $A = 625\cdot cm^2$

We also have $\delta_1 = 20\cdot mm$ $\delta_2 = 30\cdot mm$

Hence $$\delta_{disp} = \int_0^\delta\left(1-\frac{u}{U}\right)dy = \int_0^\delta\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]dy = \delta\cdot\int_0^1\left(1-\eta^{\frac{1}{7}}\right)d\eta$$ where $\eta = \frac{y}{\delta}$ $\delta_{disp} = \frac{\delta}{8}$

Hence at the inlet and exit

$\delta_{disp1} = \frac{\delta_1}{8}$ $\delta_{disp1} = 2.5\cdot mm$ $\delta_{disp2} = \frac{\delta_2}{8}$ $\delta_{disp2} = 3.75\cdot mm$

Hence the areas are $A_1 = \left(h-2\cdot\delta_{disp1}\right)^2$ $A_1 = 600\cdot cm^2$

$A_2 = \left(h-2\cdot\delta_{disp2}\right)^2$ $A_2 = 588\cdot cm^2$

Applying mass conservation between Points 1 and 2

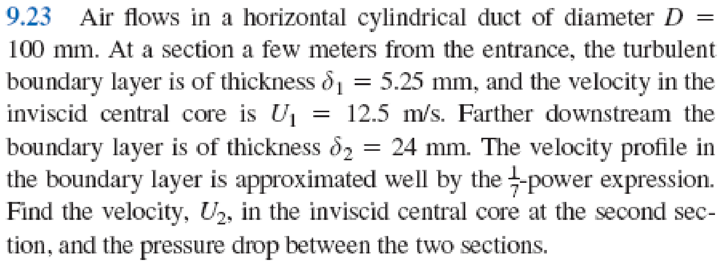
$$\left(-\rho\cdot U_1\cdot A_1\right)+\left(\rho\cdot U_2\cdot A_2\right) = 0 \quad \text{or} \quad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 25.52\frac{m}{s}$$

The pressure change is found from Bernoulli $\frac{p_1}{\rho}+\frac{U_1^2}{2} = \frac{p_2}{\rho}+\frac{U_2^2}{2}$ with $\rho = 1.21\cdot\frac{kg}{m^3}$

Hence $\Delta p = \frac{\rho}{2}\cdot\left(U_1^2-U_2^2\right)$ $\Delta p = -15.8\,Pa$ The pressure drops slightly through the test section

Problem 9.23

[2]



9.23 Air flows in a horizontal cylindrical duct of diameter $D = 100$ mm. At a section a few meters from the entrance, the turbulent boundary layer is of thickness $\delta_1 = 5.25$ mm, and the velocity in the inviscid central core is $U_1 = 12.5$ m/s. Farther downstream the boundary layer is of thickness $\delta_2 = 24$ mm. The velocity profile in the boundary layer is approximated well by the $\frac{1}{7}$-power expression. Find the velocity, U_2, in the inviscid central core at the second section, and the pressure drop between the two sections.

Given: Data on boundary layer in a cylindrical duct

Find: Velocity U_2 in the inviscid core at location 2; Pressure drop

Solution:

The solution involves using mass conservation in the inviscid core, allowing for the fact that as the boundary layer grows it reduces the siz the core. One approach would be to integrate the 1/7 law velocity profile to compute the mass flow in the boundary layer; an easier approa is to simply use the displacement thickness!

The given or available data (from Appendix A) is

$\rho = 1.23\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad U_1 = 12.5\cdot\frac{\text{m}}{\text{s}}$ $\quad D = 100\cdot\text{mm}$ $\quad \delta_1 = 5.25\cdot\text{mm}$ $\quad \delta_2 = 24\cdot\text{mm}$

Governing equations:

Mass
$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\mathcal{V} + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Bernoulli
$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad (4.24)$$

The displacement thicknesses can be computed from boundary layer thicknesses using Eq. 9.1

$$\delta_{disp} = \int_0^{\delta}\left(1 - \frac{u}{U}\right)dy = \delta\cdot\int_0^1\left(1 - \eta^{\frac{1}{7}}\right)d\eta = \frac{\delta}{8}$$

Hence at locations 1 and 2 $\quad \delta_{disp1} = \frac{\delta_1}{8} \quad \delta_{disp1} = 0.656\cdot\text{mm} \quad \delta_{disp2} = \frac{\delta_2}{8} \quad \delta_{disp2} = 3\cdot\text{mm}$

Applying mass conservation at locations 1 and 2 $\quad \left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \quad$ or $\quad U_2 = U_1\cdot\frac{A_1}{A_2}$

The two areas are given by the duct cross section area minus the displacement boundary layer

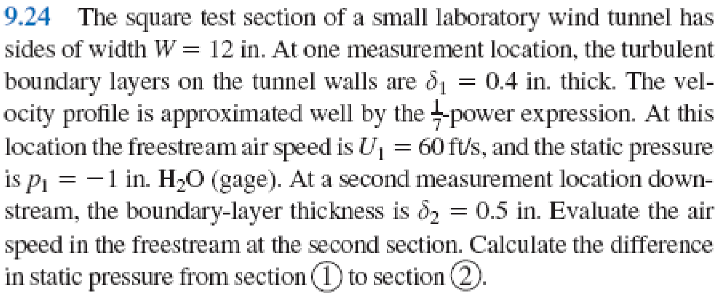
$$A_1 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp1}\right)^2 \quad A_1 = 7.65\times10^{-3}\,\text{m}^2 \qquad A_2 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp2}\right)^2 \quad A_2 = 6.94\times10^{-3}\,\text{m}^2$$

Hence $\quad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 13.8\,\frac{\text{m}}{\text{s}}$

For the pressure drop we can apply Bernoulli to locations 1 and 2 to find $\quad p_1 - p_2 = \Delta p = \frac{\rho}{2}\cdot\left(U_2^{\,2} - U_1^{\,2}\right) \quad \Delta p = 20.6\,\text{Pa}$

Problem 9.24

[2]



9.24 The square test section of a small laboratory wind tunnel has sides of width $W = 12$ in. At one measurement location, the turbulent boundary layers on the tunnel walls are $\delta_1 = 0.4$ in. thick. The velocity profile is approximated well by the $\frac{1}{7}$-power expression. At this location the freestream air speed is $U_1 = 60$ ft/s, and the static pressure is $p_1 = -1$ in. H_2O (gage). At a second measurement location downstream, the boundary-layer thickness is $\delta_2 = 0.5$ in. Evaluate the air speed in the freestream at the second section. Calculate the difference in static pressure from section ① to section ②.

Given: Data on wind tunnel and boundary layers

Find: Uniform velocity at Point 2; Change in static pressure through the test section

Solution:

Basic equations $$\frac{\partial}{\partial t}\int_{CV}\rho\, d\mathcal{V} + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12) \qquad \delta_{disp} = \int_0^{\delta}\left(1 - \frac{u}{U}\right)dy \qquad \frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$ and $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

The given data is $U_1 = 60\cdot\frac{ft}{s}$ $W = 12\cdot in$ $A = W^2$ $A = 144\cdot in^2$

We also have $\delta_1 = 0.4\cdot in$ $\delta_2 = 0.5\cdot in$

Hence $$\delta_{disp} = \int_0^{\delta}\left(1 - \frac{u}{U}\right)dy = \int_0^{\delta}\left[1 - \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]dy = \delta\cdot\int_0^1\left(1 - \eta^{\frac{1}{7}}\right)d\eta$$ where $\eta = \frac{y}{\delta}$ $\delta_{disp} = \frac{\delta}{8}$

Hence at the inlet and exit

$\delta_{disp1} = \frac{\delta_1}{8}$ $\delta_{disp1} = 0.050\cdot in$ $\delta_{disp2} = \frac{\delta_2}{8}$ $\delta_{disp2} = 0.0625\cdot in$

Hence the areas are $A_1 = \left(W - 2\cdot\delta_{disp1}\right)^2$ $A_1 = 142\cdot in^2$

$A_2 = \left(W - 2\cdot\delta_{disp2}\right)^2$ $A_2 = 141\cdot in^2$

Applying mass conservation between Points 1 and 2

$$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0$$ or $U_2 = U_1\cdot\frac{A_1}{A_2}$ $U_2 = 60.25\cdot\frac{ft}{s}$

The pressure change is found from Bernoulli $\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}$ with $\rho = 0.00234\cdot\frac{slug}{ft^3}$

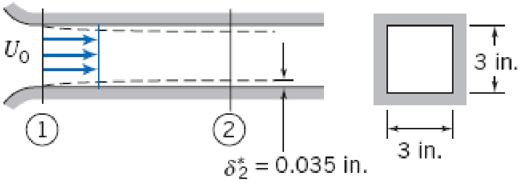
Hence $\Delta p = \frac{\rho}{2}\cdot\left(U_1^2 - U_2^2\right)$ $\Delta p = -2.47\times 10^{-4}\cdot psi$ $\Delta p = -0.0356\cdot\frac{lbf}{ft^2}$

In terms of inches of water $\rho_{H2O} = 1.94\cdot\frac{slug}{ft^3}$ $\Delta h = \frac{\Delta p}{\rho_{H2O}\cdot g}$ $\Delta h = -0.00684\cdot in$

Problem 9.25

[2]

9.25 Air flows in the entrance region of a square duct, as shown. The velocity is uniform, $U_0 = 100$ ft/s, and the duct is 3 in. square. At a section 1 ft downstream from the entrance, the displacement thickness, δ^*, on each wall measures 0.035 in. Determine the pressure change between sections ① and ②.



Given: Data on wind tunnel and boundary layers

Find: Pressure change between points 1 and 2

Solution:

Basic equations $$\frac{\partial}{\partial t}\int_{CV}\rho\, dV\!\!\!\!- + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12) \qquad \frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$

The given data is $U_0 = 100\cdot\frac{\text{ft}}{\text{s}}$ $\quad U_1 = U_0$ $\quad h = 3\cdot\text{in}$ $\quad A_1 = h^2$ $\quad A_1 = 9\cdot\text{in}^2$

We also have $\delta_{disp2} = 0.035\cdot\text{in}$

Hence at the Point 2 $A_2 = \left(h - 2\cdot\delta_{disp2}\right)^2$ $\qquad A_2 = 8.58\cdot\text{in}^2$

Applying mass conservation between Points 1 and 2

$$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \qquad \text{or} \qquad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 105\cdot\frac{\text{ft}}{\text{s}}$$

The pressure change is found from Bernoulli $\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}$ with $\rho = 0.00234\cdot\frac{\text{slug}}{\text{ft}^3}$

Hence $\Delta p = \frac{\rho}{2}\cdot\left(U_1^2 - U_2^2\right)$ $\qquad \Delta p = -8.05\times 10^{-3}\cdot\text{psi}$ $\qquad \Delta p = -1.16\cdot\frac{\text{lbf}}{\text{ft}^2}$

The pressure drops by a small amount as the air accelerates

Problem 9.26

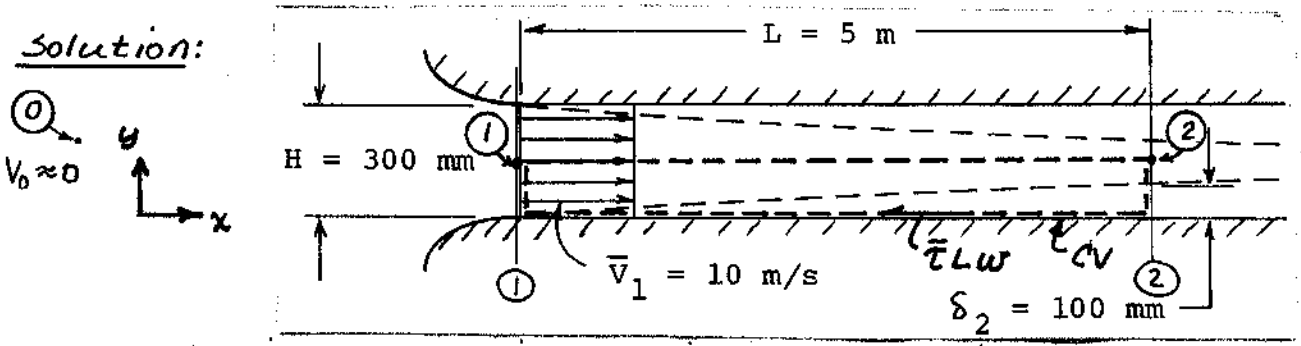
[3]

Given: Developing flow of air in flat horizontal duct.

Assume $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ in boundary layers.

Find: (a) Show $\delta^*/\delta = 1/8$
(b) Evaluate p_2
(c) Calculate average wall shear stress.

Solution:



Assume flow is steady and incompressible, and that frictional effects are negligible outside the boundary layers.

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta\int_0^1\left(1-\frac{u}{U}\right)d\eta = \delta\int_0^1\left(1-\eta^{1/7}\right)d\eta = \delta\left(\eta - \frac{7}{8}\eta^{8/7}\right)\Big|_0^1 = \delta/8$$ δ^*

From continuity $V_1A_1 = V_1 w H = V_2A_2 = V_2 w(H-2\delta_2^*)$

$$V_2 = V_1\frac{H}{H-2\delta_2^*} = 10\,\frac{m}{s}\,\frac{300\ mm}{(300-25)\,mm} = 10.9\ m/s$$

From Bernoulli, since z = constant, $\frac{p_0}{\rho} + \cancel{\frac{V_0^2}{2}} = \frac{p}{\rho} + \frac{V^2}{2}$ along a streamline.

$$p_{1g} = p_1 - p_0 = -\frac{1}{2}\rho V_1^2 = -\frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(10)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = -61.5\ Pa$$

$$p_{2g} = p_2 - p_0 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(10.9)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = -73.1\ Pa$$ p_{2g}

Apply momentum using CV shown:

$$\text{BE}\quad F_{S_x} + \overset{=0}{\cancel{F_{B_x}}} = \overset{=0}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

$$(p_1-p_2)w\frac{H}{2} - \bar{\tau}wL = \overline{V}_1\left\{-\rho\overline{V}_1\frac{H}{2}w\right\} + \underbrace{\int_0^{\delta_2} u\rho u w\,dy}_{\rho V_2^2\delta_2 w\int_0^1\eta^{2/7}d\eta \,=\, \rho V_2^2\left(\frac{7}{9}\delta_2\right)w} + V_2\left\{+\rho V_2\left(\frac{H}{2}-\delta_2\right)w\right\}$$

$$\bar{\tau}\cancel{w}L = (p_1-p_2)\cancel{w}\frac{H}{2} + \rho V_1^2\frac{H}{2}\cancel{w} - \rho V_2^2\left(\frac{H}{2}-\frac{2}{9}\delta_2\right)\cancel{w}$$

$$\bar{\tau} = \frac{1}{L}\left[(p_1-p_2)\frac{H}{2} + \rho V_1^2\frac{H}{2} - \rho V_2^2\left(\frac{H}{2}-\frac{2}{9}\delta_2\right)\right]$$

$$= \frac{1}{5\,m}\left[11.6\,\frac{N}{m^2}\times 0.15\,m + 1.23\,\frac{kg}{m^3}\times(10)^2\frac{m^2}{s^2}\times 0.15\,m\times\frac{N\cdot s^2}{kg\cdot m} - 1.23\,\frac{kg}{m^3}\times(10.9)^2\frac{m^2}{s^2}(0.15-0.0222)\,m\times\frac{N\cdot s^2}{kg\cdot m}\right.$$

$$\bar{\tau} = 0.300\ N/m^2$$ $\bar{\tau}$

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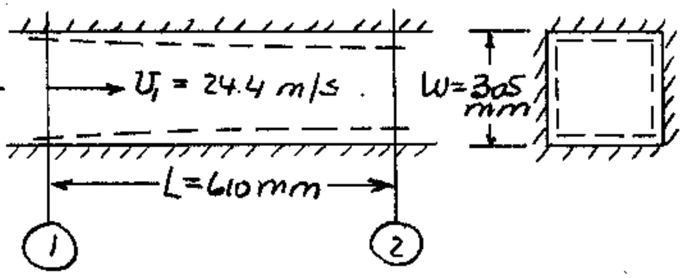
Problem 9.27

Given: Air flow in laboratory wind tunnel test section.

$h_{l_{0-1}} = -6.5\ \text{mm}\ H_2O$



$\delta_1 = 20.3\ mm$

$\delta_2 = 25.4\ mm$

$\frac{\bar{u}}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ in BLs

Find: (a) Freestream speed at exit, U_2.
(b) Pressure at exit, p_2.

Solution: Apply displacement thickness, continuity, Bernoulli eqs.

Computing equations: $\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) dy$; $0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} + g\overset{(8)}{\cancel{z_1}} = \frac{p_2}{\rho} + \frac{U_2^2}{2} + g\overset{(8)}{\cancel{z_2}}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) No friction (outside BL)
(4) Along a streamline
(5) Uniform flow (outside BL)
(6) Same BL on four walls
(7) Neglect corner effects
(8) Neglect Δz

Then

$$\frac{\delta^*}{\delta} = \int_0^1 (1 - \lambda^{1/7})\, d\lambda = \left(\lambda - \frac{7}{8}\lambda^{8/7}\right)\Big|_0^1 = 1 - \frac{7}{8} = \frac{1}{8} \quad (\lambda = y/\delta)$$

$$\delta_1^* = \frac{1}{8}\delta_1 = \frac{1}{8} \times 20.3\ mm = 2.54\ mm\ ;\ \delta_2^* = \frac{1}{8}\delta_2 = \frac{1}{8} \times 25.4\ mm = 3.18\ mm$$

From continuity, $U_1 A_1 = U_1 (W - 2\delta_1^*)^2 = U_2 A_2 = U_2 (W - 2\delta_2^*)^2$

$$U_2 = U_1 \left(\frac{W - 2\delta_1^*}{W - 2\delta_2^*}\right)^2 = 24.4\,\frac{m}{s}\left[\frac{300 - 2(2.54)}{300 - 2(3.18)}\right]^2 = 24.6\ m/s$$

From Bernoulli, $\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}$

$$p_2 - p_1 = \frac{\rho}{2}(U_1^2 - U_2^2) = \frac{1}{2} \times 1.23\,\frac{kg}{m^3}\left[(24.4)^2 - (24.6)^2\right]\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = -6.03\ N/m^2$$

Since $\Delta p = \rho g \Delta h$, then $\Delta h = \Delta p / \rho g$

$$p_2 - p_1 = -6.03\,\frac{N}{m^2} \times \frac{m^3}{999\ kg} \times \frac{s^2}{9.81\ m} \times \frac{kg \cdot m}{N \cdot s^2} = -0.615\ mm\ H_2O$$

From ambient to ①: $\left(\frac{\overset{p_{atm}}{\cancel{p_0}}}{\cancel{\rho}} + \frac{\overset{\approx 0}{\cancel{U_0^2}}}{\cancel{2}}\right) - \left(\frac{p_1}{\rho} + \frac{U_1^2}{2}\right) = h_{l_T}$

$$p_1 = -\rho h_{l_T} - \frac{1}{2}\rho U_1^2$$

$$p_1 = -6.5\ mm\ H_2O - \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (24.4)^2\frac{m^2}{s^2} \times \frac{m^3}{999\ kg} \times \frac{s^2}{9.81\ m} = -43.9\ mm\ H_2O$$

$$p_2 = p_1 + (p_2 - p_1) = -43.9 - 0.615\ mm\ H_2O = -44.5\ mm\ H_2O$$

p_2

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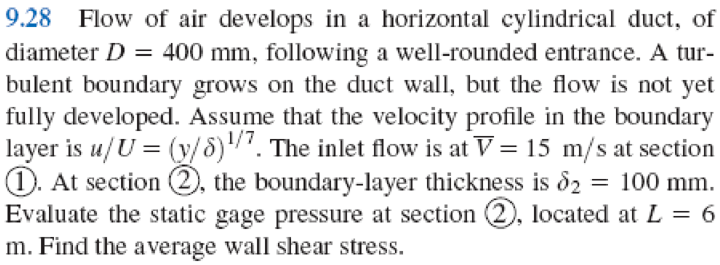
Problem 9.28

[3]



9.28 Flow of air develops in a horizontal cylindrical duct, of diameter $D = 400$ mm, following a well-rounded entrance. A turbulent boundary grows on the duct wall, but the flow is not yet fully developed. Assume that the velocity profile in the boundary layer is $u/U = (y/\delta)^{1/7}$. The inlet flow is at $\overline{V} = 15$ m/s at section ①. At section ②, the boundary-layer thickness is $\delta_2 = 100$ mm. Evaluate the static gage pressure at section ②, located at $L = 6$ m. Find the average wall shear stress.

Given: Data on fluid and boundary layer geometry

Find: Gage pressure at location 2; average wall stress

Solution:

The solution involves using mass conservation in the inviscid core, allowing for the fact that as the boundary layer grows it reduces the size core. One approach would be to integrate the 1/7 law velocity profile to compute the mass flow in the boundary layer; an easier approach i simply use the displacement thickness!

The average wall stress can be estimated using the momentum equation for a CV

The given and available (from Appendix A) data is

$$\rho = 1.23\cdot\frac{kg}{m^3} \qquad U_1 = 15\cdot\frac{m}{s} \qquad L = 6\cdot m \qquad D = 400\cdot mm \qquad \delta_2 = 100\cdot mm$$

Governing equations:

Mass $$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Momentum $$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV}\vec{V}\,\rho\, d\forall + \int_{CS}\vec{V}\,\rho\vec{V}\bullet d\vec{A} \qquad (4.17a)$$

Bernoulli $$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad (4.24)$$

Assumptions: (1) Steady flow (2) No pressure force (3) No body force in *x* direction

The displacement thickness at location 2 can be computed from boundary layer thickness using Eq. 9.1

$$\delta_{disp2} = \int_0^{\delta_2}\left(1 - \frac{u}{U}\right)dy = \delta_2\cdot\int_0^1\left(1 - \eta^{\frac{1}{7}}\right)d\eta = \frac{\delta_2}{8}$$

Hence $$\delta_{disp2} = \frac{\delta_2}{8} \qquad \delta_{disp2} = 12.5\,mm$$

Applying mass conservation at locations 1 and 2 $$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \qquad \text{or} \qquad U_2 = U_1\cdot\frac{A_1}{A_2}$$

$$A_1 = \frac{\pi}{4}\cdot D^2 \qquad A_1 = 0.126\,m^2$$

The area at location 2 is given by the duct cross section area minus the displacement boundary layer

$$A_2 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp2}\right)^2 \qquad A_2 = 0.11\,m^2$$

Hence $U_2 = U_1 \cdot \frac{A_1}{A_2}$ $\qquad U_2 = 17.1\frac{m}{s}$

For the pressure change we can apply Bernoulli to locations 1 and 2 to find

$$p_1 - p_2 = \Delta p = \frac{\rho}{2}\cdot\left(U_2^{\,2} - U_1^{\,2}\right) \qquad \Delta p = 40.8\,\text{Pa} \qquad p_2 = -\Delta p$$

Hence $p_2(\text{gage}) = p_1(\text{gage}) - \Delta p$ $\qquad p_2 = -40.8\,\text{Pa}$

For the average wall shear stress we use the momentum equation, simplified for this problem

$$\Delta p\cdot A_1 - \tau\cdot\pi\cdot D\cdot L = -\rho\cdot U_1^{\,2}\cdot A_1 + \rho\cdot U_2^{\,2}\cdot\frac{\pi}{4}\cdot\left(D - 2\cdot\delta_2\right)^2 + \rho\cdot\int_{\frac{D}{2}-\delta_2}^{\frac{D}{2}} 2\cdot\pi\cdot r\cdot u^2\,dr$$

where $u(r) = U_2\cdot\left(\frac{y}{\delta_2}\right)^{\frac{1}{7}}$ and $r = \frac{D}{2} - y$ $\qquad dr = -dy$

The integral is

$$\rho\cdot\int_{\frac{D}{2}-\delta_2}^{\frac{D}{2}} 2\cdot\pi\cdot r\cdot u^2\,dr = -2\cdot\pi\cdot\rho\cdot U_2^{\,2}\cdot\int_{\delta_2}^{0}\left(\frac{D}{2} - y\right)\cdot\left(\frac{y}{\delta_2}\right)^{\frac{2}{7}}dy$$

$$\rho\cdot\int_{\frac{D}{2}-\delta_2}^{\frac{D}{2}} 2\cdot\pi\cdot r\cdot u^2\,dr = 7\cdot\pi\cdot\rho\cdot U_2^{\,2}\cdot\delta_2\cdot\left(\frac{D}{9} - \frac{\delta_2}{8}\right)$$

Hence

$$\tau = \frac{\Delta p\cdot A_1 + \rho\cdot U_1^{\,2}\cdot A_1 - \rho\cdot U_2^{\,2}\cdot\frac{\pi}{4}\cdot\left(D - 2\cdot\delta_2\right)^2 - 7\cdot\pi\cdot\rho\cdot U_2^{\,2}\cdot\delta_2\cdot\left(\frac{D}{9} - \frac{\delta_2}{8}\right)}{\pi\cdot D\cdot L}$$

$$\tau = 0.461\,\text{Pa}$$

Problem 9.29

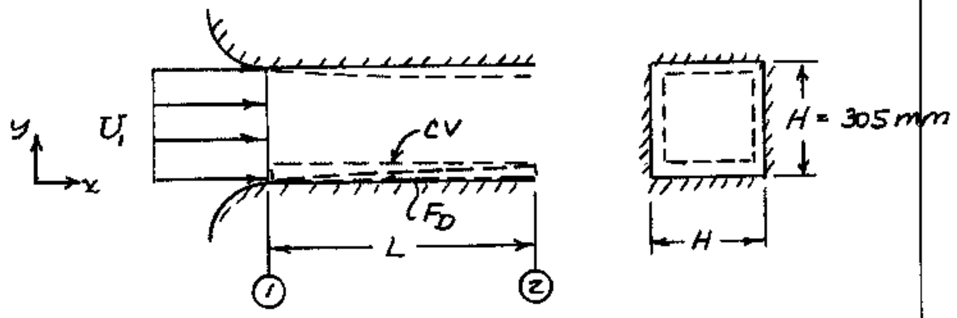
Given: Air flow into wind tunnel contraction and test section as shown.

$U_1 = 50.2\ m/s$

$\delta_2 = 20.3\ mm$

$H = 305\ mm$

$L = 609\ mm$



Find: (a) δ_2^*

(b) $p_2 - p_1$

(c) Estimate total drag force caused by friction on each wall.

Solution: Assume turbulent BL with 1/7-power profile, $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \lambda^{1/7}$

By definition $\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 (1 - \lambda^{1/7})\,d\lambda = \lambda - \frac{7}{8}\lambda^{8/7} = 1/8$

Thus $\delta_2^* = \frac{1}{8}\delta_2 = \frac{1}{8} \times 20.3\ mm = 2.54\ mm$ $\longleftarrow$ δ_2^*

From continuity, $U_1 A_1 = U_1 H^2 = U_2 A_2 = U_2 (H - 2\delta_2^*)^2$

$$U_2 = U_1 \frac{H^2}{(H - 2\delta_2^*)^2} = 50.2\ \frac{m}{s} \times \frac{(305)^2}{(305 - 2 \times 2.54)^2} = 51.9\ m/s$$

Apply the Bernoulli equation to the steady, incompressible, frictionless flow along a streamline outside the boundary layers:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2\ ;\ z_2 = z_1,\ so\ p_2 - p_1 = \frac{\rho}{2}(U_1^2 - U_2^2)$$

So

$$p_2 - p_1 = \frac{1}{2} \times 1.23 \frac{kg}{m^3} \left[(50.2)^2 - (51.9)^2\right] \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = -107\ N/m^2$$ $\longleftarrow$ $p_2 - p_1$

Choose the CV shown to evaluate drag caused by friction

Basic equation: $F_{S_x} + \overset{=0(1)}{\cancel{F_{B_x}}} = \overset{=0(2)}{\cancel{\frac{\partial}{\partial t}}} \int_{CV} u \rho\, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$

Assume: (1) Horizontal, so $F_{B_x} = 0$

(2) Steady flow

Then

$$p_1 H\delta_2 - F_D - p_2 H\delta_2 = U_1\{-\rho U_1 H\delta_2\} + \bar{U}\{+\dot{m}_{top}\} + \int_0^{\delta_2} u \rho u H\, dy$$

or $F_D = (p_1 - p_2) H\delta_2 + \rho U_1^2 H\delta_2 - \bar{U}\dot{m}_{top} - \int_0^{\delta_2} u \rho u H\, dy$ (1)

From conservation of mass, $\dot{m}_{top} = \dot{m}_1 - \dot{m}_2 = \rho U_1 H\delta_2 - \int_0^{\delta_2} \rho u H\, dy$ (2)

Evaluating the integrals, $\int_0^{\delta_2} \rho u H\, dy = \rho U_2 H\delta_2 \int_0^1 \lambda^{1/7} d\lambda = \frac{7}{8} \rho U_2 H\delta_2$

and $\int_0^{\delta_2} u \rho u H\, dy = \rho U_2^2 H\delta_2 \int_0^1 \lambda^{2/7} d\lambda = \frac{7}{9} \rho U_2^2 H\delta_2$

Thus $\int_0^{\delta_1} \rho u H dy = \frac{7}{8}\rho U_2 H \delta_2 = \frac{7}{8} \times 1.23 \frac{kg}{m^3} \times 51.4 \frac{m}{s} \times 0.305 m \times 0.0203 m = 0.346$ kg/s

$\rho U_1 H \delta_2 = 1.23 \frac{kg}{m^3} \times 50.2 \frac{m}{s} \times 0.305 m \times 0.0203 m = 0.382$ kg/s

From Eq. 2, $\dot{m}_{top} = (0.382 - 0.346) \frac{kg}{s} = 0.0364$ kg/s

Also $\int_0^{\delta_2} u \rho u H dy = \frac{7}{9}\rho U_2^2 H \delta_2 = \frac{7}{9} \times 1.23 \frac{kg}{m^3} \times (51.9)^2 \frac{m^2}{s^2} \times 0.305 m \times 0.0203 m \times \frac{N \cdot s^2}{kg \cdot m} = 16.0$ N

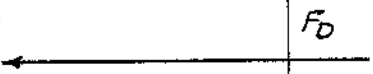
and $\bar{U} \approx \frac{1}{2}(U_1 + U_2) = \frac{1}{2}(50.2 + 51.9) \frac{m}{s} = 51.1$ m/s

From Eq. 1, $(p_1 - p_2) H\delta_2 = 107 \frac{N}{m^2} \times 0.305 m \times 0.0203 m = 0.662$ N

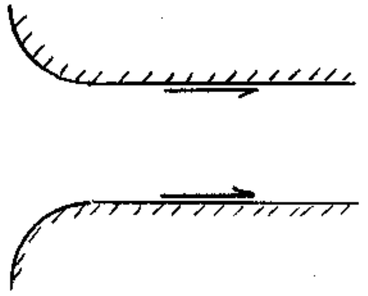
$\rho U_1^2 H \delta_2 = 1.23 \frac{kg}{m^3} \times (50.2)^2 \frac{m^2}{s^2} \times 0.305 m \times 0.0203 m \times \frac{N \cdot s^2}{kg \cdot m} = 19.2$ N

Finally, substituting into Eq. 1

$$F_D = 0.662\,N + 19.2\,N - 51.1 \frac{m}{s} \times 0.0364 \frac{kg}{s} \times \frac{N \cdot s^2}{kg \cdot m} - 16.0\,N = 2.00\,N$$



The viscous drag force acts on the CV in the direction shown. The viscous drag force on the wall of the test section is equal and opposite:



Viscous drag forces on walls of test section

13-782 500 SHEETS, FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
National® Brand

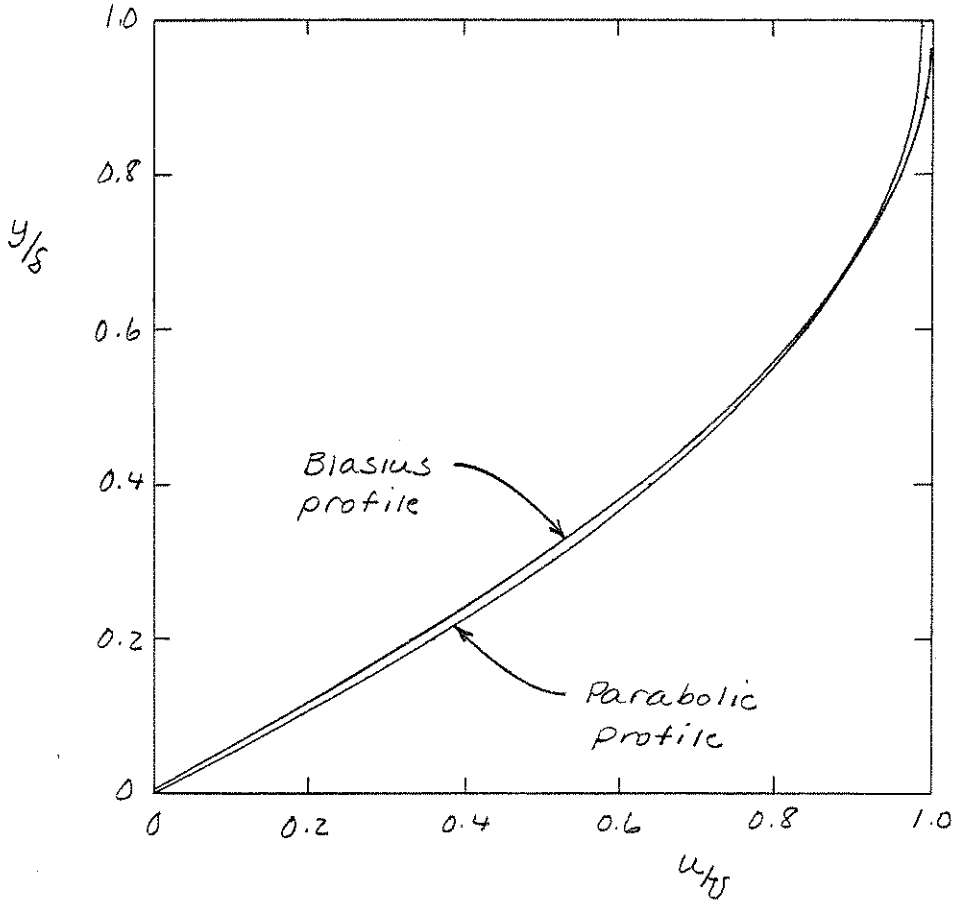
Problem *9.30

[2]

Given: Blasius exact solution for laminar boundary-layer flow.

Find: Plot and compare to parabolic profile, $\frac{u}{U} = 2\eta - \eta^2$.

Solution: The Blasius solution is given in Table 9.1; it is plotted below.



42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem *9.31

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find:
(a) Evaluate τ distribution.
(b) Plot τ/τ_w versus y/δ.
(c) Compare results from the sinusoidal profile, $\frac{u}{U} = \sin(\frac{\pi}{2}\frac{y}{\delta})$

NATIONAL 42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE

Solution: For the Blasius solution, $u = U\frac{df}{d\eta} = Uf'(\eta)$, and $\eta = y\sqrt{\frac{U}{\nu x}}$

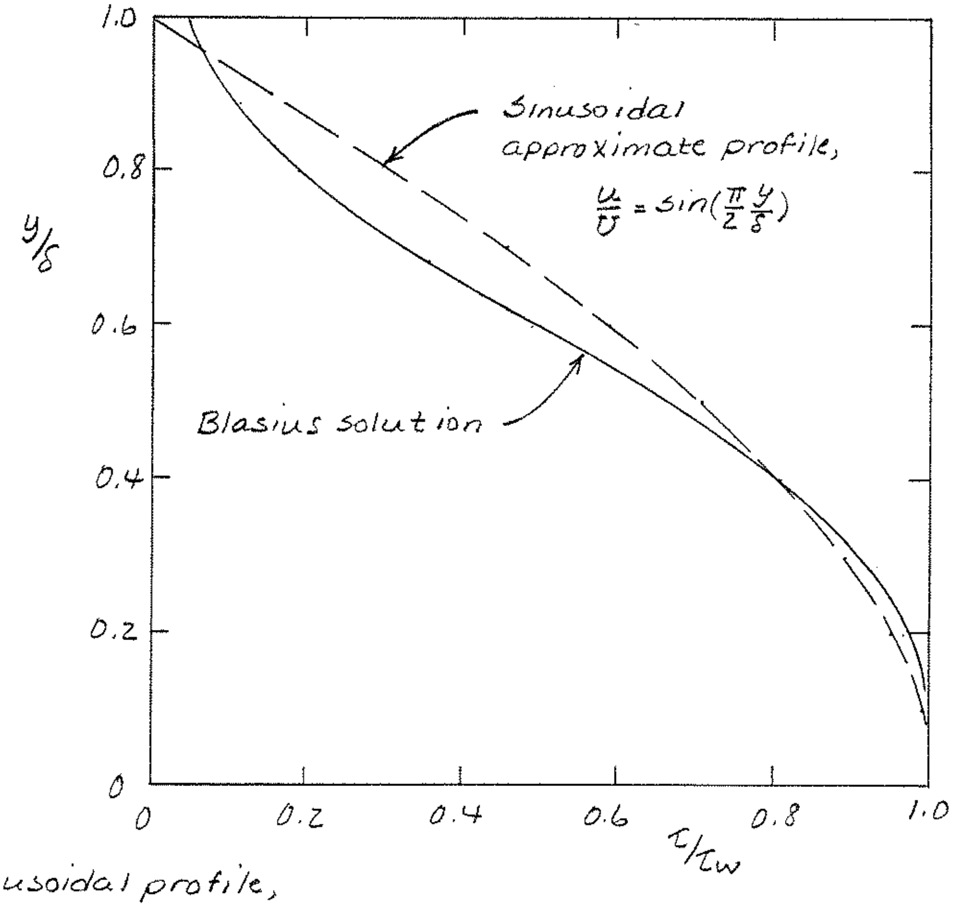
Computing equation: $\tau = \mu\frac{\partial u}{\partial y}$

For the Blasius profile, $\tau = \mu\frac{\partial}{\partial y}\left[U\frac{df}{d\eta}\right] = \mu U\frac{d^2f}{d\eta^2}\frac{\partial\eta}{\partial y} = \mu U f''(\eta)\sqrt{\frac{U}{\nu x}}$

Thus $\frac{\tau}{\rho U^2} = \frac{\mu}{\rho U}f''(\eta)\sqrt{\frac{U}{\nu x}} = \frac{f''(\eta)}{\sqrt{Re_x}}$; $\tau \sim f''(\eta)$ $\longleftarrow$ τ

From the above equation, $\frac{\tau}{\tau_w} = \frac{f''(\eta)}{f''(0)} = \frac{f''(\eta)}{0.33206}$

Since $y = \delta$ at $\eta = 5$, then $y/\delta = \eta/5$. Plotting:



For the sinusoidal profile,

$$\tau = \mu\frac{\partial u}{\partial y} = \frac{\mu U}{\delta}\frac{d(u/U)}{d(y/\delta)} = \frac{\mu U}{\delta}\frac{\pi}{2}\cos(\frac{\pi}{2}\frac{y}{\delta})\ ;\ \tau_w = \frac{\mu U}{\delta}\frac{\pi}{2}$$

Thus

$\tau/\tau_w = \cos(\frac{\pi}{2}\frac{y}{\delta})$ (sinusoidal profile) $\longleftarrow$ τ/τ_w

Problem *9.32

[3]

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find: (a) Evaluate τ distribution.
(b) Plot τ/τ_w versus y/δ.
(c) Compare results from the parabolic profile, $\frac{u}{U} = 2\eta - \eta^2$

Solution: For the Blasius solution, $u = U\frac{df}{d\eta} = Uf'(\eta)$, and $\eta = y\sqrt{\frac{U}{\nu x}}$

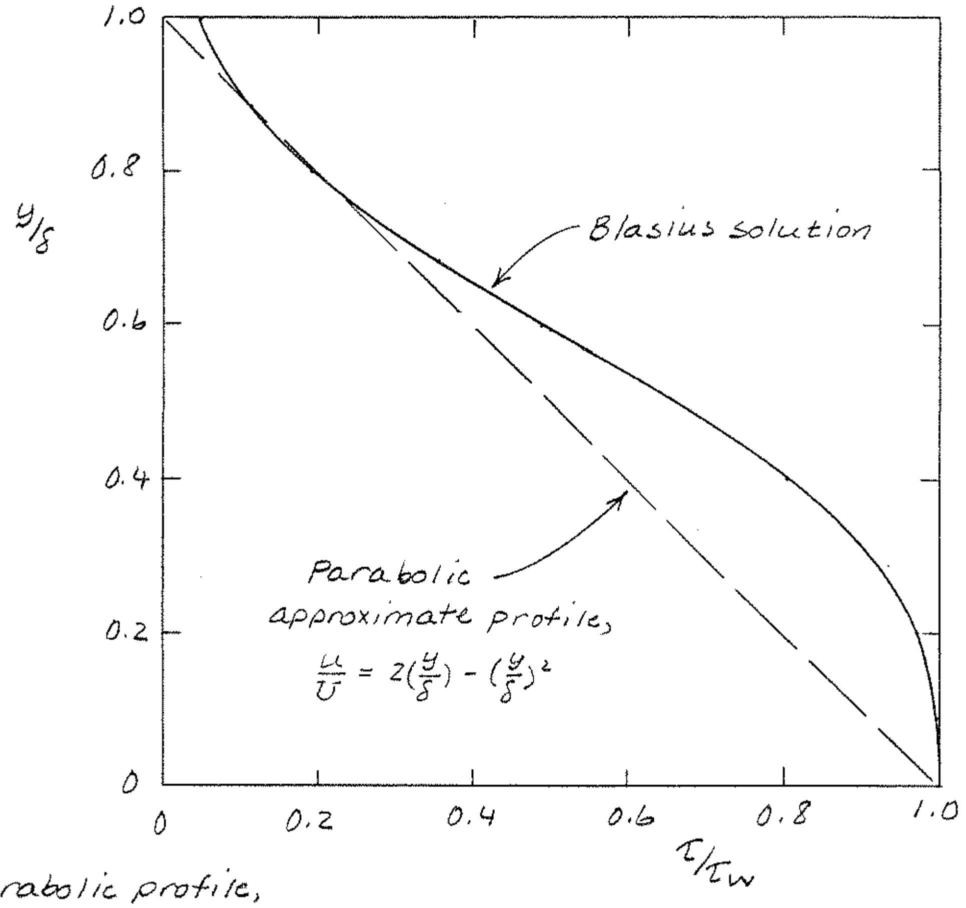
Computing equation: $\tau = \mu \frac{\partial u}{\partial y}$

For the Blasius profile, $\tau = \mu \frac{\partial}{\partial y}\left[U\frac{df}{d\eta}\right] = \mu U \frac{d^2 f}{d\eta^2}\frac{\partial \eta}{\partial y} = \mu U f''(\eta)\sqrt{\frac{U}{\nu x}}$

Thus $\frac{\tau}{\rho U^2} = \frac{\mu}{\rho U} f''(\eta)\sqrt{\frac{U}{\nu x}} = \frac{f''(\eta)}{\sqrt{Re_x}}$; $\tau \sim f''(\eta)$ $\longleftarrow$ τ

From above equation, $\frac{\tau}{\tau_w} = \frac{f''(\eta)}{f''(0)} = \frac{f''(\eta)}{0.33206}$

Since $y = \delta$ at $\eta = 5$, then $y/\delta = \eta/5$. Plotting:



For the parabolic profile,

$$\tau = \mu\frac{\partial u}{\partial y} = \frac{\mu U}{\delta}\frac{d(u/U)}{d(y/\delta)} = \frac{\mu U}{\delta}\left[2 - 2\left(\frac{y}{\delta}\right)\right] ; \tau_w = \frac{\mu U}{\delta}[2]$$

Thus

$\tau/\tau_w = 1 - \frac{y}{\delta}$ (parabolic profile) $\longleftarrow$ τ/τ_w

Problem *9.33

[3]

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find: Plot v/U versus y/δ for $Re_x = 10^5$.

Solution: For the Blasius solution, $\psi = \sqrt{U\nu x}\, f(\eta)$ and $\eta = y\sqrt{\frac{U}{\nu x}}$

From the stream function, $v = -\frac{\partial \psi}{\partial x} = -\left[\frac{1}{2}\sqrt{\frac{\nu U}{x}}\, f(\eta) + \sqrt{U\nu x}\,\frac{df}{d\eta}\frac{\partial \eta}{\partial x}\right]$

But $\frac{\partial \eta}{\partial x} = -\frac{1}{2}\frac{y}{x}\sqrt{\frac{U}{\nu x}} = -\frac{1}{2}\frac{\eta}{x}$

Thus $v = -\frac{1}{2}\sqrt{\frac{U\nu}{x}}\, f(\eta) - \sqrt{U\nu x}\,\frac{df}{d\eta}\left(-\frac{1}{2}\frac{\eta}{x}\right) = \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left[\eta f'(\eta) - f(\eta)\right]$

and

$$\frac{v}{U} = \frac{1}{2}\sqrt{\frac{\nu}{Ux}}\left[\eta f'(\eta) - f(\eta)\right] = \frac{\eta f'(\eta) - f(\eta)}{2\sqrt{Re_x}} \longleftarrow \frac{v}{U}$$

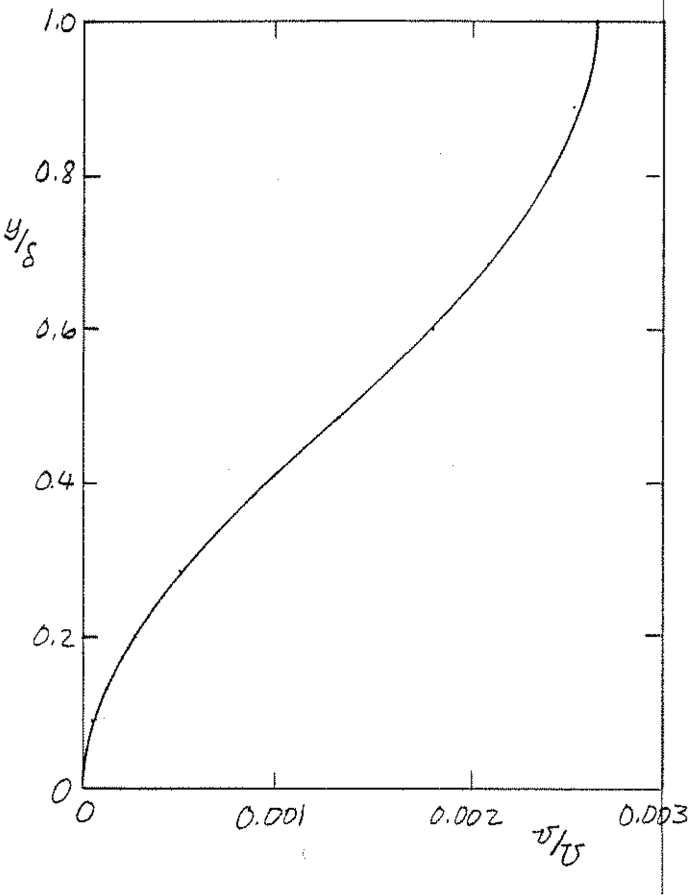
Also

$$\frac{y}{\delta} = \frac{y}{5\sqrt{\frac{\nu x}{U}}} = \frac{\eta}{5}$$

Tabulate from Table 9.1:

η	$\eta f'(\eta) - f(\eta)$	v/U
0	0	0
0.4	0.0265	4.20×10^{-5}
1.0	0.164	2.60×10^{-4}
1.4	0.316	4.99×10^{-4}
2.0	0.610	9.64×10^{-4}
2.4	0.827	1.31×10^{-3}
3.0	1.14	1.80×10^{-3}
3.4	1.32	2.09×10^{-3}
4.0	1.52	2.40×10^{-3}
4.4	1.60	2.53×10^{-3}
5.0	1.67	2.65×10^{-3}

Plot:



42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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Problem *9.34

[4]

Given: Blasius solution to boundary layer equations gives

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta f' - f\right)$$

where $\psi = f(\eta)\sqrt{\nu x U}$ and $\eta = y\sqrt{\frac{U}{\nu x}}$

Find: (a) Verify expression for v

(b) Obtain an expression for a_x

Plot: a_x vs η to determine maximum a_x for given x.

Solution: From the definition of ψ, $v = -\partial\psi/\partial x$

$$v = -\frac{\partial \psi}{\partial x} = -\left[\sqrt{\nu x U}\,\frac{\partial f}{\partial x} + f\,\frac{1}{2}\sqrt{\frac{\nu U}{x}}\right] = -\left[\sqrt{\nu x U}\,\frac{df}{d\eta}\frac{\partial \eta}{\partial x} + f\,\frac{1}{2}\sqrt{\frac{\nu U}{x}}\right]$$

$$\frac{\partial \eta}{\partial x} = y\sqrt{\frac{U}{\nu}}\left(-\frac{1}{2x^{3/2}}\right) = -\frac{y}{2x}\sqrt{\frac{U}{\nu x}} = -\frac{\eta}{2x}$$

$$\therefore v = -\left[\sqrt{\nu x U}\,f'\left(-\frac{\eta}{2x}\right) + f\,\frac{1}{2}\sqrt{\frac{\nu U}{x}}\right] = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta f' - f\right) \longleftarrow v$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad \text{where } u = Uf'$$

$$\frac{\partial u}{\partial x} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial x} = Uf''\left(-\frac{\eta}{2x}\right) = -\frac{1}{2}\frac{\eta U f''}{x}$$

$$\frac{\partial u}{\partial y} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial y} = Uf''\sqrt{\frac{U}{\nu x}}$$

Then

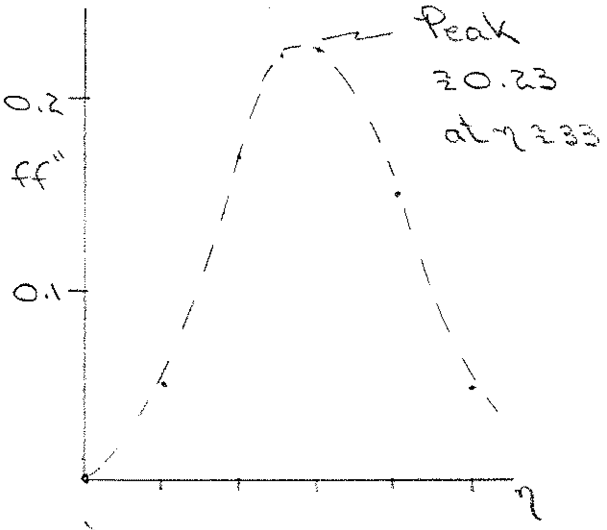
$$a_x = Uf'\left(-\frac{1}{2}\frac{\eta U f''}{x}\right) + \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta f' - f\right)Uf''\sqrt{\frac{U}{\nu x}}$$

$$= \cancel{-\frac{1}{2}\frac{U^2}{x}\eta f' f''} + \frac{1}{2}\frac{U^2}{x}\left(\cancel{\eta f' f''} - ff''\right)$$

$$a_x = -\frac{U^2}{2x}ff'' \longleftarrow a_x$$

For given x, $a_{x\,max}$ for max ff''

η	f	f''	ff''
0	0	0.3321	0
1	0.1656	0.3230	0.053
2	0.6500	0.2668	0.173
3	1.3968	0.1614	0.225
4	2.3057	0.0642	0.148
5	3.2833	0.0159	0.052

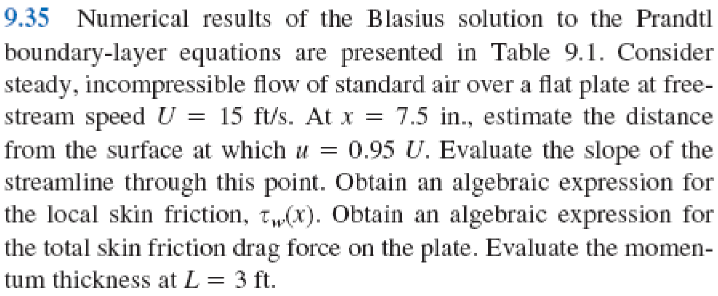


From plot $ff''\big|_{max} \approx 0.23$

$$\therefore a_{x\,max} \approx -0.115\frac{U^2}{x} \quad (\text{at } \eta \approx 3) \longleftarrow a_{x\,max}$$

Problem 9.35

[4]



9.35 Numerical results of the Blasius solution to the Prandtl boundary-layer equations are presented in Table 9.1. Consider steady, incompressible flow of standard air over a flat plate at free-stream speed $U = 15$ ft/s. At $x = 7.5$ in., estimate the distance from the surface at which $u = 0.95\,U$. Evaluate the slope of the streamline through this point. Obtain an algebraic expression for the local skin friction, $\tau_w(x)$. Obtain an algebraic expression for the total skin friction drag force on the plate. Evaluate the momentum thickness at $L = 3$ ft.

Given: Blasius solution for laminar boundary layer

Find: Point at which u = 0.95U; Slope of streamline; expression for skin friction coefficient and total drag; Momentum thicknes

Solution:

Basic equation: Use results of Blasius solution (Table 9.1 on the web), and $\eta = y\cdot\sqrt{\frac{\nu\cdot x}{U}}$

$$f' = \frac{u}{U} = 0.9130 \quad \text{at} \quad \eta = 3.5$$

$$f' = \frac{u}{U} = 0.9555 \quad \text{at} \quad \eta = 4.0$$

Hence by linear interpolation, when $f' = 0.95$ $\quad \eta = 3.5 + \frac{(4-3.5)}{(0.9555-0.9310)}\cdot(0.95-0.9310) \qquad \eta = 3.89$

From Table A.9 at 68ºF $\quad \nu = 1.62\times10^{-4}\cdot\frac{ft^2}{s}$ and $U = 15\cdot\frac{ft}{s}$ $\qquad x = 7.5\cdot in$

Hence $\quad y = \eta\cdot\sqrt{\frac{\nu\cdot x}{U}} \qquad y = 0.121\,in$

The streamline slope is given by $\quad \frac{dy}{dx} = \frac{v}{u}$ where $u = U\cdot f'$ and $v = \frac{1}{2}\cdot\sqrt{\frac{\nu\cdot U}{x}}\cdot(\eta\cdot f' - f)$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\sqrt{\frac{\nu\cdot U}{x}}\cdot(\eta\cdot f' - f)\cdot\frac{1}{U\cdot f'} = \frac{1}{2}\cdot\sqrt{\frac{\nu}{U\cdot x}}\cdot\frac{(\eta\cdot f' - f)}{f'} = \frac{1}{2\cdot\sqrt{Re_x}}\cdot\frac{(\eta\cdot f' - f)}{f'}$$

We have $\quad Re_x = \frac{U\cdot x}{\nu} \qquad Re_x = 5.79\times10^4$

From the Blasius solution (Table 9.1 on the web)

$$f = 1.8377 \quad \text{at} \quad \eta = 3.5$$

$$f = 2.3057 \quad \text{at} \quad \eta = 4.0$$

Hence by linear interpolation $\quad f = 1.8377 + \frac{(2.3057-1.8377)}{(4.0-3.5)}\cdot(3.89-3.5) \qquad f = 2.2027$

$$\frac{dy}{dx} = \frac{1}{2\cdot\sqrt{Re_x}}\cdot\frac{(\eta\cdot f' - f)}{f'} = 0.00326$$

The shear stress is $\quad \tau_w = \mu\cdot\left(\frac{\partial}{\partial y}u + \frac{\partial}{\partial x}v\right) = \mu\cdot\frac{\partial}{\partial y}u$ at y = 0 (v = 0 at the wall for all x, so the derivative is zero there)

$\tau_w = \mu\cdot U\cdot\sqrt{\frac{U}{\nu\cdot x}}\cdot\frac{d^2f}{d\eta^2}$ $\quad$ and at η = 0 $\quad \frac{d^2f}{d\eta^2} = 0.3321$ $\quad$ (from Table 9.1)

$$\tau_w = 0.3321\cdot U\cdot\sqrt{\frac{\rho\cdot U\cdot\mu}{x}} \qquad \tau_w = 0.3321\cdot\rho\cdot U^2\cdot\sqrt{\frac{\mu}{\rho\cdot U\cdot x}} = 0.3321\cdot\frac{\rho\cdot U^2}{\sqrt{Re_x}}$$

The friction drag is $F_D = \int \tau_w\,dA = \int_0^L \tau_w\cdot b\,dx$ where b is the plate width

$$F_D = \int_0^L 0.3321\cdot\frac{\rho\cdot U^2}{\sqrt{Re_x}}\cdot b\,dx = 0.3321\cdot\rho\cdot U^2\cdot\sqrt{\frac{\nu}{U}}\cdot\int_0^L \frac{1}{x^{\frac{1}{2}}}\,dx$$

$$F_D = 0.3321\cdot\rho\cdot U^2\cdot\sqrt{\frac{\nu}{U}}\cdot b\cdot 2\cdot L^{\frac{1}{2}} \qquad F_D = \rho\cdot U^2\cdot b\cdot L\cdot\frac{0.6642}{\sqrt{Re_L}}$$

For the momentum integral $\frac{\tau_w}{\rho\cdot U^2} = \frac{d\theta}{dx}$ or $d\theta = \frac{\tau_w}{\rho\cdot U^2}\cdot dx$

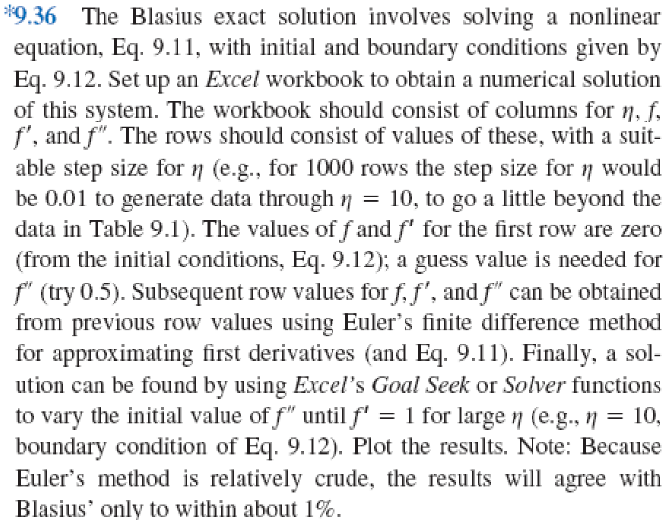
$$\theta_L = \frac{1}{\rho\cdot U^2}\cdot\int_0^L \tau_w\,dx = \frac{1}{\rho\cdot U^2}\cdot\frac{F_D}{b} = \frac{0.6642\cdot L}{\sqrt{Re_L}}$$

We have $L = 3\cdot ft$ $\qquad Re_L = \frac{U\cdot L}{\nu}$ $\qquad Re_L = 2.78\times 10^5$

$$\theta_L = \frac{0.6642\cdot L}{\sqrt{Re_L}} \qquad \theta_L = 0.0454\,in$$

Problem *9.36

[5]



*9.36 The Blasius exact solution involves solving a nonlinear equation, Eq. 9.11, with initial and boundary conditions given by Eq. 9.12. Set up an *Excel* workbook to obtain a numerical solution of this system. The workbook should consist of columns for η, f, f', and f''. The rows should consist of values of these, with a suitable step size for η (e.g., for 1000 rows the step size for η would be 0.01 to generate data through $\eta = 10$, to go a little beyond the data in Table 9.1). The values of f and f' for the first row are zero (from the initial conditions, Eq. 9.12); a guess value is needed for f'' (try 0.5). Subsequent row values for f, f', and f'' can be obtained from previous row values using Euler's finite difference method for approximating first derivatives (and Eq. 9.11). Finally, a solution can be found by using *Excel*'s *Goal Seek* or *Solver* functions to vary the initial value of f'' until $f' = 1$ for large η (e.g., $\eta = 10$, boundary condition of Eq. 9.12). Plot the results. Note: Because Euler's method is relatively crude, the results will agree with Blasius' only to within about 1%.

Given: Blasius nonlinear equation

Find: Blasius solution using Excel

Solution:

The equation to be solved is

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \qquad (9.11)$$

The boundary conditions are

$$f = 0 \quad \text{and} \quad \frac{df}{d\eta} = 0 \quad \text{at} \quad \eta = 0$$

$$f' = \frac{df}{d\eta} = 1 \quad \text{at} \quad \eta \to \infty \qquad (9.12)$$

Recall that these somewhat abstract variables are related to physically meaningful variables:

$$\frac{u}{U} = f'$$

and

$$\eta = y\sqrt{\frac{U}{\nu x}} \propto \frac{y}{\delta}$$

Using Euler's numerical method

$$f_{n+1} \approx f_n + \Delta\eta\, f'_n \qquad (1)$$

$$f'_{n+1} \approx f'_n + \Delta\eta\, f''_n \qquad (2)$$

$$f''_{n+1} \approx f''_n + \Delta\eta\, f'''_n$$

In these equations, the subscripts refer to the n^{th} discrete value of the variables, and $\Delta\eta = 10/N$ is the step size for η (N is the total number of steps).

But from Eq. 9.11

$$f''' = -\frac{1}{2} f\, f''$$

so the last of the three equations is

$$f''_{n+1} \approx f''_n + \Delta\eta\left(-\frac{1}{2} f_n\, f''_n\right) \qquad (3)$$

Equations 1 through 3 form a complete set for computing f, f', f''. All we need is the starting condition for each. From Eqs. 9.12

$$f_0 = 0 \quad \text{and} \quad f'_0 = 0$$

We do NOT have a starting condition for f''! Instead we must choose (using *Solver*) f''_0 so that the last condition of Eqs. 9.12 is met:

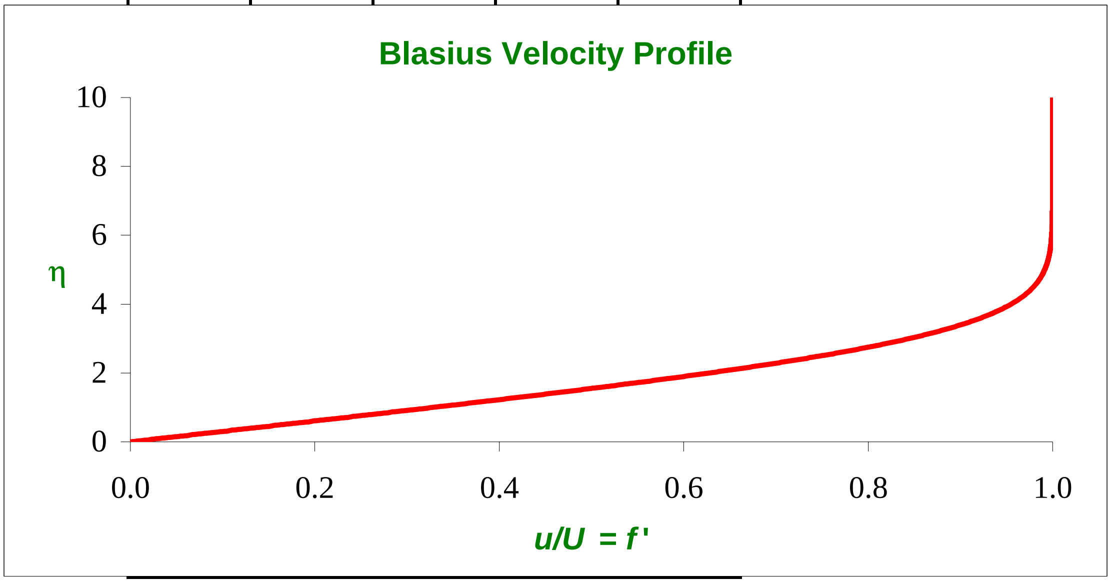
$$f'_N = 1$$

Computations (only the first few lines of 1000 are shown):

$\Delta\eta =$ 0.01

Make a guess for the first *f*"; use Solver to vary it until *f*'N = 1

Count	**η**	***f***	***f'***	***f''***
0	0.00	0.0000	0.0000	0.3303
1	0.01	0.0000	0.0033	0.3303
2	0.02	0.0000	0.0066	0.3303
3	0.03	0.0001	0.0099	0.3303
4	0.04	0.0002	0.0132	0.3303
5	0.05	0.0003	0.0165	0.3303



Problem 9.37

[2]

9.37 Consider flow of air over a flat plate. On one graph, plot the laminar boundary-layer thickness as a function of distance along the plate (up to transition) for freestream speeds $U = 1$ m/s, 2 m/s, 3 m/s, 4 m/s, 5 m/s, and 10 m/s.

Given: Data on flow over flat plate

Find: Plot of laminar thickness at various speeds

Solution:

Governing equations:

$$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \quad (9.21) \qquad \text{and} \qquad Re_x = \frac{U \cdot x}{\nu} \qquad \text{so} \qquad \delta = 5.48 \cdot \sqrt{\frac{\nu \cdot x}{U}}$$

The critical Reynolds number is $Re_{crit} = 500000$

Hence, for velocity U the critical length x_{crit} is $x_{crit} = 500000 \cdot \frac{\nu}{U}$

Tabulated or graphical data:

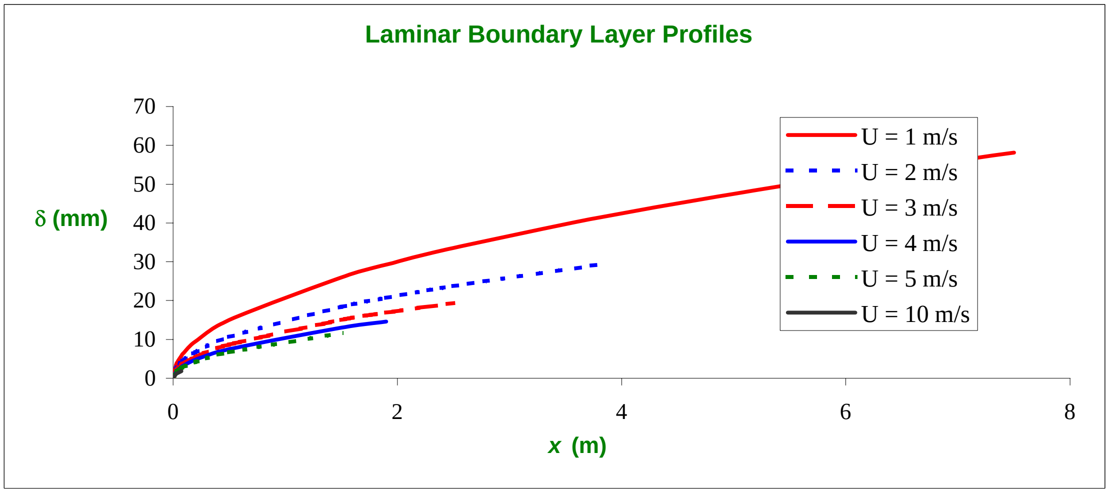
$\nu =$ 1.50E-05 m^2/s

(Table A.10, 20°C)

Computed results:

U **(m/s)**	1	2	3	4	5	10
x_{crit} **(m)**	7.5	3.8	2.5	1.9	1.5	0.75

x **(m)**	**δ (mm)**	**δ (mm)**	**δ (mm)**	**δ (mm)**	**δ (mm)**	**δ (mm)**
0.000	0.00	0.00	0.00	0.00	0.00	0.00
0.025	3.36	2.37	1.94	1.68	1.50	1.06
0.050	4.75	3.36	2.74	2.37	2.12	1.50
0.075	5.81	4.11	3.36	2.91	2.60	1.84
0.100	6.71	4.75	3.87	3.36	3.00	
0.2	9.49	6.71	5.48	4.75	4.24	
0.5	15.01	10.61	8.66	7.50	6.71	
1.5	25.99	18.38	15.01	13.00	11.62	
1.9	29.26	20.69	16.89	14.63		
2.5	33.56	23.73	19.37			
3.8	41.37	29.26				
5.0	47.46					
6.0	51.99					
7.5	58.12					



Problem 9.38

[2]

*9.38 A thin flat plate, $L = 0.25$ m long and $b = 1$ m wide, is installed in a water tunnel as a splitter. The freestream speed is $U = 1.75$ m/s and the velocity profile in the boundary layer is approximated as parabolic. Plot δ, δ^*, and τ_w versus x/L for the plate.

Given: Parabolic solution for laminar boundary layer

Find: Plot of δ, δ^*, and τ_w versus x/L

Solution:

Basic equations:

$$\frac{u}{U} = 2\cdot\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \qquad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \qquad c_f = \frac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2} = \frac{0.730}{\sqrt{Re_x}}$$

Hence

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \delta\int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \delta\int_0^1 \left(1 - 2 + \eta^2\right) d\eta = \delta\left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1 = \frac{\delta}{3}$$

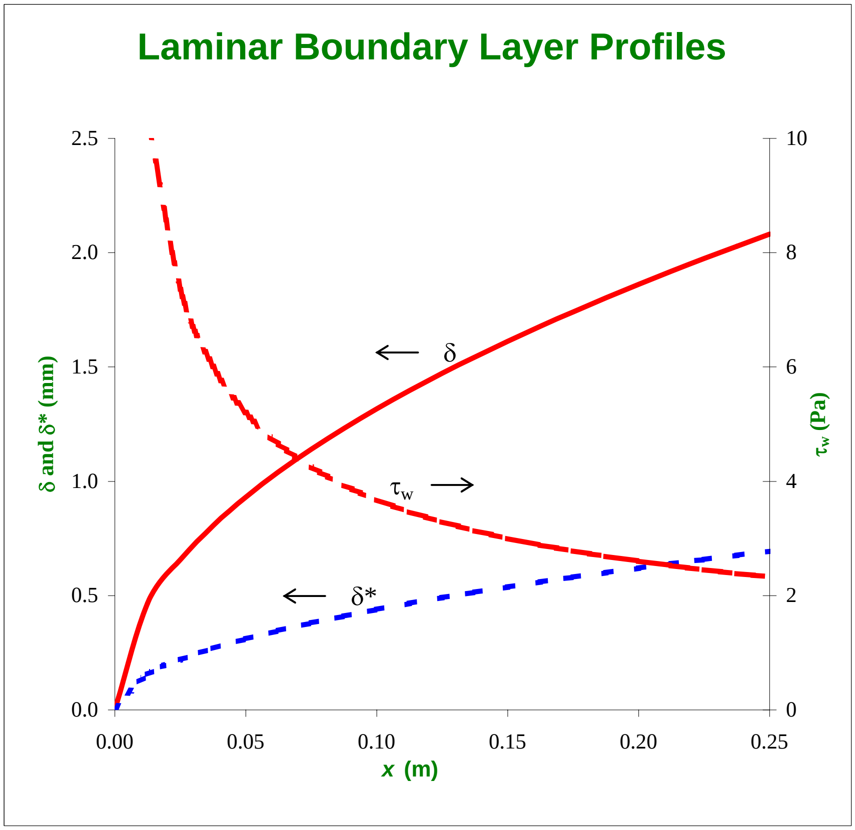
Tabulated or graphical data: Given data:

$\nu =$ 1.01E-06 m^2/s $L =$ 0.25 m

(Table A.8, 20°C) $U =$ 1.75 m/s

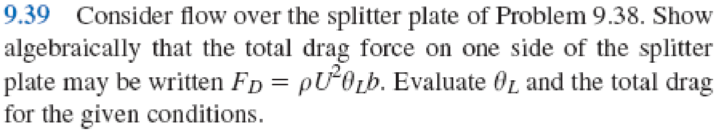
Computed results:

x (m)	*Re*$_x$	δ (mm)	δ* (mm)	τ_w (Pa)
0.0000	0.00.E+00	0.000	0.000	
0.0125	2.17.E+04	0.465	0.155	10.40
0.0250	4.33.E+04	0.658	0.219	7.36
0.0375	6.50.E+04	0.806	0.269	6.01
0.0500	8.66.E+04	0.931	0.310	5.20
0.0625	1.08.E+05	1.041	0.347	4.65
0.0750	1.30.E+05	1.140	0.380	4.25
0.0875	1.52.E+05	1.231	0.410	3.93
0.1000	1.73.E+05	1.317	0.439	3.68
0.1125	1.95.E+05	1.396	0.465	3.47
0.1250	2.17.E+05	1.472	0.491	3.29
0.1375	2.38.E+05	1.544	0.515	3.14
0.1500	2.60.E+05	1.612	0.537	3.00
0.1625	2.82.E+05	1.678	0.559	2.89
0.1750	3.03.E+05	1.742	0.581	2.78
0.1875	3.25.E+05	1.803	0.601	2.69
0.2000	3.47.E+05	1.862	0.621	2.60
0.2125	3.68.E+05	1.919	0.640	2.52
0.2250	3.90.E+05	1.975	0.658	2.45
0.2375	4.12.E+05	2.029	0.676	2.39
0.2500	4.33.E+05	**2.082**	**0.694**	**2.33**



Problem 9.39

[2]



9.39 Consider flow over the splitter plate of Problem 9.38. Show algebraically that the total drag force on one side of the splitter plate may be written $F_D = \rho U^2 \theta_L b$. Evaluate θ_L and the total drag for the given conditions.

Given: Parabolic solution for laminar boundary layer

Find: Derivation of F_D; Evaluate F_D and θ_L

Solution:

Basic equations: $\frac{u}{U} = 2\cdot\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ $\qquad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$ $\qquad \frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$

$L = 0.25\cdot m \qquad b = 1\cdot m \qquad U = 1.75\cdot\frac{m}{s} \qquad \rho = 1000\cdot\frac{kg}{m^3}$

Assumptions: 1) Flat plate so $\frac{\partial}{\partial x}p = 0$, and U = const 2) δ is a function of x only 3) Incompressible

The momentum integral equation then simplifies to $\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\cdot\theta\right)$ where $\theta = \int_0^\delta \frac{u}{U}\cdot\left(1 - \frac{u}{U}\right) dy$

For U = const $\quad \tau_w = \rho\cdot U^2\cdot\frac{d\theta}{dx}$

The drag force is then $\quad F_D = \int \tau_w \, dA = \int_0^L \tau_w\cdot b\,dx = \int_0^L \rho\cdot U^2\cdot\frac{d\theta}{dx}\cdot b\,dx = \rho\cdot U^2\cdot b\cdot\int_0^{\theta_L} 1\,d\theta \qquad F_D = \rho\cdot U^2\cdot b\cdot\theta_L$

For the given profile $\quad \frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\cdot\left(1 - \frac{u}{U}\right) d\eta = \int_0^1 \left(2\cdot\eta - \eta^2\right)\cdot\left(1 - 2\cdot\eta + \eta^2\right) d\eta = \int_0^1 \left(2\cdot\eta - 5\cdot\eta^2 + 4\cdot\eta^3 - \eta^4\right) d\eta = \frac{2}{15}$

$\theta = \frac{2}{15}\cdot\delta$

From Table A.8 at 20ºC $\quad \nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s} \qquad Re_L = \frac{U\cdot L}{\nu} \qquad Re_L = 4.332\times 10^5$

$\delta_L = L\cdot\frac{5.48}{\sqrt{Re_L}} \qquad \delta_L = 2.08\,mm$

$\theta_L = \frac{2}{15}\cdot\delta_L \qquad \theta_L = 0.278\,mm$

$F_D = \rho\cdot U^2\cdot b\cdot\theta_L \qquad F_D = 0.850\,N$

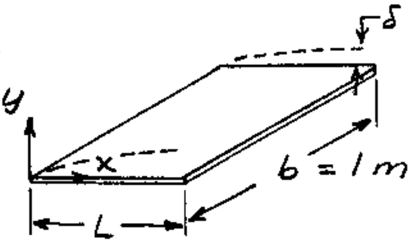
Problem 9.40

[2]

Given: Thin flat plate in water tunnel.

$U = 1.6\ m/s \longrightarrow$

$L = 0.3\ m$



Velocity profile is $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Find: Total drag force on plate due to skin friction.

Solution: Check $Re_L = \frac{UL}{\nu} = 1.6\frac{m}{s} \times 0.3\ m \times \frac{s}{10^{-6}\ m^2} = 4.8 \times 10^5$, so laminar.

Viscous drag for two sides of plate is (Assume T = 20°C.)

$$Drag = 2\int_0^L \tau_w\, b\, dx \qquad (1)$$

From the definition, $\tau_w = \mu \frac{\partial u}{\partial y}\Big)_{y=0} = \frac{\mu U}{\delta}\, \frac{d(u/U)}{d(y/\delta)}\Big]_{\eta=0} = \frac{\mu U}{\delta}\left(2 - 2\eta\right)\Big]_{\eta=0} = \frac{2\mu U}{\delta}$

Since $\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$, $\delta = \frac{5.48\, x}{\sqrt{Re_x}} = \frac{5.48}{\sqrt{U/\nu}}\, x^{1/2}$ $\qquad \mu = \rho\nu$

Substituting into Eq. 1,

$$F_D = 2\int_0^L \frac{2\mu U}{\delta}\, b\, dx = \frac{4}{5.48}\, b\mu U\sqrt{\frac{U}{\nu}}\int_0^L \frac{dx}{x^{1/2}} = \frac{4b\mu U}{5.48}\sqrt{\frac{U}{\nu}}\left[2x^{1/2}\right]_0^L = \frac{8b\mu U}{5.48}\sqrt{\frac{UL}{\nu}}$$

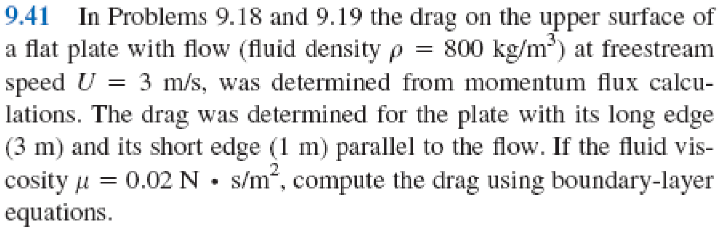
$$= \frac{8}{5.48} \times 1.0\ m \times 999\frac{kg}{m^3} \times 10^{-6}\frac{m^2}{s} \times 1.6\frac{m}{s} \times \left[1.6\frac{m}{s} \times 0.3\ m \times \frac{s}{10^{-6}\ m^2}\right]^{1/2} \frac{N \cdot s^2}{kg \cdot m}$$

$F_D = 1.62\ N$ $\longleftarrow$ F_D

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 9.41

[2]



9.41 In Problems 9.18 and 9.19 the drag on the upper surface of a flat plate with flow (fluid density $\rho = 800\ \text{kg/m}^3$) at freestream speed $U = 3$ m/s, was determined from momentum flux calculations. The drag was determined for the plate with its long edge (3 m) and its short edge (1 m) parallel to the flow. If the fluid viscosity $\mu = 0.02\ \text{N}\cdot\text{s/m}^2$, compute the drag using boundary-layer equations.

Given: Data on fluid and plate geometry

Find: Drag at both orientations using boundary layer equation

Solution:

The given data is $\rho = 800\cdot\frac{\text{kg}}{\text{m}^3}$ $\mu = 0.02\cdot\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ $U = 3\cdot\frac{\text{m}}{\text{s}}$ $L = 3\cdot\text{m}$ $b = 1\cdot\text{m}$

First determine the nature of the boundary layer $\text{Re}_L = \frac{\rho\cdot U\cdot L}{\mu}$ $\text{Re}_L = 3.6\times10^5$

The maximum Reynolds number is less than the critical value of 5×10^5

Hence:

Governing equations: $c_f = \frac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ (9.22) $c_f = \frac{0.730}{\sqrt{\text{Re}_x}}$ (9.23)

The drag (one side) is $F_D = \int_0^L \tau_w\cdot b\,dx$

Using Eqs. 9.22 and 9.23 $F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot b\cdot\int_0^L \frac{0.73}{\sqrt{\frac{\rho\cdot U\cdot x}{\mu}}}\,dx$

$F_D = 0.73\cdot b\cdot\sqrt{\mu\cdot L\cdot\rho\cdot U^3}$ $F_D = 26.3\,\text{N}$ (Compare to 30 N for Problem 9.18)

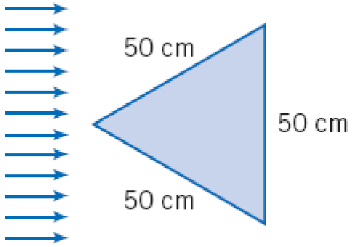
Repeating for $L = 1\cdot\text{m}$ $b = 3\cdot\text{m}$

$F_D = 0.73\cdot b\cdot\sqrt{\mu\cdot L\cdot\rho\cdot U^3}$ $F_D = 45.5\,\text{N}$ (Compare to 50.4 N for Problem 9.19)

Problem 9.42

[3]

9.42 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5-m/s air flow. The air is at 20°C and 1 atm.



Given: Triangular plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$L = 0.50\cdot cm\cdot\dfrac{\sqrt{3}}{2}$ $\quad L = 0.433\cdot cm$ $\quad W = 50\cdot cm$ $\quad U = 5\cdot\dfrac{m}{s}$

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 1443$ $\quad$ so definitely laminar

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\, dA$ $\quad F_D = \displaystyle\int_0^L \tau_w\cdot w(x)\, dx$ $\quad w(x) = W\cdot\dfrac{x}{L}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot\frac{W}{L}\cdot\int_0^L \frac{0.730\cdot x}{\sqrt{\frac{U\cdot x}{\nu}}}\, dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot\frac{W}{L}\cdot\sqrt{\nu}\cdot\int_0^L x^{\frac{1}{2}}\, dx$$

The integral is $\quad \displaystyle\int_0^L x^{\frac{1}{2}}\, dx = \frac{2}{3}\cdot L^{\frac{3}{2}}$ $\quad$ so $\quad F_D = 0.243\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\quad F_D = 4.19\times 10^{-4} N$

Note: For two-sided solution $\quad 2\cdot F_D = 8.38\times 10^{-4} N$

Problem 9.43

[3]

9.43 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5-m/s air flow, except that the base rather than the tip faces the flow. Would you expect this to be larger than, the same as, or lower than the drag for Problem 9.42?

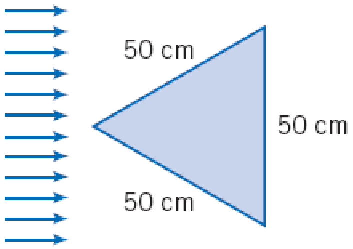


Plate is reversed from this!

Given: Triangular plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$L = 0.50\cdot\text{cm}\cdot\dfrac{\sqrt{3}}{2}$ $\qquad L = 0.433\cdot\text{cm}$ $\qquad W = 50\cdot\text{cm}$ $\qquad U = 5\cdot\dfrac{\text{m}}{\text{s}}$

From Table A.10 at 20ºC $\quad \nu = 1.50\times10^{-5}\cdot\dfrac{\text{m}^2}{\text{s}}$ $\qquad \rho = 1.21\cdot\dfrac{\text{kg}}{\text{m}^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\qquad Re_L = 1443$ $\qquad$ so definitely laminar

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\,dA$ $\qquad F_D = \displaystyle\int_0^L \tau_w\cdot w(x)\,dx$ $\qquad w(x) = W\cdot\left(1-\dfrac{x}{L}\right)$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730\cdot\left(1-\frac{x}{L}\right)}{\sqrt{\frac{U\cdot x}{\nu}}}\,dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L\left(x^{-\frac{1}{2}}-\frac{x^{\frac{1}{2}}}{L}\right)dx$$

The integral is

$$\int_0^L\left(x^{-\frac{1}{2}}-\frac{x^{\frac{1}{2}}}{L}\right)dx = 2\cdot L^{\frac{1}{2}}-\frac{2}{3}\cdot\frac{L^{\frac{3}{2}}}{L} = \frac{4}{3}\cdot\sqrt{L}$$

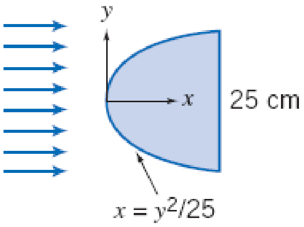
$F_D = 0.487\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\qquad F_D = 8.40\times10^{-4}\,\text{N}$

Note: For two-sided solution $\quad 2\cdot F_D = 1.68\times10^{-3}\,\text{N}$

The drag is much higher (twice as much) compared to Problem 9.42. This is because τ_w is largest near the leading edge and falls off rapidly; in this problem the widest area is also at the front

Problem 9.44

9.44 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 7.5-m/s air flow. The air is at 20°C and 1 atm. (Note that the shape is given by $x = y^2/25$, where x and y are in cm.)



Given: Parabolic plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$W = 25\cdot cm \qquad L = \dfrac{\left(\frac{W}{2}\right)^2}{25\cdot cm} \qquad L = 6.25\cdot cm \qquad U = 7.5\cdot\dfrac{m}{s}$

Note: "y" is the equation of the upper and lower surfaces, so y = W/2 at x = L

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s} \qquad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu} \qquad Re_L = 3.12\times 10^4$ so just laminar

The drag (one side) is $\quad F_D = \int \tau_w\, dA \qquad F_D = \int_0^L \tau_w\cdot w(x)\, dx \qquad w(x) = W\cdot\sqrt{\dfrac{x}{L}}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence $\quad F_D = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\displaystyle\int_0^L \dfrac{0.730\cdot\sqrt{\frac{x}{L}}}{\sqrt{\frac{U\cdot x}{\nu}}}\, dx = \dfrac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\dfrac{\nu}{L}}\cdot\int_0^L 1\, dx$

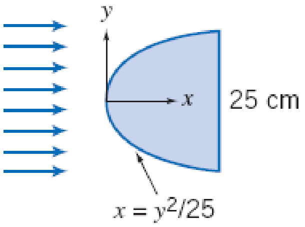
$F_D = 0.365\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3} \qquad F_D = 2.20\times 10^{-3}\, N$

Note: For two-sided solution $\quad 2\cdot F_D = 4.39\times 10^{-3}\, N$

Problem 9.45

[4]

9.45 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 7.5-m/s air flow, except that the base rather than the tip faces the flow. Would you expect this to be larger than, the same as, or lower than the drag for Problem 9.44?



Note: Plate is now reversed!

Given:

Parabolic plate

Find:

Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$W = 25\cdot cm \qquad L = \dfrac{\left(\frac{W}{2}\right)^2}{25\cdot cm} \qquad L = 6.25\cdot cm \qquad U = 7.5\cdot\dfrac{m}{s}$

Note: "y" is the equation of the upper and lower surfaces, so y = W/2 at x = 0

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s} \qquad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu} \qquad Re_L = 3.12\times 10^4 \quad$ so just laminar

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\,dA \qquad F_D = \int_0^L \tau_w\cdot w(x)\,dx \qquad w(x) = W\cdot\sqrt{1-\frac{x}{L}}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730\cdot\sqrt{1-\frac{x}{L}}}{\sqrt{\frac{U\cdot x}{\nu}}}\,dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx$$

The tricky integral is (this might be easier to do numerically!)

$$\int \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx = \sqrt{x-\frac{x^2}{L}} - \frac{i}{2}\cdot\sqrt{L}\cdot\ln\left(\frac{\sqrt{-L-x}-\sqrt{x}}{\sqrt{-L-x}+\sqrt{x}}\right) \qquad \text{so} \qquad \int_0^L \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx = 0.393\cdot\sqrt{m}$$

$$F_D = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx \qquad F_D = 3.45\times 10^{-3}\,N$$

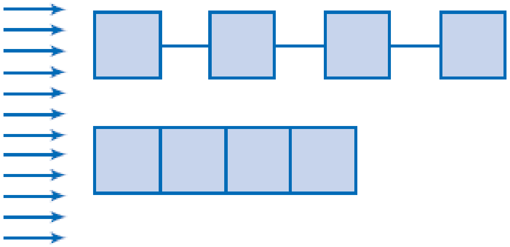
Note: For two-sided solution $\quad 2\cdot F_D = 6.9\times 10^{-3}\,N$

The drag is much higher compared to Problem 9.44. This is because τ_w is largest near the leading edge and falls off rapidly; in this problem the widest area is also at the front

Problem 9.46

[3]

9.46 Assume laminar boundary-layer flow to estimate the drag on four square plates (each 7.5 cm × 7.5 cm) placed parallel to a 1-m/s water flow, for the two configurations shown. Before calculating, which configuration do you expect to experience the lowest drag? Assume the plates attached with string are far enough apart for wake effects to be negligible, and that the water is at 20°C.



Given: Pattern of flat plates

Find: Drag on separate and composite plates

Solution:

Basic equations: $c_f = \frac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \frac{0.730}{\sqrt{Re_x}}$

For separate plates $L = 7.5\cdot cm$ $\quad W = 7.5\cdot cm$ $\quad U = 1\cdot\frac{m}{s}$

From Table A.8 at 20ºC $\nu = 1.01\times10^{-6}\cdot\frac{m^2}{s}$ $\quad \rho = 998\cdot\frac{kg}{m^3}$

First determine the nature of the boundary layer $Re_L = \frac{U\cdot L}{\nu}$ $\quad Re_L = 7.43\times10^4$ so definitely laminar

The drag (one side) is $F_D = \int \tau_w\,dA$ $\quad F_D = \int_0^L \tau_w\cdot W\,dx$

We also have $\tau_w = c_f\cdot\frac{1}{2}\cdot\rho\cdot U^2 = \frac{1}{2}\cdot\rho\cdot U^2\cdot\frac{0.730}{\sqrt{Re_x}}$

Hence $$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730}{\sqrt{\frac{U\cdot x}{\nu}}}\,dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L x^{-\frac{1}{2}}\,dx$$

The integral is $\int_0^L x^{-\frac{1}{2}}\,dx = 2\cdot L^{\frac{1}{2}}$ so $F_D = 0.730\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\quad F_D = 0.0150\,N$

This is the drag on one plate. The total drag is then $F_{Total} = 4\cdot F_D$ $\quad F_{Total} = 0.0602\,N$

For both sides: $2\cdot F_{Total} = 0.120\,N$

For the composite plate $L = 4\times7.5\cdot cm$ $\quad L = 0.30\,m$

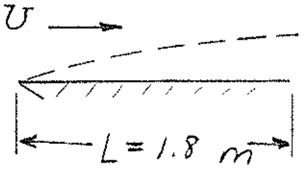
$F_{Composite} = 0.730\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\quad F_{Composite} = 0.0301\,N$

For both sides: $2\cdot F_{Composite} = 0.0602\,N$

The drag is much lower on the composite compared to the separate plates. This is because τ_w is largest near the leading edges and falls off rapidly; in this problem the separate plates experience leading edges four times!

Problem 9.47

Given: Laminar boundary layer in std. air, sinusoidal profile.
Conditions: $U = 3.2\ \text{m/s}$, $L = 1.8\ \text{m}$, $b = 0.9\ \text{m}$.



Find: Plot δ, δ^*, and τ_w vs. x/L.

Solution: Apply momentum integral equation for steady, incompressible flow.

Computing equations: $\tau_w = \rho U^2 \dfrac{d\theta}{dx}$, $\dfrac{\delta^*}{\delta} = \displaystyle\int_0^1 \left(1 - \frac{u}{U}\right) d\lambda$, $\dfrac{\theta}{\delta} = \displaystyle\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\lambda$

For the sinusoidal profile,

$$\frac{\theta}{\delta} = \int_0^1 \sin\frac{\pi}{2}\lambda\left(1 - \sin\frac{\pi}{2}\lambda\right) d\lambda = \int_0^1 \left(\sin\frac{\pi}{2}\lambda - \sin^2\frac{\pi}{2}\lambda\right) d\lambda$$

$$= \int_0^1 \left(\sin\frac{\pi}{2}\lambda - \frac{1 - \cos\pi\lambda}{2}\right) d\lambda = \left[-\frac{2}{\pi}\cos\frac{\pi}{2}\lambda - \frac{\lambda}{2} + \frac{1}{2\pi}\sin\pi\lambda\right]_0^1$$

$$\frac{\theta}{\delta} = -\frac{2}{\pi}(0 - 1) - \frac{1}{2} + \frac{1}{2\pi}(0 - 0) = \frac{2}{\pi} - \frac{1}{2} = \frac{4 - \pi}{2\pi} = 0.137$$

Also $\tau_w = \mu \left.\dfrac{du}{dy}\right)_{y=0} = \mu U \dfrac{d}{dy} \sin\left(\dfrac{\pi}{2}\dfrac{y}{\delta}\right)\Big]_{y=0} = \dfrac{\pi \mu U}{2\delta}\left(\cos\dfrac{\pi}{2}\dfrac{y}{\delta}\right)\Big]_{y=0} = \dfrac{\pi \mu U}{2\delta}$

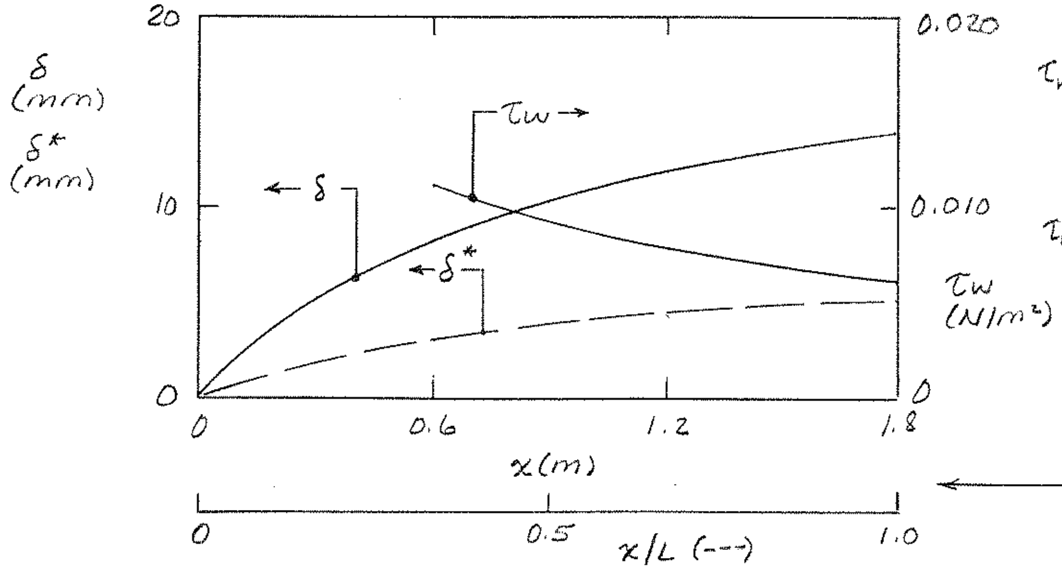
Thus $\tau_w = \rho U^2 \dfrac{d\theta}{dx} = \dfrac{\pi \mu U}{2\delta} = \rho U^2 \left(\dfrac{4 - \pi}{2\pi}\right) \dfrac{d\delta}{dx}$ or $\delta\, d\delta = \dfrac{\pi^2}{4 - \pi} \dfrac{\mu}{\rho U} dx$

Integrating, $\dfrac{\delta^2}{2} = \dfrac{\pi^2}{4 - \pi} \dfrac{\mu}{\rho U} x$ so $\dfrac{\delta}{x} = \sqrt{\dfrac{2\pi^2}{4 - \pi}} \sqrt{\dfrac{\mu}{\rho U x}} = \dfrac{4.80}{\sqrt{Re_x}}$ ← δ/x

Also $\dfrac{\delta^*}{\delta} = \displaystyle\int_0^1 \left(1 - \frac{u}{U}\right) d\lambda = \int_0^1 \left(1 - \sin\frac{\pi}{2}\lambda\right) d\lambda = \left(\lambda + \frac{2}{\pi}\cos\frac{\pi}{2}\lambda\right)\Big|_0^1 = 1 - \frac{2}{\pi} = 0.363$ ← δ^*/δ

Tabulate: $Re_x = \dfrac{Ux}{\nu} = 3.2\,\dfrac{m}{s} \cdot x\, m \times \dfrac{s}{1.46 \times 10^{-5}\, m^2} = 2.19 \times 10^5\, x$

x (m)	Re_x (---)	$1/\sqrt{Re_x}$ (---)	δ (mm)	δ^* (mm)	τ_w (N/m²)
0.6	1.31×10^5	0.00276	7.95	2.89	0.0114
1.2	2.63×10^5	0.00195	11.2	4.07	0.00804
1.8	3.94×10^5	0.00159	13.7	4.97	0.00656



$$\tau_w = \frac{\pi}{2}\frac{\mu}{\rho U \delta}\rho U^2$$

$$= \frac{1}{2}\rho U^2 \frac{\mu}{\rho U x}\frac{x}{\delta}\pi$$

$$\tau_w = \frac{\pi}{4.80\sqrt{Re_x}}\,\frac{1}{2}\rho U^2$$

← Plot

Problem 9.48

[3]

Given: Laminar boundary layer flow with velocity profile, $\frac{u}{U} = \frac{y}{\delta} = \eta$.

Find: Expressions for δ/x, c_f, using the momentum integral equation.

Solution: The momentum integral equation is

Computing equation: $$-\delta \overset{\approx 0(1)}{\cancel{\frac{\partial p}{\partial x}}} - \tau_w = \frac{\partial}{\partial x}\int_0^\delta u \rho u \, dy - U \frac{\partial}{\partial x}\int_0^\delta \rho u \, dy$$

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$
(2) δ is a function of x only, and $\delta = 0$ at $x = 0$
(3) Incompressible flow

Then
$$\tau_w = U\frac{\partial}{\partial x}\int_0^\delta \rho u \, dy - \frac{\partial}{\partial x}\int_0^\delta u \rho u \, dy = \frac{\partial}{\partial x}\int_0^\delta \rho u (U - u)\, dy$$

or
$$\tau_w = \rho U^2 \frac{\partial \delta}{\partial x}\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta = \rho U^2 \beta \frac{d\delta}{dx}$$

Now use the given velocity profile:
$$\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\eta = \int_0^1 \eta(1-\eta)\, d\eta = \left[\tfrac{1}{2}\eta^2 - \tfrac{1}{3}\eta^3\right]_0^1 = \frac{1}{6} = \beta$$
$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U}{\delta} \left.\frac{\partial(u/U)}{\partial(y/\delta)}\right]_{y/\delta = 0} = \frac{\mu U}{\delta}\left.\frac{d(u/U)}{d\eta}\right]_{\eta = 0} = \frac{\mu U}{\delta}$$

Substituting for β and τ_w,
$$\frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right) \quad \text{or} \quad \delta \, d\delta = \frac{6\mu}{\rho U}\, dx$$

Integrating, $\frac{\delta^2}{2} = \frac{6\mu}{\rho U}x + c$, but $c = 0$ since $\delta = 0$ at $x = 0$. Thus
$$\delta = \sqrt{\frac{12\mu}{\rho U}x}$$

or
$$\frac{\delta}{x} = \sqrt{\frac{12\mu}{\rho U x}} = \frac{3.46}{\sqrt{Re_x}}$$

$\frac{\delta}{x}$

Also
$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{\mu \frac{U}{\delta}}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho U \delta} = 2 \times \frac{\mu}{\rho U x}\,\frac{x}{\delta} = \frac{2}{Re_x}\,\frac{\sqrt{Re_x}}{3.46}$$
$$c_f = \frac{0.577}{\sqrt{Re_x}}$$

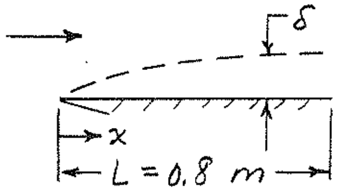
c_f

Problem 9.49

[3]

Given: Laminar boundary-layer flow of air, conditions as shown.

Assume $\frac{u}{U} = \frac{y}{\delta}$ $U = 5.3\ m/s$



Assume STP.

Find: Plot δ, δ*, and τ_w vs. x/L.

Solution: Apply the momentum integral equation, Eq. 9.19.

Computing equations: $\tau_w = \rho U^2 \frac{d\delta}{dx}\beta$ $\tau_w = \mu \frac{\partial u}{\partial y}\Big)_{y=0}$

Assume: (1) steady flow, (2) Incompressible flow, (3) zero pressure gradient

Thus $\tau_w = \mu \frac{\partial u}{\partial y}\Big)_{y=0} = \frac{\mu U}{\delta}\frac{d(u/U)}{d(y/\delta)}\Big]_{y=0} = \frac{\mu U}{\delta}(1) = \frac{\mu U}{\delta}$; $\frac{\delta^*}{\delta} = \int_0^1 (1-\lambda)\,d\lambda = \frac{1}{2}$

$$\beta = \frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \lambda(1-\lambda)\,d\lambda = \left[\frac{\lambda^2}{2}-\frac{\lambda^3}{3}\right]_0^1 = \frac{1}{6}$$

Substituting into the MIE, $\frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right)$ so $\delta\, d\delta = 6\frac{\mu}{\rho U}dx$

Integrating, $\delta^2 = 12\frac{\mu}{\rho U}x + c$ so $\frac{\delta}{x} = \frac{\sqrt{12}}{\sqrt{Re_x}} = \frac{3.46}{\sqrt{Re_x}}$; $\frac{\delta^*}{x} = \frac{1.73}{\sqrt{Re_x}}$

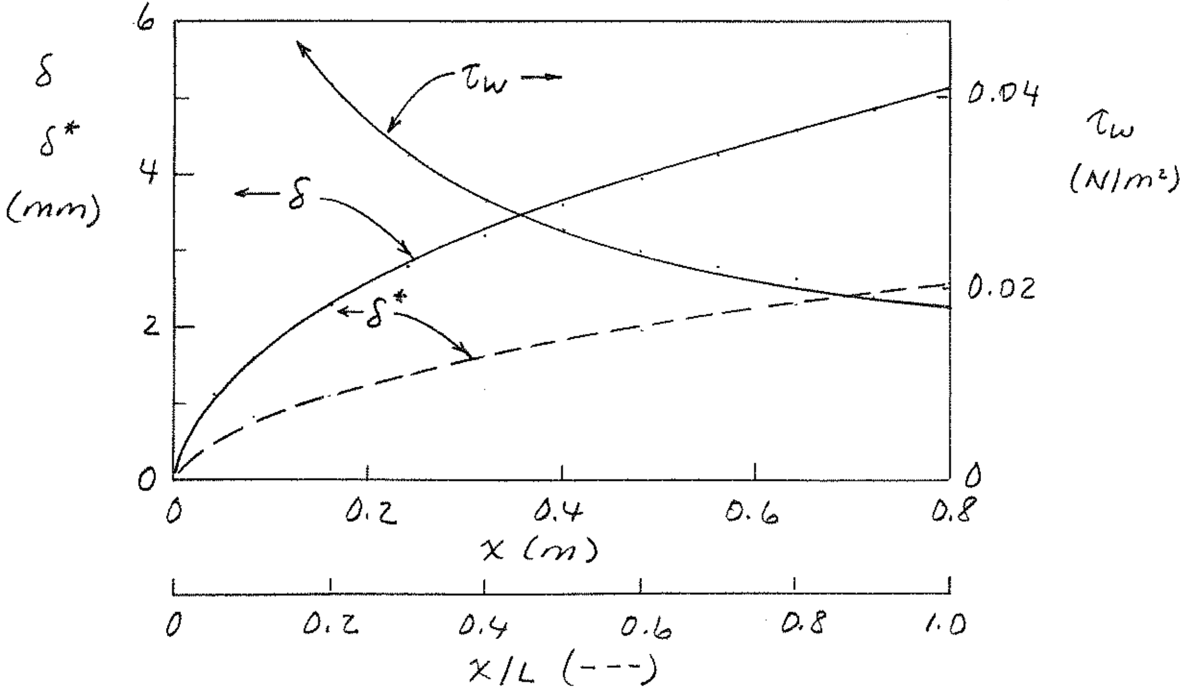
$\tau_w = \frac{\mu U}{\delta} = \mu U \frac{\sqrt{Re_x}}{3.46x} = \frac{\rho \nu U}{x}\frac{\sqrt{Re_x}}{3.46}$; $C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.578}{\sqrt{Re_x}}$

At $x = L$, $Re_L = \frac{UL}{\nu} = 5.3\frac{m}{s}\times 0.8\,m\times\frac{s}{1.46\times10^{-5}\,m^2} = 2.90\times10^5$, so laminar

$\frac{\delta}{L} = \frac{3.46}{\sqrt{2.92\times10^5}} = 0.00643$; $\delta_L = 0.00643L = 0.00643\times800\,mm = 5.14\,mm$ ← δ_L

$\delta^* = \frac{1}{2}\delta$, so $\delta^*_L = \frac{1}{2}\times5.14\,mm = 2.57\,mm$ ← δ^*_L

$\tau_w(x=L) = \frac{0.578}{\sqrt{2.90\times10^5}}\times\frac{1}{2}\times1.23\frac{kg}{m^3}(5.3)^2\frac{m^2}{sec^2}\times\frac{N\cdot sec^2}{kg\cdot m} = 0.0185\ N/m^2$ ← τ_{wL}



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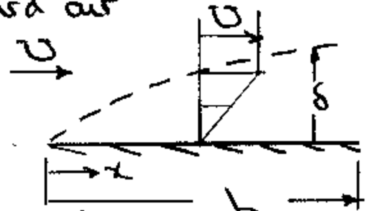
Problem 9.50

Given: Laminar boundary layer forms on flat plate of length $L = 0.8\,m$ and width $b = 1.9\,m$. Free stream velocity, $U = 5.3\,m/s$. Velocity profile in the boundary layer is linear. Standard air

Find: (a) algebraic expression for $\tau_w(x)$
(b) algebraic expression for F_D
(c) magnitude of F_D.



Solution: Apply the momentum integral equation, Eq 9.19

Computing eqs. $\tau_w = \rho U^2 \frac{d\theta}{dx}$, $\theta = \delta \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right)$

For a linear profile $\theta = \delta \int_0^1 \eta(1-\eta)\,d\eta = \delta\left[\frac{\eta^2}{2} - \frac{\eta^3}{3}\right]_0^1 = \frac{1}{6}\delta$

$\therefore \tau_w = \mu \frac{du}{dy}\Big)_{y=0} = \mu \frac{U}{\delta} \frac{d(u/U)}{d\eta}\Big]_{\eta=0} = \frac{\mu U}{\delta} = \rho U^2 \frac{1}{6}\frac{d\delta}{dx}$

Separating variables and integrating gives

$$\delta\, d\delta = 6 \frac{\mu}{\rho U} dx$$

Then $\frac{\delta^2}{2} = 6\frac{\mu}{\rho U} x + c$. Since $\delta = 0$ at $x = 0$ then

$$\delta = \left[\frac{12 \mu x}{\rho U}\right]^{1/2} = 3.46\left[\frac{\mu x}{\rho U}\right]^{1/2} \quad \text{or} \quad \frac{\delta}{x} = \frac{3.46}{\sqrt{Re_x}}$$

$\tau_w = \mu \frac{U}{\delta} = \frac{1}{3.46} \frac{\mu^{1/2} \rho^{1/2} U^{3/2}}{x^{1/2}} = 0.289 \frac{U\mu}{x}\sqrt{Re_x} \longleftarrow$ $\tau_w(x)$

The drag force is given by

$$F_D = \int \tau_w\, dA = \int_0^L \tau_w\, b\,dx = b\int_0^L \rho U^2 \frac{d\theta}{dx}dx = b\int_0^{\theta_L} \rho U^2 d\theta$$

$F_D = \rho U^2 b\,\theta_L \longleftarrow$ F_D

where $\theta_L = \frac{1}{6}\delta_L$ and $\delta_L = \frac{3.46\,L}{\sqrt{Re_L}}$

For given conditions

$Re_L = \frac{UL}{\nu} = 5.3\frac{m}{s} \times 0.8\,m \times \frac{s}{1.46\times10^{-5}\,m^2} = 2.90\times10^5$

$\delta_L = \frac{3.46\,L}{\sqrt{Re_L}} = 3.46\times0.8\,m \times \frac{1}{(2.90\times10^5)^{1/2}} = 5.14\,mm$

$\theta_L = \frac{1}{6}\delta_L = 0.857\,mm$

$F_D = \rho U^2 b\,\theta_L = 1.23\,\frac{kg}{m^3} \times (5.3)^2 \frac{m^2}{s^2} \times 1.9\,m \times 0.854\times10^{-3}\,m \times \frac{N\cdot s^2}{kg\cdot m}$

$F_D = 5.63\times10^{-2}\,N \longleftarrow$ F_D

Problem 9.51

[3]

9.51 Water at 10°C flows over a flat plate at a speed of 0.8 m/s. The plate is 0.35 m long and 1 m wide. The boundary layer on each surface of the plate is laminar. Assume that the velocity profile may be approximated as linear. Determine the drag force on the plate.

Given: Water flow over flat plate

Find: Drag on plate for linear boundary layer

Solution:

Basic equations: $F_D = 2 \cdot \int \tau_w \, dA$ $\quad \tau_w = \mu \cdot \frac{du}{dy}$ at y = 0, and also $\tau_w = \rho \cdot U^2 \cdot \frac{d\delta}{dx} \cdot \int_0^1 \frac{u}{U} \cdot \left(1 - \frac{u}{U}\right) d\eta$

$L = 0.35 \cdot m \qquad W = 1 \cdot m \qquad U = 0.8 \cdot \frac{m}{s}$

From Table A.8 at 10ºC $\quad \nu = 1.30 \times 10^{-6} \cdot \frac{m^2}{s} \qquad \rho = 1000 \cdot \frac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \frac{U \cdot L}{\nu} \qquad Re_L = 2.15 \times 10^5$ $\quad$ so laminar

The velocity profile is $\quad u = U \cdot \frac{y}{\delta} = U \cdot \eta$

Hence $\quad \tau_w = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{U}{\delta} \quad (1)$ $\quad$ but we need δ(x)

We also have

$$\tau_w = \rho \cdot U^2 \cdot \frac{d\delta}{dx} \cdot \int_0^1 \frac{u}{U} \cdot \left(1 - \frac{u}{U}\right) d\eta = \rho \cdot U^2 \cdot \frac{d\delta}{dx} \cdot \int_0^1 \eta \cdot (1 - \eta) \, d\eta$$

The integral is $\quad \int_0^1 \left(\eta - \eta^2\right) dx = \frac{1}{6}$ $\quad$ so $\quad \tau_w = \rho \cdot U^2 \cdot \frac{d\delta}{dx} = \frac{1}{6} \cdot \rho \cdot U^2 \cdot \frac{d\delta}{dx} \; (2)$

Comparing Eqs 1 and 2 $\quad \tau_w = \mu \cdot \frac{U}{\delta} = \frac{1}{6} \cdot \rho \cdot U^2 \cdot \frac{d\delta}{dx}$

Separating variables $\quad \delta \cdot d\delta = \frac{6 \cdot \mu}{\rho \cdot U} \cdot dx$ $\quad$ or $\quad \frac{\delta^2}{2} = \frac{6 \cdot \mu}{\rho \cdot U} \cdot x + c$ $\quad$ but δ(0) = 0 so c = 0

Hence $\quad \delta = \sqrt{\frac{12 \cdot \mu}{\rho \cdot U} \cdot x}$ $\quad$ or $\quad \frac{\delta}{x} = \sqrt{\frac{12}{Re_x}} = \frac{3.46}{Re_x}$

Then

$$F_D = 2 \cdot \int \tau_w \, dA = 2 \cdot W \cdot \int_0^L \mu \cdot \frac{U}{\delta} \, dx = 2 \cdot W \cdot \int_0^L \mu \cdot U \cdot \sqrt{\frac{\rho \cdot U}{12 \cdot \mu}} \cdot x^{-\frac{1}{2}} \, dx = \frac{\mu \cdot W \cdot U}{\sqrt{3}} \cdot \sqrt{\frac{U}{\nu}} \cdot \int_0^L x^{-\frac{1}{2}} \, dx$$

The integral is $\quad \int_0^L x^{-\frac{1}{2}} \, dx = 2 \cdot \sqrt{L}$ $\quad$ so $\quad F_D = \frac{2 \cdot \mu \cdot W \cdot U}{\sqrt{3}} \cdot \sqrt{\frac{U \cdot L}{\nu}}$

$F_D = \frac{2}{\sqrt{3}} \cdot \rho \cdot W \cdot \sqrt{\nu \cdot L \cdot U^3} \qquad F_D = 0.557 \, N$

Problem 9.52

[3]

9.52 Standard air flows from the atmosphere into the wide, flat channel shown. Laminar boundary layers form on the top and bottom walls of the channel (ignore boundary-layer effects on the side walls). Assume the boundary layers behave as on a flat plate, with linear velocity profiles. At any axial distance from the inlet, the static pressure is uniform across the channel. Assume uniform flow at section ①. Indicate where the Bernoulli equation can be applied in this flow field. Find the static pressure (gage) and the displacement thickness at section ②. Plot the stagnation pressure (gage) across the channel at section ②, and explain the result. Find the static pressure (gage) at section ① and compare to the static pressure (gage) at section ②.

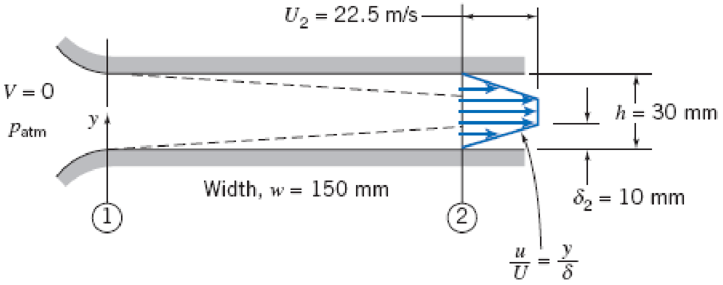
Given: Data on flow in a channel

Find: Static pressures; plot of stagnation pressure

Solution:

The given data is $h = 30\cdot\text{mm}$ $\delta_2 = 10\cdot\text{mm}$ $U_2 = 22.5\cdot\frac{\text{m}}{\text{s}}$ $w = 1\cdot\text{m}$ (Arbitrary)

Appendix A $\rho = 1.23\cdot\frac{\text{kg}}{\text{m}^3}$



Governing equations

Mass $$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Before entering the duct, and in the the inviscid core, the Bernoulli equation holds

$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad (4.24)$$

Assumptions: (1) Steady flow (2) No body force in *x* direction

For a linear velocity profile, from Table 9.2 the displacement thickness at location 2 is

$$\delta_{disp2} = \frac{\delta_2}{2} \qquad \delta_{disp2} = 5\,\text{mm}$$

From the definition of the displacement thickness, to compute the flow rate, the uniform flow at location 2 is assumed to take place in the entire duct, minus the displacement thicknesses at top and bottom

$$A_2 = w\cdot\left(h - 2\cdot\delta_{disp2}\right) \qquad A_2 = 0.02\,\text{m}^2$$

Then $$Q = A_2\cdot U_2 \qquad Q = 0.45\,\frac{\text{m}^3}{\text{s}}$$

Mass conservation (Eq. 4.12) leads to U_2

$$U_1 \cdot A_1 = U_2 \cdot A_2 \qquad \text{where} \qquad A_1 = w \cdot h \qquad A_1 = 0.03\,m^2$$

$$U_1 = \frac{A_2}{A_1} \cdot U_2 \qquad U_1 = 15\,\frac{m}{s}$$

The Bernoull equation applied between atmosphere and location 1 is

$$\frac{p_{atm}}{\rho} = \frac{p_1}{\rho} + \frac{U_1^{\,2}}{2}$$

or, working in gage pressures

$$p_1 = -\frac{1}{2} \cdot \rho \cdot U_1^{\,2} \qquad p_1 = -138\,Pa$$

(Static pressure)

Similarly, between atmosphere and location 2 (gage pressures)

$$p_2 = -\frac{1}{2} \cdot \rho \cdot U_2^{\,2} \qquad p_2 = -311\,Pa$$

(Static pressure)

The static pressure falls continuously in the entrance region as the fluid in the central core accelerates into a decreasing core

The stagnation pressure at location 2 (measured, e.g., with a Pitot tube as in Eq. 6.12), is indicated by an application of the Bernoulli equation at a point

$$\frac{p_t}{\rho} = \frac{p}{\rho} + \frac{u^2}{2}$$

where p_t is the total or stagnation pressure, $p = p_2$ is the static pressure, and u is the local velocity, given by

$$\frac{u}{U_2} = \frac{y}{\delta_2} \qquad y \le \delta_2$$

$$u = U_2 \qquad \delta_2 < y \le \frac{h}{2}$$

(Flow and pressure distibutions are symmetric about centerline)

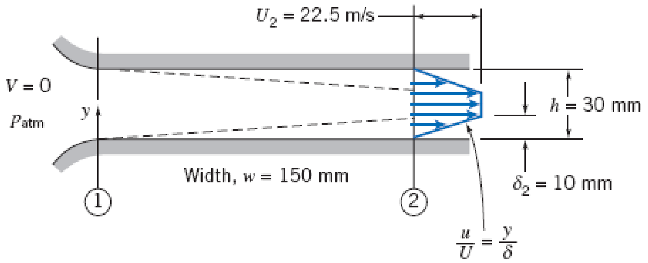
Hence $$p_t = p_2 + \frac{1}{2} \cdot \rho \cdot u^2$$

The plot of stagnation pressure is shown in the associated *Excel* workbook

Problem 9.52 (In Excel)

[3]

9.52 Standard air flows from the atmosphere into the wide, flat channel shown. Laminar boundary layers form on the top and bottom walls of the channel (ignore boundary-layer effects on the side walls). Assume the boundary layers behave as on a flat plate, with linear velocity profiles. At any axial distance from the inlet, the static pressure is uniform across the channel. Assume uniform flow at section ①. Indicate where the Bernoulli equation can be applied in this flow field. Find the static pressure (gage) and the displacement thickness at section ②. Plot the stagnation pressure (gage) across the channel at section ②, and explain the result. Find the static pressure (gage) at section ① and compare to the static pressure (gage) at section ②.



Given: Data on flow in a channel

Find: Static pressures; plot of stagnation pressure

Solution:

Given data:

$h =$ 30 mm
$U_2 =$ 22.5 m/s
$\delta_2 =$ 10 mm
$\rho =$ 1.23 kg/m^3
$p_2 =$ -311 Pa

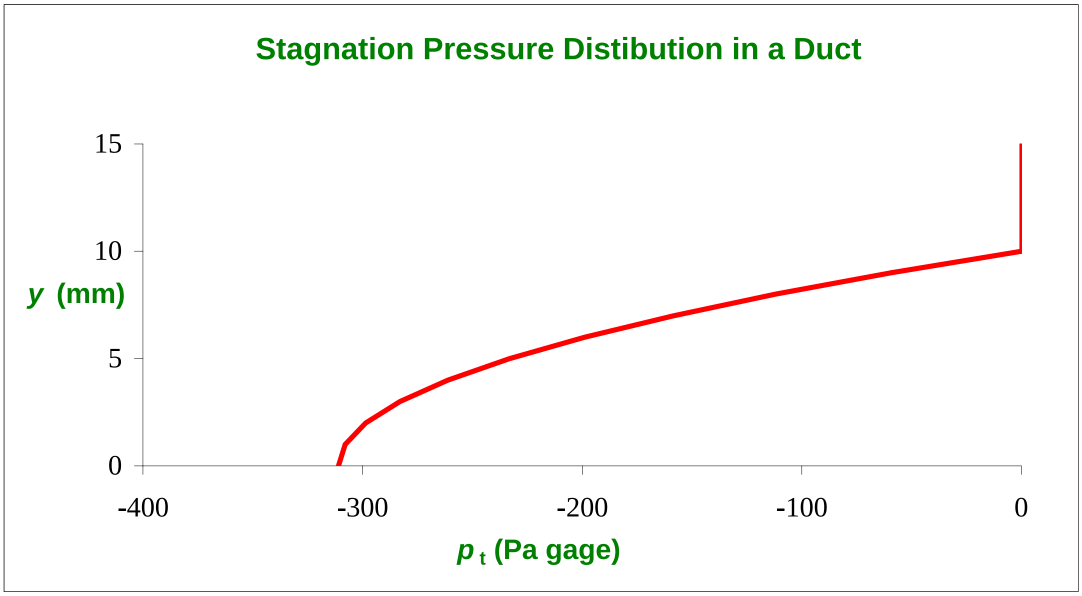
The relevant equations are:

$$\frac{u}{U_2} = \frac{y}{\delta_2} \qquad y \le \delta_2$$

$$u = U_2 \qquad \delta_2 < y \le \frac{h}{2}$$

$$p_t = p_2 + \frac{1}{2}\cdot\rho\cdot u^2$$

y (mm)	u (m/s)	p_t (Pa)
0.0	0.00	-311.00
1.0	2.25	-307.89
2.0	4.50	-298.55
3.0	6.75	-282.98
4.0	9.00	-261.19
5.0	11.25	-233.16
6.0	13.50	-198.92
7.0	15.75	-158.44
8.0	18.00	-111.74
9.0	20.25	-58.81
10.0	22.50	0.34
11.0	22.50	0.34
12.0	22.50	0.34
13.0	22.50	0.34
14.0	22.50	0.34
15.0	22.50	0.34



The stagnation pressure indicates total mechanical energy - the curve indicates significant loss close to the walls and no loss of energy in the central core.

Problem 9.53

[3]

9.53 Consider flow of air over a flat plate of length 5 m. On one graph, plot the boundary-layer thickness as a function of distance along the plate for free-stream speed $U = 10$ m/s assuming (a) a completely laminar boundary layer, (b) a completely turbulent boundary layer, and (c) a laminar boundary layer that becomes turbulent at $Re_x = 5 \times 10^5$. Use *Excel's Goal Seek* or *Solver* to find the speeds U for which transition occurs at the trailing edge, and at x = 4 m, 3 m, 2 m, and 1 m.

Given: Data on flow over a flat plate

Find: Plot of laminar and turbulent boundary layer; Speeds for transition at trailing edge

Solution:

For laminar flow

$$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \quad (9.21) \quad \text{and} \quad Re_x = \frac{U \cdot x}{\nu} \quad \text{so} \quad \delta = 5.48 \cdot \sqrt{\frac{\nu \cdot x}{U}} \quad (1)$$

The critical Reynolds number is $Re_{crit} = 500000$

Hence, for velocity U the critical length x_{crit} is

$$x_{crit} = 500000 \cdot \frac{\nu}{U} \quad (2)$$

For turbulent flow

$$\frac{\delta}{x} = \frac{0.382}{Re_x^{\frac{1}{5}}} \quad (9.26) \quad \text{so} \quad \delta = 0.382 \cdot \left(\frac{\nu}{U}\right)^{\frac{1}{5}} \cdot x^{\frac{4}{5}} \quad (3)$$

For (a) completely laminar flow Eq. 1 holds; for (b) completely turbulent flow Eq. 3 holds; for (c) transitional flow Eq.1 or 3 holds depending on x_{crit} in Eq. 2

Given data:

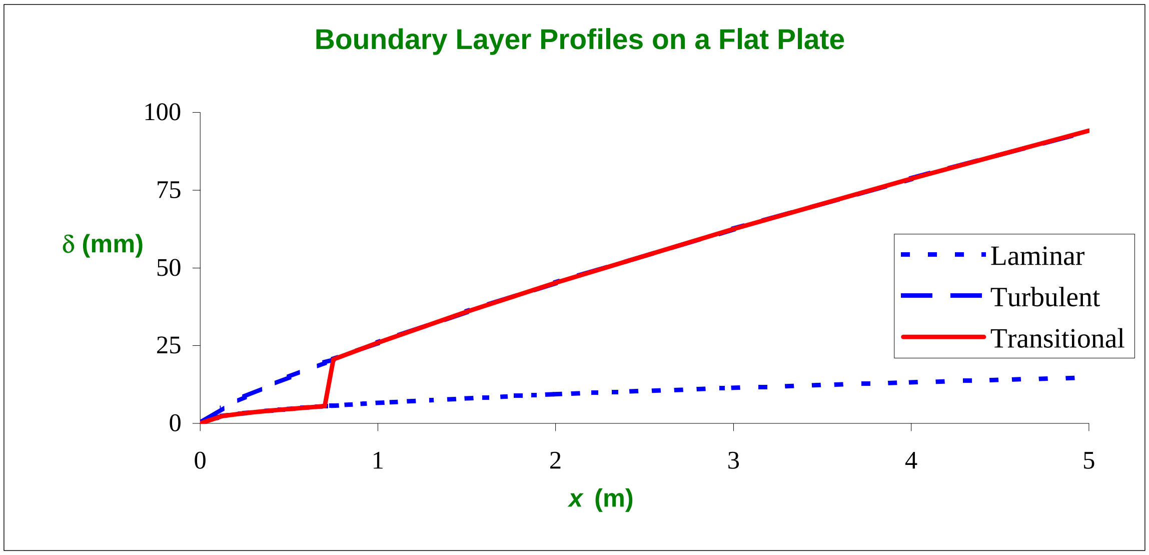
U =	10	m/s
L =	5	m

Tabulated data:

ν = 1.45E-05 m^2/s
(Table A.10)

Computed results:

x (m)	Re_x	(a) Laminar δ (mm)	(b) Turbulent δ (mm)	(c) Transition δ (mm)
0.00	0.00E+00	0.00	0.00	0.00
0.125	8.62E+04	2.33	4.92	2.33
0.250	1.72E+05	3.30	8.56	3.30
0.375	2.59E+05	4.04	11.8	4.04
0.500	3.45E+05	4.67	14.9	4.67
0.700	4.83E+05	5.52	19.5	5.5
0.75	5.17E+05	5.71	20.6	20.6
1.00	6.90E+05	6.60	26.0	26.0
1.50	1.03E+06	8.08	35.9	35.9
2.00	1.38E+06	9.3	45.2	45.2
3.00	2.07E+06	11.4	62.5	62.5
4.00	2.76E+06	13.2	78.7	78.7
5.00	3.45E+06	14.8	94.1	94.1



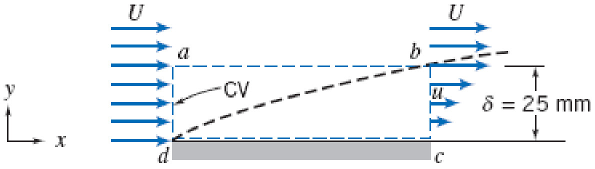
The speeds *U* at which transition occurs at specific points are shown below

x_{trans} (m)	*U* (m/s)
5	1.45
4	1.81
3	2.42
2	3.63
1	7.25

Problem 9.54

[3]

*9.54 A developing boundary layer of standard air on a flat plate is shown in Fig. P9.18. The free-stream flow outside the boundary layer is undisturbed with $U = 165$ ft/s. The plate is 10 ft wide perpendicular to the diagram. Assume flow in the boundary layer is turbulent, with a $\frac{1}{7}$-power velocity profile, and that $\delta = 0.75$ in. at surface *bc*. Calculate the mass flow rate across surface *ad* and the mass flux across surface *ab*. Evaluate the *x* momentum flux across surface *bc*. Determine the drag force exerted on the flat plate between *d* and *c*. Estimate the distance from the leading edge at which transition from laminar to turbulent flow may be expected.



Note: Figure data applies to problem 9.18 only

Given: Data on fluid and turbulent boundary layer

Find: Mass flow rate across *ab*; Momentum flux across *bc*; Distance at which turbulence occurs

Solution:

Basic equations: Mass $$\frac{\partial}{\partial t}\int_{CV}\rho\, d\rlap{/}V + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$$

Momentum $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\rlap{/}V + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) No pressure force 3) No body force in *x* direction 4) Uniform flow at *ab*

The given or available data (Table A.9) is

$U = 165\cdot\frac{ft}{s}$ $\quad\delta = 0.75\cdot in$ $\quad b = 10\cdot ft$ $\quad\rho = 0.00234\cdot\frac{slug}{ft^3}$ $\quad\nu = 1.62\times 10^{-4}\cdot\frac{ft^2}{s}$

Consider CV *abcd* $\quad m_{ad} = -\rho\cdot U\cdot b\cdot\delta$ $\quad m_{ad} = -0.241\,\frac{slug}{s}$ $\quad$ (Note: Software cannot render a dot)

Mass $\quad m_{ad} + \int_0^{\delta}\rho\cdot u\cdot b\,dy + m_{ab} = 0$ $\quad$ and in the boundary layer $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} = \eta^{\frac{1}{7}}$ $\quad dy = d\eta\cdot\delta$

Hence $\quad m_{.ab} = \rho\cdot U\cdot b\cdot\delta - \int_0^1 \rho\cdot U\cdot\eta^{\frac{1}{7}}\cdot\delta\,d\eta = \rho\cdot U\cdot b\cdot\delta - \frac{7}{8}\cdot\rho\cdot U\cdot b\cdot\delta$ $\quad m_{ab} = \frac{1}{8}\cdot\rho\cdot U\cdot b\cdot\delta$ $\quad m_{ab} = 0.0302\,\frac{slug}{s}$

The momentum flux across *bc* is $\quad mf_{bc} = \int_0^{\delta} u\cdot\rho\cdot\vec{V}\,d\vec{A} = \int_0^{\delta} u\cdot\rho\cdot u\cdot b\,dy = \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^{\frac{2}{7}}\,d\eta = \rho\cdot U^2\cdot b\cdot\delta\cdot\frac{7}{9}$

$mf_{bc} = \frac{7}{9}\cdot\rho\cdot U^2\cdot b\cdot\delta$ $\quad mf_{bc} = 31\,\frac{slug\cdot ft}{s^2}$

From momentum $\quad -R_x = U\cdot(-\rho\cdot U\cdot\delta) + m_{ab}\cdot u_{ab} + mf_{bc}$ $\quad R_x = \rho\cdot U^2\cdot b\cdot\delta - m_{ab}\cdot U - mf_{bc}$ $\quad R_x = 3.87\,lbf$

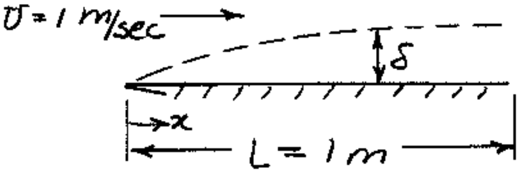
Transition occurs at $\quad Re_x = 5\times 10^5$ $\quad$ and $\quad Re_x = \frac{U\cdot x}{\nu}$ $\quad x_{trans} = \frac{Re_x\cdot\nu}{U}$ $\quad x_{trans} = 0.491\,ft$

Problem 9.55

[3]

Given: Turbulent boundary-layer flow of water, $\frac{1}{7}$-power profile.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$



{ Conditions of Example Problem 9.4. }

Find: (a) Expression for wall shear stress, τ_w.
(b) Integrate to obtain expression for drag force, F_D.
(c) Evaluate for the conditions shown.

Solution: Apply results from the momentum integral equation.

Computing equation: $$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{(Re_x)^{1/5}} \qquad (9.27)$$

Solving for τ_w,

$$\tau_w = \frac{1}{2}\rho U^2 \frac{0.0594}{(Re_x)^{1/5}} = 0.0594\left(\frac{1}{2}\rho U^2\right)\left(\frac{U}{\nu}\right)^{-1/5} x^{-1/5}$$

$\tau_w(x$

Integrating to obtain F_D,

$$F_D = \int_0^L \tau_w\, b\, dx = \int_0^L \underbrace{0.0594\left(\frac{1}{2}\rho U^2\right)\left(\frac{U}{\nu}\right)^{-1/5}}_{C} x^{-1/5}\, b\, dx = Cb\int_0^L x^{-1/5} dx$$

$$= Cb\,\frac{5}{4}x^{4/5}\Big]_0^L = \frac{5}{4}CbL^{4/5} = \frac{5}{4}CbL\,L^{-1/5}$$

$$F_D = \frac{5}{4}(0.0594)\,\frac{1}{2}\rho U^2 bL\left(\frac{U}{\nu}\right)^{-1/5}L^{-1/5} = \frac{1}{2}\rho U^2 bL\,\frac{0.0721}{(Re_L)^{1/5}}$$

F_D

Evaluating, with $b = 1$ m,

$$Re_L = \frac{UL}{\nu} = 1\,\frac{m}{s} \times 1\,m \times \frac{s}{1\times10^{-6}\,m^2} = 1.00\times10^6 \quad (T = 20^\circ C,\ \text{Table A.8})$$

$$F_D = \frac{1}{2}\times 999\,\frac{kg}{m^3}\times(1)^2\frac{m^2}{s^2}\times 1\,m\times 1\,m\times\frac{0.0743}{(10^6)^{1/5}}\times\frac{N\cdot s^2}{kg\cdot m} = 2.34\ N$$

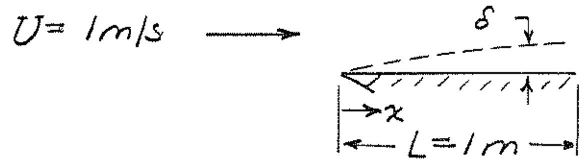
F_D

Problem 9.56

[3]

Given: Turbulent boundary-layer flow of water, conditions of Example Problem 9.4.

Assume $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$



Find: Plot δ, δ^*, and τ_w versus distance.

Solution: Apply the results of Example Problem 9.4.

Computing equations: $\frac{\delta}{x} = \frac{0.382}{(Re_x)^{1/5}}$; $\frac{\delta^*}{\delta} = \frac{1}{8}$; $C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{(Re_x)^{1/5}}$

Assume: (1) BL turbulent from $x = 0$ (i.e. tripped)

For conditions given: $Re_L = \frac{UL}{\nu} = \frac{1\,m}{s} \times 1\,m \times \frac{s}{1\times10^{-6}\,m^2} = 10^6$

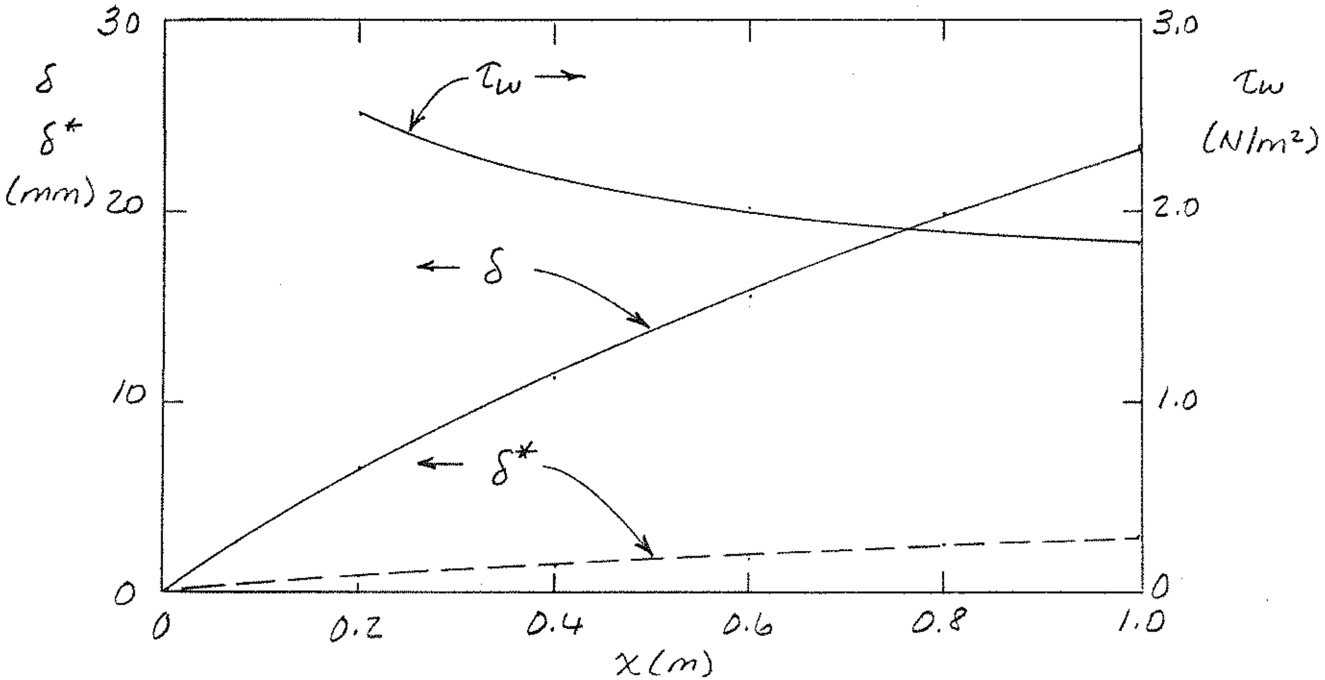
$$q = \tfrac{1}{2}\rho U^2 = \tfrac{1}{2} \times 999\,\frac{kg}{m^3} \times (1)^2\,\frac{m^2}{s^2} \times \frac{N\cdot s^2}{kg\cdot m} = 500\,N/m^2$$

$$\tau_w = 0.0594 \times 500\,\frac{N}{m^2} \times \frac{1}{(Re_x)^{1/5}} = \frac{29.7}{(Re_x)^{1/5}}$$

Tabulate:

x (m)	Re_x (---)	$Re_x^{1/5}$ (---)	δ (mm)	δ^* (mm)	τ_w (N/m²)
0.2	2×10^5	11.5	6.64	0.830	2.58
0.4	4×10^5	13.2	11.6	1.45	2.25
0.6	6×10^5	14.3	16.0	2.00	2.08
0.8	8×10^5	15.2	20.1	2.51	1.95
1.0	10×10^5	15.8	24.2	3.03	1.88

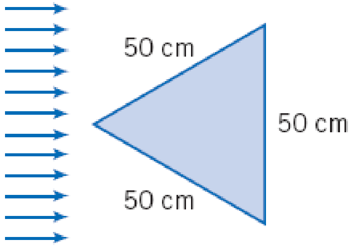
Plot:



Problem 9.57

[3]

9.57 Repeat Problem 9.42, except that the air flow is now at 25 m/s (assume turbulent boundary-layer flow).



Given: Triangular plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

$L = 0.50\cdot cm\cdot\dfrac{\sqrt{3}}{2}$ $\quad L = 0.433\cdot cm$ $\quad W = 50\cdot cm$ $\quad U = 25\cdot\dfrac{m}{s}$

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 7217$ $\quad$ so definitely still laminar, but we are told to assume turbulent!

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\, dA$ $\quad F_D = \displaystyle\int_0^L \tau_w\cdot w(x)\, dx$ $\quad w(x) = W\cdot\dfrac{x}{L}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

Hence

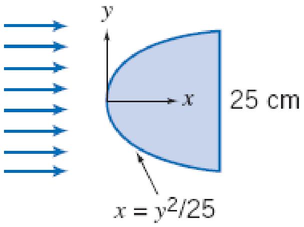
$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot\frac{W}{L}\cdot\int_0^L \frac{0.0594\cdot x}{\left(\frac{U\cdot x}{\nu}\right)^{\frac{1}{5}}}\, dx = \frac{0.0594}{2}\cdot\rho\cdot U^{\frac{9}{5}}\cdot\frac{W}{L}\cdot\nu^{\frac{1}{5}}\cdot\int_0^L x^{\frac{4}{5}}\, dx$$

The integral is $\quad \displaystyle\int_0^L x^{\frac{4}{5}}\, dx = \frac{5}{9}\cdot L^{\frac{9}{5}}$ $\quad$ so $\quad F_D = 0.0165\cdot\rho\cdot W\cdot\left(L^4\cdot\nu\cdot U^9\right)^{\frac{1}{5}}$ $\quad F_D = 4.57\times 10^{-3}\,N$

Note: For two-sided solution $\quad 2\cdot F_D = 9.14\times 10^{-3}\,N$

Problem 9.58

9.58 Repeat Problem 9.44, except that the air flow is now at 25 m/s (assume turbulent boundary-layer flow).



Given: Parabolic plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

$W = 25\cdot\text{cm}$ $\quad L = \dfrac{\left(\frac{W}{2}\right)^2}{25\cdot\text{cm}}$ $\quad L = 6.25\cdot\text{cm}$ $\quad U = 25\cdot\dfrac{\text{m}}{\text{s}}$

Note: "y" is the equation of the upper and lower surfaces, so y = W/2 at x = L

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{\text{m}^2}{\text{s}}$ $\quad \rho = 1.21\cdot\dfrac{\text{kg}}{\text{m}^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 1.04\times 10^5$ so still laminar, but we are told to assume turbulent!

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\, dA$ $\quad F_D = \displaystyle\int_0^L \tau_w\cdot w(x)\, dx$ $\quad w(x) = W\cdot\sqrt{\dfrac{x}{L}}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

Hence

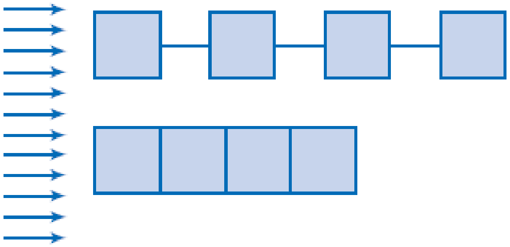
$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.0594\cdot\sqrt{\frac{x}{L}}}{\left(\frac{U\cdot x}{\nu}\right)^{\frac{1}{5}}}\, dx = \frac{0.0594}{2}\cdot\rho\cdot U^{\frac{9}{5}}\cdot W\cdot L^{-\frac{1}{2}}\cdot\nu^{\frac{1}{5}}\cdot\int_0^L x^{\frac{3}{10}}\, dx$$

$$F_D = 0.0228\cdot\rho\cdot W\cdot\left(\nu\cdot L^4\cdot U^9\right)^{\frac{1}{5}}$$ $\quad F_D = 0.0267\,\text{N}$

Note: For two-sided solution $\quad 2\cdot F_D = 0.0534\,\text{N}$

Problem 9.59

9.59 Repeat Problem 9.46, except that the water flow is now at 10 m/s (assume turbulent boundary-layer flow).



Given: Pattern of flat plates

Find: Drag on separate and composite plates

Solution:

Basic equations: $c_f = \frac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \frac{0.0594}{Re_x^{\frac{1}{5}}}$

For separate plates $L = 7.5\cdot cm$ $\qquad W = 7.5\cdot cm$ $\qquad U = 10\cdot\frac{m}{s}$

From Table A.8 at 20ºC $\nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\qquad \rho = 998\cdot\frac{kg}{m^3}$

First determine the nature of the boundary layer $Re_L = \frac{U\cdot L}{\nu}$ $\qquad Re_L = 7.43\times 10^5$ so turbulent

The drag (one side) is $F_D = \int \tau_w\, dA$ $\qquad F_D = \int_0^L \tau_w\cdot W\, dx$

We also have $\tau_w = c_f\cdot\frac{1}{2}\cdot\rho\cdot U^2 = \frac{1}{2}\cdot\rho\cdot U^2\cdot\frac{0.0594}{Re_x^{\frac{1}{5}}}$

Hence
$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.0594}{\left(\frac{U\cdot x}{\nu}\right)^{\frac{1}{5}}}\, dx = \frac{0.0594}{2}\cdot\rho\cdot U^{\frac{9}{5}}\cdot W\cdot\nu^{\frac{1}{5}}\cdot\int_0^L x^{-\frac{1}{5}}\, dx$$

The integral is $\int_0^L x^{-\frac{1}{5}}\, dx = \frac{5}{4}\cdot L^{\frac{4}{5}}$ so $F_D = 0.371\cdot\rho\cdot W\cdot\left(\nu\cdot L^4\cdot U^9\right)^{\frac{1}{5}}$ $\qquad F_D = 13.9\,N$

This is the drag on one plate. The total drag is then $F_{Total} = 4\cdot F_D$ $\qquad F_{Total} = 55.8\,N$

For both sides: $2\cdot F_{Total} = 112\,N$

For the composite plate $L = 4 \times 7.5 \cdot \mathrm{cm}$ $L = 0.30\,\mathrm{m}$

$$F_{Composite} = 0.371 \cdot \rho \cdot W \cdot \left(\nu \cdot L^{4} \cdot U^{9}\right)^{\frac{1}{5}} \qquad F_{Composite} = 42.3\,\mathrm{N}$$

For both sides: $2 \cdot F_{Composite} = 84.6\,\mathrm{N}$

The drag is much lower on the composite compared to the separate plates. This is because τ_w is largest near the leading edges and falls off rapidly; in this problem the separate plates experience leading edges four times!

Problem 9.60

[3]

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Given: Turbulent boundary layer with velocity profile, $\frac{u}{U} = \eta^{1/6}$; $\eta = \frac{y}{\delta}$.

Find: Expressions for δ/x, C_f, using momentum integral equation. Compare with results of "$1/7$-power" profile, Section 9-5.2.

Solution: The momentum integral equation is

Computing equation:
$$-\delta \overset{=0(1)}{\frac{\partial p}{\partial x}} - \tau_w = \frac{\partial}{\partial x}\int_0^\delta u\rho u\,dy - U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy$$

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$

(2) δ is a function of x only; $\delta = 0$ at $x = 0$

(3) Incompressible flow

(4) $\tau_w = 0.0233\,\rho U^2 (\nu/U\delta)^{1/4}$

Then
$$\tau_w = U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy - \int_0^\delta u\rho u\,dy = \frac{\partial}{\partial x}\int_0^\delta \rho u(U-u)\,dy$$

or
$$\tau_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \rho U^2 \frac{d\delta}{dx}\beta$$

Evaluating β,
$$\beta = \int_0^1 \eta^{1/6}(1-\eta^{1/6})\,d\eta = \left[\frac{6}{7}\eta^{7/6} - \frac{6}{8}\eta^{8/6}\right]_0^1 = \frac{6}{56}$$

Substituting
$$0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} = \rho U^2\frac{d\delta}{dx}\beta \quad \text{or} \quad \delta^{1/4}d\delta = \frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}dx$$

Integrating
$$\frac{4}{5}\delta^{5/4} = \frac{0.0225}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x + c\text{, but } c = 0\text{, since } \delta = 0 \text{ at } x = 0.$$

Thus
$$\delta = \left[\frac{5}{4}\,\frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x\right]^{4/5} = 0.353\left(\frac{\nu}{U}\right)^{\frac{1}{5}}x^{\frac{4}{5}}$$

and
$$\frac{\delta}{x} = 0.353\left(\frac{\nu}{Ux}\right)^{\frac{1}{5}} = \frac{0.353}{(Re_x)^{1/5}}$$

$\frac{\delta}{x}$

Also
$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}}{\frac{1}{2}\rho U^2} = 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{x}{\delta}\right)^{\frac{1}{4}}$$

$$= 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{1}{0.353}\right)^{\frac{1}{4}}\left[\left(\frac{Ux}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{1}{4}}$$

$$C_f = \frac{0.0605}{(Re_x)^{1/5}}$$

C_f

Comparing:

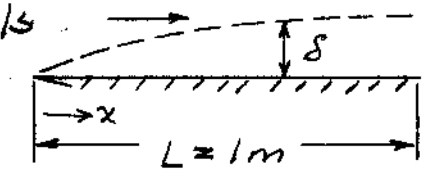
	$\delta/x\,(Re_x)^{1/5}$	$C_f\,(Re_x)^{1/5}$
$1/6$-power	0.353	0.0605
$1/7$-power	0.382	0.0594

Problem 9.61

[3]

Given: Turbulent boundary-layer flow of water, $\frac{1}{6}$-power profile.

$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/6}$ $\quad U = 1\ \text{m/s}$



{Flow conditions of Example Problem 9.4.}

Find: (a) Expression for the ratio, δ/x

(b) Expression for the skin friction coefficient, C_f.

(c) Evaluate F_D for the conditions shown.

Solution: Apply the momentum integral equation.

Computing equation: $\tau_w = \rho U^2 \frac{d\theta}{dx} = \rho U^2 \frac{d\delta}{dx}\beta$ (9.19)

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$

(2) $\delta = \delta(x)$ only; $\delta = 0$ at $x = 0$

(3) Incompressible flow

(4) Wall shear correlation, $\tau_w = 0.0233\,\rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}$ (9.25)

Now $\beta = \frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \lambda^{1/6}(1-\lambda^{1/6})\,d\lambda = \int_0^1 (\lambda^{1/6} - \lambda^{2/6})\,d\lambda \ ; \ \lambda = \frac{y}{\delta}$

$$\beta = \frac{6}{7}\lambda^{7/6} - \frac{6}{8}\lambda^{8/6}\Big]_0^1 = \frac{6(8)-6(7)}{7(8)} = \frac{6}{56} = 0.107$$

Substituting into the MIE,

$$0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4} = \rho U^2 \frac{d\delta}{dx}(0.107) \quad \text{or} \quad \delta^{1/4} d\delta = 0.218\left(\frac{\nu}{U}\right)^{1/4} dx$$

Integrating, $\frac{4}{5}\delta^{5/4} = 0.218\left(\frac{\nu}{U}\right)^{1/4} x + c$, but $c = 0$, since $\delta(0) = 0$.

Thus $\delta = \left[0.218\left(\frac{5}{4}\right)\left(\frac{\nu}{U}\right)^{1/4} x\right]^{4/5} = 0.353\left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$

and $\frac{\delta}{x} = 0.353\left(\frac{\nu}{U}\right)^{1/5} x^{-1/5} = \frac{0.353}{(Re_x)^{1/5}}$ $\quad \delta/x$

From the wall shear correlation,

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0233\rho U^2}{\frac{1}{2}\rho U^2}\left(\frac{\nu}{U\delta}\right)^{1/4} = 0.0466\left(\frac{\nu}{U\delta}\right)^{1/4} = 0.0466\left(\frac{\nu}{Ux}\right)^{1/4}\left(\frac{x}{\delta}\right)^{1/4}$$

$C_f = 0.0466\left(\frac{\nu}{Ux}\right)^{1/4}\left[\frac{(Ux/\nu)^{1/5}}{0.353}\right]^{1/4} = \frac{0.0612}{(Re_x)^{1/5}}$ $\quad C_f$

The drag force is $F_D = \int_0^L \tau_w\, b\,dx = \int_0^L 0.0612\ \frac{1}{2}\rho U^2\left(\frac{\nu}{U}\right)^{1/5} x^{-1/5}\, b\,dx$

$$F_D = 0.0612\ \frac{1}{2}\rho U^2\left(\frac{\nu}{U}\right)^{1/5} b\int_0^L x^{-1/5}dx = 0.0765\ \frac{1}{2}\rho U^2 bL\left(\frac{\nu}{UL}\right)^{1/5}$$

$F_D = \frac{0.0765}{2} \times 999\ \frac{\text{kg}}{\text{m}^3} (1)^2\frac{\text{m}^2}{\text{s}^2} \times 1\,\text{m} \times 1\,\text{m} \times \left(\frac{1}{10^6}\right)^{1/5} \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 2.41\ \text{N}$ $\quad F_D$

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 9.62

[3]

Given: Turbulent boundary layer with velocity profile, $\frac{u}{U} = \eta^{1/8}$; $\eta = \frac{y}{\delta}$.

Find: Expressions for δ/x, C_f, using momentum integral equation.
Compare with results of "1/7-power" profile, section 9-5.2.

Solution: The momentum integral equation is

Computing equation: $$-\delta \frac{\partial p}{\partial x} - \tau_w = \frac{\partial}{\partial x}\int_0^\delta u\rho u\,dy - U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy$$

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$
(2) δ is a function of x only; $\delta = 0$ at $x = 0$
(3) Incompressible flow
(4) $\tau_w = 0.0233\,\rho U^2 (\nu/U\delta)^{1/4}$

Then
$$\tau_w = U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy - \int_0^\delta u\rho u\,dy = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta = \rho U^2 \frac{d\delta}{dx}\beta$$

Evaluating β,
$$\beta = \int_0^1 \eta^{1/8}(1-\eta^{1/8})\,d\eta = \left[\frac{8}{9}\eta^{9/8} - \frac{8}{10}\eta^{10/8}\right]_0^1 = \frac{8}{90}$$

Substituting,
$$0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} = \rho U^2\frac{d\delta}{dx}\beta \quad \text{or} \quad \delta^{1/4}d\delta = \frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}dx$$

Integrating $$\frac{4}{5}\delta^{5/4} = \frac{0.0225}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x + c, \text{ but } c=0, \text{ since } \delta=0 \text{ at } x=0.$$

Thus $$\delta = \left[\frac{5}{4}\,\frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x\right]^{4/5} = 0.410\left(\frac{\nu}{U}\right)^{\frac{1}{5}}x^{4/5}$$

and $$\frac{\delta}{x} = 0.410\left(\frac{\nu}{Ux}\right)^{1/5} = \frac{0.410}{(Re_x)^{1/5}}$$

$\frac{\delta}{x}$

Also
$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}}{\frac{1}{2}\rho U^2} = 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{x}{\delta}\right)^{\frac{1}{4}}$$

$$C_f = 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{1}{0.410}\right)^{\frac{1}{4}}\left[\left(\frac{Ux}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{1}{4}} = \frac{0.0582}{(Re_x)^{1/5}}$$

C_f

Comparing:

Profile	$\frac{\delta}{x}(Re_x)^{1/5}$	$C_f\,(Re_x)^{1/5}$
1/8-power	0.410	0.0582
1/7-power	0.382	0.0594

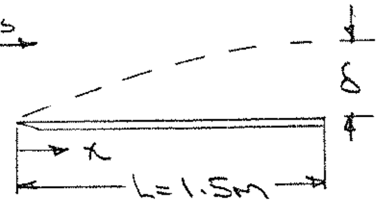
Problem 9.63

[3]

Given: Air flow over smooth flat plate as shown; width $b = 0.8\,m$.

BL tripped, so turbulent $U = 20\ m/s$

Velocity profile is 1/7-power.



Find: (a) δ at $x=L$, (b) τ_w at $x=L$

(c) Drag on portion $0.5\,m < x < L$.

Solution:

Computing equations: $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$ $c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}}$

Assumptions: (1) steady flow, (2) incompressible flow
(3) zero pressure gradient
(4) standard air ($T = 15°C$)

$$Re_L = \frac{UL}{\nu} = 20\,\frac{m}{s} \times 1.5\,m \times 1.46\times10^{-5}\,\frac{s}{m^2} = 2.06\times10^6$$

$$\delta_L = \frac{0.382\,L}{Re_x^{1/5}} = 0.382 \times 1.5\,m \times \frac{1}{(2.06\times10^6)^{0.2}} = 31.3\ mm \longleftarrow \delta_L$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}} \qquad \therefore\ \tau_w = \frac{1}{2}\rho U^2\,\frac{0.0594}{Re_x^{1/5}}$$

$$\tau_{w_L} = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (20)^2\,\frac{m^2}{s^2} \times \frac{0.0594}{(2.06\times10^6)^{0.2}} \times \frac{N\cdot s^2}{kg\cdot m} = 0.798\ N/m^2 \longleftarrow \tau_{w_L}$$

The drag force is given by:

$$F_D = \int_{x_1}^{L} \tau_w b\,dx = \frac{1}{2}\rho U^2 b \int_{x_1}^{L} \frac{0.0594}{Re_x^{1/5}}\,dx = \frac{1}{2}\rho U^2 b\,\frac{0.0594}{(U/\nu)^{1/5}} \int_{x_1}^{L} \frac{dx}{x^{1/5}}$$

$$= \frac{1}{2}\rho U^2 b\,\frac{0.0594}{(U/\nu)^{1/5}} \left[\frac{5}{4}x^{4/5}\right]_{x_1}^{L} = \frac{1}{2}\rho U^2 b\,\frac{0.0594}{(U/\nu)^{1/5}}\,\frac{5}{4}\left[\frac{x}{x^{1/5}}\right]_{x_1}^{L}$$

$$F_D = \frac{1}{2}\rho U^2 b\,\frac{5}{4}\left[L\,c_{f_L} - x_1 c_{f_{x_1}}\right]$$

For $x_1 = 0.5\,m$, $Re_{x_1} = 6.85\times10^5$, and $c_{f_{x_1}} = 0.00404$

For $x = L = 1.5\,m$, $Re_L = 2.06\times10^6$, and $c_{f_L} = 0.00324$

Substituting.

$$F_D = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (20)^2\,\frac{m^2}{s^2} \times 0.8\,m \times \frac{5}{4} \times \frac{N\cdot s^2}{kg\cdot m}\left[1.5\,m \times 0.00324 - 0.5\,m \times 0.00404\right]$$

$$F_D = 0.700\ N \longleftarrow F_D$$

Alternate solution: $F_D = \rho U^2 b(\theta_L - \theta_{x_1})$

For 1/7-power, $\theta = \frac{7}{72}\delta$. $\theta_L = 3.04\ mm$; $\delta_{x_1} = 13\,mm$, $\theta_{x_1} = 1.26\ mm$

$$F_D = 1.23\,\frac{kg}{m^3} \times (20)^2\,\frac{m^2}{s^2} \times 0.8\,m \times \frac{N\cdot s^2}{kg\cdot m}\,(0.00304 - 0.00126)\,m = 0.700\,N$$

Problem 9.64

[3]

Given: Air at standard conditions flows at 10 m/s over a flat plate.

Find: δ and τ_w at a point 1 m from leading edge for
(a) completely laminar flow (parabolic velocity profile)
(b) completely turbulent flow (1/7-power velocity profile)

Solution

	Laminar Flow	Turbulent Flow
Computing equations:	$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$	$\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$
	$C_f = \frac{0.730}{\sqrt{Re_x}}$	$C_f = \frac{0.0594}{Re_x^{1/5}}$

For standard air, $\rho = 1.23\ kg/m^3$, $\nu = 1.46\times10^{-5}\ m^2/s$ (Table A.10)
the Reynolds number is

$$Re_x = \frac{Ux}{\nu} = 10\frac{m}{s}\times 1m \times 1.46\times10^{-5}\frac{s}{m^2} = 6.85\times10^5$$

For laminar flow

$$\delta = \frac{5.48x}{\sqrt{Re_x}} = 5.48\times 1m\times\frac{1}{(6.85\times10^5)^{1/2}} = 6.62\ mm \longleftarrow \delta_{lam}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.730}{\sqrt{Re_x}} \quad \therefore\ \tau_w = \frac{1}{2}\rho U^2\,\frac{0.730}{\sqrt{Re_x}}$$

$$\tau_w = \frac{1}{2}\times 1.23\frac{kg}{m^3}\times(10)^2\frac{m^2}{s^2}\times\frac{0.730}{(6.85\times10^5)^{1/2}}\times\frac{N\cdot s^2}{kg\cdot m} = 0.054\ N/m^2 \longleftarrow \tau_{lam}$$

For turbulent flow

$$\delta = \frac{0.382x}{Re_x^{1/5}} = 0.382\times 1m\times\frac{1}{(6.85\times10^5)^{0.2}} = 26.0\ mm \longleftarrow \delta_{turb}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}} \quad \therefore\ \tau_w = \frac{1}{2}\rho U^2\,\frac{0.0594}{Re_x^{1/5}}$$

$$\tau_w = \frac{1}{2}\times 1.23\frac{kg}{m^3}\times(10)^2\frac{m^2}{s^2}\times\frac{0.0594}{(6.85\times10^5)^{0.2}}\times\frac{N\cdot s^2}{kg\cdot m} = 0.249\ N/m^2 \longleftarrow \tau_{turb}$$

Comparing,

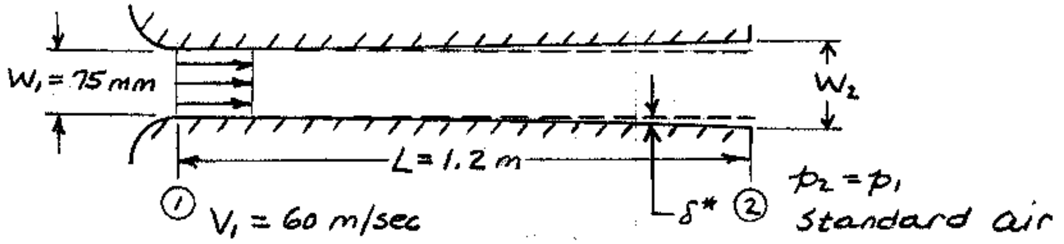
$$\frac{\delta_{turb}}{\delta_{lam}} = 3.93 \quad\text{and}\quad \frac{\tau_{w\,turb}}{\tau_{w,lam}} \approx 4.58$$

Thus the turbulent boundary layer has a much larger skin friction which causes it to grow more rapidly.

Problem 9.65

[3]

Given: Incompressible flow of air through a plane-wall diffuser. Diffuser walls diverge slightly to accommodate the boundary layer development, so there is no pressure gradient. Assume flat plate boundary layer development.



Find: (a) Explain why Bernoulli equation is applicable to this flow.
(b) Exit width, W_2.

Solution: The Bernoulli equation may be applied along a streamline in any steady, incompressible flow in the absence of friction. The given flow is steady and incompressible. Frictional effects are confined to the thin wall boundary layers. Therefore the Bernoulli equation may be applied along any streamline in the core flow outside the boundary layers. (Since there is no streamline curvature, the pressure is uniform across sections ① and ②.)

Basic equations: $$0 = \cancelto{0(1)}{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\cancel{\frac{p_1}{\rho}} + \frac{V_1^2}{2} + \cancelto{(3)}{g z_1} = \cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2} + \cancelto{(3)}{g z_2}$$

Assumptions: (1) Steady flow
(2) Turbulent, "1/7-power" boundary layer from entrance
(3) $z_1 = z_2$
(4) $p_1 = p_2$

Then from Bernoulli, $V_1 = V_2$, and from continuity,

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_{2,eff}|\} \text{ or } A_{2,eff} = (W_2 - 2\delta_2^*)b = W_1 b$$

Use the analysis of Section 9-5.2: $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$ or $\frac{\delta}{L} = \frac{0.382}{Re_L^{1/5}}$

$$Re_L = \frac{\rho V L}{\mu} = \frac{VL}{\nu} = 60\,\frac{m}{s} \times 1.2\,m \times \frac{s}{1.46 \times 10^{-5}\,m^2} = 4.93 \times 10^6; \quad \frac{\delta_2}{L} = 0.0175$$

or $\delta_2 = 0.017\ L = (0.0171)\,1.2\,m = 0.0205\,m$, or 20.5 mm. $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$, so

$$\delta_2^* = \delta_2 \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \delta \int_0^1 \left(1 - \eta^{1/7}\right) d\eta = \delta\left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}\delta = 2.63\,mm$$

Then $W_2 = W_1 + 2\delta_2^* = 75\,mm + 2(2.63\,mm) = 80.3\,mm$

W_2

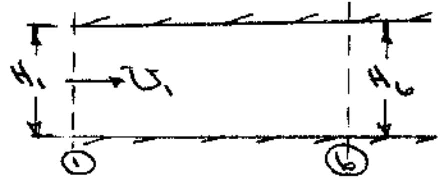
Problem 9.66

[3]

Given: Wind tunnel with flexible upper wall and constant width $W_1 = 305\,mm$. Tunnel height adjusted to give zero pressure gradient. Wall boundary layers represented by 1/7 power profile. At two sections in the tunnel

① $H_1 = 305\,mm$, $\delta_1 = 12.2\,mm$
$U_1 = 26.5\,m/s$

② $\delta_6 = 16.6\,mm$



Find: (a) the height, H_6
(b) equivalent length of flat plate to give $\delta_1 = 12.2\,mm$
(c) estimate the distance between sections ① and ⑥

Solution: To determine the height H_6 use the continuity equation and the concept of δ^*

From continuity, $A_1U_1 = A_6U_6$ where A is the effective flow area. Since $U_1 = U_6$ for zero pressure gradient, $A_1 = A_6$

$$\therefore (W - \delta_1^*)(H_1 - \delta_1^*) = (W - \delta_6^*)(H_6 - \delta_6^*)$$

and $$H_6 = \frac{(W-\delta_1^*)(H_1-\delta_1^*)}{(W-\delta_6^*)} + \delta_6^*$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta\int_0^1\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \delta\int_0^1\left(1-\frac{u}{U}\right)d\eta$$

$$\delta^* = \delta\int_0^1\left(1-\eta^{1/7}\right)d\eta = \delta\left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}$$

Substituting into the expression for H_6

$$H_6 = \frac{(305 - \frac{1}{8}\times 12.2)(305 - \frac{1}{8}\times 12.2)\,mm}{(305 - \frac{1}{8}\times 16.6)} + 16.6\,mm = 321\,mm \longleftarrow H_6$$

For flat plate turbulent boundary with 1/7 power law profile

$$\frac{\delta}{x} = \frac{0.370}{Re_x^{1/5}} \quad (9.26) \qquad \therefore\ \delta = 0.370\left(\frac{\nu}{U}\right)^{1/5}x^{4/5}$$

Then $$x = \left[\frac{\delta}{0.370}\right]^{5/4}\left(\frac{U}{\nu}\right)^{1/4}$$

At section ①, $\delta = 12.2\,mm$

$$x_1 = \left[\frac{0.0122\,m}{0.370}\right]^{1.25}\left(26.5\,\frac{m}{s}\times\frac{s}{1.45\times10^{-5}\,m^2}\right)^{0.25} = 0.517\,m \longleftarrow L_{eq}$$

At section ②, $\delta = 16.6\,mm$

$$x_6 = \left[\frac{0.0166\,m}{0.370}\right]^{1.25}\left(26.5\,\frac{m}{s}\times\frac{s}{1.45\times10^{-5}\,m^2}\right)^{0.25} = 0.759\,m$$

Approximate distance $L_6 = x_6 - x_1$

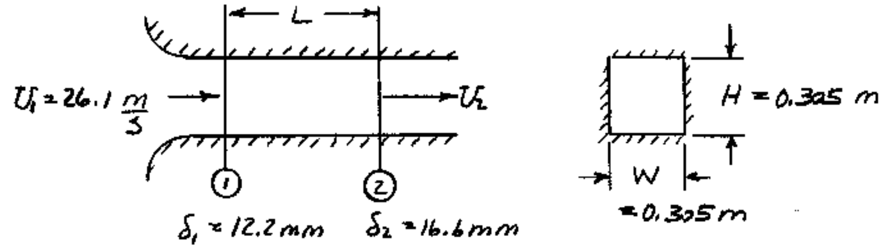
$$= 0.759\,m - 0.517\,m = 0.242\,m \longleftarrow L_6$$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 9.67

[3]

Given: Small laboratory wind tunnels with square test sections.



Boundary layers are turbulent with 1/7-power velocity profiles.

Find: (a) Change in static pressure between ① and ②.
(b) Estimate of distance L.

Solution: For a 1/7-power profile in the turbulent boundary layer, $\delta^* = \delta/8$. Therefore $\delta_1^* = 12.2/8 = 1.53$ mm and $\delta_2^* = 16.6/8 = 2.08$ mm.

Apply conservation of mass: $U_1 A_1 = U_2 A_2$; $A = W - 2\delta^*$

$$U_2 = U_1\left[\frac{305 - 2(1.53)}{305 - 2(2.08)}\right]^2 = 1.00733\,U_1$$

From Bernoulli (for steady, incompressible, inviscid flow along a streamline),

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}\ ;\quad p_1 - p_2 = \frac{\rho}{2}(U_2^2 - U_1^2) = \frac{\rho U_1^2}{2}\left[(1.00733)^2 - 1\right]$$

$$p_1 - p_2 = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (26.1)^2\frac{m^2}{s^2}\left[(1.00733)^2 - 1\right]\frac{N \cdot s^2}{kg \cdot m} = 6.16\ N/m^2$$

$p_1 - p$

For a turbulent boundary layer, $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}} = \frac{0.382}{\left(\frac{Ux}{\nu}\right)^{1/5}} = 0.382\left(\frac{\nu}{U}\right)^{1/5}\frac{1}{x^{1/5}}$

Thus $\delta = 0.382\left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$ or $x = \left(\frac{\delta}{0.382}\right)^{5/4}\left(\frac{U}{\nu}\right)^{1/4}$

For standard air $\nu = 1.46 \times 10^{-5}\ m^2/s$ (Table A.10). Thus

$$x_1 = \left(\frac{0.0122\ m}{0.382}\right)^{5/4}\left(26.1\,\frac{m}{s} \times \frac{s}{1.46 \times 10^{-5}\ m^2}\right)^{1/4} = 0.494\ m$$

$$x_2 = \left(\frac{0.0166\ m}{0.382}\right)^{5/4}\left(26.3\,\frac{m}{s} \times \frac{s}{1.46 \times 10^{-5}\ m^2}\right)^{1/4} = 0.727\ m$$

Then

$$L_{12} = x_2 - x_1 = (0.727 - 0.494)\ m = 0.233\ m \quad \text{or} \quad 233\ mm$$

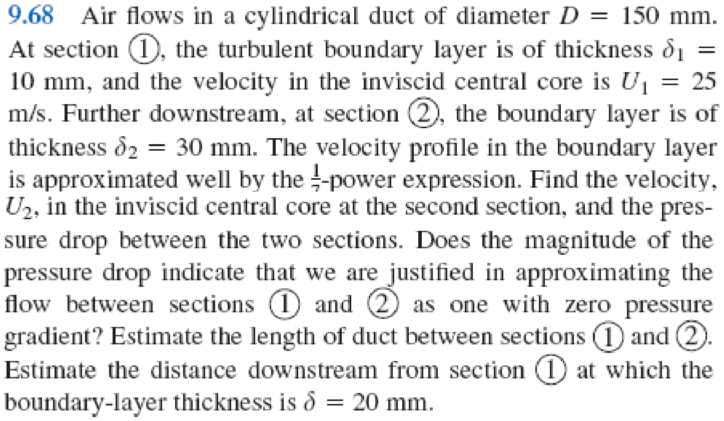
L_{12}

{Note that 6 significant figures were carried in the factor 1.00733 so that 3 would remain in 1.00733 − 1 = 0.733.}

13-782 500 SHEETS, FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
National Brand

Problem 9.68

[3]



9.68 Air flows in a cylindrical duct of diameter $D = 150$ mm. At section ①, the turbulent boundary layer is of thickness $\delta_1 = 10$ mm, and the velocity in the inviscid central core is $U_1 = 25$ m/s. Further downstream, at section ②, the boundary layer is of thickness $\delta_2 = 30$ mm. The velocity profile in the boundary layer is approximated well by the $\frac{1}{7}$-power expression. Find the velocity, U_2, in the inviscid central core at the second section, and the pressure drop between the two sections. Does the magnitude of the pressure drop indicate that we are justified in approximating the flow between sections ① and ② as one with zero pressure gradient? Estimate the length of duct between sections ① and ②. Estimate the distance downstream from section ① at which the boundary-layer thickness is $\delta = 20$ mm.

Given:
Data on flow in a duct

Find:
Velocity at location 2; pressure drop; length of duct; position at which boundary layer is 20 mm

Solution:

The given data is $D = 150 \cdot \text{mm}$ $\delta_1 = 10 \cdot \text{mm}$ $\delta_2 = 30 \cdot \text{mm}$ $U_1 = 25 \cdot \frac{\text{m}}{\text{s}}$

Table A.10 $\rho = 1.23 \cdot \frac{\text{kg}}{\text{m}^3}$ $\nu = 1.45 \times 10^{-5} \cdot \frac{\text{m}^2}{\text{s}}$

Governing equations

Mass $$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad (4.12)$$

In the boundary layer $$\frac{\delta}{x} = \frac{0.382}{Re_x^{\frac{1}{5}}} \qquad (9.26)$$

In the the inviscid core, the Bernoulli equation holds

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad (4.24)$$

Assumptions: (1) Steady flow (2) No body force (gravity) in *x* direction

For a 1/7-power law profile, from Example 9.4 the displacement thickness is $\delta_{disp} = \frac{\delta}{8}$

Hence $\delta_{disp1} = \frac{\delta_1}{8}$ $\delta_{disp1} = 1.25\,\text{mm}$

$\delta_{disp2} = \frac{\delta_2}{8}$ $\delta_{disp2} = 3.75\,\text{mm}$

From the definition of the displacement thickness, to compute the flow rate, the uniform flow at locations 1 and 2 is assumed to take place in the entire duct, minus the displacement thicknesses

$A_1 = \frac{\pi}{4} \cdot \left(D - 2 \cdot \delta_{disp1}\right)^2$ $A_1 = 0.0171\,\text{m}^2$

$$A_2 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp2}\right)^2 \qquad A_2 = 0.0159\,m^2$$

Mass conservation (Eq. 4.12) leads to U_2

$$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \qquad \text{or} \qquad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 26.8\,\frac{m}{s}$$

The Bernoulli equation applied between locations 1 and 2 is

$$\frac{p_1}{\rho} + \frac{U_1^{\,2}}{2} = \frac{p_2}{\rho} + \frac{U_2^{\,2}}{2}$$

or the pressure drop is $p_1 - p_2 = \Delta p = \frac{\rho}{2}\cdot\left(U_2^{\,2} - U_1^{\,2}\right)$ $\qquad \Delta p = 56.9\,Pa$ $\qquad$ (Depends on ρ value selected)

The static pressure falls continuously in the entrance region as the fluid in the central core accelerates into a decreasing core.

If we assume the stagnation pressure is atmospheric, a change in pressure of about 60 Pa is not significant; in addition, the velocity change about 5%, again not a large change to within engineering accuracy

To compute distances corresponding to boundary layer thicknesses, rearrange Eq.9.26

$$\frac{\delta}{x} = \frac{0.382}{Re_x^{\frac{1}{5}}} = 0.382\cdot\left(\frac{\nu}{U\cdot x}\right)^{\frac{1}{5}} \qquad \text{so} \qquad x = \left(\frac{\delta}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U}{\nu}\right)^{\frac{1}{4}}$$

Applying this equation to locations 1 and 2 (using $U = U_1$ or U_2 as approximations)

$$x_1 = \left(\frac{\delta_1}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U_1}{\nu}\right)^{\frac{1}{4}} \qquad x_1 = 0.382\,m$$

$$x_2 = \left(\frac{\delta_2}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U_2}{\nu}\right)^{\frac{1}{4}} \qquad x_2 = 1.533\,m$$

$x_2 - x_1 = 1.15\,m$ $\qquad$ (Depends on ν value selected)

For location 3 $\qquad \delta_3 = 20\cdot mm \qquad \delta_{disp3} = \frac{\delta_3}{8} \qquad \delta_{disp3} = 2.5\,mm$

$$A_3 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp3}\right)^2 \qquad A_3 = 0.017\,m^2$$

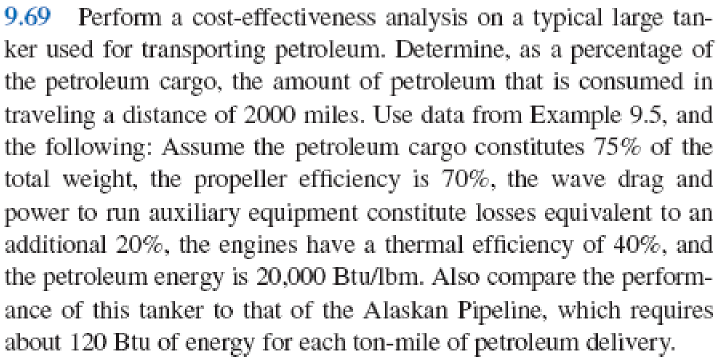
$$U_3 = U_1\cdot\frac{A_1}{A_3} \qquad U_3 = 25.9\,\frac{m}{s}$$

$$x_3 = \left(\frac{\delta_3}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U_2}{\nu}\right)^{\frac{1}{4}} \qquad x_3 = 0.923\,m$$

$x_3 - x_1 = 0.542\,m$ $\qquad$ (Depends on ν value selected)

Problem 9.69

[3]



9.69 Perform a cost-effectiveness analysis on a typical large tanker used for transporting petroleum. Determine, as a percentage of the petroleum cargo, the amount of petroleum that is consumed in traveling a distance of 2000 miles. Use data from Example 9.5, and the following: Assume the petroleum cargo constitutes 75% of the total weight, the propeller efficiency is 70%, the wave drag and power to run auxiliary equipment constitute losses equivalent to an additional 20%, the engines have a thermal efficiency of 40%, and the petroleum energy is 20,000 Btu/lbm. Also compare the performance of this tanker to that of the Alaskan Pipeline, which requires about 120 Btu of energy for each ton-mile of petroleum delivery.

Given: Data on a large tanker

Find: Cost effectiveness of tanker; compare to Alaska pipeline

Solution:

The given data is $L = 360\cdot m$ $B = 70\cdot m$ $D = 25\cdot m$ $\rho = 1020\cdot\frac{kg}{m^3}$ $U = 6.69\cdot\frac{m}{s}$ $x = 2000\cdot mi$

$P = 9.7\cdot MW$ $P = 1.30\times 10^4\,hp$ (Power consumed by drag)

The power to the propeller is $P_{prop} = \frac{P}{70\cdot\%}$ $P_{prop} = 1.86\times 10^4\,hp$

The shaft power is $P_s = 120\%\cdot P_{prop}$ $P_s = 2.23\times 10^4\,hp$

The efficiency of the engines is $\eta = 40\cdot\%$

Hence the heat supplied to the engines is $Q = \frac{P_s}{\eta}$ $Q = 1.42\times 10^8\,\frac{BTU}{hr}$

The journey time is $t = \frac{x}{U}$ $t = 134\,hr$

The total energy consumed is $Q_{total} = Q\cdot t$ $Q_{total} = 1.9\times 10^{10}\,BTU$

From buoyancy the total ship weight equals the displaced seawater volume

$M_{ship}\cdot g = \rho\cdot g\cdot L\cdot B\cdot D$ $M_{ship} = \rho\cdot L\cdot B\cdot D$ $M_{ship} = 1.42\times 10^9\,lb$

Hence the mass of oil is $M_{oil} = 75\%\cdot M_{ship}$ $M_{oil} = 1.06\times 10^9\,lb$

The chemical energy stored in the petroleum is $q = 20000\cdot\frac{BTU}{lb}$

The total chemical energy is $E = q\cdot M_{oil}$ $E = 2.13\times 10^{13}\,BTU$

The equivalent percentage of petroleum cargo used is then $\frac{Q_{total}}{E} = 0.089\,\%$

The Alaska pipeline uses $e_{pipeline} = 120\cdot\frac{BTU}{ton\cdot mi}$ but for the ship $e_{ship} = \frac{Q_{total}}{M_{oil}\cdot x}$ $e_{ship} = 17.8\,\frac{BTU}{ton\cdot mi}$

The ship uses only about 15% of the energy of the pipeline!

Problem 9.70

[3]

9.70 Consider the linear, sinusoidal, and parabolic laminar boundary-layer approximations of Problem 9.10. Compare the momentum fluxes of these profiles. Which is most likely to separate first when encountering an adverse pressure gradient?

Given: Linear, sinusoidal and parabolic velocity profiles

Find: Momentum fluxes

Solution:

The momentum flux is given by $$mf = \int_0^{\delta} \rho \cdot u^2 \cdot w \, dy$$

where *w* is the width of the boundary layer

For a linear velocity profile $$\frac{u}{U} = \frac{y}{\delta} = \eta \qquad (1)$$

For a sinusoidal velocity profile $$\frac{u}{U} = \sin\left(\frac{\pi}{2} \cdot \frac{y}{\delta}\right) = \sin\left(\frac{\pi}{2} \cdot \eta\right) \qquad (2)$$

For a parabolic velocity profile $$\frac{u}{U} = 2 \cdot \left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2 \cdot \eta - (\eta)^2 \qquad (3)$$

For each of these $$u = U \cdot f(\eta) \qquad\qquad y = \delta \cdot \eta$$

Using these in the momentum flux equation $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 f(\eta)^2 \, d\eta \qquad (4)$$

For the linear profile Eqs. 1 and 4 give $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 \eta^2 \, d\eta \qquad mf = \frac{1}{3} \cdot \rho \cdot U^2 \cdot \delta \cdot w$$

For the sinusoidal profile Eqs. 2 and 4 give $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 \sin\left(\frac{\pi}{2} \cdot \eta\right)^2 \, d\eta \qquad mf = \frac{1}{2} \cdot \rho \cdot U^2 \cdot \delta \cdot w$$

For the parabolic profile Eqs. 3 and 4 give $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 \left[2 \cdot \eta - (\eta)^2\right]^2 \, d\eta \qquad mf = \frac{8}{15} \cdot \rho \cdot U^2 \cdot \delta \cdot w$$

The linear profile has the smallest momentum, so would be most likely to separate

Problem *9.71

9.71 Table 9.1 shows the numerical results obtained from Blasius exact solution of the laminar boundary-layer equations. Plot the velocity distribution (note that from Eq. 9.13 we see that $\eta \approx 5.0\frac{y}{\delta}$). On the same graph, plot the turbulent velocity distribution given by the $\frac{1}{7}$-power expression of Eq. 9.24. Which is most likely to separate first when encountering an adverse pressure gradient? To justify your answer, compare the momentum fluxes of these profiles (the laminar data can be integrated using a numerical method such as Simpson's rule).

Given: Laminar (Blasius) and turbulent (1/7 - power) velocity distributions

Find: Plot of distributions; momentum fluxes

Solution:

The momentum flux is given by $mf = \int_0^{\delta} \rho \cdot u^2\, dy$ per unit width of the boundary layer

Using the substitutions $\frac{u}{U} = f(\eta)$ $\frac{y}{\delta} = \eta$

the momentum flux becomes $mf = \rho \cdot U^2 \cdot \delta \cdot \int_0^1 f(\eta)^2\, d\eta$

For the Blasius solution a numerical evaluation (a Simpson's rule) of the integral is needed

$$mf_{lam} = \rho \cdot U^2 \cdot \delta \cdot \frac{\Delta\eta}{3} \cdot \left(f\left(\eta_0\right)^2 + 4 \cdot f\left(\eta_1\right)^2 + 2 \cdot f\left(\eta_2\right)^2 + f\left(\eta_N\right)^2\right)$$

where $\Delta\eta$ is the step size and N the number of steps

The result for the Blasius profile is $mf_{lam} = 0.525 \cdot \rho \cdot U^2 \cdot \delta$

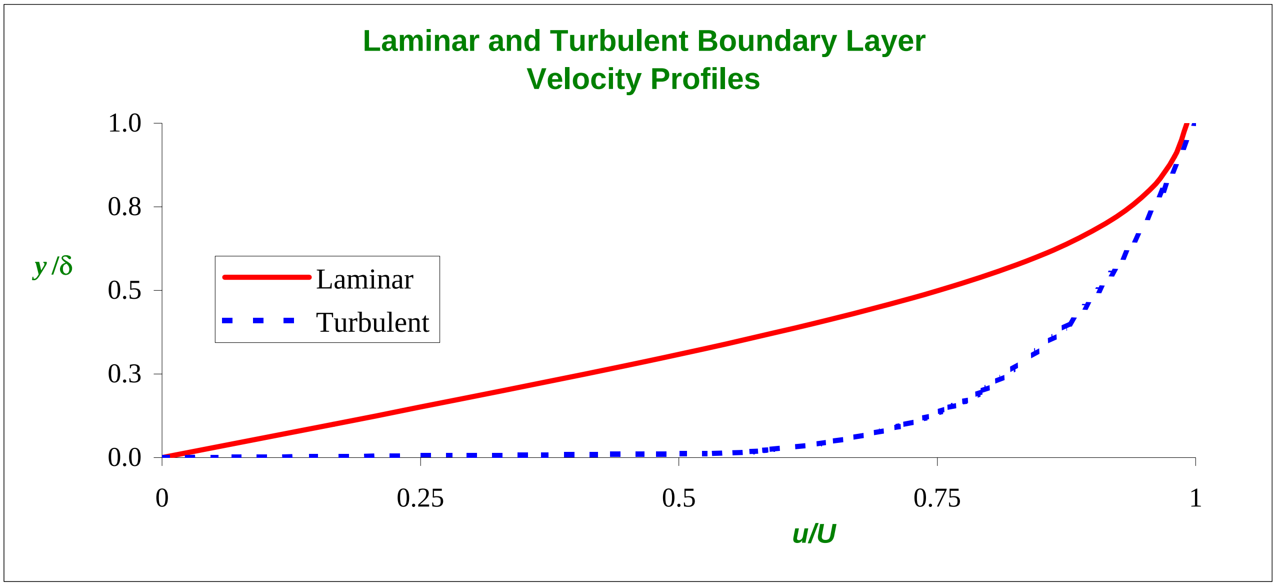
For a 1/7 power velocity profile $mf_{turb} = \rho \cdot U^2 \cdot \delta \cdot \int_0^1 \eta^{\frac{2}{7}}\, d\eta$ $mf_{turb} = \frac{7}{9} \cdot \rho \cdot U^2 \cdot \delta$

The laminar boundary has less momentum, so will separate first when encountering an adverse pressure gradient

Computed results:

	(Table 9.1)	(Simpsons Rule)	
η	**Laminar *u/U***	**Weight *w***	**Weight x $(u/U)^2$**
0.0	0.000	1	0.00
0.5	0.166	4	0.11
1.0	0.330	2	0.22
1.5	0.487	4	0.95
2.0	0.630	2	0.79
2.5	0.751	4	2.26
3.0	0.846	2	1.43
3.5	0.913	4	3.33
4.0	0.956	2	1.83
4.5	0.980	4	3.84
5.0	0.992	1	0.98
		Simpsons':	**0.525**

$y/\delta = \eta$	**Turbulent *u/U***
0.0	0.00
0.0125	0.53
0.025	0.59
0.050	0.65
0.10	0.72
0.15	0.76
0.2	0.79
0.4	0.88
0.6	0.93
0.8	0.97
1.0	1.00



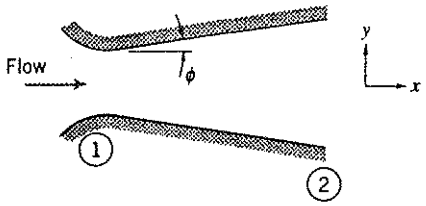
Problem 9.72

Given: Flow through plane-wall diffuser, as shown. We wish to compare the behavior of inviscid and viscous fluids

Find: (a) For an inviscid fluid, describe flow pattern and pressure distribution as ϕ is increased from $\phi = 0$

(b) Include viscous (boundary layer effects)

(c) Which fluid will have the highest exit pressure?



Solution:

For the inviscid fluid,

- with $\phi = 0$ (straight channel) there will be no change in velocity, and hence no pressure gradient.
- as ϕ is increased, the velocity decreases and hence the pressure increases (from the Bernoulli equation along the channel.

For the viscous fluid:

- with $\phi = 0$, boundary layers will form along the channel walls reducing the effective flow area. Thus to satisfy continuity for incompressible flow, the centerline velocity must increase and the pressure will drop along the channel.
- as ϕ is increased, the adverse pressure gradient increases. This causes an increased rate of boundary layer growth. If ϕ is too large, the flow will separate from one (or both) walls.

The inviscid fluid will have the highest exit pressure. (The pressure gradient with the real fluid is reduced by boundary layer development for all values of ϕ.)

Problem 9.73

Given: Laminar boundary layer with velocity profile

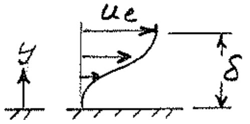
$$\frac{u}{u_e} = a + b\eta + c\eta^2 + d\eta^3 \;;\; \eta = \frac{y}{\delta}$$

Separation occurs when τ_w becomes zero.

Find: (a) Four boundary conditions for this laminar velocity profile.
(b) Evaluate constants a, b, c, and d.
(c) Calculate shape parameter, H, at separation.

Plot: Profile and compare with parabolic.

Solution: The profile shape will be:



Boundary conditions are:

$$y = 0: \; u = 0 \text{ and } \tau = \mu\frac{du}{dy} = 0 \;;\; y = \delta: \; u = u_e \text{ and } \tau = \mu\frac{du}{dy} = 0$$ BC

Applying boundary conditions:

$$y = 0 : \eta = 0 \qquad \frac{u}{u_e} = 0 = (a + b\eta + c\eta^2 + d\eta^3)_{\eta=0} = a \qquad a = 0$$ a

$$\frac{du}{dy} = \frac{u_e}{\delta}\frac{d(u/u_e)}{d\eta} = 0 = (b + 2c\eta + 3c\eta^2)_{\eta=0} = b \qquad b = 0$$ b

$$y = \delta : \eta = 1 \qquad \frac{u}{u_e} = 1 = (c\eta^2 + d\eta^3)_{\eta=1} \;;\; c + d = 1$$

$$\frac{d(u/u_e)}{d\eta} = 0 = (2c\eta + 3d\eta^2)_{\eta=1} \;;\; 2c + 3d = 0$$

Solving

$$d = 1 - c \;;\; 2c + 3(1-c) = 2c + 3 - 3c = 3 - c = 0 \qquad c = 3$$ c

$$d = 1 - c = 1 - 3 = -2 \qquad d = -2$$ d

The velocity profile is $\frac{u}{u_e} = 3\eta^2 - 2\eta^3$. $H \equiv \delta^*/\theta = \frac{\delta^*}{\delta}\frac{\delta}{\theta}$, so

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{u_e}\right)d\eta = \int_0^1 (1 - 3\eta^2 + 2\eta^3)\,d\eta = \left[\eta - \eta^3 + \frac{2}{4}\eta^4\right]_0^1 = \frac{1}{2}$$

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)d\eta = \int_0^1 (3\eta^2 - 2\eta^3)(1 - 3\eta^2 + 2\eta^3)\,d\eta$$

$$= \int_0^1 (3\eta^2 - 2\eta^3 - 9\eta^4 + 12\eta^5 - 4\eta^6)\,d\eta$$

$$\frac{\theta}{\delta} = \left[\eta^3 - \frac{2}{4}\eta^4 - \frac{9}{5}\eta^5 + \frac{12}{6}\eta^6 - \frac{4}{7}\eta^7\right]_0^1 = \frac{9}{70}$$

Thus

$$H = \frac{1}{2} \times \frac{70}{9} = \frac{70}{18} = \frac{35}{9} = 3.89$$ H

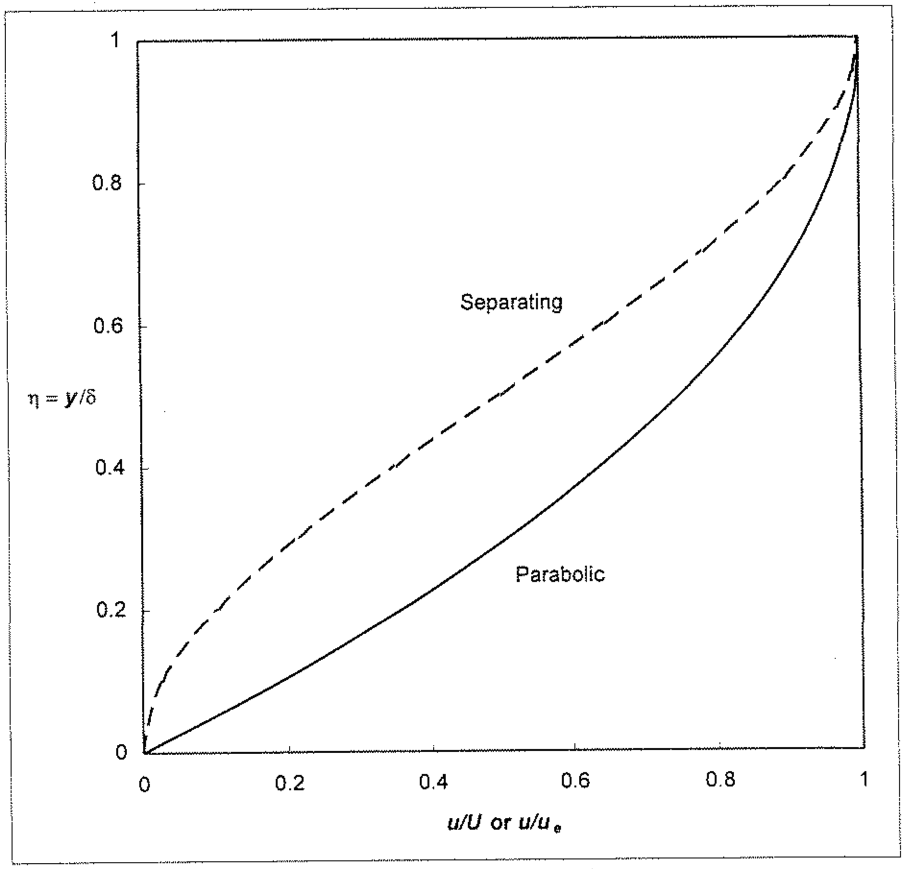
Separating:

$\frac{u}{u_e} = 3\eta^2 - 2\eta^3$

Flat plate:

$\frac{u}{U} = 2\eta - \eta^2$

η (---)	$\frac{u}{u_e}$ (---)	$\frac{u}{U}$ (---)
1.0	1.00	1.00
0.9	0.972	0.990
0.8	0.896	0.960
0.7	0.784	0.910
0.6	0.648	0.840
0.5	0.500	0.750
0.4	0.352	0.640
0.3	0.216	0.510
0.2	0.104	0.360
0.1	0.028	0.190



Problem 9.74

Open-Ended Problem Statement: For flow over a flat plate with zero pressure gradient, will the shear stress increase, decrease, or remain constant along the plate? Justify your answer. Does the momentum flux increase, decrease, or remain constant as the flow proceeds along the plate? Justify your answer. Compare the behavior of laminar flow and turbulent flow (both from the leading edge) over a flat plate. At a given distance from the leading edge, which flow will have the larger boundary-layer thickness? Does your answer depend on the distance along the plate? How would you justify your answer?

Discussion: Shear stress decreases along the plate because the freestream flow speed remains constant while the boundary-layer thickness increases.

The momentum flux decreases as the flow proceeds along the plate. Momentum thickness θ (actually proportional to the defect in momentum within the boundary layer) increases, showing that momentum flux decreases. The force that must be applied to hold the plate stationary reduces the momentum flux of the stream and boundary layer.

The laminar boundary layer has less shear stress than the turbulent boundary layer. Therefore laminar boundary-layer flow from the leading edge produces a thinner boundary layer and less shear stress everywhere along the plate than a turbulent boundary layer from the leading edge.

Since both boundary layers continue to grow with increasing distance from the leading edge, and the turbulent boundary layer continues to grow more rapidly because of its higher shear stress, this comparison will be the same no matter the distance from the leading edge.

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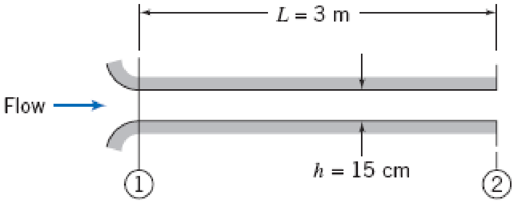
Problem 9.75

[5]

9.75 Cooling air is supplied through the wide, flat channel shown. For minimum noise and disturbance of the outlet flow, laminar boundary layers must be maintained on the channel walls. Estimate the maximum inlet flow speed at which the outlet flow will be laminar. Assuming parabolic velocity profiles in the laminar boundary layers, evaluate the pressure drop, $p_1 - p_2$. Express your answer in inches of water.



Given: Channel flow with laminar boundary layers

Find: Maximum inlet speed for laminar exit; Pressure drop for parabolic velocity in boundary layers

Solution:

Basic equations: $Re_{trans} = 5\times10^5$ $\qquad \dfrac{\delta}{x} = \dfrac{5.48}{\sqrt{Re_x}}$ $\qquad \dfrac{p}{\rho} + \dfrac{V^2}{2} + g\cdot z = const$

Assumptions: 1) Steady flow 2) Incompressible 3) z = constant

From Table A.10 at 20ºC $\quad \nu = 1.50\times10^{-5}\cdot\dfrac{m^2}{s} \qquad \rho = 1.21\cdot\dfrac{kg}{m^3} \qquad L = 3\cdot m \qquad h = 15\cdot cm$

Then $\quad Re_{trans} = \dfrac{U_{max}\cdot L}{\nu} \qquad U_{max} = \dfrac{Re_{trans}\cdot\nu}{L} \qquad U_{max} = 2.50\dfrac{m}{s} \qquad U_1 = U_{max} \qquad U_1 = 2.50\dfrac{m}{s}$

For $\quad Re_{trans} = 5\times10^5 \qquad \delta_2 = L\cdot\dfrac{5.48}{\sqrt{Re_{trans}}} \qquad \delta_2 = 0.0232\,m$

For a parabolic profile $\quad \dfrac{\delta_{disp}}{\delta} = \displaystyle\int_0^1 \left(1-\frac{u}{U}\right)d\lambda = \int_0^1 \left(1-2\cdot\lambda+\lambda^2\right)d\lambda = \frac{1}{3}$ $\quad$ where δ_{trans} is the displacement thickness

$$\delta_{disp2} = \frac{1}{3}\cdot\delta_2 \qquad \delta_{disp2} = 0.00775\,m$$

From continuity $\quad U_1\cdot w\cdot h = U_2\cdot w\cdot\left(h-2\cdot\delta_{disp2}\right) \qquad U_2 = U_1\cdot\dfrac{h}{h-2\cdot\delta_{disp2}} \qquad U_2 = 2.79\dfrac{m}{s}$

Since the boundary layers do not meet Bernoulli applies in the core

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2} \qquad \Delta p = p_1 - p_2 = \frac{\rho}{2}\cdot\left(U_2^2 - U_1^2\right)$$

$$\Delta p = \frac{\rho}{2}\cdot\left(U_2^2 - U_1^2\right) \qquad \Delta p = 0.922\,Pa$$

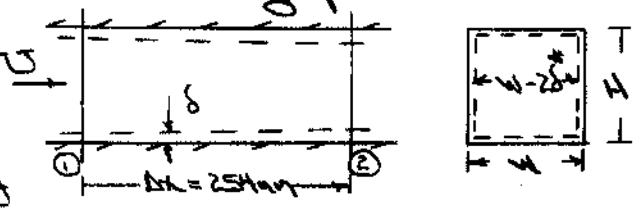
From hydrostatics $\quad \Delta p = \rho_{H2O}\cdot g\cdot\Delta h \quad$ with $\quad \rho_{H2O} = 1000\cdot\dfrac{kg}{m^3}$

$$\Delta h = \frac{\Delta p}{\rho_{H2O}\cdot g} \qquad \Delta h = 0.0940\,mm \qquad \Delta h = 0.00370\,in$$

Problem 9.76

Given: Wind tunnel with square cross section, $W_1 = H_1 = 305\text{ mm}$.
At inlet section ①, $U_1 = 24.5\text{ m/s}$, $\delta_1 = 9.75\text{ mm}$
with a "1/7-power" turbulent velocity profile

$$\left.\frac{dp}{dx}\right)_1 = -0.035\,\frac{\text{mm H}_2\text{O}}{\text{mm}}$$



Find: (a) reduction in flow area at ① caused by boundary layer
(b) $d\theta/dx$ at ①
(c) estimate of θ_2

Solution: Apply continuity and the momentum integral eqs.

Computing eqs.: $\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - (H+2)\frac{\theta}{U}\frac{dU}{dx}$; $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \eta^{1/7}$

$$H = \frac{\delta^*}{\theta}\ ,\quad \frac{\tau_w}{\rho U^2} = 0.0233\left(\frac{\nu}{U\delta}\right)^{1/4}$$

Assumptions: (1) steady flow
(2) uniform flow outside boundary layer.
(3) standard air

$$\%\text{ reduction in flow area} = \frac{A_{eff} - A}{A} = \frac{(W-2\delta^*)(H-2\delta^*) - WH}{WH}$$

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \eta^{1/7}\right) d\eta = \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}$$

$$\therefore\ \delta_1^* = \frac{1}{8}\delta_1 = \frac{1}{8} \times 9.75\text{ mm} = 1.22\text{ mm}$$

$$\therefore\ \text{area reduction} = \left(1 - \frac{2\delta^*}{W}\right)\left(1 - \frac{2\delta^*}{H}\right) - 1 = \left[1 - \frac{2\times 1.22}{305}\right]\left[1 - \frac{2\times 1.22}{305}\right] - 1$$

area reduction $= -0.0159\ (-1.59\%) \longleftarrow$ %

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

$$\frac{\tau_w}{\rho U^2} = 0.0233\left(\frac{\nu}{U\delta}\right)^{1/4} = 0.0233\left[1.45\times 10^{-5}\,\frac{\text{m}^2}{\text{s}} \times \frac{\text{s}}{24.5\text{ m}} \times \frac{1}{9.75\times 10^{-3}\text{ m}}\right]^{0.25}$$

$\frac{\tau_w}{\rho U^2} = 0.00206 \leftarrow - -$

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \eta^{1/7}\left(1 - \eta^{1/7}\right) d\eta = \left[\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}\right]_0^1 = \frac{7}{72}$$

$\theta = \frac{7}{72}\delta = \frac{7}{72} \times 9.75\text{ mm} = 0.948\text{ mm} \leftarrow - -$

$H = \frac{\delta^*}{\theta} = \frac{\frac{1}{8}\delta}{7/72\,\delta} = \frac{9}{7} = 1.29$ − − −

Outside the boundary layer, $p + \frac{1}{2}\rho U^2 = \text{constant}$

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Problem 9.76

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Then, $\frac{dp}{dx} = -\rho U \frac{dU}{dx}$ and $\frac{1}{U}\frac{dU}{dx} = -\frac{1}{\rho U^2}\frac{dp}{dx} = -\frac{1}{\rho U^2}\frac{d\,\rho_w g h}{dx}$

$$\frac{1}{U}\frac{dU}{dx} = -\frac{m^3}{1.23\,kg} \times \frac{1}{(24.5)^2}\frac{s^2}{m^2} \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times (-0.035)\frac{mm}{mm}$$

$$\frac{1}{U}\frac{dU}{dx} = 0.465\ m^{-1} \longleftarrow -\,-$$

Substituting

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

$$= 0.00206 - (1.29+2)\,0.948\,mm \times \frac{0.465}{m} \times \frac{m}{10^3\,mm}$$

$$\frac{d\theta}{dx} = 0.00206 - 0.00145 = 0.00061\ mm/mm$$

$d\theta/_{dx} = 0.61\ mm/m$ ______ $d\theta/_{dx}$

$$\theta_2 \approx \theta_1 + \left.\frac{d\theta}{dx}\right)_1 \Delta x$$

$$= 0.948 + 0.61\frac{mm}{m} \times 0.254\,m$$

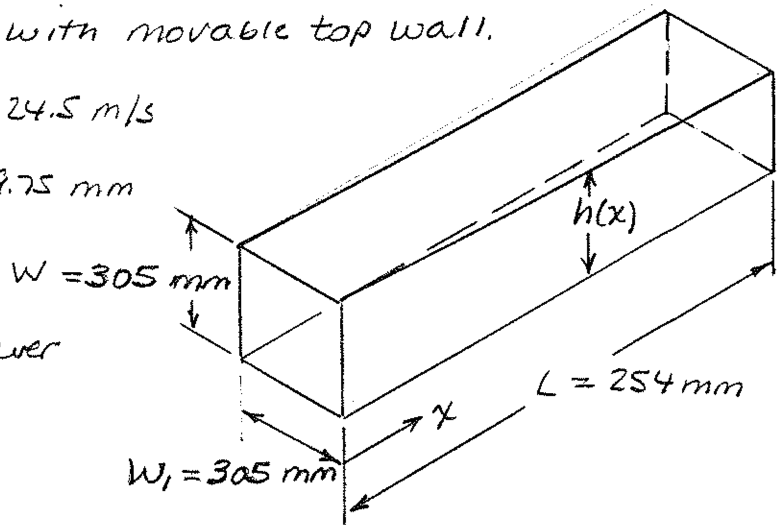
$\theta_2 \approx 1.10\ mm$ ______ θ_2

Problem 9.77

Given: Wind tunnel with movable top wall.

At inlet: $U_1 = 24.5\ m/s$

$\delta_1 = 9.75\ mm$



Assume 1/7-power turbulent BL development,

$\delta = \text{constant}$

Find: (a) Velocity distribution to make $\delta = \text{const}$, (b) $h(x)$ from 0 to L.

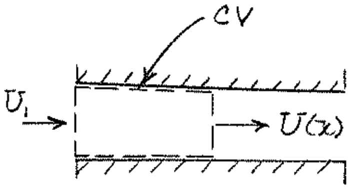
Solution: Calculate required pressure gradient ($dp/dx = -\rho U\, dU/dx$) from the momentum integral equation, then integrate to find $U(x)$, use continuity to solve for $h(x)$:

Computing equations:

$$\frac{\tau_w}{\rho U^2} = \cancelto{0(4)}{\frac{d\theta}{dx}} + (H+2)\frac{\theta}{U}\frac{dU}{dx} \qquad (9.27)$$

$$0 = \cancelto{0(1)}{\frac{\partial}{\partial t}}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow outside BL
(4) Constant BL thickness:



$$\delta^* = \frac{1}{8}\delta \text{ and } \theta = \frac{7}{72}\delta;\ H = \frac{\delta^*}{\theta} = \frac{72}{(7)8} = 1.29 \text{ for } 1/7\text{-power profile}$$

Then from the momentum integral equation

$$\frac{\tau_w}{\rho U^2} = (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

To integrate, we must make an assumption about τ_w.

Case 1: Assume $\tau_w = \text{constant}$, and rearrange

Then $U\,dU = \frac{\tau_w}{\rho\theta(H+2)}\,dx$

Integrating, $\frac{1}{2}U^2\Big]_1^2 = \frac{\tau_w}{\rho\theta(H+2)}x$ or $\frac{1}{2}U_2^2 - \frac{1}{2}U_1^2 = \frac{\tau_w}{\rho\theta(H+2)}x$

so $\frac{U}{U_1} = \left[1 + \frac{2\tau_w}{\rho U_1^2}\frac{x}{\theta(H+2)}\right]^{\frac{1}{2}} = \left[1 + \frac{C_f}{\theta(H+2)}x\right]^{\frac{1}{2}}$ $\longleftarrow$ U(x)

Case 2: Assume $\tau_w \neq \text{constant}$

Then $\tau_w = 0.0233\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$ (Eq. 9.25)

Substituting and rearranging

$$\frac{\tau_w}{\rho U^2} = 0.0233\left(\frac{\nu}{\delta}\right)^{1/4}\frac{1}{U^{1/4}} = (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

or

$$\frac{dU}{U^{0.75}} = 0.0233\left(\frac{\nu}{\delta}\right)^{1/4}\frac{dx}{(H+2)\theta}$$

Integrating,

$$4U^{1/4}\Big]_1^2 = 0.0233\left(\frac{\nu}{\delta}\right)^{1/4}\frac{x}{(H+2)\theta}$$

or

$$\frac{U}{U_1} = \left[1+0.00583\left(\frac{\nu}{U_1\delta}\right)^{\frac{1}{4}}\frac{x}{(H+2)\theta}\right]^4 \longleftarrow U(x)$$

From continuity $U_1A_1 = UA = U_1(W_1-2\delta_1^*)(H_1-2\delta_1^*) = U(W_1-2\delta_1^*)(h-2\delta_1^*)$

Thus $A/A_1 = (U/U_1)^{-1}$ and $\frac{h-2\delta_1^*}{H-2\delta_1^*} = \frac{U_1}{U} = \frac{h/H - 2\delta_1^*/H}{1-2\delta_1^*/H}$

$$\frac{h}{W} = \left(1-2\frac{\delta_1^*}{h_1}\right)\frac{U_1}{U} + \frac{2\delta_1^*}{h_1} \longleftarrow h(x)$$

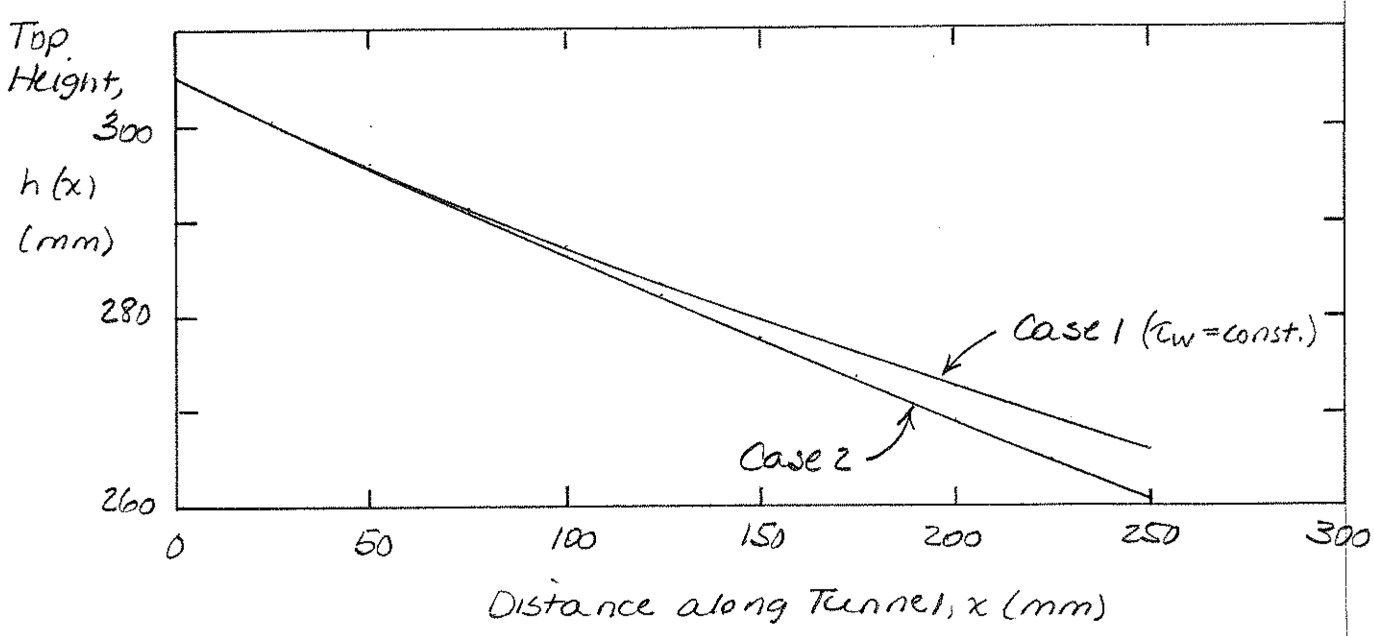
Evaluating: $\delta_1 = 9.75\ \text{mm}$, $\delta_1^* = \frac{1}{8}\delta_1 = 1.22\ \text{mm}$, $\theta_1 = \frac{7}{72}\delta_1 = 0.948\ \text{mm}$

$$\frac{1}{2}\rho U_1^2 = \frac{1}{2}\times 1.23\frac{\text{kg}}{\text{m}^3}\times(24.5)^2\frac{\text{m}^2}{\text{s}^2}\times\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 369\ \text{N/m}^2$$

$$Re_{\delta_1} = \frac{U_1\delta_1}{\nu} = 24.5\frac{\text{m}}{\text{s}}\times 0.00975\ \text{m}\times\frac{\text{s}}{1.46\times10^{-5}\ \text{m}^2} = 1.64\times10^4$$

$$C_f = 0.0466\ Re_{\delta_1}^{-1/4} = 0.0466(0.0883) = 0.00411$$

Plot results:

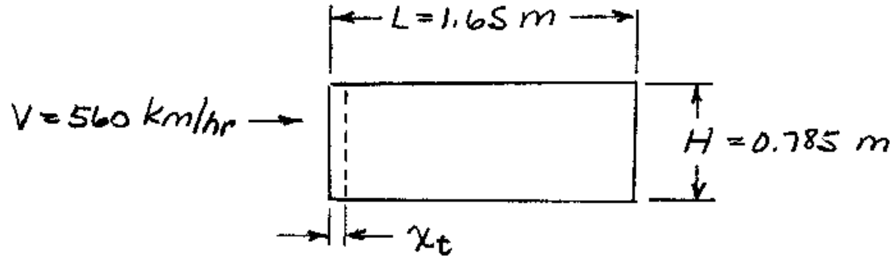


Problem 9.78

[2]

Given: Stabilizing fin on Bonneville land speed record auto.

$z = 1{,}340\ m$



Find: (a) Evaluate Re_L
(b) Location of x_t
(c) Power to overcome skin friction drag on fin.

Solution: Assume standard atmosphere, so $T = 279\ K$, $\rho/\rho_0 = 0.877$ (Table A.3); $\mu = 1.79 \times 10^{-5}\ kg/m\cdot s$ (Table A.7). Then

$$Re_L = \frac{\rho V L}{\mu} = (0.877)\,1.23\,\frac{kg}{m^3} \times 560\times10^3 \frac{m}{hr} \times 1.65\,m \times \frac{m\cdot s}{1.79\times10^{-5}\,kg} \times \frac{hr}{3600\,s}$$

$$Re_L = 1.55 \times 10^7$$

Re_L

Assume transition occurs at $Re_x = 500{,}000$. Then

$$\frac{x_t}{L} = \frac{Re_{x_t}}{Re_L} = \frac{500{,}000}{1.55\times10^7} = 0.0323$$

$$x_t = 0.0323\,L = 0.0323 \times 1.65\,m = 0.0532\,m = 53.2\,mm$$

x_t

Calculate drag force using C_D from Fig. 9.8: $F_D = C_D A \frac{1}{2}\rho V^2$

$C_D = 0.0029$ (Fig. 9.8); $A = 2LH = 2 \times 1.65\,m \times 0.785\,m = 2.59\,m^2$ (2 sides)

$$\frac{1}{2}\rho V^2 = \frac{1}{2} \times (0.877)\,1.23\,\frac{kg}{m^3}\left(560\times10^3 \frac{m}{hr} \times \frac{hr}{3600\,s}\right)^2 \frac{N\cdot s^2}{kg\cdot m} = 1.31\times10^4\,N/m^2$$

$$F_D = 0.0029 \times 2.59\,m^2 \times 1.31\times10^4 \frac{N}{m^2} = 98.4\,N \quad \text{(skin friction drag on fin)}$$

The power required is

$$P = F_D V = 98.4\,N \times 560\times10^3 \frac{m}{hr} \times \frac{hr}{3600\,s} \times \frac{W\cdot s}{N\cdot m} = 15.3\,kW$$

P

Check using Eq. 9.37b:

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1610}{Re_L} = 0.00270$$

This is slightly less than from the graph, but reasonable agreement.

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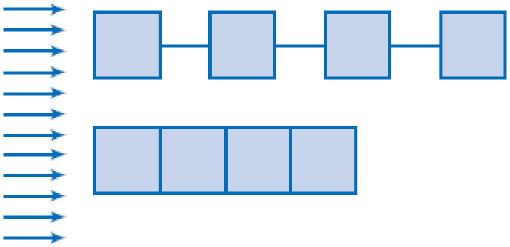
Problem 9.79

[3]

9.79 Repeat Problem 9.46, except that the water flow is now at 10 m/s (use formulas for C_D from Section 9-7).



Given: Pattern of flat plates

Find: Drag on separate and composite plates

Solution:

Basic equations: $$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$$

For separate plates $L = 7.5\cdot cm$ $W = 7.5\cdot cm$ $A = W\cdot L$ $A = 5.625\times 10^{-3}\,m^2$ $V = 10\cdot\frac{m}{s}$

From Table A.8 at 20ºC $\nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\rho = 998\cdot\frac{kg}{m^3}$

First determine the Reynolds number $Re_L = \frac{V\cdot L}{\nu}$ $Re_L = 7.43\times 10^5$ so use Eq. 9.34

$$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} \qquad C_D = 0.00497$$

The drag (one side) is then $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $F_D = 1.39\,N$

This is the drag on one plate. The total drag is then $F_{Total} = 4\cdot F_D$ $F_{Total} = 5.58\,N$

For both sides: $2\cdot F_{Total} = 11.2\,N$

For the composite plate $L = 4\times 7.5\cdot cm$ $L = 0.300\,m$ $A = W\cdot L$ $A = 0.0225\,m^2$

First determine the Reylolds number $Re_L = \frac{V\cdot L}{\nu}$ $Re_L = 2.97\times 10^6$ so use Eq. 9.34

$$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} \qquad C_D = 0.00377$$

The drag (one side) is then $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $F_D = 4.23\,N$ For both sides: $2\cdot F_D = 8.46\,N$

The drag is much lower on the composite compared to the separate plates. This is because τ_w is largest near the leading edges and falls off rapidly; in this problem the separate plates experience leading edges four times!

Problem 9.80

Given: Towboat model at 1:13.5 scale to be tested in towing tank.
Dimensions are:
Length, 11.1 ft; Beam, 3.11 ft; Draft, 0.62 ft

Model displacement in fresh water is 1200 lbf.

Find: (a) Estimate average length of wetted surface on hull.
(b) Skin friction drag force on prototype at V = 8 mph.

Solution: Represent the towboat as a rectangular solid of length, $\bar{L}$, with the displacement of the boat. From buoyancy,

$$W = \rho g \forall = \rho g \bar{L} B D \quad \text{or} \quad \bar{L} = \frac{W}{\rho g B D}$$

$$\bar{L} = 1200\,\text{lbf} \times \frac{\text{ft}^3}{1.94\,\text{slug}} \times \frac{\text{sec}^2}{32.2\,\text{ft}} \times \frac{1}{3.11\,\text{ft}} \times \frac{1}{0.62\,\text{ft}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{sec}^2} = 9.96\,\text{ft} \longleftarrow \quad \bar{L}$$

For the prototype,

$$\bar{L}_p = 13.5\,\bar{L}_m = 13.5 \times 9.96\,\text{ft} = 134\,\text{ft}$$

The Reynolds number is (Table A.7)

$$Re_L = \frac{VL}{\nu} = 8\,\frac{\text{mi}}{\text{hr}} \times 134\,\text{ft} \times \frac{\text{s}}{1.08 \times 10^{-5}\,\text{ft}^2} \times 5280\,\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}} = 1.46 \times 10^8 \qquad (T = 68^\circ\text{F})$$

Thus flow is predominantly turbulent. Apply turbulent flow analysis.

Computing equations: $F_D = C_D A \frac{1}{2} \rho U^2 \qquad C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1610}{Re_L} \qquad (9.37b)$

The wetted area is $A = \bar{L}(B + 2D)$.

$$C_D = \frac{0.455}{(\log_{10} 1.46 \times 10^8)^{2.55}} - \frac{1610}{1.46 \times 10^8} = 0.00214$$

Substituting,

$$F_D = 0.00214 \times 134\,\text{ft} \times 13.5(3.11 + 2 \times 0.62)\,\text{ft} \times \frac{1}{2} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \left(8\,\frac{\text{mi}}{\text{hr}} \times 5280\,\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}} \right)^2 \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$F_D = 2250\,\text{lbf} \;\; (\text{skin friction only}) \longleftarrow \quad F_D$$

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Problem 9.81

9.81 A jet transport aircraft cruises at 40,000 ft altitude in steady level flight at 500 mph. Model the aircraft fuselage as a circular cylinder with diameter $D = 12$ ft and length $L = 125$ ft. Neglecting compressibility effects, estimate the skin friction drag force on the fuselage. Evaluate the power needed to overcome this force.

Given: Aircraft cruising at 40,000 ft

Find: Skin friction drag force; Power required

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$

We "unwrap" the cylinder to obtain an equivalent flat plate

$L = 125\cdot ft$ $\quad D = 12\cdot ft$ $\quad A = L\cdot\pi\cdot D$ $\quad A = 4712\cdot ft^2$ $\quad V = 500\cdot mph$

From Table A.3, with $z = 40000\cdot ft$ $\quad z = 12192\,m$

For $z = 12000\cdot m$ $\quad \dfrac{\rho}{\rho_{SL}} = 0.2546$ $\quad$ with $\quad \rho_{SL} = 0.002377\cdot\dfrac{slug}{ft^3}$

$z = 13000\cdot m$ $\quad \dfrac{\rho}{\rho_{SL}} = 0.2176$

Hence at $z = 12192\,m$ $\quad \dfrac{\rho}{\rho_{SL}} = 0.2546 + \dfrac{(0.2176 - 0.2546)}{(1300 - 12000)}\cdot(12192 - 12000) = 0.255$

$\rho = 0.255\cdot\rho_{SL}$ $\quad \rho = 0.000606\cdot\dfrac{slug}{ft^3}$ $\quad$ and also $\quad T = 216.7\cdot K$

From Appendix A-3 $\quad \mu = \dfrac{b\cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}}$ $\quad$ with $\quad b = 1.458\times 10^{-6}\cdot\dfrac{kg}{m\cdot s\cdot K^{\frac{1}{2}}}$ $\quad S = 110.4\cdot K$

Hence $\quad \mu = \dfrac{b\cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}}$ $\quad \mu = 1.42\times 10^{-5}\cdot\dfrac{N\cdot s}{m^2}$ $\quad \mu = 2.97\times 10^{-7}\cdot\dfrac{lbf\cdot s}{ft^2}$

Next we need the Reynolds number $\quad Re_L = \dfrac{\rho\cdot V\cdot L}{\mu}$ $\quad Re_L = 1.87\times 10^8$ $\quad$ so use Eq. 9.35

$C_D = \dfrac{0.455}{\log\left(Re_L\right)^{2.58}}$ $\quad C_D = 0.00195$

The drag is then $\quad F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $\quad F_D = 1500\cdot lbf$

The power consumed is $\quad P = F_D\cdot V$ $\quad P = 1.100\times 10^6\cdot\dfrac{ft\cdot lbf}{s}$ $\quad P = 1999\cdot hp$

Problem 9.82

[3]

Given: Model barge tested at 1:13.5 scale; prototype speed, $V_p = 8\,mph$.

Dimensions are: Length, $L = 22\,ft$
Beam, $B = 4.0\,ft$
Draft, $D = 0.667\,ft$

Find: (a) Model test speed, V_m
(b) Boundary layers laminar or turbulent on prototype?
(c) Where position BL trips on model?
(d) Estimate skin friction drag for model and prototype.

Solution: Test should be run so $Fr_m = Fr_p = V_m / \sqrt{g L_m} = V_p / \sqrt{g L_p}$

Thus $V_m = V_p \sqrt{L_m / L_p} = 8\,mph \sqrt{1/13.5} = 2.18\,mph$ — V_m

$$Re_p = \frac{V_p L_p}{\nu} = (8)(1.47)\frac{ft}{s} \times (22)(13.5)\,ft \times \frac{s}{1.08 \times 10^{-5}\,ft^2} = 3.24 \times 10^8 \quad (T = 68°F,\ Table\ A.7)$$

Therefore boundary-layer flow is turbulent. Transition would occur at $Re_{x_t} \approx 5 \times 10^5$, so

$$x_t / L = \frac{5 \times 10^5}{3.24 \times 10^8} = 0.00154 \,; \quad x_t = 0.00154\, L_m = 0.0339\ ft \text{ from front}$$

The wetted area is $A = L(B + 2D)$. Assume for turbulent BL flow

$$C_D = 1.25\, C_f = 1.25 \times \frac{0.0594}{(Re_L)^{1/5}} = \frac{0.0743}{(Re_L)^{1/5}}$$

For the model, $L = 22\,ft$, and

$$Re_m = \frac{V_m L_m}{\nu} = (2.18)(1.47)\frac{ft}{sec} \times 22\,ft \times \frac{s}{1.08 \times 10^{-5}\,ft^2} = 6.53 \times 10^6$$

$$C_{D_m} = \frac{0.0743}{(6.53 \times 10^6)^{1/5}} = 0.00322$$

For the model

$$F_{D_m} = C_{D_m} \tfrac{1}{2} \rho V_m^2 A_m = \frac{0.00322}{2} \times 1.94 \frac{slug}{ft^3} \times [2.18(1.47)]^2 \frac{ft^2}{s^2} \times [22(4 + 2(0.667))]\,ft^2 \times \frac{lbf\, s^2}{slug \cdot ft}$$

$F_{D_m} = 3.77\ lbf$ — F_{D_m}

For the prototype,

$Re_p = 3.24 \times 10^8$, $C_{D_p} = 0.00148$, $A_p = (13.5)^2 A_m = 21{,}400\ ft^2$

$$F_{D_p} = C_{D_p} \tfrac{1}{2} \rho V_p^2 A_p = \frac{0.00148}{2} \times 1.94 \frac{slug}{ft^3} \times [(8)(1.47)]^2 \frac{ft^2}{s^2} \times 21{,}400\,ft^2 \times \frac{lbf\, s^2}{slug \cdot ft} = 4{,}250\ lbf$$ — F_{D_p}

Problem 9.83

[3]

9.83 A flat-bottomed barge, 80 ft long and 35 ft wide, submerged to a depth of 5 ft, is to be pushed up a river (the river water is at 60°F). Estimate and plot the power required to overcome skin friction for speeds ranging up to 15 mph.

Given: Barge pushed upriver

Find: Power required to overcome friction; Plot power versus speed

Solution:

Basic equations:

$$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot U^2\cdot A} \qquad (9.32) \qquad C_D = \frac{0.455}{\left(\log\left(Re_L\right)\right)^{2.58}} - \frac{1610}{Re_L} \qquad (9.37b) \qquad Re_L = \frac{U\cdot L}{\nu}$$

From Eq. 9.32 $F_D = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^2$ and $A = L\cdot(B + 2\cdot D)$

The power consumed is $P = F_D\cdot U$ $\qquad P = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^3$

Given data:

L = 80 ft

B = 35 ft

D = 5 ft

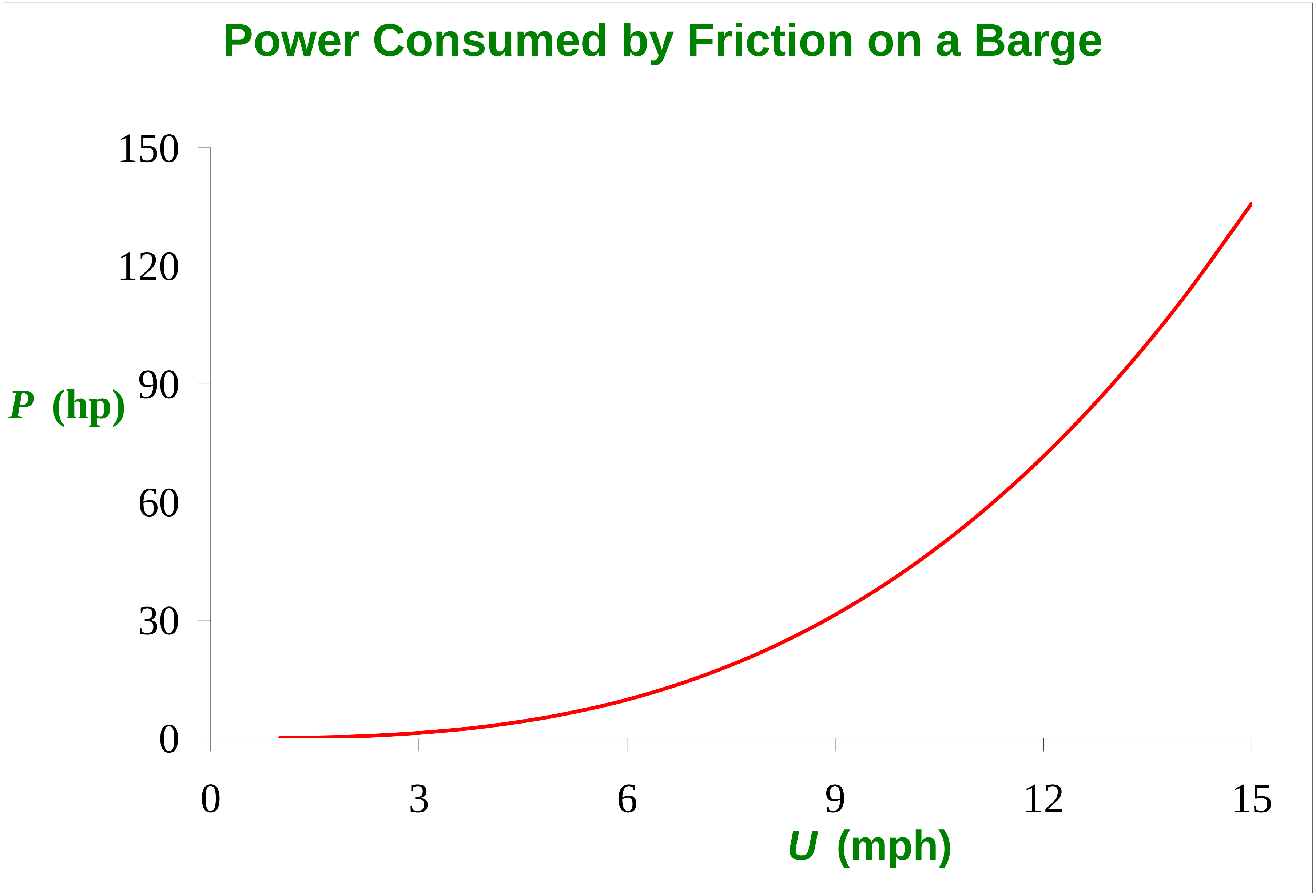
ν = 1.21E-05 ft^2/s (Table A.7)

ρ = 1.94 $slug/ft^3$ (Table A.7)

Computed results:

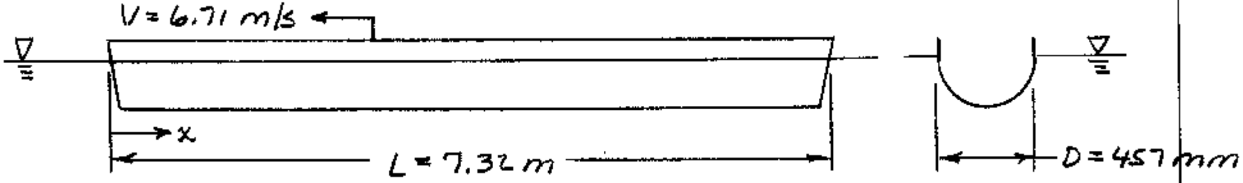
A = 3600 ft^2

U **(mph)**	***Re*** $_L$	***C*** $_D$	***P*** **(hp)**
1	9.70E+06	0.00285	0.0571
2	1.94E+07	0.00262	0.421
3	2.91E+07	0.00249	1.35
4	3.88E+07	0.00240	3.1
5	4.85E+07	0.00233	5.8
6	5.82E+07	0.00227	9.8
7	6.79E+07	0.00222	15
8	7.76E+07	0.00219	22
9	8.73E+07	0.00215	31
10	9.70E+07	0.00212	42
11	1.07E+08	0.00209	56
12	1.16E+08	0.00207	72
13	1.26E+08	0.00205	90
14	1.36E+08	0.00203	111
15	1.45E+08	0.00201	136


Power Consumed by Friction on a Barge
P (hp)
150
120
90
60
30
0
0
3
6
9
12
15
U (mph)

Problem 9.84

Given: Racing shell of Purdue crew, approximated as half a cylinder.



Find: (a) Location of transition in boundary layers on hull.
(b) Thickness of TBL at rear of hull.
(c) Total skin friction drag on hull.

Solution: Assume flow behaves as on a flat plate, with $Re_{x,t} = 500{,}000$.

$$Re_{x,t} = \frac{V x_t}{\nu} = 500{,}000 \; ; \; x_t = \frac{500{,}000\,\nu}{V} = 5\times10^5 \times 1\times10^{-6}\,\frac{m^2}{s}\times\frac{s}{6.71\,m} = 0.0745\ m$$ x_t

($\nu = 1\times10^{-6}\ m^2/s$ for water at 20°C, Table A.8). Thus LBL is only 1% of L.

For the turbulent boundary layer $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$, so $\delta = \frac{0.382}{Re_L^{1/5}}\,L$

$$Re_L = \frac{VL}{\nu} = 6.71\,\frac{m}{s}\times 7.32\,m\times\frac{s}{1.00\times10^{-6}\,m^2} = 4.91\times10^7$$

$$\delta = 0.382\times\frac{1}{(4.91\times10^7)^{1/5}}\times 7.32\,m = 0.0810\ m$$ δ

The drag force is $F_D = C_D A \frac{1}{2}\rho V^2$.

$$A \approx wL = \frac{\pi D}{2}L = \frac{\pi}{2}\times 0.457\,m\times 7.32\,m = 5.25\ m^2$$

Since $10^7 \le Re_L < 10^9$, then $C_D = \frac{0.455}{(\log Re_L)^{2.58}} = 0.00237$

Then

$$F_D = 0.00237\times 5.25\,m^2\times\frac{1}{2}\times 999\,\frac{kg}{m^3}\times(6.71)^2\,\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = 280\ N$$ F_D

$$\left\{ \begin{array}{l} \text{Note the rowers must produce an average power of} \\ \mathcal{P} = F_D V = 280\,N\times 6.71\,\frac{m}{s}\times\frac{W\cdot s}{N\cdot m} = 1.88\ kW \\ \text{to move the shell at this speed.} \end{array} \right\}$$

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Problem 9.85

[4]

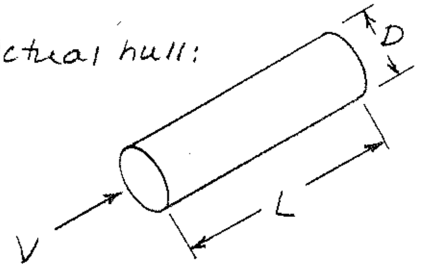
Given: Nuclear submarine, cruising submerged at $V = 27$ kt.

Assume hull is a circular cylinder, $D = 11.0$ m, and $L = 107$ m.

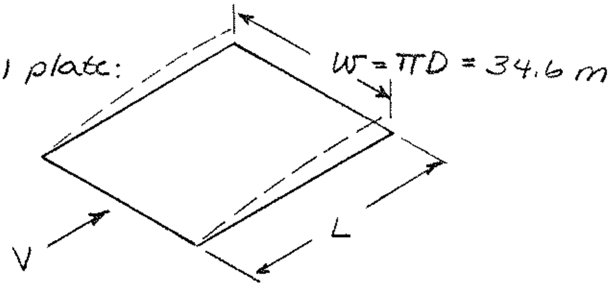
Find: (a) Estimate percentage of hull length with laminar BL.
(b) Calculate drag due to skin friction.
(c) Estimate power consumed

Solution: Treat hull as a flat plate with same wetted area.

Actual hull:



Model plate:



Computing equations: $Re_{x_t} = 500{,}000$ $\quad C_D = \dfrac{0.455}{(\log_{10} Re_L)^{2.58}}$

For seawater, $\nu = 1.05 \times 10^{-6}$ m²/sec (Table A.2), so $\quad (T = 20°C)$

$$Re_L = \frac{VL}{\nu} = 27\,\frac{nm}{hr} \times 6076\,\frac{ft}{nm} \times 0.305\,\frac{m}{ft} \times \frac{hr}{3600\,s} \times 107\,m \times \frac{s}{1.05 \times 10^{-6}\,m^2} = 1.42 \times 10^9$$

Thus $\quad \dfrac{x_t}{L} = \dfrac{Re_{x_t}}{Re_L} = \dfrac{500{,}000}{1.42 \times 10^9} = 3.52 \times 10^{-4}$ or $x_t = 0.0352\%$ of L $\longleftarrow$ %L

Neglect laminar BL; assume flow is completely turbulent.

$$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}} = \frac{0.455}{(9.15)^{2.58}} = 0.00150 \;;\; A = wL = 34.6\,m \times 107\,m = 3.70 \times 10^3\,m^2$$

$$q = \tfrac{1}{2}\rho V^2 = \tfrac{1}{2} \times 1025\,\frac{kg}{m^3}\left(\frac{27(6076)(0.305)}{3600}\right)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 99.0\ kPa$$

$$F_D = C_D\, q\, A = 0.00150 \times 99.0 \times 10^3\,\frac{N}{m^2} \times 3.70 \times 10^3\,m^2 = 5.49 \times 10^5\ N$$ $\longleftarrow$ F_D

$$\mathcal{P} = F_D V = 5.49 \times 10^5\,N \times 27\,\frac{nm}{hr} \times 6076\,\frac{ft}{nm} \times 0.305\,\frac{m}{ft} \times \frac{hr}{3600\,s} = 7.63\ MW$$

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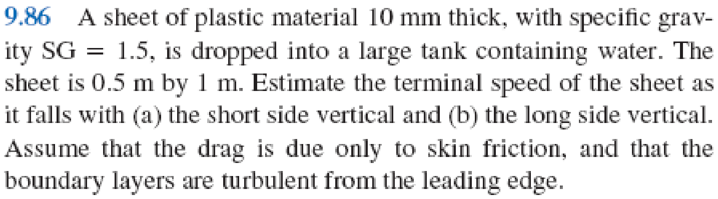
Problem 9.86

[3]



9.86 A sheet of plastic material 10 mm thick, with specific gravity SG = 1.5, is dropped into a large tank containing water. The sheet is 0.5 m by 1 m. Estimate the terminal speed of the sheet as it falls with (a) the short side vertical and (b) the long side vertical. Assume that the drag is due only to skin friction, and that the boundary layers are turbulent from the leading edge.

Given: Plastic sheet falling in water

Find: Terminal speed both ways

Solution:

Basic equations: $\Sigma F_y = 0$ for terminal speed $\quad C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A} \quad C_D = \dfrac{0.0742}{Re_L^{\frac{1}{5}}}$ (9.34) (assuming $5 \times 10^5 < Re_L < 10^7$)

$h = 10\cdot mm \qquad W = 1\cdot m \qquad L = 0.5\cdot m \qquad A = W\cdot L \qquad SG = 1.5$

From Table A.8 at 20ºC $\quad \nu = 1.01 \times 10^{-6}\cdot\dfrac{m^2}{s} \qquad \rho = 998\cdot\dfrac{kg}{m^3}$ for water

Hence $\quad F_D + F_{buoyancy} - W = 0 \qquad F_D = W - F_{buoyancy} = \rho\cdot g\cdot h\cdot A\cdot(SG - 1)$

Also
$$F_D = 2\cdot C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot V^2 = 2\cdot\frac{0.0742}{Re_L^{\frac{1}{5}}}\cdot A\cdot\frac{1}{2}\cdot\rho\cdot V^2 = \frac{0.0742}{\left(\frac{V\cdot L}{\nu}\right)^{\frac{1}{5}}}\cdot W\cdot L\cdot\rho\cdot V^2 = 0.0742\cdot W\cdot L^{\frac{4}{5}}\cdot\nu^{\frac{1}{5}}\cdot\frac{1}{2}\cdot\rho\cdot V^{\frac{9}{5}}$$

Note that we double F_D because we have two sides!

Hence
$$\rho_{H2O}\cdot g\cdot h\cdot W\cdot L\cdot(SG - 1) = 0.0742\cdot W\cdot L^{\frac{4}{5}}\cdot\nu^{\frac{1}{5}}\cdot\frac{1}{2}\cdot\rho\cdot V^{\frac{9}{5}}$$

Solving for V $\quad V = \left[\dfrac{g\cdot h\cdot(SG - 1)}{0.0742}\cdot\left(\dfrac{L}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{5}{9}} \qquad V = 3.41\dfrac{m}{s}$

Check the Reynolds number $Re_L = \dfrac{V\cdot L}{\nu} \qquad Re_L = 1.69 \times 10^6$ Hence Eq. 9.34 is reasonable

Repeating for $\quad L = 1\cdot m \qquad V = \left[\dfrac{g\cdot h\cdot(SG - 1)}{0.0742}\cdot\left(\dfrac{L}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{5}{9}} \qquad V = 3.68\dfrac{m}{s}$

Check the Reynolds number $Re_L = \dfrac{V\cdot L}{\nu} \qquad Re_L = 3.65 \times 10^6$ Eq. 9.34 is still reasonable

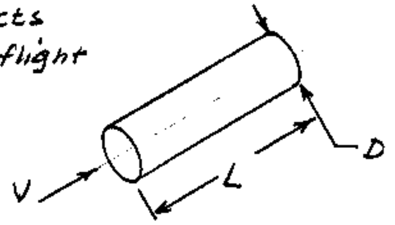
The short side vertical orientation falls more slowly because the largest friction is at the region of the leading edge (τ tails off as the boundary layer progresses); its leading edge area is larger. Note that neither orientation is likely - the plate will flip around in a chaotic manner

Problem 9.87

Given: 600-seat jet transport proposed by Airbus Industrie. Fuselage has length $L = 70$ m and diameter $D = 7.5$ m. Aircraft operates 14 hr per day, 6 days per week, cruising at $V = 257$ m/s ($M = 0.87$) at $z = 12$ km. The thrust specific fuel consumption (TSFC) is 0.06 kg/N·hr.

Find: (a) Estimate of skin friction drag on fuselage.
(b) Annual fuel saved by 1% reduction in drag by modifying surface.

Solution: Assume: (1) BL behaves as though on flat plate, $A = \pi D L = 1650\ m^2$
(2) Neglect compressibility effects
(3) All fuel consumed in cruise flight



Need Reynolds number

$$Re_L = \frac{\rho V L}{\mu}$$

From Table A.3, $T = 216.7\ K$ and $\rho/\rho_{SL} = 0.2546$; $\rho = 0.2546 \times 1.23 \frac{kg}{m^3} = 0.313\ kg/m^3$

From Eq. A.1, $\mu = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times (216.7)^{3/2} K^{3/2} \times \frac{1}{(110.4 + 216.7)K} = 1.42 \times 10^{-5} \frac{kg}{m \cdot s}$

Thus

$$Re_L = 0.313 \frac{kg}{m^3} \times 257 \frac{m}{s} \times 70\ m \times \frac{m \cdot s}{1.42 \times 10^{-5}\ kg} = 3.97 \times 10^8$$

From Eq. 9.35 ($Re_L < 10^9$),

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}} = \frac{0.455}{(\log 3.97 \times 10^8)^{2.58}} = 0.0177$$

and

$$F_D = C_D A \tfrac{1}{2} \rho V^2 = \frac{1}{2} \times 0.00177 \times 1650\ m^2 \times 0.313 \frac{kg}{m^3} \times (257)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 3.02 \times 10^4\ N$$

F_D

Then $\Delta F_D = 0.01\ F_D = 3.02 \times 10^2\ N = 302\ N$

and $\Delta F_C = \Delta F_D \times TSFC \times t$

$$\Delta F_C = 302\ N \times 0.06 \frac{kg}{N \cdot hr} \times 365 \frac{day}{yr} \times 14 \frac{hr}{day} \times \frac{6}{7} = 7.94 \times 10^4\ kg/yr$$

ΔF_C

The specific gravity of jet fuel (kerosine) is about 0.82 (Table A.2). Thus

$$\Delta \forall = \frac{\Delta F_C}{\rho} = \frac{\Delta F_C}{SG\ \rho_{H_2O}}$$

$$\Delta \forall = 7.94 \times 10^4 \frac{kg}{yr} \times \frac{m^3}{(0.82)\ 1000\ kg} \times \frac{ft^3}{(0.305)^3\ m^3} \times 7.48 \frac{gal}{ft^3} = 2.55 \times 10^4\ gal/yr$$

$$\Delta C \approx 2.55 \times 10^4 \frac{gal}{yr} \times \frac{\$1}{gal} \approx \$25{,}000$$

This is a substantial saving per aircraft. The cost saving for a fleet would be impressive.

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Problem 9.88

[4]

Given: Supertanker with 600,000 metric ton displacement.

Length, $L = 300\text{ m}$; Beam, $b = 80\text{ m}$; Draft, $D = 25\text{ m}$

Ship steams at 14 kt in seawater at 4°C

Estimate: (a) BL thickness at stern of ship.

(b) total skin-friction drag.

(c) power required to overcome skin-friction drag.

Solution:

Apply results of momentum integral analysis (Section 9-5.2) and correlations for drag coefficient (Section 9-7.1)

Computing equations:

$$\delta/x = \frac{0.382}{Re_x^{1/5}} \qquad (9.26)$$

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1610}{Re_L} \qquad (9.37b)$$

Assumptions: (1) boundary layers behave as on a flat plate.

(2) 1/7-power turbulent velocity profiles

For seawater at 4°C, $SG = 1.025$, $\nu = 1.05\,\nu_{water}$ (Table A.2)

At 4°C, $\nu_{water} = 1.55 \times 10^{-6}\text{ m}^2/\text{s}$ (Table A.8).

$$V = 14\,\frac{\text{nm}}{\text{hr}} \times \frac{1852\text{ m}}{\text{nm}} \times \frac{\text{hr}}{3600\text{ s}} = 7.20\text{ m/s}.$$

$$Re_L = \frac{VL}{\nu} = 7.20\,\frac{\text{m}}{\text{s}} \times 300\text{ m} \times \frac{1}{1.05 \times 1.55 \times 10^{-6}}\,\frac{\text{s}}{\text{m}^2} = 1.33 \times 10^{9}$$

Thus

$$\delta_L = \frac{0.382 \times 300\text{ m}}{(1.33 \times 10^9)^{0.20}} = 1.72\text{ m} \longleftarrow \delta_L$$

Model drag area as in Example Problem 9.5

$$A = (b + 2D)L = [80 + 2(25)]\text{ m} \times 300\text{ m} = 3.90 \times 10^4\text{ m}^2$$

$$C_D = \frac{0.455}{(\log 1.33 \times 10^9)^{2.58}} - \frac{1610}{Re_L} = 0.00151$$

$$F_D = C_D A \tfrac{1}{2}\rho V^2 = 0.00151 \times 3.90 \times 10^4\text{ m}^2 \times \tfrac{1}{2} \times (1.025) \times 10^3\,\frac{\text{kg}}{\text{m}^3} \times (7.20)^2\,\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$F_D = 1.56\text{ MN} \longleftarrow F_D$$

(This is skin-friction drag; wave drag cannot be estimated.)

The power is

$$P = F_D V = 1.56 \times 10^6\text{ N} \times 7.20\,\frac{\text{m}}{\text{s}} \times \frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}} = 11.2\text{ MW} \quad P$$

Problem 9.89

[4]

9.89 In Section 7-6 the wave resistance and viscous resistance on a model and prototype ship were discussed. For the prototype, L = 409 ft and A = 19,500 ft^2. From the data of Figs. 7.2 and 7.3, plot on one graph the wave, viscous, and total resistance (lbf) experienced by the prototype, as a function of speed. Plot a similar graph for the model. Discuss your results. Finally, plot the power (hp) required for the prototype ship to overcome the total resistance.

Given: "Resistance" data on a ship

Find: Plot of wave, viscous and total drag (protoype and model); Power required by prototype

Solution:

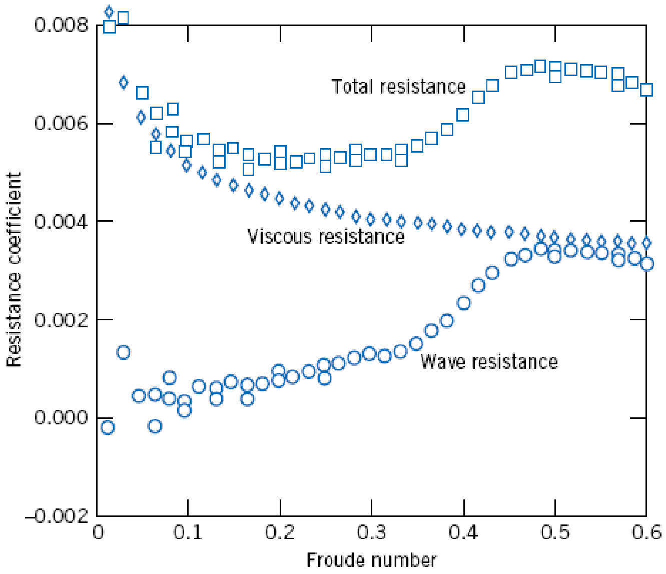


Fig. 7.2 Data from test of 1:80 scale model of U.S. Navy guided missile frigate *Oliver Hazard Perry* (FFG-7). (Data from U.S. Naval Academy Hydromechanics Laboratory, courtesy of Professor Bruce Johnson.)

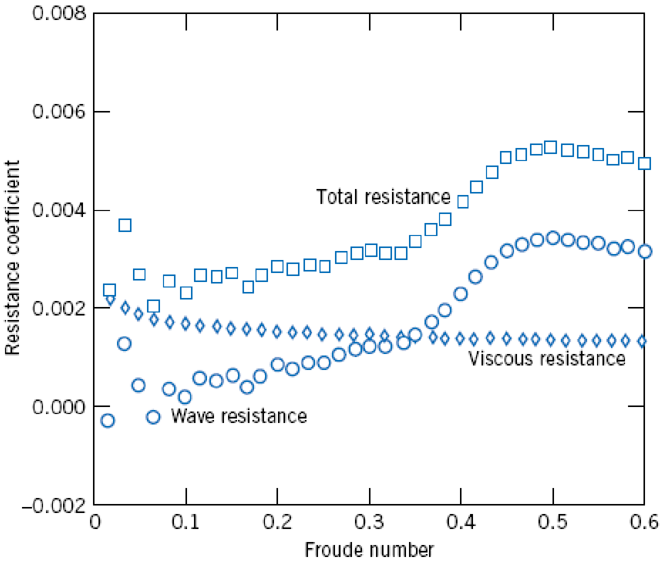


Fig. 7.3 Resistance of full-scale ship predicted from model test results. (Data from U.S. Naval Academy Hydromechanics Laboratory, courtesy of Professor Bruce Johnson.)

Governing equation: $$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot U^2\cdot A} \qquad (9.32)$$

$$Fr = \frac{U}{\sqrt{gL}}$$

From Eq. 9.32 $$F_D = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^2$$

This applies to each component of the drag (wave and viscous) as well as to the total

The power consumed is $$P = F_D\cdot U \qquad P = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^3$$

From the Froude number $$U = Fr\cdot\sqrt{gL}$$

The solution technique is: For each speed *Fr* value from the graph, compute *U*; compute the drag from the corresponding "resistance" value from the graph

Given data:

L_p = 409 ft

A_p = 19500 ft^2

L_m = 5.11 ft (1/80 scale)

A_m = 3.05 ft^2

SG = 1.025 (Table A.2)

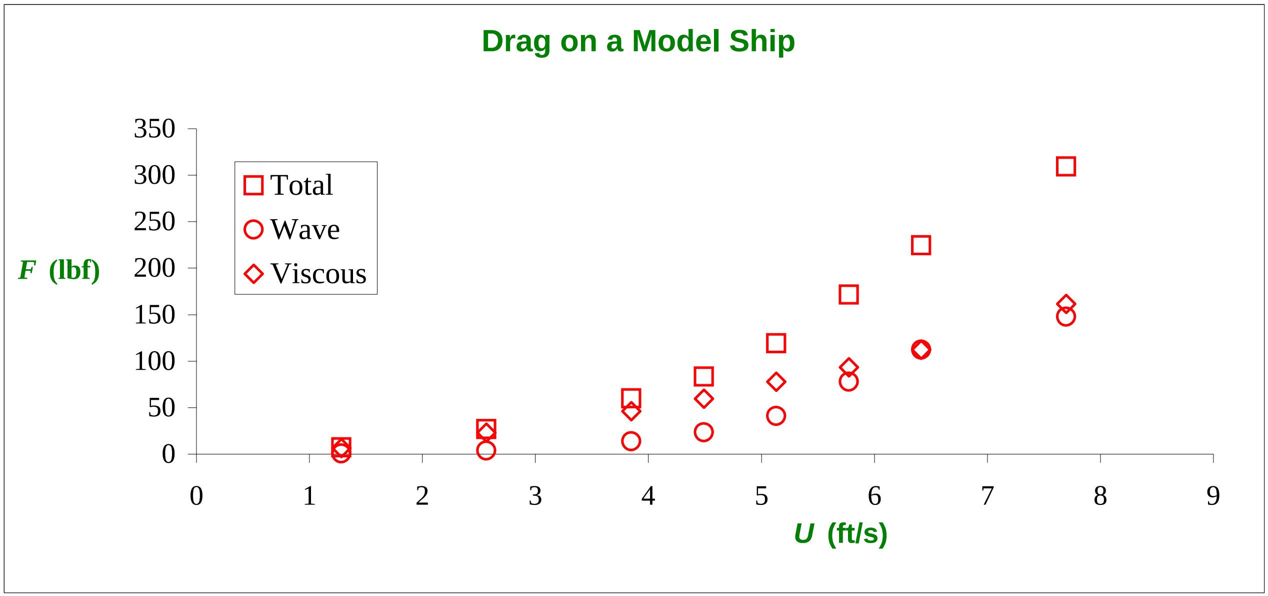
μ = 2.26E-05 $lbf.s/ft^2$ (Table A.2)

ρ = 1023 $slug/ft^3$

Computed results:

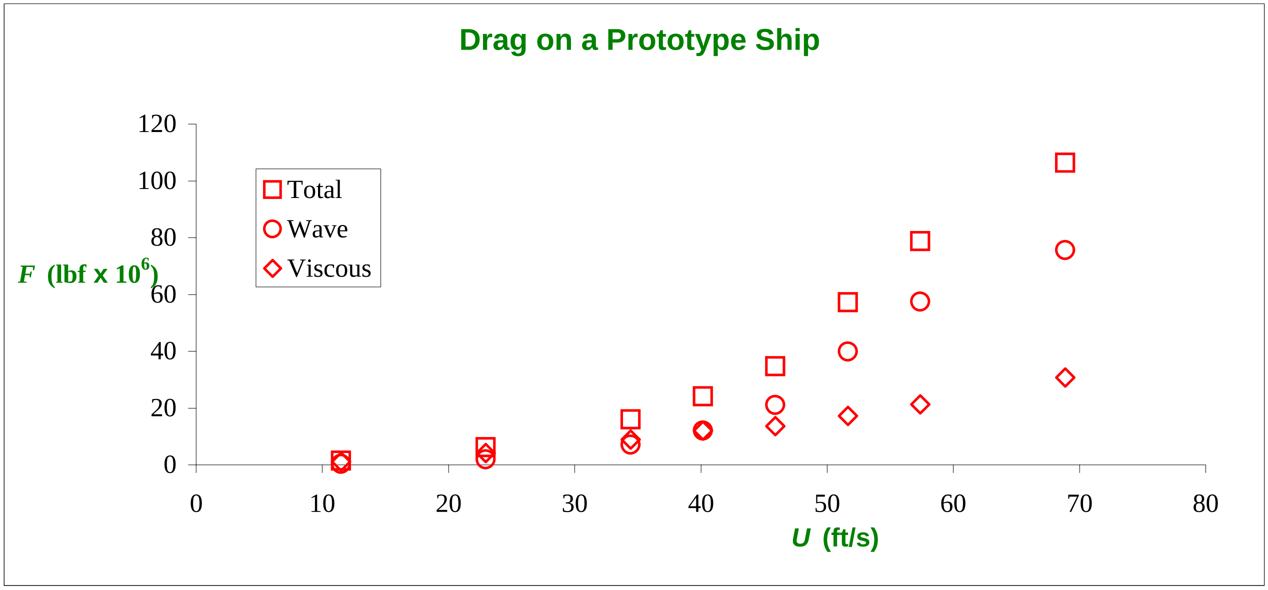
Model

Fr	**Wave "Resistance"**	**Viscous "Resistance"**	**Total "Resistance"**	***U* (ft/s)**	**Wave Drag (lbf)**	**Viscous Drag (lbf)**	**Total Drag (lbf)**
0.10	0.00050	0.0052	0.0057	1.28	0.641	6.67	7.31
0.20	0.00075	0.0045	0.0053	2.57	3.85	23.1	26.9
0.30	0.00120	0.0040	0.0052	3.85	13.9	46.2	60.0
0.35	0.00150	0.0038	0.0053	4.49	23.6	59.7	83.3
0.40	0.00200	0.0038	0.0058	5.13	41.0	78.0	119
0.45	0.00300	0.0036	0.0066	5.77	77.9	93.5	171
0.50	0.00350	0.0035	0.0070	6.42	112	112	224
0.60	0.00320	0.0035	0.0067	7.70	148	162	309



Prototype

Fr	Wave "Resistance"	Viscous "Resistance"	Total "Resistance"	*U* (ft/s)	Wave Drag (lbf x 10^6)	Viscous Drag (lbf x 10^6)	Total Drag (lbf x 10^6)
0.10	0.00050	0.0017	0.0022	11.5	0.328	1.12	1.44
0.20	0.00075	0.0016	0.0024	23.0	1.97	4.20	6.17
0.30	0.00120	0.0015	0.0027	34.4	7.09	8.87	16.0
0.35	0.00150	0.0015	0.0030	40.2	12.1	12.1	24.1
0.40	0.00200	0.0013	0.0033	45.9	21.0	13.7	34.7
0.45	0.00300	0.0013	0.0043	51.6	39.9	17.3	57.2
0.50	0.00350	0.0013	0.0048	57.4	57.5	21.3	78.8
0.60	0.00320	0.0013	0.0045	68.9	75.7	30.7	106



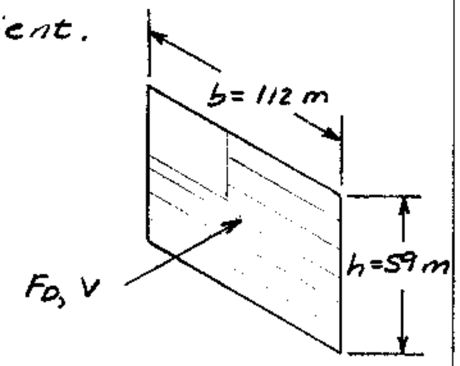
For the prototype wave resistance is a much more significant factor at high speeds!

Problem 9.90

Given: Flag, 59 m high and 112 m wide, mounted vertically.

Find: Force on flag in 16 km/hr wind. Was failure a surprise?

Solution: Apply definition of drag coefficient.



Computing equation: $F_D = C_D \frac{1}{2}\rho V^2 A$

Assumptions: (1) Flag acts as flat plate
(2) Standard air

The aspect ratio is $\frac{b}{h} = \frac{112\,m}{59\,m} = 1.9.$

From Fig. 9.10, $C_D \simeq 1.15$. Then

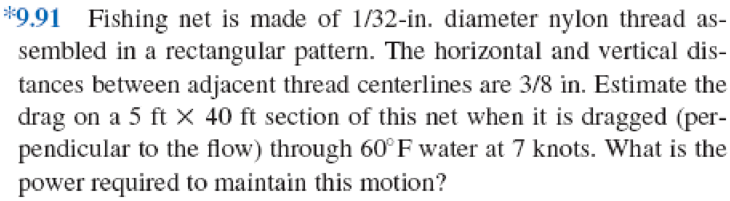
$$F_D = 1.15 \times \frac{1}{2} \times 1.23\,\frac{kg}{m^3}\left(16\times10^3\,\frac{m}{hr}\times\frac{hr}{3600\,s}\right)^2 112\,m \times 59\,m \times \frac{N\cdot s^2}{kg\cdot m}$$

$$F_D = 92.3\ kN \longleftarrow F_D$$

The flag failure should have been expected. This is a large force.

Problem 9.91

[3]



*9.91 Fishing net is made of 1/32-in. diameter nylon thread assembled in a rectangular pattern. The horizontal and vertical distances between adjacent thread centerlines are 3/8 in. Estimate the drag on a 5 ft × 40 ft section of this net when it is dragged (perpendicular to the flow) through 60°F water at 7 knots. What is the power required to maintain this motion?

Given: Fishing net

Find: Drag; Power to maintain motion

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$

We convert the net into an equivalent cylinder (we assume each segment does not interfere with its neighbors)

$L = 40\cdot ft$ $W = 5\cdot ft$ $d = \dfrac{1}{32}\cdot in$ Spacing: $D = \dfrac{3}{8}\cdot in$ $V = 7\cdot knot$ $V = 11.8\,\dfrac{ft}{s}$

Total number of threads of length L is $n_1 = \dfrac{W}{D}$ $n_1 = 160$ Total length $L_1 = n_1\cdot L$ $L_1 = 6400\,ft$

Total number of threads of length W is $n_2 = \dfrac{L}{D}$ $n_2 = 1280$ Total length $L_2 = n_2\cdot W$ $L_2 = 6400\,ft$

Total length of thread $L_T = L_1 + L_2$ $L_T = 12800\,ft$ $L_T = 2.42\,mile$ A lot!

The frontal area is then $A = L_T\cdot d$ $A = 33.3\,ft^2$ Note that $L\cdot W = 200\,ft^2$

From Table A.7 $\rho = 1.94\cdot\dfrac{slug}{ft^3}$ $\nu = 1.21\times 10^{-5}\cdot\dfrac{ft^2}{s}$

The Reynolds number is $Re_d = \dfrac{V\cdot d}{\nu}$ $Re_d = 2543$

For a cylinder in a crossflow at this Reynolds number, from Fig. 9.13, approximately $C_D = 0.8$

Hence $F_D = C_D\cdot\dfrac{1}{2}\cdot\rho\cdot V^2\cdot A$ $F_D = 3611\,lbf$

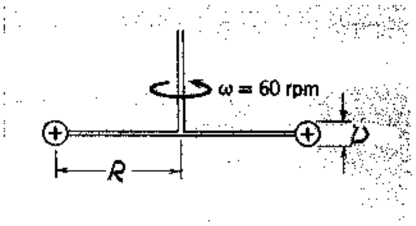
The power required is $P = F_D\cdot V$ $P = 42658\,\dfrac{ft\cdot lbf}{s}$ $P = 77.6\,hp$

Problem 9.92

[2]

Given: Rotary mixer, constructed as shown:
The mixer is rotated in brine, SG = 1.1

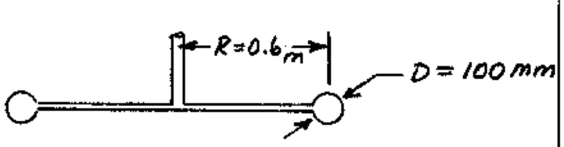
Neglect motion of liquid and drag on rods.



Find: (a) Torque
(b) Horsepower required to drive mixer.

Solution: Use drag coefficient data from Table 9.2, and

Basic equations: $T = 2RF_D$



$$\mathbb{P} = T\omega$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

Thus $V = R\omega = 0.6\,m \times 60\frac{rev}{min} \times 2\pi\frac{rad}{rev} \times \frac{min}{60\,s} = 3.77\ m/s$

From Table 9.2, $C_D = 1.17$ for a disk, so neglecting drag of rods,

$$F_D = \frac{1}{2}\rho V^2 \frac{\pi D^2}{4} C_D = \frac{\pi}{8} \times (1.1)999\frac{kg}{m^3} \times (3.77)^2\frac{m^2}{s^2} \times (0.1)^2 m^2 \times 1.17 \times \frac{N\cdot s^2}{kg\cdot m} = 71.8\ N$$

Then

$$T = 2RF_D = 2 \times 0.6\,m \times 71.8\ N = 86.2\ N\cdot m \longleftarrow$$ T

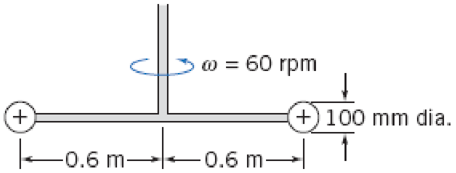
and

$$\mathbb{P} = T\omega = 86.2\ N\cdot m \times 60\frac{rev}{min} \times 2\pi\frac{rad}{rev} \times \frac{min}{60\,s} \times \frac{W\cdot s}{N\cdot m} = 542\ W \longleftarrow$$ $\mathbb{P}$

Problem 9.93

[3]

9.93 As a young design engineer you decide to make the rotary mixer look more "cool" by replacing the disks with rings. The rings may have the added benefit of making the mixer mix more effectively. If the mixer absorbs 350 W at 60 rpm, redesign the device. There is a design constraint that the outer diameter of the rings not exceed 125 mm.



Given: Data on a rotary mixer

Find: New design dimensions

Solution:

The given data or available data is

$R = 0.6\cdot m$ $\quad P = 350\cdot W$ $\quad \omega = 60\cdot rpm$ $\quad \rho = 1099\cdot\frac{kg}{m^3}$

For a ring, from Table 9.3 $\quad C_D = 1.2$

The torque at the specified power and speed is

$$T = \frac{P}{\omega} \qquad T = 55.7\,N\cdot m$$

The drag on *each* ring is then $\quad F_D = \frac{1}{2}\cdot\frac{T}{R} \qquad F_D = 46.4\,N$

The linear velocity of each ring is $\quad V = R\cdot\omega \qquad V = 3.77\,\frac{m}{s}$

The drag and velocity of each ring are related using the definition of drag coefficient

$$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$$

Solving for the ring area $\quad A = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot C_D} \qquad A = 4.95\times 10^{-3}\,m^2$

But $\quad A = \frac{\pi}{4}\cdot\left(d_o{}^2 - d_i{}^2\right)$

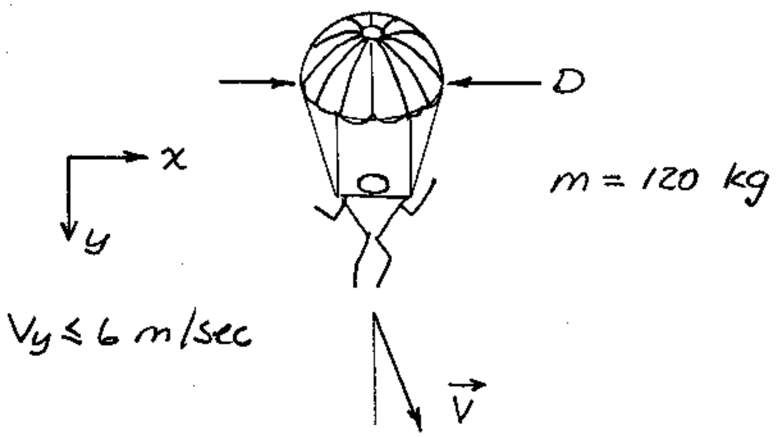
The outer diameter is $\quad d_o = 125\cdot mm$

Hence the inner diameter is $\quad d_i = \sqrt{d_o{}^2 - \frac{4\cdot A}{\pi}} \qquad d_i = 96.5\,mm$

Problem 9.94

[2]

Given: Parachute and man with total mass, $m = 120$ kg.



Find: Minimum diameter, D.

Solution: Apply Newton's second law of motion, definition of drag coefficient.

Computing equations: $\Sigma F_y = mg - F_D = ma_y$; $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

Assumptions: (1) Standard air
(2) Parachute behaves as open hemisphere
(3) V_y = constant; $a_y = 0$

Then $\Sigma F_y = mg - F_D = 0$ or $F_D = C_D A \frac{1}{2}\rho V_y^2 = mg$

$$A = \frac{\pi D^2}{4} \quad \text{so} \quad mg = C_D \frac{\pi D^2}{8} \rho V_y^2$$

$$D^2 = \frac{8mg}{\pi C_D \rho V_y^2} \quad \text{and} \quad D = \sqrt{\frac{8mg}{\pi C_D \rho V_y^2}}$$

From Table 9.3, $C_D = 1.42$ for an open hemisphere.

$$D = \left[\frac{8}{\pi} \times 120\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times \frac{1}{1.42} \times \frac{\text{m}^3}{1.23\,\text{kg}} \times \frac{\text{s}^2}{(6)^2\,\text{m}^2}\right]^{1/2}$$

$D = 6.90$ m

D

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Problem 9.95

[3]

9.95 An emergency braking parachute system on a military aircraft consists of a large parachute of diameter 6 m. If the airplane mass is 8500 kg, and it lands at 400 km/hr, find the time and distance at which the airplane is slowed to 100 km/hr by the parachute alone. Plot the aircraft speed versus distance and versus time. What is the maximum "g-force" experienced? An engineer proposes that less space would be taken up by replacing the large parachute with three non-interfering parachutes each of diameter 3.75 m. What effect would this have on the time and distance to slow to 100 km/hr?

Given: Data on airplane and parachute

Find: Time and distance to slow down; plot speed against distance and time; maximum "g"'s

Solution:

Newton's second law for the aircraft is $$M\cdot\frac{dV}{dt} = -C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$$

where A and C_D are the single parachute area and drag coefficient

Separating variables $$\frac{dV}{V^2} = -\frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot dt$$

Integrating, with IC $V = V_i$ $$V(t) = \frac{V_i}{1+\frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t} \qquad (1)$$

Integrating again with respect to t $$x(t) = \frac{2\cdot M}{C_D\cdot\rho\cdot A}\cdot\ln\left(1+\frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t\right) \qquad (2)$$

Eliminating t from Eqs. 1 and 2 $$x = \frac{2\cdot M}{C_D\cdot\rho\cdot A}\cdot\ln\left(\frac{V_i}{V}\right) \qquad (3)$$

To find the time and distance to slow down to 100 km/hr, Eqs. 1 and 3 are solved with V = 100 km/hr (or use *Goal Seek*)

The "g"'s are given by $$\frac{\frac{dV}{dt}}{g} = \frac{-C_D\cdot\rho\cdot A\cdot V^2}{2\cdot M\cdot g}$$ which has a maximum at the initial instant ($V = V_i$)

For three parachutes, the analysis is the same except A is replaced with $3A$. leading to

$$V(t) = \frac{V_i}{1+\frac{3\cdot C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t}$$

$$x(t) = \frac{2\cdot M}{3\cdot C_D\cdot\rho\cdot A}\cdot\ln\left(1+\frac{3\cdot C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t\right)$$

Given data:

M =	8500	kg
V_i =	400	km/hr
V_f =	100	km/hr
C_D =	1.42	(Table 9.3)
ρ =	1.23	kg/m^3
Single: D =	6	m

Triple: D = 3.75 m

Computed results:

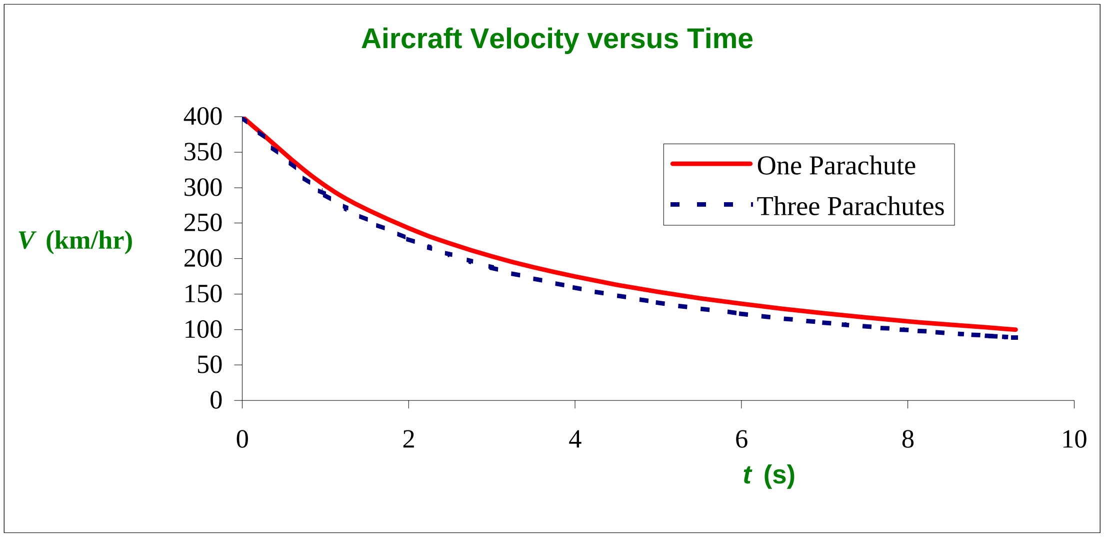
A = 28.3 m^2

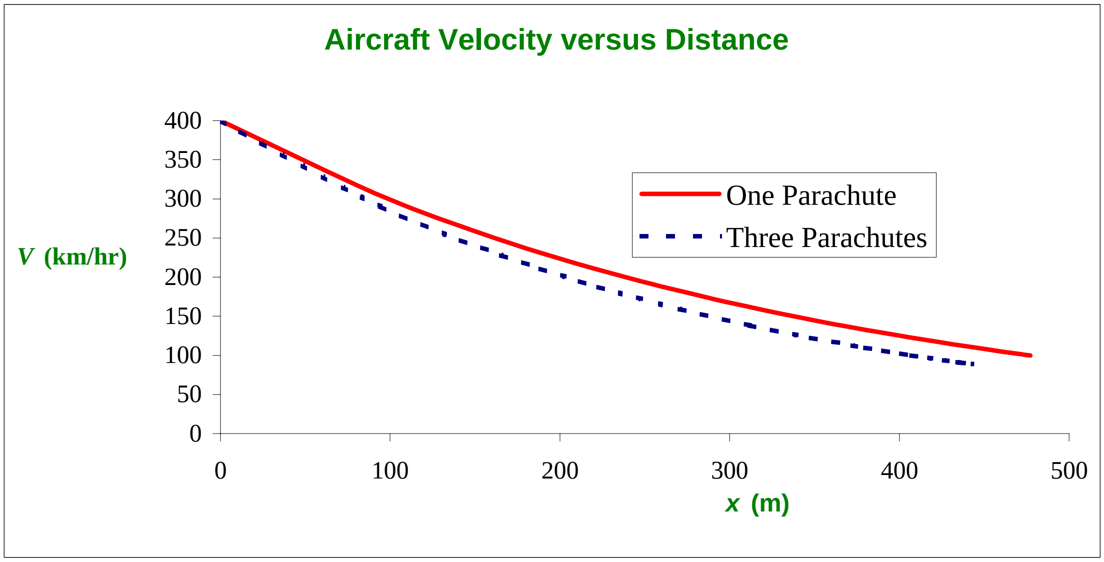
t (s)	x (m)	V (km/hr)
0.0	0.0	400
1.0	96.3	302
2.0	171	243
3.0	233	203
4.0	285	175
5.0	331	153
6.0	371	136
7.0	407	123
8.0	439	112
9.0	469	102
9.29	477	100

A = 11.0 m^2

t (s)	x (m)	V (km/hr)
0.0	0.0	400
1.0	94.2	290
2.0	165	228
3.0	223	187
4.0	271	159
5.0	312	138
6.0	348	122
7.0	380	110
7.93	407	100
9.0	436	91
9.3	443	89

"*g* "'s = -3.66 Max





Problem 9.96

[3]

9.96 As a young design engineer you are asked to design an emergency braking parachute system for use with a military aircraft of mass 9500 kg. The plane lands at 350 km/hr, and the parachute system alone must slow the airplane to 100 km/hr in less than 1200 m. Find the minimum diameter required for a single parachute, and for three non-interfering parachutes. Plot the airplane speed versus distance and versus time. What is the maximum "g-force" experienced?

Given: Data on airplane landing

Find: Single and three-parachute sizes; plot speed against distance and time; maximum "g"s

Solution:

Newton's second law for the aircraft is $M\cdot\frac{dV}{dt} = -C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$

where A and CD are the single parachute area and drag coefficient

Separating variables $\frac{dV}{V^2} = -\frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot dt$

Integrating, with IC $V = V_i$ $$V(t) = \frac{V_i}{1 + \frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t} \qquad (1)$$

Integrating again with respect to t $$x(t) = \frac{2\cdot M}{C_D\cdot\rho\cdot A}\cdot\ln\left(1 + \frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t\right) \qquad (2)$$

Eliminating t from Eqs. 1 and 2 $$x = \frac{2\cdot M}{C_D\cdot\rho\cdot A}\cdot\ln\left(\frac{V_i}{V}\right) \qquad (3)$$

To find the minimum parachute area we must solve Eq 3 for A with $x = x_f$ when $V = V_f$

$$A = \frac{2\cdot M}{C_D\cdot\rho\cdot x_f}\cdot\ln\left(\frac{V_i}{V_f}\right) \qquad (4)$$

For three parachutes, the analysis is the same except A is replaced with $3A$, leading to

$$A = \frac{2\cdot M}{3\cdot C_D\cdot\rho\cdot x_f}\cdot\ln\left(\frac{V_i}{V_f}\right) \qquad (5)$$

The "g"'s are given by $$\frac{\frac{dV}{dt}}{g} = \frac{-C_D\cdot\rho\cdot A\cdot V^2}{2\cdot M\cdot g}$$

which has a maximum at the initial instant ($V = V_i$)

Given data:

$M =$ 9500 kg

$V_i =$ 350 km/hr

$V_f =$ 100 km/hr

$x_f =$ 1200 m

$C_D =$ 1.42 (Table 9.3)

$\rho =$ 1.23 kg/m^3

Computed results:

Single:

A = 11.4 m^2

D = 3.80 m

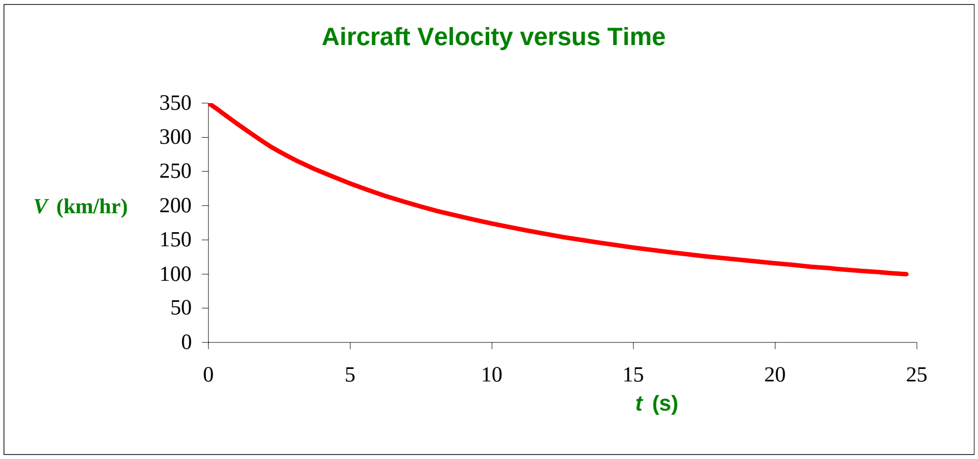
Triple:

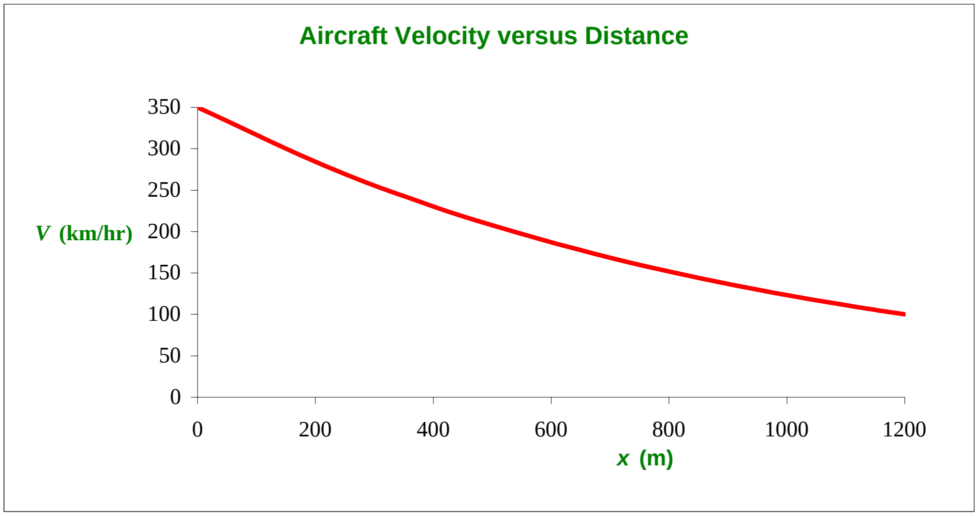
A = 3.8 m^2

D = 2.20 m

"*g* "'s = -1.01 Max

t (s)	*x* (m)	*V* (km/hr)
0.00	0.0	350
2.50	216.6	279
5.00	393.2	232
7.50	542.2	199
10.0	671.1	174
12.5	784.7	154
15.0	886.3	139
17.5	978.1	126
20.0	1061.9	116
22.5	1138.9	107
24.6	1200.0	100


Aircraft Velocity versus Time


Aircraft Velocity versus Distance

Problem 9.97

[2]

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Given: Windmills to be made from surplus 55-gal oil drums.

For a drum, $D = 24$ in., $H = 29$ in.

Find: Which configuration would be better, why, and by how much?

Solution: Sum moments about pivot, neglecting friction, interference.

Configuration A:

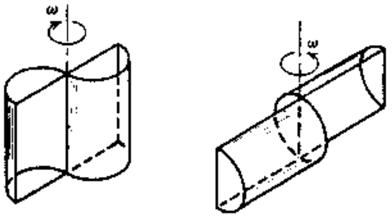
$$\Sigma M = \frac{D}{2} F_u - \frac{D}{2} F_d = \frac{D}{2}(F_u - F_d)$$

$$\Sigma M = \frac{D}{2}(C_{Du} - C_{Dd}) A \frac{1}{2} \rho V^2$$

Configuration B:

$$\Sigma M = \frac{H}{2} F_u - \frac{H}{2} F_d = \frac{H}{2}(F_u - F_d)$$

$$\Sigma M = \frac{H}{2}(C_{Du} - C_{Dd}) A \frac{1}{2} \rho V^2$$



Configuration Ⓐ Ⓑ

Performance of B will be better because $H > D$.

$$\frac{H - D}{D} = \frac{29 - 24}{24} = 0.208 \quad \text{or} \quad 20.8 \text{ percent improvement!}$$

Problem 9.98

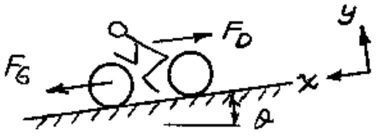
[2]

Given: Bike and rider with $M = 100\ \text{kg}$, $A = 0.46\ \text{m}^2$, and negligible rolling resistance, has terminal speed, $V_t = 15\ \text{m/s}$, on a hill with 8 percent grade. Drag coefficient estimated as $C_D \approx 1.2$.

Find: (a) Verify this calculation of drag coefficient.

(b) Distance for bike and rider to slow from 15 to 10 m/sec after reaching level road.

Solution: Treat the bike and rider as a system. From a free-body diagram,



$$\Sigma F_x = F_G - F_D = mg\sin\theta - C_D A \tfrac{1}{2}\rho V^2 = ma_x \qquad \theta = \tan^{-1}(0.08) = 4.57°$$

At terminal speed, $a_x = 0$. Then $mg\sin\theta = C_D A \frac{1}{2}\rho V_t^2$, so

$$C_D = \frac{2mg\sin\theta}{A\rho V_t^2} = 2 \times 100\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times \sin 4.57° \times \frac{1}{0.46\,\text{m}^2} \times \frac{\text{m}^3}{1.23\,\text{kg}} \times \frac{\text{s}^2}{(15)^2\,\text{m}^2} = 1.23$$

C_D ⟵

Thus $C_D \approx 1.2$ is correct.

On a flat surface, $\Sigma F_x = -F_D = -C_D A \frac{1}{2}\rho V^2 = m\frac{dV}{dt} = mV\frac{dV}{ds}$

Thus $\quad mV\frac{dV}{ds} = -C_D A \frac{1}{2}\rho V^2$

or

$$ds = -\frac{2m}{C_D\rho A}\frac{dV}{V}$$

Integrating

$$s - \cancel{s_0}^{\,0} = \int_{s_0}^{s} ds = -\frac{2m}{C_D\rho A}\int_{V_0}^{V}\frac{dV}{V} = -\frac{2m}{C_D\rho A}\ln V\Big]_{V_0}^{V} = -\frac{2m}{C_D\rho A}\ln\left(\frac{V}{V_0}\right)$$

$$s = -2 \times 100\,\text{kg} \times \frac{1}{1.23} \times \frac{\text{m}^3}{1.23\,\text{kg}} \times \frac{1}{0.46\,\text{m}^2}\ln\frac{10}{15} = 117\ \text{m}$$

s ⟵

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Problem 9.99

[2]

Given: Ballistic data for 44 magnum revolver bullet:

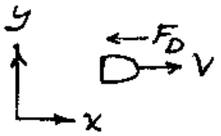
$$\left.\begin{array}{l} V_i = 250 \text{ m/s} \\ V_f = 210 \text{ m/s} \end{array}\right\} \text{over } \Delta x = 150 \text{ m} \qquad \begin{array}{l} D = 11.2 \text{ mm} \\ m = 15.6 \text{ g} \end{array}$$

Find: Evaluate average drag coefficient.

Solution: Apply Newton's second law, definition of drag coefficient.

Computing equation: $F_D = \bar{C}_D A \frac{1}{2} \rho V^2$

Basic equation: $\Sigma F_x = m a_x = m \frac{dV}{dt} = m V \frac{dV}{dx}$



From the free-body diagram, $\Sigma F_x = -F_D$, so

$$m V \frac{dV}{dx} = -F_D = -\bar{C}_D A \frac{1}{2} \rho V^2$$

Thus

$$\frac{dV}{V} = -\frac{\bar{C}_D A \rho}{2m} dx$$

Integrating

$$\int_{V_i}^{V_f} \frac{dV}{V} = \ln V \Big]_{V_i}^{V_f} = \ln \frac{V_f}{V_i} = -\frac{\bar{C}_D A \rho}{2m} \Delta x$$

Solving, using density of standard air,

$$\bar{C}_D = -\frac{2m}{\rho A \Delta x} \ln \frac{V_f}{V_i}$$

$$= -2 \times 0.0156 \text{ kg} \times \frac{\text{m}^3}{1.23 \text{ kg}} \times \frac{4}{\pi (0.0112)^2 \text{ m}^2} \times \frac{1}{150 \text{ m}} \ln \left(\frac{210}{250}\right)$$

$$\bar{C}_D = 0.299$$

$\bar{C}_D$

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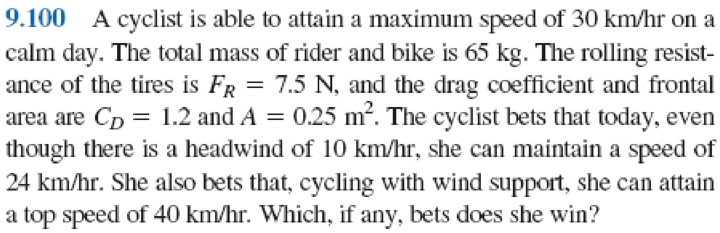
Problem 9.100

[2]



9.100 A cyclist is able to attain a maximum speed of 30 km/hr on a calm day. The total mass of rider and bike is 65 kg. The rolling resistance of the tires is $F_R = 7.5$ N, and the drag coefficient and frontal area are $C_D = 1.2$ and $A = 0.25$ m^2. The cyclist bets that today, even though there is a headwind of 10 km/hr, she can maintain a speed of 24 km/hr. She also bets that, cycling with wind support, she can attain a top speed of 40 km/hr. Which, if any, bets does she win?

Given: Data on cyclist performance on a calm day

Find: Performance hindered and aided by wind

Solution:

The given data or available data is

$F_R = 7.5\cdot\text{N}$ $\quad M = 65\cdot\text{kg}$ $\quad A = 0.25\cdot\text{m}^2$

$C_D = 1.2$ $\quad \rho = 1.23\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad V = 30\cdot\frac{\text{km}}{\text{hr}}$

The governing equation is $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 12.8\,\text{N}$

The power steady power generated by the cyclist is

$$P = \left(F_D + F_R\right)\cdot V \qquad P = 169\,\text{W} \qquad P = 0.227\,\text{hp}$$

Now, with a headwind we have $\quad V_w = 10\cdot\frac{\text{km}}{\text{hr}}$ $\quad V = 24\cdot\frac{\text{km}}{\text{hr}}$

The aerodynamic drag is greater because of the greater effective wind speed

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D \qquad F_D = 16.5\,\text{N}$$

The power required is that needed to overcome the total force $F_D + F_R$, moving at the cyclist's speed

$$P = V\cdot\left(F_D + F_R\right) \qquad P = 160\,\text{W}$$

This is less than the power she can generate — She wins the bet!

With the wind supporting her the effective wind speed is substantially lower

$$V_W = 10\cdot\frac{\text{km}}{\text{hr}} \qquad V = 40\cdot\frac{\text{km}}{\text{hr}}$$

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_W\right)^2\cdot C_D \qquad F_D = 12.8\,\text{N}$$

The power required is that needed to overcome the total force $F_D + F_R$, moving at the cyclist's speed

$$P = V\cdot\left(F_D + F_R\right) \qquad P = 226\,\text{W}$$

This is more than the power she can generate — She loses the bet

Problem 9.101

9.101 Consider the cyclist in Problem 9.100. Determine the maximum speeds she is actually able to attain today (with the 10 km/hr wind) cycling into the wind, and cycling with the wind. If she were to replace the tires with high-tech ones that had a rolling resistance of only 3.5 N, determine her maximum speed on a calm day, cycling into the wind, and cycling with the wind. If she in addition attaches an aerodynamic fairing that reduces the drag coefficient to $C_D = 0.9$, what will be her new maximum speeds?

Given:

Data on cyclist performance on a calm day

Find:

Performance hindered and aided by wind; repeat with high-tech tires; with fairing

Solution:

The given data or available data is

$F_R = 7.5\cdot N$ $\quad M = 65\cdot kg$ $\quad A = 0.25\cdot m^2$

$C_D = 1.2$ $\quad \rho = 1.23\cdot\frac{kg}{m^3}$ $\quad V = 30\cdot\frac{km}{hr}$

The governing equation is $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \quad F_D = 12.8\,N$

Power steady power generated by the cyclist is $\quad P = \left(F_D + F_R\right)\cdot V \quad P = 169\,W \quad P = 0.227\cdot hp$

Now, with a headwind we have $\quad V_w = 10\cdot\frac{km}{hr}$

The aerodynamic drag is greater because of the greater effective wind speed

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D \qquad (1)$$

The power required is that needed to overcome the total force $F_D + F_R$, moving at the cyclist's speed is

$$P = V\cdot\left(F_D + F_R\right) \qquad (2)$$

Combining Eqs 1 and 2 we obtain an expression for the cyclist's maximum speed V cycling into a headwind (where $P = 169$ W is the cyclist's power)

Cycling into the wind: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D\right]\cdot V \qquad (3)$$

This is a cubic equation for V; it can be solved analytically, or by iterating. It is convenient to use *Excel*'s *Goal Seek* (or *Solver*). From the associated *Excel* workbook

From *Solver* $\quad V = 24.7\cdot\frac{km}{hr}$

By a similar reasoning:

Cycling with the wind: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D\right]\cdot V \qquad (4)$$

From *Solver* $V = 35.8 \cdot \frac{km}{hr}$

With improved tires $F_R = 3.5 \cdot N$

Maximum speed on a calm day is obtained from $P = \left(F_R + \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D\right) \cdot V$

This is a again a cubic equation for *V*; it can be solved analytically, or by iterating. It is convenient to use *Excel*'s *Goal Seek* (or *Solver*). From the associated *Excel* workbook

From *Solver* $V = 32.6 \cdot \frac{km}{hr}$

Equations 3 and 4 are repeated for the case of improved tires

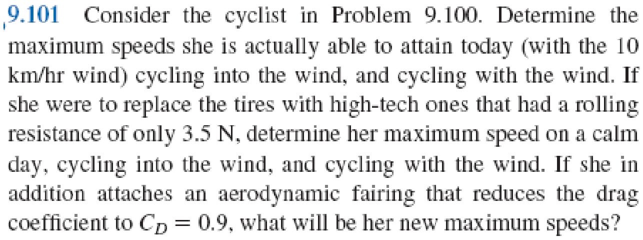
From *Solver* Against the wind $V = 26.8 \cdot \frac{km}{hr}$ With the wind $V = 39.1 \cdot \frac{km}{hr}$

For improved tires and fairing, from *Solver*

$V = 35.7 \cdot \frac{km}{hr}$ Against the wind $V = 29.8 \cdot \frac{km}{hr}$ With the wind $V = 42.1 \cdot \frac{km}{hr}$

Problem 9.101 (In Excel)

[3]



9.101 Consider the cyclist in Problem 9.100. Determine the maximum speeds she is actually able to attain today (with the 10 km/hr wind) cycling into the wind, and cycling with the wind. If she were to replace the tires with high-tech ones that had a rolling resistance of only 3.5 N, determine her maximum speed on a calm day, cycling into the wind, and cycling with the wind. If she in addition attaches an aerodynamic fairing that reduces the drag coefficient to $C_D = 0.9$, what will be her new maximum speeds?

Given: Data on cyclist performance on a calm day

Find: Performance hindered and aided by wind; repeat with high-tech tires; with fairing

Solution:

Given data:

$F_R =$ 7.5 N

$M =$ 65 kg

$A =$ 0.25 m^2

$C_D =$ 1.2

$\rho =$ 1.23 kg/m^3

$V =$ 30 km/hr

$V_w =$ 10 km/hr

Computed results:

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad F_D = 12.8 \text{ N}$$

$$P = \left(F_D + F_R\right)\cdot V \qquad P = 169 \text{ W}$$

Cycling into the wind: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D\right]\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	24.7

Cycling with the wind: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D\right]\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	35.8

With improved tires:

F_R = 3.5 N

$$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D\right)\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	32.6

Cycling into the wind:

$$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D\right]\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	26.8

Cycling with the wind:

$$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D\right]\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	39.1

With improved tires and fairing:

F_R = 3.5 N

C_D = 0.9

$$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D\right)\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	35.7

Cycling into the wind:

$$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D\right]\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	29.8

Cycling with the wind:

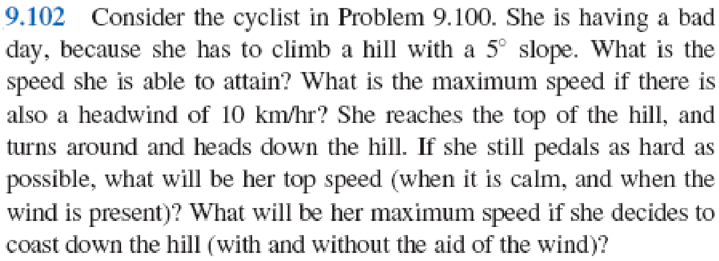
$$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D\right]\cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	42.1

Problem 9.102

[3]



9.102 Consider the cyclist in Problem 9.100. She is having a bad day, because she has to climb a hill with a 5° slope. What is the speed she is able to attain? What is the maximum speed if there is also a headwind of 10 km/hr? She reaches the top of the hill, and turns around and heads down the hill. If she still pedals as hard as possible, what will be her top speed (when it is calm, and when the wind is present)? What will be her maximum speed if she decides to coast down the hill (with and without the aid of the wind)?

Given: Data on cyclist performance on a calm day

Find: Performance on a hill with and without wind

Solution:

The given data or available data is

$F_R = 7.5\cdot N$ $\qquad M = 65\cdot kg$ $\qquad A = 0.25\cdot m^2$

$C_D = 1.2$ $\qquad \rho = 1.23\cdot\frac{kg}{m^3}$ $\qquad V = 30\cdot\frac{km}{hr}$

The governing equation is $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad F_D = 12.8\,N$

Power steady power generated by the cyclist is $\quad P = \left(F_D + F_R\right)\cdot V \qquad P = 169\,W \qquad P = 0.227\,hp$

Riding up the hill (no wind) $\quad \theta = 5\cdot deg$

For steady speed the cyclist's power is consumed by working against the net force (rolling resistance, darg, and gravity)

Cycling up the hill: $\quad P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D + M\cdot g\cdot\sin(\theta)\right)\cdot V$

This is a cubic equation for the speed which can be solved analytically, or by iteration, or using *Excel*'s *Goal Seek* or *Solver*. The solution is obtained from the associated *Excel* workbook

From *Solver* $\quad V = 9.47\cdot\frac{km}{hr}$

Now, with a headwind we have $\quad V_w = 10\cdot\frac{km}{hr}$

The aerodynamic drag is greater because of the greater effective wind speed

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D$$

The power required is that needed to overcome the total force (rolling resistance, drag, and gravity) moving at the cyclist's speed is

Uphill against the wind: $\quad P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D + M\cdot g\cdot\sin(\theta)\right]\cdot V$

This is again a cubic equation for *V*

From *Solver* $\quad V = 8.94\cdot\frac{km}{hr}$

Pedalling downhill (no wind) gravity helps increase the speed; the maximum speed is obtained from

Cycling down the hill: $$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D - M\cdot g\cdot \sin(\theta)\right)\cdot V$$

This cubic equation for *V* is solved in the associated *Excel* workbook

From *Solver* $$V = 63.6\cdot\frac{km}{hr}$$

Pedalling downhill (wind assisted) gravity helps increase the speed; the maximum speed is obtained from

Wind-assisted downhill: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D - M\cdot g\cdot \sin(\theta)\right]\cdot V$$

This cubic equation for *V* is solved in the associated *Excel* workbook

From *Solver* $$V = 73.0\cdot\frac{km}{hr}$$

Freewheeling downhill, the maximum speed is obtained from the fact that the net force is zero

Freewheeling downhill: $$F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D - M\cdot g\cdot \sin(\theta) = 0$$

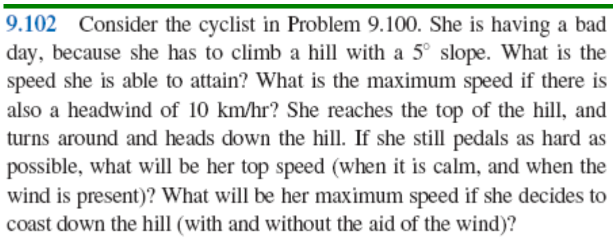
$$V = \sqrt{\frac{M\cdot g\cdot \sin(\theta) - F_R}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}} \qquad V = 58.1\frac{km}{hr}$$

Wind assisted: $$F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D - M\cdot g\cdot \sin(\theta) = 0$$

$$V = V_w + \sqrt{\frac{M\cdot g\cdot \sin(\theta) - F_R}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}} \qquad V = 68.1\frac{km}{hr}$$

Problem 9.102 (In Excel)

[3]



9.102 Consider the cyclist in Problem 9.100. She is having a bad day, because she has to climb a hill with a 5° slope. What is the speed she is able to attain? What is the maximum speed if there is also a headwind of 10 km/hr? She reaches the top of the hill, and turns around and heads down the hill. If she still pedals as hard as possible, what will be her top speed (when it is calm, and when the wind is present)? What will be her maximum speed if she decides to coast down the hill (with and without the aid of the wind)?

Given: Data on cyclist performance on a calm day

Find: Performance on a hill with and without wind

Solution:

Given data:

$F_R =$ 7.5 N
$M =$ 65 kg
$A =$ 0.25 m^2
$C_D =$ 1.2
$\rho =$ 1.23 kg/m^3
$V =$ 30 km/hr
$V_w =$ 10 km/hr
$\theta =$ 5 deg

Computed results:

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$$

$F_D =$ 12.8 N

$$P = \left(F_D + F_R\right)\cdot V$$

$P =$ 169 W

Cycling up the hill: $$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D + M\cdot g\cdot\sin(\theta)\right)\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	9.47

Uphill against the wind: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V + V_w\right)^2\cdot C_D + M\cdot g\cdot\sin(\theta)\right]\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	8.94

Cycling down the hill: $$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D - M\cdot g\cdot\sin(\theta)\right)\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	63.6

Wind-assisted downhill: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D - M\cdot g\cdot\sin(\theta)\right]\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	73.0

Problem *9.103

[3]

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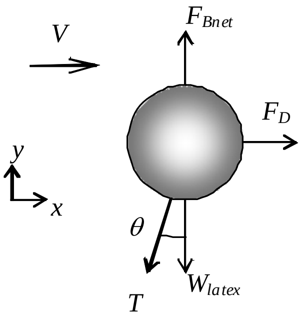
*9.103 At a surprise party for a friend you've tied a series of 9-in. diameter helium balloons to a flagpole, each tied with a short string. The first one is tied 3 ft above the ground, and the other eight are tied at 3 ft. spacings, so the last is tied at a height of 63 ft. Being quite a nerdy engineer, you notice that in the steady wind, each balloon is blown by the wind so it looks like the angles the strings make with the vertical are about 5°, 10°, 20°, 30°, 35°, 45°, 50°, 60° and 65°. Estimate and plot the wind velocity profile for the 63 ft. range. Assume the helium is at 70°F and 1.5 psig, and that each balloon is made of 1/10 oz. of latex.



Given: Series of party balloons

Find: Wind velocity profile; Plot

Note: Flagpole is actually 27 ft tall, not 63 ft!

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\quad F_B = \rho_{air}\cdot g\cdot Vol \quad \Sigma\vec{F} = 0$

The above figure applies to each balloon

For the horizontal forces $F_D - T\cdot\sin(\theta) = 0$ (1)

For the vertical forces $-T\cdot\cos(\theta) + F_{Bnet} - W_{latex} = 0$ (2)

Here $F_{Bnet} = F_B - W = \left(\rho_{air} - \rho_{He}\right)\cdot g\cdot\dfrac{\pi\cdot D^3}{6}$

$D = 9\cdot in$ $\quad M_{latex} = \dfrac{1}{10}\cdot oz \quad W_{latex} = M_{latex}\cdot g \quad W_{latex} = 0.00625\,lbf$

We have (Table A.6) $R_{He} = 386.1\cdot\dfrac{ft\cdot lbf}{lbm\cdot R} \quad p_{He} = 16.2\cdot psi \quad T_{He} = 530\cdot R \quad \rho_{He} = \dfrac{p_{He}}{R_{He}\cdot T_{He}} \quad \rho_{He} = 0.000354\dfrac{slug}{ft^3}$

$R_{air} = 53.33\cdot\dfrac{ft\cdot lbf}{lbm\cdot R} \quad p_{air} = 14.7\cdot psi \quad T_{air} = 530\cdot R \quad \rho_{air} = \dfrac{p_{air}}{R_{air}\cdot T_{air}} \quad \rho_{air} = 0.00233\dfrac{slug}{ft^3}$

$F_{Bnet} = \left(\rho_{air} - \rho_{He}\right)\cdot g\cdot\dfrac{\pi\cdot D^3}{6} \quad F_{Bnet} = 0.0140\,lbf$

Applying Eqs 1 and 2 to the top balloon, for which $\theta = 65\cdot deg$

$$F_D = T\cdot\sin(\theta) = \frac{F_{Bnet} - W_{latex}}{\cos(\theta)}\cdot\sin(\theta)$$

Hence $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta) \quad F_D = 0.0167\,lbf$

But we have $F_D = C_D\cdot\frac{1}{2}\cdot\rho_{air}\cdot V^2\cdot A = C_D\cdot\frac{1}{2}\cdot\rho_{air}\cdot V^2\cdot\dfrac{\pi\cdot D^2}{4}$ with $C_D = 0.4$ from Fig. 9.11 (we will check Re later)

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}} \quad V = 9.00\dfrac{ft}{s}$

From Table A.9 $\nu = 1.63\times 10^{-4}\cdot\dfrac{ft^2}{s}$ The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu} \quad Re_d = 4.14\times 10^4$ We are okay!

For the next balloon $\theta = 60\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.0135\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 8.09\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 3.72\times 10^4$ We are okay!

For the next balloon $\theta = 50\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.00927\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 6.71\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 3.09\times 10^4$ We are okay!

For the next balloon $\theta = 45\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.00777\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 6.15\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 2.83\times 10^4$ We are okay!

For the next balloon $\theta = 35\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.00544\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 5.14\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 2.37\times 10^4$ We are okay!

For the next balloon $\theta = 30\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.00449\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 4.67\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 2.15\times 10^4$ We are okay!

For the next balloon $\theta = 20\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.00283\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 3.71\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 1.71\times 10^4$ We are okay!

For the next balloon $\theta = 10\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.00137\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 2.58\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $Re_d = 1.19\times 10^4$ We are okay!

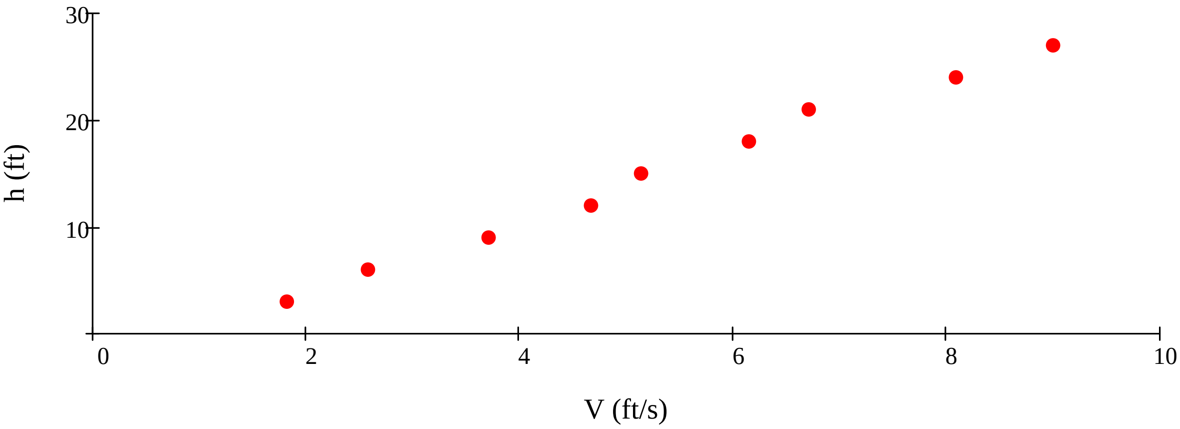
For the next balloon $\theta = 5\cdot\text{deg}$ $F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $F_D = 0.000680\,\text{lbf}$ with $C_D = 0.4$

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $V = 1.82\,\dfrac{\text{ft}}{\text{s}}$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 8367.80$ We are okay!

In summary we have $V = (1.82\ \ 2.58\ \ 3.71\ \ 4.67\ \ 5.14\ \ 6.15\ \ 6.71\ \ 8.09\ \ 9.00) \cdot \frac{ft}{s}$

$h = (3\ \ 6\ \ 9\ \ 12\ \ 15\ \ 18\ \ 21\ \ 24\ \ 27) \cdot ft$

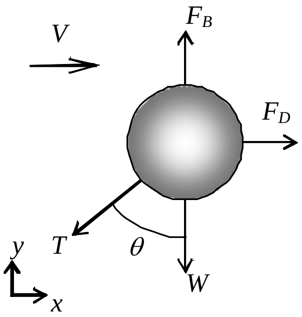


This problem is ideal for computing and plotting in *Excel*

Problem 9.104

[2]

9.104 A 1-ft diameter hollow plastic sphere containing pollution test equipment is being dragged through the Hudson River in New York by a diver riding an underwater jet device. The sphere (with an effective specific gravity of SG = 0.25) is fully submerged, and is tethered to the diver by a thin 4-ft long wire. What is the relative velocity of the diver and sphere if the angle the wire makes with the horizontal is 45°? The water is at 50°F.



Given: Sphere dragged through river

Find: Relative velocity of sphere

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\quad F_B = \rho\cdot g\cdot Vol \quad \Sigma\vec{F} = 0$

The above figure applies to the sphere

For the horizontal forces $F_D - T\cdot\sin(\theta) = 0 \qquad (1)$

For the vertical forces $-T\cdot\cos(\theta) + F_B - W = 0 \qquad (2)$

Here $D = 1\cdot ft \quad SG = 0.25$ and from Table A.7 $\nu = 1.41\times 10^{-5}\cdot\frac{ft^2}{s} \quad \rho = 1.94\cdot\frac{slug}{ft^3}$

Applying Eqs 1 and 2 to the sphere, for which $\theta = 45\cdot deg$

$$F_D = T\cdot\sin(\theta) = \frac{F_B - W}{\cos(\theta)}\cdot\sin(\theta) = \rho\cdot g\cdot Vol\cdot(1-SG)\cdot\tan(\theta)$$

Hence $F_D = \rho\cdot g\cdot\frac{\pi\cdot D^3}{6}\cdot(1-SG)\cdot\tan(\theta) \qquad F_D = 24.5\cdot lbf$

But we have $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot\frac{\pi\cdot D^2}{4}$ with $C_D = 0.4$ from Fig. 9.11 (we will check Re later)

$$V = \sqrt{\frac{8\cdot F_D}{C_D\cdot\rho\cdot\pi\cdot D^2}} \qquad V = 8.97\cdot\frac{ft}{s}$$

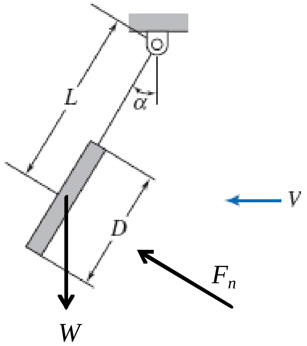
The Reynolds number is $Re_d = \frac{V\cdot D}{\nu} \qquad Re_d = 6.36\times 10^5$ A bit off from Fig 9.11

Try $C_D = 0.15 \qquad V = \sqrt{\frac{8\cdot F_D}{C_D\cdot\rho\cdot\pi\cdot D^2}} \qquad V = 14.65\cdot\frac{ft}{s}$

The Reynolds number is $Re_d = \frac{V\cdot D}{\nu} \qquad Re_d = 1.04\times 10^6$ A good fit with Fig 9.11 (extreme right of graph)

Problem 9.105

***9.105** A circular disk is hung in an air stream from a pivoted strut as shown. In a wind-tunnel experiment, performed in air at 15 m/s with a 25-mm diameter disk, α was measured at 10°. For these conditions determine the mass of the disk. Assume the drag coefficient for the disk applies when the component of wind speed normal to the disk is used. Assume drag on the strut and friction in the pivot are negligible. Plot a theoretical curve of α as a function of air speed.



Given: Circular disk in wind

Find: Mass of disk; Plot α versus V

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\qquad \Sigma\vec{M} = 0$

Summing moments at the pivot $W\cdot L\cdot\sin(\alpha) - F_n\cdot L = 0$ and $F_n = \frac{1}{2}\cdot\rho\cdot V_n^{\,2}\cdot A\cdot C_D$

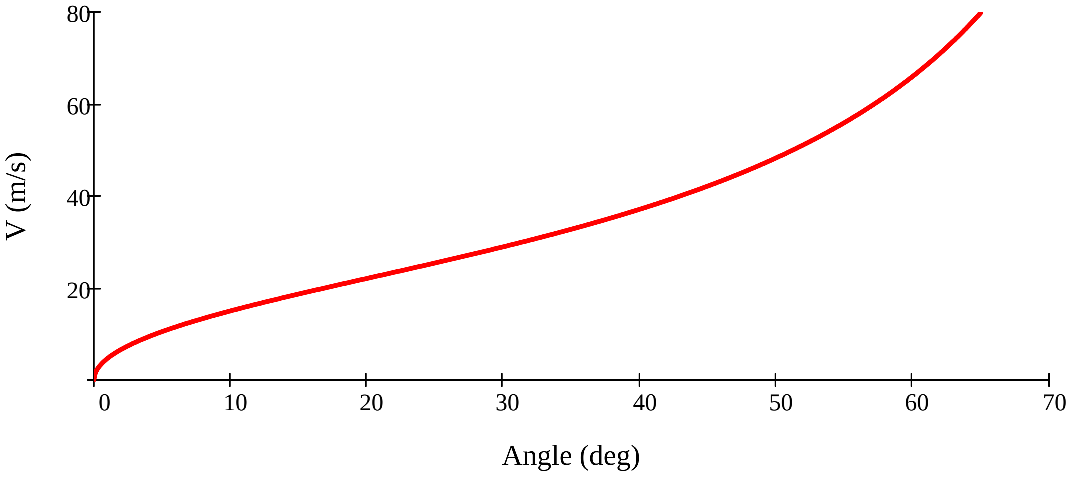
Hence $M\cdot g\cdot\sin(\alpha) = \frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\dfrac{\pi\cdot D^2}{4}\cdot C_D$

The data is $\rho = 1.225\cdot\dfrac{kg}{m^3}$ $\quad V = 15\cdot\dfrac{m}{s}$ $\quad D = 25\cdot mm$ $\quad \alpha = 10\cdot deg$ $\quad C_D = 1.17$ (Table 9.3)

$$M = \frac{\pi\cdot\rho\cdot V^2\cdot\cos(\alpha)^2\cdot D^2\cdot C_D}{8\cdot g\cdot\sin(\alpha)} \qquad M = 0.0451\,kg$$

Rearranging $V = \sqrt{\dfrac{8\cdot M\cdot g}{\pi\cdot\rho\cdot D^2\cdot C_D}}\cdot\sqrt{\dfrac{\tan(\alpha)}{\cos(\alpha)}}$ $\qquad V = 35.5\cdot\dfrac{m}{s}\cdot\sqrt{\dfrac{\tan(\alpha)}{\cos(\alpha)}}$

We can plot this by choosing α and computing V

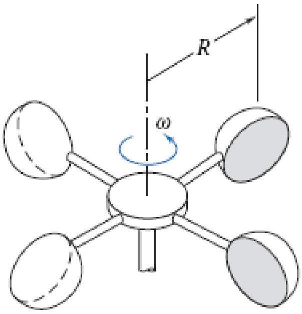


This graph can be easily plotted in *Excel*

Problem 9.106

[3]

9.106 An anemometer to measure wind speed is made from four hemispherical cups of 50 mm diameter, as shown. The center of each cup is placed at $R = 80$ mm from the pivot. Find the theoretical calibration constant k in the calibration equation $V = k\omega$, where V (km/hr) is the wind speed and ω (rpm) is the rotation speed. In your analysis base the torque calculations on the drag generated at the instant when two of the cups are orthogonal, and the other two cups are parallel, and ignore friction in the bearings. Explain why, in the absence of friction, at any given wind speed, the anemometer runs at constant speed rather than accelerating without limit. If the actual anemometer bearing has (constant) friction such that the anemometer needs a minimum wind speed of 1 km/hr to begin rotating, compare the rotation speeds with and without friction for $V = 10$ km/hr.



Given: Data on dimensions of anemometer

Find: Calibration constant; compare to actual with friction

Solution:

The given data or available data is $D = 50\cdot\text{mm}$ $\quad R = 80\cdot\text{mm}$ $\quad \rho = 1.23\cdot\frac{\text{kg}}{\text{m}^3}$

The drag coefficients for a cup with open end facing the airflow and a cup with open end facing downstream are, respectively, from Table

$$C_{Dopen} = 1.42 \qquad C_{Dnotopen} = 0.38$$

The equation for computing drag is $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ (1)

where $A = \frac{\pi\cdot D^2}{4}$ $\qquad A = 1.96\times 10^{-3}\,\text{m}^2$

Assuming steady speed ω at steady wind speed *V* the sum of moments will be zero. The two cups that are momentarily parallel to the flow will exert no moment; the two cups with open end facing and not facing the flow will exert a moment beacuse of their drag forces. For eac the drag is based on Eq. 1 (with the *relative* velocity used!). In addition, friction of the anemometer is neglected

$$\Sigma M = 0 = \left[\frac{1}{2}\cdot\rho\cdot A\cdot(V - R\cdot\omega)^2\cdot C_{Dopen}\right]\cdot R - \left[\frac{1}{2}\cdot\rho\cdot A\cdot(V + R\cdot\omega)^2\cdot C_{Dnotopen}\right]\cdot R$$

or $$(V - R\cdot\omega)^2\cdot C_{Dopen} = (V + R\cdot\omega)^2\cdot C_{Dnotopen}$$

This indicates that the anemometer reaches a steady speed even in the abscence of friction because it is the *relative* velocity on each cup that matters: the cup that has a higher drag coefficient has a lower relative velocity

Rearranging for $k = \frac{V}{\omega}$ $\qquad \left(\frac{V}{\omega} - R\right)^2\cdot C_{Dopen} = \left(\frac{V}{\omega} + R\right)^2\cdot C_{Dnotopen}$

Hence $k = \dfrac{\left(1 + \sqrt{\dfrac{C_{Dnotopen}}{C_{Dopen}}}\right)}{\left(1 - \sqrt{\dfrac{C_{Dnotopen}}{C_{Dopen}}}\right)} \cdot R$ $k = 0.251\,m$ $k = 0.0948 \dfrac{\frac{km}{hr}}{rpm}$

For the actual anemometer (with friction), we first need to determine the torque produced when the anemometer is stationary but about to rotate

Minimum wind for rotation is $V_{min} = 1 \cdot \dfrac{km}{hr}$

The torque produced at this wind speed is

$$T_f = \left(\frac{1}{2} \cdot \rho \cdot A \cdot V_{min}{}^2 \cdot C_{Dopen}\right) \cdot R - \left(\frac{1}{2} \cdot \rho \cdot A \cdot V_{min}{}^2 \cdot C_{Dnotopen}\right) \cdot R$$

$$T_f = 7.75 \times 10^{-6} N \cdot m$$

A moment balance at wind speed *V*, including this friction, is

$$\Sigma M = 0 = \left[\frac{1}{2} \cdot \rho \cdot A \cdot (V - R \cdot \omega)^2 \cdot C_{Dopen}\right] \cdot R - \left[\frac{1}{2} \cdot \rho \cdot A \cdot (V + R \cdot \omega)^2 \cdot C_{Dnotopen}\right] \cdot R - T_f$$

or $(V - R \cdot \omega)^2 \cdot C_{Dopen} - (V + R \cdot \omega)^2 \cdot C_{Dnotopen} = \dfrac{2 \cdot T_f}{R \cdot \rho \cdot A}$

This quadratic equation is to be solved for ω when $V = 10 \cdot \dfrac{km}{hr}$

After considerable calculations $\omega = 104\,rpm$

This must be compared to the rotation for a frictionless model, given by

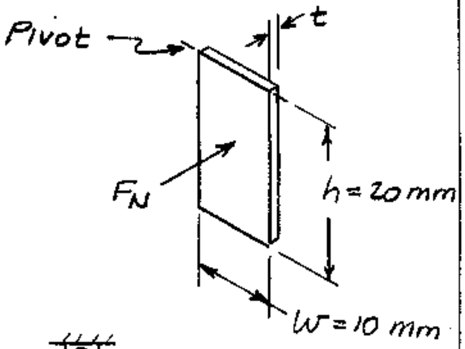
$\omega_{frictionless} = \dfrac{V}{k}$ $\omega_{frictionless} = 105\,rpm$

The error in neglecting friction is $\left|\dfrac{\omega - \omega_{frictionless}}{\omega}\right| = 1.12\%$

Problem 9.107

[2]

Given: Single-vane anemometer made from brass plate, t thick, with $h = 20$ mm and $w = 10$ mm.

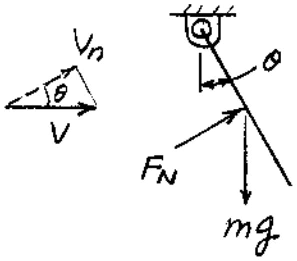


Find: (a) Relationship for wind speed as a function of deflection angle, θ.

(b) Plate thickness to give $\theta = 30°$ at $V = 10$ m/s.

Solution: Sum moments about pivot.

$$\Sigma M = F_N \frac{h}{2} - mg\frac{h}{2}\sin\theta = 0$$



$$F_N = C_D A \tfrac{1}{2}\rho V_n^2 = mg\sin\theta$$

$$C_D A \tfrac{1}{2}\rho V^2\cos^2\theta = mg\sin\theta \qquad (1)$$

$$V = \left[\frac{2mg\sin\theta}{C_D A\rho\cos^2\theta}\right]^{1/2}$$

V

From plate geometry, $m = \rho wht = SG\rho_{H_2O}wht$. From Eq. 1,

$$SG\rho_{H_2O}gwht\sin\theta = C_D A\tfrac{1}{2}\rho V^2\cos^2\theta \qquad \{\text{From Table A.1, } SG = 8.55 \text{ for brass.}\}$$

$$t = \frac{C_D A\rho V^2\cos^2\theta}{2SG\rho_{H_2O}wh\sin\theta g} = \frac{C_D\rho V^2\cos^2\theta}{2SG\rho_{H_2O}\sin\theta g} \quad \text{since } A = wh$$

From Fig. 9.10, $C_D = 1.2$ at $b/h = 2.0$, so

$$t = \frac{1.2}{8.55}\times\frac{1.23}{2}\frac{\text{kg}}{\text{m}^3}\times(10)^2\frac{\text{m}^2}{\text{s}^2}\times\cos^2(30°)\times\frac{\text{m}^3}{999\text{ kg}}\times\frac{1}{\sin(30°)}\times\frac{\text{s}^2}{9.81\text{ m}}\times 1000\frac{\text{mm}}{\text{m}}$$

$$t = 1.30 \text{ mm}$$

t

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Problem 9.108

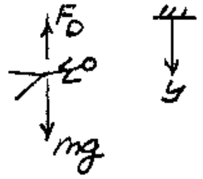
[2]

Given: Experimental data for a sky diver with $M = 75\ kg$:

Prone, spread-eagled	$C_DA = 0.85\ m^2$
Vertical fall	$C_DA = 0.11\ m^2$

Find: Estimate time and distance needed to reach 95 percent of terminal speed at 3000 m altitude on a standard day.

Solution: From Table A.3, $\rho/\rho_{SL} = 0.7423$ at 3000 m altitude.
Consider free-body diagram of sky diver:



$$\Sigma F_y = mg - F_D = mg - C_DA\tfrac{1}{2}\rho V^2 = ma_y = m\frac{dV}{dt} = mV\frac{dV}{dy}$$

At terminal speed, $a_y = 0$. Then $mg = C_DA\frac{1}{2}\rho V_t^2$

so $V_t^2 = \dfrac{2mg}{C_DA\rho}$. From above, $\dfrac{1}{g}\dfrac{dV}{dt} = \dfrac{V_t}{g}\dfrac{d(V/V_t)}{dt} = 1 - \dfrac{C_DA\rho V^2}{2mg} = 1 - \left(\dfrac{V}{V_t}\right)^2$

Thus $$\frac{d(V/V_t)}{1-(V/V_t)^2} = \frac{g}{V_t}dt$$

Integrating $$\int_0^{0.95}\frac{d(V/V_t)}{1-(V/V_t)^2} = \tanh^{-1}\left(\frac{V}{V_t}\right)\Big]_0^{0.95} = 1.83 = \frac{gt}{V_t}\ ;\ t = \frac{1.83\,V_t}{g}$$ $\leftarrow$ t

Also $1-\left(\dfrac{V}{V_t}\right)^2 = \dfrac{V}{g}\dfrac{dV}{dy} = \dfrac{V_t^2}{g}\left(\dfrac{V}{V_t}\right)\dfrac{d(V/V_t)}{dy}$ or $\dfrac{(V/V_t)\,d(V/V_t)}{1-(V/V_t)^2} = \dfrac{g}{V_t^2}dy$

Integrating, $$\int_0^{0.95}\frac{(V/V_t)\,d(V/V_t)}{1-(V/V_t)^2} = -\frac{1}{2}\ln\left[1-(V/V_t)^2\right]_0^{0.95} = 1.16 = \frac{g}{V_t^2}y\ ;\ y = \frac{1.16\,V_t^2}{g}$$ $\leftarrow$ y

Calculating for $C_DA = 0.85\ m^2$:

$$V_t = \left[2 \times 75\ kg \times 9.81\frac{m}{s^2} \times \frac{1}{0.85\ m^2} \times \frac{m^3}{(0.7423)\,1.23\ kg}\right]^{1/2} = 43.5\ m/s$$

$$t = \frac{1.83\,V_t}{g} = 1.83 \times 43.5\frac{m}{s} \times \frac{s^2}{9.81\ m} = 8.11\ s$$

$$y = 1.16\frac{V_t^2}{g} = 1.16\,(43.5)^2\frac{m^2}{s^2} \times \frac{s^2}{9.81\ m} = 224\ m$$

Tabulating:

Position	V_t (m/s)	t (s)	y (m)
Prone	43.5	8.11	224
Vertical*	121	22.6	1,730

* These are estimates; density would vary significantly during this fall.

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Problem 9.109

[3]

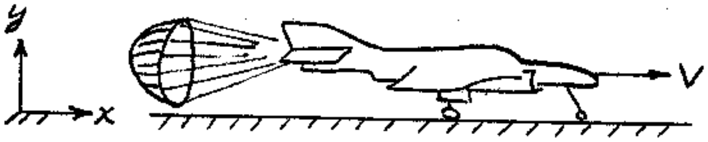
Given: F-4 aircraft slowed by dual parachutes, each 12 ft in diameter. Craft weighs 32,000 lbf, lands at 160 kt. Neglect drag of aircraft; brakes not applied.

Find: Time required to decelerate to 100 kt.

Solution: Apply Newton's second law of motion, definition of C_D.

Basic equations: $\Sigma F_x = ma_x$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



Then

$$\Sigma F_x = -2F_D = -C_D\rho V^2 A = ma_x = \frac{W}{g}\frac{dV}{dt} \qquad (1)$$

or $\dfrac{dV}{V^2} = -\dfrac{C_D\rho g A}{W}\,dt$

Integrating,

$$\int_{V_i}^{V_f}\frac{dV}{V^2} = -\frac{1}{V}\Big]_{V_i}^{V_f} = \frac{1}{V_i} - \frac{1}{V_f} = \int_0^t -\frac{C_D\rho g A}{W}\,dt = -\frac{C_D\rho g A}{W}t$$

or

$$t = \frac{W}{C_D\rho g A}\left[\frac{1}{V_f} - \frac{1}{V_i}\right]$$

Since two chutes (assume hemispheres), $A = 2\left(\frac{\pi D^2}{4}\right) = \frac{\pi D^2}{2}$

From Table 9.3, $C_D = 1.42$ for hemisphere facing stream. For standard air, $\rho g = \gamma \simeq 0.075\ \text{lbf/ft}^3$, and

$$t = 32{,}000\,\text{lbf} \times \frac{1}{1.42} \times \frac{\text{ft}^3}{0.075\,\text{lbf}} \times \frac{2}{\pi} \times \frac{1}{(12)^2\text{ft}^2}\left[\frac{1}{100} - \frac{1}{160}\right]\frac{\text{hr}}{\text{nm}} \times 3600\frac{\text{s}}{\text{hr}} \times \frac{\text{nm}}{6080\,\text{ft}}$$

or

$$t = 2.45\ \text{s}$$ t

To find distance, set $a_x = \dfrac{dV}{dt} = V\dfrac{dV}{dx}$. Then, from Eq. 1,

$$-2C_D\rho\frac{V^2}{2}A = \frac{W}{g}V\frac{dV}{dx}$$

and $\dfrac{dV}{V} = -\dfrac{C_D A\rho g}{W}\,dx$

Integrating,

$$\int_{V_i}^{V_f}\frac{dV}{V} = \ln V\Big]_{V_i}^{V_f} = \ln\frac{V_f}{V_i} = -\frac{C_D A\rho g}{W}x \quad \text{or} \quad x = -\frac{W}{C_D A\rho g}\ln\frac{V_f}{V_i}$$

Thus

$$x = -\frac{1}{1.42} \times 32{,}000\,\text{lbf} \times \frac{2}{\pi(12)^2\text{ft}^2} \times \frac{\text{ft}^3}{0.075\,\text{lbf}} \times \ln\left(\frac{100}{160}\right) = 624\ \text{ft}$$ x

Problem 9.110

[3]

Given: Land speed record vehicle at Bonneville Salt Flats, elevation 4400 ft. Engine power, $\mathbb{P} = 500$ hp, frontal area, $A = 15\ \text{ft}^2$, and $C_D = 0.15$.

Find: Theoretical maximum speed (a) in still air, (b) 20 mph head wind.

Solution: Apply definitions of power, drag coefficient.

Computing equations: $\mathbb{P} = F_D V$, $C_D = \dfrac{F_D}{\frac{1}{2}\rho(V+V_w)^2 A}$

Assumptions: (1) Neglect rolling drag

(2) $\rho \simeq 0.878\,\rho_0$ (Table A.3)

For no wind case, $V_w = 0$, and

$$\mathbb{P} = F_D V = C_D \tfrac{1}{2}\rho V^2 A V = C_D \tfrac{1}{2}\rho V^3 A$$

$$V = \left[\frac{2\mathbb{P}}{\rho C_D A}\right]^{1/3} = \left[2 \times 500\ \text{hp} \times \frac{\text{ft}^3}{(0.878)0.00238\ \text{slug}} \times \frac{1}{0.15} \times \frac{1}{15\ \text{ft}^2} \times \frac{550\ \text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/3}$$

$V = 489$ ft/s (333 mph) V

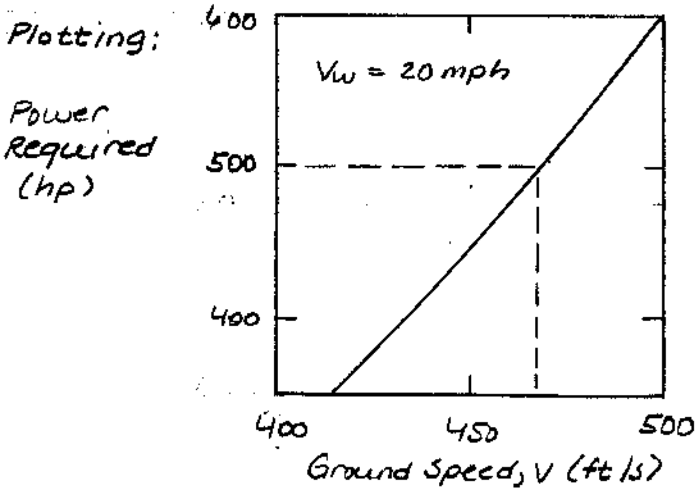
With a head wind,

$$\mathbb{P} = F_D V = C_D \tfrac{1}{2}\rho (V+V_w)^2 A V \quad \text{or} \quad \mathbb{P}(\text{hp}) = 4.27\times10^{-6}(V+V_w)^2 V\ (\text{ft}^3/\text{s}^3)$$

This can be solved by iteration. Using $V_w = 20$ mph or 29.3 ft/s,

V (ft/s)	$\mathbb{P}$ (hp)
400	315
470	501
500	599

Plotting:



From the plot, $V \simeq 468$ ft/s (319 mph) V

{ Note that the maximum speed is not reduced by 20 mph when wind is present, because drag is nonlinear. }

Problem 9.111

[2]

9.111 Compare and plot the power (hp) required by a typical large American sedan of the 1970s and a current midsize sedan to overcome aerodynamic drag versus road speed in standard air, for a speed range of 20 mph to 100 mph. Use the following as representative values:

	Weight (lbf)	Drag Coefficient	Frontal Area (ft^2)
1970's Sedan	4500	0.5	24
Current Sedan	3500	0.3	20

If rolling resistance is 1.5 percent of curb weight, determine for each vehicle the speed at which the aerodynamic force exceeds frictional resistance.

Given: Data on 1970's and current sedans

Find: Plot of power versus speed; Speeds at which aerodynamic drag exceeds rolling drag

Solution:

Basic equation: $$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$$

The aerodynamic drag is $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ The rolling resistance is $F_R = 0.015\cdot W$

Total resistance $F_T = F_D + F_R$

	1970's Sedan		Current Sedan	
W =	4500	lbf	3500	lbf
C_D =	0.5		0.3	
A =	24	ft^2	20	ft^2
ρ =	0.00234	$slug/ft^3$	(Table A.9)	

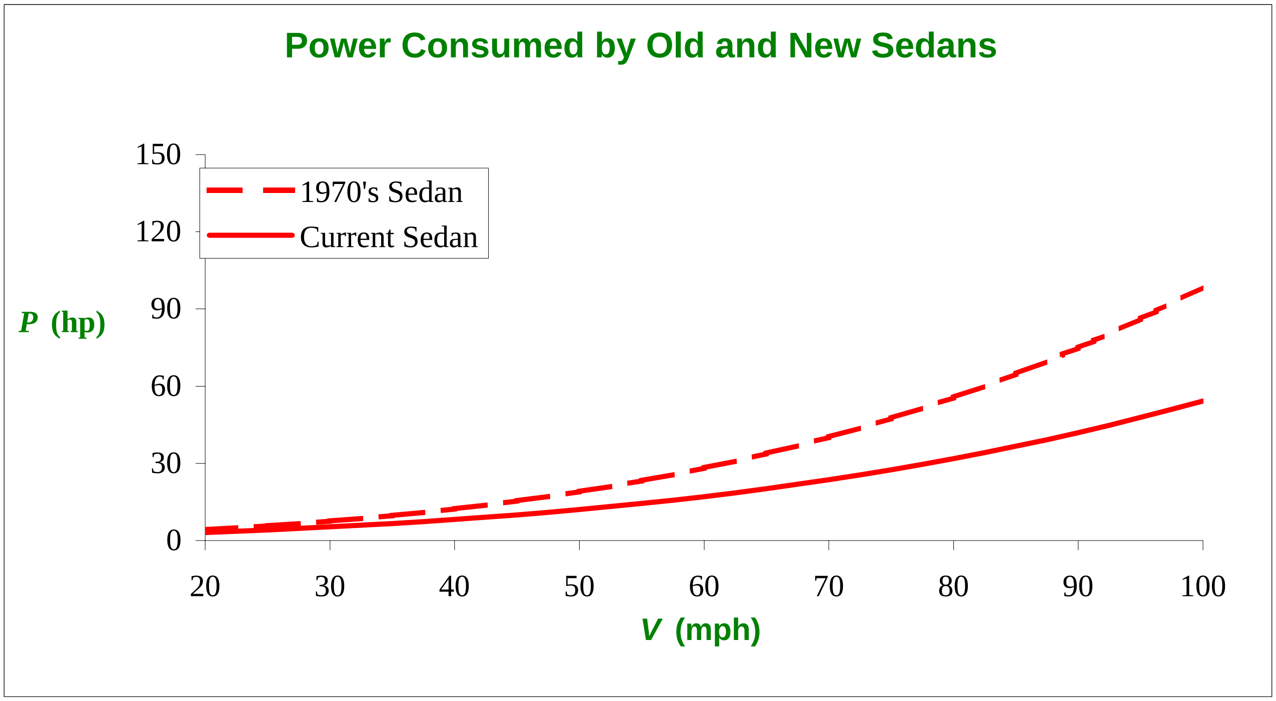
Computed results:

	1970's Sedan			Current Sedan		
V (mph)	F_D (lbf)	F_T (lbf)	P (hp)	F_D (lbf)	F_T (lbf)	P (hp)
20	12.1	79.6	4.24	6.04	58.5	3.12
25	18.9	86.4	5.76	9.44	61.9	4.13
30	27.2	94.7	7.57	13.6	66.1	5.29
35	37.0	104	9.75	18.5	71.0	6.63
40	48.3	116	12.4	24.2	76.7	8.18
45	61.2	129	15.4	30.6	83.1	10.0
50	75.5	143	19.1	37.8	90.3	12.0
55	91.4	159	23.3	45.7	98.2	14.4
60	109	176	28.2	54.4	107	17.1
65	128	195	33.8	63.8	116	20.2
70	148	215	40.2	74.0	126	23.6
75	170	237	47.5	84.9	137	27.5
80	193	261	55.6	96.6	149	31.8
85	218	286	64.8	109	162	36.6
90	245	312	74.9	122	175	42.0
95	273	340	86.2	136	189	47.8
100	302	370	98.5	151	204	54.3

V (mph)	F_D (lbf)	F_R (lbf)
47.3	67.5	67.5

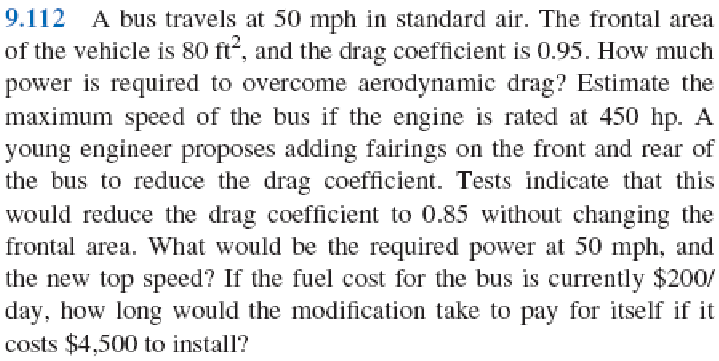
V (mph)	F_D (lbf)	F_R (lbf)
59.0	52.5	52.5

The two speeds above were obtained using *Solver*



Problem 9.112

[3]



9.112 A bus travels at 50 mph in standard air. The frontal area of the vehicle is 80 ft^2, and the drag coefficient is 0.95. How much power is required to overcome aerodynamic drag? Estimate the maximum speed of the bus if the engine is rated at 450 hp. A young engineer proposes adding fairings on the front and rear of the bus to reduce the drag coefficient. Tests indicate that this would reduce the drag coefficient to 0.85 without changing the frontal area. What would be the required power at 50 mph, and the new top speed? If the fuel cost for the bus is currently \$200/day, how long would the modification take to pay for itself if it costs \$4,500 to install?

Given: Data on a bus

Find: Power to overcome drag; Maximum speed; Recompute with new fairing; Time for fairing to pay for itself

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad P = F_D\cdot V$

The given data or available data is $V = 50\cdot\text{mph} \qquad V = 73.3\frac{\text{ft}}{\text{s}} \qquad A = 80\cdot\text{ft}^2 \qquad C_D = 0.95 \qquad \rho = 0.00234\cdot\frac{\text{slug}}{\text{ft}^3}$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad F_D = 478\,\text{lbf} \qquad P = F_D\cdot V \qquad P = 3.51\times 10^4\frac{\text{ft}\cdot\text{lbf}}{\text{s}} \qquad P = 63.8\,\text{hp}$

The power available is $P_{max} = 450\cdot\text{hp}$

The maximum speed corresponding to this maximum power is obtained from

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot A\cdot V_{max}{}^2\cdot C_D\right)\cdot V_{max} \qquad \text{or} \qquad V_{max} = \left(\frac{P_{max}}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}\right)^{\frac{1}{3}} \qquad V_{max} = 141\frac{\text{ft}}{\text{s}} \qquad V_{max} = 95.9\,\text{mph}$$

We repeat these calculations with the new fairing, for which $C_D = 0.85$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad F_D = 428\,\text{lbf} \qquad P_{new} = F_D\cdot V \qquad P_{new} = 3.14\times 10^4\frac{\text{ft}\cdot\text{lbf}}{\text{s}} \qquad P_{new} = 57.0\,\text{hp}$

The maximum speed is now $V_{max} = \left(\frac{P_{max}}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}\right)^{\frac{1}{3}} \qquad V_{max} = 146\frac{\text{ft}}{\text{s}} \qquad V_{max} = 99.5\,\text{mph}$

The initial cost of the fairing is $\text{Cost} = 4500\cdot\text{dollars}$ The fuel cost is $\text{Cost}_{day} = 200\cdot\frac{\text{dollars}}{\text{day}}$

The cost per day is reduced by improvement in the bus performance at 50 mph $\text{Gain} = \frac{P_{new}}{P} \qquad \text{Gain} = 89.5\%$

The new cost per day is then $\text{Cost}_{daynew} = \text{Gain}\cdot\text{Cost}_{day} \qquad \text{Cost}_{daynew} = 179\frac{\text{dollars}}{\text{day}}$

Hence the savings per day is $\text{Saving} = \text{Cost}_{day} - \text{Cost}_{daynew} \qquad \text{Saving} = 21.1\frac{\text{dollars}}{\text{day}}$

The initial cost will be paid for in $\tau = \frac{\text{Cost}}{\text{Saving}} \qquad \tau = 7.02\,\text{month}$

Problem 9.113

[3]

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Given: Tractor-trailer rig, with $A = 102\ \text{ft}^2$; $C_D = 0.9$. Rolling resistance is 6 lbf per 1000 lbf; $W = 72{,}000$ lbf. BSFC is 0.34 lbm/hp·hr, $\eta_d = 0.92$, and $\rho = 6.9$ lbm/gal. Truck travels 120,000 mi/yr.

Find: (a) Estimate fuel economy at 55 mph.
(b) Fuel saved by air deflector that reduces C_D by 15 percent.

Solution: Tractive force is $F_T = F_R + F_D$ ← aerodynamic force
↖ rolling resistance force

Engine power is $P_e = \dfrac{P_T}{\eta_d} = \dfrac{F_T V}{\eta_d}$

Thus

$$F_R = C_R W = 0.006 \times 72{,}000\,\text{lbf} = 432\ \text{lbf}$$

$$F_D = C_D A \tfrac{1}{2}\rho V^2 \qquad V = 55\,\frac{\text{mi}}{\text{hr}} \times 5280\,\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}} = 80.7\ \text{ft/s}$$

$$F_D = 0.9 \times 102\,\text{ft}^2 \times \frac{1}{2} \times 0.00238\,\frac{\text{slug}}{\text{ft}^3} \times (80.7)^2\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 711\ \text{lbf}$$

so

$$F_T = F_R + F_D = 432 + 711 = 1140\ \text{lbf}$$

$$P_e = 1140\,\text{lbf} \times 80.7\,\frac{\text{ft}}{\text{s}} \times \frac{1}{0.92} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 182\ \text{hp}$$

Finally

$$FE = \frac{\rho V}{P_e\,\text{BSFC}} = 6.9\,\frac{\text{lbm}}{\text{gal}} \times 55\,\frac{\text{mi}}{\text{hr}} \times \frac{1}{182\,\text{hp}} \times \frac{\text{hp}\cdot\text{hr}}{0.34\,\text{lbm}} = 6.13\ \text{mi/gal}$$

FE ←

With the air deflector,

$$F_D = (1-0.15)\,711\,\text{lbf} = 604\ \text{lbf}$$

$$F_T = F_R + F_D = 432 + 604 = 1040\ \text{lbf}$$

$$P_e = 1040\,\text{lbf} \times 80.7\,\frac{\text{ft}}{\text{s}} \times \frac{1}{0.92} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 166\ \text{hp}$$

and $$FE = 6.9\,\frac{\text{lbm}}{\text{gal}} \times 55\,\frac{\text{mi}}{\text{hr}} \times \frac{1}{166\,\text{hp}} \times \frac{\text{hp}\cdot\text{hr}}{0.34\,\text{lbm}} = 6.72\ \text{mi/gal}$$

The fuel saving would be

$$\Delta Q = \left(\frac{1}{FE}\Big|_{\text{without}} - \frac{1}{FE_{\text{with}}}\right)\text{mileage} = \left(\frac{1}{6.13} - \frac{1}{6.72}\right)\frac{\text{gal}}{\text{mi}} \times 120{,}000\,\frac{\text{mi}}{\text{yr}} = 1720\ \text{gal/yr}$$

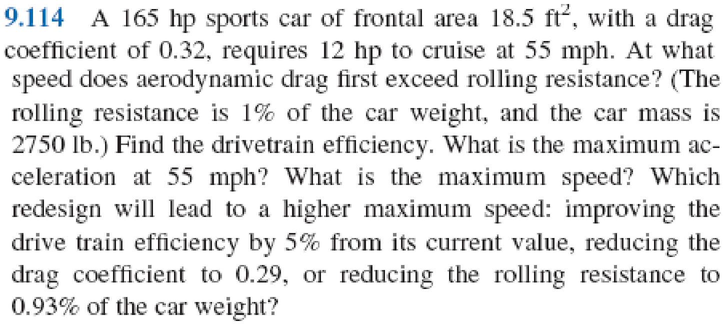
Q ←

The percentage saving would be

$$\frac{\Delta Q}{Q} = \frac{\left(\dfrac{1}{FE_{\text{without}}} - \dfrac{1}{FE_{\text{with}}}\right)}{\dfrac{1}{FE}\Big|_{\text{without}}} = 0.0878 \text{ or } 8.78 \text{ percent savings}$$

Problem 9.114

[4]



9.114 A 165 hp sports car of frontal area 18.5 ft^2, with a drag coefficient of 0.32, requires 12 hp to cruise at 55 mph. At what speed does aerodynamic drag first exceed rolling resistance? (The rolling resistance is 1% of the car weight, and the car mass is 2750 lb.) Find the drivetrain efficiency. What is the maximum acceleration at 55 mph? What is the maximum speed? Which redesign will lead to a higher maximum speed: improving the drive train efficiency by 5% from its current value, reducing the drag coefficient to 0.29, or reducing the rolling resistance to 0.93% of the car weight?

Given: Data on a sports car

Find: Speed for aerodynamic drag to exceed rolling resistance; maximum speed & acceleration at 55 mph; Redesign change that has greatest effect

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad P = F_D\cdot V$

The given data or available data is $M = 2750\cdot\text{lbm}$ $\quad A = 18.5\cdot\text{ft}^2$ $\quad C_D = 0.32$

$P_{engine} = 165\cdot\text{hp}$ $\quad F_R = 0.01\times M\cdot g$ $\quad \rho = 0.00234\cdot\frac{\text{slug}}{\text{ft}^3}$

The rolling resistance is then $F_R = 27.5\,\text{lbf}$

To find the speed at which aerodynamic drag first equals rolling resistance, set the two forces equal $\frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D = F_R$

Hence $V = \sqrt{\frac{2\cdot F_R}{\rho\cdot A\cdot C_D}}$ $\quad V = 63.0\,\frac{\text{ft}}{\text{s}}$ $\quad V = 43.0\,\text{mph}$

To find the drive train efficiency we use the data at a speed of 55 mph $V = 55\cdot\text{mph}$ $\quad V = 80.7\,\frac{\text{ft}}{\text{s}}$ $\quad P_{engine} = 12\cdot\text{hp}$

The aerodynamic drag at this speed is $F_D = \frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D$ $\quad F_D = 45.1\,\text{lbf}$

The power consumed by drag and rolling resistance at this speed is $P_{used} = \left(F_D + F_R\right)\cdot V$ $\quad P_{used} = 10.6\,\text{hp}$

Hence the drive train efficiency is $\eta = \frac{P_{used}}{P_{engine}}$ $\quad \eta = 88.7\,\%$

The acceleration is obtained from Newton's second lav $M\cdot a = \Sigma F = T - F_R - F_D$

where *T* is the thrust produced by the engine, given by $T = \frac{P}{V}$

The maximum acceleration at 55 mph is when we have maximum thrust, when full engine power is used. $P_{engine} = 165\cdot\text{hp}$

Because of drive train inefficiencies the maximum power at the wheels $P_{max} = \eta\cdot P_{engine}$ $\quad P_{max} = 146\,\text{hp}$

Hence the maximum thrust is $T_{max} = \frac{P_{max}}{V}$ $\quad T_{max} = 998\,\text{lbf}$

The maximum acceleration at 55 mph is then $a_{max} = \frac{T_{max} - F_D - F_R}{M}$ $\quad a_{max} = 10.8\,\frac{\text{ft}}{\text{s}^2}$

The maximum speed is obtained when the maximum engine power is just balanced by power consumed by drag and rolling resistance

For maximum speed: $P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}{}^{2}\cdot A\cdot C_D + F_R\right)\cdot V_{max}$

This is a cubic equation that can be solved by iteration or by using *Excel*'s *Goal Seek* or *Solver* $V_{max} = 150\,mph$

We are to evaluate several possible improvements:

For improved drive train $\eta = \eta + 5\cdot\%$ $\eta = 93.7\,\%$ $P_{max} = \eta\cdot P_{engine}$ $P_{max} = 155\,hp$

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}{}^{2}\cdot A\cdot C_D + F_R\right)\cdot V_{max}$$

Solving the cubic (using *Solver*) $V_{max} = 153\,mph$

Improved drag coefficient: $C_{Dnew} = 0.29$

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}{}^{2}\cdot A\cdot C_{Dnew} + F_R\right)\cdot V_{max}$$

Solving the cubic (using *Solver*) $V_{max} = 158\,mph$ This is the best option!

Reduced rolling resistance: $F_{Rnew} = 0.93\cdot\%\cdot M\cdot g$ $F_{Rnew} = 25.6\,lbf$

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}{}^{2}\cdot A\cdot C_D + F_{Rnew}\right)\cdot V_{max}$$

Solving the cubic (using *Solver*) $V_{max} = 154\,mph$

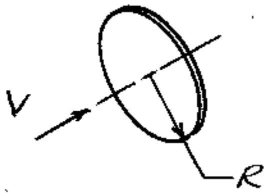
Problem 9.115

[4]

Given: Round, thin disk of radius, R, with pressure data:

$C_p = 1-\left(\frac{r}{R}\right)^6$ (front)

$C_p = -0.42$ (rear)



Find: Calculate the drag coefficient, C_D.

Solution: Computing equations are

$$C_p = \frac{p-p_\infty}{\frac{1}{2}\rho V^2} \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} \qquad A = \pi R^2$$

Assumptions: (1) Steady, incompressible flow
(2) Neglect skin friction drag (disk thin, edge area small)

Then $$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{\int_A (p_f - p_r)\,dA}{\frac{1}{2}\rho V^2 \pi R^2} = \frac{\int_0^R (p_f - p_r)\,2\pi r\,dr}{\frac{1}{2}\rho V^2 \pi R^2}$$

From defn of C_p, $p_f = p_\infty + C_{p_f}\frac{1}{2}\rho V^2$, $p_r = p_\infty + C_{p_r}\frac{1}{2}\rho V^2$

and $$p_f - p_r = (C_{p_f} - C_{p_r})\frac{1}{2}\rho V^2$$

Substituting,

$$C_D = \frac{\frac{1}{2}\rho V^2 \int_0^R (C_{p_f} - C_{p_r})\,2\pi r\,dr}{\frac{1}{2}\rho V^2 \pi R^2} = \frac{2}{R^2}\int_0^R \left[1-\left(\frac{r}{R}\right)^6 + 0.42\right] r\,dr$$

$$= 2\int_0^1 \left[1.42 - \left(\frac{r}{R}\right)^7\right]\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2\left[\frac{1.42}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{8}\left(\frac{r}{R}\right)^8\right]_0^1$$

$$= 2\,(0.710 - 0.125)$$

$$C_D = 1.17 \longleftarrow$$

C_D

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42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
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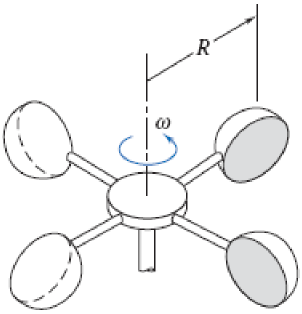
Problem 9.116

[5]

9.116 Repeat the analysis for the frictionless anemometer of Problem 9.106, except this time base the torque calculations on the more realistic model that the average torque is obtained by integrating, over one revolution, the instantaneous torque generated by each cup (i.e., as the cup's orientation to the wind varies).



Given: Data on dimensions of anemometer

Find: Calibration constant

Solution:

The given data or available data is $D = 50\cdot mm$ $R = 80\cdot mm$ $\rho = 1.23\cdot\frac{kg}{m^3}$

The drag coefficients for a cup with open end facing the airflow and a cup with open end facing downstream are, respectively, from Table

$C_{Dopen} = 1.42$ $C_{Dnotopen} = 0.38$

Assume the anemometer achieves steady speed ω due to steady wind speed *V*

The goal is to find the calibration constant *k*, defined by $k = \frac{V}{\omega}$

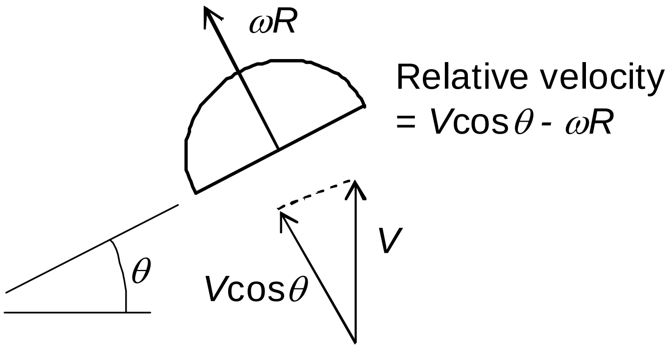
We will analyse each cup separately, with the following assumptions

1) Drag is based on the instantaneous normal component of velocity (we ignore possible effects on drag coefficient of velocity component parallel to the cup)

2) Each cup is assumed unaffected by the others - as if it were the only object present

3) Swirl is neglected

4) Effects of struts is neglected



In this more sophisticated analysis we need to compute the instantaneous normal relative velocity. From the sketch, when a cup is at angle θ, the normal component of relative velocity is

$$V_n = V\cdot\cos(\theta) - \omega\cdot R \qquad (1)$$

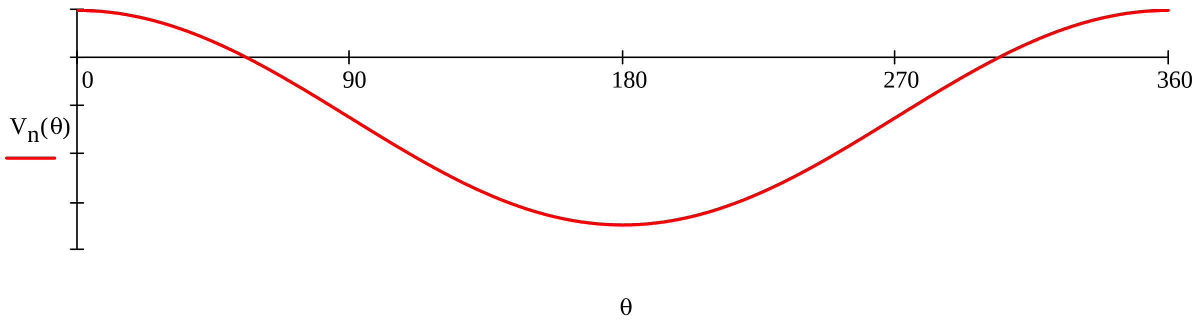
The relative velocity is sometimes positive sometimes negatiive. From Eq. 1, this is determined by

$$\theta_c = \operatorname{acos}\left(\frac{\omega\cdot R}{V}\right) \qquad (2)$$

For $0 < \theta < \theta_c$ $\quad V_n > 0$

$\theta_c < \theta < 2\cdot\pi - \theta_c$ $\quad V_n < 0$

$\theta_c < \theta < 2\cdot\pi$ $\quad V_n > 0$



The equation for computing drag is $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\,2}\cdot C_D \qquad (3)$

where $\quad A = \frac{\pi\cdot D^2}{4} \qquad A = 1.96\times10^{-3}\,m^2$

In Eq. 3, the drag coefficient, and whether the drag is postive or negative, depend on the sign of the relative velocity

For $0 < \theta < \theta_c$ $\quad C_D = C_{Dopen}$ $\quad F_D > 0$

$\theta_c < \theta < 2\cdot\pi - \theta_c$ $\quad C_D = C_{Dnotopen}$ $\quad F_D < 0$

$\theta_c < \theta < 2\cdot\pi$ $\quad C_D = C_{Dopen}$ $\quad F_D > 0$

The torque is $\quad T = F_D\cdot R = \frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\,2}\cdot C_D\cdot R$

The average torque is $\quad T_{av} = \frac{1}{2\cdot\pi}\cdot\int_{\theta}^{2\cdot\pi} T\,d\theta = \frac{1}{\pi}\cdot\int_{\theta}^{\pi} T\,d\theta$

where we have taken advantage of symmetry

Evaluating this, allowing for changes when θ = θ_c $\quad T_{av} = \frac{1}{\pi}\cdot\int_{\theta}^{\theta_c}\frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\,2}\cdot C_{Dopen}\cdot R\,d\theta - \frac{1}{\pi}\cdot\int_{\theta_c}^{\pi}\frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\,2}\cdot C_{Dnotopen}\cdot R\,d\theta$

Using Eq. 1
$$T_{av} = \frac{\rho\cdot A\cdot R}{2\cdot\pi}\cdot\left[C_{Dopen}\cdot\int_{\theta}^{\theta_c}(V\cdot\cos(\theta)-\omega\cdot R)^2\,d\theta - C_{Dnotopen}\cdot\int_{\theta_c}^{\pi}(V\cdot\cos(\theta)-\omega\cdot R)^2\,d\theta\right]$$

$$T_{av} = \frac{\rho\cdot A\cdot R\cdot\omega^2}{2\cdot\pi}\cdot\left[C_{Dopen}\cdot\int_{\theta}^{\theta_c}\left(\frac{V}{\omega}\cdot\cos(\theta)-R\right)^2 d\theta - C_{Dnotopen}\cdot\int_{\theta_c}^{\pi}\left(\frac{V}{\omega}\cdot\cos(\theta)-R\right)^2 d\theta\right]$$

and note that $\frac{V}{\omega} = k$

The integral is
$$\int (k\cdot\cos(\theta)-R)^2\,d\theta = k^2\cdot\left(\frac{1}{2}\cdot\cos(\theta)\cdot\sin(\theta)+\frac{1}{2}\cdot\theta\right)-2\cdot k\cdot R\cdot\sin(\theta)+R^2\cdot\theta$$

For convenience define
$$f(\theta) = k^2\cdot\left(\frac{1}{2}\cdot\cos(\theta)\cdot\sin(\theta)+\frac{1}{2}\cdot\theta\right)-2\cdot k\cdot R\cdot\sin(\theta)+R^2\cdot\theta$$

Hence
$$T_{av} = \frac{\rho\cdot A\cdot R}{2\cdot\pi}\cdot\left[C_{Dopen}\cdot f\left(\theta_c\right)-C_{Dnotopen}\cdot\left(f(\pi)-f\left(\theta_c\right)\right)\right]$$

For steady state conditions the torque (of each cup, and of all the cups) is zero. Hence

$$C_{Dopen}\cdot f\left(\theta_c\right)-C_{Dnotopen}\cdot\left(f(\pi)-f\left(\theta_c\right)\right) = 0$$

or
$$f\left(\theta_c\right) = \frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot f(\pi)$$

Hence
$$k^2\cdot\left(\frac{1}{2}\cdot\cos\left(\theta_c\right)\cdot\sin\left(\theta_c\right)+\frac{1}{2}\cdot\theta_c\right)-2\cdot k\cdot R\cdot\sin\left(\theta_c\right)+R^2\cdot\theta_c = \frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot\left(k^2\cdot\frac{\pi}{2}+R^2\cdot\pi\right)$$

Recall from Eq 2 that $\theta_c = \operatorname{acos}\left(\frac{\omega\cdot R}{V}\right)$ or $\theta_c = \operatorname{acos}\left(\frac{R}{k}\right)$

Hence
$$k^2\cdot\left(\frac{1}{2}\cdot\frac{R}{k}\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+\frac{1}{2}\cdot\operatorname{acos}\left(\frac{R}{k}\right)\right)-2\cdot k\cdot R\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+R^2\cdot\operatorname{acos}\left(\frac{R}{k}\right) = \frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot\left(k^2\cdot\frac{\pi}{2}+R^2\cdot\pi\right)$$

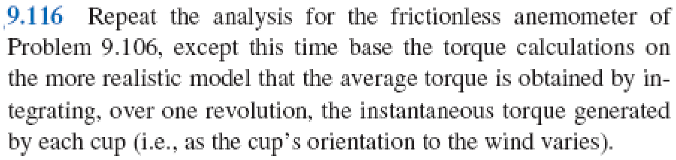
This equation is to be solved for the coefficient *k*. The equation is highly nonlinear; it can be solved by iteration or using *Excel*'s *Goal Seek* or *Solver*

From the associated *Excel* workbook

$$k = 0.316\cdot m \qquad k = 0.119\cdot\frac{\frac{km}{hr}}{rpm}$$

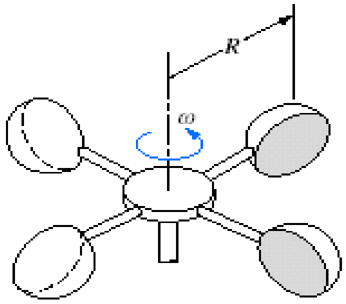
Problem 9.116 (In Excel)

[5]



9.116 Repeat the analysis for the frictionless anemometer of Problem 9.106, except this time base the torque calculations on the more realistic model that the average torque is obtained by integrating, over one revolution, the instantaneous torque generated by each cup (i.e., as the cup's orientation to the wind varies).

Given: Data on dimensions of anemometer



Find: Calibration constant

Solution:

Given data:

D = 50 mm

R = 80 mm

C_{Dopen} = 1.42

$C_{Dnotopen}$ = 0.38

$$k^2\cdot\left(\frac{1}{2}\cdot\frac{R}{k}\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+\frac{1}{2}\cdot\operatorname{acos}\left(\frac{R}{k}\right)\right)-2\cdot k\cdot R\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+R^2\cdot\operatorname{acos}\left(\frac{R}{k}\right)=\frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot\left(k^2\cdot\frac{\pi}{2}+R^2\cdot\pi\right)$$

Use *Solver* to find *k* to make the error zero!

***k* (mm)**	**Left**	**Right**	**Error**
315.85	37325.8	37326	0%

k = 0.316 m

k = 0.119 km/hr/rpm

Problem 9.117

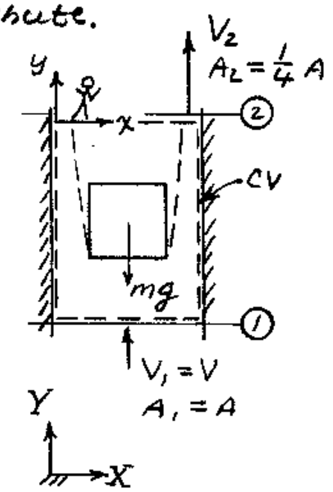
42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Object of mass, m, falling in air down mail chute.

Motion is steady.

Wake area at ② is 3/4 of chute area.

Pressure is constant in wake.



Find: Expression for terminal speed of object.

Solution: Choose a CV moving with the object.
Apply continuity, Bernoulli, and y momentum.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g\cancel{z_2}$$

$$F_{S_y} + F_{B_y} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v\rho\, d\forall + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow relative to CV
(2) Incompressible flow
(3) Neglect friction
(4) Flow along a streamline
(5) Uniform flow and pressures at ① and ②.
(6) Neglect Δz
(7) No net flow in wake

From continuity,

$$0 = \{-\rho V_1 A_1\} + \{\rho V_2 A_2\} \quad \text{so} \quad V_2 = V_1\frac{A_1}{A_2} = V\frac{A}{A_2}$$

From Bernoulli

$$p_1 - p_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V^2\left[\left(\frac{V_2}{V}\right)^2 - 1\right] = \frac{1}{2}\rho V^2\left[\left(\frac{A}{A_2}\right)^2 - 1\right]$$

From momentum

$$p_1 A - p_2 A - mg = v_1\{-\rho V_1 A_1\} + v_2\{+\rho V_2 A_2\} = \rho V A(V_2 - V) = \rho V^2 A\left(\frac{V_2}{V} - 1\right)$$

$$v_1 = V \qquad v_2 = V_2$$

or

$$(p_1 - p_2)A - mg = \rho V^2 A\left(\frac{A}{A_2} - 1\right)$$

Substituting for $p_1 - p_2$

$$\frac{1}{2}\rho V^2 A\left[\left(\frac{A}{A_2}\right)^2 - 1\right] - mg = \rho V^2 A\left(\frac{A}{A_2} - 1\right) \quad \text{or} \quad mg = \frac{1}{2}\rho V^2 A\left[\left(\frac{A}{A_2}\right)^2 - 2\left(\frac{A}{A_2}\right) + 1\right]$$

Thus

$$V = \left[\frac{2mg}{\rho A}\,\frac{1}{\left(\frac{A}{A_2}\right)^2 - 2\left(\frac{A}{A_2}\right) + 1}\right]^{1/2} \longleftarrow$$

V

where A_2 is the net flow area at Section ②.

Problem 9.118

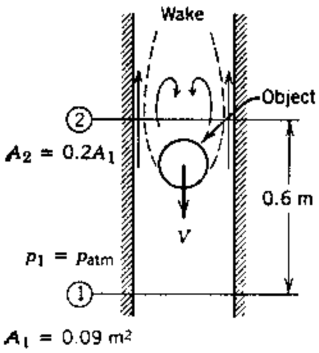
[4]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

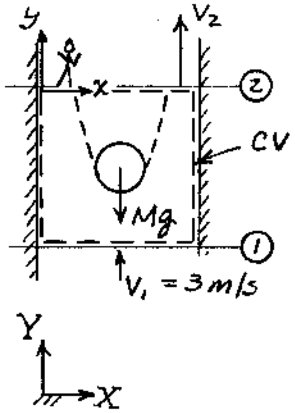
Given: Object falling in air down chute.

$V = 3\ m/s$

Frictional effects negligible.



Find: (a) Flow speed, V_2, relative to object.
(b) Static pressure, p_2.
(c) Mass of object.



Solution: Choose a CV moving with object. Apply continuity, Bernoulli, and y momentum.

Basic equations:

$$0 = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV}\rho\,d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g\cancel{z_2}$$

$$F_{S_y} + F_{B_y} = \overset{=0(1)}{\cancel{\frac{\partial}{\partial t}}}\int_{CV} v\rho\,d\forall + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$$

Assumptions: (1) Steady flow relative to CV (7) No net flow in wake
(2) Incompressible flow
(3) Neglect friction
(4) Flow along a streamline
(5) Neglect Δz
(6) Uniform flow and pressures at ① and ②.

Then from continuity,

$$0 = \{-\rho V_1 A_1\} + \{+\rho V_2 A_2\} \quad \text{so} \quad V_2 = V_1\frac{A_1}{A_2} = 3\,\frac{m}{s}\times\frac{1}{0.2} = 15\ m/s$$ V_2

From Bernoulli,

$$p_2 = p_1 + \tfrac{1}{2}\rho V_1^2 - \tfrac{1}{2}\rho V_2^2 = p_1 + \tfrac{1}{2}\rho V_1^2\left[1 - \left(\frac{V_2}{V_1}\right)^2\right]$$

$$p_2(\text{gage}) = \frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(3)^2\frac{m^2}{s^2}\left[1-\left(\frac{1}{0.2}\right)^2\right]\frac{N\cdot s^2}{kg\cdot m} = -133\ N/m^2\ (\text{gage})$$ p_2

From momentum

$$p_1A_1 - p_2A_2 - Mg = v_1\{-\rho V_1A_1\} + v_2\{+\rho V_2A_2\} = \rho V_1A_1(V_2 - V_1) = \rho V_1^2A_1\left(\frac{V_2}{V_1} - 1\right)$$

$$v_1 = V_1 \qquad v_2 = V_2$$

Thus

$$M = \frac{A_1}{g}\left[p_1 - p_2 - \rho V_1^2\left(\frac{V_2}{V_1} - 1\right)\right]$$

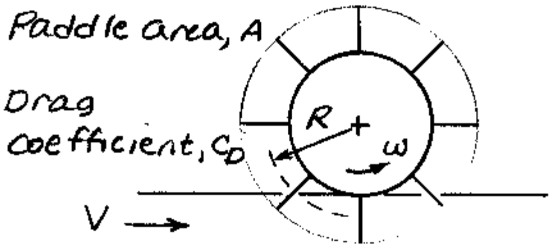
$$M = 0.09\ m^2\times\frac{s^2}{9.81\ m}\left[133\,\frac{N}{m^2}\times\frac{kg\cdot m}{N\cdot s^2} - 1.23\,\frac{kg}{m^3}\times(3)^2\frac{m^2}{s^2}\left(\frac{1}{0.2} - 1\right)\right] = 0.814\ kg$$ M

Problem 9.119

[5]

Given: Paddle wheel immersed in river current as shown.

Assume only one paddle equivalent is immersed at a time.



Find: Expressions for (a) force, (b) torque, and (c) power produced by the wheel, and find optimum angular speed.

Solution: Computing equations $F_D = C_D A \frac{1}{2}\rho V_{rel}^2$

$$T = F_D R \quad , \quad P = T\omega$$

Assumptions: (1) Neglect air resistance, since $\rho_{air} \ll \rho_{water}$
(2) Use velocity relative to the paddle

Thus $V_{rel} = V - U = V - R\omega$

$$F_D = C_D A \frac{1}{2}\rho V_{rel}^2 = C_D A \frac{1}{2}\rho (V-U)^2 \longleftarrow$$

F_D

The torque is

$$T = F_D R = C_D A \frac{1}{2}\rho (V-U)^2 R \longleftarrow$$

T

The power is

$$P = T\omega = C_D A \frac{1}{2}\rho (V-U)^2 R\omega = C_D A \frac{1}{2}\rho (V-U)^2 U \longleftarrow$$

P

To optimize power, set $\frac{dP}{dU} = 0$

$$\frac{dP}{dU} = C_D A \frac{1}{2}\rho \left[2(V-U)U(-1) + (V-U)^2\right] = 0$$

Cancelling a factor $(V-U)$ gives $-2U + V - U = 0$ or $V - 3U = 0$

Thus $U = R\omega = \frac{V}{3}$ so

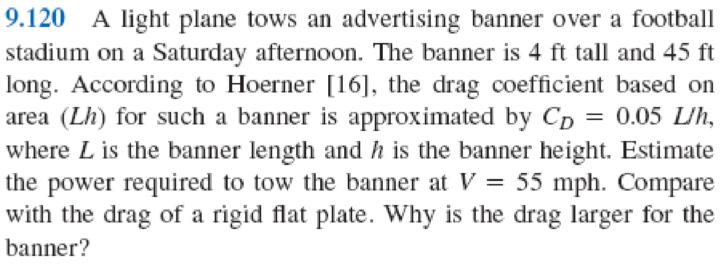
$$\omega_{opt} = \frac{V}{3R} \longleftarrow$$

ω_{opt}

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 9.120

[2]



9.120 A light plane tows an advertising banner over a football stadium on a Saturday afternoon. The banner is 4 ft tall and 45 ft long. According to Hoerner [16], the drag coefficient based on area (Lh) for such a banner is approximated by $C_D = 0.05\ L/h$, where L is the banner length and h is the banner height. Estimate the power required to tow the banner at $V = 55$ mph. Compare with the drag of a rigid flat plate. Why is the drag larger for the banner?

Given: Data on advertising banner

Find: Power to tow banner; Compare to flat plate; Explain discrepancy

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad P = F_D\cdot V$

The given data or available data is $V = 55\cdot\text{mph}$ $\quad V = 80.7\cdot\frac{\text{ft}}{\text{s}}$ $\quad L = 45\cdot\text{ft}$ $\quad h = 4\cdot\text{ft}$ $\quad \rho = 0.00234\cdot\frac{\text{slug}}{\text{ft}^3}$

$A = L\cdot h$ $\quad A = 180\cdot\text{ft}^2$ $\quad C_D = 0.05\cdot\frac{L}{h}$ $\quad C_D = 0.563$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 771\cdot\text{lbf}$ $\quad P = F_D\cdot V$ $\quad P = 6.22\times 10^4\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $\quad P = 113\cdot\text{hp}$

For a flate plate, check Re $\quad \nu = 1.62\times 10^{-4}\cdot\frac{\text{ft}^2}{\text{s}}$ $\quad$ (Table A.9, 69ºF)

$Re_L = \frac{V\cdot L}{\nu}$ $\quad Re_L = 2.241\times 10^7$ $\quad$ so flow is fully turbulent. Hence use Eq 9.37b

$C_D = \frac{0.455}{\log\left(Re_L\right)^{2.58}} - \frac{1610}{Re_L}$ $\quad C_D = 0.00258$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 3.53\cdot\text{lbf}$

This is the drag on one side. The total drag is then $2\cdot F_D = 7.06\cdot\text{lbf}$. This is VERY much less than the banner drag. The banner drag allows for banner flutter and other secondary motion which induces significant form drag.

Problem 9.121

[1]

9.121 The antenna on a car is 10 mm in diameter and 1.8 m long. Estimate the bending moment that tends to snap it off if the car is driven at 120 km/hr on a standard day.

Given: Data on car antenna

Find: Bending moment

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

The given or available data is $V = 120\cdot\frac{km}{hr}$ $\quad V = 33.3\cdot\frac{m}{s}$ $\quad L = 1.8\cdot m$ $\quad D = 10\cdot mm$

$A = L\cdot D$ $\quad A = 0.018\,m^2$

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ $\quad$ (Table A.10, 20ºC)

For a cylinder, check Re $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 2.22\times 10^4$

From Fig. 9.13 $\quad C_D = 1.0$ $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 12.3\,N$

The bending moment is then $\quad M = F_D\cdot\frac{L}{2}$ $\quad M = 11.0\cdot N\cdot m$

Problem 9.122

[1]

9.122 A large three-blade horizontal axis wind turbine (HAWT) can be damaged if the wind speed is too high. To avoid this, the blades of the turbine can be oriented so that they are parallel to the flow. Find the bending moment at the base of each blade when the wind speed is 45 m/s. Model each blade as a flat plate 35 m wide and 0.45 m long.

Given: Data on wind turbine blade

Find: Bending moment

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

The given or available data is $V = 45\cdot\frac{m}{s}$ $\quad L = 0.45\cdot m$ $\quad W = 35\cdot m$

$A = L\cdot W$ $\quad A = 15.75\, m^2$

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ $\quad$ (Table A.10, 20ºC)

For a flat plate, check Re $\quad Re_L = \frac{V\cdot L}{\nu}$ $\quad Re_L = 1.35\times 10^6$ $\quad$ so use Eq. 9.37a

$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} - \frac{1740}{Re_L}$ $\quad C_D = 0.00312$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 61.0\, N$

The bending moment is then $\quad M = F_D\cdot\frac{W}{2}$ $\quad M = 1067\cdot N\cdot m$

Problem 9.123

[4]

9.123 The HAWT of Problem 9.122 is not self-starting. The generator is used as an electric motor to get the turbine up to the operating speed of 20 rpm. To make this easier, the blades are aligned so they lie in the plane of rotation. Assuming an overall efficiency of motor and drive train of 65%, find the power required to maintain the turbine at the operating speed. As an approximation, model each blade as a series of flat plates (the outer region of each blade moves at a significantly higher speed than the inner region).

Given: Data on wind turbine blade

Find: Power required to maintain operating speed

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

The given or available data is $\omega = 20\cdot\text{rpm}$ $\quad L = 0.45\cdot m$ $\quad w = 35\cdot m$

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ (Table A.10, 20ºC)

The velocity is a function of radial position, $V(r) = r\cdot\omega$, so Re varies from 0 to $Re_{max} = \frac{V(w)\cdot L}{\nu}$ $\qquad Re_{max} = 2.20\times 10^6$

The transition Reynolds number is 500,000 which therefore occurs at about 1/4 of the maximum radial distance; the boundary layer is laminar for the first quarter of the blade. We approximate the entire blade as turbulent - the first 1/4 of the blade will not exert much moment in any event

Hence $Re(r) = \frac{L}{\nu}\cdot V(r) = \frac{L\cdot\omega}{\nu}\cdot r$

Using Eq. 9.37a

$$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} - \frac{1740}{Re_L} = \frac{0.0742}{\left(\frac{L\cdot\omega}{\nu}\cdot r\right)^{\frac{1}{5}}} - \frac{1740}{\frac{L\cdot\omega}{\nu}\cdot r} = 0.0742\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot r^{-\frac{1}{5}} - 1740\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot r^{-1}$$

The drag on a differential area is $dF_D = \frac{1}{2}\cdot\rho\cdot dA\cdot V^2\cdot C_D = \frac{1}{2}\cdot\rho\cdot L\cdot V^2\cdot C_D\cdot dr$ $\qquad$ The bending moment is then $dM = dF_D\cdot r$

Hence $M = \int 1\,dM = \int_0^w \frac{1}{2}\cdot\rho\cdot L\cdot V^2\cdot C_D\cdot r\,dr$

$$M = \int_0^w \frac{1}{2}\cdot\rho\cdot L\cdot\omega^2\cdot r^3\cdot\left[0.0742\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot r^{-\frac{1}{5}} - 1740\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot r^{-1}\right]dr$$

$$M = \frac{1}{2}\cdot\rho\cdot L\cdot\omega^2\cdot\int_0^w \left[0.0742\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot r^{\frac{14}{5}} - 1740\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot r^2\right]dr$$

$$M = \frac{1}{2}\cdot\rho\cdot L\cdot\omega^2\cdot\left[\frac{5\cdot 0.0742}{19}\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot w^{\frac{19}{5}} - \frac{1740}{3}\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot w^3\right]$$

$M = 1.43\cdot kN\cdot m$ $\qquad$ Hence the power is $P = M\cdot\omega$ $\qquad P = 3.00\,kW$

Problem 9.124

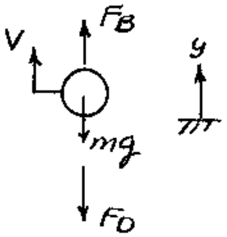
[2]

Given: Small droplets of oil (SG = 0.85) rising in water.

Find: (a) Relationship for terminal speed, V_t (m/s), as a function of droplet diameter, D(in mm).
(b) Range of D for which Stokes flow is a reasonable assumption.

Solution: Draw free-body diagram of droplet, apply Newton's second law.

Basic equation: $\Sigma F_y = -mg + F_B - F_D = ma_y$



Assume: Stokes' drag law, $F_D = 3\pi\mu VD$, for $Re < 10$

Then $-\rho\forall g + \rho_{H_2O}\forall g - 3\pi\mu V_t D = 0$ at terminal speed, V_t.

Solving, $V_t = \frac{(\rho_{H_2O} - \rho_o)\forall g}{3\pi\mu D} = \rho_{H_2O}(1-SG_o)\frac{\pi D^3}{6}\frac{g}{3\pi\mu D} = \frac{(1-SG_o)D^2 g}{18\,\nu}$

Evaluating,

$$V_t(m/s) = \frac{(1-0.85)}{18} \times D^2\,mm^2 \times 9.81\,\frac{m}{s} \times \frac{s}{1.00\,10^{-6}\,m^2} \times \frac{m^2}{10^6\,mm^2} \qquad (T = 20°C)$$

$$V_t(m/s) = 0.0818\left[D(mm)\right]^2$$ V_t

For Stokes flow, $Re < 1$, so

$$Re = \frac{\rho V_t D}{\mu} = \frac{V_t D}{\nu} = \frac{(1-SG_o)D^3 g}{18\,\nu^2} \leq 1$$

Thus

$$D^3 \leq \frac{18\,\nu^2}{(1-SG_o)g} \quad \text{or} \quad D \leq \left[\frac{18\,\nu^2}{(1-SG_o)g}\right]^{1/3}$$

Evaluating,

$$D \leq \left[\frac{18}{(1-0.85)}(1.00\times10^{-6})^2\frac{m^4}{s^2}\times\frac{s^2}{9.81\,m}\right]^{1/3} = 2.31\times10^{-4}\,m \;(0.231\,mm)$$ D

Thus Stokes' flow will be a valid assumption for D < 0.231 mm.

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Problem 9.125

[2]

Given: Wind tunnel with standard air drawn in.

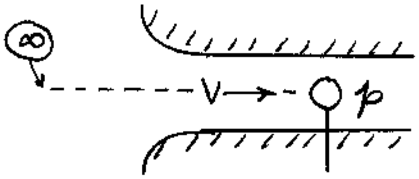
Sphere with $D = 30$ mm on a force balance.

Static pressure in tunnel, $p = -40$ mm (oil, SG = 0.85)

Find: (a) Freestream air speed

(b) Reynolds number for flow over sphere

(c) Drag force on sphere.



Solution: Apply Bernoulli

$$p_\infty + \frac{1}{2}\rho \overset{\approx 0(6)}{\cancel{V_\infty^2}} + \rho g \overset{(5)}{\cancel{z_\infty}} = p + \frac{1}{2}\rho V^2 + \rho g \overset{(5)}{\cancel{z}}$$

Assume: (1) Steady flow
(2) Incompressible flow
(3) Flow along a streamline
(4) No friction (neglect honeycomb and/or screens
(5) Neglect z
(6) $V_\infty \approx 0$

Then $p_\infty = p + \frac{1}{2}\rho V^2$ or $V = \sqrt{\dfrac{2(p_\infty - p)}{\rho}}$

But $p_\infty - p = -\rho_{oil}\, g\, \Delta h = -SG\, \rho_{H_2O}\, g\, \Delta h$

$$V = \left[\frac{-2\, SG\, \rho_{H_2O}\, g\, \Delta h}{\rho}\right]^{\frac{1}{2}}$$

$$V = \left[-2(0.85) \times 1000\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (-0.04\,m) \times \frac{m^3}{1.23\,kg}\right]^{\frac{1}{2}} = 23.3\ m/s$$

V

and

$$Re = \frac{VD}{\nu} = 23.3\,\frac{m}{s} \times 0.03\,m \times \frac{s}{1.45 \times 10^{-5}\,m^2} = 48{,}200$$

Re

Re is subcritical; BLs are laminar, and $C_D = 0.47$

$$F_D = C_D A \frac{1}{2}\rho V^2 \qquad A = \frac{\pi D^2}{4} = 7.07 \times 10^{-4}\ m^2$$

$$= 0.47 \times 7.07 \times 10^{-4}\,m^2 \times \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (23.3)^2\,\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$F_D = 0.111\ N$$

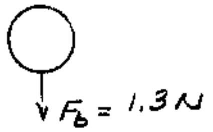
F_D

Problem 9.126

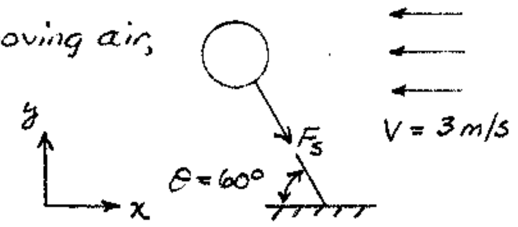
[2]

Given: Spherical hydrogen-filled balloon, $D = 0.6\ m$, in standard air.

In stationary air,



In moving air,



Find: Drag coefficient of balloon.

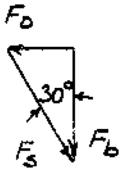
Solution: Apply Newton's second law of motion, definition of C_D.

Basic equations: $\Sigma F_x = m\overset{=0(1)}{\cancel{a_x}}$, $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

Assumptions: (1) Balloon is stationary

(2) F_b (buoyancy of balloon) $= 1.3\ N$

Then a free body diagram gives



From the diagram, $F_D = F_s \tan 30°$

$$F_D = 1.3\ N \times 0.577 = 0.750\ N$$

For standard air, $\rho = 1.23\ kg/m^3$. $A = \pi D^2/4$, so

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{8}{\pi}\frac{F_D}{\rho V^2 D^2}$$

$$C_D = \frac{8}{\pi} \times 0.750\ N \times \frac{m^3}{1.23\ kg} \times \frac{s^2}{(3)^2 m^2} \times \frac{1}{(0.6)^2 m^2} \times \frac{kg \cdot m}{N \cdot s^2} = 0.479$$

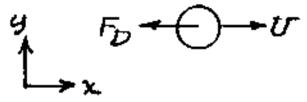
C_D

Problem 9.127

Given: Field hockey ball with $D = 73$ mm and $m = 160$ g, leaving stick at $U_0 = 50$ m/s. Ball is smooth sphere.

Find: Estimate distance traveled in horizontal flight to reduce speed of ball 10 percent.

Solution: Apply Newton's second law of motion:



Basic equation: $\Sigma F_x = m a_x = m \dfrac{dU}{dt} = mU \dfrac{dU}{dx}$

Thus $-F_D = -C_D A \frac{1}{2} \rho U^2 = mU \dfrac{dU}{dx}$ or $dx = -\dfrac{2m}{C_D A \rho} \dfrac{dU}{U}$

Check Re to find C_D (use ν at $T = 15°C$ from Table A.10):

$$Re \leq \frac{U_0 D}{\nu} = 50 \frac{m}{s} \times 0.075\, m \times \frac{s}{1.46 \times 10^{-5}\, m^2} = 2.57 \times 10^5 \qquad \text{(standard air)}$$

From Fig. 9.11, flow is subcritical and $C_D \simeq 0.47 =$ constant.

Thus
$$x = \int_0^x dx = \int_{U_0}^{U} -\frac{2m}{C_D A \rho} \frac{dU}{U} = -\frac{2m}{C_D A \rho} \ln U \Big]_{U_0}^{U} = -\frac{2m}{C_D A \rho} \ln\left(\frac{U}{U_0}\right)$$

or
$$x = -2 \times 0.160\, kg \times \frac{1}{0.47} \times \frac{4}{\pi (0.073)^2 m^2} \times \frac{m^3}{1.23\, kg} \ln(0.9) = 13.9\, m \longleftarrow x$$

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Problem 9.128

[2]

9.128 Compute the terminal speed of a 3-mm diameter raindrop (assume spherical) in standard air.

Given: 3 mm raindrop

Find: Terminal speed

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\Sigma F = 0$

Given or available data is $D = 3\cdot mm$ $\rho_{H2O} = 1000\cdot\frac{kg}{m^3}$ $\rho_{air} = 1.225\cdot\frac{kg}{m^3}$ $\nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ (Table A.10, 20ºC)

Summing vertical forces $M\cdot g - F_D = M\cdot g - \frac{1}{2}\cdot\rho_{air}\cdot A\cdot V^2\cdot C_D = 0$ Buoyancy is negligible

$M = \rho_{H2O}\cdot\frac{\pi\cdot D^3}{6}$ $M = 1.41\times 10^{-5} kg$ $A = \frac{\pi\cdot D^2}{4}$ $A = 7.07\times 10^{-6} m^2$

Assume the drag coefficient is in the flat region of Fig. 9.11 and verify Re later $C_D = 0.4$

$V = \sqrt{\frac{2\cdot M\cdot g}{C_D\cdot\rho_{air}\cdot A}}$ $V = 8.95\frac{m}{s}$

Check Re $Re = \frac{V\cdot D}{\nu}$ $Re = 1.79\times 10^3$ which does place us in the flat region of the curve

Actual raindrops are not quite spherical, so their speed will only be approximated by this result

Problem 9.129

Given: Small sphere falling through castor oil at 20 C; D = 6 mm.

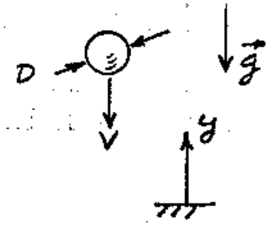
Terminal speed is 60 mm/s.

Find: (a) Compute C_D for given sphere.
(b) Density of sphere.
(c) Compare terminal speed in water.

Solution: Apply Newton's second law of motion, definition of C_D.

Basic equations: $\Sigma F_y = ma_y$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



Assume $Re < 1$, so Stokes flow, $F_D = 3\pi\mu D V$.

From the definition, noting A is the frontal area, $A = \frac{\pi D^2}{4}$,

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{3\pi\mu D V}{\frac{1}{2}\rho V^2 \frac{\pi D^2}{4}} = \frac{24\mu}{\rho V D} = \frac{24}{Re}$$

For castor oil at 20 C, $\mu = 0.9\ \text{N}\cdot\text{s}/\text{m}^2$ (Fig. A.2) and $SG \approx 0.97$ (Table A.2).

$$Re = \frac{SG\,\rho_{H_2O} V D}{\mu} = (0.97)\,999\,\frac{\text{kg}}{\text{m}^3} \times 0.06\,\frac{\text{m}}{\text{s}} \times 0.006\,\text{m} \times \frac{\text{m}^2}{0.9\,\text{N}\cdot\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 0.388 < 1 \ \checkmark\checkmark$$

$$C_D = \frac{24}{Re} = \frac{24}{0.388} = 61.9 \longleftarrow$$ C_D

From Newton's second law, $F_D + F_{buoyancy} - mg = 0$ or $F_D = mg - F_b$. Thus

$$F_D = C_D \tfrac{1}{2}\rho V^2 A = mg - F_b = (SG_s - SG_o)\rho_{H_2O} g \forall \text{ or } SG_s = SG_o + \frac{C_D \frac{1}{2}\rho V^2 A}{\rho_{H_2O} g \forall} = SG_o\left(1 + \frac{C_D V^2 A}{2g\forall}\right)$$

But $\frac{A}{\forall} = \frac{\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{4r} = \frac{3}{2D}$, so $SG_s = SG_o\left(1 + \frac{3}{4}\frac{C_D V^2}{gD}\right)$

$$SG_s = 0.97\left(1 + \frac{3}{4} \times 61.9 \times (0.06)^2\frac{\text{m}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{9.81\,\text{m}} \times \frac{1}{0.006\,\text{m}}\right) = 3.72, \text{ so } \rho_s = 3720\,\frac{\text{kg}}{\text{m}^3} \longleftarrow$$ ρ_s

In water, the net weight and drag must balance, or

$$F_D = \tfrac{1}{2}\rho V^2 C_D A = W - F_b = \gamma_{H_2O}(SG_s - 1)\forall \quad \text{or} \quad V = \left[\frac{4(SG-1)gD}{3C_D}\right]^{\frac{1}{2}}$$

However, C_D is a function of Re, so iteration is needed. From Fig. 9.11, $C_D \approx 0.4$ over a range of Re. Using $C_D = 0.4$,

$$V = \left[\frac{4}{3}\left(\frac{2.72}{0.4}\right) 9.81\,\frac{\text{m}}{\text{s}^2} \times 0.006\,\text{m}\right]^{\frac{1}{2}} = 0.731\ \text{m/s} \longleftarrow$$ V

Check Re:

$$Re = \frac{VD}{\nu} = 0.731\,\frac{\text{m}}{\text{s}} \times 0.006\,\text{m} \times \frac{\text{s}}{1.1\times10^{-6}\,\text{m}^2} = 3990 \quad \text{ok!}$$

Problem 9.130

Given: Curve fit for drag coefficient of a sphere versus Re:

$C_D = 24/Re \qquad Re \le 1$

$C_D = 24/Re^{0.646} \qquad 1 < Re \le 400$

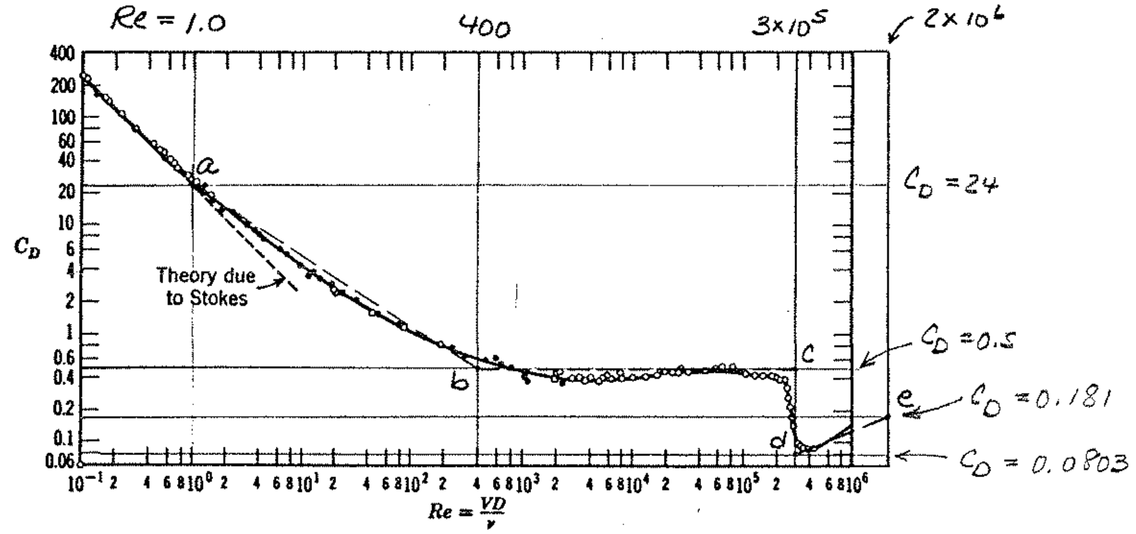
$C_D = 0.5 \qquad 400 < Re \le 3\times10^5$

$C_D = 0.000366\, Re^{0.4275} \qquad 3\times10^5 < Re \le 2\times10^6$

$C_D = 0.18 \qquad Re > 2\times10^6$

Find: Use data from Fig. 9.11 to evaluate the maximum error between the curve fit and experimental data.

Solution: The curve-fit segments are plotted on Fig. 9.11 below:



The maximum significant error occurs in the region where C_D is modeled as equal to the constant value, $C_D = 0.5$. The curve fit appears to be about 10 percent high in the region from $Re \simeq 10^3$ to $Re \simeq 10^4$.

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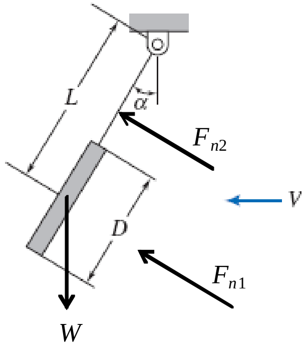
Problem 9.131

[3]

9.131 Problem 9.105 showed a circular disk hung in an air stream from a cylindrical strut. Assume the strut is $L = 40$ mm long and $d = 3$ mm in diameter. Solve Problem 9.105 including the effect of drag on the support.



Given: Circular disk in wind

Find: Mass of disk; Plot α versus V

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\qquad \Sigma \vec{M} = 0$

Summing moments at the pivot $W\cdot L\cdot\sin(\alpha) - F_{n1}\cdot L - \frac{1}{2}\cdot\left(L - \frac{D}{2}\right)\cdot F_{n2} = 0$ (1) and for each normal drag $F_n = \frac{1}{2}\cdot\rho\cdot {V_n}^2\cdot A\cdot C_D$

Assume 1) No pivot friction 2) C_D is valid for $V_n = V\cos(\alpha)$

The data is $\rho = 1.225\cdot\dfrac{kg}{m^3}$ $\qquad \mu = 1.8\times 10^{-5}\cdot\dfrac{N\cdot s}{m^2}$ $\qquad V = 15\cdot\dfrac{m}{s}$

$D = 25\cdot mm$ $\qquad d = 3\cdot mm$ $\qquad L = 40\cdot mm$ $\qquad \alpha = 10\cdot deg$

$C_{D1} = 1.17$ (Table 9.3) $\qquad Re_d = \dfrac{\rho\cdot V\cdot d}{\mu}$ $\qquad Re_d = 3063$ so from Fig. 9.13 $\qquad C_{D2} = 0.9$

Hence $F_{n1} = \frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\dfrac{\pi\cdot D^2}{4}\cdot C_{D1}$ $\qquad F_{n1} = 0.077\,N$

$F_{n2} = \frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\left(L - \frac{D}{2}\right)\cdot d\cdot C_{D2}$ $\qquad F_{n2} = 0.00992\,N$

The drag on the support is much less than on the disk (and moment even less), so results will not be much different from those of Problem

Hence Eq. 1 becomes

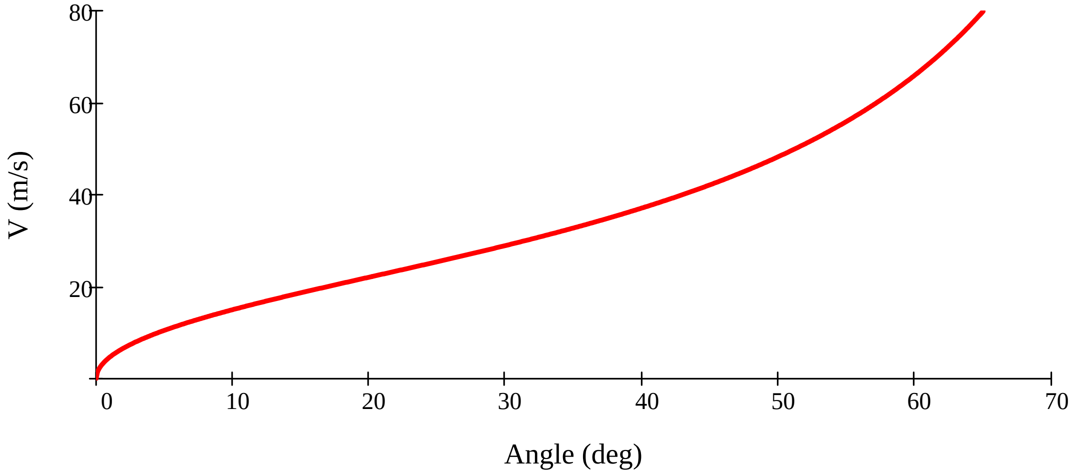
$$M\cdot L\cdot g\cdot\sin(\alpha) = L\cdot\frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\frac{\pi\cdot D^2}{4}\cdot C_{D1} + \frac{1}{2}\cdot\left(L - \frac{D}{2}\right)\cdot\left[\frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\left(L - \frac{D}{2}\right)\cdot d\cdot C_{D2}\right]$$

$$M = \frac{\rho\cdot V^2\cdot\cos(\alpha)^2}{4\cdot g\cdot\sin(\alpha)}\cdot\left[\frac{1}{2}\cdot\pi\cdot D^2\cdot C_{D1} + \left(1 - \frac{D}{2\cdot L}\right)\cdot\left(L - \frac{D}{2}\right)\cdot d\cdot C_{D2}\right]$$

$M = 0.0471\,kg$

Rearranging
$$V = \sqrt{\frac{4 \cdot M \cdot g}{\rho}} \cdot \sqrt{\frac{\tan(\alpha)}{\cos(\alpha)}} \cdot \frac{1}{\sqrt{\left[\frac{1}{2} \cdot \pi \cdot D^2 \cdot C_{D1} + \left(1 - \frac{D}{2 \cdot L}\right) \cdot \left(L - \frac{D}{2}\right) \cdot d \cdot C_{D2}\right]}} \qquad V = 35.5 \cdot \frac{m}{s} \cdot \sqrt{\frac{\tan(\alpha)}{\cos(\alpha)}}$$

We can plot this by choosing α and computing V



This graph can be easily plotted in *Excel*

Problem 9.132

[3]

9.132 A tennis ball with a mass of 57 g and diameter of 64 mm is dropped in standard sea level air. Calculate the terminal velocity of the ball. Assuming as an approximation that the drag coefficient remains constant at its terminal-velocity value, estimate the time and distance required for the ball to reach 95% of its terminal speed.

Given: Data on a tennis ball

Find: Terminal speed time and distance to reach 95% of terminal speed

Solution:

The given data or available data is $M = 57 \cdot gm$ $\quad D = 64 \cdot mm$ $\quad \nu = 1.45 \cdot 10^{-5} \cdot \frac{m^2}{s}$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$

Then $A = \frac{\pi \cdot D^2}{4}$ $\quad A = 3.22 \times 10^{-3} m^2$

Assuming high Reynolds number $C_D = 0.5$ (from Fig. 9.11)

At terminal speed drag equals weight $F_D = M \cdot g$

The drag at speed *V* is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$

Hence the terminal speed is $V_t = \sqrt{\frac{M \cdot g}{\frac{1}{2} \cdot \rho \cdot A \cdot C_D}}$ $\quad V_t = 23.8 \frac{m}{s}$

Check the Reynolds number $Re = \frac{V_t \cdot D}{\nu}$ $\quad Re = 1.05 \times 10^5$ Check!

For motion before terminal speed Newton's second law applies

$$M \cdot a = M \cdot \frac{dV}{dt} = M \cdot g - \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D \quad \text{or} \quad \frac{d}{dt} V = g - k \cdot V^2 \quad \text{where} \quad k = \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \quad k = 0.0174 \frac{1}{m}$$

Separating variables $\int_0^V \frac{1}{g - k \cdot V^2} dV = t$ $\quad \int \frac{1}{g - k \cdot V^2} dV = \frac{1}{\sqrt{g \cdot k}} \cdot \operatorname{atanh}\left(\sqrt{\frac{k}{g}} \cdot V\right)$

Hence $V(t) = \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right)$

Evaluating at $V = 0.95V_t$ $\quad 0.95 \cdot V_t = \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right)$ $\quad t = \frac{1}{\sqrt{g \cdot k}} \cdot \operatorname{atanh}\left(0.95 \cdot V_t \cdot \sqrt{\frac{k}{g}}\right)$ $\quad t = 4.44 s$

For distance *x* versus time, integrate $\frac{dx}{dt} = \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right)$ $\quad x = \int_0^t \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right) dt$

Note that $$\int \tanh(a\cdot t)\,dt = \frac{1}{a}\cdot\ln(\cosh(a\cdot t))$$

Hence $$x(t) = \frac{1}{k}\cdot\ln\left(\cosh\left(\sqrt{g\cdot k}\cdot t\right)\right)$$

Evaluating at $V = 0.95V_t$ $\quad t = 4.44\,s \quad$ so $\quad x(t) = 67.1\,m$

Problem 9.133

9.133 A model airfoil of chord 15 cm and span 60 cm is placed in a wind tunnel with an air flow of 30 m/s (the air is at 20°C). It is mounted on a cylindrical support rod 2 cm in diameter and 25 cm tall. Instruments at the base of the rod indicate a vertical force of 50 N and a horizontal force of 6 N. Calculate the lift and drag coefficients of the airfoil.

Given: Data on model airfoil

Find: Lift and drag coefficients

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ where A is plan area for airfoil, frontal area for rod

Given or available data is $D = 2\cdot cm$ $L = 25\cdot cm$ (Rod) $b = 60\cdot cm$ $c = 15\cdot cm$ (Airfoil)

$V = 30\cdot\dfrac{m}{s}$ $F_L = 50\cdot N$ $F_H = 6\cdot N$

Note that the horizontal force F_H is due to drag on the airfoil AND on the rod

$\rho = 1.225\cdot\dfrac{kg}{m^3}$ $\nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ (Table A.10, 20ºC)

For the rod $Re_{rod} = \dfrac{V\cdot D}{\nu}$ $Re_{rod} = 4\times 10^4$ so from Fig. 9.13 $C_{Drod} = 1.0$

$A_{rod} = L\cdot D$ $A_{rod} = 5\times 10^{-3}\,m^2$

$F_{Drod} = C_{Drod}\cdot\frac{1}{2}\cdot\rho\cdot A_{rod}\cdot V^2$ $F_{Drod} = 2.76\,N$

Hence for the airfoil $A = b\cdot c$ $F_D = F_H - F_{Drod}$ $F_D = 3.24\,N$

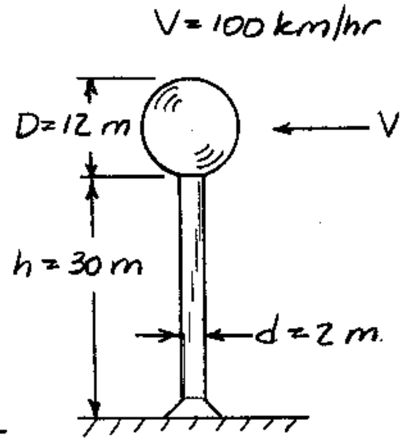
$C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $C_D = 0.0654$ $C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $C_L = 1.01$ $\dfrac{C_L}{C_D} = 15.4$

Problem 9.134

[3]

Given: Water tower as shown.

Standard day.



Find: Estimate bending moment at base of tower.

Solution: Apply definition of C_D, sum moments about base.

Computing equations: $C_D \equiv \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$; $\sum \overset{\curvearrowleft +}{M} = \sum FL$

Assumptions: (1) F_{D_S} acts at center of sphere; F_{D_C} at center of cylinder

(2) Neglect interference between sphere and cylinder.

Then $M = F_{D_S}\left(h + \frac{D}{2}\right) + F_{D_C}\left(\frac{h}{2}\right)$

$A_S = \frac{\pi D^2}{4} = \frac{\pi}{4}(12)^2\,m^2 = 113\,m^2$

$A_C = hd = 30\,m \times 2\,m = 60\,m^2$

$$V = 100\,\frac{km}{hr} \times 1000\,\frac{m}{km} \times \frac{hr}{3600\,s} = 27.8\,m/s$$

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (27.8)^2\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 475\,N/m^2$$

$C_D = C_D(Re)$. For standard air (Table A.10), $\nu = 1.46 \times 10^{-5}\,m^2/s$, so

$$Re_S = \frac{VD}{\nu} = 27.8\,\frac{m}{s} \times 12\,m \times \frac{s}{1.46 \times 10^{-5}\,m^2} = 2.28 \times 10^7$$

This Re is too large for Fig. 9.11. Thus guess $C_{D_S} = 0.18$ (Problem 9.125).

$$F_{D_S} = C_{D_S}\,q\,A_S = 0.18 \times 475\,\frac{N}{m^2} \times 113\,m^2 = 9.66\,kN$$

$$Re_C = \frac{Vd}{\nu} = 27.8\,\frac{m}{s} \times 2.0\,m \times \frac{s}{1.46 \times 10^{-5}\,m^2} = 3.81 \times 10^6$$

This Re is too large for Fig. 9.13. Thus guess $C_{D_C} \simeq 0.4$.

$$F_{D_C} = C_{D_C}\,q\,A_C = 0.4 \times 475\,\frac{N}{m^2} \times 60\,m^2 = 11.4\,kN$$

The moment is

$$M = 9.66\,kN\left(30\,m + \frac{12\,m}{2}\right) + 11.4\,kN\left(\frac{30\,m}{2}\right) = 519\,kN \cdot m$$

M

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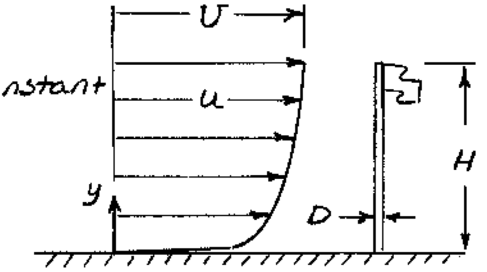
Problem 9.135

[4]

Given: Cylindrical flag pole of height, H.

Wind-speed profile, $\frac{u}{U} = \left(\frac{y}{H}\right)^{1/7}$; C_D constant



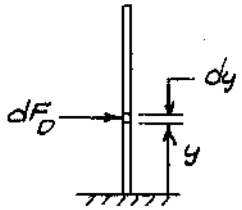
Find: (a) Drag force
(b) Bending moment
(c) Compare with values for uniform profile, U.

Solution: Apply definition of drag coefficient, $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$

Assume: (1) C_D = constant
(2) C_D same as circular cylinder

On an element of the pole,

$$dF_D = C_D \tfrac{1}{2}\rho u^2 dA = C_D \tfrac{1}{2}\rho U^2 \left(\frac{y}{H}\right)^{2/7} D\,dy$$



Thus

$$F_D = \int_0^H dF_D = \int_0^H C_D \tfrac{1}{2}\rho U^2 \left(\frac{y}{H}\right)^{2/7} D\,dy$$

$$F_D = C_D \tfrac{1}{2}\rho U^2 DH \int_0^1 \left(\frac{y}{H}\right)^{2/7} d\left(\frac{y}{H}\right) = C_D \tfrac{1}{2}\rho U^2 DH \left[\frac{7}{9}\left(\frac{y}{H}\right)^{9/7}\right]_0^1 = \frac{7}{9} C_D \tfrac{1}{2}\rho U^2 DH$$ F_D

On an element of the pole,

$$dM = y\,dF_D = y\,C_D \tfrac{1}{2}\rho u^2 dA = y\,C_D \tfrac{1}{2}\rho u^2 D\,dy$$

Thus

$$M = \int_0^H dM = \int_0^H y\,C_D \tfrac{1}{2}\rho U^2 \left(\frac{y}{H}\right)^{2/7} D\,dy = C_D \tfrac{1}{2}\rho U^2 DH^2 \int_0^1 \left(\frac{y}{H}\right)\left(\frac{y}{H}\right)^{2/7} d\left(\frac{y}{H}\right)$$

$$M = C_D \tfrac{1}{2}\rho U^2 DH^2 \int_0^1 \left(\frac{y}{H}\right)^{9/7} d\left(\frac{y}{H}\right) = C_D \tfrac{1}{2}\rho U^2 DH^2 \left[\frac{7}{16}\left(\frac{y}{H}\right)^{16/7}\right]_0^1 = \frac{7}{16} C_D \tfrac{1}{2}\rho U^2 DH^2$$ M

Comparing,

$$\frac{F_D(1/7\text{-profile})}{F_D(\text{uniform})} = \frac{\frac{7}{9} C_D \frac{1}{2}\rho U^2 DH}{C_D \frac{1}{2}\rho U^2 DH} = \frac{7}{9}$$

$$\frac{M(1/7\text{-profile})}{M(\text{uniform})} = \frac{\frac{7}{16} C_D \frac{1}{2}\rho U^2 DH^2}{C_D \frac{1}{2}\rho U^2 DH\,\frac{H}{2}} = \frac{7/16}{1/2} = \frac{7}{8}$$ $\frac{M}{M_U}$

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Problem 9.136

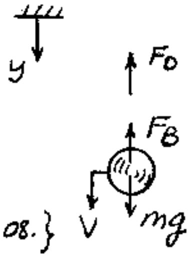
[3]

Given: Cast-iron "12-pounder" ($m = 12$ lbm) cannon ball rolls off ship and sinks in ocean where depth is $d = 1000$ m.

Find: Estimate time elapsed before cannon ball hits sea bottom.

Solution: Apply Newton's second law of motion, definition of C_D.

Computing equations:
$$\sum F_y = m a_y \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



First find diameter of ball. In air,
$$W = mg = \rho \forall g = SG\,\rho_{H_2O}\frac{\pi D^3}{6} g = 12 \text{ lbf} \quad \{\text{From Table A.1, } SG = 7.08.\}$$

Thus
$$D = \left[\frac{6W}{\pi SG\,\rho_{H_2O}\, g}\right]^{1/3} = \left[\frac{6}{\pi} \times 12 \text{ lbf} \times \frac{1}{7.08} \times \frac{ft^3}{1.94\, slug} \times \frac{s^2}{32.2\, ft} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/3} = 0.373 \text{ ft}$$

or
$$D = 0.373 \text{ ft} \times 0.3048 \frac{m}{ft} = 0.114 \text{ m}$$

D

At terminal speed, $V = V_t$, and $a_y = 0$. Summing forces,
$$mg - F_B - F_D = SG\,\rho_{H_2O} \forall g - SG_{sw}\,\rho_{H_2O}\forall g - C_D A \tfrac{1}{2}\rho V_t^{\,2} = 0$$

or
$$V_t = \left[\frac{2(SG_{ci} - SG_{sw})\rho_{H_2O}\forall g}{C_D\, SG_{sw}\,\rho_{H_2O}\, A}\right]^{1/2}$$

Introducing $\forall = \frac{\pi D^3}{6}$ and $A = \frac{\pi D^2}{4}$, then
$$V_t = \left[\frac{4}{3}\frac{(SG_{ci}/SG_{sw} - 1)\, D g}{C_D}\right]^{1/2} = \left[\frac{4}{3}(7.08/1.025 - 1)\, 0.114 m \times 9.81 \frac{m}{s^2} \times \frac{1}{C_D}\right]^{1/2} = \frac{2.97}{\sqrt{C_D}} \text{ m/s}$$

Choose $C_D = 0.47$ from flat range of curve:
$$V_t = \frac{2.97}{\sqrt{0.47}} \text{ m/sec} = 4.33 \text{ m/s} \qquad \{\text{At } T = 20°C,\ \nu_{sw} = 1.05\,\nu_{H_2O} \text{ (Table A.2).}\}$$

Then
$$Re = \frac{\rho V_t D}{\mu} = \frac{V_t D}{\nu_{sw}} = 4.33 \frac{m}{s} \times 0.114 m \times \frac{s}{1.05 \times 10^{-6} m^2} = 4.70 \times 10^5$$

This is a supercritical Re, so choose $C_D \approx 0.09$ (Fig. 9.11). Then
$$V_t = \frac{2.97}{\sqrt{0.09}} \text{ m/s} = 9.90 \text{ m/s}$$

V_t

Then
$$Re = 9.90 \frac{m}{s} \times 0.114 m \times \frac{s}{1.05 \times 10^{-6} m^2} = 1.07 \times 10^6$$

From Fig. 9.11, $C_D \approx 0.14$. Therefore $V_t = \frac{2.97}{\sqrt{0.14}} = 7.94$ m/s
$$t = \frac{d}{V_t} = 1000 m \times \frac{s}{7.94 m} = 126 \text{ s}$$

t

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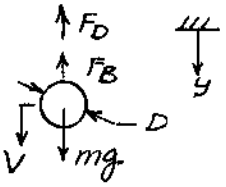
Problem 9.137

[4]

Given: Stokes drag law for smooth spheres, $F_D = 3\pi\mu V D$, to be verified experimentally by dropping steel balls in glycerin.

Find: (a) Largest steel ball for which $Re < 1$.
(b) Height of glycerine column needed to reach 95 percent of terminal speed.

Solution: Draw free-body diagram of ball, apply Newton's second law.



Basic equation: $\Sigma F_y = mg - F_B - F_D = m\frac{dV}{dt} = mV\frac{dV}{dy}$ (1)

$$\forall = \frac{\pi D^3}{6} \qquad m = \rho_s \forall \qquad F_B = \rho_g \forall g \qquad F_D = 3\pi\mu V D$$

At terminal speed, V_t, acceleration is zero. Thus

$$\rho_s \forall g - \rho_g \forall g - 3\pi\mu V_t D = 0 \quad \text{or} \quad V_t = (\rho_s - \rho_g)\frac{\pi D^3}{6} g \frac{1}{3\pi\mu D} = \frac{(\rho_s - \rho_g) D^2 g}{18\mu}$$

$$\text{or} \quad V_t = \frac{(\rho_s/\rho_g - 1)\rho_g D^2 g}{18\mu} = \frac{(SG_s/SG_g - 1) D^2 g}{18\nu} = \frac{(7.8/1.26 - 1) D^2 g}{18\nu} = 0.288\, D^2 g/\nu$$

(from Table A.2, $SG_g = 1.26$). Stokes' drag law holds for $Re < 1$. Thus

$$Re = \frac{\rho V_t D}{\mu} = \frac{V_t D}{\nu} = \frac{0.288 D^3 g}{\nu^2} \leq 1 \quad \text{or} \quad D^3 \leq \frac{1}{0.288}\frac{\nu^2}{g} \quad \text{or} \quad D \leq \left[\frac{3.47\nu^2}{g}\right]^{1/3}$$

Assuming $T = 20°C$, then from Fig. A.3, $\nu = 0.0012\ m^2/s$, so

$$D \leq \left[3.47 \times (0.0012)^2 \frac{m^4}{s^2} \times \frac{s^2}{9.81\,m}\right]^{1/3} = 0.00799\,m \quad (7.99\,mm) \longleftarrow$$ D

From Eq. 1,

$$\rho_s \forall g - \rho_g \forall g - 3\pi\mu V D = \rho_s \forall V \frac{dV}{dy}$$

Dividing by $(\rho_s - \rho_g)\forall g$ gives

$$1 - \frac{3\pi\mu D}{(\rho_s - \rho_g)\frac{\pi D^3}{6} g} V = 1 - \frac{V}{V_t} = \frac{\rho_s \forall}{(\rho_s - \rho_g)\forall g} V\frac{dV}{dy} = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V}{g}\frac{dV}{dy} = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g}\frac{V}{V_t}\frac{d(V/V_t)}{dy}$$

Separating variables, $dy = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g}\frac{(V/V_t)\,d(V/V_t)}{1 - V/V_t} = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g}\frac{r\,dr}{1 - r}$

Integrating, $\int_0^{0.95}\frac{r\,dr}{1-r} = \int_1^{0.05}\frac{(1-x)(-dx)}{x} = \int_1^{0.05} dx - \int\frac{dx}{x} = x - \ln x\Big]_1^{0.05} = -0.95 - \ln(0.05)$

Thus

$$y = 2.05\left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g} = 2.05\left(\frac{SG_s}{SG_s - SG_g}\right)\frac{V_t^2}{g}$$

But $V_t = 0.288\frac{D^2 g}{\nu} = 0.288 \times (0.0172)^2 m^2 \times 9.81\frac{m}{s^2} \times \frac{s}{0.0012\,m^2} = 0.697\ m/s$

so

$$y = 2.05 \times \frac{7.8}{(7.8 - 1.26)} \times (0.697)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\,m} = 0.121\ m \quad (121\,mm) \longleftarrow$$ y

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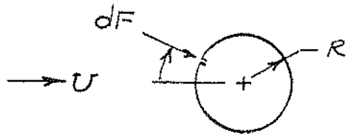
Problem 9.138

[4]

Given: Measured data for pressure difference versus angle for flow around a circular cylinder at $Re = 80{,}000$.

Find: (a) Estimate C_D for this flow.
(b) Compare with data from Fig. 9.13; explain any difference.

Solution: Consider the geometry sketched.
Apply the definition of drag coefficient.



Computing equation: $C_D = \dfrac{F_D}{\frac{1}{2}\rho U^2 A}$

Assumption; Neglect viscous force; $dF_D = dF\cos\theta = p\,dA\cos\theta = pwR\,d\theta\cos\theta$

Then $F_D = \int_A dF_D = \int_0^{2\pi} pwR\,d\theta\cos\theta = 2\int_0^{\pi} pwR\,d\theta\cos\theta = \int_0^{\pi} p\cos\theta\,(w2R)\,d\theta$

Since $\int_0^{\pi} p_\infty\cos\theta\,d\theta = 0$, then $F_D = \int_0^{\pi}(p-p_\infty)\cos\theta\,(w2R)\,d\theta$

The stagnation pressure is $p(0) - p_\infty = \frac{1}{2}\rho U^2$; so

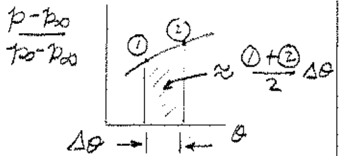
$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{\int_0^{\pi}(p-p_\infty)\cos\theta\,(w2R)\,d\theta}{(p_0-p_\infty)(w2R)} = \int_0^{\pi}\left(\frac{p-p_\infty}{p_0-p_\infty}\right)\cos\theta\,d\theta$$

Tabulating:

θ (deg)	$p-p_\infty$ (in. H_2O)	$\frac{p-p_\infty}{p_0-p_\infty}$	$\left(\frac{p-p_\infty}{p_0-p_\infty}\right)\cos\theta$
0	0.52	1.00	1.00
10	0.46	0.885	0.871
20	0.33	0.635	0.596
30	0.13	0.250	0.217
40	-0.13	-0.250	-0.192
50	-0.40	-0.769	-0.494
60	-0.57	-1.10	-0.548
70	-0.60	-1.15	-0.395
80	-0.55	-1.06	-0.184
90	-0.52	-1.00	0.00
100	-0.51	-0.981	0.170
110	-0.52	-1.00	0.342
120	-0.52	-1.00	0.500
130	-0.52	-1.00	0.643
140	-0.52	-1.00	0.766
150	-0.52	-1.00	0.866
160	-0.54	-1.04	0.976
170	-0.54	-1.04	1.02
180	-0.54	-1.04	1.04

$\Sigma = 7.194$

Trapezoidal rule:


$\frac{p-p_\infty}{p_0-p_\infty}$
①
②
$\approx \frac{①+②}{2}\Delta\theta$
$\Delta\theta$
θ

Half of the end points are subtracted to avoid double counting.

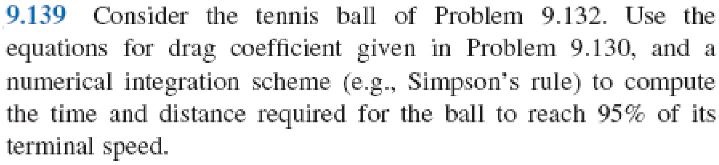
$$C_D \approx \left\{\Sigma\left(\frac{p-p_\infty}{p_0-p_\infty}\right)\cos\theta - \frac{1}{2}(\,)_0 - \frac{1}{2}(\,)_{180}\right\}\Delta\theta = \left\{7.194 - \frac{1}{2}(1.00+1.04)\right\} 10\text{ deg} \times \frac{\pi\text{ rad}}{180\text{ deg}} = 1.08$$

C_D

From Fig. 9.13, $C_D \approx 1.2$. The difference is due to skin friction effects.

Problem 9.139

[4]



9.139 Consider the tennis ball of Problem 9.132. Use the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., Simpson's rule) to compute the time and distance required for the ball to reach 95% of its terminal speed.

Given: Data on a tennis ball

Find: Terminal speed time and distance to reach 95% of terminal speed

Solution:

The given data or available data is $M = 57 \cdot gm$ $D = 64 \cdot mm$ $\nu = 1.45 \cdot 10^{-5} \cdot \frac{m^2}{s}$ $\rho = 1.23 \cdot \frac{kg}{m^3}$

Then $A = \frac{\pi \cdot D^2}{4}$ $A = 3.22 \times 10^{-3} m^2$

From Problem 9.130 $C_D = \frac{24}{Re}$ $Re \le 1$

$C_D = \frac{24}{Re^{0.646}}$ $1 < Re \le 400$

$C_D = 0.5$ $400 < Re \le 3 \times 10^5$

$C_D = 0.000366 \cdot Re^{0.4275}$ $3 \times 10^5 < Re \le 2 \times 10^6$

$C_D = 0.18$ $Re > 2 \times 10^6$

At terminal speed drag equals weight $F_D = M \cdot g$

The drag at speed *V* is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$

Assume $C_D = 0.5$

Hence the terminal speed is $V_t = \sqrt{\frac{M \cdot g}{\frac{1}{2} \cdot \rho \cdot A \cdot C_D}}$ $V_t = 23.8 \frac{m}{s}$

Check the Reynolds number $Re = \frac{V_t \cdot D}{\nu}$ $Re = 1.05 \times 10^5$

This is consistent with the tabulated C_D values!

For motion before terminal speed, Newton's second law is $M \cdot a = M \cdot \frac{dV}{dt} = M \cdot g - \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D$

Hence the time to reach 95% of terminal speed is obtained by separating variables and integrating

$$t = \int_0^{0.95 \cdot V_t} \frac{1}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV$$

For the distance to reach terminal speed Newton's second law is written in the form

$$M \cdot a = M \cdot V \cdot \frac{dV}{dx} = M \cdot g - \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D$$

Hence the distance to reach 95% of terminal speed is obtained by separating variables and integrating

$$x = \int_0^{0.95 \cdot V_t} \frac{V}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV$$

These integrals are quite difficult because the drag coefficient varies with Reynolds number, which varies with speed. They are best evaluated numerically. A form of Simpson's Rule is

$$\int f(V) \, dV = \frac{\Delta V}{3} \cdot \left(f\left(V_0\right) + 4 \cdot f\left(V_1\right) + 2 \cdot f\left(V_2\right) + 4 \cdot f\left(V_3\right) + f\left(V_N\right)\right)$$

where ΔV is the step size, and V_0, V_1 etc., are the velocities at points 0, 1, ... N.

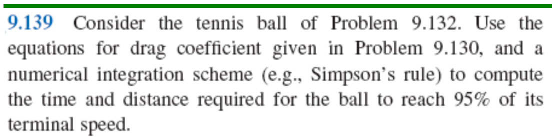
Here $V_0 = 0$ $V_N = 0.95 \cdot V_t$ $\Delta V = \frac{0.95 \cdot V_t}{N}$

From the associated *Excel* workbook $t = 4.69 \cdot s$ $x = 70.9 \cdot m$

These results compare to 4.44 s and 67.1 m from Problem 9.132, which assumed the drag coefficient was constant and analytically integrated. Note that the drag coefficient IS essentially constant, so numerical integration was not really necessary!

Problem 9.139 (In Excel)

[4]



9.139 Consider the tennis ball of Problem 9.132. Use the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., Simpson's rule) to compute the time and distance required for the ball to reach 95% of its terminal speed.

Given: Data on a tennis ball

Find: Terminal speed time and distance to reach 95% of terminal speed

Solution:

$$t = \int_{0}^{0.95 \cdot V_t} \frac{1}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV \qquad x = \int_{0}^{0.95 \cdot V_t} \frac{V}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV$$

Given data:

$M =$ 57 gm

$\rho =$ 1.23 kg/m^3

$D =$ 64 mm

$C_D =$ 0.5 (Fig. 9.11)

$\nu =$ 1.45E-05 m^2/s

$$C_D = \frac{24}{Re} \qquad Re \le 1$$

$$C_D = \frac{24}{Re^{0.646}} \qquad 1 < Re \le 400$$

$$C_D = 0.5 \qquad 400 < Re \le 3 \times 10^5$$

$$C_D = 0.000366 \cdot Re^{0.4275} \qquad 3 \times 10^5 < Re \le 2 \times 10^6$$

$$C_D = 0.18 \qquad Re > 2 \times 10^6$$

Computed results:

$A =$ 0.00322 m^2

$V_t =$ 23.8 m/s

$N =$ 20

$\Delta V =$ 1.19 m/s

For the time:

V (m/s)	Re	C_D	W	$f(V)$	$W \times f(V)$
0	0	5438	1	0.102	0.102
1.13	4985	0.500	4	0.102	0.409
2.26	9969	0.500	2	0.103	0.206
3.39	14954	0.500	4	0.104	0.416
4.52	19938	0.500	2	0.106	0.212
5.65	24923	0.500	4	0.108	0.432
6.78	29908	0.500	2	0.111	0.222
7.91	34892	0.500	4	0.115	0.458
9.03	39877	0.500	2	0.119	0.238
10.2	44861	0.500	4	0.125	0.499
11.3	49846	0.500	2	0.132	0.263
12.4	54831	0.500	4	0.140	0.561
13.6	59815	0.500	2	0.151	0.302
14.7	64800	0.500	4	0.165	0.659
15.8	69784	0.500	2	0.183	0.366
16.9	74769	0.500	4	0.207	0.828
18.1	79754	0.500	2	0.241	0.483
19.2	84738	0.500	4	0.293	1.17
20.3	89723	0.500	2	0.379	0.758
21.5	94707	0.500	4	0.550	2.20
22.6	99692	0.500	1	1.05	1.05

For the distance:

$f(V)$	$W \times f(V)$
0.00	0.000
0.115	0.462
0.232	0.465
0.353	1.41
0.478	0.955
0.610	2.44
0.752	1.50
0.906	3.62
1.08	2.15
1.27	5.07
1.49	2.97
1.74	6.97
2.05	4.09
2.42	9.68
2.89	5.78
3.51	14.03
4.36	8.72
5.62	22.5
7.70	15.4
11.8	47.2
23.6	23.6

Total time: **4.69** **s**

(This compares to 4.44s for the exact result)

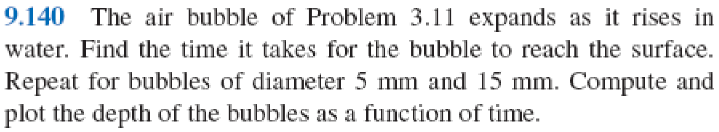
Total distance: **70.9** **m**

(This compares to 67.1 m for the exact result)

Note that C_D is basically constant, so analytical result of Problem 9.132 is accurate!

Problem 9.140

[4]



9.140 The air bubble of Problem 3.11 expands as it rises in water. Find the time it takes for the bubble to reach the surface. Repeat for bubbles of diameter 5 mm and 15 mm. Compute and plot the depth of the bubbles as a function of time.

Given: Data on an air bubble

Find: Time to reach surface; plot depth as function of time; repeat for different sizes

Solution:

The given data or available data is $d_0 = 0.3\cdot in$ $\quad h = 100\cdot ft$ $\quad \rho_w = 1000\cdot\frac{kg}{m^3}$ $\quad SG = 1.025$ (Table A.2)

$\rho = SG\cdot\rho_w$ $\quad \nu = 1.05\times 8.03\times 10^{-7}\cdot\frac{m^2}{s}$ (Tables A.2 & A.8) $\quad p_{atm} = 101\cdot kPa$

The density of air is negligible compared to that of water, so Newton's second law is applicable with negligible *MdV/dt*

$$M\cdot\frac{dV}{dt} = 0 = \Sigma F = F_B - F_D \quad \text{or} \quad F_B = F_D \qquad (1)$$

where F_B is the buoyancy force and F_D is the drag (upwards is positive x)

$$F_B = \rho\cdot Vol\cdot g \qquad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad (2)$$

For a sphere, assuming high Reynolds number, from Fig. 9.11 $C_D = 0.5$

The volume of the sphere increases as the bubble rises and experiences decreased pressure. Assuming the air is an isothermal idea gas

$$p_0\cdot Vol_0 = p\cdot Vol$$

where p_0 and Vol_0 are the initial pressure and volume (at depth h), and p and Vol are the pressure and volume at any depth

$$p_0 = p_{atm} + \rho\cdot g\cdot h \qquad p = p_{atm} + \rho\cdot g\cdot(h - x)$$

Hence

$$\left(p_{atm} + \rho\cdot g\cdot h\right)\cdot\frac{\pi}{6}\cdot d_0^{\,3} = \left[p_{atm} + \rho\cdot g\cdot(h - x)\right]\cdot\frac{\pi}{6}\cdot d^3$$

$$d = d_0\cdot\sqrt[3]{\frac{\left(p_{atm} + \rho\cdot g\cdot h\right)}{\left[p_{atm} + \rho\cdot g\cdot(h - x)\right]}} \qquad (3)$$

For example, at the free surface ($x = h$) $\quad d = 12.1\,mm$

Combining Eqs. 1, 2 and 3 $\quad \rho\cdot\frac{\pi}{6}\cdot d^3 = \frac{1}{2}\cdot\rho\cdot\frac{\pi}{4}\cdot d^2\cdot V^2\cdot C_D$

$$V = \sqrt{\frac{4\cdot g\cdot d}{3\cdot C_D}} \qquad V = \sqrt{\frac{4\cdot g\cdot d_0}{3\cdot C_D}\cdot\left[\frac{\left(p_{atm} + \rho\cdot g\cdot h\right)}{\left[p_{atm} + \rho\cdot g\cdot(h - x)\right]}\right]^{\frac{1}{6}}}$$

Strictly speaking, to obtain x as a function of t we would have to integrate this expression ($V = dx/dt$).

However, evaluating *V* at depth *h* (*x* = 0) and at the free surface (*x* = *h*)

$x = 0$ $\qquad$ $V_0 = 0.446\,\frac{m}{s}$

$x = h$ $\qquad$ $V = 0.563\,\frac{m}{s}$

we see that the velocity varies slightly. Hence, instead of integrating we use the approximation dx = Vdt where *dx* is an increment of displacement and *dt* is an increment of time. (This amounts to numerically integrating)

Note that the Reynolds number at the initial depth (the smallest *Re*) is $\quad Re_0 = \dfrac{V_0 \cdot d_0}{\nu} \qquad Re_0 = 4034$

so our use of $C_D = 0.5$ from Fig. 9.11 is reasonable

The plots of depth versus time are shown in the associated *Excel* workbook

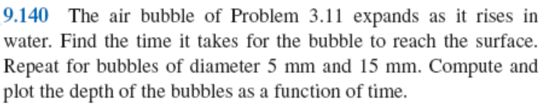
The results are $\quad d_0 = 0.3 \cdot in \qquad t = 63.4 \cdot s$

$d_0 = 5 \cdot mm \qquad t = 77.8 \cdot s$

$d_0 = 15 \cdot mm \qquad t = 45.1 \cdot s$

Problem 9.140 (In Excel)

[4]



9.140 The air bubble of Problem 3.11 expands as it rises in water. Find the time it takes for the bubble to reach the surface. Repeat for bubbles of diameter 5 mm and 15 mm. Compute and plot the depth of the bubbles as a function of time.

Given: Data on an air bubble

Find: Time to reach surface; plot depth as function of time; repeat for different sizes

Solution:

The equation is $dx = V \cdot dt$ where $V = \sqrt{\frac{4 \cdot g \cdot d_0}{3 \cdot C_D}} \cdot \left[\frac{\left(p_{atm} + \rho \cdot g \cdot h\right)}{\left[p_{atm} + \rho \cdot g \cdot (h - x)\right]}\right]^{\frac{1}{6}}$

Given data:

h = 100 ft

h = 30.5 m

ρ_w = 1000 kg/m^3

SG = 1.025 Table A.2)

C_D = 0.5 Fig. 9.11)

ρ = 1025 kg/m^3

p_{atm} = 101 kPa

Computed results:

d_0 = 0.3 in

d_0 = 7.62 mm

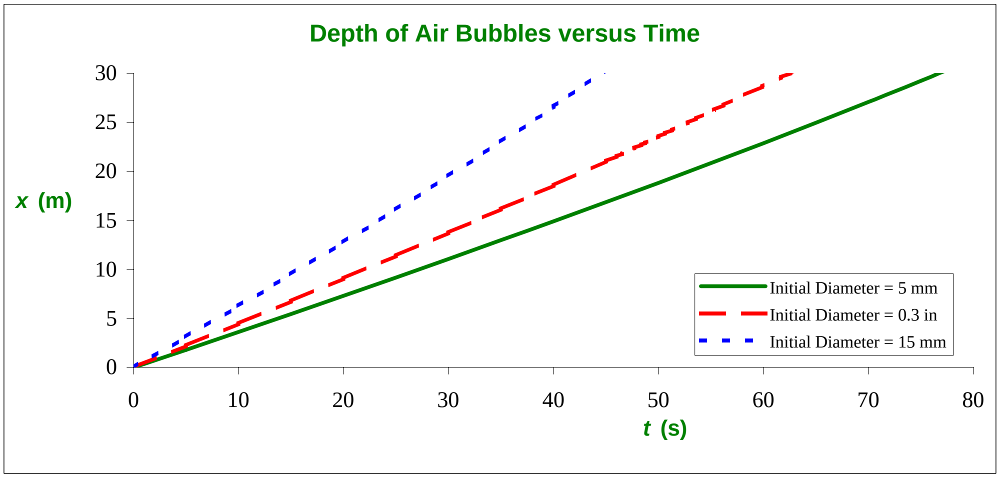
t (s)	*x* (m)	*V* (m/s)
0	0	0.446
5	2.23	0.451
10	4.49	0.455
15	6.76	0.460
20	9.1	0.466
25	11.4	0.472
30	13.8	0.478
35	16.1	0.486
40	18.6	0.494
45	21.0	0.504
50	23.6	0.516
63.4	30.5	0.563

d_0 = 5 mm

t (s)	*x* (m)	*V* (m/s)
0	0	0.362
5	1.81	0.364
10	3.63	0.367
15	5.47	0.371
20	7.32	0.374
25	9.19	0.377
30	11.1	0.381
35	13.0	0.386
40	14.9	0.390
45	16.9	0.396
50	18.8	0.401
55	20.8	0.408
60	22.9	0.415
65	25.0	0.424
70	27.1	0.435
75	29.3	0.448
77.8	30.5	0.456

d_0 = 15 mm

t (s)	*x* (m)	*V* (m/s)
0.0	0	0.626
5.0	3.13	0.635
10.0	6.31	0.644
15.0	9.53	0.655
20.0	12.8	0.667
25.0	16.1	0.682
30.0	19.5	0.699
35.0	23.0	0.721
40.0	26.6	0.749
45.1	30.5	0.790



Problem 9.141

9.141 Consider the tennis ball of Problem 9.132. Suppose it is hit so that it has an initial upward speed of 50 m/s. Estimate the maximum height of the ball, assuming (a) a constant drag coefficient and (b) using the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., a Simpson's rule).

Given: Data on a tennis ball

Find: Maximum height

Solution:

The given data or available data is $M = 57\cdot gm$ $\quad D = 64\cdot mm$ $\quad V_i = 50\cdot\frac{m}{s}$ $\quad \nu = 1.45\cdot10^{-5}\cdot\frac{m^2}{s}$ $\quad \rho = 1.23\cdot\frac{kg}{m^3}$

Then $A = \frac{\pi\cdot D^2}{4}$ $\quad A = 3.22\times10^{-3}m^2$

From Problem 9.130

$C_D = \frac{24}{Re}$ $\quad Re \le 1$

$C_D = \frac{24}{Re^{0.646}}$ $\quad 1 < Re \le 400$

$C_D = 0.5$ $\quad 400 < Re \le 3\times10^5$

$C_D = 0.000366\cdot Re^{0.4275}$ $\quad 3\times10^5 < Re \le 2\times10^6$

$C_D = 0.18$ $\quad Re > 2\times10^6$

The drag at speed *V* is given by $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

For motion before terminal speed, Newton's second law (*x* upwards) is $M\cdot a = M\cdot\frac{dV}{dt} = -\frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D - M\cdot g$

For the maximum height Newton's second law is written in the form $M\cdot a = M\cdot V\cdot\frac{dV}{dx} = -\frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D - M\cdot g$

Hence the maximum height is

$$x_{max} = \int_{V_i}^{0}\frac{V}{-\frac{\rho\cdot A\cdot C_D}{2\cdot M}\cdot V^2 - g}\,dV = \int_{0}^{V_i}\frac{V}{\frac{\rho\cdot A\cdot C_D}{2\cdot M}\cdot V^2 + g}\,dV$$

This integral is quite difficult because the drag coefficient varies with Reynolds number, which varies with speed. It is best evaluated numerically. A form of Simpson's Rule is

$$\int f(V)\,dV = \frac{\Delta V}{3}\cdot\left(f\left(V_0\right) + 4\cdot f\left(V_1\right) + 2\cdot f\left(V_2\right) + 4\cdot f\left(V_3\right) + f\left(V_N\right)\right)$$

where ΔV is the step size, and V_0, V_1 etc., are the velocities at points 0, 1, ... *N*.

Here $V_0 = 0$ $V_N = V_i$ $\Delta V = \frac{V_i}{N}$

From the associated *Excel* workbook $x_{max} = 48.7 \cdot m$

If we assume $C_D = 0.5$

the integral

$$x_{max} = \int_0^{V_i} \frac{V}{\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 + g} \, dV$$

becomes

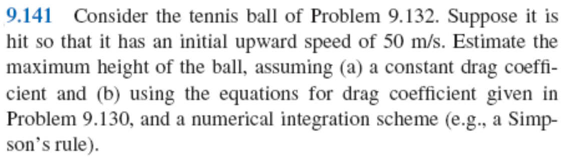
$$x_{max} = \frac{M}{\rho \cdot A \cdot C_D} \cdot \ln\left(\frac{\rho \cdot A \cdot C_D}{2 \cdot M \cdot g} \cdot V_i^2 + 1\right)$$

$x_{max} = 48.7\,m$

The two results agree very closely! This is because the integrand does not vary much after the first few steps so the numerical integral is accurate, and the analytic solution assumes $C_D = 0.5$, which it essentially does!

Problem 9.141 (In Excel)

[4]



9.141 Consider the tennis ball of Problem 9.132. Suppose it is hit so that it has an initial upward speed of 50 m/s. Estimate the maximum height of the ball, assuming (a) a constant drag coefficient and (b) using the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., a Simpson's rule).

Given: Data on a tennis ball

Find: Maximum height

Solution:

The equation is

$$x_{max} = \int_{V_i}^{0} \frac{V}{-\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 - g}\, dV = \int_{0}^{V_i} \frac{V}{\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 + g}\, dV$$

Given data:

M = 57 gm

V_0 = 50.0 m/s

ρ = 1.23 kg/m^3

D = 64 mm

C_D = 0.5 (Fig. 9.11)

ν = 1.45E-05 m^2/s

$$C_D = \frac{24}{Re} \qquad Re \le 1$$

$$C_D = \frac{24}{Re^{0.646}} \qquad 1 < Re \le 400$$

$$C_D = 0.5 \qquad 400 < Re \le 3 \times 10^5$$

$$C_D = 0.000366 \cdot Re^{0.4275} \qquad 3 \times 10^5 < Re \le 2 \times 10^6$$

$$C_D = 0.18 \qquad Re > 2 \times 10^6$$

Computed results:

A = 0.00322 m^2

N = 20

ΔV = 2.50 m/s

V (m/s)	Re	C_D	W	$f(V)$	$W \times f(V)$
0.0	0	0.000	1	0.000	0.000
2.5	11034	0.500	4	0.252	1.01
5.0	22069	0.500	2	0.488	0.976
7.5	33103	0.500	4	0.695	2.78
10.0	44138	0.500	2	0.866	1.73
12.5	55172	0.500	4	1.00	3.99
15.0	66207	0.500	2	1.09	2.19
17.5	77241	0.500	4	1.16	4.63
20.0	88276	0.500	2	1.19	2.39
22.5	99310	0.500	4	1.21	4.84
25.0	110345	0.500	2	1.21	2.42
27.5	121379	0.500	4	1.20	4.80
30.0	132414	0.500	2	1.18	2.36
32.5	143448	0.500	4	1.15	4.62
35.0	154483	0.500	2	1.13	2.25
37.5	165517	0.500	4	1.10	4.38
40.0	176552	0.500	2	1.06	2.13
42.5	187586	0.500	4	1.03	4.13
45.0	198621	0.500	2	1.00	2.00
47.5	209655	0.500	4	0.970	3.88
50.0	220690	0.500	1	0.940	0.940

Maximum height: 48.7 m

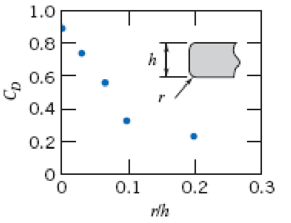
(This is the same as the exact result)

Note that C_D is basically constant, so analytical result of Problem 9.132 is accurate!

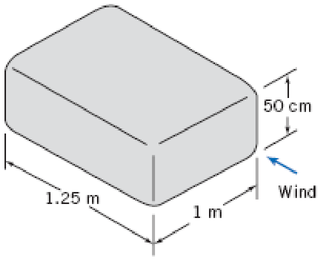
Problem 9.142

[3]

9.142 Approximate dimensions of a rented rooftop carrier are shown. Estimate the drag force on the carrier (r = 10 cm) at 100 km/hr. If the drivetrain efficiency of the vehicle is 0.85 and the brake specific fuel consumption of its engine is 0.3 kg/(kW • hr), estimate the additional rate of fuel consumption due to the carrier. Compute the effect on fuel economy if the auto achieves 12.75 km/L without the carrier. The rental company offers you a cheaper, square-edged carrier at a price $5 less than the current carrier. Estimate the extra cost of using this carrier instead of the round-edged one for a 750 km trip, assuming fuel is $3.50 per gallon. Is the cheaper carrier really cheaper?



Drag coefficient v. radius ratio [37]



Given: Data on rooftop carrier

Find: Drag on carrier; Additional fuel used; Effect on economy; Effect of "cheaper" carrier

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$

Given or available data is $w = 1\cdot m$ $\quad h = 50\cdot cm$ $\quad r = 10\cdot cm$ $\quad \eta_d = 85\cdot\%$

$V = 100\cdot\dfrac{km}{hr}$ $\quad V = 27.8\dfrac{m}{s}$ $\quad FE = 12.75\cdot\dfrac{km}{L}$ $\quad FE = 30.0\dfrac{mi}{gal}$

$\rho_{H2O} = 1000\cdot\dfrac{kg}{m^3}$ $\quad A = w\cdot h$ $\quad A = 0.5\,m^2$ $\quad BSFC = 0.3\cdot\dfrac{kg}{kW\cdot hr}$

$\rho = 1.225\cdot\dfrac{kg}{m^3}$ $\quad \nu = 1.50\times10^{-5}\cdot\dfrac{m^2}{s}$ $\quad$ (Table A.10, 20ºF)

From the diagram $\dfrac{r}{h} = 0.2$ so $C_D = 0.25$ $\quad F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$ $\quad F_D = 59.1\,N$

Additional power is $\Delta P = \dfrac{F_D\cdot V}{\eta_d}$ $\quad \Delta P = 1.93\,kW$

Additional fuel is $\Delta FC = BSFC\cdot\Delta P$ $\quad \Delta FC = 1.61\times10^{-4}\dfrac{kg}{s}$ $\quad \Delta FC = 0.00965\dfrac{kg}{min}$

Fuel consumption of the car only is (with $SG_{gas} = 0.72$ from Table A.2)

$FC = \dfrac{V}{FE}\cdot SG_{gas}\cdot\rho_{H2O}$ $\quad FC = 1.57\times10^{-3}\dfrac{kg}{s}$ $\quad FC = 0.0941\dfrac{kg}{min}$

The total fuel consumption is then $FC_T = FC + \Delta FC$ $\quad FC_T = 1.73\times10^{-3}\dfrac{kg}{s}$ $\quad FC_T = 0.104\dfrac{kg}{min}$

Fuel economy with the carrier is $FE = \dfrac{V}{FC_T}\cdot SG_{gas}\cdot\rho_{H2O}$ $\quad FE = 11.6\dfrac{km}{L}$ $\quad FE = 27.2\dfrac{mi}{gal}$

For the square-edged: $\dfrac{r}{h} = 0$ so $C_D = 0.9$ $\quad F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$ $\quad F_D = 213\,N$

Additional power is $\Delta P = \dfrac{F_D\cdot V}{\eta_d}$ $\quad \Delta P = 6.95\,kW$

Additional fuel is $\Delta FC = BSFC \cdot \Delta P$ $\Delta FC = 5.79 \times 10^{-4} \frac{kg}{s}$ $\Delta FC = 0.0348 \frac{kg}{min}$

The total fuel consumption is then $FC_T = FC + \Delta FC$ $FC_T = 2.148 \times 10^{-3} \frac{kg}{s}$ $FC_T = 0.129 \frac{kg}{min}$

Fuel economy withy the carrier is now $FE = \frac{V}{FC_T} \cdot SG_{gas} \cdot \rho_{H2O}$ $FE = 9.3 \frac{km}{L}$ $FE = 21.9 \frac{mi}{gal}$

The cost of the trip of distance $d = 750 \cdot km$ for fuel costing $p = \frac{\$ \cdot 3.50}{gal}$ with a rental $discount = \$ \cdot 5$ less than the rounded carrier is then

$Cost = \frac{d}{FE} \cdot p - discount$ $Cost = 69.47 \$$ plus the rental fee

The cost of the trip of with the rounded carrier $\left(FE = 11.6 \cdot \frac{km}{L}\right)$ is then

$Cost = \frac{d}{FE} \cdot p$ $Cost = 59.78 \$$ plus the rental fee

Hence the "cheaper" carrier is more expensive (AND the environment is significantly more damaged!)

Problem 9.143

[4]

Given: Coastdown test data from level road, calm day, measured for vehicle with $W = 25{,}000$ lbf and $A = 79\ ft^2$. $F_D(5\ mph) \ll F_D(55\ mph)$.

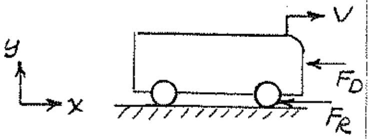
$V\ (mph)$	5	55
$\frac{dV}{dt}\ (mph/s)$	-0.150	-0.475

Find: Aerodynamic drag coefficient for this vehicle.
Speed at which F_D first exceeds F_R

Solution: Apply Newton's second law of motion, definition of C_D.

Computing equations:

$$\Sigma F_x = m a_x \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



Summing forces, $-F_R - F_D = m a_x$

At 55 mph $\qquad -F_R - F_{D55} = m a_{x55}$

At 5 mph $\qquad -F_R - \overset{\approx 0}{\cancel{F_{D5}}} = m a_{x5}$

Subtracting, obtain $\quad -F_{D55} = m(a_{x55} - a_{x5}) = m\left[\frac{dV}{dt}\Big)_{55} - \frac{dV}{dt}\Big)_5\right] = -C_D A \rho \frac{V^2}{2}$

Thus

$$C_D = \frac{m\left[\frac{dV}{dt}\Big)_5 - \frac{dV}{dt}\Big)_{55}\right]}{\frac{1}{2}\rho V^2 A} = \frac{2W\left[\frac{dV}{dt}\Big)_5 - \frac{dV}{dt}\Big)_{55}\right]}{\rho g V^2 A}$$

Evaluating, assuming standard air, with $\rho g = 0.0765\ lbf/ft^3$

$$C_D = 2 \times 25000\ lbf \times \frac{ft^3}{0.0765\ lbf} \times \frac{hr^2}{(55)^2\ mi^2} \times \frac{1}{79\ ft^2} \times [-0.150 - (-0.475)]\frac{mi}{hr \cdot s}$$
$$\times \frac{mi}{5280\ ft} \times 3600\frac{s}{hr}$$

$$C_D = 0.606$$

C_D

At $V = 5$ mph, $F_D \approx 0 \quad \therefore \quad -F_R = m a_{x5}$

$$F_R = -m a_{x5} = -25000\,lbf\,\frac{s^2}{32.2\,ft} \times -0.150\,\frac{mi}{hr \cdot s} \times \frac{hr}{3600\,s} \times \frac{5280\,ft}{mi} = 171\,lbf$$

For $F_R = F_D = C_D \frac{1}{2}\rho V^2 A$, then $V = \left[\frac{2F_R}{\rho A C_D}\right]^{1/2}$

$$V = \left[2 \times 171\,lbf \times \frac{ft^3}{0.00238\,slug} \times \frac{1}{79\,ft^2} \times \frac{1}{0.606} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2}$$

$$V = 54.8\,\frac{ft}{s} = 37.4\ mph.$$

$V_{F_R=F_D}$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 9.144

[4]

Given: Spherical sonar transducer with $D = 0.375$ m, to be towed in seawater, fully submerged, at $V = 31$ kt.

To avoid cavitation, minimum pressure on transducer surface must be > 30 kPa (abs).

Find: (a) Hydrodynamic drag force on transducer.

(b) Minimum depth of submergence.

Solution: $V = 31.1 \frac{nm}{hr} \times 1852 \frac{m}{nm} \times \frac{hr}{3600\,s} = 16.0 \ m/s$

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times (1.025)\,1000 \frac{kg}{m^3} \times (16.0)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{Pa \cdot m^2}{N} = 131 \ kPa$$

$$Re = \frac{VD}{\nu} = 16.0 \frac{m}{s} \times 0.375\,m \times \frac{1}{(1.08)\,1 \times 10^{-6}\,m^2} = 5.56 \times 10^6$$

Therefore flow over sphere is supercritical; from Fig. 9.11, $C_D \approx 0.18$

$$F_D = C_D A \tfrac{1}{2}\rho V^2 \qquad A = \frac{\pi D^2}{4} = \frac{\pi}{4} \times (0.375)^2 m^2 = 0.110 \ m^2$$

$$F_D = 0.18 \times 0.110 \ m^2 \times 131 \times 10^3 \frac{N}{m^2} = 2.59 \ kN$$

F_D

From Fig. 9.12, the minimum pressure on a sphere with supercritical flow is $C_p \approx -1.2$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2} = \frac{p - p_\infty}{q} = -1.2$$

or $p_\infty = p - C_p q$

$= 30$ kPa (abs) $- (-1.2)\,131$ kPa

p_∞ (abs) $= 187$ kPa (abs)

Thus

p_∞ (gage) $= p_\infty$ (abs) $- p_{atm} = (187 - 101)$ kPa $= 86.2$ kPa

But p_∞ (gage) $= \rho g h$, so

$$h = \frac{p_\infty (gage)}{\rho g} = 86.2 \times 10^3 \frac{N}{m^2} \times \frac{m^3}{(1.025)\,1000\,kg} \times \frac{s^2}{9.81\,m} \times \frac{kg \cdot m}{N \cdot s^2} = 8.57 \ m$$

h

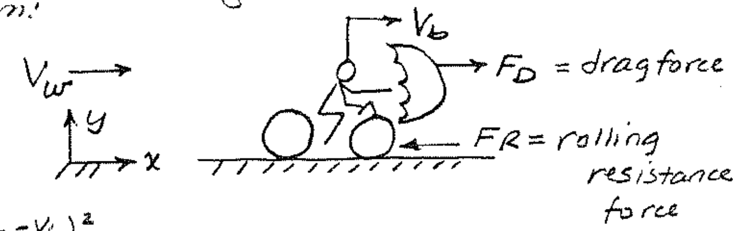
Problem 9.145

[4]

Open-Ended Problem Statement: While walking across campus one windy day, Floyd Fluids speculates about using an umbrella as a "sail" to propel a bicycle along the sidewalk. Develop an algebraic expression for the speed a bike could reach on level ground with the umbrella "propulsion system." The frontal area of the bike and rider is estimated as 0.3 m^2. $C_D = 1.2$ Evaluate the bike speed that could be achieved with an umbrella 1.22 m in diameter in a wind that blows at 24 km/hr. Discuss the practicality of this propulsion system.

Assume rolling resistance is 0.75% of weight ($m = 75$ kg)

Analysis: Draw a free-body diagram:



Sum forces in x direction:

$$\sum F_x = F_D - F_R = 0$$

But $F_D = (C_{Du} A_u + C_{Db} A_b)\frac{1}{2}\rho(V_w - V_b)^2$

$$A_u = \frac{\pi D_u^2}{4} = \frac{\pi}{4}(1.22)^2 m^2 = 1.17\ m^2$$

$$F_R = C_R mg$$

Choose $C_{Du} = 1.42$ (Table 9.3).

$$C_R = 0.75\%,\ m = 75\ kg,\ so\ F_R = 0.0075 \times 75\ kg \times 9.81\frac{m}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 5.52\ N$$

Then $V_b = V_w - \left[\dfrac{2F_R}{\rho(C_{Du} A_u + C_{Db} A_b)}\right]^{\frac{1}{2}}$ $\longleftarrow$ V_b

But

$$V_w = 24\frac{km}{hr} \times 1000\frac{m}{km} \times \frac{hr}{3600\ s} = 6.67\ m/s$$

$$V_b = 6.67\frac{m}{s} - \left[2 \times 5.52\ N \times \frac{m^3}{1.23\ kg}\ \frac{1}{(1.42)1.17\ m^2 + (1.20)0.3\ m^2} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}}$$

$V_b = 6.67\dfrac{m}{s} - 2.11\dfrac{m}{s} = 4.56\dfrac{m}{s}$ or $16.4\dfrac{km}{hr}$ $\longleftarrow$ V_b

Thus Floyd's bicycle (with the umbrella propelling it) travels at 68.3% wind speed.

{Without the umbrella, $V_b = 1.68\frac{m}{s}$ or $6.04\frac{km}{hr}$, by setting $C_{Du} = 0$ above.}

Discussion: Floyd is confused about his fluid mechanics principles if he thinks he can exceed the wind speed. It is impossible to obtain a propulsive force from aerodynamic drag unless the bicycle is moving more slowly than the wind. The drag force must be sufficient to overcome the rolling resistance of the bike and rider. At equilibrium speed the drag force and rolling resistance force must be equal and opposite.

The only benefit could be achieved by adding drag force more rapidly than rolling resistance. An umbrella, with its relatively high drag and low weight, is ideal for this purpose.

However, one would somehow have to hold the umbrella perpendicular to the wind while riding the bike. This would be dangerous at best, especially if the bike had hand-activated brakes.

Since the umbrella must be held perpendicular to the wind, it would be very effective at blocking the rider's view of the road ahead!

In summary, this "system" of propulsion appears quite impractical.

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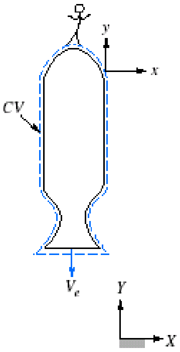
Problem 9.146

[4]

9.146 Motion of a small rocket was analyzed in Example 4.12 assuming negligible aerodynamic drag. This was not realistic at the final calculated speed of 369 m/s. Use Euler's finite difference method for approximating the first derivatives, in an *Excel* workbook, to solve the equation of motion for the rocket. Plot the rocket speed as a function of time, assuming $C_D = 0.3$ and a rocket diameter of 700 mm. Compare with the results for $C_D = 0$.



Given: Data on a rocket

Find: Plot of rocket speed with and without drag

Solution:

From Example 4.12, with the addition of drag the momentum equation becomes

$$F_{B_y} + F_{S_y} - \int_{\mathrm{CV}} a_{rf_y}\,\rho\, dV\!\!\!/ = \frac{\partial}{\partial t}\int_{\mathrm{CV}} v_{xyz}\,\rho\, dV\!\!\!/ + \int_{\mathrm{CV}} v_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A}$$

where the surface force is

$$F_{S_y} = -\frac{1}{2}\rho A V^2 C_{\mathrm{D}}$$

Following the analysis of the example problem, we end up with

$$\frac{dV_{\mathrm{CV}}}{dt} = \frac{V_e \dot{m}_e - \frac{1}{2}\rho A V_{\mathrm{CV}}{}^2 C_{\mathrm{D}}}{M_0 - \dot{m}_e t} - g$$

This can be written (dropping the subscript for convenience)

$$\frac{dV}{dt} = f(V,t) \qquad (1)$$

where

$$f(V,t) = \frac{V_e \dot{m}_e - \frac{1}{2}\rho A V^2 C_{\mathrm{D}}}{M_0 - \dot{m}_e t} - g \qquad (2)$$

Equation 1 is a differential equation for speed *V*.

It can be solved using Euler's numerical method

$$V_{\mathrm{n+1}} \approx V_{\mathrm{n}} + \Delta t\, f_{\mathrm{n}}$$

where $V_{\mathrm{n+1}}$ and V_{n} are the n + 1th and nth values of *V*, f_{n} is the function given by Eq. 2 evaluated at the nth step, and Δt is the time step.

The initial condition is $V_0 = 0 \text{ at } t = 0$

Given or available data:

M_0 = 400 kg
m_e = 5 kg/s
V_e = 3500 m/s
ρ = 1.23 kg/m^3
D = 700 mm
C_D = 0.3

Computed results:

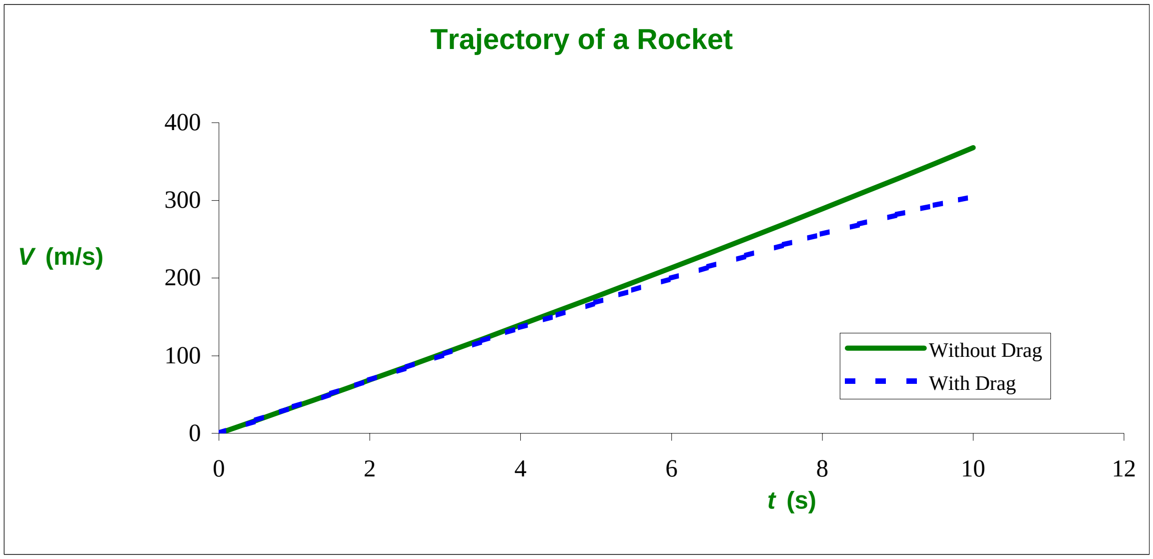
A = 0.385 m^2
N = 20
Δt = 0.50 s

With drag:

n	t_n (s)	V_n (m/s)	f_n	V_{n+1} (m/s)
0	0.0	0.0	33.9	17.0
1	0.5	17.0	34.2	34.1
2	1.0	34.1	34.3	51.2
3	1.5	51.2	34.3	68.3
4	2.0	68.3	34.2	85.5
5	2.5	85.5	34.0	102
6	3.0	102	33.7	119
7	3.5	119	33.3	136
8	4.0	136	32.8	152
9	4.5	152	32.2	168
10	5.0	168	31.5	184
11	5.5	184	30.7	200
12	6.0	200	29.8	214
13	6.5	214	28.9	229
14	7.0	229	27.9	243
15	7.5	243	26.9	256
16	8.0	256	25.8	269
17	8.5	269	24.7	282
18	9.0	282	23.6	293
19	9.5	293	22.5	305
20	10.0	305	21.4	315

Without drag:

V_n (m/s)	f_n	V_{n+1} (m/s)
0.0	33.9	17.0
17.0	34.2	34.1
34.1	34.5	51.3
51.3	34.8	68.7
68.7	35.1	86.2
86.2	35.4	104
104	35.6	122
122	35.9	140
140	36.2	158
158	36.5	176
176	36.9	195
195	37.2	213
213	37.5	232
232	37.8	251
251	38.1	270
270	38.5	289
289	38.8	308
308	39.1	328
328	39.5	348
348	39.8	368
368	40.2	388

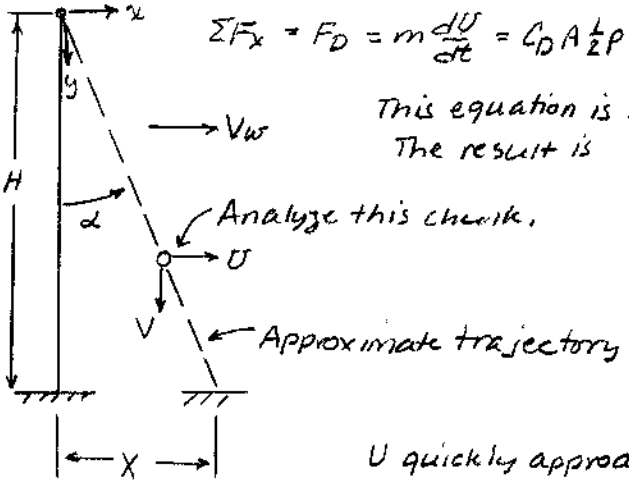


Problem 9.147

[5]

Open-Ended Problem Statement: Towers for television transmitters may be up to 500 m in height. In the winter, ice forms on structural members. When the ice thaws, chunks break off and fall to the ground. How far from the base of a tower would you recommend placing a fence to limit danger to pedestrians from falling ice chunks?

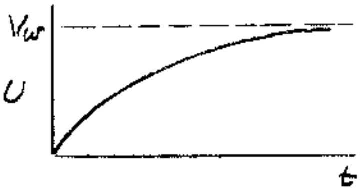
Analysis: An ice chunk detaching from a tower starts at rest, falls by gravity, and simultaneously is blown sideways by wind. Because drag is proportional to relative speed squared, it may be treated in separate x and y components.



$$\Sigma F_x = F_D = m\frac{dU}{dt} = C_D A \tfrac{1}{2}\rho (V_w - U)^2 = m\frac{dU}{dt} = -m\frac{d(V_w - U)}{dt} \qquad (1)$$

This equation is solved in Example Problem 4.11. The result is

$$\frac{U}{V_w} = \frac{bt}{1+bt}$$



$$b = \frac{\rho C_D A}{2m} V_w$$

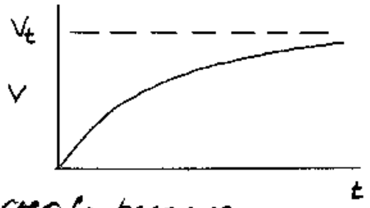
U quickly approaches V_w

$$\Sigma F_y = mg - F_D = m\frac{dV}{dt} = mV\frac{dV}{dt} = mg - C_D A \tfrac{1}{2}\rho V^2 = mg - kV^2 \qquad (2)$$

This equation is solved in Example Problem 1.2. The result is

$$V = \left\{\frac{mg}{k}\left(1 - e^{-\frac{2k}{m}y}\right)\right\}^{1/2} \quad \text{or} \quad \frac{V}{V_t} = 1 - e^{-2\frac{k}{m}y}$$

$$V_t = \left[\frac{mg}{k}\right]^{\frac{1}{2}}$$



Model the chunk as a geometric sphere, but with larger C_D because of jagged edges. Both b and V_t depend on diameter; $V_t = [8.11\, D(\text{mm})]^{\frac{1}{2}}$, or 11.0 m/s for D = 15 mm.

Thus a reasonable approximation is falling at V_t and moving sideways at V_w. For V_w = 10 mph (4.47 m/s), and V_t = 11.0 m/s, then $\alpha = \tan^{-1}(4.47/11.0) = 22.1°$, and $X = H \tan 22.1° = 203$ m.

X

To more precisely compute distance, obtain $x(t)$ and $y(t)$ by solving Eqs. 1 and 2 numerically, and plot the particle path.

Discussion: Because towers may be very tall, ice chunks can travel long distances from the base even in moderate winds. Considerable area around the base of a tower must be fenced to keep personnel on the ground safe from falling ice.

The analysis in this problem would be accurate if the drag-area product $C_D A$ for an ice chunk were known precisely. However, the size of the structural members and the thickness of the ice coating are both unknown. Therefore it is difficult to choose the most probable drag-area product. We recommend you bracket the sizes of known ice chunks, pick a reasonable range of drag coefficients, and then use the analysis to develop guidelines for the safety of personnel.

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Problem 9.148

Open-Ended Problem Statement: Wiffle™ balls made from light plastic with numerous holes are used to practice baseball and golf. Explain the purpose of the holes and why they work. Explain how you could test your hypothesis experimentally.

Discussion: The basic concept of the Wiffle ball is a low-mass, high-drag configuration that can be hit or struck with full force, but will not fly fast or far. Thus the Wiffle ball can be used for practice in a limited space.

The low mass is achieved by making the ball of relatively thin plastic material. This gives it low mass for its size, and is a step toward making the drag force relatively high compared to the weight of the ball.

Even higher drag force is achieved by perforating the surface of the Wiffle ball with numerous large holes. These holes further reduce the mass of the Wiffle ball.

In the sub-critical flow regime (below $Re_D \approx 2 \times 10^5$) skin friction drag accounts for less than 5 percent of the total drag of a sphere. The holes increase the skin friction drag of the ball by allowing boundary-layer fluid to escape into the interior of the ball. Each new bit of surface then sees essentially a new boundary layer developing, with attendant high shear stress.

Pressure drag accounts for the majority of the drag of a sphere at any Reynolds number above about 1000. The holes disrupt the flow pattern around the ball and probably trigger early separation. This ensures that the ball remains in the high-drag sub-critical flow regime no matter what its actual Reynolds number.

This hypothesis could be tested experimentally by comparing the performance of two balls, one with holes and one without. (The balls should have nearly the same mass and diameter.) With the help of an assistant, drop the balls from some height (for example, down a stairwell). After each ball has reached terminal speed, measure the time required for it to fall through a fixed distance. Then calculate and compare the drag coefficients for the two balls. If the drag coefficient for the ball with holes is significantly larger than for the ball without holes, the hypothesis is confirmed.

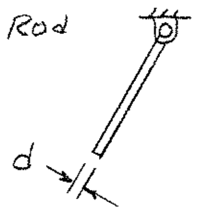
Several balls of each type might be evaluated experimentally to obtain an idea of the Reynolds number dependence of the results.

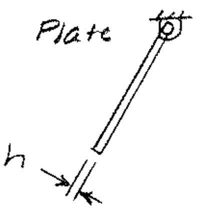
Problem 9.149

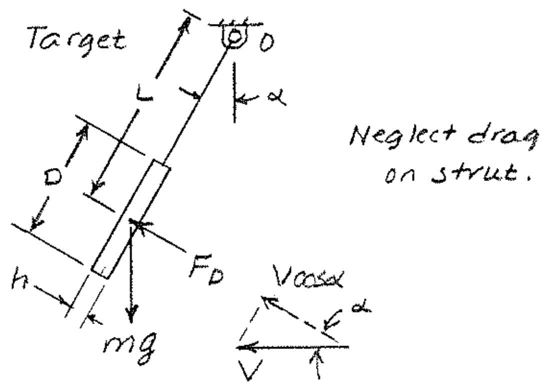
[5] Part 1/2

Open-Ended Problem Statement: Design a wind anemometer that uses aerodynamic drag to move or deflect a member or linkage, producing an output that can be related to wind speed, for the range from 1 to 10 m/s in standard air. Consider three alternative design concepts. Select the best concept and prepare a detailed design. Specify the shape, size, and material for each component. Quantify the relation between wind speed and anemometer output. Present results as a "calibration curve" of anemometer output versus wind speed. Discuss reasons why you rejected the alternative designs and chose your final design concept.

Analysis: The "target" concept was chosen for analysis. The drag force acting on the target is calculated, then moments are summed about the pivot (see Problem 9.105). The results are:







$$\overset{\curvearrowleft +}{\sum} M_O = mg\cancel{L}\sin\alpha - F_D\cancel{L} = 0$$

$$F_D = C_D A \tfrac{1}{2}\rho (V\cos\alpha)^2 = C_D A \tfrac{1}{2}\rho V^2 \cos^2\alpha$$

So $C_D A \tfrac{1}{2}\rho V^2 \cos^2\alpha = mg\sin\alpha = \rho_m \forall_m g \sin\alpha = SG\,\rho_{H_2O} A h g \sin\alpha$ (1)

Assume $\alpha = 60°$ at highest wind speed (when $V = 10$ m/s; $q = \tfrac{1}{2}\rho V^2 = 61.5\ \text{N/m}^2$).

Then $\dfrac{\sin\alpha}{\cos^2\alpha} = \dfrac{C_D A q}{SG\,\rho_{H_2O} A g h} = \dfrac{C_D q}{SG\,\rho_{H_2O} g h} = f(\alpha)$ At $\alpha = 60°$, $f(\alpha) = 3.46$

$$h = \frac{C_D q}{SG\,\rho_{H_2O}\, g f(\alpha)} = 1.2 \times 61.5\,\frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{(SG)\,1000\,\text{kg}} \times \frac{\text{s}^2}{9.81\,\text{m}} \times \frac{1}{3.46} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = \frac{0.00217\,\text{m}}{SG} = \frac{2.17\ \text{mm}}{SG}$$

Choose aluminum, with $SG = 2.64$ (Table A.1). Then

$h = \dfrac{2.17\ \text{mm}}{2.64} = 0.822$ mm ⟵ h

Solve Eq. 1 for velocity

$V = \left[\dfrac{2\,SG\,\rho_{H_2O}\, h g}{\rho C_D} \times f(\alpha)\right]^{\frac{1}{2}}$ (see plot) ⟵ V

Discussion: Concepts considered included a manometer that sensed stagnation pressure, a parallelogram linkage supporting a vertical target, and bending of a thin member in the air stream. The three final concepts chosen were variations on the theme of a single hanging member supported from a single pivot, and were chosen for their simplicity.

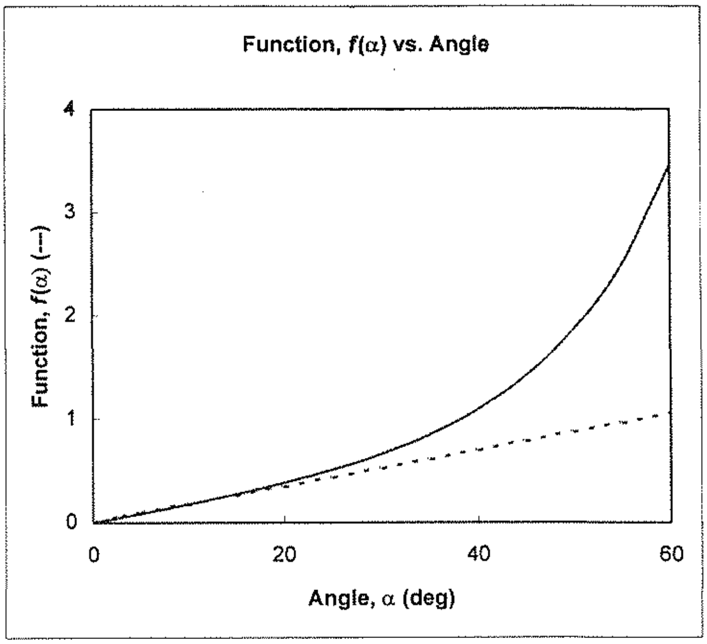
The major advantage of the target concept is that different materials can be used for the rod and target; this concept can be tailored to give the largest deflection angle for a given wind speed. Therefore this device should be capable of the most accurate indication at low wind speeds.

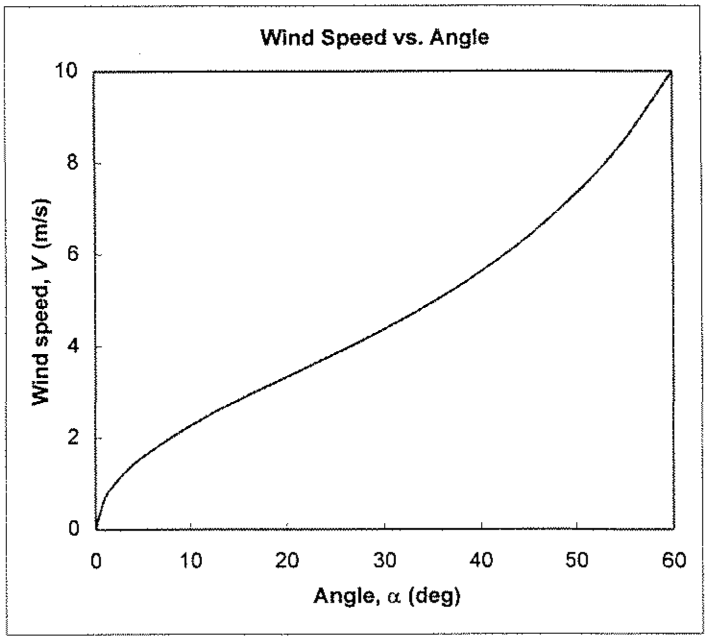
Drag force on the target is assumed to depend on the component of wind velocity acting normal to the target. This model could be improved by using actual experimental data for the drag coefficient of a disk at angle of attack.

Problem 9.149

[5] Part 2/2

α (deg)	α (rad)	$f(\alpha)$ (---)	V (m/s)
0	0	0	0
1	0.0175	0.0175	0.71
2	0.0349	0.0349	1.00
3	0.0524	0.0525	1.23
5	0.0873	0.0878	1.59
10	0.175	0.179	2.27
15	0.262	0.277	2.83
20	0.349	0.387	3.34
25	0.436	0.515	3.85
30	0.524	0.667	4.39
35	0.611	0.855	4.97
40	0.698	1.10	5.62
45	0.785	1.41	6.39
50	0.873	1.85	7.31
55	0.960	2.49	8.47
60	1.05	3.46	10.0





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Problem 9.150

Open-Ended Problem Statement: The "shot tower," used to produce spherical lead shot, has been recognized as a mechanical engineering landmark. In a shot tower, molten lead is dropped from a high tower; as the lead solidifies, surface tension pulls each shot into a spherical shape. Discuss the possibility of increasing the "hang time," or of using a shorter tower, by dropping molten lead into an air stream that is moving upward. Support your discussion with appropriate calculations.

Analysis: This problem may be analyzed parametrically, in terms of shot diameter. Consider the range from "bird shot" of about 1 mm to musket balls of about 15 mm diameter to illustrate the results.

Analysis with still air: Terminal speed is reached when aerodynamic drag force exactly equals the weight of the shot. The first plot shows terminal speed versus diameter of lead shot.

The solution for shot speed versus distance traveled, with no upward air movement, parallels the solution of Example Problem 1.2, which gives the fraction of terminal speed reached in a tower of specified height. For any tower height (choose 50 m to illustrate the results) the fraction of terminal speed reached decreases with increasing shot diameter (see the second plot).

The solution for hang time versus diameter is shown in the third plot.

Analysis with upward flow of air: The solution for shot speed versus distance traveled is more complex when air in the shot tower flows upward. Introducing upward air flow in the tower increases drag force compared to shot weight. Therefore the shot accelerate more slowly in the upward flow. It is possible to obtain an analytical solution, but the result is so complex that it is difficult to interpret. Results can be obtained for specific cases by integrating the differential equations numerically.

The solution for shot speed versus time also is more complex when air flows upward. Again numerical integration can be used to obtain results for specific cases.

Outline of Procedure: Derive a differential equation for shot acceleration from a free-body diagram. Integrate once to obtain shot velocity as a function of time. Integrate again to obtain shot position as a function of time.

From the results of the second integration, identify the "hang time" when the shot reaches the bottom of the tower. Plot hang time versus diameter and compare with results for the case with no upward air flow.

Set the upward air flow velocity to zero and compare numerical results with the analytical results for the case without flow to validate your model.

Discussion: The terminal speed reached by small shot in still air is quite low. Therefore, the "hang time" of small shot can be increased significantly by providing upward flow of air at reasonable speed in the tower.

Larger shot have higher terminal speeds. However, the higher terminal speed does not reduce hang time much because the large shot reach only a smaller fraction of their terminal speed in the 50 m tower height.

Introducing upward flow of air in the shot tower increases the drag force and results in slower acceleration of the shot. Therefore the hang time is increased. The increase in hang time allows more time for cooling, and should result in the production of more nearly spherical shot.

Problem 9.150

Input data:

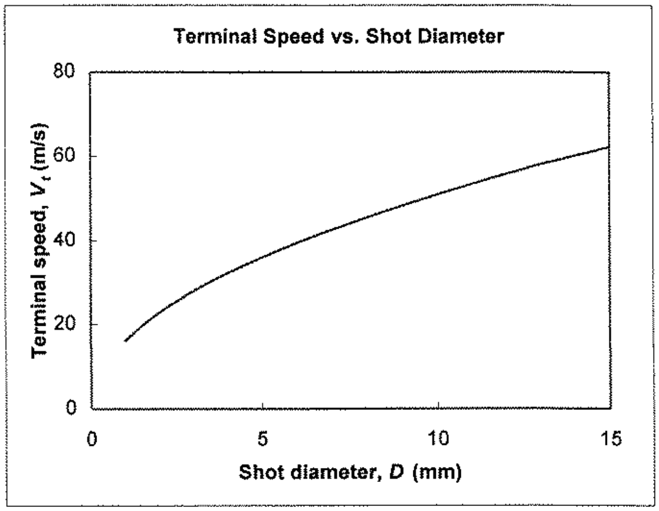
C_D =	0.47	(---)	Drag coefficient of sphere
SG_s =	11.4	(---)	Specific gravity of (lead) shot
Δz =	50	m	Height of shot tower
ρ_{air} =	1.23	kg/m^3	Density of air

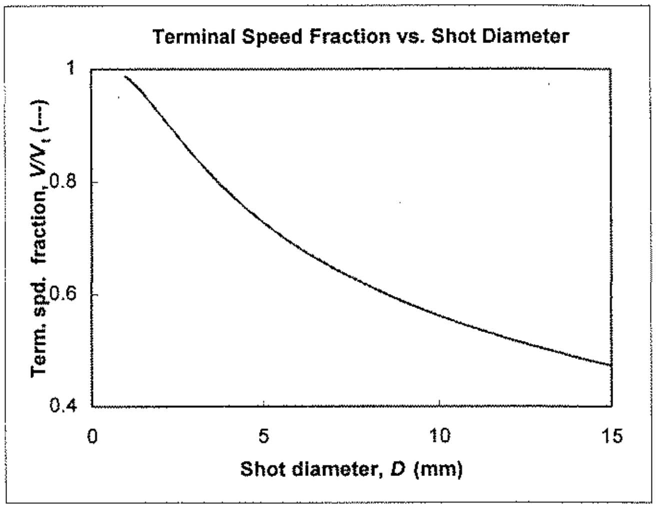
Calculated parameters:

k/D^2 =	2.27E-07	$kg/m\text{-}mm^2$	Drag factor, $F_D = kV^2$
m/D^3 =	5.969E-06	kg/mm^3	Mass of shot

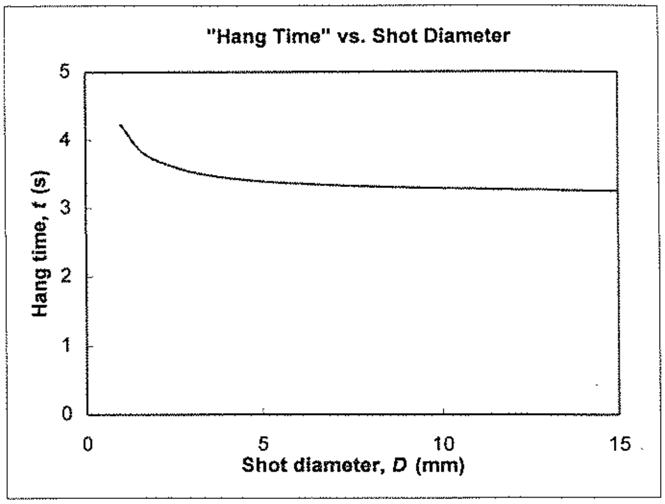
(1) Shot falling in still air:

D (mm)	V_t (m/s)	V/V_t (---)
1	16.1	0.989
1.5	19.7	0.960
2	22.7	0.922
3	27.8	0.848
4	32.1	0.783
5	35.9	0.730
6	39.3	0.685
7	42.5	0.647
8	45.4	0.615
9	48.2	0.587
10	50.8	0.562
11	53.3	0.541
12	55.6	0.521
13	57.9	0.504
14	60.1	0.488
15	62.2	0.473



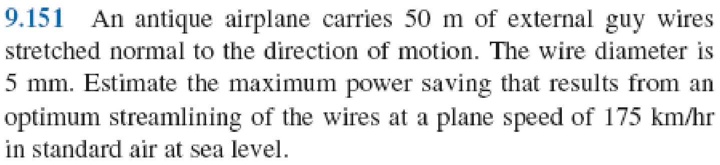


D (mm)	V_t (m/s)	V/V_t (---)	t (s)
1	16.1	0.989	4.24
1.5	19.7	0.960	3.89
2	22.7	0.922	3.71
3	27.8	0.848	3.54
4	32.1	0.783	3.45
5	35.9	0.730	3.40
6	39.3	0.685	3.36
7	42.5	0.647	3.34
8	45.4	0.615	3.32
9	48.2	0.587	3.31
10	50.8	0.562	3.29
11	53.3	0.541	3.29
12	55.6	0.521	3.28
13	57.9	0.504	3.27
14	60.1	0.488	3.27
15	62.2	0.473	3.26



Problem 9.151

[2]



9.151 An antique airplane carries 50 m of external guy wires stretched normal to the direction of motion. The wire diameter is 5 mm. Estimate the maximum power saving that results from an optimum streamlining of the wires at a plane speed of 175 km/hr in standard air at sea level.

Given: Antique airplane guy wires

Find: Maximum power saving using optimum streamlining

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\qquad P = F_D\cdot V$

Given or available data is $L = 50\cdot m$ $\qquad D = 5\cdot mm$ $\qquad V = 175\cdot\dfrac{km}{hr}$ $\qquad V = 48.6\dfrac{m}{s}$

$A = L\cdot D$ $\qquad A = 0.25\,m^2$

$\rho = 1.21\cdot\dfrac{kg}{m^3}$ $\qquad \nu = 1.50\times10^{-5}\cdot\dfrac{m^2}{s}$ $\qquad$ (Table A.10, 20ºC)

The Reynolds number is $Re = \dfrac{V\cdot D}{\nu}$ $\qquad Re = 1.62\times10^{4}$ $\qquad$ so from Fig. 9.13 $\quad C_D = 1.0$

Hence $P = \left(C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2\right)\cdot V$ $\qquad P = 17.4\cdot kW$ $\qquad$ with standard wires

Figure 9.19 suggests we could reduce the drag coefficient to $C_D = 0.06$

Hence $\qquad P_{faired} = \left(C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2\right)\cdot V$ $\qquad P_{faired} = 1.04\cdot kW$

The maximum power saving is then $\qquad \Delta P = P - P_{faired}$ $\qquad \Delta P = 16.3\cdot kW$

Thus $\qquad \dfrac{\Delta P}{P} = 94\cdot\%$ $\qquad$ which is a HUGE savings! It's amazing the antique planes flew!

Problem 9.152

Open-Ended Problem Statement: Why do modern guns have rifled barrels?

Discussion: Almost all projectiles fired by modern guns have smoothly rounded noses and abruptly tapered ("boat-tailed") or square rear ends. The minimum drag for these shapes is obtained when the projectile travels with its axis parallel to the direction of motion and its nose pointed forward.

Rifling in a gun barrel imparts spin about the longitudinal axis of the projectile. This rotation about the longitudinal axis causes the projectile to act as a gyroscope and stabilizes it during flight to keep its nose pointed in the direction of motion.

Early smoothbore guns primarily used ball projectiles. The balls were spherical and molded from lead. Since the ball shape was spherical and had no preferred orientation, no benefit would have been achieved from rifling that caused spin. Therefore the gun barrels were bored smooth, i.e., without rifling grooves, hence these guns were called "smoothbore" guns.

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Problem 9.153

Open-Ended Problem Statement: Why is it possible to kick a football farther in a spiral motion than in an end-over-end tumbling motion?

Discussion: A football has a prolate spheroid shape. It is almost circular when viewed from the front (parallel to the major axis), and longer and more elliptical when viewed from the side (along a minor axis). The football has more frontal area when traveling with the major axis perpendicular to the motion that when it is "spiraling" with the major axis parallel to the direction of travel.

The drag coefficient of the ball when parallel to the flow in spiral motion undoubtedly is less than when perpendicular to the flow. As a rough approximation, the perpendicular drag coefficient might be similar to that of a cylinder ($C_D = 1.2$), whereas the spiral drag coefficient probably is less (perhaps $C_D \approx 0.2 - 0.3$) than that of a sphere ($C_D = 0.5$). Thus the drag coefficient when traveling with the long axis perpendicular to the flow may be 4 to 6 times as large as when traveling in spiral motion with the long axis parallel to the flow. The difference in the drag-area product C_DA will be even larger.

In tumbling motion the drag-area product varies cyclically between the two extremes we have discussed. On average the drag-area product for the tumbling ball is considerably larger, perhaps 2 to 3 times as large, as when the ball is in spiral motion. Therefore the maximum range (travel distance) that can be achieved with tumbling motion is much less than that for spiral motion.

Also, a well kicked or thrown spiral is a thing of beauty. Perhaps function follows form here!

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Problem 9.154

Given: Aircraft with NACA 23012 section airfoils and effective lift area, $A = 25\ m^2$. Maximum flap setting corresponds to condition ② in Fig. 9.23. Takeoff speed is 150 kph. Neglect added lift due to ground effect.

Find: (a) Maximum gross mass at takeoff speed in Denver ($z = 1.61\ km$).
(b) Minimum takeoff speed in Denver

Solution: Apply definition of lift coefficient.

Basic equation: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A_p}$

Assumption: Lift force must equal gravity force at takeoff.

$$F_L = mg = C_D A_p \tfrac{1}{2}\rho V^2$$

For maximum mass, need maximum lift, so use $C_{L,max}$:

$$m_{max} = \frac{C_{L,max} A_p \rho V^2}{2g}$$

From Fig. 9.23, $C_{L,max} = 2.67$ for condition ②. Then for std. air,

$$m_{max} = \frac{2.67}{2} \times 25\ m^2 \times 1.23 \frac{kg}{m^3} \left(150 \times 10^3 \frac{m}{hr} \times \frac{hr}{3600\ s}\right)^2 \frac{s^2}{9.81\ m}$$

$$m_{max} = 7260\ kg \qquad m_{max}$$

{This represents the maximum mass theoretically possible when the aircraft is on the verge of stalling. To attempt takeoff at such a large mass would be ill-advised.}

(b) In Denver, $z = 1.61\ km$. From Table A.3, at $z = 1.61\ km$, $\rho/\rho_0 = 0.855$.

At the same gross mass, the lift force remains the same. Thus

$$F_{L_0} = C_D A \tfrac{1}{2}\rho_0 V_0^2 = F_{L_D} = C_D A \tfrac{1}{2}\rho_D V_D^2 \quad \text{or} \quad \rho_0 V_0^2 = \rho_D V_D^2$$

and

$$V_D = V_0\left(\frac{\rho_0}{\rho_D}\right)^{1/2} = 150\ kph\left(\frac{1}{0.855}\right)^{1/2} = 162\ kph \qquad V_D$$

{The takeoff speed must increase about 8 percent.}

Problem 9.155

Open-Ended Problem Statement: How do cab-mounted wind deflectors for tractor-trailer trucks work? Explain using diagrams of the flow pattern around the truck and pressure distribution on the surface of the truck.

Discussion: Consider both the cab and the trailer flow patterns and pressure distributions in no-wind and crosswind situations.

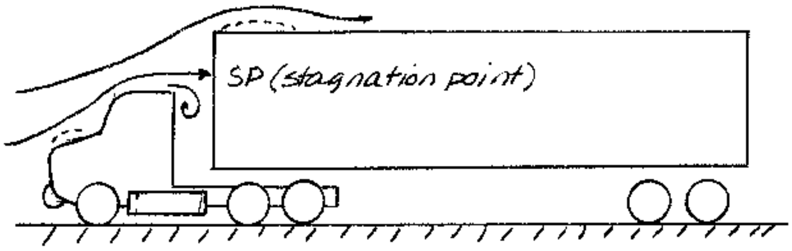
No-wind situation: Without the deflector, flow separation from the roof and sides of the tractor creates a low-pressure wake and high drag force on the tractor. (Flow patterns and pressure distributions on the tractor and the front of the trailer are sketched below.) The cab-mounted deflector reduces the pressures on the front of the tractor, thus reducing the aerodynamic drag force on the tractor.

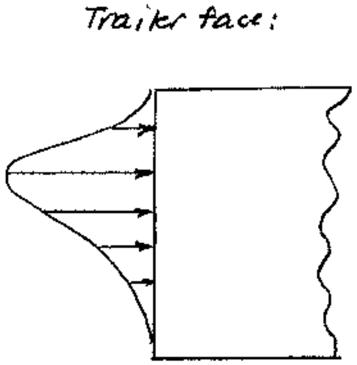
Without the deflector, high-speed air separates from the roof of the tractor and impinges on the vertical front face of the trailer. The cab-mounted deflector reduces the amount of high-speed air hitting the front of the trailer, reducing the net aerodynamic drag force on the trailer. (Ideally air from the deflector flows smoothly along the top and sides of the trailer.)

Crosswind situation: Without the deflector, flow separates from the lee side of the tractor, altering the pressure field and increasing the drag on the tractor. The cab-mounted deflector, especially in combination with side seals, minimizes the increase in drag by reducing the amount of separation around the tractor.

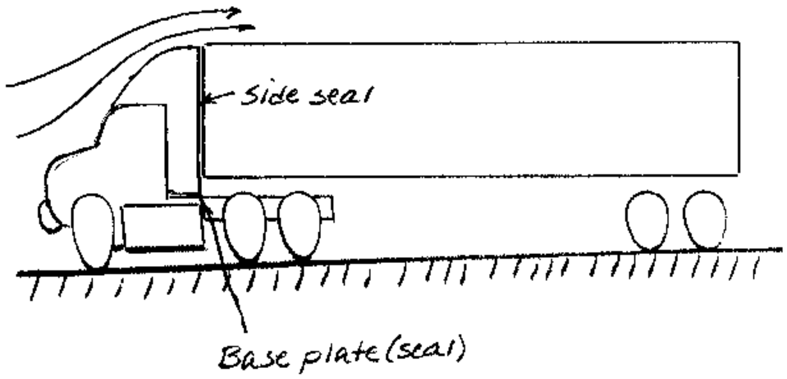
Without the deflector, the front face of the trailer is impacted by the high-speed air from the freestream flow. Massive separation occurs on the lee side of the trailer, thus altering the pressure field and increasing the drag on the trailer. With the cab-mounted deflector, the amount of high-speed air impacting the trailer is markedly reduced. This alters the flow pattern and minimizes the increase in drag caused by the crosswind.

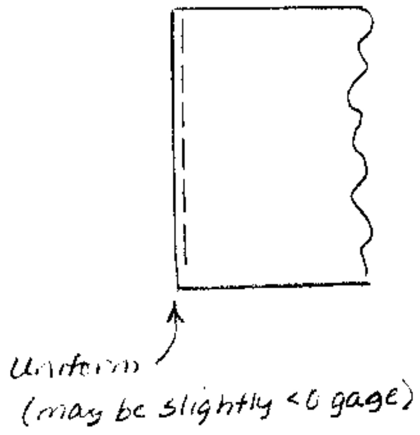
Without cab-mounted wind deflector:





With cab-mounted wind deflector:





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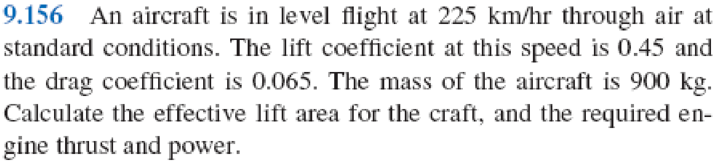
Problem 9.156

[1]



9.156 An aircraft is in level flight at 225 km/hr through air at standard conditions. The lift coefficient at this speed is 0.45 and the drag coefficient is 0.065. The mass of the aircraft is 900 kg. Calculate the effective lift area for the craft, and the required engine thrust and power.

Given: Aircraft in level flight

Find: Effective lift area; Engine thrust and power

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad P = T\cdot V$

For level, constant speed $F_D = T$ $\quad F_L = W$

Given or available data is $V = 225\cdot\dfrac{km}{hr}$ $\quad V = 62.5\dfrac{m}{s}$ $\quad C_L = 0.45$ $\quad C_D = 0.065$ $\quad M = 900\cdot kg$

$\rho = 1.21\cdot\dfrac{kg}{m^3}$ (Table A.10, 20ºC)

Hence $F_L = C_L\cdot\dfrac{1}{2}\cdot\rho\cdot A\cdot V^2 = M\cdot g$ $\quad A = \dfrac{2\cdot M\cdot g}{C_L\cdot\rho\cdot V^2}$ $\quad A = 8.30\,m^2$

Also $\dfrac{F_L}{F_D} = \dfrac{C_L}{C_D}$ $\quad F_L = M\cdot g$ $\quad F_L = 8826\,N$ $\quad F_D = F_L\cdot\dfrac{C_D}{C_L}$ $\quad F_D = 1275\,N$

$T = F_D$ $\quad T = 1275\,N$

The power required is then $P = T\cdot V$ $\quad P = 79.7\,kW$

Problem 9.157

[2]

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Given: Hydrofoil craft with effective foil area, $A = 0.7\ m^2$ and mass, $m = 1800\ kg$. Foils have $C_L = 1.6$ and $C_D = 0.5$. Neglect induced drag.

Find: (a) Minimum speed to support craft on foils.
(b) Power required at this speed.
(c) Maximum speed if 110 kW is available.

Solution: Apply definitions of lift, drag coefficients and power.

Computing equations: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$; $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$; $\mathscr{P} = F_D V$

Assumptions: (1) Lift force equals gravity force.
(2) Neglect induced drag.

Then

$$F_L = mg = C_L A \tfrac{1}{2}\rho V^2 \quad \text{so} \quad V = \left[\frac{2mg}{C_L \rho A}\right]^{1/2}$$

Minimum speed is

$$V_{min} = \left[\frac{2}{1.6} \times 1800\ kg \times 9.81\,\frac{m}{s^2} \times \frac{m^3}{999\ kg} \times \frac{1}{0.7\ m^2}\right] = 5.62\ m/s \quad (10.9\ kt)$$ V_{min}

The drag force at any speed is

$$F_D = C_D A \tfrac{1}{2}\rho V^2 \quad \text{so} \quad F_D = \frac{C_D}{C_L} F_L = \frac{C_D}{C_L} mg$$

and

$$\mathscr{P} = F_D V = \frac{C_D}{C_L} mg V$$

The minimum power is

$$\mathscr{P}_{min} = \frac{C_D}{C_L} mg V_{min} = \frac{0.5}{1.6} \times 1800\ kg \times 9.81\,\frac{m}{s^2} \times 5.62\,\frac{m}{s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$\mathscr{P}_{min} = 31.0 \times 10^3\,\frac{N \cdot m}{s} \times \frac{W \cdot s}{N \cdot m} = 31.0\ kW$$ $\mathscr{P}_{min}$

As speed increases, the craft will ride higher in the water, decreasing the lifting area such that $F_L = mg$. Thus

$$\mathscr{P}_{max} = \frac{C_D}{C_L} mg V_{max} \quad \text{or} \quad V_{max} = \frac{C_L}{C_D} \frac{\mathscr{P}_{max}}{mg}$$

Assuming C_D/C_L remains constant,

$$V_{max} = \frac{1.6}{0.5} \times 110 \times 10^3\ W \times \frac{1}{1800\ kg} \times \frac{s^2}{9.81\ m} \times \frac{N \cdot m}{W \cdot s} \times \frac{kg \cdot m}{N \cdot s^2} = 19.9\ m/s \quad (38.7\ kt)$$ V_{max}

Problem 9.158

[2]

9.158 A high school project involves building a model ultralight airplane. Some of the students propose making an airfoil from a sheet of plastic 1.5 m long by 2 m wide at an angle of attack of 12°. At this airfoil's aspect ratio and angle of attack the lift and drag coefficients are $C_L = 0.72$ and $C_D = 0.17$. If the airplane is designed to fly at 12 m/s, what is the maximum total payload? What will be the required power to maintain flight? Does this proposal seem feasible?

Given: Data on an airfoil

Find: Maximum payload; power required

Solution:

The given data or available data is $\rho = 1.23 \cdot \frac{kg}{m^3}$ $L = 1.5 \cdot m$ $w = 2 \cdot m$ $V = 12 \cdot \frac{m}{s}$ $C_L = 0.72$ $C_D = 0.17$

Then $A = w \cdot L$ $A = 3 m^2$

The governing equations for steady flight are $W = F_L$ and $T = F_D$

where *W* is the model total weight and *T* is the thrust

The lift is given by $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_L$ $F_L = 191 N$ $F_L = 43 \cdot lbf$

The payload is then given by $W = M \cdot g = F_L$

or $M = \frac{F_L}{g}$ $M = 19.5 kg$ $M = 43 \cdot lb$

The drag is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 45.2 N$ $F_D = 10.2 \cdot lbf$

Engine thrust required $T = F_D$ $T = 45.2 N$

The power required is $P = T \cdot V$ $P = 542 W$ $P = 0.727 \cdot hp$

The model ultralight is just feasible: it is possible to find an engine that can produce about 1 hp that weighs less than about 45 lb

Problem 9.159

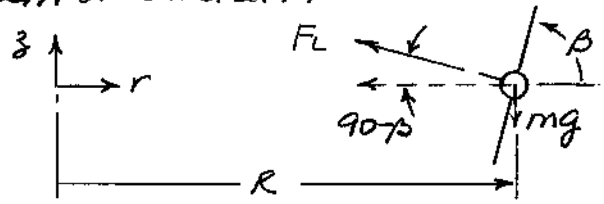
[3]

Given: USAF F-16 with $A_p = 27.9\ m^2$ and $C_{L,max} = 1.6$, at maximum gross mass of $M = 11{,}600\ kg$. Turn flown level with aircraft banked.

Find: (a) Minimum speed in standard air for $a_t = 5g$.
(b) Corresponding radius.
(c) Discuss effect of altitude.

Solution: Draw free-body diagram of aircraft:

Computing equations:



$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A_p} \qquad a_r = -\frac{V^2}{R}$$

Assume: (1) Standard air, $\rho = 1.23\ kg/m^3$
(2) Pilot feels a_t along F_L

Then $F_L = m a_t = C_L A \frac{1}{2}\rho V^2$ or $V = \sqrt{\frac{2 m a_t}{C_L A \rho}}$ is minimum at $C_{L,max}$.

$$V_{min} = \left[2 \times 11{,}600\ kg \times (5)\, 9.81\ \frac{m}{s^2} \times \frac{1}{1.6} \times \frac{1}{27.9\ m^2} \times \frac{m^3}{1.23\ kg}\right]^{1/2}$$

$V_{min} = 144\ m/s$ (minimum speed) ← V_{min}

Need a_r to find V. Sum forces vertically

$$\Sigma F_z = F_L \sin(90-\beta) - mg = m a_z = 0$$

$$\sin(90-\beta) = \cos\beta = \frac{mg}{F_L} = \frac{mg}{5mg} = \frac{1}{5}\ ;\ \beta = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°$$

Sum forces radially:

$$\Sigma F_r = -F_L \cos(90-\beta) = m a_r = m\left(-\frac{V^2}{R}\right)$$

$$R = \frac{mV^2}{F_L \sin\beta} = \frac{mV^2}{5mg\sin\beta} = \frac{V^2}{5g\sin\beta}$$

$$= \frac{1}{5} \times (144)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\ m} \times \frac{1}{\sin 78.5°}$$

$R = 431\ m$ ← R

As altitude increases, density decreases, and V is raised. This also increases R. At $z = 15\ km$, $\rho/\rho_0 = 0.159$. Thus

$$\frac{V}{V_0} = \sqrt{\frac{\rho_0}{\rho}} = 2.51 \quad \text{and} \quad \frac{R}{R_0} = \frac{V^2}{V_0^2} = 6.29$$

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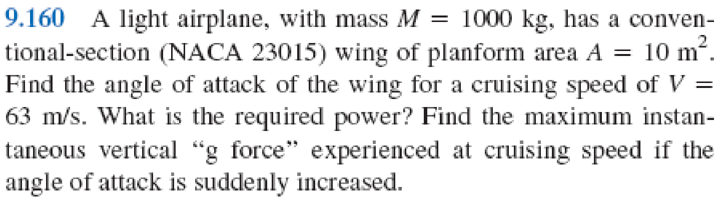
Problem 9.160

[3]



9.160 A light airplane, with mass $M = 1000$ kg, has a conventional-section (NACA 23015) wing of planform area $A = 10\ \text{m}^2$. Find the angle of attack of the wing for a cruising speed of $V = 63$ m/s. What is the required power? Find the maximum instantaneous vertical "g force" experienced at cruising speed if the angle of attack is suddenly increased.

Given: Data on a light airplane

Find: Angle of attack of wing; power required; maximum "g" force

Solution:

The given data or available data is $\rho = 1.23 \cdot \frac{\text{kg}}{\text{m}^3}$ $\quad M = 1000 \cdot \text{kg}$ $\quad A = 10 \cdot \text{m}^2$

$V = 63 \cdot \frac{\text{m}}{\text{s}}$ $\quad C_L = 0.72$ $\quad C_D = 0.17$

The governing equations for steady flight are $W = M \cdot g = F_L$ $\quad T = F_D$

where *W* is the weight *T* is the engine thrust

The lift coeffcient is given by $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_d$

Hence the required lift coefficient is $C_L = \dfrac{M \cdot g}{\frac{1}{2} \cdot \rho \cdot A \cdot V^2}$ $\quad C_L = 0.402$

From Fig 9.17, for at this lift coefficient $\alpha = 3 \cdot \text{deg}$

and the drag coefficient at this angle of attack is $C_D = 0.0065$

(Note that this does NOT allow for aspect ratio effects on lift and drag!)

Hence the drag is $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $\quad F_D = 159\,\text{N}$

and $T = F_D$ $\quad T = 159\,\text{N}$

The power required is then $P = T \cdot V$ $\quad P = 10\,\text{kW}$

The maximum "g"'s occur when the angle of attack is suddenly increased to produce the maximum lift

From Fig. 9.17 $C_{L.max} = 1.72$

$F_{Lmax} = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_{L.max}$ $\quad F_{Lmax} = 42\,\text{kN}$

The maximum "g"s are given by application of Newton's second law

$$M \cdot a_{perp} = F_{Lmax}$$

where a_{perp} is the acceleration perpendicular to the flight direction

Hence $a_{perp} = \frac{F_{Lmax}}{M}$ $a_{perp} = 42\,\frac{m}{s^2}$

In terms of "g"s $\frac{a_{perp}}{g} = 4.28$

Note that this result occurs when the airplane is banking at 90^o, i.e, when the airplane is flying momentarily in a circular flight path in the horizontal plane. For a straight horizontal flight path Newton's second law is

$$M \cdot a_{perp} = F_{Lmax} - M \cdot g$$

Hence $a_{perp} = \frac{F_{Lmax}}{M} - g$ $a_{perp} = 32.2\,\frac{m}{s^2}$

In terms of "g"s $\frac{a_{perp}}{g} = 3.28$

Problem 9.161

[3]

9.161 The teacher of the students designing the airplane of Problem 9.158 is not happy with the idea of using a sheet of plastic for the airfoil. He asks the students to evaluate the expected maximum total payload, and required power to maintain flight, if the sheet of plastic is replaced with a conventional section (NACA 23015) airfoil with the same aspect ratio and angle of attack. What are the results of the analysis?

Given: Data on an airfoil

Find: Maximum payload; power required

Solution:

The given data or available data is $V = 12 \cdot \frac{m}{s}$ $\rho = 1.23 \cdot \frac{kg}{m^3}$ $c = 1.5 \cdot m$ $b = 2 \cdot m$

Then the area is $A = b \cdot c$ $A = 3\,m^2$

and the aspect ratio is $ar = \frac{b}{c}$ $ar = 1.33$

The governing equations for steady flight are

$W = F_L$ and $T = F_D$

where *W* is the model total weight and *T* is the thrust

At a 12^o angle of attack, from Fig. 9.17 $C_L = 1.4$ $C_{Di} = 0.012$

where C_{Di} is the section drag coefficient

The wing drag coefficient is given by Eq. 9.42 $C_D = C_{Di} + \frac{C_L^{\ 2}}{\pi \cdot ar}$ $C_D = 0.48$

The lift is given by $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_L$ $F_L = 372\,N$ $F_L = 83.6\,lbf$

The payload is then given by $W = M \cdot g = F_L$

or $M = \frac{F_L}{g}$ $M = 37.9\,kg$ $M = 83.6\,lb$

The drag is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 127.5\,N$ $F_D = 28.7\,lbf$

Engine thrust required $T = F_D$ $T = 127.5\,N$

The power required is $P = T \cdot V$ $P = 1.53\,kW$ $P = 2.05\,hp$

NOTE: Strictly speaking we have TWO extremely stubby wings, so a recalculation of drag effects (lift is unaffected) gives

$b = 1 \cdot m$ $c = 1.5\,m$

and $A = b \cdot c$ $A = 1.5\,m^2$ $ar = \frac{b}{c}$ $ar = 0.667$

so the wing drag coefficient is $C_D = C_{Di} + \frac{C_L{}^2}{\pi \cdot ar}$ $C_D = 0.948$

The drag is $F_D = 2 \cdot \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 252\,N$ $F_D = 56.6\,lbf$

Engine thrust is $T = F_D$ $T = 252\,N$

The power required is $P = T \cdot V$ $P = 3.02\,kW$ $P = 4.05\,hp$

Problem 9.162

[3]

Given: Light plane with NACA 23015 airfoil, $S = 10\text{ m}$, $c = 1.8\text{ m}$, cruises at $V = 225\text{ km/hr}$ near sea level on a standard day.

Find: Determine cruise speed with NACA 66_2-215 section airfoil.

Solution: Apply definitions of coefficients, use data from Fig. 9.19.

Computing equations: $C_D = C_{D,\infty} + C_{D,i} = C_{D,\infty} + \dfrac{C_L^2}{\pi ar}$

From a free-body diagram $F_L = W$, $\mathcal{P} = F_D V/\eta_p$

From Fig. 9.19, recognize airfoils should operate near design lift coefficients. Thus assume:

Section	C_L	$C_{D,\infty}$
23015	0.3	0.0062
66_2-215	0.2	0.0031

$ar = \dfrac{S}{c} = \dfrac{10\text{ m}}{1.8\text{ m}} = 5.56$

Thus $\quad C_{D\,old} \approx 0.0062 + \dfrac{(0.3)^2}{\pi(5.56} = 0.0062 + 0.00515 = 0.0114$

$$C_{D\,new} \approx 0.0031 + \frac{(0.2)^2}{\pi(5.56)} = 0.0031 + 0.00229 = 0.00539$$

Since for level flight,

$$\mathcal{P} = F_D V/\eta_p = \frac{C_D A \frac{1}{2}\rho V^2 V}{\eta_p}, \text{ then } V = \left[\frac{2\eta_p \mathcal{P}}{C_D A}\right]^{\frac{1}{3}}$$

Assuming $\eta_p \mathcal{P}$ remains constant, then $V_{new} = V_{old}\left[\dfrac{C_{D,old}}{C_{D,new}}\right]^{\frac{1}{3}}$

$$V_{new} \approx 225\,\frac{\text{km}}{\text{hr}}\left[\frac{0.0114}{0.00539}\right]^{\frac{1}{3}} = 289\text{ km/hr} \longleftarrow$$

V_{new}

Check assumption on C_L: Since $F_L = W = C_L A \frac{1}{2}\rho V^2$, then

$$C_{L\,new} \approx C_{L\,old}\left[\frac{V_{old}}{V_{new}}\right]^2 = 0.3\left[\frac{225}{289}\right]^2 = 0.182$$

Therefore the above estimate for new cruise speed is probably conservative.

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Problem 9.163

[3]

Given: Boeing 727 aircraft, with NACA 23012 section, $A_p = 1600 \text{ ft}^2$, and effective aspect ratio, $ar = 6.5$. Aircraft flies at $V = 150$ kt, with $W = 175{,}000$ lbf.

Find: Estimate thrust needed to maintain steady, level flight.

Solution: For steady, level flight, thrust equals drag and lift equals weight.

Computing equations:

$$F_L = W = C_L \tfrac{1}{2}\rho V^2 A \qquad (1)$$

$$F_D = T = C_D \tfrac{1}{2}\rho V^2 A \qquad (2)$$

$$C_D = C_{D,\infty} + C_{D,i} = C_{D,0} + \frac{C_L^2}{\pi ar}$$

Assumptions: (1) Standard air
(2) Data from Fig. 9.23 apply

$$V = 150 \frac{\text{nm}}{\text{hr}} \times 6076 \frac{\text{ft}}{\text{nm}} \times \frac{\text{hr}}{3600 \text{ sec}} = 253 \text{ ft/sec}$$

$$q = \tfrac{1}{2}\rho V^2 = \frac{1}{2} \times 0.00238 \frac{\text{slug}}{\text{ft}^3} \times (253)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 76.2 \text{ lbf/ft}^2$$

From Eq. 1, $C_L = \dfrac{W}{qA} = 175000 \text{ lbf} \times \dfrac{\text{ft}^2}{76.2 \text{ lbf}} \times \dfrac{1}{1600 \text{ ft}^2} = 1.44$

From Fig. 9.23, this corresponds to operation with a single slot open, and $C_{D,0} \cong 0.04$. Thus

$$C_D = C_{D,0} + \frac{C_L^2}{\pi ar} = 0.04 + \frac{(1.44)^2}{\pi(6.5)} = 0.142$$

To find thrust, note

$$\frac{T}{F_L} = \frac{C_D}{C_L}\frac{qA}{qA} = \frac{C_D}{C_L} = \frac{0.142}{1.44} = 0.0986$$

Thus

$$T = F_L \frac{C_D}{C_L} = W \frac{C_D}{C_L} = 175{,}000 \text{ lbf} \times 0.0986 = 17{,}300 \text{ lbf} \longleftarrow T$$

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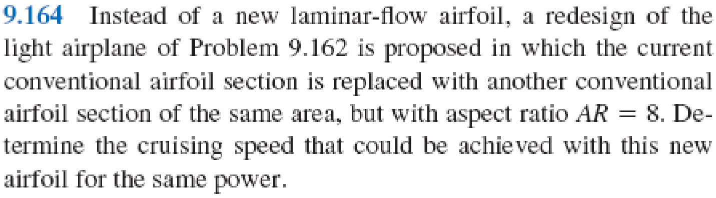
NATIONAL

Problem 9.164

[3]



9.164 Instead of a new laminar-flow airfoil, a redesign of the light airplane of Problem 9.162 is proposed in which the current conventional airfoil section is replaced with another conventional airfoil section of the same area, but with aspect ratio $AR = 8$. Determine the cruising speed that could be achieved with this new airfoil for the same power.

Given: Data on an airfoil

Find: Maximum payload; power required

Solution:

The given data or available data is $V_{old} = 225\cdot\frac{m}{s}$ $\rho = 1.23\cdot\frac{kg}{m^3}$ $A = 180\cdot m^2$ $ar_{old} = \frac{10}{1.8}$ $ar_{old} = 5.56$

Assuming the old airfoil operates at close to design lift, from Fig. 9.19 $C_L = 0.3$ $C_{Di} = 0.0062$ (C_{Di} is the old airfoil's section drag coefficient)

Then $C_{Dold} = C_{Di} + \frac{C_L^2}{\pi\cdot ar_{old}}$ $C_{Dold} = 0.0114$

The new wing aspect ratio is $ar_{new} = 8$

Hence $C_{Dnew} = C_{Di} + \frac{C_L^2}{\pi\cdot ar_{new}}$ $C_{Dnew} = 0.00978$

The power required is $P = T\cdot V = F_D\cdot V = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D\cdot V$

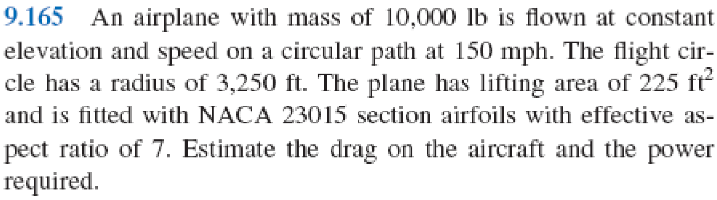
If the old and new designs have the same available power, then

$$\frac{1}{2}\cdot\rho\cdot A\cdot V_{new}^{\ 2}\cdot C_{Dnew}\cdot V_{new} = \frac{1}{2}\cdot\rho\cdot A\cdot V_{old}^{\ 2}\cdot C_{Dold}\cdot V_{old}$$

or $V_{new} = V_{old}\cdot\sqrt[3]{\frac{C_{Dold}}{C_{Dnew}}}$ $V_{new} = 236\frac{m}{s}$

Problem 9.165

[3]



9.165 An airplane with mass of 10,000 lb is flown at constant elevation and speed on a circular path at 150 mph. The flight circle has a radius of 3,250 ft. The plane has lifting area of 225 ft^2 and is fitted with NACA 23015 section airfoils with effective aspect ratio of 7. Estimate the drag on the aircraft and the power required.

Given: Aircraft in circular flight

Find: Drag and power

Solution:

Basic equations:

$$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2} \qquad C_L = \frac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2} \qquad P = F_D\cdot V \qquad \Sigma\vec{F} = M\cdot\vec{a}$$

The given data or available data are

$$\rho = 0.002377\cdot\frac{\text{slug}}{\text{ft}^3} \qquad R = 3250\cdot\text{ft} \qquad M = 10000\cdot\text{lbm} \qquad M = 311\cdot\text{slug}$$

$$V = 150\cdot\text{mph} \qquad V = 220\cdot\frac{\text{ft}}{\text{s}} \qquad A = 225\cdot\text{ft}^2 \qquad ar = 7$$

Assuming the aircraft is flying banked at angle β, the vertical force balance is

$$F_L\cdot\cos(\beta) - M\cdot g = 0 \qquad \text{or} \qquad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\cos(\beta) = M\cdot g \qquad (1)$$

The horizontal force balance is

$$-F_L\cdot\sin(\beta) = M\cdot a_r = -\frac{M\cdot V^2}{R} \qquad \text{or} \qquad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\sin(\beta) = \frac{M\cdot V^2}{R} \qquad (2)$$

Equations 1 and 2 enable the bank angle β to be found

$$\tan(\beta) = \frac{V^2}{R\cdot g} \qquad \beta = \operatorname{atan}\left(\frac{V^2}{R\cdot g}\right) \qquad \beta = 24.8\cdot\text{deg}$$

Then from Eq 1

$$F_L = \frac{M\cdot g}{\cos(\beta)} \qquad F_L = 1.10\times10^4\cdot\text{lbf}$$

Hence

$$C_L = \frac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2} \qquad C_L = 0.851$$

For the section, $C_{Dinf} = 0.0075$ at $C_L = 0.851$ (from Fig. 9.19), so

$$C_D = C_{Dinf} + \frac{C_L^{\ 2}}{\pi\cdot ar} \qquad C_D = 0.040$$

Hence

$$F_D = F_L\cdot\frac{C_D}{C_L} \qquad F_D = 524\cdot\text{lbf}$$

The power is

$$P = F_D\cdot V \qquad P = 1.15\times10^5\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}} \qquad P = 209\cdot\text{hp}$$

Problem 9.166

9.166 Find the minimum and maximum speeds at which the airplane of Problem 9.165 can fly on a 3,250 ft radius circular flight path, and estimate the drag on the aircraft and power required at these extremes.

Given: Aircraft in circular flight

Find: Maximum and minimum speeds; Drag and power at these extremes

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad P = F_D\cdot V$ $\quad \Sigma\cdot\vec{F} = M\cdot\vec{a}$

The given data or available data are

$\rho = 0.002377\cdot\dfrac{\text{slug}}{\text{ft}^3}$ $\quad R = 3250\cdot\text{ft}$ $\quad M = 10000\cdot\text{lbm}$ $\quad M = 311\cdot\text{slug}$

$A = 225\cdot\text{ft}^2$ $\quad ar = 7$

The minimum velocity will be when the wing is at its maximum lift condition. From Fig . 9. 17 or Fig. 9.19

$C_L = 1.72$ $\quad C_{Dinf} = 0.02$

where C_{Dinf} is the section drag coefficient

The wing drag coefficient is then $\quad C_D = C_{Dinf} + \dfrac{C_L^{\,2}}{\pi\cdot ar}$ $\quad C_D = 0.155$

Assuming the aircraft is flying banked at angle β, the vertical force balance is

$F_L\cdot\cos(\beta) - M\cdot g = 0$ $\quad$ or $\quad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\cos(\beta) = M\cdot g$ $\quad$ (1)

The horizontal force balance is

$-F_L\cdot\sin(\beta) = M\cdot a_r = -\dfrac{M\cdot V^2}{R}$ $\quad$ or $\quad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\sin(\beta) = \dfrac{M\cdot V^2}{R}$ $\quad$ (2)

Equations 1 and 2 enable the bank angle β and the velocity V to be determined

$$\sin(\beta)^2 + \cos(\beta)^2 = \left(\frac{\frac{M\cdot V^2}{R}}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L}\right)^2 + \left(\frac{M\cdot g}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L}\right)^2 = 1$$

or $\quad \dfrac{M^2\cdot V^4}{R^2} + M^2\cdot g^2 = \dfrac{\rho^2\cdot A^2\cdot V^4\cdot C_L^{\,2}}{4}$

$V = \sqrt[4]{\dfrac{M^2\cdot g^2}{\dfrac{\rho^2\cdot A^2\cdot C_L^{\,2}}{4} - \dfrac{M^2}{R^2}}}$ $\quad V = 149\,\dfrac{\text{ft}}{\text{s}}$ $\quad V = 102\,\text{mph}$

$\tan(\beta) = \dfrac{V^2}{R\cdot g}$ $\quad \beta = \text{atan}\left(\dfrac{V^2}{R\cdot g}\right)$ $\quad \beta = 12.0\,\text{deg}$

The drag is then $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $F_D = 918\,\text{lbf}$

The power required to overcome drag is $P = F_D\cdot V$ $P = 1.37\times 10^5\,\frac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $P = 249\,\text{hp}$

The analysis is repeated for the maximum speed case, when the lift/drag coefficient is at its minimum value. From Fig. 9.19, reasonable values are

$C_L = 0.3$ $C_{Dinf} = \frac{C_L}{47.6}$ corresponding to α = 2º (Fig. 9.17)

The wing drag coefficient is then $C_D = C_{Dinf} + \frac{C_L^{\,2}}{\pi\cdot ar}$ $C_D = 0.0104$

From Eqs. 1 and 2 $V = \sqrt[4]{\frac{M^2\cdot g^2}{\frac{\rho^2\cdot A^2\cdot C_L^{\,2}}{4} - \frac{M^2}{R^2}}}$ $V = (309.9 + 309.9i)\,\frac{\text{ft}}{\text{s}}$

Obviously unrealistic (lift is just too low, and angle of attack is too low to generate sufficient lift)

We try instead a larger, more reasonable, angle of attack

$C_L = 0.55$ $C_{Dinf} = 0.0065$ corresponding to α = 4º (Fig. 9.17)

The wing drag coefficient is then $C_D = C_{Dinf} + \frac{C_L^{\,2}}{\pi\cdot ar}$ $C_D = 0.0203$

From Eqs. 1 and 2 $V = \sqrt[4]{\frac{M^2\cdot g^2}{\frac{\rho^2\cdot A^2\cdot C_L^{\,2}}{4} - \frac{M^2}{R^2}}}$ $V = 91.2\,\frac{\text{m}}{\text{s}}$ $V = 204\,\text{mph}$

$\tan(\beta) = \frac{V^2}{R\cdot g}$ $\beta = \operatorname{atan}\left(\frac{V^2}{R\cdot g}\right)$ $\beta = 40.6\,\text{deg}$

The drag is then $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $F_D = 485\,\text{lbf}$

The power required to overcome drag is $P = F_D\cdot V$ $P = 1.45\times 10^5\,\frac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $P = 264\,\text{hp}$

Problem 9.167

[4]

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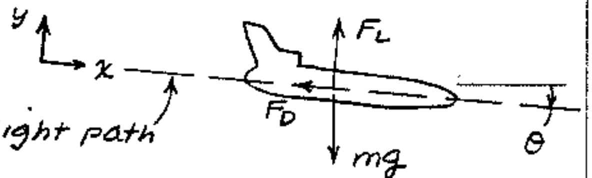
Given: Unpowered flight with lift, drag, and weight in equilibrium.

Find: (a) Show glide slope angle is $\tan\theta = C_D/C_L$
(b) Evaluate minimum glide slope angle for Boeing 727-200 of Example Problem 9.8.
(c) Glide distance from altitude of 10 km on a standard day.

Solution: Consider free-body diagram:



Sum forces along (x) and normal to (y) flight path:

$$\Sigma F_x = -F_D + mg\sin\theta = 0 \qquad mg\sin\theta = F_D$$
$$\Sigma F_y = F_L - mg\cos\theta = 0 \qquad mg\cos\theta = F_L$$
$$\tan\theta = \frac{F_D}{F_L} = \frac{C_D}{C_L}$$

tanθ

Use relationships from Section 9-8:

Computing equation: $C_D = C_{D,0} + \dfrac{C_L^2}{\pi ar} = C_{D,0} + C_{D,i}$

Thus $$\frac{C_D}{C_L} = \frac{C_{D,0}}{C_L} + \frac{C_L}{\pi ar} \qquad (1)$$

To minimize, set $d(C_D/C_L)/dC_L = 0$

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L}\right) = (-1)\frac{C_{D,0}}{C_L^2} + \frac{1}{\pi ar} = 0 \quad \text{when} \quad C_{D,0} = \frac{C_L^2}{\pi ar} = C_{D,i}$$

From Example Problem 9.8, $C_{D,0} = 0.0182$ and $ar = 6.5$. Thus optimum is

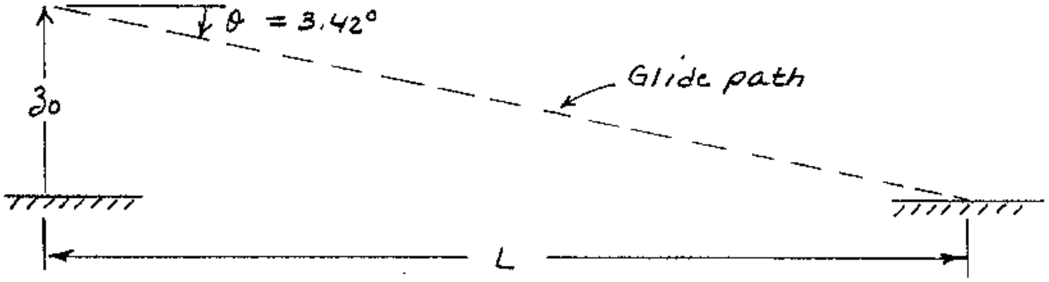
$$C_L = (\pi ar\, C_{D,0})^{1/2} = [\pi(6.5)0.0182]^{1/2} = 0.610$$

and from Eq. 1,

$$\frac{C_D}{C_L} = \frac{0.0182}{0.61} + \frac{0.61}{\pi(6.5)} = 0.0597 = \tan\theta\ ;\ \theta = \tan^{-1}(0.0597) = 3.42°$$

θ

Note θ is independent of atmospheric conditions. Thus θ = constant



$$\frac{z_0}{L} = \tan\theta\ ;\ L = \frac{z_0}{\tan\theta} = \frac{10\text{ km}}{0.0597} = 168\text{ km}$$

L

Problem 9.168

[3]

Given: Chaparral 2F with rear-mounted airfoil having span, $s = 6$ ft, and chord, $c = 1$ ft. Lift and drag coefficients same as conventional section in Fig. 9.17. Consider $V = 120$ mph (calm day), $\alpha = -12°$ (down).

Find: (a) Maximum downward force.
(b) Maximum increase in deceleration force.

Solution: Apply definitions of C_L and C_D.

Computing equations: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$ $\quad C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A} = C_{D,\infty} + \dfrac{C_L^2}{\pi ar}$ $\quad ar = \dfrac{s}{c}$

From Fig. 9.17, at $\alpha = 12°$, $C_L = 1.4$ and $C_{D,\infty} = 0.013$. Thus, since $\alpha = -12°$,

$$F_L = -C_L A \tfrac{1}{2}\rho V^2 \qquad A = sc = 6\,\text{ft}^2$$

$$= -1.4 \times 6\,\text{ft}^2 \times \frac{1}{2} \times 0.00238\,\frac{\text{slug}}{\text{ft}^3} \times \left(120\,\frac{\text{mi}}{\text{hr}} \times 5280\,\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}}\right)^2 \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$F_L = -310\ \text{lbf}\ \text{(downward force)}$$ $\xleftarrow{\hspace{3em}}$ F_L

Then $F_D = F_L \dfrac{C_D}{C_L} = \dfrac{0.013 + \frac{(1.4)^2}{\pi(6)}}{1.4} \times 310\ \text{lbf} = 25.9\ \text{lbf}$

Braking thrust increases as drag increases and as normal force increases tire adhesion (friction). Thus

$$\Delta F_B = \mu_k F_L + F_D$$

For $\mu_k = 1.0$ (probably conservative for racing tires),

$$\Delta F_B = 1.0 \times 310\ \text{lbf} + 25.9\ \text{lbf} = 336\ \text{lbf}$$ $\xleftarrow{\hspace{3em}}$ ΔF_B

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Problem 9.169

[4]

9.169 Some cars come with a "spoiler," a wing section mounted on the rear of the vehicle that salespeople sometimes claim significantly increases traction of the tires at highway speeds. Investigate the validity of this claim. Are these devices really just cosmetic?

Given: Car spoiler

Find: Whether they are effective

Solution:

To perform the investigation, consider some typical data

For the spoiler, assume $b = 4 \cdot ft$ $c = 6 \cdot in$ $\rho = 1.23 \cdot \frac{kg}{m^3}$ $A = b \cdot c$ $A = 2\,ft^2$

From Fig. 9.17 a reasonable lift coefficient for a conventional airfoil section is $C_L = 1.4$

Assume the car speed is $V = 55 \cdot mph$

Hence the "negative lift" is $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_L$ $F_L = 21.7\,lbf$

This is a relatively minor negative lift force (about four bags of sugar); it is not likely to produce a noticeable difference in car traction

The picture gets worse at 30 mph: $F_L = 6.5\,lbf$

For a race car, such as that shown on the cover of the text, typical data might be

$b = 5 \cdot ft$ $c = 18 \cdot in$ $A = b \cdot c$ $A = 7.5\,ft^2$ $V = 200 \cdot mph$

In this case: $F_L = 1078\,lbf$

Hence, for a race car, a spoiler can generate very significant negative lift!

Problem 9.170

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Given: Man-powered aircraft, the Gossamer Condor:

$W/A = 0.4 \text{ lbf/ft}^2 \quad W = 200 \text{ lbf} \quad ar = 17 \quad F_D = 6 \text{ lbf at } 12 \text{ mph}$

Pilot could sustain 0.39 hp for 2 hr.

Find: (a) Minimum power to fly aircraft.
(b) Compare to pilot output capability.

Solution: Apply relationships from Section 9-8:

Computing equations: $W = F_L = C_L A \frac{1}{2}\rho V^2 \qquad P = VF_D$

$$T = F_D = C_D A \frac{1}{2}\rho V^2 \; ; \; C_D = C_{D,0} + \frac{C_L^2}{\pi ar}$$

The task is to find V to minimize P:

$$P = VF_D = V(C_D A \tfrac{1}{2}\rho V^2) = \left(C_{D,0} + \frac{C_L^2}{\pi ar}\right) A \tfrac{1}{2}\rho V^3 \qquad (1)$$

But C_L varies with aircraft speed:

$$F_L = W = C_L A \tfrac{1}{2}\rho V^2 \; ; \quad C_L = \frac{2W}{\rho V^2 A} \; ; \quad C_L^2 = \left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^4}$$

Substituting into Eq. 1,

$$P = \left[C_{D,0} + \frac{1}{\pi ar}\left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^4}\right] A \tfrac{1}{2}\rho V^3$$

To minimize power, set $dP/dV = 0$. Then

$$\frac{dP}{dV} = C_{D,0} A \tfrac{1}{2}\rho V^2(3) + (-1)\frac{1}{\pi ar}\left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^2} A \tfrac{1}{2}\rho = 0$$

Thus at minimum power,

$$3C_{D,0} = \frac{1}{\pi ar}\left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^4} = \frac{C_L^2}{\pi ar} = C_{D,i} \quad \text{or} \quad C_{D,i} = 3C_{D,0} \quad \xleftarrow{\text{minimum power}}$$

From given data, at 12 mph (17.6 ft/s):

$$\tfrac{1}{2}\rho V^2 = \tfrac{1}{2} \times 0.00238 \frac{\text{slug}}{\text{ft}^3} \times (17.6)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 0.369 \text{ lbf/ft}^2$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{W/A}{\frac{1}{2}\rho V^2} = 0.4 \frac{\text{lbf}}{\text{ft}^2} \times \frac{\text{ft}^2}{0.369 \text{ lbf}} = 1.08 \; ; \; A = \frac{W}{W/A} = 200 \text{ lbf} \times \frac{\text{ft}^2}{0.4 \text{ lbf}} = 500 \text{ ft}^2$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = 6 \text{ lbf} \times \frac{\text{ft}^2}{0.369 \text{ lbf}} \times \frac{1}{500 \text{ ft}^2} = 0.0325$$

$$C_{D,0} = C_D - C_{D,i} = C_D - \frac{C_L^2}{\pi ar} = 0.0325 - \frac{(1.08)^2}{\pi(17)} = 0.0107 \qquad (C_{D,0} = \text{constant})$$

At flight speed for minimum power, $C_{D,i} = 3C_{D,0}$

Problem 9.170

[5] Part 2/2

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Thus at minimum power

$$C_{D,i} = \frac{C_L^2}{\pi ar} = 3C_{D,0} = 3(0.0107) = 0.0321$$

so

$$C_L = (0.0321\,\pi ar)^{1/2} = 1.31 \quad \text{(minimum power)}$$

since

$$F_L = W = C_L A \tfrac{1}{2}\rho V^2$$

then

$$V = \sqrt{\frac{2W}{C_L \rho A}} \quad \text{and} \quad \frac{V_{min}}{V} = \sqrt{\frac{C_L}{C_{L\,min}}}$$

Thus at minimum power

$$V_{min} = 12\text{ mph}\sqrt{\frac{1.08}{1.31}} = 10.9\text{ mph } (16.0\text{ ft/s}) \quad \text{(minimum power)} \longleftarrow V_{min}$$

The power requirement would be

$$P_{pilot} = \frac{P_{flight}}{\eta_{drive}\,\eta_{prop}}$$

$$P_{flight} = VF_D = V C_D A \tfrac{1}{2}\rho V^2 = (C_{D,0} + 3C_{D,0}) A \tfrac{1}{2}\rho V^3 = 2C_{D,0} A \rho V^3$$

$$= 2(0.0107)\,500\text{ ft}^2 \times 0.00238\,\frac{\text{slug}}{\text{ft}^3} \times (16.0)^3\frac{\text{ft}^3}{\text{s}^3} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{hp}\cdot\text{s}}{550\text{ ft}\cdot\text{lbf}}$$

$$P_{flight} = 0.190\text{ hp} \quad \text{(power for flight)}$$

If $\eta_{drive} = 0.9$ and $\eta_{prop} = 0.7$,

$$P_{pilot} \approx \frac{0.190}{(0.9)(0.7)} = 0.302\text{ hp} \quad \text{(minimum power)} \longleftarrow P_{pilot}$$

Thus $P_{pilot} < 0.39$ hp !

Problem 9.171

[5]

Open-Ended Problem Statement: How does a Frisbee™ fly? What causes it to curve left or right? What is the effect of spin on its flight?

Discussion: When viewed from the side, the Frisbee shape has a rounded upper surface and a flat bottom surface. Such a shape is capable of generating lift as it travels through air.

When a Frisbee is not spinning, the lift vector probably acts slightly forward of the maximum thickness on the profile. When spinning, the motion of the surface likely affects the development and separation of the boundary layers. This may displace the center of lift slightly to the right or left of center, depending on the direction of spin.

A Frisbee is not stable when thrown without spin: it will tend to tumble as it moves through the air. Spin is used to stabilize the motion (just as a spinning gyroscope tends to remain upright). The combination of spin and the off-center lift vector cause the Frisbee to precess as a gyroscope. Therefore its spin axis can change from vertical while in flight, causing the flight path to curve right or left.

The Frisbee also can be thrown intentionally to curve right or left. This is done by inclining the spin axis so that it is not vertical at launch. When the spin axis is inclined to the left (as seen by the thrower), the Frisbee drifts to the left along a more-or-less constant radius path. Inclining the spin axis to the right causes the opposite effect.

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Problem 9.172

[5]

Open-Ended Problem Statement: Roadside signs tend to oscillate in a twisting motion when a strong wind blows. Discuss the phenomena that must occur to cause this behavior.

Discussion: Many roadside signs are mounted on a single post formed from stamped steel. The post has an open "C" cross-section, which provides little torsional rigidity. Wind gusts can excite oscillations in a sign, which acts as a flat plate at an angle of attack relative to the oncoming wind. When at an angle of attack, a plate develops both a lift force and a moment that tends to twist the sign farther from its equilibrium position. While the sign twists, the post provides a resisting torque as a result of being twisted from its equilibrium position.

As the sign twists, the angle of attack relative to the oncoming air increases. An overshoot phenomenon called dynamic stall allows the flow to remain attached and the angle of attack to grow larger before stall occurs than if the change in angle of attack had been slow and gradual. Once stall occurs, the lift force and moment decrease, and the motion is no longer forced. Then the sign tends to return to its undisturbed position.

The moment of inertia of the sign causes it to overshoot the equilibrium position. The sign continues beyond equilibrium and develops a lift force and a moment tending to move it farther past equilibrium. The process repeats, with growing amplitude, until a more-or-less steady-state oscillation is reached.

The sign and post form a spring-mass-damper mechanical system. The sign is the mass, the post is the spring, and hysteresis and aerodynamic resistance to oscillation provide the damping. The "steady" oscillation occurs near the natural frequency of the system.

At steady state, the rate at which energy is added to the sign by the gusting wind exactly balances the rate at which energy is dissipated by hysteresis in the sign motion and its supporting post. The oscillations can continue almost indefinitely, and with considerable amplitude, as can be observed on a windy day. In some cases the oscillations lead to fatigue failure of the sign post.

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Problem 9.173

Open-Ended Problem Statement: An automobile travels down the road with a bicycle attached to a carrier across the rear of the trunk. The bicycle wheels rotate slowly. Explain why and in what direction the rotation occurs.

Discussion: All objects moving in ground effect generate lift (the air flows over the top faster than over the bottom because of the shape of the automobile). Any object that produces lift carries with it a bound vortex that creates circulation about the profile that accompanies lift.

The bound vortex creates two trailing vortices, one on each side of the car, which rotate in opposite directions as they follow in the wake of the automobile. When viewed from the rear of the auto, the left side trailing vortex rotates clockwise and the right side trailing vortex rotates counterclockwise.

The swirl in the trailing vortex motion is responsible for the motion of the bicycle wheels (check it out on your next auto trip during the summer months!). The swirl causes shear stresses that tend to rotate the bicycle wheels in the same senses as the trailing vortices. Again viewing from the rear of the auto, the left wheel rotates clockwise and the right wheel counterclockwise.

(Sometimes the rear wheel of the bicycle cannot freewheel. In this case only the front wheel turns slowly as the car drives down the road.)

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Problem 9.174

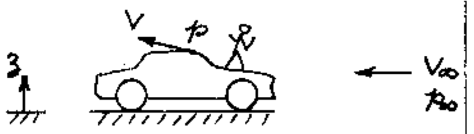
[2]

Given: Air moving over automobile, as shown in Fig. 9.25.

Find: (a) Estimate pressure reduction in car when a window is "cracked" while traveling at $V_\infty = 100$ km/hr.
(b) Air speed in freestream near window opening.

Solution: Apply the Bernoulli equation and pressure coefficient definition.

Basic equations: $\dfrac{p_\infty}{\rho} + \dfrac{V_\infty^2}{2} + g\cancel{z_\infty} = \dfrac{p}{\rho} + \dfrac{V^2}{2} + g\cancel{z}$ $\quad C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$



Assumptions: (1) Steady flow seen from auto
(2) Incompressible flow
(3) No friction
(4) Flow along a streamline
(5) Neglect changes in elevation

From Fig. 9.25, C_p near driver's window ranges between -1.23 and -0.40.

At $V_\infty = 100$ km/hr,

$$q = \frac{1}{2}\rho V_\infty^2 = \frac{1}{2} \times 1.23\,\frac{\text{kg}}{\text{m}^3} \times \left[100\,\frac{\text{km}}{\text{hr}} \times 1000\,\frac{\text{m}}{\text{km}} \times \frac{\text{hr}}{3600\,\text{s}}\right]^2 \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 475\ \text{N/m}^2$$

Thus the pressures outside may be between

$$p - p_\infty = C_p \tfrac{1}{2}\rho V_\infty^2 = -1.23 \times 475\,\frac{\text{N}}{\text{m}^2} = -584\ \text{N/m}^2\ \text{(gage)}$$

and $\quad p - p_\infty = -0.40 \times 475\,\dfrac{\text{N}}{\text{m}^2} = -190\ \text{N/m}^2\ \text{(gage)}$ $\quad p$

From the Bernoulli equation,

$$\frac{V^2}{2} = \frac{V_\infty^2}{2} + \frac{p_\infty - p}{\rho} = \frac{V_\infty^2}{2}\left(1 - \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}\right) = \frac{V_\infty^2}{2}(1 - C_p)$$

Thus

$$V = V_\infty\sqrt{1 - C_p}$$

The local flow speeds range from

$$V = V_\infty\sqrt{1-(-1.23)} = 100\,\frac{\text{km}}{\text{hr}}\sqrt{2.23} = 149\ \text{km/hr}\ \ (41.5\ \text{m/s})$$

$$V = V_\infty\sqrt{1-(-0.40)} = 100\,\frac{\text{km}}{\text{hr}}\sqrt{1.40} = 118\ \text{km/hr}\ \ (32.9\ \text{m/s})$$ $\quad V$

Thus local flow speeds are significantly higher than V_∞.

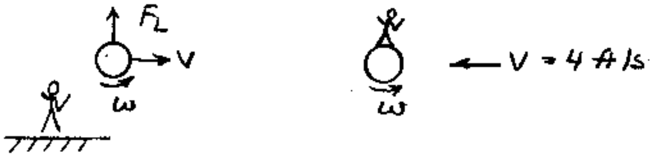
Problem 9.175

[2]

Given: Classroom demonstration of lift on spinning cylinder.

$L = 10$ in., $D = 2$ in.

$\omega = 300$ rpm



Find: Estimate lift force acting on cylinder.

Solution: Apply definition of lift coefficient, data from Fig. 9.29.

Computing equation: $F_L = C_D A \frac{1}{2} \rho V^2$

From Fig. 9.29, $C_L = C_L(\omega D/2V)$

$$\omega = 300 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\text{ s}} = 31.4 \text{ rad/s}$$

$$\frac{\omega D}{2V} = \frac{1}{2} \times 31.4 \frac{\text{rad}}{\text{s}} \times 2 \text{ in.} \times \frac{\text{s}}{4 \text{ ft}} \times \frac{\text{ft}}{12 \text{ in.}} = 0.654$$

There is a data band in Fig. 9.29. The highest value is $C_L \approx 1.1$

$$A = DL = 2 \text{ in.} \times 10 \text{ in.} \times \frac{\text{ft}^2}{144 \text{ in.}^2} = 0.139 \text{ ft}^2$$

For standard atmosphere conditions

$$F_L = 1.1 \times 0.139 \text{ ft}^2 \times \frac{1}{2} \times 0.00238 \frac{\text{slug}}{\text{ft}^3} \times (4)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = 0.00291 \text{ lbf}$$

F_L

{This is quite a small force, but the speed is low.}

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Problem 9.176

Given: Golf ball with mass, $m = 48\,g$, and diameter, $D = 43\,mm$, is hit from a sand trap with speed, $V = 20\,m/sec$, and backspin, $\omega = 2000\,rpm$.

Find: (a) Lift and drag forces acting on ball.
(b) Express as fractions of mg.

Solution: Use data from Fig. 9.28 for lift and drag coefficients.

Computing equations: $F_L = C_L A \frac{1}{2}\rho V^2 \qquad F_D = C_D A \frac{1}{2}\rho V^2$

At $V = 20\,m/sec$, then $q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.23 \frac{kg}{m^3} \times (20)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 246\,N/m^2$

From Fig. 9.28, $C_L = C_L\left(Re, \frac{\omega D}{2V}\right)$

$$Re = \frac{VD}{\nu} = 20\frac{m}{s} \times 0.043\,m \times \frac{s}{1.45 \times 10^{-5}\,m^2} = 5.93 \times 10^4 \text{ (assume close to } 1.26 \times 10^5\text{)}$$

$$\frac{\omega D}{2V} = \frac{1}{2} \times 2000\,\frac{rev}{min} \times 0.043\,m \times \frac{s}{20\,m} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} = 0.225$$

Then $C_L \simeq 0.23$ and $F_L = C_L A \frac{1}{2}\rho V^2 = C_L A q$

$$F_L = 0.23 \times \frac{\pi (0.043)^2}{4} m^2 \times 246 \frac{N}{m^2} = 0.0822\,N$$

F_L

From Fig. 9.28 $C_D = 0.31$, so

$$F_D = \frac{C_D}{C_L} F_L = \frac{0.31}{0.23} \times 0.0822\,N = 0.111\,N$$

F_D

For the ball, $mg = 0.048\,kg \times 9.81 \frac{m}{sec^2} \times \frac{N \cdot sec^2}{kg \cdot m} = 0.471\,N$

Thus $$\frac{F_L}{mg} = \frac{0.0822\,N}{0.471\,N} = 0.175$$

$$\frac{F_D}{mg} = \frac{0.111\,N}{0.471\,N} = 0.236$$

Ratios

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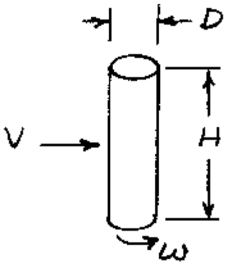
Problem 9.177



Given: Rotating cylinders for ship propulsion.

$D = 3\ m,\ H = 15\ m,\ N \leq 750\ rpm$

Find: (a) Calculate maximum lift and drag forces on each cylinder at $V = 50\ km/hr$.
(b) Compare to force at optimum L/D.
(c) Estimate power needed to spin rotor at $N = 750\ rpm$.

Solution: Apply definitions of coefficients, data from Fig. 9.29.

Computing equations: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A} \qquad C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A} \qquad A = DH$

Coefficients are functions of spin ratio, $\omega D/2V$.

$$\omega = 750\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} = 78.5\ rad/s\ ; \quad V = 50\,\frac{km}{hr} \times 1000\,\frac{m}{km} \times \frac{hr}{3600\,s} = 13.9\,\frac{m}{s}$$

$$\frac{\omega D}{2V} = \frac{1}{2} \times 78.5\,\frac{rad}{s} \times 3\ m \times \frac{s}{13.9\ m} = 8.47\ ; \quad q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (13.9)^2\,\frac{m^2}{s^2} = 119\ N/m^2$$

This is above the range of data shown in Fig. 9.29, so ω could be less.
Choose $C_L \simeq 9.5,\ C_D \simeq 3.5.\ A = DH = 45\ m^2$

$$F_L = 9.5 \times 45\ m^2 \times 119\,\frac{N}{m^2} = 50.9\ kN$$ F_L

$$F_D = 3.5 \times 45\ m^2 \times 119\,\frac{N}{m^2} = 18.7\ kN$$ F_D

From inspection of Fig. 9.29, this appears close to the optimum L/D.
Then

$$F = \left[F_L^2 + F_D^2\right]^{\frac{1}{2}} = \left[(50.9)^2 + (18.7)^2\right]^{\frac{1}{2}} kN = 54.2\ kN$$ F

Torque will be product of mean shear stress, $\bar{\tau}$, area, and radius.

$$T = \bar{\tau} A R = \bar{\tau}(\pi D H) R = \frac{\pi \bar{\tau} D^2 H}{2}$$

Estimate (very roughly) $\bar{\tau}$ from flat plate correlation. Assume

$$Re = \frac{(V+U)D}{\nu} = \frac{(V+\omega R)D}{\nu} = (13.9 + 118)\,\frac{m}{s} \times 3\ m \times \frac{s}{1.45 \times 10^{-5}\ m^2} = 2.73 \times 10^7$$

From Fig. 9.8, $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A} = 0.003\ ; \quad \bar{\tau} = \dfrac{F_D}{A} = C_D\, q = 0.003 \times 119\,\dfrac{N}{m^2} = 0.357\,\dfrac{N}{m^2}$

Then

$$T = \frac{\pi}{2} \times 0.357\,\frac{N}{m^2} \times (3)^2\,m^2 \times 15\ m = 75.7\ N \cdot m$$

and

$$\mathscr{P} = \omega T = 78.5\,\frac{rad}{s} \times 75.7\ N \cdot m \times \frac{W \cdot s}{N \cdot m} = 5.94\ kW$$ $\mathscr{P}$

{ The power estimate is gross only. More specific information is needed to design with confidence. }

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Problem 9.178

[3]

Given: American and British golf balls (Problems 1.11 and 1.12), dimensions below. Hit from tee at $V = 85$ m/sec, with backspin, $N = 9000$ rpm.

Find: (a) Evaluate lift and drag forces on each ball (express as fractions of body force).

(b) Estimate radius of curvature of trajectory.

(c) Which ball would have the longer range?

Solution: Apply definitions of lift and drag coefficients, data from Fig. 9.28.

Computing equations: $C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{F_L}{qA}$; $C_D = \frac{F_D}{qA}$; $q = \frac{1}{2}\rho V^2$; $A = \frac{\pi D^2}{4}$

The parameters are Re_D and $\omega D/2V$; tabulate results:

Ball	m (oz)	D (in.)	$\frac{\omega D}{2V}$ (---)	Re (---)	C_L (---)	C_D (---)	F_L (N)	F_D (N)
American	1.62	1.68	0.236	2.50×10^5	0.27	0.32	1.71	2.03
British	1.62	1.62	0.228	2.41×10^5	0.26	0.31	1.53	1.83

Taking ratios for the American ball: British ball:

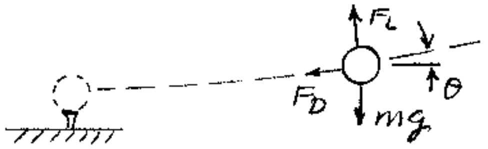
$$F_L/mg = 1.71\text{ N} \times \frac{1}{1.62\text{ oz}} \times \frac{16\text{ oz}}{\text{lbf}} \times \frac{\text{lbf}}{4.448\text{ N}} = 3.80 \qquad 3.40$$

F_L/mg

$$F_D/mg \qquad = 4.51 \qquad 4.07$$

F_D/mg

Draw FBD to compute trajectory:



$$\Sigma F_{\perp \text{ to path}} = F_L - mg\cos\theta = m\frac{V^2}{R}$$

Assume θ small, so $\cos\theta \approx 1$. Then

$$R \approx \frac{mV^2}{F_L - mg} = \frac{V^2/g}{F_L/mg - 1} = \frac{1}{3.80-1} \times (85)^2\frac{m^2}{s^2} \times \frac{s^2}{9.81\,m} = 263 \text{ m (American)}$$

$$= \frac{1}{3.40-1} \times (85)^2\frac{m^2}{s^2} \times \frac{s^2}{9.81\,m} = 307 \text{ m (British)}$$

R

(Note because $F_L/mg > 1$, the balls actually rise!)

Drag probably is more important than lift in affecting range of a drive. Therefore one probably would expect the British ball to carry farther.

Range

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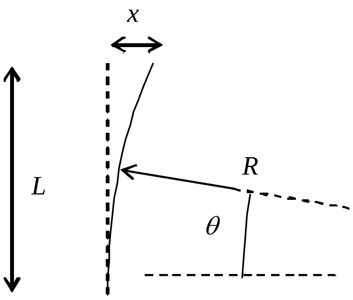
Problem 9.179

[4]

9.179 A baseball pitcher throws a ball at 80 mph. Home plate is 60 ft away from the pitcher's mound. What spin should be placed on the ball for maximum horizontal deviation from a straight path? (A baseball has a mass of 5 oz and a circumference of 9 in.) How far will the ball deviate from a straight line?



Given:

Baseball pitch

Find:

Spin on the ball

Solution:

Basic equations: $C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\qquad \Sigma\cdot\vec{F} = M\cdot\vec{a}$

The given or available data is $\rho = 0.00234\cdot\dfrac{\text{slug}}{\text{ft}^3}$ $\qquad \nu = 1.62\times 10^{-4}\cdot\dfrac{\text{ft}^2}{\text{s}}$ $\qquad L = 60\cdot\text{ft}$

$M = 5\cdot\text{oz}$ $\qquad C = 9\cdot\text{in}$ $\qquad D = \dfrac{C}{\pi}$ $\qquad D = 2.86\,\text{in}$ $\qquad A = \dfrac{\pi\cdot D^2}{4}$ $\qquad A = 6.45\,\text{in}^2$ $\qquad V = 80\cdot\text{mph}$

Compute the Reynolds number $Re = \dfrac{V\cdot D}{\nu}$ $\qquad Re = 1.73\times 10^5$

This Reynolds number is slightly beyond the range of Fig. 9.27; we use Fig. 9.27 as a rough estimate

The ball follows a trajectory defined by Newton's second law. In the horizontal plane (*x* coordinate)

$$F_L = M\cdot a_R = M\cdot a_x = M\cdot\frac{V^2}{R} \qquad \text{and} \qquad F_L = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L$$

where R is the instantaneous radius of curvature of the trajectory

From Eq 1 we see the ball trajectory has the smallest radius (i.e. it curves the most) when CL is as large as possible.
From Fig. 9.27 we see this is when $C_L = 0.4$

Solving for R $\qquad R = \dfrac{2\cdot M}{C_L\cdot A\cdot\rho}$ $\qquad (1)$ $\qquad R = 463.6\,\text{ft}$

Also, from Fig. 9.27 $\qquad \dfrac{\omega\cdot D}{2\cdot V} = 1.5$ $\qquad$ to $\qquad \dfrac{\omega\cdot D}{2\cdot V} = 1.8$ $\qquad$ defines the best range

Hence $\qquad \omega = 1.5\cdot\dfrac{2\cdot V}{D}$ $\qquad \omega = 14080\,\text{rpm}$ $\qquad \omega = 1.8\cdot\dfrac{2\cdot V}{D}$ $\qquad \omega = 16896\,\text{rpm}$

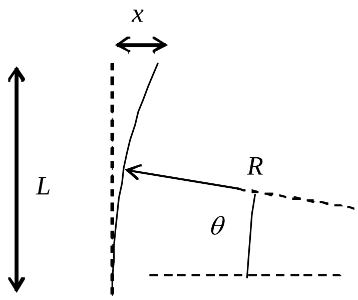
From the trajectory geometry $\qquad x + R\cdot\cos(\theta) = R$ $\qquad$ where $\qquad \sin(\theta) = \dfrac{L}{R}$

Hence $\qquad x + R\cdot\sqrt{1-\left(\dfrac{L}{R}\right)^2} = R$

Solving for x $\qquad x = R - R\cdot\sqrt{1-\left(\dfrac{L}{R}\right)^2}$ $\qquad x = 3.90\,\text{ft}$

Problem 9.180

9.180 A soccer player takes a free kick. Over a distance of 10 m, the ball veers to the right by about 1 m. Estimate the spin the player's kick put on the ball if its speed is 30 m/s. The ball has a mass of 420 gm and has a circumference of 70 cm.



Given: Soccer free kick

Find: Spin on the ball

Solution:

Basic equations: $C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad \Sigma\cdot\vec{F} = M\cdot\vec{a}$

The given or available data is $\rho = 1.21\cdot\dfrac{kg}{m^3}$ $\quad \nu = 1.50\cdot 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad L = 10\cdot m$ $\quad x = 1\cdot m$

$M = 420\cdot gm$ $\quad C = 70\cdot cm$ $\quad D = \dfrac{C}{\pi}$ $\quad D = 22.3\,cm$ $\quad A = \dfrac{\pi\cdot D^2}{4}$ $\quad A = 0.0390\,m^2$ $\quad V = 30\cdot\dfrac{m}{s}$

Compute the Reynolds number $Re = \dfrac{V\cdot D}{\nu}$ $\quad Re = 4.46\times 10^5$

This Reynolds number is beyond the range of Fig. 9.27; however, we use Fig. 9.27 as a rough estimate

The ball follows a trajectory defined by Newton's second law. In the horizontal plane (*x* coordinate)

$$F_L = M\cdot a_R = M\cdot a_x = M\cdot\frac{V^2}{R} \quad \text{and} \quad F_L = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L$$

where R is the instantaneous radius of curvature of the trajectory

Hence, solving for R $\quad R = \dfrac{2\cdot M}{C_L\cdot A\cdot\rho}$ $\quad$ (1)

From the trajectory geometry $\quad x + R\cdot\cos(\theta) = R$ $\quad$ where $\quad \sin(\theta) = \dfrac{L}{R}$

Hence $\quad x + R\cdot\sqrt{1 - \left(\dfrac{L}{R}\right)^2} = R$

Solving for *R* $\quad R = \dfrac{\left(L^2 + x^2\right)}{2\cdot x}$ $\quad R = 50.5\,m$

Hence, from Eq 1 $\quad C_L = \dfrac{2\cdot M}{R\cdot A\cdot\rho}$ $\quad C_L = 0.353$

For this lift coefficient, from Fig. 9.27 $\quad \dfrac{\omega\cdot D}{2\cdot V} = 1.2$

Hence $\quad \omega = 1.2\cdot\dfrac{2\cdot V}{D}$ $\quad \omega = 3086\,rpm$

(And of course, Beckham still kind of rules!)

Problem 10.1

[2]

Given: Impeller dimensions of Example Problem 10.1:

		Inlet①	Outlet②
$Q = 150$ gpm	Radius, r (in.)		2.0
$N = 3450$ rpm	Blade width, b (in.)		0.383
	Blade angle, β (deg)		60°, 70°, 80°, 85°

Construct: velocity diagram for exit flow leaving tangent to the blade.

Find: (a) ideal head rise, (b) mechanical power input

Solution:

Computing equations:

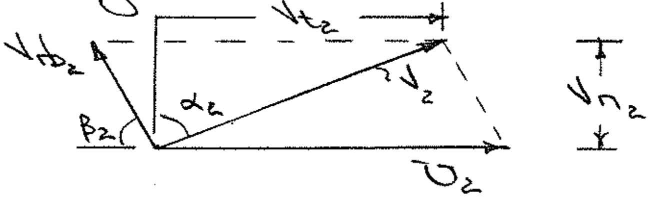
$$\dot{W}_m = (U_2 V_{t_2} - \cancel{U_1 V_{t_1}}^{\,0(1)})\dot{m} \qquad (10.2b)$$

$$H = \frac{1}{g}(U_2 V_{t_2} - \cancel{U_1 V_{t_1}}^{\,0(1)}) \qquad (10.2c)$$

Assumptions: (1) axial inlet flow (given) so $V_{t_1} = 0$

(2) at blade outlet, flow is uniform and leaves tangent to blade

Exit velocity diagram:



From continuity, $V_{n_2} = \dfrac{Q}{2\pi r_2 b_2} = V_{tb_2}\sin\beta_2 \qquad \therefore V_{tb_2} = \dfrac{V_{n_2}}{\sin\beta_2}$

From geometry, $V_{t_2} = U_2 - V_{tb_2}\cos\beta_2 = U_2 - \dfrac{V_{n_2}}{\sin\beta_2}\cos\beta_2 = U_2 - V_{n_2}\cot\beta_2$

Substituting numerical values, for $\beta = 60°$

$$U_2 = \omega r_2 = 3450\,\frac{\text{rev}}{\text{min}} \times 2\pi\,\frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{s}} \times 2.0\,\text{in} \times \frac{\text{ft}}{12\,\text{in}} = 60.2\ \text{ft/s}$$

$$V_{n_2} = \frac{Q}{2\pi r_2 b_2} = \frac{1}{2\pi} \times 150\,\frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60\,\text{s}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{1}{2.0\,\text{in}} \times \frac{1}{0.383\,\text{in}} \times \frac{144\,\text{in}^2}{\text{ft}^2} = 10.0\ \text{ft/s}$$

$$V_{t_2} = U_2 - V_{n_2}\cot\beta_2 = 60.2\,\frac{\text{ft}}{\text{s}} - 10.0\,\frac{\text{ft}}{\text{s}}\cot 60° = 54.4\ \text{ft/s}$$

$$\dot{m} = \rho Q = 1.94\,\frac{\text{slug}}{\text{ft}^3} \times 150\,\frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60\,\text{s}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} = 0.648\ \text{slug/s}$$

$$H = \frac{1}{g}U_2 V_{t_2} = \frac{\text{s}^2}{32.2\,\text{ft}} \times 60.2\,\frac{\text{ft}}{\text{s}} \times 54.4\,\frac{\text{ft}}{\text{s}} = 102\ \text{ft} \longleftarrow H \quad \beta = 60°$$

$$\dot{W}_m = \dot{m} U_2 V_{t_2} = \dot{m} g H = 0.648\,\frac{\text{slug}}{\text{s}} \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times 102\,\text{ft} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}}$$

$$\dot{W}_m = 3.87\ \text{hp} \longleftarrow \dot{W}_m \quad \beta = 60°$$

For $\beta = 70°$, $V_{t_2} = 56.6$ ft/s, $H = 106$ ft, $\dot{W}_m = 4.02$ hp ←

$\beta = 80°$, $V_{t_2} = 58.4$ ft/s, $H = 109$ ft, $\dot{W}_m = 4.15$ hp ←

$\beta = 85°$, $V_{t_2} = 59.3$ ft/s, $H = 111$ ft, $\dot{W}_m = 4.21$ hp ←

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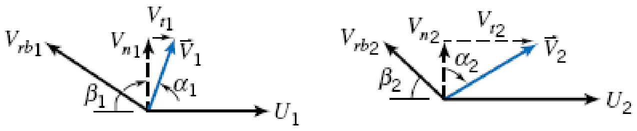
Problem 10.2

[2]

10.2 The geometry of a centrifugal water pump is $r_1 = 4$ in., $r_2 = 7.5$ in., $b_1 = b_2 = 1.5$ in., $\beta_1 = 30°$, $\beta_2 = 20°$, and it runs at speed 1500 rpm. Estimate the discharge required for axial entry, the horsepower generated in the water, and the head produced.



Given: Geometry of centrifugal pump

Find: Estimate discharge for axial entry; Head

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$$\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3} \qquad r_1 = 4\cdot\text{in} \qquad r_2 = 7.5\cdot\text{in} \qquad b_1 = 1.5\cdot\text{in} \qquad b_2 = 1.5\cdot\text{in}$$

$$\omega = 1500\cdot\text{rpm} \qquad \beta_1 = 30\cdot\text{deg} \qquad \beta_2 = 20\cdot\text{deg}$$

From continuity $\quad V_n = \dfrac{Q}{2\cdot\pi\cdot r\cdot b} = V_{rb}\cdot\sin(\beta) \qquad V_{rb} = \dfrac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb}\cdot\cos(\beta) = U - \dfrac{V_n}{\sin(\beta)}\cdot\cos(\beta) = U - \dfrac{Q}{2\cdot\pi\cdot r\cdot b}\cdot\cot(\beta)$

For an axial entry $\quad V_{t1} = 0 \qquad$ so $\qquad U_1 - \dfrac{Q}{2\cdot\pi\cdot r_1\cdot b_1}\cdot\cot(\beta_1) = 0$

Using given data $\quad U_1 = \omega\cdot r_1 \qquad U_1 = 52.4\cdot\dfrac{\text{ft}}{\text{s}}$

Hence $\quad Q = 2\cdot\pi\cdot r_1\cdot b_1\cdot U_1\cdot\tan(\beta_1) \qquad Q = 7.91\cdot\dfrac{\text{ft}^3}{\text{s}} \qquad Q = 3552\cdot\text{gpm}$

To find the power we need U_2, V_{t2}, and m_{rate}

The mass flow rate is $\quad m_{rate} = \rho\cdot Q \qquad m_{rate} = 15.4\cdot\dfrac{\text{slug}}{\text{s}}$

$$U_2 = \omega\cdot r_2 \qquad U_2 = 98.2\cdot\frac{\text{ft}}{\text{s}}$$

$$V_{t2} = U_2 - \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot(\beta_2) \qquad V_{t2} = 53.9\cdot\frac{\text{ft}}{\text{s}}$$

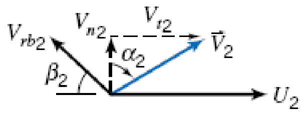
Hence $\quad W_m = (U_2\cdot V_{t2} - U_1\cdot V_{t1})\cdot m_{rate} \qquad W_m = 81212\cdot\dfrac{\text{ft}\cdot\text{lbf}}{\text{s}} \qquad W_m = 148\cdot\text{hp}$

The head is $\quad H = \dfrac{W_m}{m_{rate}\cdot g} \qquad H = 164\cdot\text{ft}$

Problem 10.3

[2]

10.3 A centrifugal pump running at 3500 rpm pumps water at a rate of 150 gpm. The water enters axially, and leaves the impeller at 17.5 ft/s relative to the blades, which are radial at the exit. If the pump requires 6.75 hp, and is 67 percent efficient, estimate the basic dimensions (impeller exit diameter and width), using the Euler turbomachine equation.



Given: Data on centrifugal pump

Find: Estimate basic dimensions

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b, directly derived from the Euler turbomachine equation)

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $Q = 150 \cdot \text{gpm}$ $Q = 0.334 \frac{\text{ft}^3}{\text{s}}$ $W_{in} = 6.75 \cdot \text{hp}$ $\eta = 67 \cdot \%$

$\omega = 3500 \cdot \text{rpm}$ $V_{rb2} = 17.5 \cdot \frac{\text{ft}}{\text{s}}$ $\beta_2 = 90 \cdot \text{deg}$

For an axial inlet $V_{t1} = 0$

From the outlet geometry $V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = U_2$ and $U_2 = r_2 \cdot \omega$

Hence, in Eq. 10.2b $W_m = U_2^{\ 2} \cdot m_{rate} = r_2^{\ 2} \cdot \omega^2 \cdot m_{rate}$

with $W_m = \eta \cdot W_{in}$ $W_m = 4.52\,\text{hp}$

and $m_{rate} = \rho \cdot Q$ $m_{rate} = 0.648 \frac{\text{slug}}{\text{s}}$

Hence $r_2 = \sqrt{\frac{W_m}{m_{rate} \cdot \omega^2}}$ $r_2 = 0.169\,\text{ft}$ $r_2 = 2.03\,\text{in}$

Also $V_{n2} = V_{rb2} \cdot \sin(\beta_2)$ $V_{n2} = 17.5 \frac{\text{ft}}{\text{s}}$

From continuity $V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$

Hence $b_2 = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot V_{n2}}$ $b_2 = 0.0180\,\text{ft}$ $b_2 = 0.216\,\text{in}$

Problem 10.4

[2]

Given: Dimensions of a centrifugal pump impeller:
$N = 750$ rpm

	Inlet ①	Outlet ②
Radius, r (mm)	175	500
Blade width, b (mm)	50	30
Blade angle, β (deg)	65	70

Find: (a) theoretical head, (b) mechanical power input for water flow rate of $Q = 0.75\ m^3/s$

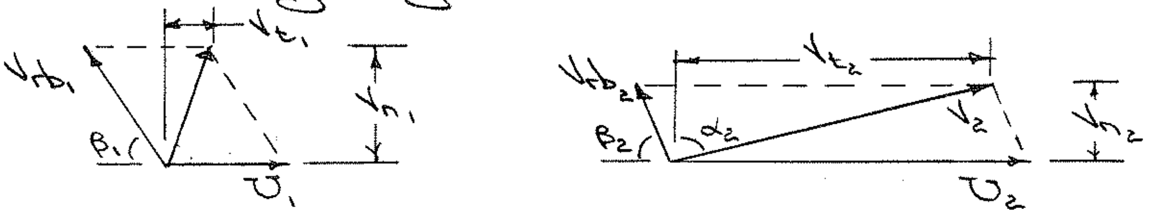
Solution:

Computing equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (10.2b)

$H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$ (10.2c)

Assume: (1) uniform flow at blade inlet and outlet
(2) flow enters and leaves tangent to the blade

Draw velocity diagrams:



From continuity, $V_n = \frac{Q}{2\pi r b} = V_{rb} \sin\beta \quad \therefore V_{rb} = \frac{V_n}{\sin\beta}$

From geometry, $V_t = U - V_{rb}\cos\beta = U - \frac{V_n}{\sin\beta}\cos\beta = U - \frac{Q}{2\pi r b}\cot\beta$

Substituting numerical values,

$\omega = 750 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{min}{60\ s} = 78.5\ rad/s$

$U_1 = \omega r_1 = 78.5 \frac{rad}{s} \times 0.175\ m = 13.7\ m/s \ ; \ U_2 = 39.3\ m/s$

$V_{t_1} = U_1 - \frac{Q}{2\pi r_1 b_1}\cot\beta_1 = 13.7 \frac{m}{s} - \frac{0.75}{2\pi} \frac{m^3}{s} \times \frac{1}{0.175\ m} \times \frac{\cot 65^\circ}{0.05\ m} = 7.34\ m/s$

$V_{t_2} = 39.3 \frac{m}{s} - \frac{0.75}{2\pi} \frac{m^3}{s} \times \frac{1}{0.50\ m} \times \frac{\cot 70^\circ}{0.03\ m} = 36.4\ m/s$

$H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) = \frac{s^2}{9.81\ m}\left(39.3 \frac{m}{s} \times 36.4 \frac{m}{s} - 13.7 \frac{m}{s} \times 7.34 \frac{m}{s}\right) = 135\ m \quad \underline{H}$

$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} = gH\rho Q = 9.81 \frac{m}{s^2} \times 135\ m \times 999 \frac{kg}{m^3} \times 0.75 \frac{m^3}{s} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{W \cdot s}{N \cdot m}$

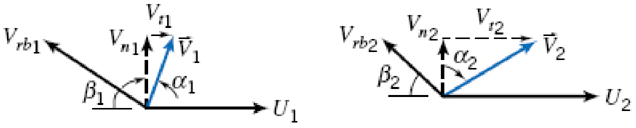
$\dot{W}_m = 994\ kW \longleftarrow \dot{W}_m$

Problem 10.5

[2]

10.5 Dimensions of a centrifugal pump impeller are

Parameter	Inlet, Section ①	Outlet, Section ②
Radius, r (in.)	3	9.75
Blade width, b (in.)	1.5	1.125
Blade angle, β (deg)	60	70



The pump is driven at 1250 rpm while pumping water. Calculate the theoretical head and mechanical power input if the flow rate is 1500 gpm.

Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations:

$$\dot{W}_m = (U_2V_{t_2} - U_1V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2V_{t_2} - U_1V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$$\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3} \qquad r_1 = 3\cdot\text{in} \qquad r_2 = 9.75\cdot\text{in} \qquad b_1 = 1.5\cdot\text{in} \qquad b_2 = 1.125\cdot\text{in}$$

$$\omega = 1250\cdot\text{rpm} \qquad \beta_1 = 60\cdot\text{deg} \qquad \beta_2 = 70\cdot\text{deg} \qquad Q = 1500\cdot\text{gpm} \qquad Q = 3.34\frac{\text{ft}^3}{\text{s}}$$

From continuity $\qquad V_n = \frac{Q}{2\cdot\pi\cdot r\cdot b} = V_{rb}\cdot\sin(\beta) \qquad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\qquad V_t = U - V_{rb}\cdot\cos(\beta) = U - \frac{V_n}{\sin(\beta)}\cdot\cos(\beta) = U - \frac{Q}{2\cdot\pi\cdot r\cdot b}\cdot\cot(\beta)$

Using given data $\qquad U_1 = \omega\cdot r_1 \qquad U_1 = 32.7\frac{\text{ft}}{\text{s}} \qquad U_2 = \omega\cdot r_2 \qquad U_2 = 106.4\frac{\text{ft}}{\text{s}}$

$$V_{t1} = U_1 - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}\cdot\cot\left(\beta_1\right) \qquad V_{t1} = 22.9\frac{\text{ft}}{\text{s}}$$

$$V_{t2} = U_2 - \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot\left(\beta_2\right) \qquad V_{t2} = 104\frac{\text{ft}}{\text{s}}$$

The mass flow rate is $\qquad m_{rate} = \rho\cdot Q \qquad m_{rate} = 6.48\frac{\text{slug}}{\text{s}}$

Hence $\qquad W_m = \left(U_2\cdot V_{t2} - U_1\cdot V_{t1}\right)\cdot m_{rate} \qquad W_m = 66728\frac{\text{ft}\cdot\text{lbf}}{\text{s}} \qquad W_m = 121\,\text{hp}$

The head is $\qquad H = \frac{W_m}{m_{rate}\cdot g} \qquad H = 320\,\text{ft}$

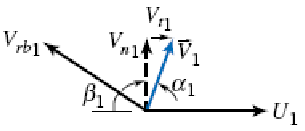
Problem 10.6

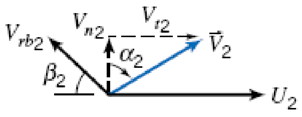
[2]

10.6 Dimensions of a centrifugal pump impeller are

Parameter	Inlet, Section ①	Outlet, Section ②
Radius, *r* (in.)	15	45
Blade width, *b* (in.)	4.75	3.25
Blade angle, *β* (deg)	40	60

The pump is driven at 575 rpm and the fluid is water. Calculate the theoretical head and mechanical power if the flow rate is 80,000 gpm.





Given:

Geometry of centrifugal pump

Find:

Theoretical head; Power input for given flow rate

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 15 \cdot \text{in}$ $\quad r_2 = 45 \cdot \text{in}$ $\quad b_1 = 4.75 \cdot \text{in}$ $\quad b_2 = 3.25 \cdot \text{in}$

$\omega = 575 \cdot \text{rpm}$ $\quad \beta_1 = 40 \cdot \text{deg}$ $\quad \beta_2 = 60 \cdot \text{deg}$ $\quad Q = 80000 \cdot \text{gpm}$ $\quad Q = 178 \frac{\text{ft}^3}{\text{s}}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

Using given data $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 75.3 \frac{\text{ft}}{\text{s}}$ $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 226 \frac{\text{ft}}{\text{s}}$

$V_{t1} = U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1)$ $\quad V_{t1} = 6.94 \frac{\text{ft}}{\text{s}}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 210 \frac{\text{ft}}{\text{s}}$

The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 346 \frac{\text{slug}}{\text{s}}$

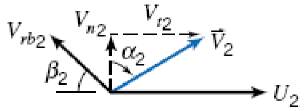
Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate}$ $\quad W_m = 1.62 \times 10^7 \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $\quad W_m = 2.94 \times 10^4 \text{hp}$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 1455 \, \text{ft}$

Problem 10.7

[2]

10.7 A centrifugal water pump, with 15 cm diameter impeller and axial inlet flow, is driven at 1750 rpm. The impeller vanes are backward-curved ($\beta_2 = 65°$) and have axial width $b_2 = 2$ cm. For a volume flow rate of 225 m^3/hr determine the theoretical head rise and power input to the pump.



Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b)

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$ (Eq. 10.2c)

The given or available data is

$\rho = 1000 \cdot \frac{\text{kg}}{\text{m}^3}$ $\quad r_2 = 7.5 \cdot \text{cm}$ $\quad b_2 = 2 \cdot \text{cm}$ $\quad \beta_2 = 65 \cdot \text{deg}$

$\omega = 1750 \cdot \text{rpm}$ $\quad Q = 225 \cdot \frac{\text{m}^3}{\text{hr}}$ $\quad Q = 0.0625 \frac{\text{m}^3}{\text{s}}$

From continuity $\quad V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$ $\quad V_{n2} = 6.63 \frac{\text{m}}{\text{s}}$

From geometry $\quad V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = U_2 - \frac{V_{n2}}{\sin(\beta_2)} \cdot \cos(\beta_2)$

Using given data $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 13.7 \frac{\text{m}}{\text{s}}$

Hence $\quad V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 10.7 \frac{\text{m}}{\text{s}}$ $\quad V_{t1} = 0$ (axial inlet)

The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 62.5 \frac{\text{kg}}{\text{s}}$

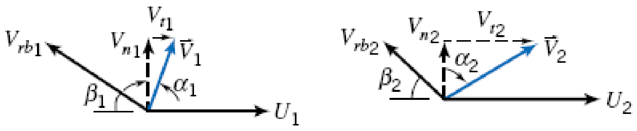
Hence $\quad W_m = U_2 \cdot V_{t2} \cdot m_{rate}$ $\quad W_m = 9.15 \, \text{kW}$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 14.9 \, \text{m}$

Problem 10.8

[2]

10.8 For the impeller of Problem 10.5, determine the rotational speed for which the tangential component of the inlet velocity is zero if the volume flow rate is 4000 gpm. Calculate the theoretical head and mechanical power input.



Given: Geometry of centrifugal pump

Find: Rotational speed for zero inlet velocity; Theoretical head; Power input

Solution:

Basic equations: $$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 3 \cdot \text{in}$ $\quad r_2 = 9.75 \cdot \text{in}$ $\quad b_1 = 1.5 \cdot \text{in}$ $\quad b_2 = 1.125 \cdot \text{in}$

$\beta_1 = 60 \cdot \text{deg}$ $\quad \beta_2 = 70 \cdot \text{deg}$ $\quad Q = 4000 \cdot \text{gpm}$ $\quad Q = 8.91 \cdot \frac{\text{ft}^3}{\text{s}}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

For $V_{t1} = 0$ we get $\quad U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$ $\quad$ or $\quad \omega \cdot r_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$

Hence, solving for ω $\quad \omega = \frac{Q}{2 \cdot \pi \cdot r_1^{\,2} \cdot b_1} \cdot \cot(\beta_1)$ $\quad \omega = 105 \frac{\text{rad}}{\text{s}}$ $\quad \omega = 1001\,\text{rpm}$

We can now find U_2 $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 85.2 \cdot \frac{\text{ft}}{\text{s}}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 78.4 \cdot \frac{\text{ft}}{\text{s}}$

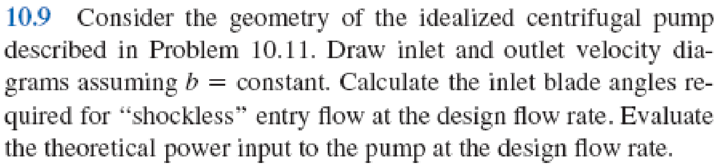
The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 17.3 \cdot \frac{\text{slug}}{\text{s}}$

Hence Eq 10.2b becomes $\quad W_m = U_2 \cdot V_{t2} \cdot m_{rate}$ $\quad W_m = 1.15 \times 10^5 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $\quad W_m = 210 \cdot \text{hp}$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 208 \cdot \text{ft}$

Problem 10.9

[2]



10.9 Consider the geometry of the idealized centrifugal pump described in Problem 10.11. Draw inlet and outlet velocity diagrams assuming b = constant. Calculate the inlet blade angles required for "shockless" entry flow at the design flow rate. Evaluate the theoretical power input to the pump at the design flow rate.

Given: Geometry of centrifugal pump

Find: Draw inlet and exit velocity diagrams; Inlet blade angle; Power

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ $\quad V_n = \frac{Q}{2\cdot\pi\cdot r\cdot b}$

The given or available data is

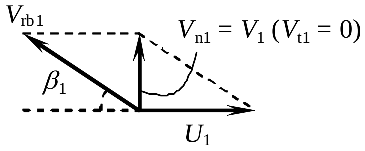
$R_1 = 1\cdot in$ $\quad R_2 = 7.5\cdot in$ $\quad b_2 = 0.375\cdot in$ $\quad \omega = 2000\cdot rpm$

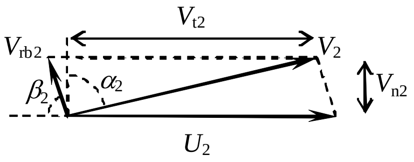
$\rho = 1.94\cdot\frac{slug}{ft^3}$ $\quad Q = 800\cdot gpm$ $\quad Q = 1.8\cdot\frac{ft^3}{s}$ $\quad \beta_2 = 75\cdot deg$

$U_1 = \omega\cdot R_1$ $\quad U_1 = 17.5\cdot\frac{ft}{s}$ $\quad U_2 = \omega\cdot R_2$ $\quad U_2 = 131\cdot\frac{ft}{s}$

$V_{n2} = \frac{Q}{2\cdot\pi\cdot R_2\cdot b_2}$ $\quad V_{n2} = 14.5\cdot\frac{ft}{s}$ $\quad V_{n1} = \frac{R_2}{R_1}\cdot V_{n2}$ $\quad V_{n1} = 109\cdot\frac{ft}{s}$

Velocity diagrams:





Then $\quad \beta_1 = \mathrm{atan}\left(\frac{V_{n1}}{U_1}\right)$ $\quad \beta_1 = 80.9\cdot deg$ $\quad$ (Essentially radial entry)

From geometry $\quad V_{t1} = U_1 - V_{n1}\cdot\cos(\beta_1)$ $\quad V_{t1} = 0.2198\cdot\frac{ft}{s}$ $\quad V_{t2} = U_2 - V_{n2}\cdot\cos(\beta_2)$ $\quad V_{t2} = 127.1\cdot\frac{ft}{s}$

Then $\quad W_m = (U_2\cdot V_{t2} - U_1\cdot V_{t1})\cdot\rho\cdot Q$ $\quad W_m = 5.75\times 10^4\cdot\frac{ft\cdot lbf}{s}$ $\quad W_m = 105\cdot hp$

Note: In earlier printings the flow rate was given as 8000 gpm not 800 gpm; water at 1089 ft/s would be quite dangerous!

Problem 10.10

Given: Dimensions of a centrifugal pump impeller:
$N = 750$ rpm

	Inlet ①	Outlet ②
Radius, r (mm)	175	500
Blade width, b (mm)	50	30
Blade angle, β (deg)	65	70

Find: (a) volume flow rate Q for which $V_{t_1} = 0$, (b) theoretical head, and (c) mechanical power input

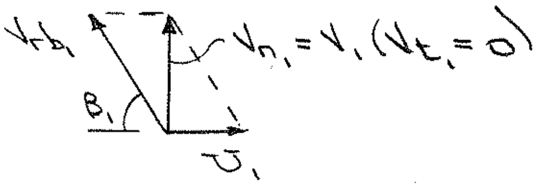
Solution:

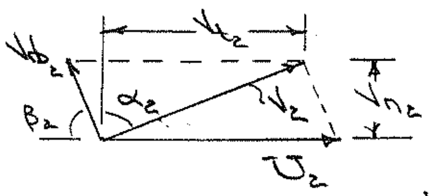
Computing equations: $\dot{W}_m = (U_2 V_{t_2} - \overset{0(3)}{\cancel{U_1 V_{t_1}}})\dot{m}$ (10.2b)

$H = \frac{1}{g}(U_2 V_{t_2} - \overset{0(3)}{\cancel{U_1 V_{t_1}}})$ (10.2c)

Assume: (1) uniform flow at blade inlet and outlet
(2) flow enters and leaves tangent to the blade
(3) $V_{t_1} = 0$ (given)

Draw velocity diagrams:





From continuity, $V_n = \frac{Q}{2\pi r b} = V_{rb}\sin\beta \quad \therefore V_{rb} = \frac{V_n}{\sin\beta}$

From geometry, $V_t = U - V_{rb}\cos\beta = U - \frac{V_n}{\sin\beta}\cos\beta = U - \frac{Q}{2\pi r b}\cot\beta$

For $V_{t_1} = 0$, then $U_1 - \frac{Q}{2\pi r_1 b_1}\cot\beta_1 = 0$ and $Q = \frac{2\pi r_1 b_1 U_1}{\cot\beta_1}$

Substituting numerical values

$U_1 = \omega_1 r_1 = 750\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} \times 0.175\,m = 13.7\,m/s;\; U_2 = 39.3\,m/s$

$Q = \frac{2\pi}{\cot 65^\circ} \times 0.175\,m \times 0.050\,m \times 13.7\,\frac{m}{s} = 1.62\,m^3/s \longleftarrow Q$

$V_{t_2} = U_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2 = 39.3\,\frac{m}{s} - 1.62\,\frac{m^3}{s} \times \frac{1}{2\pi} \times \frac{1}{0.50\,m} \times \frac{\cot 70^\circ}{0.03\,m} = 33.0\,m/s$

$H = \frac{1}{g}U_2 V_{t_2} = \frac{s^2}{9.81\,m} \times 39.3\,\frac{m}{s} \times 33.0\,\frac{m}{s} = 132\,m \longleftarrow H$

$\dot{W}_m = \dot{m} U_2 V_{t_2} = \rho Q g H = 999\,\frac{kg}{m^3} \times 1.62\,\frac{m^3}{s} \times 9.81\,\frac{m}{s^2} \times 132\,m \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{W \cdot s}{N \cdot m}$

$\dot{W}_m = 2{,}100\,kW \longleftarrow$

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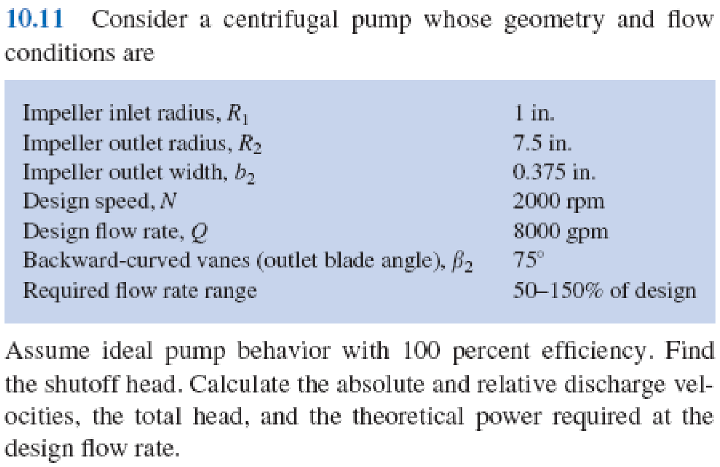
Problem 10.11

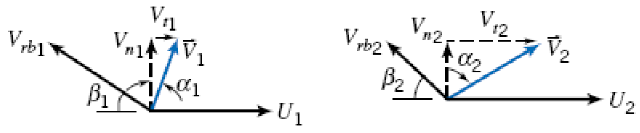
[3]



10.11 Consider a centrifugal pump whose geometry and flow conditions are

Impeller inlet radius, R_1	1 in.
Impeller outlet radius, R_2	7.5 in.
Impeller outlet width, b_2	0.375 in.
Design speed, N	2000 rpm
Design flow rate, Q	8000 gpm
Backward-curved vanes (outlet blade angle), β_2	75°
Required flow rate range	50–150% of design

Assume ideal pump behavior with 100 percent efficiency. Find the shutoff head. Calculate the absolute and relative discharge velocities, the total head, and the theoretical power required at the design flow rate.



Note: Earlier printings had 8000 gpm; it is actually 800 gpm!

Given: Geometry of centrifugal pump

Find: Shutoff head; Absolute and relative exit velocitiesTheoretical head; Power input

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}$ $\quad R_1 = 1\cdot\text{in}$ $\quad R_2 = 7.5\cdot\text{in}$ $\quad b_2 = 0.375\cdot\text{in}$

$\omega = 2000\cdot\text{rpm}$ $\quad \beta_2 = 75\cdot\text{deg}$ $\quad Q = 800\cdot\text{gpm}$ $\quad Q = 1.8\cdot\frac{\text{ft}^3}{\text{s}}$

At the exit $\quad U_2 = \omega\cdot R_2 \quad U_2 = 131\cdot\frac{\text{ft}}{\text{s}}$

At shutoff $\quad V_{t2} = U_2 \quad V_{t2} = 131\cdot\frac{\text{ft}}{\text{s}} \quad H_0 = \frac{1}{g}\cdot\left(U_2\cdot V_{t2}\right) \quad H_0 = 533\cdot\text{ft}$

At design. from continuity $\quad V_{n2} = \frac{Q}{2\cdot\pi\cdot R_2\cdot b_2} \quad V_{n2} = 15\cdot\frac{\text{ft}}{\text{s}}$

From the velocity diagram $\quad V_{n2} = V_{rb2}\cdot\sin\left(\beta_2\right) \quad V_{rb2} = \frac{V_{n2}}{\sin\left(\beta_2\right)} \quad V_{rb2} = 15.0\cdot\frac{\text{ft}}{\text{s}}$

$V_{t2} = U_2 - V_{n2}\cdot\cot\left(\beta_2\right) \quad V_{t2} = 127.0\cdot\frac{\text{ft}}{\text{s}}$

Hence we obtain $\quad V_2 = \sqrt{{V_{n2}}^2 + {V_{t2}}^2} \quad V_2 = 128\cdot\frac{\text{ft}}{\text{s}}$

with (see sketch above) $\quad \alpha_2 = \text{atan}\left(\frac{V_{t2}}{V_{n2}}\right) \quad \alpha_2 = 83.5\cdot\text{deg}$

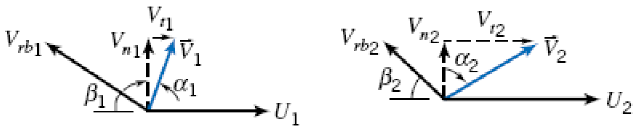
For $V_{t1} = 0$ we get $\quad W_m = U_2\cdot V_{t2}\cdot\rho\cdot Q \quad W_m = 5.75\times10^4\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}} \quad W_m = 105\cdot\text{hp}$

$H = \frac{W_m}{\rho\cdot Q\cdot g} \quad H = 517\cdot\text{ft}$

Problem 10.12

[2]

10.12 For the impeller of Problem 10.6, determine the inlet blade angle for which the tangential component of the inlet velocity is zero if the volume flow rate is 125,000 gpm. Calculate the theoretical head and mechanical power input.



Given: Geometry of centrifugal pump

Find: Inlet blade angle for no tangential inlet velocity at 125,000 gpm; Head; Power

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 15 \cdot \text{in}$ $\quad r_2 = 45 \cdot \text{in}$ $\quad b_1 = 4.75 \cdot \text{in}$ $\quad b_2 = 3.25 \cdot \text{in}$

$\omega = 575 \cdot \text{rpm}$ $\quad \beta_2 = 60 \cdot \text{deg}$ $\quad Q = 125000 \cdot \text{gpm}$ $\quad Q = 279 \frac{\text{ft}^3}{\text{s}}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

For $V_{t1} = 0$ we obtain $\quad U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$ $\quad$ or $\quad \cot(\beta_1) = \frac{2 \cdot \pi \cdot r_1 \cdot b_1 \cdot U_1}{Q}$

Using given data $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 75.3 \cdot \frac{\text{ft}}{\text{s}}$

Hence $\quad \beta_1 = \text{acot}\left(\frac{2 \cdot \pi \cdot r_1 \cdot b_1 \cdot U_1}{Q}\right)$ $\quad \beta_1 = 50\,\text{deg}$

Also $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 226 \cdot \frac{\text{ft}}{\text{s}}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 201 \cdot \frac{\text{ft}}{\text{s}}$

The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 540 \cdot \frac{\text{slug}}{\text{s}}$

Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate}$ $\quad W_m = 2.45 \times 10^7 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $\quad W_m = 44497 \cdot \text{hp}$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 1408 \cdot \text{ft}$

Problem 10.13

[3]

Given: Impeller dimensions of Example Problem 10.1: $Q = 150$ gpm

$D_1 = 1.25$ in.

$b = 0.383$ in.

$N = 3450$ rpm

Find: Construct velocity diagram for shockless flow at the impeller inlet.

Investigate effects on inlet flow angle of:
(a) variations in impeller width
(b) variations in inlet swirl velocity

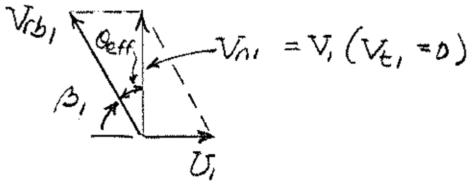
Solution: $Q = 150\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} = 0.334\ ft^3/sec$; $r_1 = 0.0521$ ft

$b = 0.0319$ ft ; $\omega = 3450\frac{rev}{min} \times 2\pi\frac{rad}{rev} \times \frac{min}{60\,s} = 361$ rad/s

From continuity, $V_{n1} = \frac{Q}{2\pi r_1 b_1} = \frac{1}{2\pi} \times 0.334\frac{ft^3}{s} \times \frac{1}{0.0521\,ft} \times \frac{1}{0.0319\,ft} = 32.0$ ft/s

$U_1 = \omega r_1 = 361\frac{rad}{s} \times 0.0521\ ft = 18.8$ ft/s

$\beta_1 = \tan^{-1}\frac{V_{n1}}{U_1} = \tan^{-1}\left(\frac{32.0}{18.8}\right) = 59.6°$



Thus for radial vanes,

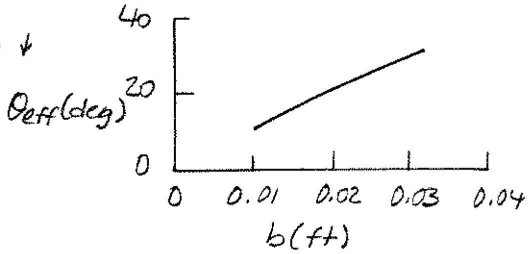
$\theta_{eff} = \frac{\pi}{2} - \beta_1 = 90° - 59.6° = 30.4°$

θ_{eff}

To change θ_{eff}: (a) Vary b with no inlet swirl: $V_1 = V_{n1} = \frac{Q}{2\pi r_1 b_1}$

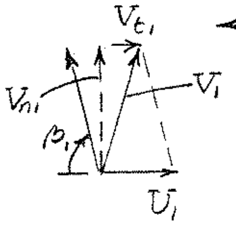
$\beta_1 = \tan^{-1}\frac{Q}{2\pi r_1 b_1 U_1}$ so $\beta_1 \uparrow$ as $b_1 \downarrow$

$\theta_{eff} = 90° - \beta_1$



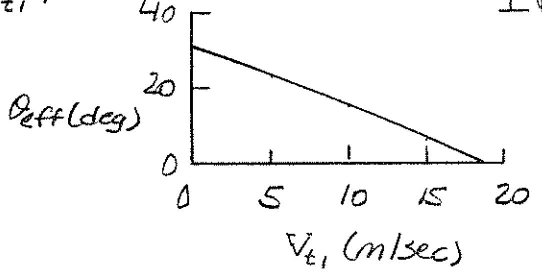
b

(b) Vary inlet swirl (V_{t1}) with $b = 0.0319$ ft:



$\beta_1 = \tan^{-1}\frac{V_{n1}}{U_1 - V_{t1}}$ so $\beta_1 \uparrow$ as $V_{t1} \uparrow$

$\theta_{eff} = 90° - \beta_1$



V_{t1}

Problem 10.14

10.14 A centrifugal pump runs at 1750 rpm while pumping water at a rate of 50 L/s. The water enters axially, and leaves tangential to the impeller blades. The impeller exit diameter and width are 300 mm and 10 mm, respectively. If the pump requires 45 kW, and is 75 percent efficient, estimate the exit angle of the impeller blades.

Given: Data on a centrifugal pump

Find: Estimate exit angle of impeller blades

Solution:

The given or available data is $\rho = 999 \cdot \frac{kg}{m^3}$ $Q = 50 \cdot \frac{L}{s}$ $W_{in} = 45 \cdot kW$ $\eta = 75 \cdot \%$

$\omega = 1750 \cdot rpm$ $b_2 = 10 \cdot mm$ $D = 300 \cdot mm$

The governing equation (derived directly from the Euler turbomachine equation) is $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (10.2b)

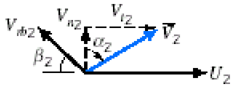
For an axial inlet $V_{t1} = 0$ hence $V_{t2} = \frac{W_m}{U_2 \cdot \rho \cdot Q}$

We have $U_2 = \frac{D}{2} \cdot \omega$ $U_2 = 27.5 \frac{m}{s}$ and $W_m = \eta \cdot W_{in}$ $W_m = 33.8 \, kW$

Hence $V_{t2} = \frac{W_m}{U_2 \cdot \rho \cdot Q}$ $V_{t2} = 24.6 \frac{m}{s}$

From continuity $V_{n2} = \frac{Q}{\pi \cdot D \cdot b_2}$ $V_{n2} = 5.31 \frac{m}{s}$

With the exit velocities determined, β can be determined from exit geometry



$\tan(\beta) = \frac{V_{n2}}{U_2 - V_{t2}}$ or $\beta = \operatorname{atan}\left(\frac{V_{n2}}{U_2 - V_{t2}}\right)$ $\beta = 61.3 \, deg$

Problem 10.15

Given: Centrifugal water-pump designed for $N = 1300$ rpm; $\eta = 0.75$, $Q = 35$ L/s.

$r_1 = 100$ mm $r_2 = 175$ mm

$b_1 = 10$ mm $b_2 = 7.5$ mm

$\beta_2 = 40°$

(a) Draw the inlet and outlet velocity diagrams
(b) Find inlet blade angle so $V_{t_1} = 0$
(c) Determine the outlet absolute flow angle (measured w.r.t. normal).
(d) Evaluate hydraulic power and head.

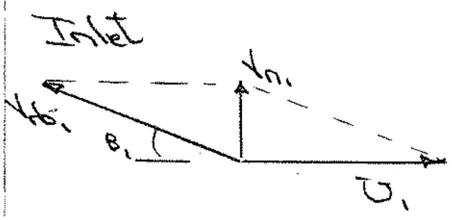
Solution: Apply continuity and the Euler turbomachine equation.

Computing equations: $V_n = \frac{Q}{2\pi r b}$ $\dot{W}_m = \rho Q(U_2 V_{t_2} - U_1 V_{t_1})$

$\omega = 1300 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{min}{60 s} = 136\ rad/s$; $U_1 = 13.6\ m/s$, $U_2 = 23.8\ m/s$

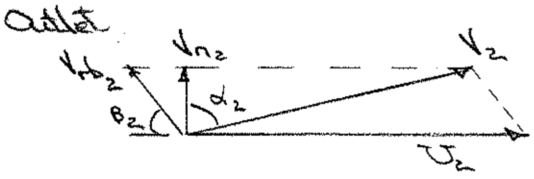
$V_{n_1} = \frac{Q}{2\pi r_1 b_1} = \frac{1}{2\pi} \times 35 \frac{L}{s} \times \frac{m^3}{10^3 L} \times \frac{1}{0.1 m} \times \frac{1}{0.01 m} = 5.57\ m/s.$

$V_{n_2} = \frac{r_1 b_1}{r_2 b_2} V_{n_1} = \frac{100}{175} \times \frac{10}{7.5} \times 5.57\ m/s = 4.24\ m/s.$



$\tan\beta_1 = \frac{V_{n_1}}{U_1}$

$\beta_1 = \tan^{-1}\left(\frac{5.57}{13.6}\right)$



$\beta_1 = 22.3°$ ←——— β_1

From the outlet diagram, $V_{t_2} = U_2 - V_{n_2}\cot\beta_2 = 23.8 \frac{m}{s} - 4.24 \frac{m}{s} \times \frac{1}{\tan 40°}$

$V_{t_2} = 18.8\ m/s$

$\alpha_2 = \tan^{-1} \frac{V_{t_2}}{V_{n_2}} = \tan^{-1}\left(\frac{18.8}{4.24}\right) = 77.3°$ ——— α_2

$\dot{W}_m = \rho Q(U_2 V_{t_2} - U_1 V_{t_1}) = 999 \frac{kg}{m^3} \times 35 \frac{L}{s} \times \frac{m^3}{10^3 L}\left[23.8 \frac{m}{s} \times 18.8 \frac{m}{s} - 0\right] \frac{N \cdot s^2}{kg \cdot m} \times 0.75$

$\dot{W}_m = 11.7$ kW ←——— $\dot{W}_m$

$H = \frac{\dot{W}_m}{\rho g Q} = \frac{U_2 V_{t_2} - U_1 V_{t_1}}{g} = 23.8 \frac{m}{s} \times 18.8 \frac{m}{s} \times \frac{s}{9.81 m} \times 0.75 = 34.2 m$ ←— H

Problem 10.16

10.16 Repeat the analysis for determining the optimum speed for an impulse turbine of Example 10.5, using the Euler turbomachine equation.

Given: Impulse turbibe

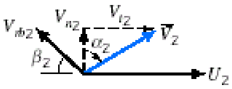
Find: Optimum speed using the Euler turbomachine equation

Solution:

The governing equation is the Euler turbomachine equation

$$T_{\text{shaft}} = (r_2 V_{t_2} - r_1 V_{t_1})\dot{m} \qquad (10.1c)$$

In terms of the notation of Example 10.5, for a stationary CV



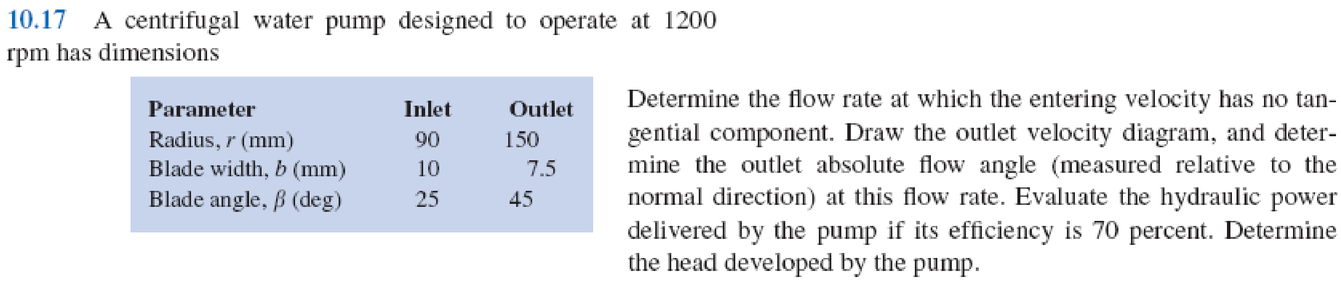
$r_1 = r_2 = R$ $\qquad U_1 = U_2 = U$ $\qquad V_{t1} = V - U$ $\qquad V_{t2} = (V - U)\cdot\cos(\theta)$ and $\quad m_{flow} = \rho\cdot Q$

Hence $\qquad T_{shaft} = [R\cdot(V - U)\cdot\cos(\theta) - R\cdot(V - U)]\cdot\rho\cdot Q \qquad T_{out} = T_{shaft} = \rho\cdot Q\cdot R\cdot(V - U)\cdot(1 - \cos(\theta))$

The power is $\qquad W_{out} = \omega\cdot T_{out} = \rho\cdot Q\cdot R\cdot\omega\cdot(V - U)\cdot(1 - \cos(\theta)) \qquad W_{out} = \rho\cdot Q\cdot U\cdot(V - U)\cdot(1 - \cos(\theta))$

These results are identical to those of Example 10.5. The proof that maximum power is when *U* = *V*/2 is hence also the same and will not be repeated here.

Problem 10.17

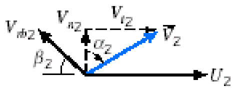


10.17 A centrifugal water pump designed to operate at 1200 rpm has dimensions

Parameter	**Inlet**	**Outlet**
Radius, *r* (mm)	90	150
Blade width, *b* (mm)	10	7.5
Blade angle, *β* (deg)	25	45

Determine the flow rate at which the entering velocity has no tangential component. Draw the outlet velocity diagram, and determine the outlet absolute flow angle (measured relative to the normal direction) at this flow rate. Evaluate the hydraulic power delivered by the pump if its efficiency is 70 percent. Determine the head developed by the pump.

Given: Data on a centrifugal pump



Find: Flow rate for zero inlet tangential velocity; outlet flow angle; power; head developed

Solution:

The given or available data is $\rho = 999\cdot\frac{kg}{m^3}$ $\omega = 1200\cdot rpm$ $\eta = 70\cdot\%$

$r_1 = 90\cdot mm$ $b_1 = 10\cdot mm$ $\beta_1 = 25\cdot deg$ $r_2 = 150\cdot mm$ $b_2 = 7.5\cdot mm$ $\beta_2 = 45\cdot deg$

The governing equations (derived directly from the Euler turbomachine equation) are

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad (10.2b)$$

$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad (10.2c)$$

We also have from geometry $\alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ (1)

From geometry $V_{t1} = 0 = U_1 - V_{rb1}\cdot\cos(\beta_1) = r_1\cdot\omega\cdot - \frac{V_{n1}}{\sin(\beta_1)}\cdot\cos(\beta_1)$

and from continuity $V_{n1} = \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}$

Hence $r_1\cdot\omega - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1\cdot\tan(\beta_1)} = 0$ $Q = 2\cdot\pi\cdot r_1^{\,2}\cdot b_1\cdot\omega\cdot\tan(\beta_1)$ $Q = 29.8\frac{L}{s}$ $Q = 0.0298\frac{m^3}{s}$

The power, head and absolute angle α at the exit are obtained from direct computation using Eqs. 10.2b, 10.2c, and 1 above

$U_1 = r_1\cdot\omega$ $U_1 = 11.3\frac{m}{s}$ $U_2 = r_2\cdot\omega$ $U_2 = 18.8\frac{m}{s}$ $V_{t1} = 0\cdot\frac{m}{s}$

From geometry $V_{t2} = U_2 - V_{rb2}\cdot\cos(\beta_2) = r_2\cdot\omega\cdot - \frac{V_{n2}}{\sin(\beta_2)}\cdot\cos(\beta_2)$

and from continuity $V_{n2} = \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}$ $V_{n2} = 4.22\frac{m}{s}$

Hence $V_{t2} = r_2 \cdot \omega - \frac{V_{n2}}{\tan(\beta_2)}$ $V_{t2} = 14.6 \frac{m}{s}$

Using these results in Eq. 1 $\alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ $\alpha_2 = 73.9\,deg$

Using them in Eq. 10.2b $W_m = \left(U_2 \cdot V_{t2} - U_1 \cdot V_{t1}\right) \cdot \rho \cdot Q$ $W_m = 8.22\,kW$

Using them in Eq. 10.2c $H = \frac{1}{g} \cdot \left(U_2 \cdot V_{t2} - U_1 \cdot V_{t1}\right)$ $H = 28.1\,m$

This is the power and head assuming no inefficiency; with $\eta = 70\%$, we have (from Eq. 10.8c)

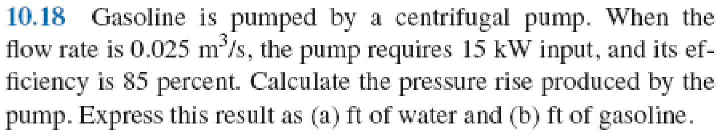
$W_h = \eta \cdot W_m$ $W_h = 5.75\,kW$

$H_p = \eta \cdot H$ $H_p = 19.7\,m$

(This last result can also be obtained from Eq. 10.8a $W_h = \rho \cdot Q \cdot g \cdot H_p$)

Problem 10.18

[1]



10.18 Gasoline is pumped by a centrifugal pump. When the flow rate is 0.025 m^3/s, the pump requires 15 kW input, and its efficiency is 85 percent. Calculate the pressure rise produced by the pump. Express this result as (a) ft of water and (b) ft of gasoline.

Given: Data on centrifugal pump

Find: Pressure rise; Express as ft of water and gasoline

Solution:

Basic equations: $\eta = \dfrac{\rho \cdot Q \cdot g \cdot H}{W_m}$

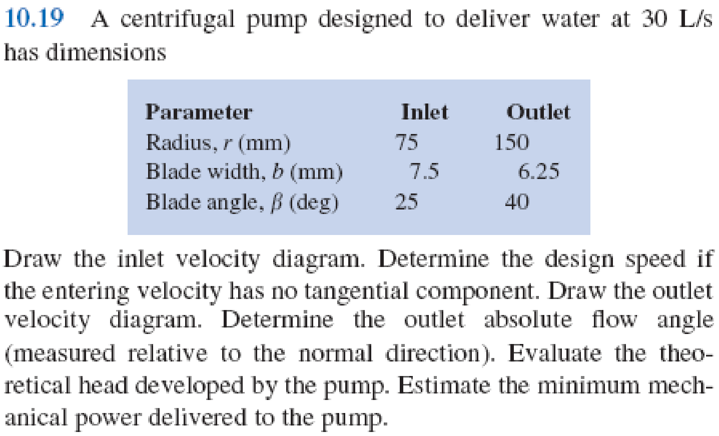
The given or available data is $\rho_w = 1000 \cdot \dfrac{kg}{m^3}$ $\quad Q = 0.025 \cdot \dfrac{m^3}{s}$ $\quad W_m = 15 \cdot kW$ $\quad \eta = 85 \cdot \%$

Solving for H $\quad H = \dfrac{\eta \cdot W_m}{\rho_w \cdot Q \cdot g}$ $\quad H = 52.0\,m$ $\quad H = 171\,ft$

For gasoline, from Table A.2 $\quad SG = 0.72$ $\quad H_g = \dfrac{\eta \cdot W_m}{SG \cdot \rho_w \cdot Q \cdot g}$ $\quad H_g = 72.2\,m$ $\quad H_g = 237\,ft$

Problem 10.19

[3]



10.19 A centrifugal pump designed to deliver water at 30 L/s has dimensions

Parameter	Inlet	Outlet
Radius, r (mm)	75	150
Blade width, b (mm)	7.5	6.25
Blade angle, β (deg)	25	40

Draw the inlet velocity diagram. Determine the design speed if the entering velocity has no tangential component. Draw the outlet velocity diagram. Determine the outlet absolute flow angle (measured relative to the normal direction). Evaluate the theoretical head developed by the pump. Estimate the minimum mechanical power delivered to the pump.

Given: Geometry of centrifugal pump

Find: Draw inlet velocity diagram; Design speed for no inlet tangential velocity; Outlet angle; Head; Power

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b)

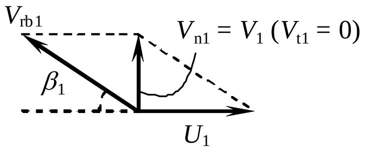
$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \quad \text{(Eq. 10.2c)}$$

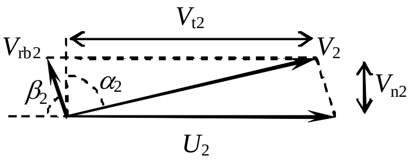
The given or available data is

$r_1 = 75\cdot\text{mm}$ $\quad r_2 = 150\cdot\text{mm}$ $\quad b_1 = 7.5\cdot\text{mm}$ $\quad b_2 = 6.25\cdot\text{mm}$ $\quad \beta_1 = 25\cdot\text{deg}$ $\quad \beta_2 = 40\cdot\text{deg}$

$\rho = 1000\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad Q = 30\cdot\frac{\text{L}}{\text{s}}$ $\quad Q = 0.030\cdot\frac{\text{m}^3}{\text{s}}$

Velocity diagrams:





From continuity $\quad V_n = \frac{Q}{2\cdot\pi\cdot r\cdot b} = V_{rb}\cdot\sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$ $\quad \frac{V_{n1}}{V_{n2}} = \frac{A_1}{A_2} = \frac{r_1\cdot b_1}{r_2\cdot b_2}$

From geometry $\quad V_t = U - V_{rb}\cdot\cos(\beta) = U - \frac{V_n}{\sin(\beta)}\cdot\cos(\beta) = U - \frac{Q}{2\cdot\pi\cdot r\cdot b}\cdot\cot(\beta)$

For $V_{t1} = 0$ we obtain $\quad U_1 - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}\cdot\cot(\beta_1) = 0$ $\quad$ or $\quad \omega\cdot r_1 - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}\cdot\cot(\beta_1) = 0$

Solving for ω $\quad \omega = \frac{Q}{2\cdot\pi\cdot r_1^{\,2}\cdot b_1}\cdot\cot(\beta_1)$ $\quad \omega = 243\,\frac{\text{rad}}{\text{s}}$ $\quad \omega = 2318\,\text{rpm}$

Hence $\quad U_1 = \omega\cdot r_1$ $\quad U_1 = 18.2\,\frac{\text{m}}{\text{s}}$ $\quad U_2 = \omega\cdot r_2$ $\quad U_2 = 36.4\,\frac{\text{m}}{\text{s}}$

$V_{n2} = \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}$ $\quad V_{n2} = 5.09\,\frac{\text{m}}{\text{s}}$ $\quad V_{t2} = U_2 - \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot(\beta_2)$ $\quad V_{t2} = 30.3\,\frac{\text{m}}{\text{s}}$

From the sketch $\alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ $\alpha_2 = 80.5\,\text{deg}$

Hence $W_m = U_2 \cdot V_{t2} \cdot \rho \cdot Q$ $W_m = 33.1 \cdot \text{kW}$

The head is $H = \frac{W_m}{\rho \cdot Q \cdot g}$ $H = 113\,\text{m}$

Problem 10.20

[4]

Given: Centrifugal pump operating with water, at shutoff.

Actual head rise is 70 percent of theoretical.

Find: (a) Prepare log-log plot of impeller radius versus theoretical head rise, with standard motor speeds as parameters.
(b) Explain how this plot might be used for preliminary design.

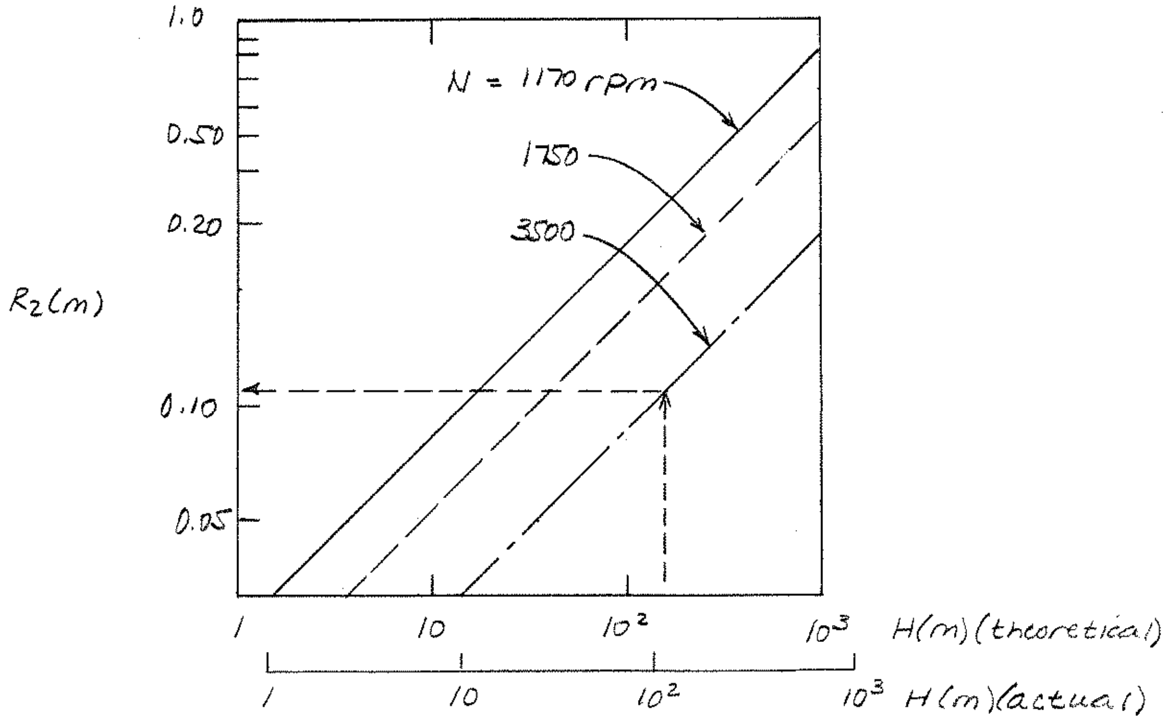
Solution: Apply the Euler turbomachine equation.

Computing equation: $H = \frac{1}{g}(U_2 V_{t_2} - U_1 \overset{\approx 0 (2)}{\cancel{V_{t_1}}})$

Assumptions: (1) No through flow, (2) Neglect V_{t_1}

Then $H = \frac{1}{g}(\omega R_2 \omega R_2) = \frac{\omega^2 R_2^2}{g}$ or $\log H = 2 \log \omega + 2 \log R_2 - \log g$

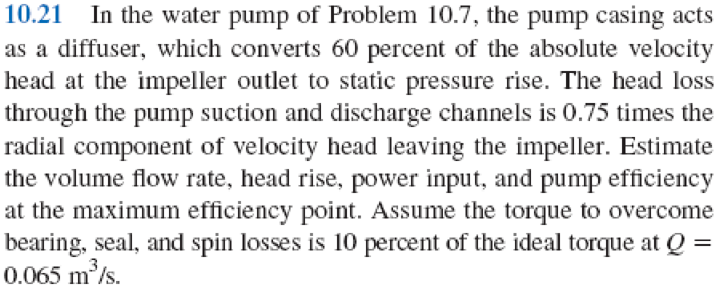
These will be straight lines on a plot of $\log R_2$ vs. $\log H$ (at constant ω):



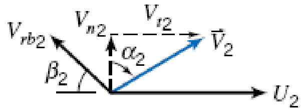
For a given application, enter the abscissa with the desired head, move up to the desired driver speed, then move left to the ordinate and read the required impeller radius. The example (--- line) illustrates.

Problem 10.21

[4]



10.21 In the water pump of Problem 10.7, the pump casing acts as a diffuser, which converts 60 percent of the absolute velocity head at the impeller outlet to static pressure rise. The head loss through the pump suction and discharge channels is 0.75 times the radial component of velocity head leaving the impeller. Estimate the volume flow rate, head rise, power input, and pump efficiency at the maximum efficiency point. Assume the torque to overcome bearing, seal, and spin losses is 10 percent of the ideal torque at $Q = 0.065\ \text{m}^3/\text{s}$.



Given: Geometry of centrifugal pump with diffuser casing

Find: Flow rate; Theoretical head; Power; Pump efficiency at maximum efficiency point

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1000 \cdot \frac{\text{kg}}{\text{m}^3}$ $\quad r_2 = 7.5 \cdot \text{cm}$ $\quad b_2 = 2 \cdot \text{cm}$ $\quad \beta_2 = 65 \cdot \text{deg}$

$\omega = 1750 \cdot \text{rpm}$ $\quad \omega = 183 \cdot \frac{\text{rad}}{\text{s}}$

Using given data $\quad U_2 = \omega \cdot r_2 \quad U_2 = 13.7 \frac{\text{m}}{\text{s}}$

Illustrate the procedure with $\quad Q = 0.065 \cdot \frac{\text{m}^3}{\text{s}}$

From continuity $\quad V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \quad V_{n2} = 6.9 \frac{\text{m}}{\text{s}}$

From geometry $\quad V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = U_2 - \frac{V_{n2}}{\sin(\beta_2)} \cdot \cos(\beta_2)$

Hence $\quad V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2) \quad V_{t2} = 10.5 \frac{\text{m}}{\text{s}} \quad V_{t1} = 0$ (axial inlet)

$V_2 = \sqrt{V_{n2}^2 + V_{t2}^2} \quad V_2 = 12.6 \frac{\text{m}}{\text{s}}$

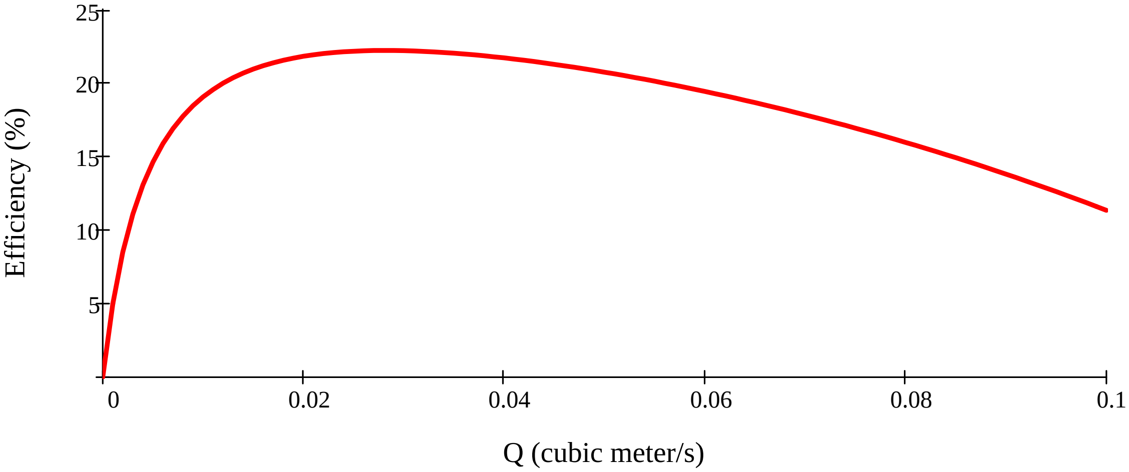
$H_{ideal} = \frac{U_2 \cdot V_{t2}}{g} \quad H_{ideal} = 14.8 \cdot \text{m}$

$$T_{friction} = 10 \cdot \% \cdot \frac{W_{mideal}}{\omega} = 10 \cdot \% \cdot \frac{\rho \cdot Q \cdot H_{ideal}}{\omega}$$

$T_{friction} = 10 \cdot \% \cdot \frac{Q \cdot \rho \cdot g \cdot H_{ideal}}{\omega} \quad T_{friction} = 5.13 \text{N} \cdot \text{m}$

$$H_{actual} = 60\cdot\%\cdot\frac{V_2^{\ 2}}{2\cdot g} - 0.75\cdot\frac{V_{n2}^{\ 2}}{2\cdot g} \qquad H_{actual} = 3.03\,m$$

$$\eta = \frac{Q\cdot\rho\cdot g\cdot H_{actual}}{Q\cdot\rho\cdot g\cdot H_{ideal} + \omega\cdot T_{friction}} \qquad \eta = 18.7\,\%$$



The above graph can be plotted in Excel. In addition, Solver can be used to vary Q to maximize η. The results are

$$Q = 0.0282\,\frac{m^3}{s} \qquad \eta = 22.2\,\% \qquad H_{ideal} = 17.3\,m \qquad H_{actual} = 4.60\,m$$

$$W_m = Q\cdot\rho\cdot g\cdot H_{ideal} + \omega\cdot T_{friction} \qquad W_m = 5.72\,kW$$

Problem 10.22

Given: Performance curves (Appendix D) for Peerless 4AE12 pump at 1750 and 3550 nominal rpm, with a 12.12 in. impeller.

Find: Obtain and plot curve-fits for total head vs. delivery at each speed for this pump.

Solution: Tabulate data from Figs. D.4 (1750 rpm) and D.5 (3550 rpm):

1750 rpm:	Q (gpm)	0	200	400	600
	H (ft)	155	150	137	106

Curve-fit: $\hat{H}(ft) = 156 - 1.36 \times 10^{-4}[Q(gpm)]^2$; $r^2 = 0.994$

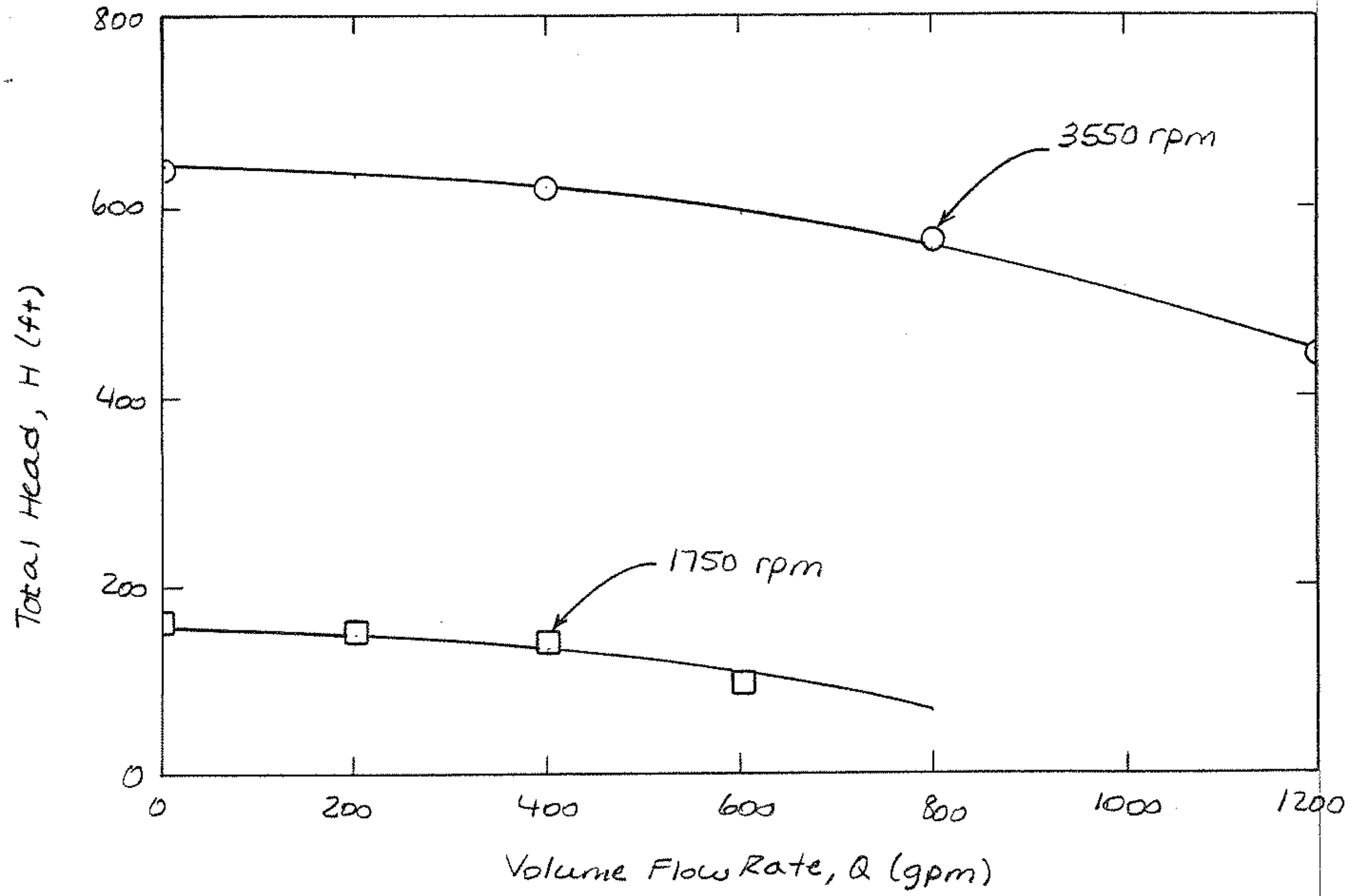
	$\hat{H}$ (ft)	156	151	134	107

3550 rpm:	Q (gpm)	0	400	800	1200
	H (ft)	635	620	565	445

Curve-fit: $\hat{H}(ft) = 641 - 1.33 \times 10^{-4}[Q(gpm)]^2$; $r^2 = 0.994$

	$\hat{H}$ (ft)	641	619	556	449

Plot:



Problem 10.23

[2]

Given: Performance curves (Appendix D) for Peerless 16A 18B pump at 705 and 880 nominal rpm, with an 18.0 in. diameter impeller.

Find: Obtain and plot curve-fits for total head versus delivery at each speed for this pump.

Solution: Tabulate data from Figs. D.9 (705 rpm) and D.10 (880 rpm):

705 rpm:	Q(gpm)	0	2000	4000	6000	8000
	H (ft)	59	56	50	43	32

Curve-fit: $\hat{H}(ft) = 57.8 - 4.09 \times 10^{-7} [Q(gpm)]^2$; $r^2 = 0.994$

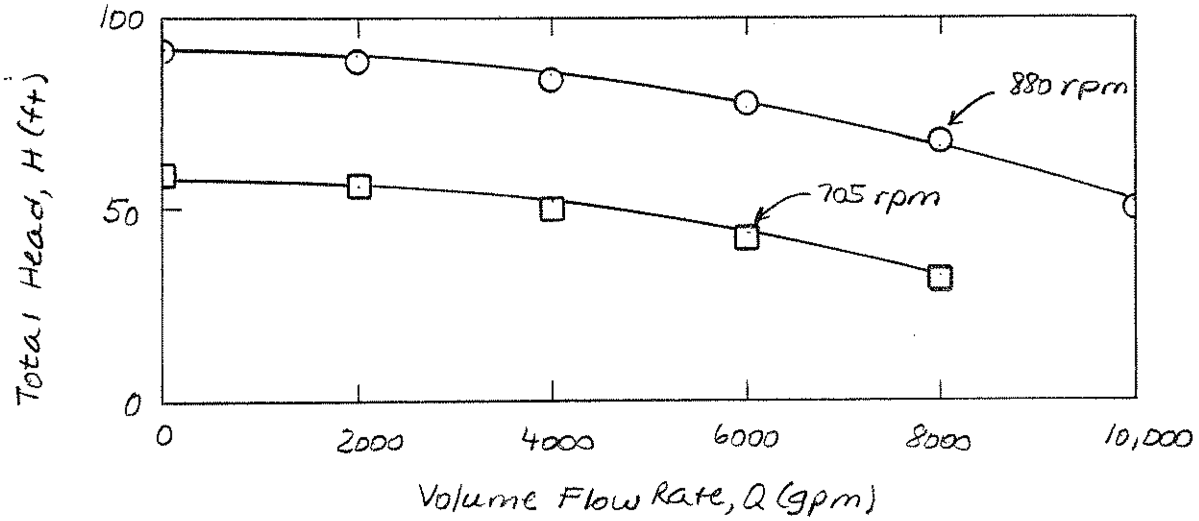
$\hat{H}(ft)$	57.8	56.2	51.3	43.1	31.6

880 rpm:	Q(gpm)	0	2000	4000	6000	8000	10,000
	H (ft)	92	89	84	78	68	50

Curve-fit: $\hat{H}(ft) = 91.5 - 4.01 \times 10^{-7} [Q(gpm)]^2$; $r^2 = 0.992$

$\hat{H}(ft)$	91.5	89.9	85.1	77.1	65.9	51.5

Plot:



Problem 10.24

[3]

10.24 Data from tests of a suction pump operated at 1750 rpm with a 35-cm diameter impeller are

Flow rate, Q ($\mathrm{m^3/s} \times 10^3$)	17	26	38	45	63
Total head, H (m)	60	59	54	50	37
Power input, $\mathscr{P}$ (kW)	19	22	26	30	34

Plot the performance curves for this pump; include a curve of efficiency versus volume flow rate. Locate the best efficiency point and specify the pump rating at this point.

Given: Data on suction pump

Find: Plot of performance curves; Best effiiciency point

Solution:

Basic equations: $\eta_p = \frac{P_h}{P_m}$ $P_h = \rho\cdot Q\cdot g\cdot H$ (Note: Software cannot render a dot!)

$\rho = 1000 \quad \mathrm{kg/m^3}$

Fitting a 2nd order polynomial to each set of data we find

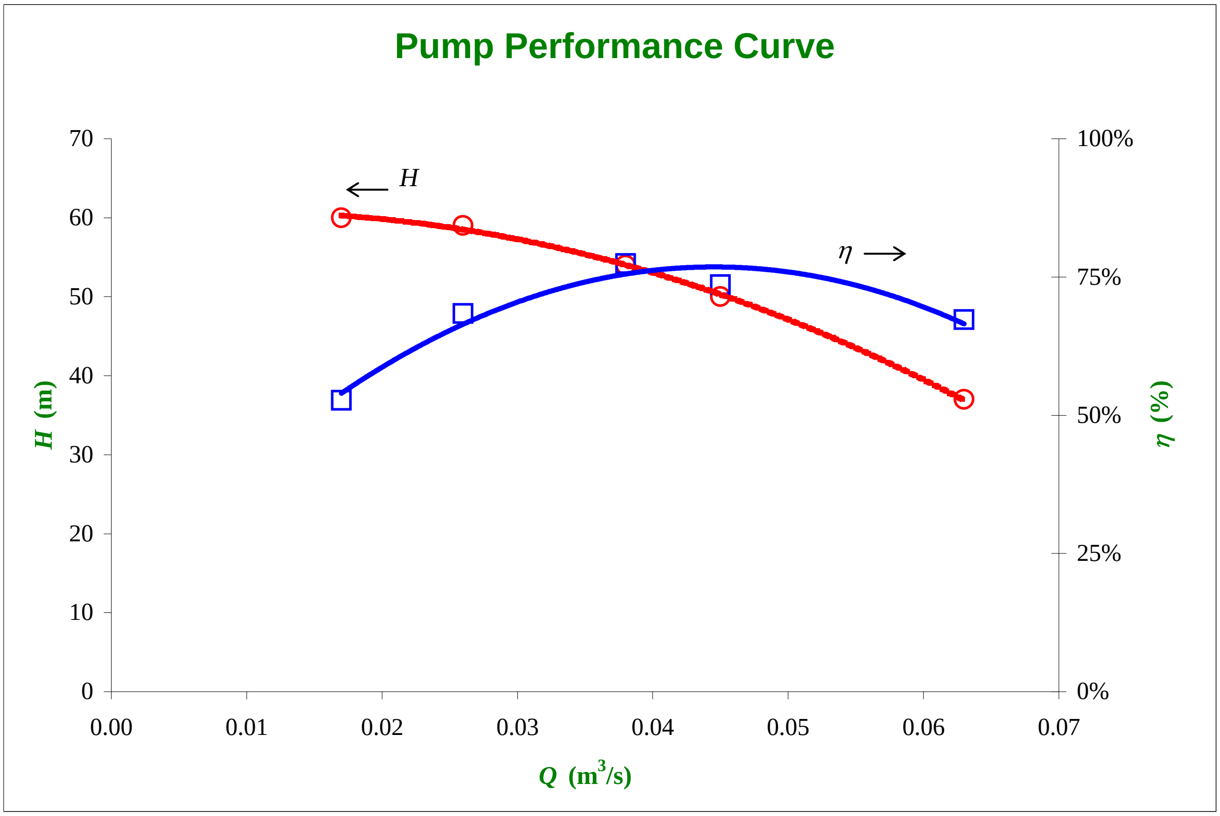
$H = -8440Q^2 + 167Q + 59.9$

$\eta = -302Q^2 + 26.9Q + 0.170$

Q ($\mathrm{m^3/s}$)	H (m)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
0.017	60	19	10.0	52.7%
0.026	59	22	15.0	68.4%
0.038	54	26	20.1	77.4%
0.045	50	30	22.1	73.6%
0.063	37	34	22.9	67.3%

Finally, we use Solver to maximize η by varying Q:

Q ($\mathrm{m^3/s}$)	H (m)	η (%)
0.045	50.6	76.9%



Problem 10.25

[3]

10.25 Data from tests of a suction pump operated at 1750 rpm with a 35-cm diameter impeller are

Flow rate, Q ($m^3/s \times 10^3$)	18	28	35	50	58	81
Total head, H (m)	62	62	61	57	53	41
Power input, $\mathscr{P}$ (kW)	22	26	30	34	37	45

Plot the performance curves for this pump; include a curve of efficiency versus volume flow rate. Locate the best efficiency point and specify the pump rating at this point.

Given: Data on suction pump

Find: Plot of performance curves; Best efficiency point

Solution:

Basic equations: $\eta_p = \frac{P_h}{P_m}$ $P_h = \rho \cdot Q \cdot g \cdot H$ (Note: Software cannot render a dot!)

$\rho = 1000$ kg/m^3

Fitting a 2nd order polynomial to each set of data we find

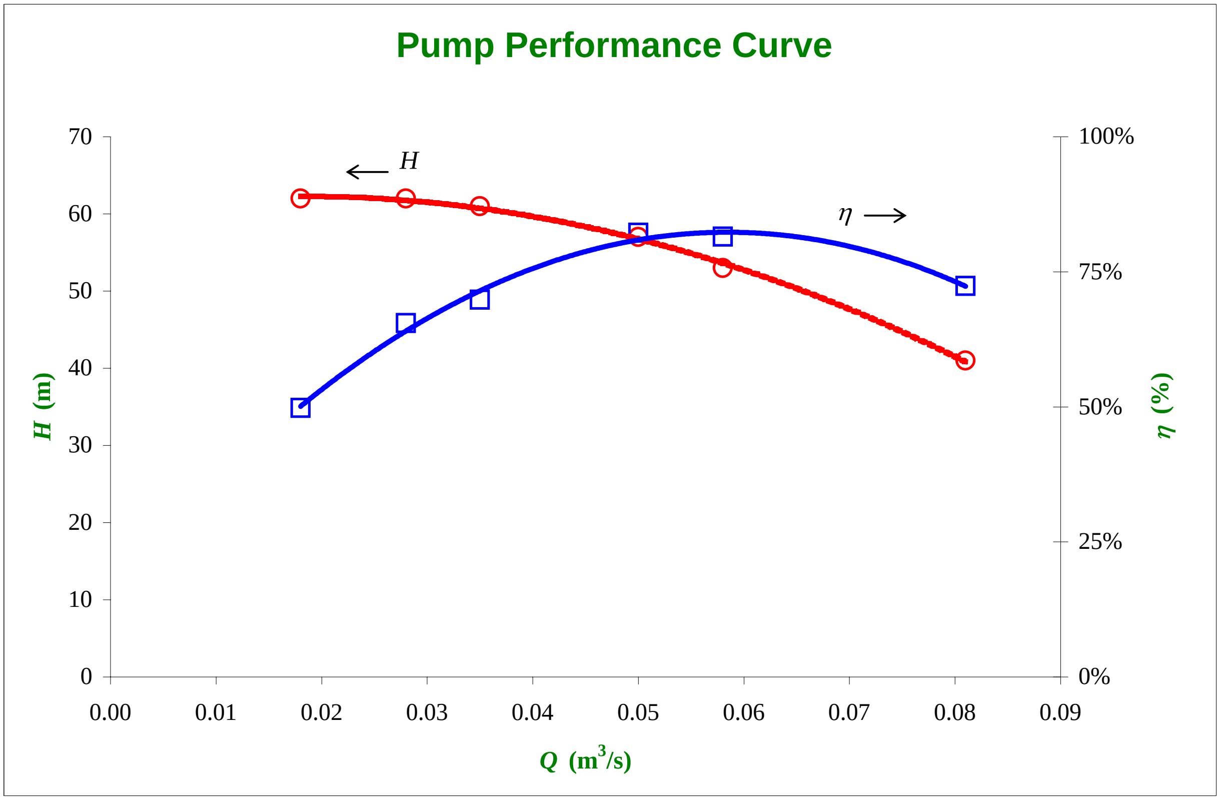
$H = -5404Q^2 + 194Q + 60.5$

$\eta = -197Q^2 + 23.0Q + 0.150$

Q (m^3/s)	H (m)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
0.018	62	22	10.9	49.8%
0.028	62	26	17.0	65.5%
0.035	61	30	20.9	69.8%
0.050	57	34	28.0	82.2%
0.058	53	37	30.2	81.5%
0.081	41	45	32.6	72.4%

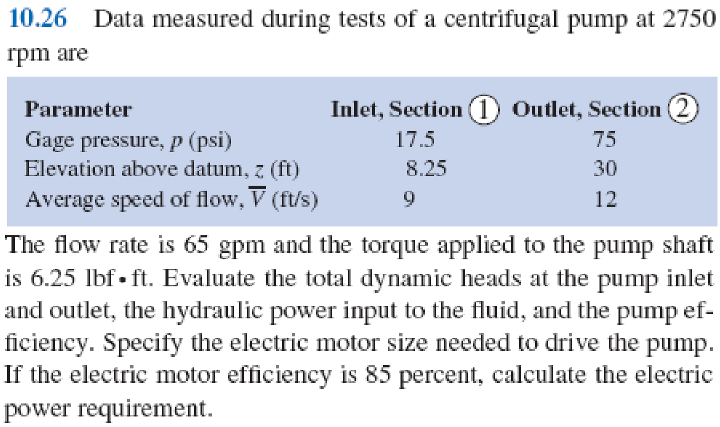
Finally, we use Solver to maximize η by varying Q:

Q (m^3/s)	H (m)	η (%)
0.058	53.4	82.1%



Problem 10.26

[3]



10.26 Data measured during tests of a centrifugal pump at 2750 rpm are

Parameter	Inlet, Section ①	Outlet, Section ②
Gage pressure, p (psi)	17.5	75
Elevation above datum, z (ft)	8.25	30
Average speed of flow, $\overline{V}$ (ft/s)	9	12

The flow rate is 65 gpm and the torque applied to the pump shaft is 6.25 lbf • ft. Evaluate the total dynamic heads at the pump inlet and outlet, the hydraulic power input to the fluid, and the pump efficiency. Specify the electric motor size needed to drive the pump. If the electric motor efficiency is 85 percent, calculate the electric power requirement.

Given: Data on centrifugal pump

Find: Dynamic head at inlet and exit; Hydraulic power input; Pump efficiency; Motor size; Electric power required

Solution:

Basic equations: $\dot{W}_h = \rho Q g H_p$ (Eq. 10.8a)

$$H_p = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{discharge}} - \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{suction}} \quad \text{(Eq. 10.8b)}$$

$$\eta_p = \frac{\dot{W}_h}{\dot{W}_m} = \frac{\rho Q g H_p}{\omega T} \quad \text{(Eq. 10.8c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\omega = 2750 \cdot \text{rpm}$ $\eta_e = 85 \cdot \%$ $Q = 65 \cdot \text{gpm}$ $Q = 0.145 \cdot \frac{\text{ft}^3}{\text{s}}$ $T = 6.25 \cdot \text{lbf} \cdot \text{ft}$

$p_1 = 17.5 \cdot \text{psi}$ $z_1 = 8.25 \cdot \text{ft}$ $V_1 = 9 \cdot \frac{\text{ft}}{\text{s}}$ $p_2 = 75 \cdot \text{psi}$ $z_2 = 30 \cdot \text{ft}$ $V_2 = 12 \cdot \frac{\text{ft}}{\text{s}}$

Then $H_{p1} = \frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + z_1$ $H_{p1} = 49.9 \cdot \text{ft}$ $H_{p2} = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + z_2$ $H_{p2} = 205 \cdot \text{ft}$

Also, from Eq. 10.8a $W_h = \rho \cdot g \cdot Q \cdot \left(H_{p2} - H_{p1}\right)$ $W_h = 1405 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_h = 2.55 \cdot \text{hp}$

The mechanical power in is $W_m = \omega \cdot T$ $W_m = 1800 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_m = 3.27 \cdot \text{hp}$

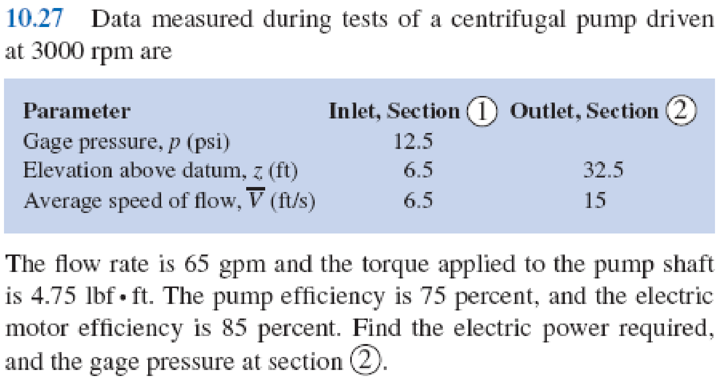
We need a 3.5 hp motor

From Eq. 10.8c $\eta_p = \frac{W_h}{W_m}$ $\eta_p = 78.0 \cdot \%$

The input power is then $W_e = \frac{W_m}{\eta_e}$ $W_e = 2117 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_e = 3.85 \cdot \text{hp}$ $W_e = 2.87 \cdot \text{kW}$

Problem 10.27

[3]



10.27 Data measured during tests of a centrifugal pump driven at 3000 rpm are

Parameter	Inlet, Section ①	Outlet, Section ②
Gage pressure, p (psi)	12.5	
Elevation above datum, z (ft)	6.5	32.5
Average speed of flow, $\overline{V}$ (ft/s)	6.5	15

The flow rate is 65 gpm and the torque applied to the pump shaft is 4.75 lbf • ft. The pump efficiency is 75 percent, and the electric motor efficiency is 85 percent. Find the electric power required, and the gage pressure at section ②.

Given:

Data on centrifugal pump

Find:

Electric power required; gage pressure at exit

Solution:

Basic equations: $\dot{W}_h = \rho Q g H_p$ (Eq. 10.8a)

$$H_p = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{discharge}} - \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{suction}} \qquad \text{(Eq. 10.8b)} \qquad \eta_p = \frac{\dot{W}_h}{\dot{W}_m} = \frac{\rho Q g H_p}{\omega T} \qquad \text{(Eq. 10.8c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\omega = 3000 \cdot \text{rpm}$ $\eta_p = 75 \cdot \%$ $\eta_e = 85 \cdot \%$ $Q = 65 \cdot \text{gpm}$ $Q = 0.145 \cdot \frac{\text{ft}^3}{\text{s}}$

$T = 4.75 \cdot \text{lbf} \cdot \text{ft}$ $p_1 = 12.5 \cdot \text{psi}$ $z_1 = 6.5 \cdot \text{ft}$ $V_1 = 6.5 \cdot \frac{\text{ft}}{\text{s}}$ $z_2 = 32.5 \cdot \text{ft}$ $V_2 = 15 \cdot \frac{\text{ft}}{\text{s}}$

From Eq. 10.8c $H_p = \frac{\omega \cdot T \cdot \eta_p}{\rho \cdot Q \cdot g}$ $H_p = 124 \cdot \text{ft}$

Hence, from Eq. 10.8b $p_2 = p_1 + \frac{\rho}{2} \cdot \left(V_1^{\,2} - V_2^{\,2}\right) + \rho \cdot g \cdot \left(z_1 - z_2\right) + \rho \cdot g \cdot H_p$ $p_2 = 53.7 \cdot \text{psi}$

Also $W_h = \rho \cdot g \cdot Q \cdot H_p$ $W_h = 1119 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_h = 2.03 \cdot \text{hp}$

The shaft work is then $W_m = \frac{W_h}{\eta_p}$ $W_m = 1492 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_m = 2.71 \cdot \text{hp}$

Hence, electrical input is $W_e = \frac{W_m}{\eta_e}$ $W_e = 1756 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_e = 2.38 \cdot \text{kW}$

Problem 10.28

[2]

Write the turbine specific speed in terms of the power coefficient and the head coefficient

Solution:

$N_s = \omega P^{1/2} / \rho^{1/2} h^{5/4}$ 10.18a

Power coefficient $\pi_3 = \frac{P}{\rho \omega^3 D^5}$

Head coefficient $\pi_2 = \frac{h}{\omega^2 D^2}$

Then

$$N_s = \left[\frac{P}{\rho \omega^3 D^5}\right]^{1/2} \left[\frac{\omega^2 D^2}{h}\right]^{5/4} = \frac{P^{1/2}}{\rho^{1/2} \omega^{3/2} D^{5/2}} \times \frac{\omega^{5/2} D^{5/2}}{h^{5/4}} = \frac{P^{1/2} \omega}{\rho^{1/2} h^{5/4}}$$

$N_s = \pi_3^{1/2} / \pi_2^{5/4}$ ← N_s

Problem 10.29

Write the pump specific speed in terms of the flow coefficient and the head coefficient

Solution:

$$N_s = \frac{\omega Q^{1/2}}{h^{3/4}} \quad \text{.......} \quad (7.6a)$$

Flow coefficient $\pi_1 = \frac{Q}{\omega D^3}$

Head coefficient $\pi_2 = \frac{h}{\omega^2 D^2}$

Then

$$N_s = \left[\frac{Q}{\omega D^3}\right]^{1/2} \cdot \left[\frac{\omega^2 D^2}{h}\right]^{3/4} = \frac{Q^{1/2}}{\omega^{1/2}\cancel{D^{3/2}}} \cdot \frac{\omega^{3/2}\cancel{D^{3/2}}}{h^{3/4}} = \frac{\omega Q^{1/2}}{h^{3/4}}$$

$$N_s = \pi_1^{1/2} / \pi_2^{3/4} \longleftarrow N_s$$

National® Brand
[illegible]
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.

Problem 10.30

[2]

Given: Kilogram force ≡ force exerted on 1 kg in standard gravity.

Metric horsepower (hpm) ≡ 75 m·kgf/s.

Find: (a) Develop a conversion relating hpm to U.S. hp.
(b) Relate specific speed for a hydraulic turbine -- expressed in units of rpm, hpm, and m -- to the specific speed calculated in U.S. customary units.

Solution:

$$1\ \text{hp (U.S.)} = 550\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} \times 0.305\ \frac{\text{m}}{\text{ft}} \times 0.4536\ \frac{\text{kgf}}{\text{lbf}} \times \frac{\text{hpm}\cdot\text{s}}{75\ \text{m}\cdot\text{kgf}} = 1.01\ \text{hpm}$$

hpm

$$N_{s_{cu}} = \frac{N(\mathscr{P})^{1/2}}{(h)^{5/4}} = \frac{N(\text{rpm})[\mathscr{P}(\text{hpUS})]^{1/2}}{[h(\text{ft})]^{5/4}}$$

$$= N(\text{rpm})\frac{[\mathscr{P}(\text{hpUS})]^{1/2}}{[\mathscr{P}(\text{hpm})]^{1/2}} \times [\mathscr{P}(\text{hpm})]^{\frac{1}{2}} \times \frac{[h(\text{m})]^{5/4}}{[h(\text{ft})]^{5/4}} \times \frac{1}{[h(\text{m})]^{5/4}}$$

$$= N(\text{rpm})\frac{[\mathscr{P}(\text{hpm})]^{1/2}}{[h(\text{m})]^{5/4}} \times \left[\frac{\mathscr{P}(\text{hpUS})}{\mathscr{P}(\text{hpm})}\right]^{1/2} \times \left[\frac{h(\text{m})}{h(\text{ft})}\right]^{5/4}$$

$$= N_s(\text{rpm}, \text{hpm}, \text{m}) \times (1.01)^{1/2} \times (0.305)^{5/4}$$

$$N_{s_{cu}} = 0.228\ N_s(\text{rpm}, \text{hpm}, \text{m})$$

Ns

Check: $N = 1\ \text{rpm},\ \mathscr{P} = 1\ \text{hp},\ h = 1\ \text{ft};\ N_s\ (\text{USCS}) = 1$

$$N = 1\ \text{rpm},\ \mathscr{P} = 1\ \text{hpm},\ h = 1\ \text{m};\ N_s = \frac{(1)\left(\frac{1}{1.01}\right)^{1/2}}{(0.305)^{5/4}} = 4.39$$

$$\frac{N_s(\text{USCS})}{N_s(\text{rpm}, \text{hpm}, \text{m})} = \frac{1}{4.39} = 0.228\ \checkmark\checkmark$$

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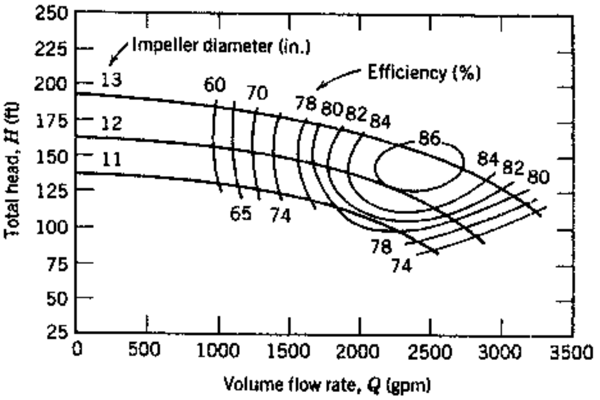
Problem 10.31

Given: Typical performance curves for a centrifugal pump, tested with three different impeller diameters:



Find: (a) Specify the flow rate and head at the best efficiency point (BEP) with a 12 in. diameter impeller.
(b) Scale these data to predict the BEP for 11 in. and 13 in. diameter impellers.
(c) Comment on the accuracy of the scaling procedure.

Solution: From the graph, BEP occurs for the 12 in. impeller at

$Q \simeq 2200 \text{ gpm}$ and $H \simeq 130 \text{ ft}$ $\longleftarrow$ BEP_{12}

From Section 10-4.3, scaling rules are

$$Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^3 ; \quad Q_{11} = 2200 \text{ gpm}\left(\frac{11}{12}\right)^3 = 1690 \text{ gpm}$$

$$Q_{13} = 2200 \text{ gpm}\left(\frac{13}{12}\right)^3 = 2800 \text{ gpm}$$

$$H_2 = H_1\left(\frac{D_2}{D_1}\right)^2 ; \quad H_{11} = 130 \text{ ft}\left(\frac{11}{12}\right)^2 = 109 \text{ ft}$$

$$H_{13} = 130 \text{ ft}\left(\frac{13}{12}\right)^2 = 153 \text{ ft}$$

Thus BEP_{11} is at $Q = 1690 \text{ gpm}$, $H = 109 \text{ ft}$ BEP_{11}

BEP_{13} is at $Q = 2800 \text{ gpm}$, $H = 153 \text{ ft}$ $\longleftarrow$ BEP_{13}

The complete scaling rules tend to move the volume flow rate too far. Accuracy would be improved using $Q_2 = Q_1(D_2/D_1)^2$, since the impeller width does not change, and $H_2 = H_1(D_2/D_1)^2$, since $H \simeq V^2$. With these modified rules

$(Q_{11}, H_{11}) = 1850 \text{ gpm}, 109 \text{ ft}$ and $(Q_{13}, H_{13}) = 2580 \text{ gpm}, 153 \text{ ft}$

These modified scaling points are closer to the measured BEPs.

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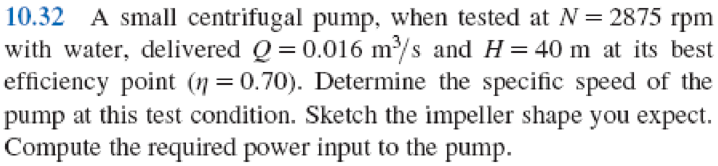
Problem 10.32

[2]



10.32 A small centrifugal pump, when tested at $N = 2875$ rpm with water, delivered $Q = 0.016\ \mathrm{m^3/s}$ and $H = 40$ m at its best efficiency point ($\eta = 0.70$). Determine the specific speed of the pump at this test condition. Sketch the impeller shape you expect. Compute the required power input to the pump.

Given: Data on small centrifugal pump

Find: Specific speed; Sketch impeller shape; Required power input

Solution:

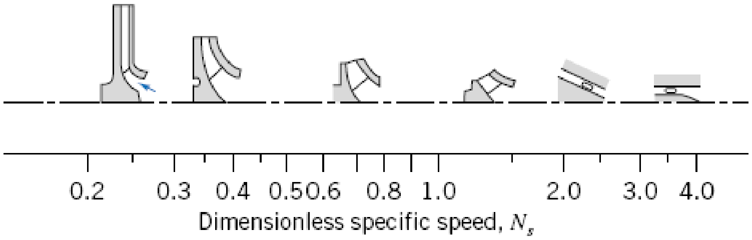
Basic equation: $N_S = \dfrac{\omega Q^{1/2}}{h^{3/4}}$ (Eq. 10.22b) $\quad \eta_p = \dfrac{\dot{W}_h}{\dot{W}_m} = \dfrac{\rho Q g H_p}{\omega T}$ (Eq. 10.8c)

The given or available data is

$\rho = 1000 \cdot \dfrac{\mathrm{kg}}{\mathrm{m}^3}$ $\quad \omega = 2875 \cdot \mathrm{rpm}$ $\quad \eta_p = 70 \cdot \%$ $\quad Q = 0.016 \cdot \dfrac{\mathrm{m}^3}{\mathrm{s}}$ $\quad H = 40 \cdot \mathrm{m}$

Hence $\quad h = g \cdot H \quad h = 392 \dfrac{\mathrm{m}^2}{\mathrm{s}^2} \quad$ (H is energy/weight. h is energy/mass)

Then $\quad N_S = \dfrac{\omega \cdot Q^{\frac{1}{2}}}{h^{\frac{3}{4}}} \quad N_S = 0.432$

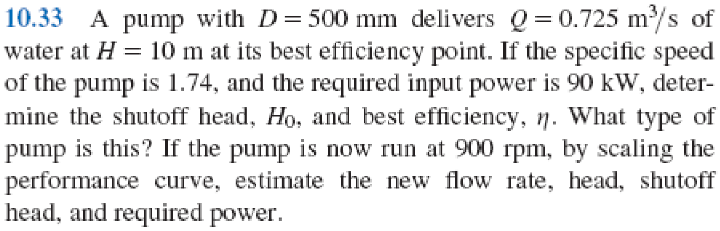


From the figure we see the impeller will be centrifugal

The power input is (from Eq. 10.8c) $\quad W_m = \dfrac{W_h}{\eta_p} \quad W_m = \dfrac{\rho \cdot Q \cdot g \cdot H}{\eta_p} \quad W_m = 8.97\,\mathrm{kW}$

Problem 10.33

[3]



10.33 A pump with $D = 500$ mm delivers $Q = 0.725\ \text{m}^3/\text{s}$ of water at $H = 10$ m at its best efficiency point. If the specific speed of the pump is 1.74, and the required input power is 90 kW, determine the shutoff head, H_0, and best efficiency, η. What type of pump is this? If the pump is now run at 900 rpm, by scaling the performance curve, estimate the new flow rate, head, shutoff head, and required power.

Given: Data on a pump

Find: Shutoff head; best efficiency; type of pump; flow rate, head, shutoff head and power at 900 rpm

Solution:

The given or available data is

$\rho = 999\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad N_s = 1.74$ $\quad D = 500\cdot\text{mm}$ $\quad Q = 0.725\cdot\frac{\text{m}^3}{\text{s}}$ $\quad H = 10\cdot\text{m}$ $\quad W_m = 90\cdot\text{kW}$ $\quad \omega' = 900\cdot\text{rpm}$

The governing equations are $\quad W_h = \rho\cdot Q\cdot g\cdot H \quad$ (10.8a)

$$N_s = \omega\cdot Q^{\frac{1}{2}} h^{\frac{3}{4}} \quad \text{(7.16a)}$$

$$H_0 = C_1 = \frac{U_2^{\,2}}{g} \quad \text{(From Eq. 10.7b)}$$

Similarity rules

$$\frac{Q_1}{\omega_1\cdot D_1^{\,3}} = \frac{Q_2}{\omega_2\cdot D_2^{\,3}} \quad \text{(10.19a)} \qquad \frac{h_1}{\omega_1^{\,2}\cdot D_1^{\,2}} = \frac{h2}{\omega_2^{\,2}\cdot D_2^{\,2}} \quad \text{(10.19b)} \qquad \frac{P_1}{\rho_1\cdot\omega_1^{\,3}\cdot D_1^{\,5}} = \frac{P_2}{\rho_2\cdot\omega_2^{\,3}\cdot D_2^{\,5}} \quad \text{(10.19a)}$$

$h = g\cdot H \qquad h = 98.1\frac{\text{J}}{\text{kg}}$

Hence from Eq. 7.16a $\quad \omega = \dfrac{N_s\cdot h^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad \omega = 608\,\text{rpm} \qquad \omega = 63.7\frac{\text{rad}}{\text{s}}$

From Eq. 10.8a $\quad W_h = \rho\cdot Q\cdot g\cdot H \qquad W_h = 71\,\text{kW}$

$$\eta_p = \frac{W_h}{W_m} \qquad \eta_p = 78.9\,\%$$

The shutoff head is given by $\quad H_0 = \dfrac{U_2^{\,2}}{g} \quad$ (From Eq. 10.7b)

$U_2 = \frac{D}{2} \cdot \omega \qquad U_2 = 15.9 \frac{m}{s}$

Hence $\quad H_0 = \frac{U_2^{\,2}}{g} \qquad H_0 = 25.8\,m$

From Eq. 10.19a (with $D_1 = D_2$) $\quad \frac{Q_1}{\omega_1} = \frac{Q_2}{\omega_2}$ or $\frac{Q}{\omega} = \frac{Q'}{\omega'} \qquad Q' = Q \cdot \frac{\omega'}{\omega} \qquad Q' = 1.07 \frac{m^3}{s}$

From Eq. 10.19b (with $D_1 = D_2$) $\quad \frac{h_1}{\omega_1^{\,2}} = \frac{h_2}{\omega_2^{\,2}}$ or $\frac{H}{\omega^2} = \frac{H'}{\omega'^2} \qquad H' = H \cdot \left(\frac{\omega'}{\omega}\right)^2 \qquad H' = 21.9\,m$

Also $\quad \frac{H_0}{\omega^2} = \frac{H'_0}{\omega'^2} \qquad H'_0 = H_0 \cdot \left(\frac{\omega'}{\omega}\right)^2 \qquad H'_0 = 56.6\,m$

(Alternatively, we could have used $H'_0 = \frac{U'_2{}^2}{g}$)

From Eq. 10.19c (with $D_1 = D_2$) $\quad \frac{P_1}{\rho \cdot \omega_1^{\,3}} = \frac{P_2}{\rho \cdot \omega_2^{\,3}}$ or $\frac{W_m}{\omega^3} = \frac{W'_m}{\omega'^3} \qquad W'_m = W_m \cdot \left(\frac{\omega'}{\omega}\right)^3 \qquad W'_m = 292\,kW$

Problem 10.34

[3]

Given: Mixed-flow pump at BEP ($\eta = 0.85$) has $D = 400$ mm, and delivers $Q = 1.20\ m^3/s$ at $H = 50$ m when operating at $N = 1500$ rpm.

(a) Calculate the specific speed of this pump
(b) Estimate the required power input
(c) Determine the curve-fit parameters using BEP and shutoff points.
(d) Scale the performance curve to estimate the flow, head, efficiency, and power required at 750 rpm.

Solution: In SI units, $N = 157$ rad/s, $Q = 1.20\ m^3/s$, and $h = gH = 490\ m^2/s^2$

$$N_s = \frac{\omega Q^{1/2}}{h^{3/4}} = 157\frac{rad}{s} \times (1.20)^{1/2}\frac{m^{3/2}}{s^{1/2}} \times \frac{s^{3/2}}{(490)^{3/4}\,m^{3/2}} = 1.65 \qquad N_s$$

$$\dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho g Q H}{\eta} = \frac{\rho Q h}{\eta}$$

$$\dot{W}_m = \frac{999}{0.85}\frac{kg}{m^3} \times 1.20\frac{m^3}{s} \times 490\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{kW \cdot s}{10^3\,N \cdot m} = 691\ kW \qquad \dot{W}_m$$

At shutoff, $V_{t_2} = U_2$, so $H_0 = \frac{U_2^2}{g} = \frac{(\omega R_2)^2}{g}$

$$H_0 = \left(157\frac{rad}{s} \times \frac{0.40}{2}m\right)^2 \times \frac{s^2}{9.81\,m} = 100\ m$$

Thus, $H = H_0 - AQ^2$ or $A = (H_0 - H)/Q^2$

$$A = (100 - 50)\,m \times \frac{1}{(1.20)^2}\frac{s^2}{m^6} = 34.7\ m^{-5}s^2$$

or

$$H_m = 100 - 34.7\left[Q(m^3/s)\right]^2 \qquad (1500\ rpm) \qquad H$$

At 750 rpm, $H_0' = \left(\frac{N'}{N}\right)^2 H_0 = \left(\frac{750}{1500}\right)^2 100 = 25\ m$, and $A' = A$

Thus

$$H'(m) = 25 - 34.7\left[Q'(m^3/s)\right]^2 \qquad (750\ rpm) \qquad H$$

At BEP,
$$Q' = Q\left(\frac{\omega'}{\omega}\right) = 1.20\frac{m^3}{s}\left(\frac{750}{1500}\right) = 0.60\ m^3/s$$

$$H' = H\left(\frac{\omega'}{\omega}\right)^2 = 50\,m\left(\frac{75}{1500}\right)^2 = 12.5\ m$$

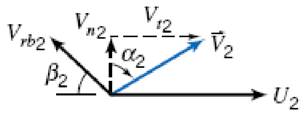
$$\eta' = \eta = 0.85$$

$$P' = P\left(\frac{\omega'}{\omega}\right)^3 = 691\,kW\left(\frac{750}{1500}\right)^3 = 86.4\ kW \qquad 750\ rpm$$

Problem 10.35

[2]

10.35 A centrifugal water pump operates at 1750 rpm; the impeller has backward-curved vanes with $\beta_2 = 60°$ and $b_2 = 1.25$ cm. At a flow rate of 0.025 m^3/s, the radial outlet velocity is $V_{n_2} = 3.5$ m/s. Estimate the head this pump could deliver at 1150 rpm.



Given:

Data on centrifugal pump

Find:

Head at 1150 rpm

Solution:

Basic equation: $$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}\left(U_2 V_{t_2} - U_1 V_{t_1}\right) \quad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1000\cdot\frac{kg}{m^3}$ $\quad Q = 0.025\cdot\frac{m^3}{s}$ $\quad \beta_2 = 60\cdot deg$ $\quad b_2 = 1.25\cdot cm$

$\omega = 1750\cdot rpm$ $\quad \omega' = 1150\cdot rpm$ $\quad V_{n2} = 3.5\cdot\frac{m}{s}$

From continuity $\quad V_{n2} = \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}$

Hence $\quad r_2 = \frac{Q}{2\cdot\pi\cdot b_2\cdot V_{n2}}$ $\quad r_2 = 0.0909\,m$ $\quad r_2 = 9.09\,cm$

Then $\quad V'_{n2} = \frac{\omega'}{\omega}\cdot V_{n2}$ $\quad V'_{n2} = 2.30\,\frac{m}{s}$

Also $\quad U'_2 = \omega'\cdot r_2$ $\quad U'_2 = 11.0\,\frac{m}{s}$

From the outlet geometry $\quad V'_{t2} = U'_2 - V'_{n2}\cdot\cos(\beta_2)$ $\quad V'_{t2} = 9.80\,\frac{m}{s}$

Finally $\quad H' = \frac{U'_2\cdot V'_{t2}}{g}$ $\quad H' = 10.9\,m$

Problem 10.36

[3]

10.36 A pumping system must be specified for a lift station at a wastewater treatment facility. The average flow rate is 110 million liters per day and the required lift is 10 m. Non-clogging impellers must be used; about 65 percent efficiency is expected. For convenient installation, electric motors of 37.5 kW or less are desired. Determine the number of motor/pump units needed and recommend an appropriate operating speed.

Given: Data on pumping system

Find: Number of pumps needed; Operating speed

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H$ $\qquad \eta_p = \dfrac{W_h}{W_m}$

The given or available data is

$\rho = 1000 \cdot \dfrac{kg}{m^3}$ $\qquad Q_{total} = 110 \times 10^6 \cdot \dfrac{L}{day}$ $\qquad Q_{total} = 1.273 \dfrac{m^3}{s}$ $\qquad H = 10 \cdot m$ $\qquad \eta = 65 \cdot \%$

Then for the system $\quad W_h = \rho \cdot Q_{total} \cdot g \cdot H \qquad W_h = 125 \cdot kW$

The required total power is $\quad W_m = \dfrac{W_h}{\eta} \qquad W_m = 192 \cdot kW$

Hence the total number of pumps must be $\dfrac{192}{37.5} = 5.12$, or at least six pumps

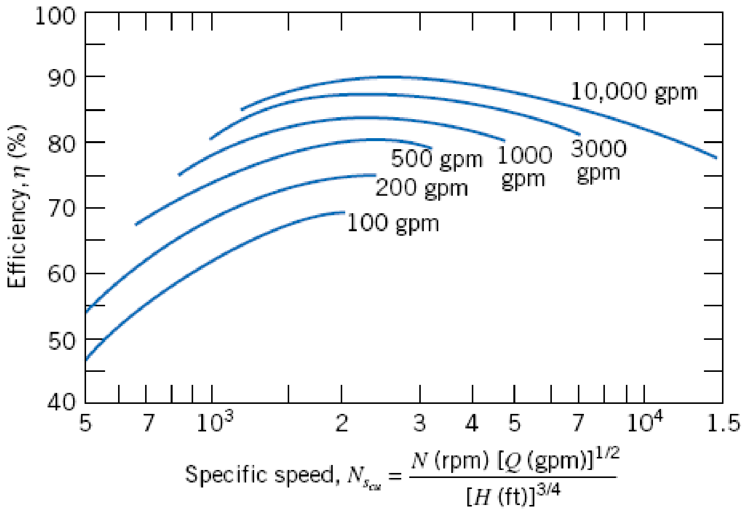
The flow rate per pump will then be $\quad Q = \dfrac{Q_{total}}{6} \qquad Q = 0.212 \dfrac{m^3}{s} \qquad Q = 212 \cdot \dfrac{L}{s}$

From Fig. 10.15 the peak effiiciency is at a specific speed of about

$N_{Scu} = 2000$

We also need $\quad H = 32.8 \cdot ft \qquad Q = 3363 \cdot gpm$

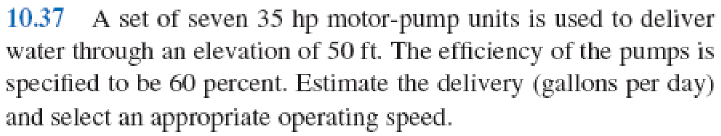
Hence $\quad N = N_{Scu} \cdot \dfrac{H^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad N = 473$



The nearest standard speed to $N = 473$ rpm should be used

Problem 10.37

[3]



10.37 A set of seven 35 hp motor-pump units is used to deliver water through an elevation of 50 ft. The efficiency of the pumps is specified to be 60 percent. Estimate the delivery (gallons per day) and select an appropriate operating speed.

Given: Data on pumping system

Find: Total delivery; Operating speed

Solution:

Basic equations: $W_h = \rho\cdot Q\cdot g\cdot H \qquad \eta_p = \dfrac{W_h}{W_m}$

The given or available data is

$$\rho = 1000\cdot\frac{kg}{m^3} \qquad W_m = 35\cdot hp \qquad H = 50\cdot ft \qquad \eta = 60\cdot\%$$

Then for the system $W_{mTotal} = 7\cdot W_m \qquad W_{mTotal} = 245\cdot hp$

The hydraulic total power is $W_{hTotal} = \dfrac{W_{mTotal}}{\eta} \qquad W_{hTotal} = 304\cdot kW$

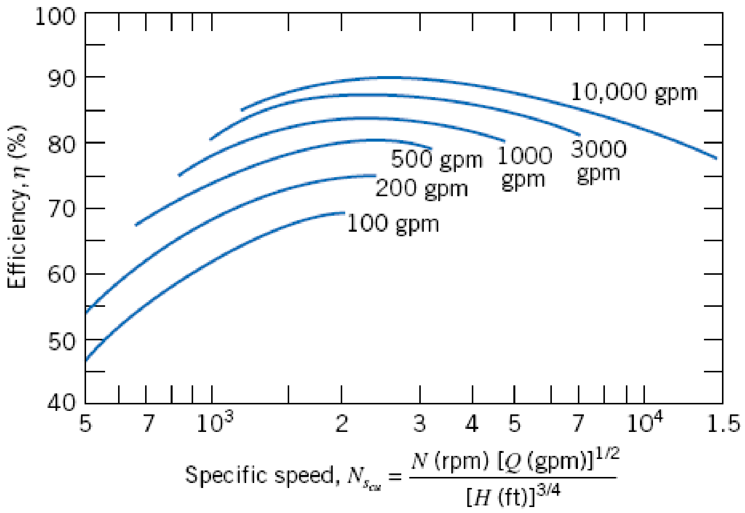
The total flow rate will then be $Q_{Total} = \dfrac{W_{hTotal}}{\rho\cdot g\cdot H} \qquad Q_{Total} = 71.95\cdot\dfrac{ft^3}{s} \qquad Q_{Total} = 32293\cdot gpm$

The flow rate per pump is $Q = \dfrac{Q_{Total}}{6} \qquad Q = 12.0\cdot\dfrac{ft^3}{s} \qquad Q = 5382\cdot gpm$

From Fig. 10.15 the peak effiiciency is at a specific speed of about

$$N_{Scu} = 2500$$

Hence $N = N_{Scu}\cdot\dfrac{H^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad N = 641$



The nearest standard speed to $N = 641$ rpm should be used

Problem 10.38

[3]

10.38 Appendix D contains area bound curves for pump model selection and performance curves for individual pump models. Use these data and the similarity rules to predict and plot the curves of head *H* (ft) versus *Q* (gpm) of a Peerless Type 10AE12 pump, with impeller diameter *D* = 12 in., for nominal speeds of 1000, 1200, 1400, and 1600 rpm.

Given: Data on Peerless Type 10AE12 pump at 1760 rpm

Find: Data at speeds of 1000, 1200, 1400, and 1600 rpm

Solution:

The governing equations are the similarity rules

$$\frac{Q_1}{\omega_1 \cdot D_1^{\,3}} = \frac{Q_2}{\omega_2 \cdot D_2^{\,3}} \qquad (10.19a)$$

$$\frac{h_1}{\omega_1^{\,2} \cdot D_1^{\,2}} = \frac{h2}{\omega_2^{\,2} \cdot D_2^{\,2}} \qquad (10.19b)$$

where $h = g \cdot H$

For scaling from speed ω_1 to speed ω_2, with $D_1 = D_2$ from Eq. 10.19a

$$Q_2 = Q_1 \cdot \frac{\omega_2}{\omega_1}$$

and from Eq. 10.19b $H_2 = H_1 \cdot \left(\frac{\omega_2}{\omega_1}\right)^2$

Speed (rpm) = 1760

Q (gal/min)	Q^2	H (ft)	H (fit)
0	0	170	161
500	250000	160	160
1000	1000000	155	157
1500	2250000	148	152
2000	4000000	140	144
2500	6250000	135	135
3000	9000000	123	123
3500	12250000	110	109
4000	16000000	95	93

Speed (rpm) = 1000

Q (gal/min)	H (ft)
0	52.0
284	51.7
568	50.7
852	49.0
1136	46.6
1420	43.5
1705	39.7
1989	35.3
2273	30.2

Speed (rpm) = 1200

Q (gal/min)	H (ft)
0	74.9
341	74.5
682	73.0
1023	70.5
1364	67.1
1705	62.6
2045	57.2
2386	50.8
2727	43.5

Speed (rpm) = 1400

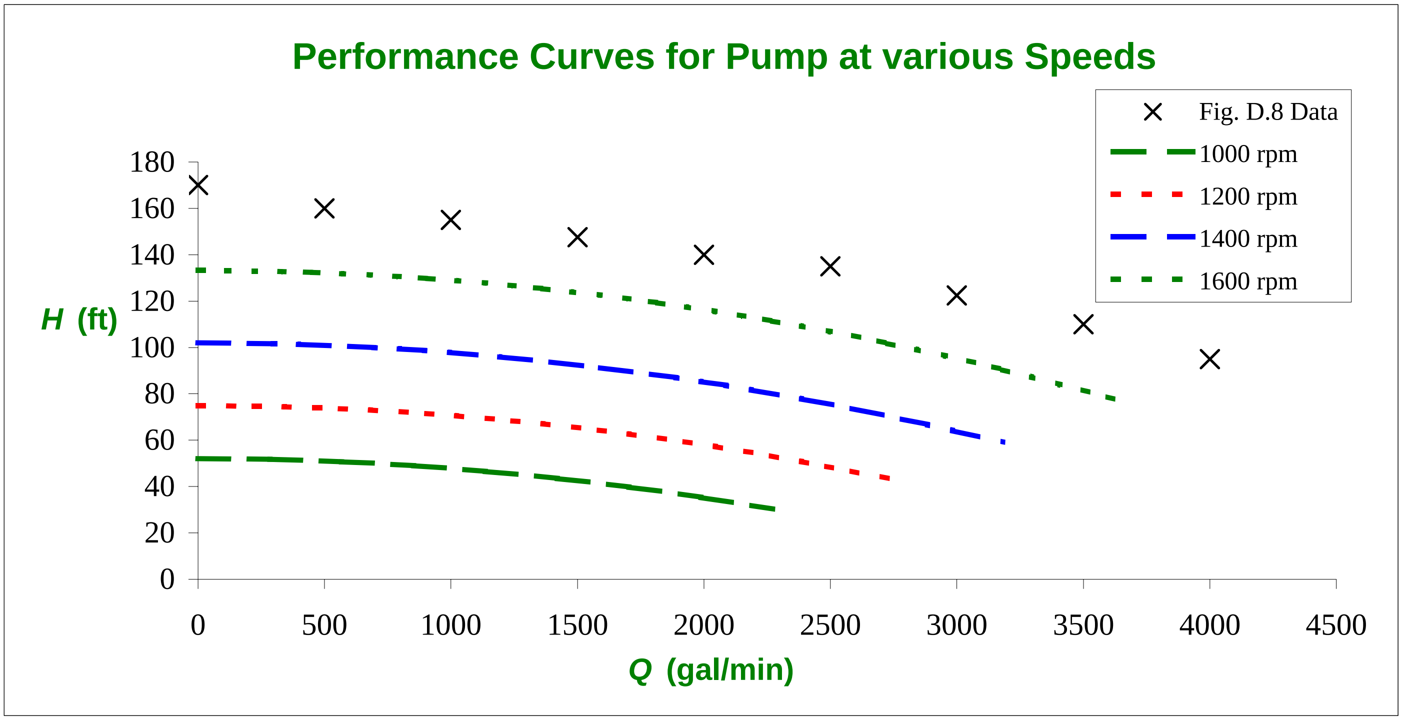
Q (gal/min)	H (ft)
0	102.0
398	101.3
795	99.3
1193	96.0
1591	91.3
1989	85.3
2386	77.9
2784	69.2
3182	59.1

Speed (rpm) = 1600

Q (gal/min)	H (ft)
0	133.2
455	132.4
909	129.7
1364	125.4
1818	119.2
2273	111.4
2727	101.7
3182	90.4
3636	77.2

Data from Fig. D.8 is "eyeballed"
The fit to data is obtained from a least squares fit to $H = H_0 - AQ^2$

$H_0 =$	161	ft
$A =$	4.23E-06	ft/(gal/min)



Problem 10.39

[2]

Given: Area bound curves for pump model selection and performance curves for individual pump models, Appendix D.

Find: Use these data to verify the similarity rules for a Peerless Type 4AE12 pump operated at 1750 and 3550 nominal rpm, with 11.00 in. impeller.

Solution: From Figs. D.4 and D.5, at the best efficiency point (BEP):

N (rpm)	Q (gpm)	H (ft)	$\dot{W}_m$ (hp)	η (%)
1750	470	104	17	73^+
3550	970	430	135	74^+

The similarity rules are

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}, \quad \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}, \quad \frac{\mathscr{P}_1}{\omega_1^3 D_1^5} = \frac{\mathscr{P}_2}{\omega_2^3 D_2^5}, \text{ and } \eta_1 = \eta_2$$

Evaluating, with $D_1 = D_2$,

$$Q_1 = Q_2 \frac{\omega_1}{\omega_2} = 970 \text{ gpm} \, \frac{1750 \text{ rpm}}{3550 \text{ rpm}} = 478 \text{ gpm}$$

$$H_1 = H_2 \left(\frac{\omega_1}{\omega_2}\right)^2 = 430 \text{ ft} \left(\frac{1750 \text{ rpm}}{3550 \text{ rpm}}\right)^2 = 104 \text{ ft}$$

$$\mathscr{P}_1 = \mathscr{P}_2 \left(\frac{\omega_1}{\omega_2}\right)^3 = 135 \text{ hp} \left(\frac{1750 \text{ rpm}}{3550 \text{ rpm}}\right)^3 = 16.2 \text{ hp}$$

$$\eta_1 = \eta_2 = 0.74^+$$

Comparing shows excellent agreement.

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Problem 10.40

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Given: Data from Appendix D for Peerless Type 4AE12 pump.

Find: Verify the similarity rules for the effects of diameter change at 1750 and 3550 nominal rpm.

Solution: From Figs. D.4 and D.5 at the best efficiency point (BEP):

N (rpm)	D (in.)	Q (gpm)	H (ft)	$\dot{W}_m$ (hp)	η (%)
1750	10.00	455	87	13	73^+
	11.00	470	104	17	73^+
	12.12	500	123	22	73^+
3550	10.00	930	360	115	74^+
	11.00	970	430	135	74^+
	12.12	1030	500	180	74^+

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \;;\; \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2} \;,\; \frac{\mathcal{P}_1}{\omega_1^3 D_1^5} = \frac{\mathcal{P}_2}{\omega_2^3 D_2^5} \;, \text{ and } \eta_1 = \eta_2 .$$

Evaluating, with $\omega_1 = \omega_2 = 1750$ rpm,

$$Q_{10} = Q_{11}\left(\frac{10}{11}\right)^3 = 353 \;;\; Q_{12} = Q_{11}\left(\frac{12.12}{11}\right)^3 = 629 \;;\; H_{10} = 86.0 \text{ ft}, \; H_{12} = 126 \text{ ft}$$

$$\mathcal{P}_{10} = 10.6 \text{ hp}, \; \mathcal{P}_{12} = 27.6 \text{ hp} \;;\; \eta = \text{constant}$$

Evaluating, with $\omega_1 = \omega_2 = 3550$ rpm,

$$Q_{10} = 729 \text{ gpm}, \; Q_{12} = 1300 \text{ gpm} ; \; H_{10} = 355 \text{ ft}, \; H_{12} = 522 \text{ ft} \;;\; \mathcal{P}_{10} = 83.8 \text{ hp},$$

$$\mathcal{P}_{12} = 219 \text{ hp} \;;\; \eta = \text{constant}$$

Comparing results with data shows:

(1) flow rate is scaled poorly

(2) head is scaled well

(3) power is scaled poorly (because flow rate is scaled poorly)

Better results are obtained using the modified scaling rules (see p. 526); then

$Q \sim D^2$ so $Q_{10} = 388$ gpm and $\mathcal{P} \sim D^4$ so $\mathcal{P}_{10} = 11.6$ hp at 1750 rpm

and $Q_{10} = 802$ gpm and $\mathcal{P}_{10} = 92.2$ hp at 3550 rpm.

Problem 10.41

[3]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Data in Appendix D for Peerless Type 16A 18 B pump.

Find: Verify the similarity rules for (a) impeller diameter change and (b) speed change.

Solution: From Figs. D.9 and D.10 at the best efficiency point (BEP):

N (rpm)	D (in.)	Q (gpm)	H (ft)	Wm (hp)	η (%)
705	18.0	6250	42	76	86+
	17.0	5850	37	63	86+
	16.0	5600	32	54	86
880	18.0	7900	69	155	87+
	17.0	7400	59	125	87
	16.0	7100	50	105	85

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}, \quad \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}, \quad \frac{\mathcal{P}_1}{\omega_1^3 D_1^5} = \frac{\mathcal{P}_2}{\omega_2^3 D_2^5}, \text{ and } \eta_1 = \eta_2$$

Evaluating with $\omega_1 = \omega_2 = 705$ rpm,

$Q_{17} = Q_{18}\left(\frac{17}{18}\right)^3 = 5270$ gpm, $Q_{16} = 4390$ gpm; $H_{17} = H_{18}\left(\frac{17}{18}\right)^2 = 37.5$ ft,

$H_{16} = 33.2$ ft; $\mathcal{P}_{17} = \mathcal{P}_{18}\left(\frac{17}{18}\right)^5 = 57.1$ hp, $\mathcal{P}_{16} = 42.2$ hp; η = constant

At 880 rpm, $Q_{17} = 6660$ gpm, $Q_{16} = 5550$ gpm; $H_{17} = 61.5$ ft, $H_{16} = 54.5$ ft;

$\mathcal{P}_{17} = 116$ hp, $\mathcal{P}_{16} = 86.0$ hp; η = constant

Evaluating with $D_1 = D_2 = 18.0$ in.,

$Q_{705} = Q_{880}\left(\frac{705}{880}\right) = 6330$ gpm; $H_{705} = H_{880}\left(\frac{705}{880}\right)^2 = 44.3$ ft;

$\mathcal{P}_{705} = \mathcal{P}_{880}\left(\frac{705}{880}\right)^3 = 79.7$ hp; η = constant

Comparing results with data shows at constant speed:

(1) flow rate scales poorly, (2) head scales well, (3) power scales poorly with changes in diameter.

Comparing results with data shows at constant diameter:

all quantities scale well with changes in speed.

Flow rate scaling may be improved using the modified procedure discussed on page 526, in which $Q \sim D^2$ and $\mathcal{P} \sim D^4$.

Problem 10.42

Given: Performance curves for Peerless Type 16 A 18B pump, Appendix D (with $D = 18.0$ in. impeller).

Find: (a) Develop and plot curve-fits for 705 and 880 nominal rpm.
(b) Verify the effect of pump speed on scaling pump curves using the procedure of Example Problem 10.7.

Solution: Tabulate performance data and curve-fits:

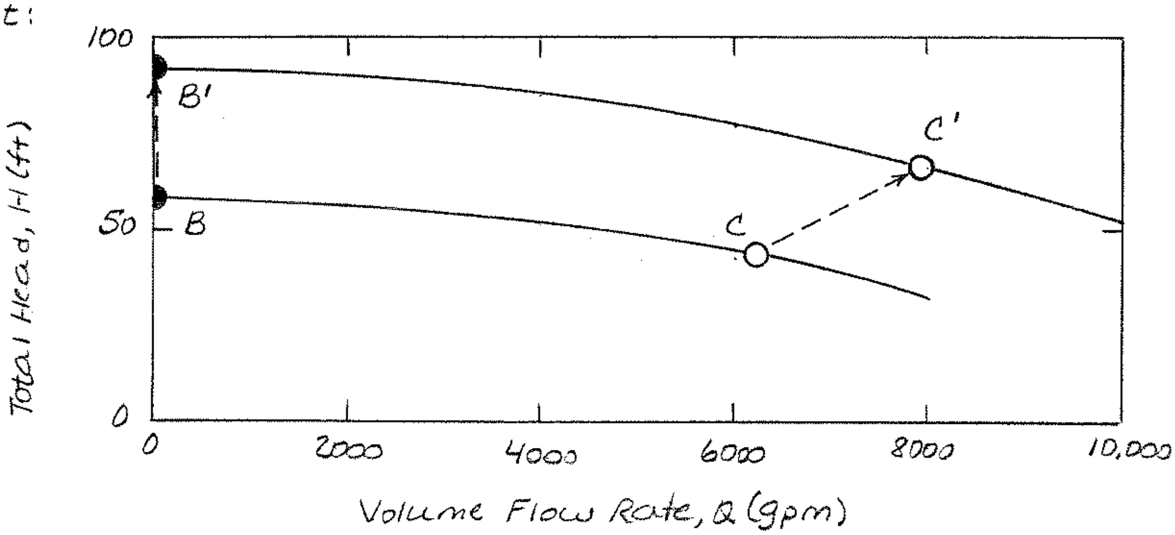
705 rpm:								
	Q (gpm)	0	2000	4000	6000	8000	BEP:	6250
	H (ft)	59	56	50	43	32		42

Curve-fit: $\hat{H}(ft) = 57.8 - 4.09 \times 10^{-7}[Q(gpm)]^2$; $r^2 = 0.994$

880 rpm:								
	Q (gpm)	0	2000	4000	6000	8000	BEP:	7900
	H (ft)	92	89	84	78	68		69

Curve-fit: $\hat{H}(ft) = 91.5 - 4.01 \times 10^{-7}[Q(gpm)]^2$; $r^2 = 0.992$

Plot:



Using the procedure of Example Problem 10.7:

$$Q_{B'} = Q_B = 0;\ H_{B'} = H_B\left(\frac{\omega'}{\omega}\right)^2 = 59\,ft\left(\frac{880}{705}\right)^2 = 91.9\ ft$$

$$Q_{C'} = Q_C\left(\frac{\omega'}{\omega}\right) = 6250\,gpm\left(\frac{880}{705}\right) = 7800\ gpm$$

$$H_{C'} = H_C\left(\frac{\omega'}{\omega}\right)^2 = 42\,ft\left(\frac{880}{705}\right)^2 = 65.4\ ft$$

Comparing the curve-fit parameters shows good agreement:

$\hat{H} = H_0 - AQ^2$ $\quad H_0 = 91.5$ ft compared to $H_{B'} = 91.9$ ft

$$\left.\begin{aligned} A' &= 4.01 \times 10^{-7}\ ft/(gpm)^2 \\ A' &= 4.09 \times 10^{-7}\ ft/(gpm)^2 \end{aligned}\right\}\ \text{within } 2.0\%$$

Problem 10.43

Given: Performance curves for Peerless Type 10AE12 pump, Appendix D.

Find: (a) Develop and plot a curve-fit for 1760 nominal rpm.
(b) Scale the curve-fit to a pump speed of 1150 nominal rpm, using the procedure of Example Problem 10.7.

Solution: Tabulate performance data and curve-fit, for $D = 12$ in. diameter impeller at 1760 nominal rpm:

Q(gpm)	1500	2000	2500	3000	3500	4000
H(ft)	148	141	133	123	110	95
$\hat{H}$(ft)	148	141	133	122	110	95.5

Curve-fit: $\hat{H}(ft) = 157 - 3.83\times10^{-6}[Q(gpm)]^2$; $r^2 = 0.999$

or $\hat{H} = H_0 - AQ^2$

The similarity rules are

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \; ; \; \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}$$

The pump diameter stays constant, so

$$Q_2 = Q_1\left(\frac{\omega_2}{\omega_1}\right) \text{ and } H_2 = H_1\left(\frac{\omega_2}{\omega_1}\right)^2 = H_1\left(\frac{1150}{1760}\right)^2 = 0.427\, H_1$$

Following the procedure of Example Problem 10.7, then at 1150 rpm,

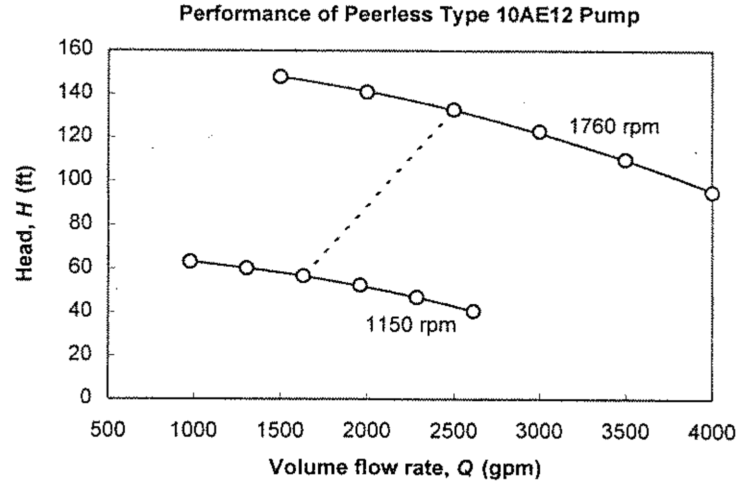
$$\hat{H}(ft) = 0.427\, H_0 - AQ^2$$

$$= (0.427)\, 157\ ft - 3.83\times10^{-6}[Q(gpm)]^2$$

$$\hat{H}(ft) = 67.0\ ft - 3.83\times10^{-6}[Q(gpm)]^2$$

$\hat{H}$(ft) (1150 rpm)

The plot is:



42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 10.44

Open-Ended Problem Statement: Problem 10.20 suggests that pump head at best efficiency is typically about 70 percent of shutoff head. Use pump data from Appendix D to evaluate this suggestion. A further suggestion in Section 10-4 is that the appropriate scaling for tests of a pump casing with different impeller diameters is $Q \propto D^2$. Use pump data to evaluate this suggestion.

Discussion: Data selected from pump performance curves in Appendix D is tabulated and plotted on the next page. Data were selected at the maximum efficiency point for the largest ($D_{\max}$) and smallest ($D_{\min}$) diameter impellers with which each pump was tested.

The head at the best efficiency point with the largest impeller was selected to compare with the shutoff head for the same impeller. These data are shown in the first graph, where they are compared to the average ratio, $H_{\mathrm{BEP}} = 0.766\ H_0$. There is some scatter, but the trend of agreement is fairly clear. The actual values suggest a higher ratio than the 0.7 mentioned in Problem 10.20.

The flow rate ratio $Q_{\max}/Q_{\min}$ was compared with the square of the impeller diameter ratio $(D_{\max}/D_{\min})^2$. These data ratios are shown in the second graph, where they are compared to the correlation line. Agreement is not perfect, but the trend supports a positive correlation of 0.751. The predicted relationship between diameter and flow rate is $Q_{\max}/Q_{\min} = 0.751\ (D_{\max}/D_{\min})^2$.

(Use of three significant figures probably is not justified in this problem. The data are read from small graphs in the Appendix that have already been smoothed by the manufacturer. Also there is some uncertainty in selecting the best efficiency point on each curve.)

Problem 10.44

Sample	Fig.	Model	Speed (rpm)	H_0 (ft)	H_{BEP} (ft)	H_{BEP}/H_0 (---)
1	D.3	4AE11	1750	113	95	0.84
2	D.5	4AE12	3550	636	500	0.79
3	D.6	6AE14	1750	209	160	0.77
4	D.7	8AE20G	1770	430	365	0.85
5	D.8	10AE12	1760	170	112	0.66
6	D.9	16A18B	705	59	42	0.71
7	D.10	16A18B	880	92	69	0.75
					Average:	0.766

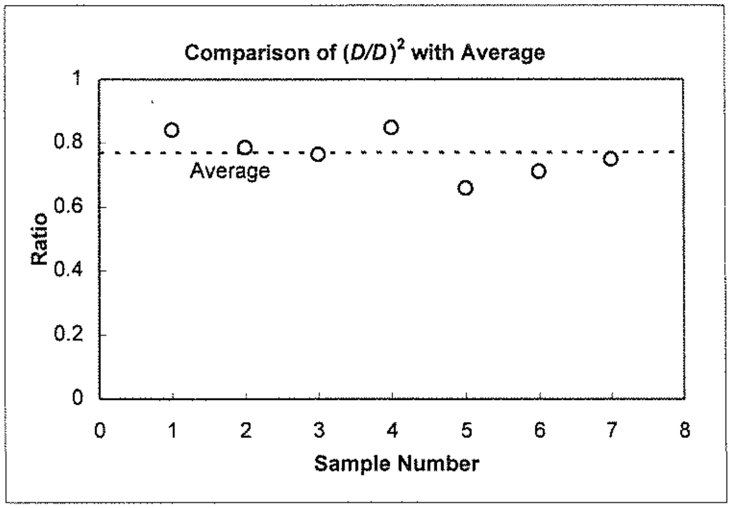
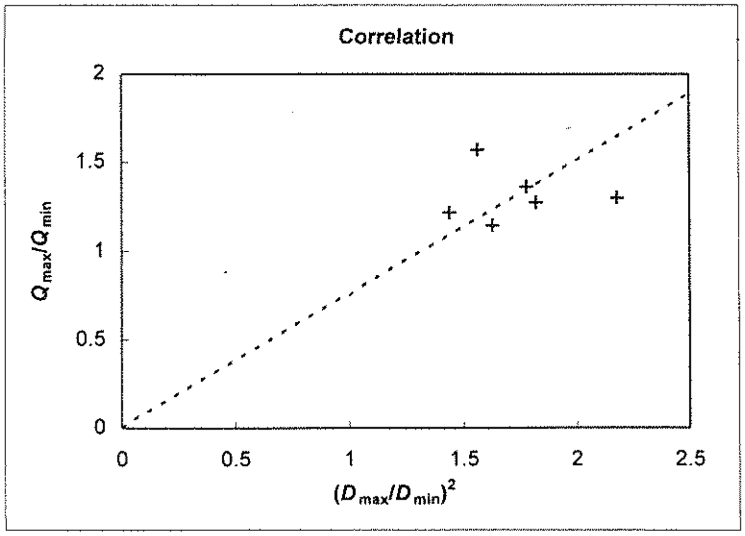


Fig.	Model	Speed (rpm)	D_{min} (in.)	Q_{BEP} (gpm)	D_{max} (in.)	Q_{BEP} (gpm)	$(D_{max}/D_{min})^2$ (---)	Q_{max}/Q_{min} (---)
D.3	4AE11	1750	7.62	740	11.25	960	2.2	1.3
D.5	4AE12	3550	9.5	910	12.12	1040	1.6	1.1
D.6	6AE14	1750	10.38	1375	14.0	1750	1.8	1.3
D.7	8AE20G	1770	16.0	2200	20.0	3450	1.6	1.6
D.8	10AE12	1760	9.0	2500	12.0	3400	1.8	1.4
D.9	16A18B	705	15.0	5100	18.0	6200	1.4	1.2
D.10	16A18B	880	15.0	6500	18.0	7900	1.4	1.2
						Correlation:		0.751



Problem 10.45

[4]

Given: Catalog data for centrifugal pump at design conditions:

$$Q = 250 \text{ gpm} \quad \Delta p = 18.6 \text{ psi} \quad N = 1750 \text{ rpm}$$

Laboratory flume requires $Q_f = 200$ gpm at $H_f = 32$ ft; the only available motor develops 3 hp at 1750 rpm.

Find: (a) Is motor suitable?
(b) How might the pump/motor match be improved?

Solution: To obtain efficiency and pump power requirement, find specific speed.

$$H = \frac{\Delta p}{\rho g} = 18.6 \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{62.4 \text{ lbf}} \times 144 \frac{\text{in}^2}{\text{ft}^2} = 42.9 \text{ ft} \quad ; Q = \frac{250 \text{ gal}}{\text{min}} = 0.557 \text{ cfs}$$

$$N_{S_{cu}} = \frac{N Q^{1/2}}{H^{3/4}} = \frac{1750 \text{ rpm} (250 \text{ gpm})^{1/2}}{(42.9 \text{ ft})^{3/4}} = 1650$$

From Fig. 10.15, $\eta \approx 0.73$. Thus

$$\dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho Q g H}{\eta} = \frac{1}{0.73} \times 62.4 \frac{\text{lbf}}{\text{ft}^3} \times 0.557 \frac{\text{ft}^3}{\text{s}} \times 42.9 \text{ ft} \times \frac{\text{hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lbf}} = 3.71 \text{ hp}$$

The motor is not suitable to drive the pump directly.

The pump at 1750 rpm produces more head and flow than necessary. It may be run at reduced speed, e.g., by using a belt drive.

To produce $Q_f = 200$ gpm, solve $\frac{Q_p}{\omega_p D_p^3} = \frac{Q_f}{\omega_f D_f^3}$; $\omega_f = \frac{200}{250} \times 1750 = 1400$ rpm

To produce $H_f = 32$ ft, solve $\frac{H_p}{\omega_p^2 D_p^2} = \frac{H_f}{\omega_f^2 D_f^2}$; $\omega_f = \sqrt{\frac{H_f}{H_p}}\,\omega_p = \sqrt{\frac{32}{42.9}} \times 1750 = 1510$ rpm

At 1510 rpm the power requirement will be given by $\frac{\mathcal{P}_p}{\omega_p^3 D_p^5} = \frac{\mathcal{P}_f}{\omega_f^3 D_f^5}$, so

$$\mathcal{P}_f = \mathcal{P}_p \left(\frac{\omega_f}{\omega_p}\right)^3 = 3.71 \text{ hp} \left(\frac{1510}{1750}\right)^3 = 2.38 \text{ hp}$$

This is well within the capability of the 3 hp motor. Therefore run pump at 1510 rpm.

Problem 10.46

[3]

10.46 A reaction turbine is designed to produce 17.5 MW at 120 rpm under 45 m of head. Laboratory facilities are available to provide 10 m of head and to absorb 35 kW from the model turbine. Assume comparable efficiencies for the model and prototype turbines. Determine the appropriate model test speed, scale ratio, and volume flow rate.

Given: Data on turbine system

Find: Model test speed; Scale; Volume flow rate

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H$ $\quad \eta = \dfrac{W_{mech}}{W_h}$ $\quad N_S = \dfrac{\omega \cdot P^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h^{\frac{5}{4}}}$

The given or available data is

$\rho = 1000 \cdot \dfrac{kg}{m^3}$ $\quad W_p = 17.5 \cdot MW$ $\quad H_p = 45 \cdot m$ $\quad \omega_p = 120 \cdot rpm$ $\quad H_m = 10 \cdot m$ $\quad W_m = 35 \cdot kW$

where sub p stands for prototype and sub m stands for model

Note that we need h (energy/mass), not H (energy/weight) $\quad h_p = H_p \cdot g$ $\quad h_p = 441 \dfrac{m^2}{s^2}$ $\quad h_m = H_m \cdot g$ $\quad h_m = 98.1 \dfrac{m^2}{s^2}$

Hence for the prototype $\quad N_S = \dfrac{\omega_p \cdot W_p^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h_p^{\frac{5}{4}}}$ $\quad N_S = 0.822$

Then for the model $\quad N_S = \dfrac{\omega_m \cdot W_m^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h_m^{\frac{5}{4}}}$ $\quad \omega_m = N_S \cdot \dfrac{\rho^{\frac{1}{2}} \cdot h_m^{\frac{5}{4}}}{W_m^{\frac{1}{2}}}$ $\quad \omega_m = 42.9 \dfrac{rad}{s}$ $\quad \omega_m = 409\, rpm$

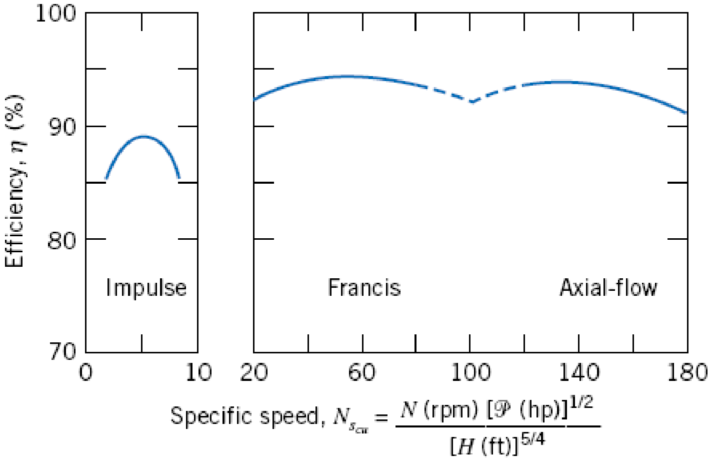
For dynamically similar conditions $\quad \dfrac{H_p}{\omega_p^{\,2} \cdot D_p^{\,2}} = \dfrac{H_m}{\omega_m^{\,2} \cdot D_m^{\,2}}$ $\quad$ so $\quad \dfrac{D_m}{D_p} = \dfrac{\omega_p}{\omega_m} \cdot \sqrt{\dfrac{H_m}{H_p}} = 0.138$

Also $\quad \dfrac{Q_p}{\omega_p \cdot D_p^{\,3}} = \dfrac{Q_m}{\omega_m \cdot D_m^{\,3}}$ $\quad$ so $\quad Q_m = Q_p \cdot \dfrac{\omega_m}{\omega_p} \cdot \left(\dfrac{D_m}{D_p}\right)^3$

To find Q_p we need efficiency. At $W_p = 17.5\, MW$ or $W_p = 23468\, hp$ and $H_p = 45\, m$ or $H_p = 148\, ft$

from F ig. 10.17 we find (see below), for

$$N_{Scu} = \frac{N(rpm) \cdot P(hp)^{\frac{1}{2}}}{H(ft)^{5.4}} = 35.7 \qquad \eta = 93 \cdot \%$$

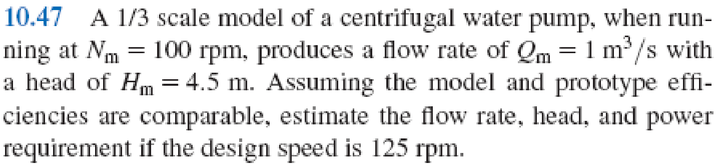


Hence from $\eta = \frac{W_{mech}}{W_h} = \frac{W_{mech}}{\rho \cdot Q \cdot g \cdot H}$ $\quad Q_p = \frac{W_p}{\rho \cdot g \cdot H_p \cdot \eta}$ $\quad Q_p = 42.6 \frac{m^3}{s}$

and also $Q_m = \frac{W_m}{\rho \cdot g \cdot H_m \cdot \eta}$ $\quad Q_m = 0.384 \frac{m^3}{s}$

Problem 10.47

[3]



10.47 A 1/3 scale model of a centrifugal water pump, when running at $N_m = 100$ rpm, produces a flow rate of $Q_m = 1\ m^3/s$ with a head of $H_m = 4.5$ m. Assuming the model and prototype efficiencies are comparable, estimate the flow rate, head, and power requirement if the design speed is 125 rpm.

Given: Data on a model pump

Find: Prototype flow rate, head, and power at 125 rpm

Solution:

Basic equation: $W_h = \rho \cdot Q \cdot g \cdot H$ and similarity rules

$$\frac{Q_1}{\omega_1 \cdot D_1^3} = \frac{Q_2}{\omega_2 \cdot D_2^3} \quad (10.19a) \qquad \frac{h_1}{\omega_1^2 \cdot D_1^2} = \frac{h2}{\omega_2^2 \cdot D_2^2} \quad (10.19b) \qquad \frac{P_1}{\rho_1 \cdot \omega_1^3 \cdot D_1^5} = \frac{P_2}{\rho_2 \cdot \omega_2^3 \cdot D_2^5} \quad (10.19a)$$

The given or available data is $N_m = 100 \cdot rpm$ $\quad N_p = 125 \cdot rpm$ $\quad \rho = 1000 \cdot \frac{kg}{m^3}$

$$Q_m = 1 \cdot \frac{m^3}{s} \qquad H_m = 4.5 \cdot m$$

From Eq. 10.8a $\quad W_{hm} = \rho \cdot Q_m \cdot g \cdot H_m \qquad W_{hm} = 44.1 \cdot kW$

From Eq. 10.19a (with *D*$_m$/*D*$_p$ = 1/3)

$$\frac{Q_p}{\omega_p \cdot D_p^3} = \frac{Q_m}{\omega_m \cdot D_m^3} \quad \text{or} \quad Q_p = Q_m \cdot \frac{\omega_p}{\omega_m} \cdot \left(\frac{D_p}{D_m}\right)^3 = 3^3 \cdot Q_m \cdot \frac{\omega_p}{\omega_m}$$

$$Q_p = 27 \cdot Q_m \cdot \frac{N_p}{N_m} \qquad Q_p = 33.8 \frac{m^3}{s}$$

From Eq. 10.19b (with *D*$_m$/*D*$_p$ = 1/3)

$$\frac{h_p}{\omega_p^2 \cdot D_p^2} = \frac{h_m}{\omega_m^2 \cdot D_m^2} \quad \text{or} \quad \frac{g \cdot H_p}{\omega_p^2 \cdot D_p^2} = \frac{g \cdot H_m}{\omega_m^2 \cdot D_{pm}^2}$$

$$H_p = H_m \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \cdot \left(\frac{D_p}{D_m}\right)^2 = 3^2 \cdot H_m \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \qquad H_p = 9 \cdot H_m \cdot \left(\frac{N_p}{N_m}\right)^2 \qquad H_p = 63.3\,m$$

From Eq. 10.19c (with *D*$_m$/*D*$_p$ = 1/3)

$$\frac{P_p}{\rho \cdot \omega_p^3 \cdot D_p^5} = \frac{P_m}{\rho \cdot \omega_m^3 \cdot D_m^5} \quad \text{or} \quad W_{hp} = W_{hm} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3 \cdot \left(\frac{D_p}{D_m}\right)^5 = 3^5 \cdot W_{hm} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3$$

$$W_{hp} = 243 \cdot W_{hm} \cdot \left(\frac{N_p}{N_m}\right)^3 \qquad W_{hp} = 20.9 \cdot MW$$

Problem 10.48

[4]

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Pump to operate at $Q = 250$ cfs, $H = 400$ ft, and $N = 870$ rpm.

Model test to be run in facility where $Q \leq 5$ cfs and a 300 hp dynamometer is available. Assume model and prototype efficiencies are comparable.

Find: Appropriate model test speed and scale ratio.

Solution: To obtain homologous operating points, run model test at same specific speed as prototype.

$$Q = 250\,\frac{\text{ft}^3}{\text{sec}} \times 7.48\,\frac{\text{gal}}{\text{ft}^3} \times 60\,\frac{\text{sec}}{\text{min}} = 112{,}000 \text{ gpm}$$

$$N_{s_{cu}} = \frac{NQ^{1/2}}{H^{3/4}} = \frac{870 \text{ rpm}\,(112{,}000 \text{ gpm})^{1/2}}{(400 \text{ ft})^{3/4}} = 3260$$

(The dimensionless specific speed is $N_{s,nd} = 3260/2733 = 1.19$. Figure 10.14 indicates a mixed-flow geometry.) Figure 10.15 indicates $\eta = 0.92$ at this N_s. Thus

$$\dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho Q g H}{\eta} = \frac{1}{0.92} \times 62.4\,\frac{\text{lbf}}{\text{ft}^3} \times 250\,\frac{\text{ft}^3}{\text{s}} \times 400 \text{ ft} \times \frac{\text{hp}\cdot\text{s}}{550 \text{ ft}\cdot\text{lbf}} = 12{,}300 \text{ hp}$$

For the model,

$$H_m = \eta \dot{W}_m / \rho Q g = 0.92 \times 300 \text{ hp} \times \frac{\text{ft}^3}{62.4 \text{ lbf}} \times \frac{\text{s}}{5 \text{ ft}^3} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 487 \text{ ft}$$

To match specific speeds, then

$$N_{m_{cu}} = N_{s_{cu}} \frac{H_m^{3/4}}{Q_m^{1/2}} = 3260 \frac{(487 \text{ ft})^{3/4}}{(2240 \text{ gpm})^{1/2}} = 7140 \text{ rpm} \longleftarrow N_m$$

The scale ratio may be obtained from the scaling laws. For example, since

$$\frac{Q_m}{\omega_m D_m^3} = \frac{Q_p}{\omega_p D_p^3} \qquad \frac{D_m}{D_p} = \left[\frac{Q_m}{Q_p}\,\frac{\omega_p}{\omega_m}\right]^{\frac{1}{3}} = \left[\frac{1}{50} \times \frac{870}{7140}\right]^{\frac{1}{3}} = 0.135 \longleftarrow D_m/D_p$$

Thus $D_m = 0.135\,D_p$ (scale ratio is $1/0.135 = 7.43$ to 1)

Check using the head ratio, $\dfrac{H_m}{\omega_m^2 D_m^2} = \dfrac{H_p}{\omega_p^2 D_p^2}$

$$H_m = H_p \left(\frac{\omega_m}{\omega_p}\right)^2 \left(\frac{D_m}{D_p}\right)^2 = 400 \text{ ft} \left(\frac{7140}{870}\right)^2 (0.135)^2 = 491 \text{ ft} \approx 487 \text{ ft}$$

This is acceptable agreement, considering roundoff error.

{Great care would be needed to avoid cavitation in the model pump at speeds above 7000 rpm.}

Problem 10.49

[5]

Given: Model efficiency using curve-fit, $\eta = aQ - bQ^3$, where a and b are constants.

Find: (a) Describe a procedure to evaluate a and b from data.
(b) Evaluate using data for Peerless Type 10AE12 pump, with impeller diameter $D = 12.0$ in., operating at 1760 rpm.

Solution: From Fig. D.8, data are:

η (%)	70	75	80	84	86	86	84
Q (gpm)	1850	2100	2400	2780	3100	3700	4075

Two equations are needed to solve for constants a and b directly. A second equation may be obtained by differentiating. At peak efficiency,

$$\frac{d\eta}{dQ} = a - 3bQ^2 = 0$$

Assume peak efficiency is 87 percent at 3400 gpm. Then

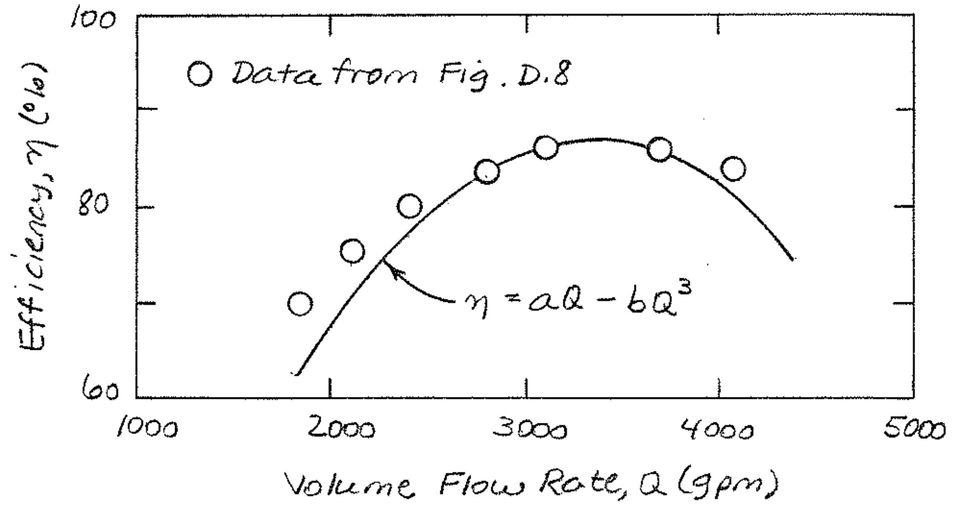
$$\eta_{max} = aQ - bQ^3$$
$$0 = a - 3bQ^2$$

Substituting from the second equation into the first gives

$$\eta_{max} = 3bQ^3 - bQ^3 = 2bQ^3 \; ; \; b = \frac{\eta_{max}}{2Q^3} = 1.11 \times 10^{-9} \; ; \; a = 3bQ^2 = 0.0384$$

a, b

Plotting:

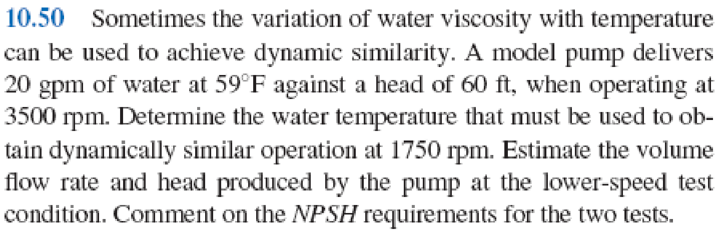


The curve-fit does a good job near peak efficiency, but tends to underestimate the measured data elsewhere.

{ An alternative curve-fit procedure is to plot η/Q versus $a - bQ^2$, then do a least-squares fit (using all the data) to obtain a and b. Then $a = 0.0426\,(\text{gpm})^{-1}$, $b = -1.56 \times 10^{-9}\,(\text{gpm})^{-2}$; $r^2 = 0.996$. This underestimates η at $Q > 3500$ gpm. }

Problem 10.50

[2]



10.50 Sometimes the variation of water viscosity with temperature can be used to achieve dynamic similarity. A model pump delivers 20 gpm of water at 59°F against a head of 60 ft, when operating at 3500 rpm. Determine the water temperature that must be used to obtain dynamically similar operation at 1750 rpm. Estimate the volume flow rate and head produced by the pump at the lower-speed test condition. Comment on the *NPSH* requirements for the two tests.

Given: Data on a model pump

Find: Temperature for dynamically similar operation at 1750 rpm; Flow rate and head; Comment on NPSH

Solution:

Basic equation: $\mathrm{Re}_1 = \mathrm{Re}_2$ and similarity rules $\dfrac{Q_1}{\omega_1 \cdot D_1^{\,3}} = \dfrac{Q_2}{\omega_2 \cdot D_2^{\,3}}$ $\quad \dfrac{H_1}{\omega_1^{\,2} \cdot D_1^{\,2}} = \dfrac{H2}{\omega_2^{\,2} \cdot D_2^{\,2}}$

The given or available data is $\omega_1 = 3500 \cdot \mathrm{rpm}$ $\quad \omega_2 = 1750 \cdot \mathrm{rpm}$ $\quad Q_1 = 20 \cdot \mathrm{gpm}$ $\quad H_1 = 60 \cdot \mathrm{ft}$

From Table A.7 at 59ºF $\quad \nu_1 = 1.23 \times 10^{-5} \cdot \dfrac{\mathrm{ft}^2}{\mathrm{s}}$

For D = constant $\quad \mathrm{Re}_1 = \dfrac{V_1 \cdot D}{\nu_1} = \dfrac{\omega_1 \cdot D \cdot D}{\nu_1} = \mathrm{Re}_2 = \dfrac{\omega_2 \cdot D \cdot D}{\nu_2}$ $\quad$ or $\quad \nu_2 = \nu_1 \cdot \dfrac{\omega_2}{\omega_1}$ $\quad \nu_2 = 6.15 \times 10^{-6} \cdot \dfrac{\mathrm{ft}^2}{\mathrm{s}}$

From Table A.7, at $\nu_2 = 6.15 \times 10^{-6} \cdot \dfrac{\mathrm{ft}^2}{\mathrm{s}}$, we find, by linear interpolation

$T_2 = 110 + \dfrac{(120 - 110)}{(6.05 - 6.68)} \cdot (6.15 - 6.68)$ $\quad T_2 = 118$ degrees F

From similar operation $\quad \dfrac{Q_1}{\omega_1 \cdot D^3} = \dfrac{Q_2}{\omega_2 \cdot D^3}$ $\quad$ or $\quad Q_2 = Q_1 \cdot \dfrac{\omega_2}{\omega_1}$ $\quad Q_2 = 10 \cdot \mathrm{gpm}$

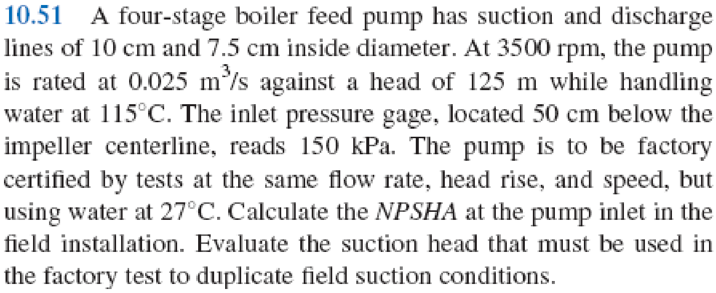
and also $\quad \dfrac{H_1}{\omega_1^{\,2} \cdot D^2} = \dfrac{H_2}{\omega_2^{\,2} \cdot D^2}$ $\quad$ or $\quad H_2 = H_1 \cdot \left(\dfrac{\omega_2}{\omega_1}\right)^2$ $\quad H_2 = 15 \cdot \mathrm{ft}$

The water at 118ºF is closer to boiling. The inlet pressure would have to be changed to avoid cavitation. The increase between runs 1 and 2 would have to be $\Delta p = p_{v2} - p_{v1}$ where p_{v2} and p_{v1} are the vapor pressures at T_2 and T_1. From the steam tables (find them by Googling!)

$p_{v1} = 0.247 \cdot \mathrm{psi}$ $\quad p_{v2} = 1.603 \cdot \mathrm{psi}$ $\quad \Delta p = p_{v2} - p_{v1}$ $\quad \Delta p = 1.36 \cdot \mathrm{psi}$

Problem 10.51

[3]



10.51 A four-stage boiler feed pump has suction and discharge lines of 10 cm and 7.5 cm inside diameter. At 3500 rpm, the pump is rated at 0.025 m^3/s against a head of 125 m while handling water at 115°C. The inlet pressure gage, located 50 cm below the impeller centerline, reads 150 kPa. The pump is to be factory certified by tests at the same flow rate, head rise, and speed, but using water at 27°C. Calculate the *NPSHA* at the pump inlet in the field installation. Evaluate the suction head that must be used in the factory test to duplicate field suction conditions.

Given: Data on a boiler feed pump

Find: NPSHA at inlet for field temperature water; Suction head to duplicate field conditions

Solution:

Basic equation:

$$NPSHA = p_t - p_v = p_g + p_{atm} + \frac{1}{2}\cdot\rho\cdot V^2 - p_v$$

Given or available data is

$$D_s = 10\cdot cm \qquad D_d = 7.5\cdot cm \qquad H = 125\cdot m \qquad Q = 0.025\cdot\frac{m^3}{s}$$

$$p_{inlet} = 150\cdot kPa \qquad p_{atm} = 101\cdot kPa \qquad z_{inlet} = -50\cdot cm \qquad \rho = 1000\cdot\frac{kg}{m^3} \qquad \omega = 3500\cdot rpm$$

For field conditions

$$p_g = p_{inlet} + \rho\cdot g\cdot z_{inlet} \qquad p_g = 145\,kPa$$

From continuity

$$V_s = \frac{4\cdot Q}{\pi\cdot D_s^{\,2}} \qquad V_s = 3.18\,\frac{m}{s}$$

From steam tables (try Googling!) at 115ºC $p_v = 169\cdot kPa$

Hence

$$NPSHA = p_g + p_{atm} + \frac{1}{2}\cdot\rho\cdot V_s^{\,2} - p_v \qquad NPSHA = 82.2\,kPa$$

Expressed in meters or feet of water

$$\frac{NPSHA}{\rho\cdot g} = 8.38\,m \qquad \frac{NPSHA}{\rho\cdot g} = 27.5\,ft$$

In the laboratory we must have the same NPSHA. From Table A.8 (or steam tables - try Googling!) at 27ºC $p_v = 3.57\cdot kPa$

Hence

$$p_g = NPSHA - p_{atm} - \frac{1}{2}\cdot\rho\cdot V_s^{\,2} + p_v \qquad p_g = -20.3\,kPa$$

The absolute pressure is $p_g + p_{atm} = 80.7\,kPa$

Problem 10.52

[2]

10.52 Data from tests of a pump operated at 1500 rpm, with a 30-cm diameter impeller, are

Flow rate, *Q* ($m^3/s \times 10^3$)	**10**	**20**	**30**	**40**	**50**	**60**	**70**
Net positive suction head required, *NPSR* (m)	2.2	2.4	2.6	3.1	3.6	4.1	5.1

Develop and plot a curve-fit equation for *NPSHR* versus volume flow rate in the form $NPSHR = a + bQ^2$, where a and b are constants. If the $NPSHA = 6$ m, estimate the maximum allowable flow rate of this pump.

Given: Data on a NPSHR for a pump

Find: Curve fit; Maximum allowable flow rate

Solution:

Q ($m^3/s \times 10^3$)	Q^2	*NPSHR* (m)	*NPSHR* (fit)
10	1.00E+02	2.2	2.2
20	4.00E+02	2.4	2.4
30	9.00E+02	2.6	2.7
40	1.60E+03	3.1	3.1
50	2.50E+03	3.6	3.6
60	3.60E+03	4.1	4.2
70	4.90E+03	5.1	5.0

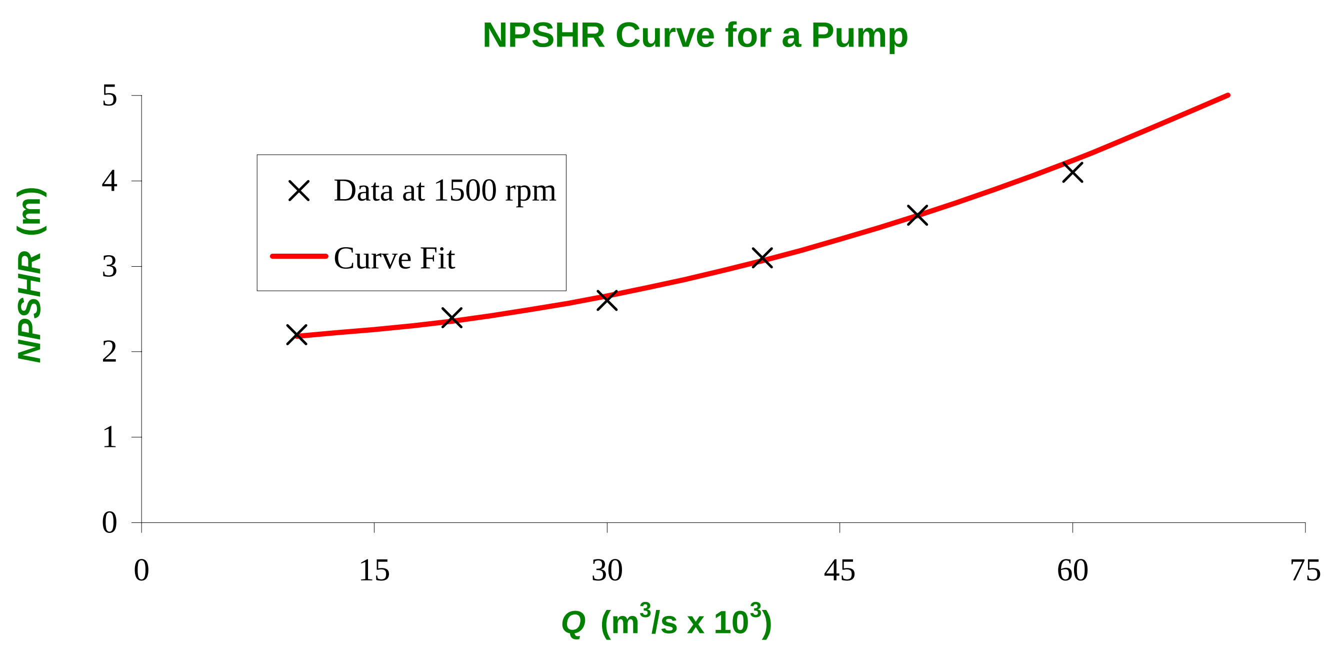
The fit to data is obtained from a least squares fit to $NPSHR = a + bQ^2$

$a =$ 2.12 m

$b =$ 5.88E-04 $m/(m^3/s \times 10^3)^2$

Q ($m^3/s \times 10^3$)	*NPSHR* (m)
81.2	6.00

Use *Goal Seek* to find *Q*!



Problem 10.53

Open-Ended Problem Statement: A large deep fryer at a snack-food plant contains hot oil that is circulated through a heat exchanger by pumps. Solid particles and water droplets coming from the food product are observed in the flowing oil. What special factors must be considered in specifying the operating conditions for the pumps?

Discussion: Any solid particles must be able to pass through the pumps without clogging. If the particles are large, this may require larger than normal clearances within the pumps.

If the water droplets flashed to steam, they would form local pockets of water vapor. The pockets of water vapor would disrupt the flow patterns in the pumps in the same way as cavitation in a homogeneous liquid. To prevent this "cavitation" from occurring, static pressure everywhere in the flow circuit must be maintained above the saturation pressure of the water droplets at the temperature of the flowing oil.

The net positive suction head at the pump inlets must be sufficiently high to prevent any problems from occurring within the pumps themselves.

The solid particles may act as nucleation sites, which would foster the development of vapor pockets in the flow. This might increase the net positive suction head required by the pump above that measured in tests using water. The system must be sized to maintain a large net positive suction head at the design flow rate.

Finally, the viscosity of the oil must be considered. If viscosity is high, pump performance will be degraded compared to pumping water. Then a larger pump must be specified to handle the flow requirement of the hot oil circulation system.

[illegible] 50 SHEETS FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
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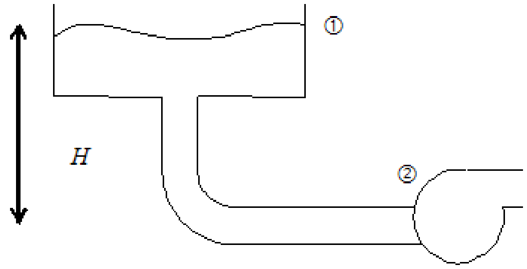
Problem 10.54

[3]

10.54 The net positive suction head required (*NPSHR*) by a pump may be expressed approximately as a parabolic function of volume flow rate. The *NPSHR* for a particular pump operating at 1750 rpm is given as $H_r = H_0 + AQ^2$, where $H_0 = 3$ m of water and $A = 3000\ \text{m}/(\text{m}^3/\text{s})^2$. Assume the pipe system supplying the pump suction consists of a reservoir, whose surface is 6 m above the pump centerline, a square entrance, 6 m of 15 cm cast-iron pipe, and a 90° elbow. Calculate the maximum volume flow rate at 20°C for which the suction head is sufficient to operate this pump without cavitation.



Given:

Pump and supply pipe system

Find:

Maximum operational flow rate

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}$$

$$h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

L_e for the elbow, and K for the square entrance

$$NPSHA=\frac{p_t-p_v}{\rho\cdot g}$$

$$H_r=H_0+A\cdot Q^2$$

Assumptions: 1) $p_1 = 0$ 2) $V_1 = 0$ 3) $\alpha_2 = 0$ 4) $z_2 = 0$

We must match the NPSHR (=H_r) and NPSHA

From the energy equation

$$g\cdot H-\left(\frac{p_2}{\rho}+\frac{V^2}{2}\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

$$\frac{p_2}{\rho\cdot g}=H-\frac{V^2}{2\cdot g}\cdot\left[1+f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]$$

$$NPSHA=\frac{p_t-p_v}{\rho\cdot g}=\frac{p_2}{\rho\cdot g}+\frac{p_{atm}}{\rho\cdot g}+\frac{V_2^{\,2}}{2\cdot g}-\frac{p_v}{\rho\cdot g}$$

$$NPSHA=H-\frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]+\frac{\left(p_{atm}-p_v\right)}{\rho\cdot g}$$

Given data:

$L =$ 6 m

$e =$ 0.26 mm

$D =$ 15 cm

$K_{ent} =$ 0.5

$L_e/D =$ 30

$H_0 =$ 3 m

$A =$ 3000 $\text{m}/(\text{m}^3/\text{s})^2$

$H =$ 6 m

$p_{atm} =$ 101 kPa

$p_v =$ 2.34 kPa

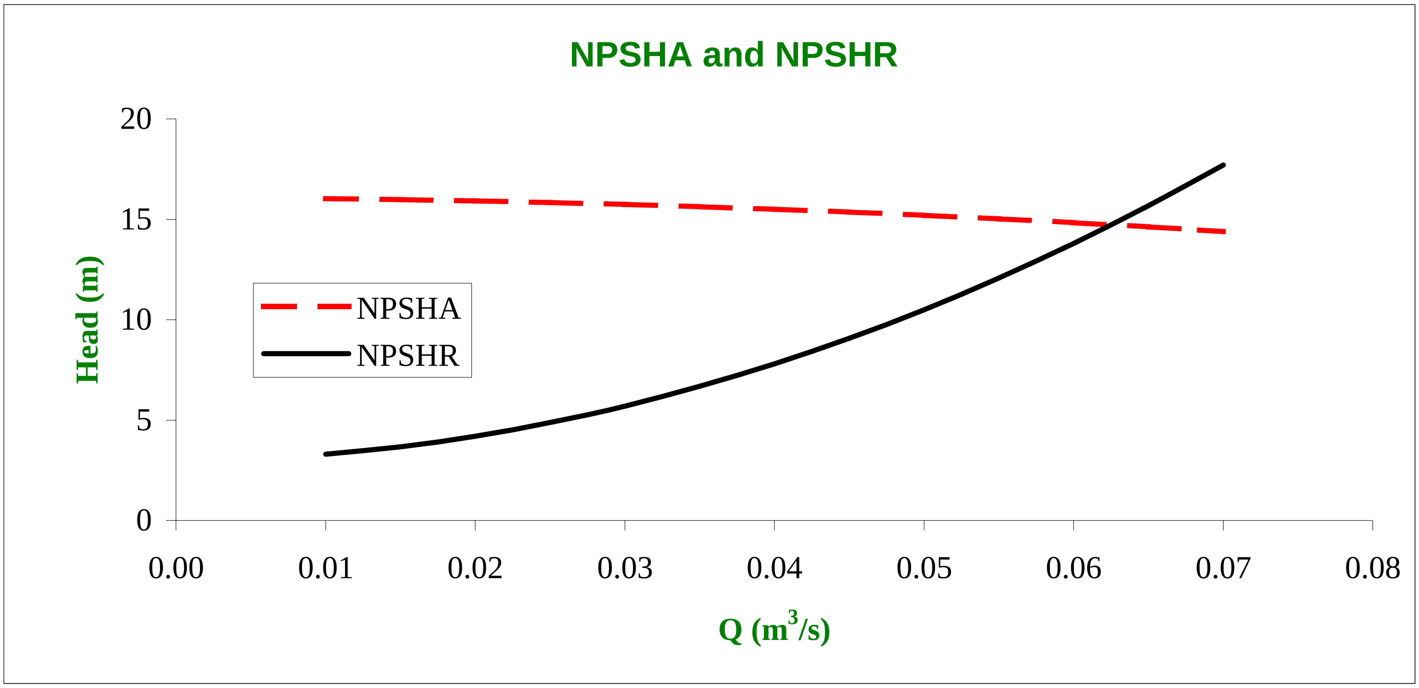
$\rho =$ 1000 kg/m^3

$\nu =$ 1.01E-06 m^2/s

Computed results:

Q **(m^3/s)**	***V*** **(m/s)**	***Re***	***f***	***NPSHA*** **(m)**	***NPSHR*** **(m)**
0.010	0.566	8.40E+04	0.0247	16.0	3.30
0.015	0.849	1.26E+05	0.0241	16.0	3.68
0.020	1.13	1.68E+05	0.0237	15.9	4.20
0.025	1.41	2.10E+05	0.0235	15.8	4.88
0.030	1.70	2.52E+05	0.0233	15.7	5.70
0.035	1.98	2.94E+05	0.0232	15.6	6.68
0.040	2.26	3.36E+05	0.0232	15.5	7.80
0.045	2.55	3.78E+05	0.0231	15.4	9.08
0.050	2.83	4.20E+05	0.0230	15.2	10.5
0.055	3.11	4.62E+05	0.0230	15.0	12.1
0.060	3.40	5.04E+05	0.0230	14.8	13.8
0.065	3.68	5.46E+05	0.0229	14.6	15.7
0.070	3.96	5.88E+05	0.0229	14.4	17.7

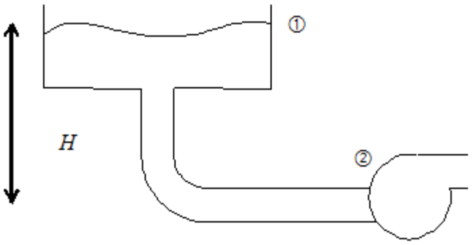
						Error
0.0625	3.54	5.25E+05	0.0229	14.7	14.7	0.00


NPSHA and NPSHR
Head (m)
20
15
10
5
0
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Q (m3/s)
NPSHA
NPSHR

Problem 10.55

[5]

10.55 For the pump and flow system of Problem 10.54, calculate the maximum flow rate for hot water at various temperatures and plot versus water temperature. (Be sure to consider the density variation as water temperature is varied.)



Given: Pump and supply pipe system

Find: Maximum operational flow rate as a function of temperature

Solution:

Basic equations:
$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

L_e for the elbow, and K for the square entrance

$$\text{NPSHA}=\frac{p_t-p_v}{\rho\cdot g} \qquad H_r=H_0+A\cdot Q^2$$

Assumptions: 1) $p_1 = 0$ 2) $V_1 = 0$ 3) $\alpha_2 = 0$ 4) $z_2 = 0$

We must match the NPSHR (=H_r) and NPSHA

From the energy equation
$$g\cdot H-\left(\frac{p_2}{\rho}+\frac{V^2}{2}\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2} \qquad \frac{p_2}{\rho\cdot g}=H-\frac{V^2}{2\cdot g}\cdot\left[1+f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]$$

$$\text{NPSHA}=\frac{p_t-p_v}{\rho\cdot g}=\frac{p_2}{\rho\cdot g}+\frac{p_{atm}}{\rho\cdot g}+\frac{V_2^{\,2}}{2\cdot g}-\frac{p_v}{\rho\cdot g} \qquad \text{NPSHA}=H-\frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]+\frac{(p_{atm}-p_v)}{\rho\cdot g}$$

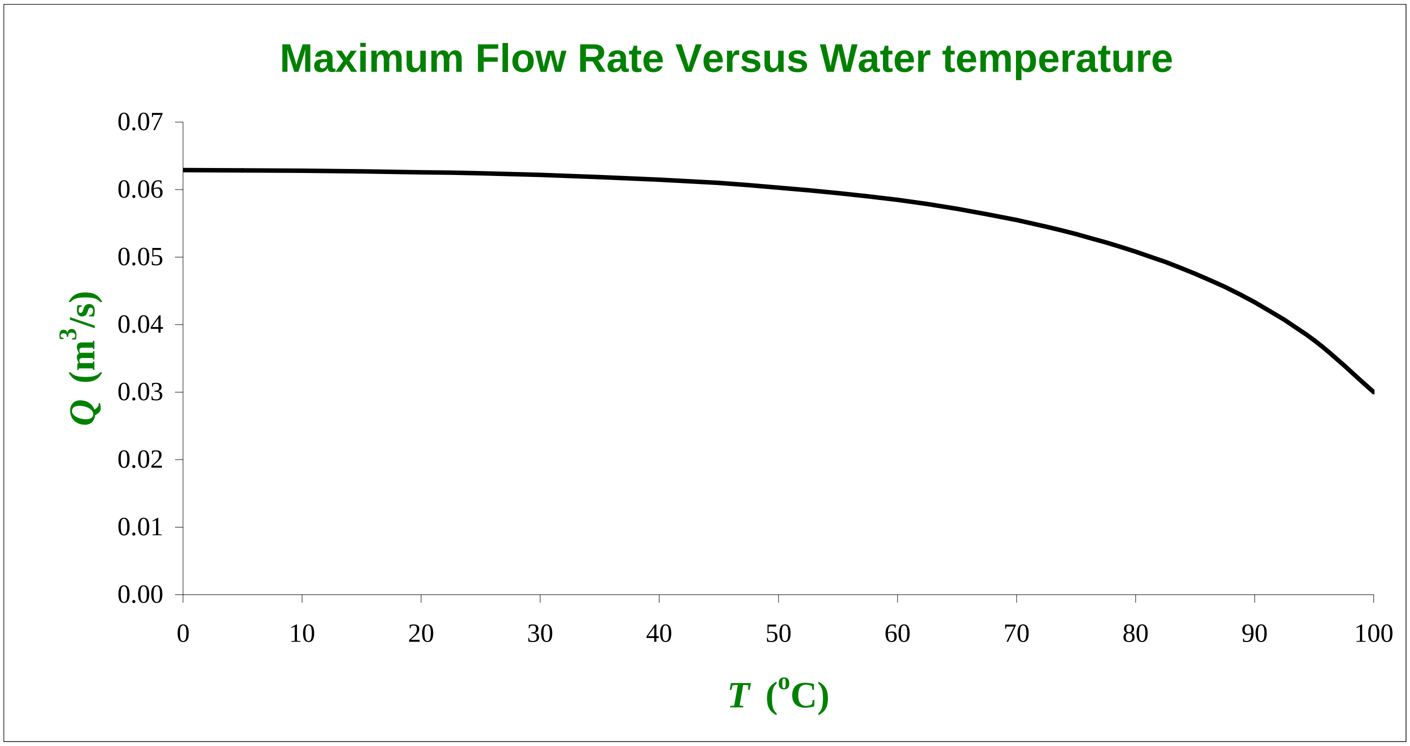
Given data:

L =	6	m
e =	0.26	mm
D =	15	cm
K_{ent} =	0.5	
L_e/D =	30	
H_0 =	3	m
A =	3000	m/(m^3/s)^2
H =	6	m
p_{atm} =	101	kPa
ρ =	1000	kg/m^3
ν =	1.01E-06	m^2/s

Computed results:

T (°C)	p_v (kPa)	ρ (kg/m^3)	ν (m^3/s)	Q (m^3/s)	V (m/s)	Re	f	$NPSHA$ (m)	$NPSHR$ (m)	$Error$
0	0.661	1000	1.76E-06	0.06290	3.56	3.03E+05	0.0232	14.87	14.87	0.00
5	0.872	1000	1.51E-06	0.06286	3.56	3.53E+05	0.0231	14.85	14.85	0.00
10	1.23	1000	1.30E-06	0.06278	3.55	4.10E+05	0.0230	14.82	14.82	0.00
15	1.71	999	1.14E-06	0.06269	3.55	4.67E+05	0.0230	14.79	14.79	0.00
20	2.34	998	1.01E-06	0.06257	3.54	5.26E+05	0.0229	14.75	14.75	0.00
25	3.17	997	8.96E-07	0.06240	3.53	5.91E+05	0.0229	14.68	14.68	0.00
30	4.25	996	8.03E-07	0.06216	3.52	6.57E+05	0.0229	14.59	14.59	0.00
35	5.63	994	7.25E-07	0.06187	3.50	7.24E+05	0.0228	14.48	14.48	0.00
40	7.38	992	6.59E-07	0.06148	3.48	7.92E+05	0.0228	14.34	14.34	0.00
45	9.59	990	6.02E-07	0.06097	3.45	8.60E+05	0.0228	14.15	14.15	0.00
50	12.4	988	5.52E-07	0.06031	3.41	9.27E+05	0.0228	13.91	13.91	0.00
55	15.8	986	5.09E-07	0.05948	3.37	9.92E+05	0.0228	13.61	13.61	0.00
60	19.9	983	4.72E-07	0.05846	3.31	1.05E+06	0.0228	13.25	13.25	0.00
65	25.0	980	4.40E-07	0.05716	3.23	1.10E+06	0.0227	12.80	12.80	0.00
70	31.2	978	4.10E-07	0.05548	3.14	1.15E+06	0.0227	12.24	12.24	0.00
75	38.6	975	3.85E-07	0.05342	3.02	1.18E+06	0.0227	11.56	11.56	0.00
80	47.4	972	3.62E-07	0.05082	2.88	1.19E+06	0.0227	10.75	10.75	0.00
85	57.8	969	3.41E-07	0.04754	2.69	1.18E+06	0.0227	9.78	9.78	0.00
90	70.1	965	3.23E-07	0.04332	2.45	1.14E+06	0.0227	8.63	8.63	0.00
95	84.6	962	3.06E-07	0.03767	2.13	1.05E+06	0.0228	7.26	7.26	0.00
100	101	958	2.92E-07	0.02998	1.70	8.71E+05	0.0228	5.70	5.70	0.00
Use *Solver* to make the sum of absolute errors between *NPSHA* and *NPSHR* zero by varying the *Q* 's										0.00

NPSHR increases with temperature because the p_v increases; NPHSA decreases because ρ decreases and p_v increases



Problem 10.56

[2]

Given: Centrifugal pump, operating at $N = 2265$ rpm, lifts water between two reservoirs connected by two cast-iron pipes in series.

$L_1 = 300$ ft, $D_1 = 6$ in.; $L_2 = 100$ ft, $D_2 = 3$ in.; $\Delta z = 25$ ft.

Find: (a) head requirement, (b) power need, and (c) hourly cost of electrical energy, for $Q = 200$ gpm if electricity costs 12¢/kw.hr and $\eta_m = 0.85$

Solution:

Apply the energy equation to the total system for steady, incompressible flow using ③ and ④ at reservoir surfaces.

Computing eq.: $\cancel{\frac{p_3}{\rho g}} + \alpha_3 \cancel{\frac{V_3^2}{2g}} + z_3 + H_a = \cancel{\frac{p_4}{\rho g}} + \alpha_4 \cancel{\frac{V_4^2}{2g}} + z_4 + \frac{h_{l_T}}{g}$ (10.24b)

$$h_{l_T} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2}$$

Assumptions: (1) $p_3 = p_4 = p_{atm}$, $V_3 = V_4 \approx 0$
(2) neglect minor losses

Then

$H_a = z_4 - z_3 + f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$ ......(1)

$V_1 = \frac{Q}{A_1} = 200 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times \frac{4}{\pi (0.50)^2 ft^2} = 2.27$ ft/s

$V_2 = 9.08$ ft/s. For water at 59°F, $\nu = 1.23\times10^{-5}$ ft²/s (Table A.7)

$Re_1 = \frac{V_1 D_1}{\nu} = 2.27 \frac{ft}{s} \times \frac{6}{12} ft \times 1.23\times10^{-5} \frac{s}{ft^2} = 9.23\times10^4$

$Re_2 = 1.85\times10^5$. From Table 8.1 for cast iron, $e = 0.00085$ ft

$e/D_1 = 0.0017$, $e/D_2 = 0.0034$

From Eq. 8.37, $f_1 = 0.0244$, $f_2 = 0.0278$. Substituting into (1)

$H_a = 25\,ft + 0.0244 \times \frac{300\times12}{6} \times \frac{(2.27)^2}{2} \frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft} + 0.0278 \times \frac{100\times12}{3} \times \frac{(9.08)^2}{2} \frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft}$

$H_a = 40.4$ ft $\longleftarrow H_a$

Specific speed $N_{S_{cu}} = \frac{N Q^{1/2}}{H^{3/4}} = \frac{2265\,(200)^{1/2}}{(40.4)^{3/4}} = 2000$

From Fig 10.15, $\eta_p \approx 0.75$. Then $\mathcal{P}_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho Q g H}{\eta_p}$

$\mathcal{P}_m = \frac{1}{0.75} \times 200 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times 62.4 \frac{lbf}{ft^3} \times 40.4\,ft \times \frac{hp\cdot s}{550\,ft\cdot lbf} = 2.72$ hp $\longleftarrow \mathcal{P}_m$

Cost $= c\,\mathcal{P}_e$. Since $c = \$0.12$/kw.hr & $\eta_m = 0.85$

Then $\mathcal{P}_e = \frac{\mathcal{P}_m}{\eta_m}$ and

Cost $= c \frac{\mathcal{P}_m}{\eta_m} = \frac{\$0.12}{kw\cdot hr} \times \frac{2.72\,hp}{0.85} \times 0.746 \frac{kw}{hp} = 28.7¢$/hr $\longleftarrow C$

Problem 10.57

Given: Water supply for Grand Canyon National Park,
$L = 13{,}200$ ft, $Q = 600$ gpm
Location 1: Colorado River, $z_1 = 3734$ ft
Location 2: South Rim, $z_2 = 7022$ ft in storage tank.
Head loss due to friction is $h_{l_T}/g = 290$ ft.

Find: (a) estimate diameter of commercial steel pipe.
(b) pumping power if $\eta_p = 0.61$.

Solution:

Apply the energy equation to the total system for steady incompressible flow using ① and ② at inlet and reservoir surface respectively.

Computing eq.: $\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + H_a = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2} + z_2 + \frac{h_{l_T}}{g}$ (10.24b)

$h_{l_T} = f \frac{L}{D} \frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$, $\bar{V}_1 \approx \bar{V}_2 \approx 0$
(2) neglect minor losses.

Then $H_a = z_2 - z_1 + \frac{h_{l_T}}{g}$ ......(1) and $\frac{h_{l_T}}{g} = f \frac{L}{D} \frac{\bar{V}^2}{2g} = 290$ ft

Since $f = f(Re, e/D)$ and D is unknown, we must iterate.

For commercial steel, $e = 0.00015$ ft

The procedure is
- assume D, calculate V, Re; determine f (Eqs 8.37a,b); calculate h_{l_T}/g and compare to value of 290 ft.

D (in.)	V (ft/s)	Re	f_0	$f^{0.5}$	f	h_{lT}/g (ft)
12	1.70	1.46E+06	0.0139	8.517	0.0138	8.2
10	2.45	1.75E+06	0.0141	8.433	0.0141	20.8
8	3.83	2.19E+06	0.0146	8.306	0.0145	65.3
6	6.81	2.92E+06	0.0153	8.111	0.0152	289

Thus $D = 6.0$ in ⟵ D

The total pump head is $H_a = 7022 - 3734 + 290 = 3578$ ft.

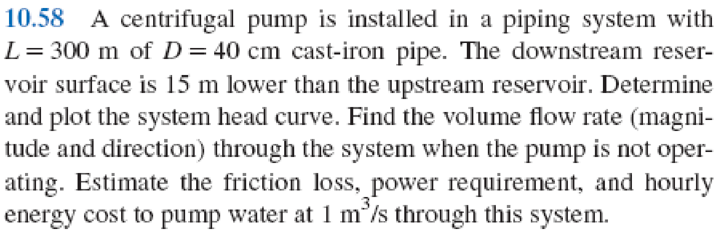
The pump power is $\mathcal{P}_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho g Q H}{\eta_p}$

$$\mathcal{P}_m = \frac{1}{0.61} \times 62.4 \frac{\text{lbf}}{\cancel{\text{ft}^3}} \times 600 \frac{\cancel{\text{gal}}}{\cancel{\text{min}}} \times \frac{\text{ft}^3}{7.48 \cancel{\text{gal}}} \times \frac{\cancel{\text{min}}}{60 \text{s}} \times 3578 \text{ ft} \times \frac{\text{hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lbf}}$$

$\mathcal{P}_m = 890$ hp ⟵ $\mathcal{P}_m$

Problem 10.58

[3]



10.58 A centrifugal pump is installed in a piping system with $L = 300$ m of $D = 40$ cm cast-iron pipe. The downstream reservoir surface is 15 m lower than the upstream reservoir. Determine and plot the system head curve. Find the volume flow rate (magnitude and direction) through the system when the pump is not operating. Estimate the friction loss, power requirement, and hourly energy cost to pump water at 1 m^3/s through this system.

Given: Pump and reservoir system

Find: System head curve; Flow rate when pump off; Loss, Power required and cost for 1 m^3/s flow rate

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}-h_p \qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+\Sigma\cdot K\cdot\frac{V^2}{2}\ \text{(K for the exit)}$$

where points 1 and 2 are the reservoir free surfaces, and h_p is the pump head

Note also $H=\frac{h}{g}$ Pump efficiency: $\eta_p=\frac{W_h}{W_m}$

Assumptions: 1) $p_1 = p_2 = p_{atm}$ 2) $V_1 = V_2 = 0$ 3) $\alpha_2 = 0$ 4) $z_1 = 0,\ z_2 = -15\cdot m$ 4) $K = K_{ent} + K_{ent} = 1.5$

From the energy equatio $-g\cdot z_2 = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}-h_p+K\cdot\frac{V^2}{2}$ $\qquad h_p = g\cdot z_2+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$ $\qquad H_p = z_2+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+K\cdot\frac{V^2}{2\cdot g}$

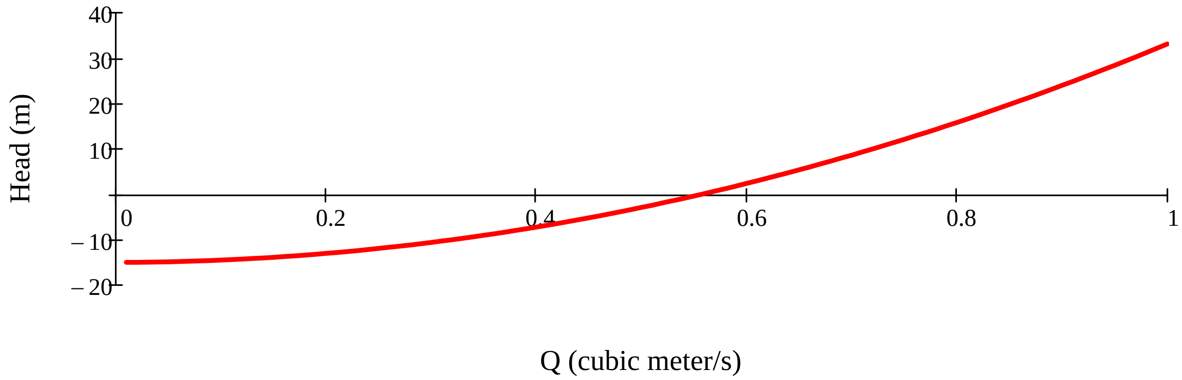
Given or available data $L = 300\cdot m$ $\qquad D = 40\cdot cm$ $\qquad e = 0.26\cdot mm$ (Table 8.1)

$\rho = 1000\cdot\frac{kg}{m^3}$ $\qquad \nu = 1.01\times10^{-6}\cdot\frac{m^2}{s}$ (Table A.8)

The set of equations to solve for each flow rate Q are

$$V=\frac{4\cdot Q}{\pi\cdot D^2} \qquad Re=\frac{V\cdot D}{\nu} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad H_p=z_2+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+K\cdot\frac{V^2}{2\cdot g}$$

For example, for $Q = 1\cdot\frac{m^3}{s}$ $\qquad V = 7.96\cdot\frac{m}{s}$ $\qquad Re = 3.15\times10^6$ $\qquad f = 0.0179$ $\qquad H_p = 33.1\cdot m$



The above graph can be plotted in Excel. In Excel, Solver can be used to find Q for $H_p = 0$ $Q = 0.557 \frac{m^3}{s}$ (Zero power rate)

At $Q = 1 \cdot \frac{m^3}{s}$ we saw that $H_p = 33.1 \cdot m$

Assuming optimum efficiency at $Q = 1.59 \times 10^4 \cdot gpm$ from Fig. 10.15 $\eta_p = 92 \cdot \%$

Then the hydraulic power is $W_h = \rho \cdot g \cdot H_p \cdot Q$ $W_h = 325 \cdot kW$

The pump power is then $W_m = \frac{W_h}{\eta_p}$ $W_m \cdot 2 = 706 \cdot kW$

If electricity is 10 cents per kW-hr then the hourly cost is about \$35

If electricity is 15 cents per kW-hr then the hourly cost is about \$53

If electricity is 20 cents per kW-hr then the hourly cost is about \$71

Given: Peerless horizontal split-case 4AE12 pump with 11 in diameter impeller, operating at 1750 rpm, lifts water between two reservoirs connected by two cast-iron pipes in series.

$L_1 = 200 \text{ ft}, D_1 = 4 \text{ in.}; \; L_2 = 200 \text{ ft}, D_2 = 3 \text{ in.}; \; \Delta z = 10 \text{ ft}$

Plot the system head curve and determine the pump operating point.

Solution:

Apply the energy equation to the total system for steady, incompressible flow using ③ and ④ at reservoir surfaces.

Computing eq.: $\cancel{\frac{p_3}{\rho g}} + \cancel{\alpha_3 \frac{\bar{V}_3^2}{2g}} + z_3 + H_a = \cancel{\frac{p_4}{\rho g}} + \cancel{\alpha_4 \frac{\bar{V}_4^2}{2g}} + z_4 + \frac{h_{\ell_T}}{g}$ (10.24b)

$$h_{\ell_T} = f_1 \frac{L_1}{D_1} \frac{\bar{V}_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{\bar{V}_2^2}{2g}$$

Assumptions: (1) $p_3 = p_4 = p_{atm}$, $V_3 = V_4 \approx 0$

(2) neglect minor losses

then

$$H_a = z_4 - z_3 + f_1 \frac{L_1}{D_1} \frac{\bar{V}_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{\bar{V}_2^2}{2g} \quad \text{.......(1)}$$

Express V as a function of Q

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times \left(\frac{12}{4}\right)^2 \frac{1}{ft^2} Q \left(\frac{gal}{min}\right) \times \frac{ft^3}{7.48 gal} \times \frac{min}{60s} = 0.0255 Q(gpm)$$

$V_2 = 0.0454 \, Q(gpm)$

The friction factor is determined from the Colebrook eq.

$$\frac{1}{f^{0.5}} = -2.0 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re \, f^{0.5}}\right) \quad (8.37a)$$

using the equation of Miller for the original estimate

$$f_0 = 0.25 \left[\log\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2} \quad (8.37b)$$

Assuming $T = 59°F$, $\nu = 1.23 \times 10^{-5} \, ft^2/s$ (Table A.7)

$Re_1 = \frac{D\bar{V}_1}{\nu} = \frac{4}{12} \times \frac{10^5}{1.23} V_1 = 2.71 \times 10^4 V_1$, $Re_2 = 2.03 \times 10^4 V_2$

For cast iron, $e = 0.00085$ ft (Table 8.1)

$e/D_1 = 0.00255$ $\quad e/D_2 = 0.00340$

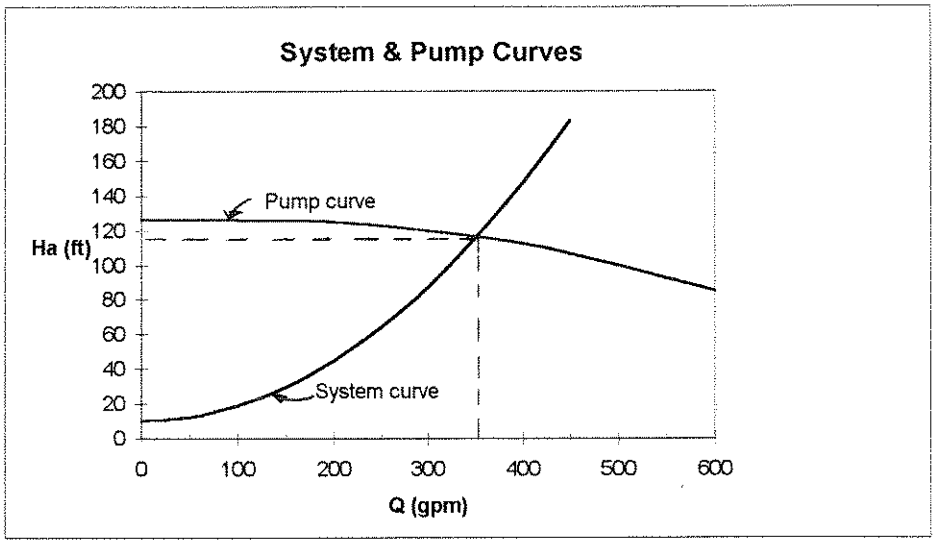
$$H_a = 10 ft + f_1 \times 9.317 V_1^2 + f_2 \times 12.42 V_2^2$$

The pump curve is obtained from Fig. D.4

Problem 10.59

Q (gpm)	V_1 (ft/s)	Re_1	$f_0)_1$	$f^{-0.5})_1$	f_1	V_2 (ft/s)	Re_2	$f_0)_2$	$f^{-0.5})_2$	f_2	H_a (ft)
0	0.0	0.00E+00				0					10.0
50	1.3	3.46E+04	0.0290	5.905	0.0287	2.3	4.61E+04	0.0299	5.817	0.0295	12
100	2.6	6.91E+04	0.0273	6.083	0.0270	4.5	9.22E+04	0.0287	5.933	0.0284	19
150	3.8	1.04E+05	0.0266	6.154	0.0264	6.8	1.38E+05	0.0282	5.977	0.0280	30
200	5.1	1.38E+05	0.0263	6.193	0.0261	9.1	1.84E+05	0.0280	6.000	0.0278	45
250	6.4	1.73E+05	0.0261	6.217	0.0259	11.4	2.30E+05	0.0278	6.014	0.0277	64
300	7.7	2.07E+05	0.0259	6.233	0.0257	13.6	2.76E+05	0.0277	6.023	0.0276	88
350	8.9	2.42E+05	0.0258	6.245	0.0256	15.9	3.23E+05	0.0276	6.030	0.0275	115
400	10.2	2.76E+05	0.0257	6.255	0.0256	18.2	3.69E+05	0.0276	6.036	0.0275	147
450	11.5	3.11E+05	0.0256	6.262	0.0255	20.4	4.15E+05	0.0275	6.040	0.0274	183

Q (gpm)	H_p (ft)
0	126
100	126
200	125
300	120
400	113
500	100
600	85



Pump operates at Q ≈ 350 gpm.
H ≈ 115 ft

Problem 10.60

[3]

10.60 A pump transfers water from one reservoir to another through two cast-iron pipes in series. The first is 3000 ft of 9 in. pipe and the second is 1000 ft of 6 in. pipe. A constant flow rate of 75 gpm is tapped off at the junction between the two pipes. Obtain and plot the system head versus flow rate curve. Find the delivery if the system is supplied by the pump of Example 10.7, operating at 1750 rpm.

Given: Data on pump and pipe system

Find: Delivery through system

Solution:

Given or available data:

$L_1 =$	3000	ft	$\nu =$	1.23E-05	ft^2/s (Table A.7)
$D_1 =$	9	in	$K_{ent} =$	0.5	(Fig. 8.14)
$L_2 =$	1000	ft	$K_{exp} =$	1	
$D_2 =$	6	in	$Q_{loss} =$	75	gpm
$e =$	0.00085	ft (Table 8.1)			

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

and the pump head (in energy/mass) is given by (from Example 10.7)

$$H_{pump}(ft) = 55.9 - 3.44\times10^{-5}\cdot Q(gpm)^2$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = z_2$) we have

$$0 = h_{lT} - \Delta h_{pump}$$

$$h_{lT} = g\cdot H_{system} = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} = H_{pump} \qquad (1)$$

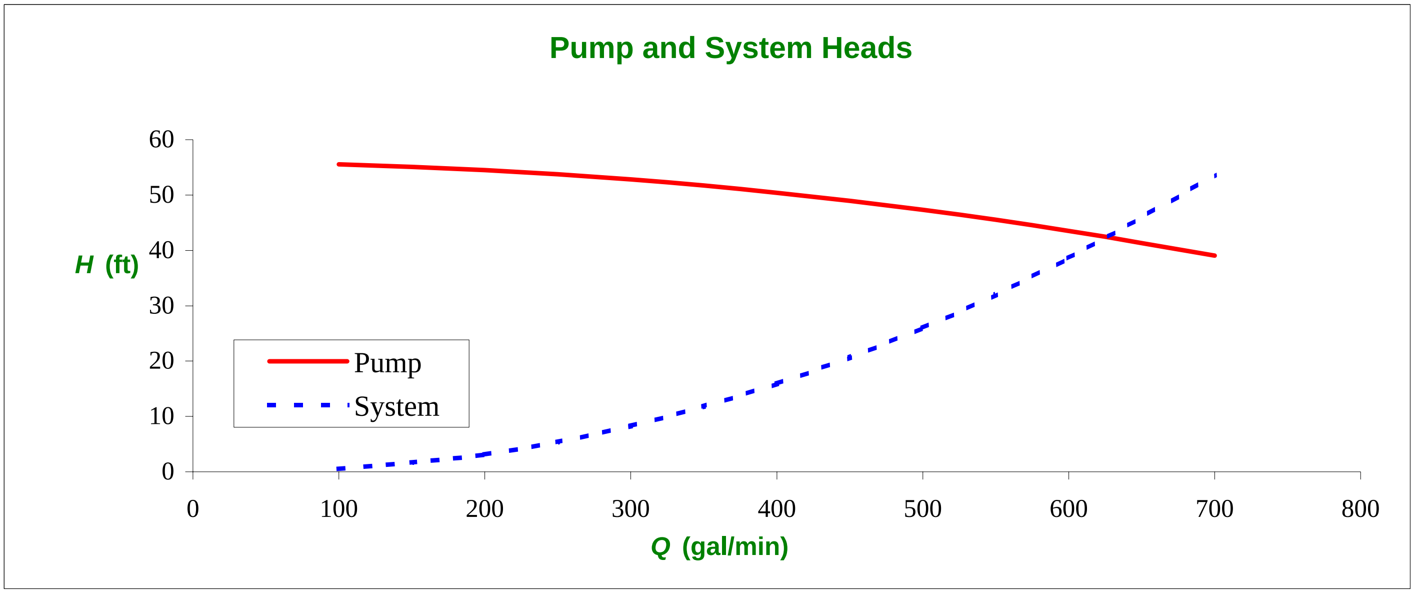
where

$$H_{lT} = \left(f_1\cdot\frac{L_1}{D_1}+K_{ent}\right)\cdot\frac{V_1^2}{2\cdot g}+\left(f_2\cdot\frac{L_2}{D_2}+K_{exit}\right)\cdot\frac{V_2^2}{2}$$

The system and pump heads are computed and plotted below.
To find the operating condition, *Goal Seek* is used to vary Q_1
so that the error between the two heads is zero.

Q_1 (gpm)	Q_2 (gpm)	V_1 (ft/s)	V_2 (ft/s)	Re_1	Re_2	f_1	f_2	H_{IT} (ft)	H_{pump} (ft)
100	25	0.504	0.284	30753	11532	0.0262	0.0324	0.498	55.6
200	125	1.01	1.42	61506	57662	0.0238	0.0254	3.13	54.5
300	225	1.51	2.55	92260	103792	0.0228	0.0242	8.27	52.8
400	325	2.02	3.69	123013	149922	0.0222	0.0237	15.9	50.4
500	425	2.52	4.82	153766	196052	0.0219	0.0234	26.0	47.3
600	525	3.03	5.96	184519	242182	0.0216	0.0233	38.6	43.5
700	625	3.53	7.09	215273	288312	0.0215	0.0231	53.6	39.0

Q_1 (gpm)	Q_2 (gpm)	V_1 (ft/s)	V_2 (ft/s)	Re_1	Re_2	f_1	f_2	H_{IT} (ft)	H_{pump} (ft)	Error)
627	552	3.162	6.263	192785	254580	0.0216	0.0232	42.4	42.4	0%



Problem 10.61

[3]

10.61 Performance data for a pump are

H (m)	27.5	27	25	22	18	13	6.5
Q (m^3/s)	0	0.025	0.050	0.075	0.100	0.125	0.150

The pump is to be used to move water between two open reservoirs with an elevation increase of 7.5 m. The connecting pipe system consists of 500 m of commercial steel pipe containing two 90° elbows and an open gate valve. Find the flow rate if we use a) 20 cm, b) 30 cm, and c) 40 cm pipe.

Given: Pump and reservoir/pipe system

Find: Flow rate using different pipe sizes

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}-h_p$$

$$h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+\Sigma f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+\Sigma K\cdot\frac{V^2}{2}$$

L_e for the elbows, and K for the square entrance and exit

and also $H=\dfrac{h}{g}$

Assumptions: 1) $p_1 = p_2 = p_{atm}$ 2) $V_1 = V_2 = 0$ 3) $\alpha = 0$ 4) $z_1 = 0,\ z_2 = 7.5\cdot m$ 4) $K = K_{ent} + K_{ent}$ 5) $\dfrac{L_e}{D}$ is for two elbows

Hence

$$h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

and also $-z_2 = h_{lT} - h_p$ or $h_{1T} = h_p - z_2$

We want to find a flow that satisfies these equations, rewritten as energy/weight rather than energy/mass

$$H_{lT}=\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]\cdot\frac{V^2}{2\cdot g}\qquad H_{1T}+z_2=H_p$$

Given or available data **(Note: final results will vary depending on fluid data selected)**:

$L =$ 500 m

$e =$ 0.046 mm (Table 8.1)

$D =$ 20 cm

$\nu =$ 1.01E-06 m^2/s (Table A.8)

$z_2 =$ 7.5 m

$K_{ent} =$ 0.5 (Fig. 8.14)

$K_{exp} =$ 1

$L_e/D_{elbow} =$ 60 (Two)

$L_e/D_{valve} =$ 8 (Table 8.4)

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

Q (m^3/s)	Q^2	H_p (m)
0.000	0.00000	27.5
0.025	0.00063	27.0
0.050	0.00250	25.0
0.075	0.00563	22.0
0.100	0.01000	18.0
0.125	0.01563	13.0
0.150	0.02250	6.5

V (m/s)	Re	f
0.00	0	0.0000
0.80	157579	0.0179
1.59	315158	0.0164
2.39	472737	0.0158
3.18	630317	0.0154
3.98	787896	0.0152
4.77	945475	0.0150

H_p (fit)	$H_{lT} + z_2$ (m)
27	7.5
27	9.0
25	13.1
22	19.7
18	28.7
12.9	40.2
6.5	54.1

H_0 = 27 m
A = 9.30E+02 /(m^3/s)^2

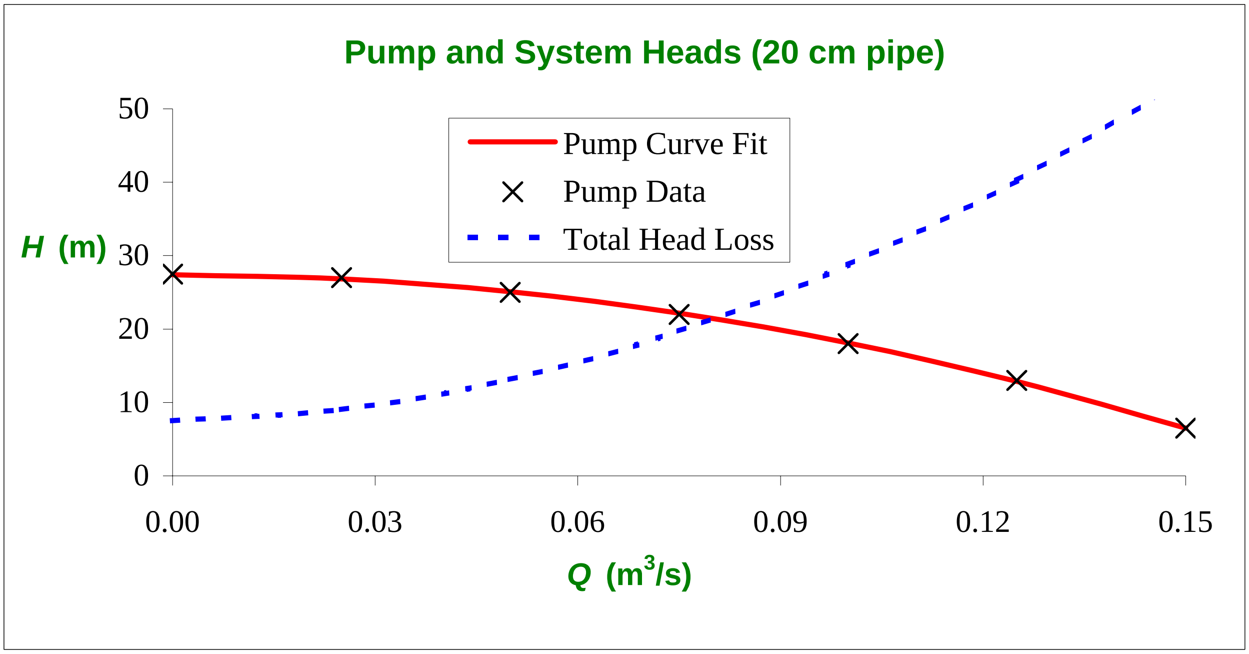
Q (m^3/s)	V (m/s)	Re	f	H_p (fit)	$H_{lT} + z_2$ (m)	Error)
0.0803	2.56	506221	0.0157	21.4	21.4	0.00%

Repeating for: D = 30 cm

Q (m^3/s)	V (m/s)	Re	f	H_p (fit)	$H_{lT} + z_2$ (m)	Error)
0.1284	1.82	539344	0.0149	12.1	12.1	0.00%

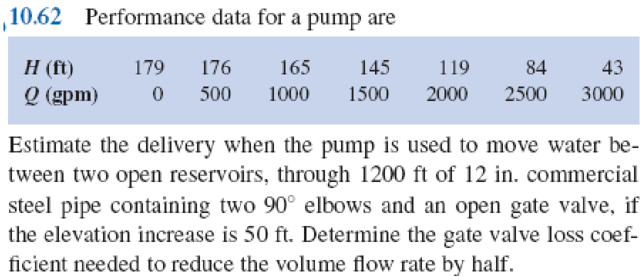
Repeating for: D = 40 cm

Q (m^3/s)	V (m/s)	Re	f	H_p (fit)	$H_{lT} + z_2$ (m)	Error)
0.1413	1.12	445179	0.0148	8.9	8.9	0.00%



Problem 10.62

[3]



10.62 Performance data for a pump are

H (ft)	179	176	165	145	119	84	43
Q (gpm)	0	500	1000	1500	2000	2500	3000

Estimate the delivery when the pump is used to move water between two open reservoirs, through 1200 ft of 12 in. commercial steel pipe containing two 90° elbows and an open gate valve, if the elevation increase is 50 ft. Determine the gate valve loss coefficient needed to reduce the volume flow rate by half.

Given: Data on pump and pipe system

Find: Delivery through system; valve position to reduce delivery by half

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

$L =$ 1200 ft

$D =$ 12 in

$e =$ 0.00015 ft (Table 8.1)

$\nu =$ 1.23E-05 ft^2/s (Table A.7)

$\Delta z =$ -50 ft

$K_{ent} =$ 0.5 (Fig. 8.14)

$K_{exp} =$ 1

$L_e/D_{elbow} =$ 30

$L_e/D_{valve} =$ 8 (Table 8.4)

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D} + 2\cdot\frac{L_e}{D_{elbow}} + \frac{L_e}{D_{valve}}\right) + K_{ent} + K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

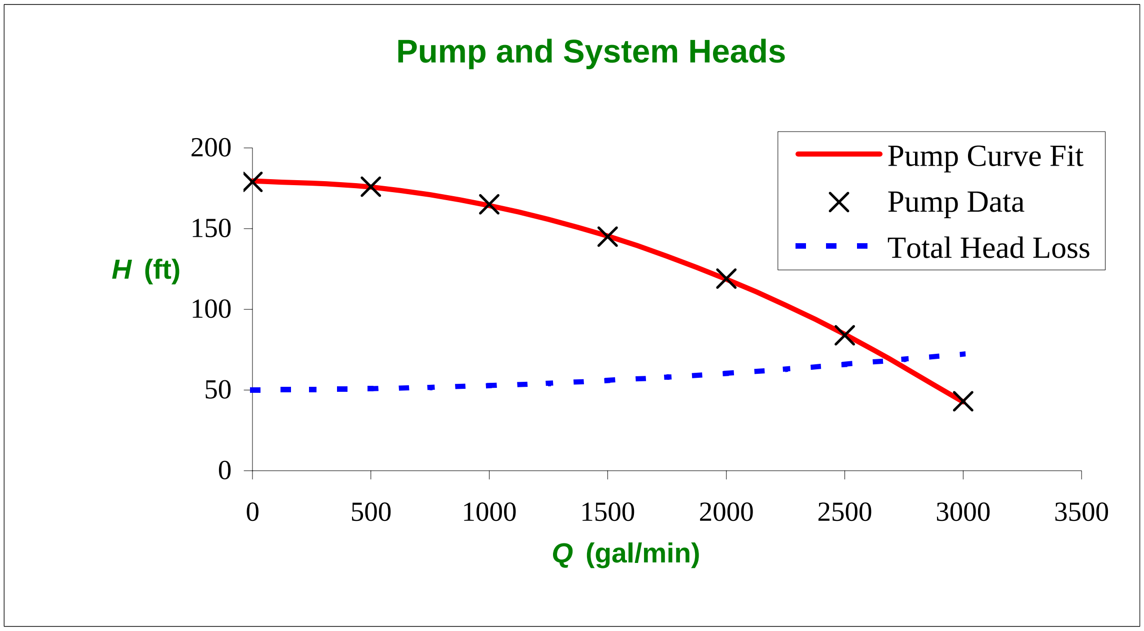
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)
0	0	179
500	250000	176
1000	1000000	165
1500	2250000	145
2000	4000000	119
2500	6250000	84
3000	9000000	43

V (ft/s)	*Re*	f
0.00	0	0.0000
1.42	115325	0.0183
2.84	230649	0.0164
4.26	345974	0.0156
5.67	461299	0.0151
7.09	576623	0.0147
8.51	691948	0.0145

H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)
180	50.0
176	50.8
164	52.8
145	56.0
119	60.3
84.5	65.8
42.7	72.4

H_0 = 180 ft
A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	*Re*	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2705	7.67	623829	0.0146	68.3	68.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 26858

Q (gpm)	V (ft/s)	*Re*	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
1352	3.84	311914	0.0158	151.7	151.7	0%

Problem 10.63

[3]

10.63 Consider again the pump and piping system of Problem 10.62. Determine the volume flow rate and gate valve loss coefficient for the case of two identical pumps installed in *series*.

Given: Data on pump and pipe system

Find: Delivery through series pump system; valve position to reduce delivery by half

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

L =	1200	ft	K_{ent} =	0.5	(Fig. 8.14)
D =	12	in	K_{exp} =	1	
e =	0.00015	ft (Table 8.1)	L_e/D_{elbow} =	30	
ν =	1.23E-05	ft^2/s (Table A.7)	L_e/D_{valve} =	8	(Table 8.4)
Δz =	-50	ft			

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in series

$$H_{pump} = 2\cdot H_0 - 2\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

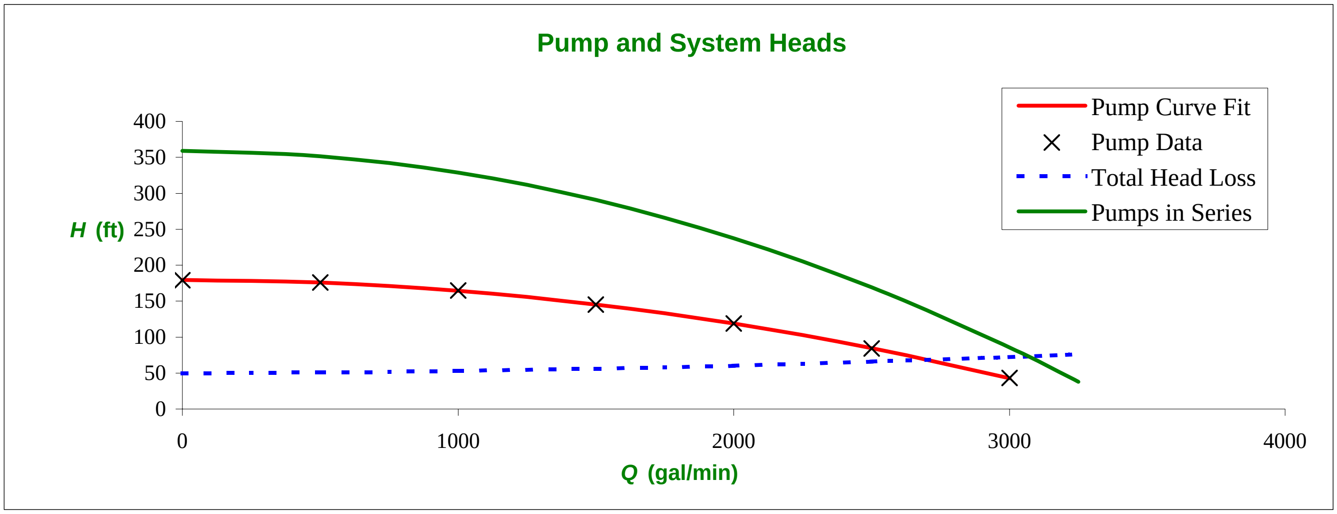
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	*Re*	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3250				9.22	749610	0.0144

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
359	50.0
351	50.8
329	52.8
291	56.0
237	60.3
169	65.8
85	72.4
38	76.1

H_0 = 180 ft

A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	*Re*	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
3066	8.70	707124	0.0145	73.3	73.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 50723

Q (gpm)	V (ft/s)	*Re*	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
1533	4.35	353562	0.0155	287.7	287.7	0%

Problem 10.64

[3]

10.64 Consider again the pump and piping system of Problem 10.62. Determine the volume flow rate and gate valve loss coefficient for the case of two identical pumps installed in *parallel*.

Given: Data on pump and pipe system

Find: Delivery through parallel pump system; valve position to reduce delivery by half

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

L = 1200 ft

D = 12 in

e = 0.00015 ft (Table 8.1)

ν = 1.23E-05 ft^2/s (Table A.7)

Δz = -50 ft

K_{ent} = 0.5 (Fig. 8.14)

K_{exp} = 1

L_e/D_{elbow} = 30

L_e/D_{valve} = 8 (Table 8.4)

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in parallel

$$H_{pump} = H_0 - \frac{1}{4}\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

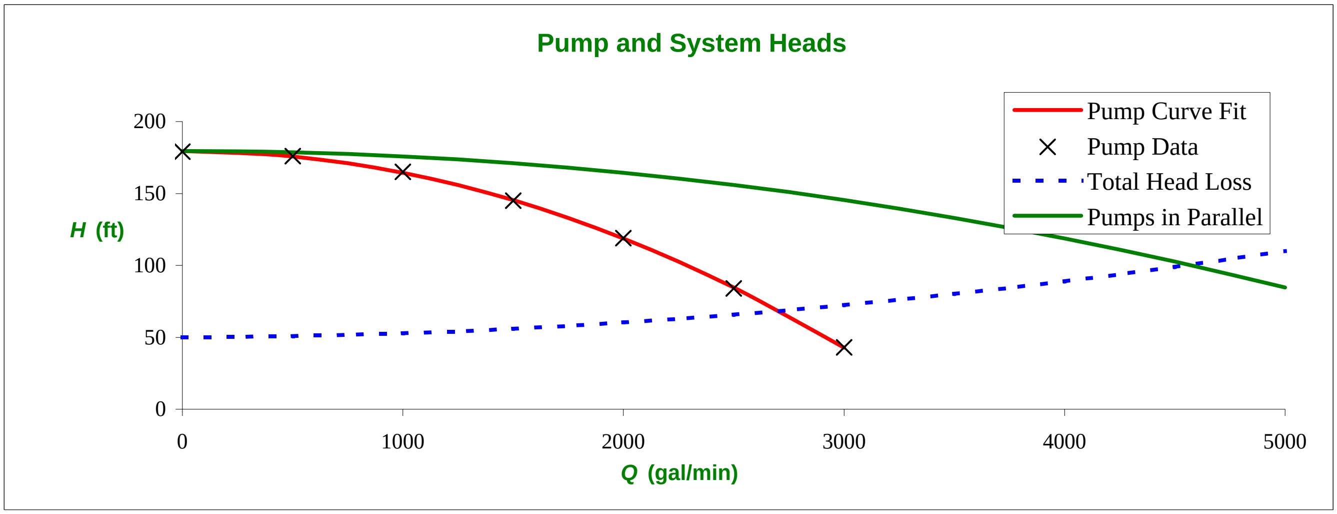
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3500				9.93	807273	0.0143
4000				11.35	922597	0.0142
4500				12.77	1037922	0.0141
5000				14.18	1153247	0.0140

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
180	50.0
179	50.8
176	52.8
171	56.0
164	60.3
156	65.8
145	72.4
133	80.1
119	89.0
103	98.9
85	110.1

$H_0 =$ 180 ft

$A =$ 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
4565	12.95	1053006	0.0141	100.3	100.3	0%



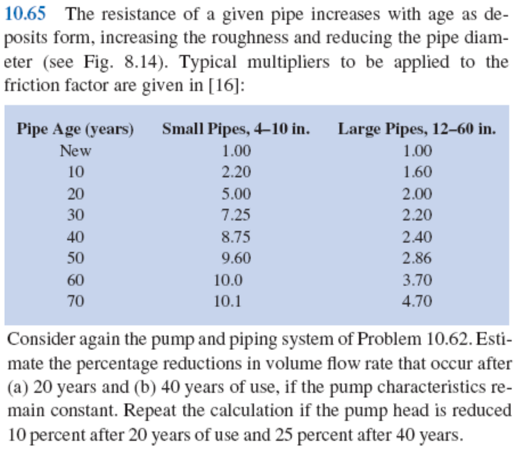
For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

$L_e/D_{valve} =$ 9965

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
2283	6.48	526503	0.0149	159.7	159.7	0%

Problem 10.65

[4]



10.65 The resistance of a given pipe increases with age as deposits form, increasing the roughness and reducing the pipe diameter (see Fig. 8.14). Typical multipliers to be applied to the friction factor are given in [16]:

Pipe Age (years)	Small Pipes, 4–10 in.	Large Pipes, 12–60 in.
New	1.00	1.00
10	2.20	1.60
20	5.00	2.00
30	7.25	2.20
40	8.75	2.40
50	9.60	2.86
60	10.0	3.70
70	10.1	4.70

Consider again the pump and piping system of Problem 10.62. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years.

Given: Data on pump and pipe system, and their aging

Find: Reduction in delivery through system after 20 and 40 years (aging and non-aging pumps)

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

L = 1200 ft
D = 12 in
e = 0.00015 ft (Table 8.1)
ν = 1.23E-05 ft^2/s (Table A.7)
Δz = -50 ft

K_{ent} = 0.5 (Fig. 8.14)
K_{exp} = 1
L_e/D_{elbow} = 30
L_e/D_{valve} = 8 (Table 8.4)

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

New System:

Q (gpm)	Q^2 (gpm)	H_{pump} (ft)
0	0	179
500	250000	176
1000	1000000	165
1500	2250000	145
2000	4000000	119
2500	6250000	84
3000	9000000	43

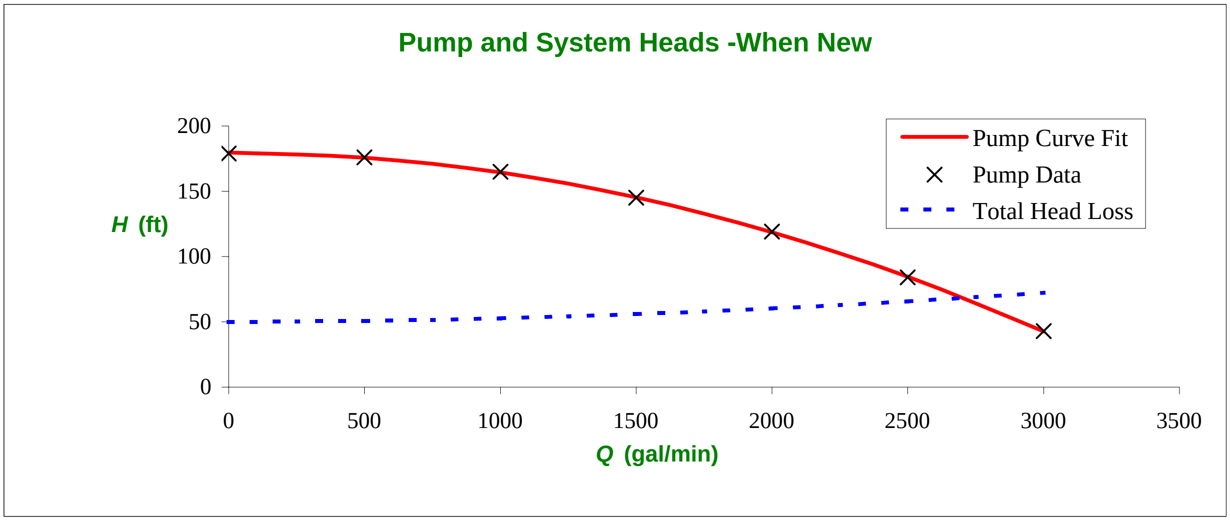
V (ft/s)	Re	f
0.00	0	0.0000
1.42	115325	0.0183
2.84	230649	0.0164
4.26	345974	0.0156
5.67	461299	0.0151
7.09	576623	0.0147
8.51	691948	0.0145

H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)
180	50.0
176	50.8
164	52.8
145	56.0
119	60.3
84.5	65.8
42.7	72.4

$H_0 =$ 180 ft

$A =$ 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2705	7.67	623829	0.0146	68.3	68.3	0%



20-Year Old System:

$f = 2.00\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2541	7.21	586192	0.0295	81.4	81.4	0%

Flow reduction:

163 gpm
6.0% Loss

40-Year Old System:

$f = 2.40\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2484	7.05	572843	0.0354	85.8	85.8	0%

Flow reduction:

221 gpm
8.2% Loss

20-Year Old System and Pump:

$f = 2.00\, f_{new}$ $H_{pump} = 0.90\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2453	6.96	565685	0.0296	79.3	79.3	0%

Flow reduction:

252 gpm
9.3% Loss

40-Year Old System and Pump:

$f = 2.40\, f_{new}$ $H_{pump} = 0.75\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2214	6.28	510754	0.0358	78.8	78.8	0%

Flow reduction:

490 gpm
18.1% Loss

Problem 10.66

[4]

10.66 Consider again the pump and piping system of Problem 10.63. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years. (Use the data of Problem 10.65 for increase in pipe friction factor with age.)

Given: Data on pump and pipe system

Find: Delivery through series pump system; reduction after 20 and 40 years

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

L = 1200 ft

D = 12 in

e = 0.00015 ft (Table 8.1)

ν = 1.23E-05 ft^2/s (Table A.7)

Δz = -50 ft

K_{ent} = 0.5 (Fig. 8.14)

K_{exp} = 1

L_e/D_{elbow} = 30

L_e/D_{valve} = 8 (Table 8.4)

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in series

$$H_{pump} = 2\cdot H_0 - 2\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.

The system and pump heads are computed and plotted below.

To find the operating condition, *Solver* is used to vary Q so that the error between the two heads is minimized.

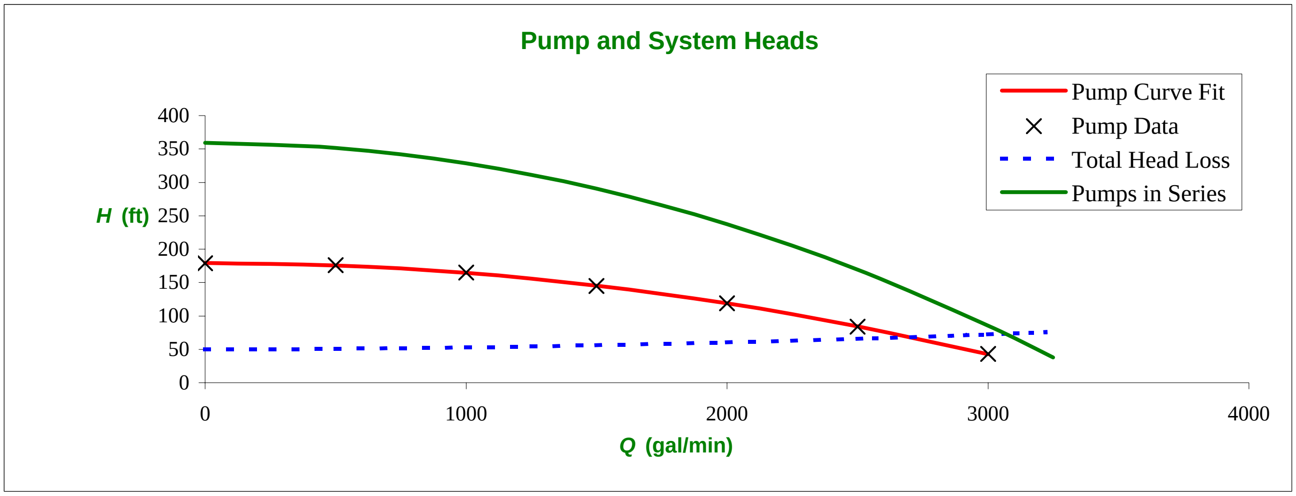
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3250				9.22	749610	0.0144

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
359	50.0
351	50.8
329	52.8
291	56.0
237	60.3
169	65.8
85	72.4
38	76.1

H_0 = 180 ft

A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
3066	8.70	707124	0.0145	73.3	73.3	0%



20-Year Old System:

f = 2.00 f_{new}

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
2964	8.41	683540	0.0291	92.1	92.1	0%

Flow reduction:

102 gpm
3.3% Loss

40-Year Old System:

f = 2.40 f_{new}

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2925	8.30	674713	0.0349	98.9	98.9	0%

Flow reduction:

141 gpm
4.6% Loss

20-Year Old System and Pumps:

f = 2.00 f_{new} H_{pump} = 0.90 H_{new}

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2915	8.27	672235	0.0291	90.8	90.8	0%

Flow reduction:

151 gpm
4.9% Loss

40-Year Old System and Pumps:

f = 2.40 f_{new} H_{pump} = 0.75 H_{new}

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2772	7.86	639318	0.0351	94.1	94.1	0%

Flow reduction:

294 gpm
9.6% Loss

Problem 10.67

[4]

10.67 Consider again the pump and piping system of Problem 10.64. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years. (Use the data of Problem 10.65 for increase in pipe friction factor with age.)

Given: Data on pump and pipe system

Find: Delivery through parallel pump system; reduction in delivery after 20 and 40 years

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

L = 1200 ft

D = 12 in

e = 0.00015 ft (Table 8.1)

ν = 1.23E-05 $\mathrm{ft^2/s}$ (Table A.7)

Δz = -50 ft

K_{ent} = 0.5 (Fig. 8.14)

K_{exp} = 1

L_e/D_{elbow} = 30

L_e/D_{valve} = 8 (Table 8.4)

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{\text{pump}} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in parallel

$$H_{pump} = H_0 - \frac{1}{4}\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.

The system and pump heads are computed and plotted below.

To find the operating condition, *Solver* is used to vary Q so that the error between the two heads is minimized.

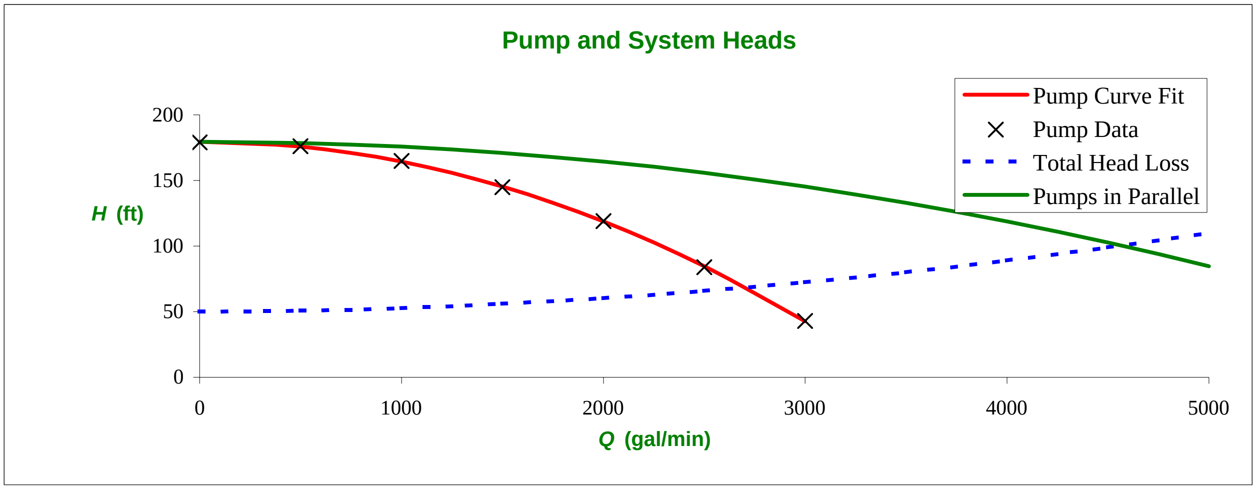
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3500				9.93	807273	0.0143
4000				11.35	922597	0.0142
4500				12.77	1037922	0.0141
5000				14.18	1153247	0.0140

H_{pumps} (par)	$H_{\text{IT}} + \Delta z$ (ft)
180	50.0
179	50.8
176	52.8
171	56.0
164	60.3
156	65.8
145	72.4
133	80.1
119	89.0
103	98.9
85	110.1

$H_0 =$ 180 ft

$A =$ 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{\text{IT}} + \Delta z$ (ft)	Error)
4565	12.95	1053006	0.0141	100.3	100.3	0%



20-Year Old System:

$f = 2.00\, f_{\text{new}}$

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{\text{IT}} + \Delta z$ (ft)	Error)
3906	11.08	900891	0.0284	121.6	121.6	0%

Flow reduction:

660 gpm

14.4% Loss

40-Year Old System:

$f = 2.40\, f_{\text{new}}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{\text{IT}} + \Delta z$ (ft)	Error)
3710	10.52	855662	0.0342	127.2	127.2	0%

Flow reduction:

856

18.7%

20-Year Old System and Pumps:

$f = 2.00\, f_{\text{new}}$ $H_{\text{pump}} = 0.90\, H_{\text{new}}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{\text{IT}} + \Delta z$ (ft)	Error)
3705	10.51	854566	0.0285	114.6	114.6	0%

Flow reduction:

860 gpm

18.8% Loss

40-Year Old System and Pumps:

$f = 2.40\, f_{\text{new}}$ $H_{\text{pump}} = 0.75\, H_{\text{new}}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{\text{IT}} + \Delta z$ (ft)	Error)
3150	8.94	726482	0.0347	106.4	106.4	0%

Flow reduction:

1416

31.0%

Problem 10.68

Given: Water supply for Englewood, Colorado; $L = 5800$ ft, $D = 27$ in., steel.

Section 1: North Platte River at $z_1 = 5280$ ft.

Section 2: Reservoir at $z_2 = 5310$ ft

Design flow rates: $Q = 31$ cfs (initially), 38 cfs (ultimately).

Find: (a) Calculate and plot the system resistance curve.
(b) Specify an appropriate pumping system.
(c) Estimate power required for steady-state operation at both flow rates.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\cancel{\frac{p_1}{\rho g}} + \alpha_1 \overset{\approx 0(2)}{\cancel{\frac{\bar{V}_1^2}{2g}}} + z_1 + H_p = \cancel{\frac{p_2}{\rho g}} + \alpha_2 \overset{\approx 0(2)}{\cancel{\frac{\bar{V}_2^2}{2g}}} + z_2 + \frac{h_{\ell T}}{g}$; $h_{\ell T} = \left[f\left(\frac{L}{D} + \overset{=0(4)}{\cancel{\frac{L_e}{D}}}\right) + K\right]\frac{\bar{V}^2}{2g}$

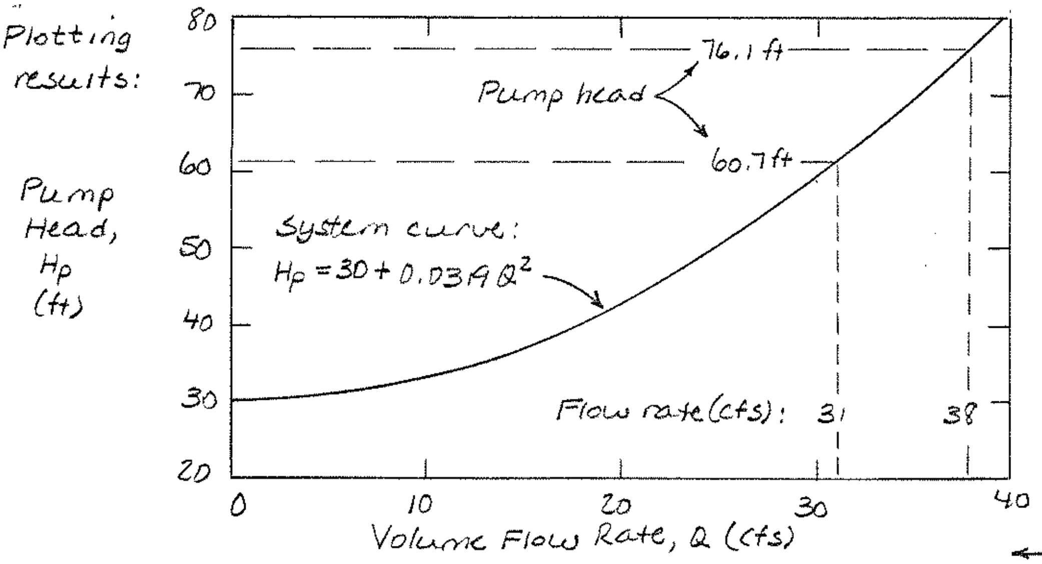
Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 \approx \bar{V}_2 \approx 0$, (3) $z_1 = 5280$ ft, $z_2 = 5310$ ft, (4) $K_{ent} = 0$, $K_{exit} = 1$, $\frac{L_e}{D} = 0$

Sample calculation at $Q = 31$ cfs (13,900 gpm), with $T = 70°F$, $e = 0.00015$ ft (Table 8.1)

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 31 \frac{ft^3}{s} \times \left(\frac{12}{27}\right)^2 \frac{1}{ft^2} = 7.80 \text{ ft/s}; \quad Re = \frac{\bar{V}D}{\nu} = 1.64 \times 10^6; \quad \frac{e}{D} = 6.67 \times 10^{-5}; \quad f = 0.0124$$

$$H_p = z_2 - z_1 + \left(f\frac{L}{D} + K\right)\frac{\bar{V}^2}{2g} = (5{,}310 - 5{,}280)\text{ ft} + \left(0.0124\frac{5800}{(27/12)} + 1\right)\frac{1}{2} \times (7.80)^2 \frac{ft^2}{s^2} \times \frac{s^2}{32.2\text{ ft}} = 61.1 \text{ ft}$$

Plotting results:



Plot

The maximum flow rate is $Q_{max} = 38 \frac{ft^3}{s} \times 7.48 \frac{gal}{ft^3} \times 60 \frac{s}{min} = 17{,}100$ gpm.

Could choose two Peerless 16A 18B pumps at 880^+ rpm (Fig. D.10) or three 10AE 14(G) pumps at 1750 rpm (Fig. D.2). Efficiency (Fig. 11.15) might be $\eta_p = 0.91$.

n, n

Pump power is $\dot{W}_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho Q g H}{\eta_p} = 361$ hp (at $Q = 38$ cfs), 235 hp (at 31 cfs)

$\dot{W}_m$

Problem 10.69

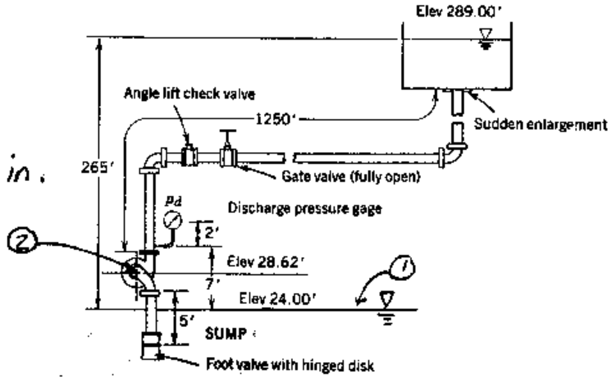
[3]

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Given: Flow system shown.

Design flow rate: $Q = 200\,gpm$

Pipe is commercial steel, $D = 4\ in.$



Find: (a) Calculate the NPSHA.
(b) Select a suitable pump.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_i}{\rho g} + \alpha_i \overset{\approx 0(2)}{\frac{\bar{V}_i^2}{2g}} + z_i + H_p = \frac{p_j}{\rho g} + \alpha_j \overset{\approx 0(2)}{\frac{\bar{V}_j^2}{2g}} + z_j + \frac{h_{\ell T}}{g}$; $h_{\ell T} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) Reservoirs open to atmosphere, (2) Reservoir velocities negligible.
(3) $T = 70°F$ ($p_v = 0.363\,psia$, Table A.7)

At $Q = 200\,gpm\ (0.446\,ft^3/s)$, so $\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.446\,\frac{ft^3}{s} \times \left(\frac{12}{4.026}\right)^2 \frac{1}{ft^2} = 5.04\,ft/s$; $e = 0.00015\,ft$

Thus $Re = \frac{\bar{V}D}{\nu} = 1.58 \times 10^5$; $\frac{e}{D} = 0.000447$; $f = 0.0190$; $\frac{\bar{V}^2}{2g} = \frac{1}{2} \times (5.04)^2 \frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft} = 0.394\,ft$

For the inlet, $\frac{p_2}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} = z_1 - z_2 - \left[f\left(\frac{L}{D} + \overset{\text{foot valve, 75}}{\frac{L_e}{D}}\right) + \underset{\text{entrance, 0.78}}{K}\right]\frac{\bar{V}^2}{2g} = -5.60\,ft$, since $\bar{V}_1 \approx 0$

Then $NPSHA = \frac{p_{t,abs} - p_v}{\rho g} = \frac{p_t}{\rho g} + \frac{p_{atm}}{\rho g} - \frac{p_v}{\rho g} = (-5.60 + 33.9 - 0.838)\,ft = 27.5\,ft$ ← NPSHA

For the complete system, $L/D = (1250+5)\,ft \times \left(\frac{12}{4.026}\right)\frac{1}{ft} = 3740$, $\frac{L_e}{D} = 75 + \overset{\text{check valve}}{55} + 8 \xleftarrow{\text{gate valve}} = 138$

$$H_p = z_2 - z_1 + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g} = (289 - 24)\,ft + \left[0.019(3740 + 138) + 0.78 + 1\right]0.394\,ft$$

$H_p = 295\,ft$ at $Q = 200\,gpm$ ← H_p

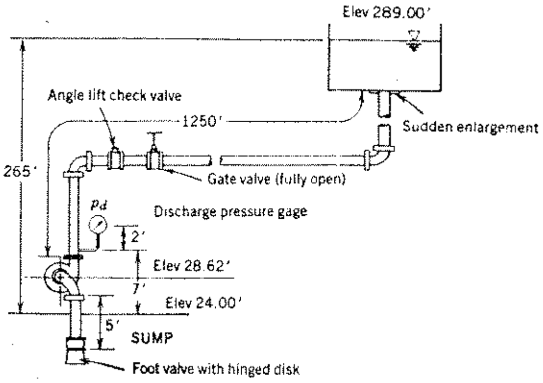
From Fig. D.1, a 4AE12 pump (4 in. discharge line) would fit the application. This pump could produce the required head at a speed between 1750 and 3550 rpm (Figs. D.4 and D.5), but the efficiency may not be acceptable.

{Consult a complete catalog to make a better selection.}

Problem 10.70

Given: Flow system and data of Problem 10.69.

Data for pipe aging from Problem 10.65.



Find: (a) Select pumps that will maintain system flow rate for 10 and 20 years.
(b) Compare delivery to that with pump sized for new pipes only.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\cancel{\frac{p_1}{\rho g}} + \overset{\approx 0(2)}{\cancel{\alpha_1 \frac{\bar{V}_1^2}{2g}}} + z_1 + H_p = \cancel{\frac{p_2}{\rho g}} + \overset{\approx 0(3)}{\cancel{\alpha_2 \frac{\bar{V}_2^2}{2g}}} + z_2 + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 = \bar{V}_2 \approx 0$, (3) Minor losses as in Problem 10.61

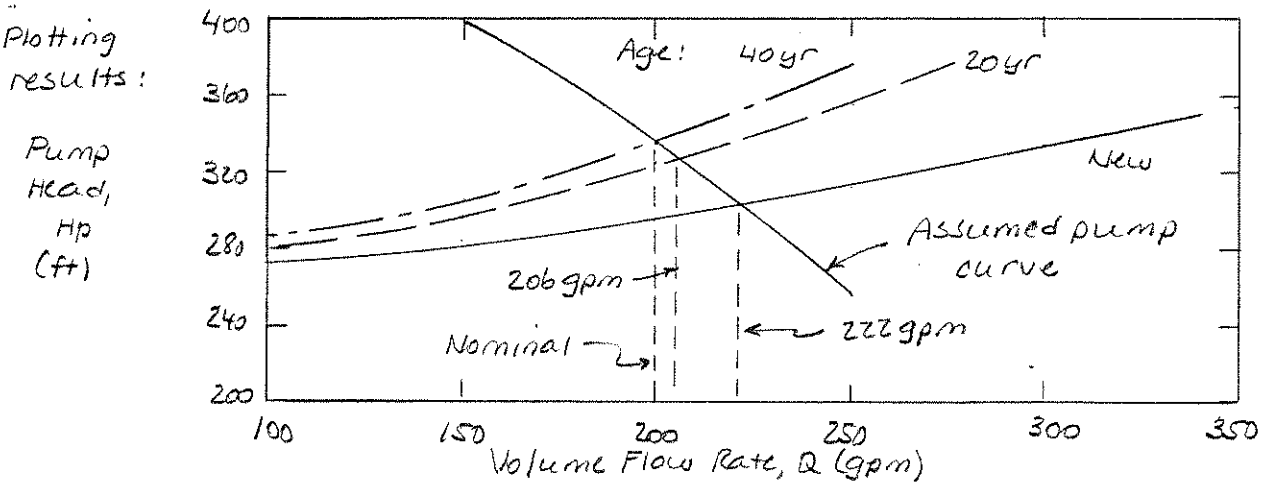
Sample calculations at $Q = 200\,\text{gpm}\,(0.446\,\text{ft}^3/\text{s})$ for new pipe, $e = 0.00015$ ft (Table 8.1):

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.446 \frac{\text{ft}^3}{\text{s}} \times \left(\frac{12}{4.026}\right)^2 \frac{1}{\text{ft}^2} = 5.04\ \text{ft/s}\,; \quad Re = 1.58 \times 10^5; \ \frac{e}{D} = 0.000447; \ f = 0.0190$$

For the complete system, $\frac{L}{D} = (1250 + 5)\,\text{ft} \times \left(\frac{12}{4.026}\right)\frac{1}{\text{ft}} = 3740$; $\frac{L_e}{D} = 138$; $K = 1.28$

$$H_p = z_2 - z_1 + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g} = (289 - 24)\,\text{ft} + \left[0.019(3740 + 138) + 1.28\right]0.394\ \text{ft} = 295\ \text{ft}$$

Plotting results:



Assume H at 200 gpm is 70% of H_0. Then $H_{200} = 336$ ft, $H_0 = \frac{1}{0.7} 336\ \text{ft} = 480$ ft and $H = H_0 - AQ^2$; $A = (H_0 - H)/Q^2 = 0.3\,H_0/Q^2 = 0.3 \times 480\ \text{ft}/(200\ \text{gpm})^2 = 3.60 \times 10^{-3}\ \text{ft}/(\text{gpm})^2$.

Sizing the pump for 200 gpm at 40 years would (assuming no change in pump characteristics) produce 206 gpm at 20 years and 222 gpm in the new system.

The extra head (336 ft, compared to 295 ft) at 200 gpm could be obtained by increasing impeller diameter about 7-10% compared to the pump of Problem 10.69

Problem 10.71

[3]

Given: Flow system of Problem 8.155

Location	Elevation	Pressure
Entrance	50.0 ft	20 psig
Discharge	94.0 ft	0 psig

$D = 2.5$ in. (nominal), $L = 290$ ft

2 open gate values, 1 open angle valve, 7 standard 90° elbows
1 square-edged entrance from a reservoir, 1 free discharge

$Q = 0.439$ ft³/s $\quad$ ($Q = 197$ gpm) $\quad$ Galvanized pipe

Find: (a) Select an appropriate pump.
(b) Check the NPSHR vs. the NPSHA for this system.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\frac{p_1}{\rho g} + \frac{\alpha_1 \overset{\approx 0(1)}{\bar{V}_1^2}}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}\,; \quad h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$$

Assumptions: (1) $\bar{V}_1 \approx 0$, (2) $K_{ent} = 0.5$, (3) $\frac{L_e}{D} = 2(8) + 1(150) + 7(30) = 376$; $D = 2.47$ in.

(4) Galvanized pipe, $e = 0.0005$ ft; $\frac{e}{D} = \frac{0.0005 \text{ ft}}{2.47 \text{ in.}} \times 12 \frac{\text{in.}}{\text{ft}} = 0.00243$

Then
$$H_p = \frac{p_2 - p_1}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} + z_2 - z_1 + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g}\,; \quad \bar{V} = \frac{Q}{A} = 13.2 \text{ ft/s}\,; \quad Re = \frac{\bar{V}D}{\nu} = 2.54\times 10^5; f = 0.02$$

$$H_p = (0-20)\frac{\text{lbf}}{\text{in.}^2} \times 144\frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{ft}^3}{62.4 \text{ lbf}} + \frac{1}{2} \times (13.2)^2\frac{\text{ft}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{32.2 \text{ ft}} + (94.0 - 50.0)\text{ ft}$$
$$+ \left[0.025\left(\frac{290}{2.47/12} + 376\right) + 0.5\right]\frac{1}{2} \times (13.2)^2\frac{\text{ft}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{32.2 \text{ ft}} = 123 \text{ ft}$$

H_p

The pump requirement is $Q = 197$ gpm at $H = 123$ ft. This could be supplied by a Peerless Type 4AE12 pump, with impeller $D = 11$ in., operating at 1750 rpm.

(This pump may be slightly too large, since this operating point is at a flow rate below that for best efficiency.)

The NPSHR for this pump at $Q = 197$ gpm is about 5 ft.

The NPSHA is $\frac{p_1}{\rho g} + \frac{p_{atm}}{\rho g} + \frac{\alpha_1 \bar{V}_1^2}{2g} - p_v = 46.2 + 33.9 + 2.71 - 0.782 \text{ ft} = 82.0 \text{ ft}$

Thus NPSHA >> NPSHR

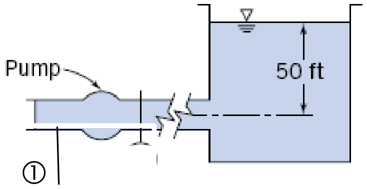
NPSH

Cavitation-free operation is assured.

Problem 10.72

[3]

10.72 Consider the flow system shown in Problem 8.110. Assume the minimum *NPSHR* at the pump inlet is 15 ft of water. Select a pump appropriate for this application. Use the data for increase in friction factor with pipe age given in Problem 10.65 to determine and compare the system flow rate after 10 years of operation.



Given: Flow from pump to reservoir

Find: Select a pump to satisfy NPSHR

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}-h_p \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_1^2}{2}+K_{exit}\cdot\frac{V_1^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 is approximately 1 4) $V_2 <<$

Note that we compute head per unit weight, H, not head per unit mass, h, so the energy equation between Point 1 and the free surface (Point 2) becomes
$$\left(\frac{p_1}{\rho\cdot g}+\frac{V^2}{2\cdot g}\right)-\left(z_2\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+K_{exit}\cdot\frac{V^2}{2\cdot g}-H_p$$

Solving for H_p
$$H_p=z_2-\frac{p_1}{\rho\cdot g}-\frac{V^2}{2\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+K_{exit}\cdot\frac{V^2}{2\cdot g}$$

From Table A.7 (68°F) $\rho = 1.94\cdot\frac{slug}{ft^3}$ $\quad \nu = 1.08\times 10^{-5}\cdot\frac{ft^2}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 6.94\times 10^5$

For commercial steel pipe $e = 0.00015\cdot ft$ (Table 8.1) so $\frac{e}{D} = 0.0002$

Flow is turbulent: Given
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0150$$

For the exit $K_{exit} = 1.0$ so we find $H_p=z_2-\frac{p_1}{\rho\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$

Note that for an NPSHR of 15 ft this means $\frac{p_1}{\rho\cdot g}=15\cdot ft$ $\quad H_p=z_2-\frac{p_1}{\rho\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$ $\quad H_p = 691\,ft$

Note that $Q=\frac{\pi\cdot D^2}{4}\cdot V$ $\quad Q = 4.42\frac{ft^3}{s}$ $\quad Q = 1983\,gpm$

For this combination of Q and Hp, from Fig. D.11 the best pump appears to be a Peerless two-stage 10TU22C operating at 1750 rpm

After 10 years, from Problem 10.65, the friction factor will have increased by a factor of 2.2 $f = 2.2\times 0.150$ $\quad f = 0.330$

We now need to solve $H_p=z_2-\frac{p_1}{\rho\cdot g}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$ for the new velocity V

$$V=\sqrt{\frac{2\cdot D\cdot g}{f\cdot L}\cdot\left(H_p-z_2+\frac{p_1}{\rho\cdot g}\right)} \qquad V = 2.13\frac{ft}{s}$$

and f will still be 2.2×0.150

$Q=\frac{\pi\cdot D^2}{4}\cdot V$ $\quad Q = 0.94\frac{ft^3}{s}$ $\quad Q = 423\,gpm$ Much less!

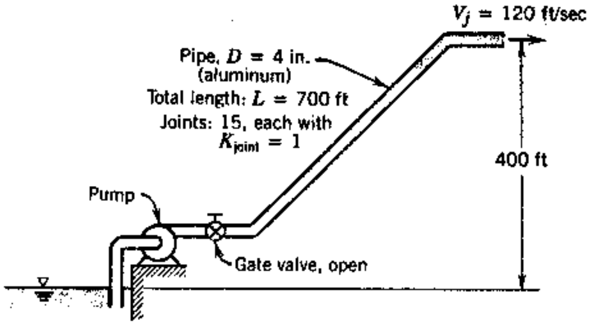
Problem 10.73

[3]

Given: Flow system of Problem 8.156:

$Q = 600\ \text{gpm}$

$\eta_p \approx 0.70$



Find: (a) select an appropriate pump.
(b) Compare pump efficiency with estimate in problem.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\cancel{\frac{p_1}{\rho g}} + \alpha_1 \overset{\approx 0(2)}{\cancel{\frac{\bar{V}_1^2}{2g}}} + z_1 + H_p = \cancel{\frac{p_2}{\rho g}} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}\ ; \quad h_{lT} = \left[f\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]\frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 \approx 0$, (3) Neglect elbow and nozzle losses, (4) $\frac{L_e}{D}$(valve) $= 8$

$$\bar{V} = \frac{Q}{A} = 600\frac{\text{gal}}{\text{min}} \times \frac{4}{\pi}\left(\frac{12}{4}\right)^2\frac{1}{\text{ft}^2} \times \frac{\text{ft}^3}{7.48\ \text{gal}} \times \frac{\text{min}}{60\ \text{s}} = 15.3\ \frac{\text{ft}}{\text{s}}\ ; \quad \frac{\bar{V}^2}{2g} = \frac{1}{2} \times \frac{(15.3)^2\ \text{ft}^2}{\text{s}^2} \times \frac{\text{s}^2}{32.2\ \text{ft}} = 3.63\ \text{ft}$$

$$Re = \frac{\bar{V}D}{\nu} = 15.3\frac{\text{ft}}{\text{s}} \times \left(\frac{4}{12}\right)\text{ft} \times \frac{\text{s}}{1.08 \times 10^{-5}\ \text{ft}^2} = 4.77 \times 10^5\ ;\ \text{smooth};\ f = 0.013 \quad (T = 68°\text{F})$$

$$H_p = z_2 - z_1 + \alpha_2\frac{\bar{V}_2^2}{2g} + \frac{h_{lT}}{g} = z_2 - z_1 + \frac{\bar{V}_2^2}{2g} + \left[f\left(\frac{L}{D}+\frac{L_e}{D}\right)+15K_J\right]\frac{\bar{V}^2}{2g}$$

$$H_p = 400\ \text{ft} + \frac{1}{2} \times \frac{(120)^2\ \text{ft}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{32.2\ \text{ft}} + \left[0.013\left(\frac{700}{4/12}+8\right)+15\right]3.63\ \text{ft} = 778\ \text{ft}$$

Thus the pump requirement is $H_p = 778$ ft at $Q = 600$ gpm. This head is too great to be developed by a single-stage pump (Fig. D.1). From Fig. D.12, the flow could be supplied by a 5TUT168 3-stage pump, driven at 1750 rpm. ←

Single-stage pumps have peak efficiencies of 86 percent at 1750 rpm (Fig. D.8). Thus 70 percent efficiency might be reasonable for a 3-stage pump, since $(0.86)^3 = 0.636$. ←

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

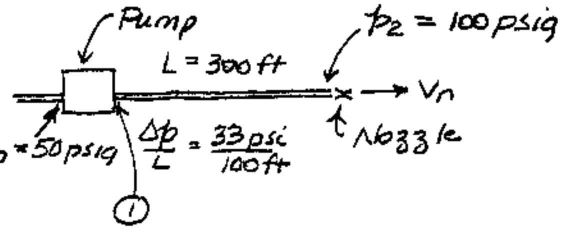
Problem 10.74

[3]

Given: Fire hose and nozzle as shown.

Canvas hose: $D = 3\,in.$

$e = 0.001\,ft$



Find: (a) Design flow rate

(b) Maximum nozzle exit speed

(c) Pump selection

(d) Efficiency and power

Solution: Apply the energy equation for pipe flow:

Computing equation: $$\left(\frac{p_1}{\rho} + \cancel{\alpha_1 \frac{\bar{V}_1^2}{2}} + \cancel{gz_1}\right) - \left(\frac{p_2}{\rho} + \cancel{\alpha_2 \frac{\bar{V}_2^2}{2}} + \cancel{gz_2}\right) = h_{lT} = h_l + \cancel{h_{lm}}$$

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$; (2) $z_1 = z_2$, (3) $h_{lm} = 0$

Then $$\frac{p_1}{\rho} - \frac{p_2}{\rho} = \frac{\Delta p}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2}\ ;\ \bar{V} = \left[\frac{2D\Delta p}{f\rho L}\right]^{\frac{1}{2}}\ ;\ \frac{e}{D} = \frac{0.001\,ft}{3\,in.} \times 12\frac{in.}{ft} = 0.004\ ;\ f = 0.028$$ (fully rough)

$$\bar{V} = \left[\frac{2}{0.028} \times 3\,in. \times 3(33)\frac{lbf}{in.^2} \times \frac{ft^3}{1.94\,slug} \times \frac{1}{300\,ft} \times 12\frac{in.}{ft} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}} = 20.9\,ft/s$$

$$Q = \bar{V}A\ ;\ A = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(\frac{3}{12}\right)^2 ft^2 = 0.0491\,ft^2$$

$$Q = 20.9\frac{ft}{s} \times 0.0491\,ft^2 \times 7.48\frac{gal}{ft^3} \times \frac{60\,s}{min} = 461\,gpm$$ (design flow rate)

Q

Apply Bernoulli to nozzle.

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2} = \frac{\cancelto{0}{p_{atm}}}{\rho} + \frac{V_n^2}{2} + \cancel{gz_n}\ ;\ V_n = \left[\frac{2p_2}{\rho} + V_2^2\right]^{\frac{1}{2}}$$

$$V_n = \left[2 \times 100\frac{lbf}{in.^2} \times \frac{ft^3}{1.94\,slug} \times 144\frac{in.^2}{ft^2} + (20.9)^2\frac{ft^2}{s^2}\right]^{\frac{1}{2}} = 124\,ft/s$$

V_n

The pump head requirement (neglecting V and z) will be

$$H_p = \frac{p_1 - p_0}{\rho g} = [100 + 3(33) - 50]\frac{lbf}{in.^2} \times \frac{ft^3}{62.4\,lbf} \times 144\frac{in.^2}{ft^2} = 344\,ft$$

H_p

From the pump selector chart (Fig. D.1) choose 3AE9G or 4AE10 pump, 3500 rpm.

Based on 4AE12 at 3550 rpm (Fig. D.5), expect $\eta \approx 0.75$

η

$$P = \frac{Q\Delta p}{\eta} = \frac{Q\rho g \Delta h}{\eta} = \frac{1}{0.75} \times 20.9\frac{ft}{s} \times 0.0491\,ft^2 \times 62.4\frac{lbf}{ft^3} \times 344\,ft \times \frac{hp \cdot s}{550\,ft \cdot lbf} = 53.4\,hp$$

P

13-782 500 SHEETS, FILLER 5 SQUARE
42-381 50 SHEETS EYE-EASE® 5 SQUARE
42-382 100 SHEETS EYE-EASE® 5 SQUARE
42-389 200 SHEETS EYE-EASE® 5 SQUARE
42-392 100 RECYCLED WHITE 5 SQUARE
42-399 200 RECYCLED WHITE 5 SQUARE
Made in U.S.A.
National® Brand

Problem 10.75

[3]

10.75 Consider the pipe network of Problem 8.168. Select a pump suitable to deliver a total flow rate of 300 gpm through the pipe network.

Given: Water pipe system

Find: Pump suitable for 300 gpm

Solution:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) = h_l \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad \text{(Turbulent)}$$

The energy equation can be simplified to $\Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$

This can be written for each pipe section

Pipe A (first section) $\Delta p_A = \rho \cdot f_A \cdot \frac{L_A}{D_A} \cdot \frac{V_A^{\,2}}{2}$ (1)

Pipe B (1.5 in branch) $\Delta p_B = \rho \cdot f_B \cdot \frac{L_B}{D_B} \cdot \frac{V_B^{\,2}}{2}$ (2)

Pipe C (1 in branch) $\Delta p_C = \rho \cdot f_C \cdot \frac{L_C}{D_C} \cdot \frac{V_C^{\,2}}{2}$ (3)

Pipe D (last section) $\Delta p_D = \rho \cdot f_D \cdot \frac{L_D}{D_D} \cdot \frac{V_D^{\,2}}{2}$ (4)

In addition we have the following contraints

$Q_A = Q_D = Q$ (5)

$Q = Q_B + Q_C$ (6)

$\Delta p = \Delta p_A + \Delta p_B + \Delta p_D$ (7)

$\Delta p_B = \Delta p_C$ (8)

We have 2 unknown flow rates (or, equivalently, velocities); We solve the above eight equations simultaneously

Once we compute the flow rates and pressure drops, we can compute data for the pump

$\Delta p_{pump} = \Delta p$ and $Q_{pump} = Q_A$ $\qquad W_{pump} = \Delta p_{pump} \cdot Q_{pump}$

Pipe Data:

Pipe	*L* (ft)	*D* (in)	*e* (ft)
A	150	1.5	0.00085
B	150	1.5	0.00085
C	150	1	0.00085
D	150	1.5	0.00085

Fluid Properties:

$\rho =$ 1.94 slug/ft^3

$\mu =$ 2.10E-05 lbf·s/ft^2

Flow Rate:

$Q =$ 300 gpm

= 0.668 ft^3/s

Flows:

Q_A (ft^3/s)	Q_B (ft^3/s)	Q_C (ft^3/s)	Q_D (ft^3/s)
0.668	0.499	0.169	0.668

V_A (ft/s)	V_B (ft/s)	V_C (ft/s)	V_D (ft/s)
54.47	40.67	31.04	54.47

Re_A	Re_B	Re_C	Re_D
6.29E+05	4.70E+05	2.39E+05	6.29E+05

f_A	f_B	f_C	f_D
0.0335	0.0336	0.0384	0.0335

Heads:

Δp_A (psi)	Δp_B (psi)	Δp_C (psi)	Δp_D (psi)
804.0	448.8	448.8	804.0

Constraints:

(6) $Q = Q_B + Q_C$	(8) $\Delta p_B = \Delta p_C$
0.00%	**0.00%**

Error: **0.00%**

Vary Q_B and Q_C using *Solver* to minimize total error

For the pump:

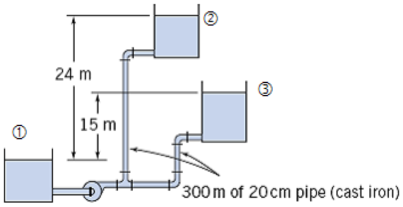
Δp (psi)	Q (gpm)	$\mathscr{P}$ (hp)
2057	300	360

This is a very high pressure; a sequence of pumps would be needed

Problem 10.76

[4]

10.76 A pumping system with two different static lifts is shown. Each reservoir is supplied by a line consisting of 300 m of 20 cm cast-iron pipe. Evaluate and plot the system head versus flow curve. Explain what happens when the pump head is less than the height of the upper reservoir. Calculate the flow rate delivered at a pump head of 26 m.



Given: Pump and supply pipe system

Find: Head versus flow curve; Flow for a head of 26 m

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}-h_{pump} \qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

Applying to the 24 m branch (branch a)

$$-g\cdot H_a=f\cdot\frac{L}{D}\cdot\frac{V_a^{\,2}}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V_a^{\,2}}{2}+K\cdot\frac{V_a^{\,2}}{2}-g\cdot H_{pump}$$

where $H_a = 24\cdot m$ and $\frac{L_{ea}}{D}$ is due to a standard T branch (= 60) and a standard elbow (= 30) from Table 8.4, and $K = K_{ent}+K_{exit} = 1.5$ from Fig. 8.14

$$H_{pump}=H_a+\left[f\cdot\left(\frac{L}{D}+\frac{L_{ea}}{D}\right)+K\right]\cdot\frac{V_a}{2\cdot g} \qquad (1)$$

Applying to the 15 m branch (branch b)

$$H_{pump}=H_b+\left[f\cdot\left(\frac{L}{D}+\frac{L_{eb}}{D}\right)+K\right]\cdot\frac{V_b}{2\cdot g} \qquad (2)$$

where $H_b = 15\cdot m$ and $\frac{L_{eb}}{D}$ is due to a standard T run (= 20) and two standard elbows (= 60), and $K = K_{ent}+K_{exit} = 1.5$

Given data:

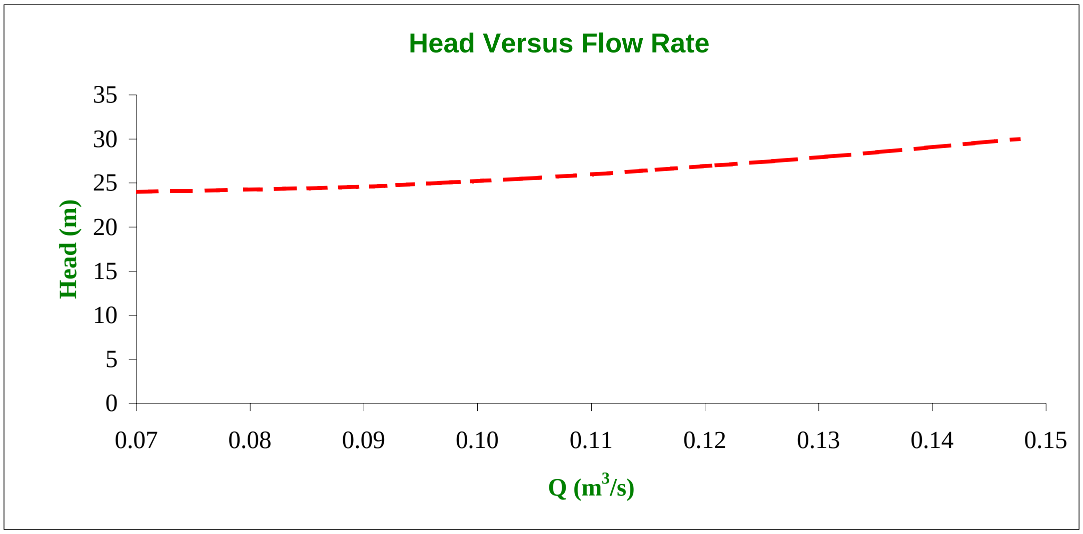
L = 300 m
e = 0.26 mm
D = 20 cm
K = 1.5
L_{ea}/D = 90
L_{eb}/D = 80
H_a = 24 m
H_b = 15 m
ρ = 1000 kg/m^3
ν = 1.01E-06 m^2/s

Computed results: Set up Solver so that it varies all flow rates to make the total head error zero

H_{pump} (m)	Q (m^3/s)	Q_a (m^3/s)	V_a (m/s)	Re_a	f_a	H_{pump} (Eq. 1)	Q_b (m^3/s)	V_b (m/s)	Re_b	f_b	H_{pump} (Eq. 2)	H (Errors)
24.0	0.070	0.000	0.000	8.62E+00	7.4264	24.0	0.070	2.230	4.42E+05	0.0215	24.0	0.00
24.5	0.088	0.016	0.506	1.00E+05	0.0231	24.5	0.072	2.292	4.54E+05	0.0215	24.5	0.00
25.0	0.097	0.023	0.72	1.44E+05	0.0225	25.0	0.074	2.35	4.66E+05	0.0215	25.0	0.00
25.5	0.104	0.028	0.89	1.77E+05	0.0223	25.5	0.076	2.41	4.78E+05	0.0215	25.5	0.00
26.0	0.110	0.033	1.03	2.05E+05	0.0221	26.0	0.078	2.47	4.89E+05	0.0215	26.0	0.00
26.5	0.116	0.036	1.16	2.30E+05	0.0220	26.5	0.079	2.52	5.00E+05	0.0215	26.5	0.00
27.0	0.121	0.040	1.27	2.52E+05	0.0219	27.0	0.081	2.58	5.11E+05	0.0214	27.0	0.00
27.5	0.126	0.043	1.38	2.73E+05	0.0218	27.5	0.083	2.63	5.21E+05	0.0214	27.5	0.00
28.0	0.131	0.046	1.47	2.92E+05	0.0218	28.0	0.084	2.69	5.32E+05	0.0214	28.0	0.00
28.5	0.135	0.049	1.56	3.10E+05	0.0217	28.5	0.086	2.74	5.42E+05	0.0214	28.5	0.00
29.0	0.139	0.052	1.65	3.27E+05	0.0217	29.0	0.088	2.79	5.52E+05	0.0214	29.0	0.00
29.5	0.144	0.054	1.73	3.43E+05	0.0217	29.5	0.089	2.84	5.62E+05	0.0214	29.5	0.00
30.0	0.148	0.057	1.81	3.59E+05	0.0216	30.0	0.091	2.89	5.72E+05	0.0214	30.0	0.00

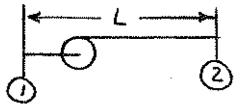
For the pump head less than the upper reservoir head flow will be out of the reservoir (into the lower one)

Total Error: 0.00



Problem 10.77

Given: Chilled water circulation system of Problem 8.158:



$L = 3$ mi (15,800 ft), $D = 2$ ft (steel), $Q = 11{,}200$ gpm, Loop configuration

Find: (a) Select suitable pumps for parallel operation.
(b) Calculate power for 3 pumps in parallel.
(c) Calculate volume flow rate and power if 1 or 2 pumps operate.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{\alpha_1 \bar{V}_1^2}{2g}} + \cancel{z_1} + H_a = \cancel{\frac{p_2}{\rho g}} + \cancel{\frac{\alpha_2 \bar{V}_2^2}{2g}} + \cancel{z_2} + \frac{h_{LT}}{g}$; $h_{LT} = \left[f\left(\frac{L}{D} + \cancel{\frac{L_e}{D}}\right) + \cancel{K}\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2$, (2) $\alpha_1 \bar{V}_1^2 = \alpha_2 \bar{V}_2^2$, (3) $z_1 = z_2$, (4) Neglect minor losses, $\frac{L_e}{D} \approx 0$, $K \approx 0$

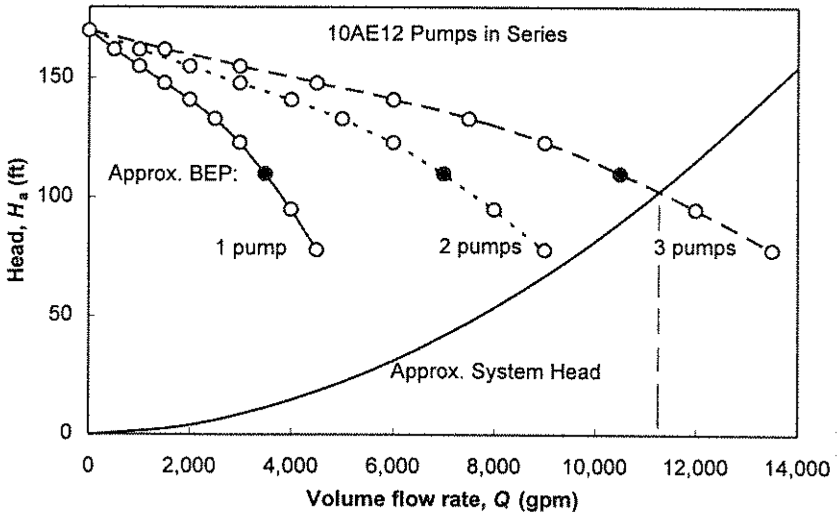
$$\bar{V} = \frac{Q}{A} = 11{,}200\,\frac{\text{gal}}{\text{min}} \times \frac{4}{\pi (2)^2\,\text{ft}^2} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{min}}{60\,\text{sec}} = 7.94\ \text{ft/s}; \quad \frac{\bar{V}^2}{2g} = \frac{1}{2} \times \frac{(7.94)^2\,\text{ft}^2}{\text{s}^2} \times \frac{\text{s}^2}{32.2\,\text{ft}} = 0.979\ \text{ft}$$

Assume $T = 40$ F, so $\nu = 1.64 \times 10^{-5}$ ft²/s ; $Re = \frac{\bar{V}D}{\nu} = 9.68 \times 10^5$; $\frac{e}{D} = 7.5 \times 10^{-5}$; $f = 0.013$

$$H_a = f\frac{L}{D}\frac{\bar{V}^2}{2g} = 0.013 \times \frac{3(5280)\,\text{ft}}{2\,\text{ft}} \times 0.979\ \text{ft} = 101\ \text{ft}$$ H_p

For three pumps in parallel, each will operate at $Q/3 = 3730$ gpm. The requirement for each pump is $H = 101$ ft at $Q = 3730$ gpm. This can be supplied by Peerless Type 10AE12 pumps with impellers of $D = 12$ in. diameter, operating at $N = 1760$ nominal rpm. The efficiency at this operating point is $\eta \approx 0.85$. 3 pump.

Find operating points graphically for 1, 2, and 3 pumps:



The graphical solution is shown.

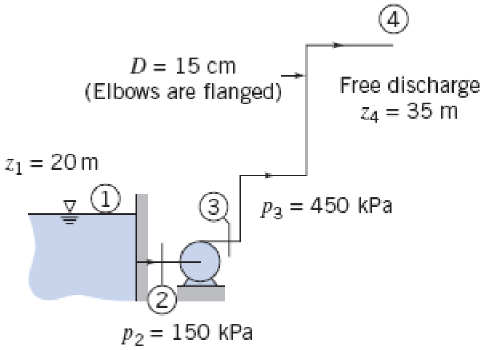
Q_1 = not satisfactory, $Q_2 = 9400$ gpm (marginal), $Q_3 = 11{,}200$ gpm (OK) Q

Assuming $\eta_p \approx 0.7$, then $\dot{W}_{m_1} = \frac{\rho Q g H}{\eta} \approx 78$ hp, $\dot{W}_{m_2} \approx 241$ hp, and $\dot{W}_{m_3} \approx 409$ hp

Problem 10.78

[4]

10.78 Consider the flow system shown in Problem 8.76. Evaluate the *NPSHA* at the pump inlet. Select a pump appropriate for this application. Use the data on pipe aging from Problem 10.65 to estimate the reduction in flow rate after 10 years of operation.



Given: Data on flow from reservoir/pump

Find: Appropriate pump; Reduction in flow after 10 years

Solution:

Basic equation: $\left(\frac{p_1}{\rho\cdot g}+\alpha\cdot\frac{V_1^{\,2}}{2\cdot g}+z_1\right)-\left(\frac{p_4}{\rho\cdot g}+\alpha\cdot\frac{V_4^{\,2}}{2\cdot g}+z_4\right)=H_{lT}-H_p$ for flow from 1 to 4

$$H_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2\cdot g}+K\cdot\frac{V^2}{2\cdot g}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) $V_2 = V_3 = V_4$ (constant area pipe)

Given or available data $\rho = 1000\cdot\frac{kg}{m^3}$ $\nu = 1.01\times10^{-6}\cdot\frac{m^2}{s}$ $p_v = 2.34\cdot kPa$ (Table A.8)

$p_2 = 150\cdot kPa$ $p_3 = 450\cdot kPa$ $D = 15\cdot cm$ $e = 0.046\cdot mm$ $Q = 0.075\cdot\frac{m^3}{s}$

$z_1 = 20\cdot m$ $z_4 = 35\cdot m$ $V = \frac{4\cdot Q}{\pi\cdot D^2}$ $V = 4.24\frac{m}{s}$

For minor losses we have Four elbows: $\frac{L_e}{D} = 4\times12 = 48$ (Fig. 8.16) Square inlet: $K_{ent} = 0.5$

At the pump inlet $NPSHA = \frac{p_2+\frac{1}{2}\cdot\rho\cdot V^2-p_v}{\rho\cdot g}$ $NPSHA = 16.0\,m$

The head rise through the pump is $H_p = \frac{p_3-p_2}{\rho\cdot g}$ $H_p = 30.6\,m$

Hence for a flow rate of $Q = 0.075\frac{m^3}{s}$ or $Q = 1189\,gpm$ and $H_p = 30.6\,m$ or $H_p = 100\,ft$, from Appendix D. Fig. D3 a Peerless4AE11 would suffice

We do not know the pipe length L! Solving the energy equation for it $z_1 - z_4 = H_{lT} - H_p = f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2\cdot g}+K_{ent}\cdot\frac{V^2}{2\cdot g}-H_p$

For f $Re = \frac{V\cdot D}{\nu}$ $Re = 6.303\times10^5$ and $\frac{e}{D} = 3.07\times10^{-4}$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0161$

Hence, substituting values $L = \frac{2 \cdot g \cdot D}{f \cdot V^2} \cdot \left(z_1 - z_4 + H_p\right) - D \cdot \left(\frac{L_e}{D}\right) - \frac{K_{ent} \cdot D}{f}$ $L = 146\,m$

From Problem 10.65, for a pipe $D = 0.15\,m$ or $D = 5.91\,in$, the aging over 10 years leads to $f_{worn} = 2.2 \cdot f$

We need to solve the energy equation for a new V

$$V_{worn} = \sqrt{\frac{2 \cdot g \cdot \left(z_1 - z_4 \cdot\right.}{f_{worn} \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right) + K_{ent}}} \quad V_{worn} = 2.88\,\frac{m}{s}$$

Hence $Q_{worn} = \frac{\pi \cdot D^2}{4} \cdot V_{worn}$ $Q_{worn} = 0.0510\,\frac{m^3}{s}$

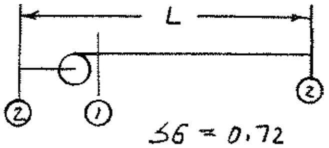
$\Delta Q = Q_{worn} - Q$ $\Delta Q = -0.0240\,\frac{m^3}{s}$ $\frac{\Delta Q}{Q} = -32.0\,\%$

Check f $Re_{worn} = \frac{V_{worn} \cdot D}{\nu}$ Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re_{worn} \cdot \sqrt{f}}\right)$ $f = 0.0165$

Hence using 2.2 x 0.0161 is close enough to using 2.2 x 0.0165

Problem 10.79

Given: Gasoline pipeline of Problem 8.124:



$L = 13\ km$, $D = 0.6\ m$, $\Delta p_{12} = 1.4\ MPa$ $\qquad SG = 0.72$

Roughness equivalent to galvanized iron ($e = 0.15\ mm = 0.00015\ m$)

Find: (a) Select suitable pumps for parallel operation.
(b) Calculate the power required for 4 pumps in parallel.
(c) Calculate volume flow rate and power with 1, 2, and 3 pumps.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_i}{\rho g} + \alpha_i \frac{\bar V_i^2}{2g} + z_i + H_p = \frac{p_j}{\rho g} + \alpha_j \frac{\bar V_j^2}{2g} + z_j + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar V^2}{2}$

Assumptions: (1) $\bar V_1 = \bar V_2$, $\alpha_1 = \alpha_2$, (2) $z_1 = z_2$, (3) Neglect minor losses, $\frac{L_e}{D} \approx 0$, $K \approx 0$

Find flow rate to size pump. From 1 to 2, $H_p = 0$, so $\qquad D = 0.6\,m \times \frac{ft}{0.305\,m} = 1.97\ ft$

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + \frac{h_{lT}}{g};\ \Delta p = \rho h_{lT} = \rho f \frac{L}{D}\frac{\bar V^2}{2};\quad \bar V = \left[\frac{2\Delta p D}{\rho f L}\right]^{\frac{1}{2}} \qquad \frac{e}{D} = \frac{0.00015\,m}{0.6\,m} = 2.5\times10^{-4}$$

But $f = f(Re, e/D)$; Re is not known. Choose f from fully-rough region, $f = 0.014$.

$$\Delta p = 1.4\times10^6\,Pa \times \frac{14.7\,psi}{101\times10^3\,Pa} = 204\ psi;\ \rho = 0.72\rho_{H_2O} = 0.72\times1.94\frac{slug}{ft^3} = 1.40\ slug/ft^3$$

$$\bar V = \left[2\times 204\frac{lbf}{in.^2}\times 1.97\,ft\times\frac{ft^3}{1.40\,slug}\times\frac{1}{0.014}\times\frac{1}{42{,}600\,ft}\times\frac{slug\cdot ft}{lbf\cdot s^2}\times 144\frac{in.^2}{ft^2}\right]^{\frac{1}{2}} = 11.8\ ft/s$$

Check: $Re = \frac{\bar V D}{\nu} = 11.8\frac{ft}{s}\times 1.97\,ft\times\frac{s}{8.6\times10^{-6}\,ft^2} = 2.70\times10^6$; $f = 0.0146$ (See Note below.)

$$\bar V = \left[\frac{0.014}{0.0146}\right]^{1/2} 11.8\frac{ft}{s} = 11.6\ ft/s, \qquad Q = \bar V A = 35.3\frac{ft^3}{s} = 15{,}700\ gpm$$ Q

For parallel operation with four pumps, each must supply $\frac{Q}{4} = 3930\ gpm$.

The head requirement is $H_p = \frac{p_1 - p_2}{\rho g} = 204\frac{lbf}{in.^2}\times\frac{ft^3}{(0.72)62.4\,lbf}\times 144\frac{in.^2}{ft^2} = 654\ ft$ (gasoline)

This combination of head and flow rate cannot be supplied by a single-stage pump. From Fig. D.11, the two-stage Peerless Type 10TU22C pump may be chosen. Pump

The input power requirement is $\dot W_m = \frac{\rho Q g H}{\eta_p}$. Assuming $\eta_p = 0.65$,

$$\dot W_m = \frac{1}{0.65}\times 15{,}700\frac{gal}{min}\times 204\frac{lbf}{in.^2}\times\frac{ft^3}{7.48\,gal}\times\frac{min}{60\,s}\times 144\frac{in.^2}{ft^2}\times\frac{hp\cdot s}{550\,ft\cdot lbf} = 2870\ hp\ (total)$$ $\dot W_m$

Flow rates with fewer pumps operating may be found from a plot. The pump characteristic may be approximated as (assume $H = 0.7\,H_0$):

$$\hat H_p \approx 934 - 1.81\times10^{-5} Q^2 \qquad \left\{H_0 = \frac{654}{0.7} = 934;\ B = \frac{H_0 - H}{Q^2} = \frac{(0.3)934}{(3930)^2} = 1.81\times10^{-5}\right\}$$

{Note: gasoline is between octane and heptane, Fig. A.3. For $T = 15°C$, $\nu \approx 8\times10^{-7}\frac{m^2}{s} = 8.6\times10^{-6}\frac{ft^2}{s}$.}

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Problem 10.79

For two pumps in parallel, $\hat{H} = H_0 - A\left(\frac{Q}{2}\right)^2 \approx 934 - 4.54 \times 10^{-6} Q^2$

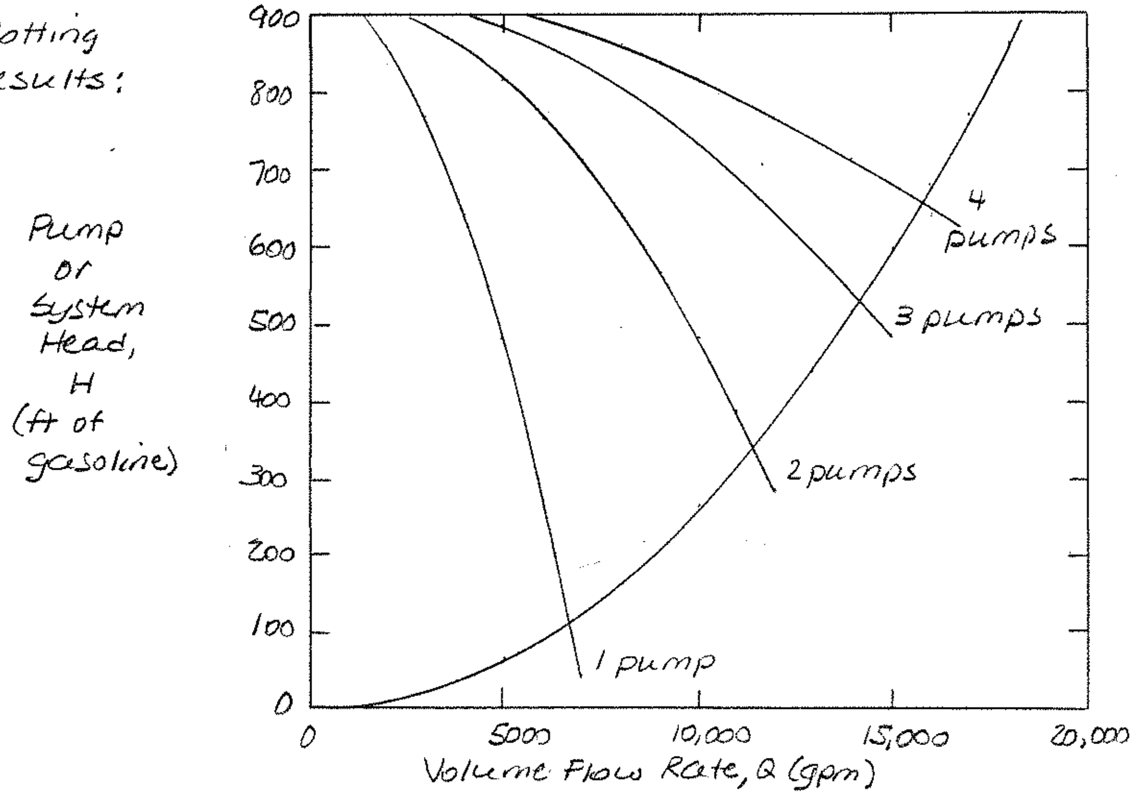
For three pumps in parallel, $\hat{H} = H_0 - A\left(\frac{Q}{3}\right)^2 \approx 934 - 2.02 \times 10^{-6} Q^2$

For four pumps in parallel, $\hat{H} = H_0 - A\left(\frac{Q}{4}\right)^2 \approx 934 - 1.13 \times 10^{-6} Q^2$

The pipe system characteristic is approximately given by

$$\hat{H}_s \approx BQ^2 \; ; \; B = \frac{\hat{H}_s}{Q^2} = \frac{654}{(15{,}700)^2} = 2.65 \times 10^{-6}\ \text{ft/gpm}^2, \text{ so } \hat{H}_s = 2.65 \times 10^{-6} Q^2$$

Plotting results:



The approximate volume flow rates, heads, and power requirements (assuming $\eta_p = 0.65$) are:

Number of Pumps	1	2	3	4
Flow Rate (gpm)	6710	11,400	14,200	15,700
Head (ft gasoline)	119	345	531	654
Power (hp)	224	1100	2110	2880

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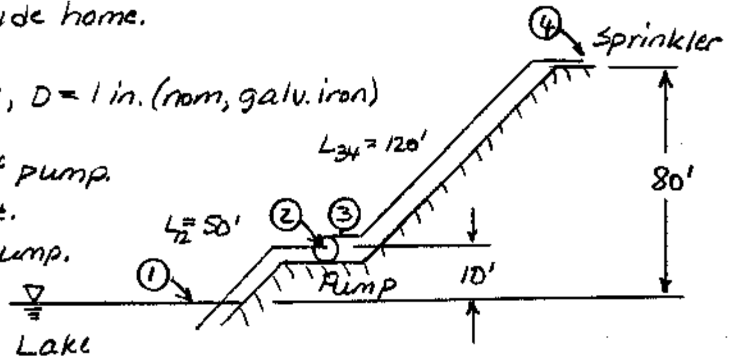
Given: Sprinkler system for lakeside home.

$Q = 10$ gpm, $p_4 = 50$ psig, $D = 1$ in. (nom, galv. iron)



Find:
(a) Head loss on suction side of pump.
(b) Gage pressure at pump inlet.
(c) Hydralic power req'mt for pump.
(d) Change to $D = 1.5$ in.?
(e) Pump halfway up hill?

Solution: Assume $T = 68°F$, so $\nu = 1.08 \times 10^{-5}\ ft^2/s$ (Table A.7).

For 1 in. (nominal) pipe, $D = 1.049$ in. (Table 8.5). $\bar V = \frac{Q}{A}$; $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\left(\frac{1.049}{12}\right)^2 ft^2 = 0.00600\ ft^2$

$$\bar V = \frac{Q}{A} = 10\,\frac{gal}{min} \times \frac{1}{0.00600\ ft^2} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} = 3.71\ ft/s;\quad \frac{\bar V^2}{2} = \frac{1}{2} \times (3.71)^2 \frac{ft^2}{s^2} = 6.90\ ft^2/s^2$$

$$Re = \frac{\bar V D}{\nu} = 3.71\,\frac{ft}{s} \times 1.049\ in. \times \frac{s}{1.08 \times 10^{-5}\ ft^2} \times \frac{ft}{12\ in.} = 3.00 \times 10^4;\ e = 0.0005\ ft\ \text{(Table 8.1)}$$

Therefore $\frac{e}{D} = 0.0005\ ft \times \frac{1}{1.049\ in.} \times 12\,\frac{in.}{ft} = 0.00572$ and $f \approx 0.034$ (Fig. 8.13).

Apply energy equation for steady incompressible pipe flow:

$$\left(\overset{\approx 0(1)}{\cancel{\frac{p_1}{\rho g}}} + \alpha_1 \overset{\approx 0(2)}{\cancel{\frac{\bar V_1^2}{2g}}} + \overset{=0(3)}{\cancel{z_1}}\right) - \left(\frac{p_2}{\rho g} + \alpha_2 \frac{\bar V_2^2}{2g} + z_2\right) = \frac{h_{l_{T_{12}}}}{g} = \left(f\frac{L_{12}}{D} + \underbrace{K_{ent}}_{=0.78} + f\underbrace{\frac{L_e}{D_{45°}}}_{=16} + f\underbrace{\frac{L_e}{D_{90°}}}_{=30}\right)\frac{\bar V^2}{2g}$$

(Table 8.4)

(Table 8.2)

Assumptions:
(1) $p_1 = p_{atm} = 0$ (gage)
(2) $V_1 = 0$
(3) $z_1 = 0$
(4) $\alpha_2 \approx 1$

$$\frac{L_{12}}{D} = \frac{50\ ft}{1.049\ in.} \times 12\,\frac{in.}{ft} = 572$$

$$H_{l_{T_{12}}} = \left[0.034(572 + 30 + 16) + 0.78\right] 6.88\,\frac{ft^2}{s^2} \times \frac{s^2}{32.2\ ft}$$

$$H_{l_{T_{12}}} = 4.67\ ft$$

$h_{l_{T_{12}}}$

$$p_2(gage) = -\rho g\left(\frac{\bar V_2^2}{2g} + z_2 + H_{l_T}\right)$$

$$p_2(gage) = -62.4\,\frac{lbf}{ft^3}\left(6.88\,\frac{ft^2}{s^2} \times \frac{s^2}{32.2\ ft} + 10\ ft + 4.67\ ft\right)\frac{ft^2}{144\ in.^2} = -6.45\ psig$$

p_2

To find hydraulic power, must know p_3. Apply energy equation:

$$\left(\frac{p_3}{\rho g} + \alpha_3 \cancel{\frac{\bar V_3^2}{2g}} + z_3\right) - \left(\frac{p_4}{\rho g} + \alpha_4 \cancel{\frac{\bar V_4^2}{2g}} + z_4\right) = h_{l_{T_{34}}} = \left(f\frac{L_{34}}{D} + 2K_{45°}\right)\frac{\bar V^2}{2g}$$

Assumptions: (1) $\bar V_3 = \bar V_4 = \bar V = \bar V_2$

$$\frac{L_{34}}{D} = \frac{120\ ft}{1.049\ in.} \times 12\,\frac{in.}{ft} = 1{,}370$$

$$p_3 = p_4 + \rho g\left[z_4 - z_3 + \left(f\frac{L_{34}}{D} + 2f\frac{L_e}{D_{45°}}\right)\frac{\bar V^2}{2g}\right]$$

$$p_3 = 50\ psig + 62.4\,\frac{lbf}{ft^3}\left[(80-10)ft + (0.034 \times (1370 + 2 \times 16))6.90\,\frac{ft^2}{s^2} \times \frac{s^2}{32.2\ ft}\right]\frac{ft^2}{144\ in.^2} = 84.8\ psig$$

Thus the pump head is $H_p = \frac{p_3 - p_2}{\rho g} = [84.8 - (-6.45)]\,\frac{lbf}{in.^2} \times \frac{ft^3}{62.4\ lbf} \times 144\,\frac{in.^2}{ft^2} = 211\ ft$

since $\bar V_2 = \bar V_3$ and the elevations are corrected to the pump centerline ($z_2 = z_3$).

Problem 10.80

[4] Part 2/2

From Eq. 10.8a, $\dot{W}_h = \rho Q g H_p$. Thus $\dot{W}_h = \rho g Q H_p$

$$\dot{W}_h = 62.4\,\frac{lbf}{ft^3} \times 10\,\frac{gal}{min} \times 211\,ft \times \frac{ft^3}{7.48\,gal} \times \frac{hp \cdot min}{33{,}000\,ft \cdot lbf} = 0.533\ hp$$ $\dot{W}_h$

Changing to $D = 1.5$ in. (nominal) pipe would reduce the mean velocity and hence the head loss and the minor loss. For this pipe $D = 1.610$ in. (Table 8.4).

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(\frac{1.610}{12}\right)^2 ft^2 = 0.0141\ ft^2;\ \text{so}\ \bar{V} = \frac{Q}{A} = 10\,\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times \frac{1}{0.0141\,ft^2} = 1.58\ ft/s$$

$$Re = \frac{\bar{V}D}{\nu} = 1.58\,\frac{ft}{s} \times \frac{1.61}{12}\,ft \times \frac{s}{1.08 \times 10^{-5}\,ft^2} = 1.96 \times 10^4;\ \frac{e}{D} = 0.00373;\ f \approx 0.032\ (\text{Fig. } 8.13)$$

Thus $$\frac{\bar{V}_2^2}{2g} = \frac{1}{2} \times (1.58)^2\,\frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft} = 0.0388\ ft$$

$$H_{l_{T_{12}}} = \left(\left(0.031 \times \frac{50}{(1.61/12)} + 30 + 16\right) + 0.78\right) 0.0388\ ft = 0.534\ ft$$

$$H_{l_{T_{34}}} = \left(0.031 \times \left(\frac{120}{(1.61/12)} + 2 \times 16\right)\right) 0.0388\ ft = 1.11\ ft$$

$$p_2(gage) = -\rho g\left(\frac{\bar{V}_2^2}{2g} + z_2 + H_{l_{T_{12}}}\right)$$

$$p_2(gage) = -62.4\,\frac{lbf}{ft^3}(0.0388 + 10 + 0.534)\,ft \times \frac{ft^2}{144\,in.^2} = -4.58\ psig$$

and

$$p_3(gage) = p_4 + \rho g(z_4 - z_3 + H_{l_{T_{34}}})$$

$$p_3(gage) = 50\ psig + 62.4\,\frac{lbf}{ft^3}(80 - 10 + 1.11)\,ft \times \frac{ft^2}{144\,in.^2} = 80.8\ psig$$

The pump head is $$H_p = \frac{p_3 - p_2}{\rho g} = (80.8 - (-4.58))\,\frac{lbf}{in.^2} \times \frac{ft^3}{62.4\,lbf} \times 144\,\frac{in.^2}{ft^2} = 197\ ft$$

Thus the pump power is $\dot{W}_h = \rho g Q H_p = 0.498\ hp$

The power reduction is $$\Delta\dot{W}_h = \frac{\dot{W}_h(1.5\,in.) - \dot{W}_h(1\,in.)}{\dot{W}_h(1\,in.)} = -6.7\ \text{percent}$$ $\Delta\dot{W}_h$

The pump should not be moved up the hill. The NPSHA now is

$$NPSHA = \frac{1}{\rho g}\left[p(abs) + \frac{\rho \bar{V}_2^2}{2} - p_v\right] = \frac{1}{\rho g}\left[p_{atm} + p(gage) + \frac{\rho \bar{V}_2^2}{2} - p_v\right]$$

At $T = 68°F$, $p_v = 0.339$ psia (Table A.7).

$$NPSHA = \frac{ft^3}{62.4\,lbf}\left[14.7\,\frac{lbf}{in.^2} + (-6.45)\,\frac{lbf}{in.^2} + 1.94\,\frac{slug}{ft^3} \times 6.90\,\frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{ft^2}{144\,in.^2} - 0.339\,\frac{lbf}{in.^2}\right] 144\,\frac{in.^2}{ft^2}$$

$NPSHA = 18.5\ ft$ (1 in. pipe) 22.6 ft (1.5 in. pipe) NPSHA

This should not be reduced further by raising the pump. If anything, the pump should be lowered to increase NPSHA.

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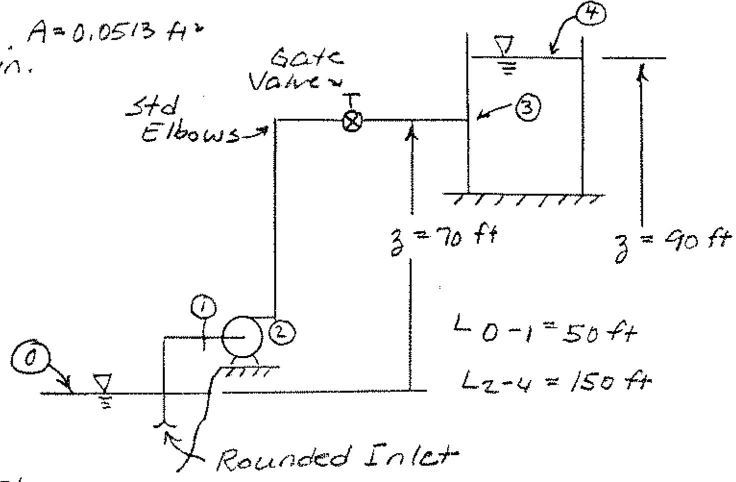
Problem 10.81

Given: Pump and piping system at lakeside home, as shown.

Galvanized iron pipe. $A = 0.0513\ ft^2$

$D = 3$ in. (nominal) $= 3.068$ in.



Find: (a) System head-flow curve

(b) System operating point

(c) Power input if $\eta_p = 0.8$

(d) Sketch of system curve when $z_4 = 90$ ft

(e) Sketch of system curve when $z_3 = 75$ ft, valve part closed, and $Q = 0.1\ ft^3/s$.

(f) Which case has higher pump efficiency?

Solution: Apply the energy equation for pipe flow. The pump must overcome the gravity lift plus the head losses in the pipe and fittings.

Assume: (1) Nominal speed is $\bar{V} = 12$ ft/s, $T = 60°F$, $\nu = 1.21\times10^{-5}\ ft^2/s$ (Table A.7)

(2) Flow in fully rough zone ($e = 0.0005$ ft (Table 8.1), $e/D = 0.002$, $f \approx 0.024$)

(3) Cases: (1) Water in tank below ③ } Valve open

(2) Water in tank at $z = 90$ ft }

(3) Valve closed so $Q = 0.1\ ft^3/s$, valve part closed.

Then

$$H_{lT} = \frac{h_{lT}}{g} = \left[K_{ent} + f\left(\frac{L}{D} + 3\frac{L_e}{D}(\text{elbow}) + \frac{L_e}{D}(\text{gate valve}\right) + K_{exit}\right]\frac{\bar{V}^2}{2g}$$

$$H_{lT} = \left[0.04 + 0.024\left(\frac{200\ ft}{3.068\ in.}\times 12\frac{in}{ft} + 3(30) + 8\right) + 1\right]\frac{\bar{V}^2}{2g} = 22.2\ \frac{\bar{V}^2}{2g}$$

and $$H_s = z_{end} + 22.2\frac{\bar{V}^2}{2g}$$ ← H_s

Assume $\bar{V} = 12$ ft/s, $Q = 276$ gpm, $\frac{\bar{V}^2}{2g} = 2.24$ ft, $H_s = 70 + 22.2(2.24) = 120$ ft

Case 1: $z_{end} = 70$ ft Operating point: $Q = 276$ gpm, $H_p = H_s = 120$ ft ← Oper. Pt.

$$P = \frac{\rho g Q H}{\eta_p} = 62.4\frac{lbf}{ft^3}\times 12.0\frac{ft}{s}\times 0.0513\ ft^2\times 120\ ft\times\frac{1}{0.8}\times\frac{hp\cdot s}{550\ ft\cdot lbf} = 10.5\ hp$$ ← P

Case 2: $z_{end} = 90$ ft; $H_s = 90 + 22.2(2.24) = 140$ ft

$$H_s = 90 + 50.0\frac{[Q(gpm)]^2}{(276)^2(gpm)^2} = 90 + 6.56\times10^{-4}[Q(gpm)]^2$$ ← H_s

Case 3: $Q = 0.1\ ft^3/s = 44.9$ gpm; $H_s = H_p$; Assume $H_{BEP} = 0.7\ H_0$

$$H_p = H_0 + \frac{(H_0 - H_{op})}{Q_{op}^2}Q^2 = \frac{120}{0.7} - \frac{(120/0.7 - 120)}{(276)^2}Q^2 = 169 - 6.75\times10^{-4}Q^2$$

$H_p = 168$ ft at $Q = 44.9$ gpm ← H_p

over

Problem 10.81

Water pumped from lake to storage tank on bluff:

Input Data:

Friction factor:	$f =$	0.024	(---)
Pipe diameter:	$D =$	3.068	in.

Calculated Results:

Pipe area:	$A =$	0.0513	ft^2

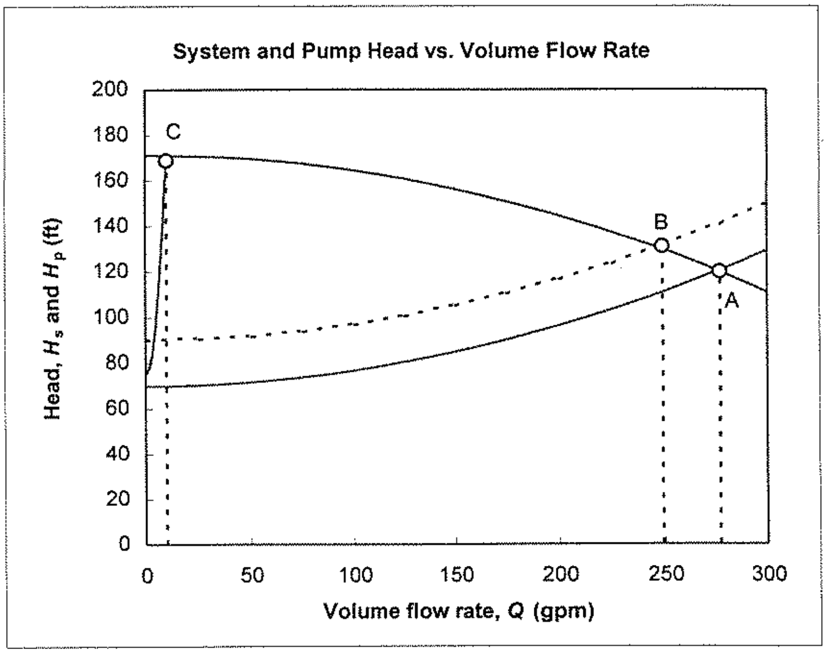
System Curves for Various Conditions:

			Case 1:	Case 2:
			H_s	H_s
Q	V	$V^2/2g$	(z_3 = 70 ft)	(z_3 = 90 ft)
(gpm)	(ft/s)	(ft)	(ft)	(ft)
0	0	0.00	70.0	90.0
25	1.09	0.02	70.4	90.4
50	2.17	0.07	71.7	91.7
75	3.26	0.16	73.7	93.7
100	4.34	0.29	76.6	96.6
125	5.43	0.46	80.3	100
150	6.51	0.66	84.9	105
175	7.60	0.90	90.2	110
200	8.68	1.17	96.4	116
225	9.77	1.48	103	123
249.3	10.8	1.82	111	131
277	12.0	2.24	121	141
300	13.0	2.63	129	149

Case 3: Valve partially closed	
	H_s
Q	(z_3 = 75 ft)
(gpm)	(ft)
0	75.0
2	78.8
4	90.0
6	109
8	135
10	169

Pump Head Curve:

Q	H_p
(gpm)	(ft)
0	171
50	170
100	165
150	156
200	145
250	130
277	120
300	111



Problem 10.82

[4]

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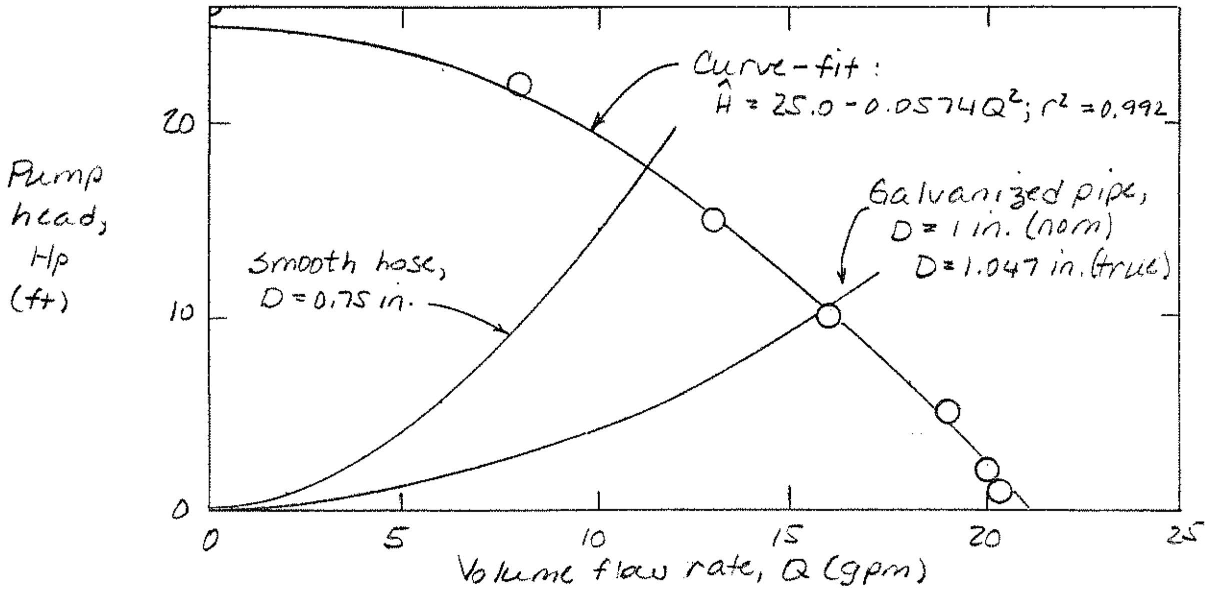
Given: Manufacturer data for submersible pump:

Discharge Height (ft)	1	2	5	10	15	20	26.3
Water Flow Rate (gpm)	20.4	20	19	16	13	8	0

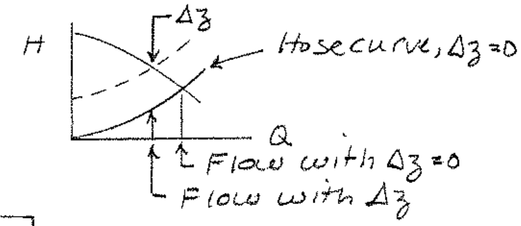
Find: (a) Plot a performance curve for this pump.
(b) Develop and show a curve-fit to the data.
(c) Calculate and plot the pump delivery versus discharge height for (1) 50' of 3/4 in. garden hose, (2) 50' of 1 in. pipe.

Solution: The performance data and curve-fit are plotted below:

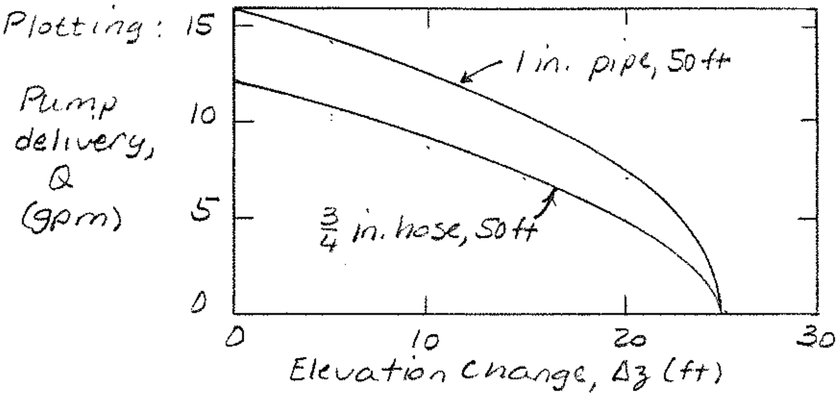


The energy equation for steady, incompressible pipe flow was used to develop the system curves shown above, for $\Delta z = 0$. To obtain delivery versus height, add the elevation change to the head for $\Delta z = 0$ to find the new intersection with the pump curve:

The same result is obtained when the difference between the pump curve and the hose, at a given flow rate, equals Δz.



Plotting:



Problem 10.83

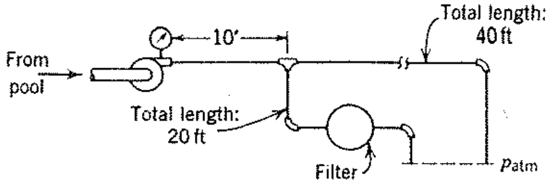
[4]

Given: Swimming pool filtration system of Problem 8.169.

Assume pipe used is 3/4 in. (nominal) smooth PVC plastic.

Pump delivers $Q = 30$ gpm of water at 75°F.

Filter pressure drop is $\Delta p = 0.6\,Q^2$, where Δp in psi and Q in gpm.



Find: (a) Specify speed and impeller diameter of suitable pump.
(b) Estimate pump efficiency.

Solution: Need head to specify pump. Using FM, with $Q = 30$ gpm (0.0668 cfs) and $D = 0.824$ in. (0.0687 ft), $\Delta p_{12} = 5.56$ psi.

Flow must split to give same pressure drop in each branch. Assuming L_e/D for each elbow is 30, iteration gives:

$Q_{24} = 24.8$ gpm (0.0553 cfs) and $\Delta p = 16.69$ psi

$Q_{23} = 5.2$ gpm (0.0116 cfs) and $\Delta p = 16.2 + 0.6023 = 16.8$ psi

Neglecting any pressure at the pump inlet, the pump must supply

$$\Delta p_{pump} = \Delta p_{12} + \Delta p_{23} = 5.56 + 16.7 = 22.3 \text{ psi}$$

The pump head is $H = \dfrac{\Delta p}{\rho g} = 22.3\,\dfrac{\text{lbf}}{\text{in.}^2} \times \dfrac{\text{ft}^3}{62.4\text{ lbf}} \times 144\,\dfrac{\text{in.}^2}{\text{ft}^2} = 51.5$ ft H

This pump is too small to be found in Fig. D.1. Therefore approximate its characteristics using N_s and H_0. Assume $N = 3500$ rpm:

$$N_{s_{cu}} = \frac{N Q^{1/2}}{H^{3/4}} = \frac{3500\,(30)^{1/2}}{(51.5)^{3/4}} = 997$$

From Fig. 10.15, $\eta \approx 0.62$ or less. Assuming $H = 0.7\,H_0$, then

$$H = 0.7\,H_0 = 0.7\,\frac{(\omega R)^2}{g}\;;\quad R = \frac{1}{\omega}\left(\frac{gH}{0.7}\right)^{\frac{1}{2}} = \frac{\text{s}}{367\text{ rad}}\left[32.2\,\frac{\text{ft}}{\text{s}^2} \times 51.5\text{ ft} \times \frac{1}{0.7}\right]^{\frac{1}{2}} = 0.133\text{ ft}$$

The impeller diameter is approximately

$D = 2R = 2 \times 0.133\text{ ft} \times 12\,\dfrac{\text{in.}}{\text{ft}} = 3.18$ in. D

The pump power requirement is

$$\dot{W}_m = \frac{\rho Q g H}{\eta_p} = \frac{1}{0.6} \times 0.0668\,\frac{\text{ft}^3}{\text{s}} \times 62.4\,\frac{\text{lbf}}{\text{ft}^3} \times 51.5\text{ ft} \times \frac{\text{hp}\cdot\text{s}}{550\text{ ft}\cdot\text{lbf}} = 0.651\text{ hp}$$

A 3/4 horsepower motor should be used. $\dot{W}_m$

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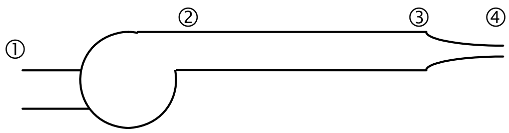
Problem 10.84

[4]

10.84 Consider the fire hose and nozzle of Problem 8.159. Specify an appropriate pump and impeller diameter to supply four such hoses simultaneously. Calculate the power input to the pump.



Given: Fire nozzle/pump system

Find: Appropriate pump; Impeller diameter; Pump power input needed

Solution:

Basic equations $$\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)-\left(\frac{p_3}{\rho}+\alpha\cdot\frac{V_3^{\,2}}{2}+g\cdot z_3\right)=h_l \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V_2^{\,2}}{2}$$ for the hose

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 2 and 3 is approximately 1 4) No minor loss

$$\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)-\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)=h_{pump}$$ for the pump

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) No minor loss

The first thing we need is the flow rate. Below we repeat Problem 8.159 calculations

Hence for the hose $$\frac{\Delta p}{\rho}=\frac{p_2-p_3}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad \text{or} \qquad V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$$

We need to iterate to solve this for V because f is unknown until Re is known. This can be done using *Excel*'s Solver, but here:

$\Delta p = 750\cdot\text{kPa}$ $\quad L = 100\cdot\text{m}$ $\quad e = 0$ $\quad D = 3.5\cdot\text{cm}$ $\quad \rho = 1000\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad \nu = 1.01\times10^{-6}\cdot\frac{\text{m}^2}{\text{s}}$

Make a guess for f $\quad f = 0.01$ $\quad V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 7.25\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re = 2.51\times10^{5}$

Given $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0150$$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 5.92\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re = 2.05\times10^{5}$

Given $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0156$$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 5.81\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re = 2.01\times10^{5}$

Given $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0156$$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 5.80\frac{\text{m}}{\text{s}}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re = 2.01\times10^{5}$

$Q = \frac{\pi \cdot D^2}{4} \cdot V$ $\quad Q = 5.578 \times 10^{-3} \frac{m^3}{s}$ $\quad Q = 0.335 \cdot \frac{m^3}{min}$

We have $\quad p_1 = 350 \cdot kPa$ $\quad p_2 = 700 \cdot kPa + 750 \cdot kPa$ $\quad p_2 = 1450 kPa$

For the pump

$$\left(\frac{p_2}{\rho} + \alpha \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) - \left(\frac{p_1}{\rho} + \alpha \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) = h_{pump}$$

so $\quad h_{pump} = \frac{p_2 - p_1}{\rho}$ $\quad$ or $\quad H_{pump} = \frac{p_2 - p_1}{\rho \cdot g}$ $\quad H_{pump} = 112 m$

We need a pump that can provide a flow of $Q = 0.335 \frac{m^3}{min}$ or $Q = 88.4 gpm$, with a head of $H_{pump} = 112 m$ or $H_{pump} = 368 ft$

From Appendix D, Fig. D.1 we see that a Peerless 2AE11 can provide this kind of flow/head combination; it could also handle four such hoses (the flow rate would be $4 \cdot Q = 354 gpm$). An impeller diameter could be chosen from proprietary curves.

The required power input is $\quad W_m = \frac{W_h}{\eta_p}$ $\quad$ where we choose $\eta_p = 75 \cdot \%$ from Fig. 10.15

$W_m = \frac{\rho \cdot Q \cdot g \cdot H_{pump}}{\eta_p}$ $\quad W_m = 8.18 kW$ for one hose or $4 \cdot W_m = 32.7 kW$ for four

$P_{required} = \frac{P_{pump}}{\eta}$ $\quad P_{required} = \frac{6.14 \cdot kW}{70 \cdot \%}$ $\quad P_{required} = 8.77 \cdot kW$ or $4 \cdot P_{required} = 35.1 kW$ for four

Problem 10.85

[3]

10.85 Performance data for a centrifugal fan of 1 m diameter, tested at 650 rpm, are

Volume flow rate Q (m^3/s)	3	4	5	6	7	8
Static pressure rise, Δp (mm H_2O)	53	51	45	35	23	11
Power output $\mathscr{P}$ (kW)	2.05	2.37	2.60	2.62	2.61	2.4

Plot the performance data versus volume flow rate. Calculate static efficiency and show the curve on the plot. Find the best efficiency point and specify the fan rating at this point.

Given: Data on centrifugal fan

Find: Plot of performance curves; Best efficiency point

Solution:

Basic equations:
$$\eta_p = \frac{W_h}{W_m} \qquad W_h = Q\cdot\Delta p \qquad \Delta p = \rho_w\cdot g\cdot\Delta h$$
(Note: Software cannot render a dot!)

$\rho_w = 1000 \quad kg/m^3$

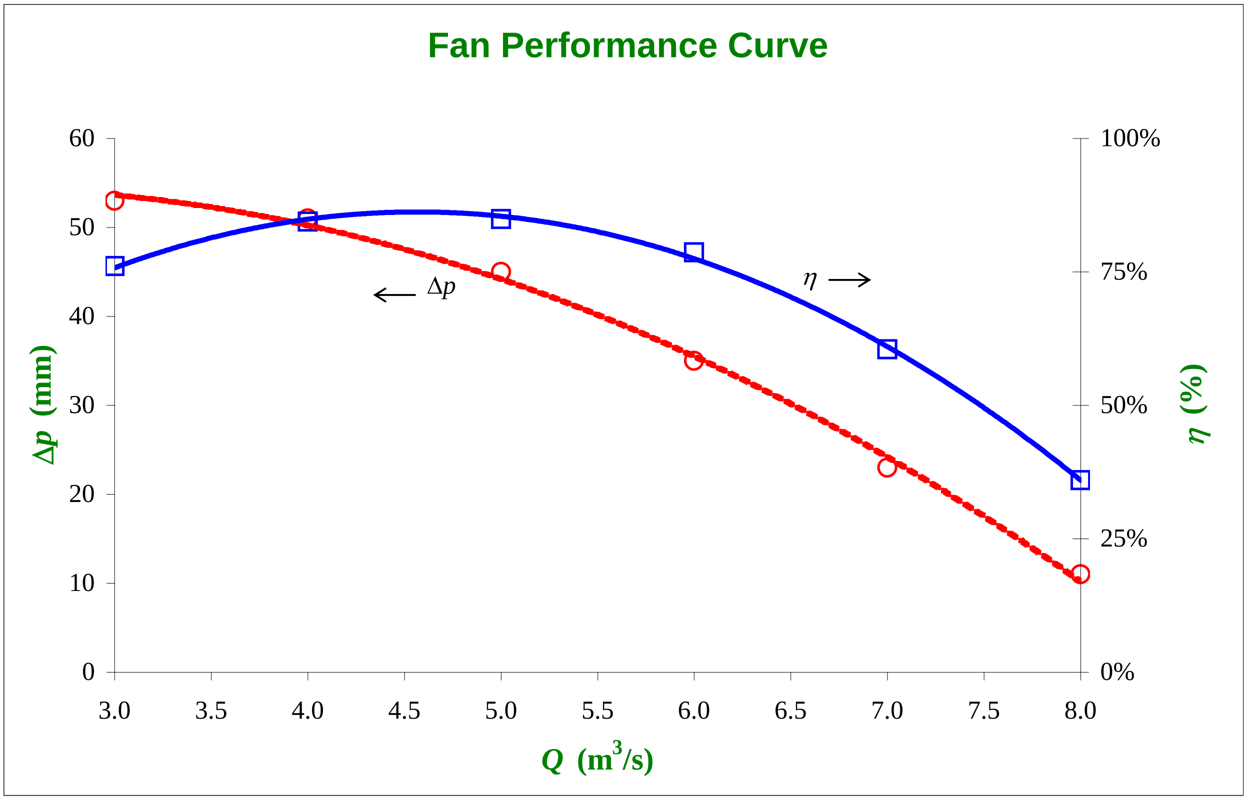
Q (m^3/s)	Δp (mm)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
3	53	2.05	1.56	76.1%
4	51	2.37	2.00	84.4%
5	45	2.60	2.21	84.9%
6	35	2.62	2.06	78.6%
7	23	2.61	1.58	60.5%
8	11	2.40	0.86	36.0%

Fitting a 2nd order polynomial to each set of data we find

$$\Delta p = -1.32Q^2 + 5.85Q + 48.0$$
$$\eta = -0.0426Q^2 + 0.389Q - 0.0267$$

Finally, we use Solver to maximize η by varying Q:

Q (m^3/s)	Δp (mm)	η (%)
4.57	47.2	86.1%



Problem 10.86

[3]

10.86 Using the fan of Problem 10.85 determine the minimum-size square sheet-metal duct that will carry a flow of 5.75 m^3/s over a distance of 15 m. Estimate the increase in delivery if the fan speed is increased to 800 rpm.

Given: Data on centrifugal fan and square metal duct

Find: Minimum duct geometry for flow required; Increase if fan speed is increased

Solution:

Basic equations: $\eta_p = \frac{W_h}{W_m}$ $\quad W_h = Q\cdot\Delta p$ $\quad \Delta p = \rho_w\cdot g\cdot\Delta h$ (Note: Software cannot render a dot!)

and for the duct $\Delta p = \rho_{air}\cdot f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2}$ $\quad D_h = \frac{4\cdot A}{P} = \frac{4\cdot H^2}{4\cdot H} = H$

and fan scaling $Q = 5.75\cdot\frac{m^3}{s}$ $\quad \omega = 650\cdot rpm$ $\quad \omega' = 800\cdot rpm$ $\quad Q' = \frac{\omega'}{\omega}\cdot Q$ $\quad Q' = 7.08\,\frac{m^3}{s}$

$\rho_w =$ 1000 kg/m^3

$\rho_{air} =$ 1.225 kg/m^3

$\nu_{air} =$ 1.50.E-05 m^2/s

$L =$ 15 m

Assume smooth ducting

Fitting a 2nd order polynomial to each set of data we find

$\Delta p = -1.32Q^2 + 5.85Q + 48.0$

User Solver to vary H so the error in Δp is zero

	Fan
Q (m^3/s)	Δp (mm)
7.08	23.3

Note: Efficiency curve not needed for this problem

Q (m^3/s)	Δp (mm)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
3	53	2.05	1.56	76.1%
4	51	2.37	2.00	84.4%
5	45	2.60	2.21	84.9%
6	35	2.62	2.06	78.6%
7	23	2.61	1.58	60.5%
8	11	2.40	0.86	36.0%

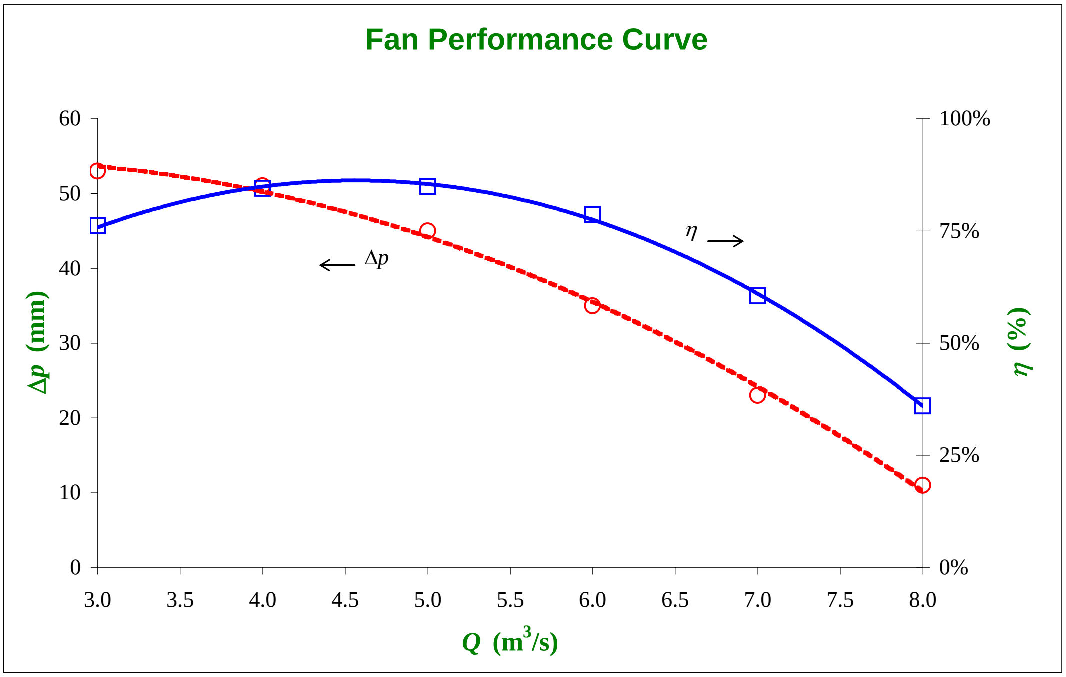
				Duct
H (m)	V (m/s)	Re	f	Δp (mm)
0.472	31.73	9.99.E+05	0.0116	23.3

Error in Δp 0.00%

Answers:

Q (m^3/s)	H (m)
5.75	0.394

Q (m^3/s)	H (m)
7.08	0.472



Problem 10.87

[3]

10.87 Consider the fan and performance data of Problem 10.85. At $Q = 5.75\ \mathrm{m^3/s}$, the dynamic pressure is equivalent to 4 mm of water. Evaluate the fan outlet area. Plot total pressure rise and input horsepower for this fan versus volume flow rate. Calculate the fan total efficiency and show the curve on the plot. Find the best efficiency point and specify the fan rating at this point.

Given: Data on centrifugal fan

Find: Fan outlet area; Plot total pressure rise and power; Best effiiciency point

Solution:

Basic equations:

$$\eta_p = \frac{W_h}{W_m} \qquad W_h = Q\cdot\Delta p_t \qquad \Delta p = \rho_w\cdot g\cdot\Delta h_t$$

(Note: Software cannot render a dot!)

$$p_{dyn} = \frac{1}{2}\cdot\rho_{air}\cdot V^2$$

At $Q = 5.75\cdot\frac{m^3}{s}$ we have $h_{dyn} = 4\cdot mm$ $\qquad Q = V\cdot A \qquad$ and $\qquad h_{dyn} = \frac{p_{dyn}}{\rho_w\cdot g} = \frac{\rho_{air}}{\rho_w}\cdot\frac{V^2}{2}$

Hence $\qquad V = \sqrt{\frac{\rho_w}{\rho_{air}}\cdot 2\cdot g\cdot h_{dyn}} \qquad$ and $\qquad A = \frac{Q}{V}$

The velocity V is directly proportional to Q, so the dynamic pressure at any flow rate Q is $\qquad h_{dyn} = 4\cdot mm\cdot\left(\frac{Q}{5.75\cdot\frac{m^3}{s}}\right)^2$

The total pressure Δh_t will then be $\qquad \Delta h_t = \Delta h + h_{dyn} \qquad$ Δh is the tabulated static pressure rise

At $Q =$ 5.75 $\mathrm{m^3/s}$

$h_{dyn} =$ 4 mm $\qquad$ Hence $\qquad V =$ 8.00 m/s

$A =$ 0.71838 $\mathrm{m^2}$

$\rho_w =$ 1000 $\mathrm{kg/m^3}$

$\rho_{air} =$ 1.225 $\mathrm{kg/m^3}$

Fitting a 2nd order polynomial to each set of data we find

$$h_t = -0.12Q^2 + 0.585Q + 4.7986$$

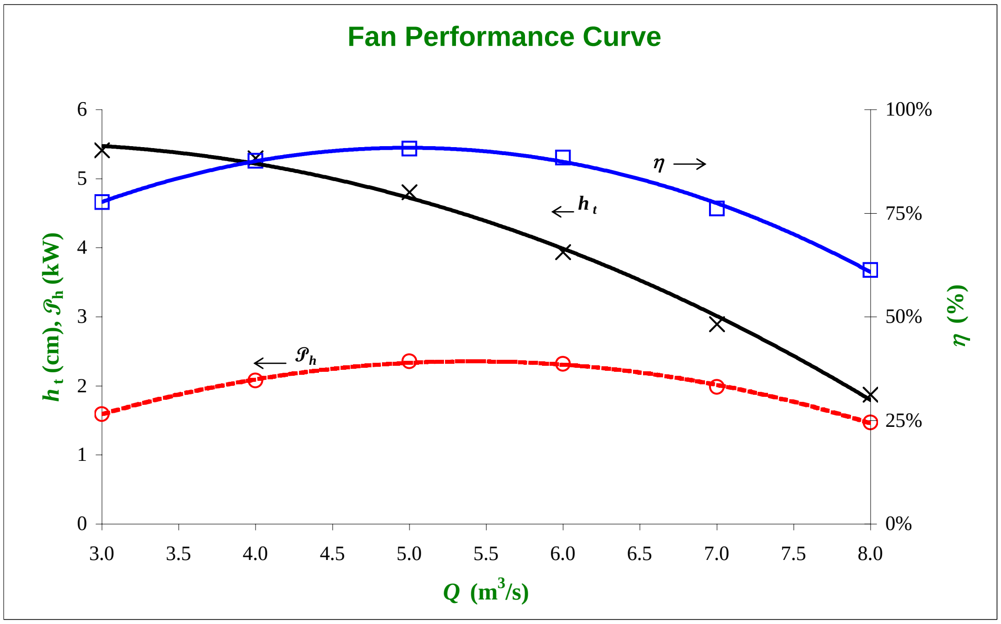
$$\mathscr{P}_h = -0.133Q^2 + 1.43Q - 1.5202$$

$$\eta = -0.0331Q^2 + 0.330Q + 0.0857$$

Q (m^3/s)	Δp (mm)	$\mathscr{P}_m$ (kW)	h_{dyn} (mm)	h_t (cm)	$\mathscr{P}_h$ (kW)	η (%)
3	53	2.05	1.09	5.41	1.59	77.7%
4	51	2.37	1.94	5.29	2.08	87.6%
5	45	2.60	3.02	4.80	2.36	90.6%
6	35	2.62	4.36	3.94	2.32	88.4%
7	23	2.61	5.93	2.89	1.99	76.1%
8	11	2.40	7.74	1.87	1.47	61.3%

Finally, we use Solver to maximize η by varying Q:

Q (m^3/s)	h_t (cm)	$\mathscr{P}_h$ (kW)	η (%)
4.98	4.73	2.30	90.8%



Problem 10.88

[3]

10.88 The performance data of Problem 10.85 are for a 1-m diameter fan wheel. This fan also is manufactured with 1.025, 1.125-, 1.250-, and 1.375-m diameter wheels. Pick a standard fan to deliver 14 m^3/s against a 25-mm H_2O static pressure rise. Assume standard air at the fan inlet. Determine the required fan speed and the input power needed.

Volume flow rate Q (m^3/s)	3	4	5	6	7	8
Static pressure rise, Δp (mm H_2O)	53	51	45	35	23	11
Power output $\mathcal{P}$ (kW)	2.05	2.37	2.60	2.62	2.61	2.4

Given: Data on centrifugal fan and various sizes

Find: Suitable fan; Fan speed and input power

Solution:

Basic equations: $\frac{Q'}{Q} = \left(\frac{\omega'}{\omega}\right)\cdot\left(\frac{D'}{D}\right)^3$ $\quad\frac{h'}{h} = \left(\frac{\omega'}{\omega}\right)^2\cdot\left(\frac{D'}{D}\right)^2$ $\quad\frac{P'}{P} = \left(\frac{\omega'}{\omega}\right)^3\cdot\left(\frac{D'}{D}\right)^5$

We choose data from the middle of the table above as being in the region of the best efficiency

$Q = 5\cdot\frac{m^3}{s}$ $\quad h = 45\cdot mm$ $\quad P = 2.62\cdot kW$ and $\omega = 650\cdot rpm$ $\quad D = 1\cdot m$

The flow and head are $\quad Q' = 14\cdot\frac{m^3}{s}$ $\quad h' = 25\cdot mm$

These equations are the scaling laws for scaling from the table data to the new fan. Solving for scaled fan speed, and diameter using the first two equations

$\omega' = \omega\cdot\left(\frac{Q}{Q'}\right)^{\frac{1}{2}}\cdot\left(\frac{h'}{h}\right)^{\frac{3}{4}}$ $\quad\omega' = 250\,rpm$ $\quad D' = D\cdot\left(\frac{Q'}{Q}\right)^{\frac{1}{2}}\cdot\left(\frac{h}{h'}\right)^{\frac{1}{4}}$ $\quad D' = 1.938\,m$

This size is too large; choose (by trial and error)

$Q = 7\cdot\frac{m^3}{s}$ $\quad h = 23\cdot mm$ $\quad P = 2.61\cdot kW$

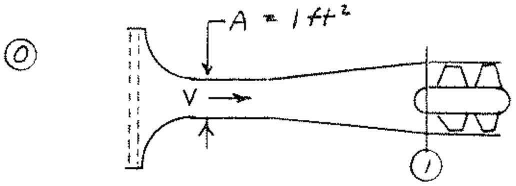
$\omega' = \omega\cdot\left(\frac{Q}{Q'}\right)^{\frac{1}{2}}\cdot\left(\frac{h'}{h}\right)^{\frac{3}{4}}$ $\quad\omega' = 489\,rpm$ $\quad D' = D\cdot\left(\frac{Q'}{Q}\right)^{\frac{1}{2}}\cdot\left(\frac{h}{h'}\right)^{\frac{1}{4}}$ $\quad D' = 1.385\,m$

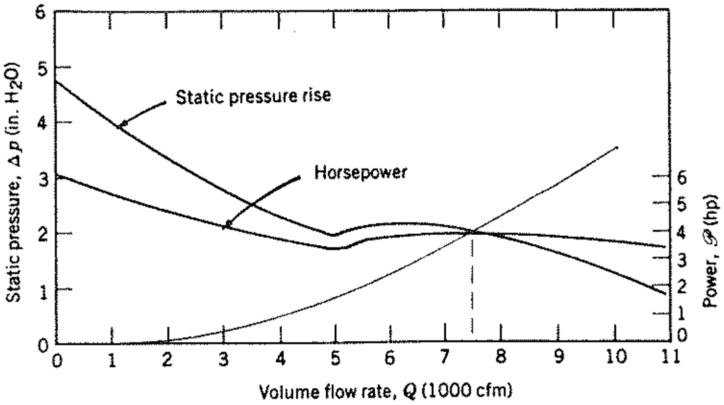
Hence it looks like the largest fan (1.375 m) will be the only fit; it must run at about 500 rpm. Note that it will NOT be running at best efficiency. The power will be

$P' = P\cdot\left(\frac{\omega'}{\omega}\right)^3\cdot\left(\frac{D'}{D}\right)^5$ $\quad P' = 5.67\,kW$

Problem 10.89

Given: Wind tunnel, with 1 foot square test section, powered by fan below. Tunnel contains two screens, each with $K = 0.12$, and a diffuser between the test section and the 24 in. ϕ fan inlet.





Find: (a) Calculate and plot Δp versus Q.

(b) Estimate the maximum air speed.

Solution: Apply energy

Computing equation: $\frac{p_0}{\rho} + \overset{\approx 0(2)}{\cancel{\frac{V_0^2}{2}}} + \cancel{g z_0} = \frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + \cancel{g z_1} + h_{lT}$; $h_{lT} = \left[f\left(\cancel{\frac{L}{D}} + \cancel{\frac{L_e}{D}}\right) + K \right] \frac{V^2}{2}$

Assumptions: (1) $p_0 = p_{atm}$, (2) $V_0 \approx 0$, $\alpha_1 \approx 1$, (3) $z_0 = z_1$, (4) Losses in diffuser, screens

$$\frac{\Delta p_{fan}}{\rho} = \frac{p_{atm} - p_1}{\rho} = \frac{V_1^2}{2} + h_{lT} = \frac{V_1^2}{2} + (2K_{screen} + K_{diffuser})\frac{V^2}{2} = \left[2K_s + K_d + \left(\frac{A}{A_1}\right)^2\right]\frac{Q^2}{2A^2}$$

From continuity, $V_1 A_1 = VA$; $V_1^2 = V^2\left(\frac{A}{A_1}\right)^2$; $V = \frac{Q}{A}$; $V^2 = \frac{Q^2}{A^2}$; $A = 1\ ft^2$; $A_1 = \frac{\pi}{4} D_1^2 = 3.14\ ft^2$

From Fig. 8.19, $K_d = C_{p_i} - C_p = 1 - \left(\frac{1}{AR}\right)^2 - 0.70 = 1 - \left(\frac{1}{3.14}\right)^2 - 0.70 = 0.199$

$$\Delta p_{fan} = [2(0.12) + 0.199 + 0.101]\,\frac{1}{2} \times Q^2 \frac{ft^6}{min^2} \times \frac{1}{(1)^2 ft^4} \times 0.00238 \frac{slug}{ft^3} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{min^2}{3600\ s^2}$$

$$\Delta h_{fan} = \frac{\Delta p}{\rho g} = 1.79 \times 10^{-7} [Q(cfm)^2] \frac{lbf}{ft^2} \times \frac{ft^3}{62.4\ lbf} \times 12 \frac{in.}{ft} = 3.43 \times 10^{-8} [Q(cfm)^2]$$

The resulting curve is plotted above; computed values are tabulated below.

The system will operate where the fan curve and system curve cross. The approximate operating point is $Q = 7400$ cfm at $h = 1.9$ in. H_2O.

The test section speed is

$$V = \frac{Q}{A} = 7400 \frac{ft^3}{min} \times \frac{1}{1\ ft^2} \times \frac{min}{60\ s} = 123\ ft/s$$

V

Q (1000 cfm)	1	2	3	4	5	6	7	8	9	10
Δp (in. H_2O)	0.03	0.14	0.31	0.55	0.86	1.24	1.68	2.20	2.78	3.43

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 10.90

Given: Axial-flow fan and wind tunnel of Problem 10.89.

Find: (a) Scale performance of fan as it varies with operating speed.
(b) Develop and plot a "calibration curve" showing test section flow speed (m/sec) versus fan speed (rpm).

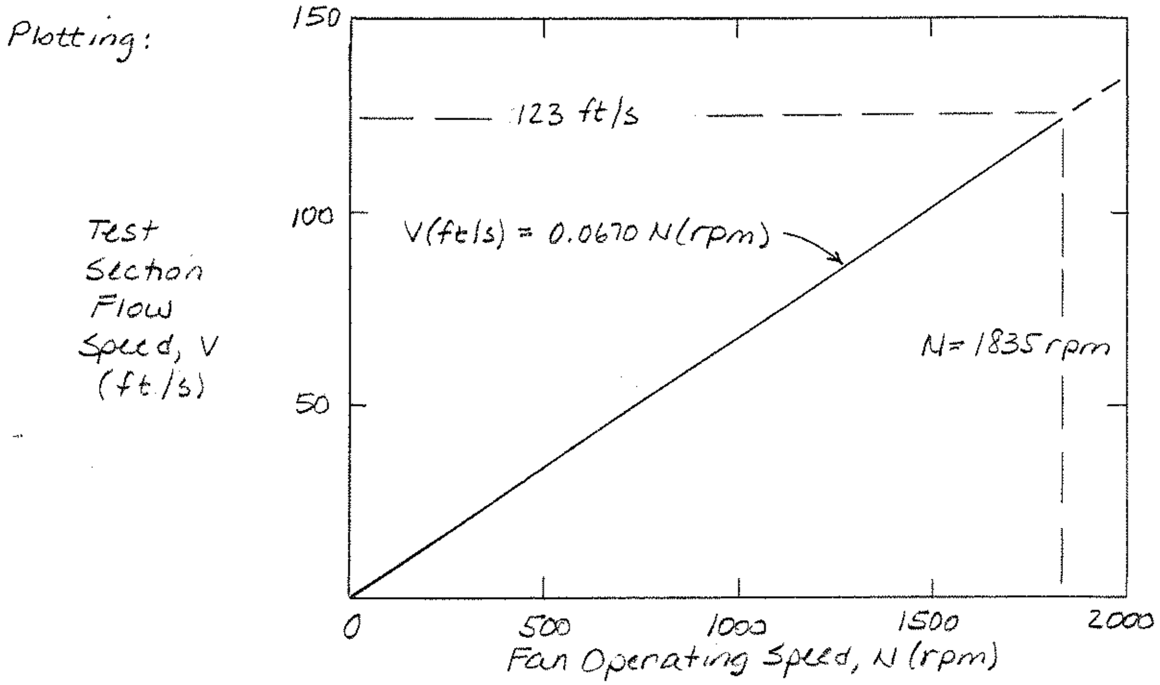
Solution: From the solution to Problem 10.89, $\Delta h_{fan} \propto Q^2 \propto V^2$.

The scaling laws for varying fan speed suggest $Q \propto \omega$ and $p \propto \omega^2$.

Thus $\Delta h_{fan} \propto Q^2 \propto p \propto \omega^2$ or $Q \propto \omega$. The volume flow rate (and the test section flow speed) should vary directly with ω.

From the results of Problem 10.89, $V = 123$ ft/s when $N = 1835$ rpm.

Plotting:



The slope of the linear relationship is $\frac{V}{N} = 123 \frac{ft}{s} \times \frac{1}{1835\ rpm} = 0.0670$ ft/s/rpm.

Thus $V(ft/s) = 0.0670\ N(rpm)$ $\longleftarrow$ V

Problem 10.91

[3]

Given: Experimental test data for aircraft engine fuel pump:

Pump supplies fuel at 450 pph, 150 psig to the controller.

Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph*)	Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph)	Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph)
	200	1810		200	1730		200	89
4536	300	1810	4355	300	1750	453	250	73
(100%)	400	1810	(96%)	400	1735	(10%)	300	58.5
	500	1790		500	1720		350	45
	900	1720		900	1635		400	30

* Fuel flow rate measured in pounds per hour (pph).

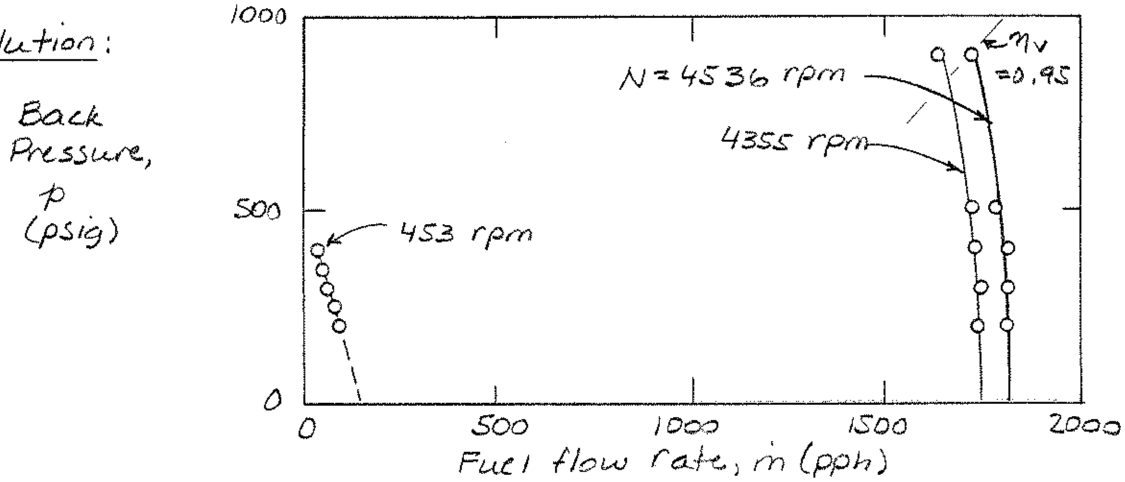
Find: (a) Plot curves of fuel pressure versus delivery at the three constant speeds.

(b) Estimate the pump displacement volume per revolution.

(c) Calculate volumetric efficiency at each point, sketch η_v contours.

(d) Evaluate energy loss due to throttling at 100% speed, full delivery.

Solution:



For the pump, $\dot{m} = \rho \forall N$, so $\forall = \frac{\dot{m}}{\rho N}$. Analyzing the 4536 rpm case,

$$\forall \approx 1810 \frac{\text{lbm}}{\text{hr}} \times \frac{\text{gal}}{6.8 \text{ lbm}} \times \frac{\text{min}}{4536 \text{ rev}} \times \frac{\text{ft}^3}{7.48 \text{ gal}} \times 1728 \frac{\text{in.}^3}{\text{ft}^3} \times \frac{\text{hr}}{60 \text{ min}} = 0.226 \text{ in.}^3/\text{rev}$$ $\forall$

At constant speed, $\eta_v = \frac{\forall_{actual}}{\forall_{geometric}} = \frac{\dot{m}}{\dot{m}(p=0)}$. Calculation shows η_v decreases as speed is reduced, see below.

Energy loss is $\dot{W}_L = \left(\frac{\dot{m}_p - \dot{m}_L}{\rho}\right) p_L$

$$= (1810 - 450) \frac{\text{lbm}}{\text{hr}} \times \frac{\text{gal}}{6.8 \text{ lbm}} \times 150 \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{7.48 \text{ gal}} \times 144 \frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{hp}\cdot\text{s}}{550 \text{ ft}\cdot\text{lbf}} \times \frac{\text{hr}}{3600 \text{ s}}$$

$\dot{W}_L = 0.292$ hp $\dot{W}_L$

At 453 rpm, the best volumetric efficiency is

$$\eta_v \approx \frac{\dot{m}}{\dot{m}(p=0)} \times \frac{4536}{453} \approx \frac{89 \text{ pph}}{1810 \text{ pph}} \times \frac{4536}{453} = 0.0492, \text{ or about } 5\%$$ η_v

At 4355 rpm,

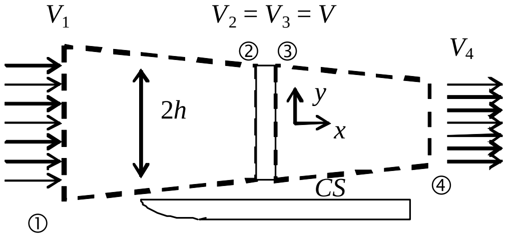
$$\eta_v \approx \frac{1730 \text{ pph}}{1810 \text{ pph}} \times \frac{4536}{4355} = 0.996, \text{ or more than } 99\% \text{ (this is doubtful).}$$ η_v

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 10.92

[3]

10.92 The propeller on an airboat used in the Florida Everglades moves air at the rate of 90 lbm/s. When at rest, the speed of the slipstream behind the propeller is 90 mph at a location where the pressure is atmospheric. Calculate (a) the propeller diameter, (b) the thrust produced at rest, and (c) the thrust produced when the airboat is moving ahead at 30 mph if the mass flow rate through the propeller remains constant.



Given: Data on boat and propeller

Find: Propeller diameter; Thrust at rest; Thrust at 30 mph

Solution:

Basic equation:

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, d\forall + \int_{CS} \vec{V}_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A} \qquad (4.26)$$

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum is then $T = u_1\cdot\left(-m_{rate}\right) + u_4\cdot\left(m_{rate}\right) = \left(V_4 - V_1\right)\cdot m_{rate}$ where $m_{rate} = 90\cdot\frac{\text{lbm}}{\text{s}}$ is the mass flow rate

It can be shown (see Example 10.13) that $V = \frac{1}{2}\cdot\left(V_4 + V_1\right)$

For the static case $V_1 = 0\cdot\text{mph}$ $V_4 = 90\cdot\text{mph}$ so $V = \frac{1}{2}\cdot\left(V_4 + V_1\right)$ $V = 45\,\text{mph}$

From continuity $m_{rate} = \rho\cdot V\cdot A = \rho\cdot V\cdot\frac{\pi\cdot D^2}{4}$ with $\rho = 0.002377\cdot\frac{\text{slug}}{\text{ft}^3}$

Hence $D = \sqrt{\frac{4\cdot m_{rate}}{\rho\cdot\pi\cdot V}}$ $D = 4.76\,\text{ft}$

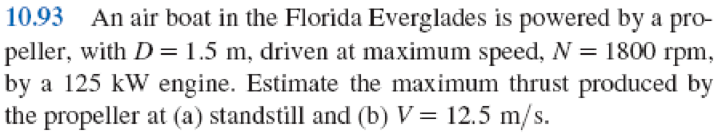
For $V_1 = 0$ $T = m_{rate}\cdot\left(V_4 - V_1\right)$ $T = 369\,\text{lbf}$

When in motion $V_1 = 30\cdot\text{mph}$ and $V = \frac{1}{2}\cdot\left(V_4 + V_1\right)$ so $V_4 = 2\cdot V - V_1$ $V_4 = 60\,\text{mph}$

Hence for $V_1 = 30$ mph $T = m_{rate}\cdot\left(V_4 - V_1\right)$ $T = 123\,\text{lbf}$

Problem 10.93

[3]



10.93 An air boat in the Florida Everglades is powered by a propeller, with $D = 1.5$ m, driven at maximum speed, $N = 1800$ rpm, by a 125 kW engine. Estimate the maximum thrust produced by the propeller at (a) standstill and (b) $V = 12.5$ m/s.

Given: Data on air boat and propeller

Find: Thrust at rest; Thrust at 12.5 m/s

Solution:

Assume the aircraft propeller coefficients in Fi.g 10.40 are applicable to this propeller.

At V = 0, J = 0. Extrapolating from Fig. 10.40b $C_F = 0.16$

We also have $D = 1.5\cdot m$ $n = 1800\cdot rpm$ $n = 30\cdot\frac{rev}{s}$ and $\rho = 1.225\cdot\frac{kg}{m^3}$

The thrust at standstill (J = 0) is found from $F_T = C_F\cdot\rho\cdot n^2\cdot D^4$ (Note: n is in rev/s) $F_T = 893\,N$

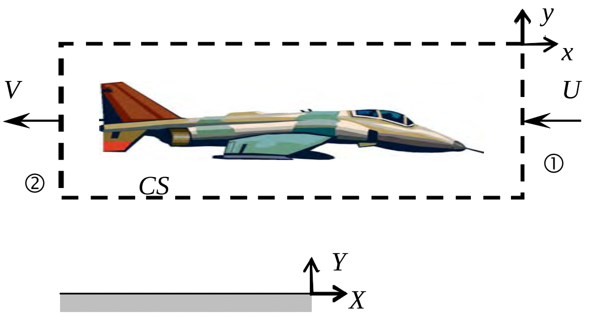
At a speed $V = 12.5\cdot\frac{m}{s}$ $J = \frac{V}{n\cdot D}$ $J = 0.278$ and so from Fig. 10.40b $C_P = 0.44$ and $C_F = 0.145$

The thrust and power at this speed can be found $F_T = C_F\cdot\rho\cdot n^2\cdot D^4$ $F_T = 809\,N$ $P = C_P\cdot\rho\cdot n^3\cdot D^5$ $P = 111\,kW$

Problem 10.94

[3]

10.94 A jet-propelled aircraft traveling at 450 mph takes in 90 lbm/s of air and discharges it at 1200 mph relative to the aircraft. Determine the propulsive efficiency (defined as the ratio of the useful work output to the mechanical energy input to the fluid) of the aircraft.



Given: Data on jet-propelled aircraft

Find: Propulsive efficiency

Solution:

Basic equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, dV\!\!\!/ + \int_{CS} \vec{V}_{xyz}\,\rho \vec{V}_{xyz} \cdot d\vec{A} \qquad (4.26)$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\, dV\!\!\!/ + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} \qquad (4.56)$$

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum is then $-F_D = u_1\cdot\left(-m_{rate}\right) + u_4\cdot\left(m_{rate}\right) = (-U)\cdot\left(-m_{rate}\right) + (-V)\cdot\left(m_{rate}\right)$

or $F_D = m_{rate}\cdot(V - U)$ where $m_{rate} = 90\cdot\frac{lbm}{s}$ is the mass flow rate

The useful work is then $F_D\cdot U = m_{rate}\cdot(V - U)\cdot U$

The energy equation simplifies to $-W = \left(\frac{U^2}{2}\right)\cdot\left(-m_{rate}\right) + \left(\frac{V^2}{2}\right)\cdot\left(m_{rate}\right) = \frac{m_{rate}}{2}\cdot\left(V^2 - U^2\right)$

Hence $\eta = \dfrac{m_{rate}\cdot(V - U)\cdot U}{\frac{m_{rate}}{2}\cdot\left(V^2 - U^2\right)} = \dfrac{2\cdot(V - U)\cdot U}{\left(V^2 - U^2\right)} = \dfrac{2}{1 + \frac{V}{U}}$

With $U = 450\cdot mph$ and $V = 1200\cdot mph$ $\eta = \dfrac{2}{1 + \frac{V}{U}}$ $\eta = 54.5\%$

Problem 10.95

Given: Ship drag data (Figs. 7.2 and 7.3) and dimensions (Problem 9.89).

Performance characteristics of marine propeller (Fig. 10.40). Propeller operates at maximum efficiency when ship steams at maximum speed, $V = 37.6$ kt.

Find: Calculate size, operating speed, and power input for a single propeller to propel this vessel.

Solution: From Problem 9.89, $L = 409$ ft and $A = 19{,}500\ \text{ft}^2$. At maximum speed,

$$V = 37.6\,\frac{\text{nm}}{\text{hr}} \times 6076\,\frac{\text{ft}}{\text{nm}} \times \frac{\text{hr}}{3600\,\text{s}} = 63.5\ \text{ft/s}$$

The Froude number is

$$Fr = \frac{V}{\sqrt{gL}} = 63.5\,\frac{\text{ft}}{\text{s}} \times \left[\frac{\text{s}^2}{32.2\,\text{ft}} \times \frac{1}{409\,\text{ft}}\right]^{\frac{1}{2}} = 0.553$$

From Fig. 7.2, $C_D \approx 0.0054$. The definition is $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$, so

$$F_D = C_D A \tfrac{1}{2}\rho V^2\ ;\quad \tfrac{1}{2}\rho V^2 = \tfrac{1}{2} \times (1.025)\,1.94\,\frac{\text{slug}}{\text{ft}^3} \times (63.5)^2\,\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 4010\ \text{lbf/ft}^2$$

$$F_D = 0.0054 \times 19{,}500\,\text{ft}^2 \times 4010\,\frac{\text{lbf}}{\text{ft}^2} = 422{,}000\ \text{lbf}$$

From Fig. 10.40(a), the maximum efficiency is $\eta = 0.67$ at $J = 0.85$. Then

$$nD = \frac{V}{J} = 63.5\,\frac{\text{ft}}{\text{s}} \times \frac{1}{0.85} = 74.7\ \text{ft/s} \qquad (1)$$

Since $C_F = 0.11 = \dfrac{F_D}{\rho n^2 D^4} = \dfrac{F_D}{\rho (n^2 D^2) D^2} = \dfrac{F_D}{\rho (nD)^2 D^2}$, then

$$D = \left[\frac{F_D}{\rho (nD)^2 C_F}\right]^{\frac{1}{2}} = \left[422{,}000\,\text{lbf} \times \frac{\text{ft}^3}{(1.025)\,1.94\,\text{slug}} \times \frac{\text{s}^2}{(74.7)^2\,\text{ft}^2} \times \frac{1}{0.11} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{\frac{1}{2}} = 18.6\ \text{ft}$$

D

From Eq. 1,

$$n = \frac{nD}{D} = 74.7\,\frac{\text{ft}}{\text{s}} \times \frac{1}{18.6\,\text{ft}} = 4.02\ \text{rev/s} \qquad (241\ \text{rpm})$$

n

The input power would be

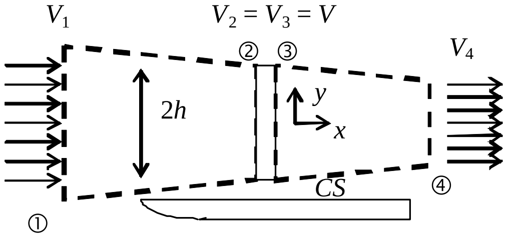
$$\mathcal{P}_{in} = \frac{\mathcal{P}_{out}}{\eta} = \frac{F_D V}{\eta} = 422{,}000\,\text{lbf} \times 63.5\,\frac{\text{ft}}{\text{s}} \times \frac{1}{0.67} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 72{,}700\ \text{hp}$$

$\mathcal{P}_{in}$

Problem 10.96

[4]

10.96 The propulsive efficiency, η, of a propeller is defined as the ratio of the useful work produced to the mechanical energy input to the fluid. Determine the propulsive efficiency of the moving airboat of Problem 10.92. What would be the efficiency if the boat were not moving?



Given: Definition of propulsion efficiency η

Find: η for moving and stationary boat

Solution:

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum (Example 10.3): $T = u_1 \cdot (-m_{rate}) + u_4 \cdot (m_{rate}) = m_{rate} \cdot (V_4 - V_1)$

Applying the energy equation to steady, incompressible, uniform flow through the moving CV gives the minimum power input requiremen

$$P_{min} = m_{rate} \cdot \left(\frac{V_4^{\,2}}{2} - \frac{V_1^{\,2}}{2}\right)$$

On the other hand, useful work is done at the rate of

$$P_{useful} = V_1 \cdot T = V_1 \cdot m_{rate} \cdot (V_4 - V_1)$$

Combining these expressions

$$\eta = \frac{V_1 \cdot m_{rate} \cdot (V_4 - V_1)}{m_{rate} \cdot \left(\frac{V_4^{\,2}}{2} - \frac{V_1^{\,2}}{2}\right)} = \frac{V_1 \cdot (V_4 - V_1)}{\frac{1}{2} \cdot (V_4 - V_1) \cdot (V_4 + V_1)}$$

or $$\eta = \frac{2 \cdot V_1}{V_1 + V_4}$$

When in motion $V_1 = 30 \cdot mph$ and $V_4 = 90 \cdot mph$ $\eta = \frac{2 \cdot V_1}{V_1 + V_4}$ $\eta = 50\%$

For the stationary case $V_1 = 0 \cdot mph$ $\eta = \frac{2 \cdot V_1}{V_1 + V_4}$ $\eta = 0\%$

Problem 10.97

[4]

Given: Propeller for Gossamer Condor human-powered aircraft has $D = 12$ ft and rotates at $N = 107$ rpm.

Additional information on the aircraft in Problem 9.170;
$W = 200$ lbf, $W/A = 0.4$ lbf/ft², $ar = 17$, $\mathcal{P}_{pilot} = 0.39$ hp, $F_D = 6$ lbf at $V = 12$ mph

Find: Estimate the dimensionless performance characteristics and efficiency of this propeller at cruise conditions.

Solution: From the solution to Problem 9.170, minimum power to propel the aircraft occurs at $V = 10.7$ mph (16.0 ft/s).
Assume this is the cruise condition.

From the given data, at 12 mph (17.6 ft/s), $F_D = 6$ lbf

$$\frac{1}{2}\rho V^2 = \frac{1}{2} \times 0.00238\,\frac{\text{slug}}{\text{ft}^3} \times (17.6)^2\,\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 0.369\ \text{lbf/ft}^2$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{W}{qA} = \frac{W/A}{q} = \frac{0.4\ \text{lbf/ft}^2}{0.369\ \text{lbf/ft}^2} = 1.08$$

$$C_D = C_L\frac{F_D}{F_L} = 1.08\,\frac{6\ \text{lbf}}{200\ \text{lbf}} = 0.0324$$

$$C_{D,0} = C_D - C_{D,i} = C_D - \frac{C_L^2}{\pi ar} = 0.0324 - \frac{(1.08)^2}{\pi(17)} = 0.0106$$

At $V = 10.7$ mph (16.0 ft/s), $q = 0.305$ lbf/ft²

$$C_L = \frac{W}{qA} = \frac{W/A}{q} = 0.4\,\frac{\text{lbf}}{\text{ft}^2} \times \frac{\text{ft}^2}{0.305\ \text{lbf}} = 1.31\ ;\quad C_{D,i} = \frac{C_L^2}{\pi ar} = 0.0321$$

$$C_D = C_{D,0} + C_{D,i} = 0.0106 + 0.0321 = 0.0427;\quad F_D = F_L\frac{C_D}{C_L} = 200\ \text{lbf}\ \frac{0.0427}{1.31} = 6.52\ \text{lbf}$$

For the propeller,

$$J = \frac{V}{nD} = 16.0\,\frac{\text{ft}}{\text{s}} \times \left(\frac{60}{107}\right)\frac{\text{s}}{\text{rev}} \times \frac{1}{12.0\ \text{ft}} = 0.748$$

J

$$C_F = \frac{F_D}{\rho n^2 D^4} = 6.52\ \text{lbf} \times \frac{\text{ft}^3}{0.00238\ \text{slug}} \times \left(\frac{60}{107}\right)^2 \frac{\text{s}^2}{\text{rev}^2} \times \frac{1}{(12.0)^4\ \text{ft}^4} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 0.0415$$

C_F

Assume a 30 percent reserve for climbing and maneuvers. Then if $\eta_d = 0.9$,

$$\eta_p = \frac{\mathcal{P}_{out}}{\mathcal{P}_{in}\eta_d} = \frac{F_D V}{(0.7)\,0.39\ \text{hp}\,(0.9)} = \frac{1}{0.246\ \text{hp}} \times 6.52\ \text{lbf} \times 16.0\,\frac{\text{ft}}{\text{s}} \times \frac{\text{hp}\cdot\text{s}}{550\ \text{ft}\cdot\text{lbf}} = 0.771$$

η

Finally, $\mathcal{P}_{prop} = \eta_d \mathcal{P}_{in} = 0.246\ \text{hp} = \omega T\ ;\ T = \frac{0.246\ \text{hp}}{\omega} = 12.1\ \text{ft}\cdot\text{lbf}$

$$C_T = \frac{T}{\rho n^2 D^5} = 12.1\ \text{ft}\cdot\text{lbf} \times \frac{\text{ft}^3}{0.00238\ \text{slug}} \times \left(\frac{60}{107}\right)^2 \frac{\text{s}^2}{\text{rev}^2} \times \frac{1}{(12.0)^5\ \text{ft}^5} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 0.00642$$

C_T

$$C_P = \frac{C_T}{\eta} = 0.00642 \times \frac{60}{107} = 0.0036$$

C_P

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem 10.98

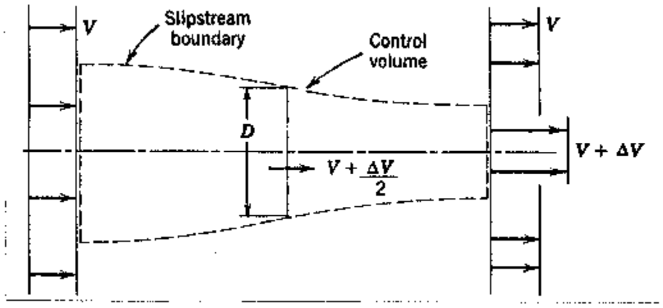
[5]

Given: Equations for thrust, power, and efficiency of propulsion devices derived in Section 10-5.

Find: (a) Show that for constant thrust, $\eta = \dfrac{2}{1+\left(1+\dfrac{F_T}{\rho\frac{V^2}{2}\frac{\pi D^2}{2}}\right)^{\frac{1}{2}}}$

(b) Interpret physically.

Solution: Apply 1-D forms of momentum and energy to CV of Fig. 10.38:



Computing equations:

Continuity $\dot{m} = \rho\left(V+\frac{\Delta V}{2}\right)\frac{\pi D^2}{4}$

Momentum $F_T = \dot{m}\,\Delta V$ (10.28)

Energy $\mathcal{P}_{in} = \dot{m}V\Delta V\left(1+\frac{\Delta V}{2V}\right)$ (10.29)

Assumptions: (1) Steady flow, (2) Incompressible flow, (3) Uniform flow, (4) Frictionless flow

Propulsion efficiency is $\eta_p = \dfrac{\mathcal{P}_{out}}{\mathcal{P}_{in}} = \dfrac{F_T V}{\dot{m}V\Delta V\left(1+\frac{\Delta V}{2V}\right)} = \dfrac{\dot{m}V\Delta V}{\dot{m}V\Delta V\left(1+\frac{\Delta V}{2V}\right)} = \dfrac{1}{1+\frac{\Delta V}{2V}}$ (1)

F_T may be written using continuity as

$$F_T = \dot{m}\,\Delta V = \rho\left(V+\frac{\Delta V}{2}\right)\frac{\pi D^2}{4}\Delta V = 2\rho V^2\frac{\pi D^2}{4}\left(1+\frac{\Delta V}{2V}\right)\left(\frac{\Delta V}{2V}\right) = 2\rho V^2\frac{\pi D^2}{4}(1+\lambda)\lambda$$

where $\lambda = \frac{\Delta V}{2V}$. For constant F_T,

$$\lambda^2 + \lambda - \frac{F_T}{2\rho V^2\frac{\pi D^2}{4}} = 0$$

Solving via the quadratic equation, and choosing the positive root

$$\lambda = \frac{-1 \pm \sqrt{1 + 4\dfrac{F_T}{2\rho V^2\frac{\pi D^2}{4}}}}{2} = \frac{1}{2}\left\{-1 + \sqrt{1 + \frac{F_T}{\rho\frac{V^2}{2}\,\frac{\pi D^2}{4}}}\right\}$$

From Eq. 1, $\eta_p = \dfrac{1}{1+\lambda} = \dfrac{1}{1+\frac{1}{2}\{\ \}} = \dfrac{2}{2+\{\ \}} = \dfrac{2}{1+\left(1+\dfrac{F_T}{\rho\frac{V^2}{2}\frac{\pi D^2}{4}}\right)^{\frac{1}{2}}}$ η_p

The ratio, $F_T/\frac{\pi D^2}{4}$, may be interpreted as the disk loading: the force developed per unit area of the actuator disk. Note $\eta_p \to 1$ as $\frac{\pi D^2}{4}$ increases.

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 10.99

[2]

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Given: Preliminary calculations for a hydroelectric power generation site show a net head, $H = 2350$ ft, is available at water flow rate, $Q = 75$ ~~ft~~ ft³/s.

Find: Compare the geometry and efficiency of Pelton wheels designed to run at (a) 450 rpm and (b) 600 rpm.

Solution: Apply specific speed equation to classify performance.

Computing equation: $N_{s_{cu}} = \dfrac{N\,\mathcal{P}^{1/2}}{H^{5/4}}$ (rpm, hp, and ft units)

From Fig. 10.17, $\eta_{max} \approx 0.89$ at $N_{s_{cu}} = 5$. The output power (used to define $N_{s_{cu}}$) is

$$\mathcal{P}_{out} = \eta\rho Q g H = 0.89 \times 62.4\,\frac{\text{lbf}}{\text{ft}^3} \times 75\,\frac{\text{ft}^3}{\text{s}} \times 2350\text{ ft} \times \frac{\text{hp}\cdot\text{s}}{550\text{ ft}\cdot\text{lbf}} = 17{,}800\text{ hp}$$

At $N = 450$ rpm

$$N_{s_{cu}} = \frac{450\text{ rpm}(17{,}800\text{ hp})^{\frac{1}{2}}}{(2350)^{5/4}} = 3.67\,,\ \text{so } \eta \approx 0.88$$

Neglect nozzle losses and elevation above the tailrace. Then

$$V_j \approx \sqrt{2gH} = \left[2 \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times 2350\text{ ft}\right]^{\frac{1}{2}} = 389\text{ ft/s}$$

From Fig. 10.10, $U = R\omega \approx 0.47\,V_j = 183$ ft/s. Thus

$$D = 2R = \frac{2(0.47\,V_j)}{\omega} = 2 \times 183\,\frac{\text{ft}}{\text{s}} \times \frac{\text{s}}{47.1\text{ rad}} = 7.77\text{ ft}$$

The jet diameter is found from $Q = V_j A_j = \pi V_j D_j^2/4$, so

$$D_j = \sqrt{\frac{4Q}{\pi V_j}} = \left[\frac{4}{\pi} \times 75\,\frac{\text{ft}^3}{\text{s}} \times \frac{\text{s}}{389\text{ ft}}\right]^{\frac{1}{2}} = 0.495\text{ ft (5.95 in.)}$$

The ratio of jet diameter to wheel diameter is

$$r = \frac{D_j}{D} = \frac{0.495\text{ ft}}{7.77\text{ ft}} = 0.0637 \quad \text{or} \quad 1:15.7 \quad \text{(this is reasonable)}$$

Results from similar computations at $N = 600$ rpm are:

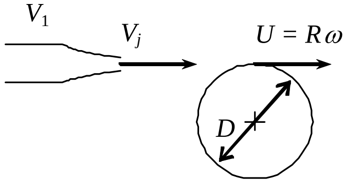
N (rpm)	$N_{s_{cu}}$ (USCS)	D (ft)	D_j/D (---)	η (---)
450	3.67	7.77	1:15.7	0.88
600	4.89	5.83	1:11.8	0.89

The unit operating at 600 rpm is closer to $N_{s_{cu}} = 5$, where peak hydraulic efficiency is expected.

Problem 10.100

[2]

10.100 Conditions at the inlet to the nozzle of a Pelton wheel are $p = 700$ psig and $V = 15$ mph. The jet diameter is $d = 7.5$ in. and the nozzle loss coefficient is $K_{\text{nozzle}} = 0.04$. The wheel diameter is $D = 8$ ft. At this operating condition, $\eta = 0.86$. Calculate (a) the power output, (b) the normal operating speed, (c) the approximate runaway speed, (d) the torque at normal operating speed, and (e) the approximate torque at zero speed.



Given: Pelton turbine

Find: 1) Power 2) Operating speed 3) Runaway speed 4) Torque 5) Torque at zero speed

Solution:

Basic equations

$$\left(\frac{p_1}{\rho\cdot g}+\alpha\cdot\frac{V_1^2}{2\cdot g}+z_1\right)-\left(\frac{p_j}{\rho\cdot g}+\alpha\cdot\frac{V_j^2}{2\cdot g}+z_j\right)=\frac{h_{lT}}{g} \qquad h_{lT}=h_l+h_{lm}=K\cdot\frac{V^2}{2}$$

and from Example 10.5 $\quad T_{ideal}=\rho\cdot Q\cdot R\cdot\left(V_j-U\right)\cdot(1-\cos(\theta)) \qquad \theta = 165\cdot\text{deg}$

Assumptions: 1) $p_j = p_{amt}$ 2) Incompressible flow 3) α at 1 and j is approximately 1 4) Only minor loss at nozzle 5) $z_1 = z_j$

Given data $\quad p_{1g} = 700\cdot\text{psi} \qquad V_1 = 15\cdot\text{mph} \qquad V_1 = 22\frac{\text{ft}}{\text{s}} \qquad \eta = 86\cdot\%$

$d = 7.5\cdot\text{in} \qquad D = 8\cdot\text{ft} \qquad R = \frac{D}{2} \qquad K = 0.04 \qquad \rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}$

Then $\quad \frac{p_{1g}}{\rho\cdot g}+\frac{V_1^2}{2\cdot g}-\frac{V_j^2}{2\cdot g}=\frac{K}{g}\cdot\frac{V_j^2}{2} \quad$ or $\quad V_j=\sqrt{\frac{2\cdot\left(\frac{p_{1g}}{\rho}+\frac{V_1^2}{2}\right)}{1+K}} \qquad V_j = 317\frac{\text{ft}}{\text{s}}$

and $\quad Q = V_j\cdot\frac{\pi\cdot d^2}{4} \qquad Q = 97.2\frac{\text{ft}^3}{\text{s}} \qquad H = \frac{p_{1g}}{\rho\cdot g}+\frac{V_1^2}{2\cdot g} \qquad H = 1622\,\text{ft}$

Hence $\quad P = \eta\cdot\rho\cdot Q\cdot g\cdot H \qquad P = 15392\,\text{hp}$

From Fig. 10.10, normal operating speed is around $U = 0.47\cdot V_j \qquad U = 149\frac{\text{ft}}{\text{s}} \qquad \omega = \frac{U}{R} \qquad \omega = 37.2\frac{\text{rad}}{\text{s}} \qquad \omega = 356\,\text{rpm}$

At runaway $\quad U_{run} = V_j \qquad \omega_{run} = \frac{U_{run}}{\left(\frac{D}{2}\right)} \qquad \omega_{run} = 79.2\frac{\text{rad}}{\text{s}} \qquad \omega_{run} = 756\,\text{rpm}$

From Example 10.5 $\quad T_{ideal} = \rho\cdot Q\cdot R\cdot\left(V_j-U\right)\cdot(1-\cos(\theta)) \qquad T_{ideal} = 2.49\times10^5\,\text{ft}\cdot\text{lbf}$

Hence $\quad T = \eta\cdot T_{ideal} \qquad T = 2.14\times10^5\,\text{ft}\cdot\text{lbf}$

Stall occurs when $\quad U = 0 \qquad T_{stall} = \eta\cdot\rho\cdot Q\cdot R\cdot V_j\cdot(1-\cos(\theta)) \qquad T_{stall} = 4.04\times10^5\,\text{ft}\cdot\text{lbf}$

Problem 10.101

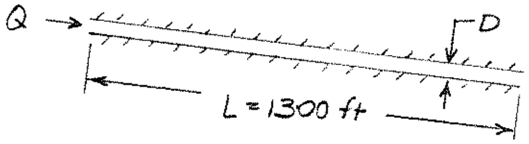
[2]

Given: Francis (reaction) turbines at Niagara Falls:

$D_0 = 176$ in., $\mathcal{P} = 72{,}500$ hp, $N = 107$ rpm, $\eta = 0.938$, $H = 214$ ft (net)

Penstock has $L = 1300$ ft; $H_{net} = 0.85\,H_{gross}$.

Find: (a) Calculate specific speed.
(b) Evaluate volume flow rate.
(c) Estimate penstock size.



Solution: $$N_{s_{cu}} = \frac{N\mathcal{P}^{\frac{1}{2}}}{H^{\frac{5}{4}}} = \frac{107\text{ rpm}\,(72{,}500\text{ hp})^{\frac{1}{2}}}{(214\text{ ft})^{\frac{5}{4}}} = 35.1$$ N_s

Efficiency is defined as $\eta = \mathcal{P}/\rho Q g H$, so

$$Q = \frac{\mathcal{P}}{\eta \rho g H} = \frac{1}{0.938} \times 72{,}500\text{ hp} \times \frac{\text{ft}^3}{62.4\text{ lbf}} \times \frac{1}{214\text{ ft}} \times \frac{550\text{ ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 31800\text{ ft}^3/\text{s}$$ Q

Apply the definition of head loss: $h_{\ell T} = g H_{\ell T} = f \frac{L}{D} \frac{\bar{V}^2}{2}$

$$H_{\ell T} = 0.15\,H_{gross} = 0.15\,\frac{H_{net}}{0.85} = 0.176\,H_{net} = 0.176 \times 214\text{ ft} = 37.7\text{ ft}$$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2}\ ;\ \bar{V}^2 = \frac{16Q^2}{\pi^2 D^4}\ ;\ g\Delta H = f\frac{L}{D}\frac{\bar{V}^2}{2} = f\frac{L}{D}\frac{8Q^2}{\pi^2 D^4} = \frac{8fLQ^2}{\pi^2 D^5}$$

Solving, $$D = \left[\frac{8fLQ^2}{\pi^2 g \Delta H}\right]^{\frac{1}{5}}$$ (Assume T = 50°F)

This system is very large, so it is difficult to estimate f. Assuming concrete-lined penstocks, $e = 0.01$ ft (Table 8.1). Start with $f = 0.01$ to get by iteration (see below),

$$D = 26.8\text{ ft}$$

The flow properties are

$$\bar{V} = 56.8\text{ ft/sec},\ Re = 1.18 \times 10^8, \text{ and } f = 0.0157$$

Flow is in the fully rough zone with $e/D = 0.01/26.8 = 0.00037$.

Using $e = 0.02$ gives $D = 28.1$ ft, so $D \approx 27 - 28$ ft D

The iterations are:

Assumed D (ft)	V (ft/s)	Re	f_0	$f^{-0.5}$	f	Calculated D (ft)
					0.01	24.5
24.5	67.5	1.18E+08	0.0160	7.91	0.0160	26.9
26.9	56.0	1.08E+08	0.0157	7.99	0.0156	26.8
26.80	56.4	1.08E+08	0.0157	7.99	0.0157	26.77
26.77	56.5	1.08E+08	0.0157	7.99	0.0157	26.77
					0.02	28.1

Problem 10.102

[2]

Given: Francis turbine Units 19, 20, and 21 at Grand Coulee Dam.

Rated conditions: $\mathcal{P} = 820{,}000\ \text{hp}$, $N = 72\ \text{rpm}$, $H = 285\ \text{ft}$, $\eta = 0.95$

Turbines operate from $220 < H < 355\ \text{ft}$.

Find: (a) Calculate specific speed at rated conditions.
(b) Estimate maximum water flow rate through each turbine.

Solution: Apply definitions of specific speed and efficiency.

Computing equations: $N_{s_{cu}} = \dfrac{N\mathcal{P}^{1/2}}{H^{5/4}}$ $\qquad \eta = \dfrac{\mathcal{P}}{\rho Q g H}$

Thus

$$N_{s_{cu}} = \frac{72\ \text{rpm}\,(820{,}000\ \text{hp})^{\frac{1}{2}}}{(285\ \text{ft})^{5/4}} = 55.7$$

$N_{s_{cu}}$

From η,

$$Q = \frac{\mathcal{P}}{\eta \rho g H}$$

so Q is maximum at minimum head. Assuming $\eta = 0.95$, the

$$Q \approx \frac{1}{0.95} \times 820{,}000\ \text{hp} \times \frac{\text{ft}^3}{62.4\ \text{lbf}} \times \frac{1}{220\ \text{ft}} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 34{,}600\ \text{ft}^3/\text{s} \quad (\text{max})$$

Q

{This is an estimate because η may not be constant, nor may it be possible to develop full power at $H = 220\ \text{ft}$.}

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Problem 10.103

[3]

Given: Measured data for reaction turbines at Shasta Dam, Fig. 10.13.

Each turbine is rated at $\mathcal{P} = 103{,}000$ hp at $N = 138.6$ rpm, under a net head of $H = 380$ ft.

Find: (a) Specific speed at rated conditions.

(b) Shaft torque at rated conditions.

(c) Calculate the water flow rate per turbine needed to produce rated output power; plot versus head.

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Solution: Apply the definitions of specific speed and efficiency; use data from Fig. 10.13:

Computing equations: $N_{s_{cu}} = \dfrac{N \mathcal{P}^{\frac{1}{2}}}{H^{\frac{5}{4}}}$ $\eta = \dfrac{\mathcal{P}}{\rho Q g H}$ $\mathcal{P} = \omega T$

At rated conditions, $N_{s_{cu}} = \dfrac{(138.6\,\text{rpm})(103{,}000\,\text{hp})^{\frac{1}{2}}}{(380\,\text{ft})^{\frac{5}{4}}} = 26.5$ $\longleftarrow$ N_s

$$T = \frac{\mathcal{P}}{\omega} = 103{,}000\,\text{hp} \times \frac{\text{s}}{14.5\,\text{rad}} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 3.91 \times 10^{6}\ \text{ft}\cdot\text{lbf} \longleftarrow$$

Find Q from definition of η; at rated conditions, $\eta \approx 0.93$ (Fig. 10.13):

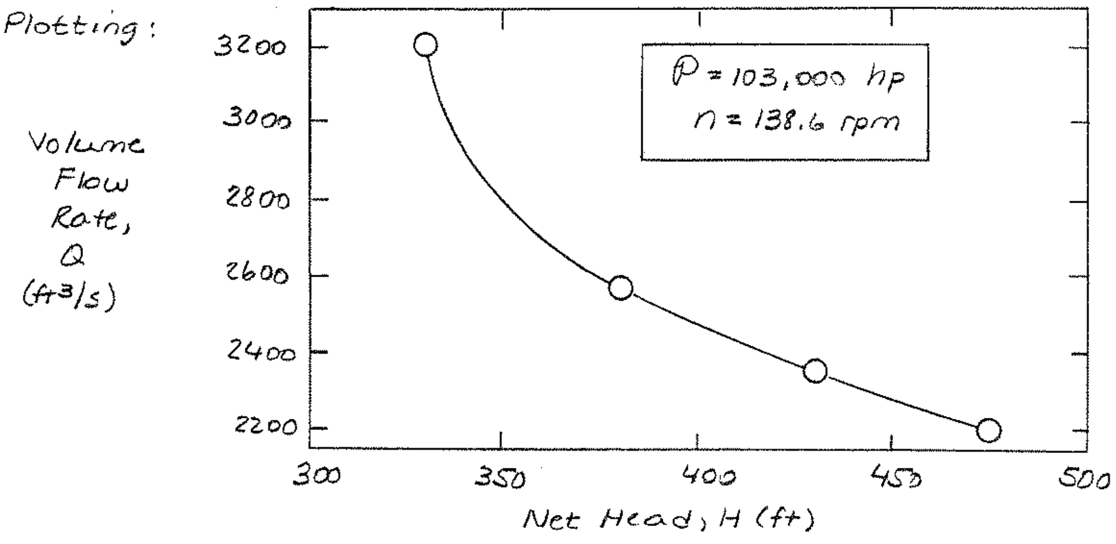
$$Q = \frac{\mathcal{P}}{\eta \rho g H} = \frac{1}{0.93} \times 103{,}000\,\text{hp} \times \frac{\text{ft}^3}{62.4\,\text{lbf}} \times \frac{1}{380\,\text{ft}} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 2570\ \text{ft}^3/\text{s}$$

Tabulating similar calculations:

H (ft)	$\mathcal{P}$ (hp)	η (---)	Q (ft³/s)
238	*	–	–
280	*	–	–
330	103,000	0.86	3200
380	103,000	0.93	2570
430	103,000	0.90	2350
475	103,000	0.87	2200

* cannot produce rated power at this head

Plotting:



Problem 10.104

Given: Data in Fig. 10.11 for Pelton wheel in PG&E Tiger Creek plant.

Rating is $\mathcal{P} = 36{,}000$ hp at $N = 225$ rpm under $H = 1190$ ft (net)

Assume reasonable flow angles and nozzle loss coefficient.

(a) Determine rotor diameter.
(b) Estimate jet diameter.
(c) Compute volume flow rate of water.

Solution: From Bernoulli, the ideal jet velocity would be $V_i = \sqrt{2gH}$. Assuming $C_V = 0.98$ (4 percent loss in nozzle), then

$$V_j = C_V V_i = C_V\sqrt{2gH} = 0.98\left[2 \times 32.2\,\frac{ft}{s^2} \times 1190\,ft\right]^{\frac{1}{2}} = 271\ ft/s$$

From Fig. 10.10, $U = R\omega = 0.47\,V_j$ at optimum conditions. Then

$$D = 2R = \frac{2(0.47)V_j}{\omega} = 0.94 \times 271\,\frac{ft}{s} \times \frac{s}{23.6\ rad} = 10.8\ ft$$

D

From Fig. 11.11, $\eta = 0.86$ at full load. Thus

$$\eta = \frac{\mathcal{P}}{\rho Q g H}\ ;\ Q = \frac{\mathcal{P}}{\eta \rho g H}\ ;\ Q = V_j A_j = V_j\,\frac{\pi D_j^2}{4}$$

$$Q = \frac{1}{0.86} \times 36{,}000\ hp \times \frac{ft^3}{62.4\ lbf} \times \frac{1}{1190\ ft} \times 550\,\frac{ft\cdot lbf}{hp\cdot s} = 310\ ft^3/s$$

Q

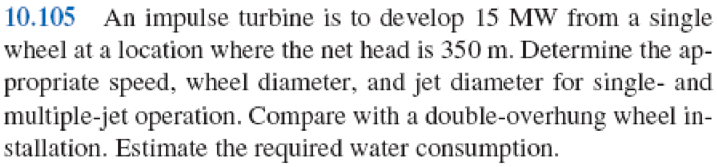
$$D_j = \left[\frac{4Q}{\pi V_j}\right]^{\frac{1}{2}} = \left[\frac{4}{\pi} \times 310\,\frac{ft^3}{s} \times \frac{s}{271\ ft}\right]^{\frac{1}{2}} = 1.21\ ft\ (14.5\ in.)$$

D_j

Note $D_j/D = 1.21/10.8 = 0.112$ (1:8.93).

Problem 10.105

[3]



10.105 An impulse turbine is to develop 15 MW from a single wheel at a location where the net head is 350 m. Determine the appropriate speed, wheel diameter, and jet diameter for single- and multiple-jet operation. Compare with a double-overhung wheel installation. Estimate the required water consumption.

Given: Impulse turbine requirements

Find: 1) Operating speed 2) Wheel diameter 4) Jet diameter 5) Compare to multiple-jet and double-overhung

Solution:

Basic equations: $V_j = \sqrt{2\cdot g\cdot H}$ $\quad N_S = \dfrac{\omega\cdot P^{\frac{1}{2}}}{\rho^{\frac{1}{2}}\cdot h^{\frac{5}{4}}}$ $\quad \eta = \dfrac{P}{\rho\cdot Q\cdot g\cdot H}$ $\quad Q = V_j\cdot A_j$

Model as optimum. This means. from Fig. 10.10 $\quad U = 0.47\cdot V_j$ $\quad$ and from Fig. 10.17 $\quad N_{Scu} = 5$ $\quad$ with $\quad \eta = 89\cdot\%$

Given or available data $\quad H = 350\cdot m$ $\quad P = 15\cdot MW$ $\quad \rho = 1.94\cdot\dfrac{slug}{ft^3}$

Then $\quad V_j = \sqrt{2\cdot g\cdot H}$ $\quad V_j = 82.9\,\dfrac{m}{s}$ $\quad U = 0.47\cdot V_j$ $\quad U = 38.9\,\dfrac{m}{s}$

We need to convert from N_{Scu} (from Fig. 10.17) to N_S (see discussion after Eq. 10.18b). $\quad N_S = \dfrac{N_{Scu}}{43.46}$ $\quad N_S = 0.115$

The water consumption is $\quad Q = \dfrac{P}{\eta\cdot\rho\cdot g\cdot H}$ $\quad Q = 4.91\,\dfrac{m^3}{s}$

For a single jet $\quad \omega = N_S\cdot\dfrac{\rho^{\frac{1}{2}}\cdot(g\cdot H)^{\frac{5}{4}}}{P^{\frac{1}{2}}}$ $\quad$ (1) $\quad \omega = 236\,rpm$ $\quad D_j = \sqrt{\dfrac{4\cdot Q}{\pi\cdot V_j}}$ $\quad$ (2) $\quad D_j = 0.275\,m$

The wheel radius is $\quad D = \dfrac{2\cdot U}{\omega}$ $\quad$ (3) $\quad D = 3.16\,m$

For multiple (n) jets, we use the power and flow per jet

From Eq 1 $\quad \omega_n = \omega\cdot\sqrt{n}$ $\quad$ From Eq. 2 $\quad D_{jn} = \dfrac{D_j}{\sqrt{n}}$ $\quad$ and $\quad D_n = \dfrac{D}{\sqrt{n}}$ $\quad$ from Eq. 3

Results:

$n =$	$\omega_n(n) =$ (rpm)	$D_{jn}(n) =$ (m)	$D_n(n) =$ (m)
1	236	0.275	3.16
2	333	0.194	2.23
3	408	0.159	1.82
4	471	0.137	1.58
5	527	0.123	1.41

A double-hung wheel is equivalent to having a single wheel with two jets

Problem 10.106

[2]

10.106 Tests of a model impulse turbine under a net head of 20 m produced the following results:

Wheel Speed (rpm)	**No-Load Discharge (m^3/hr)**	**Net Brake Scale Reading (N) (R = 2 m)**					
300	10	33	72	107	140	194	233
325	11.4	29	63	96	124	175	213
	Discharge (m^3/hr)	44	86	124	157	211	257

Calculate and plot the machine power output and efficiency versus water flow rate.

Given:

Data on impulse turbine

Find:

Plot of power and efficiency curves

Solution:

Basic equations: $T = F \cdot R$ $\quad P = \omega \cdot T$ $\quad \eta = \dfrac{P}{\rho \cdot Q \cdot g \cdot H}$

$H = 25$ m

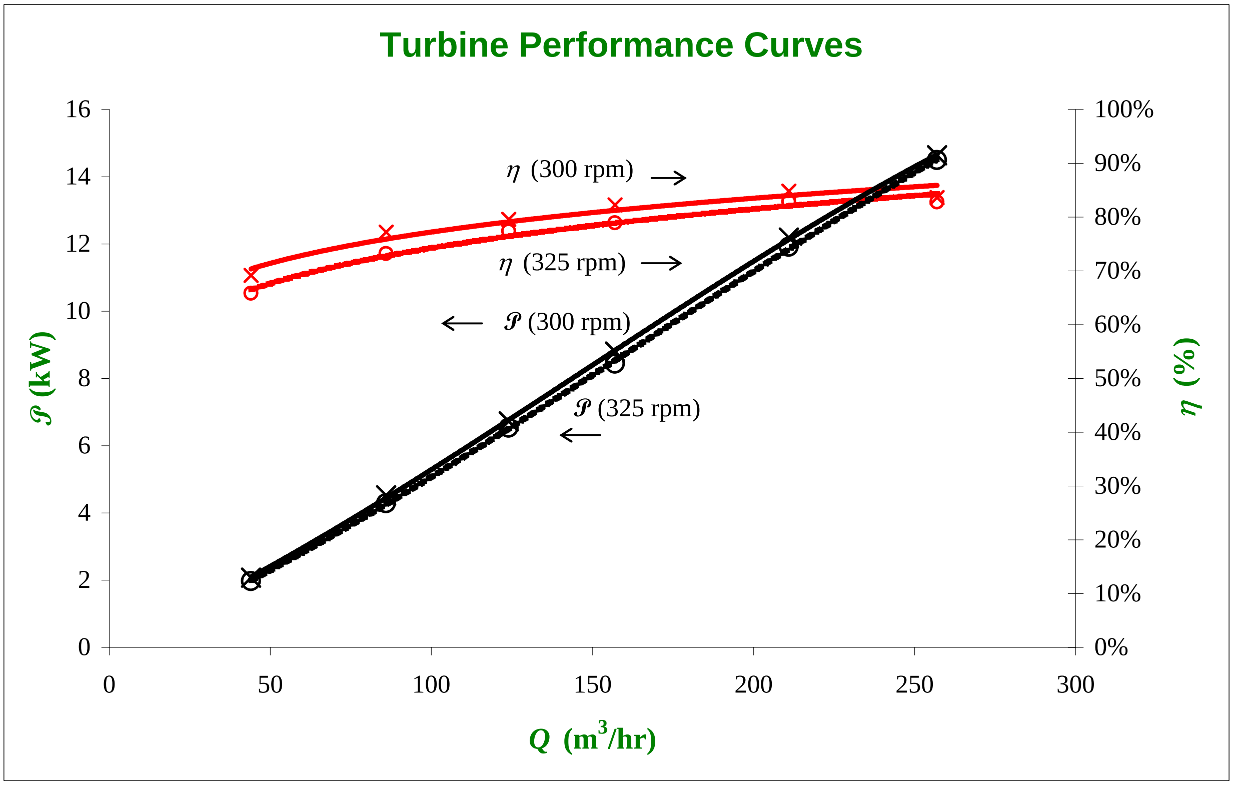
NOTE: Earlier printings had H incorrectly as 20 m, which gives efficiencies > 100%

$\rho = 1000$ kg/m^3

$R = 2.00$ m

ω = 300 rpm				
Q (m^3/hr)	**F (N)**	**T (N·m)**	**$\mathscr{P}$ (kW)**	**η (%)**
44	33	66	2.07	69.2%
86	72	144	4.52	77.2%
124	107	214	6.72	79.6%
157	140	280	8.80	82.2%
211	194	388	12.19	84.8%
257	233	466	14.64	83.6%

ω = 325 rpm				
Q (m^3/hr)	**F (N)**	**T (N·m)**	**$\mathscr{P}$ (kW)**	**η (%)**
44	29	58	1.97	65.9%
86	63	126	4.29	73.2%
124	96	192	6.53	77.4%
157	124	248	8.44	78.9%
211	175	350	11.91	82.9%
257	213	426	14.50	82.8%



Problem 10.107

[4]

Given: Definition of specific speed for a hydraulic turbine in U.S. units:

$$N_{s_{cu}} = \frac{N(\text{rpm})[\mathscr{P}(\text{hp})]^{\frac{1}{2}}}{[h(\text{ft})]^{5/4}}$$

Impulse turbine: $N = 400$ rpm, $H = 1190$ ft, $\eta = 0.86$, with $D_j = 6$ in.

Find: (a) Develop a conversion from $N_{s_{cu}}$ to a dimensionless N_s in SI units.
(b) Evaluate N_s of turbine in U.S. and S.I. units.
(c) Estimate the wheel diameter.

Solution: Apply definitions of specific speed and efficiency.

Computing equations: $N_{s_{cu}} = \dfrac{N\mathscr{P}^{1/2}}{H^{5/4}}$ $\quad \eta \equiv \dfrac{\mathscr{P}}{\rho Q g H}$ $\quad N_s = \dfrac{\omega \mathscr{P}^{1/2}}{\rho^{1/2} h^{5/4}}$

From dimensional analysis, recall that for pumps, $N_s = \dfrac{NQ^{1/2}}{H^{3/4}}$ was dimensionless only when H is expressed as gH, energy per unit mass. Thus the dimensions are

$$[N_s] = \left[\frac{NQ^{1/2}}{(gH)^{3/4}}\right] = \frac{1}{t}\left(\frac{L^3}{t}\right)^{\frac{1}{2}}\left(\frac{t^2}{L^2}\right)^{\frac{3}{4}} = \frac{1}{t}\,\frac{L^{3/2}}{t^{1/2}}\,\frac{t^{3/2}}{L^{3/2}} = 1 \quad \checkmark\checkmark$$

To form the N_s for turbines, Q must be multiplied by $\rho g H$ to obtain power. Thus for turbines, the dimensionless specific speed is

$$[N_s] = \left[\frac{NQ^{1/2}}{(gH)^{3/4}} \times \frac{(\rho g H)^{1/2}}{(\rho g H)^{1/2}}\right] = \left[\frac{N\mathscr{P}^{1/2}}{\rho^{1/2}(gH)^{5/4}}\right] = \frac{1}{t}\left(\frac{FL}{t} \times \frac{ML}{Ft^2}\right)^{\frac{1}{2}}\left(\frac{L^3}{M}\right)^{\frac{1}{2}}\left(\frac{t^2}{L^2}\right)^{\frac{5}{4}} = \frac{M^{\frac{1}{2}} L^{\frac{5}{2}} t^{\frac{5}{2}}}{M^{\frac{1}{2}} L^{5/2} t^{5/2}} = 1 \quad \checkmark\checkmark$$

The simplest way to convert is to evaluate each specific speed, then take the ratio.

The jet speed will be approximately $V_j = \sqrt{2gH} = \left[2 \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times 1190\,\text{ft}\right]^{\frac{1}{2}} = 277$ ft/s

and $Q = V_j \dfrac{\pi D_j^2}{4} = 277\,\dfrac{\text{ft}}{\text{s}} \times \dfrac{\pi}{4}\left(\dfrac{1}{2}\right)^2 \text{ft}^2 = 54.4\,\text{ft}^3/\text{s}$. Thus

$$\mathscr{P} = \eta \rho Q g H = 0.86 \times 62.4\,\frac{\text{lbf}}{\text{ft}^3} \times 54.4\,\frac{\text{ft}^3}{\text{s}} \times 1190\,\text{ft} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 6320\,\text{hp}\ (4.71\ \text{MW})$$

For the wheel, $U = 0.47 V_j = R\omega$; $R = 3.10$ ft; $D = 6.20$ ft $\leftarrow$ D

In U.S. units, $N_{s_{cu}} = \dfrac{400\,\text{rpm}\,(6320\,\text{hp})^{\frac{1}{2}}}{(1190\,\text{ft})^{5/4}} = 4.55$ $\leftarrow$ N_s

In S.I. units, $\omega = 41.9$ rad/s and $gH = 3560\ \text{m}^2/\text{s}^2$, so

$$N_s = \frac{\omega \mathscr{P}^{1/2}}{\rho^{1/2}(gH)^{5/4}} = 41.9\,\frac{\text{rad}}{\text{s}} \times (4.71\times 10^6\,\text{W})^{\frac{1}{2}} \times \left(\frac{\text{m}^3}{999\,\text{kg}}\right)^{\frac{1}{2}}\left(\frac{\text{s}^2}{3560\,\text{m}^2}\right)^{\frac{5}{4}} = 0.105$$ $\leftarrow$ N_s

The conversion is $\quad \dfrac{N_{s\,cu}}{N_s} = \dfrac{4.55}{0.105} = 43.5$ $\leftarrow$ Ratio

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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42-389 200 SHEETS 5 SQUARE
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Problem 10.108

[3]

Given: Published data for PG&E Tiger Creek Power Plant:

$H_{gross} = 1219$ ft, $Q = 750$ ft^3/s, $\mathcal{P} = 58$ MW (rating is 60 MW)

Plant is claimed to produce 968 kW·hr/acre·ft of water and 336.4×10^6 kW·hr/yr of operation.

Find: (a) Estimate the net head at the site, the turbine specific speed, and the turbine efficiency.

(b) Comment on the internal consistency of the published data.

Solution: Apply definitions of specific speed and efficiency.

Computing equations: $N_{s_{cu}} = \dfrac{N \mathcal{P}^{1/2}}{H^{5/4}}$ $\qquad \eta = \dfrac{\mathcal{P}}{\rho Q g H_{net}}$

Use the operating point to estimate net head. From Fig. 10.11, assume $\eta = 0.87$: ← η

$$H_{net} = \frac{\mathcal{P}}{\eta \rho Q g} = \frac{1}{0.87} \times 58 \times 10^6\,\text{W} \times \frac{\text{ft}^3}{62.4\,\text{lbf}} \times \frac{\text{s}}{750\,\text{ft}^3} \times \frac{\text{hp}}{746\,\text{W}} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 1050\,\text{ft}$$ ← H_{net}

Thus

$H_{net}/H_{gross} = 1050/1219 = 0.861$ or 86.1 percent (Reasonable)

The specific speed should be $N_{s_{cu}} \approx 5$. Checking, ← N_s

$$N = \frac{N_s H^{5/4}}{\mathcal{P}^{1/2}} = \frac{5\,(1050\,\text{ft})^{5/4}}{(77{,}700\,\text{hp})^{1/2}} = 107$$

This is too low, so the plant must have several turbines. Reducing $\mathcal{P}$ to the output per turbine would raise N.

Check data consistency:

$$58 \times 10^6\,\text{W} \times 24\,\frac{\text{hr}}{\text{day}} \times 365\,\frac{\text{day}}{\text{yr}} = 508 \times 10^6\,\text{kW}\cdot\text{hr/yr}$$

$$60 \times 10^6\,\text{W} \qquad = 526 \times 10^6\,\text{kW}\cdot\text{hr/yr}$$

} Both values are ~50% higher than quoted.

$$58 \times 10^6\,\text{W} \times \frac{\text{s}}{750\,\text{ft}^3} \times \frac{\text{hr}}{3600\,\text{s}} \times \frac{43{,}600\,\text{ft}^3}{\text{acre}\cdot\text{ft}} = 937\,\text{kW}\cdot\text{hr/acre}\cdot\text{ft}$$

$$60 \times 10^6\,\text{W} \qquad = 969\,\text{kW}\cdot\text{hr/acre}\cdot\text{ft}$$

} Excellent agreement

Problem 10.109

[4]

10.109 Design the piping system to supply a water turbine from a mountain reservoir. The reservoir surface is 1000 ft above the turbine site. The turbine efficiency is 80 percent, and it must produce 35 hp of mechanical power. Define the minimum standard-size pipe required to supply water to the turbine and the required volume flow rate of water. Discuss the effects of turbine efficiency, pipe roughness, and installing a diffuser at the turbine exit on the performance of the installation.

Given: Hydraulic turbine site

Find: Minimum pipe size; Fow rate; Discuss

Solution:

Basic equations: $H_l = \frac{h_l}{g} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g}$ and also, from Example 10.15 the optimum is when $H_l = \frac{\Delta z}{3}$

As in Fig. 10.41 we assume $L = 2 \cdot \Delta z$ and $f = 0.02$

Then, for a given pipe diameter D $V = \sqrt{\frac{2 \cdot g \cdot D \cdot H_l}{f \cdot L}} = \sqrt{\frac{g \cdot D}{3 \cdot f}}$

Also $Q = V \cdot \frac{\pi \cdot D^2}{4}$ $P_h = \rho \cdot Q \cdot \frac{V^2}{2}$ $P_m = \eta \cdot P_h$

$f =$ 0.02

$\rho =$ 1.94 slug/ft^3

$R =$ 2.00 m

$\eta =$ 80%

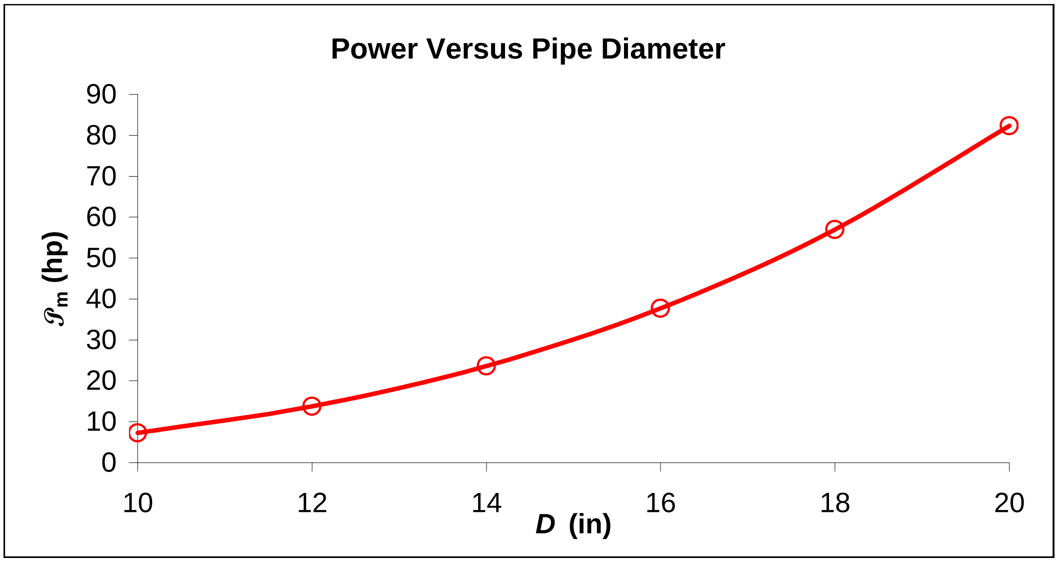
D (in)	V (m/s)	Q (ft^3/s)	$\mathscr{P}_h$ (hp)	$\mathscr{P}_m$ (hp)
10	21.1	11.5	9.10	7.28
12	23.2	18.2	17.22	13.78
14	25.0	26.7	29.54	23.63
16	26.7	37.3	47.13	37.71
18	28.4	50.1	71.18	56.95
20	29.9	65.2	102.93	82.34

Turbine efficiency varies with specific speed (Fig. 10.17).
Pipe roughness appears to the 1/2 power, so has a secondary effect.
A 20% error in f leads to a 10% change in water speed and 30% change in power.
A Pelton wheel is an impulse turbine that does not flow full of water; it directs the stream with open buckets.
A diffuser could not be used with this system.

15.7	26.5	35.4	43.75	35.00

Use Goal Seek or Solver to vary D to make $\mathscr{P}_m$ 35 hp!

The smallest standard size is 16 in.



Problem 10.110

Given: NASA-DOE wind turbine generator at Plum Brook, Ohio.

Two blades, $D = 38$ m; delivers $\mathcal{P} = 100$ kW when $V \geq 29$ km/hr, operating at 40 rpm, with powertrain efficiency, $\eta = 0.75$.

Find: For the maximum power condition, estimate the rotor tip speed and power coefficient.

Solution: Apply definitions

Computing equations: $U = \omega R \qquad X = \omega R / V \qquad C_p = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 \pi R^2}$

At $N = 40$ rpm,

$$\omega = 40\,\frac{\text{rev}}{\text{min}} \times 2\pi\,\frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{s}} = 4.19\ \text{rad/s}$$

$$U = 4.19\,\frac{\text{rad}}{\text{s}} \times \left(\frac{38}{2}\right)\text{m} = 79.6\ \text{m/s}$$

U

$$V = 29\,\frac{\text{km}}{\text{hr}} \times 1000\,\frac{\text{m}}{\text{km}} \times \frac{\text{hr}}{3600\,\text{s}} = 8.06\ \text{m/s}$$

$$X = U/V = 79.6/8.06 = 9.88$$

(Obviously X decreases as wind speed goes up.)

$$\mathcal{P}_m = \frac{\mathcal{P}_e}{0.75} = \frac{1}{0.75} \times 100\ \text{kW} = 133\ \text{kW}$$

$$\frac{1}{2}\rho V^3 \pi R^2 = \frac{\pi}{2} \times 1.23\,\frac{\text{kg}}{\text{m}^3} \times (8.06)^3\,\frac{\text{m}^3}{\text{s}^3} \times \left(\frac{38}{2}\right)^2 \text{m}^2 \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \times \frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}} = 365\ \text{kW}$$

$$C_p = \frac{\mathcal{P}_m}{\frac{1}{2}\rho V^3 \pi R^2} = \frac{133\ \text{kW}}{365\ \text{kW}} = 0.364$$

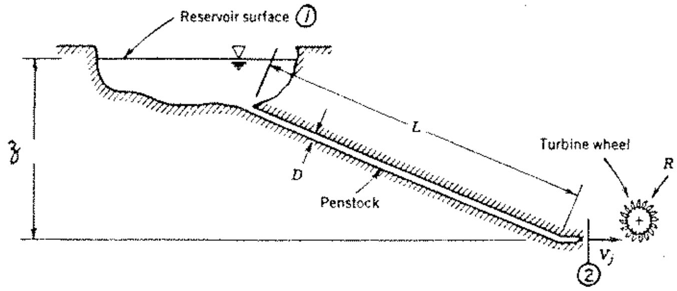
C_p

Problem 10.111

[4]

Given: Small hydraulic impulse turbine installation as shown.

$H = 300$ ft
$L = 1000$ ft (welded steel)
$D = 6$ in., $d_j = 2$ in.
$K_{ent} = 0.5$
$K_{nozzle} = 0.04$



(a) Find V_j, Q, and $\mathcal{P}_h$
(b) Plot $\mathcal{P}_h$ vs. d_j to find optimum d_j.
(c) Then explore effects of varying loss coefficients and pipe roughness.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation:

$$\frac{\cancel{p_1}}{\rho g}+\alpha_1\frac{\overset{\approx 0(2)}{\cancel{\bar V_1^2}}}{2g}+z_1=\frac{\cancel{p_2}}{\rho g}+\frac{\alpha_2\bar V_2^2}{2g}+z_2+\frac{h_{\ell T}}{g};\quad h_{\ell T}=\left[f\left(\frac{L}{D}+\cancel{\frac{L_e}{D}}\right)+K\right]\frac{\bar V^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar V_1 \approx 0$, $\alpha_2 = 1$, (3) $L_e/D = 0$, (4) K_{nozzle} based on V_j^2

Then

$$H = \frac{V_j^2}{2g} + \left(f\frac{L}{D} + K_{entrance}\right)\frac{\bar V^2}{2g} + K_{nozzle}\frac{V_j^2}{2g}$$

From continuity, $\bar V A = V_j A_j$, so $\bar V = V_j\, A_j/A = V_j (d/D)^2$; $\bar V^2 = V_j^2 (d/D)^4$, and

$$H = \left[\left(f\frac{L}{D} + K_{ent}\right)\left(\frac{d}{D}\right)^4 + 1 + K_{nozzle}\right]\frac{V_j^2}{2g};\quad V_j = \left[\frac{2gH}{\left(f\frac{L}{D}+K_{ent}\right)\left(\frac{d}{D}\right)^4+1+K_n}\right]^{\frac{1}{2}}$$

Assume $e = 0.00015$ ft (Table 8.1), so $e/D = 0.0003$. From Fig. 8.13, in the fully rough zone, $f = 0.015$. Then for $d_j = 2$ in.

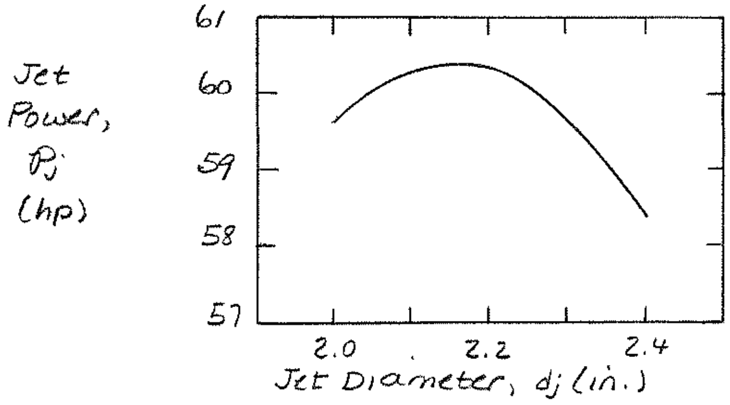
$$V_j = \left[2 \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times 300\,\text{ft} \times \frac{1}{\left(0.015\,\frac{1000\,\text{ft}}{0.5\,\text{ft}} + 0.5\right)\left(\frac{2}{6}\right)^4 + 1 + 0.04}\right]^{\frac{1}{2}} = 117\ \text{ft/sec}$$

($\bar V = 13.0$ ft/s, $Re = \bar V D/\nu = 6.05 \times 10^5$, so $f = 0.016$, which makes $V_j = 116$ ft/s.)

The jet flow rate is $Q = V_j A_j = 116\,\frac{\text{ft}}{\text{s}} \times \frac{\pi}{4}\left(\frac{2}{12}\right)^2 \text{ft}^2 = 2.53\ \text{ft}^3/\text{s}$, and the jet power is

$$\mathcal{P}_h = \rho Q\frac{V_j^2}{2} = \frac{1}{2} \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times (116)^2\,\frac{\text{ft}^2}{\text{s}^2} \times 2.53\,\frac{\text{ft}^3}{\text{s}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 60.0\ \text{hp}$$

Repeating these calculations using a computer program gives:



Peak power, $\mathcal{P}_j \approx 60.3$ hp, occurs for $2.15 < d < 2.20$ in.

Opt

Loss coefficients have a minor effect. Making both K_{ent} and K_n zero increases $\mathcal{P}_j$ by 4.8 percent.

Pipe roughness causes larger changes; $\mathcal{P}_j$ increased 12.8 percent with $e = 0$ (smooth).

Problem 10.112

[2]

10.112 A model of an American multiblade farm windmill is to be built for display. The model, with $D = 1$ m, is to develop full power at $V = 10$ m/s wind speed. Calculate the angular speed of the model for optimum power generation. Estimate the power output.

Given: Model of farm windmill

Find: Angular speed for optimum power; Power output

Solution:

Basic equations: $C_P = \dfrac{P}{\frac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2}$ $\quad X = \dfrac{\omega\cdot R}{V}$ $\quad$ and we have $\rho = 1.225\cdot\dfrac{kg}{m^3}$

From Fig. 10.45 $\quad C_{Pmax} = 0.3$ $\quad$ at $\quad X = 0.8$ $\quad$ and $\quad D = 1\cdot m$ $\quad R = \dfrac{D}{2}$ $\quad R = 0.5\,m$

Hence, for $\quad V = 10\cdot\dfrac{m}{s}$ $\quad \omega = \dfrac{X\cdot V}{R}$ $\quad \omega = 16\,\dfrac{rad}{s}$ $\quad \omega = 153\,rpm$

Also $\quad P = C_{Pmax}\cdot\dfrac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2$ $\quad P = 144\,W$

Problem 10.113

[3]

Given: Typical American multiblade farm windmill, $D = 7$ ft, designed to produce maximum power in winds with $V = 15$ mph.

Find: Estimate the rate of water delivery, as a function of the height to which the water is pumped, for this windmill.

Solution: Assume the efficiency trends shown in Fig. 10.45.

Computing equations: $C_P = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 \pi R^2}$ $\qquad X = \omega R / V$

From Fig. 10.45, $C_{P\,max} \approx 0.3$ at $X = 0.8$. $V = 15$ mph (22.0 ft/s). Then the power developed is

$$\mathcal{P} = \frac{\pi}{2} \times 0.3 \times 0.00238 \frac{slug}{ft^3} \times (22.0)^3 \frac{ft^3}{s^3} \times \left(\frac{7}{2}\right)^2 ft^2 \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\, ft \cdot lbf} = 0.266\ hp$$

Converting this mechanical power to pumping gives hydraulic power as

$$\mathcal{P}_h = \rho Q g h = \eta \mathcal{P}_m$$

Thus $Qh = \dfrac{\eta \mathcal{P}_m}{\rho g} = 0.7 \times 0.266\ hp \times \dfrac{ft^3}{62.4\ lbf} \times 550 \dfrac{ft \cdot lbf}{hp \cdot s} \times 7.48 \dfrac{gal}{ft^3} \times 60 \dfrac{s}{min}$

$$Qh = 737\ gpm \cdot ft$$

Qh

Q varies inversely with the distance lifted, h. The volume flow rate actually delivered would be less, due to suction lift, pipe friction, and minor losses.

Problem 10.114

Given: Largest known Darrieus vertical-axis wind turbine built by DoE near Sandia, New Mexico, is 60 ft tall and 30 ft diameter; the rotor swept area is $A \approx 1200\ \text{ft}^2$.

Find: Estimate the maximum power this windmill can produce in a wind with $V = 20$ mph (29.3 ft/s).

Solution: Assume the efficiency trends shown in Fig. 10.45.

Computing equations: $C_P = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 \pi R^2}$ $\quad X = \omega R/V$

From Fig. 10.45, $C_{P\max} \approx 0.34$ at $X = 5.3$. Use swept area in place of πR^2:

$$\mathcal{P} = \frac{1}{2} \times 0.34 \times 0.00238\ \frac{\text{slug}}{\text{ft}^3} \times (29.3)^3 \frac{\text{ft}^3}{\text{s}^3} \times 1200\ \text{ft}^2 \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{hp}\cdot\text{s}}{550\ \text{ft}\cdot\text{lbf}} = 22.2\ \text{hp}$$

$\mathcal{P}$

To generate maximum power, the windmill must rotate at

$$\omega = \frac{XV}{R} = 5.3 \times 29.3\ \frac{\text{ft}}{\text{s}} \times \frac{1}{15\ \text{ft}} = 10.4\ \text{rad/s} \quad (98.9\ \text{rpm})$$

ω

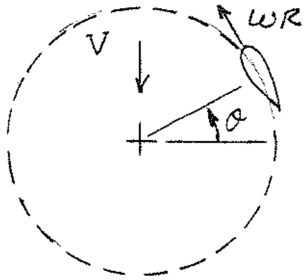
Problem 10.115

Given: Section lift and drag coefficient data for a NACA 0012 section, tested at $Re \approx 6 \times 10^6$ with standard roughness:

Angle of attack, α (deg)	0	2	4	6	8	10	12
Lift coefficient, C_L (−)	0	0.23	0.45	0.68	0.82	0.94	1.02
Drag coefficient, C_D (−)	0.0098	0.0100	0.0119	0.0147	0.0194	–	–

Find: (a) Analyze air flow relative to a blade element in a Darrieus rotor.
(b) Develop a numerical model for a blade element.
(c) Calculate the power coefficient as a function of tip speed ratio.
(d) Compare with the trend shown in Fig. 10.45.

Solution: Consider plan view of rotor element, absolute velocities:



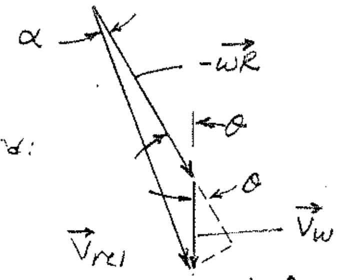
Computing equations: $F_L = C_L \frac{1}{2} \rho V_r^2 A_p$, V_r = relative velocity

$F_D = C_D \frac{1}{2} \rho V_r^2 A_p$ A_p = planform, A_s = swept

$C_P = \dfrac{P}{\frac{1}{2} \rho V^3 A_s}$ V = wind velocity

Resolve to relative velocity, for position shown:

$$\vec{V}_{abs} = \vec{V}_{blade} + \vec{V}_{rel} \; ; \; \vec{V}_{rel} = \vec{V}_{abs} - \vec{V}_{blade}$$



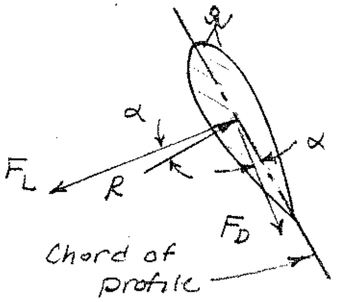
To compute V_{rel}, resolve into components along (a) and transverse (t) to the airfoil chord:

$$V_{rel})_a = \omega R + V_w \cos\theta$$

$$V_{rel})_t = V_w \sin\theta$$

$$V_{rel} = \left[V_{rel})_a^2 + V_{rel})_t^2 \right]^{\frac{1}{2}}$$

$$\alpha = \tan^{-1}\left[V_{rel})_t / V_{rel})_a \right]$$



Lift force (F_L) is normal to $\vec{V}_{rel}$ and drag force (F_D) is parallel to $\vec{V}_{rel}$. Thus

$$T = R(F_L \sin\alpha - F_D \cos\alpha) \quad (\text{torque, } T > 0 \text{ when } F_L/F_D > \cot\alpha)$$

Both C_L and C_D must be modeled as functions of angle of attack, α. From a graph of C_L and C_D versus α, a satisfactory representation is

$$C_L = 0.12\alpha - 0.0026|\alpha|\alpha, \quad -12 < \alpha < 12 \text{ degrees}; \; C_L = 0, \; |\alpha| > 12 \text{ degrees}$$

$$C_D = 0.00952 + 1.52 \times 10^{-4}\alpha^2; \quad -12 < \alpha < 12 \text{ degrees}; \; C_D = 0.0314, \; |\alpha| > 12 \text{ degrees}$$

(The models in the stalled region, $|\alpha| > 12$ degrees, obviously are crude.)

Problem 10.115

Sample calculation: Choose $R = 10\,ft$, $c = 0.5\,ft$, $w = 1\,ft$, $X = 5$, $V_w = 20\,mph$

At $\theta = 30°$, with $V_w = 20\,mph\,(29.3\,ft/s)$

$$X = \omega R/V_w\;;\quad \omega = \frac{X V_w}{R} = 5 \times 29.3\,\frac{ft}{s} \times \frac{1}{10\,ft} = 14.7\,rad/s \quad (N = 140\,rpm)$$

$$\omega R = 14.7\,\frac{rad}{s} \times 10\,ft = 147\,ft/s$$

$$V_{rel})_a = \omega R + V_w \cos\theta = 147 + 29.3 \cos 30° = 172\,ft/s$$

$$V_{rel})_t = V_w \sin\theta = 29.3 \sin\theta = 14.7\,ft/s$$

$$V_{rel} = \left[V_{rel})_a^2 + V_{rel})_t^2\right]^{\frac{1}{2}} = \left[(172)^2 + (14.7)^2\right]^{\frac{1}{2}} = 173\,ft/s$$

$$\alpha = \tan^{-1}\left[V_{rel})_t / V_{rel})_a\right] = \tan^{-1}(14.7/172) = 4.88\ \text{degrees}$$

$$q = \frac{1}{2}\rho V_{rel}^2 = \frac{1}{2} \times 0.00238\,\frac{slug}{ft^3} \times (173)^2\,\frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} = 35.6\,lbf/ft^2$$

A_p (projected area of airfoil section) $= cw = 0.5\,ft \times 1\,ft = 0.5\,ft^2$

$$C_L = 0.12\alpha - 0.0026|\alpha|\alpha = 0.12 \times 4.88 - 0.0026|4.88|4.88 = 0.524$$

$$C_D = 0.00952 + 1.52 \times 10^{-4}\alpha^2 = 0.00952 + 1.52 \times 10^{-4}(4.88)^2 = 0.0131$$

$$\left.\begin{aligned} F_L &= C_L q A_p = 0.524 \times 35.6\,\frac{lbf}{ft^2} \times 0.5\,ft^2 = 9.33\,lbf \\ F_D &= C_D q A_p = 0.0131 \times 35.6\,\frac{lbf}{ft^2} \times 0.5\,ft^2 = 0.233\,lbf \end{aligned}\right\} F_L/F_D = 40.0$$

$$T = R(F_L \sin\alpha - F_D \cos\alpha) = 10\,ft\,(9.33 \sin(4.88°) - 0.233 \cos(4.88°))\,lbf = 5.62\,ft \cdot lbf$$

$$\mathcal{P} = \omega T = 14.7\,\frac{rad}{s} \times 5.62\,ft \cdot lbf = 82.6\,\frac{ft \cdot lbf}{s}\ (0.150\,hp)$$

$$C_p = \frac{\mathcal{P}}{\frac{1}{2}\rho V_w^3 A_s}\;;\ A_s = \text{area swept by element} = 2Rw = 2 \times 10\,ft \times 1\,ft = 20\,ft^2$$

$$C_p = 82.6\,\frac{ft \cdot lbf}{s} \times \frac{ft^3}{(\frac{1}{2})\,0.00238\,slug} \times \frac{s^3}{(29.3)^3\,ft^3} \times \frac{1}{20\,ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 0.138\ (\text{at } \theta = 15°)$$

Obtain $\bar{C}_p$ for a complete rotor revolution by integrating numerically. Such results are presented on the next page, and plotted versus tip speed ratio, $X = \omega R/V_w$.

From the plot, $\bar{C}_p$ is small at low X. It increases as X is raised, then peaks and decreases again. Comparison with Fig. 10.45 shows the trends are similar, but the model predicts useful power at larger X than observed experimentally. Blade elements at smaller radii on the rotor would produce less power, since ω = constant along rotor. $\bar{C}_p$ at large X is also sensitive to C_D.

Low $\bar{C}_p$ at small X occurs because the airfoil is stalled.

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42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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Problem 10.115

Computed results:

```
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*                                                      *


Airfoil:       NACA 0012 Section; Chord, c = 6 in.

Blade element: Span, w = 1 ft; Radius, R = 10 ft

Input data:    Tip speed ratio, X =  5.0 (---)
               Wind speed, Vw = 20 mph (29.3 ft/sec)

Calculated:    Rotor speed, omega = 140.4 rpm

----------------------------------------------------------------------
theta  Vrel   alpha    Cl      Cd       Fl      Fd       T       Cp
(deg) (ft/s)  (deg)   (--)    (--)    (lbf)   (lbf)  (ft-lbf)  (--)
----- ------ ------- ------ ------- ------- ------- ------- -------
  0    176     0.00   0.00   0.010     0.0   0.176    -1.8  -0.043
 30    173     4.87   0.52   0.013     9.3   0.233     5.6   0.136
 60    164     8.95   0.87   0.022    13.8   0.343    18.1   0.439
 90    150    11.31   1.02   0.029    13.7   0.384    23.1   0.562
120    135    10.89   1.00   0.027    10.8   0.295    17.5   0.425
150    122     6.90   0.70   0.017     6.3   0.148     6.1   0.147
180    118    -0.00  -0.00   0.010    -0.0   0.078    -0.8  -0.019
210    122    -6.90  -0.70   0.017    -6.3   0.148     6.1   0.147
240    135   -10.89  -1.00   0.027   -10.8   0.295    17.5   0.425
270    150   -11.31  -1.02   0.029   -13.7   0.384    23.1   0.562
300    164    -8.95  -0.87   0.022   -13.8   0.343    18.1   0.439
330    173    -4.87  -0.52   0.013    -9.3   0.233     5.6   0.136
360    176     0.00   0.00   0.010     0.0   0.176    -1.8  -0.043
----------------------------------------------------------------------

 Average power coefficient for complete revolution: Cp,bar =  0.280
```
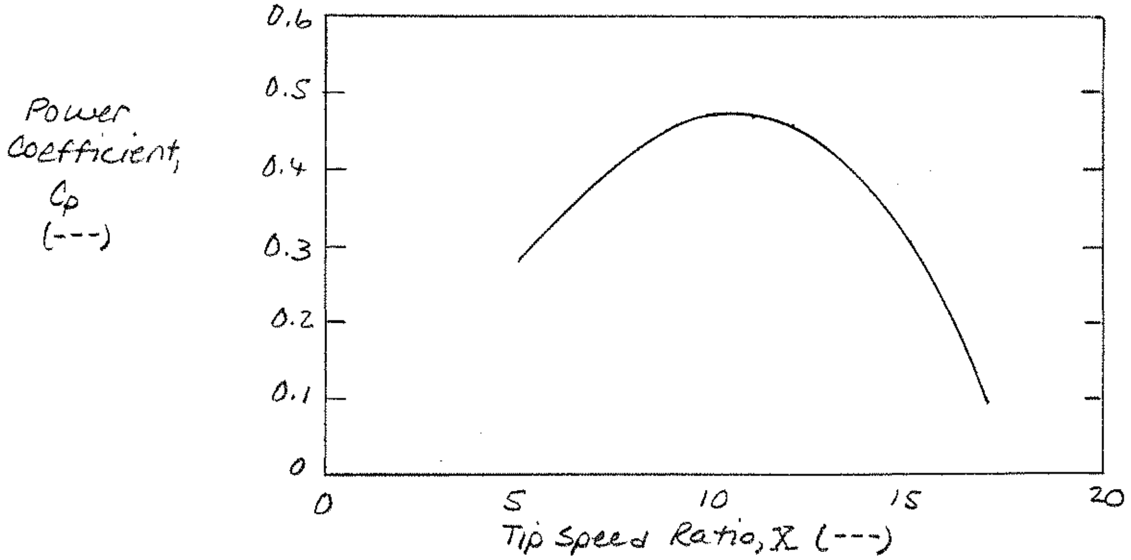

Plotting results of similar calculations at various tip speed ratios give:



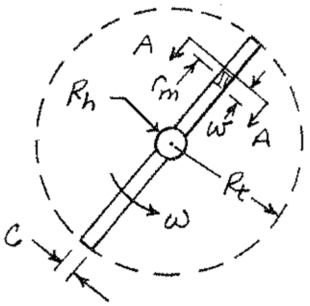
Problem 10.116

Given: Lift and drag data for NACA 23015 airfoil section, Fig. 9.17.

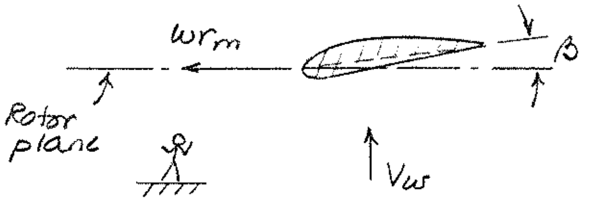
Consider two-blade, horizontal-axis wind turbine with this section.

Find: (a) Analyze air flow relative to a blade element in rotating turbine.
(b) Develop numerical model for blade element.
(c) Calculate power coefficient as a function of tip speed ratio.
(d) Compare with the trend shown in Fig. 10.45.

Solution: Front view of rotor; blade element shown cross-hatched:

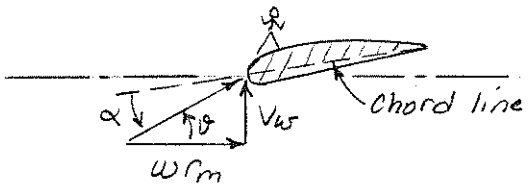


Section A-A:
(absolute velocities)

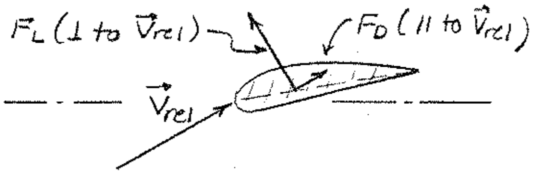


β = blade twist angle

(relative velocities)



α = angle of attack



Resolve to relative velocity:

$$\vec{V}_{abs} = \vec{V}_{blade\ element} + \vec{V}_{rel}$$

$$\vec{V}_{rel} = \vec{V}_w - \vec{V}_{blade} = \vec{V}_w + (-\vec{V}_{blade})$$

Computing equations:

$F_L = C_L \frac{1}{2}\rho V_r^2 A_p$; V_r = relative velocity

$F_D = C_D \frac{1}{2}\rho V_r^2 A_p$; A_p = planform area

$T = n_b r_m (F_L \sin\theta - F_D \cos\theta)$; n_b = number of blades (2)

$V_{rel} = \left[(\omega r_m)^2 + V_w^2\right]^{\frac{1}{2}}$; $\theta = \tan^{-1}(V_w/\omega r_m)$; $\alpha = \theta - \beta$

{Note: $\beta > 0$ is required for windmill to self start.}

Both C_L and C_D must be modeled as functions of angle of attack, α. From Fig. 9.17, satisfactory representations are:

$0 \le \alpha < 12°$ $C_L = 0.12 + 0.107\alpha$ $0 \le \alpha < 4°$ $C_D = 0.0065 + 4.55\times10^{-5}\alpha$

$12° \le \alpha < 18°$ $C_L = 0.12 + 0.107\alpha - 0.00852(\alpha - 12)^2$ $4° \le \alpha < 16°$ C_D = above $+ 7.72\times10^{-5}(\alpha - 4)^2$

$18° \le \alpha$ $C_L = 0.2$ $16° \le \alpha$ $C_D = 0.02$

{Obviously both models are crude for $C_L(\alpha > 18°)$, $C_D(\alpha > 16°)$.} Choose $R_t = 10$ ft, $r_m = 5.5$ ft, $c = 6$ in., $w = 1$ ft, $V_w = 20$ mph (29.4 ft/s), $X = 5$, and $\beta = 5°$. Then

$$\omega = \frac{X V_w}{R_t} = 5.0 \times 29.4 \frac{ft}{s} \times \frac{1}{10\ ft} = 14.7\ rad/s\ \ (140\ rpm);\ \omega r_m = 80.9\ ft/s$$

Problem 10.116

[5] Part 2/3

$V_{rel} = [(80.9)^2 + (29.4)^2]^{\frac{1}{2}} = 86.1 \text{ ft/s}$; $q = \frac{1}{2}\rho V_{rel}^2 = 8.82 \text{ lbf/ft}^2$; $A_p = wc = 0.5 \text{ ft}^2$

$\theta = \tan^{-1}(29.4/80.9) = 20.0°$; $\alpha = \theta - \beta = 20.0 - 5.0 = 15.0°$; $C_L = 1.65$; $C_D = 0.017$

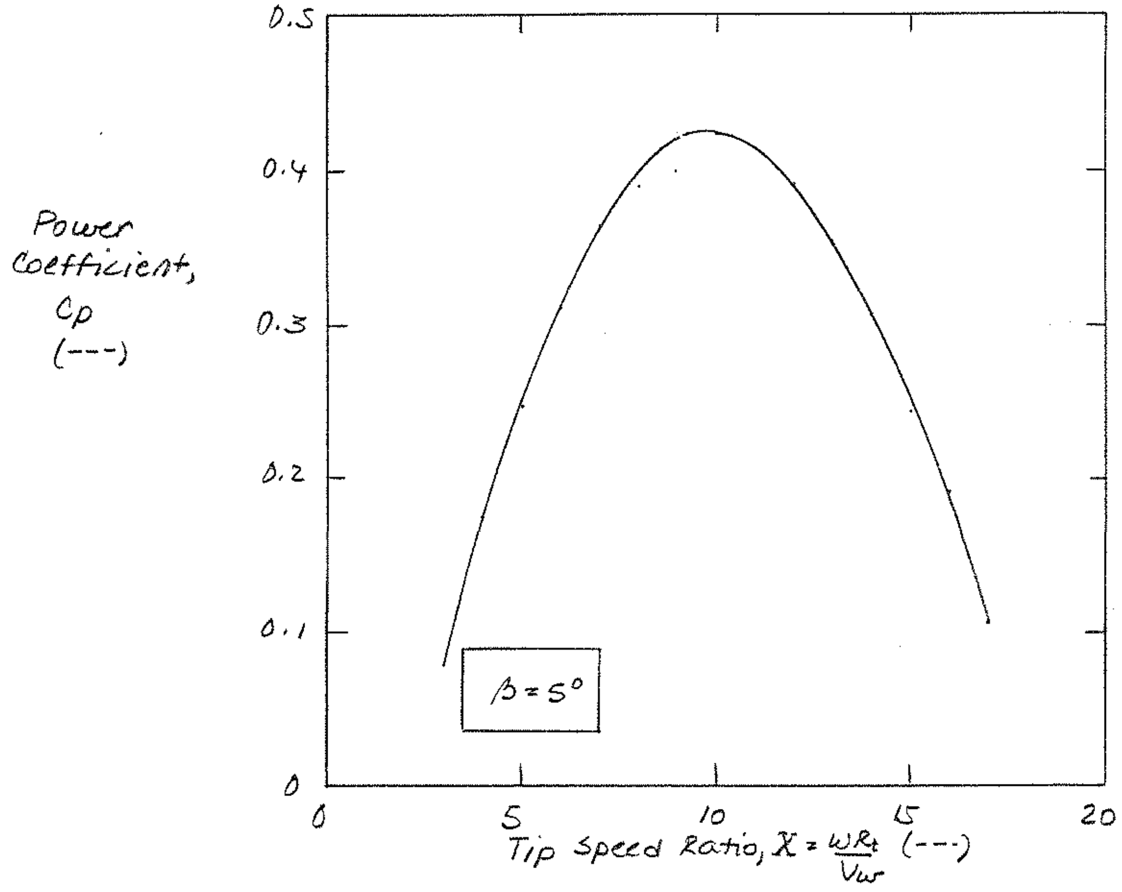
$F_L = 1.65 \times 8.82 \frac{\text{lbf}}{\text{ft}^2} \times 0.5 \text{ ft}^2 = 7.28 \text{ lbf}$; $F_D = 0.017 \times 8.82 \frac{\text{lbf}}{\text{ft}^2} \times 0.5 \text{ ft}^2 = 0.075 \text{ lbf}$

$T = 2 \times 5.5 \text{ ft} (7.28 \times \sin 20° - 0.075 \cos 20°) \text{ lbf} = 26.6 \text{ ft} \cdot \text{lbf}$

Similar calculations for the other blade elements show that torque for the complete propeller is $T_p = 159 \text{ ft} \cdot \text{lbf}$. The power coefficient is

$$C_p = \frac{\mathcal{P}}{\frac{1}{2}\rho V_w^3 \pi R_t^2} = \frac{\omega T_p}{\frac{1}{2}\rho V_w^3 \pi R_t^2} = \frac{2}{\pi} \times 14.7 \frac{\text{rad}}{\text{s}} \times 159 \text{ ft} \cdot \text{lbf} \times \frac{\text{ft}^3}{0.00238 \text{ slug}} \times \frac{\text{s}^3}{(29.4)^3 \text{ ft}^3} \times \frac{1}{(10)^2 \text{ ft}^2} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} = 0.246$$

Calculated results are tabulated on the next page, plotted and discussed below:



Trends shown are similar to Fig. 10.45. At small X, the blade is entirely stalled so useful output is low. At large X, α becomes negative near the tips, reducing output.

This model does not include: (1) axial interference that reduces normal velocity below V_w as loading increases, or (2) swirl introduce by blade drag. Both these effects reduce performance. For more details, see Division L, Section XI of [30].

Computed results:

```
*****************************************************
*                                                   *



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*    West Lafayette, Indiana 47907                  *



Airfoil:       NACA 23015 Section; Chord, c = 6 in.
               Tip radius, Rt = 10 ft
               Twist angle, beta =  5 degrees

Blade element: Delta r, dr = 1.00 ft

Input data:    Tip speed ratio, X =  5.0 (---)
               Wind speed, Vw = 20 mph (29.4 ft/s)

Calculated:    Rotor speed, omega = 140.4 rpm

------------------------------------------------------------
  Rm    Vrel   alpha     Cl      Cd       Fl       Fd      T
 (ft)  (ft/s)  (deg)    (--)    (--)     (lbf)    (lbf) (ft-lbf)
------ ------ ------- ------ ------- ------- ------- -------
 1.50    22    48.13   0.20   0.020    0.06   0.006   0.13
 2.50    37    33.66   0.20   0.020    0.16   0.016   0.44
 3.50    51    24.74   0.20   0.020    0.32   0.032   0.90
 4.50    66    18.96   0.20   0.020    0.52   0.052   1.48
 5.50    81    14.98   1.65   0.017    6.41   0.067  23.39
 6.50    96    12.10   1.41   0.013    7.69   0.069  28.53
 7.50   110     9.93   1.18   0.010    8.55   0.074  31.99
 8.50   125     8.24   1.00   0.009    9.31   0.081  34.90
 9.50   140     6.89   0.86   0.008    9.95   0.091  37.25
------------------------------------------------------------

     Torque for complete propeller:  T  = 159.0 ft-lbf

     Power coefficient for windmill: Cp =  0.246 (---)
```

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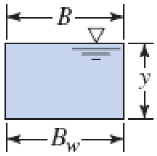
Problem 11.1

11.1 A 2-m-wide rectangular channel with a bed slope of 0.0005 has a depth of flow of 1.5 m. Manning's roughness coefficient is 0.015. Determine the steady uniform discharge in the channel.



Given: Rectangular channel flow

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 2\cdot m$ and depth $y = 1.5\cdot m$ we find from Table 11.2

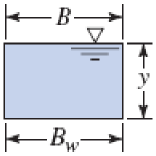
$A = B_w\cdot y$ $\qquad A = 3.00\cdot m^2$ $\qquad R = \frac{B_w\cdot y}{B_w + 2\cdot y}$ $\qquad R = 0.600\cdot m$

Manning's roughness coefficient is $n = 0.015$ and $S_0 = 0.0005$

$Q = \frac{1.49}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$ $\qquad Q = 3.18\cdot\frac{m^3}{s}$

Problem 11.2

11.2 Determine the uniform flow depth in a rectangular channel 2.5 m wide with a discharge of 3 m^3/s. The slope is 0.0004 and Manning's roughness factor is 0.015.



Given: Data on rectangular channel

Find: Depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 2.5 \cdot m$ and flow rate $Q = 3 \cdot \frac{m^3}{s}$ we find from Table 11.2 $A = B_w \cdot y$ $R = \frac{B_w \cdot y}{B_w + 2 \cdot y}$

Manning's roughness coefficient is $n = 0.015$ and $S_0 = 0.0004$

Hence the basic equation becomes $Q = \frac{1}{n} \cdot B_w \cdot y \cdot \left(\frac{B_w \cdot y}{B_w + 2 \cdot y}\right)^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Solving for y $y \cdot \left(\frac{B_w \cdot y}{B_w + 2 \cdot y}\right)^{\frac{2}{3}} = \frac{Q \cdot n}{B_w \cdot S_0^{\frac{1}{2}}}$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below, to make the left side evaluate to $\frac{Q \cdot n}{B_w \cdot S_0^{\frac{1}{2}}} = 0.900$.

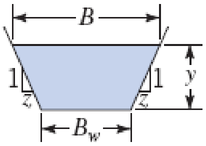
For $y = 1$ (m) $y \cdot \left(\frac{B_w \cdot y}{B_w + 2 \cdot y}\right)^{\frac{2}{3}} = 0.676$ For $y = 1.2$ (m) $y \cdot \left(\frac{B_w \cdot y}{B_w + 2 \cdot y}\right)^{\frac{2}{3}} = 0.865$

For $y = 1.23$ (m) $y \cdot \left(\frac{B_w \cdot y}{B_w + 2 \cdot y}\right)^{\frac{2}{3}} = 0.894$ For $y = 1.24$ (m) $y \cdot \left(\frac{B_w \cdot y}{B_w + 2 \cdot y}\right)^{\frac{2}{3}} = 0.904$

The solution to three figures is $y = 1.24$ (m)

Problem 11.3

11.3 Determine the uniform flow depth in a trapezoidal channel with a bottom width of 8 ft and side slopes of 1 vertical to 2 horizontal. The discharge is 100 ft^3/s. Manning's roughness factor is 0.015 and the channel bottom slope is 0.0004.



Given: Data on trapzoidal channel

Find: Depth of flow

Solution:

Basic equation:
$$Q = \frac{1.49}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 8\cdot ft$ $\quad z = 2$ $\quad Q = 100\cdot\frac{ft^3}{s}$ $\quad S_0 = 0.0004$

$n = 0.015$

Hence from Table 11.2 $\quad A = \left(B_w + z\cdot y\right)\cdot y = (8 + 2\cdot y)\cdot y \quad R = \frac{\left(B_w + z\cdot y\right)\cdot y}{B_w + 2\cdot y\cdot\sqrt{1+z^2}} = \frac{(8+2\cdot y)\cdot y}{8 + 2\cdot y\cdot\sqrt{5}}$

Hence
$$Q = \frac{1.49}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1.49}{0.015}\cdot(8+2\cdot y)\cdot y\cdot\left[\frac{(8+2\cdot y)\cdot y}{8+2\cdot y\cdot\sqrt{5}}\right]^{\frac{2}{3}}\cdot 0.0004^{\frac{1}{2}} = 100$$
(Note that we don't use units!)

Solving for y
$$\frac{[(8+2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(8+2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 50.3$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 2$ (ft) $\quad \frac{[(8+2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(8+2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 30.27$ $\qquad$ For $y = 3$ (ft) $\quad \frac{[(8+2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(8+2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 65.8$

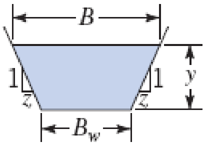
For $y = 2.6$ (ft) $\quad \frac{[(8+2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(8+2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 49.81$ $\qquad$ For $y = 2.61$ (ft) $\quad \frac{[(8+2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(8+2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 50.18$

The solution to three figures is $\quad y = 2.61$ (ft)

Problem 11.4

[3]

11.4 Determine the uniform flow depth in a trapezoidal channel with a bottom width of 2.5 m and side slopes of 1 vertical to 2 horizontal with a discharge of 3 m^3/s. The slope is 0.0004 and Manning's roughness factor is 0.015.



Given: Data on trapezoidal channel

Find: Depth of flow

Solution:

Basic equation: $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.5\cdot m$ $\quad z = 2 \quad Q = 3\cdot\frac{m^3}{s} \quad S_0 = 0.0004$

$n = 0.015$

Hence from Table 11.2 $$A = \left(B_w + z\cdot y\right)\cdot y = (8 + 2\cdot y)\cdot y \qquad R = \frac{\left(B_w + z\cdot y\right)\cdot y}{B_w + 2\cdot y\cdot\sqrt{1+z^2}} = \frac{(2.5 + 2\cdot y)\cdot y}{2.5 + 2\cdot y\cdot\sqrt{5}}$$

Hence $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1}{0.015}\cdot(2.5 + 2\cdot y)\cdot y\cdot\left[\frac{(2.5+2\cdot y)\cdot y}{2.5 + 2\cdot y\cdot\sqrt{5}}\right]^{\frac{2}{3}}\cdot 0.0004^{\frac{1}{2}} = 3$$ (Note that we don't use units!)

Solving for y $$\frac{[(2.5 + 2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(2.5 + 2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 2.25$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 1$ (m) $$\frac{[(2.5 + 2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(2.5 + 2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 3.36$$

For $y = 0.8$ (m) $$\frac{[(2.5 + 2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(2.5 + 2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 2.17$$

For $y = 0.81$ (m) $$\frac{[(2.5 + 2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(2.5 + 2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 2.23$$

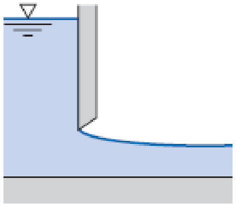
For $y = 0.815$ (m) $$\frac{[(2.5 + 2\cdot y)\cdot y]^{\frac{5}{3}}}{\left(2.5 + 2\cdot y\cdot\sqrt{5}\right)^{\frac{2}{3}}} = 2.25$$

The solution to three figures is $y = 0.815$ (m)

Problem 11.5

[2]

11.5 A partially open sluice gate in a 3-m-wide rectangular channel carries water at 8.5 m^3/sec. The upstream depth is 2 m. Find the downstream depth and Froude number.



Given: Data on sluice gate

Find: Downstream depth; Froude number

Solution:

Basic equation: $\frac{p_1}{\rho \cdot g} + \frac{V_1^{\,2}}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^{\,2}}{2 \cdot g} + y_2 + h$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$\frac{V_1^{\,2}}{2 \cdot g} + y_1 = \frac{V_2^{\,2}}{2 \cdot g} + y_2$$

The given data is $b = 3 \cdot m$ $\quad y_1 = 2 \cdot m$ $\quad Q = 8.5 \cdot \frac{m^3}{s}$

For mass flow $Q = V \cdot A$ so $V_1 = \frac{Q}{b \cdot y_1}$ and $V_2 = \frac{Q}{b \cdot y_2}$

Using these in the Bernoulli equation $\frac{\left(\frac{Q}{b \cdot y_1}\right)^2}{2 \cdot g} + y_1 = \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2$ (1)

The only unknown on the right is y_2. The left side evaluates to $\frac{\left(\frac{Q}{b \cdot y_1}\right)^2}{2 \cdot g} + y_1 = 2.10 \, m$

To find y_2 we need to solve the non-linear equation. We must do this numerically; we may use the Newton method or similar, or *Excel*'s *Solver* or *Goal Seek*. Here we interate manually, starting with an arbitrary value less than y_1.

For $y_2 = 0.5 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.14 \, m$ $\quad$ For $y_2 = 0.51 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.08 \, m$

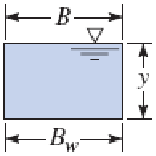
For $y_2 = 0.505 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.11 \, m$ $\quad$ For $y_2 = 0.507 \cdot m$ $\quad \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2 = 2.10 \, m$

Hence $y_2 = 0.507 \, m$

Then $V_2 = \frac{Q}{b \cdot y_2}$ $\quad V_2 = 5.59 \frac{m}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $\quad Fr_2 = 2.51$

Problem 11.6

11.6 A rectangular flume built of concrete, with 1 ft per 1000 ft slope, is 6 ft wide. Water flows at a normal depth of 3 ft. Compute the discharge.



Given: Data on flume

Find: Discharge

Solution:

Basic equation: $Q = \frac{1.49}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 6\cdot ft$ and depth $y = 3\cdot ft$ we find from Table 11.2

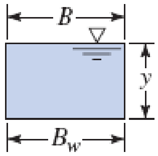
$A = B_w\cdot y$ $\quad A = 18\cdot ft^2$ $\quad R = \frac{B_w\cdot y}{B_w + 2\cdot y}$ $\quad R = 1.50\cdot ft$

For concrete (Table 11.1) $\quad n = 0.013$ and $S_0 = \frac{1\cdot ft}{1000\cdot ft}$ $\quad S_0 = 0.001$

$Q = \frac{1.49}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$ $\quad Q = 85.5\cdot \frac{ft^3}{s}$

Problem 11.7

11.7 A rectangular flume built of timber is 3 ft wide. The flume is to handle a flow of 90 ft^3/sec at a normal depth of 6 ft. Determine the slope required.



Given: Data on flume

Find: Slope

Solution:

Basic equation: $$Q = \frac{1.49}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 3\cdot\mathrm{ft}$ and depth $y = 6\cdot\mathrm{ft}$ we find

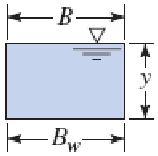
$$A = B_w\cdot y \qquad A = 18\cdot\mathrm{ft}^2 \qquad R = \frac{B_w\cdot y}{B_w + 2\cdot y} \qquad R = 1.20\cdot\mathrm{ft}$$

For wood (not in Table 11.1) a Google search finds n = 0.012 to 0.017; we use $n = 0.0145$ with $Q = 90\cdot\frac{\mathrm{ft}^3}{\mathrm{s}}$

$$S_0 = \left(\frac{n\cdot Q}{1.49\cdot A\cdot R^{\frac{2}{3}}}\right)^2 \qquad S_0 = 1.86\times 10^{-3}$$

Problem 11.8

11.8 A channel with square cross section is to carry 20 m^3/sec of water at normal depth on a slope of 0.003. Compare the dimensions of the channel required for (a) concrete and (b) soil cement.



Given: Data on square channel

Find: Dimensions for concrete and soil cement

Solution:

Basic equation: $Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a square channel of width B_w we find $A = B_w^{\,2}$ $\quad R = \frac{B_w\cdot y}{B_w + 2\cdot y} = \frac{B_w^{\,2}}{B_w + 2\cdot B_w} = \frac{B_w}{3}$

Hence $Q = \frac{1}{n}\cdot B_w^{\,2}\cdot\left(\frac{B_w}{3}\right)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{S_0^{\frac{1}{2}}}{n\cdot 3^{\frac{2}{3}}}\cdot B_w^{\frac{8}{3}}$ or $B_w = \left(\frac{3^{\frac{2}{3}}\cdot Q}{S_0^{\frac{1}{2}}}\cdot n\right)^{\frac{3}{8}}$

The given data is $Q = 20\cdot\frac{m^3}{s}$ $\quad S_0 = 0.003$

For concrete, from Table 11.1 (assuming large depth) $\quad n = .013$

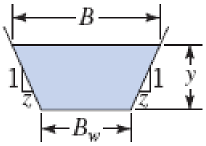
$B_w = 2.36\,m$

For soil cement from Table 11.1 (assuming large depth) $\quad n = .020$

$B_w = 2.77\,m$

Problem 11.9

11.9 Water flows in a trapezoidal channel at a normal depth of 1.2 m. The bottom width is 2.4 m and the sides slope at 1:1 (45°). The flow rate is 7.1 m^3/sec. The channel is excavated from bare soil. Find the bed slope.



Given: Data on trapezoidal channel

Find: Bed slope

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.4 \cdot m$ $\quad z = 1$ $\quad y = 1.2 \cdot m$ $\quad Q = 7.1 \cdot \frac{m^3}{s}$

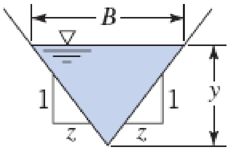
For bare soil (Table 11.1) $n = 0.020$

Hence from Table 11.2 $A = \left(B_w + z \cdot y\right) \cdot y$ $\quad A = 4.32\,m^2$ $\quad R = \frac{\left(B_w + z \cdot y\right) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$ $\quad R = 0.746\,m$

Hence $S_0 = \left(\frac{Q \cdot n}{A \cdot R^{\frac{2}{3}}}\right)^2$ $\quad S_0 = 1.60 \times 10^{-3}$

Problem 11.10

11.10 A triangular channel with side angles of 45° is to carry 10 m^3/sec at a slope of 0.001. The channel is concrete. Find the required dimensions.



Given: Data on triangular channel

Find: Required dimensions

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the triangular channel we have $z = 1$ $\quad S_0 = 0.001$ $\quad Q = 10 \cdot \frac{m^3}{s}$

For concrete (Table 11.1) $n = 0.013$ (assuming y > 60 cm: verify later)

Hence from Table 11.2 $A = z \cdot y^2 = y^2$ $\quad R = \frac{z \cdot y}{2 \cdot \sqrt{1 + z^2}} = \frac{y}{2 \cdot \sqrt{2}}$

Hence

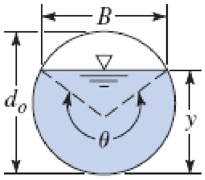
$$Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{n} \cdot y^2 \cdot \left(\frac{y}{2 \cdot \sqrt{2}}\right)^{\frac{2}{3}} \cdot S_0 = \frac{1}{n} \cdot y^{\frac{8}{3}} \cdot \left(\frac{1}{8}\right)^{\frac{1}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{2 \cdot n} \cdot y^{\frac{8}{3}} \cdot S_0^{\frac{1}{2}}$$

Solving for y $\quad y = \left(\frac{2 \cdot n \cdot Q}{\sqrt{S_0}}\right)^{\frac{3}{8}}$ $\quad y = 2.20\,m$ (The assumption that y > 60 cm is verified)

Problem 11.11

[2]

11.11 A semicircular trough of corrugated steel, with diameter $D = 1$ m, carries water at depth $y_n = 0.25$ m. The slope is 0.01. Find the discharge.



Given: Data on semicircular trough

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $d_0 = 1 \cdot m$ $\quad y = 0.25 \cdot m$ $\quad S_0 = 0.01$

Hence, from geometry $\theta = 2 \cdot \text{asin}\left(\frac{y - \frac{d_0}{2}}{\frac{d_0}{2}}\right) + 180 \cdot deg$ $\quad \theta = 120 \cdot deg$

For corrugated steel, a Google search leads to $n = 0.022$

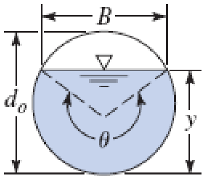
Hence from Table 11.2 $A = \frac{1}{8} \cdot (\theta - \sin(\theta)) \cdot d_0^{\;2}$ $\quad A = 0.154\, m^2$

$R = \frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right) \cdot d_0$ $\quad R = 0.147\, m$

Then the discharge is $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} \cdot \frac{m^3}{s}$ $\quad Q = 0.194 \frac{m^3}{s}$

Problem 11.12

11.12 Find the discharge at which the channel of Problem 11.11 flows full.



Given:

Data on semicircular trough

Find:

Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $d_0 = 1 \cdot m$ $\theta = 180 \cdot deg$ $S_0 = 0.01$

For corrugated steel, a Google search leads to (Table 11.1) $n = 0.022$

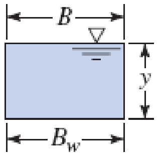
Hence from Table 11.2 $A = \frac{1}{8} \cdot (\theta - \sin(\theta)) \cdot d_0^2$ $A = 0.393\,m^2$

$R = \frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right) \cdot d_0$ $R = 0.25\,m$

Then the discharge is $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} \cdot \frac{m^3}{s}$ $Q = 0.708\,\frac{m^3}{s}$

Problem 11.13

11.13 The flume of Problem 11.6 is fitted with a new plastic film liner ($n = 0.010$). Find the new depth of flow if the discharge remains constant at 85.5 $\mathrm{ft^3/sec}$.



Given: Data on flume with plastic liner

Find: Depth of flow

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 6 \cdot ft$ and depth y we find from Table 11.2

$$A = B_w \cdot y = 6 \cdot y \qquad R = \frac{B_w \cdot y}{B_w + 2 \cdot y} = \frac{6 \cdot y}{6 + 2 \cdot y}$$

and also $n = 0.010$ and $S_0 = \frac{1 \cdot ft}{1000 \cdot ft}$ $S_0 = 0.001$

Hence $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1.49}{0.010} \cdot 6 \cdot y \cdot \left(\frac{6 \cdot y}{6 + 2 \cdot y}\right)^{\frac{2}{3}} \cdot 0.001^{\frac{1}{2}} = 85.5$ (Note that we don't use units!)

Solving for y $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = \frac{85.5 \cdot 0.010}{1.49 \cdot .001^{\frac{1}{2}} \cdot 6 \cdot 6^{\frac{2}{3}}}$ or $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.916$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with Problem 11.6's depth

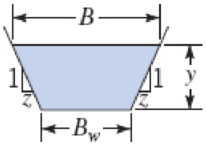
For $y = 3$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 1.191$ For $y = 2$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.684$

For $y = 2.5$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.931$ For $y = 2.45$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.906$

For $y = 2.47$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.916$ $y = 2.47$ (feet)

Problem 11.14

11.14 Discharge through the channel of Problem 11.9 is increased to 15 m^3/sec. Find the corresponding normal depth, if the bed slope is 0.00193.



Given: Data on trapzoidal channel

Find: New depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.4 \cdot m$ $z = 1$ $Q = 15 \cdot \frac{m^3}{s}$ $S_0 = 0.00193$

For bare soil (Table 11.1) $n = 0.020$

Hence from Table 11.2 $A = (B_w + z \cdot y) \cdot y = (2.4 + y) \cdot y$ $R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(2.4 + y) \cdot y}{2.4 + 2 \cdot y \cdot \sqrt{2}}$

Hence $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.020} \cdot (2.4 + y) \cdot y \cdot \left[\frac{(2.4 + y) \cdot y}{2.4 + 2 \cdot y \cdot \sqrt{2}}\right]^{\frac{2}{3}} \cdot 0.00193^{\frac{1}{2}} = 15$ (Note that we don't use units!)

Solving for y $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 6.83$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a larger depth than Problem 11.9's.

For $y = 1.5$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 5.37$ For $y = 1.75$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 7.2$

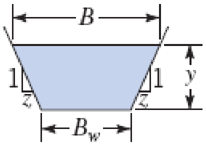
For $y = 1.71$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 6.89$ For $y = 1.70$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 6.82$

The solution to three figures is $y = 1.70$ (m)

Problem 11.15

[3]

11.15 The channel of Problem 11.9 has 0.00193 bed slope. Find the normal depth for the given discharge after a new plastic liner ($n = 0.010$) is installed.



Given: Data on trapzoidal channel

Find: New depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.4 \cdot m$ $\quad z = 1$ $\quad Q = 7.1 \cdot \frac{m^3}{s}$ $\quad S_0 = 0.00193$

For bare soil (Table 11.1) $n = 0.010$

Hence from Table 11.2 $A = \left(B_w + z \cdot y\right) \cdot y = (2.4 + y) \cdot y$ $\quad R = \frac{\left(B_w + z \cdot y\right) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(2.4 + y) \cdot y}{2.4 + 2 \cdot y \cdot \sqrt{2}}$

Hence $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.010} \cdot (2.4 + y) \cdot y \cdot \left[\frac{(2.4 + y) \cdot y}{2.4 + 2 \cdot y \cdot \sqrt{2}}\right]^{\frac{2}{3}} \cdot 0.00193^{\frac{1}{2}} = 7.1$ (Note that we don't use units!)

Solving for y $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.62$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a shallower depth than that of Problem 11.9.

For $y = 1$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 2.55$ For $y = 0.75$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.53$

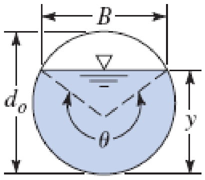
For $y = 0.77$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.60$ For $y = 0.775$ (m) $\frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{\left(2.4 + 2 \cdot y \cdot \sqrt{2}\right)^{\frac{2}{3}}} = 1.62$

The solution to three figures is $y = 0.775$ (m)

Problem 11.16

[3]

11.16 Consider again the semicircular channel of Problem 11.11. Find the normal depth that corresponds to a discharge of 0.3 m^3/sec.



Given: Data on semicircular trough

Find: New depth of flow

Solution:

Basic equation: $Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $d_0 = 1\cdot m$ $S_0 = 0.01$ $Q = 0.3\cdot\frac{m^3}{s}$

For corrugated steel, a Google search leads to (Table 11.1) $n = 0.022$

From Table 11.2 $A = \frac{1}{8}\cdot(\theta - \sin(\theta))\cdot d_0^{\ 2} = \frac{1}{8}\cdot(\theta - \sin(\theta))$

$R = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)\cdot d_0 = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)$

Hence $Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1}{0.022}\cdot\left[\frac{1}{8}\cdot(\theta - \sin(\theta))\right]\cdot\left[\frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)\right]^{\frac{2}{3}}\cdot 0.01^{\frac{1}{2}} = 0.3$ (Note that we don't use units!

Solving for θ $\theta^{-\frac{2}{3}}\cdot(\theta - \sin(\theta))^{\frac{5}{3}} = 1.33$

This is a nonlinear implicit equation for θ and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a half-full channe

For $\theta = 180\cdot deg$ $\theta^{-\frac{2}{3}}\cdot(\theta - \sin(\theta))^{\frac{5}{3}} = 3.14$ For $\theta = 140\cdot deg$ $\theta^{-\frac{2}{3}}\cdot(\theta - \sin(\theta))^{\frac{5}{3}} = 1.47$

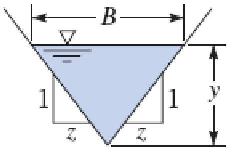
For $\theta = 135\cdot deg$ $\theta^{-\frac{2}{3}}\cdot(\theta - \sin(\theta))^{\frac{5}{3}} = 1.30$ For $\theta = 136\cdot deg$ $\theta^{-\frac{2}{3}}\cdot(\theta - \sin(\theta))^{\frac{5}{3}} = 1.33$

The solution to three figures is $\theta = 136\cdot deg$

From geometry $y = \frac{d_0}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$ $y = 0.313\,m$

Problem 11.17

11.17 Consider a symmetric open channel of triangular cross section. Show that for a given flow area, the wetted perimeter is minimized when the sides meet at a right angle.



Given: Triangular channel

Find: Proof that wetted perimeter is minimized when sides meet at right angles

Solution:

From Table 11.2 $A = z\cdot y^2$ $P = 2\cdot y\cdot\sqrt{1+z^2}$

We need to vary z to minimize P while keeping A constant, which means thε $y = \sqrt{\frac{A}{z}}$ with A = constant

Hence we eliminate y in the expression for P $P = 2\cdot\sqrt{\frac{A}{z}}\cdot\sqrt{1+z^2} = 2\cdot\sqrt{\frac{A\cdot\left(1+z^2\right)}{z}}$

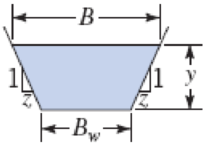
For optimizing P $\frac{dP}{dz} = \frac{z^2-1}{z}\cdot\sqrt{\frac{A}{z\cdot\left(z^2+1\right)}} = 0$ or $z = 1$

For z = 1 we find from the figure that we have the case where the sides are inclined at 45º, so meet at 90º. Note that we have only proved that this is a minimum OR maximum of P! It makes sense that it's the minimum, as, for constant A, we get a huge P if we set z to a large number (almost vertical walls); taking the second derivative at z = 1 results in a value of $\sqrt{2\cdot A}$, which is positive, so we DO have a minimum.

Problem 11.18

[3]

11.18 A trapezoidal channel with a bottom width of 20 ft, side slopes of 1 to 2, channel bottom slope of 0.0016, and a Manning's *n* of 0.025 carries a discharge of 400 cfs. Compute the critical depth and velocity of this channel.



Given: Data on trapezoidal channel

Find: Critical depth and velocity

Solution:

Basic equation: $E = y + \dfrac{V^2}{2\cdot g}$

The given data is: $B_w = 20\cdot ft$ $\quad z = \dfrac{1}{2}$ $\quad S_0 = 0.0016$ $\quad n = 0.025$ $\quad Q = 400\cdot\dfrac{ft^3}{s}$

In terms of flow rate $E = y + \dfrac{Q^2}{2\cdot A^2\cdot g}$ where (Table 11.2) $A = \left(B_w + z\cdot y\right)\cdot y$

Hence in terms of y $E = y + \dfrac{Q^2}{2\cdot\left(B_w + z\cdot y\right)^2\cdot y^2\cdot g}$

For critical conditions $\dfrac{dE}{dy} = 0 = 1 - \dfrac{Q^2\cdot z}{g\cdot y^2\cdot\left(B_w + y\cdot z\right)^3} - \dfrac{Q^2}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^2} = 1 - \dfrac{B_w\cdot Q^2}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3}$

Hence $g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = B_w\cdot Q^2$

The only unknown on the right is y. The right side evaluates to $B_w\cdot Q^2 = 3.20\times 10^6\dfrac{ft^7}{s^2}$

To find y we need to solve the non-linear equation. We must do this numerically; we may use the Newton method or similar, or *Excel*'s *Solver* or *Goal Seek*. Here we interate manually, starting with an arbitrary value

For $y = 1\cdot ft$ $\quad g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = 2.77\times 10^5\dfrac{ft^7}{s^2}$ $\qquad$ For $y = 2\cdot ft$ $\quad g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = 2.38\times 10^6\dfrac{ft^7}{s^2}$

For $y = 2.5\cdot ft$ $\quad g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = 4.82\times 10^6\dfrac{ft^7}{s^2}$ $\qquad$ For $y = 2.2\cdot ft$ $\quad g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = 3.22\times 10^6\dfrac{ft^7}{s^2}$

For $y = 2.19\cdot ft$ $\quad g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = 3.17\times 10^6\dfrac{ft^7}{s^2}$ $\qquad$ For $y = 2.20\cdot ft$ $\quad g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3 = 3.22\times 10^6\dfrac{ft^7}{s^2}$

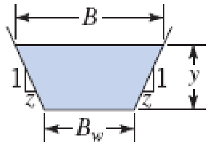
Hence the critical depth is $y = 2.20\,ft$

Also $A = \left(B_w + z\cdot y\right)\cdot y$ $\quad A = 46.4\,ft^2$ so critical speed is $V = \dfrac{Q}{A}$ $\quad V = 8.62\dfrac{ft}{s}$

Problem 11.19

[3]

11.19 Compute the normal depth and velocity of the channel of Problem 11.18.



Given: Data on trapezoidal channel

Find: Normal depth and velocity

Solution:

Basic equation: $$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 20 \cdot \text{ft}$ $\quad z = \frac{1}{2}$ $\quad Q = 400 \cdot \frac{\text{ft}^3}{\text{s}}$ $\quad S_0 = 0.0016$ $\quad n = 0.025$

Hence from Table 11.2 $$A = \left(B_w + z \cdot y\right) \cdot y = \left(20 + \frac{1}{2} \cdot y\right) \cdot y \qquad R = \frac{\left(B_w + z \cdot y\right) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{\left(20 + \frac{1}{2} \cdot y\right) \cdot y}{20 + y \cdot \sqrt{5}}$$

Hence $$Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.025} \cdot \left(20 + \frac{1}{2} \cdot y\right) \cdot y \cdot \left[\frac{\left(20 + \frac{1}{2} \cdot y\right) \cdot y}{20 + y \cdot \sqrt{5}}\right]^{\frac{2}{3}} \cdot 0.0016^{\frac{1}{2}} = 400$$ (Note that we don't use units!)

Solving for y $$\frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 250$$

This is a nonlinear implicit equation for y and must be solved numerically. We can one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with an arbitrary depth

For $y = 5$ (ft) $$\frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 265$$

For $y = 4.9$ (ft) $$\frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 256$$

For $y = 4.85$ (ft) $$\frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 252$$

For $y = 4.83$ (ft) $$\frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 250$$

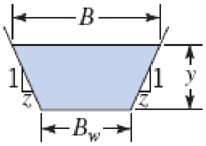
The solution to three figures is $y = 4.83 \cdot \text{ft}$ $\qquad$ Then $A = \left(B_w + z \cdot y\right) \cdot y$ $\quad A = 108 \cdot \text{ft}^2$

Finally, the normal velocity is $V = \frac{Q}{A}$ $\qquad V = 3.69 \cdot \frac{\text{ft}}{\text{s}}$

Problem 11.20

[3]

11.20 Derive an expression for the hydraulic radius of a trapezoidal channel with bottom width B_w, liquid depth y, and side slope angle θ. Verify the equation given in Table 11.2. Plot the ratio R/y for $B_w = 2$ m with side slope angles of 30° and 60° for $0.5 < y < 3$ m.



Given: Trapezoidal channel

Find: Derive expression for hydraulic radius; Plot R/y versus y for two different side slopes

Solution:

The area is (from simple geometry or Table 11.2) $A = B_w \cdot y + 2 \cdot \frac{1}{2} \cdot y \cdot y \cdot z = \left(B_w + z \cdot y\right) \cdot y$

The wetted perimeter is (from simple geometry or Table 11.2) $P = B_w + 2 \cdot y \cdot \sqrt{1 + z^2}$

Hence the hydraulic radius is $R = \frac{A}{P} = \frac{\left(B_w + z \cdot y\right) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$ which is the same as that listed in Table 11.2

We are to plot $\frac{R}{y} = \frac{\left(B_w + z \cdot y\right)}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$ with $B_w = 2 \cdot m$ for θ = 30º and 60º, and 0.5 < y < 3 m.

Note: For θ = 30º $z = \frac{1}{\tan(30 \cdot deg)}$ $z = 1.73$

Note: For θ = 60º $z = \frac{1}{\tan(60 \cdot deg)}$ $z = 0.577$

The graph is plotted in the associated *Excel* workbook

Problem 11.20

[3]

11.20 Derive an expression for the hydraulic radius of a trapezoidal channel with bottom width B_w, liquid depth y, and side slope angle θ. Verify the equation given in Table 11.2. Plot the ratio R/y for $B_w = 2$ m with side slope angles of 30° and 60° for $0.5 < y < 3$ m.

Given: Trapezoidal channel

Find: Derive expression for hydraulic radius; Plot R/y versus y for two different side slopes

Solution:

Given data: $B_w = 2\ \text{m}$

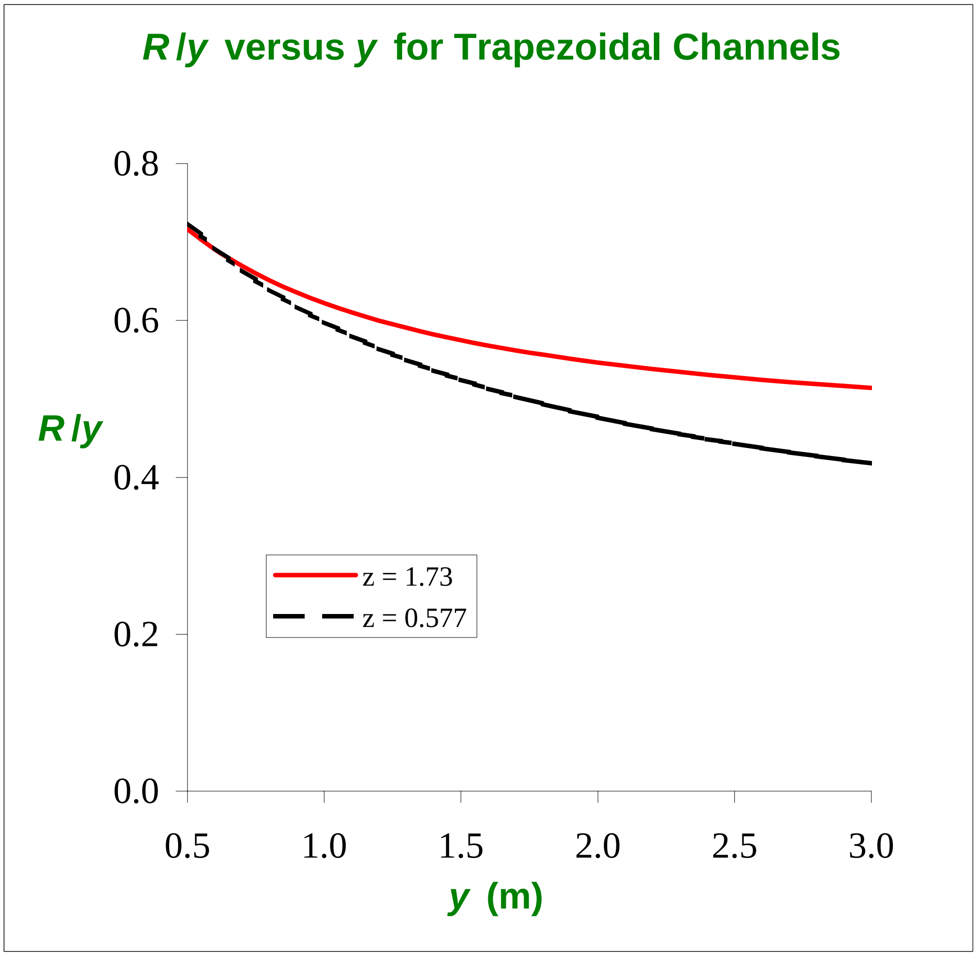
We are to plot $\frac{R}{y} = \frac{(B_w + z\cdot y)}{B_w + 2\cdot y\cdot\sqrt{1+z^2}}$ with $B_w = 2\cdot\text{m}$ for $\theta = 30°$ and $60°$, and $0.5 < y < 3$ m.

Note: For $\theta = 30°$ $z = \frac{1}{\tan(30\cdot\text{deg})}$ $z = 1.73$

Note: For $\theta = 60°$ $z = \frac{1}{\tan(60\cdot\text{deg})}$ $z = 0.577$

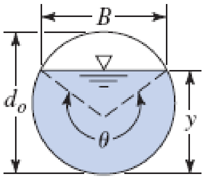
Computed results:

	θ = 30°	θ = 60°
	z = 1.73	*z* = 0.577
y (m)	*R*/*y*	*R*/*y*
0.5	0.717	0.725
0.6	0.691	0.693
0.7	0.669	0.665
0.8	0.651	0.640
0.9	0.636	0.618
1.0	0.622	0.598
1.1	0.610	0.580
1.2	0.600	0.564
1.3	0.591	0.550
1.4	0.582	0.537
1.5	0.575	0.524
1.6	0.568	0.513
1.7	0.562	0.503
1.8	0.556	0.494
1.9	0.551	0.485
2.0	0.546	0.477
2.1	0.542	0.469
2.2	0.538	0.462
2.3	0.534	0.455
2.4	0.531	0.449
2.5	0.527	0.443
2.6	0.524	0.437
2.7	0.522	0.432
2.8	0.519	0.427
2.9	0.516	0.422
3.0	0.514	0.418



Problem 11.21

11.21 Verify the equation given in Table 11.2 for the hydraulic radius of a circular channel. Evaluate and plot the ratio R/d_o, for liquid depths between 0 and d_o.



Given: Circular channel

Find: Derive expression for hydraulic radius; Plot R/d_0 versus d_0 for a range of depths

Solution:

The area is (from simple geometry or Table 11.2)

$$A = \frac{d_0^2}{8}\cdot\theta + 2\cdot\frac{1}{2}\cdot\frac{d_0}{2}\cdot\sin\left(\pi - \frac{\theta}{2}\right)\cdot\frac{d_0}{2}\cdot\cos\left(\pi - \frac{\theta}{2}\right) = \frac{d_0^2}{8}\cdot\theta + \frac{d_0^2}{4}\cdot\sin\left(\pi - \frac{\theta}{2}\right)\cdot\cos\left(\pi - \frac{\theta}{2}\right)$$

$$A = \frac{d_0^2}{8}\cdot\theta + \frac{d_0^2}{8}\cdot\sin(2\cdot\pi - \theta) = \frac{d_0^2}{8}\cdot\theta - \frac{d_0^2}{8}\cdot\sin(\theta) = \frac{d_0^2}{8}\cdot(\theta - \sin(\theta))$$

The wetted perimeter is (from simple geometry or Table 11.2) $P = \frac{d_0}{2}\cdot\theta$

Hence the hydraulic radius is $R = \frac{A}{P} = \frac{\frac{d_0^2}{8}\cdot(\theta - \sin(\theta))}{\frac{d_0}{2}\cdot\theta} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)\cdot d_0$ which is the same as that listed in Table 11.2

We are to plot $\frac{R}{d_0} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)$

We will need y as a function of θ: $y = \frac{d_0}{2} + \frac{d_0}{2}\cdot\cos\left(\pi - \frac{\theta}{2}\right) = \frac{d_0}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$ or $\frac{y}{d_0} = \frac{1}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$

The graph is plotted in the associated *Excel* workbook

Problem 11.21

[3]

11.21 Verify the equation given in Table 11.2 for the hydraulic radius of a circular channel. Evaluate and plot the ratio R/d_o, for liquid depths between 0 and d_o.

Given: Circular channel

Find: Derive expression for hydraulic radius; Plot R/d_0 versus d_0

Solution:

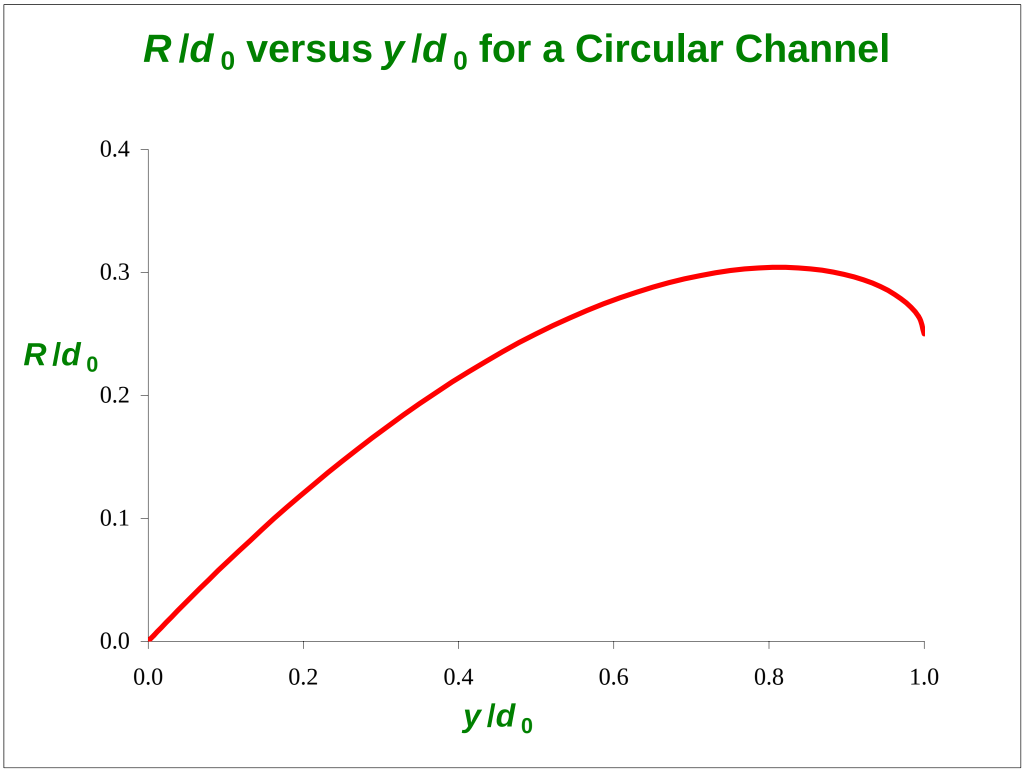
Given data

The hydraulic radius is $$R = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)\cdot d_0$$

We are to plot $$\frac{R}{d_0} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)$$

We will need y as a function of θ: $$y = \frac{d_0}{2} + \frac{d_0}{2}\cdot\cos\left(\pi - \frac{\theta}{2}\right) = \frac{d_0}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$$ or $$\frac{y}{d_0} = \frac{1}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$$

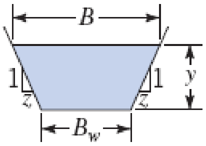
θ (°)	y/d_0	R/d_0
0	0.000	0.000
20	0.008	0.005
40	0.030	0.020
60	0.067	0.043
80	0.117	0.074
100	0.179	0.109
120	0.250	0.147
140	0.329	0.184
160	0.413	0.219
180	0.500	0.250
200	0.587	0.274
220	0.671	0.292
240	0.750	0.302
260	0.821	0.304
280	0.883	0.300
300	0.933	0.291
320	0.970	0.279
340	0.992	0.264
360	1.000	0.250



Problem 11.22

[5]

11.22 Determine the cross-section of the greatest hydraulic efficiency for a trapezoidal channel with side slope of 1 vertical to 2 horizontal if the design discharge is 10 m^3/s. The channel slope is 0.001 and Manning's roughness factor is 0.020.



Given: Data on trapezoidal channel

Find: Geometry for greatest hydraulic efficiency

Solution:

Basic equation: $$Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $z = 2$ $\quad Q = 10 \cdot \frac{m^3}{s}$ $\quad S_0 = 0.001$ $\quad n = 0.020$

From Table 11.2 $\quad A = \left(B_w + z \cdot y\right) \cdot y$ $\quad P = B_w + 2 \cdot y \cdot \sqrt{1 + z^2}$

We need to vary B_w and y to obtain optimum conditions. These are when the area and perimeter are optimized. Instead of two independent variables B_w and y, we eliminate B_w by doing the following

$$B_w = \frac{A}{y} - z \cdot y \qquad \text{and so} \qquad P = \frac{A}{y} - z \cdot y + 2 \cdot y \cdot \sqrt{1 + z^2}$$

Taking the derivative w.r.t. y $$\frac{\partial}{\partial y} P = \frac{1}{y} \cdot \frac{\partial}{\partial y} A - \frac{A}{y^2} - z + 2 \cdot \sqrt{1 + z^2}$$

But at optimum conditions $$\frac{\partial}{\partial y} P = 0 \qquad \text{and} \qquad \frac{\partial}{\partial y} A = 0$$

Hence $$0 = -\frac{A}{y^2} - z + 2 \cdot \sqrt{1 + z^2} \qquad \text{or} \qquad A = 2 \cdot y^2 \cdot \sqrt{1 + z^2} - z \cdot y^2$$

Comparing to $$A = \left(B_w + z \cdot y\right) \cdot y \qquad \text{we find} \qquad A = \left(B_w + z \cdot y\right) \cdot y = 2 \cdot y^2 \cdot \sqrt{1 + z^2} - z \cdot y^2$$

Hence $$B_w = 2 \cdot y \cdot \sqrt{1 + z^2} - 2 \cdot z \cdot y$$

Then $$A = \left(B_w + z \cdot y\right) \cdot y = y^2 \cdot \left(2 \cdot \sqrt{1 + z^2} - z\right)$$

$$P = B_w + 2 \cdot y \cdot \sqrt{1 + z^2} = 4 \cdot y \cdot \sqrt{1 + z^2} - 2 \cdot z \cdot y$$

and $$R = \frac{A}{P} = \frac{y^2\cdot\left(2\cdot\sqrt{1+z^2}-z\right)}{4\cdot y\cdot\sqrt{1+z^2}-2\cdot z\cdot y} = \frac{\left(2\cdot\sqrt{1+z^2}-z\right)}{4\cdot\sqrt{1+z^2}-2\cdot z}\cdot y$$

Hence $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1}{n}\cdot\left[y^2\cdot\left(2\cdot\sqrt{1+z^2}-z\right)\right]\cdot\left[\frac{\left(2\cdot\sqrt{1+z^2}-z\right)}{4\cdot\sqrt{1+z^2}-2\cdot z}\cdot y\right]^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

$$Q = \frac{\left(2\cdot\sqrt{1+z^2}-z\right)^{\frac{5}{3}}\cdot S_0^{\frac{1}{2}}}{n\cdot\left(4\cdot\sqrt{1+z^2}-2\cdot z\right)^{\frac{2}{3}}}\cdot y^{\frac{8}{3}}$$

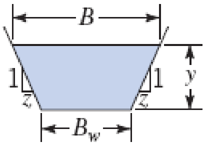
Solving for y $$y = \left[\frac{n\cdot\left(4\cdot\sqrt{1+z^2}-2\cdot z\right)^{\frac{2}{3}}}{\left(2\cdot\sqrt{1+z^2}-z\right)^{\frac{5}{3}}\cdot S_0^{\frac{1}{2}}}\cdot Q\right]^{\frac{3}{8}}$$ $y = 1.69$ (m)

Finally $$B_w = 2\cdot y\cdot\sqrt{1+z^2}-2\cdot z\cdot y$$ $B_w = 0.799$ (m)

Problem 11.23

[3]

11.23 For a trapezoidal shaped channel ($n = 0.014$ and slope $S_o = 0.0002$ with a 20-ft bottom width and side slopes of 1 vertical to 1.5 horizontal), determine the normal depth for a discharge of 1000 cfs.



Given:

Data on trapezoidal channel

Find:

Normal depth

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 20 \cdot ft$ $\quad z = 1.5$ $\quad Q = 1000 \cdot \frac{ft^3}{s}$ $\quad S_0 = 0.0002$

$n = 0.014$

Hence from Table 11.2 $\quad A = \left(B_w + z \cdot y\right) \cdot y = (20 + 1.5 \cdot y) \cdot y$ $\quad R = \frac{\left(B_w + z \cdot y\right) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(20 + 1.5 \cdot y) \cdot y}{20 + 2 \cdot y \cdot \sqrt{3.25}}$

Hence $\quad Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1.49}{0.014} \cdot (20 + 1.5 \cdot y) \cdot y \cdot \left[\frac{(20 + 1.5 \cdot y) \cdot y}{20 + 2 \cdot y \cdot \sqrt{3.25}}\right]^{\frac{2}{3}} \cdot 0.0002^{\frac{1}{2}} = 1000$ (Note that we don't use units!

Solving for y $\quad \frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 664$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 7.5$ (ft) $\quad \frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 684$ $\quad$ For $y = 7.4$ (ft) $\quad \frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 667$

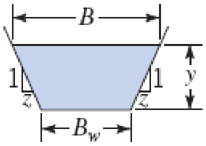
For $y = 7.35$ (ft) $\quad \frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 658$ $\quad$ For $y = 7.38$ (ft) $\quad \frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{\left(20 + 2 \cdot y \cdot \sqrt{3.25}\right)^{\frac{2}{3}}} = 663$

The solution to three figures is $\quad y = 7.38$ (ft)

Problem 11.24

[5]

11.24 Show that the best hydraulic trapezoidal section is one-half of a hexagon.



Given: Trapezoidal channel

Find: Geometry for greatest hydraulic efficiency

Solution:

From Table 11.2 $A = \left(B_w + z\cdot y\right)\cdot y$ $\qquad P = B_w + 2\cdot y\cdot\sqrt{1+z^2}$

We need to vary B_w and y (and then z!) to obtain optimum conditions. These are when the area and perimeter are optimized. Instead of two independent variables B_w and y, we eliminate B_w by doing the following

$$B_w = \frac{A}{y} - z\cdot y \qquad \text{and so} \qquad P = \frac{A}{y} - z\cdot y + 2\cdot y\cdot\sqrt{1+z^2}$$

Taking the derivative w.r.t. y $\quad \frac{\partial}{\partial y}P = \frac{1}{y}\cdot\frac{\partial}{\partial y}A - \frac{A}{y^2} - z + 2\cdot\sqrt{1+z^2}$

But at optimum conditions $\quad \frac{\partial}{\partial y}P = 0 \qquad \text{and} \qquad \frac{\partial}{\partial y}A = 0$

Hence $\quad 0 = -\frac{A}{y^2} - z + 2\cdot\sqrt{1+z^2} \qquad \text{or} \qquad A = 2\cdot y^2\cdot\sqrt{1+z^2} - z\cdot y^2 \qquad (1)$

Now we optimize A w.r.t. z $\quad \frac{\partial}{\partial z}A = \frac{2\cdot y^2\cdot z}{\sqrt{z^2+1}} - y^2 = 0 \qquad \text{or} \qquad 2\cdot z = \sqrt{z^2+1}$

Hence $\quad 4\cdot z^2 = z^2 + 1 \qquad \text{or} \qquad z = \frac{1}{\sqrt{3}}$

We can now evaluate A from Eq 1 $\quad A = 2\cdot y^2\cdot\sqrt{1+z^2} - z\cdot y^2 = 2\cdot y^2\cdot\sqrt{1+\frac{1}{3}} - \frac{1}{3}\cdot y^2 = \left(\frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)\cdot y^2 = \sqrt{3}\cdot y^2$

But for a trapezoid $\quad A = \left(B_w + z\cdot y\right)\cdot y = \left(B_w + \frac{1}{\sqrt{3}}\cdot y\right)\cdot y$

Comparing the two A expressions $\quad A = \left(B_w + \frac{1}{\sqrt{3}}\cdot y\right)\cdot y = \sqrt{3}\cdot y^2 \qquad \text{we find} \qquad B_w = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\cdot y = \frac{2}{\sqrt{3}}\cdot y$

But the perimeter is $\quad P = B_w + 2\cdot y\cdot\sqrt{1+z^2} = B_w + 2\cdot y\cdot\sqrt{1+\frac{1}{3}} = B_w + \frac{4}{\sqrt{3}}\cdot y = B_w + 2\cdot B_w = 3\cdot B_w$

In summary we have $\quad z = \frac{1}{\sqrt{3}} \qquad \theta = \operatorname{atan}\left(\frac{1}{z}\right) \qquad \theta = 60\,\text{deg}$ $\quad$ where θ is the angle the sides make with the vertical

and $\quad B_w = \frac{1}{3}\cdot P$ $\quad$ so each of the symmetric sides is $\quad \frac{P - \frac{1}{3}\cdot P}{2} = \frac{1}{3}\cdot P$

We have proved that the optimum shape is equal side and bottom lengths, with 60 angles i.e., half a hexagon!

Problem 11.25

[3]

11.25 Solve Example 11.4 for discharges of 0, 25, 75, 125, and 200 ft^3/s.

Given: Rectangular channel

Find: Plot of specific energy curves; Critical depths; Critical specific energy

Solution:

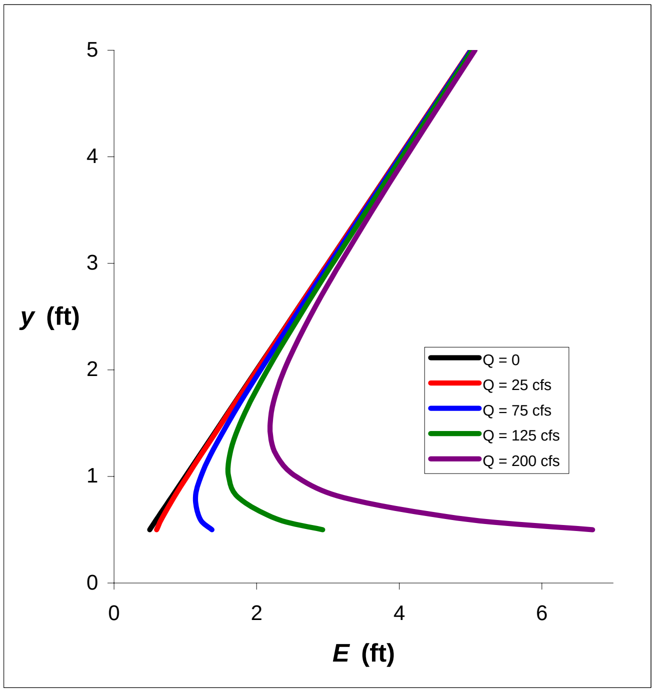
Given data: $B =$ 20 ft

Specific energy: $$E = y + \left(\frac{Q^2}{2gB^2}\right)\frac{1}{y^2}$$

Critical depth: $$y_c = \left(\frac{Q^2}{gB^2}\right)^{\frac{1}{3}}$$

	Specific Energy, *E* (ft·lb/lb)				
***y* (ft)**	***Q* = 0**	***Q* = 25**	***Q* = 75**	***Q* = 125**	***Q* = 200**
0.5	0.50	0.60	1.37	2.93	6.71
0.6	0.60	0.67	1.21	2.28	4.91
0.8	0.80	0.84	1.14	1.75	3.23
1.0	1.00	1.02	1.22	1.61	2.55
1.2	1.20	1.22	1.35	1.62	2.28
1.4	1.40	1.41	1.51	1.71	2.19
1.6	1.60	1.61	1.69	1.84	2.21
1.8	1.80	1.81	1.87	1.99	2.28
2.0	2.00	2.01	2.05	2.15	2.39
2.2	2.20	2.21	2.25	2.33	2.52
2.4	2.40	2.40	2.44	2.51	2.67
2.6	2.60	2.60	2.63	2.69	2.83
2.8	2.80	2.80	2.83	2.88	3.00
3.0	3.00	3.00	3.02	3.07	3.17
3.5	3.50	3.50	3.52	3.55	3.63
4.0	4.00	4.00	4.01	4.04	4.10
4.5	4.50	4.50	4.51	4.53	4.58
5.0	5.00	5.00	5.01	5.02	5.06

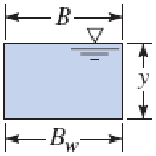
y_c (ft)	**0.365**	**0.759**	**1.067**	**1.46**
E_c (ft)	**0.547**	**1.14**	**1.60**	**2.19**



Problem 11.26

[2]

11.26 Rework Example 11.5 for a 30-cm-high hump and a side wall constriction that reduces the channel width to 1.6 m.



Given: Rectangular channel flow with hump and/or side wall restriction

Find: Whether critical flow occurs

Solution:

Basic equations: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $E = y + \frac{Q^2}{2 \cdot g \cdot A^2}$ $A = B_w \cdot y$ $E_{min} = \frac{3}{2} \cdot y_c$ (From Example 11.5)

Given data: $B_w = 2 \cdot m$ $y = 1 \cdot m$ $\Delta z = 30 \cdot cm$ $B = 1.6 \cdot m$ $Q = 2.4 \cdot \frac{m^3}{s}$

(a) For a hump with $\Delta z = 30\,cm$ $E = y + \frac{Q^2}{2 \cdot g \cdot B_w^{\ 2}} \cdot \frac{1}{y^2}$ $E = 1.07\,m$

$$y_c = \left[\frac{\left(\frac{Q}{B_w}\right)^2}{g}\right]^{\frac{1}{3}} \qquad y_c = 0.528\,m \qquad E_{min} = \frac{3}{2} \cdot y_c \qquad E_{min} = 0.791\,m$$

$$\Delta z_{crit} = E - E_{min} \qquad \Delta z_{crit} = 0.282\,m$$

Hence we have $\Delta z = 0.3\,m$ $> \Delta z_{crit} = 0.282\,m$ so the hump IS sufficient for critical flow

(b) For the sidewall restriction with $B = 1.6\,m$

$$y_c = \left[\frac{\left(\frac{Q}{B}\right)^2}{g}\right]^{\frac{1}{3}} \qquad y_c = 0.612\,m \qquad E_{min} = \frac{3}{2} \cdot y_c \qquad E_{min} = 0.918\,m$$

Hence we have $E = 1.073\,m$ $> E_{min} = 0.918\,m$ so the restriction is insufficient for critical flow

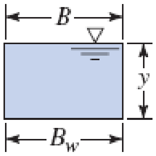
(a) For both, we can use the minimum energy from case (b) $E_{min} = 0.918\,m$

$$\Delta z_{crit} = E - E_{min} \qquad \Delta z_{crit} = 0.155\,m$$

Hence we have $\Delta z = 0.3\,m$ $> \Delta z_{crit} = 0.155\,m$ so in this case the conditions ARE sufficient for critical flow

Problem 11.27

11.27 Compute the critical depth for the channel in Problem 11.1.



Given: Rectangular channel flow

Find: Critical depth

Solution:

Basic equations: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\quad Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

For a rectangular channel of width $B_w = 2 \cdot m$ and depth $y = 1.5 \cdot m$ we find from Table 11.2

$A = B_w \cdot y$ $\quad A = 3.00 \cdot m^2$ $\quad R = \frac{B_w \cdot y}{B_w + 2 \cdot y}$ $\quad R = 0.600 \cdot m$

Manning's roughness coefficient is $n = 0.015$ and $S_0 = 0.0005$

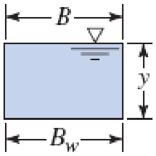
$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$ $\quad Q = 3.18 \cdot \frac{m^3}{s}$

Hence $q = \frac{Q}{B_w}$ $\quad q = 1.59 \frac{m^2}{s}$ $\quad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\quad y_c = 0.637 m$

Problem 11.28

[1]

11.28 Compute the critical depth for the channel in Problem 11.2.



Given: Rectangular channel flow

Find: Critical depth

Solution:

Basic equations: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

Given data: $B_w = 2.5 \cdot m$ $\quad Q = 3 \cdot \frac{m^3}{s}$

Hence $q = \frac{Q}{B_w}$ $\quad q = 1.2 \frac{m^2}{s}$ $\quad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\quad y_c = 0.528\,m$

Problem 11.29

[2]

11.29 Rework Example 11.6 with discharges of 0, 25, 75, 125, and 200 cfs.

Given: Rectangular channel

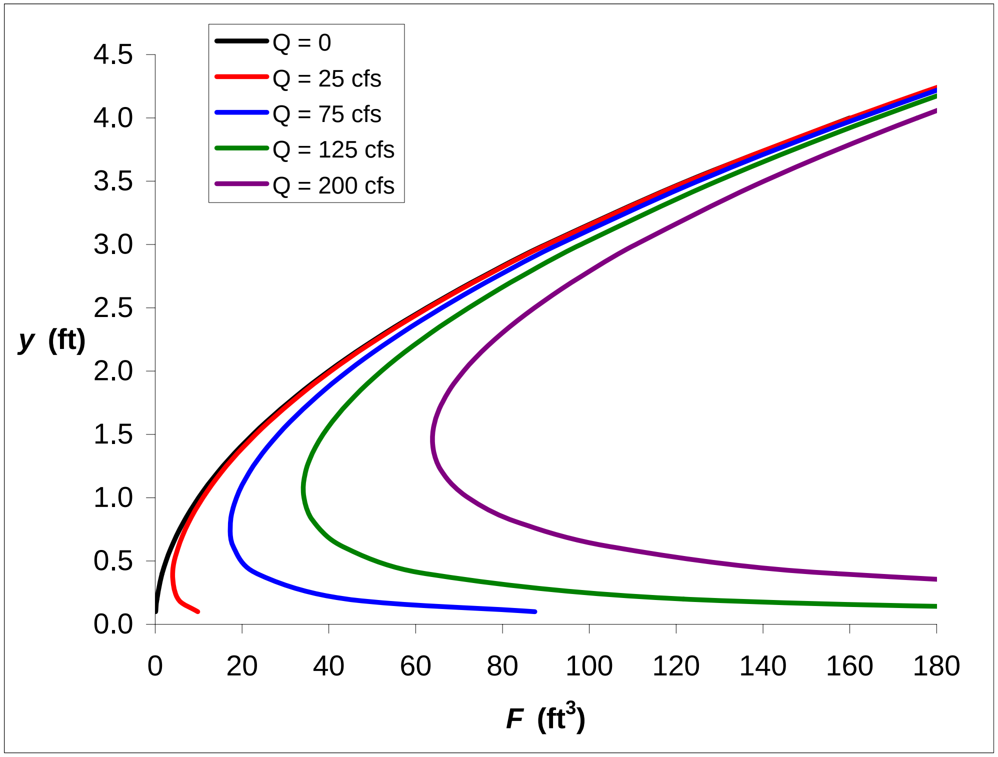
Find: Plot of specific force curves

Solution:

Given data: $B =$ 20 ft

Specific force: $$F = \frac{Q^2}{gBy} + \frac{By^2}{2}$$

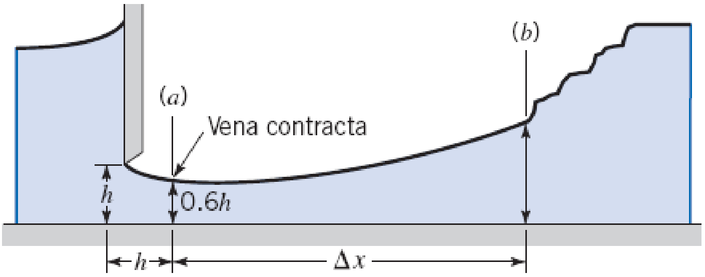
	Specific Force, F (ft^3)				
y (ft)	Q = 0	Q = 25	Q = 75	Q = 125	Q = 200
0.1	0.10	9.80	87.44	242.72	621.22
0.2	0.40	5.25	44.07	121.71	310.96
0.4	1.60	4.03	23.44	62.26	156.88
0.6	3.60	5.22	18.16	44.04	107.12
0.8	6.40	7.61	17.32	36.73	84.04
1.0	10.00	10.97	18.73	34.26	72.11
1.2	14.40	15.21	21.68	34.62	66.16
1.4	19.60	20.29	25.84	36.93	63.97
1.6	25.60	26.21	31.06	40.76	64.42
1.8	32.40	32.94	37.25	45.88	66.91
2.0	40.00	40.49	44.37	52.13	71.06
2.2	48.40	48.84	52.37	59.43	76.63
2.4	57.60	58.00	61.24	67.71	83.48
2.6	67.60	67.97	70.96	76.93	91.49
2.8	78.40	78.75	81.52	87.07	100.58
3.0	90.00	90.32	92.91	98.09	110.70
3.5	122.50	122.78	125.00	129.43	140.25
4.0	160.00	160.24	162.18	166.07	175.53
4.5	202.50	202.72	204.44	207.89	216.30
5.0	250.00	250.19	251.75	254.85	262.42



Problem 11.30

[2]

11.30 Resolve Example 11.7 for a channel bed slope of 0.003.



Flow downstream of a sluice gate in a wide rectangular channel.

Given: Vena contracta at a sluice gate

Find: Distance from vena contracta at which depth is 0.5 m

Solution:

Basic equations: $E = y + \frac{V^2}{2\cdot g}$ $\quad R = y \quad$ (Wide channel) $\quad S_f = \left(\frac{V_{ave}\cdot n}{R_{ave}^{\frac{2}{3}}}\right)^2$ $\quad \Delta x = \frac{E_a - E_b}{S_f - S_0}$

(Some equations from Example 11.7)

Given data: $q = 4.646\cdot\frac{\frac{m^3}{s}}{m}$ $\quad y_a = 0.457\cdot m$ $\quad y_b = 0.5\cdot m$ $\quad n = 0.020$ $\quad S_0 = 0.003$

Hence we find $V_a = \frac{q}{y_a}$ $\quad V_a = 10.2\frac{m}{s}$ $\quad V_b = \frac{q}{y_b}$ $\quad V_b = 9.29\frac{m}{s}$

Then $E_a = y_a + \frac{V_a^2}{2\cdot g}$ $\quad E_a = 5.73\,m$ $\quad E_b = y_b + \frac{V_b^2}{2\cdot g}$ $\quad E_b = 4.90\,m$

and $V_{ave} = \frac{V_a + V_b}{2}$ $\quad V_{ave} = 9.73\frac{m}{s}$

$R_a = y_a$ $\quad R_b = y_b$ $\quad R_{ave} = \frac{R_a + R_b}{2}$ $\quad R_a = 0.457\,m$

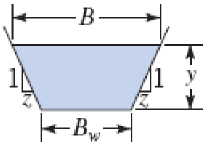
Then $S_f = \left(\frac{V_{ave}\cdot n}{R_{ave}^{\frac{2}{3}}}\right)^2$ $\quad S_f = 0.101$

Finally $\Delta x = \frac{E_a - E_b}{S_f - S_0}$ $\quad \Delta x = 8.40\,m$

Problem 11.31

[4]

11.31 Once again consider the trapezoidal channel in Problem 11.8 with a dam placed in the channel so that water backs up to a depth of 5 ft immediately behind the dam. How far upstream would you expect the depth to be 4.80 ft? Consider an energy correction coefficient of 1.1.



Given: Data on trapezoidal channel and dam

Find: Location upstream at which depth is 4.80 ft

Solution:

Basic equations:

From Example 11.7 $$\Delta x = \frac{\Delta y + \left(\frac{V_1^2}{2\cdot g} - \frac{V_2^2}{2\cdot g}\right)}{S_0 - S_f}$$ and $$S_f = \left(\frac{n\cdot V}{1.49\cdot R^{\frac{2}{3}}}\right)^2$$ (note the factor 1.49 because this is not SI units)

The given data is: $B_w = 20\cdot ft$ $\quad z = \frac{1}{2}$ $\quad S_0 = 0.0016$ $\quad n = 0.025$ $\quad Q = 400\cdot\frac{ft^3}{s}$ $\quad y_1 = 5\cdot ft$ $\quad y_2 = 4.80\cdot ft$

We need to modify the specific energy equation to allow for the emergy correction coefficient (Section 8-6): instead of $\frac{V^2}{2\cdot g}$, the kinetic energy per unit weight is $\alpha\cdot\frac{V^2}{2\cdot g}$ where $\alpha = 1.1$

Hence $$\Delta x = \frac{\Delta y + \alpha\cdot\left(\frac{V_1^2}{2\cdot g} - \frac{V_2^2}{2\cdot g}\right)}{S_0 - S_f}$$

We need to obtain terms on the right $\quad S_0 = 0.0016 \quad \alpha = 1.1 \quad \Delta y = y_1 - y_2 \quad \Delta y = 0.200\,ft$

We will need (Table 11.2) $A = \left(B_w + z\cdot y\right)\cdot y$ $\qquad R = \frac{\left(B_w + z\cdot y\right)\cdot y}{B_w + 2\cdot y\cdot\sqrt{1+z^2}}$

Then $V_1 = \frac{Q}{A_1}$ $\qquad V_1 = \frac{Q}{\left(B_w + z\cdot y_1\right)\cdot y_1}$ $\qquad V_1 = 3.56\frac{ft}{s}$

$V_2 = \frac{Q}{A_2}$ $\qquad V_2 = \frac{Q}{\left(B_w + z\cdot y_2\right)\cdot y_2}$ $\qquad V_2 = 3.72\frac{ft}{s}$

For S_f we use averages for V and R (as in Example 11.7)

and $V_{ave} = \frac{V_1 + V}{2}$ $V_{ave} = 1.11\frac{m}{s}$

$R_1 = \frac{\left(B_w + z\cdot y_1\right)\cdot y_1}{B_w + 2\cdot y_1\cdot\sqrt{1+z^2}}$ $R_1 = 3.61\,ft$ $\qquad R_2 = \frac{\left(B_w + z\cdot y_2\right)\cdot y_2}{B_w + 2\cdot y_2\cdot\sqrt{1+z^2}}$ $\qquad R_2 = 3.50\,ft$

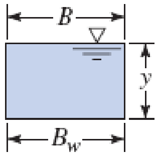
$$R_{ave} = \frac{R_1 + R_2}{2} \qquad R_{ave} = 3.55 \cdot ft$$

Then $$S_f = \left(\frac{V_{ave} \cdot n}{1.49 \cdot R_{ave}^{\frac{2}{3}}} \right)^2 \qquad S_f = 0.000687$$

Finally $$\Delta x = \frac{\Delta y + \alpha \cdot \left(\frac{V_1^{\,2}}{2 \cdot g} - \frac{V_2^{\,2}}{2 \cdot g} \right)}{S_0 - S_f} \qquad \Delta x = 197\,ft$$

Problem 11.32

11.32 A rectangular channel carries a discharge of 10 ft^3/sec per foot of width. Determine the minimum specific energy possible for this flow. Compute the corresponding flow depth and speed.



Given:
Data on rectangular channel

Find:
Minimum specific energy; Flow depth; Speed

Solution:

Basic equation: $E = y + \frac{V^2}{2\cdot g}$ (11.14)

In Section 11-2 we prove that the minimum specific energy is when we have critical flow; here we rederive the minimum energy point

For a rectangular channel $Q = V\cdot B_w\cdot y$ or $V = \frac{Q}{B_w\cdot y}$ with $\frac{Q}{B_w} = 10\cdot\frac{\frac{ft^3}{s}}{ft} = \text{constant}$

Hence, using this in Eq. 11.14 $E = y + \left(\frac{Q}{B_w\cdot y}\right)^2\cdot\frac{1}{2\cdot g} = y + \left(\frac{Q^2}{2\cdot B_w{}^2\cdot g}\right)\cdot\frac{1}{y^2}$

E is a minimum when $\frac{dE}{dy} = 1 - \left(\frac{Q^2}{B_w{}^2\cdot g}\right)\cdot\frac{1}{y^3} = 0$ or $y = \left(\frac{Q^2}{B_w{}^2\cdot g}\right)^{\frac{1}{3}}$ $y = 1.46\cdot ft$

Note that from Eq. 11.22 we have $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{Q^2}{B_w{}^2\cdot g}\right)^{\frac{1}{3}}$ which is the same result we derived

The speed is then given by $V = \frac{Q}{B_w\cdot y}$ $V = 6.85\cdot\frac{ft}{s}$

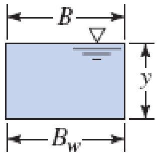
Note that from Eq. 11.20 we also have $V = \sqrt{g\cdot D}$ where D is the hydraulic depth $D = y$ $V = \sqrt{g\cdot D}$ $V = 6.85\cdot\frac{ft}{s}$

The minimum energy is then $E_{min} = y + \frac{V^2}{2\cdot g}$ $E_{min} = 2.19\cdot ft$

Problem 11.33

[3]

11.33 Flow in the channel of Problem 11.32 ($E_{\min} = 2.19$ ft) is to be at twice the minimum specific energy. Compute the alternate depths for this E.



Given: Data on rectangular channel

Find: Depths for twice the minimum energy

Solution:

Basic equation: $E = y + \frac{V^2}{2 \cdot g}$ (11.14)

For a rectangular channel $Q = V \cdot B_w \cdot y$ or $V = \frac{Q}{B_w \cdot y}$ with $\frac{Q}{B_w} = 10 \cdot \frac{\frac{ft^3}{s}}{ft} = \text{constant}$

Hence, using this in Eq. 11.14 $E = y + \left(\frac{Q}{B_w \cdot y}\right)^2 \cdot \frac{1}{2 \cdot g} = y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2}$ and $E = 2 \cdot 2.19 \cdot ft$ $E = 4.38 \cdot ft$

We have a nonlinear implicit equation for y $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = E$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a y larger than the critical, and evaluate the left side of the equation so that it is equal to $E = 4.38 ft$

For $y = 2 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 2.39 \cdot ft$

For $y = 4 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.10 \cdot ft$

For $y = 4.5 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.58 \cdot ft$

For $y = 4.30 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.38 \cdot ft$

Hence $y = 4.3 \cdot ft$

For the shallow depth

For $y = 1 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 2.55 \cdot ft$

For $y = 0.5 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 6.72 \cdot ft$

For $y = 0.6 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.92 \cdot ft$

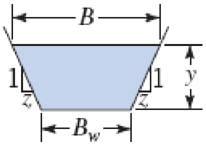
For $y = 0.65 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.33 \cdot ft$

For $y = 0.645 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\,2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.38 \cdot ft$

Hence $y = 0.645 \cdot ft$

Problem 11.34

11.34 Water flows at 300 ft^3/sec in a trapezoidal channel with bottom width of 8 ft. The sides are sloped at 2:1. Find the critical depth for this channel.



Given: Data on trapezoidal channel

Find: Critical depth

Solution:

Basic equation: $E = y + \frac{V^2}{2\cdot g}$ (11.14)

In Section 11-2 we prove that the minimum specific energy is when we have critical flow; here we rederive the minimum energy point

For a trapezoidal channel (Table 11.2) $A = \left(B_w + z\cdot y\right)\cdot y$ and $B_w = 8\cdot ft$ $z = 0.5$

Hence for V $V = \frac{Q}{A} = \frac{Q}{\left(B_w + z\cdot y\right)\cdot y}$ and $Q = 300\cdot\frac{ft^3}{s}$

Using this in Eq. 11.14 $E = y + \left[\frac{Q}{\left(B_w + z\cdot y\right)\cdot y}\right]^2\cdot\frac{1}{2\cdot g}$

E is a minimum when $\frac{dE}{dy} = 1 - \frac{Q^2\cdot z}{g\cdot y^2\cdot\left(B_w + y\cdot z\right)^3} - \frac{Q^2}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^2} = 0$

Hence we obtain for y $\frac{Q^2\cdot z}{g\cdot y^2\cdot\left(B_w + y\cdot z\right)^3} + \frac{Q^2}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^2} = 1$ or $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 1$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below, to make the left side equal unity

$y = 1\cdot ft$ $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 41$

$y = 5\cdot ft$ $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 0.251$

$y = 3\cdot ft$ $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 1.33$

$y = 3.5\cdot ft$ $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 0.809$

$y = 3.25\cdot ft$ $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 1.03$

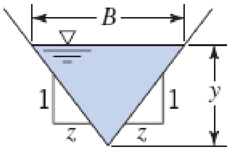
$y = 3.28\cdot ft$ $\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 0.998$

The critical depth is $y = 3.28\cdot ft$

Problem 11.35

[2]

11.35 For a channel of nonrectangular cross section, critical depth occurs at minimum specific energy. Obtain a general equation for critical depth in a triangular channel in terms of Q, g, and z.



Given: Triangular channel

Find: Critcal depth

Solution:

Basic equation: $E = y + \frac{V^2}{2\cdot g}$ (11.14)

For a triangular channel (Table 11.2) $A = z\cdot y^2$

Hence for V $V = \frac{Q}{A} = \frac{Q}{z\cdot y^2}$

Using this in Eq. 11.14 $E = y + \left(\frac{Q}{z\cdot y^2}\right)^2\cdot\frac{1}{2\cdot g}$

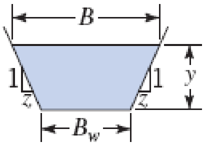
E is a minimum when $\frac{dE}{dy} = 1 - 4\cdot\frac{Q^2}{z^2\cdot y^5}\cdot\frac{1}{2\cdot g} = 0$

Hence we obtain for y $y = \left(\frac{2\cdot Q^2}{z^2\cdot g}\right)^{\frac{1}{5}}$

Problem 11.36

[2]

11.36 For a channel of nonrectangular cross section, critical depth occurs at minimum specific energy. Obtain a general equation for critical depth in a channel of trapezoidal section in terms of Q, g, B_w, and z.



Given: Trapezoidal channel

Find: Critcal depth

Solution:

Basic equation: $E = y + \frac{V^2}{2 \cdot g}$ (11.14)

The critical depth occurs when the specific energy is minimized

For a trapezoidal channel (Table 11.2) $A = \left(B_w + z \cdot y\right) \cdot y$ and $B_w = 8 \cdot ft$ $z = 0.5$

Hence for V $V = \frac{Q}{A} = \frac{Q}{\left(B_w + z \cdot y\right) \cdot y}$ and $Q = 300 \cdot \frac{ft^3}{s}$

Using this in Eq. 11.14 $E = y + \left[\frac{Q}{\left(B_w + z \cdot y\right) \cdot y}\right]^2 \cdot \frac{1}{2 \cdot g}$

E is a minimum when $\frac{dE}{dy} = 1 - \frac{Q^2 \cdot z}{g \cdot y^2 \cdot \left(B_w + y \cdot z\right)^3} - \frac{Q^2}{g \cdot y^3 \cdot \left(B_w + y \cdot z\right)^2} = 0$

Hence we obtain for y $\frac{Q^2 \cdot z}{g \cdot y^2 \cdot \left(B_w + y \cdot z\right)^3} + \frac{Q^2}{g \cdot y^3 \cdot \left(B_w + y \cdot z\right)^2} = 1$

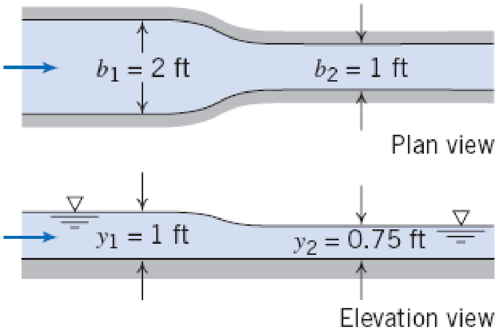
This can be simplified to $\frac{Q^2 \cdot \left(B_w + 2 \cdot y \cdot z\right)}{g \cdot y^3 \cdot \left(B_w + y \cdot z\right)^3} = 1$

This expression is the simplest one for y; it is implicit

Problem 11.37

[2]

11.37 Consider the Venturi flume shown. The bed is horizontal and flow may be considered frictionless. The upstream depth is 1 ft and the downstream depth is 0.75 ft. The upstream breadth is 2 ft and the breadth of the throat is 1 ft. Estimate the flow rate through the flume.



Given: Data on venturi flume

Find: Flow rate

Solution:

Basic equation:
$$\frac{p_1}{\rho\cdot g}+\frac{V_1^2}{2\cdot g}+y_1=\frac{p_2}{\rho\cdot g}+\frac{V_2^2}{2\cdot g}+y_2$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow

At each section $Q = V\cdot A = V\cdot b\cdot y$ or $V = \frac{Q}{b\cdot y}$

The given data is $b_1 = 2\cdot ft$ $y_1 = 1\cdot ft$ $b_2 = 1\cdot ft$ $y_2 = 0.75\cdot ft$

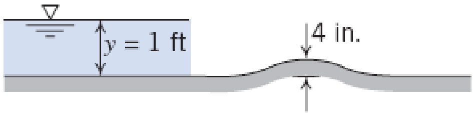
Hence the Bernoulli equation becomes (with $p_1 = p_2 = p_{atm}$)
$$\frac{\left(\frac{Q}{b_1\cdot y_1}\right)^2}{2\cdot g}+y_1=\frac{\left(\frac{Q}{b_2\cdot y_2}\right)^2}{2\cdot g}+y_2$$

Solving for Q
$$Q=\sqrt{\frac{2\cdot g\cdot\left(y_1-y_2\right)}{\left(\frac{1}{b_2\cdot y_2}\right)^2-\left(\frac{1}{b_1\cdot y_1}\right)^2}}$$
$$Q = 3.24\cdot\frac{ft^3}{s}$$

Problem 11.38

[3]

11.38 A rectangular channel 10 ft wide carries 100 cfs on a horizontal bed at 1.0 ft depth. A smooth bump across the channel rises 4 in. above the channel bottom. Find the elevation of the liquid free surface above the bump.



Given:

Data on rectangular channel and a bump

Find:

Elevation of free surface above the bump

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^{\ 2}}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^{\ 2}}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $y_1 = 1 \cdot ft$ $h = 4 \cdot in$ $Q = 100 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $V_1 = 10 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^{\ 2}}{2 \cdot g} + y_1$ $E_1 = 2.554 \cdot ft$

Hence $E_1 = E_2 + h = \frac{V_2^{\ 2}}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 + h$ or $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 2.22 \cdot ft$

For $y_2 = 1 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 2.55 \cdot ft$ For $y_2 = 1.5 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 2.19 \cdot ft$

For $y_2 = 1.4 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 2.19 \cdot ft$ For $y_2 = 1.3 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 2.22 \cdot ft$

Hence $y_2 = 1.30 \cdot ft$

Note that $V_2 = \frac{Q}{b \cdot y_2}$ $V_2 = 7.69 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $Fr_1 = 1.76$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $Fr_2 = 1.19$

Problem 11.39

11.39 A rectangular channel 10 ft wide carries a discharge of 20 ft^3/sec at 0.9 ft depth. A smooth bump 0.2 ft high is placed on the floor of the channel. Estimate the local change in flow depth caused by the bump.

Given: Data on rectangular channel and a bump

Find: Local change in flow depth caused by the bump

Solution:

Basic equation: $$\frac{p_1}{\rho\cdot g}+\frac{V_1^2}{2\cdot g}+y_1=\frac{p_2}{\rho\cdot g}+\frac{V_2^2}{2\cdot g}+y_2+h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is y_2 + h, where h is the bump height

Recalling the specific energy $E=\frac{V^2}{2\cdot g}+y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V\cdot A = V\cdot b\cdot y$ or $V=\frac{Q}{b\cdot y}$

The given data is $b = 10\cdot ft$ $y_1 = 0.9\cdot ft$ $h = 0.2\cdot ft$ $Q = 20\cdot\frac{ft^3}{s}$

Hence we find $V_1=\frac{Q}{b\cdot y_1}$ $V_1 = 2.22\cdot\frac{ft}{s}$

and $E_1=\frac{V_1^2}{2\cdot g}+y_1$ $E_1 = 0.977\cdot ft$

Hence $E_1 = E_2+h=\frac{V_2^2}{2\cdot g}+y_2+h=\frac{Q^2}{2\cdot g\cdot b^2\cdot y_2^2}+y_2+h$ or $\frac{Q^2}{2\cdot g\cdot b^2\cdot y_2^2}+y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 0.777\cdot ft$

For $y_2 = 0.9\cdot ft$ $\frac{Q^2}{2\cdot g\cdot b^2\cdot y_2^2}+y_2 = 0.977\cdot ft$ For $y_2 = 0.5\cdot ft$ $\frac{Q^2}{2\cdot g\cdot b^2\cdot y_2^2}+y_2 = 0.749\cdot ft$

For $y_2 = 0.6\cdot ft$ $\frac{Q^2}{2\cdot g\cdot b^2\cdot y_2^2}+y_2 = 0.773\cdot ft$ For $y_2 = 0.61\cdot ft$ $\frac{Q^2}{2\cdot g\cdot b^2\cdot y_2^2}+y_2 = 0.777\cdot ft$

Hence $y_2 = 0.61\cdot ft$ and $\frac{y_2 - y_1}{y_1} = -32.2\%$

Note that $V_2=\frac{Q}{b\cdot y_2}$ $V_2 = 3.28\cdot\frac{ft}{s}$

so we have $Fr_1=\frac{V_1}{\sqrt{g\cdot y_1}}$ $Fr_1 = 0.41$ and $Fr_2=\frac{V_2}{\sqrt{g\cdot y_2}}$ $Fr_2 = 0.74$

Problem 11.40

11.40 At a section of a 10-ft-wide rectangular channel, the depth is 0.3 ft for a discharge of 20 ft^3/sec. A smooth bump 0.1 ft high is placed on the floor of the channel. Determine the local change in flow depth caused by the bump.

Given: Data on rectangular channel and a bump

Find: Local change in flow depth caused by the bump

Solution:

Basic equation: $\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $\quad y_1 = 0.3 \cdot ft$ $\quad h = 0.1 \cdot ft$ $\quad Q = 20 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $\quad V_1 = 6.67 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $\quad E_1 = 0.991 \cdot ft$

Hence $E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 + h$ or $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 0.891 \cdot ft$

For $y_2 = 0.3 \cdot ft$ $\quad \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.991 \cdot ft$ $\qquad$ For $y_2 = 0.35 \cdot ft$ $\quad \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.857 \cdot ft$

For $y_2 = 0.33 \cdot ft$ $\quad \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.901 \cdot ft$ $\qquad$ For $y_2 = 0.334 \cdot ft$ $\quad \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.891 \cdot ft$

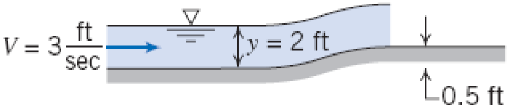
Hence $y_2 = 0.334 \cdot ft$ and $\frac{y_2 - y_1}{y_1} = 11.3\%$

Note that $V_2 = \frac{Q}{b \cdot y_2}$ $\quad V_2 = 5.99 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $\quad Fr_1 = 2.15$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $\quad Fr_2 = 1.83$

Problem 11.41

11.41 Water, at 3 ft/sec and 2 ft depth, approaches a smooth rise in a wide channel. Estimate the stream depth after the 0.5 ft rise.



Given: Data on wide channel

Find: Stream depth after rise

Solution:

Basic equation: $$\frac{p_1}{\rho\cdot g}+\frac{V_1^2}{2\cdot g}+y_1=\frac{p_2}{\rho\cdot g}+\frac{V_2^2}{2\cdot g}+y_2+h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2\cdot g}+y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V\cdot A = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2$ $\qquad V_2 = V_1\cdot\frac{y_1}{y_2}$

The given data is $y_1 = 2\cdot ft$ $\qquad V_1 = 3\cdot\frac{ft}{s}$ $\qquad h = 0.5\cdot ft$

Hence $E_1 = \frac{V_1^2}{2\cdot g}+y_1$ $\qquad E_1 = 2.14\cdot ft$

Then $E_1 = E_2 + h = \frac{V_2^2}{2\cdot g}+y_2+h = \frac{V_1^2\cdot y_1^2}{2\cdot g}\cdot\frac{1}{y_2^2}+y_2+h$ or $\frac{V_1^2\cdot y_1^2}{2\cdot g}\cdot\frac{1}{y_2^2}+y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 1.64\cdot ft$

For $y_2 = 2\cdot ft$ $\qquad \frac{V_1^2\cdot y_1^2}{2\cdot g}\cdot\frac{1}{y_2^2}+y_2 = 2.14\cdot ft$ $\qquad$ For $y_2 = 1.5\cdot ft$ $\qquad \frac{V_1^2\cdot y_1^2}{2\cdot g}\cdot\frac{1}{y_2^2}+y_2 = 1.75\cdot ft$

For $y_2 = 1.3\cdot ft$ $\qquad \frac{V_1^2\cdot y_1^2}{2\cdot g}\cdot\frac{1}{y_2^2}+y_2 = 1.63\cdot ft$ $\qquad$ For $y_2 = 1.31\cdot ft$ $\qquad \frac{V_1^2\cdot y_1^2}{2\cdot g}\cdot\frac{1}{y_2^2}+y_2 = 1.64\cdot ft$

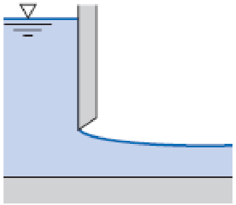
Hence $y_2 = 1.31\cdot ft$

Note that $V_2 = V_1\cdot\frac{y_1}{y_2}$ $\qquad V_2 = 4.58\cdot\frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\qquad Fr_1 = 0.37$ $\qquad$ and $\qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\qquad Fr_2 = 0.71$

Problem 11.42

11.42 Water issues from a sluice gate at 0.6 m depth. The discharge per unit width is 6.0 m^3/sec/m. Estimate the water level far upstream where the flow speed is negligible. Calculate the maximum rate of flow per unit width that could be delivered through the sluice gate.



Given: Data on sluice gate

Find: Water level upstream; Maximum flow rate

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^{\ 2}}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^{\ 2}}{2 \cdot g} + y_2 + h$$ The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, and V_1 is approximately zero (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$y_1 = \frac{V_2^{\ 2}}{2 \cdot g} + y_2$$

The given data is $\frac{Q}{b} = 6.0 \frac{m^2}{s}$ $\qquad y_2 = 0.6 \cdot m$

Hence $Q = V_2 \cdot A_2 = V_2 \cdot b \cdot y_2$ or $V_2 = \frac{Q}{b \cdot y_2}$ $\qquad V_2 = 10 \frac{m}{s}$

Then upstream $y_1 = \left(\frac{V_2^{\ 2}}{2 \cdot g} + y_2 \right)$ $\qquad y_1 = 5.70 m$

The maximum flow rate occurs at critical conditions (see Section 11-2), for constant specific energy

In this case $V_2 = V_c = \sqrt{g \cdot y_c}$

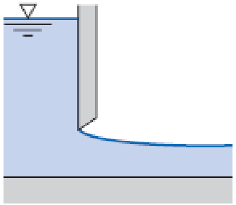
Hence we find $$y_1 = \frac{V_c^{\ 2}}{2 \cdot g} + y_c = \frac{g \cdot y_c}{2 \cdot g} + y_c = \frac{3}{2} \cdot y_c$$

Hence $y_c = \frac{2}{3} \cdot y_1$ $\qquad y_c = 3.80 m$ $\qquad V_c = \sqrt{g \cdot y_c}$ $\qquad V_c = 6.10 \frac{m}{s}$

$\frac{Q}{b} = V_c \cdot y_c$ $\qquad \frac{Q}{b} = 23.2 \frac{\frac{m^3}{s}}{m}$ (Maximum flow rate)

Problem 11.43

11.43 A horizontal rectangular channel 3 ft wide contains a sluice gate. Upstream of the gate the depth is 6 ft; the depth downstream is 0.9 ft. Estimate the volume flow rate in the channel.



Given: Data on sluice gate

Find: Flow rate

Solution:

Basic equation: $$\frac{p_1}{\rho\cdot g}+\frac{V_1^2}{2\cdot g}+y_1=\frac{p_2}{\rho\cdot g}+\frac{V_2^2}{2\cdot g}+y_2+h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$\frac{V_1^2}{2\cdot g}+y_1=\frac{V_2^2}{2\cdot g}+y_2$$

The given data is $b = 3\cdot ft$ $\quad y_1 = 6\cdot ft$ $\quad y_2 = 0.9\cdot ft$

Also $Q = V\cdot A$ so $V_1 = \frac{Q}{b\cdot y_1}$ and $V_2 = \frac{Q}{b\cdot y_2}$

Using these in the Bernoulli equation $$\frac{\left(\frac{Q}{b\cdot y_1}\right)^2}{2\cdot g}+y_1=\frac{\left(\frac{Q}{b\cdot y_2}\right)^2}{2\cdot g}+y_2$$

Solving for Q $$Q=\sqrt{\frac{2\cdot g\cdot b^2\cdot y_1^2\cdot y_2^2}{y_1+y_2}}\qquad Q = 49.5\frac{ft^3}{s}$$

Note that $V_1 = \frac{Q}{b\cdot y_1}$ $\quad V_1 = 2.75\frac{ft}{s}$ $\quad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 0.198$

$V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 18.3\frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 3.41$

Problem 11.44

[4]

11.44 Consider a 2.45-m-wide rectangular channel with a bed slope of 0.0004 and a Manning's roughness factor of 0.015. A weir is placed in the channel and the depth upstream of the weir is 1.52 m for a discharge of 5.66 m^3/s. Determine if a hydraulic jump forms upstream of the weir.

Given: Data on rectangular channel and weir

Find: If a hydraulic jump forms upstream of the weir

Solution:

Basic equations: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$ $\qquad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

Note that the Q equation is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 2.45 \cdot m$ and depth y we find from Table 11.2

$$A = B_w \cdot y = 2.45 \cdot y \qquad R = \frac{B_w \cdot y}{B_w + 2 \cdot y} = \frac{2.45 \cdot y}{2.45 + 2 \cdot y}$$

and also $\quad n = 0.015 \quad$ and $\quad S_0 = 0.0004 \quad Q = 5.66 \cdot \frac{m^3}{s}$

Hence $\quad Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.015} \cdot 2.45 \cdot y \cdot \left(\frac{2.45 \cdot y}{2.45 + 2 \cdot y}\right)^{\frac{2}{3}} \cdot 0.0004^{\frac{1}{2}} = 5.66 \quad$ (Note that we don't use units!)

Solving for y $\quad \frac{y^{\frac{5}{3}}}{(2.45 + 2 \cdot y)^{\frac{2}{3}}} = \frac{5.66 \cdot 0.015}{.0004^{\frac{1}{2}} \cdot 2.54 \cdot 2.54^{\frac{2}{3}}} \quad$ or $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2 \cdot y)^{\frac{2}{3}}} = 0.898$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with the given depth

For $\quad y = 1.52 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2 \cdot y)^{\frac{2}{3}}} = 0.639 \qquad$ For $\quad y = 2 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2 \cdot y)^{\frac{2}{3}}} = 0.908$

For $\quad y = 1.95 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2 \cdot y)^{\frac{2}{3}}} = 0.879 \qquad$ For $\quad y = 1.98 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2 \cdot y)^{\frac{2}{3}}} = 0.896$

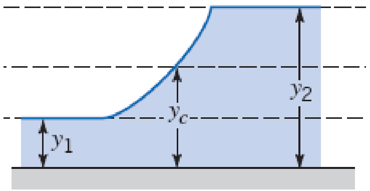
$y = 1.98 \quad$ (m) $\qquad$ This is the normal depth.

We also have the critical depth: $q = \dfrac{Q}{B_w}$ $q = 2.31\,\dfrac{m^2}{s}$ $y_c = \left(\dfrac{q^2}{g}\right)^{\frac{1}{3}}$ $y_c = 0.816\,m$

Hence the given depth is 1.52 m > y_c, but 1.52 m < y_n, the normal depth. This implies the flow is subcritical (far enough upstream it is dep 1.98 m), and that it draws down to 1.52 m as it gets close to the wier. There is no jump.

Problem 11.45

11.45 A hydraulic jump occurs in a rectangular channel 4.0 m wide. The water depth before the jump is 0.4 m and after the jump is 1.7 m. Compute the flow rate in the channel, the critical depth, and the headloss in the jump.



Given: Data on rectangular channel and hydraulic jump

Find: Flow rate; Critical depth; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^2}\right)$ $\quad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2\cdot g}\right) - \left(y_2 + \frac{V_2^2}{2\cdot g}\right)$ $\quad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

The given data is $b = 4\cdot m$ $\quad y_1 = 0.4\cdot m$ $\quad y_2 = 1.7\cdot m$

We can solve for Fr_1 from the basic equation $\sqrt{1+8\cdot Fr_1^2} = 1 + 2\cdot\frac{y_2}{y_1}$

$$Fr_1 = \sqrt{\frac{\left(1+2\cdot\frac{y_2}{y_1}\right)^2 - 1}{8}} \qquad Fr_1 = 3.34 \qquad \text{and} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$$

Hence $V_1 = Fr_1\cdot\sqrt{g\cdot y_1}$ $\quad V_1 = 6.62\frac{m}{s}$

Then $Q = V_1\cdot b\cdot y_1$ $\quad Q = 10.6\cdot\frac{m^3}{s}$ $\quad q = \frac{Q}{b}$ $\quad q = 2.65\frac{m^2}{s}$

The critical depth is $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\quad y_c = 0.894\,m$

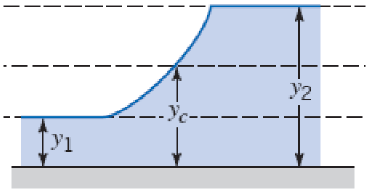
Also $V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 1.56\frac{m}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 0.381$

The energy loss is $H_l = \left(y_1 + \frac{V_1^2}{2\cdot g}\right) - \left(y_2 + \frac{V_2^2}{2\cdot g}\right)$ $\quad H_l = 0.808\,m$

Note that we could use the result of Example 11.9 $H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 0.808\,m$

Problem 11.46

11.46 A wide channel carries 20 ft^3/sec per foot of width at a depth of 1 ft at the toe of a hydraulic jump. Determine the depth of the jump and the head loss across it.



Given: Data on wide channel and hydraulic jump

Find: Jump depth; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^{\,2}}\right)$ $\quad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $\frac{Q}{b} = 20\frac{\frac{ft^3}{s}}{ft}$ $\quad y_1 = 1\cdot ft$

Also $Q = V\cdot A = V\cdot b\cdot y$

Hence $V_1 = \frac{Q}{b\cdot y_1}$ $\quad V_1 = 20.0\frac{ft}{s}$ $\quad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 3.53$

Then $y_2 = \frac{y_1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^{\,2}}\right)$ $\quad y_2 = 4.51\,ft$

$V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 4.43\frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 0.368$

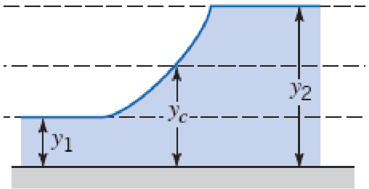
The energy loss is $H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$ $\quad H_l = 2.40\,ft$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 2.40\,ft$

Problem 11.47

[1]

11.47 A hydraulic jump occurs in a wide horizontal channel. The discharge is 30 ft^3/sec per foot of width. The upstream depth is 1.3 ft. Determine the depth of the jump.



Given: Data on wide channel and hydraulic jump

Find: Jump depth

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\ 2}}\right)$

The given data is $\frac{Q}{b} = 30\frac{\frac{ft^3}{s}}{ft}$ $\qquad y_1 = 1.3\cdot ft$

Also $Q = V\cdot A = V\cdot b\cdot y$

Hence $V_1 = \frac{Q}{b\cdot y_1}$ $\qquad V_1 = 23.1\frac{ft}{s}$ $\qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\qquad Fr_1 = 3.57$

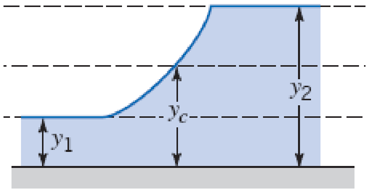
Then $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\ 2}}\right)$ $\qquad y_2 = 5.94\,ft$

Note: $V_2 = \frac{Q}{b\cdot y_2}$ $\qquad V_2 = 5.05\frac{ft}{s}$ $\qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\qquad Fr_2 = 0.365$

Problem 11.48

[2]

11.48 A hydraulic jump occurs in a rectangular channel. The flow rate is 200 ft³/sec, and the depth before the jump is 1.2 ft. Determine the depth behind the jump and the head loss, if the channel is 10 ft wide.



Given: Data on wide channel and hydraulic jump

Find: Jump depth; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $Q = 200\cdot\frac{ft^3}{s}$ $\quad b = 10\cdot ft$ $\quad y_1 = 1.2\cdot ft$

Also $Q = V\cdot A = V\cdot b\cdot y$

Hence $V_1 = \frac{Q}{b\cdot y_1}$ $\quad V_1 = 16.7\cdot\frac{ft}{s}$ $\quad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 2.68$

Then $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad y_2 = 3.99\cdot ft$

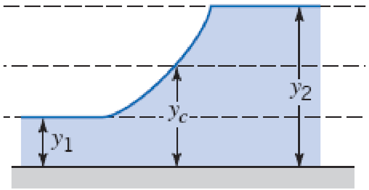
$V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 5.01\cdot\frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 0.442$

The energy loss is $H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$ $\quad H_l = 1.14\cdot ft$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{\left(y_2 - y_1\right)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 1.14\cdot ft$

Problem 11.49

11.49 The hydraulic jump may be used as a crude flow meter. Suppose that in a horizontal rectangular channel 5 ft wide the observed depths before and after a hydraulic jump are 0.66 and 3.0 ft. Find the rate of flow and the head loss.



Given: Data on wide channel and hydraulic jump

Find: Flow rate; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $b = 5\cdot ft$ $\quad y_1 = 0.66\cdot ft$ $\quad y_2 = 3.0\cdot ft$

We can solve for Fr_1 from the basic equation $\sqrt{1 + 8\cdot Fr_1^{\,2}} = 1 + 2\cdot\frac{y_2}{y_1}$

$$Fr_1 = \sqrt{\frac{\left(1 + 2\cdot\frac{y_2}{y_1}\right)^2 - 1}{8}} \qquad Fr_1 = 3.55 \qquad \text{and} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$$

Hence $V_1 = Fr_1\cdot\sqrt{g\cdot y_1}$ $\quad V_1 = 16.4\cdot\frac{ft}{s}$

Then $Q = V_1\cdot b\cdot y_1$ $\quad Q = 54.0\frac{ft^3}{s}$

Also $V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 3.60\cdot\frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 0.366$

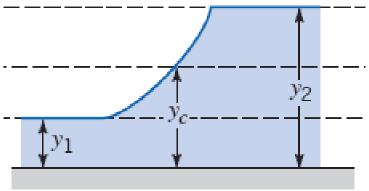
The energy loss is $H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$ $\quad H_l = 1.62\cdot ft$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{\left(y_2 - y_1\right)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 1.62\cdot ft$

Problem 11.50

[2]

11.50 A hydraulic jump occurs on a horizontal apron downstream from a wide spillway, at a location where depth is 0.9 m and speed is 25 m/sec. Estimate the depth and speed downstream from the jump. Compare the specific energy downstream of the jump to that upstream.



Given: Data on wide spillway flow

Find: Depth after hydraulic jump; Specific energy change

Solution:

Basic equations:

$$\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^{\,2}}\right) \qquad H_l = E_1 - E_2 = \left(y_1+\frac{V_1^{\,2}}{2\cdot g}\right)-\left(y_2+\frac{V_2^{\,2}}{2\cdot g}\right)$$

The given data is

$$y_1 = 0.9\cdot m \qquad V_1 = 25\frac{m}{s}$$

Then Fr_1 is

$$Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}} \qquad Fr_1 = 8.42$$

Hence

$$y_2 = \frac{y_1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^{\,2}}\right) \qquad y_2 = 10.3\,m$$

Then

$$Q = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2 \qquad V_2 = V_1\cdot\frac{y_1}{y_2} \qquad V_2 = 2.19\frac{m}{s}$$

For the specific energies

$$E_1 = y_1+\frac{V_1^{\,2}}{2\cdot g} \qquad E_1 = 32.8\,m$$

$$E_2 = y_2+\frac{V_2^{\,2}}{2\cdot g} \qquad E_2 = 10.5\,m \qquad \frac{E_2}{E_1} = 0.321$$

The energy loss is

$$H_l = E_1 - E_2 \qquad H_l = 22.3\,m$$

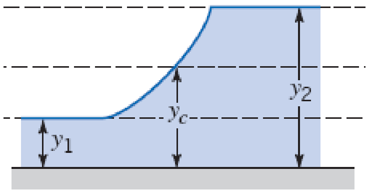
Note that we could use the result of Example 11.9

$$H_l = \frac{\left(y_2-y_1\right)^3}{4\cdot y_1\cdot y_2} \qquad H_l = 22.3\cdot m$$

Problem 11.51

[2]

11.51 A hydraulic jump occurs in a rectangular channel. The flow rate is 6.5 m^3/sec and the depth before the jump is 0.4 m. Determine the depth after the jump and the head loss, if the channel is 1 m wide.



Given: Data on rectangular channel flow

Find: Depth after hydraulic jump; Specific energy change

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\qquad$ $H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $y_1 = 0.4\cdot m$ $\qquad$ $b = 1\cdot m$ $\qquad$ $Q = 6.5\frac{m^3}{s}$

Then $Q = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2$ $\qquad$ $V_1 = \frac{Q}{b\cdot y_1}$ $\qquad$ $V_1 = 16.3\frac{m}{s}$

Then Fr_1 is $Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\qquad$ $Fr_1 = 8.20$

Hence $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\qquad$ $y_2 = 4.45\,m$

and $V_2 = \frac{Q}{b\cdot y_2}$ $\qquad$ $V_2 = 1.46\frac{m}{s}$

For the specific energies $E_1 = y_1 + \frac{V_1^{\,2}}{2\cdot g}$ $\qquad$ $E_1 = 13.9\,m$

$E_2 = y_2 + \frac{V_2^{\,2}}{2\cdot g}$ $\qquad$ $E_2 = 4.55\,m$

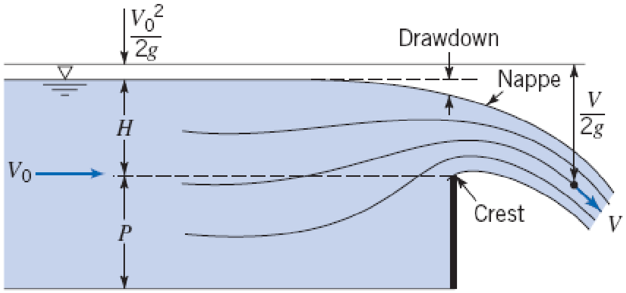
The energy loss is $H_l = E_1 - E_2$ $\qquad$ $H_l = 9.31\,m$

Note that we could use the result of Example 11.9 $\qquad$ $H_l = \frac{\left(y_2 - y_1\right)^3}{4\cdot y_1\cdot y_2}$ $\qquad$ $H_l = 9.31\cdot m$

Problem 11.52

[3]

11.52 A rectangular, sharp-crested weir with end contraction is 1.6 m long. How high should it be placed in a channel to maintain an upstream depth of 2.5 m for 0.5 m^3/s flow rate?



Given: Data on rectangular, sharp-crested weir

Find: Required weir height

Solution:

Basic equations: $Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}}$ where $C_d = 0.62$ and $B' = B - 0.1 \cdot n \cdot H$ with $n = 2$

Given data: $B = 1.6 \cdot m$ $Q = 0.5 \cdot \frac{m^3}{s}$

Hence we find

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}} = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}}$$

Rearranging $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = \frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d}$

This is a nonlinear implicit equation for H and must be solved numerically. We can use one of a number of numerical root finding techniq such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

The right side evaluates to $\frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d} = 0.273\,m^{\frac{5}{2}}$

For $H = 1 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 1.40\,m^{\frac{5}{2}}$ For $H = 0.5 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.530\,m^{\frac{5}{2}}$

For $H = 0.3 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.253\,m^{\frac{5}{2}}$ For $H = 0.35 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.317\,m^{\frac{5}{2}}$

For $H = 0.31 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.265\,m^{\frac{5}{2}}$ For $H = 0.315 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.272\,m^{\frac{5}{2}}$

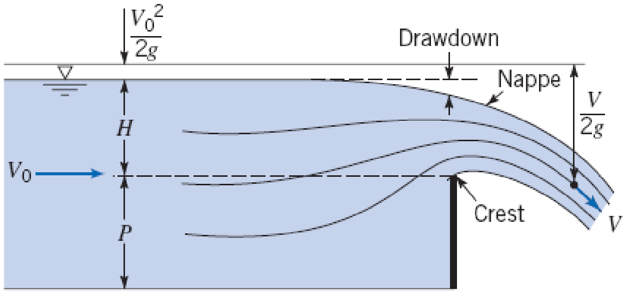
For $H = 0.316 \cdot m$ $(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.273\,m^{\frac{5}{2}}$ $H = 0.316\,m$

But from the figure $H + P = 2.5 \cdot m$ $P = 2.5 \cdot m - H$ $P = 2.18\,m$

Problem 11.53

[1]

11.53 For a sharp-crested suppressed weir ($C_w = 3.33$) of length $B = 8.0$ ft, $P = 2.0$ ft, and $H = 1.0$ ft, determine the discharge over the weir. Neglect the velocity of approach head.



Given: Data on rectangular, sharp-crested weir

Find: Discharge

Solution:

Basic equation: $Q = C_w \cdot B \cdot H^{\frac{3}{2}}$ where $C_w = 3.33$ and $B = 8 \cdot ft$ $P = 2 \cdot ft$ $H = 1 \cdot ft$

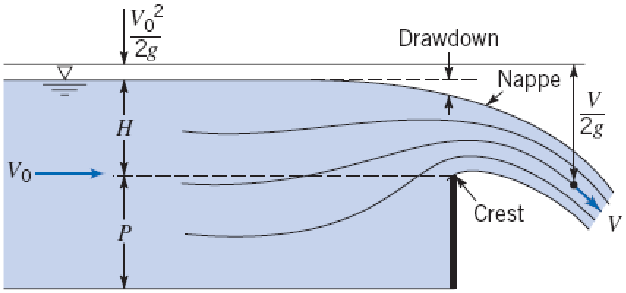
Note that this is an "engineering" equation, to be used without units!

$$Q = C_w \cdot B \cdot H^{\frac{3}{2}} \qquad Q = 26.6 \, \frac{ft^3}{s}$$

Problem 11.54

[3]

11.54 A rectangular sharp-crested weir with end contractions is 1.5 m long. How high should the weir crest be placed in a channel to maintain an upstream depth of 2.5 m for 0.5 m^3/s flow rate?



Given: Data on rectangular, sharp-crested weir

Find: Required weir height

Solution:

Basic equations: $Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}}$ where $C_d = 0.62$ and $B' = B - 0.1 \cdot n \cdot H$ with $n = 2$

Given data: $B = 1.5 \cdot m$ $\quad Q = 0.5 \cdot \frac{m^3}{s}$

Hence we find

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}} = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}}$$

Rearranging

$$(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = \frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d}$$

This is a nonlinear implicit equation for H and must be solved numerically. We can use one of a number of numerical root finding techniq such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

The right side evaluates to $\frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d} = 0.273 \cdot m^{\frac{5}{2}}$

For $H = 1 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 1.30 \cdot m^{\frac{5}{2}}$ $\qquad$ For $H = 0.5 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.495 \cdot m^{\frac{5}{2}}$

For $H = 0.3 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.237 \cdot m^{\frac{5}{2}}$ $\qquad$ For $H = 0.35 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.296 \cdot m^{\frac{5}{2}}$

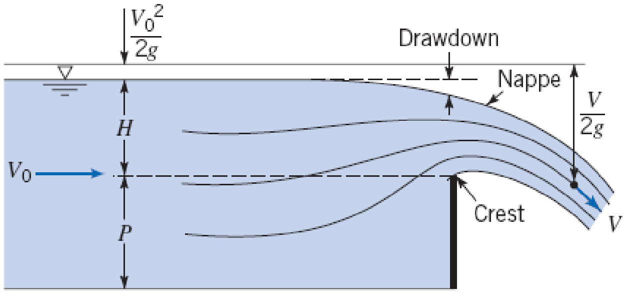
For $H = 0.34 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.284 \cdot m^{\frac{5}{2}}$ $\qquad$ For $H = 0.33 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.272 \cdot m^{\frac{5}{2}}$

For $H = 0.331 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.273 \cdot m^{\frac{5}{2}}$ $\qquad$ $H = 0.331 m$

But from the figure $\quad H + P = 2.5 \cdot m$ $\qquad P = 2.5 \cdot m - H$ $\qquad P = 2.17 m$

Problem 11.55

11.55 Determine the head on a 60° V-notch weir for a discharge of 150 l/s. Take $C_d = 0.58$.



Given: Data on V-notch weir

Find: Flow head

Solution:

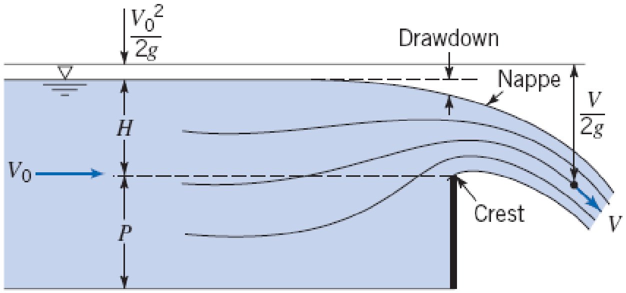
Basic equation: $Q = C_d \cdot \frac{8}{15} \cdot \sqrt{2 \cdot g} \cdot \tan\left(\frac{\theta}{2}\right) \cdot H^{\frac{5}{2}}$ where $C_d = 0.58$ $\theta = 60 \cdot \text{deg}$ $Q = 150 \cdot \frac{L}{s}$

$$H = \left(\frac{Q}{C_d \cdot \frac{8}{15} \cdot \sqrt{2 \cdot g} \cdot \tan\left(\frac{\theta}{2}\right)} \right)^{\frac{2}{5}}$$

$H = 0.514\,\text{m}$

Problem 11.56

11.56 The head on a 90° V-notch weir is 1.5 ft. Determine the discharge.



Given: Data on V-notch weir

Find: Discharge

Solution:

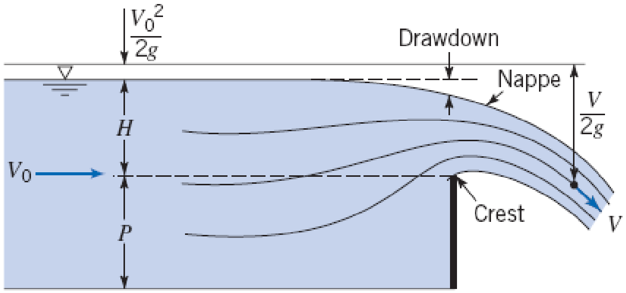
Basic equation: $Q = C_w \cdot H^{\frac{5}{2}}$ where $H = 1.5 \cdot ft$ $C_w = 2.50$ for $\theta = 90 \cdot deg$

Note that this is an "engineering" equation in which we ignore units!

$$Q = C_w \cdot H^{\frac{5}{2}} \qquad Q = 6.89 \frac{ft^3}{s}$$

Problem 11.57

11.57 Determine the weir coefficient of a 90° V-notch weir for a head of 180 mm for a flow rate of 20 l/s.



Given: Data on V-notch weir

Find: Weir coefficient

Solution:

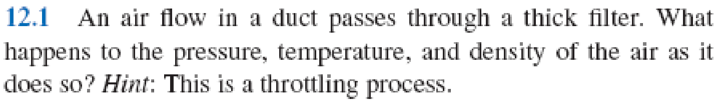
Basic equation: $Q = C_w \cdot H^{\frac{5}{2}}$ where $H = 180 \cdot \text{mm}$ $Q = 20 \cdot \frac{L}{s}$

Note that this is an "engineering" equation in which we ignore units!

$$C_w = \frac{Q}{H^{\frac{5}{2}}} \qquad C_w = 1.45$$

Problem 12.1

[2]



12.1 An air flow in a duct passes through a thick filter. What happens to the pressure, temperature, and density of the air as it does so? *Hint*: This is a throttling process.

Given: Air flow through a filter

Find: Change in p, T and ρ

Solution:

Basic equations: $h_2 - h_1 = c_p \cdot (T_2 - T_1)$ $p = \rho \cdot R \cdot T$

Assumptions: 1) Ideal gas 2) Throttling process

In a throttling process enthalpy is constant. Hence $h_2 - h_1 = 0$ so $T_2 - T_1 = 0$ or $T = \text{constant}$

The filter acts as a resistance through which there is a pressure drop (otherwise there would be no flow. Hence $p_2 < p_1$

From the ideal gas equation $\dfrac{p_1}{p_2} = \dfrac{\rho_1 \cdot T_1}{\rho_2 \cdot T_2}$ so $\rho_2 = \rho_1 \cdot \left(\dfrac{T_1}{T_2}\right) \cdot \left(\dfrac{p_2}{p_1}\right) = \rho_1 \cdot \left(\dfrac{p_2}{p_1}\right)$ Hence $\rho_2 < \rho_1$

The governing equation for entropy is $\Delta s = c_p \cdot \ln\left(\dfrac{T_2}{T_1}\right) - R \cdot \ln\left(\dfrac{p_2}{p_1}\right)$

Hence $\Delta s = -R \cdot \ln\left(\dfrac{p_2}{p_1}\right)$ and $\dfrac{p_2}{p_1} < 1$ so $\Delta s > 0$

Entropy increases because throttling is an irreversible adiabatic process

Problem 12.2

[2]

Given: Steady flow through a turbine. Air expands from $T_1 = 1300°C$, $p_1 = 2.0$ MPa (abs) to $T_2 = 500°C$, $p_2 = 101$ kPa.

Find: (a) $u_2 - u_1$ (b) $h_2 - h_1$ (c) $s_2 - s_1$
(d) show process on a Ts diagram

Solution:

$\Delta u = u_2 - u_1 = c_v (T_2 - T_1) = 717.4 \frac{J}{kg \cdot K} (500 - 1300) K = -574 \text{ kJ/kg}$ ← $u_2 - u_1$

$\Delta h = h_2 - h_1 = c_p (T_2 - T_1) = 1004 \frac{J}{kg \cdot K} (-800 K) = -803 \text{ kJ/kg}$ ← $h_2 - h_1$

To calculate the entropy change, we use the Tds equation

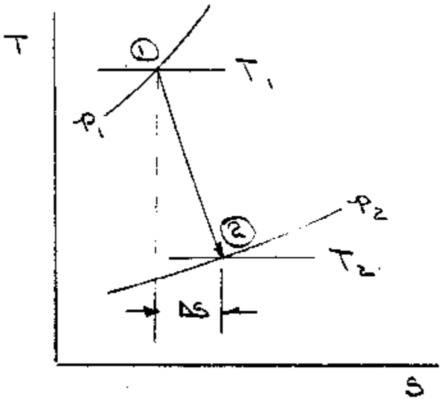
$T ds = dh - v\,dp = c_p dT - RT \frac{dp}{p}$

$\therefore \quad ds = c_p \frac{dT}{T} - R \frac{dp}{p}$

$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

$= 1004 \frac{J}{kg \cdot K} \ln \frac{500 + 273}{1300 + 273} - 286.9 \frac{J}{kg \cdot K} \ln \frac{0.101}{2.0}$

$s_2 - s_1 = (-713.3 + 856.6) \text{ J/kg} \cdot K = 143 \text{ J/kg} \cdot K$ ← $s_2 - s_1$



Problem 12.3

12.3 A vendor claims that an adiabatic air compressor takes in air at standard atmosphere conditions and delivers the air at 650 kPa (gage) and 285°C. Is this possible? Justify your answer by calculation. Sketch the process on a *Ts* diagram.

Given: Data on an air compressor

Find: Whether or not the vendor claim is feasible

Solution:

Basic equation: $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

The data provided, or available in the Appendices, is:

$p_1 = 101 \cdot \text{kPa}$ $\qquad T_1 = (20 + 273) \cdot \text{K}$

$p_2 = (650 + 101) \cdot \text{kPa}$ $\qquad T_2 = (285 + 273) \cdot \text{K}$

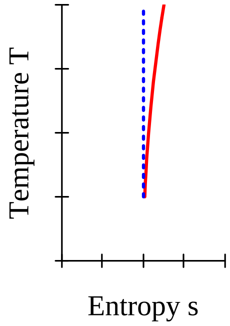
$c_p = 1004 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}}$ $\qquad R = 287 \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}}$

Then $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ $\qquad \Delta s = 71.0 \frac{\text{J}}{\text{kg} \cdot \text{K}}$

The second law of thermodynamics states that, for an adiabatic process

$\Delta s \geq 0$ or for all real processes $\Delta s > 0$

Hence the process is feasible!



Problem 12.4

12.4 What is the lowest possible delivery temperature generated by an adiabatic air compressor, starting with standard atmosphere conditions and delivering the air at 100 psig? Sketch the process on a *Ts* diagram.

Given: Adiabatic air compressor

Find: Lowest delivery temperature; Sketch the process on a Ts diagram

Solution:

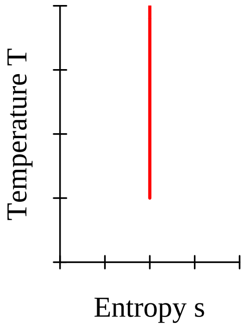
Basic equation: $$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

The lowest temperature implies an ideal (reversible) process; it is also adiabatic, so Δs = 0, and $$T_2 = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$$

The data provided, or available in the Appendices, is $p_1 = 14.7 \cdot \text{psi}$ $p_2 = (100 + 14.7) \cdot \text{psi}$ $T_1 = (68 + 460) \cdot R$ $k = 1.4$

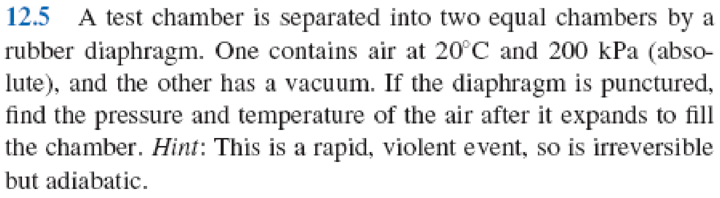
Hence $$T_2 = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$$ $T_2 = 950\,R$ $T_2 = 490\,°F$

The process is



Problem 12.5

[2]



12.5 A test chamber is separated into two equal chambers by a rubber diaphragm. One contains air at 20°C and 200 kPa (absolute), and the other has a vacuum. If the diaphragm is punctured, find the pressure and temperature of the air after it expands to fill the chamber. *Hint*: This is a rapid, violent event, so is irreversible but adiabatic.

Given: Test chamber with two chambers

Find: Pressure and temperature after expansion

Solution:

Basic equation: $p = \rho \cdot R \cdot T$ $\quad \Delta u = q - w$ (First law - closed system) $\quad \Delta u = c_v \cdot \Delta T$

Assumptions: 1) Ideal gas 2) Adiabatic 3) No work

For no work and adiabatic the first law becomes $\Delta u = 0$ or for an Ideal gas $\Delta T = 0$ $\quad T_2 = T_1$

We also have $M = \rho \cdot Vol = const$ and $Vol_2 = 2 \cdot Vol_1$ so $\rho_2 = \frac{1}{2} \cdot \rho_1$

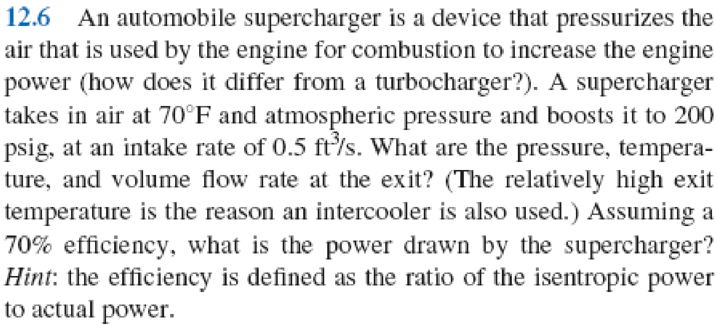
From the ideal gas equation $\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_2}{T_1} = \frac{1}{2}$ so $p_2 = \frac{1}{2} \cdot p_1$

Hence $T_2 = 20\,°F$ $\quad p_2 = \frac{200 \cdot kPa}{2}$ $\quad p_2 = 100 \cdot kPa$

Note that $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) = -R \cdot \ln\left(\frac{1}{2}\right) = 0.693 \cdot R$ so entropy increases (irreversible adiabatic)

Problem 12.6

[2]



12.6 An automobile supercharger is a device that pressurizes the air that is used by the engine for combustion to increase the engine power (how does it differ from a turbocharger?). A supercharger takes in air at 70°F and atmospheric pressure and boosts it to 200 psig, at an intake rate of 0.5 ft^3/s. What are the pressure, temperature, and volume flow rate at the exit? (The relatively high exit temperature is the reason an intercooler is also used.) Assuming a 70% efficiency, what is the power drawn by the supercharger? *Hint*: the efficiency is defined as the ratio of the isentropic power to actual power.

Given: Supercharger

Find: Pressure, temperature and flow rate at exit; power drawn

Solution:

Basic equation: $p = \rho \cdot R_{air} \cdot T$ $\qquad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$\Delta h = q - w$ (First law - open system) $\qquad \Delta h = c_p \cdot \Delta T$

Assumptions: 1) Ideal gas 2) Adiabatic

In an ideal process (reversible and adiabatic) the first law becomes $\Delta h = w$ or for an Ideal gas $w_{ideal} = c_p \cdot \Delta T$

For an isentropic process $\Delta s = 0 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ or $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$

The given or available data is $T_1 = (70 + 460) \cdot R$ $\qquad p_1 = 14.7 \cdot psi$ $\qquad p_2 = (200 + 14.7) \cdot psi$ $\qquad \eta = 70 \cdot \%$

$Q_1 = 0.5 \cdot \frac{ft^3}{s}$ $\qquad k = 1.4$ $\qquad c_p = 0.2399 \cdot \frac{Btu}{lbm \cdot R}$ $\qquad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Hecne $T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \cdot T_1$ $\qquad T_2 = 1140 \cdot R$ $\qquad T_2 = 681 \cdot °F$ $\qquad p_2 = 215 \cdot psi$

We also have $m_{rate} = \rho_1 \cdot Q_1 = \rho_2 \cdot Q_2$ $\qquad Q_2 = Q_1 \cdot \frac{\rho_1}{\rho_2}$ $\qquad Q_2 = Q_1 \cdot \frac{p_1}{p_2} \cdot \frac{T_2}{T_1}$ $\qquad Q_2 = 0.0737 \cdot \frac{ft^3}{s}$

For the power we use $P_{ideal} = m_{rate} \cdot w_{ideal} = \rho_1 \cdot Q_1 \cdot c_p \cdot \Delta \cdot T$

From the ideal gas equation $\rho_1 = \frac{p_1}{R_{air} \cdot T_1}$ $\qquad \rho_1 = 0.00233 \cdot \frac{slug}{ft^3}$ or $\rho_1 = 0.0749 \cdot \frac{lbm}{ft^3}$

Hence $P_{ideal} = \rho_1 \cdot Q_1 \cdot c_p \cdot (T_2 - T_1)$ $\qquad P_{ideal} = 5.78 \cdot kW$

The actual power needed is $P_{actual} = \frac{P_{ideal}}{\eta}$ $\qquad P_{actual} = 8.26 \cdot kW$

A supercharger is a pump that forces air into an engine, but generally refers to a pump that is driven directly by the engine, as opposed to a turbocharger that is driven by the pressure of the exhaust gases.

Problem 12.7

[2]

12.7 Five kilograms of air is cooled in a closed tank from 250 to 50°C. The initial pressure is 3 MPa. Compute the changes in entropy, internal energy, and enthalpy. Show the process state points on a *Ts* diagram.

Given: Cooling of air in a tank

Find: Change in entropy, internal energy, and enthalpy

Solution:

Basic equation: $p = \rho \cdot R \cdot T$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$\Delta u = c_v \cdot \Delta T$ $\quad \Delta h = c_p \cdot \Delta T$

Assumptions: 1) Ideal gas 2) Constant specific heats

Given or available data $M = 5 \cdot kg$ $\quad T_1 = (250 + 273) \cdot K$ $\quad T_2 = (50 + 273) \cdot K$ $\quad p_1 = 3 \cdot MPa$

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = 717.4 \cdot \frac{J}{kg \cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.4$ $\quad R = c_p - c_v$ $\quad R = 287 \cdot \frac{J}{kg \cdot K}$

For a constant volume process the ideal gas equation gives $\frac{p_2}{p_1} = \frac{T_2}{T_1}$ $\quad p_2 = \frac{T_2}{T_1} \cdot p_1$ $\quad p_2 = 1.85 \cdot MPa$

Then $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ $\quad \Delta s = -346 \cdot \frac{J}{kg \cdot K}$

$\Delta u = c_v \cdot \left(T_2 - T_1\right)$ $\quad \Delta u = -143 \cdot \frac{kJ}{kg}$

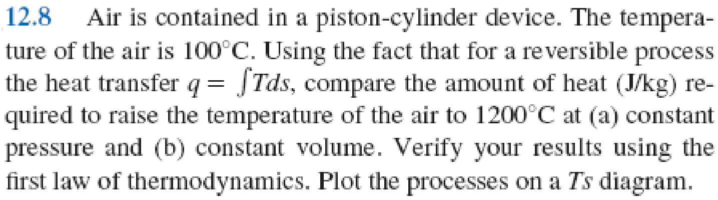
$\Delta h = c_p \cdot \left(T_2 - T_1\right)$ $\quad \Delta h = -201 \cdot \frac{kJ}{kg}$

Total amounts are $\Delta S = M \cdot \Delta s$ $\quad \Delta S = -1729 \cdot \frac{J}{K}$

$\Delta U = M \cdot \Delta u$ $\quad \Delta U = -717 \cdot kJ$

$\Delta H = M \cdot \Delta h$ $\quad \Delta H = -1004 \cdot kJ$

Problem 12.8



12.8 Air is contained in a piston-cylinder device. The temperature of the air is 100°C. Using the fact that for a reversible process the heat transfer $q = \int T ds$, compare the amount of heat (J/kg) required to raise the temperature of the air to 1200°C at (a) constant pressure and (b) constant volume. Verify your results using the first law of thermodynamics. Plot the processes on a *Ts* diagram.

Given: Air in a piston-cylinder

Find: Heat to raise temperature to 1200ºC at a) constant pressure and b) constant volume

Solution:

The data provided, or available in the Appendices, is:

$T_1 = (100 + 273) \cdot K$ $\quad T_2 = (1200 + 273) \cdot K$ $\quad R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 717 \cdot \frac{J}{kg \cdot K}$

a) For a constant pressure process we start with $\quad T \cdot ds = dh - v \cdot dp$

Hence, for *p* = const. $\quad ds = \frac{dh}{T} = c_p \cdot \frac{dT}{T}$

But $\quad \delta q = T \cdot ds$

Hence $\quad \delta q = c_p \cdot dT \quad q = \int c_p\, dT \quad q = c_p \cdot (T_2 - T_1) \quad q = 1104 \cdot \frac{kJ}{kg}$

b) For a constant volume process we start $\quad T \cdot ds = du + p \cdot dv$

Hence, for v = const. $\quad ds = \frac{du}{T} = c_v \cdot \frac{dT}{T}$

But $\quad \delta q = T \cdot ds$

Hence $\quad \delta q = c_v \cdot dT \quad q = \int c_v\, dT \quad q = c_v \cdot (T_2 - T_1) \quad q = 789 \cdot \frac{kJ}{kg}$

Heating to a higher temperature at constant pressure requires more heat than at constant volume: some of the heat is used to do work in expanding the gas; hence for constant pressure less of the heat is available for raising the temperature.

From the first law: Constant pressure: $q = \Delta u + w$ Constant volume: $q = \Delta u$

The two processes can be plotted using Eqs. 11.11b and 11.11a, simplified for the case of constant pressure and constant volume.

a) For constant pressure $\quad s_2 - s_1 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ so $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right)$

b) For constant volume $\quad s_2 - s_1 = c_v \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{v_2}{v_1}\right)$ so $\quad \Delta s = c_v \cdot \ln\left(\frac{T_2}{T_1}\right)$

The processes are plotted in the associated *Excel* workbook

Problem 12.8 (In Excel)

[3]

12.8 Air is contained in a piston-cylinder device. The temperature of the air is 100°C. Using the fact that for a reversible process the heat transfer $q = \int T ds$, compare the amount of heat (J/kg) required to raise the temperature of the air to 1200°C at (a) constant pressure and (b) constant volume. Verify your results using the first law of thermodynamics. Plot the processes on a *Ts* diagram.

Given: Air in a piston-cylinder

Find: Heat to raise temperature to 1200^{o}C at a) constant pressure and b) constant volume; plot

Solution:

The given or available data is:

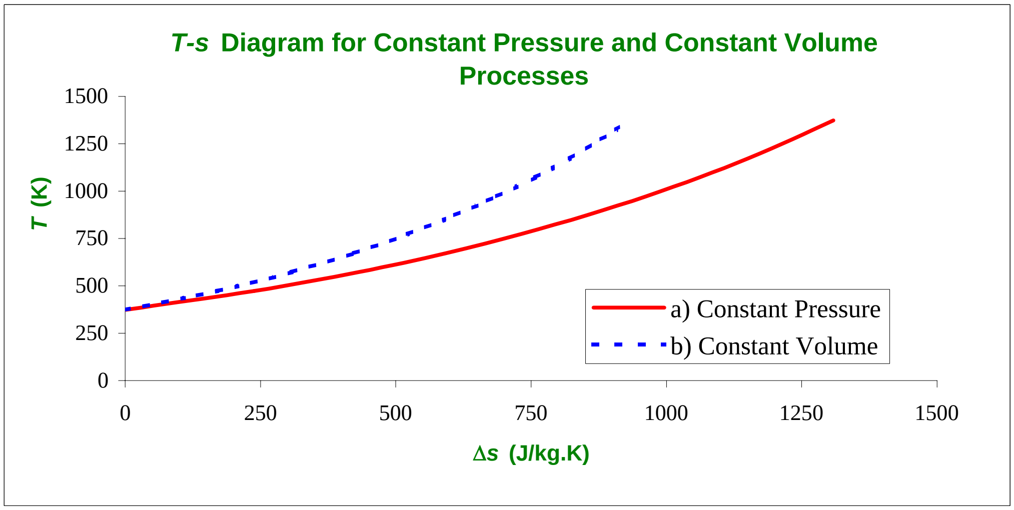
$T_1 =$	100	^{o}C
$T_2 =$	1200	^{o}C
$R =$	287	J/kg.K
$c_p =$	1004	J/kg.K
$c_v =$	717	J/kg.K

The equations to be plotted are:

a) For constant pressure
$$s_2 - s_1 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

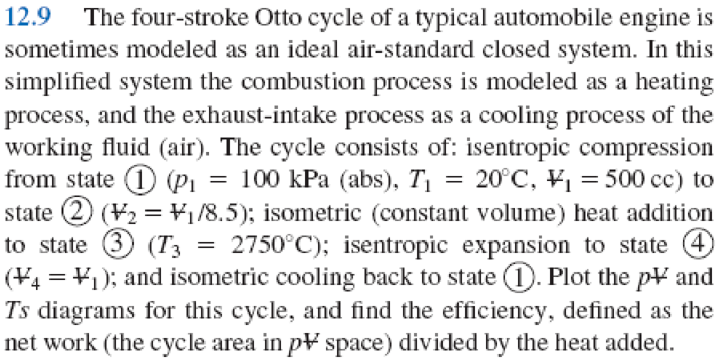
b) For constant volume
$$s_2 - s_1 = c_v \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{v_2}{v_1}\right)$$

T (K)	a) Δ*s* J/kg·K)	b) Δ*s* J/kg·K)
373	0	0
473	238	170
573	431	308
673	593	423
773	732	522
873	854	610
973	963	687
1073	1061	758
1173	1150	821
1273	1232	880
1373	1308	934



Problem 12.9

[4]



12.9 The four-stroke Otto cycle of a typical automobile engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① ($p_1 = 100$ kPa (abs), $T_1 = 20°\text{C}$, $V_1 = 500$ cc) to state ② ($V_2 = V_1/8.5$); isometric (constant volume) heat addition to state ③ ($T_3 = 2750°\text{C}$); isentropic expansion to state ④ ($V_4 = V_1$); and isometric cooling back to state ①. Plot the pV and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in pV space) divided by the heat added.

Given: Data on Otto cycle

Find: Plot of *pV* and *Ts* diagrams; efficiency

Solution:

The data provided, or available in the Appendices, is:

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 717 \cdot \frac{J}{kg \cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.4$

$p_1 = 100 \cdot kPa$ $\quad T_1 = (20 + 273) \cdot K$ $\quad T_3 = (2750 + 273) \cdot K$ $\quad V_1 = 500 \cdot cc$ $\quad V_2 = \frac{V_1}{8.5}$ $\quad V_2 = 58.8 \cdot cc$

$V_4 = V_1$

Computed results: $\quad M = \frac{p_1 \cdot V_1}{R \cdot T_1}$ $\quad M = 5.95 \times 10^{-4} kg$

For process 1-2 we have isentropic behavior $\quad T \cdot v^{k-1} = \text{constant}$ $\quad p \cdot v^k = \text{constant}$ $\quad$ (12.12 a and 12.12b)

Hence $\quad T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{k-1}$ $\quad T_2 = 690\,K$ $\quad p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^k$ $\quad p_2 = 2002 \cdot kPa$

The process from 1 -2 is $\quad p(V) = p_1 \cdot \left(\frac{V_1}{V}\right)^k$ $\quad$ and $\quad s = \text{constant}$

The work is $\quad W_{12} = \left(\int_{V_1}^{V_2} p(V)\, dV = \frac{p_1 \cdot V_1 - p_2 \cdot V_2}{k - 1}\right)$ $\quad W_{12} = -169\,J$ $\quad Q_{12} = 0 \cdot J$ $\quad$ (Isentropic)

For process 2 - 3 we have constant volume $\quad V_3 = V_2$ $\quad V_3 = 58.8 \cdot cc$

Hence $\quad p_3 = p_2 \cdot \frac{T_3}{T_2}$ $\quad p_3 = 8770 \cdot kPa$

The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_2}\right)$ $W_{23} = 0 \cdot J$

(From 12.11a)

$Q_{23} = M \cdot \Delta u = M \cdot \int c_v \, dT$ $Q_{23} = M \cdot c_v \cdot \left(T_3 - T_2\right)$ $Q_{23} = 995 J$

For process 3 - 4 we again have isentropic behavior

Hence $T_4 = T_3 \cdot \left(\frac{V_3}{V_4}\right)^{k-1}$ $T_4 = 1284 K$ $p_4 = p_3 \cdot \left(\frac{V_3}{V_4}\right)^{k}$ $p_4 = 438 \cdot kPa$

The process from 3 - 4 is $p(V) = p_3 \cdot \left(\frac{V_3}{V}\right)^{k}$ and $s = \text{constant}$

The work is $W_{34} = \frac{p_3 \cdot V_3 - p_4 \cdot V_4}{k - 1}$ $W_{34} = 742 J$ $Q_{34} = 0 \cdot J$

For process 4-1 we again have constant volume

The process from 4 -1 is $V = V_4 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_4}\right)$ $W_{41} = 0 \cdot J$

(From 12.11a)

$Q_{41} = M \cdot c_v \cdot \left(T_1 - T_4\right)$ $Q_{41} = -422 J$

The net work is $W_{net} = W_{12} + W_{23} + W_{34} + W_{41}$ $W_{net} = 572 J$

The efficiency is $\eta = \frac{W_{net}}{Q_{23}}$ $\eta = 57.5 \cdot \%$

This is consistent with the expression for the Otto efficiency $\eta_{Otto} = 1 - \frac{1}{r^{k-1}}$

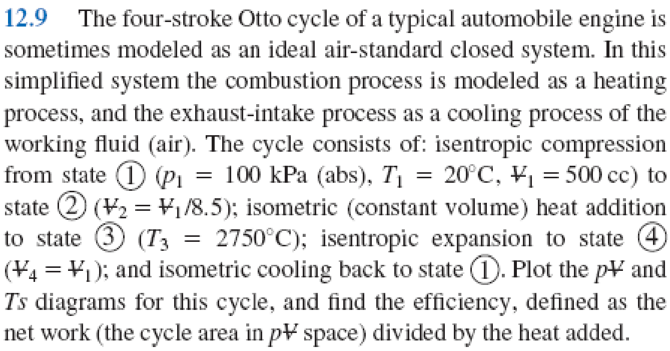
where *r* is the compression ratio $r = \frac{V_1}{V_2}$ $r = 8.5$

$\eta_{Otto} = 57.5 \cdot \%$

Plots of the cycle in *pV* and *Ts* space are shown in the associated *Excel* workbook

Problem 12.9 (In Excel)

[4]



12.9 The four-stroke Otto cycle of a typical automobile engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① (p_1 = 100 kPa (abs), T_1 = 20°C, $\mathcal{V}_1 = 500$ cc) to state ② ($\mathcal{V}_2 = \mathcal{V}_1/8.5$); isometric (constant volume) heat addition to state ③ (T_3 = 2750°C); isentropic expansion to state ④ ($\mathcal{V}_4 = \mathcal{V}_1$); and isometric cooling back to state ①. Plot the $p\mathcal{V}$ and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in $p\mathcal{V}$ space) divided by the heat added.

Given: Data on Otto cycle

Find: Plot of pV and Ts diagrams; efficiency

Solution:

The given, available, or computed data is:

R =	287	J/kg.K
c_p =	1004	J/kg.K
c_v =	717	J/kg.K
k =	1.4	

T_1 =	293	K	p_1 =	100	kPa	V_1 =	500	cc
T_2 =	690	K	p_2 =	2002	kPa	V_2 =	58.8	cc
T_3 =	3023	K	p_3 =	8770	kPa	V_3 =	58.8	cc
T_4 =	1284	K	p_4 =	438	kPa	V_4 =	500	cc

The process from 1 -2 is $p(V) = p_1 \cdot \left(\frac{V_1}{V}\right)^k$ and $s = \text{constant}$

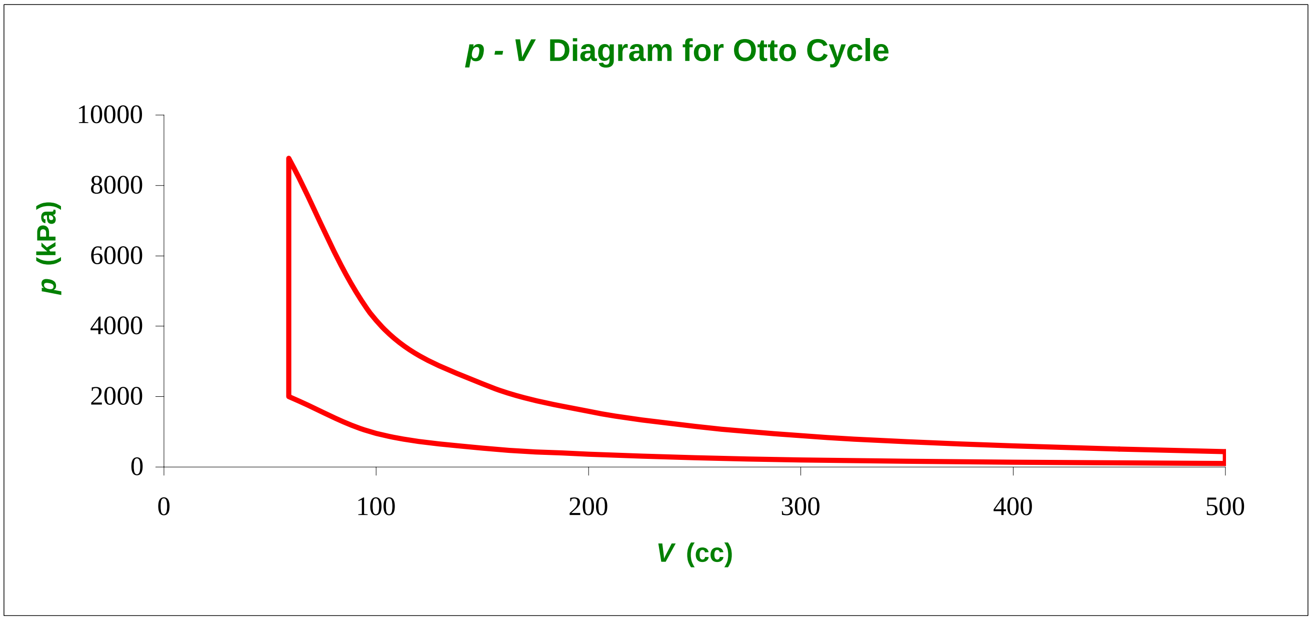
The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_2}\right)$

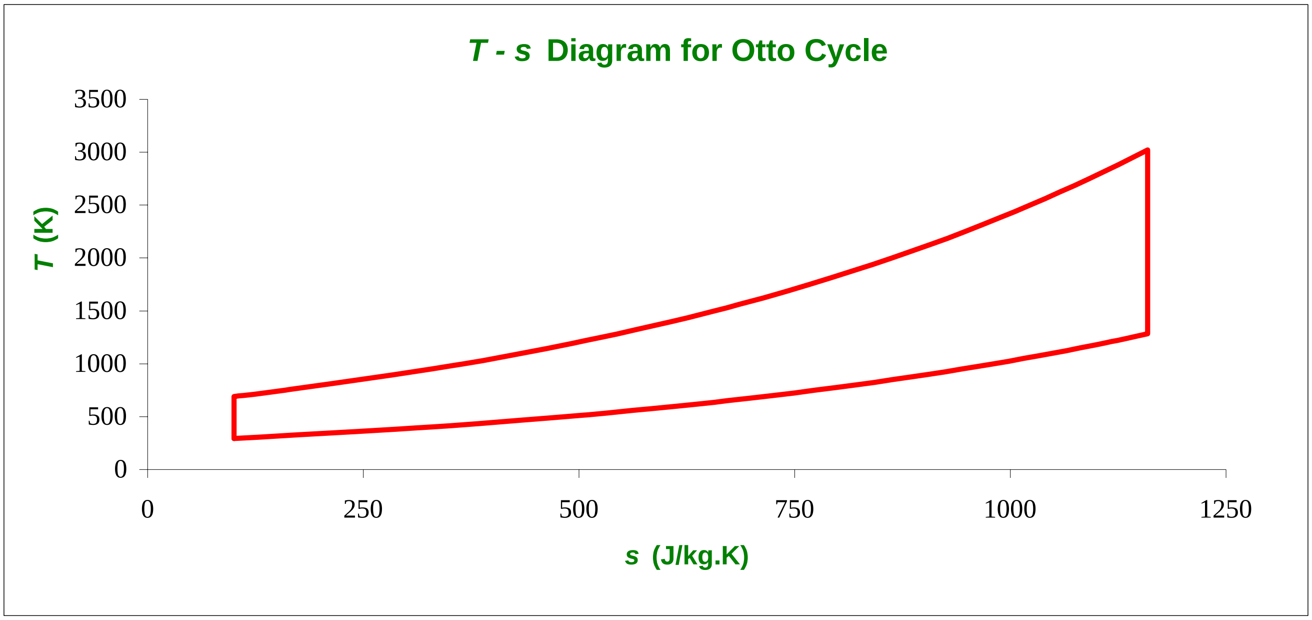
The process from 3 - 4 is $p(V) = p_3 \cdot \left(\frac{V_3}{V}\right)^k$ and $s = \text{constant}$

The process from 4 -1 is $V = V_4 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_4}\right)$

The computations are:

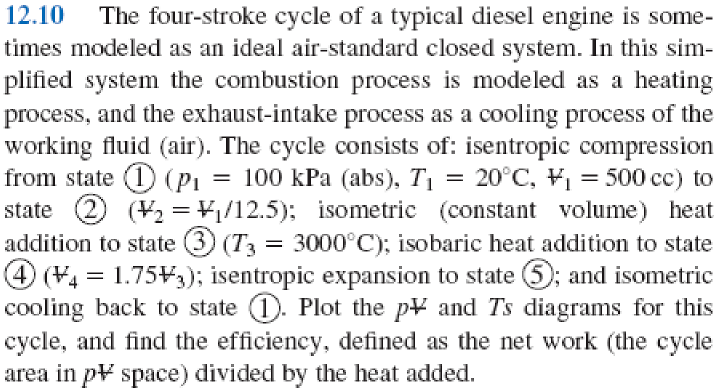
	V (cc)	p (kPa)	T (K)	s J/kg·K)	
1	500	100	293	100	Initial entropy is arbitrary
	450	116	306	100	Temperatures from Eq. 12.12b
	400	137	320	100	
	350	165	338	100	
	300	204	359	100	
	250	264	387	100	
	200	361	423	100	
	150	540	474	100	
	100	952	558	100	
2	58.8	2002	690	100	Uniform temperature steps
	58.8	2176	750	160	
	58.8	2901	1000	366	
	58.8	3626	1250	526	
	58.8	4352	1500	657	
	58.8	5077	1750	767	
	58.8	5802	2000	863	
	58.8	6527	2250	947	
	58.8	7253	2500	1023	
	58.8	7978	2750	1091	
3	58.8	8770	3023	1159	Temperatures from Eq. 12.12b
	100	4172	2445	1159	
	150	2364	2078	1159	
	200	1580	1852	1159	
	250	1156	1694	1159	
	300	896	1575	1159	
	350	722	1481	1159	
	400	599	1403	1159	
	450	508	1339	1159	
4	500	438	1284	1159	Uniform temperature steps
	500	410	1200	1111	
	500	375	1100	1049	
	500	341	1000	980	
	500	307	900	905	
	500	273	800	820	
	500	239	700	724	
	500	205	600	614	
	500	171	500	483	
	500	137	400	323	
1	500	100	293	100	


p - V Diagram for Otto Cycle
p (kPa)
10000
8000
6000
4000
2000
0
0
100
200
300
400
500
V (cc)


T - s Diagram for Otto Cycle
T (K)
3500
3000
2500
2000
1500
1000
500
0
0
250
500
750
1000
1250
s (J/kg.K)

Problem 12.10

[4]



12.10 The four-stroke cycle of a typical diesel engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① ($p_1 = 100$ kPa (abs), $T_1 = 20°\text{C}$, $\mathcal{V}_1 = 500$ cc) to state ② ($\mathcal{V}_2 = \mathcal{V}_1/12.5$); isometric (constant volume) heat addition to state ③ ($T_3 = 3000°\text{C}$); isobaric heat addition to state ④ ($\mathcal{V}_4 = 1.75\mathcal{V}_3$); isentropic expansion to state ⑤; and isometric cooling back to state ①. Plot the $p\mathcal{V}$ and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in $p\mathcal{V}$ space) divided by the heat added.

Given: Data on diesel cycle

Find: Plot of *pV* and *Ts* diagrams; efficiency

Solution:

The data provided, or available in the Appendices, is:

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 717 \frac{J}{kg \cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.4$

$p_1 = 100 \cdot kPa$ $\quad T_1 = (20 + 273) \cdot K$ $\quad T_3 = (3000 + 273) \cdot K$ $\quad V_1 = 500 \cdot cc$

$V_2 = \frac{V_1}{12.5}$ $\quad V_2 = 40\,cc$ $\quad V_3 = V_2$ $\quad V_4 = 1.75 \cdot V_3$ $\quad V_4 = 70\,cc$ $\quad V_5 = V_1$

Computed results: $\quad M = \frac{p_1 \cdot V_1}{R \cdot T_1}$ $\quad M = 5.95 \times 10^{-4} kg$

For process 1-2 we have isentropic behavior $\quad T \cdot v^{k-1} = \text{constant}$ (12.12a) $\quad p \cdot v^k = \text{constant}$ (12.12c)

Hence $\quad T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{k-1}$ $\quad T_2 = 805\,K$ $\quad p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^k$ $\quad p_2 = 3435\,kPa$

The process from 1 -2 is $\quad p(V) = p_1 \cdot \left(\frac{V_1}{V}\right)^k$ $\quad$ and $\quad s = \text{constant}$

The work is $\quad W_{12} = \int_{V_1}^{V_2} p(V)\, dV = \frac{p_1 \cdot V_1 - p_2 \cdot V_2}{k - 1}$ $\quad W_{12} = -218\,J$ $\quad Q_{12} = 0 \cdot J$ (Isentropic)

For process 2 - 3 we have constant volume $\quad V_3 = V_2$ $\quad V_3 = 40\,cc$

Hence $\quad p_3 = p_2 \cdot \frac{T_3}{T_2}$ $\quad p_3 = 13963\,kPa$

The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_2}\right)$ $W_{23} = 0 \cdot J$

(From Eq. 12.11a)

$Q_{23} = M \cdot \Delta u = M \cdot \int c_v \, dT$ $Q_{23} = M \cdot c_v \cdot \left(T_3 - T_2\right)$ $Q_{23} = 1052\,J$

For process 3 - 4 we have constant pressure $p_4 = p_3$ $p_4 = 13963\,kPa$ $T_4 = T_3 \cdot \left(\frac{V_4}{V_3}\right)$ $T_4 = 5728\,K$

The process from 3 - 4 is $p = p_3 = \text{constant}$ and $\Delta s = c_p \cdot \ln\left(\frac{T}{T_3}\right)$

(From Eq. 12.11b)

$W_{34} = p_3 \cdot \left(V_4 - V_3\right)$ $W_{34} = 419\,J$ $Q_{34} = M \cdot c_p \cdot \left(T_4 - T_3\right)$ $Q_{34} = 1465\,J$

For process 4 - 5 we again have isentropic behavior $T_5 = T_4 \cdot \left(\frac{V_4}{V_5}\right)^{k-1}$ $T_5 = 2607\,K$

Hence $p_5 = p_4 \cdot \left(\frac{V_4}{V_5}\right)^{k}$ $p_5 = 890\,kPa$

The process from 4 - 5 is $p(V) = p_4 \cdot \left(\frac{V_4}{V}\right)^{k}$ and $s = \text{constant}$

The work is $W_{45} = \frac{p_4 \cdot V_4 - p_5 \cdot V_5}{k - 1}$ $W_{45} = 1330\,J$ $Q_{45} = 0 \cdot J$

For process 5-1 we again have constant volume

The process from 5 -1 is $V = V_5 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_5}\right)$

(From Eq. 12.11a)

$Q_{51} = M \cdot c_v \cdot \left(T_1 - T_5\right)$ $Q_{51} = -987\,J$ $W_{51} = 0 \cdot J$

The net work is $W_{net} = W_{12} + W_{23} + W_{34} + W_{45} + W_{51}$ $W_{net} = 1531\,J$

The heat added is $Q_{added} = Q_{23} + Q_{34}$ $Q_{added} = 2517\,J$

The efficiency is $\eta = \frac{W_{net}}{Q_{added}}$ $\eta = 60.8\,\%$

This is consistent with the expression from thermodynamics for the diesel efficiency

$$\eta_{diesel} = 1 - \frac{1}{r^{k-1}} \cdot \left[\frac{r_c{}^k - 1}{k \cdot (r_c - 1)} \right]$$

where *r* is the compression ratio $r = \frac{V_1}{V_2}$ $r = 12.5$

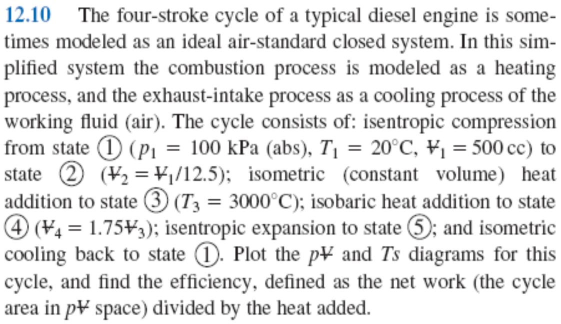
and r_c is the cutoff ratio $r_c = \frac{V_4}{V_3}$ $r_c = 1.75$

$$\eta_{diesel} = 58.8\%$$

The plots of the cycle in *pV* and *Ts* space are shown in the associated *Excel* workbook

Problem 12.10 (In Excel)

[4]



12.10 The four-stroke cycle of a typical diesel engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① (p_1 = 100 kPa (abs), T_1 = 20°C, $\mathcal{V}_1 = 500$ cc) to state ② ($\mathcal{V}_2 = \mathcal{V}_1/12.5$); isometric (constant volume) heat addition to state ③ (T_3 = 3000°C); isobaric heat addition to state ④ ($\mathcal{V}_4 = 1.75\mathcal{V}_3$); isentropic expansion to state ⑤; and isometric cooling back to state ①. Plot the $p\mathcal{V}$ and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in $p\mathcal{V}$ space) divided by the heat added.

Given: Data on diesel cycle

Find: Plot of *pV* and *Ts* diagrams; efficiency

Solution:

The given, available, or computed data is:

R =	287	J/kg.K
c_p =	1004	J/kg.K
c_v =	717	J/kg.K
k =	1.4	

T_1 =	293	K	p_1 =	100	kPa	V_1 =	500	cc
T_2 =	805	K	p_2 =	3435	kPa	V_2 =	40	cc
T_3 =	3273	K	p_3 =	13963	kPa	V_3 =	40	cc
T_4 =	5728	K	p_4 =	13963	kPa	V_4 =	70	cc
T_5 =	2607	K	p_5 =	890	kPa	V_5 =	500	cc

The process from 1 -2 is $p(V) = p_1\cdot\left(\frac{V_1}{V}\right)^k$ and $s = \text{constant}$

The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v\cdot\ln\left(\frac{T}{T_2}\right)$

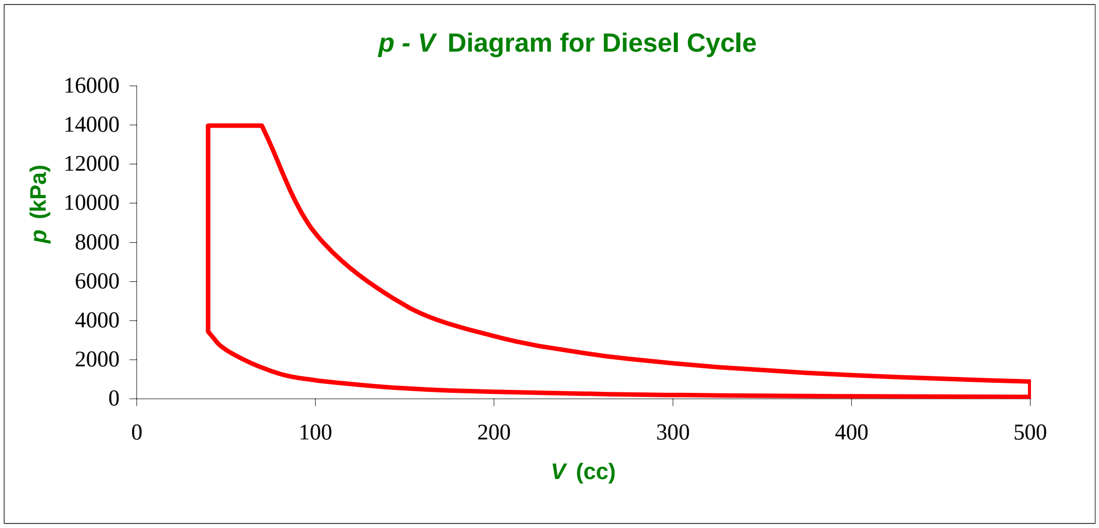
The process from 3 - 4 is $p = p_3 = \text{constant}$ and $\Delta s = c_p\cdot\ln\left(\frac{T}{T_3}\right)$

The process from 4 - 5 is $p(V) = p_4\cdot\left(\frac{V_4}{V}\right)^k$ and $s = \text{constant}$

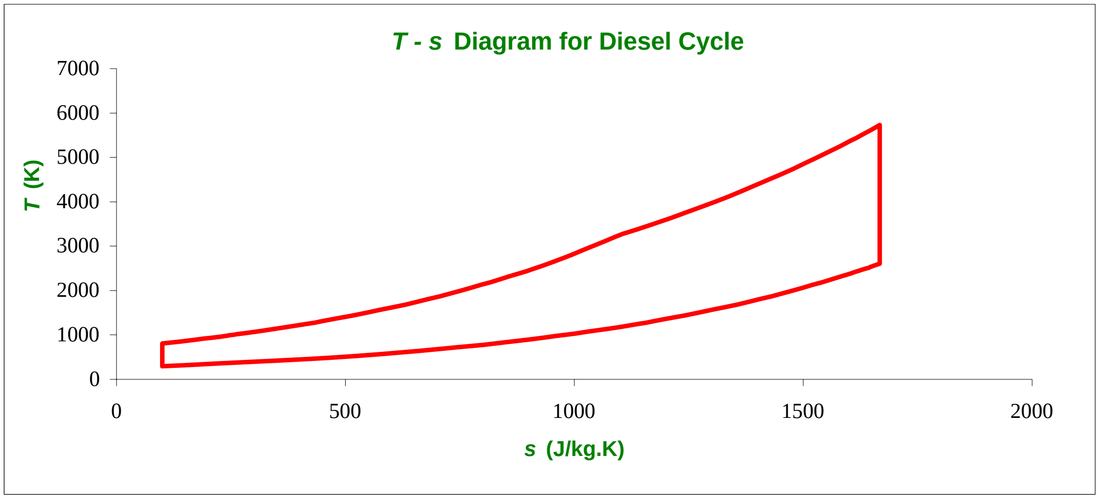
The process from 5 -1 is $V = V_5 = \text{constant}$ and $\Delta s = c_v\cdot\ln\left(\frac{T}{T_5}\right)$

The computations are:

	V (cc)	p (kPa)	T (K)	s J/kg.K)	
1	500	100	293	100	Initial entropy is arbitrary
	400	137	320	100	Temperatures from Eq. 12.12b
	300	204	359	100	
	250	264	387	100	
	200	361	423	100	
	150	540	474	100	
	100	952	558	100	
	75.0	1425	626	100	
	50.0	2514	736	100	
2	40.0	3435	805	100	Uniform temperature steps
	40.0	3840	900	180	
	40.0	4266	1000	255	
	40.0	5333	1250	415	
	40.0	6399	1500	546	
	40.0	7466	1750	657	
	40.0	8532	2000	752	
	40.0	9599	2250	837	
	40.0	10666	2500	912	
	40.0	11732	2750	981	
3	40.0	13963	3273	1105	Uniform temperature steps
	42.8	13963	3500	1173	
	45.8	13963	3750	1242	
	48.9	13963	4000	1307	
	51.9	13963	4250	1368	
	55.0	13963	4500	1425	
	58.1	13963	4750	1479	
	61.1	13963	5000	1531	
	64.2	13963	5250	1580	
	67.2	13963	5500	1627	
4	70.0	13963	5728	1667	Temperatures from Eq. 12.12b
	100	8474	4966	1667	
	150	4803	4222	1667	
	200	3210	3763	1667	
	250	2349	3441	1667	
	300	1820	3199	1667	
	350	1466	3007	1667	
	400	1216	2851	1667	
	450	1031	2720	1667	
5	500	890	2607	1667	Uniform temperature steps
	500	853	2500	1637	
	500	768	2250	1562	
	500	683	2000	1477	
	500	597	1750	1381	
	500	512	1500	1271	
	500	427	1250	1140	
	500	341	1000	980	
	500	256	750	774	
	500	171	500	483	
1	500	100	293	100	



p - V Diagram for Diesel Cycle
p (kPa)
16000
14000
12000
10000
8000
6000
4000
2000
0
0
100
200
300
400
500
V (cc)



T - s Diagram for Diesel Cycle
T (K)
7000
6000
5000
4000
3000
2000
1000
0
0
500
1000
1500
2000
s (J/kg.K)

Problem 12.11

Given: Air compressed from standard conditions to fill tank with $\forall = 10\ m^3$, $p = 4.5$ MPa (gage). Ideal gas. Reversible.

Find: (a) Energy for isothermal compression
(b) Energy for isentropic compression
(c) Energy removed by cooling from (b) to (a)
(d) Sketch Ts & pv diagrams

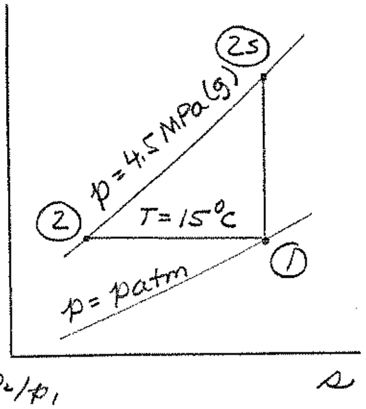
Solution: Apply ideal gas, energy, isentropic process equations.

Computing equations: $Q - W = \Delta E = m(u_2 - u_1)$, $W = -\int p\,d\forall$

$p/\rho^k = \text{constant}$, $p = \rho R T$

The tank contains $m = \rho\forall$

$$\rho = \frac{p}{RT} = (4.5 + 0.101)10^6 \frac{N}{m^2} \times \frac{kg \cdot K}{287\, N \cdot m} \times \frac{1}{(273+15)\,K} = 55.7\ \frac{kg}{m^3}$$



$$m = \rho\forall = 55.7\ \frac{kg}{m^3} \times 10\ m^3 = 557\ kg$$

For process $1 \rightarrow 2$,

$$-W = -\int p\,d\forall = -m\int p\,dv = -m\int RT\frac{dp}{p} = -mRT \ln p_2/p_1$$

$$W_{12} = -557\ kg \times 287 \frac{J}{kg \cdot K} \times (273+15)\,K \times \ln\left(\frac{4.5+0.101}{0.101}\right) = 176\ MJ$$

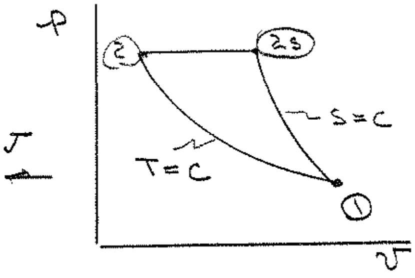
W_{12}

For process $2 \rightarrow 2s$, $\frac{p_{2s}}{p_1} = \left(\frac{\rho_{2s}}{\rho_1}\right)^k \rightarrow \frac{T_{2s}}{T_1} = \left(\frac{p_{2s}}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{4.601}{0.101}\right)^{0.286} = 2.98$

$T_{2s} = 2.98\,T_1 = 2.98(273+15)\,K = 858\ K$

T_{2s}

$$W_{12s} = m(u_{2s} - u_1) - \cancel{Q_{12s}}^{\,0} = m c_v (T_{2s} - T_1)$$



$$W_{12s} = 557\ kg \times 0.717 \frac{kJ}{kg \cdot K}(858 - 288)\,K = 228\ MJ$$

W_{12s}

Process $2s \rightarrow 2$ is at constant pressure

$$Q_{2s2} = m(u_2 - u_{2s}) - W_{2s2} = m(u_2 - u_{2s}) - mp(v_2 - v_{2s}) = m(h_2 - h_{2s})$$

$$= m c_p (T_2 - T_{2s})$$

$$Q_{2s2} = 557\ kg \times 1.00 \frac{kJ}{kg \cdot K} \times (288 - 858)\,K = -317\ MJ \quad \text{(out of air)}$$

Q_{2s2}

NATIONAL 42 381 50 SHEETS 5 SQUARE 42 382 100 SHEETS 5 SQUARE 42 389 200 SHEETS 5 SQUARE

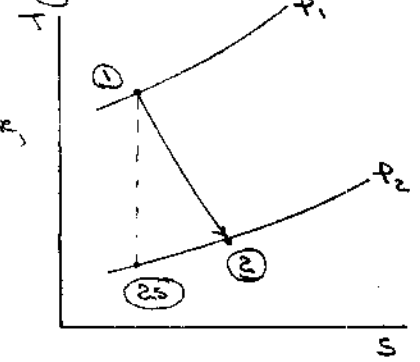
Problem 12.12

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: Steady flow of air, $\dot{m} = 0.5$ kg/s, through a turbine.
At inlet, $V_1 \approx 0$, $T_1 = 1300°C$, $p_1 = 2.0$ MPa (abs).
At outlet, $V_2 = 200$ m/s, $p_2 = 101$ kPa, $T_2 = 500°C$

Find: (a) power produced by the turbine
(b) label state points on a Ts diagram

Solution:



For an isentropic expansion through the turbine,

$$T_{2s=c} = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$T_{2s=c} = (1300+273)K \left(\frac{0.101 \text{ MPa}}{2.0 \text{ MPa}}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{2s=c} = 670 K \; (397°C)$$

Writing the first law of thermodynamics between the turbine inlet and outlet

$$\dot{W} + \cancel{\dot{Q}} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \cancel{\frac{V_1^2}{2}}^{\approx 0}\right)\right]$$

(Assume $\dot{Q} = 0$)

For an ideal gas with constant specific heats, $h_2 - h_1 = c_p(T_2 - T_1)$

$$\therefore \dot{W} = \dot{m}\left[c_p(T_2 - T_1) - \frac{V_2^2}{2}\right]$$

$$= 0.5 \frac{kg}{s}\left[1004 \frac{N \cdot m}{kg \cdot K}(773 - 1573)K - \frac{1}{2}(200)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}\right]$$

$$\dot{W} = -392 \times 10^3 \frac{N \cdot m}{s}$$ (negative sign indicates work out).

$$\therefore \dot{W}_{out} = 392 \text{ kW} \longleftarrow \dot{W}_{out}$$

Problem 12.13

[3]

Given: Natural gas (thermodynamic properties of methane) flows in a pipe of diameter, $D = 0.6\text{m}$. At compressor inlet: $T_1 = 13°C$, $V_1 = 32 \text{ m/s}$, $p_1 = 0.5 \text{ MPa (gage)}$. At compressor outlet, $p_2 = 8.0 \text{ MPa (gage)}$. The compressor efficiency $\eta_c = 0.85$.

Find: (a) $\dot{m}$ (b) T_2, V_2 (c) $\dot{W}_{in}$ d) label state points on Ts diagram

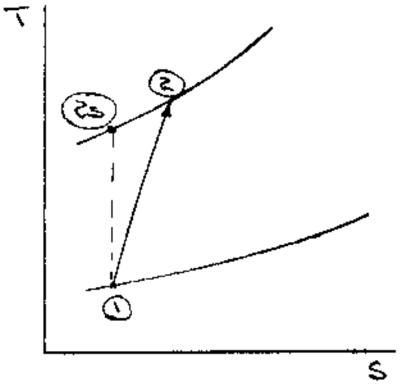
Solution:

The mass flow rate is given by $\dot{m} = \rho V A$ where $\rho = \frac{p}{RT}$

$$\therefore \dot{m} = \frac{p_1}{RT_1} V_1 \frac{\pi D^2}{4} = (500+101)10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{518.3 \, N \cdot m} \times \frac{1}{286 \, K} \times 32 \frac{m}{s} \times \frac{\pi}{4} (0.6)^2 m^2$$

$\dot{m} = 36.7 \text{ kg/s}$ $\longleftarrow$ $\dot{m}$

For an isentropic compression



$$T_{2s=c} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 286K \left(\frac{8.101 \text{ MPa}}{0.601 \text{ MPa}}\right)^{\frac{1.31-1}{1.31}} = 529 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad \therefore T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_c}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = 286K + \frac{(529-286)K}{0.85}$$

$T_2 = 572 \text{ K}$ $\longleftarrow$ T_2

From continuity, $\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$. Assuming $A_1 = A_2$, then

$$V_2 = \frac{\rho_1}{\rho_2} V_1 = \frac{p_1}{p_2} \frac{T_2}{T_1} V_1 = \frac{0.601}{8.101} \times \frac{572}{286} \times 32 \frac{m}{s} = 4.75 \text{ m/s}$$ $\longleftarrow$ V_2

Writing the first law of thermodynamics between compressor inlet + outlet

$$\dot{W} + \overset{=0}{\cancel{\dot{Q}}} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right] \qquad (\text{Assume } \dot{Q} = 0)$$

$$\dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)\right] = \dot{m}\left[c_p(T_2 - T_1) + \frac{1}{2}(V_2^2 - V_1^2)\right]$$

$$\dot{W} = 36.7 \frac{kg}{s}\left[2190 \frac{N \cdot m}{kg \cdot K}(572-286)K + \frac{1}{2}\left\{(4.75)^2 - (32)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}\right\}\right]$$

$$\dot{W} = 36.7\left[626 \times 10^3 - 501\right] \frac{N \cdot m}{s}$$

$\dot{W} = 23 \text{ MW}$ $\longleftarrow$ $\dot{W}_{in}$

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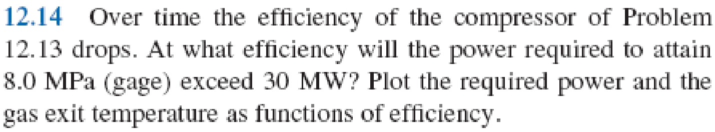
NATIONAL

Problem 12.14

[3]



12.14 Over time the efficiency of the compressor of Problem 12.13 drops. At what efficiency will the power required to attain 8.0 MPa (gage) exceed 30 MW? Plot the required power and the gas exit temperature as functions of efficiency.

Given: Data on flow through compressor

Find: Efficiency at which power required is 30 MW; plot required efficiency and exit temperature as functions of efficiency

Solution:

The data provided, or available in the Appendices, is:

$R = 518.3 \cdot \frac{J}{kg \cdot K}$ $\quad c_p = 2190 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 1672 \frac{J}{kg \cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.31$

$T_1 = (13 + 273) \cdot K$ $\quad p_1 = 0.5 \cdot MPa + 101 \cdot kPa$ $\quad V_1 = 32 \cdot \frac{m}{s}$

$p_2 = 8 \cdot MPa + 101 \cdot kPa$ $\quad W_{comp} = 30 \cdot MW$ $\quad D = 0.6 \cdot m$

The governing equation is the first law of thermodynamics for the compressor

$$M_{flow} \cdot \left[\left(h_2 + \frac{V_2^2}{2} \right) - \left(h_1 + \frac{V_1^2}{2} \right) \right] = W_{comp} \qquad \text{or} \qquad W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} \right]$$

We need to find the mass flow rate and the temperature and velocity at the exit

$$M_{flow} = \rho_1 \cdot A_1 \cdot V_1 = \frac{p_1}{R \cdot T_1} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_1 \qquad M_{flow} = \frac{p_1}{R \cdot T_1} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_1 \qquad M_{flow} = 36.7 \frac{kg}{s}$$

The exit velocity is then given by $M_{flow} = \frac{p_2}{R \cdot T_2} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_2$ $\qquad V_2 = \frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}$ $\qquad$ (1)

The exit velocity cannot be computed until the exit temperature is determined!

Using Eq. 1 in the first law

$$W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{\left(\frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2} \right)^2 - V_1^2}{2} \right]$$

In this complicated expression the only unknown is T_2, the exit temperature. The equation is a quadratic, so is solvable explicitly for T_2, but instead we use *Excel*'s *Goal Seek* to find the solution (the second solution is mathematically correct but physically unrealistic - a very large negative absolute temperature). The exit temperature is $T_2 = 660 \cdot K$

If the compressor was ideal (isentropic), the exit temperature would be given by

$$T \cdot p^{\frac{1-k}{k}} = \text{constant} \qquad (12.12b)$$

Hence $T_{2s} = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$ $T_{2s} = 529\,K$

For a compressor efficiency η, we have $\eta = \frac{h_{2s} - h_1}{h_2 - h_1}$ or $\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$ $\eta = 65.1\,\%$

To plot the exit temperature and power as a function of efficiency we use $T_2 = T_1 + \frac{T_{2s} - T_1}{\eta}$

with $V_2 = \frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}$ and $W_{comp} = M_{flow} \cdot \left[c_p \cdot \left(T_2 - T_1\right) + \frac{V_2^{\,2} - V_1^{\,2}}{2}\right]$

The dependencies of T_2 and W_{comp} on efficiency are plotted in the associated *Excel* workbook

Problem 12.14 (In Excel)

[3]

12.14 Over time the efficiency of the compressor of Problem 12.13 drops. At what efficiency will the power required to attain 8.0 MPa (gage) exceed 30 MW? Plot the required power and the gas exit temperature as functions of efficiency.

Given: Data on flow through compressor

Find: Efficiency at which power required is 30 MW; plot required efficiency and exit temperature as functions of efficiency

Solution:

The given or available data is:

R = 518.3 J/kg.K
c_p = 2190 J/kg.K
c_v = 1672 J/kg.K
k = 1.31
T_1 = 286 K
p_1 = 601 kPa
V_1 = 32 m/s
p_2 = 8101 kPa
D = 0.6 m/s
W_{comp} = 30 MW

Computed results:

$$M_{flow} = \frac{p_1}{R \cdot T_1} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_1$$

M_{flow} = 36.7 kg/s

$$W_{comp} = M_{flow} \cdot \left[c_p \cdot \left(T_2 - T_1\right) + \frac{\left(\dfrac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}\right)^2 - V_1^2}{2} \right]$$

Use *Goal Seek* to vary T_2 below so that the error between the left and right sides is zero!

T_2 = 660 K

LHS (MW)	RHS (MW)	Error
30.0	30.0	0.00%

$$T_{2s} = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$$

T_{2s} = 529 K

$$\eta = \frac{h_{2s} - h_1}{h_2 - h_1}$$

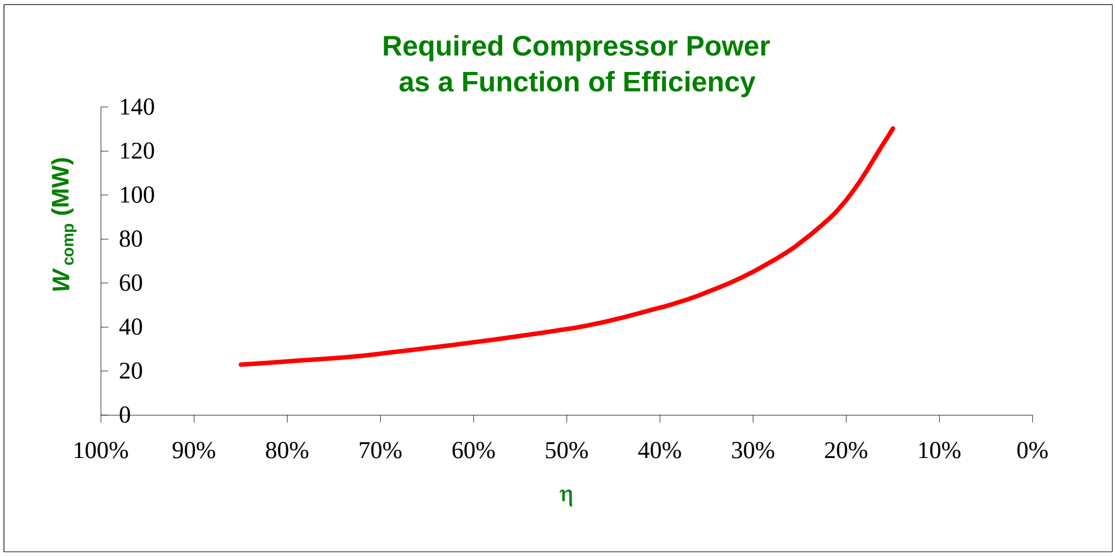
η = 65.1%

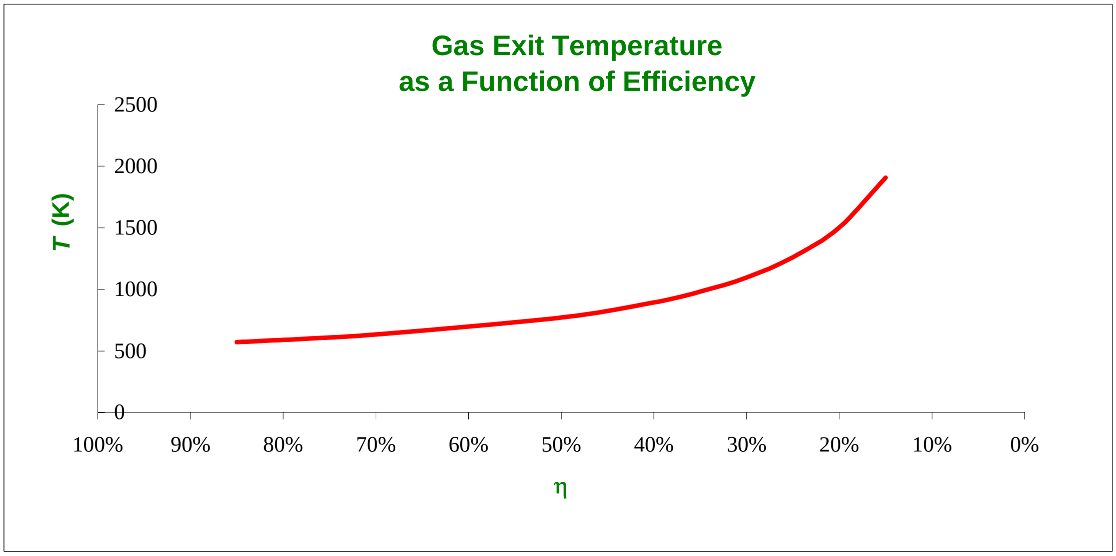
$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$V_2 = \frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}$$

$$W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{V_2^{\,2} - V_1^{\,2}}{2} \right]$$

η	T_2 (K)	V_2 (m/s)	W_{comp} (MW)
85%	572	4.75	23
80%	590	4.90	24
70%	634	5.26	28
50%	773	6.41	39
40%	894	7.42	49
35%	981	8.14	56
30%	1097	9.11	65
25%	1259	10.45	78
20%	1503	12.47	98
15%	1908	15.84	130





Problem 12.15

[4]

Given: Balloon inflated isothermally from $r = 5$ to $r = 7$ in.

Flow is $Q = 0.10$ cfm of standard air (59F, 14.7 psia)

Balloon skin tension is $\sigma = kA$, where $k = 200\ \text{lbf/ft}^3$, and A = surface area of balloon.

Find: Time required.

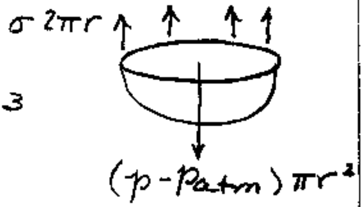
Solution: The mass flow rate is $\dot{m} = \rho_{std} Q$ = constant, so

Computing equation: $\Delta t = \dfrac{\Delta m}{\dot{m}}$ $\qquad p = \rho R T$

Assume: (1) Standard air, $\rho = 0.0765\ \text{lbm/ft}^3$; (2) Ideal gas

Then $\dot{m} = \rho Q = 0.0765 \dfrac{\text{lbm}}{\text{ft}^3} \times 0.10 \dfrac{\text{ft}^3}{\text{min}} \times \dfrac{\text{min}}{60\ \text{s}} = 1.28 \times 10^{-4}\ \text{lbm/s}$

From a force balance on the balloon:



$$(p - p_{atm})\pi r^2 = \sigma 2\pi r = k(4\pi r^2)2\pi r = 8\pi^2 k r^3$$

or $p = p_{atm} + 8\pi k r$

For $r = 5$ in., $p = 14.7 + 8\pi \times 200 \dfrac{\text{lbf}}{\text{ft}^3} \times 5\ \text{in.} \times \dfrac{\text{ft}^3}{1728\ \text{in.}^3} = 29.2$ psia

$$\rho = \frac{p}{RT} = 29.2 \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{lbm}\cdot{}^\circ\text{R}}{53.3\ \text{ft}\cdot\text{lbf}} \times \frac{1}{519^\circ\text{R}} \times 144 \frac{\text{in.}^2}{\text{ft}^2} = 0.152\ \text{lbm/ft}^3$$

$$\forall = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times (5)^3\ \text{in.}^3 \times \frac{\text{ft}^3}{1728\ \text{in.}^3} = 0.303\ \text{ft}^3$$

$$m = \rho \forall = 0.152 \frac{\text{lbm}}{\text{ft}^3} \times 0.303\ \text{ft}^3 = 0.0461\ \text{lbm}$$

For $r = 7$ in., $p = 14.7 \dfrac{\text{lbf}}{\text{in.}^2} + 8\pi \times 200 \dfrac{\text{lbf}}{\text{ft}^3} \times 7\ \text{in.} \times \dfrac{\text{ft}^3}{1728\ \text{in.}^3} = 35.1$ psia

Tabulating,

r (in.)	p (psia)	ρ (lbm/ft³)	$\forall$ (ft³)	m (lbm)
5	29.2	0.152	0.303	0.0461
7	35.1	0.183	0.831	0.152

Then $\Delta m = m_7 - m_5 = 0.152 - 0.0461\ \text{lbm} = 0.106\ \text{lbm}$

and

$$\Delta t = 0.106\ \text{lbm} \times \frac{\text{s}}{1.28 \times 10^{-4}\ \text{lbm}} = 828\ \text{s} \quad (\simeq 14\ \text{min})$$

Δt

Problem 12.16

[3]

12.16 For the balloon process of Problem 12.15 we could define a "volumetric ratio" as the ratio of the volume of standard air supplied to the volume increase of the balloon, per unit time. Plot this ratio over time as the balloon radius is increased from 5 to 7 inches.

Given: Data on flow rate and balloon properties

Find: "Volumetric efficiency" over time

Solution:

The given or available data is:

$R =$ 53.3 ft.lbf/lb°R

$T_{atm} =$ 519 R

$p_{atm} =$ 14.7 psi

$k =$ 200 $\mathrm{lbf/ft^3}$

$V_{rate} =$ 0.1 $\mathrm{ft^3/min}$

Computing equations:

Standard air density
$$\rho_{air} = \frac{p_{atm}}{R \cdot T_{atm}}$$

Mass flow rate
$$M_{rate} = V_{rate} \cdot \rho_{air}$$

From a force balance on each hemisphere
$$\left(p - p_{atm}\right) \cdot \pi \cdot r^2 = \sigma \cdot 2 \cdot \pi \cdot r \qquad \text{where} \qquad \sigma = k \cdot A = k \cdot 4 \cdot \pi \cdot r^2$$

Hence
$$p = p_{atm} + \frac{2 \cdot \sigma}{r} \qquad \text{or} \qquad p = p_{atm} + 8 \cdot \pi \cdot k \cdot r$$

Density in balloon
$$\rho = \frac{p}{R \cdot T_{air}}$$

The instantaneous volume is
$$V_{ball} = \frac{4}{3} \cdot \pi \cdot r^3$$

The instantaneous mass is
$$M_{ball} = V_{ball} \cdot \rho$$

The time to fill to radius *r* from *r* = 5 in is
$$t = \frac{M_{ball}(r) - M_{ball}(r = 5in)}{M_{rate}}$$

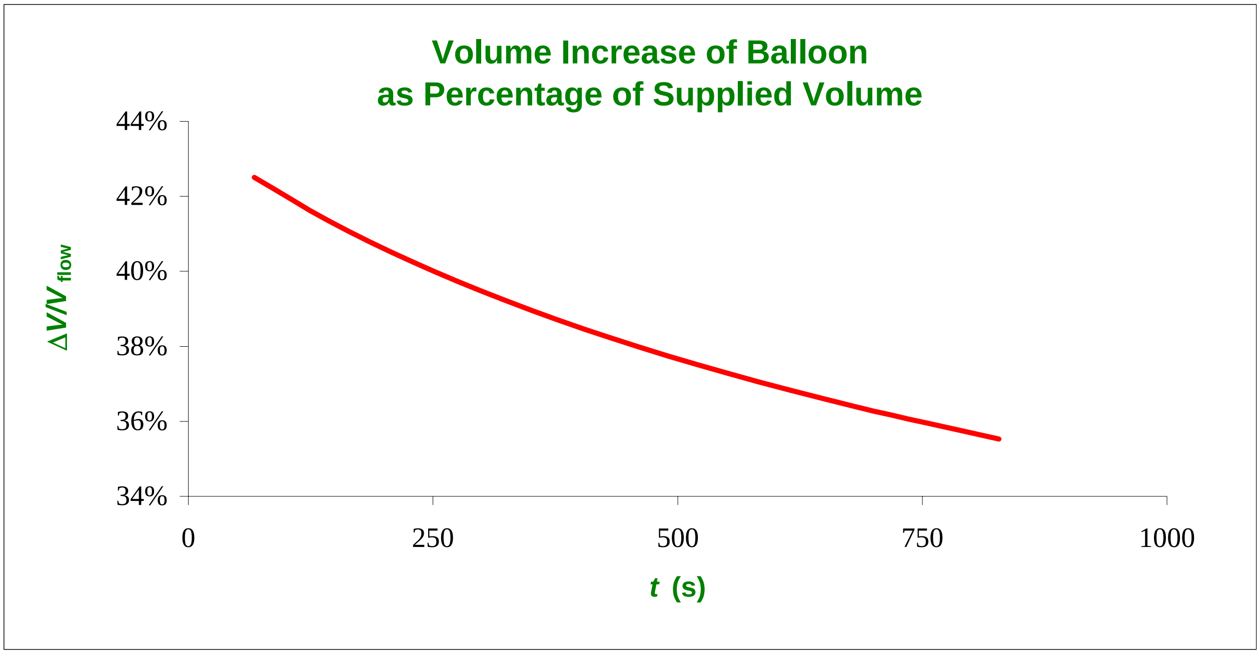
The volume change between time steps Δt is
$$\Delta V = V_{ball}(t + \Delta t) - V_{ball}(t)$$

Computed results:

$\rho_{air} =$ 0.0765 $\mathrm{lb/ft^3}$

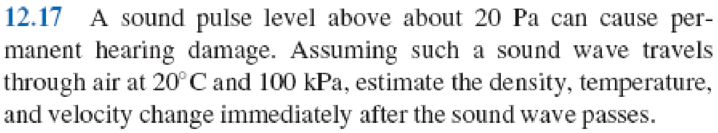
$M_{rate} =$ 0.000128 lb/s

r (in)	p (psi)	ρ (lb/ft^3)	V_{ball} (ft^3)	M_{ball} (lb)	t (s)	$\Delta V/V_{\text{rate}}$
5.00	29.2	0.152	0.303	0.0461	0.00	0.00
5.25	30.0	0.156	0.351	0.0547	67.4	42.5%
5.50	30.7	0.160	0.403	0.0645	144	41.3%
5.75	31.4	0.164	0.461	0.0754	229	40.2%
6.00	32.2	0.167	0.524	0.0876	325	39.2%
6.25	32.9	0.171	0.592	0.101	433	38.2%
6.50	33.6	0.175	0.666	0.116	551	37.3%
6.75	34.3	0.179	0.746	0.133	683	36.4%
7.00	35.1	0.183	0.831	0.152	828	35.5%



Problem 12.17

[3]



12.17 A sound pulse level above about 20 Pa can cause permanent hearing damage. Assuming such a sound wave travels through air at 20°C and 100 kPa, estimate the density, temperature, and velocity change immediately after the sound wave passes.

Given: Sound wave

Find: Estimate of change in density, temperature, and velocity after sound wave passes

Solution:

Basic equation: $p = \rho \cdot R \cdot T$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$du = c_v \cdot dT$ $\quad dh = c_p \cdot dT$

Assumptions: 1) Ideal gas 2) Constant specific heats 3) Isentropic process 4) infinitesimal changes

Given or available data

$T_1 = (20 + 273) \cdot K$ $\quad p_1 = 100 \cdot kPa$ $\quad dp = 20 \cdot Pa$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

$c = \sqrt{k \cdot R \cdot T_1}$ $\quad c = 343 \frac{m}{s}$

For small changes, from Section 11-2 $\quad dp = c^2 \cdot d\rho$ $\quad$ so $\quad d\rho = \frac{dp}{c^2}$ $\quad d\rho = 1.70 \times 10^{-4} \cdot \frac{kg}{m^3}$ $\quad$ a very small change!

The air density is $\rho_1 = \frac{p_1}{R \cdot T_1}$ $\quad \rho_1 = 1.19 \frac{kg}{m^3}$

Then $\quad dV_x = \frac{1}{\rho_1 \cdot c} \cdot dp$ $\quad dV_x = 0.049 \frac{m}{s}$ $\quad$ This is the velocity of the air after the sound wave!

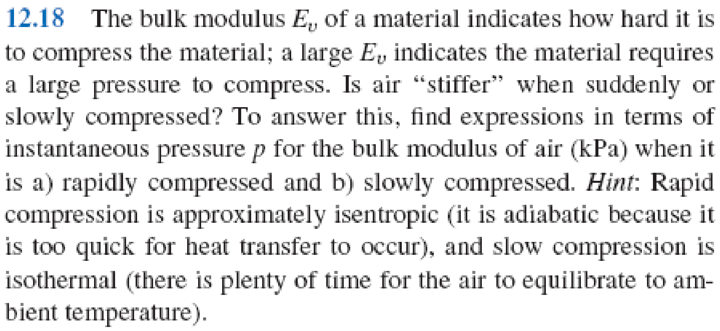
For the change in temperature we start with the ideal gas equation $\quad p = \rho \cdot R \cdot T$ $\quad$ and differentiate $\quad dp = d\rho \cdot R \cdot T + \rho \cdot R \cdot dT$

Dividing by the ideal gas equation we find $\quad \frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$

Hence $\quad dT = T_1 \cdot \left(\frac{dp}{p_1} - \frac{d\rho}{\rho_1}\right)$ $\quad dT = 0.017 K$ $\quad dT = 0.030 \cdot \Delta°F$ $\quad$ a very small change!

Problem 12.18

[3]



12.18 The bulk modulus E_v of a material indicates how hard it is to compress the material; a large E_v indicates the material requires a large pressure to compress. Is air "stiffer" when suddenly or slowly compressed? To answer this, find expressions in terms of instantaneous pressure p for the bulk modulus of air (kPa) when it is a) rapidly compressed and b) slowly compressed. *Hint*: Rapid compression is approximately isentropic (it is adiabatic because it is too quick for heat transfer to occur), and slow compression is isothermal (there is plenty of time for the air to equilibrate to ambient temperature).

Given: Sound wave

Find: Estimate of change in density, temperature, and velocity after sound wave passes

Solution:

Basic equations: $p = \rho \cdot R \cdot T$ $\qquad$ $E_v = \dfrac{dp}{\dfrac{d\rho}{\rho}}$

Assumptions: 1) Ideal gas 2) Constant specific heats 3) Infinitesimal changes

To find the bulk modulus we need $\dfrac{dp}{d\rho}$ in $\qquad$ $E_v = \dfrac{dp}{\dfrac{d\rho}{\rho}} = \rho \cdot \dfrac{dp}{d\rho}$

For rapid compression (isentropic) $\dfrac{p}{\rho^k} = \text{const}$ and so $\dfrac{dp}{d\rho} = k \cdot \dfrac{p}{\rho}$

Hence $\qquad E_v = \rho \cdot \left(k \cdot \dfrac{p}{\rho}\right) \qquad E_v = k \cdot p$

For gradual compression (isothermal) we can use the ideal gas equation $\quad p = \rho \cdot R \cdot T \quad$ so $\quad dp = d\rho \cdot R \cdot T$

Hence $\qquad E_v = \rho \cdot (R \cdot T) = p \qquad E_v = p$

We conclude that the "stiffness" (E_v) of air is equal to kp when rapidly compressed and p when gradually compressed. To give an idea of

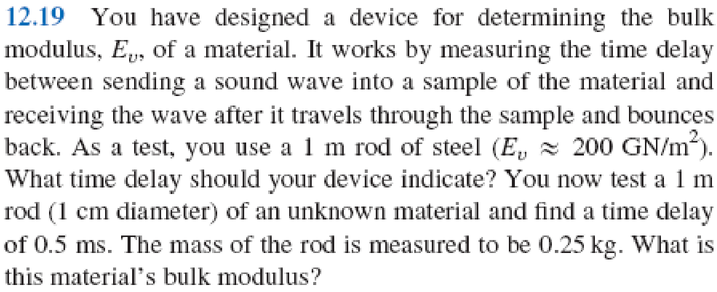
For water $\qquad E_v = 2.24 \cdot \text{GPa}$

For air ($k = 1.4$) at $p = 101 \cdot \text{kPa}$ $\quad$ Rapid compression $\quad E_v = k \cdot p \quad E_v = 141 \cdot \text{kPa}$

Gradual compression $\quad E_v = p \quad E_v = 101 \cdot \text{kPa}$

Problem 12.19

[2]



12.19 You have designed a device for determining the bulk modulus, E_v, of a material. It works by measuring the time delay between sending a sound wave into a sample of the material and receiving the wave after it travels through the sample and bounces back. As a test, you use a 1 m rod of steel ($E_v \approx$ 200 GN/m^2). What time delay should your device indicate? You now test a 1 m rod (1 cm diameter) of an unknown material and find a time delay of 0.5 ms. The mass of the rod is measured to be 0.25 kg. What is this material's bulk modulus?

Given: Device for determining bulk modulus

Find: Time delay; Bulk modulus of new material

Solution:

Basic equation: $c = \sqrt{\frac{E_v}{\rho}}$

Hence for given data $E_v = 200 \cdot \frac{GN}{m^2}$ $\quad L = 1 \cdot m$ $\quad$ and for steel $\quad SG = 7.83$ $\quad \rho_w = 1000 \cdot \frac{kg}{m^3}$

For the steel $c = \sqrt{\frac{E_v}{SG \cdot \rho_w}}$ $\quad c = 5054 \frac{m}{s}$

Hence the time to travel distance L is $\Delta t = \frac{L}{c}$ $\quad \Delta t = 1.98 \times 10^{-4} s$ $\quad \Delta t = 0.198 ms$ $\quad \Delta t = 198 \mu s$

For the unknown material $M = 0.25 \cdot kg$ $\quad D = 1 \cdot cm$ $\quad \Delta t = 0.5 \cdot ms$

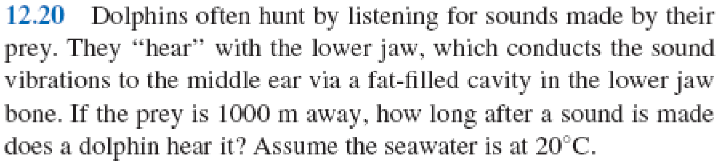
The density is then $\rho = \frac{M}{L \cdot \frac{\pi \cdot D^2}{4}}$ $\quad \rho = 3183 \frac{kg}{m^3}$

The speed of sound in it is $c = \frac{L}{\Delta t}$ $\quad c = 2000 \frac{m}{s}$

Hence th bulk modulus is $E_v = \rho \cdot c^2$ $\quad E_v = 12.7 \frac{GN}{m^2}$

Problem 12.20

[2]



12.20 Dolphins often hunt by listening for sounds made by their prey. They "hear" with the lower jaw, which conducts the sound vibrations to the middle ear via a fat-filled cavity in the lower jaw bone. If the prey is 1000 m away, how long after a sound is made does a dolphin hear it? Assume the seawater is at 20°C.

Given: Hunting dolphin

Find: Time delay before it hears prey at 1000 m

Solution:

Basic equation: $c = \sqrt{\frac{E_v}{\rho}}$

Given (and Table A.2) data $L = 1000 \cdot m$ $\quad SG = 1.025$ $\quad E_v = 2.42 \cdot \frac{GN}{m^2}$ $\quad \rho_w = 1000 \cdot \frac{kg}{m^3}$

For the seawater $c = \sqrt{\frac{E_v}{SG \cdot \rho_w}}$ $\quad c = 1537 \frac{m}{s}$

Hence the time for sound to travel distance L is $\Delta t = \frac{L}{c}$ $\quad \Delta t = 0.651 \cdot s$ $\quad \Delta t = 651 \cdot ms$

Problem 12.21

[2]

12.21 A submarine sends a sonar signal to detect the enemy. The reflected wave returns after 25 s. Estimate the separation between the submarines. (As an approximation assume the seawater is at 20°C.)

Given: Submarine sonar

Find: Separation between submarines

Solution:

Basic equation: $c = \sqrt{\frac{E_v}{\rho}}$

Given (and Table A.2) data $\Delta t = 25 \cdot s$ $\quad SG = 1.025$ $\quad E_v = 2.42 \cdot \frac{GN}{m^2}$ $\quad \rho_w = 1000 \cdot \frac{kg}{m^3}$

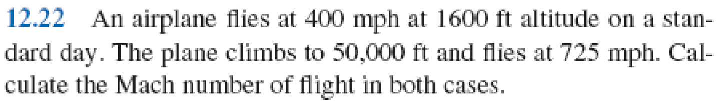
For the seawater $c = \sqrt{\frac{E_v}{SG \cdot \rho_w}}$ $\quad c = 1537 \frac{m}{s}$

Hence the distance sound travels in time Δt is $L = c \cdot \Delta t$ $\quad L = 38.4\, km$

The distance between submarines is half of this $x = \frac{L}{2}$ $\quad x = 19.2\, km$

Problem 12.22

[1]



12.22 An airplane flies at 400 mph at 1600 ft altitude on a standard day. The plane climbs to 50,000 ft and flies at 725 mph. Calculate the Mach number of flight in both cases.

Given: Airplane cruising at two different elevations

Find: Mach numbers

Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \dfrac{V}{c}$

Available data $R = 286.9 \dfrac{J}{kg \cdot K}$ $\quad k = 1.4$

At $z = 1600 \cdot ft$ $\quad z = 488\,m$ $\quad$ interpolating from Table A.3 $\quad T = 288.2 \cdot K + \dfrac{(284.9 - 288.2) \cdot K}{(500 - 0) \cdot m} \cdot (z - 0m)$

$T = 285\,K$

Hence $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 338 \dfrac{m}{s}$ $\quad c = 757\,mph$ $\quad$ and we have $\quad V = 400 \cdot mph$

The Mach number is $M = \dfrac{V}{c}$ $\quad M = 0.529$

Repeating at $z = 50000 \cdot ft$ $\quad z = 15240\,m$ $\quad T = 216.7 \cdot K$

Hence $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 295 \dfrac{m}{s}$ $\quad c = 660\,mph$ $\quad$ and we have $\quad V = 725 \cdot mph$

The Mach number is $M = \dfrac{V}{c}$ $\quad M = 1.10$

Problem 12.23

[1]

Given: The Lockheed SR-71 aircraft is thought to cruise at $M = 3.3$ at altitude $z = 85{,}000$ ft.

Find: (a) speed of sound and flight speed for these conditions.
(b) Compare speed to muzzle speed (700 m/sec) of a 30-06 rifle bullet.

Solution:

At altitude, $z = 85{,}000 \text{ ft} \times 0.3048 \frac{\text{m}}{\text{ft}} = 25.9 \text{ km}$

From Table A.3, $T = 222 \text{ K}$

$$\therefore \; c = \sqrt{kRT} = \left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 222\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 299 \text{ m/s} \longleftarrow c$$

$$V = Mc = 3.3 \times 299 \text{ m/s} = 987 \text{ m/s} \longleftarrow V$$

$$\frac{V}{V_{\text{bullet}}} = \frac{987}{700} = 1.41 \longleftarrow$$

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Problem 12.24

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Given: Boeing 727 cruises at 520 mi/hr at an altitude of 33,000 ft on a standard day

Find: (a) cruise Mach number of the aircraft
(b) flight speed corresponding to $M_{max} = 0.9$

Solution:

At 33,000 ft, $z = 10.06$ km. From Table A.3, $T = 223$ K.

Then, $c = \sqrt{kRT} = \left[1.4 \times 287 \dfrac{N \cdot m}{kg \cdot K} \times 223 K \times \dfrac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 299$ m/s

$$V = 520 \frac{mi}{hr} \times 5280 \frac{ft}{mi} \times \frac{hr}{3600 s} \times 0.3048 \frac{m}{ft} = 232 \text{ m/s}$$

$$M = \frac{V}{c} = \frac{232 \text{ m/s}}{299 \text{ m/s}} = 0.776 \quad M_{cr}$$

At $M = 0.90$

$$V = Mc = 0.90 \times 299 \text{ m/s} = 269 \text{ m/s} \quad (603 \text{ mph}) \quad V$$

Problem 12.25

12.25 Investigate the effect of altitude on Mach number by plotting the Mach number of a 500 mph airplane as it flies at altitudes ranging from sea level to 10 km.

Given: Airplane cruising at 550 mph

Find: Mach number versus altitude

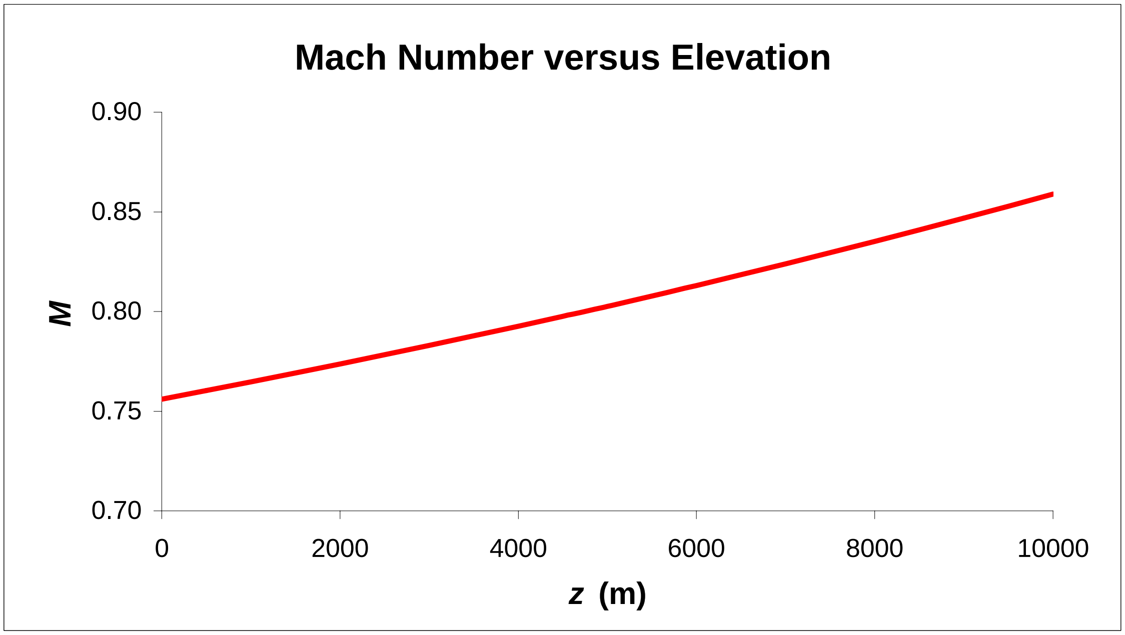
Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$ $M = \frac{V}{c}$

$V =$ 500 mph
$R =$ 286.90 J/kg·K (Table A.6)
$k =$ 1.40

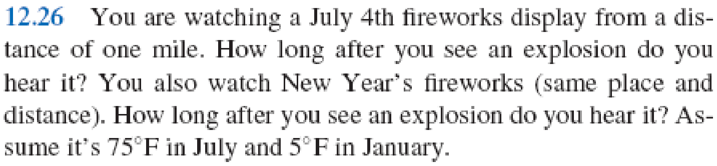
Data on temperature versus height obtained from Table A.3

z (m)	T (K)	c (m/s)	c (mph)	M
0	288.2	340	661	0.756
500	284.9	338	658	0.760
1000	281.7	336	654	0.765
1500	278.4	334	650	0.769
2000	275.2	332	646	0.774
2500	271.9	330	642	0.778
3000	268.7	329	639	0.783
3500	265.4	326	635	0.788
4000	262.2	325	631	0.793
4500	258.9	322	627	0.798
5000	255.7	320	623	0.803
6000	249.2	316	615	0.813
7000	242.7	312	607	0.824
8000	236.2	308	599	0.835
9000	229.7	304	590	0.847
10000	223.3	299	582	0.859



Problem 12.26

[2]



12.26 You are watching a July 4th fireworks display from a distance of one mile. How long after you see an explosion do you hear it? You also watch New Year's fireworks (same place and distance). How long after you see an explosion do you hear it? Assume it's 75°F in July and 5°F in January.

Given: Fireworks displays!

Find: How long after seeing them do you hear them?

Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$

Assumption: Speed of light is essentially infinite (compared to speed of sound)

The given or available data is $T_{July} = (75 + 460) \cdot R$ $\quad L = 1 \cdot mi$ $\quad k = 1.4$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Hence $c_{July} = \sqrt{k \cdot R_{air} \cdot T_{July}}$ $\quad c_{July} = 1134 \frac{ft}{s}$

Then the time is $\Delta t_{July} = \frac{L}{c_{July}}$ $\quad \Delta t_{July} = 4.66\,s$

In January $T_{Jan} = (5 + 460) \cdot R$

Hence $c_{Jan} = \sqrt{k \cdot R_{air} \cdot T_{Jan}}$ $\quad c_{Jan} = 1057 \frac{ft}{s}$

Then the time is $\Delta t_{Jan} = \frac{L}{c_{Jan}}$ $\quad \Delta t_{Jan} = 5.00\,s$

Problem 12.27

[2]

12.27 Use data for specific volume to calculate and plot the speed of sound in saturated liquid water over the temperature range from 0 to 200°C.

Given: Data on water specific volume

Find: Speed of sound over temperature range

Solution:

Basic equation: $c = \sqrt{\frac{\partial}{\partial\rho}p}$ at isentropic conditions

As an approximation for a liquid $c = \sqrt{\frac{\Delta p}{\Delta\rho}}$ using available data.

We use compressed liquid data at adjacent pressures of 5 MPa and 10 MPa, and estimate the change in density between these pressures from the corresponding specific volume changes

$$\Delta p = p_2 - p_1 \qquad \Delta\rho = \frac{1}{v_2} - \frac{1}{v_1} \qquad \text{and} \qquad c = \sqrt{\frac{\Delta p}{\Delta\rho}} \quad \text{at each temperature}$$

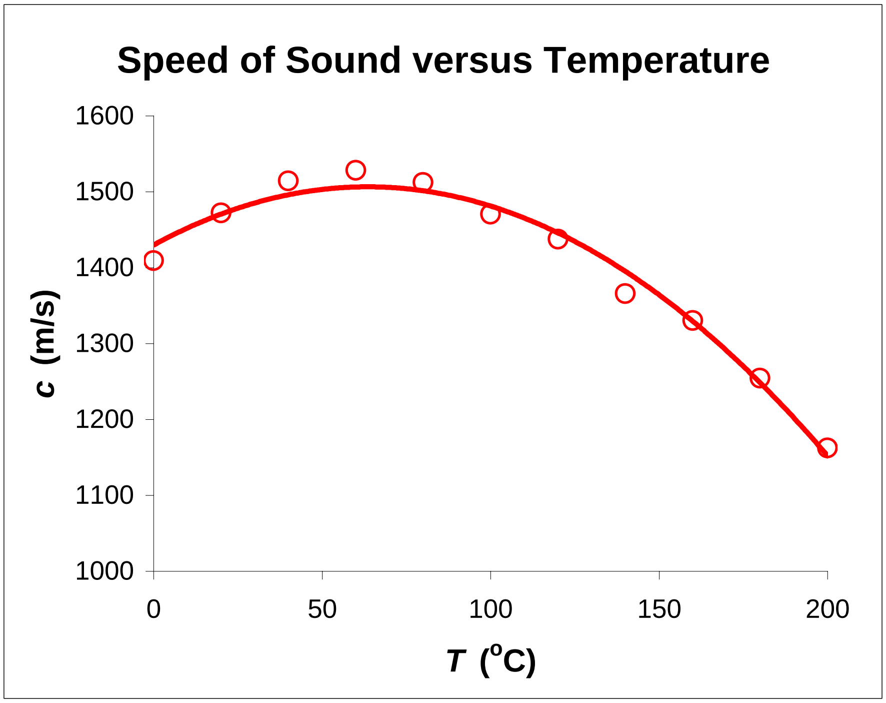
$p_2 =$ 10 MPa

$p_1 =$ 5 MPa

$\Delta p =$ 5 MPa

Data on specific volume versus temperature can be obtained fro any good thermodynamics text (try the Web!)

	p_1	p_2		
T (°C)	v (m³/kg)	v (m³/kg)	Δρ (kg/m³)	c (m/s)
0	0.0009977	0.0009952	2.52	1409
20	0.0009996	0.0009973	2.31	1472
40	0.0010057	0.0010035	2.18	1514
60	0.0010149	0.0010127	2.14	1528
80	0.0010267	0.0010244	2.19	1512
100	0.0010410	0.0010385	2.31	1470
120	0.0010576	0.0010549	2.42	1437
140	0.0010769	0.0010738	2.68	1366
160	0.0010988	0.0010954	2.82	1330
180	0.0011240	0.0011200	3.18	1254
200	0.0011531	0.0011482	3.70	1162



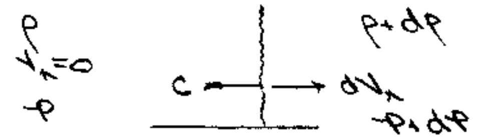
Problem 12.28

[3]

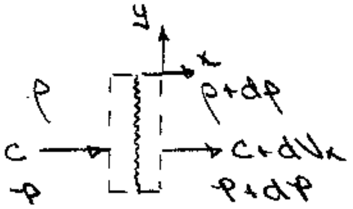
Given: Derivation of sonic speed (Eq. 12.18, Section 12-2)

Rederive assuming direction of fluid motion behind the wave is dV_x to the right.

Solution:



(a) Propagating Wave.



(b) Inertial CV moving with wave

Apply continuity to CV (b)

$$0 = \{-|\rho cA|\} + \{|(\rho+d\rho)(c+dV_x)A|\}$$

$$0 = -\cancel{\rho cA} + \cancel{\rho cA} + \rho dV_x A + cA\,d\rho + \cancel{d\rho\, dV_x}^{\approx 0}$$

then

$$\rho\, dV_x + c\, d\rho = 0 \qquad \text{or} \qquad dV_x = -\frac{c}{\rho}\, d\rho \quad \text{......(1)}$$

Applying x-momentum equation to same CV gives

$$\sum F_{s_x} = pA - (p+dp)A = \int_{CS} V_x \rho \vec{V}_{xyz}\cdot dA = c\{-|\rho Ac|\} + (c+dV_x)\{|\rho Ac|\}$$

$\{\dot{m}_{in} = \dot{m}_{out}\}$

$$\therefore\ -A\,dp = \rho Ac\, dV_x \quad \text{, and}$$

$$dV_x = -\frac{dp}{\rho c} \quad \text{.......(2)}$$

Combining equations (1) and (2) we obtain

$$-\frac{c}{\rho}\, d\rho = -\frac{dp}{\rho c}$$

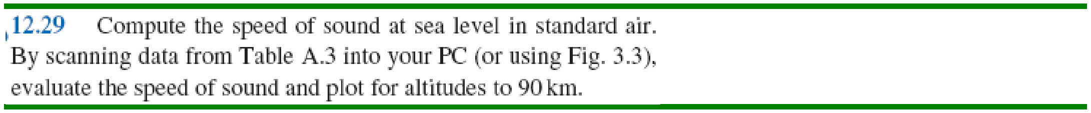
or

$$c^2 = \frac{dp}{d\rho}$$

This is the same result as obtained in the derivation of Section 12-2 with the direction of the fluid motion behind the wave to the left.

Problem 12.29

[2]



12.29 Compute the speed of sound at sea level in standard air. By scanning data from Table A.3 into your PC (or using Fig. 3.3), evaluate the speed of sound and plot for altitudes to 90 km.

Given: Data on atmospheric temperature variation with altitude

Find: Sound of speed at sea level; plot speed as function of altitude

Solution

The given or available data is:

$R =$ 286.9 J/kg.K

$k =$ 1.4

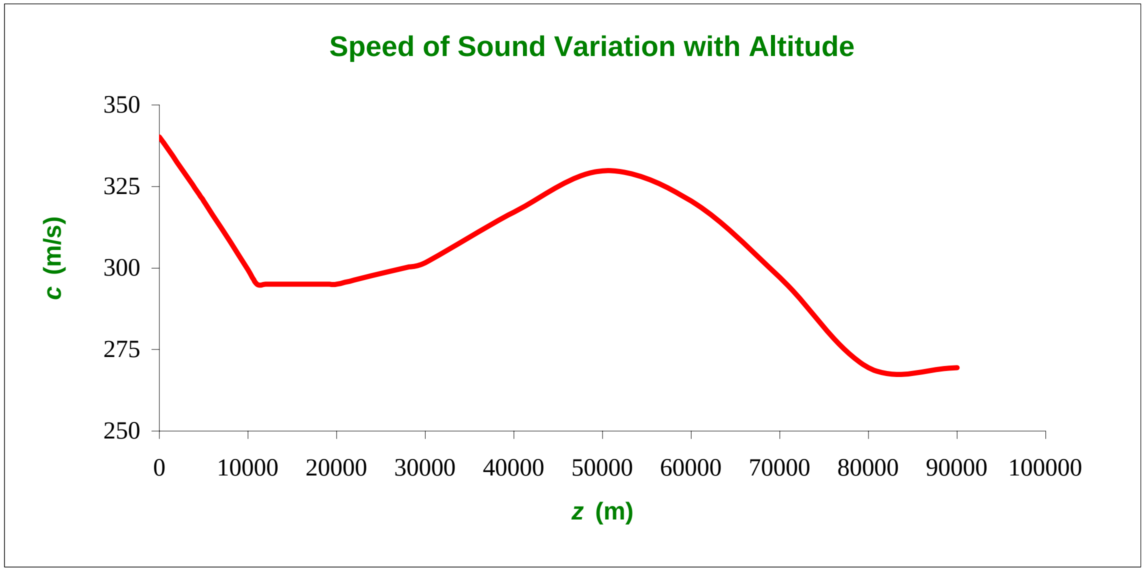
Computing equation:

$$c = \sqrt{kRT}$$

Computed results:

(Only partial data is shown in table)

z (m)	T (K)	c (m/s)
0	288.2	340
500	284.9	338
1000	281.7	336
1500	278.4	334
2000	275.2	332
2500	271.9	330
3000	268.7	329
3500	265.4	326
4000	262.2	325
4500	258.9	322
5000	255.7	320
6000	249.2	316
7000	242.7	312
8000	236.2	308
9000	229.7	304
10000	223.3	299



Problem 12.30

12.30 The temperature varies linearly from sea level to approximately 11 km altitude in the standard atmosphere. Evaluate the *lapse rate*—the rate of decrease of temperature with altitude—in the standard atmosphere. Derive an expression for the rate of change of sonic speed with altitude in an ideal gas under standard atmospheric conditions. Evaluate and plot from sea level to 10 km altitude.

Given: Data on atmospheric temperature variation with altitude

Find: Lapse rate; plot of rate of change of sonic speed with altitude

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
T_0 =	288.2	K
T_{10k} =	223.3	K

Computing equations:

For a linear temperature variation $T = T_0 + m\cdot z$

$$\frac{dT}{dz} = m = \frac{T - T_0}{z}$$ which can be evaluated at z = 10 km

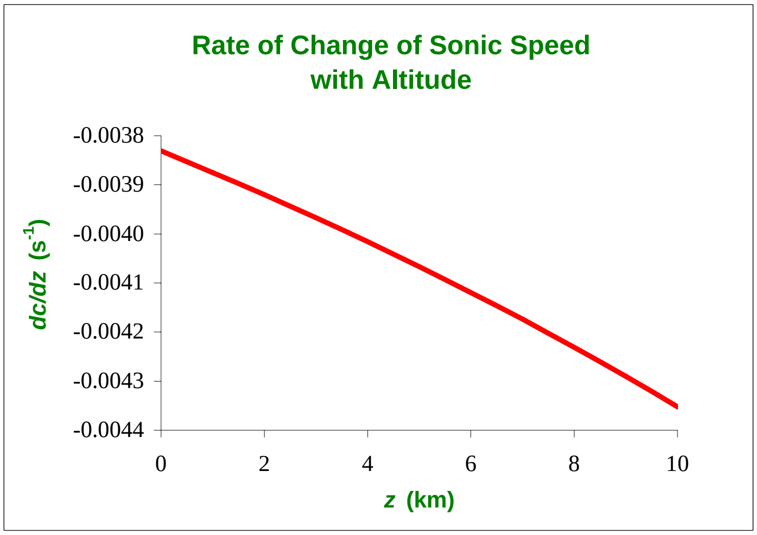
For an ideal gas $c = \sqrt{k\cdot R\cdot T} = \sqrt{k\cdot R\cdot \left(T_0 + m\cdot z\right)}$

Hence $$\frac{dc}{dz} = \frac{m\cdot k\cdot R}{2\cdot c}$$

Computed results:

m = -0.00649 K/m (Using T at z = 10 km)

z (km)	T (K)	dc/dz (s^{-1})
0	288.2	-0.00383
1	281.7	-0.00387
2	275.2	-0.00392
3	268.7	-0.00397
4	262.2	-0.00402
5	255.8	-0.00407
6	249.3	-0.00412
7	242.8	-0.00417
8	236.3	-0.00423
9	229.8	-0.00429
10	223.3	-0.00435



Problem 12.31

12.31 Air at 77°F flows at $M = 1.9$. Determine the air speed and the Mach angle.

Given: Air flow at M = 1.9

Find: Air speed; Mach angle

Solution:

Basic equations: $c = \sqrt{k\cdot R\cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \mathrm{asin}\left(\frac{1}{M}\right)$

The given or available data is $T = (77 + 460)\cdot R$ $\quad M = 1.9$ $\quad k = 1.4$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$

Hence $c = \sqrt{k\cdot R_{air}\cdot T}$ $\quad c = 1136\cdot\frac{ft}{s}$

Then the air speed is $V = M\cdot c$ $\quad V = 2158\frac{ft}{s}$ $\quad V = 1471\,mph$

The Mach angle is given by $\alpha = \mathrm{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 31.8\,deg$

Problem 12.32

[1]

Given: Gas at $P = 50$ psia $\rho = 0.27$ lbm/ft^3
Projectile fired into gas. Total angle of Mach cone is $20°$

Find: speed of projectile relative to gas

Solution:

Basic equation: $p = \rho R T$

Definitions: $\sin\alpha = \frac{1}{M}$ $\quad M = \frac{V}{c}$

Computing eq: $c = \sqrt{kRT}$

Assumption: ideal gas

$$\sin\alpha = \frac{1}{M} \qquad M = \frac{1}{\sin\alpha} = \frac{1}{\sin 10°} = 5.76$$

$$c = \left[kRT\right]^{1/2} = \left[k\frac{p}{\rho}\right]^{1/2}$$

$$= \left[1.4 \times 50\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{ft}^3}{0.27\,\text{lbm}} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \times \frac{144\,\text{in}^2}{\text{ft}^2}\right]^{1/2}$$

$$c = 1100 \text{ ft/s}$$

$$M = \frac{V}{c} \qquad \therefore \quad V = Mc = 5.76 \times 1100 \text{ ft/s} = 6320 \text{ ft/s}$$

V

Problem 12.33

Given : Photo of bullet moving through standard air shows Mach angle, $\alpha = 32°$

Find : Speed of bullet

Solution:

Computing equations, $\sin\alpha = \frac{1}{M}$ $\quad c = \sqrt{kRT}$

Assumptions: (1) air behaves as an ideal gas
(2) constant specific heats.

$M = \frac{1}{\sin\alpha}$, $M = \frac{V}{c}$ $\therefore V = cM = \frac{c}{\sin\alpha}$

Since $c = \sqrt{kRT}$, then

$$V = \frac{1}{\sin\alpha}(kRT)^{1/2} = \frac{1}{\sin 32°}\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 288\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2}$$

$V = 642$ m/s (2110 ft/s) $\longleftarrow$ V

Problem 12.34

[2]

Given: A schlieren photograph taken in the NTF shows a Mach angle, $\alpha = 57°$, at a location where $T = -270°F$ and $p = 1.3$ psia.

Find: (a) the local Mach number and flow speed
(b) the unit Reynolds number for the flow

Solution:

$$\sin\alpha = \frac{1}{M} \qquad \therefore \; M = \frac{1}{\sin\alpha} = \frac{1}{\sin 57°} = 1.19 \qquad M$$

$$c = \sqrt{kRT} = \left[1.4 \times 53.3\,\frac{ft\cdot lbf}{lbm\cdot °R} \times 32.2\,\frac{lbm}{slug} \times 190°R \times \frac{slug\cdot ft}{lbf\cdot s^2}\right]^{1/2} = 676\ ft/s$$

$$V = Mc = 1.19\,(676\ ft/s) = 804\ ft/s \qquad V$$

$$Re_x = \frac{\rho V x}{\mu}$$

$$\rho = \frac{p}{RT} = 1.3\,\frac{lbf}{in^2} \times \frac{lbm\cdot °R}{53.3\,ft\cdot lbf} \times \frac{1}{190°R} \times \frac{144\,in^2}{ft^2} = 0.0185\ lbm/ft^3$$

From Eq. A.1 (Appendix A)

$$\mu = \frac{bT^{1/2}}{1+S/T} \qquad b = 1.458\times10^{-6}\ kg/m\cdot s\cdot K^{1/2}$$

$S = 110.4\,K$

T in K

$T = -270\,F = -167\,C = 106\,K.$

$$\mu = 1.458\times10^{-6}\,\frac{kg}{m\cdot s\cdot K^{1/2}}\,(106K)^{1/2} \times \frac{1}{1+\frac{110.4}{106}} = 7.35\times10^{-6}\ kg/m\cdot s$$

$$\mu = 7.35\times10^{-6}\,\frac{\cancel{kg}}{m\cdot s} \times \frac{N\cdot s^2}{\cancel{kg}\cdot m} \times \frac{2.089\times10^{-2}\ lbf\cdot s/ft^2}{1\ N\cdot s}$$

$$\mu = 1.54\times10^{-7}\ lbf\cdot s/ft^2$$

$$\therefore \; \frac{Re}{x} = \frac{\rho V}{\mu} = 0.0185\,\frac{lbm}{\cancel{ft^3}} \times 804\,\frac{\cancel{ft}}{s} \times \frac{ft^2}{1.54\times10^{-7}\ lbf\cdot s} \times \frac{slug}{32.2\ lbm} \times \frac{lbf\cdot s^2}{slug\cdot ft}$$

$$\frac{Re}{x} = 3.00\times10^{6}\ ft^{-1} = 9.84\times10^{6}\ m^{-1} \qquad Re/x$$

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Problem 12.35

[2]

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42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

Given: An aircraft flies at $M = 1.4$ at elevation $z = 200\text{m}$
The air temperature is 35°C

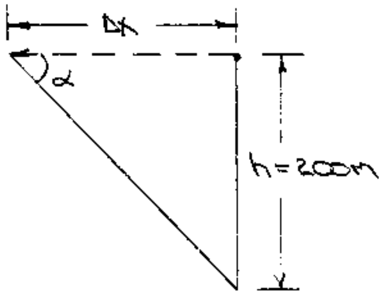
Find: (a) air speed of aircraft
(b) time between instant when aircraft passes directly overhead and instant when Mach cone passes a point on the ground.

Solution: Assume T = constant over 200m elevation.

$$T = 35\,\text{C} = 308\,\text{K}$$

$$c = \sqrt{kRT} = \left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 308\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 352\,\text{m/s}$$

$$V = Mc = 1.4 \times 352\,\text{m/s} = 493\,\text{m/s} \quad \longleftarrow V$$



From the instant the aircraft is directly overhead until the Mach cone reaches the ground, the plane travels a distance Δx at speed $V = 493\,\text{m/s}$.

$$\sin\alpha = \frac{1}{M} = \frac{1}{1.4} = 0.7143$$

$$\alpha = 45.6^\circ$$

$$\frac{h}{\Delta x} = \tan\alpha \qquad \therefore\ \Delta x = \frac{h}{\tan\alpha} = \frac{200\,\text{m}}{\tan 45.6^\circ} = 196\,\text{m}$$

Since the plane moves at constant speed V

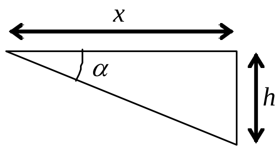
$$\Delta x = V\,\Delta t \quad \text{and} \quad \Delta t = \frac{\Delta x}{V} = \frac{196\,\text{m}}{493\,\text{m/s}}$$

$$\Delta t = 0.398\,\text{s} \quad \longleftarrow \Delta t$$

Problem 12.36

[2]

12.36 While jogging on the beach (it's a warm summer day, about 30°C) a high-speed jet flies overhead. You guesstimate it's at an altitude of about 3500 m, and count off about 5 s before you hear it. Estimate the speed and Mach number of the jet.



Given: High-speed jet flying overhead

Find: Estimate speed and Mach number of jet

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \operatorname{asin}\left(\frac{1}{M}\right)$

Given or available data $T = (30 + 273) \cdot K$ $\quad h = 3500 \cdot m$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

The time it takes to fly from directly overhead to where you hear it is $\Delta t = 5 \cdot s$

The distance traveled, moving at speed V, is $x = V \cdot \Delta t$

The Mach angle is related to height h and distance x by $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{h}{x} = \frac{h}{V \cdot \Delta t}$ (1)

and also we have $\sin(\alpha) = \frac{1}{M} = \frac{c}{V}$ (2)

Dividing Eq. 2 by Eq 1 $\cos(\alpha) = \frac{c}{V} \cdot \frac{V \cdot \Delta t}{h} = \frac{c \cdot \Delta t}{h}$

Note that we could have written this equation from geometry directly!

We have $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 349 \frac{m}{s}$ so $\alpha = \operatorname{acos}\left(\frac{c \cdot \Delta t}{h}\right)$ $\quad \alpha = 60.1 \cdot deg$

Hence $M = \frac{1}{\sin(\alpha)}$ $\quad M = 1.15$

Then the speed is $V = M \cdot c$ $\quad V = 402 \frac{m}{s}$

Note that we assume the temperature of the air is uniform. In fact the temperature will vary over 3500 m, so the Mach cone will be curved. This speed and Mach number are only rough estimates

Problem 12.37

Given: Aircraft passes overhead at an altitude of 3 km, travelling at $M = 1.35$. The air temperature is constant at $T = 30^\circ C$ and a head wind blows at $V_{air} = 10$ m/s.

Find: (a) the airspeed of the aircraft
(b) time between instant when aircraft passes directly overhead and instant when sound reaches the ground.

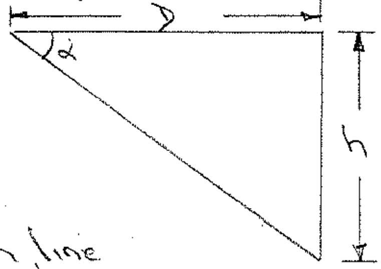
Solution:

$$T = \text{constant} = 20^\circ C = 293\ K$$

$$c = (kRT)^{1/2} = \left(1.4 \times 287\ \frac{N\cdot m}{kg\cdot K} \times 293\ K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 343\ m/s$$

$$V = Mc = 1.5 \times 343\ \frac{m}{sec} = 515\ m/s \longleftarrow \text{airspeed}$$

The airspeed is the velocity of the plane relative to the air. The ground speed is then $\vec{V}_p = \vec{V}_{air} + \vec{V}_{pla}$ or $V_p = 485$ m/sec



From the instant the aircraft is directly overhead until the Mach cone reaches the ground, the plane travels a distance, D, at speed $V_p = 485$ m/s.
The value of the time, t, is then $t = D/V_p$
Since the air temperature is constant, the Mach line is straight and $D = h/\tan\alpha$, where $\alpha = \sin^{-1}(1/M)$

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$$

Then,

$$t = \frac{D}{V_p} = \frac{h}{\tan\alpha}\frac{1}{V_p} = \frac{3000\ m}{\tan 41.8} \times \frac{s}{485\ m} = 6.92\ s \longleftarrow t$$

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42 382 100 SHEETS 5 SQUARE
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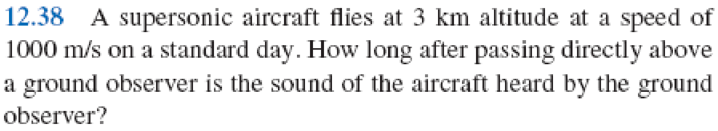
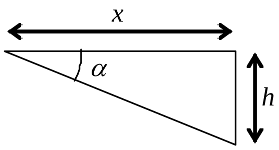
Problem 12.38

[3]



12.38 A supersonic aircraft flies at 3 km altitude at a speed of 1000 m/s on a standard day. How long after passing directly above a ground observer is the sound of the aircraft heard by the ground observer?



Given: Supersonic aircraft flying overhead

Find: Time at which airplane heard

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \operatorname{asin}\left(\frac{1}{M}\right)$

Given or available data $V = 1000 \cdot \frac{m}{s}$ $\quad h = 3 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

The time it takes to fly from directly overhead to where you hear it is $\Delta t = \frac{x}{V}$

If the temperature is constant then $x = \frac{h}{\tan(\alpha)}$

The temperature is not constant so the Mach line will not be straight. We can find a range of Δt by considering the temperature range

At $h = 3\,km$ we find from Table A.3 that $T = 268.7 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 329 \frac{m}{s}$ and $M = \frac{V}{c}$ $\quad M = 3.04$

Hence $\alpha = \operatorname{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.2\,deg$ $\quad x = \frac{h}{\tan(\alpha)}$ $\quad x = 8625\,m$ $\quad \Delta t = \frac{x}{V}$ $\quad \Delta t = 8.62\,s$

At sea level we find from Table A.3 that $T = 288.2 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340 \frac{m}{s}$ and $M = \frac{V}{c}$ $\quad M = 2.94$

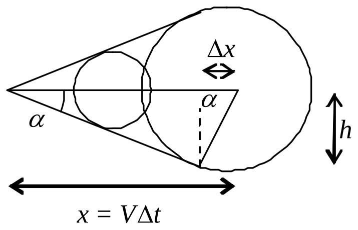
Hence $\alpha = \operatorname{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.9\,deg$ $\quad x = \frac{h}{\tan(\alpha)}$ $\quad x = 8291\,m$ $\quad \Delta t = \frac{x}{V}$ $\quad \Delta t = 8.29\,s$

Thus we conclude that the time is somwhere between 8.62 and 8.29 s. Taking an average $\Delta t = 8.55 \cdot s$

Problem 12.39

[3]

12.39 For the conditions of Problem 12.38, find the location at which the sound wave that first reaches the ground observer was emitted.



Given: Supersonic aircraft flying overhead

Find: Location at which first sound wave was emitted

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \dfrac{V}{c}$ $\quad \alpha = \mathrm{asin}\left(\dfrac{1}{M}\right)$

Given or available data $V = 1000 \cdot \dfrac{m}{s}$ $\quad h = 3 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \dfrac{J}{kg \cdot K}$

We need to find Δx as shown in the figure $\Delta x = h \cdot \tan(\alpha)$

The temperature is not constant so the Mach line will not be straight (α is not constant). We can find a range of α and Δx by considering the temperature range

At $h = 3\,km$ we find from Table A.3 that $T = 268.7 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 329 \dfrac{m}{s}$ and $M = \dfrac{V}{c}$ $\quad M = 3.04$

Hence $\alpha = \mathrm{asin}\left(\dfrac{1}{M}\right)$ $\quad \alpha = 19.2\,deg$ $\quad \Delta x = h \cdot \tan(\alpha)$ $\quad \Delta x = 1043\,m$

At sea level we find from Table A.3 that $T = 288.2 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340 \dfrac{m}{s}$ and $M = \dfrac{V}{c}$ $\quad M = 2.94$

Hence $\alpha = \mathrm{asin}\left(\dfrac{1}{M}\right)$ $\quad \alpha = 19.9\,deg$ $\quad \Delta x = h \cdot \tan(\alpha)$ $\quad \Delta x = 1085\,m$

Thus we conclude that the distance is somwhere between 1043 and 1085 m. Taking an average $\Delta x = 1064 \cdot m$

Problem 12.40

[4] Part 1/2

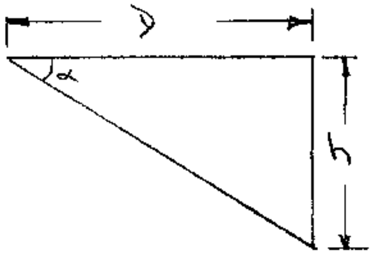
Given: Concorde supersonic transport cruises at $M = 2.2$ at an altitude, $h = 17$ km on a standard day.
At $t = 0$, plane is directly overhead.

Find: value of t when aircraft is first heard.

Solution:

At altitude, $T = 216.7\,\text{K}$ $\quad c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 216.7\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 295\,\text{m/s}$

$V = Mc = 2.2 \times 295\,\text{m/s} = 649\,\text{m/s}.$



If the speed of sound were constant all the way to the ground, the Mach line would remain straight. The Mach angle, α, would be constant with

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = 27^\circ$$

Then, from the diagram $\quad D = \dfrac{h}{\tan\alpha}$

and $\quad t = \dfrac{D}{V} = \dfrac{h}{\tan\alpha\, V} = \dfrac{17000\,\text{m}}{\tan 27^\circ} \times \dfrac{1}{649}\dfrac{\text{sec}}{\text{m}} = 51.4\,\text{s}$

However, the speed of sound varies over the altitude because the temperature varies with altitude.

At sea level $\quad T = 288.2\,\text{K}$

$$c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 288.2\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 340\,\text{m/s}$$

The corresponding value of Mach number for $V = 649\,\text{m/s}$ is

$$M = \frac{V}{c} = \frac{649}{340} = 1.91$$

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.91}\right) = 31.6^\circ$$

Thus, if the speed of sound were constant (at the sea level) value over the entire altitude, then

$$t = \frac{D}{V} = \frac{h}{\tan\alpha\, V} = \frac{17000\,\text{m}}{\tan 31.6} \times \frac{\text{s}}{649\,\text{m}} = 42.6\,\text{s}$$

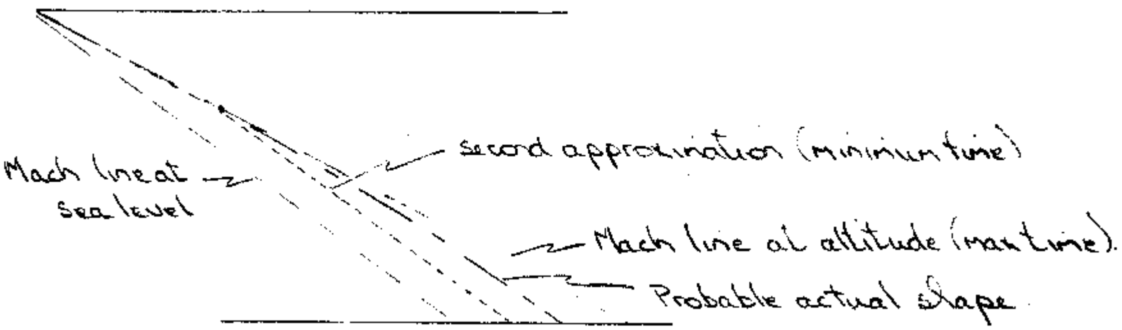
We can obtain a better approximate by considering the variation of temperature with altitude.
From Table A.3

$11\,\text{km} \le y < 20\,\text{km} \qquad T = 216.7\,\text{K}$

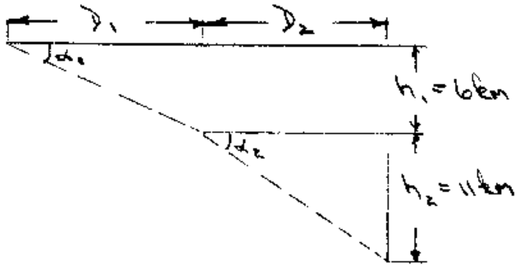
$0 < y \le 11\,\text{km} \qquad T$ varies linearly with y

$T = T_0 - by \qquad T_0 = 288.2\,\text{K}$

$b = 6.50\,\text{K/km}$


second approximation (minimum time)
Mach line at sea level
Mach line at altitude (max time).
Probable actual shape

Since T is constant for $y > y_0 = 11\ \text{km}$, the second approximation which assumes the Mach line at sea level for $0 < y \le 11\ \text{km}$ gives the minimum time



$$D_1 = \frac{h_1}{\tan\alpha_1} = \frac{6\text{km}}{\tan 27^\circ} = 11.77\ \text{km}$$

$$D_2 = \frac{h_2}{\tan\alpha_2} = \frac{11\text{km}}{\tan 31.6^\circ} = 17.88\ \text{km}$$

$$D = D_1 + D_2$$

$$t = \frac{D}{V} = \frac{29.65\ \text{km}}{649\ \text{m/s}} = 45.7\ \text{s}$$

Consequently, $45.7 \le t \le 51.4$ s

Since the two values are reasonably close, it is appropriate to take the average value and say $t \doteq 48.5$ s ⟵ t

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Problem 12.41

[2]

12.41 The airflow around an automobile is assumed to be incompressible. Investigate the validity of this assumption for an automobile traveling at 60 mph. (Relative to the automobile the minimum air velocity is zero, and the maximum is approximately 120 mph.)

Given: Speed of automobile

Find: Whether flow can be considered incompressible

Solution:

Consider the automobile at rest with 60 mph air flowing over it. Let state 1 be upstream, and point 2 the stagnation point on the automobile

The data provided, or available in the Appendices, is:

$R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad V_1 = 60 \cdot mph$ $\quad p_1 = 101 \cdot kPa$ $\quad T_1 = (20 + 273) \cdot K$

The basic equation for the density change is

$$\frac{\rho_0}{\rho} = \left[1 + \frac{(k-1)}{2} \cdot M^2\right]^{\frac{1}{k-1}} \qquad (12.20c)$$

or

$$\rho_0 = \rho_1 \cdot \left[1 + \frac{(k-1)}{2} \cdot M_1^{\,2}\right]^{\frac{1}{k-1}}$$

$$\rho_1 = \frac{p_1}{R \cdot T_1} \qquad \rho_1 = 1.201 \frac{kg}{m^3}$$

For the Mach number we need c

$$c_1 = \sqrt{k \cdot R \cdot T_1} \qquad c_1 = 343 \frac{m}{s}$$

$$V_1 = 26.8 \frac{m}{s} \qquad M_1 = \frac{V_1}{c_1} \qquad M_1 = 0.0782$$

$$\rho_0 = \rho_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\,2}\right)^{\frac{1}{k-1}} \qquad \rho_0 = 1.205 \frac{kg}{m^3}$$

The percentage change in density is $\left|\frac{\rho_0 - \rho_1}{\rho_0}\right| = 0.305\%$

This is an insignificant change, so the flow can be considered incompressible. Note that $M < 0.3$, the usual guideline for incompressibility

For the maximum speed present

$$V_1 = 120 \cdot mph \qquad V_1 = 53.6 \frac{m}{s} \qquad M_1 = \frac{V_1}{c_1} \qquad M_1 = 0.156$$

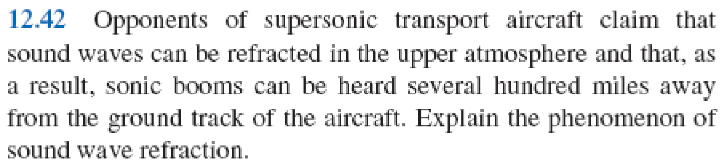
$$\rho_0 = \rho_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\,2}\right)^{\frac{1}{k-1}} \qquad \rho_0 = 1.216 \frac{kg}{m^3}$$

The percentage change in density is $\left|\frac{\rho_0 - \rho_1}{\rho_0}\right| = 1.21\%$

This is still an insignificant change, so the flow can be considered incompressible.

Problem 12.42

[5]



12.42 Opponents of supersonic transport aircraft claim that sound waves can be refracted in the upper atmosphere and that, as a result, sonic booms can be heard several hundred miles away from the ground track of the aircraft. Explain the phenomenon of sound wave refraction.

Given: Supersonic transport aircraft

Find: Explanation of sound wave refraction

Solution:

A sound wave is refracted when the speed of sound varies with altitude in the atmosphere. (The variation in sound speed is caused by temperature variations in the atmosphere, as shown in Fig. 3.3)

Imagine a plane wave front that initially is vertical. When the wave encounters a region where the temperature increase with altitude (such as between 20.1 km and 47.3 km altitude in Fig. 3.3), the sound speed increases with elevation. Therefore the upper portion of the wave travels faster than the lower portion. The wave front turns gradually and the sound wave follows a curved path through the atmosphere. Thus a wave that initially is horizontal bends and follows a curved path, tending to reach the ground some distance from the source.

The curvature and the path of the sound could be calculated for any specific temperature variation in the atmosphere. However, the required analysis is beyond the scope of this text.

Problem 12.43

[2]

12.43 Plot the percentage discrepancy between the density at the stagnation point and the density at a location where the Mach number is *M*, of a compressible flow, for Mach numbers ranging from 0.05 to 0.95. Find the Mach numbers at which the discrepancy is 1 percent, 5 percent, and 10 percent.

Given: Mach number range from 0.05 to 0.95

Find: Plot of percentage density change; Mach number for 1%, 5%, and 10% change

Solution:

The given or available data is:

$k = 1.4$

Computing equation:

$$\frac{\rho_0}{\rho} = \left[1 + \frac{(k-1)}{2} \cdot M^2\right]^{\frac{1}{k-1}} \qquad (12.20c)$$

Hence $$\frac{\Delta\rho}{\rho_0} = \frac{\rho_0 - \rho}{\rho_0} = 1 - \frac{\rho}{\rho_0}$$

so $$\frac{\Delta\rho}{\rho_0} = 1 - \left[1 + \frac{(k-1)}{2} \cdot M^2\right]^{\frac{1}{1-k}}$$

Computed results:

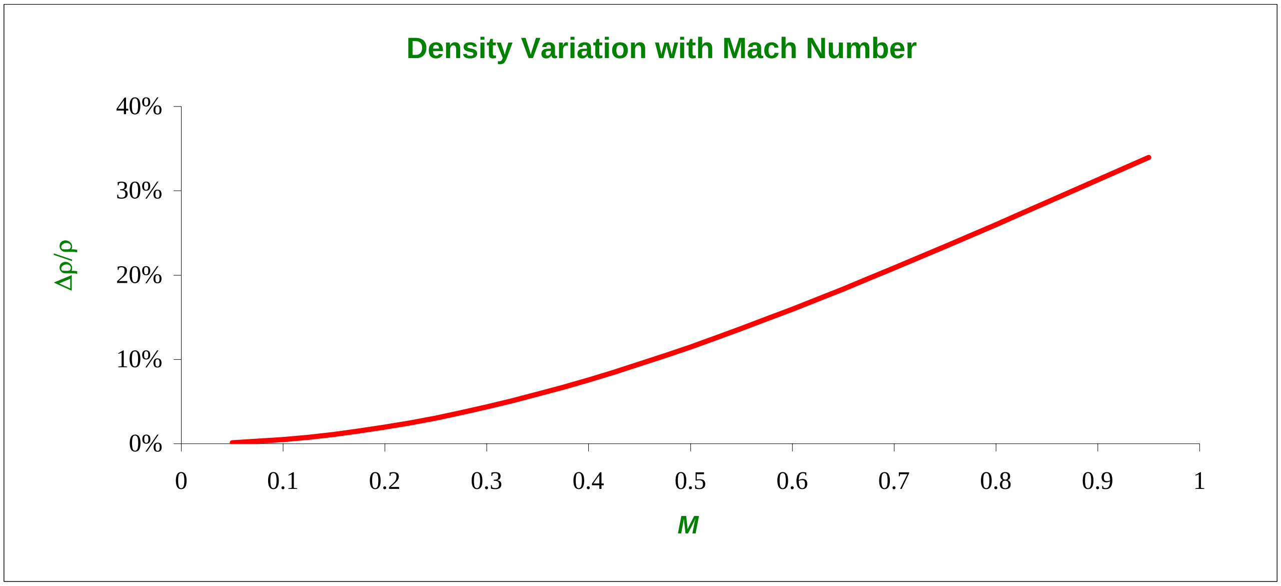
M	$\Delta\rho/\rho_o$
0.05	0.1%
0.10	0.5%
0.15	1.1%
0.20	2.0%
0.25	3.1%
0.30	4.4%
0.35	5.9%
0.40	7.6%
0.45	9.4%
0.50	11%
0.55	14%
0.60	16%
0.65	18%
0.70	21%
0.75	23%
0.80	26%
0.85	29%
0.90	31%
0.95	34%

To find *M* for specific density changes
use *Goal Seek* repeatedly

M	$\Delta\rho/\rho_o$
0.142	1%
0.322	5%
0.464	10%

Note: Based on ρ (not ρ_o) the results are:

0.142	0.314	0.441



Problem 12.44

[1]

12.44 An aircraft flies at 250 m/s in air at 28 kPa and −50°C. Find the stagnation pressure at the nose of the aircraft.

Given: Aircraft flying at 250 m/s

Find: Stagnation pressure

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \dfrac{V}{c}$ $\quad \dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $V = 250 \cdot \dfrac{m}{s}$ $\quad T = (-50 + 273) \cdot K$ $\quad p = 28 \cdot kPa$ $\quad k = 1.4$ $\quad R = 286.9 \dfrac{J}{kg \cdot K}$

First we need $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 299 \dfrac{m}{s}$ $\quad$ then $\quad M = \dfrac{V}{c}$ $\quad M = 0.835$

Finally we solve for p_0 $\quad p_0 = p \cdot \left(1 + \dfrac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 44.2\,kPa$

Problem 12.45

12.45 Compute the air density in the undisturbed air, and at the stagnation point, of Problem 12.44. What is the percentage increase in density? Can we approximate this as an incompressible flow?

Given: Pressure data on aircraft in flight

Find: Change in air density; whether flow can be considered incompressible

Solution:

The data provided, or available in the Appendices, is:

$$k = 1.4 \qquad p_0 = 48 \cdot \text{kPa} \qquad p = 27.6 \cdot \text{kPa} \qquad T = (-55 + 273) \cdot \text{K}$$

Governing equation (assuming isentropic flow):

$$\frac{p}{\rho^k} = \text{constant} \qquad (12.12c)$$

Hence

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

so

$$\frac{\Delta\rho}{\rho} = \frac{\rho_0 - \rho}{\rho} = \frac{\rho_0}{\rho} - 1 = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} - 1 \qquad \frac{\Delta\rho}{\rho} = 48.5 \cdot \%$$

NOT an incompressible flow!

Problem 12.46

12.46 Find the ratio of static to total pressure for a car moving at 55 mph at sea level and an airplane moving at 550 mph at 30,000 ft.

Given: Car at sea level and aircraft flying at 30,000 ft

Find: Ratio of static to total pressure in each case

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $M = \frac{V}{c}$ $\frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $V_{car} = 55 \cdot mph$ $V_{car} = 80.7 \frac{ft}{s}$ $V_{plane} = 550 \cdot mph$ $V_{plane} = 807 \frac{ft}{s}$

$k = 1.4$ $R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

At sea level, from Table A.3 $T = 288.2 \cdot K$ or $T = 519 R$

Hence $c = \sqrt{k \cdot R_{air} \cdot T}$ $c = 1116 \frac{ft}{s}$ $M_{car} = \frac{V_{car}}{c}$ $M_{car} = 0.0723$

The pressure ratio is $\frac{p}{p_0} = \left(1 + \frac{k-1}{2} \cdot M_{car}^{\ 2}\right)^{-\frac{k}{k-1}} = 0.996$

Note that the Bernoulli equation would give the same result!

At $h = 30000 \cdot ft$ or $h = 9144 m$,interpolating from Table A.3

$T = 229.7 \cdot K + \frac{(223.3 - 229.7) \cdot K}{(10000 - 9000)} \cdot (9144 - 9000)$ $T = 229 K$ $T = 412 R$

Hence $c = \sqrt{k \cdot R_{air} \cdot T}$ $c = 995 \frac{ft}{s}$ $M_{plane} = \frac{V_{plane}}{c}$ $M_{plane} = 0.811$

The pressure ratio is $\frac{p}{p_0} = \left(1 + \frac{k-1}{2} \cdot M_{plane}^{\ 2}\right)^{-\frac{k}{k-1}} = 0.649$

Problem 12.47

[2]

12.47 For an aircraft traveling at $M = 2$ at an elevation of 12 km, find the dynamic and stagnation pressures.

Given: Aircraft flying at 12 km

Find: Dynamic and stagnation pressures

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $M = \frac{V}{c}$ $\frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $p_{dyn} = \frac{1}{2} \cdot \rho \cdot V^2$

Given or available data $M = 2$ $h = 12 \cdot km$ $k = 1.4$ $R = 286.9 \cdot \frac{J}{kg \cdot K}$

$\rho_{SL} = 1.225 \cdot \frac{kg}{m^3}$ $p_{SL} = 101.3 \cdot kPa$

At $h = 12\,km$,from Table A.3 $\rho = 0.2546 \cdot \rho_{SL}$ $\rho = 0.312 \frac{kg}{m^3}$ $p = 0.1915 \cdot p_{SL}$ $p = 19.4\,kPa$ $T = 216.7 \cdot K$

Hence $p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $p_0 = 152\,kPa$

Also $c = \sqrt{k \cdot R \cdot T}$ $c = 295 \frac{m}{s}$ $V = M \cdot c$ $V = 590 \frac{m}{s}$

Hence $p_{dyn} = \frac{1}{2} \cdot \rho \cdot V^2$ $p_{dyn} = 54.3\,kPa$

Problem 12.48

Given: Body moving through standard air at 200 m/s

Find : Stagnation pressure, assuming
(a) incompressible flow
b) compressible flow

Solution:

For standard air , $P = 101$ kPa , $T = 15°C$

Computing equations $P_o = P + \frac{1}{2}\rho V^2$ (incompressible)

$$\frac{P_o}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \text{ (compressible).}$$

(a) Incompressible flow

$$P_o = P + \frac{1}{2}\rho V^2 = 101\text{ kPa} + \frac{1}{2} \times 1.225\,\frac{kg}{m^3} \times (200)^2\,\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{kPa}{10^3}\,\frac{m^2}{N}$$

$P_o = 125.5$ kPa ⟵ $P_{o\,in}$

b. Compressible flow

$$M = \frac{V}{c} \qquad c = (kRT)^{1/2} = \left(1.4 \times 287\,\frac{N \cdot m}{kg \cdot K} \times 288K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 340 \text{ m/s}$$

$$M = \frac{200}{340} = 0.588$$

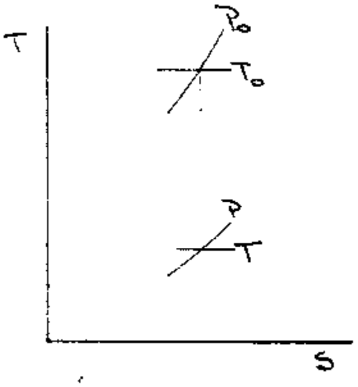
$$P_o = P\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} = 101\text{ kPa}\left[1 + 0.2(0.588)^2\right]^{3.5} = 127.6\text{ kPa}$$ ⟵ $P_{o\,con}$

Problem 12.49

Given: Flow of standard air, $V = 600\,m/s$

Find: p_0, h_0, T_0

Solution:



Computing equations: $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$c = \sqrt{kRT}$$

Assumption: air behaves as an ideal gas, $k = 1.4$

$$c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 288\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 340\ m/s$$

$$M = \frac{V}{c} = \frac{600}{340} = 1.76$$

$$p_0 = p\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} = 101\,kPa\left[1 + 0.2(1.76)^2\right]^{3.5} = 546\ kPa$$ p_0

$$T_0 = T\left[1 + \frac{k-1}{2}M^2\right] = 288\,K\left[1 + 0.2(1.76)^2\right] = 466\ K$$ T_0

$dh = c_p dT$. For c_p = constant

$$h_0 - h = \int_h^{h_0} dh = \int_T^{T_0} c_p dT = c_p(T_0 - T) = 1000 \frac{J}{kg \cdot K}(466 - 288)K$$

$$h_0 - h = 178\ kJ/kg$$ $h_0 - h$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 12.50

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Given: DC-10 aircraft cruises at altitude, $z = 12$ km on a standard day.

$p_0 = 29.6$ kPa $\quad p = 19.4$ kPa

Find: (a) M (b) V (c) T_0 (on aircraft).

Solution:

Computing equations: $\dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2}M^2\right)^{k/k-1}$ $\qquad \dfrac{T_0}{T} = 1 + \dfrac{k-1}{2}M^2$

Solving the first equation for M

$$M = \left\{\frac{2}{(k-1)}\left[\left(\frac{p_0}{p}\right)^{\frac{k-1}{k}} - 1\right]\right\} = \left\{\frac{2}{1.4-1}\left[\left(\frac{29.6}{19.4}\right)^{\frac{1.4-1}{1.4}} - 1\right]\right\}^{1/2} = 0.801 \longleftarrow M$$

At $z = 12$ km, $T = 216.7$ K (Table A.3)

$$c = (kRT)^{1/2} = \left(1.4 \times 286.9\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 216.7\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 295 \text{ m/s}$$

$$V = Mc = 0.801 \times 295 \text{ m/s} = 236 \text{ m/s} \longleftarrow V$$

$$T_0 = T\left(1 + \frac{k-1}{2}M^2\right) = 216.7\text{K}\left[1 + \frac{(1.4-1)}{2}(0.801)^2\right] = 245\text{K} \longleftarrow T_0$$

Problem 12.51

[2]

12.51 An aircraft cruises at $M = 0.65$ at 10 km altitude on a standard day. The aircraft speed is deduced from measurement of the difference between the stagnation and static pressures. What is the value of this difference? Compute the air speed from this actual difference assuming (a) compressibility and (b) incompressibility. Is the discrepancy in air-speed computations significant in this case?

Given: Mach number of aircraft

Find: Pressure difference; air speed based on a) compressible b) incompressible assumptions

Solution:

The data provided, or available in the Appendices, is:

$$R = 287 \cdot \frac{J}{kg \cdot K} \qquad c_p = 1004 \cdot \frac{J}{kg \cdot K} \qquad k = 1.4 \qquad M = 0.65$$

From Table A.3, at 10 km altitude $\quad T = 223.3 \cdot K \qquad p = 0.2615 \cdot 101 \cdot kPa \qquad p = 26.4\,kPa$

The governing equation for pressure change is:
$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}} \qquad (12.20a)$$

Hence $\quad p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}} \qquad p_0 = 35.1\,kPa$

The pressure difference is $\quad p_0 - p = 8.67\,kPa$

a) Assuming compressibility $\quad c = \sqrt{k \cdot R \cdot T} \qquad c = 300 \frac{m}{s} \qquad V = M \cdot c \qquad V = 195 \frac{m}{s}$

b) Assuming incompressibility

Here the Bernoulli equation applies in the form $\quad \frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho} \quad$ so $\quad V = \sqrt{\frac{2 \cdot (p_0 - p)}{\rho}}$

For the density $\quad \rho = \frac{p}{R \cdot T} \qquad \rho = 0.412 \frac{kg}{m^3} \qquad V = \sqrt{\frac{2 \cdot (p_0 - p)}{\rho}}$

Hence $\quad V = 205 \frac{m}{s}$

In this case the error at *M* = 0.65 in computing the speed of the aircraft using Bernoulli equation is $\quad \frac{205 - 195}{195} = 5.13\%$

Problem 12.52

Given: The Anglo-French "Concorde" cruises at $M = 2.2$ at an altitude, $z = 20$ km.

Find: (a) c (b) V (c) α
(d) Maximum T_0 on aircraft

Solution:

At $z = 20$ km, $T = 216.7$ K (Table A.3).

$c = (kRT)^{1/2} = \left[1.4 \times 286.9 \frac{N \cdot m}{kg \cdot K} \times 216.7 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 295$ m/s $\longleftarrow c$

$V = Mc = 2.2 \times 295$ m/s $= 649$ m/s $\longleftarrow V$

$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = 27.0°$ $\longleftarrow \alpha$

$T_0 = T\left(1 + \frac{k-1}{2}M^2\right) = 216.7 K\left[1 + \frac{(1.4-1)}{2}(2.2)^2\right] = 426$ K $\longleftarrow T_0$

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 12.53

[2]

12.53 Modern high-speed aircraft use "air data computers" to compute air speed from measurement of the difference between the stagnation and static pressures. Plot, as a function of actual Mach number M, for $M = 0.1$ to $M = 0.9$, the percentage error in computing the Mach number assuming incompressibility (i.e., using the Bernoulli equation), from this pressure difference. Plot the percentage error in speed, as a function of speed, of an aircraft cruising at 12 km altitude, for a range of speeds corresponding to the actual Mach number ranging from $M = 0.1$ to $M = 0.9$.

Given: Flight altitude of high-speed aircraft

Find: Mach number and aircraft speed errors assuming incompressible flow; plot

Solution:

The governing equation for pressure change is:

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad (12.20a)$$

Hence

$$\Delta p = p_0 - p = p\cdot\left(\frac{p_0}{p} - 1\right) \qquad \Delta p = p\cdot\left[\left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right] \qquad (1)$$

For each Mach number the actual pressure change can be computed from Eq. 1

Assuming incompressibility, the Bernoulli equation applies in the form

$$\frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho} \quad \text{so} \quad V = \sqrt{\frac{2\cdot(p_0 - p)}{\rho}} = \sqrt{\frac{2\cdot\Delta p}{\rho}}$$

and the Mach number based on this is

$$M_{incomp} = \frac{V}{c} = \frac{\sqrt{\frac{2\cdot\Delta p}{\rho}}}{\sqrt{k\cdot R\cdot T}} = \sqrt{\frac{2\cdot\Delta p}{k\cdot\rho\cdot R\cdot T}}$$

Using Eq. 1

$$M_{incomp} = \sqrt{\frac{2}{k}\cdot\left[\left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right]}$$

The error in using Bernoulli to estimate the Mach number is

$$\frac{\Delta M}{M} = \frac{M_{incomp} - M}{M}$$

For errors in speed:

Actual speed: $V = M\cdot c$ $\qquad V = M\cdot\sqrt{k\cdot R\cdot T}$

Speed assuming incompressible flow: $V_{inc} = M_{incomp}\cdot\sqrt{k\cdot R\cdot T}$

The error in using Bernoulli to estimate the speed from the pressure difference is

$$\frac{\Delta V}{V} = \frac{V_{incomp} - V}{V}$$

The computations and plots are shown in the associated *Excel* workbook

Problem 12.53 (In Excel)

[2]

12.53 Modern high-speed aircraft use "air data computers" to compute air speed from measurement of the difference between the stagnation and static pressures. Plot, as a function of actual Mach number *M*, for $M = 0.1$ to $M = 0.9$, the percentage error in computing the Mach number assuming incompressibility (i.e., using the Bernoulli equation), from this pressure difference. Plot the percentage error in speed, as a function of speed, of an aircraft cruising at 12 km altitude, for a range of speeds corresponding to the actual Mach number ranging from $M = 0.1$ to $M = 0.9$.

Given: Flight altitude of high-speed aircraft

Find: Mach number and aircraft speed errors assuming incompressible flow; plot

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4
$T =$ 216.7 K (At 12 km, Table A.3)

Computing equations:

$$M_{incomp} = \sqrt{\frac{2}{k}\cdot\left[\left(1+\frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}-1\right]}$$

$$\frac{\Delta M}{M} = \frac{M_{incomp}-M}{M}$$

$$V = M\cdot\sqrt{k\cdot R\cdot T}$$

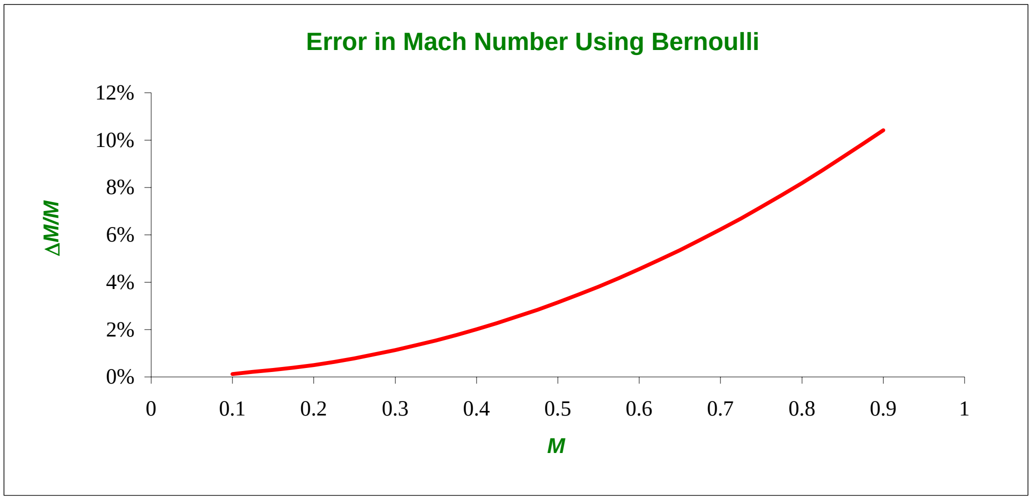
$$V_{inc} = M_{incomp}\cdot\sqrt{k\cdot R\cdot T}$$

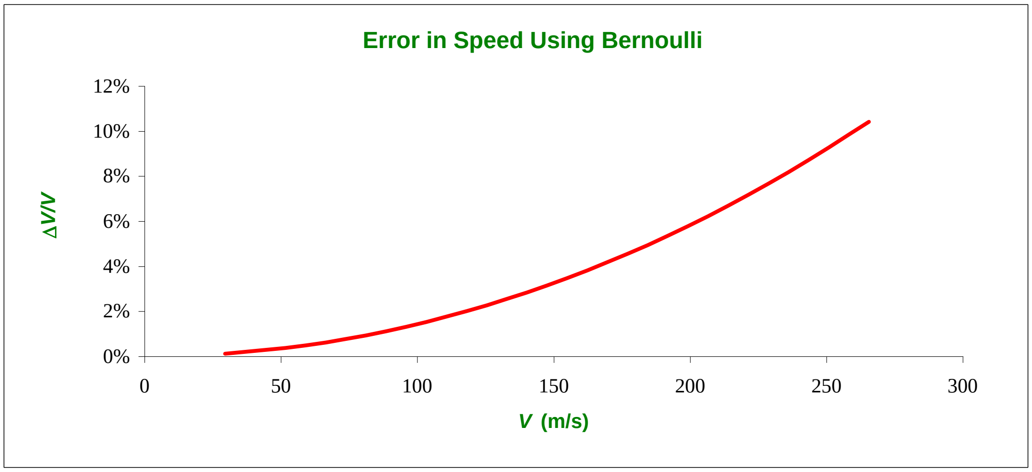
$$\frac{\Delta V}{V} = \frac{V_{incomp}-V}{V}$$

Computed results:

c = 295 m/s

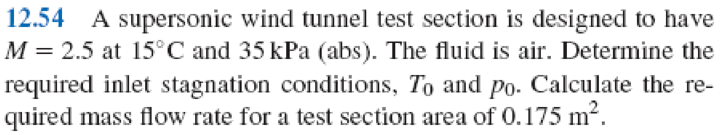
M	***M* incomp**	**Δ*M*/*M***	***V* (m/s)**	***V* incomp (m/s)**	**Δ*V*/*V***
0.1	0.100	0.13%	29.5	29.5	0.13%
0.2	0.201	0.50%	59.0	59.3	0.50%
0.3	0.303	1.1%	88.5	89.5	1.1%
0.4	0.408	2.0%	118	120	2.0%
0.5	0.516	3.2%	148	152	3.2%
0.6	0.627	4.6%	177	185	4.6%
0.7	0.744	6.2%	207	219	6.2%
0.8	0.865	8.2%	236	255	8.2%
0.9	0.994	10.4%	266	293	10.4%





Problem 12.54

[2]



12.54 A supersonic wind tunnel test section is designed to have $M = 2.5$ at 15°C and 35 kPa (abs). The fluid is air. Determine the required inlet stagnation conditions, T_0 and p_0. Calculate the required mass flow rate for a test section area of 0.175 m^2.

Given: Wind tunnel at M = 2.5

Find: Stagnation conditions; mass flow rate

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$

Given or available data $M = 2.5$ $\quad T = (15 + 273) \cdot K$ $\quad p = 35 \cdot kPa$ $\quad A = 0.175 \cdot m^2$

$k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

Then $T_0 = T \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)$ $\quad T_0 = 648\,K$ $\quad T_0 = 375 \cdot °C$

Also $p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 598 \cdot kPa$

The mass flow rate is given by $m_{rate} = \rho \cdot A \cdot V$

We need $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340 \frac{m}{s}$ $\quad V = M \cdot c$ $\quad V = 850 \frac{m}{s}$

and also $\rho = \frac{p}{R \cdot T}$ $\quad \rho = 0.424 \frac{kg}{m^3}$

Then $m_{rate} = \rho \cdot A \cdot V$ $\quad m_{rate} = 63.0 \frac{kg}{s}$

Problem 12.55

Given: Steady air flow through a constant area duct. Properties change due to friction, but flow is adiabatic.

Find: (a) Show that the energy equation reduces to

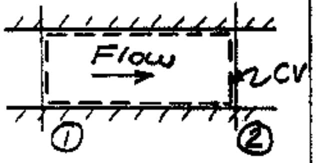
$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = \text{Constant}$$

(b) Show that for adiabatic flow $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

(c) Effects on T_0, p_0.

Solution: Apply the energy equation to the CV shown:

$$\text{BE}\quad \cancel{\dot{Q}}^{=0(1)} + \cancel{\dot{W}_s}^{=0(2)} + \cancel{\dot{W}_{shear}}^{=0(3)} = \cancel{\frac{\partial}{\partial t}\int_{CV} e\rho\, dV}^{=0(4)} + \int_{CS} (e + pv)\rho \vec{V}\cdot d\vec{A}$$



Assumptions: (1) $\dot{Q} = 0$ (adiabatic)
(2) $\dot{W}_s = 0$
(3) $\dot{W}_{shear} = 0$
(4) Steady flow
(5) Uniform flow at each section
(6) Neglect Δz

Then

$$0 = \left(u_1 + p_1 v_1 + \frac{V_1^2}{2}\right)\left\{-|\rho_1 V_1 A|\right\} + \left(u_2 + p_2 v_2 + \frac{V_2^2}{2}\right)\left\{|\rho_2 V_2 A|\right\}$$

But $h \equiv u + pv$, and $|\rho_1 V_1 A| = |\rho_2 V_2 A| = |\rho V A| = \dot{m}$, so

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h + \frac{V^2}{2} = h_0 = \text{Constant} \longleftarrow \quad h_0$$

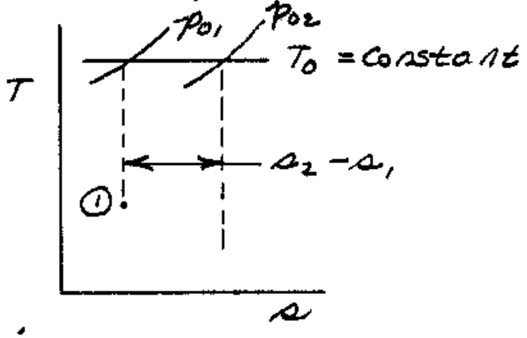
Assumption (7) Ideal gas; $h_0 - h = c_p(T_0 - T)$, $c_p = \frac{kR}{k-1}$, $c^2 = kRT$

Thus $c_p T_0 = c_p T + \frac{V^2}{2}$

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{(k-1)V^2}{2kRT} = 1 + \frac{k-1}{2}\frac{V^2}{c^2} = 1 + \frac{k-1}{2}M^2 \longleftarrow \quad \frac{T_0}{T}$$

From the energy equation, $T_{01} = T_{02} = T_0 = \text{constant} \longleftarrow \quad T_0$

The Ts diagram is



Since flow is frictional, $s_2 > s_1$. Therefore $p_{02} < p_{01} \longleftarrow \quad p_{02}$

Problem 12.56

[3]

12.56 A new design for a supersonic transport is tested in a wind tunnel at $M = 1.8$. Air is the working fluid. The stagnation temperature and pressure for the wind tunnel are 200 psia and 500°F, respectively. The model wing area is 100 in^2. The measured lift and drag are 12,000 lbf and 1600 lbf, respectively. Find the lift and drag coefficients.

Given: Wind tunnel test of supersonic transport

Find: Lift and drag coefficients

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$

$$C_L = \frac{F_L}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A} \qquad C_D = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A}$$

Given or available data $\quad M = 1.8 \quad T_0 = (500 + 460) \cdot R \quad p_0 = 200 \cdot \mathrm{psi} \quad F_L = 12000 \cdot \mathrm{lbf} \quad F_D = 1600 \cdot \mathrm{lbf}$

$$A = 100 \cdot \mathrm{in}^2 \qquad k = 1.4 \qquad R_{air} = 53.33 \cdot \frac{\mathrm{ft} \cdot \mathrm{lbf}}{\mathrm{lbm} \cdot R}$$

We need local conditions $\quad p = p_0 \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{-\frac{k}{k-1}} \qquad p = 34.8\,\mathrm{psi}$

$$T = \frac{T_0}{1 + \frac{k-1}{2} \cdot M^2} \qquad T = 583\,R \qquad T = 123\,°F$$

Then $\quad c = \sqrt{k \cdot R_{air} \cdot T} \qquad c = 1183\,\frac{\mathrm{ft}}{\mathrm{s}} \qquad c = 807\,\mathrm{mph}$

and $\quad V = M \cdot c \qquad V = 2129\,\frac{\mathrm{ft}}{\mathrm{s}} \qquad V = 1452\,\mathrm{mph}$

We also need $\quad \rho = \frac{p}{R_{air} \cdot T} \qquad \rho = 0.00501\,\frac{\mathrm{slug}}{\mathrm{ft}^3}$

Finally $\quad C_L = \frac{F_L}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A} \qquad C_L = 1.52$

$$C_D = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A} \qquad C_D = 0.203$$

Problem 12.57

[2]

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Given: For aircraft flying at supersonic speeds, the lift and drag coefficients are functions of M only.

Aircraft: $s = 75\ m$, $V = 780\ m/s$, $z = 20\ km$
standard atmosphere

Model: $s = 0.9\ m$, $T = 10°C$, $p = 10\ kPa\ (abs)$

Find:
(a) air speed for model test
(b) stagnation temperature for model test
(c) stagnation pressure for model test

Solution:

At $z = 20\ km$, $T = 216.7\ K$ (Table A.3).

$$c = (kRT)^{1/2} = \left(1.4 \times 286.9\ \frac{N \cdot m}{kg \cdot K} \times 216.7\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 295\ m/s$$

Thus for aircraft, $M = \frac{V}{c} = \frac{780}{295} = 2.64$

This Mach number must be duplicated in the model test. In the tunnel,

$$c = (kRT)^{1/2} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 283\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 337\ m/s$$

$$\therefore V = Mc = 2.64\,(337\ m/s) = 890\ m/s \longleftarrow V$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \therefore T_0 = T\left(1 + \frac{k-1}{2} M^2\right)$$

$$T_0 = 283\ K\left[1 + \frac{(1.4-1)}{2}(2.64)^2\right] = 677\ K \longleftarrow T_0$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} \qquad \therefore p_0 = p\left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$$

$$p_0 = 10\ kPa\left[1 + 0.2(2.64)^2\right]^{3.5} = 212\ kPa \longleftarrow p_0$$

Problem 12.58

NATIONAL 42 381 50 SHEETS 5 SQUARE 42 382 100 SHEETS 5 SQUARE 42 389 200 SHEETS 5 SQUARE

Given: The Lockheed "Blackbird" aircraft is thought to cruise at $M = 3.3$ at altitude, $z = 26$ km.

Because the speed is supersonic, a normal shock occurs in front of a total-head tube. The stagnation pressure decreases by 74.7 percent across the shock.

Find: (a) V (b) p_o c) p_o on aircraft d) T_o on aircraft

Solution:

At altitude $z = 26$ km, $T = 222.5$ K, $p/p_{SL} = 0.0216$ (Table A.3).

$$c = (kRT)^{1/2} = \left[1.4 \times 286.9 \frac{N \cdot m}{kg \cdot K} \times 222.5\,K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 299 \text{ m/s}$$

$$V = Mc = 3.3 \times 299 \text{ m/s} = 987 \text{ m/s} \longleftarrow V$$

The stagnation pressure ahead of the shock (designated p_{o1}) is given by

$$p_{o1} = p\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} = 0.0216\,p_{SL}\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$$

$$p_{o1} = 0.0216 \times 101.3\text{ kPa}\left[1 + \frac{(1.4-1)}{2}(3.3)^2\right]^{\frac{1.4}{0.4}} = 125 \text{ kPa} \longleftarrow p_{o1}$$

Designating the stagnation pressure behind the shock as p_{o2}, then

$$\frac{p_{o1} - p_{o2}}{p_{o1}} = 0.747 \quad \text{or} \quad p_{o2} = p_{o1} - 0.747\,p_{o1} = 0.253\,p_{o1}$$

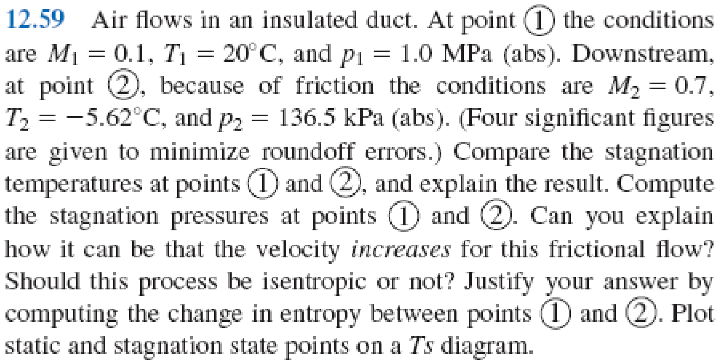
$$p_{o2} = 0.253 \times 125 \text{ kPa} = 31.6 \text{ kPa} \longleftarrow p_{o2}$$

The stagnation temperature does not change across a shock.

$$T_o = T\left[1 + \frac{k-1}{2}M^2\right] = 222.5\,K\left[1 + \frac{(1.4-1)}{2}(3.3)^2\right] = 707 \text{ K} \longleftarrow T_o$$

Problem 12.59

[2]



12.59 Air flows in an insulated duct. At point ① the conditions are $M_1 = 0.1$, $T_1 = 20°\text{C}$, and $p_1 = 1.0$ MPa (abs). Downstream, at point ②, because of friction the conditions are $M_2 = 0.7$, $T_2 = -5.62°\text{C}$, and $p_2 = 136.5$ kPa (abs). (Four significant figures are given to minimize roundoff errors.) Compare the stagnation temperatures at points ① and ②, and explain the result. Compute the stagnation pressures at points ① and ②. Can you explain how it can be that the velocity *increases* for this frictional flow? Should this process be isentropic or not? Justify your answer by computing the change in entropy between points ① and ②. Plot static and stagnation state points on a *Ts* diagram.

Given: Data on air flow in a duct

Find: Stagnation pressures and temperatures; explain velocity increase; isentropic or not?

Solution:

The data provided, or available in the Appendices, is:

$$R = 287 \cdot \frac{J}{kg \cdot K} \qquad c_p = 1004 \cdot \frac{J}{kg \cdot K} \qquad k = 1.4$$

$$M_1 = 0.1 \quad T_1 = (20 + 273) \cdot K \quad p_1 = 1000 \cdot kPa \quad M_2 = 0.7 \quad T_2 = (-5.62 + 273) \cdot K \quad p_2 = 136.5 \cdot kPa$$

For stagnation temperatures:

$$T_{01} = T_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^2\right) \qquad T_{01} = 293.6 K \qquad T_{01} = 20.6 \cdot C$$

$$T_{02} = T_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^2\right) \qquad T_{02} = 293.6 K \qquad T_{02} = 20.6 \cdot C$$

(Because the stagnation temperature is constant, the process is adiabatic)

For stagnation pressures:

$$p_{01} = p_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^2\right)^{\frac{k}{k-1}} \qquad p_{01} = 1.01 \cdot MPa$$

$$p_{02} = p_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^2\right)^{\frac{k}{k-1}} \qquad p_{02} = 189 \cdot kPa$$

The entropy change is:

$$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = 480 \cdot \frac{J}{kg \cdot K}$$

Note that

$$V_1 = M_1 \cdot \sqrt{k \cdot R \cdot T_1} \qquad V_1 = 34.3 \frac{m}{s} \qquad V_2 = M_2 \cdot \sqrt{k \cdot R \cdot T_2} \qquad V_2 = 229 \frac{m}{s}$$

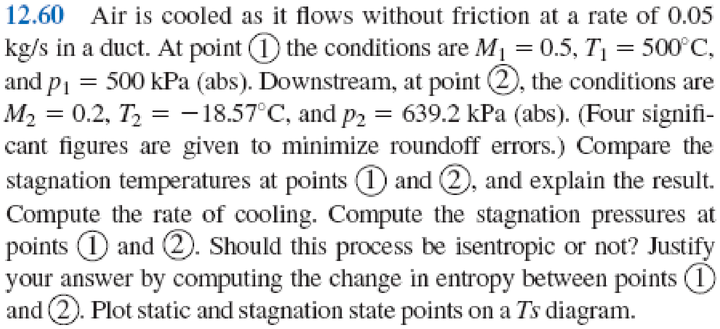
Although there is friction, suggesting the flow should decelerate, because the static pressure drops so much, the net effect is flow acceleration!

The entropy increases because the process is adiabatic but irreversible (friction).

From the second law of thermodynamics $ds \geq \frac{\delta q}{T}$: becomes $ds > 0$

Problem 12.60

[2]



12.60 Air is cooled as it flows without friction at a rate of 0.05 kg/s in a duct. At point ① the conditions are $M_1 = 0.5$, $T_1 = 500°\text{C}$, and $p_1 = 500$ kPa (abs). Downstream, at point ②, the conditions are $M_2 = 0.2$, $T_2 = -18.57°\text{C}$, and $p_2 = 639.2$ kPa (abs). (Four significant figures are given to minimize roundoff errors.) Compare the stagnation temperatures at points ① and ②, and explain the result. Compute the rate of cooling. Compute the stagnation pressures at points ① and ②. Should this process be isentropic or not? Justify your answer by computing the change in entropy between points ① and ②. Plot static and stagnation state points on a *Ts* diagram.

Given: Data on air flow in a duct

Find: Stagnation temperatures; explain; rate of cooling; stagnation pressures; entropy change

Solution:

The data provided, or available in the Appendices, is: $R = 287\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$ $c_p = 1004\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$ $k = 1.4$

$T_1 = (500 + 273)\cdot\text{K}$ $p_1 = 500\cdot\text{kPa}$ $T_2 = (-18.57 + 273)\cdot\text{K}$ $p_2 = 639.2\cdot\text{kPa}$

$M_1 = 0.5$ $M_2 = 0.2$ $M_{rate} = 0.05\cdot\frac{\text{kg}}{\text{s}}$

For stagnation temperatures: $T_{01} = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)$ $T_{01} = 811.7\,\text{K}$ $T_{01} = 539\cdot\text{C}$

$T_{02} = T_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)$ $T_{02} = 256.5\,\text{K}$ $T_{02} = -16.5\cdot\text{C}$

The fact that the stagnation temperature (a measure of total energy) decreases suggests cooling is taking place.

For the heat transfer: $Q = M_{rate}\cdot c_p\cdot\left(T_{02} - T_{01}\right)$ $Q = -27.9\,\text{kW}$

For stagnation pressures: $p_{01} = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}}$ $p_{01} = 593\,\text{kPa}$

$p_{02} = p_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)^{\frac{k}{k-1}}$ $p_{02} = 657\,\text{kPa}$

The entropy change is: $\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right)$ $\Delta s = -1186\,\frac{\text{J}}{\text{kg}\cdot\text{K}}$

The entropy decreases because the process is a cooling process (*Q* is negative).

From the second law of thermodynamics: $ds \ge \frac{\delta q}{T}$ becomes $ds \ge -ve$

Hence, if the process is reversible, the entropy must decrease; if it is irreversible, it may increase or decrease

Problem 12.61

[3]

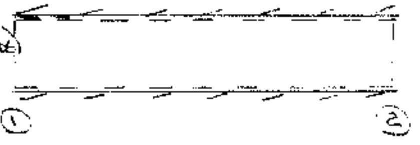
42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE

Given: Adiabatic flow through long straight pipe of cross-sectional area, $A = 0.05\ m^2$, as shown. Fluid is air

$T_1 = 60°C$

$p_1 = 200\ kPa\ (abs)$

$V_1 = 146\ m/s$



$p_2 = 95.6\ kPa\ (abs)$

$V_2 = 280\ m/s$

Find: $p_{0_1}, p_{0_2}, T_{0_1}, T_{0_2}, s_2 - s_1$

Solution:

Computing equations: $\dfrac{T_0}{T} = 1 + \dfrac{k-1}{2}M^2 \qquad \dfrac{p_0}{p} = \left[1 + \dfrac{k-1}{2}M^2\right]^{k/k-1}$

Basic equations: $0 = \overset{0(4)}{\cancel{\dfrac{\partial}{\partial t}\int_{CV}\rho\, d\forall}} + \int_{CS}\rho\vec{V}\cdot d\vec{A}$

$$\overset{0(1)}{\cancel{\dot{Q}}} + \overset{0(2)}{\cancel{\dot{W}_s}} + \overset{0(3)}{\cancel{\dot{W}_{shear}}} = \overset{0(4)}{\cancel{\frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall}} + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) $\dot{Q} = 0$ (adiabatic flow)
(2) $\dot{W}_s = 0$
(3) $\dot{W}_{shear} = 0$
(4) steady flow
(5) uniform flow at each section
(6) $\Delta z = 0$
(7) ideal gas, $k = 1.4$
(8) $A_1 = A_2 = A =$ constant

$M_1 = \dfrac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287\dfrac{N\cdot m}{kg\cdot K} \times 333K \times \dfrac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 366\ m/s$

$M_1 = \dfrac{V_1}{c_1} = \dfrac{146}{366} = 0.399 \qquad T_{0_1} = T_1\left[1 + \dfrac{k-1}{2}M_1^2\right] = 333K\left[1 + 0.2(0.399)^2\right] = 344K \longleftarrow T_{0_1}$

$p_{0_1} = p_1\left[1 + \dfrac{k-1}{2}M_1^2\right]^{k/k-1} = 200\ kPa\left[1 + 0.2(0.399)^2\right]^{3.5} = 223\ kPa \longleftarrow p_{0_1}$

From the energy equation, $0 = \left(u_1 + p_1v_1 + \dfrac{V_1^2}{2}\right)\{-|\rho_1V_1A|\} + \left(u_2 + p_2v_2 + \dfrac{V_2^2}{2}\right)\{|\rho_2V_2A|\}$

From continuity, $0 = -|\rho_1V_1A| + |\rho_2V_2A|$ or $\rho_1V_1 = \rho_2V_2$

Then, using $h = u + pv$,

$h_1 + \dfrac{V_1^2}{2} = h_{0_1} = h_2 + \dfrac{V_2^2}{2} = h_{0_2}$ or $h_{0_1} = h_{0_2}$

For an ideal gas with constant specific heats, $T_{0_2} = T_{0_1} = 344\ K \longleftarrow T_{0_2}$

From continuity, $\rho_2 = \rho_1\dfrac{V_1}{V_2} = \dfrac{p_1}{RT_1}\dfrac{V_1}{V_2} = 200\times10^3\dfrac{N}{m^2} \times \dfrac{kg\cdot K}{287\,N\cdot m} \times \dfrac{1}{333K} \times \dfrac{146}{280} = 1.09\ kg/m^3$

Then

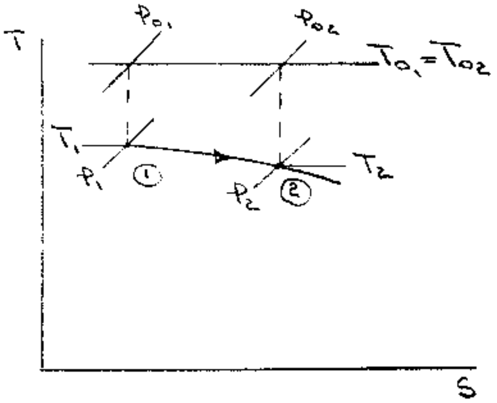
$T_2 = \dfrac{p_2}{\rho_2R} = 95.6\times10^3\dfrac{N}{m^2} \times \dfrac{m^3}{1.09\ kg} \times \dfrac{kg\cdot K}{287\,N\cdot m} = 306K\ (33°C)$

$c_2 = (kRT_2)^{1/2} = \left(1.4 \times 287\dfrac{N\cdot m}{kg\cdot K} \times 306K \times \dfrac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 351\ m/s$

$M_2 = \dfrac{V_2}{c_2} = \dfrac{280}{351} = 0.798$

$p_{0_2} = p_2\left[1 + \dfrac{k-1}{2}M^2\right]^{k/k-1} = 95.6\ kPa\left[1 + 0.2(0.798)^2\right]^{3.5}$

$p_{0_2} = 145\ kPa \longleftarrow p_{0_2}$



$T\,ds = dh - v\,dp = c_p\,dT - \dfrac{RT}{p}\,dp$

$ds = c_p\dfrac{dT}{T} - R\dfrac{dp}{p} = ds_0 = -R\dfrac{dp_0}{p_0}$

$s_{0_2} - s_{0_1} = s_2 - s_1 = -R\ln\dfrac{p_{0_2}}{p_{0_1}}$

$s_2 - s_1 = -287\dfrac{J}{kg\cdot K}\ln\dfrac{145}{223} = 0.124\dfrac{kJ}{kg\cdot K} \longleftarrow s_2 - s_1$

Problem 12.62

[3]

12.62 Air flows steadily through a constant-area duct. At section ①, the air is at 400 kPa (abs), 325 K, and 150 m/s. As a result of heat transfer and friction, the air at section ② downstream is at 275 kPa (abs), 450 K. Calculate the heat transfer per kilogram of air between sections ① and ②, and the stagnation pressure at section ②.

Given: Air flow in duct with heat transfer and friction

Find: Heat transfer; Stagnation pressure at location 2

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

$\rho \cdot V \cdot A = \text{const}$ $\quad h_1 + \frac{V_1^{\,2}}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^{\,2}}{2}$

Given or available data $p_1 = 400 \cdot kPa$ $\quad T_1 = 325 \cdot K$ $\quad V_1 = 150 \cdot \frac{m}{s}$

$p_2 = 275 \cdot kPa$ $\quad T_2 = 450 \cdot K$

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

Then $\rho_1 = \frac{p_1}{R \cdot T_1}$ $\quad \rho_1 = 4.29 \frac{kg}{m^3}$ $\quad \rho_2 = \frac{p_2}{R \cdot T_2}$ $\quad \rho_2 = 2.13 \frac{kg}{m^3}$

and from $\rho \cdot V \cdot A = \text{const}$ $\quad V_2 = V_1 \cdot \frac{\rho_1}{\rho_2}$ $\quad V_2 = 302 \frac{m}{s}$

Also $\frac{\delta Q}{dm} = q = h_2 - h_1 + \frac{V_2^{\,2} - V_1^{\,2}}{2}$

$q = c_p \cdot \left(T_2 - T_1\right) + \frac{V_2^{\,2} - V_1^{\,2}}{2}$ $\quad q = 160 \frac{kJ}{kg}$

We also have $c_2 = \sqrt{k \cdot R \cdot T_2}$ $\quad c_2 = 425 \frac{m}{s}$ so $M_2 = \frac{V_2}{c_2}$ $\quad M_2 = 0.711$

Hence $p_{02} = p_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^{\,2}\right)^{\frac{k}{k-1}}$ $\quad p_{02} = 385 \, kPa$

Problem 12.63

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
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Given: Air passes through a normal shock in a supersonic wind tunnel. Conditions upstream (state ①) and downstream (state ②) of the shock are given below.

$M_1 = 1.8$	$M_2 = 0.6165$
$T_1 = 270K$	$T_2 = 413.6K.$
$p_1 = 10\text{ kPa (abs)}$	$p_2 = 36.13\text{ kPa (abs)}$
①	②

Find: T_{01}, p_{01}, T_{02}, p_{02}, $s_2 - s_1$

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\quad \frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$

$$T_{01} = T_1\left[1+\frac{k-1}{2}M_1^2\right] = 270K\left[1+0.2(1.8)^2\right] = 445\text{ K} \longleftarrow T_{01}$$

$$p_{01} = p_1\left[1+\frac{k-1}{2}M_1^2\right]^{k/k-1} = 10.0\text{ kPa}\left[1+0.2(1.8)^2\right]^{3.5} = 57.5\text{ kPa (abs)} \longleftarrow p_{01}$$

$$T_{02} = T_2\left[1+\frac{k-1}{2}M_2^2\right] = 413.6K\left[1+0.2(0.6165)^2\right] = 445.K \longleftarrow T_{02}$$

(Flow through the shock is adiabatic, $T_{02} = T_{01}$)

$$p_{02} = p_2\left[1+\frac{k-1}{2}M_2^2\right]^{k/k-1} = 36.13\text{ kPa}\left[1+0.2(0.6165)^2\right]^{3.5} = 46.7\text{ kPa(abs)} \longleftarrow p_{02}$$

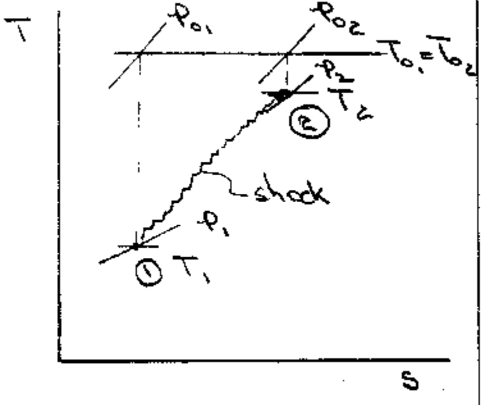
$$T\,ds = dh - v\,dp = c_p dT - RT\frac{dp}{p}$$

$$ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$s_2 - s_1 = s_{02} - s_{01} = -R\ln\frac{p_{02}}{p_{01}}$$

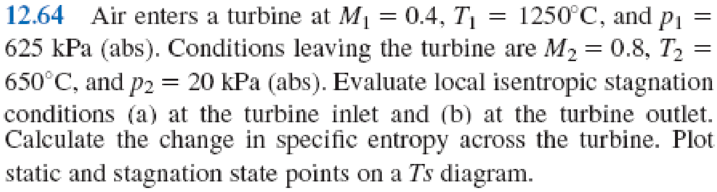
$$= -287\frac{J}{kg\cdot K}\ln\frac{46.69}{57.46}$$

$$s_2 - s_1 = 59.6\text{ J/kg}\cdot K \longleftarrow s_2 - s_1$$



Problem 12.64

[2]



12.64 Air enters a turbine at $M_1 = 0.4$, $T_1 = 1250°\text{C}$, and $p_1 = 625$ kPa (abs). Conditions leaving the turbine are $M_2 = 0.8$, $T_2 = 650°\text{C}$, and $p_2 = 20$ kPa (abs). Evaluate local isentropic stagnation conditions (a) at the turbine inlet and (b) at the turbine outlet. Calculate the change in specific entropy across the turbine. Plot static and stagnation state points on a Ts diagram.

Given: Air flow through turbine

Find: Stagnation conditions at inlet and exit; change in specific entropy; Plot on Ts diagram

Solution:

Basic equations:

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad \frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \Delta s = c_p\cdot \ln\left(\frac{T_2}{T_1}\right) - R\cdot \ln\left(\frac{p_2}{p_1}\right)$$

Given or available data

$M_1 = 0.4$ $\quad p_1 = 625\cdot\text{kPa}$ $\quad T_1 = (1250 + 273)\cdot\text{K}$

$M_2 = 0.8$ $\quad p_2 = 20\cdot\text{kPa}$ $\quad T_2 = (650 + 273)\cdot\text{K}$

$c_p = 1004\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$ $\quad k = 1.4$ $\quad R = 286.9\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$

Then

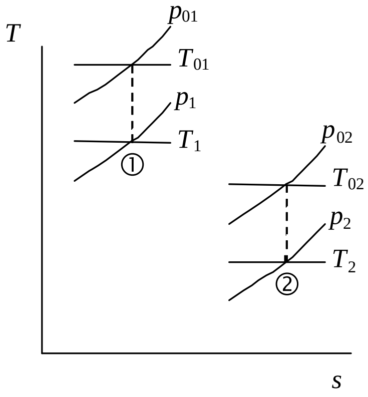
$$T_{01} = T_1\cdot\left(1 + \frac{k-1}{2}\cdot {M_1}^2\right) \qquad T_{01} = 1572\,\text{K} \qquad T_{01} = 1299\cdot°\text{C}$$

$$p_{01} = p_1\cdot\left(1 + \frac{k-1}{2}\cdot {M_1}^2\right)^{\frac{k}{k-1}} \qquad p_{01} = 698\cdot\text{kPa}$$

$$T_{02} = T_2\cdot\left(1 + \frac{k-1}{2}\cdot {M_2}^2\right) \qquad T_{02} = 1041\,\text{K} \qquad T_{02} = 768\cdot°\text{C}$$

$$p_{02} = p_2\cdot\left(1 + \frac{k-1}{2}\cdot {M_2}^2\right)^{\frac{k}{k-1}} \qquad p_{02} = 30\cdot\text{kPa}$$

$$\Delta s = c_p\cdot \ln\left(\frac{T_2}{T_1}\right) - R\cdot \ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = 485\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$$



Problem 12.65

[3]

Given: Boeing 747 cruising at $M = 0.87$ at $z = 13$ km, std. day.
Window located where $M = 0.2$ relative to surface.
Cabin pressurized to equivalent of $z = 2.5$ km, std. day.

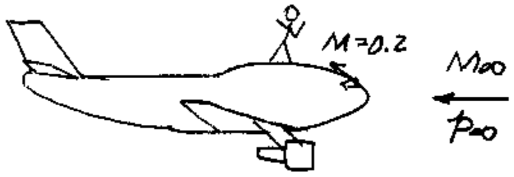
Find: Pressure difference across window.

Solution: Apply isentropic stagnation relations.

Computing equation: $p_0 = p\left(1+\frac{k-1}{2}M^2\right)^{k/k-1}$

Assumptions: (1) Ideal gas
(2) Isentropic flow

Consider observer on aircraft: air is decelerated isentropically from $M_\infty = 0.87$ to $M = 0.2$.



From Table A.3:

Calculated: $p = \left(\frac{p}{p_0}\right)p_0$

$p_0 = 101.3$ kPa

Altitude (km)	p/p_0 (---)	p (kPa)
2.5	0.7372	74.7
13.0	0.1636	16.6

For isentropic stagnation:

$$p_0 = p_\infty\left(1+\frac{k-1}{2}M_\infty^2\right)^{k/k-1} = 16.6\text{ kPa}\left(1+\frac{1.4-1}{2}(0.87)^2\right)^{3.5} = 27.2\text{ kPa (abs)}$$

From stagnation to $M = 0.2$:

$$p_{out} = \frac{p_0}{\left(1+\frac{k-1}{2}M^2\right)^{k/k-1}} = \frac{27.2\text{ kPa}}{\left(1+0.2(0.2)^2\right)^{3.5}} = 26.5\text{ kPa (abs)}$$

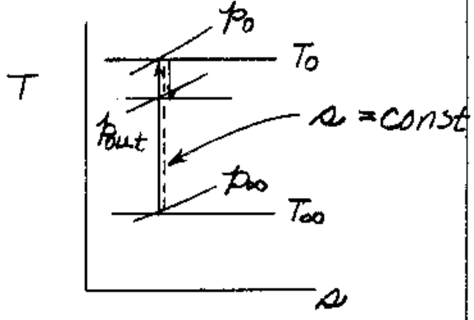
Pressure difference across window is:

$$\Delta p = p_{in} - p_{out} = (74.7 - 26.5)\text{ kPa} = 48.2\text{ kPa}$$

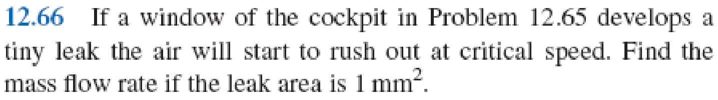
Δp

{Inside pressure is higher; window force is toward outside.}

The corresponding Ts diagram is:



Problem 12.66



12.66 If a window of the cockpit in Problem 12.65 develops a tiny leak the air will start to rush out at critical speed. Find the mass flow rate if the leak area is 1 mm^2.

Given: Air flow leak in window of airplane

Find: Mass flow rate

Solution:

Basic equations:

$$m_{rate} = \rho \cdot V \cdot A \qquad V_{crit} = \sqrt{\frac{2 \cdot k}{k+1} \cdot R \cdot T_0} \qquad \frac{\rho_0}{\rho_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}$$

The interior conditions are the stagnation conditions for the flow

Given or available data

$$T_0 = 271.9 \cdot K \qquad \rho_{SL} = 1.225 \cdot \frac{kg}{m^3} \qquad \rho_0 = 0.7812 \cdot \rho_{SL} \qquad \rho_0 = 0.957 \frac{kg}{m^3}$$

(Above data from Table A.3 at an altitude of 2500 m)

$$A = 1 \cdot mm^2 \qquad c_p = 1004 \cdot \frac{J}{kg \cdot K} \qquad k = 1.4 \qquad R = 286.9 \cdot \frac{J}{kg \cdot K}$$

Then

$$\rho_{crit} = \frac{\rho_0}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}} \qquad \rho_{crit} = 0.607 \frac{kg}{m^3} \qquad V_{crit} = \sqrt{\frac{2 \cdot k}{k+1} \cdot R \cdot T_0} \qquad V_{crit} = 302 \frac{m}{s}$$

The mass flow rate is

$$m_{rate} = \rho_{crit} \cdot V_{crit} \cdot A \qquad m_{rate} = 1.83 \times 10^{-4} \frac{kg}{s}$$

Problem 12.67

[1]

Given: A CO_2 cartridge contains gas at $p_0 = 45$ MPa (gage) and $T_0 = 25°C$

Find: T^*, p^*, V^* that correspond to these stagnation conditions

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/k-1}$

For CO_2, $k = 1.29$. At critical conditions, $M = 1$

$\frac{T_0^*}{T^*} = 1 + \frac{k-1}{2} = 1.145 \qquad \therefore T^* = \frac{T_0^*}{1.145} = \frac{298\,K}{1.145} = 260\,K \longleftarrow T^*$

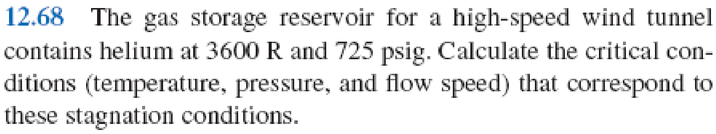
$\frac{p_0^*}{p^*} = \left[1 + \frac{k-1}{2}\right]^{k/k-1} = \left[1.145\right]^{4.448} = 1.826$

$\therefore p^* = \frac{p_0^*}{1.826} = \frac{45.101\text{ MPa}}{1.826} = 24.7\text{ MPa (abs)} \longleftarrow p^*$

$V^* = c^* = (kRT^*)^{1/2} = \left(1.29 \times 189\,\frac{N\cdot m}{kg\cdot K} \times 260\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 252\text{ m/s} \longleftarrow V^*$

Problem 12.68

[1]



12.68 The gas storage reservoir for a high-speed wind tunnel contains helium at 3600 R and 725 psig. Calculate the critical conditions (temperature, pressure, and flow speed) that correspond to these stagnation conditions.

Given: Data on helium in reservoir

Find: Critical conditions

Solution:

The data provided, or available in the Appendices, is:

$R_{He} = 386.1 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $\quad k = 1.66$ $\quad T_0 = 3600 \cdot R$ $\quad p_0 = (725 + 14.7) psi$ $\quad p_0 = 740 \, psi$

For critical conditions $\quad \frac{T_0}{T_{crit}} = \frac{k+1}{2}$ $\quad T_{crit} = \frac{T_0}{\frac{k+1}{2}}$ $\quad T_{crit} = 2707 \, R$

$$\frac{p_0}{p_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \qquad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}} \qquad p_{crit} = 361 \, psi$$

absolute

$$V_{crit} = \sqrt{k \cdot R_{He} \cdot T_{crit}} \qquad V_{crit} = 7471 \frac{ft}{s}$$

Problem 12.69

Given: Stagnation conditions in a solid propellant rocket motor are $T_0 = 3000\,K$ and $p_0 = 45\,MPa$ (gage). Assume ideal gas behavior with $R = 323\,J/kg\cdot K$, $k = 1.2$. The Mach number is unity at the throat of the nozzle.

Find: T_t, p_t, V_t

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\quad$ $\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/k-1}$

Assume flow to throat is isentropic.

At throat, $M = 1$

$$\therefore T_{0_t} = T_t\left(1 + \frac{k-1}{2}\right) = 1.1\,T_t \qquad \therefore T_t = \frac{T_0}{1.1} = \frac{30.00\,K}{1.10} = 2730\,K \longleftarrow T_t$$

$$\frac{p_{0_t}}{p_t} = \left(1 + \frac{k-1}{2}\right)^{k/k-1} = (1.1)^{\frac{1.2}{0.2}} = 1.7716$$

$$\therefore p_t = \frac{p_0}{1.7716} = \frac{45.101\,MPa}{1.7716} = 25.5\,MPa\,(abs) \longleftarrow p_t$$

$$V_t = (kRT_t)^{1/2} = \left[1.2 \times 323\,\frac{N\cdot m}{kg\cdot K} \times 2730\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 1030\,m/s \longleftarrow V_t$$

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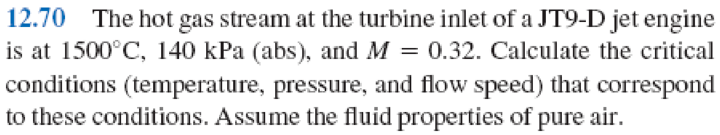
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 12.70

[1]



12.70 The hot gas stream at the turbine inlet of a JT9-D jet engine is at 1500°C, 140 kPa (abs), and $M = 0.32$. Calculate the critical conditions (temperature, pressure, and flow speed) that correspond to these conditions. Assume the fluid properties of pure air.

Given: Data on hot gas stream

Find: Critical conditions

Solution:

The data provided, or available in the Appendices, is:

$R = 287 \cdot \frac{J}{kg \cdot K}$ $\qquad k = 1.4$ $\qquad T_0 = (1500 + 273) \cdot K$ $\qquad T_0 = 1773\,K$ $\qquad p_0 = 140 \cdot kPa$

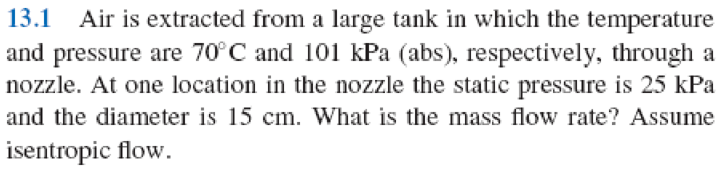
For critical conditions $\qquad \frac{T_0}{T_{crit}} = \frac{k+1}{2}$ $\qquad T_{crit} = \frac{T_0}{\frac{k+1}{2}}$ $\qquad T_{crit} = 1478\,K$

$\frac{p_0}{p_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$ $\qquad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}}$ $\qquad p_{crit} = 74.0\,kPa$ $\qquad$ absolute

$V_{crit} = \sqrt{k \cdot R \cdot T_{crit}}$ $\qquad V_{crit} = 770\,\frac{m}{s}$

Problem 13.1

[2]



13.1 Air is extracted from a large tank in which the temperature and pressure are 70°C and 101 kPa (abs), respectively, through a nozzle. At one location in the nozzle the static pressure is 25 kPa and the diameter is 15 cm. What is the mass flow rate? Assume isentropic flow.

Given: Air extracted from a large tank

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho\cdot V\cdot A$ $\quad h_1 + \frac{V_1^{\,2}}{2} = h_2 + \frac{V_2^{\,2}}{2}$ $\quad \frac{p}{\rho^k} = \text{const}$ $\quad T\cdot p^{\frac{(1-k)}{k}} = \text{const}$

Given or available data $T_0 = (70 + 273)\cdot K$ $\quad p_0 = 101\cdot kPa$ $\quad p = 25\cdot kPa$

$D = 15\cdot cm$ $\quad c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\quad k = 1.4$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$

The mass flow rate is given by $m_{rate} = \rho\cdot A\cdot V$ $\quad A = \frac{\pi\cdot D^2}{4}$ $\quad A = 0.0177\,m^2$

We need the density and velocity at the nozzle. In the tank $\rho_0 = \frac{p_0}{R\cdot T_0}$ $\quad \rho_0 = 1.026\frac{kg}{m^3}$

From the isentropic relation $\rho = \rho_0\cdot\left(\frac{p}{p_0}\right)^{\frac{1}{k}}$ $\quad \rho = 0.379\frac{kg}{m^3}$

We can apply the energy equation between the tank (stagnation conditions) and the point in the nozzle to find the velocity

$$h_0 = h + \frac{V^2}{2} \qquad V = \sqrt{2\cdot\left(h_0 - h\right)} = \sqrt{2\cdot c_p\cdot\left(T_0 - T\right)}$$

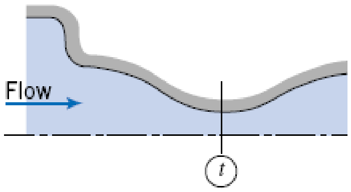
Fot T we again use insentropic relations $T = T_0\cdot\left(\frac{p_0}{p}\right)^{\frac{(1-k)}{k}}$ $\quad T = 230.167\,K$ $\quad T = -43.0\cdot{}^\circ C$

Then $V = \sqrt{2\cdot c_p\cdot\left(T_0 - T\right)}$ $\quad V = 476\frac{m}{s}$

The mass flow rate is $m_{rate} = \rho\cdot A\cdot V$ $\quad m_{rate} = 3.18\frac{kg}{s}$

Note that the flow is supersonic at this point $c = \sqrt{k\cdot}\; c = 304\frac{m}{s}$ $\quad M = \frac{V}{c}$ $\quad M = 1.57$

Hence we must have a converging-diverging nozzle



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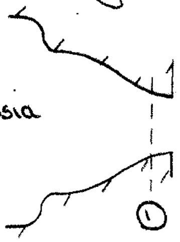
Problem 13.2

Given: Steam flows steadily and isentropically through a nozzle

$T_0 = 900°F$

$P_0 = 900$ psia



$P_1 = 600$ psia

$D_1 = 0.188$ in

(nozzle shape unspecified)

Find: V_1, M_1, $\dot{m}$, passage shape

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} = h + \frac{V^2}{2} = h_0 =$ constant

$\rho A V =$ constant $= \dot{m}$

Assumptions:
(1) steady flow
(2) isentropic flow
(3) uniform flow at a section
(4) superheated steam can be treated as an ideal gas, $R = 85.8$ ft·lbf/lbm·°R, $k = 1.28$
(5) $\Delta z = 0$

From steam tables for superheated vapor with $T_0 = 900°F$, $P_0 = 900$ psia,
$h_0 = 1451.8$ Btu/lbm $s_0 = 1.6257$ Btu/lbm·°R

From steam tables for superheated vapor with $s_1 = 1.6257$ Btu/lbm·°R, and $P_1 = 600$ psia,
$T_1 = 780°F$, $h_1 = 1396.5$ Btu/lbm, $v_1 = 1.167$ ft³/lbm

From the first law,

$$V_1 = [2(h_0 - h_1)]^{1/2} = \left[2(1451.8 - 1396.5)\frac{\text{Btu}}{\text{lbm}} \times 778\frac{\text{ft}\cdot\text{lbf}}{\text{Btu}} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/2}$$

$$V_1 = 1660 \ \text{ft/s} \longleftarrow V_1$$

$$\dot{m} = \rho A V = \frac{1}{v}\pi\frac{D^2}{4}V = \frac{\text{lbm}}{1.167\,\text{ft}^3} \times \frac{\pi}{4} \times \left(\frac{0.188}{12}\right)^2 \text{ft}^2 \times 1660\frac{\text{ft}}{\text{s}} = 0.274 \ \text{lbm/s} \longleftarrow \dot{m}$$

$$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT)^{1/2} = \left(1.30 \times 85.8\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°\text{R}} \times 1240\,\text{R} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2}$$

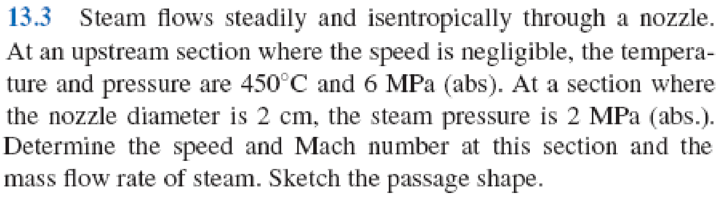
$$c_1 = 2110 \ \text{ft/s}$$

$$M_1 = \frac{V_1}{c_1} = \frac{1660}{2110} = 0.787 \longleftarrow$$

Since $M_1 < 1.0$, passage converges as shown above

Problem 13.3

[2]



13.3 Steam flows steadily and isentropically through a nozzle. At an upstream section where the speed is negligible, the temperature and pressure are 450°C and 6 MPa (abs). At a section where the nozzle diameter is 2 cm, the steam pressure is 2 MPa (abs.). Determine the speed and Mach number at this section and the mass flow rate of steam. Sketch the passage shape.

Given: Steam flow through a nozzle

Find: Speed and Mach number; Mass flow rate; Sketch the shape

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions: 1) Steady flow 2) Isentropic 3) Uniform flow 4) Superheated steam can be treated as ideal gas

Given or available data $T_0 = (450 + 273) \cdot K$ $\quad p_0 = 6 \cdot MPa$ $\quad p = 2 \cdot MPa$

$D = 2 \cdot cm$ $\quad k = 1.30$ $\quad R = 461.4 \cdot \frac{J}{kg \cdot K}$ (Table A.6)

From the steam tables (try finding interactive ones on the Web!), at stagnation conditions

$s_0 = 6720 \cdot \frac{J}{kg \cdot K}$ $\quad h_0 = 3.302 \times 10^6 \cdot \frac{J}{kg}$

Hence at the nozzle section $s = s_0 = 6720 \cdot \frac{J}{kg \cdot K}$ and $p = 2\,MPa$

From these values we find from the steam tables that $T = 289\,°C$ $\quad h = 2.997 \times 10^6 \cdot \frac{J}{kg}$ $\quad v = 0.1225 \cdot \frac{m^3}{kg}$

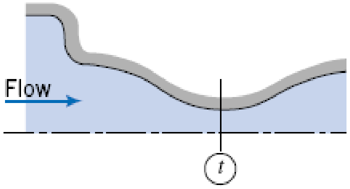
Hence the first law becomes $V = \sqrt{2 \cdot \left(h_0 - h\right)}$ $\quad V = 781 \frac{m}{s}$

The mass flow rate is given by $m_{rate} = \rho \cdot A \cdot V = \frac{A \cdot V}{v}$ $\quad A = \frac{\pi \cdot D^2}{4}$ $\quad A = 3.14 \times 10^{-4} m^2$

Hence $m_{rate} = \frac{A \cdot V}{v}$ $\quad m_{rate} = 2.00 \frac{kg}{s}$

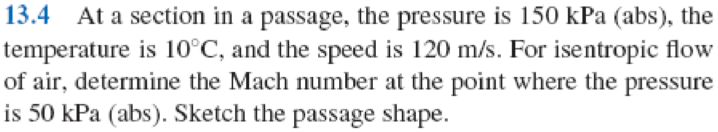
For the Mach number we need $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 581 \frac{m}{s}$ $\quad M = \frac{V}{c}$ $\quad M = 1.35$

The flow is supersonic starting from rest, so must be converging-diverging



Problem 13.4

[2]



13.4 At a section in a passage, the pressure is 150 kPa (abs), the temperature is 10°C, and the speed is 120 m/s. For isentropic flow of air, determine the Mach number at the point where the pressure is 50 kPa (abs). Sketch the passage shape.

Given: Air flow in a passage

Find: Mach number; Sketch shape

Solution:

Basic equations: $\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\qquad c = \sqrt{k\cdot R\cdot T}$

Given or available data $T_1 = (10 + 273)\cdot K$ $\qquad p_1 = 150\cdot kPa$ $\qquad V_1 = 120\cdot\frac{m}{s}$

$p_2 = 50\cdot kPa$ $\qquad k = 1.4$ $\qquad R = 286.9\cdot\frac{J}{kg\cdot K}$

The speed of sound at state 1 is $c_1 = \sqrt{k\cdot R\cdot T_1}$ $\qquad c_1 = 337\frac{m}{s}$

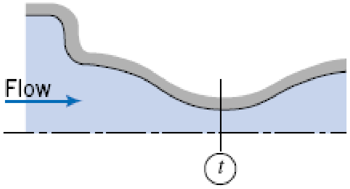
Hence $M_1 = \frac{V_1}{c_1}$ $\qquad M_1 = 0.356$

For isentropic flow stagnation pressure is constant. Hence at state 2 $\frac{p_0}{p_2} = \left(1 + \frac{k-1}{2}\cdot {M_2}^2\right)^{\frac{k}{k-1}}$

Hence $p_0 = p_1\cdot\left(1 + \frac{k-1}{2}\cdot {M_1}^2\right)^{\frac{k}{k-1}}$ $\qquad p_0 = 164\,kPa$

Solving for M_2 $\qquad M_2 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_2}\right)^{\frac{k-1}{k}} - 1\right]}$ $\qquad M_2 = 1.42$

Hence, as we go from subsonic to supersonic we must have a converging-diverging nozzle



Problem 13.5

[2]

13.5 At a section in a passage, the pressure is 30 psia, the temperature is 100°F, and the speed is 1750 ft/s. At a section downstream the Mach number is 2.5. Determine the pressure at this downstream location for isentropic flow of air. Sketch the passage shape.

Given: Data on flow in a passage

Find: Pressure at downstream location

Solution:

The given or available data is:

$R =$ 53.33 ft·lbf/lbm·°R
$k =$ 1.4
$T_1 =$ 560 °R
$p_1 =$ 30 psi
$V_1 =$ 1750 ft/s
$M_2 =$ 2.5

Equations and Computations:

From T_1 and Eq. 12.18

$$c = \sqrt{kRT}$$

$c_1 =$ 1160 ft/s

Then

$M_1 =$ 1.51

From M_1 and p_1, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)}$$

$p_{01} =$ 111 psi

For isentropic flow ($p_{01} = p_{02}$)

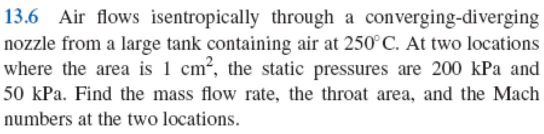
$p_{02} =$ 111 psi

From M_2 and p_{02}, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_2 =$ 6.52 psi

Problem 13.6

[3]



13.6 Air flows isentropically through a converging-diverging nozzle from a large tank containing air at 250°C. At two locations where the area is 1 cm^2, the static pressures are 200 kPa and 50 kPa. Find the mass flow rate, the throat area, and the Mach numbers at the two locations.

Given: Data on flow in a nozzle

Find: Mass flow rate; Throat area; Mach numbers

Solution:

The given or available data is:

R = 286.9 J/kg·K

k = 1.4

T_0 = 523 K

p_1 = 200 kPa $\quad$ p_2 = 50 kPa

A = 1 cm^2

Equations and Computations:

We don't know the two Mach numbers. We do know for each that Eq. 13.7a applies:

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)}$$

Hence we can write two equations, but have three unknowns (M_1, M_2, and p_0)!

We also know that states 1 and 2 have the same area. Hence we can write Eq. 13.7d twice:

$$\frac{A}{A*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2} M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

We now have four equations for four unknowns ($A*$, M_1, M_2, and p_0)!

We make guesses (using Solver) for M_1 and M_2, and make the errors in computed $A*$ and p_0 zero.

For:	M_1 =	0.512		M_2 =	1.68		**Errors**
from Eq. 13.7a:	p_0 =	239	kPa	p_0 =	239	kPa	0.00%
and from Eq. 13.7d:	$A*$ =	0.759	cm^2	$A*$ =	0.759	cm^2	0.00%
Note that the throat area is the critical area						**Sum**	0.00%

The stagnation density is then obtained from the ideal gas equation

ρ_0 = 1.59 kg/m^3

The density at critical state is obtained from Eq. 13.7a (or 12.22c)

$\rho*$ = 1.01 kg/m^3

The velocity at critical state can be obtained from Eq. 12.23)

$$V* = c* = \sqrt{\frac{2k}{k+1} R T_0}$$

$V*$ = 418 m/s

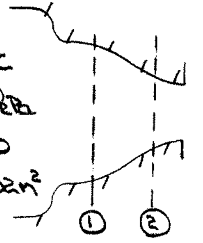
The mass flow rate is $\rho* V* A*$

m_{rate} = 0.0321 kg/s

Problem 13.7

Given: Steady, isentropic flow of air through a passage

$T_1 = 60°C$
$p_1 = 40\ kPa$
$M_1 = 2.0$
$A_1 = 0.02\ m^2$



$V_2 = 519\ m/s$

(passage shape unspecified)

Find: M_2, shape of passage

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions:
(1) steady flow
(2) isentropic flow
(3) uniform flow at a section
(4) $\Delta z = 0$
(5) ideal gas

$M_2 = \frac{V_2}{c_2}$ where $c_2 = (kRT_2)^{1/2}$. Hence T_2 must be found

$$h_2 = h_1 + \frac{1}{2}(V_1^2 - V_2^2)$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 333 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 732\ m/s$$

$$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

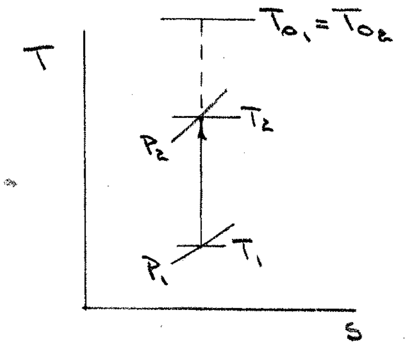
$$= 333K + [(732)^2 - (519)^2]\frac{m^2}{s^2} \times \frac{1}{2} \times \frac{kg \cdot K}{10^3\ N \cdot m} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$T_2 = 466\ K$$

$$c_2 = (kRT_2)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 466 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 433\ m/s$$

$$M_2 = \frac{V_2}{c_2} = \frac{519}{433} = 1.20 \longleftarrow M_2$$

Since $M_2 < M_1$ and $M_2 > 1.0$, then passage from ① to ② is a supersonic diffuser as shown above



Problem 13.8

[3]

13.8 Air flows steadily and isentropically through a passage at 150 lbm/s. At the section where the diameter is $D = 3$ ft, $M = 1.75$, $T = 32°$F, and $p = 25$ psia. Determine the speed and cross-sectional area downstream where $T = 225°$F. Sketch the flow passage.

Given: Air flow in a passage

Find: Speed and area downstream; Sketch flow passage

Solution:

Basic equations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad c = \sqrt{k\cdot R\cdot T} \qquad \frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Given or available data

$$T_1 = (32+460)\cdot R \qquad p_1 = 25\cdot psi \qquad M_1 = 1.75$$

$$T_2 = (225+460)\cdot R \qquad k = 1.4 \qquad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$$

$$D_1 = 3\cdot ft \qquad A_1 = \frac{\pi\cdot D_1^{\,2}}{4} \qquad A_1 = 7.07\,ft^2$$

Hence

$$T_0 = T_1\cdot\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right) \qquad T_0 = 793\,R \qquad T_0 = 334\,°F$$

For isentropic flow stagnation conditions are constant. Hence

$$M_2 = \sqrt{\frac{2}{k-1}\cdot\left(\frac{T_0}{T_2}-1\right)} \qquad M_2 = 0.889$$

We also have

$$c_2 = \sqrt{k\cdot R_{air}\cdot T_2} \qquad c_2 = 1283\,\frac{ft}{s}$$

Hence

$$V_2 = M_2\cdot c_2 \qquad V_2 = 1141\,\frac{ft}{s}$$

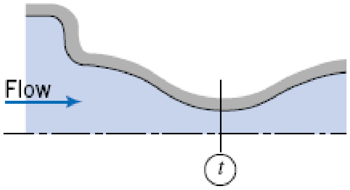
From state 1

$$A_{crit} = \frac{A_1\cdot M_1}{\left(\frac{1+\frac{k-1}{2}\cdot M_1^{\,2}}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}} \qquad A_{crit} = 5.10\,ft^2$$

Hence at state 2

$$A_2 = \frac{A_{crit}}{M_2}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M_2^{\,2}}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}} \qquad A_2 = 5.15\,ft^2$$

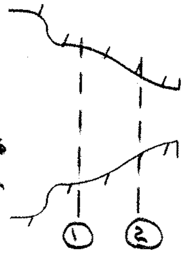
Hence, as we go from supersonic to subsonic we must have a converging-diverging diffuser



Problem 13.9

Given: Steady, isentropic flow of air through a passage.

$T_1 = 27°C$
$p_1 = 60\ kPa$
$V_1 = 486\ m/s$
$A_1 = 0.02\ m^2$



$p_2 = 78.8\ kPa$

(passage shape unspecified)

Find: M_2

Solution:

Computing equations: $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $\qquad c = \sqrt{kRT}$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

For isentropic flow, $p_{0_1} = p_{0_2} = p_0 = \text{constant}$

$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 300K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 347\ m/s$

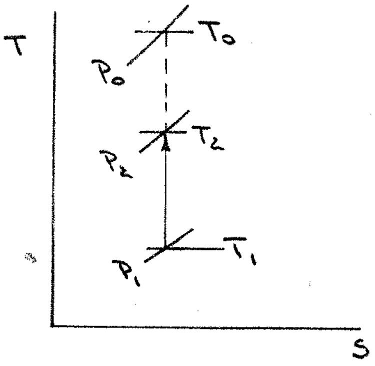
$M_1 = \frac{V_1}{c_1} = \frac{486}{347} = 1.40 \qquad p_{0_1} = p_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 60\ kPa\left[1 + 0.2(1.40)^2\right]^{3.5}$

$p_{0_1} = 191\ kPa$

$\frac{p_{0_2}}{p_2} = \left[1 + \frac{k-1}{2}M_2^2\right]^{k/k-1}, \qquad p_{0_2} = p_{0_1}$

$M_2 = \left\{\frac{2}{k-1}\left[\left(\frac{p_{0_1}}{p_2}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{191}{78.8}\right)^{0.286} - 1\right]\right\}^{1/2} = 1.20 \longleftarrow M_2$

Since $M_2 < M_1$ and $M_2 > 1.0$, then passage from ① to ② is a supersonic diffuser as shown above



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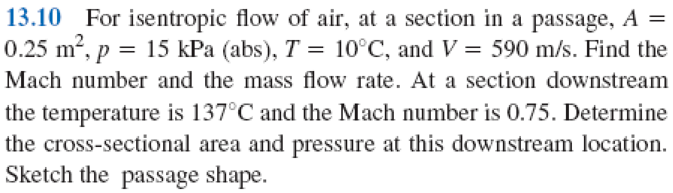
Problem 13.10

[2]



13.10 For isentropic flow of air, at a section in a passage, $A = 0.25\ \text{m}^2$, $p = 15$ kPa (abs), $T = 10°\text{C}$, and $V = 590$ m/s. Find the Mach number and the mass flow rate. At a section downstream the temperature is 137°C and the Mach number is 0.75. Determine the cross-sectional area and pressure at this downstream location. Sketch the passage shape.

Given: Data on flow in a passage

Find: Flow rate; area and pressure at downstream location; sketch passage shape

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K

$k =$ 1.4

$A_1 =$ 0.25 m^2

$T_1 =$ 283 K

$p_1 =$ 15 kPa

$V_1 =$ 590 m/s

$T_2 =$ 410

$M_2 =$ 0.75

Equations and Computations:

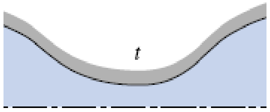
From T_1 and Eq. 12.18

$$c = \sqrt{kRT} \quad (12.18)$$

$c_1 =$ 337 m/s

Then $M_1 =$ 1.75

Because the flow decreases isentropically from supersonic to subsonic the passage shape must be convergent-divergent



From p_1 and T_1 and the ideal gas equation

$\rho_1 =$ 0.185 kg/m^3

The mass flow rate is $m_{\text{rate}} = \rho_1 A_1 V_1$

$m_{\text{rate}} =$ 27.2 kg/s

From M_1 and A_1, and Eq. 13.7d
(using built-in function *IsenA* (M ,k))

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (13.7d)$$

$A^* =$ 0.180 m^2

From M_2 and A^*, and Eq. 13.7d
(using built-in function *IsenA* (M ,k))

$A_2 =$ 0.192 m^2

From M_1 and p_1, and Eq. 13.7a
(using built-in function *Isenp* (M ,k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{01} =$ 79.9 kPa

For isentropic flow ($p_{01} = p_{02}$)

$p_{02} =$ 79.9 kPa

From M_2 and p_{02}, and Eq. 13.7a
(using built-in function *Isenp* (M ,k))

$p_2 =$ 55.0 kPa

Problem 13.11

[3]

13.11 Atmospheric air (101 kPa and 20°C) is drawn into a receiving pipe via a converging nozzle. The throat cross-section diameter is 1 cm. Plot the mass flow rate delivered for the receiving pipe pressure ranging from 100 kPa down to 5 kPa.

Given: Flow in a converging nozzle to a pipe

Find: Plot of mass flow rate

Solution:

The given or available data is

$R =$ 287 J/kg·K
$k =$ 1.4
$T_0 =$ 293 K
$p_0 =$ 101 kPa
$D_t =$ 1 cm
$A_t =$ 0.785 cm^2

Equations and Computations:

The critical pressure is given by

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22\text{a})$$

$p^* =$ 53.4 kPa

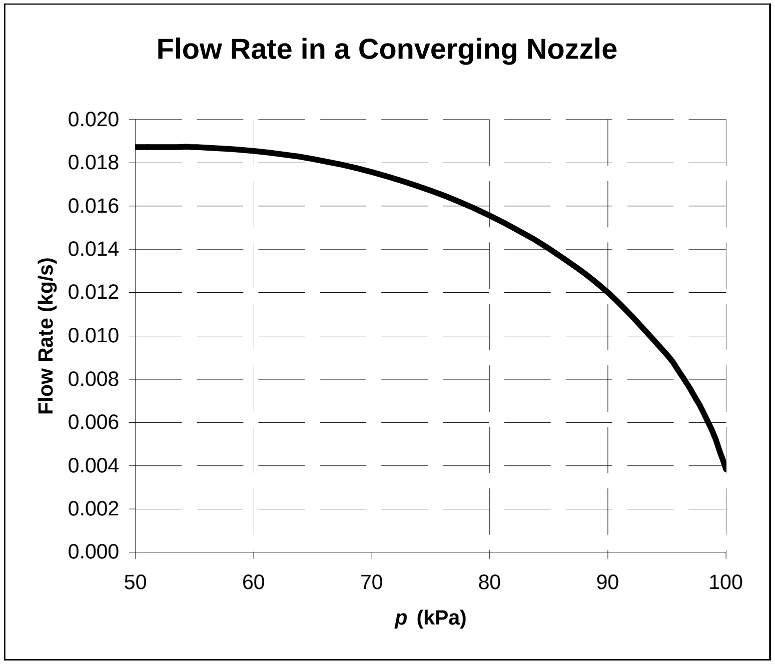
Hence for p = 100 kPa down to this pressure the flow gradually increases; then it is constant

p (kPa)	M (Eq. 13.7a)	T (K) (Eq. 13.7b)	c (m/s)	$V = M \cdot c$ (m/s)	$\rho = p/RT$ (kg/m³)	Flow (kg/s)
100	0.119	292	343	41	1.19	0.00383
99	0.169	291	342	58	1.18	0.00539
98	0.208	290	342	71	1.18	0.00656
97	0.241	290	341	82	1.17	0.00753
96	0.270	289	341	92	1.16	0.00838
95	0.297	288	340	101	1.15	0.0091
90	0.409	284	337	138	1.11	0.0120
85	0.503	279	335	168	1.06	0.0140
80	0.587	274	332	195	1.02	0.0156
75	0.666	269	329	219	0.971	0.0167
70	0.743	264	326	242	0.925	0.0176
65	0.819	258	322	264	0.877	0.0182
60	0.896	252	318	285	0.828	0.0186
55	0.974	246	315	306	0.778	0.0187
53.4	1.000	244	313	313	0.762	0.0187
53	1.000	244	313	313	0.762	0.0187
52	1.000	244	313	313	0.762	0.0187
51	1.000	244	313	313	0.762	0.0187
50	1.000	244	313	313	0.762	0.0187

Using critical conditions, and Eq. 13.9 for mass flow rate:

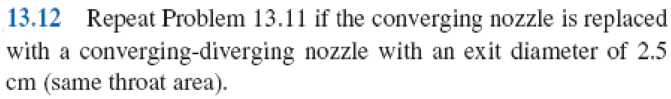
53.4	1.000	244	313	313	0.762	0.0185

(Note: discrepancy in mass flow rate is due to round-off error)



Problem 13.12

[2]



13.12 Repeat Problem 13.11 if the converging nozzle is replaced with a converging-diverging nozzle with an exit diameter of 2.5 cm (same throat area).

Given: Flow in a converging-diverging nozzle to a pipe

Find: Plot of mass flow rate

Solution:

The given or available data is

$R =$ 286.9 J/kg·K

$k =$ 1.4

$T_0 =$ 293 K

$p_0 =$ 101 kPa

$D_t =$ 1 cm $De =$ 2.5 cm

$A_t =$ 0.785 cm^2 $A_e =$ 4.909 cm^2

Equations and Computations:

The critical pressure is given by

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p^* =$ 53.4 kPa   This is the minimum throat pressure

For the CD nozzle, we can compute the pressure at the exit required for this to happen

$A^* =$ 0.785 cm^2 $(= A_t)$

$A_e/A^* =$ 6.25

$M_e =$ 0.0931 or 3.41 (Eq. 13.7d)

$p_e =$ 100.4 or 67.2 kPa (Eq. 13.7a)

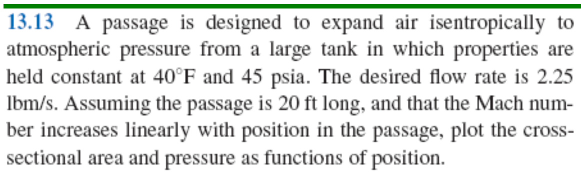
Hence we conclude flow occurs in regimes *iii* down to *v* (Fig. 13.8); the flow is ALWAYS choked!

p^* (kPa)	M 13.7a)	T^* (K) (Eq. 13.7b)	c^* (m/s)	$V^* = c^*$ (m/s)	$\rho = p/RT$ (kg/m^3)	Flow (kg/s)
53.4	1.000	244	313	313	0.762	0.0187
(Note: discrepancy in mass flow rate is due to round-off error)						0.0185

(Using Eq. 13.9)

Problem 13.13

[3]



13.13 A passage is designed to expand air isentropically to atmospheric pressure from a large tank in which properties are held constant at 40°F and 45 psia. The desired flow rate is 2.25 lbm/s. Assuming the passage is 20 ft long, and that the Mach number increases linearly with position in the passage, plot the cross-sectional area and pressure as functions of position.

Given: Data on tank conditions; isentropic flow

Find: Plot cross-section area and pressure distributions

Solution:

The given or available data is:

$R =$ 53.33 ft·lbf/lbm·^{o}R
$k =$ 1.4
$T_0 =$ 500 ^{o}R
$p_0 =$ 45 psia
$p_e =$ 14.7 psia
$m_{rate} =$ 2.25 lbm/s

Equations and Computations:

From p_0, p_e and Eq. 13.7a (using built-in function *IsenMfromp* (M,k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$M_e =$ 1.37

Because the exit flow is supersonic, the passage must be a CD nozzle
We need a scale for the area.
From p_0, T_0, m_{flow}, and Eq. 13.10c

$$\dot{m}_{\text{choked}} = 76.6\,\frac{A_t p_0}{\sqrt{T_0}} \qquad (13.10c)$$

Then $A_t = A^* =$ 0.0146 ft^2

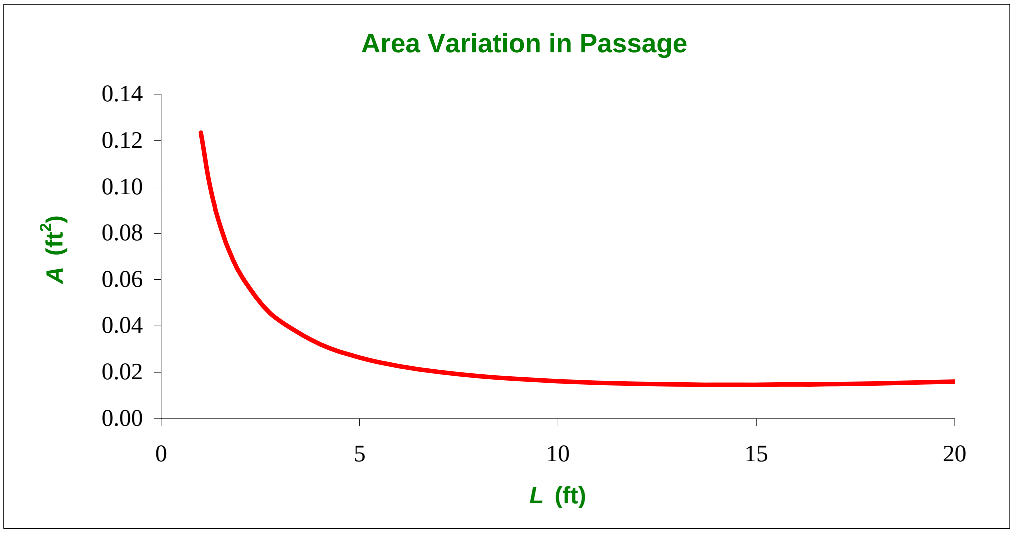
For each M, and A^*, and Eq. 13.7d
(using built-in function *IsenA* (M,k)

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (13.7d)$$

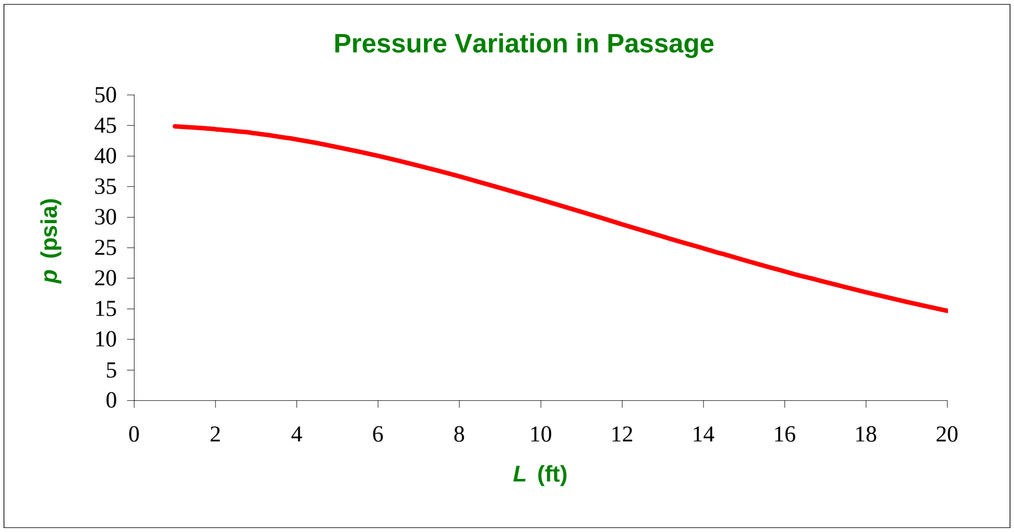
we can compute each area A.

From each M, and p_0, and Eq. 13.7a
(using built-in function *Isenp* (M,k)
we can compute each pressure p.

L (ft)	M	A (ft^2)	p (psia)
1.00	0.069	0.1234	44.9
1.25	0.086	0.0989	44.8
1.50	0.103	0.0826	44.7
1.75	0.120	0.0710	44.5
2.00	0.137	0.0622	44.4
2.50	0.172	0.0501	44.1
3.00	0.206	0.0421	43.7
4.00	0.274	0.0322	42.7
5.00	0.343	0.0264	41.5
6.00	0.412	0.0227	40.0
7.00	0.480	0.0201	38.4
8.00	0.549	0.0183	36.7
9.00	0.618	0.0171	34.8
10.00	0.686	0.0161	32.8
11.00	0.755	0.0155	30.8
12.00	0.823	0.0150	28.8
13.00	0.892	0.0147	26.8
14.00	0.961	0.0146	24.9
14.6	**1.000**	**0.0146**	**23.8**
16.00	1.098	0.0147	21.1
17.00	1.166	0.0149	19.4
18.00	1.235	0.0152	17.7
19.00	1.304	0.0156	16.2
20.00	1.372	0.0161	14.7



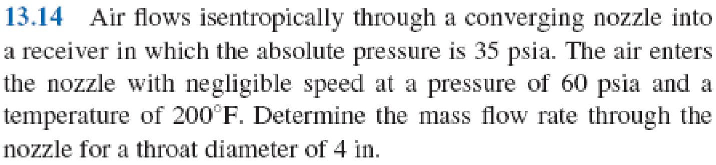
Area Variation in Passage



Pressure Variation in Passage

Problem 13.14

[2]



13.14 Air flows isentropically through a converging nozzle into a receiver in which the absolute pressure is 35 psia. The air enters the nozzle with negligible speed at a pressure of 60 psia and a temperature of 200°F. Determine the mass flow rate through the nozzle for a throat diameter of 4 in.

Given: Air flow in a converging nozzle

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad \dfrac{T_0}{T} = 1 + \dfrac{k-1}{2} \cdot M^2$ $\quad \dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $p_b = 35 \cdot psi$ $\quad p_0 = 60 \cdot psi$ $\quad T_0 = (200 + 460) \cdot R$ $\quad D_t = 4 \cdot in$

$k = 1.4$ $\quad R_{air} = 53.33 \cdot \dfrac{ft \cdot lbf}{lbm \cdot R}$ $\quad A_t = \dfrac{\pi}{4} \cdot D_t^2$ $\quad A_t = 0.0873 \cdot ft^2$

Since $\dfrac{p_b}{p_0} = 0.583$ is greater than 0.528, the nozzle is not choked and $p_t = p_b$

Hence $M_t = \sqrt{\dfrac{2}{k-1} \cdot \left[\left(\dfrac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_t = 0.912$

and $T_t = \dfrac{T_0}{1 + \dfrac{k-1}{2} \cdot M_t^2}$ $\quad T_t = 566 \cdot R$ $\quad T_t = 106 \cdot °F$

$c_t = \sqrt{k \cdot R_{air} \cdot T_t}$ $\quad V_t = c_t$ $\quad V_t = 1166 \cdot \dfrac{ft}{s}$

$\rho_t = \dfrac{p_t}{R_{air} \cdot T_t}$ $\quad \rho_t = 5.19 \times 10^{-3} \cdot \dfrac{slug}{ft^3}$

$m_{rate} = \rho_t \cdot A_t \cdot V_t$ $\quad m_{rate} = 0.528 \cdot \dfrac{slug}{s}$ $\quad m_{rate} = 17.0 \cdot \dfrac{lbm}{s}$

Problem 13.15

[2]

13.15 Air flows isentropically through a converging nozzle into a receiver where the pressure is 250 kpa (abs). If the pressure is 350 kpa (abs) and the speed is 150 m/s at the nozzle location where the Mach number is 0.5, determine the pressure, speed, and Mach number at the nozzle throat.

Given: Isentropic air flow in converging nozzle

Find: Pressure, speed and Mach number at throat

Solution:

Basic equations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$$

Given or available data

$$p_1 = 350\cdot kPa \qquad V_1 = 150\cdot\frac{m}{s} \qquad M_1 = 0.5 \qquad p_b = 250\cdot kPa$$

$$k = 1.4 \qquad R = 286.9\cdot\frac{J}{kg\cdot K}$$

The flow will be choked if $p_b/p_0 < 0.528$

$$p_0 = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^{\,2}\right)^{\frac{k}{k-1}} \qquad p_0 = 415\,kPa \qquad \frac{p_b}{p_0} = 0.602 \qquad \text{(Not choked)}$$

Hence

$$\frac{p_0}{p_t} = \left(1 + \frac{k-1}{2}\cdot M_t^{\,2}\right)^{\frac{k}{k-1}} \quad \text{where} \quad p_t = p_b \qquad p_t = 250\,kPa$$

so

$$M_t = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_t = 0.883$$

Also

$$V_1 = M_1\cdot c_1 = M_1\cdot\sqrt{k\cdot R\cdot T_1} \quad \text{or} \quad T_1 = \frac{1}{k\cdot R}\cdot\left(\frac{V_1}{M_1}\right)^2 \qquad T_1 = 224\,K \qquad T_1 = -49.1\,°C$$

Then

$$T_0 = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^{\,2}\right) \qquad T_0 = 235\,K \qquad T_0 = -37.9\,°C$$

Hence

$$T_t = \frac{T_0}{1 + \frac{k-1}{2}\cdot M_t^{\,2}} \qquad T_t = 204\,K \qquad T_t = -69.6\,°C$$

Then

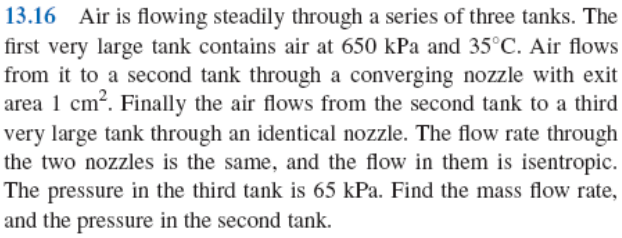
$$c_t = \sqrt{k\cdot R\cdot T_t} \qquad c_t = 286\,\frac{m}{s}$$

Finally

$$V_t = M_t\cdot c_t \qquad V_t = 252\,\frac{m}{s}$$

Problem 13.16

[3]



13.16 Air is flowing steadily through a series of three tanks. The first very large tank contains air at 650 kPa and 35°C. Air flows from it to a second tank through a converging nozzle with exit area 1 cm^2. Finally the air flows from the second tank to a third very large tank through an identical nozzle. The flow rate through the two nozzles is the same, and the flow in them is isentropic. The pressure in the third tank is 65 kPa. Find the mass flow rate, and the pressure in the second tank.

Given: Data on three tanks

Find: Mass flow rate; Pressure in second tank

Solution:

The given or available data is:

R = 286.9 J/kg.K

k = 1.4

A_t = 1 cm^2

We need to establish whether each nozzle is choked. There is a large total pressure drop so this is likely. However, BOTH cannot be choked and have the same flow rate. This is because Eq. 13.9a, below

$$\dot{m}_{\text{choked}} = 0.04\frac{A_e p_0}{\sqrt{T_0}} \qquad (13.9b)$$

indicates that the choked flow rate depends on stagnation temperature (which is constant) but also stagnation pressure, which drops because of turbulent mixing in the middle chamber. Hence BOTH nozzles cannot be choked. We assume the second one only is choked (why?) and verify later.

Temperature and pressure in tank 1: T_{01} = 308 K

p_{01} = 650 kPa

We make a guess at the pressure at the first nozzle exit: p_{e1} = 527 kPa

NOTE: The value shown is the final answer! It was obtained using *Solver* !

This will also be tank 2 stagnation pressure: p_{02} = 527 kPa

Pressure in tank 3: p_3 = 65 kPa

Equations and Computations:

From the p_{e1} guess and Eq. 13.17a: M_{e1} = 0.556

Then at the first throat (Eq.13.7b): T_{e1} = 290 K

The density at the first throat (Ideal Gas) is: ρ_{e1} = 6.33 kg/m^3

Then c at the first throat (Eq. 12.18) is: c_{e1} = 341 m/s

Then V at the first throat is: V_{e1} = 190 m/s

Finally the mass flow rate is: m_{rate} = 0.120 kg/s First Nozzle!

For the presumed choked flow at the second nozzle we use Eq. 13.9a, with $T_{01} = T_{02}$ and p_{02}:

m_{rate} = 0.120 kg/s Second Nozzle!

For the guess value for p_{e1} we compute the error between the two flow rates:

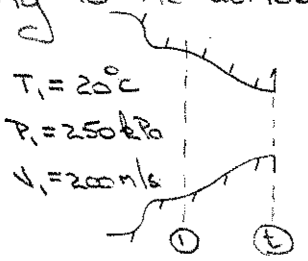
Δm_{rate} = 0.000 kg/s

Use Solver to vary the guess value for p_{e1} to make this error zero!

Note that this could also be done manually.

Problem 13.17

Given: Air flows isentropically through a converging nozzle, discharging to the atmosphere



$P_b = P_{atm} = 101\ kPa$

Find: P_t

Solution:

Computing equations: $\frac{P_o}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $\quad c = \sqrt{kRT}$

Assumptions: (1) steady flow
(2) isentropic flow in the nozzle
(3) uniform flow at a section
(4) ideal gas

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The nozzle will be choked, ie $M_t = 1.0$ if $\frac{P_b}{P_o} \leq 0.528$

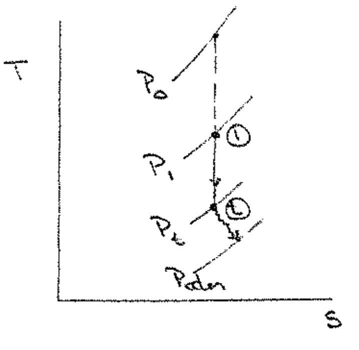
$M_1 = \frac{V_1}{c_1} \quad c_1 = \sqrt{kRT_1} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 293 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 343\ m/s$

$M_1 = \frac{V_1}{c_1} = \frac{200}{343} = 0.583$

$\frac{P_o}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \quad P_o = P_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 250\ kPa\left[1 + 0.2(0.583)^2\right]^{3.5} = 315\ kPa$

Then, $\frac{P_b}{P_o} = \frac{101}{315} = 0.321 < 0.528 \quad \therefore M_t = 1.0$

For $M_t = 1.0$, $\frac{P_t}{P_o} = 0.528 \quad P_t = 0.528\, P_o = 0.528 \times 315\ kPa = 166\ kPa \longleftarrow P_t$

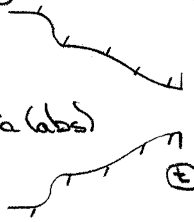


Problem 13.18

Given: Isentropic flow of air from a large tank through a converging nozzle discharges to atmosphere.

$T_0 = 550°C$

$p_0 = 650$ kPa (abs)



$p_b = p_{atm} = 101$ kPa (abs).

$A_t = 600$ mm^2

Find: $\dot{m}$

Solution:

Basic equations: $\dot{m} = \rho V A = \text{const}$ $\quad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} \qquad c = \sqrt{kRT}$

Assumptions: (1) steady flow
(2) isentropic flow in nozzle
(3) uniform flow at a section
(4) ideal gas

Since $p_b/p_0 = \frac{101}{650} = 0.155 < 0.528$, the nozzle is choked and $M_t = 1.0$

From the continuity equation $\dot{m} = \rho V A$ and hence we need to determine ρ_t and V_t

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \; ; \quad T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{823\text{ K}}{1 + 0.2(1.0)^2} = 686\text{ K}$$

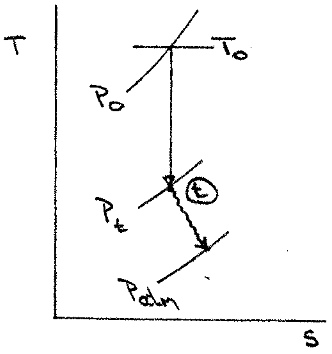
$$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0\left(1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 686\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 525 \text{ m/s}$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} ; \quad p_t = \frac{p_0}{\left[1 + \frac{k-1}{2} M_t^2\right]^{k/k-1}} = \frac{650\text{ kPa}}{\left[1 + 0.2(1.0)^2\right]^{3.5}} = 343 \text{ kPa}$$

$$\rho_t = \frac{p_t}{RT_t} = 343 \times 10^3 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\text{ N}\cdot\text{m}} \times \frac{1}{686\text{K}} = 1.74 \text{ kg/m}^3$$

Finally

$$\dot{m} = \rho_t V_t A_t = 1.74 \frac{\text{kg}}{\text{m}^3} \times 525 \frac{\text{m}}{\text{s}} \times 6 \times 10^{-4} \text{ m}^2 = 0.548 \text{ kg/s} \longleftarrow \dot{m}$$



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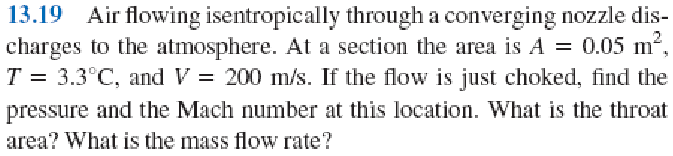
Problem 13.19

[2]



13.19 Air flowing isentropically through a converging nozzle discharges to the atmosphere. At a section the area is $A = 0.05\ \text{m}^2$, $T = 3.3°\text{C}$, and $V = 200$ m/s. If the flow is just choked, find the pressure and the Mach number at this location. What is the throat area? What is the mass flow rate?

Given: Data on converging nozzle; isentropic flow

Find: Pressure and Mach number; throat area; mass flow rate

Solution:

The given or available data is:

$R = 286.9$ J/kg.K

$k = 1.4$

$A_1 = 0.05$ m^2

$T_1 = 276.3$ K

$V_1 = 200$ m/s

$p_{atm} = 101$ kPa

Equations and Computations:

From T_1 and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_1 = 333$ m/s

Then $M_1 = 0.60$

To find the pressure, we first need the stagnation pressure.
If the flow is just choked

$p_e = p_{atm} = p^* = 101$ kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p_0 = 191$ kPa

From M_1 and p_0, and Eq. 13.7a
(using built-in function *Isenp* (*M* ,*k*)

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

Then $p_1 = 150$ kPa

The mass flow rate is $m_{\text{rate}} = \rho_1 A_1 V_1$

Hence, we need ρ_1 from the ideal gas equation.

$\rho_1 =$ 1.89 kg/m^3

The mass flow rate m_{rate} is then

$m_{\text{rate}} =$ 18.9 kg/s

The throat area $A_t = A^*$ because the flow is choked.
From M_1 and A_1, and Eq. 13.7d
(using built-in function *IsenA* (M ,k)

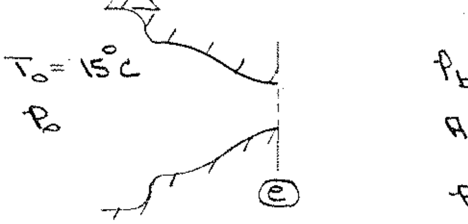
$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \tag{13.7d}$$

$A^* =$ 0.0421 m^2

Hence $A_t =$ 0.0421 m^2

Problem 13.20

Given: Flow of air from stagnation state through a converging nozzle discharges to the atmosphere. $p_e \geq 325\ \text{kPa (gage)}$



$p_b = 101\ \text{kPa}$

$A_e = 0.001\ m^2$

$p_e = 325\ \text{kPa (gage)} = 426\ \text{kPa (abs)}$

Find: p_0 , $\dot m$

Solution:

Basic equations: $\dot m = \rho A V$ $\quad p = \rho R T$

Computing equations: $\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$

Assumptions : (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas behavior

Since $p_e > p_b$, nozzle is choked and $M_e = 1.0$

$p_0 = p_e\left[1 + \frac{k-1}{2} M_e^2\right]^{k/k-1} = 426\ \text{psia}\,[1 + 0.2]^{3.5} = 806\ \text{kPa}$ $\longleftarrow p_0$

$\frac{T_0}{T_e} = 1 + \frac{k-1}{2} M_e^2 \quad \therefore \ T_e = \frac{T_0}{1.2} = \frac{288\,K}{1.2} = 240\,K$

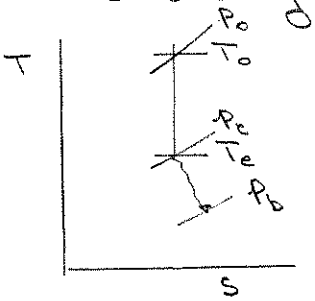
$V_e = c_e = (kRT_e)^{1/2} = \left[1.4 \times 287 \frac{J}{kg \cdot K} \times 240\,K \times \frac{N \cdot m}{J} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 311\ m/s$

$\rho_e = \frac{p_e}{RT_e} = 426 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\,J} \times \frac{1}{240\,K} \times \frac{J}{N \cdot m} = 6.18\ kg/m^3$

Then,

$\dot m = \rho_e V_e A_e = 6.18 \frac{kg}{m^3} \times 311 \frac{m}{s} \times 0.001\,m^2 = 1.92\ kg/s$

For steady flow, $\dot m = 1.92\ kg/s$ must be supplied to the tank $\longleftarrow \dot m$



The corresponding volume flow rate of standard air is

$Q = \frac{\dot m}{\rho_{stand}} = 1.92 \frac{kg}{s} \times \frac{m^3}{6.18\,kg}$

$Q = 0.31\ m^3/s$

Problem 13.21

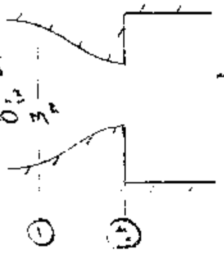
[2]

Given: Isentropic flow of air through a converging nozzle discharges to a back pressure, p_b.

$T_0 = 350\ K$ $M_1 = 0.5$

$p_0 = 650\ kPa$ $A_1 = 2.6\times10^{-3}\ m^2$

$p_b = 270\ kPa$



Find: A_t

Solution:

Computing equations: $p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

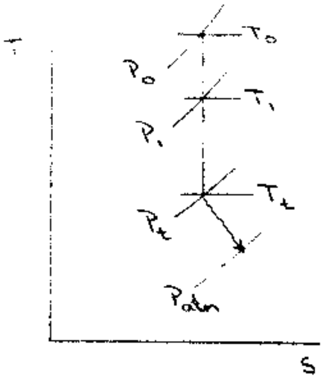
$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{1+\frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \quad (12.6)$$

Assumptions: (1) steady flow
(2) isentropic flow in nozzle
(3) uniform flow at a section
(4) ideal gas

Since $p_b/p_0 = \dfrac{270\ kPa}{650\ kPa} = 0.415 < 0.528$, the nozzle is choked and $M_t = 1.0$

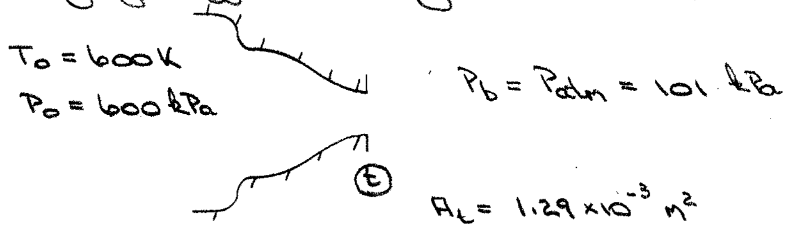
From Eq 12.6 with $M_1 = 0.5$, $A_1/A^* = 1.340$

Then $A_t = A^* = A_1/1.340 = \dfrac{2.6\times10^{-3}\ m^2}{1.340} = 1.94\times10^{-3}\ m^2 \longleftarrow A_t$



Problem 13.22

Given: Reversible, adiabatic flow of air from a large tank through a converging nozzle discharges to atmosphere.



Find: (a) range of tank pressure, p_0, for which $M_t = 1.0$
(b) $\dot{m}$ for conditions given

Solution:

Basic equations: $\dot{m} = \rho V A = \text{const}$ $\qquad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (2) isentropic flow in nozzle (3) uniform flow at a section (4) ideal gas

The nozzle will be choked, ie $M_t = 1.0$ for $p_b/p_0 \le 0.528$

Since $p_b = 101\,kPa$, nozzle is choked for

$$p_0 \ge \frac{p_b}{0.528} = \frac{101\,kPa}{0.528} = 191\,kPa \longleftarrow p_0$$

Thus for $p_0 = 600\,kPa$, $M_t = 1.0$

$$\frac{T_0}{T} = 1+\frac{k-1}{2}M^2 \;; \quad T_t = \frac{T_0}{1+\frac{k-1}{2}M_t^2} = \frac{600\,K}{1+0.2(1.0)^2} = 500\,K$$

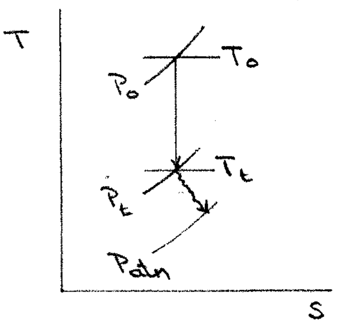
$$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0\left(1.4\times 287\frac{N\cdot m}{kg\cdot K}\times 500\,K\times\frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 448\,m/s$$

$$\frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1} ; \quad p_t = \frac{p_0}{\left[1+\frac{k-1}{2}M_t^2\right]^{k/k-1}} = \frac{600\,kPa}{\left[1+0.2(1.0)^2\right]^{3.5}} = 317\,kPa$$

$$\rho_t = \frac{p_t}{RT_t} = 317\times10^3\,\frac{N}{m^2}\times\frac{kg\cdot K}{287\,N\cdot m}\times\frac{1}{500\,K} = 2.21\,kg/m^3$$

Finally,

$$\dot{m} = \rho_t V_t A_t = 2.21\,\frac{kg}{m^3}\times 448\,\frac{m}{s}\times 1.29\times10^{-3}\,m^2 = 1.28\,kg/s \longleftarrow \dot{m}$$



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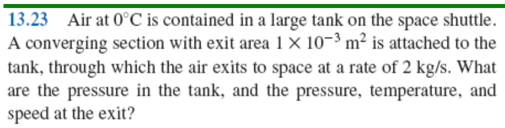
Problem 13.23

[2]



13.23 Air at 0°C is contained in a large tank on the space shuttle. A converging section with exit area 1×10^{-3} m^2 is attached to the tank, through which the air exits to space at a rate of 2 kg/s. What are the pressure in the tank, and the pressure, temperature, and speed at the exit?

Given: Temperature in and mass flow rate from a tank

Find: Tank pressure; pressure, temperature and speed at exit

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
T_0 = 273 K
A_t = 0.001 m^2
m_{rate} = 2 kg/s

Equations and Computations:

Because $p_b = 0$ $\quad p_e = p^*$
Hence the flow is choked!

Hence $\quad T_e = T^*$

From T_0, and Eq. 12.22b

$$\frac{T_0}{T^*} = \frac{k+1}{2} \qquad (12.22b)$$

T^* = 228 K

T_e = 228 K
-45.5 ^{o}C

Also $\quad M_e$ = 1
Hence $\quad V_e$ = V^* = c_e

From T_e and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

c_e = 302 m/s

Then $\quad V_e$ = 302 m/s

To find the exit pressure we use the ideal gas equation
after first finding the exit density.
The mass flow rate is $m_{\text{rate}} = \rho_e A_e V_e$

Hence $\quad \rho_e$ = 6.62 kg/m^3

From the ideal gas equation $p_e = \rho_e R T_e$

p_e = 432 kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

p_0 = 817 kPa

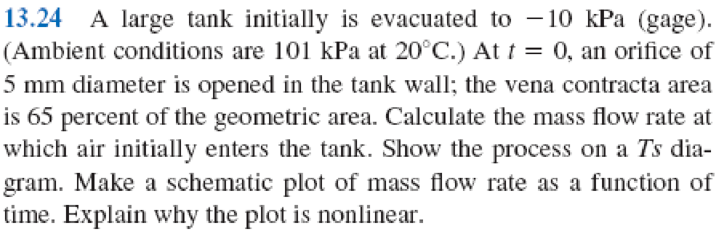
We can check our results:
From p_0, T_0, A_t, and Eq. 13.9a

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.9a)$$

Then $\quad m_{\text{choked}}$ = 2.00 kg/s
m_{choked} = m_{rate} Correct!

Problem 13.24

[2]



13.24 A large tank initially is evacuated to −10 kPa (gage). (Ambient conditions are 101 kPa at 20°C.) At *t* = 0, an orifice of 5 mm diameter is opened in the tank wall; the vena contracta area is 65 percent of the geometric area. Calculate the mass flow rate at which air initially enters the tank. Show the process on a *Ts* diagram. Make a schematic plot of mass flow rate as a function of time. Explain why the plot is nonlinear.

Given: Isentropic air flow into a tank

Find: Initial mass flow rate; Ts process; explain nonlinear mass flow rate

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\qquad m_{rate} = \rho\cdot A\cdot V$

Given or available data $\quad p_0 = 101\cdot kPa \qquad p_b = p_0 - 10\cdot kPa \qquad p_b = 91\cdot kPa \qquad T_0 = (20 + 273)\cdot K$

$k = 1.4 \qquad R = 286.9\cdot\frac{J}{kg\cdot K} \qquad D = 5\cdot mm$

Then $\quad A = \frac{\pi}{4}\cdot D^2 \qquad A_{vena} = 65\cdot\%\cdot A \qquad A_{vena} = 12.8\cdot mm^2$

The flow will be choked if $p_b/p_0 < 0.528$ $\qquad \frac{p_b}{p_0} = 0.901$ $\qquad$ (Not choked)

Hence $\quad \frac{p_0}{p_{vena}} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad$ where $\quad p_{vena} = p_b \qquad p_{vena} = 91\cdot kPa$

so $\quad M_{vena} = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_{vena}}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_{vena} = 0.389$

Then $\quad T_{vena} = \frac{T_0}{1 + \frac{k-1}{2}\cdot M_{vena}^2} \qquad T_{vena} = 284\,K \qquad T_{vena} = 11.3\cdot °C$

Then $\quad c_{vena} = \sqrt{k\cdot R\cdot T_{vena}} \qquad c_{vena} = 338\frac{m}{s}$

and $\quad V_{vena} = M_{vena}\cdot c_{vena} \qquad V_{vena} = 131\frac{m}{s}$

Also $\quad \rho_{vena} = \frac{p_{vena}}{R\cdot T_{vena}} \qquad \rho_{vena} = 1.12\frac{kg}{m^3}$

Finally $\quad m_{rate} = \rho_{vena}\cdot A_{vena}\cdot V_{vena} \qquad m_{rate} = 1.87\times 10^{-3}\frac{kg}{s}$

The Ts diagram will be a vertical line (T decreases and s = const). After entering the tank there will be turbulent mixing (s increases) and t comes to rest (T increases). The mass flow rate versus time will look like the curved part of Fig. 13.6b; it is nonlinear because V AND ρ v

Problem 13.25

[3]

13.25 A-50 cm diameter spherical cavity initially is evacuated. The cavity is to be filled with air for a combustion experiment. The pressure is to be 45 kPa (abs), measured after its temperature reaches T_{atm}. Assume the valve on the cavity is a converging nozzle with throat diameter of 1 mm, and the surrounding air is at standard conditions. For how long should the valve be opened to achieve the desired final pressure in the cavity? Calculate the entropy change for the air in the cavity.

Given:

Spherical cavity with valve

Find:

Time to reach desired pressure; Entropy change

Solution:

Basic equations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad \Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right)$$

$$p = \rho\cdot R\cdot T \qquad c = \sqrt{k\cdot R\cdot T} \qquad m_{rate} = \rho\cdot A\cdot V \qquad m_{choked} = A_t\cdot p_0\cdot\sqrt{\frac{k}{R\cdot T_0}}\cdot\left(\frac{2}{k+1}\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Given or available data $p_0 = 101\cdot kPa$ $\quad T_{atm} = (20+273)\cdot K$ $\quad T_0 = T_{atm}$ $\quad d = 1\cdot mm$ $\quad D = 50\cdot cm$

$$p_f = 45\cdot kPa \qquad T_f = T_{atm} \qquad k = 1.4 \qquad R = 286.9\cdot\frac{J}{kg\cdot K} \qquad c_p = 1004\cdot\frac{J}{kg\cdot K}$$

Then the inlet area is $A_t = \frac{\pi}{4}\cdot d^2$ $\quad A_t = 0.785\,mm^2$ $\quad$ and tank volume is $V = \frac{\pi}{3}\cdot D^3$ $\quad V = 0.131\,m^3$

The flow will be choked if $p_b/p_0 < 0.528$; the MAXIMUM back pressure is $p_b = p_f$ $\quad$ so $\quad \frac{p_b}{p_0} = 0.446$ $\quad$ (Choked)

The final density is $\rho_f = \frac{p_f}{R\cdot T_f}$ $\quad \rho_f = 0.535\frac{kg}{m^3}$ $\quad$ and final mass is $M = \rho_f\cdot V$ $\quad M = 0.0701\,kg$

Since the mass flow rate is constant (flow is always choked) $\quad M = m_{rate}\cdot\Delta t$ $\quad$ or $\quad \Delta t = \frac{M}{m_{rate}}$

We have choked flow so $m_{rate} = A_t\cdot p_0\cdot\sqrt{\frac{k}{R\cdot T_0}}\cdot\left(\frac{2}{k+1}\right)^{\frac{k+1}{2\cdot(k-1)}}$ $\quad m_{rate} = 1.873\times 10^{-4}\frac{kg}{s}$

Hence $\Delta t = \frac{M}{m_{rate}}$ $\quad \Delta t = 374\,s$ $\quad \Delta t = 6.23\,min$

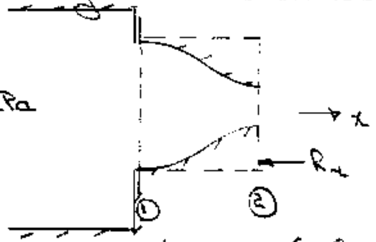
The air in the tank will be cold when the valve is closed. Because $\rho = M/V$ is constant, $p = \rho RT = \text{const} \times T$, so as the temperature rises to ambient, the pressure will rise too.

For the entropy change during the charging process is given by $\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right)$ where $T_1 = T_{atm}$ $\quad T_2 = T_{atm}$

and $\quad p_1 = p_0$ $\quad p_2 = p_f$ $\quad$ Hence $\quad \Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right)$ $\quad \Delta s = 232\frac{J}{kg\cdot K}$

Problem 13.26

Given: Isentropic flow of air from a large tank through a converging nozzle discharges to the atmosphere



$T_0 = 27°C$

$p_0 = 171$ kPa

$M_1 = 0.2$ $p_b = 101$ kPa

$A_2 = 0.015\ m^2$

Find: Magnitude and direction of force required to keep nozzle in place

Solution:

Basic equations: $F_{S_x} = p_1A_1 - p_2A_2 - p_{atm}(A_1 - A_2) - R_x = \dot m (V_2 - V_1)$

$\dot m = \rho V A = \text{const}$ $p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$M_2 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_2}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{171}{101}\right)^{0.286} - 1\right]\right\}^{1/2} = 0.901$ Hence flow not choked

$T_2 = T_0 / \left[1 + \frac{k-1}{2}M_2^2\right] = 300\,K / \left[1 + 0.2(0.901)^2\right] = 258\ K$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 0.901\left(1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 258\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 290\ m/s$

$\rho_2 = \frac{p_2}{RT_2} = 101\times10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{258\,K} = 1.36\ kg/m^3$

$\dot m = \rho_2 V_2 A_2 = 1.36 \frac{kg}{m^3} \times 290 \frac{m}{s} \times 0.015\,m^2 = 5.92\ kg/s$

$T_1 = T_0 / \left[1 + \frac{k-1}{2}M_1^2\right] = 300\,K / \left[1 + 0.2(0.2)^2\right] = 298\ K$

$V_1 = M_1 c_1 = M_1 (kRT_1) = 0.2\left(1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 298\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 69.2\ m/s$

$p_1 = p_0 / \left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 171\,kPa / \left[1 + 0.2(0.2)^2\right]^{3.5} = 166\ kPa$

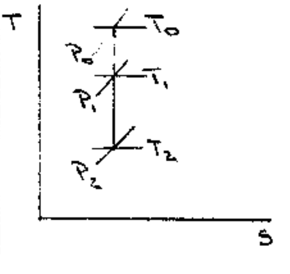
$\rho_1 = \frac{p_1}{RT_1} = 166\times10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{298\,K} = 1.94\ kg/m^3$

$A_1 = \dot m / \rho_1 V_1 = 5.92 \frac{kg}{s} \times \frac{m^3}{1.94\,kg} \times \frac{s}{69.2\,m} = 0.0441\ m^2$

$R_x = p_1A_1 - p_2A_2 - p_{atm}(A_1 - A_2) - \dot m (V_2 - V_1) = p_{1g}A_1 - \overset{0}{\cancel{p_{2g}A_2}} - \dot m (V_2 - V_1)$

$= (166 - 101)\times10^3 \frac{N}{m^2} \times 0.0441\,m^2 - 5.92 \frac{kg}{s}(290 - 69.2)\frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m}$

$R_x = 1560\ N$ (to the left) $\longleftarrow R_x$



Problem 13.27

Given: "Rocket" cart propelled by compressed air, converging nozzle.

Initially in tank, $p_0 = 1.3\ \text{MPa (abs)}$, $T_0 = 20°\text{C}$, $M_0 = 25\ \text{kg}$

$A_e = 30\ \text{mm}^2$; $F_R = 6\ \text{N}$, aerodynamic drag is negligible.

Find: (a) Pressure in exit plane.
(b) Mass flow rate of air through nozzle.
(c) Acceleration of assembly.
(d) static and stagnation states on Ts diagram.

Solution: Assume steady, one-dimensional flow of an ideal gas.

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$; $\frac{p_0}{p} = \left(1+\frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$; $c = \sqrt{kRT}$; $p = \rho RT$

Check for choking: $\frac{p_{atm}}{p_0} = \frac{101 \times 10^3}{1.3 \times 10^6} = 0.0777 < 0.528$, so choked. $M_e = 1$

Thus

$$T_e = \frac{T_0}{1+\frac{k-1}{2}M_e^2} = \frac{(273+20)\text{K}}{1+\frac{k-1}{2}} = 244\ \text{K}$$

$$p_e = \frac{p_0}{\left(1+\frac{k-1}{2}M_e^2\right)^{k/k-1}} = \frac{1.3 \times 10^6\ \text{Pa}}{\left(1+\frac{k-1}{2}\right)^{k/k-1}} = \frac{1.3\times 10^6\ \text{Pa}}{(1.2)^{3.5}} = 687\ \text{kPa (abs)}$$

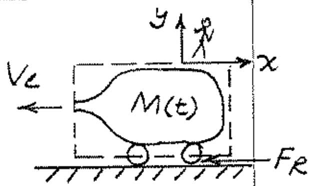
p_e

$$\dot{m} = \rho_e V_e A_e \ ;\ \rho_e = \frac{p_e}{RT_e} = 687 \times 10^3 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\ \text{N}\cdot\text{m}} \times \frac{1}{244\ \text{K}} = 9.81\ \text{kg/m}^3$$

$$V_e = M_e c_e = c_e = \sqrt{kRT_e} = \left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 244\ \text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 313\ \text{m/s}$$

$$\dot{m} = 9.81 \frac{\text{kg}}{\text{m}^3} \times 313 \frac{\text{m}}{\text{s}} \times 30 \times 10^{-6}\ \text{m}^2 = 0.0921\ \text{kg/s}$$

$\dot{m}$



Apply momentum to find acceleration of assembly.

$$\text{Basic equation:}\quad F_{S_x} + \overset{=0(4)}{F_{B_x}} - \int_{CV} a_{rf_x}\rho\, d\forall = \overset{\approx 0(5)}{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

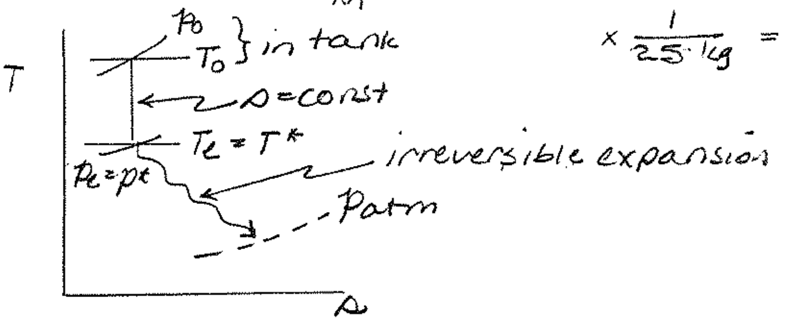
Assume: (4) Horizontal; (5) $u \approx 0$ within CV; (5) Uniform flow at exit.

$$\text{Then}\quad -F_R + (p_e - p_{atm})A_e - M a_{rf_x} = u_e\{+\dot{m}\} = -V_e \dot{m}$$

$$u_e = -V_e$$

$$a_{rf_x} = \frac{V_e\dot{m} - F_R + (p_e - p_{atm})A_e}{M} = \left[313 \frac{\text{m}}{\text{s}} \times 0.0921 \frac{\text{kg}}{\text{s}} - 6\ \text{N} + (687-101)10^3 \frac{\text{N}}{\text{m}^2} \times 30 \times 10^{-6}\ \text{m}^2\right] \times \frac{1}{25\ \text{kg}} = 1.62\ \text{m/s}^2$$

a_{rf_x}



Problem 13.28

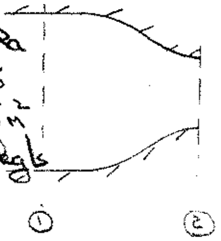
Given: Steady, isentropic flow of air. Duct area to be reduced as much as possible without reducing flowrate.

$p_1 = 300\,kPa$

$M_1 = 0.5$

$A_1 = 5\times10^{-4}\,m^2$

$\dot m = 0.25\,kg/s$



Find: (a) T_0 (b) % ΔA possible, (c) V_2 and p_2

Solution:

Basic equations: $\dot m = \rho V A$ $\quad$ $p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\quad$ $\frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{1+\frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.7d)$$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

To determine T_0, we first need to find T_1

$$\dot m = \rho_1 V_1 A_1 = \frac{p_1}{RT_1} M_1 c_1 A_1 = \frac{p_1}{RT_1} M_1 (kRT_1)^{1/2} A_1 = p_1 M_1 A_1 \left(\frac{k}{RT_1}\right)^{1/2}$$

Solving for T_1,

$$T_1 = \frac{k}{R}\left(\frac{p_1 M_1 A_1}{\dot m}\right)^2 = 1.4 \times \frac{kg\cdot K}{287\,J}\left[300\times10^3\,\frac{N}{m^2}\times 0.5\times 5\times10^{-4}\,m^2 \times \frac{s}{0.25\,kg}\times \frac{kg\cdot m}{N\cdot s^2}\right]^2 \frac{N\cdot s^2}{kg\cdot m}$$

$T_1 = 439\,K$

$T_{0_1} = T_1\left[1+\frac{k-1}{2}M_1^2\right] = 439\,K\left[1+0.2(0.5)^2\right] = 461\,K$ $\longleftarrow$ T_0

Maximum area reduction occurs where $M_2 = 1$

For $M_1 = 0.5$, from Eq 12.7d $A/A^* = 1.34$ $\therefore A^* = A_2 = \frac{A_1}{1.34} = 3.73\times10^{-4}\,m^2$

$\% \Delta A = \frac{A_2 - A_1}{A_1}\times 100 = \frac{3.73-5}{5}\times 100 = -25.4\%$ $\longleftarrow$ $\% \Delta A$

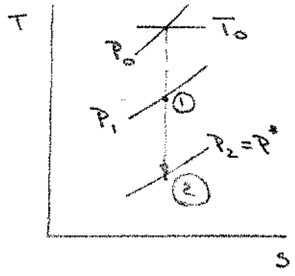
$T_2 = T_0/\left[1+\frac{k-1}{2}M_2^2\right] = 461\,K/\left[1+0.2(1)^2\right] = 384\,K$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 1.0\left(1.4\times 287\,\frac{J}{kg\cdot K}\times 384\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2}$

$V_2 = 393\,m/s$ $\longleftarrow$ V_2

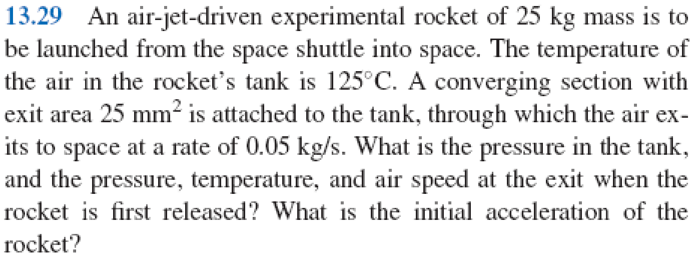
$p_{0_1} = p_{0_2} = p_1\left[1+\frac{k-1}{2}M_1^2\right]^{k/k-1} = 300\,kPa\left[1+0.2(0.5)^2\right]^{3.5} = 356\,kPa$

$p_2 = p_{0_2}/\left[1+\frac{k-1}{2}M_2^2\right]^{k/k-1} = 356\,kPa/\left[1+0.2(1)^2\right]^{3.5} = 188\,kPa$ $\longleftarrow$ p_2



Problem 13.29

[3]



13.29 An air-jet-driven experimental rocket of 25 kg mass is to be launched from the space shuttle into space. The temperature of the air in the rocket's tank is 125°C. A converging section with exit area 25 mm^2 is attached to the tank, through which the air exits to space at a rate of 0.05 kg/s. What is the pressure in the tank, and the pressure, temperature, and air speed at the exit when the rocket is first released? What is the initial acceleration of the rocket?

Given: Air-driven rocket in space

Find: Tank pressure; pressure, temperature and speed at exit; initial acceleration

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4
$T_0 =$ 398 K
$A_t =$ 25 mm^2
$M =$ 25 kg
$m_{rate} =$ 0.05 kg/s

Equations and Computations:

Because $p_b = 0$ $\quad p_e = p^*$

Hence the flow is choked!

Hence $\quad T_e = T^*$

From T_0, and Eq. 12.22b

$$\frac{T_0}{T^*} = \frac{k+1}{2} \qquad (12.22b)$$

$T^* =$ 332 K

$T_e =$ 332 K
58.7 $^{\circ}$C

Also $\quad M_e =$ 1

Hence $\quad V_e = \quad V^* = \quad c_e$

From T_e and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_e =$ 365 m/s

Then $\quad V_e =$ 365 m/s

To find the exit pressure we use the ideal gas equation
after first finding the exit density.
The mass flow rate is $m_{\text{rate}} = \rho_e A_e V_e$

Hence $\rho_e =$ 0.0548 kg/m^3

From the ideal gas equation $p_e = \rho_e R T_e$

$p_e =$ 5.21 kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p_0 =$ 9.87 kPa

We can check our results:
From p_0, T_0, A_t, and Eq. 13.9a

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.9a)$$

Then $m_{\text{choked}} =$ 0.050 kg/s
$m_{\text{choked}} = m_{\text{rate}}$ Correct!

The initial acceleration is given by:

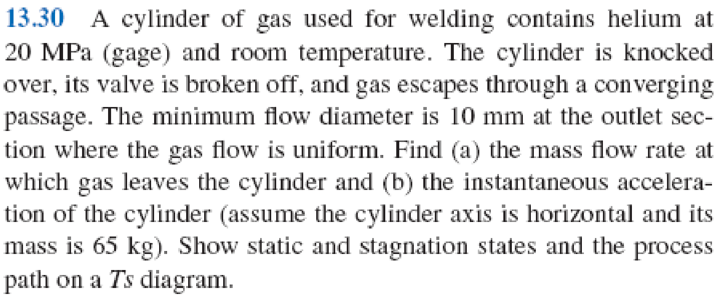
$$\vec{F} - \int_{\text{CV}} \vec{a}_{rf}\,\rho\, d\forall = \frac{\partial}{\partial t}\int_{\text{CV}} \vec{V}_{xyz}\,\rho\, d\forall + \int_{\text{CS}} \vec{V}_{xyz}\,\rho \vec{V}_{xyz} \cdot d\vec{A} \qquad (4.33)$$

which simplifies to: $p_e A_t - M a_x = m_{rate} V$ or: $a_x = \dfrac{m_{rate} V + p_e A_t}{M}$

$a_x =$ 1.25 m/s^2

Problem 13.30

[3]



13.30 A cylinder of gas used for welding contains helium at 20 MPa (gage) and room temperature. The cylinder is knocked over, its valve is broken off, and gas escapes through a converging passage. The minimum flow diameter is 10 mm at the outlet section where the gas flow is uniform. Find (a) the mass flow rate at which gas leaves the cylinder and (b) the instantaneous acceleration of the cylinder (assume the cylinder axis is horizontal and its mass is 65 kg). Show static and stagnation states and the process path on a *Ts* diagram.

Given: Gas cylinder with broken valve

Find: Mass flow rate; acceleration of cylinder

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $p = \rho\cdot R\cdot T$ $c = \sqrt{k\cdot R\cdot T}$ $m_{rate} = \rho\cdot A\cdot V$

$$\vec{F}_S + \vec{F}_B - \int_{CV} \vec{a}_{rf}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, dV + \int_{CS} \vec{V}_{xyz}\,\rho\,\vec{V}_{xyz}\cdot d\vec{A} \qquad (4.33)$$

Given or available data $p_{atm} = 101\cdot kPa$ $p_0 = 20\cdot MPa$ $T_0 = (20 + 273)\cdot K$ $k = 1.4$ $R = 286.9\cdot\frac{J}{kg\cdot K}$

$d = 10\cdot mm$ so the nozzle area is $A_e = \frac{\pi}{4}\cdot d^2$ $A_e = 78.5\cdot mm^2$ $M_{CV} = 65\cdot kg$

The flow will be choked if $p_b/p_0 < 0.528$: $p_b = p_{atm}$ so $\frac{p_b}{p_0} = 5.05\times 10^{-3}$ (Choked: Critical conditions)

The exit temperature is $T_e = \frac{T_0}{\left(1 + \frac{k-1}{2}\right)}$ $T_e = 244\,K$ $T_e = -29\cdot°C$ $c_e = \sqrt{k\cdot R\cdot T_e}$

The exit speed is $V_e = c_e$ $V_e = 313\,\frac{m}{s}$

The exit pressure is $p_e = \frac{p_0}{\left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}}$ $p_e = 10.6\cdot MPa$ and exit density is $\rho_e = \frac{p_e}{R\cdot T_e}$ $\rho_e = 151\,\frac{kg}{m^3}$

Then $m_{rate} = \rho_e\cdot A_e\cdot V_e$ $m_{rate} = 3.71\,\frac{kg}{s}$

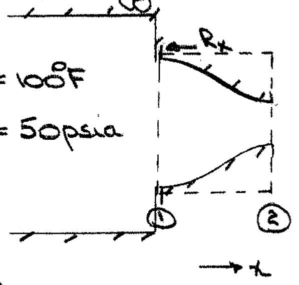
The momentum equation (Eq. 4.33) simplifies to $\left(p_e - p_{atm}\right)\cdot A_e - M_{CV}\cdot a_x = -V_e\cdot m_{rate}$

Hence $a_x = \frac{\left(p_e - p_{atm}\right)\cdot A_e + V_e\cdot m_{rate}}{M_{CV}}$ $a_x = 30.5\,\frac{m}{s^2}$

The process is isentropic, followed by nonisentropic expansion to atmospheric pressure

Problem 13.36

Given: Isentropic flow of air through a converging nozzle discharges to atmosphere; nozzle is bolted to a large tank.



$T_0 = 100°F$

$P_0 = 50\text{psia}$

$P_b = P_{atm} = 14.7 \text{ psia}$

$A_2 = 1.0 \text{ in}^2$

$A_1 = 10.0 \text{ in}^2$

Find: Force in bolts

Solution:

Basic equations: $F_{S_x} = P_1A_1 - P_2A_2 - P_{atm}(A_1 - A_2) - R_x = \dot{m}(V_2 - V_1)$

$\dot{m} = \rho V A = \text{const} \qquad P = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow
(2) isentropic flow in nozzle
(3) uniform flow at a section
(4) $F_{B_x} = 0$
(5) $V_1 \approx 0$

First check for choking

$\frac{P_b}{P_0} = \frac{14.7}{50} = 0.294 < 0.528$ and hence the nozzle is choked

$M_2 = 1.0$ and $P_2 = 0.528\, P_0 = 0.528(50 \text{ psia}) = 26.4 \text{ psia}$

$T_0/T = 1 + \frac{k-1}{2}M^2 \;; \quad T_2 = T_0/\left[1 + \frac{k-1}{2}M_2^2\right] = 560°R/\left[1 + 0.2(1.0)^2\right] = 467°R$

$\rho_2 = \frac{P_2}{RT_2} = 26.4 \frac{\text{lb}}{\text{in}^2} \times \frac{\text{lbm} \cdot °R}{53.3 \text{ ft} \cdot \text{lbf}} \times \frac{1}{467°R} \times 144 \frac{\text{in}^2}{\text{ft}^2} = 0.153 \text{ lbm/ft}^3$

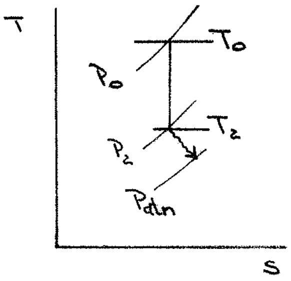
$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 1.0\left(1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot °R} \times 467°R \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 1060 \text{ ft/s}$

$\dot{m} = \rho_2 V_2 A_2 = 0.153 \frac{\text{lbm}}{\text{ft}^3} \times 1060 \frac{\text{ft}}{\text{s}} \times 1.0 \text{ in}^2 \times \frac{\text{ft}^2}{144 \text{ in}^2} = 1.13 \text{ lbm/s}$

$R_x = P_1A_1 - P_2A_2 - P_{atm}(A_1 - A_2) - \dot{m}(V_2 - \overset{\approx 0}{\cancel{V_1}}) = P_{1g}A_1 - P_{2g}A_2 - \dot{m}V_2$

$R_x = (50 - 14.7)\frac{\text{lbf}}{\text{in}^2} \times 10 \text{ in}^2 - (26.4 - 14.7)\frac{\text{lbf}}{\text{in}^2} \times 1.0 \text{ in}^2 - 1.13 \frac{\text{lbm}}{\text{s}} \times 1060 \frac{\text{ft}}{\text{s}} \times \frac{\text{slug}}{32.2 \text{ lbm}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{ft} \cdot \text{slug}}$

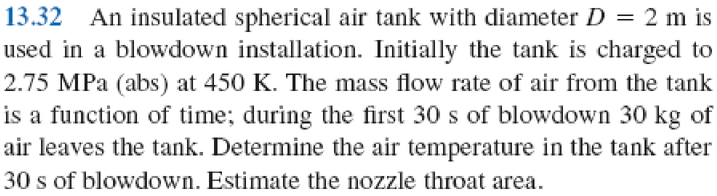
$R_x = 304 \text{ lbf}$ Since R_x acts to the left on CV, bolts are in tension $\qquad R_x$



42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 13.32

[4]



13.32 An insulated spherical air tank with diameter $D = 2$ m is used in a blowdown installation. Initially the tank is charged to 2.75 MPa (abs) at 450 K. The mass flow rate of air from the tank is a function of time; during the first 30 s of blowdown 30 kg of air leaves the tank. Determine the air temperature in the tank after 30 s of blowdown. Estimate the nozzle throat area.

Given: Spherical air tank

Find: Air temperature after 30s; estimate throat area

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\qquad \frac{p}{\rho^k} = \text{const}$ $\qquad \frac{\partial}{\partial t}\int \rho\, dV_{CV} + \int \rho\cdot\vec{V}\, d\vec{A}_{CS} = 0$ (4.12)

Assumptions: 1) Large tank (stagnation conditions) 2) isentropic 3) uniform flow

Given or available data $\quad p_{atm} = 101\cdot\text{kPa} \quad p_1 = 2.75\cdot\text{MPa} \quad T_1 = 450\cdot\text{K} \quad D = 2\cdot\text{m} \quad V = \frac{\pi}{6}\cdot D^3 \quad V = 4.19\cdot\text{m}^3$

$\Delta M = 30\cdot\text{kg} \quad \Delta t = 30\cdot\text{s} \quad k = 1.4 \quad R = 286.9\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$

The flow will be choked if $p_b/p_1 < 0.528$: $\quad p_b = p_{atm} \quad$ so $\quad \frac{p_b}{p_1} = 0.037 \quad$ (Initially choked: Critical conditions)

We need to see if the flow is still choked after 30s

The initial (State 1) density and mass are $\quad \rho_1 = \frac{p_1}{R\cdot T_1} \quad \rho_1 = 21.3\frac{\text{kg}}{\text{m}^3} \quad M_1 = \rho_1\cdot V \quad M_1 = 89.2\,\text{kg}$

The final (State 2) mass and density are then $\quad M_2 = M_1 - \Delta M \quad M_2 = 59.2\,\text{kg} \quad \rho_2 = \frac{M_2}{V} \quad \rho_2 = 14.1\frac{\text{kg}}{\text{m}^3}$

For an isentropic process $\quad \frac{p}{\rho^k} = \text{const} \quad$ so $\quad p_2 = p_1\cdot\left(\frac{\rho_2}{\rho_1}\right)^k \quad p_2 = 1.55\cdot\text{MPa} \quad \frac{p_b}{p_2} = 0.0652 \quad$ (Still choked)

The final temperature is $\quad T_2 = \frac{p_2}{\rho_2\cdot R} \quad T_2 = 382\,\text{K} \quad T_2 = 109\cdot{}^\circ\text{C}$

To estimate the throat area we use $\quad \frac{\Delta M}{\Delta t} = m_{tave} = \rho_{tave}\cdot A_t\cdot V_{tave} \quad$ or $\quad A_t = \frac{\Delta M}{\Delta t\cdot\rho_{tave}\cdot V_{tave}}$

where we use average values of density and speed at the throat.

The average stagnation temperature is $\quad T_{0ave} = \frac{T_1 + T_2}{2} \quad T_{0ave} = 416\,\text{K}$

The average stagnation pressure is $\quad p_{0ave} = \frac{p_1 + p_2}{2} \quad p_{0ave} = 2.15\cdot\text{MPa}$

Hence the average temperature and pressure (critical) at the throat are

$$T_{tave} = \frac{T_{0ave}}{\left(1 + \frac{k-1}{2}\right)} \qquad T_{tave} = 347\,K \qquad \text{and} \qquad p_{tave} = \frac{p_{0ave}}{\left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}} \qquad p_{tave} = 1.14 \cdot MPa$$

Hence $V_{tave} = \sqrt{k \cdot R \cdot T_{tave}}$ $\quad V_{tave} = 373\,\frac{m}{s}$ $\quad \rho_{tave} = \frac{p_{tave}}{R \cdot T_{tave}}$ $\quad \rho_{tave} = 11.4\,\frac{kg}{m^3}$

Finally $A_t = \frac{\Delta M}{\Delta t \cdot \rho_{tave} \cdot V_{tave}}$ $\quad A_t = 2.35 \times 10^{-4}\,m^2$ $\quad A_t = 235 \cdot mm^2$

This corresponds to a diameter

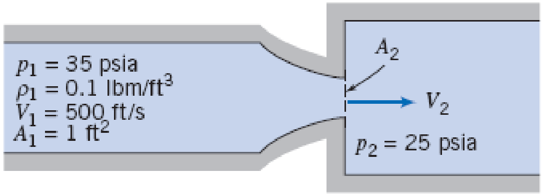
$$D_t = \sqrt{\frac{4 \cdot A_t}{\pi}} \qquad D_t = 0.0173\,m \qquad D_t = 17.3 \cdot mm$$

The process is isentropic, followed by nonisentropic expansion to atmospheric pressure

Problem 13.33

[4]

13.33 An ideal gas, with $k = 1.25$, flows isentropically through the converging nozzle shown and discharges into a large duct where the pressure is $p_2 = 25$ psia. The gas is *not* air and the gas constant, R, is unknown. Flow is steady and uniform at all cross-sections. Find the exit area of the nozzle, A_2, and the exit speed, V_2.



Given: Ideal gas flow in a converging nozzle

Find: Exit area and speed

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad \frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1 + \frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$

Given or available data $\quad p_1 = 35\cdot psi \quad \rho_1 = 0.1\cdot\frac{lbm}{ft^3} \quad V_1 = 500\cdot\frac{ft}{s} \quad A_1 = 1\cdot ft^2 \quad p_2 = 25\cdot psi \quad k = 1.25$

Check for choking: $\quad c_1 = \sqrt{k\cdot R\cdot T_1}$ or, replacing R using the ideal gas equation $\quad c_1 = \sqrt{k\cdot\frac{p_1}{\rho_1}} \quad c_1 = 1424\frac{ft}{s}$

Hence $\quad M_1 = \frac{V_1}{c_1} \quad M_1 = 0.351$

Then $\quad p_0 = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}} \quad p_0 = 37.8\,psi$

The critical pressure is then $\quad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}} \quad p_{crit} = 21.0\,psi \quad$ Hence $p_2 > p_{crit}$, so NOT choked

Then we have $\quad M_2 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \quad M_2 = 0.830$

From M_1 we find $\quad A_{crit} = \frac{M_1\cdot A_1}{\left(\frac{1 + \frac{k-1}{2}\cdot M_1^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}} \quad A_{crit} = 0.557\,ft^2 \quad A_2 = \frac{A_{crit}}{M_2}\cdot\left(\frac{1 + \frac{k-1}{2}\cdot M_2^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}} \quad A_2 = 0.573\,ft^2$

For isentropic flow $\quad p\cdot\rho^k = const \quad$ so $\quad \rho_2 = \rho_1\cdot\left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \quad \rho_2 = 0.131\frac{lbm}{ft^3}$

Finally from continuity $\quad \rho\cdot A\cdot V = const \quad$ so $\quad V_2 = V_1\cdot\frac{A_1\cdot\rho_1}{A_2\cdot\rho_2} \quad V_2 = 667\frac{ft}{s}$

Problem 13.34

Given: Jet transport aircraft cruises at 11 km. altitude. Initial cabin conditions are $T_i = 25°C$, p_i = 2.5 km altitude. Cabin volume $\forall = 25\,m^3$. Air escapes through a small hole with effective flow area, $A = 0.002\,m^2$

Find: Time required for cabin pressure to decrease by 40 percent. Plot cabin pressure as a function of time.

Solution:

Basic equations: $\frac{\partial}{\partial t}\int_{cv}\rho\,d\forall + \int_{cs}\rho\vec{V}\cdot d\vec{A} = 0$

$$\frac{p}{\rho^k} = \text{constant} \qquad p = \rho RT$$

Assumptions: (1) model flow as isentropic flow through a converging nozzle
(2) assume uniform properties within the cabin, isentropic expansion
(3) ideal gas behavior

Stagnation conditions within the cabin are
$T_i = 298\text{ K}$ $\quad p_i = p_{atm}$ at 2.5 km = 74.7 kPa (Table A.3)

$$p_f = 0.60\,p_i = 44.8\text{ kPa}$$

Back pressure $p_b = p_{atm}$ at 11 km = 22.7 kPa

Then $p_b/p_i = 0.304$ and $p_b/p_f = 0.507$. Flow is choked.

Note: conditions in cabin are stagnation conditions.

From continuity.

$$\frac{\partial}{\partial t}\int_{cv}\rho\,d\forall = -\int_{cs}\rho\vec{V}\cdot d\vec{A} = -\rho_e V_e A_e$$

$$\forall\,\frac{d\rho}{dt} = -\rho_e V_e A_e.$$

For choked flow, $M_e = 1.0$

$$\frac{\rho}{\rho_e} = \left[1+\frac{k-1}{2}M_e^2\right]^{\frac{1}{k-1}} = (1.20)^{2.5} = 1.5774 \qquad \therefore \rho_e = 0.6339\,\rho$$

$$\frac{T}{T_e} = 1+\frac{k-1}{2}M_e^2 = 1.2 \qquad \therefore T_e = 0.8333\,T$$

$$V_e = (kRT_e)^{1/2} = (kR)^{1/2}(0.8333T)^{1/2} = 0.9129\,(kR)^{1/2}T^{1/2}$$

Then

$$\forall\,\frac{d\rho}{dt} = -\rho_e V_e A_e = -0.6339\rho\,(0.9129)(kR)^{1/2}T^{1/2}A_e$$

$$\forall\,\frac{d\rho}{dt} = -0.5787\,(kR)^{1/2}A_e\,\rho T^{1/2}$$

For an isentropic expansion ρ and T can be related

$$\frac{p}{\rho^k} = \text{const} = \frac{\rho RT}{\rho^k} \qquad \therefore \rho^{(1-k)}T = \text{constant}$$

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Then, $\rho^{(1-k)}T = \rho_i^{(1-k)}T_i$ or $T = T_i\left(\frac{\rho_i}{\rho}\right)^{(1-k)}$ and $T^{1/2} = \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}}\rho^{(k-1)/2}$

Substituting we obtain

$$\forall \frac{d\rho}{dt} = -0.5787(kR)^{1/2} A_e \rho \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}}\rho^{(k-1)/2}$$

$$\frac{d\rho}{dt} = -0.5787\frac{(kR)^{1/2}}{\forall}A_e\frac{T_i^{1/2}}{\rho_i^{(k-1)/2}}\rho^{\frac{k+1}{2}} = c_1\rho^{(k+1)/2}$$

$$\frac{d\rho}{\rho^{(k+1)/2}} = -c_1\,dt \qquad \text{where } c_1 = 0.5787(kR)^{1/2}\frac{A_e}{\forall}\frac{T_i^{1/2}}{\rho_i^{(k-1)/2}}$$

To integrate, we write

$$-c_1 t = \int \rho^{-\frac{(k+1)}{2}}d\rho = \frac{1}{1-\frac{(k+1)}{2}}\rho^{1-\frac{(k+1)}{2}}\Big]_{\rho_i}^{\rho_f} = \frac{2}{(1-k)}\rho^{\frac{(1-k)}{2}}\Big]_{\rho_i}^{\rho_f}$$

$$-c_1 t = \frac{2}{(k-1)}\left[\rho_i^{(1-k)/2} - \rho_f^{(1-k)/2}\right] = \frac{2}{(k-1)}\rho_i^{(1-k)/2}\left[1-\left(\frac{\rho_f}{\rho_i}\right)^{(1-k)/2}\right]$$

$$c_1 t = \frac{2}{(k-1)}\rho_i^{(1-k)/2}\left[\left(\frac{\rho_i}{\rho_f}\right)^{(k-1)/2}-1\right]$$

$$0.5787(kR)^{1/2}\frac{A_e}{\forall}\frac{T_i^{1/2}}{\rho_i^{(k-1)/2}}\,t = \frac{2}{(k-1)}\rho_i^{(1-k)/2}\left[\left(\frac{\rho_i}{\rho_f}\right)^{(k-1)/2}-1\right]$$

$$0.5787(kR)^{1/2}\frac{A_e}{\forall}T_i^{1/2}\,t = \frac{2}{(k-1)}\left[\left(\frac{\rho_i}{\rho_f}\right)^{\frac{(k-1)}{2}}-1\right]$$

Since $p/\rho^k = \text{const}$, $\frac{\rho_i}{\rho_f} = \left(\frac{p_i}{p_f}\right)^{1/k}$

and
$$0.5787(kR)^{1/2}\frac{A_e}{\forall}T_i^{1/2}\,t = \frac{2}{(k-1)}\left[\left(\frac{p_i}{p_f}\right)^{\frac{(k-1)}{2k}}-1\right] \qquad (1)$$

Substituting numerical values

$$0.5787\left[1.4\times287\frac{N\cdot m}{kg\cdot K}\times298K\times\frac{kg\cdot m}{N\cdot s^2}\right]^{1/2}0.002m^2\times\frac{1}{25m^3}\,t$$
$$= \frac{2}{0.4}\left[\left(\frac{1}{0.6}\right)^{0.1429}-1\right]$$

$$0.01602\,t = 0.3786$$

$$t = 23.6\,s. \longleftarrow t$$

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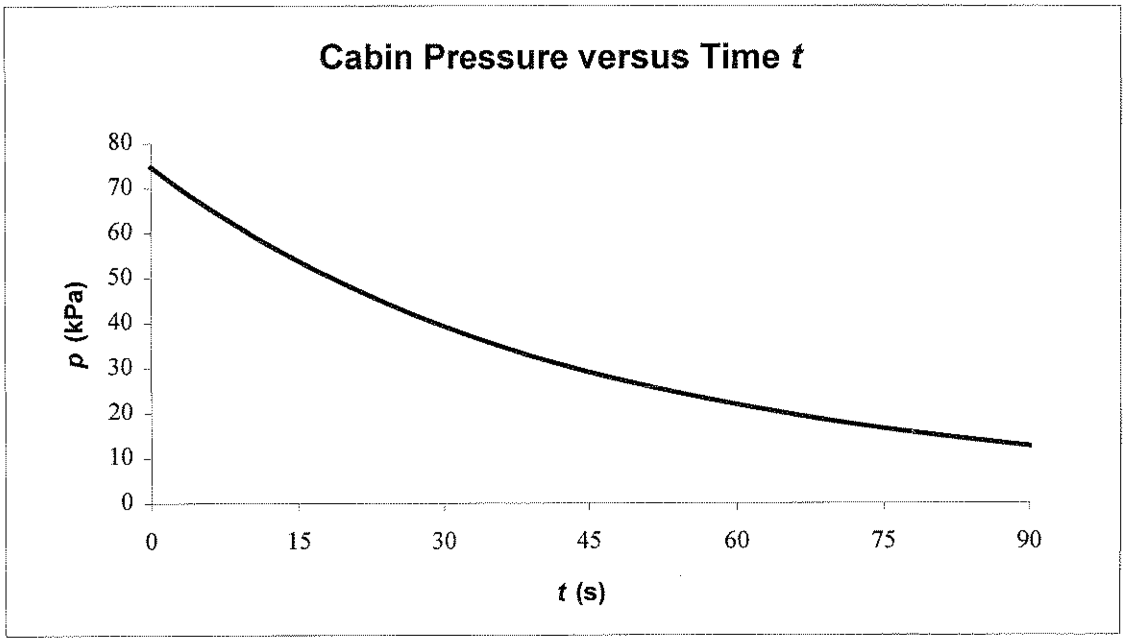
Problem 13.34

Equation 1 is plotted using *Excel*
Note that it's easier to compute t from p values!

t (s)	p (kPa)
0.000	74.7
1.03	73
2.27	71
3.56	69
4.89	67
6.26	65
7.69	63
9.17	61
10.7	59
12.3	57
14.0	55
15.7	53
17.5	51
19.4	49
21.4	47
23.6	**44.8**
25.6	43
27.9	41
30.4	39
33.0	37
35.7	35
38.6	33
41.8	31
45.2	29
48.8	27
52.8	25
57.2	23
62.0	21
67.4	19
73.5	17
80.5	15
90.0	12.7



Problem 13.35

Given: Large insulated tank, pressurized to 620 kPa (gage), supplying air to converging nozzle with discharge to atmosphere. Initial temperature in tank is 127°C.

Find: (a) Initial Mach number at nozzle exit plane.

(b) p in exit plane when flow starts.

(c) How exit plane pressure varies with time.

(d) How flow rate varies with time.

(e) Air temperature in tank when flow rate approaches zero.

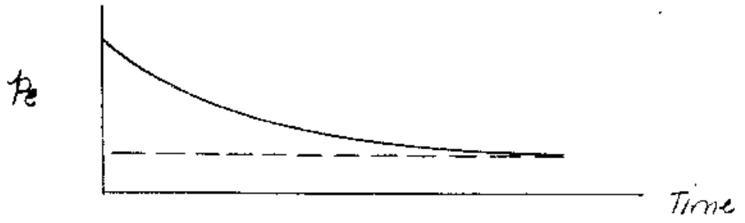
Solution: Assume stagnation conditions in tank, $p_b = p_{atm}$. Then

$$\frac{p_b}{p_0} = \frac{101.3 \text{ kPa}}{(620+101.3)\text{ kPa (abs)}} = 0.140 \ll 0.528 \text{, so flow is choked! } M_e = 1$$ M_e

At exit plane, $M_e = 1$, so

$$p_e = p_0\left(\frac{1}{1+\frac{k-1}{2}M_e^2}\right)^{\frac{k}{k-1}} = (620+101.3)\text{ kPa (abs)}\left(\frac{1}{1.2}\right)^{3.5} = 381 \text{ kPa (abs)}$$ p_e

Exit plane pressure decreases with time, asymptotically approaching p_{atm}.



Flow rate varies similarly.

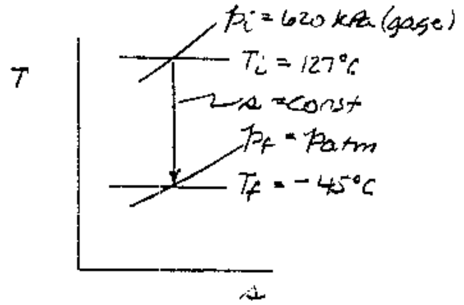
When flow rate approaches zero, $p_0 \to p_{atm}$. Assume tank behaves as a reversible adiabatic process. Thus

$$\frac{p_f}{p_0} = \left(\frac{T_f}{T_0}\right)^{\frac{k}{k-1}}$$

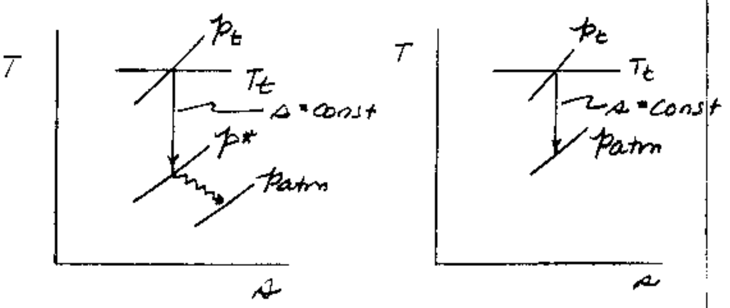
$$\text{Thus } T_f = T_0\left(\frac{p_f}{p_0}\right)^{\frac{k-1}{k}} = (273+127)\text{K}\left(\frac{101.3}{620+101.3}\right)^{0.286} = 228 \text{ K}$$

$$T_f = (228-273)°\text{C} = -45°\text{C}$$ T_f

The Ts diagram for air in the tank is:



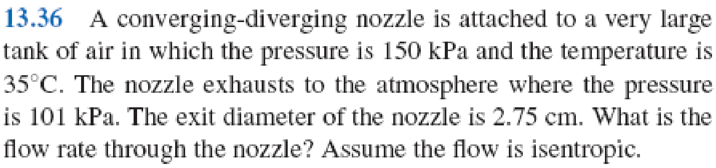
The Ts diagrams for air flowing from the tank are:



While choked ($p_t > p_{atm}/0.528$)

No longer choked ($p_t \le p_{atm}/0.528$)

Problem 13.36



13.36 A converging-diverging nozzle is attached to a very large tank of air in which the pressure is 150 kPa and the temperature is 35°C. The nozzle exhausts to the atmosphere where the pressure is 101 kPa. The exit diameter of the nozzle is 2.75 cm. What is the flow rate through the nozzle? Assume the flow is isentropic.

Given: CD nozzle attached to large tank

Find: Flow rate

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad m_{rate} = \rho\cdot V\cdot A$

Given or available data $p_0 = 150\cdot kPa$ $\quad T_0 = (35+273)\cdot K$ $\quad p_e = 101\cdot kPa$ $\quad D = 2.75\cdot cm$

$k = 1.4$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$ $\quad A_e = \frac{\pi}{4}\cdot D^2$ $\quad A_e = 5.94\,cm^2$

For isentropic flow $M_e = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_e}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_e = 0.773$

Then $T_e = \frac{T_0}{\left(1 + \frac{k-1}{2}\cdot M_e^2\right)}$ $\quad T_e = 275\,K$ $\quad T_e = 1.94\,°C$

Also $c_e = \sqrt{k\cdot R\cdot T_e}$ $\quad c_e = 332\,\frac{m}{s}$ $\quad V_e = M_e\cdot c_e$ $\quad V_e = 257\,\frac{m}{s}$

$\rho_e = \frac{p_e}{R\cdot T_e}$ $\quad \rho_e = 1.28\,\frac{kg}{m^3}$

Finally $m_{rate} = \rho_e\cdot V_e\cdot A_e$ $\quad m_{rate} = 0.195\,\frac{kg}{s}$

Problem 13.37

13.37 At the design condition of the system of Problem 13.36, the exit Mach number is $M_e = 2.0$. Find the pressure in the tank of Problem 13.36 (keeping the temperature constant) for this condition. What is the flow rate? What is the throat area?

Given: Design condition in a converging-diverging nozzle

Find: Tank pressure; flow rate; throat area

Solution:

The given or available data is:

$R = 53.33$ ft.lbf/lbm.^{o}R

$k = 1.4$

$T_0 = 560$ ^{o}R

$A_e = 1$ in^2

$p_b = 14.7$ psia

$M_e = 2$

Equations and Computations:

At design condition $p_e = p_b$

$p_e = 14.7$ psia

From M_e and p_e, and Eq. 13.7a
(using built-in function *Isenp* (M, k)

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_0 = 115$ psia

From M_e and A_e, and Eq. 13.7d
(using built-in function *IsenA* (M, k)

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (13.7d)$$

$A^* = 0.593$ in^2

Hence $A_t = 0.593$ in^2

From p_0, T_0, A_t, and Eq. 13.10a

$$\dot{m}_{\text{choked}} = A_t p_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.10a)$$

$m_{\text{choked}} = 1.53$ lb/s

Problem 13.38

Given: Air escapes from high-pressure bicycle tire through hole having $D = 0.254\,mm$, $p_1 = 620\,kPa$ (gage), and $T = 27°C$, which remains constant. Internal volume of tire is $\forall = 4.26 \times 10^{-4}\,m^3$, and also is constant.

Find: (a) Time needed for p in tire to drop to 310 kPa (gage).
(b) Entropy change of air in tire during this process.
(c) Sketch a Ts diagram showing states and process paths.
Plot pressure as a function of time.

Solution: Apply continuity equation, isentropic relationships.

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$ $\qquad \frac{T_0}{T} = \left(1 + \frac{k-1}{2} M^2\right)$; $\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$

Check for choking: $\frac{p_{atm}}{p_{min}} = \frac{101}{310+101} = 0.246 < 0.528$ so always choked.

Thus $\dot{m} = \rho^* V^* A^*$. Assume: (1) Uniform density in tire: $\int_{CV} = \rho \forall$
(2) Uniform flow at throat
(3) Isentropic process to throat.

Then

$$0 = \forall \frac{d\rho}{dt} + \rho^* V^* A_e$$

But $\rho^* = \frac{\rho}{\left(1 + \frac{k-1}{2} M_*^2\right)^{1/k-1}} = \frac{\rho}{(1.2)^{2.5}} = 0.634\rho$

so $\frac{d\rho}{\rho} = -0.634 \frac{V^* A_e}{\forall} dt$

Integrating, $\ln \frac{\rho_2}{\rho_1} = -0.634 \frac{V^* A_e}{\forall} t = \ln \frac{p_2}{p_1}$ since $T = $ constant

Thus

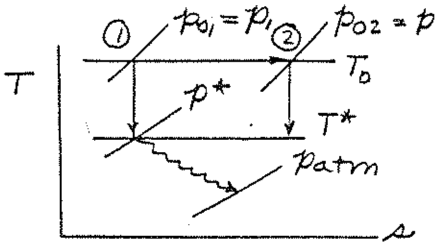
$$t = -\frac{\forall}{0.634\, V^* A_e} \ln \frac{p_2}{p_1} \qquad (1)$$

$$V^* = c^* = \sqrt{kRT^*} = \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times \frac{273+27}{1.2} K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 317\ m/s$$

$$A^* = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.000254)^2 m^2 = 5.07 \times 10^{-8}\, m^2$$

$$t = -\frac{1}{0.634} \times 4.26 \times 10^{-4} m^3 \times \frac{s}{317\, m} \times \frac{1}{5.07 \times 10^{-8} m^2} \times \ln\left(\frac{310+101}{620+101}\right) = 23.5\ s$$ $\qquad t$

Ts diagram:



Process ① → ② in tire

Process (isentropic) ① → ⊛ (moving to) ② → ⊛ in converging passage.

In tire,

$$\Delta s = c_p \ln \cancel{\frac{T_2}{T_1}} - R \ln \frac{p_2}{p_1} = -287 \frac{N \cdot m}{kg \cdot K} \times \ln \frac{(310+101)}{(620+101)} = 161\ J/(kg \cdot K)$$ $\qquad \Delta s$

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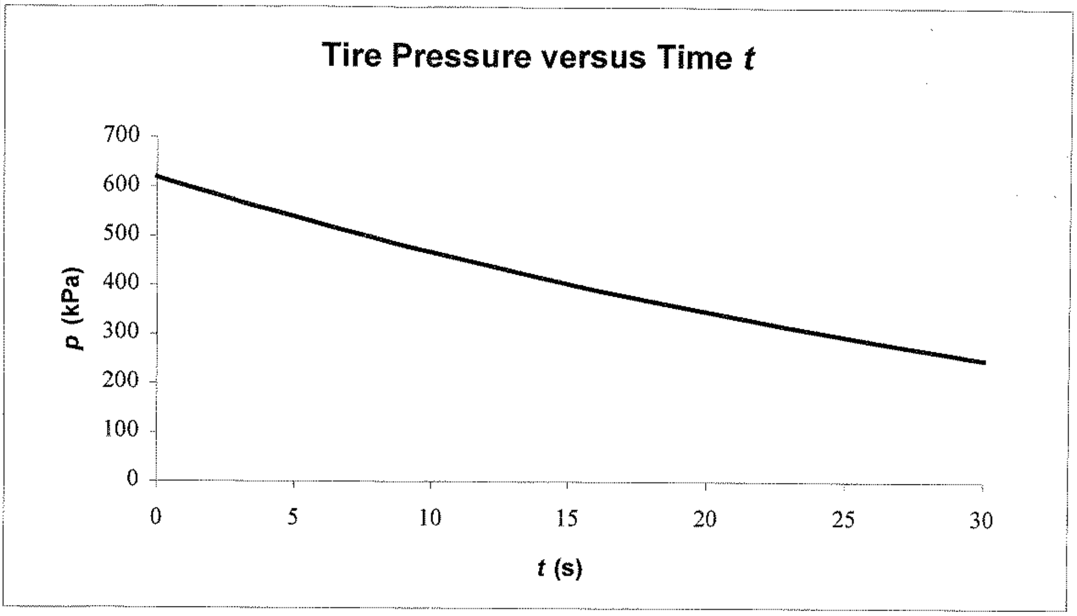
Problem 13.38

Equation 1 is plotted using *Excel*
Note that it's easier to compute *t* from *p* values!

t (s)	*p* (kPa)
0.000	620
0.584	610
1.18	600
1.78	590
2.39	580
3.00	570
3.63	560
4.27	550
4.92	540
5.57	530
6.24	520
6.92	510
7.61	500
8.31	490
9.03	480
9.75	470
10.5	460
11.2	450
12.0	440
12.8	430
13.6	420
14.4	410
15.2	400
16.1	390
16.9	380
17.8	370
18.7	360
19.6	350
20.6	340
21.5	330
22.5	320
23.5	310
24.5	300
25.6	290
26.7	280
27.8	270
28.9	260
30.0	251



Problem 13.39

Given: Converging - diverging nozzle with $M_{design} = 3.0$ and $A_e = 250\ \text{mm}^2$
Nozzle bolted to side of large tank with $p = 4.5$ MPa (gage)
and $T = 750$ K. Flow in nozzle is isentropic.

Find: (a) Pressure in nozzle exit plane.
(b) Mass flow rate in nozzle.
(c) Sketch Ts diagram: label tank, throat, exit, and ambient.

Solution: Assume stagnation conditions in tank, isentropic flow in nozzle.

Computing equations: $\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$; $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$; $c = \sqrt{kRT}$

For $M_e = 3.0$, $\frac{p_0}{p_e} = \left[1 + \frac{k-1}{2}(3.0)^2\right]^{\frac{k}{k-1}} = 36.7$

$$p_e = \frac{p_0}{36.7} = \frac{(4.5 \times 10^6 + 101 \times 10^3)\ \text{Pa}}{36.7} = 125\ \text{kPa (abs)} \quad \text{or} \quad 24\ \text{kPa (gage)}$$

p_e

Thus the nozzle is underexpanded. For steady, 1-D flow, $\dot{m} = \rho_e V_e A_e$.

$$T_e = \frac{T_0}{1 + \frac{k-1}{2}(3.0)^2} = \frac{750\ \text{K}}{1 + 0.2(3)^2} = 268\ \text{K}$$

$$c_e = \sqrt{kRT_e} = \left[1.4 \times 287\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 268\ \text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 328\ \text{m/s}$$

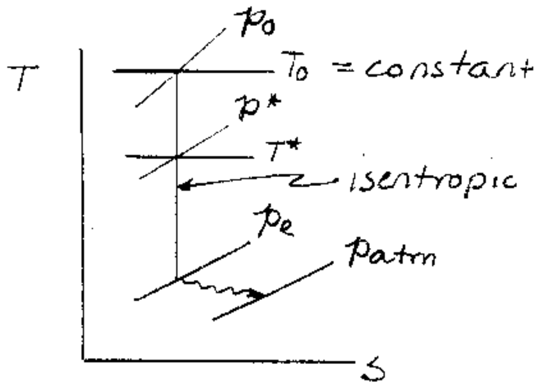
$$\rho_e = \frac{p_e}{RT_e} = 125 \times 10^3\ \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\ \text{N}\cdot\text{m}} \times \frac{1}{268\ \text{K}} = 1.63\ \text{kg/m}^3$$

Finally, $V_e = M_e c_e = 3.0 \times 328\ \text{m/s} = 984\ \text{m/s}$

$$\dot{m} = \rho_e V_e A_e = 1.63\ \frac{\text{kg}}{\text{m}^3} \times 984\ \frac{\text{m}}{\text{s}} \times 250\ \text{mm}^2 \times \frac{\text{m}^2}{10^6\ \text{mm}^2} = 0.401\ \text{kg/s}$$

$\dot{m}$

Ts diagram:



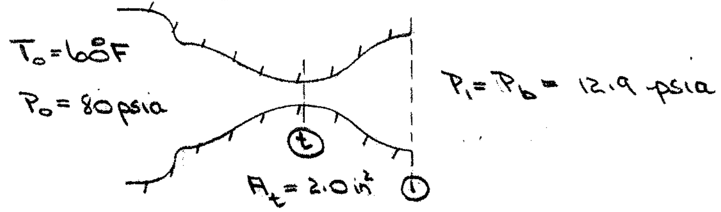
0 = tank conditions

* = throat conditions

e = exit plane conditions

Problem 13.40

Given: Isentropic flow of air from a large tank through a converging-diverging nozzle discharges to a back pressure, p_b



Find: $\dot{m}$, A_1

Solution

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$, $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

$$A/A^* = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.6)$$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$; $M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{80}{12.9}\right)^{0.286} - 1\right]\right\}^{1/2} = 1.85$

$T_0/T = 1 + \frac{k-1}{2}M^2$; $T_1 = \frac{T_0}{1 + \frac{k-1}{2}M_1^2} = \frac{520°R}{1 + 0.2(1.85)^2} = 309°R$

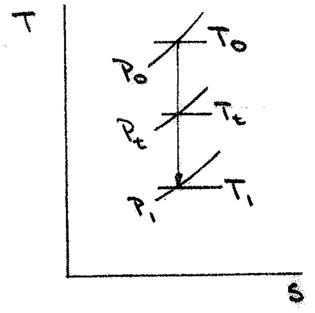
$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 1.85\left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 309°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1594\ ft/s$

$\rho_1 = \frac{p_1}{RT_1} = 12.9 \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{309°R} \times \frac{144\ in^2}{ft^2} = 0.113\ lbm/ft^3$

Since $M_1 = 1.85$, nozzle must be choked and $M_t = 1.0$; $A_t = A^*$

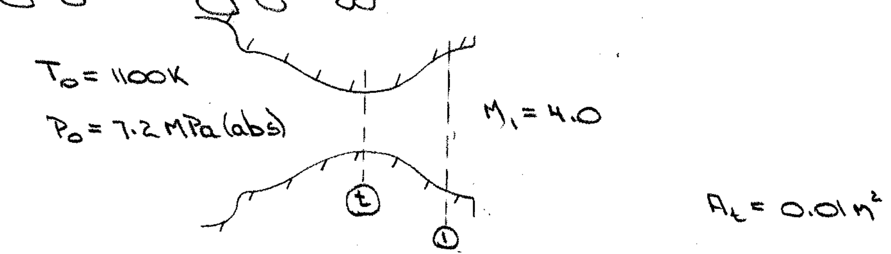
For $M_1 = 1.85$, from Eq. 12.6 (and Fig E.1), $A_1/A^* = 1.496$; $\therefore A_1 = 2.99\ in^2$ ← A_1

$\dot{m} = \rho_1 V_1 A_1 = 0.113 \frac{lbm}{ft^3} \times 1594 \frac{ft}{s} \times 2.99\ in^2 \times \frac{ft^2}{144\ in^2} = 3.74\ lbm/s$ ← $\dot{m}$



Problem 13.46

Given: Isentropic flow of air from stagnation state through a converging - diverging nozzle as shown.



Find: V_1, $\dot{m}$

Solution:

Basic equations: $\dot{m} = \rho V A$ $\quad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$ $\quad \frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \;;\; T_1 = \frac{T_0}{1 + \frac{k-1}{2} M^2} = \frac{1100\,K}{1 + 0.2\,(4.0)^2} = 262\,K$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 4.0 \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 262\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 1300\,m/s \longleftarrow V_1$$

Since $M_1 = 4.0$, nozzle must be choked and $M_t = 1.0$

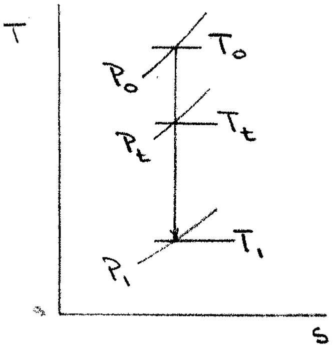
$$p_t = \frac{p_0}{\left[1 + \frac{k-1}{2} M_t^2\right]^{k/k-1}} = \frac{7.2 \times 10^6\,Pa}{\left[1 + 0.2(1.0)^2\right]^{3.5}} = 3.80\,MPa$$

$$T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{1100\,K}{1 + 0.2(1.0)^2} = 917\,K$$

$$\rho_t = \frac{p_t}{RT_t} = 3.80 \times 10^6 \frac{N}{m^2} \times \frac{kg \cdot K}{287\,N \cdot m} \times \frac{1}{917\,K} = 14.4\,kg/m^3$$

$$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0 \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 917\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 607\,m/s$$

$$\dot{m} = \rho_t V_t A_t = 14.4 \frac{kg}{m^3} \times 607 \frac{m}{s} \times 0.01\,m^2 = 87.4\,kg/s \longleftarrow \dot{m}$$



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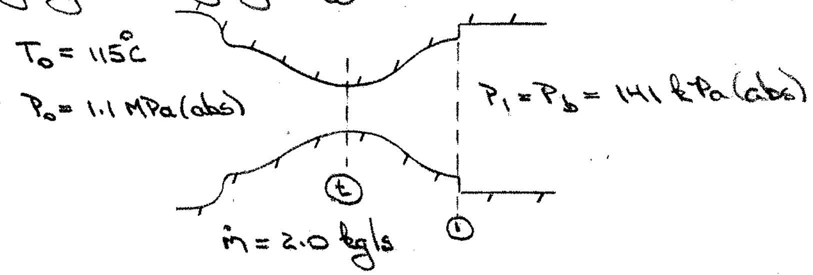
Problem 13.42

Given: Isentropic flow of air from stagnation state through a converging-diverging nozzle to a back pressure, p_b.



Find: A_t, A_1

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$, $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \quad (12.6)$$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$; $M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{1.1}{0.141}\right)^{0.286} - 1\right]\right\}^{1/2} = 2.0$

$T_0/T = 1 + \frac{k-1}{2}M^2$; $T_1 = \frac{T_0}{1 + \frac{k-1}{2}M_1^2} = \frac{388\,K}{1 + 0.2(2.0)^2} = 216\,K$

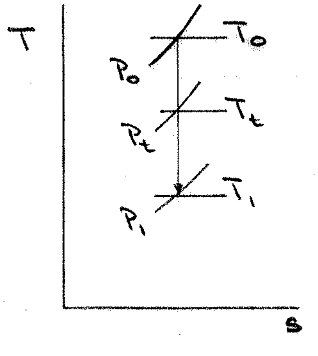
$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 216\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 589\ m/s$

$\rho_1 = \frac{p_1}{RT_1} = 141 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\,N \cdot m} \times \frac{1}{216\,K} = 2.27\ kg/m^3$

$A_1 = \frac{\dot{m}}{\rho_1 V_1} = 2.0 \frac{kg}{s} \times \frac{m^3}{2.27\,kg} \times \frac{s}{589\,m} = 1.50 \times 10^{-3}\ m^2 \longleftarrow A_1$

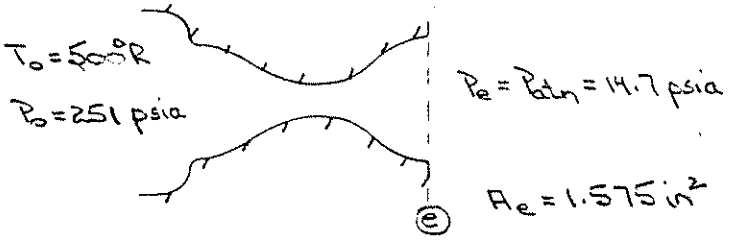
For $M_1 = 2.0$, from Eq. 12.6 and Fig. E.1, $A_1/A^* = 1.688$

Then $A_t = A^* = A_1/1.688 = 8.89 \times 10^{-4}\ m^2 \longleftarrow A_t$



Problem 13.43

Given: Isentropic flow of air from a large tank through a converging-diverging nozzle discharging to atmosphere



Find: (a) $\dot{m}$

(b) effect on $\dot{m}$ of raising T_0 to 2000°R

(c) plot $\dot{m}(T)$ for $500°R \le T \le 2000°R$

Solution:

Basic equations: $\dot{m} = \rho V A$ $\qquad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \quad ; \; M_e = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_e}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{251}{14.7}\right)^{0.286} - 1\right]\right\}^{1/2} = 2.50$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \quad ; \; T_e = \frac{T_0}{1 + \frac{k-1}{2}M_e^2} = \frac{500°R}{1 + 0.2\,(2.50)^2} = 222\,R$$

$$V_e = M_e c_e = M_e (kRT_e)^{1/2} = 2.5\left(1.4 \times 53.3\,\frac{ft \cdot lbf}{lbm \cdot °R} \times 222°R \times 32.2\,\frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1826\,ft/s$$

$$\rho_e = \frac{p_e}{RT_e} = 14.7\,\frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\,ft \cdot lbf} \times \frac{1}{222°R} \times 144\,\frac{in^2}{ft^2} = 0.179\,lbm/ft^3$$

$$\dot{m} = \rho_e V_e A_e = 0.179\,\frac{lbm}{ft^3} \times 1827\,\frac{ft}{s} \times 1.575\,in^2 \times \frac{ft^2}{144\,in^2} = 3.57\,lbm/s \longleftarrow \dot{m}$$

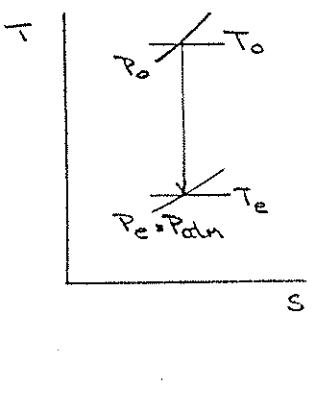
$\dot{m} = \rho_e V_e A_e$ If T_0 is increased by a factor of 4 (holding pressures constant)

(1) T_e will increase by a factor of 4 (since T_0/T = const)

(2) V_e " " " " " 2 (since $V_e \propto T_e^{1/2}$)

(3) ρ_e " decrease " " " " 4 (since $\rho_e \propto 1/T_e$)

Thus the mass flow rate, $\dot{m}$, will decrease by a factor of 2 ⟵



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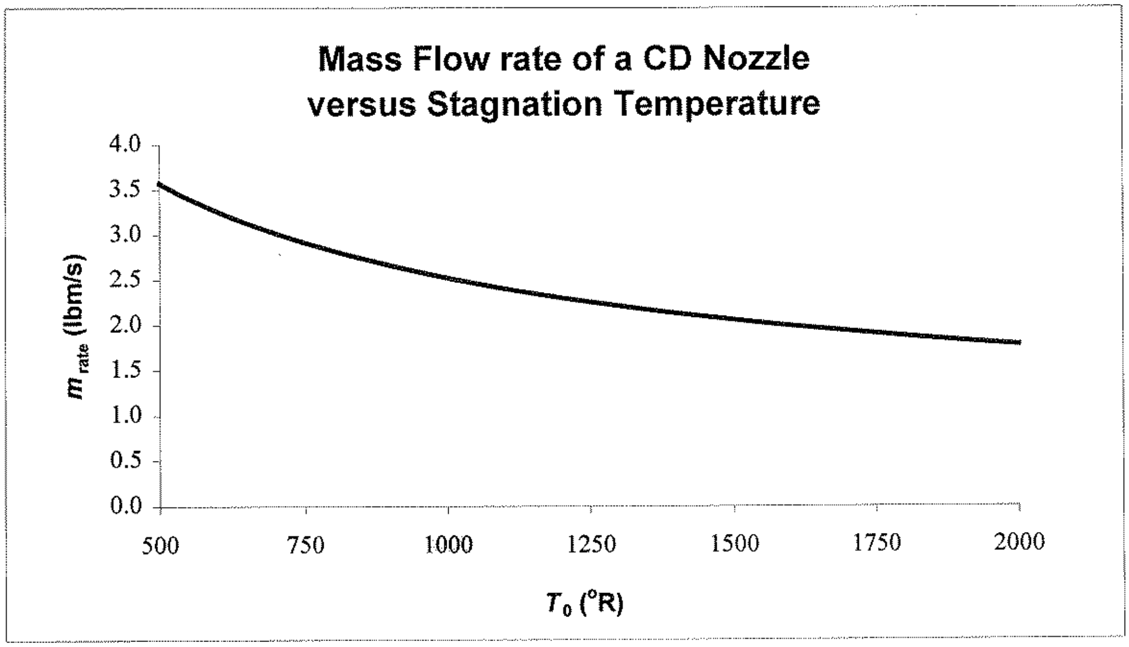
Problem 13.43

[3] Part 2/2

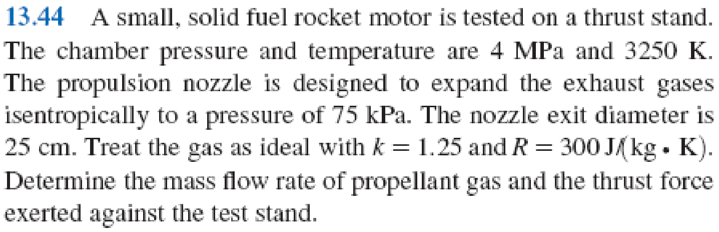
The calculations from page 1 are repeated for various T_0 values and plotted using *Excel*

T_0 (°R)	T_e (°R)	ρ_e (lbm/ft^3)	V_e (ft/s)	m_{rate} (lbm/s)
500	222	0.179	1827	3.57
550	244	0.162	1916	3.40
600	267	0.149	2001	3.26
650	289	0.137	2083	3.13
700	311	0.128	2161	3.02
750	333	0.119	2237	2.92
800	356	0.112	2311	2.82
850	378	0.105	2382	2.74
900	400	0.0993	2451	2.66
950	422	0.0941	2518	2.59
1000	444	0.0894	2583	2.52
1050	467	0.0851	2647	2.46
1100	489	0.0812	2710	2.41
1150	511	0.0777	2770	2.35
1200	533	0.0745	2830	2.30
1250	556	0.0715	2888	2.26
1300	578	0.0687	2946	2.21
1350	600	0.0662	3002	2.17
1400	622	0.0638	3057	2.13
1450	644	0.0616	3111	2.10
1500	667	0.0596	3164	2.06
1550	689	0.0577	3216	2.03
1600	711	0.0558	3268	2.00
1650	733	0.0542	3319	1.97
1700	756	0.0526	3368	1.94
1750	778	0.0511	3418	1.91
1800	800	0.0496	3466	1.88
1850	822	0.0483	3514	1.86
1900	844	0.0470	3561	1.83
1950	867	0.0458	3608	1.81
2000	889	0.0447	3654	1.79



Mass Flow rate of a CD Nozzle versus Stagnation Temperature

Problem 13.44



13.44 A small, solid fuel rocket motor is tested on a thrust stand. The chamber pressure and temperature are 4 MPa and 3250 K. The propulsion nozzle is designed to expand the exhaust gases isentropically to a pressure of 75 kPa. The nozzle exit diameter is 25 cm. Treat the gas as ideal with $k = 1.25$ and $R = 300\ \mathrm{J/(kg \cdot K)}$. Determine the mass flow rate of propellant gas and the thrust force exerted against the test stand.

Given: Rocket motor on test stand

Find: Mass flow rate; thrust force

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p = \rho\cdot R\cdot T$ $\quad c = \sqrt{k\cdot R\cdot T}$ $\quad m_{rate} = \rho\cdot A\cdot V$

$\left(p_{atm} - p_e\right)\cdot A_e + R_x = m_{rate}\cdot V_e$ Momentum for pressure p_e and velocity V_e at exit; R_x is the reaction for

Given or available data $p_e = 75\cdot kPa$ $\quad p_{atm} = 101\cdot kPa$ $\quad p_0 = 4\cdot MPa$ $\quad T_0 = 3250\cdot K$ $\quad k = 1.25$ $\quad R = 300\cdot\frac{J}{kg\cdot K}$

$d = 25\cdot cm$ so the nozzle exit area is $A_e = \frac{\pi}{4}\cdot d^2$ $\quad A_e = 491\cdot cm^2$

From the pressures $M_e = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_e}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_e = 3.12$

The exit temperature is $T_e = \frac{T_0}{\left(1 + \frac{k-1}{2}\cdot M_e^2\right)}$ $\quad T_e = 1467\,K$ $\quad c_e = \sqrt{k\cdot R\cdot T_e}$ $\quad c_e = 742\,\frac{m}{s}$

The exit speed is $V_e = M_e\cdot c_e$ $\quad V_e = 2313\,\frac{m}{s}$ and $\rho_e = \frac{p_e}{R\cdot T_e}$ $\quad \rho_e = 0.170\cdot\frac{kg}{m^3}$

Then $m_{rate} = \rho_e\cdot A_e\cdot V_e$ $\quad m_{rate} = 19.3\,\frac{kg}{s}$

The momentum equation (Eq. 4.33) simplifies to $\left(p_e - p_{atm}\right)\cdot A_e - M_{CV}\cdot a_x = -V_e\cdot m_{rate}$

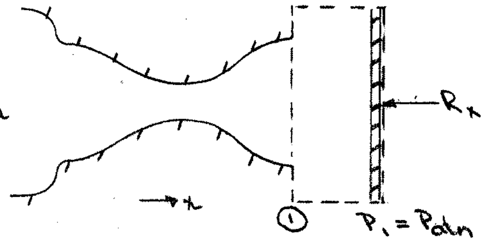
Hence $R_x = \left(p_e - p_{atm}\right)\cdot A_e + V_e\cdot m_{rate}$ $\quad R_x = 43.5\cdot kN$

Problem 13.45

Given: Flow of nitrogen through a converging-diverging nozzle leaves nozzle at atmospheric pressure, exhaust impinges on vertical flat plate.

$T_0 = 400\,\text{K}$

$p_0 = 371\,\text{kPa}$



$p_1 = p_{atm}$ $\quad A_1 = 0.003\,\text{m}^2$

Find: Force required to hold the plate.

Solution:

Basic equations: $F_{S_x} = -R_x = -\dot{m}V_1$ (momentum eq for cv shown)

$\dot{m} = \rho V A = \text{const} \qquad p = \rho R T$

Computing equations: $\dfrac{T_0}{T} = 1 + \dfrac{k-1}{2}M^2 \qquad \dfrac{p_0}{p} = \left[1 + \dfrac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow $\quad$ (4) p_{atm} over entire C.S.

(2) isentropic flow in nozzle $\quad$ (5) $F_{B_x} = 0$

(3) uniform flow at a section $\quad$ (6) ideal gas, $k = 1.40$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \; ; \quad M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{371}{101}\right)^{0.286} - 1\right]\right\}^{1/2} = 1.50$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M_1^2 \; ; \quad T_1 = \frac{T_0}{1 + \frac{k-1}{2}M_1^2} = \frac{400\,\text{K}}{1 + 0.2(1.50)^2} = 276\,\text{K}$$

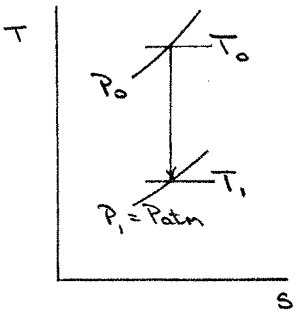
$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 1.5\left(1.4 \times 297\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 276\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 508\,\text{m/s}$$

$$\rho_1 = \frac{p_1}{RT_1} = 101\times10^3\,\frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{297\,\text{N}\cdot\text{m}} \times \frac{1}{276\,\text{K}} = 1.23\,\text{kg/m}^3$$

$$\dot{m} = \rho_1 V_1 A_1 = 1.23\,\frac{\text{kg}}{\text{m}^3} \times 508\,\frac{\text{m}}{\text{s}} \times 0.003\,\text{m}^2 = 1.87\,\text{kg/s}$$

$$R_x = \dot{m}V_1 = 1.87\,\frac{\text{kg}}{\text{s}} \times 508\,\frac{\text{m}}{\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 950\,\text{N} \leftarrow$$

(to the left as shown). R_x



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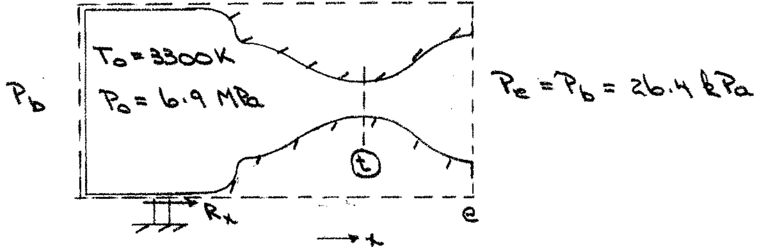
Problem 13.46

Given: Liquid rocket motor designed to expand exhaust gases isentropically to design back pressure corresponding to an altitude of 10,000 m. Thrust produced is 100 kN. Exhaust gases treated as water vapor and ideal gas behavior assumed.



Find: (a) $\dot m$ (b) A_e (c) A_e/A_t

Solution

Basic equations: $F_{S_x} = R_x = \dot m V_e$ $\quad \dot m = \rho V A \quad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \quad \frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{1+\frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.6)$$

Assumptions: (1) steady flow (4) thrust specified at altitude
(2) isentropic flow (5) $F_{B_x} = 0$
(3) uniform flow at a section (6) ideal gas, $k = 1.3$, $R = 461\,\frac{N\cdot m}{kg\cdot K}$

$p_0/p = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$; $M_e = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p}\right)^{\frac{k-1}{k}}-1\right]\right\}^{1/2} = \left\{\frac{2}{0.30}\left[\left(\frac{690}{2.64}\right)^{0.231}-1\right]\right\}^{1/2} = 4.18$

$T_0/T = 1+\frac{k-1}{2}M^2$; $T_e = \frac{T_0}{1+\frac{k-1}{2}M_e^2} = \frac{3300\,K}{1+0.15(4.18)^2} = 911\,K$

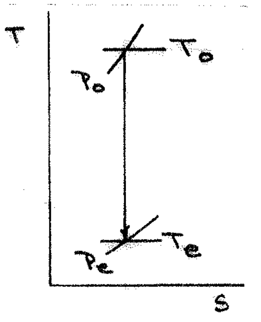
$V_e = M_e c_e = M_e(kRT_e)^{1/2} = 4.18\left[1.3 \times 461\,\frac{N\cdot m}{kg\cdot K} \times 911\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 3090\,m/s$

$R_x = \dot m V_e$; $\dot m = \frac{R_x}{V_e} = 10^5\,N \times \frac{s}{3090\,m} \times \frac{kg\cdot m}{N\cdot s^2} = 32.4\,kg/s \longleftarrow \dot m$

$\rho_e = \frac{p_e}{RT_e} = 26.4\times10^3\,\frac{N}{m^2} \times \frac{kg\cdot K}{461\,N\cdot m} \times \frac{1}{911\,K} = 6.29\times10^{-2}\,kg/m^3$

$\dot m = \rho_e V_e A_e$; $A_e = \frac{\dot m}{\rho_e V_e} = 32.4\,\frac{kg}{s} \times \frac{m^3}{6.29\times10^{-2}\,kg} \times \frac{s}{3090\,m} = 0.167\,m^2 \longleftarrow A_e$

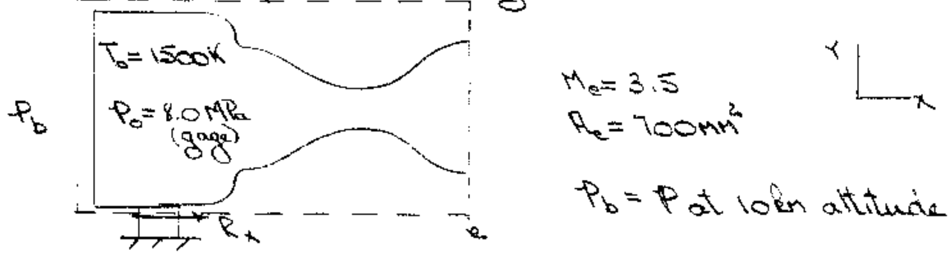
For $M_e = 4.18$, from Eq 12.6, $A_e/A^* = A_e/A_t = 19.4 \longleftarrow A_e/A_t$



Problem 13.47

[3]

Given: Small rocket motor, fueled with H_2 and O_2, is tested on thrust stand at a simulated altitude of 10 km. Combustion product is water vapor which may be treated as an ideal gas.



$M_e = 3.5$

$A_e = 700\,mm^2$

$p_b = p$ at 10 km altitude

Find: (a) p_e (b) $\dot m$ (c) force on test stand

Solution:

Basic equations: $\dot m = \rho V A$, $p = \rho R T$

Assumptions: (1) steady flow (3) ideal gas behavior
(2) isentropic flow $k = 1.3$, $R = 461\ J/kg\cdot K$

At 10 km altitude, $p_b = 26.5\ kPa$ (Table A.3)

Evaluate design pressure at exit

$$\frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1} \quad \therefore\ p_d = \frac{8.10\times10^6\,Pa}{\left[1+0.15(3.5)^2\right]^{4.333}} = 88.3\ kPa\ (abs)$$

Since $p_b < p_d$, $p_e = p_d = 88.3\ kPa$ ← p_e

$$\frac{T_0}{T_e} = 1+\frac{k-1}{2}M_e^2 \quad \therefore\ T_e = \frac{T_0}{1+\left(\frac{k-1}{2}\right)M^2} = \frac{1500\,K}{1+0.15(3.5)^2} = 529\ K$$

$$\rho_e = \frac{p_e}{RT_e} = 88.3\times10^3\,\frac{N}{m^2}\times\frac{kg\cdot K}{461\,N\cdot m}\times\frac{1}{529\,K} = 0.362\ \frac{kg}{m^3}$$

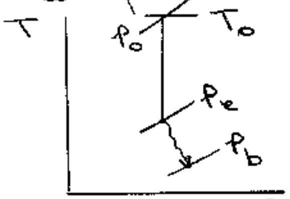
$$V_e = Mc_e = 3.5\left[1.30\times461\,\frac{N\cdot m}{kg\cdot K}\times529\,K\times\frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 1970\ m/s$$

$$\dot m = \rho_e V_e A_e = 0.362\,\frac{kg}{m^3}\times1970\,\frac{m}{s}\times700\,mm^2\times\frac{m^2}{10^6\,mm^2} = 0.499\ kg/s$$ ← $\dot m$

To determine force on test stand apply x momentum equation to CV shown

Basic equation: $F_{S_x} + \cancel{F_{B_x}}^{\,0} = \frac{\partial}{\partial t}\int_{CV} V_x\,\rho\,d\forall + \int_{CS} V_x\,\rho\vec V\cdot d\vec A$

$$R_x + p_bA_e - p_eA_e = \dot m V_e$$



Force on test stand is $K_x = -R_x$

$$\therefore\ K_x = -R_x = -\dot m V_e - A_e(p_e - p_b)$$

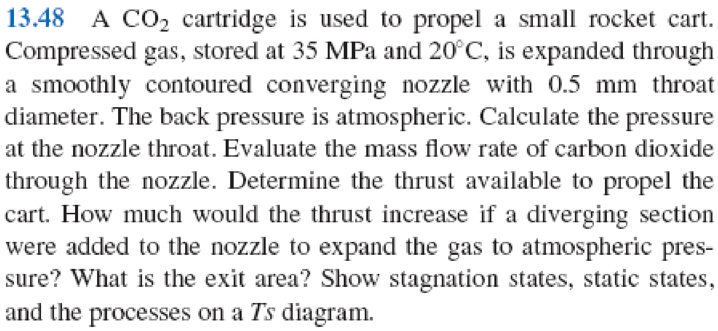
$$K_x = -0.499\,\frac{kg}{s}\times1970\,\frac{m}{s}\times\frac{N\cdot s^2}{kg\cdot m} - 700\,mm^2\times\frac{m^2}{10^6\,mm^2}(88.3-26.5)\times10^3\,\frac{N}{m^2}$$

$K_x = -1{,}026\ N$ (to left) ← K_x

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 13.48

[4]



13.48 A CO_2 cartridge is used to propel a small rocket cart. Compressed gas, stored at 35 MPa and 20°C, is expanded through a smoothly contoured converging nozzle with 0.5 mm throat diameter. The back pressure is atmospheric. Calculate the pressure at the nozzle throat. Evaluate the mass flow rate of carbon dioxide through the nozzle. Determine the thrust available to propel the cart. How much would the thrust increase if a diverging section were added to the nozzle to expand the gas to atmospheric pressure? What is the exit area? Show stagnation states, static states, and the processes on a *Ts* diagram.

Given: Compressed CO_2 in a cartridge expanding through a nozzle

Find: Throat pressure; Mass flow rate; Thrust; Thrust increase with diverging section; Exit area

Solution:

Basic equations: $F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,d\forall + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$

Assumptions: 1) Isentropic flow 2) Stagnation in cartridge 3) Ideal gas 4) Uniform flow

Given or available data: $k = 1.29$ $\quad R = 188.9\cdot\frac{J}{kg\cdot K}$ $\quad p_{atm} = 101\cdot kPa$

$p_0 = 35\cdot MPa$ $\quad T_0 = (20 + 273)\cdot K$ $\quad d_t = 0.5\cdot mm$

From isentropic relations $p_{crit} = \dfrac{p_0}{\left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}}$ $\qquad p_{crit} = 19.2\,MPa$

Since $p_b << p_{crit}$, then $p_t = p_{crit}$ $\qquad p_t = 19.2\,MPa$

Throat is critical so $m_{rate} = \rho_t\cdot V_t\cdot A_t$

$$T_t = \frac{T_0}{1 + \frac{k-1}{2}} \qquad T_t = 256\,K$$

$$V_t = \sqrt{k\cdot R\cdot T_t} \qquad V_t = 250\,\frac{m}{s}$$

$$A_t = \frac{\pi\cdot d_t^{\,2}}{4} \qquad A_t = 1.963\times 10^{-7}\,m^2$$

$$\rho_t = \frac{p_t}{R\cdot T_t} \qquad \rho_t = 396\,\frac{kg}{m^3}$$

$$m_{rate} = \rho_t\cdot V_t\cdot A_t \qquad m_{rate} = 0.0194\,\frac{kg}{s}$$

For 1D flow with no body force the momentum equation reduces to $R_x - p_{tgage} \cdot A_t = m_{rate} \cdot V_t$ $\quad p_{tgage} = p_t - p_{atm}$

$$R_x = m_{rate} \cdot V_t + p_{tgage} \cdot A_t \qquad R_x = 8.60\,\mathrm{N}$$

When a diverging section is added the nozzle can exit to atmospheric pressu $p_e = p_{atm}$

Hence the Mach number at exit is

$$M_e = \left[\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_e} \right)^{\frac{k-1}{k}} - 1 \right] \right]^{\frac{1}{2}} \qquad M_e = 4.334$$

$$T_e = \frac{T_0}{1 + \frac{k-1}{2} \cdot M_e^{\,2}} \qquad T_e = 78.7\,\mathrm{K}$$

$$c_e = \sqrt{k \cdot R \cdot T_e} \qquad c_e = 138\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$V_e = M_e \cdot c_e \qquad V_e = 600\,\frac{\mathrm{m}}{\mathrm{s}}$$

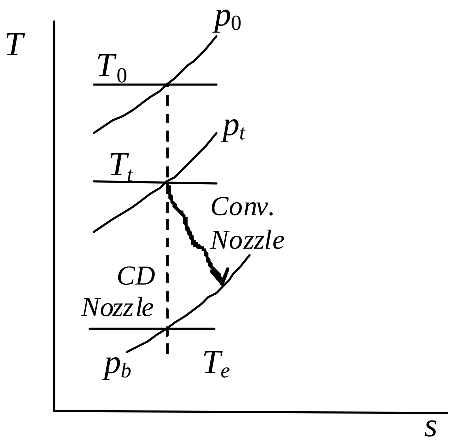
The mass flow rate is unchanged (choked flow)

From the momentum equation $R_x = m_{rate} \cdot V_e$ $\quad R_x = 11.67\,\mathrm{N}$

The percentage increase in thrust is $\dfrac{11.67 \cdot \mathrm{N} - 8.60 \cdot \mathrm{N}}{8.60 \cdot \mathrm{N}} = 35.7\,\%$

The exit area is obtained from $m_{rate} = \rho_e \cdot V_e \cdot A_e$ and $\rho_e = \dfrac{p_e}{R \cdot T_e}$ $\quad \rho_e = 6.79\,\dfrac{\mathrm{kg}}{\mathrm{m}^3}$

$$A_e = \frac{m_{rate}}{\rho_e \cdot V_e} \qquad A_e = 4.77 \times 10^{-6}\,\mathrm{m}^2 \qquad A_e = 4.77\,\mathrm{mm}^2$$



Problem 13.49

[3]

13.49 Consider the converging-diverging option of Problem 13.48. To what pressure would the compressed gas need to be raised (keeping the temperature at 20°C) to develop a thrust of 15N? (Assume isentropic flow.)

Given: CO_2 cartridge and convergent nozzle

Find: Tank pressure to develop thrust of 15 N

Solution:

The given or available data is:

$R =$ 188.9 J/kg·K

$k =$ 1.29

$T_0 =$ 293 K

$p_b =$ 101 kPa

$D_t =$ 0.5 mm

Equations and Computations:

$A_t =$ 0.196 mm^2

The momentum equation gives

$$R_x = m_{\text{flow}} V_e$$

Hence, we need m_{flow} and V_e

For isentropic flow

$p_e = p_b$

$p_e =$ 101 kPa

If we knew p_0 we could use it and p_e, and Eq. 13.7a, to find M_e.

Once M_e is known, the other exit conditions can be found.

Make a guess for p_0, and eventually use *Goal Seek* (see below).

$p_0 =$ 44.6 MPa

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$M_e =$ 4.5

From M_e and T_0 and Eq. 13.7b
(using built-in function $IsenT(M,k)$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad (13.7b)$$

$T_e =$ 74.5 K

From T_e and Eq. 12.18 $c = \sqrt{kRT}$ (12.18)

$c_e =$ 134.8 m/s

Then $V_e =$ 606 m/s

The mass flow rate is obtained from p_0, T_0, A_t, and Eq. 13.10a

$$\dot{m}_{\text{choked}} = A_t p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.10a)$$

$m_{\text{choked}} =$ 0.0248 kg/s

Finally, the momentum equation gives

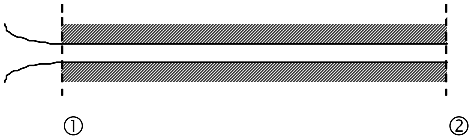
$R_x = m_{\text{flow}} V_e$
$=$ 15.0 N

We need to set R_x to 15 N. To do this use *Goal Seek*
to vary p_0 to obtain the result!

Problem 13.50

[2]

13.50 Room air is drawn into an insulated duct of constant area through a smoothly contoured converging nozzle. Room conditions are $T = 80°\text{F}$ and $p = 14.7$ psia. The duct diameter is $D = 1$ in. The pressure at the duct inlet (nozzle outlet) is $p_1 = 13$ psia. Find (a) the mass flow rate in the duct and (b) the range of exit pressures for which the duct exit flow is choked.



Given: Air flow in an insulated duct

Find: Mass flow rate; Range of choked exit pressures

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad c = \sqrt{k\cdot R\cdot T}$ $\quad \frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$

Given or available data $\quad T_0 = (80+460)\cdot R$ $\quad p_0 = 14.7\cdot psi$ $\quad p_1 = 13\cdot psi$ $\quad D = 1\cdot in$

$k = 1.4$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$ $\quad A = \frac{\pi\cdot D^2}{4}$ $\quad A = 0.785\cdot in^2$

Assuming isentropic flow, stagnation conditions are constant. Hence

$$M_1 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_1 = 0.423 \qquad T_1 = \frac{T_0}{1+\frac{k-1}{2}\cdot M_1^{\ 2}} \qquad T_1 = 521\cdot R \qquad T_1 = 61.7\cdot °F$$

$c_1 = \sqrt{k\cdot R_{air}\cdot T_1}$ $\quad c_1 = 341\frac{m}{s}$ $\quad V_1 = M_1\cdot c_1$ $\quad V_1 = 144\frac{m}{s}$

Also $\quad \rho_1 = \frac{p_1}{R_{air}\cdot T_1}$ $\quad \rho_1 = 0.0673\cdot\frac{lbm}{ft^3}$

Hence $\quad m_{rate} = \rho_1\cdot V_1\cdot A$ $\quad m_{rate} = 0.174\cdot\frac{lbm}{s}$

When flow is choked $\quad M_2 = 1$ $\quad T_2 = \frac{T_0}{1+\frac{k-1}{2}}$ $\quad T_2 = 450\cdot R$ $\quad T_2 = -9.7\cdot °F$

We also have $\quad c_2 = \sqrt{k\cdot R_{air}\cdot T_2}$ $\quad c_2 = 1040\cdot\frac{ft}{s}$ $\quad V_2 = c_2$ $\quad V_2 = 1040\cdot\frac{ft}{s}$

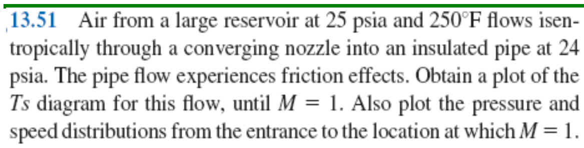
From continuity $\quad \rho_1\cdot V_1 = \rho_2\cdot V_2$ $\quad \rho_2 = \rho_1\cdot\frac{V_1}{V_2}$ $\quad \rho_2 = 0.0306\cdot\frac{lbm}{ft^3}$

Hence $\quad p_2 = \rho_2\cdot R_{air}\cdot T_2$ $\quad p_2 = 5.11\cdot psi$

The flow will therefore choke for any back pressure (pressure at the exit) less than or equal to this pressure

(From Fanno line function $\quad \frac{p_1}{p_{crit}} = 2.545$ $\quad$ at $\quad M_1 = 0.423$ $\quad$ so $\quad p_{crit} = \frac{p_1}{2.545}$ $\quad p_{crit} = 5.11\,psi$ $\quad$ Check!)

Problem 13.51



13.51 Air from a large reservoir at 25 psia and 250°F flows isentropically through a converging nozzle into an insulated pipe at 24 psia. The pipe flow experiences friction effects. Obtain a plot of the *Ts* diagram for this flow, until $M = 1$. Also plot the pressure and speed distributions from the entrance to the location at which $M = 1$.

Given: Air flow from converging nozzle into pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·°R
$k =$	1.4	
$c_p =$	0.2399	Btu/lbm·°R
	187	ft·lbf/lbm·°R
$T_0 =$	710	°R
$p_0 =$	25	psi
$p_e =$	24	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (*M*,*k*)) $M_e =$ 0.242

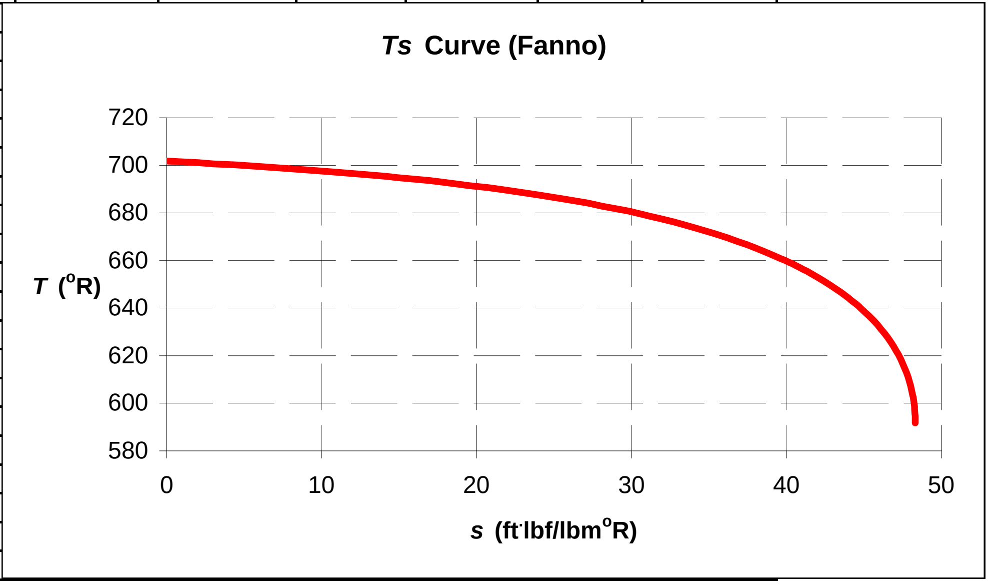
Using built-in function *IsenT* (*M*,*k*) $T_e =$ 702 °R

Using p_e, M_e, and function *Fannop* (*M*,*k*) $p^* =$ 5.34 psi

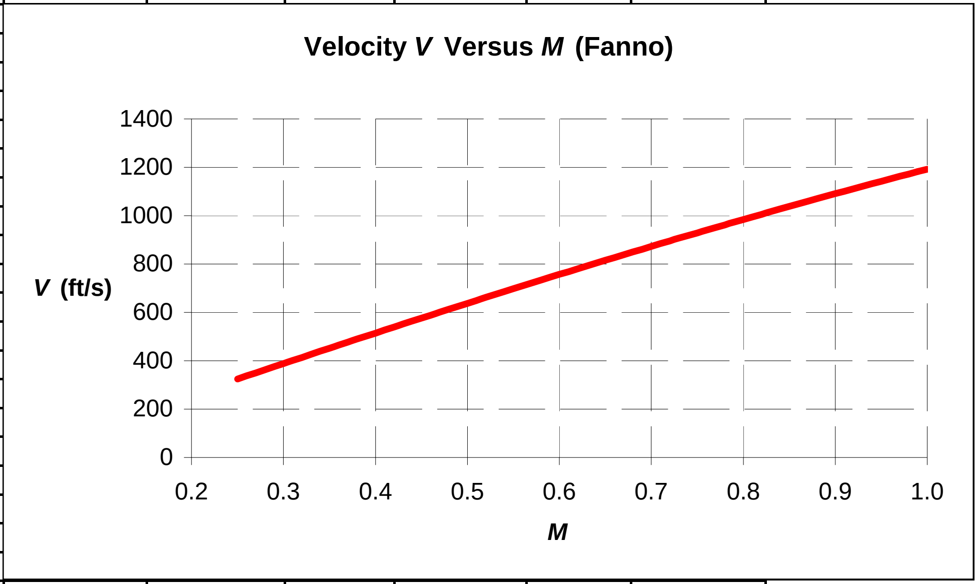
Using T_e, M_e, and function *FannoT* (*M*,*k*) $T^* =$ 592 °R

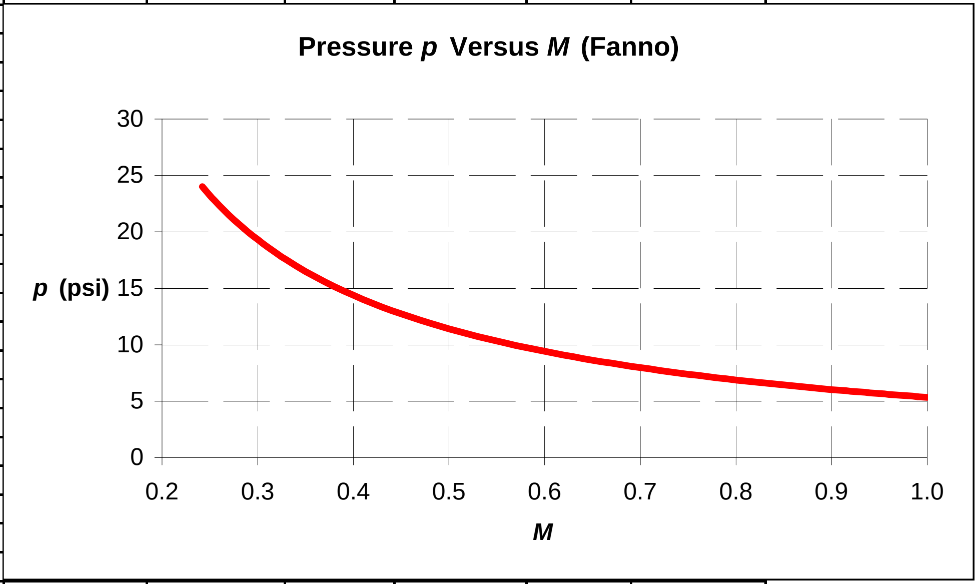
We can now use Fanno-line relations to compute values for a range of Mach numbers:

M	*T*/*T**	*T* (°R)	*c* (ft/s)	*V* (ft/s)	*p*/*p**	*p* (psi)	Δ*s* (ft·lbf/lbm·°R) Eq. (12.11b)
0.242	1.186	702	1299	315	4.50	24.0	0.00
0.25	1.185	701	1298	325	4.35	23.2	1.57
0.26	1.184						
0.27	1.183						
0.28	1.181						
0.29	1.180						
0.3	1.179						
0.31	1.177						
0.32	1.176						
0.33	1.174						
0.34	1.173						
0.35	1.171						
0.36	1.170						
0.37	1.168						
0.38	1.166						
0.39	1.165						
0.4	1.163						
0.41	1.161						
0.42	1.159						
0.43	1.157						
0.44	1.155						
0.45	1.153						



0.46	1.151	681	1280	589	2.33	12.4	29.44
0.47	1.149	680	1279	601	2.28	12.2	30.31
0.48	1.147						
0.49	1.145						
0.5	1.143						
0.51	1.141						
0.52	1.138						
0.53	1.136						
0.54	1.134						
0.55	1.132						
0.56	1.129						
0.57	1.127						
0.58	1.124						
0.59	1.122						
0.6	1.119						
0.61	1.117						
0.62	1.114						
0.63	1.112						
0.64	1.109						
0.65	1.107						
0.66	1.104						
0.67	1.101						
0.68	1.098	650	1250	850	1.54	8.2	42.77
0.69	1.096	648	1248	861	1.52	8.1	43.15
0.7	1.093						
0.71	1.090						
0.72	1.087						
0.73	1.084						
0.74	1.082						
0.75	1.079						
0.76	1.076						
0.77	1.073						
0.78	1.070						
0.79	1.067						
0.8	1.064						
0.81	1.061						
0.82	1.058						
0.83	1.055						
0.84	1.052						
0.85	1.048						
0.86	1.045						
0.87	1.042						
0.88	1.039						
0.89	1.036						
0.9	1.033	611	1212	1091	1.13	6.0	47.84
0.91	1.029	609	1210	1101	1.11	6.0	47.94
0.92	1.026	607	1208	1112	1.10	5.9	48.02
0.93	1.023	605	1206	1122	1.09	5.8	48.09
0.94	1.020	603	1204	1132	1.07	5.7	48.15
0.95	1.017	601	1202	1142	1.06	5.7	48.20
0.96	1.013	600	1201	1153	1.05	5.6	48.24
0.97	1.010	598	1199	1163	1.04	5.5	48.27
0.98	1.007	596	1197	1173	1.02	5.5	48.30
0.99	1.003	594	1195	1183	1.01	5.4	48.31
1	1.000	592	1193	1193	1.00	5.3	48.31





Problem 13.52

13.52 Repeat Problem 13.51 except the nozzle is now a converging-diverging nozzle delivering the air to the pipe at 2.5 psia.

Given: Air flow from converging-diverging nozzle into pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

$R =$ 53.33 ft·lbf/lbm·^{o}R
$k =$ 1.4
$c_p =$ 0.2399 Btu/lbm·^{o}R
187 ft·lbf/lbm·^{o}R
$T_0 =$ 710 ^{o}R
$p_0 =$ 25 psi
$p_e =$ 2.5 psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M,k)) $M_e =$ 2.16

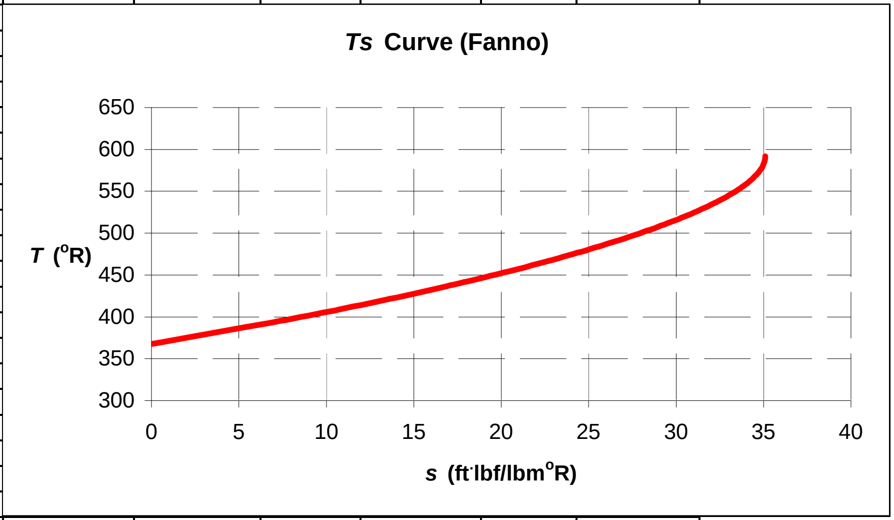
Using built-in function *IsenT* (M,k) $T_e =$ 368 ^{o}R

Using p_e, M_e, and function *Fannop* (M,k) $p^* =$ 6.84 psi

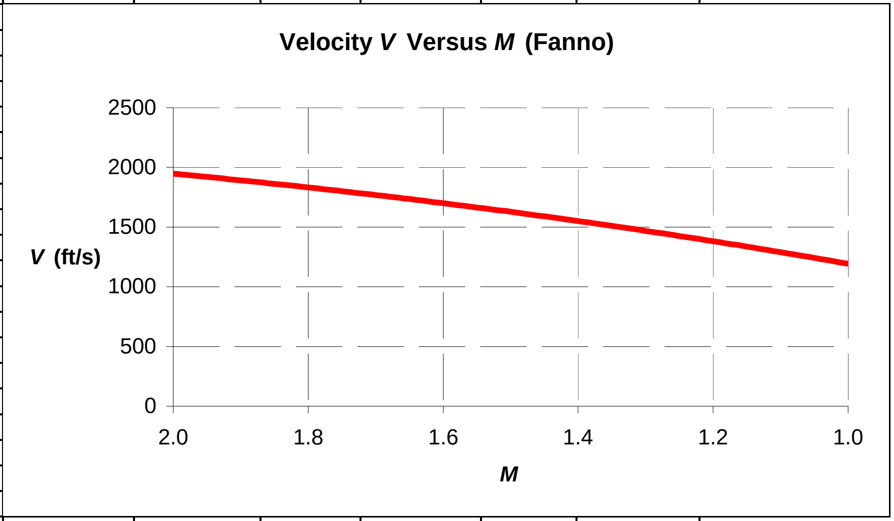
Using T_e, M_e, and function *FannoT* (M,k) $T^* =$ 592 ^{o}R

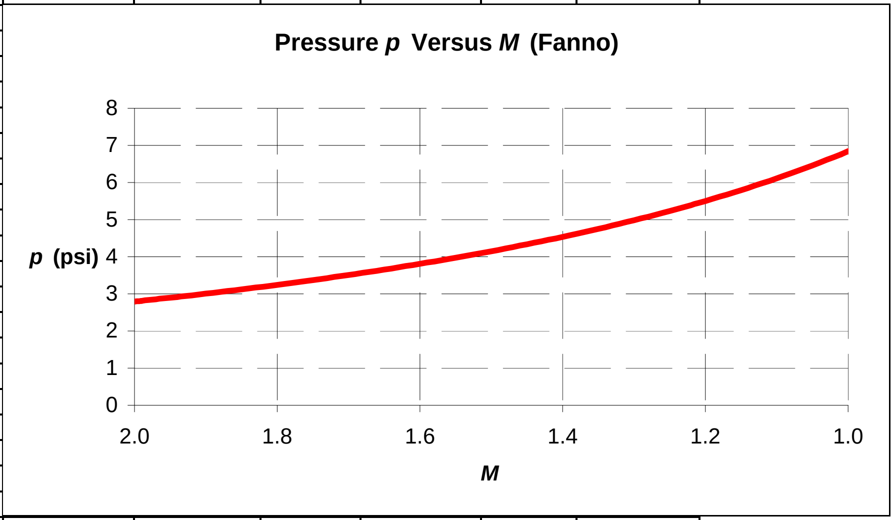
We can now use Fanno-line relations to compute values for a range of Mach numbers:

M	T/T^*	T (^{o}R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·^{o}R) Eq. (12.11b)
2.157	0.622	368	940	2028	0.37	2.5	0.00
2	0.667	394	974	1948	0.41	2.8	7.18
1.99	0.670						
1.98	0.673						
1.97	0.676						
1.96	0.679						
1.95	0.682						
1.94	0.685						
1.93	0.688						
1.92	0.691						
1.91	0.694						
1.9	0.697						
1.89	0.700						
1.88	0.703						
1.87	0.706						
1.86	0.709						
1.85	0.712						
1.84	0.716						
1.83	0.719						
1.82	0.722						
1.81	0.725						
1.8	0.728						
1.79	0.731	433	1020	1826	0.48	3.3	16.08
1.78	0.735	435	1022	1819	0.48	3.3	16.48
1.77	0.738	436	1024	1813	0.49	3.3	16.88
1.76	0.741	438	1027	1807	0.49	3.3	17.27
1.75	0.744	440	1029	1801	0.49	3.4	17.66
1.74	0.747	442	1031	1794	0.50	3.4	18.05
1.73	0.751	444	1033	1788	0.50	3.4	18.44
1.72	0.754	446	1036	1781	0.50	3.5	18.82
1.71	0.757	448	1038	1775	0.51	3.5	19.20
1.7	0.760	450	1040	1768	0.51	3.5	19.58



1.69	0.764	452	1042	1761	0.52	3.5	19.95
1.68	0.767	454	1045	1755	0.52	3.6	20.32
1.67	0.770	456	1047	1748	0.53	3.6	20.69
1.66	0.774	458	1049	1741	0.53	3.6	21.06
1.65	0.777	460	1051	1735	0.53	3.7	21.42
1.64	0.780	462	1054	1728	0.54	3.7	21.78
1.63	0.784	464	1056	1721	0.54	3.7	22.14
1.62	0.787						
1.61	0.790						
1.6	0.794						
1.59	0.797						
1.58	0.800						
1.57	0.804						
1.56	0.807						
1.55	0.811						
1.54	0.814						
1.53	0.817						
1.52	0.821						
1.51	0.824						
1.5	0.828						
1.49	0.831						
1.48	0.834						
1.47	0.838						
1.46	0.841						
1.45	0.845						
1.44	0.848						
1.43	0.852						
1.42	0.855	506	1103	1566	0.65	4.5	28.76
1.41	0.859	508	1105	1558	0.66	4.5	29.03
1.4	0.862						
1.39	0.866						
1.38	0.869						
1.37	0.872						
1.36	0.876						
1.35	0.879						
1.34	0.883						
1.33	0.886						
1.32	0.890						
1.31	0.893						
1.3	0.897						
1.29	0.900						
1.28	0.904						
1.27	0.907						
1.26	0.911						
1.25	0.914						
1.24	0.918						
1.23	0.921						
1.22	0.925						
1.21	0.928						
1.2	0.932	551	1151	1381	0.80	5.5	33.50
1.19	0.935	553	1153	1372	0.81	5.6	33.65
1.18	0.939	555	1155	1363	0.82	5.6	33.79
1.17	0.942	557	1158	1354	0.83	5.7	33.93
1.16	0.946	559	1160	1345	0.84	5.7	34.05
1.15	0.949	561	1162	1336	0.85	5.8	34.18
1.14	0.952	564	1164	1327	0.86	5.9	34.29
1.13	0.956	566	1166	1318	0.87	5.9	34.40
1.12	0.959	568	1168	1308	0.87	6.0	34.50
1.11	0.963	570	1170	1299	0.88	6.0	34.59
1.1	0.966	572	1172	1290	0.89	6.1	34.68
1.09	0.970	574	1174	1280	0.90	6.2	34.76
1.08	0.973	576	1176	1271	0.91	6.2	34.83
1.07	0.976	578	1179	1261	0.92	6.3	34.89
1.06	0.980	580	1181	1251	0.93	6.4	34.95
1.05	0.983	582	1183	1242	0.94	6.5	34.99
1.04	0.987	584	1185	1232	0.96	6.5	35.03
1.03	0.990	586	1187	1222	0.97	6.6	35.06
1.02	0.993	588	1189	1212	0.98	6.7	35.08
1.01	0.997	590	1191	1203	0.99	6.8	35.10
1	1.000	592	1193	1193	1.00	6.8	35.10





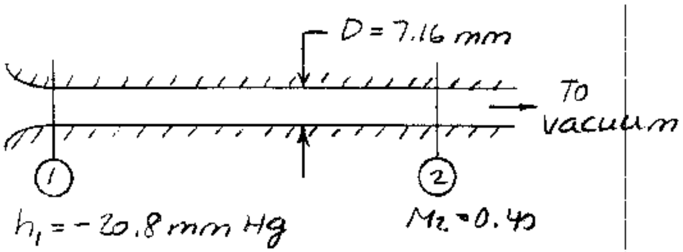
Problem 13.53

[2]

Given: Fanno line flow apparatus in laboratory, smooth brass tube fed by converging nozzle.

$T = 23°C$

$h_{barometer} = 755.1 \text{ mm Hg}$



Find: (a) M_1

(b) Mass flow rate in tube

(c) p_2

Solution: Apply equations for steady, 1-D compressible flow:

Computing equations: $T_0 = T\left(1+\frac{k-1}{2}M^2\right)$ {Entire flow adiabatic}

$p_0 = p\left(1+\frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$ {Isentropic in nozzle}

Assume: (1) Stagnation conditions in laboratory

(2) Ideal gas

Then $$\frac{p_0}{p_1} = \left(1+\frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = \frac{755.1 \text{ mm Hg}}{(755.1-20.8)\text{ mm Hg}} = 1.03$$

$$M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}}-1\right]\right\}^{1/2} = \left\{5\left[\left(\frac{755.1}{734.3}\right)^{0.286}-1\right]\right\}^{1/2} = 0.200$$

M_1

From continuity, $\dot{m} = \rho_1 V_1 A_1$; $\rho_1 = \frac{p_1}{RT_1}$

$$p_1 = \rho_{Hg} g h_1 = (13.5)1000\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 0.734\, m \times \frac{N\cdot s^2}{kg\cdot m} = 97.2 \text{ kPa (abs)}$$

$$T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{(273+23)K}{1+0.2(0.200)^2} = 294\text{ K} \; ; \; c_1 = \sqrt{kRT_1} = 344 \text{ m/s}$$

$$\rho_1 = 97.2\times10^3\frac{N}{m^2} \times \frac{kg\cdot K}{287\, N\cdot m} \times \frac{1}{294\, K} = 1.15 \text{ kg/m}^3$$

$$V_1 = M_1 c_1 = 0.200 \times 344 \text{ m/s} = 68.8 \text{ m/s}$$

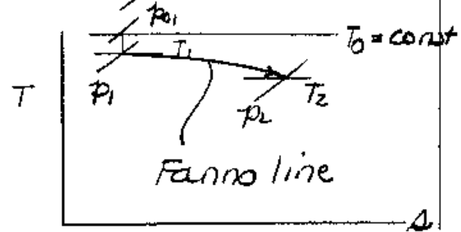
$$A_1 = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.00716)^2 m^2 = 4.03\times10^{-5} m^2$$

$$\dot{m} = 1.15\frac{kg}{m^3} \times 68.8\frac{m}{s} \times 4.03\times10^{-5} m^2 = 3.19\times10^{-3} \text{ kg/s}$$

$\dot{m}$

Since T_0 = constant, $T_2 = T_0 / \left(1+\frac{k-1}{2}M_2^2\right) = 287$ K ; $c_2 = 340$ m/s ; $V_2 = M_2 c_2 = 136$ m/s

$$\rho_2 = \rho_1\frac{V_1}{V_2} = 1.15\frac{kg}{m^3} \times \frac{68.8}{136} = 0.582 \text{ kg/m}^3$$



Then $p_2 = \rho_2 R T_2$

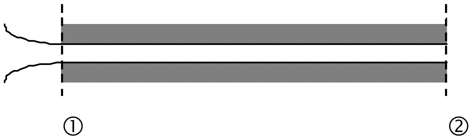
$$p_2 = 0.582\frac{kg}{m^3} \times 287\frac{N\cdot m}{kg\cdot K} \times 287\, K = 47.9 \text{ kPa (abs)}$$

p_2

Problem 13.54

[3]

13.54 Air flows steadily and adiabatically from a large tank through a converging nozzle connected to an insulated constant-area duct. The nozzle may be considered frictionless. Air in the tank is at $p = 145$ psia and $T = 250°\text{F}$. The absolute pressure at the nozzle exit (duct inlet) is 125 psia. Determine the pressure at the end of the duct, if the temperature there is 150°F. Find the entropy increase.



Given: Air flow in a converging nozzle and insulated duct

Find: Pressure at end of duct; Entropy increase

Solution:

Basic equations:
$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad \Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R_{air}\cdot\ln\left(\frac{p_2}{p_1}\right) \qquad c = \sqrt{k\cdot R\cdot T}$$

Given or available data
$$T_0 = (250 + 460)\cdot R \qquad p_0 = 145\cdot psi \qquad p_1 = 125\cdot psi \qquad T_2 = (150 + 460)\cdot R$$
$$k = 1.4 \qquad c_p = 0.2399\cdot\frac{Btu}{lbm\cdot R} \qquad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$$

Assuming isentropic flow in the nozzle
$$M_1 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_1 = 0.465 \qquad T_1 = \frac{T_0}{1 + \frac{k-1}{2}\cdot M_1^{\,2}} \qquad T_1 = 681\cdot R \qquad T_1 = 221\cdot°F$$

In the duct T_0 (a measure of total energy) is constant, so $M_2 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{T_0}{T_2}\right) - 1\right]}$ $\qquad M_2 = 0.905$

At each location
$$c_1 = \sqrt{k\cdot R_{air}\cdot T_1} \qquad c_1 = 1279\cdot\frac{ft}{s} \qquad V_1 = M_1\cdot c_1 \qquad V_1 = 595\cdot\frac{ft}{s}$$
$$c_2 = \sqrt{k\cdot R_{air}\cdot T_2} \qquad c_2 = 1211\cdot\frac{ft}{s} \qquad V_2 = M_2\cdot c_2 \qquad V_2 = 1096\cdot\frac{ft}{s}$$

Also
$$\rho_1 = \frac{p_1}{R_{air}\cdot T_1} \qquad \rho_1 = 0.4960\cdot\frac{lbm}{ft^3}$$

Hence
$$m_{rate} = \rho_1\cdot V_1\cdot A = \rho_2\cdot V_2\cdot A \qquad \text{so} \qquad \rho_2 = \rho_1\cdot\frac{V_1}{V_2} \qquad \rho_2 = 0.269\cdot\frac{lbm}{ft^3}$$

Then $p_2 = \rho_2\cdot R_{air}\cdot T_2$ $\qquad p_2 = 60.8\cdot psi$ $\qquad$ Finally $\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R_{air}\cdot\ln\left(\frac{p_2}{p_1}\right)$ $\qquad \Delta s = 0.0231\cdot\frac{Btu}{lbm\cdot R}$

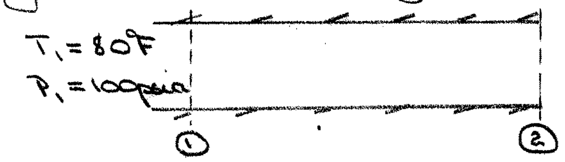
(Note: Using Fanno line relations, at $M_1 = 0.465$ $\qquad \frac{T_1}{T_{crit}} = 1.150 \qquad T_{crit} = \frac{T_1}{1.150} \qquad T_{crit} = 329\,K$

$$\frac{p_1}{p_{crit}} = 2.306 \qquad p_{crit} = \frac{p_1}{2.3060} \qquad p_{crit} = 54.2\cdot psi$$

Then $\frac{T_2}{T_{crit}} = 1.031$ $\qquad$ so $\qquad M_2 = 0.907 \qquad \frac{p_2}{p_{crit}} = 1.119 \qquad p_2 = 1.119\cdot p_{crit} \qquad p_2 = 60.7\cdot psi$ $\qquad$ Check!)

Problem 13.55

Given: Steady flow of air through an insulated pipe



$\dot{m} = 600$ lbm/min

$D_1 = D_2 = D = 4$ in

Find: (a) P_{min} (b) V_{max}, in pipe

Solution:

Basic equations: $\dot{m} = \rho V A \qquad P = \rho R T$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2$, $T_0 = $ constant

Assumptions: (1) steady flow (2) adiabatic flow (3) uniform flow at a section (4) ideal gas

$$\rho_1 = \frac{P_1}{RT_1} = 100\,\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\cdot\text{R}}{53.3\,\text{ft}\cdot\text{lbf}} \times \frac{1}{540\,\text{R}} \times \frac{144\,\text{in}^2}{\text{ft}^2} = 0.500\ \text{lbm/ft}^3$$

$$\dot{m} = \rho V_1 A \quad ; \quad A = \pi D^2/4$$

$$V_1 = \frac{4\dot{m}}{\pi D^2 \rho_1} = \frac{4}{\pi} \times 600\,\frac{\text{lbm}}{\text{min}} \times \left(\frac{12}{4}\right)^2 \frac{1}{\text{ft}^2} \times \frac{\text{ft}^3}{0.500\,\text{lbm}} \times \frac{\text{min}}{60\,\text{s}} = 229\ \text{ft/s}$$

$$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 53.3\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°\text{R}} \times 540°\text{R} \times \frac{32.2\,\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 1140\ \text{ft/s}$$

$$M_1 = \frac{229}{1140} = 0.201$$

Since $M_1 < 1.0$, the minimum pressure and max velocity occur for $M_2 = 1.0$

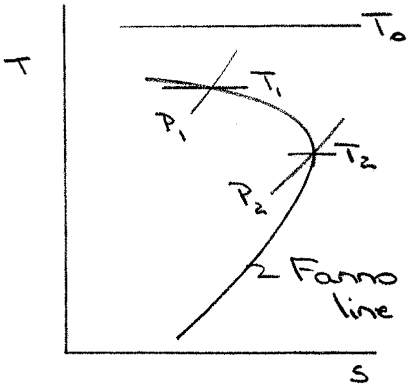
$$T_0/T = 1 + \frac{k-1}{2}M^2 \quad ; \quad \frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2} = \frac{1 + 0.2(0.201)^2}{1 + 0.2(1.0)^2} = 0.840$$

$$T_2 = 0.840(T_1) = 0.840(540°\text{R}) = 454°\text{R}$$

$$V_2 = M_2 c_2 = M_2(kRT_2)^{1/2} = 1.0\left(1.4 \times 53.3\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°\text{R}} \times 454°\text{R} \times 32.2\,\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 1040\ \text{ft/s} \longleftarrow V_{max}$$

$$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A \quad \therefore \quad \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{229}{1040} \times 0.500\,\frac{\text{lbm}}{\text{ft}^3} = 0.110\ \text{lbm/ft}^3$$

$$P_2 = \rho_2 R T_2 = 0.110\,\frac{\text{lbm}}{\text{ft}^3} \times 53.3\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°\text{R}} \times 454°\text{R} \times \frac{\text{ft}^2}{144\,\text{in}^2} = 18.5\ \text{psia} \longleftarrow P_{min}$$



*Fanno-Line Flow Functions

From Appendix E with $M_1 = 0.20$,

$T_1/T^* = 1.191$ (12.18a) $\quad \therefore T_2 = T^* = 453°\text{R}$

$p_1/p^* = 5.456$ (12.18d) $\quad \therefore p_2 = p^* = 18.3$ psia

$V_1/V^* = 0.2182$ (12.18e) $\quad \therefore V_2 = V^* = 1050$ ft/s

Problem 13.56

Given: Compressible flow through long tube (7.16 mm i.d). Air from atmosphere drawn through by vacuum pump downstream. As back pressure is lowered, pressure distribution along tube changes until $p_b = 626$ mmHg (vacuum)

Find: (a) $\dot{m}_{max}$ (b) p_{0e} (c) $s_e - s_i$

Solution:

Basic equations: $\dot{m} = \rho V A$ $\quad p = \rho R T$ $\quad T ds = dh - v dp$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$ $\quad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/k-1}$

Assumptions: (1) steady flow (2) ideal gas
(3) adiabatic flow through tube
(4) uniform flow at a section

Since pressure distribution does not change when p_b reaches 626 mmHg (vacuum), flow is choked, $M_e = 1.0$ and $p_e = p_b$

$$p_e = p_{atm} - \rho g \Delta h = 101 \times 10^3 \frac{N}{m^2} - 13.55 \times 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.626 m \times \frac{N \cdot s^2}{kg \cdot m} = 17.87 \times 10^3 \frac{N}{m^2}$$

$$T_e = \frac{T_0}{1 + \frac{k-1}{2} M_e^2} = \frac{(273+20) K}{1 + 0.2} = 244 K$$

$$\rho_e = \frac{p_e}{R T_e} = 17.87 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 N \cdot m} \times \frac{1}{244 K} = 0.255 \, kg/m^3$$

$$V_e = (k R T_e)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 244 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 313 \, m/s$$

$$\dot{m} = \rho_e V_e A = 0.255 \frac{kg}{m^3} \times 313 \frac{m}{s} \times \frac{\pi}{4} (0.00716 m)^2 = 0.00321 \, kg/s \quad \longleftarrow \dot{m}$$

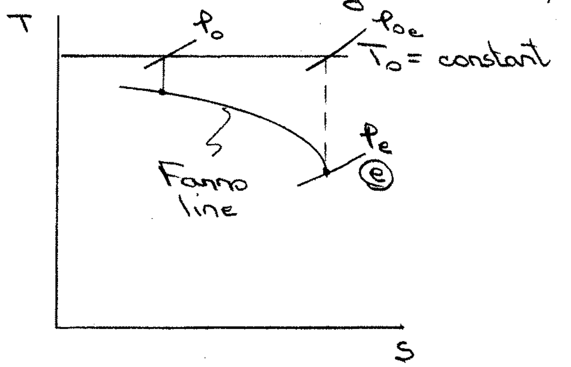
$$p_{0e} = p_e \left(1 + \frac{k-1}{2} M_e^2\right)^{k/k-1} = 17.87 \, kPa \, (1 + 0.2)^{3.5} = 33.8 \, kPa \, (abs) \quad \longleftarrow p_{0e}$$

For an ideal gas, the Tds equation can be written as

$$T ds = dh - v dp = c_p dT - RT \frac{dp}{p} \quad \therefore \quad ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

Then,
$$s_e - s_1 = s_{0e} - s_{01} = \cancel{c_p \ln \frac{T_{02}}{T_{01}}} - R \ln \frac{p_{02}}{p_{01}}$$

$$s_e - s_1 = -287 \frac{J}{kg \cdot K} \ln\left(\frac{33.8}{101}\right) = 314 \, J/kg \cdot K \quad \longleftarrow s_e - s_1$$

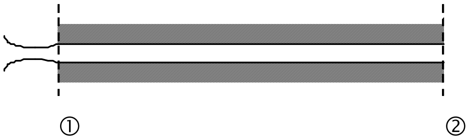


42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem 13.57

[3]

13.57 A converging-diverging nozzle discharges air into an insulated pipe with area $A = 1$ in^2. At the pipe inlet, $p = 18.5$ psia, $T = 100°$F, and $M = 2.0$. For shockless flow to a Mach number of unity at the pipe exit, calculate the exit temperature, the net force of the fluid on the pipe, and the entropy change.



Given: Air flow in a CD nozzle and insulated duct

Find: Temperature at end of duct; Force on duct; Entropy increase

Solution:

Basic equations: $F_s = p_1 \cdot A - p_2 \cdot A + R_x = m_{rate} \cdot (V_2 - V_1)$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R_{air} \cdot \ln\left(\frac{p_2}{p_1}\right)$

Given or available data $T_1 = (100 + 460) \cdot R$ $\quad p_1 = 18.5 \cdot psi$ $\quad M_1 = 2$ $\quad M_2 = 1$ $\quad A = 1 \cdot in^2$

$k = 1.4$ $\quad c_p = 0.2399 \cdot \frac{Btu}{lbm \cdot R}$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Assuming isentropic flow in the nozzle

$$\frac{T_0}{T_1} \cdot \frac{T_2}{T_0} = \frac{1 + \frac{k-1}{2} \cdot M_1^2}{1 + \frac{k-1}{2} \cdot M_2^2} \quad \text{so} \quad T_2 = T_1 \cdot \frac{1 + \frac{k-1}{2} \cdot M_1^2}{1 + \frac{k-1}{2} \cdot M_2^2} \qquad T_2 = 840 \cdot R \qquad T_2 = 380 \cdot °F$$

Also $c_1 = \sqrt{k \cdot R_{air} \cdot T_1}$ $\quad V_1 = M_1 \cdot c_1$ $\quad V_1 = 2320 \cdot \frac{ft}{s}$ $\quad c_2 = \sqrt{k \cdot R_{air} \cdot T_2}$ $\quad V_2 = M_2 \cdot c_2$ $\quad V_2 = 1421 \cdot \frac{ft}{s}$

$\rho_1 = \frac{p_1}{R_{air} \cdot T_1}$ $\quad \rho_1 = 0.0892 \cdot \frac{lbm}{ft^3}$ $\quad m_{rate} = \rho_1 \cdot V_1 \cdot A = \rho_2 \cdot V_2 \cdot A_2$ $\quad$ so $\quad \rho_2 = \rho_1 \cdot \frac{V_1}{V_2}$ $\quad \rho_2 = 0.146 \cdot \frac{lbm}{ft^3}$

$m_{rate} = \rho_1 \cdot V_1 \cdot A$ $\quad m_{rate} = 1.44 \cdot \frac{lbm}{s}$ $\quad p_2 = \rho_2 \cdot R_{air} \cdot T_2$ $\quad p_2 = 45.3 \cdot psi$

Hence $\quad R_x = (p_2 - p_1) \cdot A + m_{rate} \cdot (V_2 - V_1)$ $\quad R_x = -13.3 \cdot lbf$ $\quad$ (Force is to the right)

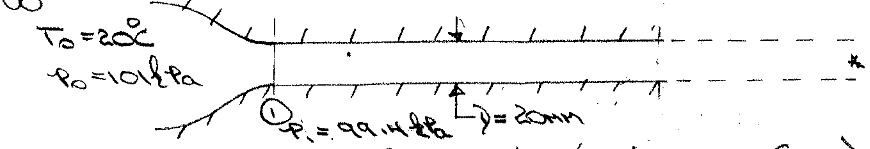
Finally $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R_{air} \cdot \ln\left(\frac{p_2}{p_1}\right)$ $\quad \Delta s = 0.0359 \cdot \frac{Btu}{lbm \cdot R}$

(Note: Using Fanno line relations, at $M_1 = 2$ $\quad \frac{T_1}{T_{crit}} = \frac{T_1}{T_2} = 0.6667$ $\quad T_2 = \frac{T_1}{0.667}$ $\quad T_2 = 840 \cdot R$

$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = 0.4083$ $\quad p_2 = \frac{p_1}{0.4083}$ $\quad p_2 = 45.3 \cdot psi$ $\quad$ Check!)

Problem 13.58

Given: Air, at 20°C and 101 kPa, is drawn through a converging nozzle into a long, 20 mm diameter, insulated tube. At the nozzle outlet (tube inlet) $p_1 = 99.4$ kPa



Find: (a) $\dot{m}$ (b) T^* and p^* for nozzle (isentropic flow)
(c) T^* and p^* for adiabatic tube flow

Solution:

Basic equations: $p = \rho RT$ $\quad \dot{m} = \rho VA$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \quad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/k-1}$

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) isentropic flow in nozzle, adiabatic flow in tube.

Since flow in nozzle is isentropic, $T_{0_1} = T_0$ and $p_{0_1} = p_0$

$\therefore T_1 = \frac{T_{0_1}}{1 + \frac{k-1}{2}M_1^2} = \frac{293\text{K}}{1 + 0.2(0.15)^2} = 291.7\text{ K}$; $M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_{0_1}}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = 0.151$

$V_1 = M_1 c_1 = M_1(kRT_1)^{1/2} = 0.151\left[1.4 \times 287\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 291.7\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 51.7\text{ m/s}$

$\rho_1 = \frac{p_1}{RT_1} = 99.4 \times 10^3\frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\text{ N}\cdot\text{m}} \times \frac{1}{291.7\text{K}} = 1.187\text{ kg/m}^3$

$\dot{m} = \rho_1 V_1 A_1 = 1.187\frac{\text{kg}}{\text{m}^3} \times 51.7\frac{\text{m}}{\text{s}} \times \frac{\pi}{4}(0.020\text{m})^2 = 0.0193\text{ kg/s}$ ← $\dot{m}$

(b) For isentropic flow, $\frac{T_0}{T^*} = 1 + \frac{k-1}{2} = 1.20 \quad \therefore T^* = \frac{T_0}{1.20} = \frac{293\text{K}}{1.20} = 244\text{K}$ ← $T^*_{s=c}$

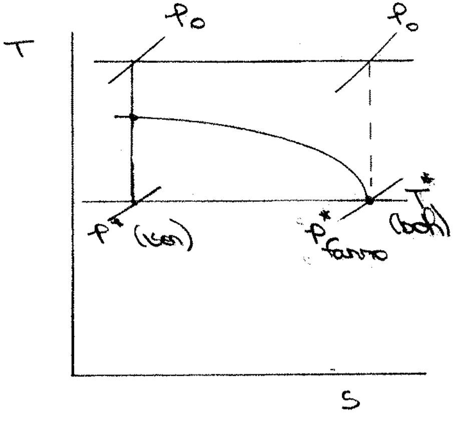
$\frac{p_0}{p^*} = \left(\frac{T_0}{T^*}\right)^{k/k-1} \quad \therefore p^* = \frac{p_0}{(T_0/T^*)^{k/k-1}} = \frac{101\text{ kPa}}{(1.2)^{3.5}} = 53.4\text{ kPa (abs)}$ ← $p^*_{s=c}$

(c) For Fanno line flow $T_0^* = T_{0_1} \quad \therefore T^* = T^*_{\text{isen}} = 244\text{ K}$ ← T^*

At *, $M = 1$, $\therefore V = c = (kRT^*)^{1/2} = \left[1.4 \times 287\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 244\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 313\text{ m/s}$

From continuity $\rho_1 V_1 = \rho^* V^* \quad \therefore \rho^* = \frac{V_1}{V^*}\rho_1 = \frac{51.4}{313} \times 1.187\frac{\text{kg}}{\text{m}^3} = 0.1949\text{ kg/m}^3$

$p^* = \rho^* RT^* = 0.1949\frac{\text{kg}}{\text{m}^3} \times 287\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 244\text{K} = 13.16\text{ kPa (abs)}$ ← p^*



* Fanno-Line Flow Functions (Appendix E.2)

For $M_1 = 0.151$, Eq 12.18d gives $p_1/p^* = 7.238$
$\therefore p^* = 13.7$ kPa ✓

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

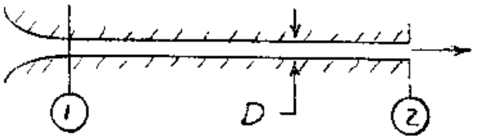
Problem 13.59

[4]

Given: Air flow through nozzle then duct, from tank at 15°C with variable pressure. The duct exhausts to atmosphere.

When $M_2 = 1$: $T_0 = 15°C$

$D = 0.249$ in.



$p_1 = 53.2$ psia $\quad p_2 = 14.7$ psia

$M_1 = 0.30 \quad M_2 = 1.0$

Find: (a) Tank pressure, p_{0_t}

(b) T_2, p_{0_2}, and $\dot{m}$

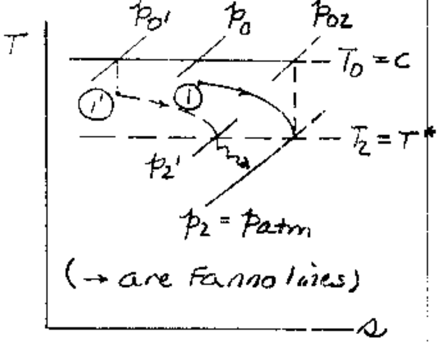
(c) Show on a Ts diagram the effect of raising to $p_0 = 100$ psia.

(d) Plot pressure distribution vs. distance.

Solution: Assume conditions in tank are stagnation properties; flow is isentropic in nozzle, Fanno flow in duct.

Basic equations: $T_0 = T(1+\frac{k-1}{2}M^2) = \text{const}$ (energy)

$$p_0 = p(1+\frac{k-1}{2}M^2)^{\frac{k}{k-1}} \quad \text{(isentropic)}$$



Assume steady, 1-D compressible flow, with constant stagnation temperature, $T_0 = 15°C$ (59°F)

$$T_0 = (460+59)°R = 519°R$$

$$p_{0_t} = p_1(1+\frac{k-1}{2}M_1^2)^{\frac{k}{k-1}} = 53.2 \text{ psia}(1+0.2(0.3)^2)^{3.5} = 56.6 \text{ psia} \quad p_{0_t}$$

At exit plane, $M_2 = 1$

$$T_2 = \frac{T_0}{(1+\frac{k-1}{2}M_2^2)} = \frac{519°R}{1+0.2(1)^2} = 433°R \quad T_2$$

$$p_{0_2} = p_2(1+\frac{k-1}{2}M_2^2)^{\frac{k}{k-1}} = 14.7 \text{ psia}(1+0.2(1)^2)^{3.5} = 27.8 \text{ psia} \quad p_{0_2}$$

At exit plane, $V = V^* = c^* = \sqrt{kRT^*} = 1020$ ft/s

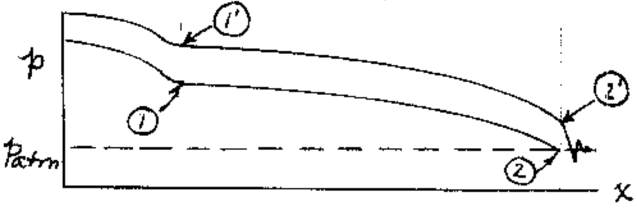
$$\rho_2 = \frac{p_2}{RT_2} = 14.7\frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{lbm}\cdot°R}{53.3\text{ ft}\cdot\text{lbf}} \times \frac{1}{433°R} \times 144\frac{\text{in.}^2}{\text{ft}^2} = 0.0917 \text{ lbm/ft}^3$$

$$\dot{m} = \rho_2 V_2 A = 0.0917\frac{\text{lbm}}{\text{ft}^3} \times 1020\frac{\text{ft}}{\text{s}} \times \frac{\pi}{4}(\frac{0.249}{12})^2\text{ft}^2 = 0.0316 \text{ lbm/s} \quad \dot{m}$$

When p_t is increased, density and mass flow rate increase. Flow will shift to a new Fanno line as shown dashed in the Ts diagram above.

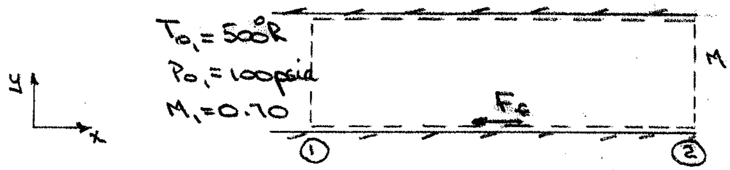
T_0 is fixed, so T^* remains fixed.

{Since ρ and $\dot{m}$ increase, $p_{2'} > p_2 = p_{atm}$. The plot of p vs. x is as shown.}



Problem 13.60

Given: Adiabatic flow of air in a constant-area duct with friction



$A = 1.0\ \text{ft}^2$

Find: Friction force exerted on the fluid by the pipe

Solution:

Basic equations: $F_{S_x} = p_1A - p_2A - F_f = \dot m(V_2 - V_1)$ $\quad \dot m = \rho V A$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2 \qquad p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow
(2) adiabatic flow, $T_0 = \text{const}$
(3) uniform flow at a section
(4) $F_{B_x} = 0$
(5) ideal gas

$$p_0/p = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1} \qquad p_1 = \frac{p_{0_1}}{\left[1+\frac{k-1}{2}M^2\right]^{k/k-1}} = \frac{100\ \text{psia}}{[1+0.2(0.70)^2]^{3.5}} = 72.1\ \text{psia}$$

$$T_0/T = 1+\frac{k-1}{2}M^2 \qquad T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{500°R}{1+0.2(0.70)^2} = 455°R$$

$$\rho_1 = \frac{p_1}{RT_1} = 72.1\ \frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\,°R}{53.3\ \text{ft}\cdot\text{lbf}} \times \frac{1}{455°R} \times \frac{144\ \text{in}^2}{\text{ft}^2} = 0.428\ \text{lbm}/\text{ft}^3$$

$$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 0.70\left(1.4 \times 53.3\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 455°R \times 32.2\ \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 732\ \text{ft}/\text{s}$$

$$\dot m = \rho_1V_1A = 0.428\ \frac{\text{lbm}}{\text{ft}^3} \times 732\ \frac{\text{ft}}{\text{s}} \times 1.0\ \text{ft}^2 = 313\ \text{lbm}/\text{s}$$

$$T_0/T = 1+\frac{k-1}{2}M^2,\quad T_{0_2} = T_{0_1},\quad T_2 = \frac{T_0}{1+\frac{k-1}{2}M_2^2} = \frac{500°R}{1+0.2(1.0)^2} = 417°R$$

$$V_2 = M_2c_2 = M_2(kRT_2)^{1/2} = 1.0\left(1.4 \times 53.3\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 417°R \times 32.2\ \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 1000\ \text{ft}/\text{s}$$

$$\dot m = \rho_1V_1A = \rho_2V_2A \qquad \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{732}{1000} \times 0.428\ \text{lbm}/\text{ft}^3 = 0.313\ \text{lbm}/\text{ft}^3$$

$$p_2 = \rho_2RT_2 = 0.313\ \frac{\text{lbm}}{\text{ft}^3} \times 53.3\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 417°R \times \frac{\text{ft}^2}{144\ \text{in}^2} = 48.3\ \text{psia}$$

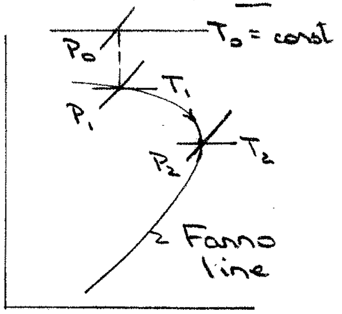
Solving the momentum equation for F_f

$$F_f = (p_1 - p_2)A - \dot m(V_2 - V_1)$$

$$= (72.1 - 48.3)\ \frac{\text{lbf}}{\text{in}^2} \times 1\ \text{ft}^2 \times 144\ \frac{\text{in}^2}{\text{ft}^2} - 313\ \frac{\text{lbm}}{\text{s}}\,(1000 - 732)\ \frac{\text{ft}}{\text{s}} \times \frac{\text{slug}}{32.2\ \text{lbm}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$F_f = 822\ \text{lbf} \qquad \overset{F_f}{\longleftarrow}$$

F_f is the force on the control volume from the surroundings. Consequently, F_f is the force on the fluid from the pipe; F_f opposes the motion



* Fanno-Line Flow Functions

From Appendix E.2 with $M_1 = 0.70$,

$p_1/p^* = 1.493$ (12.18d) $\quad \therefore p^* = p_2 = 48.3$ psia

$V_1/V^* = 0.732$ (12.18e) $\quad \therefore V^* = V_2 = 1000$ ft/s

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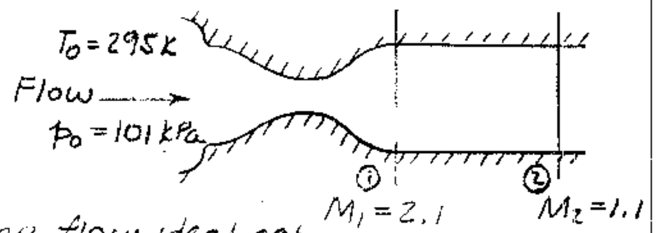
Problem 13.61

Given: Supersonic wind tunnel with conditions shown, no shocks.

System is insulated.

Find: (a) T_2, (b) p_2, (c) $s_2 - s_1$
(d) Ts diagram



Solution: Consider steady, 1-D, comp. flow, ideal gas.

Computing equations: $T_0 = T\left(1 + \frac{k-1}{2}M^2\right)$; $p_0 = p\left(1+\frac{k-1}{2}M^2\right)^{k/k-1}$ ($s =$ const)

$$T\,ds = dh - v\,dp \qquad \rho V A = \text{constant}$$

Assumptions: (1) Isentropic flow in nozzle
(2) Insulated, so $T_0 =$ constant

Then $T_2 = \dfrac{T_0}{1+\frac{k-1}{2}M_2^2} = \dfrac{295\text{ K}}{1+0.2(1.1)^2} = 238\text{ K}$ ← T_2

From continuity, $\rho_1 V_1 = \rho_2 V_2$, since $A_1 = A_2$. But $V = Mc = M\sqrt{kRT}$ and $\rho = p/RT$, so

$$\frac{p_1}{RT_1}M_1\sqrt{kRT_1} = \frac{p_2}{RT_2}M_2\sqrt{kRT_2} \;;\; \frac{p_1}{\sqrt{T_1}}M_1 = \frac{p_2}{\sqrt{T_2}}M_2 \;;\; p_2 = p_1\frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}$$

From isentropic relations,

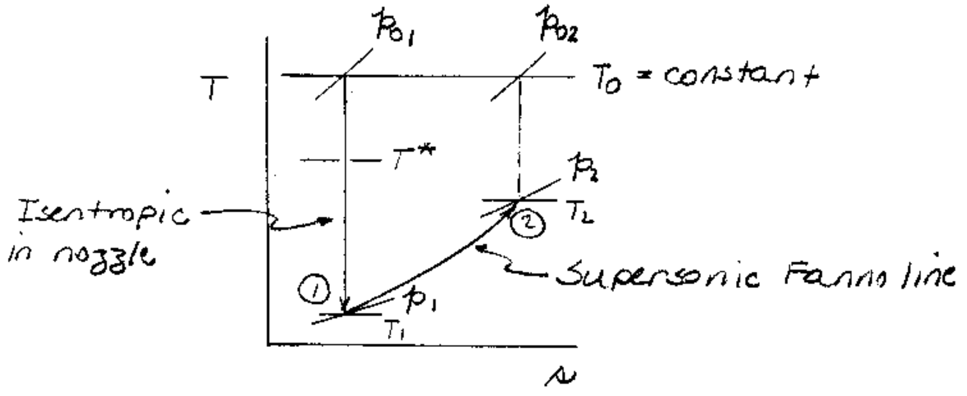
$$T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{295\text{ K}}{1+0.2(2.1)^2} = 157\text{ K} \;;\; p_1 = \frac{p_{01}}{\left(1+\frac{k-1}{2}M_1^2\right)^{k/k-1}} = \frac{101\text{ kPa}}{(1.88)^{3.5}} = 11.1\text{ kPa}$$

Thus $p_2 = \dfrac{2.1}{1.1}\sqrt{\dfrac{238\text{ K}}{157\text{ K}}}\;11.1\text{ kPa} = 26.1\text{ kPa (abs)}$ ← p_2

$$ds = \frac{dh}{T} - v\frac{dp}{T} = c_p\frac{dT}{T} - \frac{RT}{p}\frac{dp}{T} = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$\Delta s = c_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = 1004\frac{\text{J}}{\text{kg}\cdot\text{K}}\ln\left(\frac{238}{157}\right) - 287\frac{\text{J}}{\text{kg}\cdot\text{K}}\ln\left(\frac{26.1}{11.1}\right) = 172\text{ J/kg}\cdot\text{K}$$ ← Δs

The Ts diagram is



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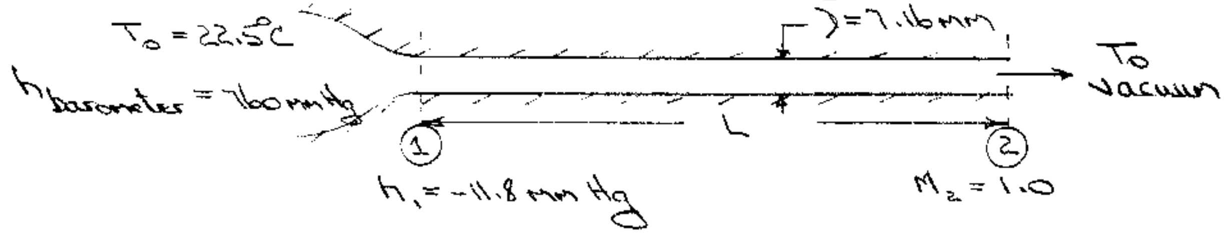
* Fanno-Line Flow Functions (Appendix E.2):

For $M_1 = 2.1$, Eq. 12.18d gives $p_1/p^* = 0.380$, so $p_2 = 26.1$ kPa (abs)
$M_2 = 1.1$ $p_2/p^* =$ [illegible]

Problem 13.62

[2]

Given: Fanno line flow apparatus in laboratory, smooth brass tube fed by converging nozzle.



Find: (a) M_1 (b) L_2 (c) T_2, p_{0_2}

Solution: * Compressible flow functions to be used

Assumptions:
(1) steady flow
(2) isentropic flow in nozzle
(3) adiabatic " in duct
(4) $\dot{W}_s = \dot{W}_{shear} = 0$
(5) $\Delta z = 0$
(6) ideal gas, air
(7) uniform flow at a section

$$\frac{p_1}{p_0} = \frac{p_1}{p_{atm}} = \frac{\rho g \Delta h_1}{\rho g \Delta h_{atm}} = \frac{(760-11.8)\,\text{mm}}{760\,\text{mm}} = 0.9845$$

From App. E.1 (Eq. 11.17a) $M_1 = 0.150 \longleftarrow M_1$

$$p_1 = \rho_{Hg}\, g h_1 = (13.5)1000\frac{\text{kg}}{\text{m}^3} \times 9.81\frac{\text{m}}{\text{sec}^2} \times 0.748\,\text{m} \times \frac{\text{N}\cdot\text{sec}^2}{\text{kg}\cdot\text{m}} = 99.1\,\text{kPa (abs)}$$

From App. E.1 (Eq. 11.17b), $T_1/T_0 = 0.9955$ $\therefore T_1 = 294$ K

From App. E.2, with $M_1 = 0.150$

$$p_1/p^* = 7.287, \quad T/T^* = 1.195, \quad \frac{\bar{f} L_{max}}{D} = 27.93$$

$$p_2 = p^* = 13.60 \text{ and } p_{0_2} = \frac{p_2}{0.5283} = 25.7\,\text{kPa}, \quad T^* = T_2 = 247\text{K} \longleftarrow p_{0_2}, T_2$$

$$f = f(Re), \quad Re = \frac{\rho V D}{\mu}$$

$$\rho_1 = \frac{p_1}{RT_1} = 99.1\times10^3 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\,\text{N}\cdot\text{m}} \times \frac{1}{294\,\text{K}} = 1.17\,\text{kg/m}^3$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.149\left(1.4 \times 287\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 294\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 51.2\,\text{m/s}$$

From Table A.10 @ $T_1 = 21°C$, $\mu = 1.82\times10^{-5}\,\text{N}\cdot\text{sec/m}^2$

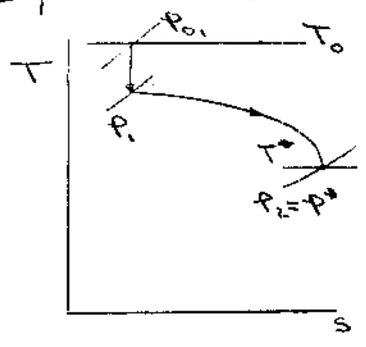
$$Re_1 = \frac{\rho V D}{\mu} = 1.17\frac{\text{kg}}{\text{m}^3} \times 51.2\frac{\text{m}}{\text{s}} \times 7.16\times10^{-3}\,\text{m} \times \frac{1}{1.82\times10^{-5}}\frac{\text{m}^2}{\text{N}\cdot\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$Re_1 = 2.36\times10^4$

From Fig. 8.13, friction factor $f = 0.0245$

$$\therefore L_{12} = 27.9\frac{D}{f} = 27.9 \times 7.16\times10^{-3}\,\text{m} \cdot \frac{1}{0.0245}$$

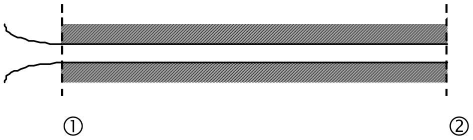
$L_{12} = 8.2\,\text{m} \longleftarrow L_{12}$



Problem 13.63

[2]

13.63 For the conditions of Problem 13.54, find the length, *L*, of commercial steel pipe of 2 in. diameter between sections ① and ②.



Given: Air flow in a converging nozzle and insulated duct

Find: Length of pipe

Solution:

Basic equations: Fanno-line flow equations, and friction factor

Given or available data

$T_0 = (250 + 460)\cdot R$ $\quad p_0 = 145\cdot psi$ $\quad p_1 = 125\cdot psi$ $\quad T_2 = (150 + 460)\cdot R$

$D = 2\cdot in$ $\quad k = 1.4$ $\quad c_p = 0.2399\cdot\frac{Btu}{lbm\cdot R}$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$

From isentropic relations

$$M_1 = \left[\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]\right]^{\frac{1}{2}} \qquad M_1 = 0.465$$

$$\frac{T_0}{T_1} = 1 + \frac{k-1}{2}\cdot M_1^{\,2} \quad \text{so} \quad T_1 = \frac{T_0}{\left(1 + \frac{k-1}{2}\cdot M_1^{\,2}\right)} \qquad T_1 = 681\cdot R \qquad T_1 = 221\cdot °F$$

Then for Fanno-line flow

$$\frac{f_{ave}\cdot L_{max1}}{D_h} = \frac{1 - M_1^{\,2}}{k\cdot M_1^{\,2}} + \frac{k+1}{2\cdot k}\cdot \ln\left[\frac{(k+1)\cdot M_1^{\,2}}{2\cdot\left(1 + \frac{k-1}{2}\cdot M_1^{\,2}\right)}\right] = 1.3923$$

$$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = \frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_1^{\,2}}\right)^{\frac{1}{2}} = 2.3044 \qquad \frac{T_1}{T_{crit}} = \frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_1^{\,2}} = 1.150 \qquad T_{crit} = \frac{T_1}{1.150}$$

$$p_{crit} = \frac{p_1}{2.3044} \qquad p_{crit} = 54.2\cdot psi \qquad T_{crit} = 592\cdot R \qquad T_{crit} = 132\cdot °F$$

Also, for $\frac{T_2}{T_{crit}} = 1.031$

$$\frac{T_2}{T_{crit}} = \frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_2^{\,2}} \quad \text{leads to} \quad M_2 = \sqrt{\frac{2}{k-1}\cdot\left(\frac{k+1}{2}\cdot\frac{T_{crit}}{T_2} - 1\right)} \qquad M_2 = 0.906$$

Then

$$\frac{f_{ave}\cdot L_{max2}}{D_h} = \frac{1 - M_2^{\,2}}{k\cdot M_2^{\,2}} + \frac{k+1}{2\cdot k}\cdot \ln\left[\frac{(k+1)\cdot M_2^{\,2}}{2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^{\,2}\right)}\right] = 0.01271$$

Also

$$\rho_1 = \frac{p_1}{R_{air}\cdot T_1} \qquad \rho_1 = 0.496\frac{lbm}{ft^3} \qquad V_1 = M_1\cdot\sqrt{k\cdot R_{air}\cdot T_1} \qquad V_1 = 595\frac{ft}{s}$$

For air at $T_1 = 221\,°F$, from Table A.9 (approximately) $\mu = 4.48 \times 10^{-7} \cdot \frac{lbf \cdot s}{ft^2}$ so $Re_1 = \frac{\rho_1 \cdot V_1 \cdot D}{\mu}$

For commercial steel pipe (Table 8.1) $e = 0.00015 \cdot ft$ $\frac{e}{D} = 9 \times 10^{-4}$ and $Re_1 = 3.41 \times 10^{6}$

Hence at this Reynolds number and roughness (Eq. 8.37) $f = 0.01924$

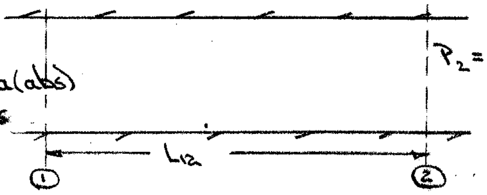
Combining results $L_{12} = \frac{D}{f} \cdot \left(\frac{f_{ave} \cdot L_{max2}}{D_h} - \frac{f_{ave} \cdot L_{max1}}{D_h} \right) = \frac{\frac{2}{12} \cdot ft}{.01924} \cdot (1.3923 - 0.01271)$ $L_{12} = 12.0 \cdot ft$

These calculations are a LOT easier using the *Excel* Add-ins!

Problem 13.64

Given: Adiabatic flow of air in a constant-area duct

$T_1 = 200°C$
$p_1 = 2.0$ MPa (abs)
$V_1 = 140$ m/s



$p_2 = 1.26$ MPa (abs)

$D = 150$ mm
$e/D = 0.0003$

Find: T_2, L_{12}

Solution:

* Compressible flow functions to be used in the solution

Assumptions: (1) steady flow (3) uniform flow at a section
(2) adiabatic flow (4) ideal gas

$M_1 = \frac{V_1}{c_1}$ $\quad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 473 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 436$ m/s

$M_1 = \frac{140}{436} = 0.321$ From Appendix E.2, with $M_1 = 0.321$,

$T_1/T^* = 1.176 \quad p_1/p^* = 3.378 \quad \bar{f}L_{max}/D]_1 = 4.409$

$\therefore T^* = 402 K \quad , \quad p^* = 0.592$ MPa

For $p_2/p^* = \frac{1.26}{0.592} = 2.128$. From App. E.2, $M_2 = 0.502$, $T_2/T^* = 1.142$, $\bar{f}\frac{L_{max}}{D}\Big]_2 = 1.053$

$\therefore T_2 = 1.142 T^* = 1.142 (402 K) = 459 K$ ← T_2

$\bar{f}\frac{L_{12}}{D} = \bar{f}\frac{L_{max}}{D}\Big]_1 - \bar{f}\frac{L_{max}}{D}\Big]_2 = 4.409 - 1.053 = 3.356$

$\rho_1 = \frac{p_1}{RT_1} = 2.0 \times 10^6 \frac{N}{m^2} \times \frac{kg \cdot K}{287 N \cdot m} \times \frac{1}{473 K} = 14.7 \; kg/m^3$

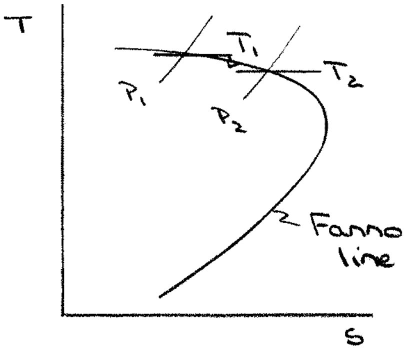
$Re = \frac{\rho_1 V_1 D}{\mu}$. To obtain μ at 200C, use Sutherland correlation (Appendix A)

$\mu = \frac{bT^{1/2}}{1 + S/T} = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times (473K)^{1/2} \times \frac{1}{\left(1 + \frac{110.4}{473}\right)} \times \frac{N \cdot s^2}{kg \cdot m} = 2.57 \times 10^{-5} \; N \cdot s/m^2$

$Re = \frac{\rho_1 V_1 D}{\mu} = 14.7 \frac{kg}{m^3} \times 140 \frac{m}{s} \times 1.5 \times 10^{-1} m \times \frac{m^2}{2.57 \times 10^{-5} N \cdot s} = 1.20 \times 10^7$

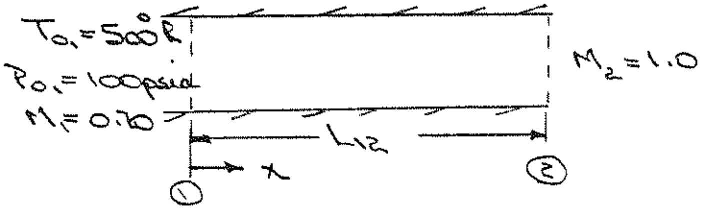
With $e/D = 0.0003$, from Fig. 8.13, friction factor, $f = 0.0147$

$\bar{f}\frac{L_{12}}{D} = 3.356 \quad . \quad L_{12} = 3.356 \frac{D}{\bar{f}} = \frac{3.356}{0.0147} \times 0.15 m = 34.2 m$ ← L_{12}



Problem 13.65

Given: Adiabatic flow of air in a constant area duct with friction



circular duct:
$A = 1.0\ ft^2$
commercial steel

Find: L_{12}

Plot: $p(x)$, $M(x)$

Solution:

Compressible flow functions to be used in the solution

Assumptions: (1) steady flow (3) uniform flow at a section
(2) adiabatic flow, $T_0 = $ const (4) ideal gas

For $M_1 = 0.70$, from Appendix E.1, $T_0/T_1 = 1.098$ (11.17b) $\therefore T_1 = 455°R$
$p_0/p_1 = 1.387$ (11.17a) $\therefore p_1 = 72.1$ psia

For $M_1 = 0.70$, from Appendix E.2, $p_1/p^* = 1.493$ (12.18d) $\therefore p^* = 48.3$ psia
$\bar{f}L_{max}/D_h = 0.2081$ (12.17)

For $M_2 = 1.0$, $\bar{f}L_{max}/D_h = 0$ $\therefore \bar{f}L_{12}/D_h = 0.2081$

$\bar{f} = \bar{f}(Re, e/D)$

$A = \frac{\pi D^2}{4}$ $\therefore D = \left(\frac{4A}{\pi}\right)^{1/2} = \left[\frac{4}{\pi} \times 1.0\ ft^2\right]^{1/2} = 1.13\ ft$

For commercial steel from Table 8.1, $e = 0.00015\ ft$ $\therefore \frac{e}{D} = 0.00013$

$\rho_1 = \frac{p_1}{RT_1} = 72.1 \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{455°R} \times 144 \frac{in^2}{ft^2} = 0.428\ lbm/ft^3$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.7\left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 455°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 732\ ft/s$

$T_1 = 455°R = -5°F = -20°C = 253\ K$.

From the Sutherland correlation, $\mu = \frac{bT^{1/2}}{1 + S/T}$ (A.1)

$\mu = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times \frac{(253\ K)^{1/2}}{1 + 110/253} = 1.62 \times 10^{-5} \frac{kg}{m \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} = 1.62 \times 10^{-5} \frac{N \cdot s}{m^2}$

$\mu = 1.62 \times 10^{-5} \frac{N \cdot s}{m^2} \times 0.0209 \frac{lbf \cdot s/ft^2}{N \cdot s/m^2} = 3.39 \times 10^{-7}\ lbf \cdot s/ft^2$

$Re = \frac{\rho_1 V_1 D}{\mu} = 0.428 \frac{lbm}{ft^3} \times 732 \frac{ft}{s} \times 1.13\ ft \times \frac{ft^2}{3.39 \times 10^{-7}\ lbf \cdot s} \times \frac{slug}{32.2\ lbm} \times \frac{lbf \cdot s^2}{ft \cdot slug}$

$Re = 3.24 \times 10^7$.

From Fig 8.13, friction factor $f = 0.0125$

$\bar{f}\frac{L_{12}}{D} = 0.2081$ $\therefore L_{12} = \frac{0.2081\, D}{f} = \frac{0.2081}{0.0125} \times 1.13\ ft = 18.8\ ft$ $\xleftarrow{\quad L_{12} \quad}$

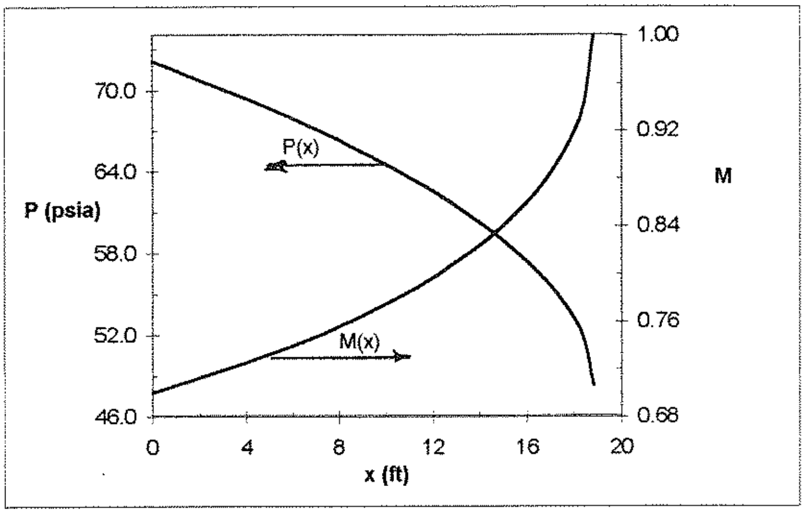
Problem 13.65

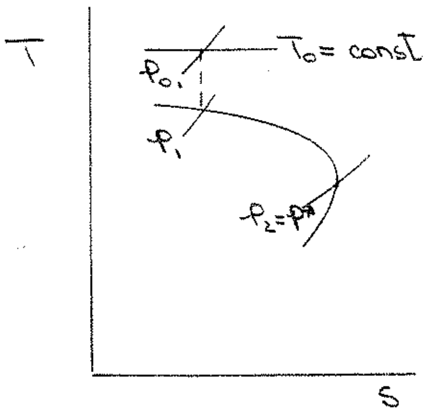
[3] Part 2/2

To plot $p(x)$, $M(x)$

- assume values of M, $0.70 \le M \le 1.00$
  - calculate corresponding fL/D from Eq. 12.17
    - solve for corresponding $\bar{f}\,\Delta L/D$ where $\Delta L = x$, assuming constant $\bar{f}$
  - calculate corresponding p/p^* from Eq. 12.18d

M	$(fL/D)_M$	$\Delta(fL/D)$	x(ft)	P/P*	P(psia)
0.70	0.2081	0	0	1.4935	72.1
0.74	0.1411	0.0670	6.1	1.4054	67.9
0.78	0.0917	0.1165	10.5	1.3261	64.0
0.82	0.0559	0.1522	13.8	1.2542	60.6
0.86	0.0310	0.1772	16.0	1.1889	57.4
0.90	0.0145	0.1936	17.5	1.1291	54.5
0.94	0.0048	0.2033	18.4	1.0743	51.9
1.0	0.0000	0.2081	18.8	1	48.3





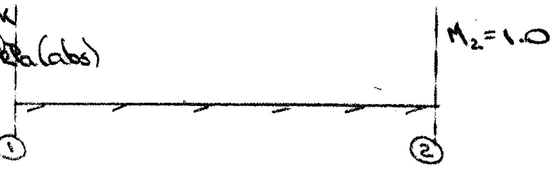
Problem 13.66

Given: Air flow in a smooth, insulated, constant-area tube. Inlet conditions as given in Example Problem 13.8

$T_{0_1} = 296\,K$ $T_1 = 294\,K$

$p_{0_1} = 101\,kPa\,(abs)$ $p_1 = 98.8\,kPa\,(abs)$

$M_1 = 0.19$



s^* is the entropy at the condition where $M = 1.0$

Plot: the Fanno line $[T/T_0 \text{ vs } (s-s^*)/c_p]$ for $0.19 \leq M \leq 1.0$

Solution:

- Compressible flow functions to be used in the solution

Basic equations: $Tds = dh - v\,dP$, $P = \rho RT$

$$Tds = dh - v\,dP = c_p\,dT - \frac{1}{\rho}dP \qquad ds = c_p\frac{dT}{T} - \frac{dP}{P}$$

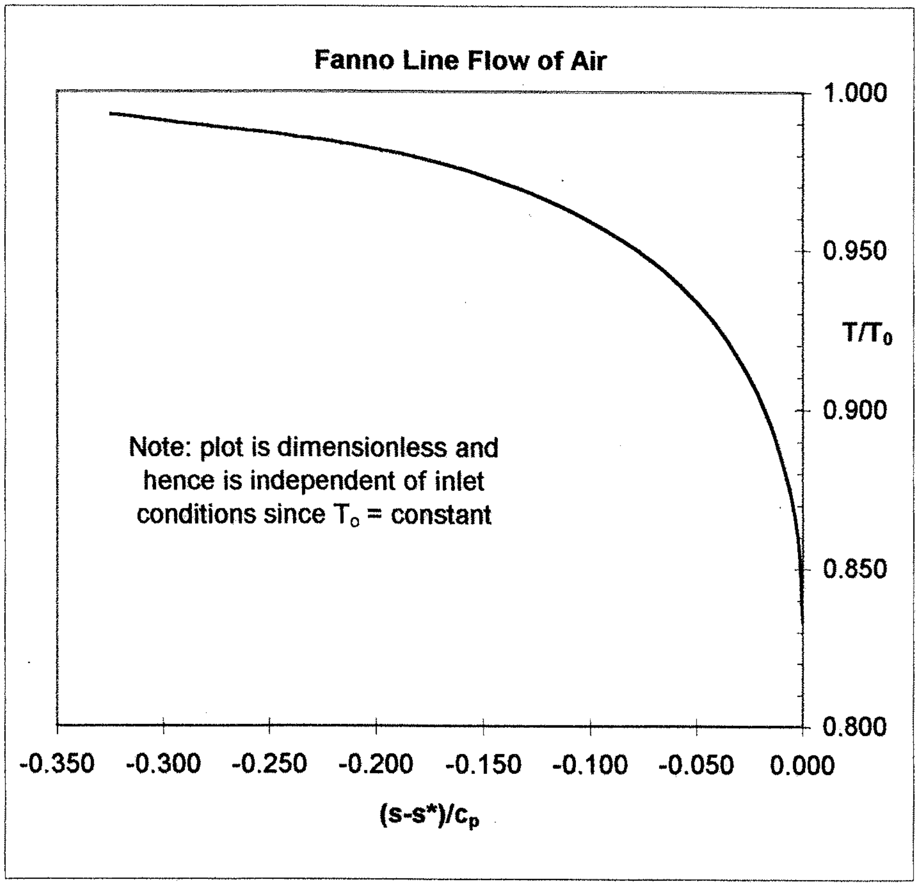
$$s - s^* = s_0 - s_0^* = c_p\int_{T_0^*}^{T_0}\frac{dT}{T} - R\int_{P_0^*}^{P_0}\frac{dP}{P} = \cancel{c_p \ln\frac{T_0}{T_0^*}}^{0} - R\ln\frac{P_0}{P_0^*} = -(c_p - c_v)\ln\frac{P_0}{P_0^*}$$

$$\frac{s-s^*}{c_p} = -\frac{(c_p - c_v)}{c_p}\ln\frac{P_0}{P_0^*} = -\frac{(k-1)}{k}\ln\frac{P_0}{P_0^*} = -\frac{0.4}{1.4}\ln\frac{P_0}{P_0^*} = -0.286\ln\frac{P_0}{P_0^*}$$

For a given value of M, T/T_0 is found from Appendix E.1 (11.17b)

P_0/P_0^* " " " " E.2 (12.18e)

M	T/T_0	P_0/P_0^*	$(s-s^*)/c_p$
0.19	0.993	3.112	-0.325
0.30	0.982	2.035	-0.203
0.40	0.969	1.590	-0.133
0.50	0.952	1.340	-0.084
0.60	0.933	1.188	-0.049
0.70	0.911	1.094	-0.026
0.80	0.887	1.038	-0.011
0.90	0.861	1.009	-0.003
1.00	0.833	1.000	0.000

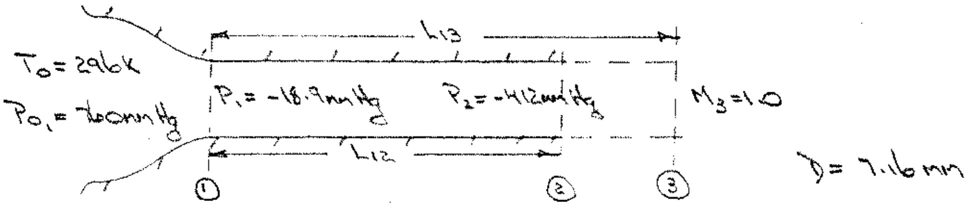


42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem 13.67

[3]

Given: Air flow in a smooth, insulated, constant-area tube, as given in Example 13.7



Plot: P(x), T(x), M(x) vs L(x/D) from pipe inlet to choked condition

Solution:

* Compressible flow functions to be used in the solution

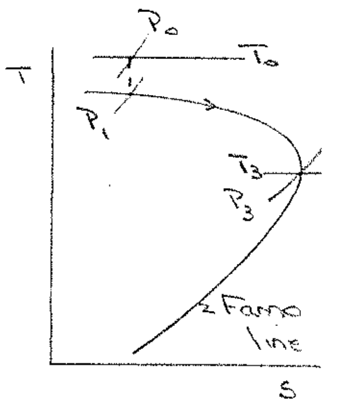
Assumptions: (1) steady flow (4) uniform flow at a section
(2) isentropic flow in nozzle (5) ideal gas
(3) $\dot{Q} = 0$ in duct

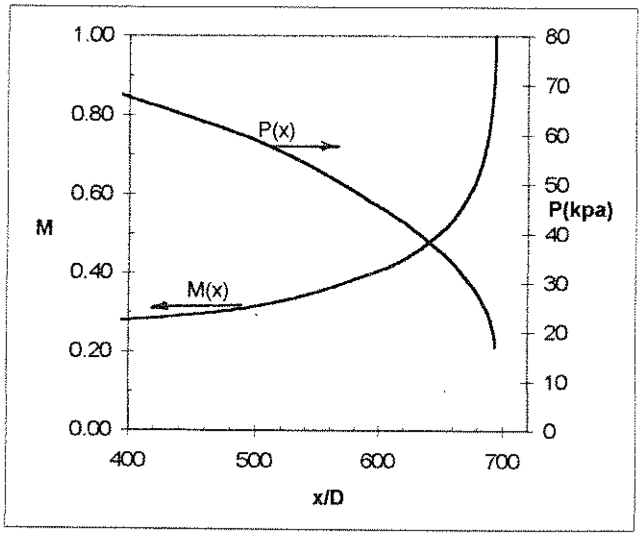
From the solution of Example 13.7 we know that

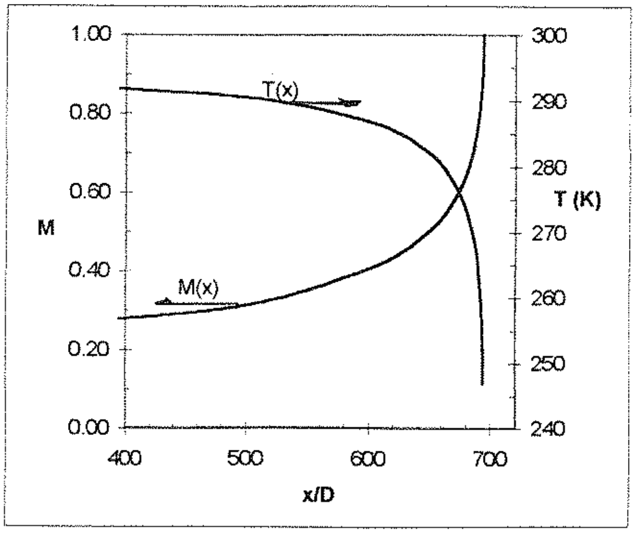
$M_1 = 0.190$, $T_1 = 294 K$, $P_1 = 98.8 kPa$, $P_{01} = 101 kPa$, $\bar{f} = 0.0235$

To determine P(x), T(x), M(x), use Appendix E.2

M	$(fL/D)_M$	$\Delta(fL/D)$	x/D	T/T*	T(K)	P/P*	P(kPa)
0.19	16.38	0	0	1.191	294	5.745	98
0.30	5.30	11.08	469	1.179	291	3.619	62
0.40	2.31	14.07	596	1.163	287	2.696	46
0.50	1.07	15.31	649	1.143	282	2.138	37
0.60	0.49	15.88	673	1.119	276	1.763	30
0.70	0.21	16.17	685	1.093	270	1.493	26
0.80	0.07	16.30	691	1.064	263	1.289	22
0.90	0.01	16.36	693	1.033	255	1.129	19
1.00	0.00	16.38	694	1.000	247	1.000	17







Problem 13.68

[3]

Given: Fanno line flow for air ; s^* is the entropy at the condition where $M = 1.0$.

Plot: Fanno line for Mach numbers in the range $0.1 < M < 3.0$, $\frac{T}{T^*}$ vs $\frac{s-s^*}{c_p}$

Solution:

* Compressible flow functions to be used in the solution.

Basic equation: $T\,ds = dh - v\,dp$, $P = \rho R T$

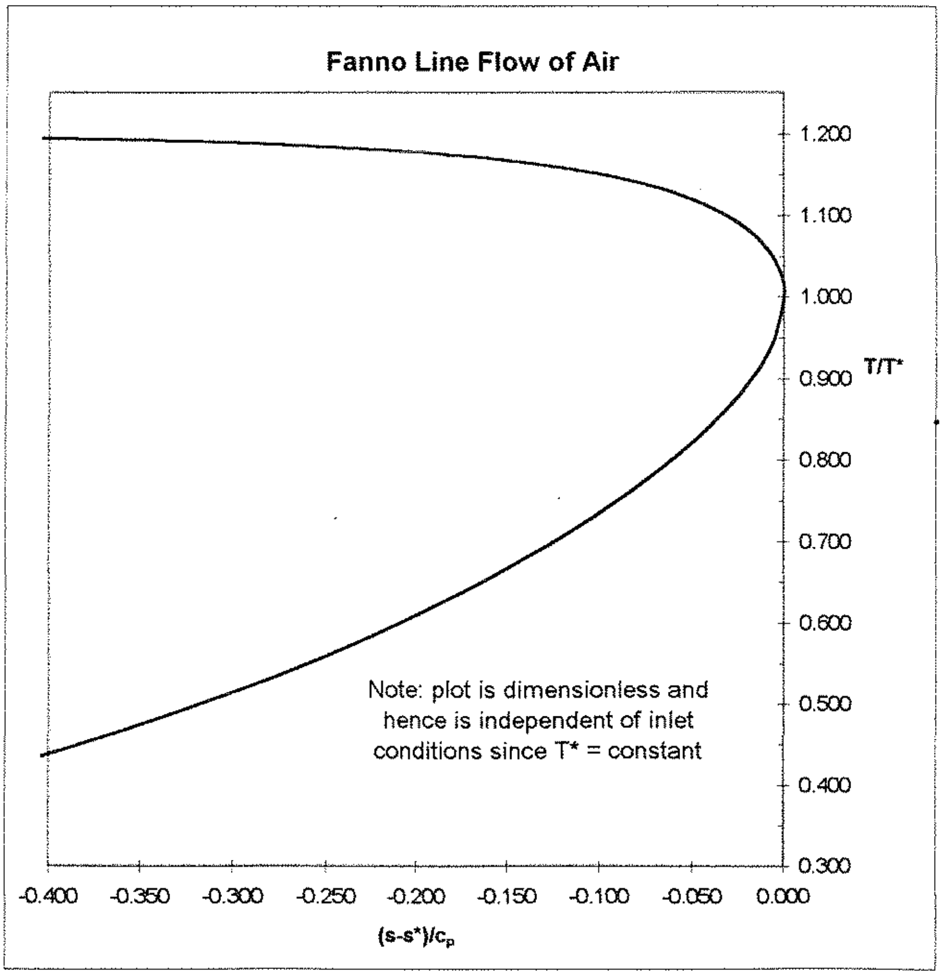
$$T\,ds = dh - v\,dP = c_p\,dT - \frac{1}{\rho}\,dP \quad ; \quad ds = c_p \frac{dT}{T} - \frac{dP}{\rho}$$

$$s - s^* = s_0 - s_0^* = c_p \int_{T_0^*}^{T_0} \frac{dT}{T} - R \int_{P_0^*}^{P_0} \frac{dP}{P} = \cancelto{0}{c_p \ln \frac{T_0}{T_0^*}} - R \ln \frac{P_0}{P_0^*} = -(c_p - c_v) \ln \frac{P_0}{P_0^*}$$

$$\frac{s-s^*}{c_p} = -\frac{(c_p - c_v)}{c_p} \ln \frac{P_0}{P_0^*} = -\frac{(k-1)}{k} \ln \frac{P_0}{P_0^*} = -\frac{(0.4)}{1.4} \ln \frac{P_0}{P_0^*} = -0.286 \ln \frac{P_0}{P_0^*}$$

For a given value of M, use Appendix E.2 to determine T/T^* (12.18a) & P_0/P_0^* (12.18e).

M	T/T^*	P_0/P_0^*	$(s-s^*)/c_p$
0.10	1.198	5.822	-0.504
0.14	1.195	4.182	-0.409
0.20	1.190	2.964	-0.311
0.30	1.179	2.035	-0.203
0.40	1.163	1.590	-0.133
0.50	1.143	1.340	-0.084
0.60	1.119	1.188	-0.049
0.70	1.093	1.094	-0.026
0.80	1.064	1.038	-0.011
0.90	1.033	1.009	-0.003
1.00	1.000	1.000	0.000
1.20	0.932	1.030	-0.009
1.40	0.862	1.115	-0.031
1.60	0.794	1.250	-0.064
1.80	0.728	1.439	-0.104
2.00	0.667	1.688	-0.150
2.20	0.610	2.005	-0.199
2.40	0.558	2.403	-0.251
2.60	0.510	2.896	-0.304
2.80	0.467	3.500	-0.358
3.00	0.429	4.235	-0.413

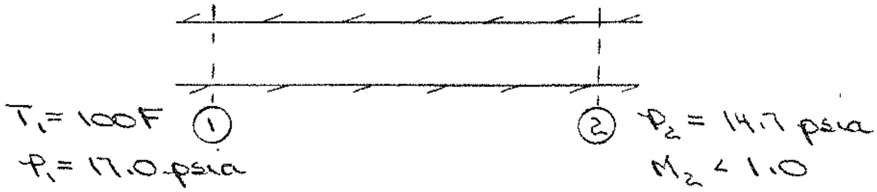


Note: plot is dimensionless and hence is independent of inlet conditions since T* = constant

Problem 13.69

[4] Part 1/2

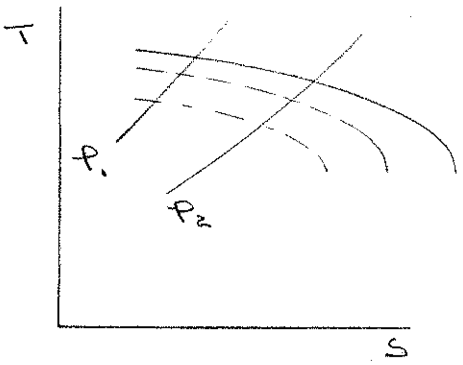
Given: Air flows through an insulated constant-area duct ($D = 2.12$ ft, $e/D = 0.002$, $L_{12} = 40$ ft) with conditions at sections ① • ② as shown.



Find: (a) Is it possible to solve for M_1 and M_2? Prove answer graphically

(b) Find $\dot{m}$ and T_2

Solution:



(a) It is possible to solve for M_1 and M_2. There is a different fanno line for each different flow rate (or M_1)

- need to find the value of M_1 that gives the pressure drop $p_1 - p_2$ over the length L_{12}

(b) Procedure for trial and error solution is to assume M_1 and calculate p_2

Use Fanno-line flow functions of Appendix E-2

Assume M_1

- determine p^*, Appendix E (12.18d)
  $\bar{f}L/D$, " " (12.17)
  $\bar{f} = \bar{f}(Re)$
- calculate $\bar{f}L_{12}/D$ and $\bar{f}L/D)_2 = \bar{f}L/D)_1 - \bar{f}L_{12}/D$
- knowing $\bar{f}L/D)_2$, iterate (12.17) to determine M_2
- determine p_2 from M_2 (12.18d) and check vs known p_2

Repeat with another assumed value of M_1

Additional computing equations:

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = M_1 \left(1.4 \times 53.3 \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot{}^\circ\text{R}} \times 560^\circ\text{R} \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 1160 M_1$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{17\,\text{lbf}}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times \frac{\text{lbm}\cdot{}^\circ\text{R}}{53.3\,\text{ft}\cdot\text{lbf}} \times \frac{1}{560^\circ\text{R}} \times \frac{\text{slug}}{32.2\,\text{lbm}} = 2.55 \times 10^{-3} \frac{\text{slug}}{\text{ft}^3}$$

From Table A.9, $\mu = 3.97 \times 10^{-7}$ lbf·s/ft²

$$Re = \frac{\rho_1 V_1 D}{\mu} = 2.55 \times 10^{-3} \frac{\text{slug}}{\text{ft}^3} \times 1160 M_1 \frac{\text{ft}}{\text{s}} \times 2.12\,\text{ft} \times \frac{\text{ft}^2}{3.97 \times 10^{-7}\,\text{lbf}\cdot\text{s}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}} = 7.45 \times 10^6 M_1$$

For $M_1 = 0.4$, $Re = 3.0 \times 10^6$. With $e/D = 0.002$, Fig. 8.13 gives $f = 0.023$

Assume f is constant

Problem 13.69

[4] Part 2/2

M_1	T_1/T^*	P_1/P^*	P^*(psia)		$(fL_m/D)_1$	V_1 (ft/s)	Re(10^6)	f	fL_{12}/D	$(fL_m/D)_2$	M_2	T_2/T^*	P_2/P^*	P_2(psia)
0.400	1.163	2.6958	6.30606	0.186	2.31	464	2.98	0.0230	0.434	1.875	0.427	1.158	2.520	15.9
0.450	1.153	2.3865	7.12347	0.234	1.57	522	3.35	0.0230	0.434	1.132	0.493	1.144	2.170	15.5
0.500	1.143	2.1381	7.95102	0.286	1.07	580	3.73	0.0230	0.434	0.635	0.568	1.127	1.869	14.9
0.510	1.141	2.0942	8.118	0.297	0.99	592	3.80	0.023	0.434	0.556	0.585	1.123	1.812	14.7

Iterate to determine M_2 for known M_1

M_1	$(fL_m/D)_2$	$(M_2)_{guess}$		$(fL_m/D)_2$
0.400	1.875	0.427	0.211	1.870
0.450	1.132	0.493	0.278	1.128
0.500	0.635	0.568	0.364	0.633
0.510	0.556	0.585	0.384	0.553

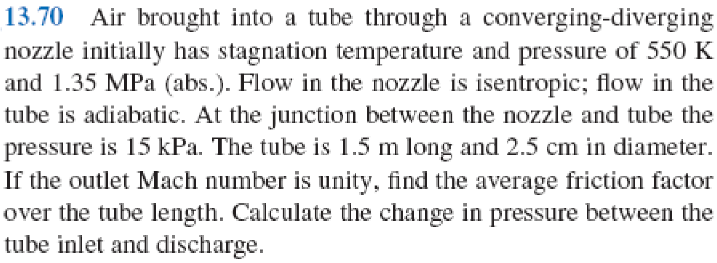
$M_1 = 0.510 \quad \frac{T_1}{T^*} = 1.141 \;, \quad \frac{T_2}{T^*} = 1.123 \;, \quad \therefore \frac{T_2}{T_1} = 0.984$

$T_2 = 551°R \longleftarrow T_2$

$\dot{m} = \rho_1 V_1 A = 2.55\times10^{-3}\,\frac{slug}{ft^3} \times 592\,\frac{ft}{s} \times \frac{\pi}{4}(2/12)^2\,ft^2 = 5.33\ slug/s \longleftarrow \dot{m}$

Problem 13.70

[2]



13.70 Air brought into a tube through a converging-diverging nozzle initially has stagnation temperature and pressure of 550 K and 1.35 MPa (abs.). Flow in the nozzle is isentropic; flow in the tube is adiabatic. At the junction between the nozzle and tube the pressure is 15 kPa. The tube is 1.5 m long and 2.5 cm in diameter. If the outlet Mach number is unity, find the average friction factor over the tube length. Calculate the change in pressure between the tube inlet and discharge.

Given: Air flow through a CD nozzle and tube.

Find: Average friction factor; Pressure drop in tube

Solution:

Assumptions: 1) Isentropic flow in nozzle 2) Adiabatic flow in tube 3) Ideal gas 4) Uniform flow

Given or available data: $k = 1.40$ $R = 286.9\cdot\frac{J}{kg\cdot K}$ $p_1 = 15\cdot kPa$ where State 1 is the nozzle exit

$p_0 = 1.35\cdot MPa$ $T_0 = 550\cdot K$ $D = 2.5\cdot cm$ $L = 1.5\cdot m$

From isentropic relations $M_1 = \left[\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]\right]^{\frac{1}{2}}$ $M_1 = 3.617$

Then for Fanno-line flow (for choking at the exit)

$$\frac{f_{ave}\cdot L_{max}}{D_h} = \frac{1 - M_1^2}{k\cdot M_1^2} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^2}{2\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)}\right] = 0.599$$

Hence $f_{ave} = \frac{D}{L}\cdot\left[\frac{1 - M_1^2}{k\cdot M_1^2} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^2}{2\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)}\right]\right]$ $f_{ave} = 0.0100$

$$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = \frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_1^2}\right)^{\frac{1}{2}} = 0.159$$

$p_2 = \frac{p_1}{\left[\frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_1^2}\right)^{\frac{1}{2}}\right]}$ $p_2 = 94.2\,kPa$

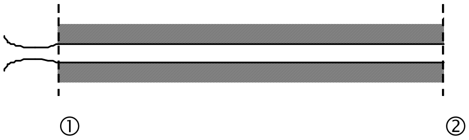
$\Delta p = p_1 - p_2$ $\Delta p = -79.2\,kPa$

These calculations are a LOT easier using the Excel Add-ins!

Problem 13.71

[3]

13.71 For the conditions of Problem 13.57, determine the duct length. Assume the duct is circular and made from commercial steel. Plot the variations of pressure and Mach number versus distance along the duct.



Given: Air flow in a CD nozzle and insulated duct

Find: Duct length; Plot of M and p

Solution:

Basic equations: Fanno-line flow equations, and friction factor

Given or available data $T_1 = (100 + 460)\cdot R$ $\quad p_1 = 18.5\cdot psi$ $\quad M_1 = 2$ $\quad M_2 = 1$ $\quad A = 1\cdot in^2$

$k = 1.4$ $\quad c_p = 0.2399\cdot\frac{Btu}{lbm\cdot R}$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$

Then for Fanno-line flow at $M_1 = 2$

$$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = \frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}\cdot M_1^{\,2}}\right)^{\frac{1}{2}} = 0.4082 \qquad \frac{f_{ave}\cdot L_{max1}}{D_h} = \frac{1-M_1^{\,2}}{k\cdot M_1^{\,2}} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^{\,2}}{2\cdot\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right)}\right] = 0.3050$$

so $p_{crit} = \frac{p_1}{0.4082}$ $\quad p_{crit} = 45.3\cdot psi$

and at $M_2 = 1$

$$\frac{f_{ave}\cdot L_{max2}}{D_h} = \frac{1-M_2^{\,2}}{k\cdot M_2^{\,2}} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_2^{\,2}}{2\cdot\left(1+\frac{k-1}{2}\cdot M_2^{\,2}\right)}\right] = 0$$

Also $\rho_1 = \frac{p_1}{R_{air}\cdot T}$ $\quad \rho_1 = 0.089\cdot\frac{lbm}{ft^3}$ $\quad V_1 = M_1\cdot\sqrt{k\cdot R_{air}\cdot T_1}$ $\quad V_1 = 2320\cdot\frac{ft}{s}$ $\quad D = \sqrt{\frac{4\cdot A}{\pi}}$ $\quad D = 1.13\cdot in$

For air at $T_1 = 100\cdot°F$, from Table A.9 $\quad \mu = 3.96\times 10^{-7}\cdot\frac{lbf\cdot s}{ft^2}$ $\quad$ so $\quad Re_1 = \frac{\rho_1\cdot V_1\cdot D}{\mu}$

For commercial steel pipe (Table 8.1) $\quad e = 0.00015\cdot ft$ $\quad \frac{e}{D} = 1.595\times 10^{-3}$ $\quad$ and $\quad Re_1 = 1.53\times 10^{6}$

Hence at this Reynolds number and roughness (Eq. 8.37) $\quad f = .02222$

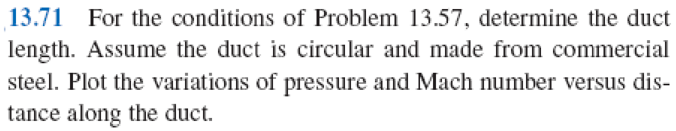
Combining results

$$L_{12} = \frac{D}{f}\cdot\left(\frac{f_{ave}\cdot L_{max2}}{D_h} - \frac{f_{ave}\cdot L_{max1}}{D_h}\right) = \frac{\frac{1.13}{12}\cdot ft}{.02222}\cdot(0.3050 - 0) \qquad L_{12} = 1.29\cdot ft \qquad L_{12} = 15.5\cdot in$$

These calculations are a LOT easier using the *Excel* Add-ins! The *M* and *p* plots are shown in the associated *Excel* workbook

Problem 13.71 (In Excel)

[3]


13.71 For the conditions of Problem 13.57, determine the duct length. Assume the duct is circular and made from commercial steel. Plot the variations of pressure and Mach number versus distance along the duct.

Given: Air flow in a CD nozzle and insulated duct

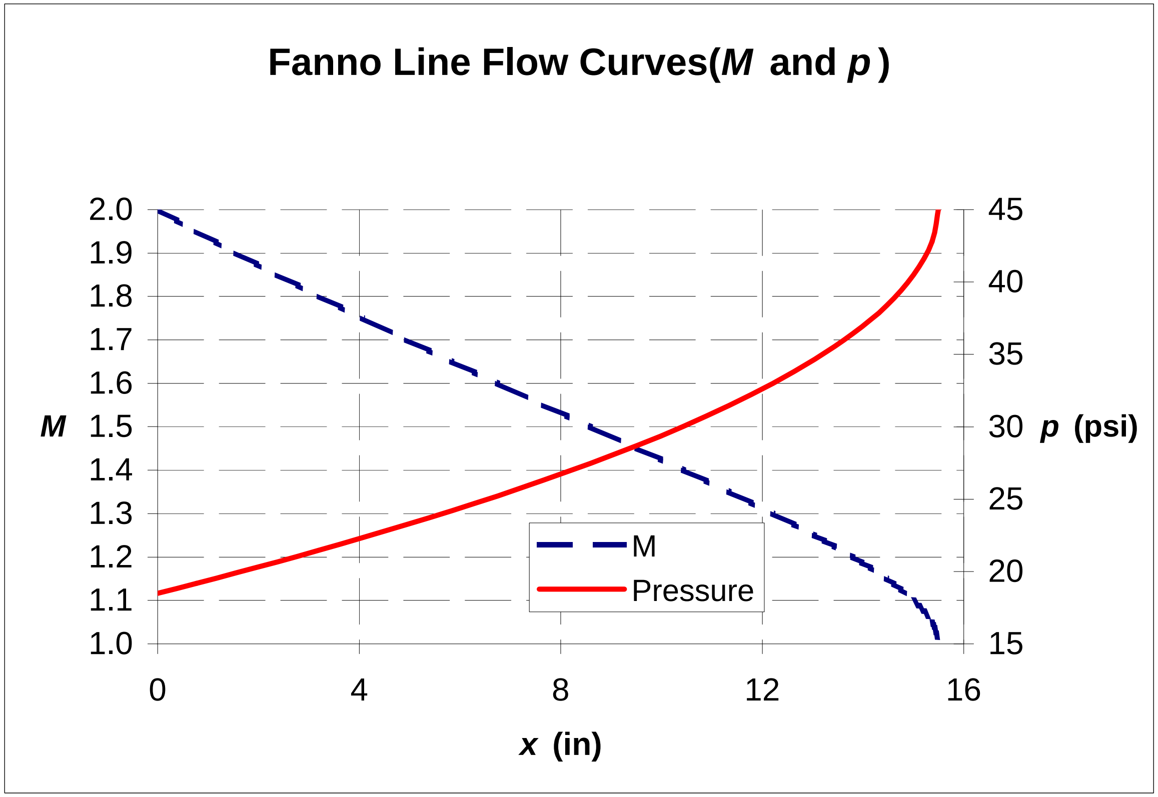
Find: Duct length; Plot of M and p

Solution:

The given or available data is:

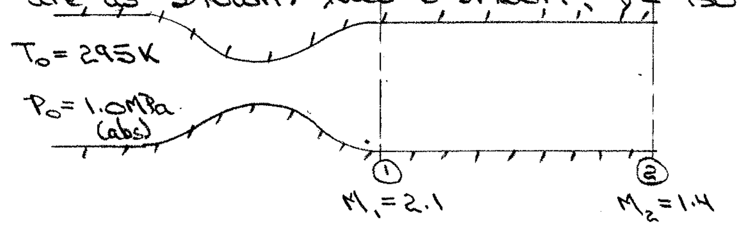
$f = 0.0222$
$p^* = 45.3$ kPa
$D = 1.13$ in

M	fL_{max}/D	$\Delta fL_{max}/D$	x (in)	p/p^*	p (psi)
2.00	0.305	0.000	0	0.408	18.49
1.95	0.290	0.015	0.8	0.423	19.18
1.90	0.274	0.031	1.6	0.439	19.90
1.85	0.258	0.047	2.4	0.456	20.67
1.80	0.242	0.063	3.2	0.474	21.48
1.75	0.225	0.080	4.1	0.493	22.33
1.70	0.208	0.097	4.9	0.513	23.24
1.65	0.190	0.115	5.8	0.534	24.20
1.60	0.172	0.133	6.7	0.557	25.22
1.55	0.154	0.151	7.7	0.581	26.31
1.50	0.136	0.169	8.6	0.606	27.47
1.45	0.118	0.187	9.5	0.634	28.71
1.40	0.100	0.205	10.4	0.663	30.04
1.35	0.082	0.223	11.3	0.695	31.47
1.30	0.065	0.240	12.2	0.728	33.00
1.25	0.049	0.256	13.0	0.765	34.65
1.20	0.034	0.271	13.8	0.804	36.44
1.15	0.021	0.284	14.5	0.847	38.37
1.10	0.010	0.295	15.0	0.894	40.48
1.05	0.003	0.302	15.4	0.944	42.78
1.00	0.000	0.305	15.5	1.000	45.30



Problem 13.72

Given: Flow of air from a large tank ($p_0 = 1.0$ MPa (abs), $T_0 = 295$ K) through a C-D nozzle to a constant area duct. Properties are as shown. Duct is smooth, $D = 150$ mm.



Find: (a) p_2 (b) L_{1-2} (c) $s_2 - s_1$

Solution: * Compressible flow functions to be used in solution.

Assumptions: (1) steady flow (2) uniform flow at a section
(3) isentropic flow in nozzle, adiabatic flow in duct
(4) ideal gas

(a)

From App. E.1 for $M_1 = 2.1$ $T/T_0 = 0.5314$ $\therefore T_1 = 156.7$ K

$p/p_0 = 0.1094$ $\therefore p_1 = 109.4$ kPa.

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.1\left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 156.7 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 527 \, m/s$$

For App. E.2 for $M = 2.1$ $p_{0_1}/p_0^* = 1.837$ $\therefore p_0^* = 544.4$ kPa

$p_1/p^* = 0.3802$ $\therefore p^* = 287.7$ kPa

for $M_2 = 1.4$ $p_2/p^* = 0.6632$ $\therefore p_2 = 190.8$ kPa ← p_2

$p_{0_2}/p_0^* = 1.115$ $\therefore p_{0_2} = 320.8$ kPa

(b) From App. E.2 for $M_1 = 2.1$, $\bar{f} L_{max}/D_h = 0.3339$

$M_2 = 1.4$, $\bar{f} L_{max}/D_h = 0.09974$

$$\therefore \bar{f} L_{12}/D_h = 0.3339 - 0.09974 = 0.2342$$

To obtain L_{12}, we need f; $f = f(Re)$; $Re = \rho \frac{VD}{\mu}$ {use conditions at ①}

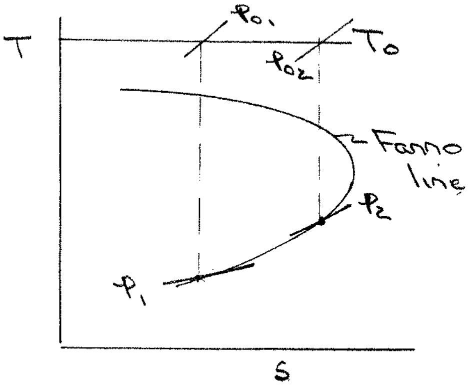
To obtain μ at 156.7K, use Sutherland correlation (Appendix A)

$$\mu = \frac{bT^{1/2}}{1 + S/T} = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times (156.7K)^{1/2} \times \frac{1}{1 + \frac{110.4}{156.7}} = 1.071 \times 10^{-5} \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho_1 V D}{\mu} = \frac{p_1 V D}{R T_1 \mu} = 109.4 \times 10^3 \frac{N}{m^2} \times 527 \frac{m}{s} \times (0.15m) \times \frac{kg \cdot K}{287 \, N \cdot m} \times \frac{1}{156.7K} \times \frac{1}{1.071 \times 10^{-5}} \frac{m^2}{N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$Re = 1.80 \times 10^7$. For smooth pipe from Fig 8.13 friction factor, $f \approx 0.007$

$\therefore 0.007 L_{12}/D_h = 0.2342$ $L_{12} = \frac{0.2342}{0.007} D = 5.02m$ ← L_{12}



From the Tds eq.

$$s_2 - s_1 = s_{0_2} - s_{0_1} = \cancel{c_p \ln \frac{T_{0_2}}{T_{0_1}}} - R \ln \frac{p_{0_2}}{p_{0_1}}$$

$$s_2 - s_1 = -287 \frac{N \cdot m}{kg \cdot K} \ln \frac{320.8}{1000} = 0.326 \frac{kJ}{kg \cdot K}$$ ← $s_2 - s_1$

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem *13.73

[2]

***13.73** In long, constant-area pipelines, as used for natural gas, temperature is constant. Assume gas leaves a pumping station at 350 kPa and 20°C at $M = 0.10$. At the section along the pipe where the pressure has dropped to 150 kPa, calculate the Mach number of the flow. Is heat added to or removed from the gas over the length between the pressure taps? Justify your answer: Sketch the process on a Ts diagram. Indicate (qualitatively) T_{0_1}, T_{0_2}, and p_{0_2}.

Given: Isothermal air flow in a duct

Find: Downstream Mach number; Direction of heat transfer; Plot of Ts diagram

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad m_{rate} = \rho\cdot V\cdot A$

Given or available data $\quad T_1 = (20 + 273)\cdot K \quad p_1 = 350\cdot kPa \quad M_1 = 0.1 \quad p_2 = 150\cdot kPa$

From continuity $\quad m_{rate} = \rho_1\cdot V_1\cdot A = \rho_2\cdot V_2\cdot A$ $\quad$ so $\quad \rho_1\cdot V_1 = \rho_2\cdot V_2$

Also $\quad p = \rho\cdot R\cdot T$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad$ or $\quad V = M\cdot c$

Hence continuity becomes $\quad \frac{p_1}{R\cdot T_1}\cdot M_1\cdot c_1 = \frac{p_2}{R\cdot T_2}\cdot M_2\cdot c_2$

Since $\quad T_1 = T_2 \quad c_1 = c_2$ $\quad$ so $\quad p_1\cdot M_1 = p_2\cdot M_2$

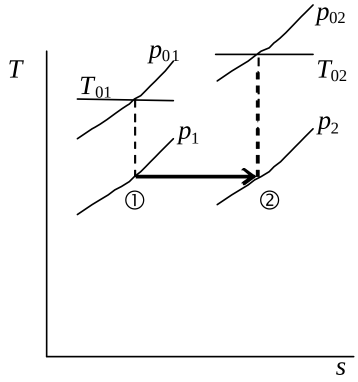
Hence $\quad M_2 = \frac{p_1}{p_2}\cdot M_1 \qquad M_2 = 0.233$

From energy $\quad \frac{\delta Q}{dm} = \left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right) = h_{02} - h_{01} = c_p\cdot\left(T_{02} - T_{01}\right)$

But at each state $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad$ or $\quad T_0 = T\cdot\left(1 + \frac{k-1}{2}\cdot M^2\right)$

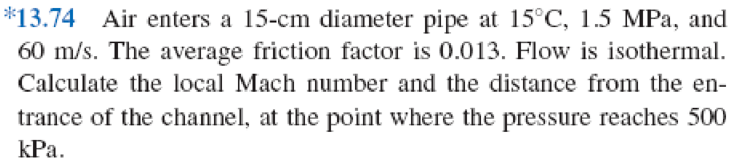
Since T = const, but $M_2 > M_1$, then $T_{02} > T_{01}$, and

$\frac{\delta Q}{dm} > 0$ $\quad$ so energy is ADDED to the system



Problem *13.74

[5]



*13.74 Air enters a 15-cm diameter pipe at 15°C, 1.5 MPa, and 60 m/s. The average friction factor is 0.013. Flow is isothermal. Calculate the local Mach number and the distance from the entrance of the channel, at the point where the pressure reaches 500 kPa.

Given: Isothermal air flow in a pipe

Find: Mach number and location at which pressure is 500 kPa

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad \dfrac{f \cdot L_{max}}{D} = \dfrac{1 - k \cdot M^2}{k \cdot M^2} + \ln\left(k \cdot M^2\right)$

Given or available data $T_1 = (15 + 273) \cdot K$ $\quad p_1 = 1.5 \cdot MPa$ $\quad V_1 = 60 \cdot \dfrac{m}{s}$ $\quad f = 0.013$ $\quad p_2 = 500 \cdot kPa$

$D = 15 \cdot cm$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \dfrac{J}{kg \cdot K}$

From continuity $\rho_1 \cdot V_1 = \rho_2 \cdot V_2$ or $\dfrac{p_1}{T_1} \cdot V_1 = \dfrac{p_2}{T_2} \cdot V_2$

Since $T_1 = T_2$ and $V = M \cdot c = M \cdot \sqrt{k \cdot R \cdot T}$ $\quad M_2 = M_1 \cdot \dfrac{p_1}{p_2}$

$c_1 = \sqrt{k \cdot R \cdot T_1}$ $\quad c_1 = 340 \dfrac{m}{s}$ $\quad M_1 = \dfrac{V_1}{c_1}$ $\quad M_1 = 0.176$

Then $M_2 = M_1 \cdot \dfrac{p_1}{p_2}$ $\quad M_2 = 0.529$

At $M_1 = 0.176$ $\quad \dfrac{f \cdot L_{max1}}{D} = \dfrac{1 - k \cdot M_1^{\ 2}}{k \cdot M_1^{\ 2}} + \ln\left(k \cdot M_1^{\ 2}\right) = 18.819$

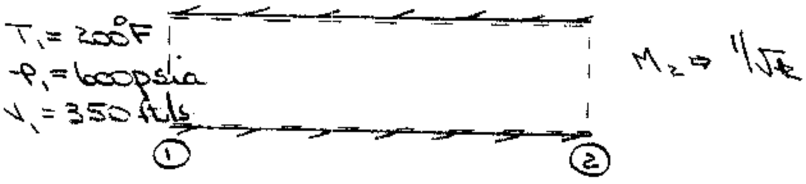
At $M_2 = 0.529$ $\quad \dfrac{f \cdot L_{max2}}{D} = \dfrac{1 - k \cdot M_2^{\ 2}}{k \cdot M_2^{\ 2}} + \ln\left(k \cdot M_2^{\ 2}\right) = 0.614$

Hence $\quad \dfrac{f \cdot L_{12}}{D} = \dfrac{f \cdot L_{max2}}{D} - \dfrac{f \cdot L_{max1}}{D} = 18.819 - 0.614 = 18.2$

$L_{12} = 18.2 \cdot \dfrac{D}{f}$ $\quad L_{12} = 210 m$

Problem *13.75

Given: Air enters a constant area channel at conditions shown and procedes to choking under isothermal flow conditions.



Find: limiting pressure, p_2
Compare with p_2 for frictional adiabatic flow.

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad \dot{m} = \rho V A$

Computing equation: $T_0/T = 1 + \frac{k-1}{2} M^2$

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section (4) $\dot{W}_s = \dot{W}_{shear} = 0$

$$c_1 = (kRT_1)^{1/2} = \left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 660°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1260 \, ft/s$$

$$M_1 = \frac{V_1}{c_1} = \frac{350}{1260} = 0.278$$

$$V_2 = M_2 c_2 = M_2 c_1 = \frac{1}{\sqrt{1.4}} \times 1260 \, ft/s = 1060 \, ft/s$$

$$\rho_1 V_1 = \rho_2 V_2 \quad \text{or} \quad \frac{p_1}{RT_1} V_1 = \frac{p_2}{RT_2} V_2 \quad \text{......(1)}$$

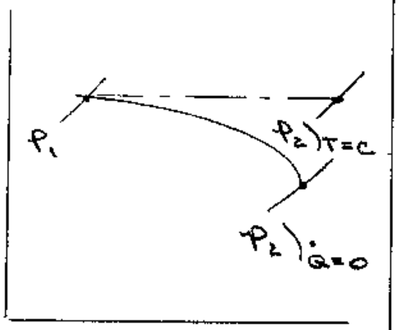
Since $T_1 = T_2$, $p_2 = p_1 \frac{V_1}{V_2} = 600 \, psia \times \frac{350}{1060} = 198 \, psia \leftarrow p_2)_{T=c}$

For adiabatic flow, T_0 = constant and $M_2 = 1.0$

$$T_{0_2} = T_{0_1} = T_1\left(1 + \frac{k-1}{2} M_1^2\right) = T_2\left(1 + \frac{k-1}{2} M_2^2\right)$$

$$\frac{T_2}{T_1} = \frac{1 + 0.2(0.278)^2}{1 + 0.2} = 0.846$$

$$T_2 = 558°R$$



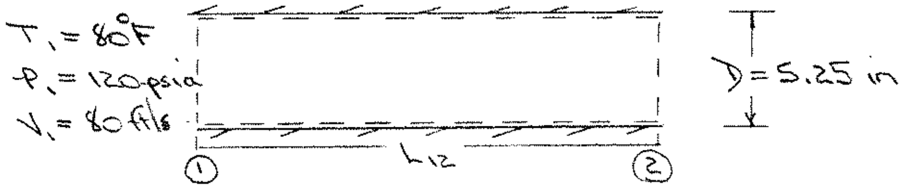
$$V_2 = c_2 = (kRT_2)^{1/2} = (1.4 \times 53.3 \times 558 \times 32.2)^{1/2} = 1160 \, ft/s$$

From continuity (Eq. 1)

$$p_2 = p_1 \frac{V_1}{V_2} \frac{T_2}{T_1} = 600 \, psia \times \frac{350}{1160} \times 0.846$$

$p_2 = 153 \, psia \leftarrow p_2)_{\dot{Q}=0}$

Given: Air enters a clean steel pipe of length $L = 950$ ft and diameter $D = 5.25$ in. at the conditions shown.



Find: pressure drop, $p_1 - p_2$, assuming (a) incompressible (b) isothermal, and (c) adiabatic flow

Solution:

Basic equations: $\dot{m} = \rho V A \qquad p = \rho R T$

Computing equations: $p_1 - p_2 = \rho f \frac{L}{D} \frac{\bar{V}^2}{2}$ (ρ = constant)

$$f \frac{L_{max}}{D} = \frac{1 - kM^2}{kM^2} + \ln kM^2 \quad (T = \text{constant})$$

$$\frac{\bar{f} L_{max}}{D} = \frac{1 - M^2}{kM^2} + \frac{k+1}{2k} \ln \left[\frac{(k+1)M^2}{2\left(1 + \frac{k-1}{2}M^2\right)} \right] \quad (\dot{Q} = 0)$$

$$\rho_1 = \frac{p_1}{RT_1} = 120 \frac{\text{lbf}}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times \frac{\text{lbm} \cdot °\text{R}}{53.3 \, \text{ft} \cdot \text{lbf}} \times \frac{1}{540°\text{R}} = 0.600 \, \text{lbm}/\text{ft}^3$$

From Table A.9 {at $T = 80°F$} $\mu = 3.86 \times 10^{-7}$ lbf·s/ft²

$$Re_1 = \frac{\rho D V}{\mu} = 0.600 \frac{\text{lbm}}{\text{ft}^3} \times \frac{5.25}{12} \text{ft} \times 80 \frac{\text{ft}}{\text{s}} \times \frac{1}{3.86 \times 10^{-7}} \frac{\text{ft}^2}{\text{lbf} \cdot \text{s}} \times \frac{\text{slug}}{32.2 \, \text{lbm}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$Re_1 = 1.69 \;10^6$

For commercial steel (Table 8.1), $e = 0.00015$ ft. $\therefore e/D = 0.00034$

From Fig. 8.13, $f = 0.0155$

(a) For incompressible flow

$$p_1 - p_2 = \rho f \frac{L}{D} \frac{\bar{V}^2}{2} = 0.60 \frac{\text{lbm}}{\text{ft}^3} \times 0.0155 \times \frac{950 \times 12}{5.25} \times \frac{(80)^2}{2} \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{slug}}{32.2 \, \text{lbm}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \frac{\text{ft}^2}{144 \, \text{in}^2}$$

$p_1 - p_2 = 13.9$ psia $\longleftarrow$ $(p_1 - p_2)_{\rho = c}$

(b) For isothermal flow

$$c_1 = (kRT_1)^{1/2} = \left(1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot °\text{R}} \times 540°\text{R} \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 1140 \, \text{ft/s}$$

$$M_1 = \frac{V_1}{c_1} = \frac{80}{1140} = 0.0702$$

At state ① $\left(f \frac{L_{max}}{D}\right)_1 = \frac{1 - kM_1^2}{kM_1^2} + \ln kM_1^2$

$$= \frac{1 - 1.4(0.0702)^2}{1.4(0.0702)^2} + \ln\left[1.4(0.0702)^2\right]$$

$$\left(f \frac{L_{max}}{D}\right)_1 = 139$$

$$f\frac{L_{12}}{D} = 0.0155 \times \frac{950\times 12}{5.25} = 33.6$$

$$\therefore\ f\frac{L_{max}}{D}\Big)_2 = 139 - 33.6 = 105 = \frac{1-kM_2^2}{kM_2^2} + \ln kM_2^2$$

Trial and error solution for M_2

M_2	$f\frac{L_{max}}{D}\Big)_2$
0.10	66.2
0.08	106
0.081	103
0.0805	105

$V_2 = M_2c_2 = M_2c_1 = 0.0805\times 1140$ ft/s $= 91.8$ ft/s

$\rho_1V_1 = \rho_2V_2$ or $\frac{p_1}{T_1}V_1 = \frac{p_2}{T_2}V_2$ ......(1)

Since $T_2 = T_1$, $p_2 = p_1\frac{V_1}{V_2} = 120$ psia $\times \frac{80}{91.8} = 105$ psia

$p_1 - p_2 = 15.0$ psia ______ $(p_2-p_1)_{T=c}$

(c) For adiabatic flow $M_1 = 0.0702$

$$f\frac{L_{max}}{D}\Big)_1 = \frac{1-(0.0702)^2}{1.4(0.0702)^2} + \frac{1.4+1}{2(1.4)}\ln\left[\frac{(2.4)(0.0702)^2}{2(1+0.2(0.0702)^2)}\right] = 139.8$$

$$\therefore\ f\frac{L_{max}}{D}\Big)_2 = 139.8 - 33.6 = 106.2 = \frac{1-M_2^2}{1.4M_2^2} + \frac{2.4}{2.8}\ln\left[\frac{2.4M_2^2}{2(1+0.2M_2^2)}\right]$$

Trial and error solution for M_2

M_2	$f\frac{L_{max}}{D}\Big)_2$
0.085	94.1
0.080	106.7
0.0802	106.2 ✓

For adiabatic flow, T_0 = constant

$$\therefore\ \frac{T_2}{T_1} = \frac{1+\frac{k-1}{2}M_1^2}{1+\frac{k-1}{2}M_2^2} = \frac{1+0.2(0.0702)^2}{1+0.2(0.0802)^2} = 1.00$$

$\therefore\ V_2 = M_2c_2 = M_2c_1 = 0.0802\times 1140$ ft/sec $= 91.4$ ft/s

From continuity (Eq. 1)

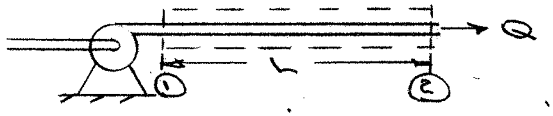
$p_2 = p_1\frac{V_1}{V_2} = 120$ psia $\times \frac{80}{91.4} = 105$ psia

$\therefore\ p_1 - p_2 = 15.0$ psia ______ $(p_2-p_1)_{Q=0}$

Note: p_2 is essentially the same for isothermal and adiabatic flow. The value is higher that for incompressible flow

Problem * 13.77

Given: Natural gas (molecular mass $M_m = 18$, $k = 1.3$) is pumped through a constant area pipe ($D = 36\,in$) for a distance $L = 40$ miles



$D = 36\,in$
$T = 78°F = \text{const.}$

$p_1 \le 90\,psig$ $\quad$ $p_2 \ge 10\,psig$

Find: Volume flowrate, Q (ft^3 day @ 70°F and 1 atm)

Solution:

Basic equations: $\quad \dot{m} = \rho V A \qquad p = \rho R T$

Computing equation: $\quad f\dfrac{L_{max}}{D} = \dfrac{1-kM^2}{kM^2} + \ln kM^2$

Then, $\quad f\dfrac{L_{12}}{D} = f\dfrac{L_{max}}{D}\Big)_1 - f\dfrac{L_{max}}{D}\Big)_2 = \dfrac{1-kM_1^2}{kM_1^2} - \dfrac{1-kM_2^2}{kM_2^2} + \ln\dfrac{M_1^2}{M_2^2}$

Since information on pressures is known, we relate p and M

From the ideal gas equation of state, for T = constant $\quad \dfrac{\rho_1}{\rho_2} = \dfrac{p_1}{p_2}$

From the continuity equation $\quad \dfrac{\rho_1}{\rho_2} = \dfrac{V_2}{V_1}$ and for T = constant, $\dfrac{V_2}{V_1} = \dfrac{M_2}{M_1}$

Hence $\quad \dfrac{p_1}{p_2} = \dfrac{M_2}{M_1}$ and $\quad M_2 = \dfrac{p_1}{p_2}M_1$

Substituting for M_2 into the equation for $f\dfrac{L_{12}}{D_h}$ and rearranging we obtain

$$f\frac{L_{12}}{D_h} = \frac{1-(p_2/p_1)^2}{kM_1^2} - \ln\left(\frac{p_1}{p_2}\right)^2$$

Solving this equation for M_1, then $\quad M_1 = \left\{\dfrac{1}{k}\left[\dfrac{1-\left(\frac{p_2}{p_1}\right)^2}{f\frac{L_{12}}{D_h} + \ln\left(\frac{p_1}{p_2}\right)^2}\right]\right\}^{1/2}$

$\dfrac{L}{D_h} = 40\,mi \times 5280\,\dfrac{ft}{mi} \times \dfrac{1}{3\,ft} = 70{,}400$

Assume pipe is commercial steel. From Table 8.1, $e = 0.00015\,ft$ and hence $e/D = 0.00005$. Assume $Re > 3.0 \times 10^7$, then $f = 0.0105$. Solving for M_1,

$$M_1 = \left\{\frac{1}{1.3}\left[\frac{1-\left(\frac{24.7}{104.7}\right)^2}{0.0105(70{,}400) + \ln\left(\frac{104.7}{24.7}\right)}\right]\right\}^{1/2} = 0.0313$$

$$R_{Nat\,gas} = \frac{R_u}{M} = 1544\,\frac{ft\cdot lbf}{lbmole\cdot°R}\,\frac{lbmole}{18\,lbm} = 85.8\,\frac{ft\cdot lbf}{lbm\cdot°R}$$

$$c_1 = (kRT_1)^{1/2} = \left(1.3 \times 85.8\,\frac{ft\cdot lbf}{lbm\cdot°R} \times 530°R \times \frac{32.2\,lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2}\right)^{1/2} = 1380\,ft/s$$

$$V_1 = M_1c_1 = 0.0313 \times 1380\,ft/s = 43.2\,ft/s$$

$$\rho_1 = \frac{p_1}{RT_1} = 104.7\,\frac{lbf}{in^2} \times \frac{lbm\cdot°R}{85.8\,ft\cdot lbf} \times \frac{1}{530°R} \times 144\,\frac{in^2}{ft^2} = 0.332\,lbm/ft^3$$

Check assumption on Re; from Fig A.2 (for methane), $\mu = 1.08 \times 10^{-5} \times 2.089 \times 10^{-2}\,lbf\cdot s/ft^2$

$$Re = \frac{\rho V D}{\mu} = 0.332\,\frac{lbm}{ft^3} \times 43.2\,\frac{ft}{s} \times 3\,ft \times \frac{1}{2.26 \times 10^{-7}}\,\frac{ft^2}{lbf\cdot s} \times \frac{slug}{32.2\,lbm} \times \frac{lbf\cdot s^2}{ft\cdot slug} = 5.91 \times 10^6$$

42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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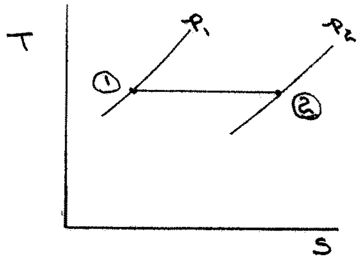
Then $f = 0.011$ and $\frac{f L_{max}}{D} = 774$ $M_1 = 0.0305$ and $V_1 = 42.1$ ft/sec

$$\dot{m} = \rho_1 V_1 A = 0.332 \frac{\text{lbm}}{\text{ft}^3} \times 42.1 \frac{\text{ft}}{\text{sec}} \times \frac{\pi}{4}(3)^2 \text{ft}^2 = 98.8 \text{ lbm/sec}$$

$$\rho_{atm} = \frac{P_{atm}}{RT_1} = 14.7 \frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\cdot\text{R}}{85.8 \text{ ft}\cdot\text{lbf}} \times \frac{1}{530\text{R}} \times \frac{144 \text{ in}^2}{\text{ft}^2} = 4.65 \times 10^{-2} \text{ lbm/ft}^3$$

Volume flowrate at atmospheric pressure

$$Q = \frac{\dot{m}}{\rho_{atm}} = 98.8 \frac{\text{lbm}}{\text{sec}} \times \frac{\text{ft}^3}{4.65 \times 10^{-2} \text{ lbm}} \times \frac{3600 \text{ sec}}{\text{hr}} \times \frac{24 \text{ hr}}{\text{day}} = 1.84 \times 10^8 \text{ ft}^3/\text{day} \longleftarrow Q$$



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Problem 13.78

13.78 Air from a large reservoir at 25 psia and 250°F flows isentropically through a converging nozzle into a frictionless pipe at 24 psia. The flow is heated as it flows along the pipe. Obtain a plot of the Ts diagram for this flow, until $M = 1$. Also plot the pressure and speed distributions from the entrance to the location at which $M = 1$.

Given: Air flow from converging nozzle into heated pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·°R
$k =$	1.4	
$c_p =$	0.2399	Btu/lbm·°R
	187	ft·lbf/lbm·°R
$T_0 =$	710	°R
$p_0 =$	25	psi
$p_e =$	24	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a (using built-in function *IsenMfromp* (M, k)) $M_e =$ 0.242

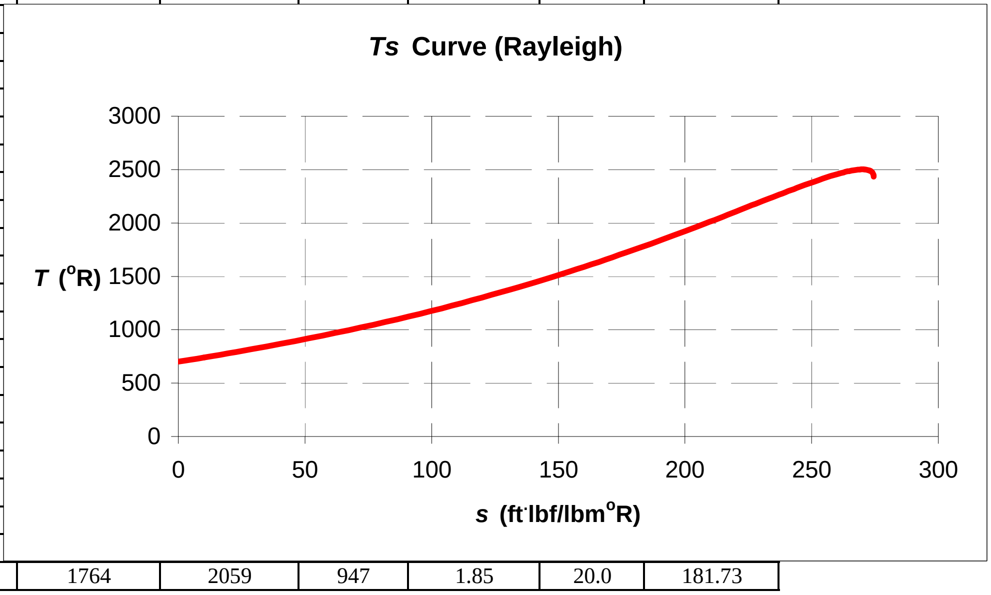
Using built-in function *IsenT* (M, k) $T_e =$ 702 °R

Using p_e, M_e, and function *Rayp* (M, k) $p^* =$ 10.82 psi

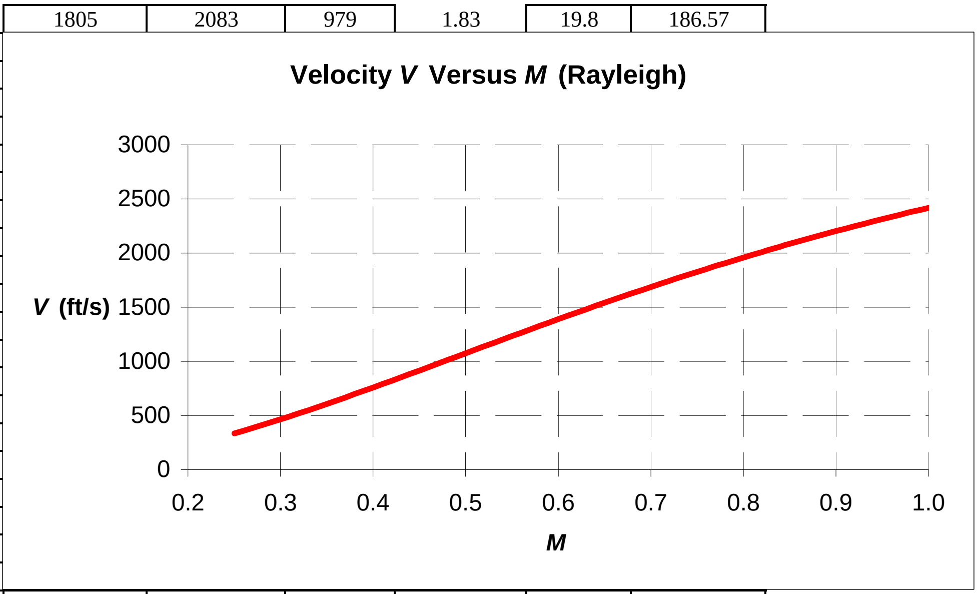
Using T_e, M_e, and function *RayT* (M, k) $T^* =$ 2432 °R

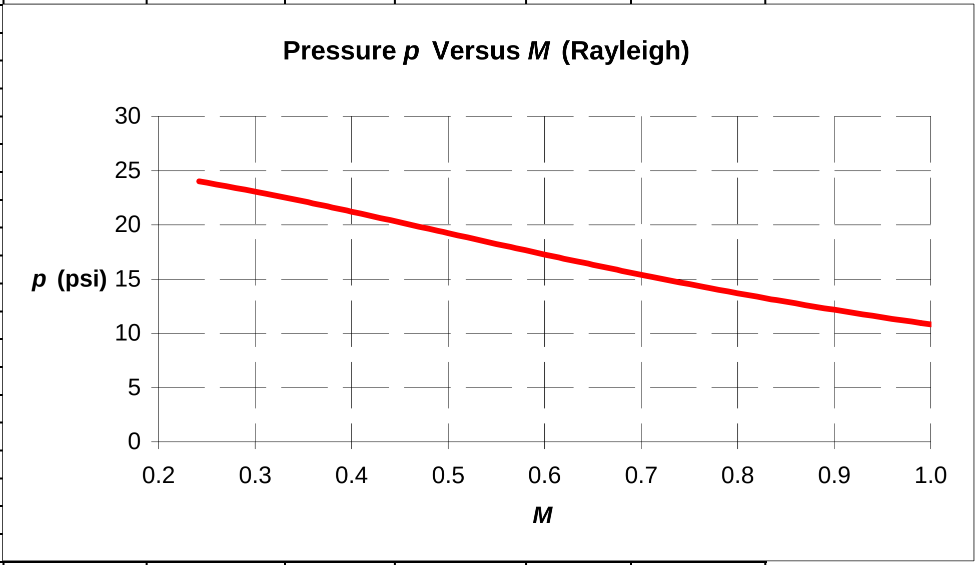
We can now use Rayleigh-line relations to compute values for a range of Mach numbers:

M	T/T^*	T (°R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·°R) Eq. (12.11b)
0.242	0.289	702	1299	315	2.22	24.0	0.00
0.25	0.304	740	1334	334	2.21	23.9	10.26
0.26	0.325						
0.27	0.346						
0.28	0.367						
0.29	0.388						
0.3	0.409						
0.31	0.430						
0.32	0.451						
0.33	0.472						
0.34	0.493						
0.35	0.514						
0.36	0.535						
0.37	0.555						
0.38	0.576						
0.39	0.595						
0.4	0.615						
0.41	0.634						
0.42	0.653						
0.43	0.672						
0.44	0.690						
0.45	0.708						
0.46	0.725	1764	2059	947	1.85	20.0	181.73



0.47	0.742	1805	2083	979	1.83	19.8	186.57
0.48	0.759						
0.49	0.775						
0.5	0.790						
0.51	0.805						
0.52	0.820						
0.53	0.834						
0.54	0.847						
0.55	0.860						
0.56	0.872						
0.57	0.884						
0.58	0.896						
0.59	0.906						
0.6	0.917						
0.61	0.927						
0.62	0.936						
0.63	0.945						
0.64	0.953						
0.65	0.961						
0.66	0.968						
0.67	0.975						
0.68	0.981	2387	2396	1629	1.46	15.8	250.96
0.69	0.987	2401	2403	1658	1.44	15.6	252.70
0.7	0.993						
0.71	0.998						
0.72	1.003						
0.73	1.007						
0.74	1.011						
0.75	1.014						
0.76	1.017						
0.77	1.020						
0.78	1.022						
0.79	1.024						
0.8	1.025						
0.81	1.027						
0.82	1.028						
0.83	1.028						
0.84	1.029						
0.85	1.029						
0.86	1.028						
0.87	1.028						
0.88	1.027						
0.89	1.026						
0.9	1.025	2492	2448	2203	1.12	12.2	272.78
0.91	1.023	2488	2446	2226	1.11	12.0	273.13
0.92	1.021	2484	2444	2248	1.10	11.9	273.43
0.93	1.019	2479	2441	2270	1.09	11.7	273.70
0.94	1.017	2474	2439	2292	1.07	11.6	273.92
0.95	1.015	2468	2436	2314	1.06	11.5	274.11
0.96	1.012	2461	2433	2335	1.05	11.3	274.26
0.97	1.009	2455	2429	2356	1.04	11.2	274.38
0.98	1.006	2448	2426	2377	1.02	11.1	274.46
0.99	1.003	2440	2422	2398	1.01	10.9	274.51
1	1.000	2432	2418	2418	1.00	10.8	274.52





Problem 13.79

13.79 Repeat Problem 13.78 except the nozzle is now a converging-diverging nozzle delivering the air to the pipe at 2.5 psia.

Given: Air flow from converging-diverging nozzle into heated pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

R =	53.33	ft·lbf/lbm·^{o}R
k =	1.4	
c_p =	0.2399	Btu/lbm·^{o}R
	187	ft·lbf/lbm·^{o}R
T_0 =	710	^{o}R
p_0 =	25	psi
p_e =	2.5	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M,k)) M_e = 2.16

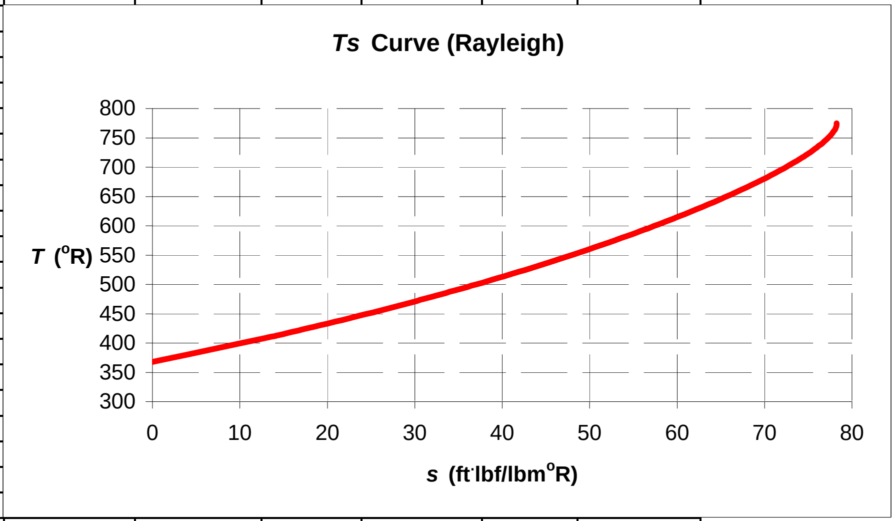
Using built-in function *IsenT* (M,k) T_e = 368 ^{o}R

Using p_e, M_e, and function *Rayp* (M,k) p^* = 7.83 psi

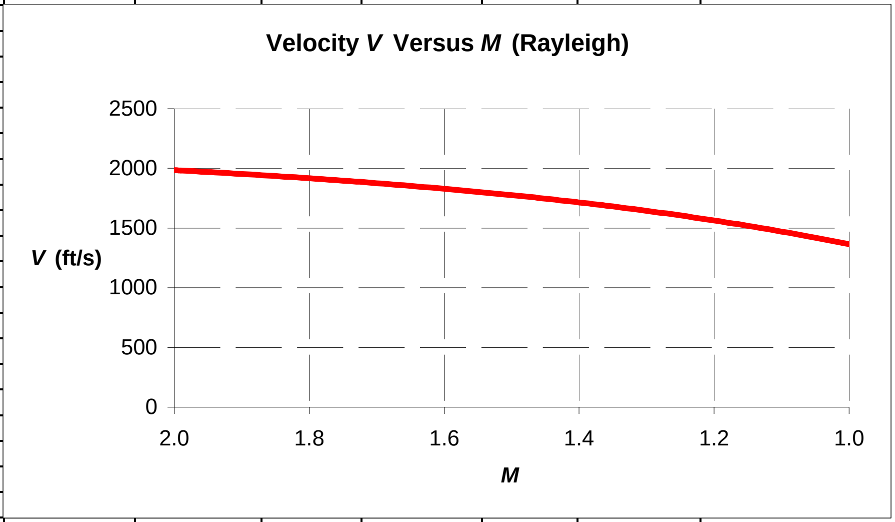
Using T_e, M_e, and function *RayT* (M,k) T^* = 775 ^{o}R

We can now use Rayleigh-line relations to compute values for a range of Mach numbers:

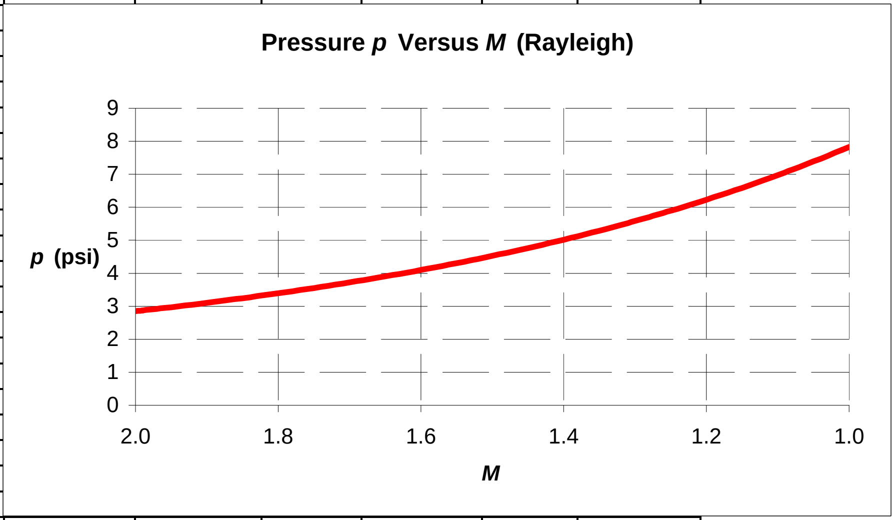
M	T/T^*	T (^{o}R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·^{o}R) Eq. (12.11b)
2.157	0.475	368	940	2028	0.32	2.5	0.00
2	0.529	410	993	1985	0.36	2.8	13.30
1.99	0.533						
1.98	0.536						
1.97	0.540						
1.96	0.544						
1.95	0.548						
1.94	0.552						
1.93	0.555						
1.92	0.559						
1.91	0.563						
1.9	0.567						
1.89	0.571						
1.88	0.575						
1.87	0.579						
1.86	0.584						
1.85	0.588						
1.84	0.592						
1.83	0.596						
1.82	0.600						
1.81	0.605						
1.8	0.609						
1.79	0.613	475	1069	1913	0.44	3.4	31.06
1.78	0.618	479	1073	1909	0.44	3.5	31.90
1.77	0.622	482	1076	1905	0.45	3.5	32.73
1.76	0.626	485	1080	1901	0.45	3.5	33.57
1.75	0.631	489	1084	1897	0.45	3.6	34.40
1.74	0.635	492	1088	1893	0.46	3.6	35.23
1.73	0.640	496	1092	1889	0.46	3.6	36.06
1.72	0.645	499	1096	1885	0.47	3.7	36.89
1.71	0.649	503	1100	1880	0.47	3.7	37.72
1.7	0.654	507	1104	1876	0.48	3.7	38.54



1.69	0.658	510	1107	1872	0.48	3.8	39.36
1.68	0.663	514	1111	1867	0.48	3.8	40.18
1.67	0.668	517	1115	1863	0.49	3.8	41.00
1.66	0.673	521	1119	1858	0.49	3.9	41.81
1.65	0.677	525	1123	1853	0.50	3.9	42.62
1.64	0.682	529	1127	1849	0.50	3.9	43.43
1.63	0.687	532	1131	1844	0.51	4.0	44.24
1.62	0.692						
1.61	0.697						
1.6	0.702						
1.59	0.707						
1.58	0.712						
1.57	0.717						
1.56	0.722						
1.55	0.727						
1.54	0.732						
1.53	0.737						
1.52	0.742						
1.51	0.747						
1.5	0.753						
1.49	0.758						
1.48	0.763						
1.47	0.768						
1.46	0.773						
1.45	0.779						
1.44	0.784						
1.43	0.789						
1.42	0.795	616	1217	1728	0.63	4.9	60.18
1.41	0.800	620	1221	1721	0.63	5.0	60.88
1.4	0.805						
1.39	0.811						
1.38	0.816						
1.37	0.822						
1.36	0.827						
1.35	0.832						
1.34	0.838						
1.33	0.843						
1.32	0.848						
1.31	0.854						
1.3	0.859						
1.29	0.865						
1.28	0.870						
1.27	0.875						
1.26	0.881						
1.25	0.886						
1.24	0.891						
1.23	0.896						
1.22	0.902						
1.21	0.907						
1.2	0.912	706	1303	1564	0.80	6.2	73.21
1.19	0.917	710	1307	1555	0.80	6.3	73.65
1.18	0.922	714	1310	1546	0.81	6.4	74.08
1.17	0.927	718	1314	1537	0.82	6.4	74.50
1.16	0.932	722	1318	1528	0.83	6.5	74.89
1.15	0.937	726	1321	1519	0.84	6.6	75.27
1.14	0.942	730	1324	1510	0.85	6.7	75.63
1.13	0.946	733	1328	1500	0.86	6.7	75.96
1.12	0.951	737	1331	1491	0.87	6.8	76.28
1.11	0.956	741	1334	1481	0.88	6.9	76.58
1.1	0.960	744	1337	1471	0.89	7.0	76.86
1.09	0.965	747	1341	1461	0.90	7.1	77.11
1.08	0.969	751	1344	1451	0.91	7.1	77.34
1.07	0.973	754	1347	1441	0.92	7.2	77.55
1.06	0.978	757	1349	1430	0.93	7.3	77.73
1.05	0.982	761	1352	1420	0.94	7.4	77.88
1.04	0.986	764	1355	1409	0.95	7.5	78.01
1.03	0.989	767	1358	1398	0.97	7.6	78.12
1.02	0.993	769	1360	1387	0.98	7.6	78.19
1.01	0.997	772	1362	1376	0.99	7.7	78.24
1	1.000	775	1365	1365	1.00	7.8	78.25



Velocity *V* Versus *M* (Rayleigh)

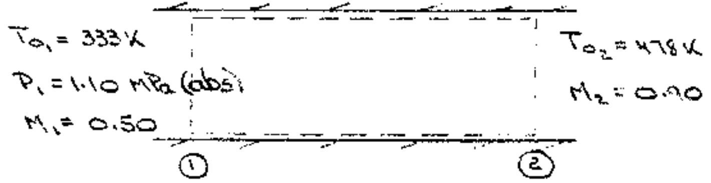


Pressure *p* Versus *M* (Rayleigh)

Problem 13.80

[2]

Given: Frictionless flow of air through a constant area duct



$T_{0_1} = 333\ \text{K}$
$P_1 = 1.10\ \text{MPa (abs)}$
$M_1 = 0.50$
$T_{0_2} = 478\ \text{K}$
$M_2 = 0.90$
①
②

Find: $\delta Q/dm$, $P_1 - P_2$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad P_1A - P_2A = \dot{m}(V_2 - V_1)$

Computing equation: $T_0/T = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow $\qquad$ (5) $F_{B_x} = 0$
(2) frictionless flow $\qquad$ (6) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section $\qquad$ (7) $\Delta z = 0$
(4) ideal gas

$h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad \frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1}) = 10^3 \frac{\text{J}}{\text{kg}\cdot\text{K}}(478 - 333)\text{K} = 145\ \frac{\text{kJ}}{\text{kg}} \longleftarrow \frac{\delta Q}{dm}$

$P_1A - P_2A = \dot{m}(V_2 - V_1) = \rho_1 V_1 A (V_2 - V_1)$ $\qquad$ and $\qquad P_1 - P_2 = \rho_1 V_1 (V_2 - V_1)$

$T_0/T = 1 + \frac{k-1}{2}M^2 \qquad T_1 = \frac{T_{0_1}}{1 + \frac{k-1}{2}M_1^2} = \frac{333\ \text{K}}{1 + 0.2(0.50)^2} = 317\ \text{K}$

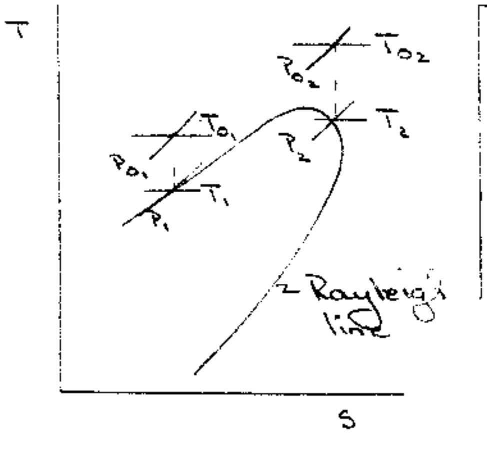
$T_2 = \frac{T_{0_2}}{1 + \frac{k-1}{2}M_2^2} = \frac{478\ \text{K}}{1 + 0.2(0.90)^2} = 411\ \text{K}$

$V_1 = M_1 c_1 = M_1(kRT_1)^{1/2} = 0.50\left(1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 317\ \text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 178\ \text{m/s}$

$V_2 = M_2 c_2 = M_2(kRT_2)^{1/2} = 0.90\left(1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 411\ \text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 366\ \text{m/s}$

$\rho_1 = \frac{P_1}{RT_1} = 1.10 \times 10^6 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg}\cdot\text{K}}{287\ \text{N}\cdot\text{m}} \times \frac{1}{317\ \text{K}} = 12.1\ \text{kg/m}^3$

$P_1 - P_2 = \rho_1 V_1 (V_2 - V_1) = 12.1 \frac{\text{kg}}{\text{m}^3} \times 178 \frac{\text{m}}{\text{s}} \times (366 - 178)\frac{\text{m}}{\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 405\ \text{kPa} \longleftarrow P_1 - P_2$



* Rayleigh-Line Flow Functions

From Appendix E.3

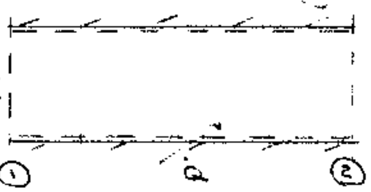
- for $M_1 = 0.5$, $p_1/p^* = 1.778$ (12.30a)
  $\therefore p^* = 619$ kPa
- for $M_2 = 0.90$, $p_2/p^* = 1.125$ (12.30a)
  $\therefore p_2 = 696$ kPa $\therefore p_1 - p_2 = 404$ kPa.

Problem 13.81

[2]

Given: Frictionless flow of air through a constant area duct.

$T_1 = 50°C$
$\rho_1 = 2.16\ \text{kg/m}^3$
$M_1 = 0.30$



$p_2 = 150\ \text{kPa}$
$M_2 = 0.60$

Find: $\delta Q/dm$, $s_2 - s_1$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad T ds = dh - v dp$

Computing equations: $T_0/T = 1 + \frac{k-1}{2} M^2$

Assumptions: (1) Steady flow (4) ideal gas
(2) frictionless flow (5) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section (6) $\Delta z = 0$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.3 (1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 323K \times \frac{kg \cdot m}{N \cdot sec^2})^{1/2} = 108\ \text{m/sec}$

$p_1 = \rho_1 R T_1 = 2.16 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 323K = 200\ \text{kPa}$

$\rho_1 V_1 = \rho_2 V_2 \qquad \rho_2 = \frac{p_2}{RT_2} \quad ; \quad V_2 = M_2 c_2 = M (kRT_2)^{1/2}$

$\therefore \rho_1 V_1 = \frac{p_2}{RT_2} M (kRT_2)^{1/2} = p_2 M_2 \left(\frac{k}{RT_2}\right)^{1/2}$. Solving for T_2,

$T_2 = \frac{k}{R}\left(\frac{p_2 M_2}{\rho_1 V_1}\right)^2 = 1.4 \times \frac{kg \cdot K}{287 N \cdot m}\left[150 \times 10^3 \frac{N}{m^2} \times 0.60 \times \frac{m^3}{2.16 kg} \times \frac{s}{108 m} \times \frac{kg \cdot m}{N \cdot s^2}\right]^2 \times \frac{N \cdot s^2}{kg \cdot m}$

$T_2 = 726\ K$.

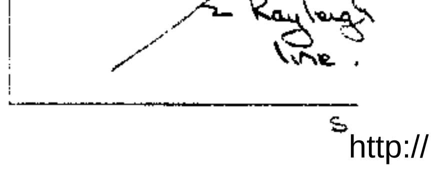
$s_2 - s_1 = \int_1^2 ds = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{p_1}^{p_2} R \frac{dp}{p} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

$s_2 - s_1 = 1.0 \frac{kJ}{kg \cdot K} \ln \frac{726}{323} - 0.287 \frac{kJ}{kg \cdot K} \ln \frac{150}{200} = 0.892\ \text{kJ/kg} \cdot K$ ——— $s_2 - s_1$

$T_{01} = T_1 \left[1 + \frac{k-1}{2} M_1^2\right] = 323K\left[1 + 0.2(0.3)^2\right] = 329K$

$T_{02} = T_2 \left[1 + \frac{k-1}{2} M_2^2\right] = 726K\left[1 + 0.2(0.6)^2\right] = 778K$

$\frac{\delta Q}{dm} = h_{02} - h_{01} = c_p (T_{02} - T_{01}) = 1.0 \frac{kJ}{kg \cdot K} (778 - 329) K = 449\ \text{kJ/kg}$ ⟵ $\frac{\delta Q}{dm}$



* Rayleigh-Line Flow Functions (App. E.3)

For $M_1 = 0.30$ $\quad T_{01}/T_0^* = 0.3469 \quad \therefore T_0^* = 948K$
$T_1/T^* = 0.4089 \quad \therefore T^* = 790K$
$p_1/p^* = 2.131 \quad \therefore p^* = 93.9\ \text{kPa}$

For $M_2 = 0.60$ $\quad T_{02}/T_0^* = 0.8189 \quad \therefore T_{02} = 776K$
$T_2/T^* = 0.9167 \quad \therefore T_2 = 724K$
$p_2/p^* = 1.596 \quad \therefore p_2 = 150\ \text{kPa}$

Note: In using the flow functions, it is not necessary to know $p_2 = 150\ \text{kPa}$

Problem 13.82

[2]

13.82 Air flows through a 5-cm-inside diameter pipe with negligible friction. Inlet conditions are $T_1 = 15°\text{C}$, $p_1 = 1$ MPa (abs), and $M_1 = 0.35$. Determine the heat exchange per pound of air required to produce $M_2 = 1.0$ at the pipe exit, where $p_2 = 500$ kPa.

Given: Frictionless air flow in a pipe

Find: Heat exchange per lb (or kg) at exit, where 500 kPa

Solution:

Basic equations: $m_{rate} = \rho\cdot V\cdot A$ $\quad p = \rho\cdot R\cdot T$ $\quad \dfrac{\delta Q}{dm} = c_p\cdot\left(T_{02} - T_{01}\right)$ (Energy) $\quad p_1 - p_2 = \rho_1\cdot V_1\cdot\left(V_2 - V_1\right)$ (Momentum)

Given or available data $T_1 = (15 + 273)\cdot K$ $\quad p_1 = 1\cdot MPa$ $\quad M_1 = 0.35$ $\quad p_2 = 500\cdot kPa$ $\quad M_2 = 1$

$D = 5\cdot cm$ $\quad k = 1.4$ $\quad c_p = 1004\cdot\dfrac{J}{kg\cdot K}$ $\quad R = 286.9\cdot\dfrac{J}{kg\cdot K}$

At section 1 $\rho_1 = \dfrac{p_1}{R\cdot T_1}$ $\quad \rho_1 = 12.1\dfrac{kg}{m^3}$ $\quad c_1 = \sqrt{k\cdot R\cdot T_1}$ $\quad c_1 = 340\dfrac{m}{s}$

$V_1 = M_1\cdot c_1$ $\quad V_1 = 119\dfrac{m}{s}$

From momentum $V_2 = \dfrac{p_1 - p_2}{\rho_1\cdot V_1} + V_1$ $\quad V_2 = 466\dfrac{m}{s}$

From continuity $\rho_1\cdot V_1 = \rho_2\cdot V_2$ $\quad \rho_2 = \rho_1\cdot\dfrac{V_1}{V_2}$ $\quad \rho_2 = 3.09\dfrac{kg}{m^3}$

Hence $T_2 = \dfrac{p_2}{\rho_2\cdot R}$ $\quad T_2 = 564\,K$ $\quad T_2 = 291\,°C$

and $T_{02} = T_2\cdot\left(1 + \dfrac{k-1}{2}\cdot M_2^{\,2}\right)$ $\quad T_{02} = 677\,K$ $\quad T_{02} = 403\,°C$

with $T_{01} = T_1\cdot\left(1 + \dfrac{k-1}{2}\cdot M_1^{\,2}\right)$ $\quad T_{01} = 295\,K$ $\quad T_{01} = 21.9\,°C$

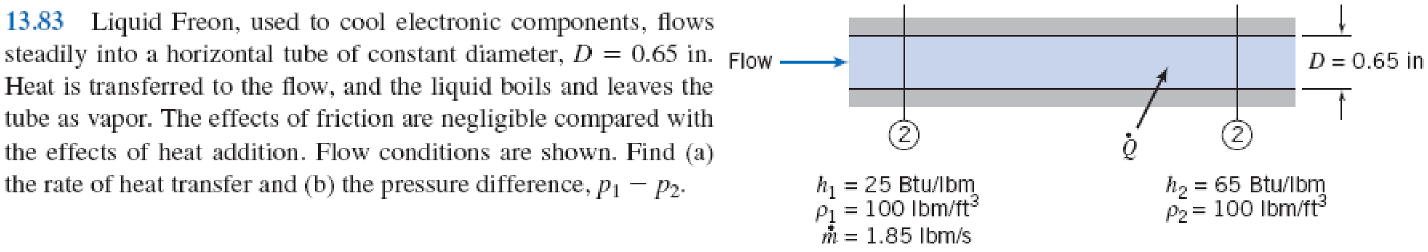
Then $\dfrac{\delta Q}{dm} = c_p\cdot\left(T_{02} - T_{01}\right) = 164\cdot\dfrac{Btu}{lbm} = 383\cdot\dfrac{kJ}{kg}$

(Note: Using Rayleigh line functions, for $M_1 = 0.35$ $\dfrac{T_0}{T_{0crit}} = 0.4389$

so $\quad T_{0crit} = \dfrac{T_{01}}{0.4389}$ $\quad T_{0crit} = 672\,K$ close to T_2 ... Check!)

Problem 13.83

[2]



13.83 Liquid Freon, used to cool electronic components, flows steadily into a horizontal tube of constant diameter, $D = 0.65$ in. Heat is transferred to the flow, and the liquid boils and leaves the tube as vapor. The effects of friction are negligible compared with the effects of heat addition. Flow conditions are shown. Find (a) the rate of heat transfer and (b) the pressure difference, $p_1 - p_2$.

Given: Frictionless flow of Freon in a tube

Find: Heat transfer; Pressure drop

NOTE: ρ_2 is NOT as stated; see below

Solution:

Basic equations: $m_{rate} = \rho\cdot V\cdot A$ $\quad p = \rho\cdot R\cdot T$ $\quad Q = m_{rate}\cdot\left(h_{02} - h_{01}\right)$ $\quad h_0 = h + \frac{V^2}{2}$ $\quad p_1 - p_2 = \rho_1\cdot V_1\cdot\left(V_2 - V_1\right)$

Given or available data $\quad h_1 = 25\cdot\frac{\text{Btu}}{\text{lbm}}$ $\quad \rho_1 = 100\cdot\frac{\text{lbm}}{\text{ft}^3}$ $\quad h_2 = 65\cdot\frac{\text{Btu}}{\text{lbm}}$ $\quad \rho_2 = 0.850\cdot\frac{\text{lbm}}{\text{ft}^3}$

$D = 0.65\cdot\text{in}$ $\quad A = \frac{\pi}{4}\cdot D^2$ $\quad A = 0.332\,\text{in}^2$ $\quad m_{rate} = 1.85\cdot\frac{\text{lbm}}{\text{s}}$

Then $\quad V_1 = \frac{m_{rate}}{\rho_1\cdot A}$ $\quad V_1 = 8.03\frac{\text{ft}}{\text{s}}$ $\quad h_{01} = h_1 + \frac{V_1^{\,2}}{2}$ $\quad h_{01} = 25.0\frac{\text{Btu}}{\text{lbm}}$

$V_2 = \frac{m_{rate}}{\rho_2\cdot A}$ $\quad V_2 = 944\frac{\text{ft}}{\text{s}}$ $\quad h_{02} = h_2 + \frac{V_2^{\,2}}{2}$ $\quad h_{02} = 82.8\frac{\text{Btu}}{\text{lbm}}$

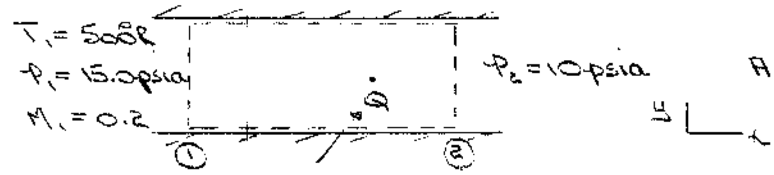
The heat transfer is $\quad Q = m_{rate}\cdot\left(h_{02} - h_{01}\right)$ $\quad Q = 107\frac{\text{Btu}}{\text{s}}$ $\quad$ (74 Btu/s with the wrong ρ_2!)

The pressure drop is $\quad \Delta p = \rho_1\cdot V_1\cdot\left(V_2 - V_1\right)$ $\quad \Delta p = 162\,\text{psi}$ $\quad$ (-1 psi with the wrong ρ_2!)

Problem 13.84

[3]

Given: Frictionless flow of air in a constant-area duct



Find: (a) V_2 and T_2 (b) $\dot{Q}$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equation: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow (2) uniform flow at a section (3) ideal gas
(4) frictionless flow (5) $F_{B_x} = 0$ (6) $\dot{W}_s = \dot{W}_{shear} = 0$

$$\rho_1 = \frac{p_1}{RT_1} = 15\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\,°R}{53.3\,\text{ft}\,\text{lbf}} \times \frac{1}{500°R} \times 144\frac{\text{in}^2}{\text{ft}^2} = 0.0811\ \text{lbm/ft}^3$$

$$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 0.2\left[1.4 \times 53.3\frac{\text{ft}\,\text{lbf}}{\text{lbm}\,°R} \times 500°R \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\,\text{ft}}{\text{lbf}\,\text{s}^2}\right]^{1/2} = 219\ \text{ft/s}$$

From the momentum eq. $(p_1 - p_2)A = \dot{m}(V_2 - V_1) = \rho_1V_1A(V_2 - V_1)$

$$\therefore V_2 = \frac{(p_1 - p_2)}{\rho_1V_1} + V_1 = (15-10)\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{ft}^3}{0.0811\,\text{lbm}} \times \frac{\text{s}}{219\,\text{ft}} \times 144\frac{\text{in}^2}{\text{ft}^2} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\,\text{ft}}{\text{lbf}\,\text{s}^2} + 219\ \text{ft/s}$$

$V_2 = 1520$ ft/s $\longleftarrow$ V_2

From continuity $\rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{219}{1520} \times 0.0811\frac{\text{lbm}}{\text{ft}^3} = 0.0117\ \text{lbm/ft}^3$

$T_2 = \frac{p_2}{\rho_2R} = 10\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{ft}^3}{0.0117\,\text{lbm}} \times \frac{\text{lbm}\,°R}{53.3\,\text{ft}\,\text{lbf}} \times 144\frac{\text{in}^2}{\text{ft}^2} = 2310°R$ $\longleftarrow$ T_2

$$T_{0_1} = T_1\left(1 + \frac{k-1}{2}M_1^2\right) = 500[1 + 0.2(0.2)^2] = 504°R$$

$$c_2 = (kRT_2)^{1/2} = \left[1.4 \times 53.3\frac{\text{ft}\,\text{lbf}}{\text{lbm}\,°R} \times 2310°R \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\,\text{ft}}{\text{lbf}\,\text{s}^2}\right]^{1/2} = 2360\ \text{ft/s}$$

$M_2 = \frac{V_2}{c_2} = \frac{1520}{2360} = 0.644$ $\qquad T_{0_2} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 2310°R[1 + 0.2(0.644)^2] = 2500°R$

Since $h_0 = h + \frac{V^2}{2}$, the energy eq. can be written as $\frac{\delta Q}{dm} = h_{0_2} - h_{0_1}$

$$\therefore \dot{Q} = \dot{m}\frac{\delta Q}{dm} = \dot{m}(h_{0_2} - h_{0_1}) = \dot{m}c_p(T_{0_2} - T_{0_1}) = \rho_1V_1Ac_p(T_{0_2} - T_{0_1})$$

$$\dot{Q} = 0.0811\frac{\text{lbm}}{\text{ft}^3} \times 219\frac{\text{ft}}{\text{s}} \times 0.087\,\text{ft}^2 \times 0.24\frac{\text{Btu}}{\text{lbm}\,°R} \times (2500 - 504)°R$$

$\dot{Q} = 740$ Btu/s $\longleftarrow$

---

* Rayleigh-Line Flow Functions (App. E.3)

At $M_1 = 0.2$

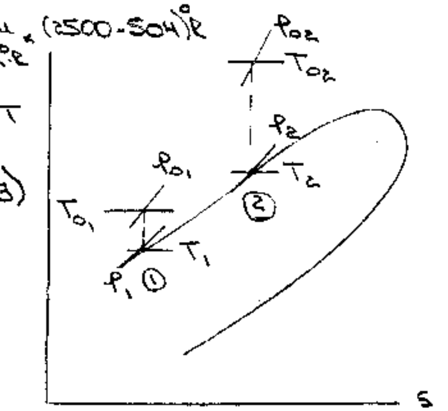
$p_1/p^* = 2.273$ (12.30a) $\therefore p^* = 6.60$ psia

$T_1/T^* = 0.2066$ (12.30b) $\therefore T^* = 2420°R$

$T_{0_1}/T_0^* = 0.1736$ (12.30d) $\therefore T_0^* = 2900°R$

At ② $p_2/p^* = 1.515$ $\therefore M_2 = 0.646$

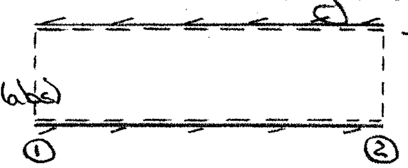
Also $T_{0_2}/T_0^* = 0.8644$ $\therefore T_{0_2} = 2510°R$ and $T_2/T^* = 0.9408$ $\therefore T_2 = 2280°R$



Problem 13.85

42.381 50 SHEETS 5 SQUARE
42.382 100 SHEETS 5 SQUARE
42.389 200 SHEETS 5 SQUARE
NATIONAL

Given: Frictionless flow of air through a constant area duct.



$T_1 = 52°C$ $\quad$ $T_2 = 45°C$ $\quad$ $\dot{m} = 1.42\ kg/s$

$P_1 = 60\ kPa(abs)$ $\quad$ $M_2 = 1.0$ $\quad$ $D = 100\ mm$

Find: $\delta Q/dm$, $s_2 - s_1$, $P_{0_1} - P_{0_2}$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad$ $Tds = dh - v\,dP$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2$ $\qquad$ $P_0/P = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow $\qquad$ (4) ideal gas

(2) frictionless flow $\qquad$ (5) $\dot{W}_s = \dot{W}_{shear} = 0$

(3) uniform flow at a section $\qquad$ (6) $\Delta z = 0$

$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.1)^2 m^2 = 7.85 \times 10^{-3}\ m^2$

$P = \rho RT \qquad \rho_1 = \frac{P_1}{RT_1} = 60 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{325\ K} = 0.643\ kg/m^3$

$\dot{m} = \rho V A \qquad V_1 = \frac{\dot{m}}{\rho_1 A} = 1.42 \frac{kg}{s} \times \frac{m^3}{0.643\ kg} \times \frac{1}{7.85 \times 10^{-3}\ m^2} = 281\ m/s$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 1.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 318\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 357\ m/s$

$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A \quad ; \quad \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{281}{357} \times 0.643\ kg/m^3 = 0.506\ kg/m^3$

$P = \rho RT \qquad P_2 = \rho_2 R T_2 = 0.506 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 318\ K = 46.2\ kPa$

$Tds = dh - v\,dP = c_p dT - \frac{1}{\rho} dP \qquad ds = c_p \frac{dT}{T} - R \frac{dP}{P}$

$s_2 - s_1 = \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{P_1}^{P_2} R \frac{dP}{P} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1.0 \frac{kJ}{kg \cdot K} \ln \frac{318}{325} - 0.287 \frac{kJ}{kg \cdot K} \ln \frac{46.2}{60}$

$s_2 - s_1 = 0.0532\ kJ/kg \cdot K \longleftarrow$ $s_2 - s_1$

$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 325\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 361\ m/s \quad , \quad M_1 = \frac{V_1}{c_1} = \frac{281}{361} = 0.778$

$T_0/T = 1 + \frac{k-1}{2}M^2 \qquad T_{0_1} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 325\ K\left[1 + 0.2(0.778)^2\right] = 364\ K$

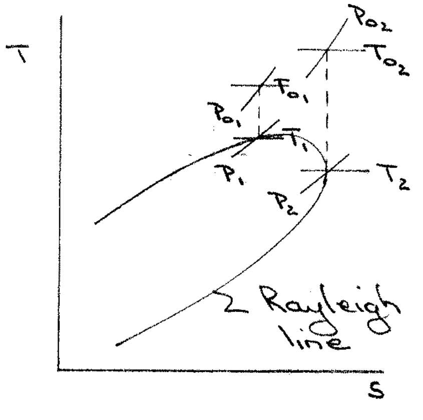
$T_{0_2} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 318\ K\left[1 + 0.2(1.0)^2\right] = 382\ K$

$h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2} \qquad \frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1}) = 1.0 \frac{kJ}{kg \cdot K}(382 - 364)K = 18\ kJ/kg \longleftarrow$ $\delta Q/dm$

$P_0/P = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \qquad P_{0_1} = P_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 60\ kPa\left[1 + 0.2(0.778)^2\right]^{3.5} = 89.5\ kPa$

$P_{0_2} = P_2\left[1 + \frac{k-1}{2}M_2^2\right]^{k/k-1} = 46.2\ kPa\left[1 + 0.2(1.0)^2\right]^{3.5} = 87.5\ kPa$

$P_{0_1} - P_{0_2} = (89.5 - 87.5)\ kPa = 2.0\ kPa \longleftarrow$ $P_{0_1} - P_{0_2}$



* Rayleigh-Line Flow Functions
From Appendix E.3 for $M_1 = 0.778$

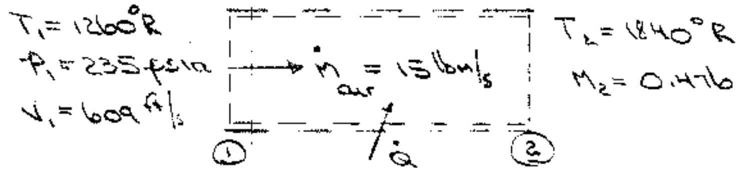
$T_{0_1}/T_0^* = 0.954$ (12.30d) $\therefore T_0^* = T_{0_2} = 381\ K$

$P_{0_1}/P_0^* = 1.024$ (12.30e) $\therefore P_0^* = P_{0_2} = 87.4\ kPa$

$P_1/P^* = 1.299$ (12.30a) $\therefore P^* = P_2 = 46.2\ kPa$

Problem 13.86

Given: Combustor modeled as frictionless flow through a constant area duct with heat transfer. Air-fuel ratio is large enough so properties are those of air



Heating value of the fuel is 18,000 Btu/lbm.

Find: (a) p_2 (b) $\dot{Q}$ (c) $\dot{m}_{fuel}$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equation: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow (2) frictionless flow
(3) ideal gas, properties are those of air
(4) uniform flow at a section
(5) $\dot{W}_s = \dot{W}_{shear} = 0$

$\rho_1 = \frac{p_1}{RT_1} = 235 \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\,ft \cdot lbf} \times \frac{1}{1260°R} \times 144 \frac{in^2}{ft^2} = 0.504\ lbm/ft^3$

$c_1 = (kRT_1)^{1/2} = \left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot R} \times 1260R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2} = 1740\ ft/s$

$M_1 = V_1/c_1 = 609/1740 = 0.350 \qquad T_{01} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 1260°R[1 + 0.2(0.35)^2] = 1290°R$

$T_{02} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 1840[1 + 0.2(0.476)^2] = 1920°R$

From energy eq. $\dot{Q} = \dot{m}(h_{02} - h_{01}) = \dot{m}c_p(T_{02} - T_{01}) = 15 \frac{lbm}{s} \times 0.24 \frac{Btu}{lbm \cdot R} \times (1920 - 1290)R$

$\dot{Q} = 2270\ Btu/s$ ← $\dot{Q}$

But $\dot{Q} = \dot{m}_{fuel} h_{fuel} \quad \therefore \dot{m}_f = \frac{\dot{Q}}{h_f} = 2270 \frac{Btu}{s} \times \frac{lbm}{18{,}000\,Btu} = 0.126\ lbm/s$ ← $\dot{m}_f$

$\dot{m}_f/\dot{m}_{air} = 0.126/15 = 0.0084$ ← $\dot{m}_f/\dot{m}_{air}$

$V_2 = M_2c_2 = M_2(kRT_2)^{1/2} = 0.476\left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot R} \times 1840R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2} = 1000\ ft/s$

From momentum

$p_2 = p_1 - \frac{\dot{m}}{A}(V_2 - V_1) = p_1 - \rho_1V_1(V_2 - V_1)$

$p_2 = 235 \frac{lbf}{in^2} - 0.504 \frac{lbm}{ft^3} \times 609 \frac{ft}{s} \times (1000 - 609)\frac{ft}{s} \times \frac{slug}{32.2\,lbm} \times \frac{lbf \cdot s^2}{ft \cdot slug} \times \frac{ft^2}{144\,in^2} = 209$ psia ← p_2

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* Rayleigh-Line Flow Functions (App. E.3)

For $M_1 = 0.35$,

$T_0/T_0^* = 0.4389$ (12.30d) $\therefore T_0^* = 2940°R$

$T_1/T^* = 0.5141$ (12.30b) $\therefore T^* = 2450°R$

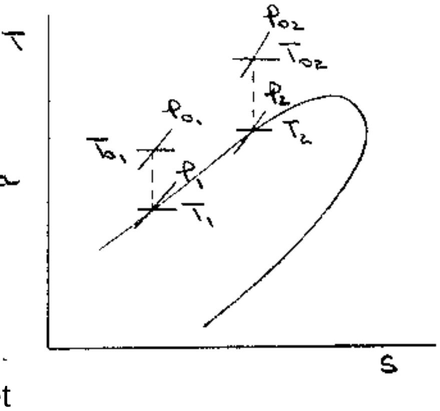
$p_1/p^* = 2.0487$ (12.30a) $\therefore p^* = 114.7$ psia

For $M_2 = 0.476$

$T_{02}/T_0^* = 0.6551 \quad \therefore T_{02} = 1930°R$

$T_2/T^* = 0.7522 \quad \therefore T_2 = 1840°R$

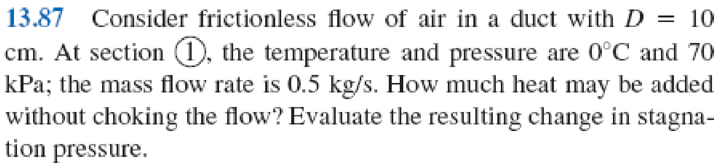
$p_2/p^* = 1.822 \quad \therefore p_2 = 209$ psia



42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE

Problem 13.87

[3]



13.87 Consider frictionless flow of air in a duct with $D = 10$ cm. At section ①, the temperature and pressure are 0°C and 70 kPa; the mass flow rate is 0.5 kg/s. How much heat may be added without choking the flow? Evaluate the resulting change in stagnation pressure.

Given: Frictionless flow of air in a duct

Find: Heat transfer without choking flow; change in stagnation pressure

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\qquad p = \rho\cdot R\cdot T$ $\qquad m_{rate} = \rho\cdot A\cdot V$

$p_1 - p_2 = \frac{m_{rate}}{A}\cdot\left(V_2 - V_1\right)$ $\qquad \frac{\delta Q}{dm} = c_p\cdot\left(T_{02} - T_{01}\right)$

Given or available data $\quad T_1 = (0 + 273)\cdot K$ $\qquad p_1 = 70\cdot kPa$ $\qquad m_{rate} = 0.5\cdot\frac{kg}{s}$ $\qquad D = 10\cdot cm$

$A = \frac{\pi}{4}\cdot D^2$ $\qquad A = 78.54\,cm^2$ $\qquad k = 1.4$ $\qquad M_2 = 1$ $\qquad c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\qquad R = 286.9\cdot\frac{J}{kg\cdot K}$

At state 1 $\quad \rho_1 = \frac{p_1}{R\cdot T_1}$ $\qquad \rho_1 = 0.894\frac{kg}{m^3}$ $\qquad c_1 = \sqrt{k\cdot R\cdot T_1}$ $\qquad c_1 = 331\frac{m}{s}$

From continuity $\quad V_1 = \frac{m_{rate}}{\rho_1\cdot A}$ $\qquad V_1 = 71.2\frac{m}{s}$ $\qquad$ then $\qquad M_1 = \frac{V_1}{c_1}$ $\qquad M_1 = 0.215$

From momentum $\quad p_1 - p_2 = \frac{m_{rate}}{A}\cdot\left(V_2 - V_1\right) = \rho_2\cdot V_2^{\,2} - \rho_1\cdot V_1^{\,2}$ $\qquad$ but $\qquad \rho\cdot V^2 = \rho\cdot c^2\cdot M^2 = \frac{p}{R\cdot T}\cdot k\cdot R\cdot T\cdot M^2 = k\cdot p\cdot M^2$

Hence $\quad p_1 - p_2 = k\cdot p_2\cdot M_2^{\,2} - k\cdot p_1\cdot M_1^{\,2}$ $\qquad$ or $\qquad p_2 = p_1\cdot\left(\frac{1 + k\cdot M_1^{\,2}}{1 + k\cdot M_2^{\,2}}\right)$ $\qquad p_2 = 31.1\,kPa$

From continuity $\quad \rho_1\cdot V_1 = \frac{p_1}{R\cdot T_1}\cdot M_1\cdot c_1 = \frac{p_1}{R\cdot T_1}\cdot M_1\cdot\sqrt{k\cdot R\cdot T_1} = \sqrt{\frac{k}{R}}\cdot\frac{p_1\cdot M_1}{\sqrt{T_1}} = \rho_2\cdot V_2 = \sqrt{\frac{k}{R}}\cdot\frac{p_2\cdot M_2}{\sqrt{T_2}}$

Hence $\quad \frac{p_1\cdot M_1}{\sqrt{T_1}} = \frac{p_2\cdot M_2}{\sqrt{T_2}}$ $\qquad T_2 = T_1\cdot\left(\frac{p_2}{p_1}\cdot\frac{M_2}{M_1}\right)^2$ $\qquad T_2 = 1161\,K$ $\qquad T_2 = 888\,°C$

Then $\quad T_{02} = T_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^{\,2}\right)$ $\qquad T_{02} = 1394\,K$ $\qquad T_{01} = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^{\,2}\right)$ $\qquad T_{01} = 276\,K$

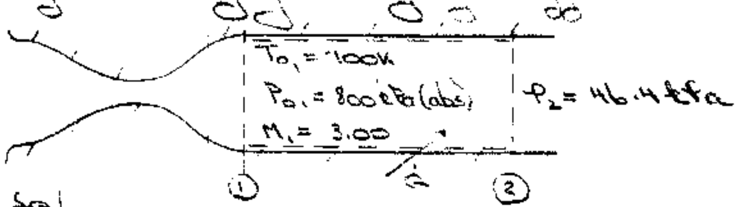
$p_{02} = p_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^{\,2}\right)^{\frac{k}{k-1}}$ $\qquad p_{02} = 58.8\,kPa$ $\qquad p_{01} = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^{\,2}\right)^{\frac{k}{k-1}}$ $\qquad p_{01} = 72.3\,kPa$

Finally $\quad \frac{\delta Q}{dm} = c_p\cdot\left(T_{02} - T_{01}\right) = 1.12\cdot\frac{MJ}{kg}$ $\qquad \Delta p_0 = p_{02} - p_{01}$ $\qquad \Delta p_0 = -13.5\,kPa$

(Using Rayleigh functions, at $M_1 = 0.215$ $\qquad \frac{T_{01}}{T_{0crit}} = \frac{T_{01}}{T_{02}} = 0.1975$ $\qquad T_{02} = \frac{T_{01}}{0.1975}$ $\qquad T_{02} = 1395\,K$ $\qquad$ and ditto for p_{02} ...Check!)

Problem 13.88

Given: Frictionless flow of air through a constant-area duct supplied by a converging-diverging nozzle



Find: V_2, M_2, $\delta Q/dm$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2$ $p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions:
(1) steady flow (5) $F_{B_x} = 0$
(2) frictionless flow (6) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section (7) $\Delta z = 0$
(4) ideal gas

$T_0/T = 1 + \frac{k-1}{2}M^2$ $T_1 = \frac{T_{0_1}}{1 + \frac{k-1}{2}M_1^2} = \frac{700\,K}{1 + 0.2(3.0)^2} = 250\,K$

$p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $p_1 = \frac{p_{0_1}}{\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1}} = \frac{800\,kPa}{\left[1 + 0.2(3.0)^2\right]^{3.5}} = 21.8\,kPa$

$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 3.0\left(1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 250\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 951\,m/s$

$p = \rho RT$ $\rho_1 = \frac{p_1}{RT_1} = 21.8 \times 10^3\frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{250\,K} = 0.304\,kg/m^3$

$(p_1 - p_2)A = \rho_1V_1A(V_2 - V_1)$ $\therefore V_2 = V_1 + \frac{(p_1 - p_2)}{\rho_1V_1}$. Solving for V_2,

$V_2 = 951\,m/s + (21.8 - 46.4)10^3\frac{N}{m^2} \times \frac{m^3}{0.304\,kg} \times \frac{s}{951\,m} \times \frac{kg\cdot m}{N\cdot s^2} = 866\,m/s$ ⟵ V_2

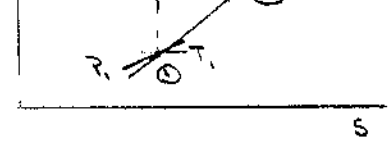
From continuity, $\rho_2 = \rho_1\frac{V_1}{V_2} = 0.304\frac{kg}{m^3} \times \frac{951}{866} = 0.334\,kg/m^3$

$T_2 = \frac{p_2}{\rho_2R} = 46.4 \times 10^3\frac{N}{m^2} \times \frac{m^3}{0.334\,kg} \times \frac{kg\cdot K}{287\,N\cdot m} = 484\,K$

$c_2 = (kRT_2)^{1/2} = \left[1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 484\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 441\,m/s$; $M_2 = \frac{V_2}{c_2} = \frac{866}{441} = 1.96$ ⟵ M_2

$T_{0_2} = T_2\left[1 + \frac{k-1}{2}M^2\right] = 484\left[1 + 0.2(1.96)^2\right] = 856\,K$

$\delta Q/dm = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1}) = 1\frac{kJ}{kg\cdot K}(856\,K - 700\,K) = 156\,kJ/kg$ ⟵ $\frac{\delta Q}{dm}$



• Rayleigh-Line Flow Functions (Appendix E.3)

For $M_1 = 3.0$, $T_{0_1}/T_0^* = 0.6540$, $\therefore T_0^* = 1070\,K$
$T_1/T^* = 0.2803$ $\therefore T^* = 892\,K$
$p_1/p^* = 0.1765$ $\therefore p^* = 124\,kPa$
$V_1/V^* = 1.588$ $\therefore V^* = 599\,m/s$

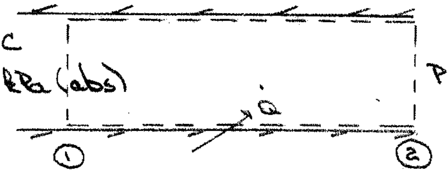
At section ② for $p_2/p^* = 0.3742$
$M_2 = 1.966$ ✓
$T_{0_2}/T_0^* = 0.800$ $\therefore T_{0_2} = 856\,K$ ✓
$T_2/T^* = 0.542$ $\therefore T_2 = 483\,K$
$V_2/V^* = 1.447$ $\therefore V_2 = 867\,m/s$

Problem 13.89

Given: Frictionless flow of air through a constant area duct

$T_1 = 260°C$

$p_1 = 126$ kPa (abs)



$p_2 = 101$ kPa (abs)

$A = 0.02\ m^2$

$\dot{m} = 1.83$ kg/s

Find: $M_2, T_2, T_{0_2}, \dot{Q}$

Solution:

* Compressible flow functions (Appendix E) to be used in solution

Basic equation: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$

Assumptions:
(1) steady flow
(2) frictionless flow
(3) uniform flow at a section
(4) ideal gas
(5) $F_{B_x} = 0$
(6) $\dot{W}_s = \dot{W}_{shear} = 0$
(7) $\Delta z = 0$

$$\rho_1 = \frac{p_1}{RT_1} = 126 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{533\ K} = 0.8237\ kg/m^3$$

$$\dot{m} = \rho_1 V_1 A \qquad V_1 = \frac{\dot{m}}{\rho_1 A} = 1.83 \frac{kg}{s} \times \frac{m^3}{0.8237\ kg} \times \frac{1}{0.02\ m^2} = 111\ m/s$$

$$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 533\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 463\ m/s$$

$$M_1 = \frac{111}{463} = 0.240$$

From App. E.1, $T_{0_1}/T_1 = 1.012$ (11.17b) $\therefore T_{0_1} = 539$ K

From App E.3, $T_{0_1}/T_0^* = 0.2395$ (12.30d), $T_1/T^* = 0.2841$ (12.30b), $p_1/p^* = 2.221$ (12.30a)

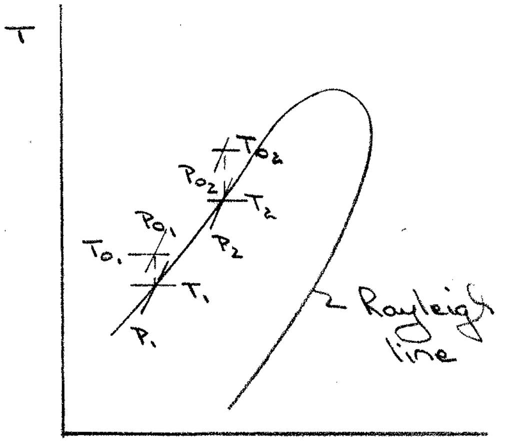
$\therefore T_0^* = 2250$ K $\qquad T^* = 1876$ K $\qquad p^* = 56.73$ kPa

At section ② $\frac{p_2}{p^*} = \frac{101}{56.73} = 1.780$ From App E.3 (Eq. 12.30a), $M = 0.50$ ⟵ M_2

Also, $T_{0_2}/T_0^* = 0.6914$ (12.30d), $T_2/T^* = 0.7901$. Therefore: $T_{0_2} = 1556$ K, $T_2 = 1480$ K ⟵ T_{0_2}, T_2

$$\dot{Q} = \dot{m}\frac{\delta Q}{dm} = \dot{m}(h_{0_2} - h_{0_1}) = \dot{m}c_p(T_{0_2} - T_{0_1}) = 1.83 \frac{kg}{s} \times 1.0 \frac{kJ}{kg \cdot K} \times (1556 - 539)\ K$$

$\dot{Q} = 1.86$ MJ/s ⟵ $\dot{Q}$



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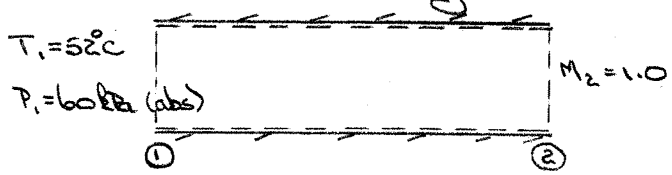
Problem 13.90

Given: Frictionless flow of air through a constant area duct.



$\dot{m} = 1.42\ kg/sec$

$D = 100\ mm$

Find: $\delta Q/dm$, fluid properties at section ②

Solution:

* Compressible flow functions (Appendix E) to be used in solution

Basic equation: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$

Assumptions:
(1) steady flow
(2) frictionless flow
(3) uniform flow at a section
(4) ideal gas
(5) $\dot{W}_s = \dot{W}_{shear} = 0$
(6) $\Delta z = 0$

$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.1)^2\ m^2 = 7.85 \times 10^{-3}\ m^2$

$p = \rho R T \qquad \rho_1 = \frac{p_1}{RT_1} = 60 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{325\ K} = 0.643\ kg/m^3$

$\dot{m} = \rho V A \qquad V_1 = \frac{\dot{m}}{\rho A} = 1.42 \frac{kg}{s} \times \frac{m^3}{0.643\ kg} \times \frac{1}{7.85 \times 10^{-3}\ m^2} = 281\ m/s$

$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 325\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 362\ m/s$

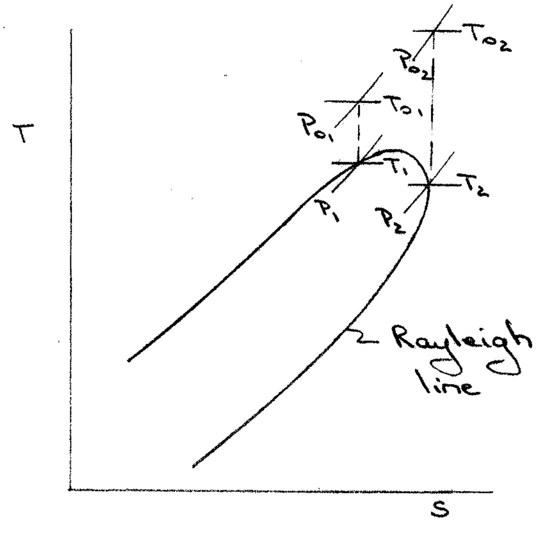
$M_1 = \frac{V_1}{c_1} = \frac{281}{362} = 0.776$

From App E.1, $T_1/T_{0_1} = 0.8925 \quad p_1/p_{0_1} = 0.6717 \quad \therefore T_{0_1} = 364\ K,\ p_{0_1} = 89.3\ kPa$

From App E.3, $\frac{T_{0_1}}{T_0^*} = 0.9535 \quad \frac{p_{0_1}}{p_0^*} = 1.024 \quad \frac{T_1}{T^*} = 1.022 \quad \frac{p_1}{p^*} = 1.302 \quad \frac{V}{V^*} = \frac{\rho^*}{\rho} = 0.7844$

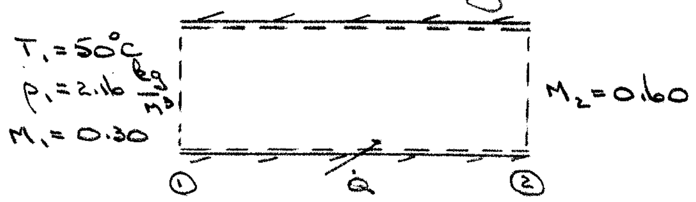
$\therefore T_{0_2} = 382\ K,\ p_{0_2} = 87.2\ kPa,\ T_2 = 318\ K,\ p_2 = 46.1\ kPa,\ \rho_2 = 0.504 \frac{kg}{m^3},\ V_2 = 358 \frac{m}{s}$ ← $(\ )_2$

$\frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1}) = 1.0 \frac{kJ}{kg \cdot K}(382 - 364)K = 18.0\ kJ/kg$ ← $\delta Q/dm$



Problem 13.91

Given: Frictionless flow of air through a constant area duct



Find: $\delta Q/dm$, $s_2 - s_1$, $p_{01} - p_{02}$

Solution:

* Compressible flow functions (Appendix E) to be used in solution

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad Tds = dh - v dP$

Assumptions: (1) steady flow
(2) frictionless flow
(3) uniform flow at a section
(4) ideal gas
(5) $\dot{W}_s = \dot{W}_{shear} = 0$
(6) $\Delta z = 0$

$p_1 = \rho_1 R T_1 = 2.16 \frac{kg}{m^3} \times 287 \frac{N\cdot m}{kg\cdot K} \times 323\,K = 200\,kPa$

$M_1 = 0.30$ From App. E.1 $T_1/T_{01} = 0.9823$ (11.17b), $p_1/p_{01} = 0.9395$ (11.17a)

$\therefore T_{01} = 329\,K \qquad p_{01} = 213\,kPa$

From App. E.3 $T_{01}/T_0^* = 0.3469$ (12.30d), $p_{01}/p_0^* = 1.199$ (12.30e)

$\therefore T_0^* = 948\,K \qquad p_0^* = 178\,kPa$

$M_2 = 0.60$ From App. E.3 $T_{02}/T_0^* = 0.8189$ (12.30d), $p_{02}/p_0^* = 1.075$ (12.30e)

$\therefore T_{02} = 776\,K \qquad p_{02} = 191\,kPa$

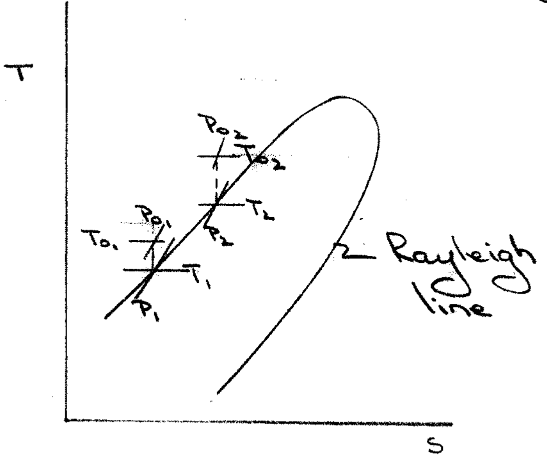
$p_{01} - p_{02} = (213 - 191)\,kPa = 22\,kPa \longleftarrow \underline{p_{01} - p_0}$

$\delta Q/dm = h_{02} - h_{01} = c_p(T_{02} - T_{01}) = 1.0 \frac{kJ}{kg\cdot K}(776 - 329)K = 447\,kJ/kg \longleftarrow \underline{\delta Q/dm}$

$Tds = dh - vdP = c_p dT - \frac{1}{\rho} dP \qquad ds = c_p \frac{dT}{T} - R\frac{dP}{P}$

$$s_2 - s_1 = s_{02} - s_{01} = \int_{s_{01}}^{s_{02}} ds = \int_{T_{01}}^{T_{02}} c_p \frac{dT}{T} - \int_{P_{01}}^{P_{02}} R\frac{dP}{P} = c_p \ln\frac{T_{02}}{T_{01}} - R\ln\frac{P_{02}}{P_{01}}$$

$s_2 - s_1 = 1.0 \frac{kJ}{kg\cdot K} \ln\frac{776}{329} - 0.287 \frac{kJ}{kg\cdot K} \ln\frac{191}{213} = 0.889\,kJ/kg\cdot K \longleftarrow \underline{s_2 - s_1}$



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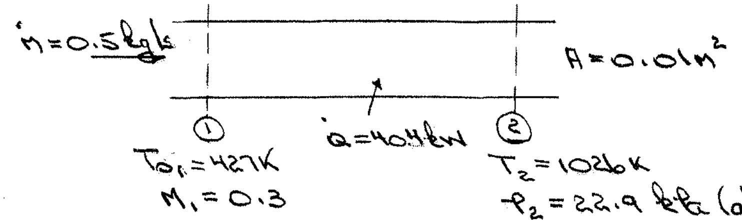
42.389 200 SHEETS 5 SQUARE
NATIONAL

Problem 13.92

Given: Air enters an engine combustion chamber where heat is added during a frictionless process in a tube with constant area, $A = 0.01\,m^2$. Conditions are as shown.



Find: (a) M_2 (b) p_1 (c) $p_{0_2} - p_{0_1}$

Solution: * Compressible flow functions (Appendix E) to be used in solution

Basic equations: $h_{0_1} + \frac{\delta Q}{dm} = h_{0_2}$ $\quad \dot{m} = \rho V A \quad p = \rho R T$

Assumptions: (1) steady flow (2) frictionless flow
(3) ideal gas (4) uniform flow at a section
(5) $\dot{W}_s = \dot{W}_{shear} = 0$

From the energy equation, $\dot{Q} = \dot{m}(h_{0_2} - h_{0_1}) = \dot{m}c_p(T_{0_2} - T_{0_1})$

$$\therefore T_{0_2} = T_{0_1} + \frac{\dot{Q}}{\dot{m}c_p} = 427K + 404\times10^3 W \times \frac{s}{0.5\,kg} \times \frac{kg\cdot K}{1004\,J} \times \frac{J}{W\cdot s} = 1232K$$

At $M_1 = 0.3$, from App. E.3, $T_{0_1}/T_0^* = 0.3469 \quad \therefore T_0^* = 1231\,K$

Since $T_{0_2} \approx T_0^*$, then $M_2 = 1.0$ ⟵ M_2

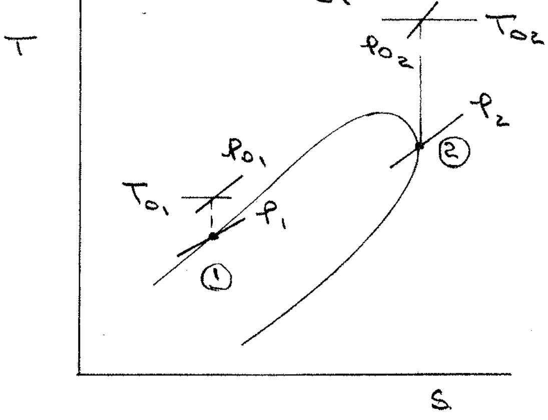
$\therefore p_2 = p^* = 22.9$ kPa

From Appendix E.3 for $M_1 = 0.3$, $p_1/p^* = 2.131 \quad \therefore p_1 = 48.8$ kPa ⟵ p_1

" " E.1 " $M_1 = 0.3$, $p_1/p_{0_1} = 0.9395 \quad \therefore p_{0_1} = 51.9$ kPa

From Appendix E.1 for $M_2 = 1.0$, $p_2/p_{0_2} = 0.5283 \quad \therefore p_{0_2} = 43.3$ kPa

$$\left.\begin{aligned} &\therefore p_{0_2} - p_{0_1} = (43.3 - 51.9)\,kPa = -8.6\,kPa \\ &\frac{\Delta p_0}{p_{0_1}} = \frac{-8.6}{51.9} = -0.166 \quad (\text{or } -16.6\%) \end{aligned}\right\} \longleftarrow \Delta p_0$$



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Problem 13.93

[3]

13.93 Flow in a gas turbine combustor is modeled as steady, one-dimensional, frictionless heating of air in a channel of constant area. For a certain process, the inlet conditions are 500°C, 1.5 MPa (abs), and $M = 0.5$. Calculate the maximum possible heat addition. Find all fluid properties at the outlet section and the reduction in stagnation pressure. Show the process path on a Ts diagram, indicating all static and stagnation state points.

Given: Data on flow through gas turbine combustor

Find: Maximum heat addition; Outlet conditions; Reduction in stagnation pressure; Plot of process

Solution:

The given or available data is:

$R =$ 286.9 J/kg·K

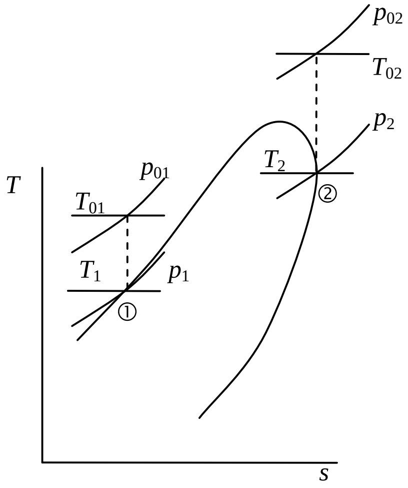
$k =$ 1.4

$c_p =$ 1004 J/kg·K

$T_1 =$ 773 K

$p_1 =$ 1.5 MPa

$M_1 =$ 0.5



Equations and Computations:

From $p_1 = \rho_1 R T_1$ $\rho_1 =$ 6.76 kg/m^3

From $V_1 = M_1\sqrt{kRT_1}$ $V_1 =$ 279 m/s

Using built-in function *IsenT* (M,k):

$T_{01}/T_1 =$ 1.05 $T_{01} =$ 812 K

Using built-in function *Isenp* (M,k):

$p_{01}/p_1 =$ 1.19 $p_{01} =$ 1.78 MPa

For maximum heat transfer: $M_2 =$ 1

Using built-in function *rayT0* (M,k), *rayp0* (M,k), *rayT* (M,k), *rayp* (M,k), *rayV* (M,k):

$T_{01}/T_0^* =$ 0.691 $T_0^* =$ 1174 K $(= T_{02})$

$p_{01}/p_0^* =$ 1.114 $p_0^* =$ 1.60 MPa $(= p_{02})$

$T/T^* =$ 0.790 $T^* =$ 978 K $(= T_{02})$

$p/p^* =$ 1.778 $p^* =$ 0.844 MPa $(= p_2)$

$\rho^*/\rho =$ 0.444 $\rho^* =$ 3.01 kg/m^3 $(= \rho_2)$

Note that at state 2 we have critical conditions!

Hence: $p_{012} - p_{01} =$ -0.182 MPa -182 kPa

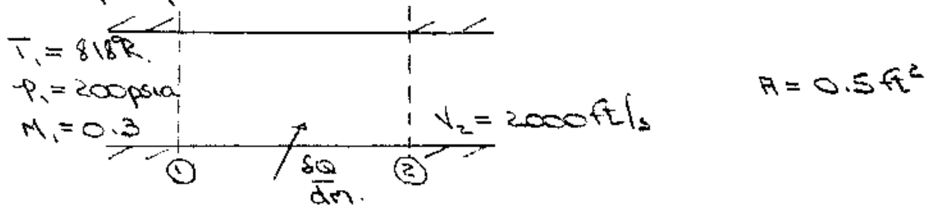
From the energy equation: $$\frac{\delta Q}{dm} = c_p\left(T_{02} - T_{01}\right)$$

$\delta Q/dm =$ 364 kJ/kg

Problem 13.94

[3]

Given: Combustor modeled as frictionless flow through a constant-area duct with heat addition. Air fuel ratio is large enough so properties are those of air



Find: (a) T_2, p_2, ρ_2, M_2 (b) $\dot{Q}$

Solution: (using compressible flow functions – Appendix E)

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad p = \rho R T \quad \dot{m} = \rho V A$

Assumptions: (1) steady flow (2) frictionless flow
(3) ideal gas, properties are those of air
(4) uniform flow at a section
(5) $\dot{W}_s = \dot{W}_{shear} = 0$

$$\rho_1 = \frac{p_1}{RT_1} = 200\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\,°\text{R}}{53.3\,\text{ft}\cdot\text{lbf}} \times \frac{1}{818°\text{R}} \times 144\frac{\text{in}^2}{\text{ft}^2} = 0.6606\ \text{lbm/ft}^3$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.3\left[1.4 \times 53.3\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°\text{R}} \times 818°\text{R} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/2} = 421\ \text{ft/s}$$

From App. E.1 $\quad M_1 = 0.3 \quad T/T_0 = 0.9823 \quad \therefore T_{0_1} = 832.7°R$
$p/p_0 = 0.9395 \quad \therefore p_{0_1} = 212.9$ psia

From continuity, $\rho_1 V_1 = \rho_2 V_2$

$\therefore \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{421}{2000} \times 0.6606\ \text{lbm/ft}^3 = 0.139\ \text{lbm/ft}^3$ $\quad \rho_2$

From App E.3

$M_1 = 0.3 \quad \frac{T_{0_1}}{T_0^*} = 0.3469 \quad \frac{p_{0_1}}{p_0^*} = 1.199 \quad \frac{T_1}{T^*} = 0.4089 \quad \frac{p_1}{p^*} = 2.131 \quad \frac{V_1}{V^*} = 0.1918$

$\therefore T_0^* = 2400R \quad p_0^* = 177.6$ psia $\quad T^* = 2000R \quad p^* = 93.85$ psia $\quad V^* = 2193\ \frac{\text{ft}}{\text{sec}}$

At section ② $\quad V_2/V^* = 2000/2193 = 0.9120$

From App. E.3 $\quad M_2 = 0.90$ $\quad M_2$

$T_{0_2}/T_0^* = 0.9921$, $p_{0_2}/p_0^* = 1.005$, $T_2/T^* = 1.025$, $p_2/p^* = 1.125$

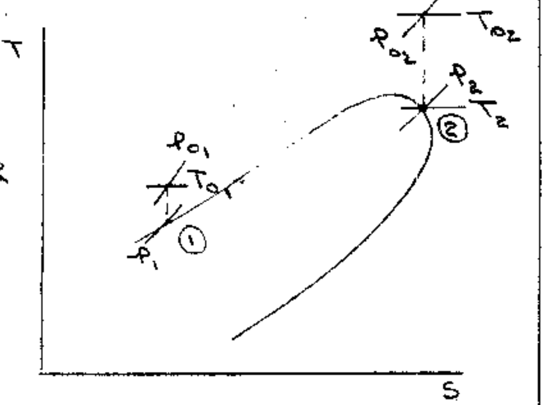
$\therefore T_{0_2} = 2380R$, $p_{0_2} = 178$ psia, $T_2 = 2050R$, $p_2 = 106$ psia $\quad T_{0_2}, p_{0_2}, T_2, p_2$

From the energy equation,

$$\dot{Q} = \dot{m}\frac{\delta Q}{dm} = \rho_1 V_1 A (h_{0_2} - h_{0_1}) = \rho_1 V_1 A c_p (T_{0_2} - T_{0_1})$$

$$\dot{Q} = 0.6606\frac{\text{lbm}}{\text{ft}^3} \times 420.6\frac{\text{ft}}{\text{s}} \times 0.5\,\text{ft}^2 \times 0.24\frac{\text{BTU}}{\text{lbm}\,°\text{R}} \times (2380 - 833)°\text{R}$$

$\dot{Q} = 5.16 \times 10^4$ Btu/s $\quad \dot{Q}$



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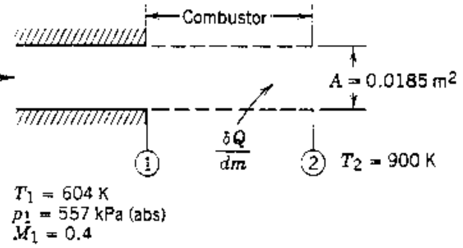
Problem 13.95

[3]

Given: Steady flow combustor operating under conditions shown. Assume thermodynamic properties are those of pure air.

Find: (a) T_{0_2}
(b) M_2
(c) $\delta Q/dm$, $\dot{Q}$
(d) $\dot{Q}/\dot{Q}_{max}$



Solution:
(using compressible flow functions - Appendix E)

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad \dot{m} = \rho V A \quad p = \rho R T$

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) $\dot{W}_s = \dot{W}_{shear} = 0$

$$\rho_1 = \frac{p_1}{RT_1} = 557\times10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287\, N\cdot m} \times \frac{1}{604\ K} = 3.21\ kg/m^3$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.4\left(1.4\times287 \frac{N\cdot m}{kg\cdot K}\times 604K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 197\ m/s$$

$$\dot{m} = \rho_1 V_1 A = 3.21 \frac{kg}{m^3} \times 197 \frac{m}{s} \times 0.0185\ m^2 = 11.7\ kg/s$$

From App. E.1 at $M_1 = 0.4$, $T_1/T_{0_1} = 0.969$ $\quad \therefore T_{0_1} = 623\ K$

From App. E.3 at $M_1 = 0.4$, $T/T^* = 0.6152$, $T_0/T_0^* = 0.5290$

$\therefore T^* = 982$ and $T_0^* = 1180\ K$

At section ② $T_2 = 900\ K$ $\quad \therefore (T/T^*)_2 = 0.916$

From App. E.3, $M_2 = 0.60$, $(T_0/T_0^*)_2 = 0.8189$

$\therefore T_{0_2} = 966\ K$, $M_2 = 0.60$ ← M_2, T_{0_2}

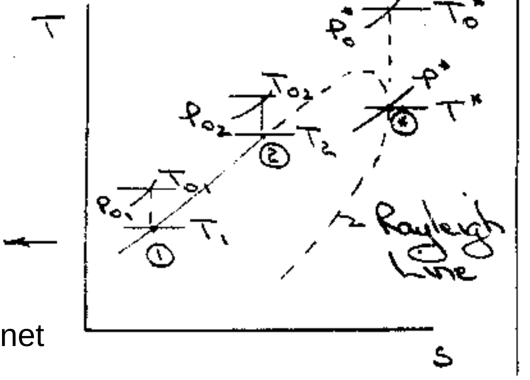
From the energy equation,

$$\frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p (T_{0_2} - T_{0_1}) = 1.00 \frac{kJ}{kg\cdot K}(966-623)K = 343 \frac{kJ}{kg}$$ ← $\delta Q/dm$

$$\dot{Q} = \dot{m}\frac{\delta Q}{dm} = 11.7 \frac{kg}{sec} \times 343 \frac{kJ}{kg} = 4010 \frac{kJ}{sec} = 4010\ kW$$ ← $\dot{Q}$

The heat transfer may be expressed as a fraction of the maximum possible heat addition

$$\frac{\delta Q/dm}{(\delta Q/dm)_{max}} = \frac{T_{0_2} - T_{0_1}}{T_0^* - T_{0_1}} = \frac{966-623}{1180-623} = 0.616$$ ←



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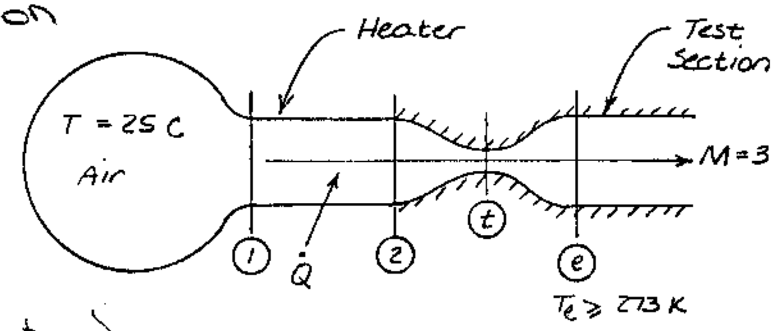
Problem 13.96

[3]

Given: Supersonic wind tunnel, with test section Mach number $M = 3$, is supplied from a high-pressure tank of air at 25°C. Air from the tank is heated in a short, constant area section upstream of the converging-diverging nozzle which feeds the test section. The heat addition, $\dot{Q} = 10\text{ kW}$ is sufficient to ensure $T > 0°C$ at the entrance to the test section

Find: (a) T_{0_2}
(b) $\dot{m}_{max}$
(c) A_e/A_t



Solution:

(using compressible flow functions)

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad \dot{m} = \rho V A \quad p = \rho R T$

Assumptions: (1) steady flow (2) uniform flow at a section
(3) frictionless flow in the heater
(4) isentropic flow through the nozzle
(5) ideal gas (6) $\dot{W}_s = \dot{W}_{shear} = 0$

$T_{0_1} = T = 298\text{ K}$

From Appendix E.1 at $M = 3$, $T/T_0 = 0.3571$, $A_e/A^* = A_e/A_t = 4.23$ ← $\frac{A}{A_t}$

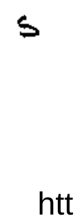
With $T_e = 273\text{K}$, $T_{0_e} = T_{0_2} = 273\text{K}/0.3571 = 764\text{ K}$ ← T_{0_2}

From the energy equation $\frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1})$

Since $\dot{Q} = \dot{m}\frac{\delta Q}{dm}$, then $\dot{m} = \frac{\dot{Q}}{c_p(T_{0_2} - T_{0_1})}$

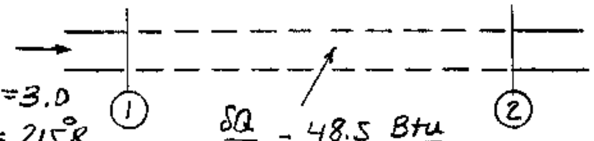
and

$$\dot{m} = 10\text{ kW} \times \frac{10^3\text{ N}\cdot\text{m}}{\text{s}\cdot\text{kW}} \times \frac{\text{kg}\cdot\text{K}}{1000\text{ N}\cdot\text{m}} \times \frac{1}{(764-298)\text{K}} = 0.0215\text{ kg/s}$$ ← $\dot{m}$



Problem 13.97

Given: Frictionless flow of air in a constant-area duct, discharging to atmospheric pressure, with the flow conditions shown.



$M_1 = 3.0$
$T_1 = 215°R$
$p_1 = 1.73$ psia

$\frac{\delta Q}{dm} = 48.5 \ \frac{Btu}{lbm}$

p_{atm} (outside)

Find: (a) Compare p_2 with atmospheric pressure.
(b) Is M_2 greater than, equal to, or less than unity?
(c) Sketch p vs. x along the channel.

Solution: Apply equations for Rayleigh line flow of an ideal gas.

Basic equations: $c_p T_{01} + \frac{\delta Q}{dm} = c_p T_{02}$ $\qquad p_1 A - p_2 A = \dot{m}(V_2 - V_1)$

Computing equation: $T_0 = T\left(1 + \frac{k-1}{2} M^2\right)$

Assumptions: (1) Steady flow
(2) Frictionless flow
(3) Uniform flow at each section
(4) Ideal gas
(5) $F_{B_x} = 0$
(6) $\dot{W}_s = \dot{W}_{shear} = 0$
(7) $\Delta z = 0$

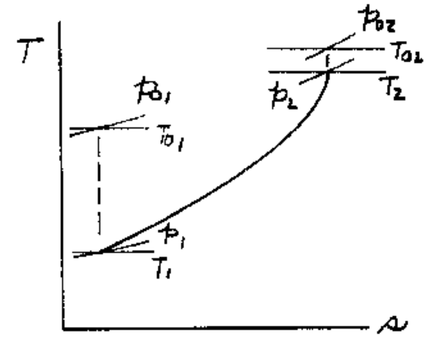
The minimum possible Mach number for supersonic flow with heating is $M_2 = 1$.

$$T_{01} = T_1\left(1 + \frac{k-1}{2} M^2\right) = 215°R\left(1 + 0.2(3.0)^2\right) = 613°R$$

$$T_{02} = T_{01} + \frac{1}{c_p}\frac{\delta Q}{dm} = 613°R + \frac{lbm \cdot °R}{0.240 \ Btu} \times 48.5 \frac{Btu}{lbm} = 815°R$$

Check for $M_2 = 1.0$: $T_2 = \frac{T_{02}}{1 + 0.2(1)^2} = \frac{815°R}{1.2} = 679°R$

$$V_2 = c_2 = \sqrt{kRT_2} = 1,280 \ ft/s$$



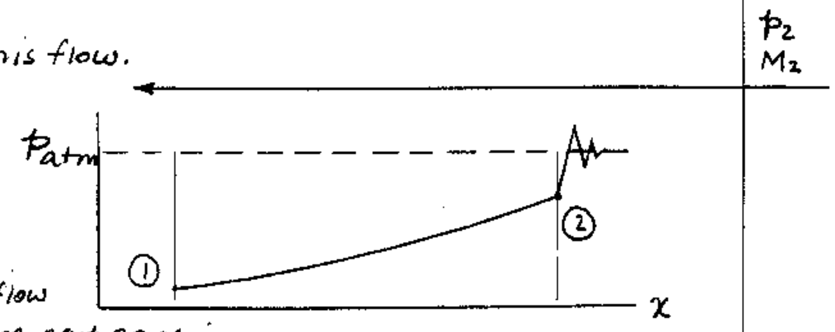
Thus $p_2 = p_1 + \frac{\dot{m}}{A}(V_1 - V_2) = p_1 + \rho_1 V_1 (V_1 - V_2)$

$V_1 = M_1 c_1 = M_1\sqrt{kRT_1} = 3.0 \times 719 \frac{ft}{s} = 2,160 \ ft/s$; $\rho_1 = \frac{p_1}{RT_1} = 0.0217 \ lbm/ft^3$

$$p_2 = 1.73 \frac{lbf}{in.^2} + 0.0217 \frac{lbm}{ft^3} \times 2160 \frac{ft}{s} (2160 - 1280)\frac{ft}{s} \times \frac{slug}{32.2 \ lbm} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{ft^2}{144 \ in.^2} = 10.6 \ psia$$

Thus if $M_2 = 1.0$, the $p_2 < p_{atm}$, which is not possible for sonic flow. Therefore $p_2 < p_{atm}$ and $M_2 > 1.0$ for this flow.

The pressure vs. distance plot is:



{This problem could be solved quantitatively, but only by either iteration or by use of compressible flow functions; that solution is presented on the next page.}

Problem 13.97

[4] Part 2/2

For $M_1 = 3.0$, from App. E.3: $\frac{T_0}{T_0^*} = 0.6540$, and $\frac{p}{p^*} = 0.1765$

Thus $T_0^* = \frac{T_{01}}{(T_0/T_0^*)_1} = \frac{613°R}{0.6540} = 937°R$; $p^* = \frac{p_1}{(p/p^*)_1} = \frac{1.73 \text{ psia}}{0.1765} = 9.80 \text{ psia}$

At section 2, $T_{02} = 815\ R$ and $\frac{T_{02}}{T_0^*} = \frac{815°R}{937°R} = 0.870$

From App. E.3 this corresponds to $M_2 = 1.74$. At this Mach number, $\frac{p}{p^*} = 0.4581$.

Thus $p_2 = p^* \left(\frac{p}{p^*}\right)_2 = 9.80 \text{ psia} \times 0.4581 = 4.49 \text{ psia}$ $\longleftarrow$ p_2

{ These calculations confirm that $M_2 > 1$ and $p_2 < p_{atm}$. }

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Problem 13.98

[4] Part 1/2

Given: Aircraft cabin pressurization system, $\dot{m} = 0.75$ lbm/s

Cruise is $M = 0.85$ at $z = 40{,}000$ ft (State 1)

Air slowed isentropically to 100 ft/s w.r.to plane (State 2)

Air compressed adiabatically to pressure equivalent to 8,000 ft altitude; $\Delta T = 170°F$ (State 3); $V_3 \simeq V_2$

Air is cooled at constant pressure, with negligible friction to 70°F (State 4); $V_4 \simeq V_3$

Find:
(a) Sketch a system diagram, labeling all components.
(b) Determine static and stagnation temperatures at each cross-section.
(c) Evaluate compressor work added and heat rejected in cooling process.

Solution: From the standard atmosphere, at $z_1 = 40{,}000$ ft, $T_1 = -70°F$ and $p_1 = 2.73$ psia

$$c_1 = \sqrt{kRT_1} = \left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times (460-70)°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}} = 968 \frac{ft}{s}$$

$$V_1 = M_1 c_1 = 0.85 \times 968 \frac{ft}{s} = 823 \ ft/s$$

For isentropic deceleration $T_{01} = T_1 + \frac{V_1^2}{2c_p} = T_{02} = T_2 + \frac{V_2^2}{2c_p}$

$$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

$$T_2 = 390°R + \frac{1}{2} \times \frac{lbm \cdot °R}{0.240 \ Btu}\left[(823)^2 - (100)^2\right]\frac{ft^2}{s^2} \times \frac{Btu}{778 \ ft \cdot lbf} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{slug}{32.2 \ lbm} = 445°R$$

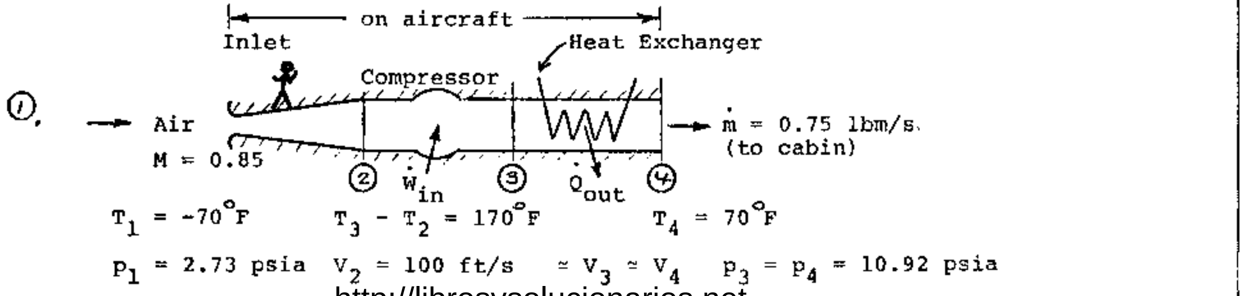
For isentropic deceleration,

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{k/k-1} = 2.73 \ psia\left(\frac{445}{390}\right)^{3.5} = 4.33 \ psia$$

The compressor raises the air the equivalent of $z_3 = z_4 = 8{,}000$ ft; from the Standard Atmosphere

$$p_3 = p_4 = 10.92 \ psia \ ; \ p_{03} = p_3\left(1 + \frac{k-1}{2}M_3^2\right)^{\frac{k}{k-1}} = 10.92\left(1 + 0.2(0.0823)^2\right)^{3.5} = 10.97 \ psia$$

Assume $V_2 \simeq V_3 \simeq V_4$ since no other data are known. The system is



Problem 13.98

Evaluating properties: $T_{01} = T_1\left(1+\frac{k-1}{2}M_1^2\right) = 390°R\left(1+0.2(0.85)^2\right) = 446°R = T_{02}$

$p_{01} = p_1\left(1+\frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = 2.73\,\text{psia}\left(1+0.2(0.85)^2\right)^{3.5} = 4.38\,\text{psia} = p_{02}$

$T_3 = T_2 + 170°R = 445 + 170°R = 615°R$; $T_{03} = T_{02} + 170°R = 616°R$ (since $V_2 \simeq$ const)

From the energy equation

$\dot{W}_{in} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2)$

$\dot{W}_{in} = 0.75\,\frac{\text{lbm}}{\text{s}} \times 0.240\,\frac{\text{Btu}}{\text{lbm}\cdot°R}\,(170)°R \times 778\,\frac{\text{ft}\cdot\text{lbf}}{\text{Btu}} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 43.3\,\text{hp}$ $\dot{W}_{in}$

$\dot{Q}_{in} = \dot{m}(h_4 - h_3) = \dot{m}c_p(T_4 - T_3)$

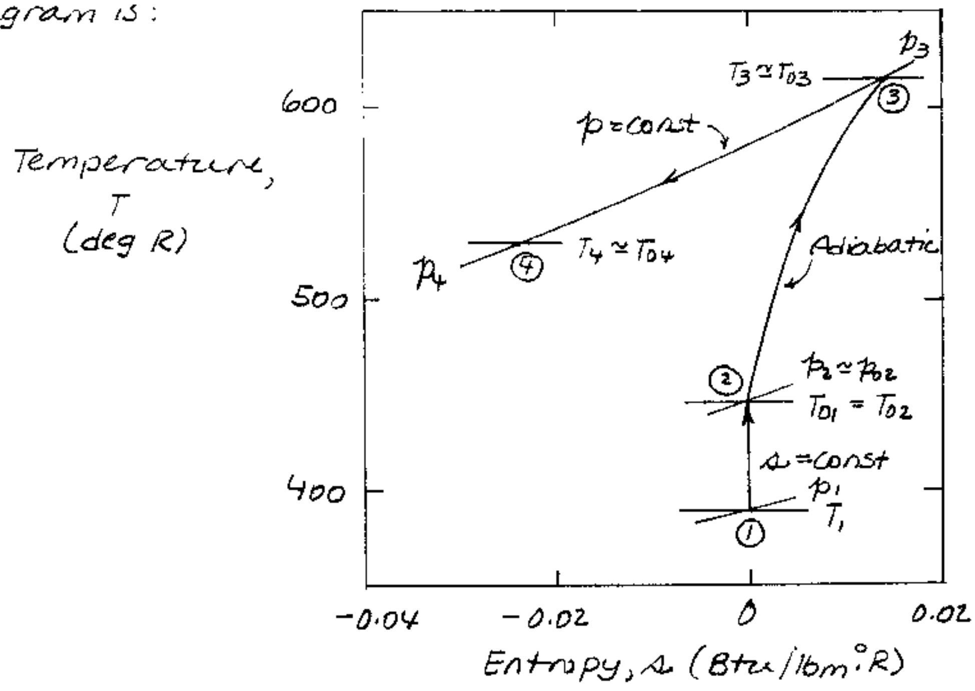
$\dot{Q}_{in} = 0.75\,\frac{\text{lbm}}{\text{s}} \cdot 0.240\,\frac{\text{Btu}}{\text{lbm}\cdot°R}(530 - 615)°R = -15.3\,\text{Btu/s}$ (out) $\dot{Q}_{in}$

The entropy changes are computed using $Tds = dh - vdp$, as

$$\Delta s_{23} = c_p \ln\frac{T_{03}}{T_{02}} - R\ln\frac{p_{03}}{p_{02}} = 0.240\,\frac{\text{Btu}}{\text{lbm}\cdot°R}\ln\left(\frac{616}{446}\right) - 53.3\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°R}\ln\left(\frac{10.97}{4.38}\right)\frac{\text{Btu}}{778\,\text{ft}\cdot\text{lbf}} = 0.0146\,\frac{\text{Btu}}{\text{lbm}\cdot°R}$$

$$\Delta s_{34} = c_p \ln\frac{T_{04}}{T_{03}} - R\ln\frac{p_{04}}{p_{03}} = 0.240\,\frac{\text{Btu}}{\text{lbm}\cdot°R}\ln\left(\frac{530}{616}\right) - \frac{53.3}{778}\,\frac{\text{Btu}}{\text{lbm}\cdot°R}\ln\left(\frac{10.97}{10.97}\right) = -0.0361\,\text{Btu/lbm}\cdot°R$$

The Ts diagram is:



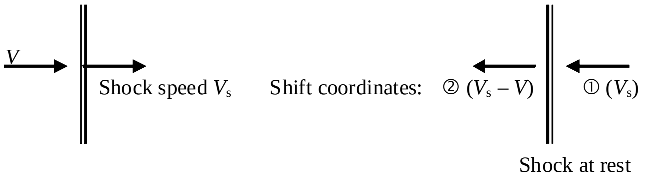
Problem 13.99

[3]

13.99 Testing of a demolition explosion is to be evaluated. Sensors indicate that the shock wave generated at the instant of explosion is 30 MPa (abs). If the explosion occurs in air at 20°C and 101 kPa, find the speed of the shock wave, and the temperature and speed of the air just after the shock passes. As an approximation assume $k = 1.4$. (Why is this an approximation?)

Given: Normal shock due to explosion

Find: Shock speed; temperature and speed after shock



Solution:

Basic equations:

$$M_2^{\,2} = \frac{M_1^{\,2} + \dfrac{2}{k-1}}{\left(\dfrac{2\cdot k}{k-1}\right)\cdot M_1^{\,2} - 1} \qquad V = M\cdot c = M\cdot\sqrt{k\cdot R\cdot T}$$

$$\frac{p_2}{p_1} = \frac{2\cdot k}{k+1}\cdot M_1^{\,2} - \frac{k-1}{k+1} \qquad \frac{T_2}{T_1} = \frac{\left(1 + \dfrac{k-1}{2}\cdot M_1^{\,2}\right)\cdot\left(k\cdot M_1^{\,2} - \dfrac{k-1}{2}\right)}{\left(\dfrac{k+1}{2}\right)^2\cdot M_1^{\,2}}$$

Given or available data $k = 1.4$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$ $\quad p_2 = 30\cdot MPa$ $\quad p_1 = 101\cdot kPa$ $\quad T_1 = (20+273)\cdot K$

From the pressure ratio

$$M_1 = \sqrt{\left(\frac{k+1}{2\cdot k}\right)\cdot\left(\frac{p_2}{p_1} + \frac{k-1}{k+1}\right)} \qquad M_1 = 16.0$$

Then we have

$$T_2 = T_1\cdot\frac{\left(1 + \dfrac{k-1}{2}\cdot M_1^{\,2}\right)\cdot\left(k\cdot M_1^{\,2} - \dfrac{k-1}{2}\right)}{\left(\dfrac{k+1}{2}\right)^2\cdot M_1^{\,2}} \qquad T_2 = 14790\,K \qquad T_2 = 14517\cdot{}^{\circ}C$$

$$M_2 = \sqrt{\frac{M_1^{\,2} + \dfrac{2}{k-1}}{\left(\dfrac{2\cdot k}{k-1}\right)\cdot M_1^{\,2} - 1}} \qquad M_2 = 0.382$$

Then the speed of the shock ($V_s = V_1$) is $\quad V_1 = M_1\cdot\sqrt{k\cdot R\cdot T_1}$ $\quad V_1 = 5475\,\frac{m}{s}$ $\quad V_s = V_1$ $\quad V_s = 5475\,\frac{m}{s}$

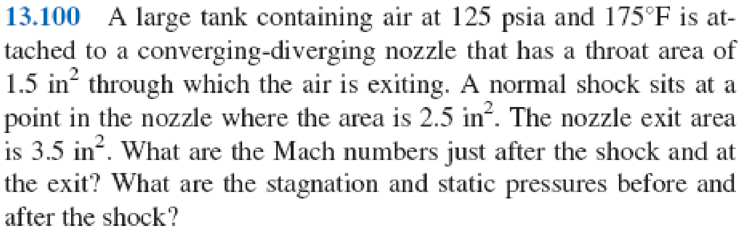
After the shock (V_2) the speed is $\quad V_2 = M_2\cdot\sqrt{k\cdot R\cdot T_2}$ $\quad V_2 = 930\,\frac{m}{s}$

But we have $\quad V_2 = V_s - V$ $\quad V = V_s - V_2$ $\quad V = 4545\,\frac{m}{s}$

These results are unrealistic because at the very high post-shock temperatures experienced, the specific heat ratio will NOT be constant! The extremely high initial air velocity and temperature will rapidly decrease as the shock wave expands in a spherical manner and thus weakens.

Problem 13.100

[3]



13.100 A large tank containing air at 125 psia and 175°F is attached to a converging-diverging nozzle that has a throat area of 1.5 in^2 through which the air is exiting. A normal shock sits at a point in the nozzle where the area is 2.5 in^2. The nozzle exit area is 3.5 in^2. What are the Mach numbers just after the shock and at the exit? What are the stagnation and static pressures before and after the shock?

Given: C-D nozzle with normal shock

Find: Mach numbers at the shock and at exit; Stagnation and static pressures before and after the shock

Solution:

Basic equations: Isentropic flow

$$\frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}} \qquad \frac{p_0}{p} = \left(1+\frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$$

Normal shock

$$M_2^{\ 2} = \frac{M_1^{\ 2}+\frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_1^{\ 2}-1} \qquad \frac{p_2}{p_1} = \frac{2\cdot k}{k+1}\cdot M_1^{\ 2}-\frac{k-1}{k+1} \qquad \frac{p_{02}}{p_{01}} = \frac{\left(\frac{\frac{k+1}{2}\cdot M_1^{\ 2}}{1+\frac{k-1}{2}\cdot M_1^{\ 2}}\right)^{\frac{k}{k-1}}}{\left(\frac{2\cdot k}{k+1}\cdot M_1^{\ 2}-\frac{k-1}{k+1}\right)^{\frac{1}{k-1}}}$$

Given or available data $k = 1.4$ $\qquad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$ $\qquad p_{01} = 125\cdot psi$ $\qquad T_0 = (175+460)\cdot R$

$A_t = 1.5\cdot in^2$ $\qquad A_s = 2.5\cdot in^2$ $\qquad$ (Shock area) $\qquad A_e = 3.5\cdot in^2$

Because we have a normal shock the CD must be accelerating the flow to supersonic so the throat is at critical state.

$$A_{crit} = A_t$$

At the shock we have $\frac{A_s}{A_{crit}} = 1.667$

At this area ratio we can find the Mach number before the shock from the isentropic relation $\frac{A_s}{A_{crit}} = \frac{1}{M_1}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M_1^{\ 2}}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$

Solving iteratively (or using *Excel*'s *Solver*, or even better the function *isenMsupfromA* from the Web site!) $\qquad M_1 = 1.985$

The stagnation pressure before the shock was given: $\qquad p_{01} = 125\,psi$

The static pressure is then $\qquad p_1 = \frac{p_{01}}{\left(1+\frac{k-1}{2}\cdot M_1^{\ 2}\right)^{\frac{k}{k-1}}}$ $\qquad p_1 = 16.4\,psi$

After the shock we have $M_2 = \sqrt{\dfrac{{M_1}^2 + \dfrac{2}{k-1}}{\left(\dfrac{2 \cdot k}{k-1}\right) \cdot {M_1}^2 - 1}}$ $M_2 = 0.580$

Also $p_{02} = p_{01} \cdot \dfrac{\left(\dfrac{\dfrac{k+1}{2} \cdot {M_1}^2}{1 + \dfrac{k-1}{2} \cdot {M_1}^2}\right)^{\frac{k}{k-1}}}{\left(\dfrac{2 \cdot k}{k+1} \cdot {M_1}^2 - \dfrac{k-1}{k+1}\right)^{\frac{1}{k-1}}}$ $p_{02} = 91.0\,\text{psi}$

and $p_2 = p_1 \cdot \left(\dfrac{2 \cdot k}{k+1} \cdot {M_1}^2 - \dfrac{k-1}{k+1}\right)$ $p_2 = 72.4\,\text{psi}$

Finally, for the Mach number at the exit, we could find the critical area change across the shock; instead we find the new critical area from isentropic conditions at state 2.

$$A_{crit2} = A_s \cdot M_2 \cdot \left(\frac{1 + \dfrac{k-1}{2} \cdot {M_2}^2}{\dfrac{k+1}{2}}\right)^{-\frac{k+1}{2 \cdot (k-1)}} \qquad A_{crit2} = 2.06\,\text{in}^2$$

At the exit we have $\dfrac{A_e}{A_{crit2}} = 1.698$

At this area ratio we can find the Mach number before the shock from the isentropic relation $\dfrac{A_e}{A_{crit2}} = \dfrac{1}{M_e} \cdot \left(\dfrac{1 + \dfrac{k-1}{2} \cdot {M_e}^2}{\dfrac{k+1}{2}}\right)^{\frac{k+1}{2 \cdot (k-1)}}$

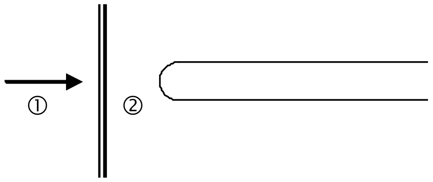
Solving iteratively (or using *Excel*'s *Solver*, or even better the function *isenMsubfromA* from the Web site!) $M_e = 0.369$

These calculations are obviously a LOT easier using the *Excel* functions available on the Web site!

Problem 13.101

[2]

13.101 A normal shock occurs when a pitot-static tube is inserted into a supersonic wind tunnel. Pressures measured by the tube are $p_{0_2} = 10$ psia and $p_2 = 8$ psia. Before the shock, $T_1 = 285°\text{R}$ and $p_1 = 1.5$ psia. Calculate the air speed in the wind tunnel.



Given: Normal shock near pitot tube

Find: Air speed

Solution:

Basic equations: $p_1 - p_2 = \rho_1 \cdot V_1 \cdot \left(V_2 - V_1\right)$ (Momentum) $\dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $T_1 = 285 \cdot R$ $p_1 = 1.75 \cdot psi$ $p_{02} = 10 \cdot psi$ $p_2 = 8 \cdot psi$

$k = 1.4$ $R_{air} = 53.33 \cdot \dfrac{ft \cdot lbf}{lbm \cdot R}$

At state 2 $M_2 = \sqrt{\dfrac{2}{k-1} \cdot \left[\left(\dfrac{p_{02}}{p_2}\right)^{\frac{k-1}{k}} - 1\right]}$ $M_2 = 0.574$

From momentum $p_1 - p_2 = \rho_2 \cdot V_2^{\,2} - \rho_1 \cdot V_1^{\,2}$ but $\rho \cdot V^2 = \rho \cdot c^2 \cdot M^2 = \dfrac{p}{R \cdot T} \cdot k \cdot R \cdot T \cdot M^2 = k \cdot p \cdot M^2$

$p_1 - p_2 = k \cdot p_2 \cdot M_2^{\,2} - k \cdot p_1 \cdot M_1^{\,2}$ or $p_1 \cdot \left(1 + k \cdot M_1^{\,2}\right) = p_2 \cdot \left(1 + k \cdot M_2^{\,2}\right)$

Hence $M_1 = \sqrt{\dfrac{1}{k} \cdot \left[\dfrac{p_2}{p_1} \cdot \left(1 + k \cdot M_2^{\,2}\right) - 1\right]}$ $M_1 = 2.01$

Also $c_1 = \sqrt{k \cdot R_{air} \cdot T_1}$ $c_1 = 827 \dfrac{ft}{s}$

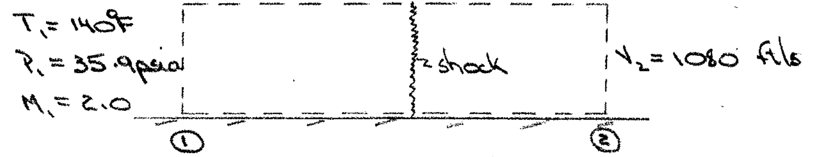
Then $V_1 = M_1 \cdot c_1$ $V_1 = 1666 \dfrac{ft}{s}$

Note: With $p_1 = 1.5$ psi we obtain $V_1 = 1822 \cdot \dfrac{ft}{s}$

(Using normal shock functions, for $\dfrac{p_2}{p_1} = 4.571$ we find $M_1 = 2.02$ $M_2 = 0.573$ Check!)

Problem 13.102

Given: Steady flow of air through a constant area duct



Find: ρ_2, M_2, sketch pressure distribution

Solution:

Basic equation: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions:
- (1) steady flow
- (2) uniform flow at a section
- (3) $\dot{Q} = \dot{W}_s = \dot{W}_{shear} = 0$
- (4) $\Delta z = 0$
- (5) ideal gas
- (6) $A_1 = A_2 = A$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0\left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 600°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 2400 \text{ ft/s}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \qquad T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

$$T_2 = 600°R + \frac{1}{2}\left[(2.4)^2 - (1.08)^2\right] \times 10^6 \frac{ft^2}{s^2} \times \frac{lbm \cdot °R}{0.24 \, Btu} \times \frac{Btu}{778 \, ft \cdot lbf} \times \frac{slug}{32.2 \, lbm} \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

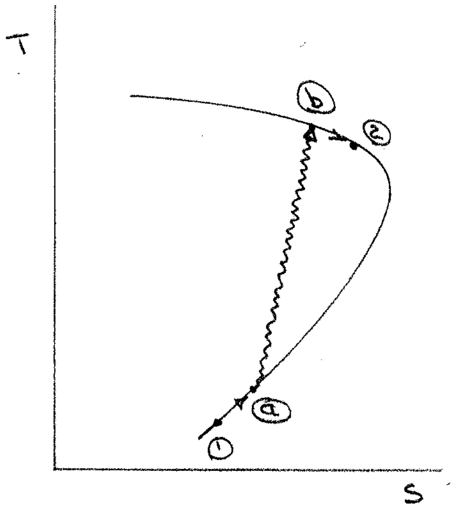
$$T_2 = 982°R$$

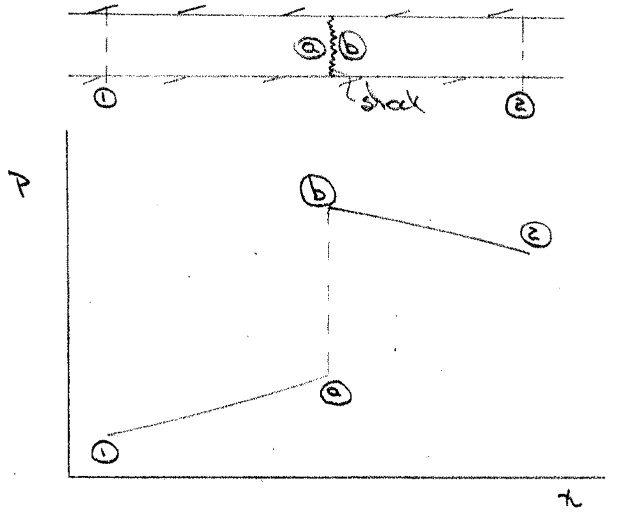
$$M_2 = \frac{V_2}{c_2} \qquad c_2 = (kRT_2)^{1/2} = \left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 982°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1540 \text{ ft/s}$$

$$M_2 = \frac{1080}{1540} = 0.701 \longleftarrow M_2$$

From continuity $\rho_1 V_1 = \rho_2 V_2 \qquad \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{V_1}{V_2}\frac{p_1}{RT_1}$

$$\rho_2 = \frac{2400}{1080} \times 35.9 \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3 \, ft \cdot lbf} \times \frac{1}{600°R} \times \frac{144 \, in^2}{ft^2} = 0.359 \text{ lbm/ft}^3 \longleftarrow \rho_2$$





42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
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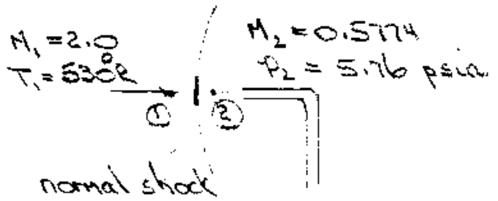
Problem 13.103

Given: A total-pressure probe is placed in a supersonic flow, $M = 2.0$. Behind the shock $M_2 = 0.5774$, $p_2 = 5.76$ psia



Find: a) T_{02}, p_{02} b) p_{01}, T_{01}, p_1, ρ_1, V_1

Solution:

Computing equations: $p(1+kM^2) = \text{const.}$ (across shock)

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/k-1}$$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) ideal gas

Across the shock $p(1+kM^2) = \text{const}$

$$\therefore p_1 = p_2\frac{(1+kM_2^2)}{(1+kM_1^2)} = 5.76 \text{ psia}\,\frac{[1+1.4(0.5774)^2]}{[1+1.4(2)^2]} = 1.28 \text{ psia} \longleftarrow p_1$$

$$\rho_1 = \frac{p_1}{RT_1} = 1.28\,\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\,R}{53.3\,\text{ft}\cdot\text{lbf}} \times \frac{1}{530°R} \times 144\,\frac{\text{in}^2}{\text{ft}^2} = 0.00653\ \text{lbm/ft}^3 \longleftarrow \rho_1$$

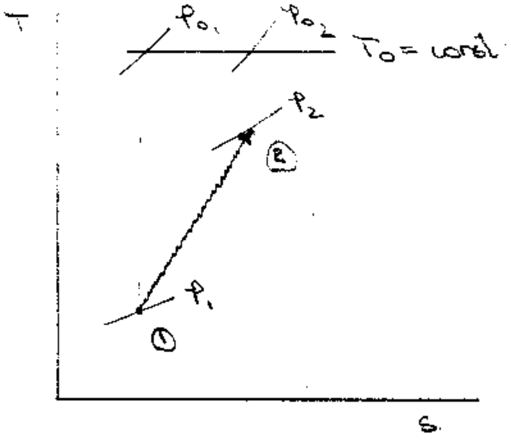
$$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 2.0\left[1.4 \times 53.3\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 530°R \times 32.2\,\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/2} = 2260\ \text{ft/s} \longleftarrow V_1$$

$$T_{01} = T_1\left(1+\frac{k-1}{2}M_1^2\right) = 530°R[1+0.2(2)^2] = 954°R \longleftarrow T_{01}$$

$$p_{01} = p_1\left(1+\frac{k-1}{2}M_1^2\right)^{k/k-1} = 1.28 \text{ psia}(1.8)^{3.5} = 10.0 \text{ psia} \longleftarrow p_{01}$$

$$p_{02} = p_2\left(1+\frac{k-1}{2}M_2^2\right)^{k/k-1} = 5.76 \text{ psia}[1+0.2(0.5774)^2]^{3.5} = 7.22 \text{ psia} \longleftarrow p_{02}$$

$$T_{02} = T_{01} = 954\,R. \longleftarrow$$



Compressible-Flow Functions (Appendix E)

For $M_1 = 2.0$, from App E.4

$M_2 = 0.577$ (12.34b)

$p_2/p_1 = 4.50$ (12.36a) $\therefore p_1 = 1.28$ psia

$p_{02}/p_{01} = 0.721$ (12.37)

For $M_1 = 2.0$, from App. E.1

$p_1/p_{01} = 0.1278$ (11.17a) $\therefore p_{01} = 10.0$ psia

and $p_{02} = 0.721\,p_{01} = 7.21$ psia

$T_1/T_0 = 0.556$ (11.17b) $\therefore T_{01} = 954°R$

Note: In using the tables it is not necessary to know the downstream Mach number.

42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
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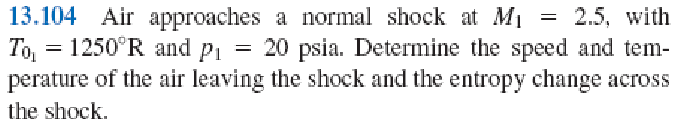
Problem 13.104

[2]



13.104 Air approaches a normal shock at $M_1 = 2.5$, with $T_{0_1} = 1250°\text{R}$ and $p_1 = 20$ psia. Determine the speed and temperature of the air leaving the shock and the entropy change across the shock.

Given: Normal shock

Find: Speed and temperature after shock; Entropy change

Solution:

The given or available data is:

$R =$ 53.33 ft·lbf/lbm·R 0.0685 Btu/lbm·R

$k =$ 1.4

$c_p =$ 0.2399 Btu/lbm·R

$T_{01} =$ 1250 $^\text{o}$R

$p_1 =$ 20 psi

$M_1 =$ 2.5

Equations and Computations:

From $p_1 = \rho_1 R T_1$ $\rho_1 =$ 300.02 kg/m^3

$V_1 =$ 764 m/s

Using built-in function *IsenT* (M,k):

$T_{01}/T_1 =$ 2.25 $T_1 =$ 556 $^\text{o}$R

96 $^\text{o}$F

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.513

Using built-in function *NormTfromM* (M,k):

$T_2/T_1 =$ 2.14 $T_2 =$ 1188 $^\text{o}$R

728 $^\text{o}$F

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 7.13 $p_2 =$ 143 psi

From $V_2 = M_2\sqrt{kRT_2}$ $V_2 =$ 867 ft/s

From $\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$

$\Delta s =$ 0.0476 Btu/lbm·R

37.1 ft·lbf/lbm·R

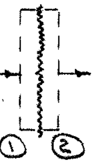
Problem 13.105

Given: Air flow through a normal shock as shown:

$T_1 = 35°C$

$p_1 = 229$ kPa (abs)

$V_1 = 704$ m/sec



Find: T_2, p_{0_2}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions:
- (1) steady flow
- (2) uniform flow at a section
- (3) $\dot{Q} = \dot{W}_s = \dot{W}_{shear} = 0$
- (4) $\Delta z = 0$
- (5) $F_{B_x} = 0$
- (6) no friction forces
- (7) ideal gas
- (8) $A_1 = A_2 = A$

$$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 308\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 352 \text{ m/s}$$

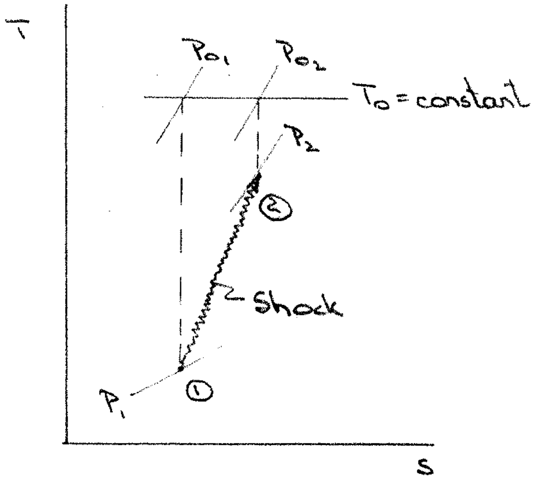
$$M_1 = \frac{704}{352} = 2.00$$

From App. E.1 $\quad p_1/p_{0_1} = 0.1278$

From App. E.4 $\quad p_{0_2}/p_{0_1} = 0.7209 \qquad T_2/T_1 = 1.687$

$$T_2 = \frac{T_2}{T_1} \times T_1 = 1.687 \times 308\,K = 520\,K \longleftarrow T_2$$

$$p_{0_2} = \frac{p_{0_2}}{p_{0_1}} \times \frac{p_{0_1}}{p_1} \times p_1 = 0.7209 \times \frac{1}{0.1278} \times 229 \text{ kPa} = 1.29 \text{ MPa (abs)} \longleftarrow p_{0_2}$$



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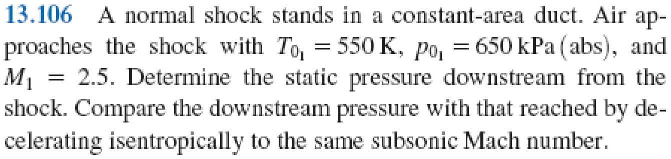
42-389 200 SHEETS 5 SQUARE

Problem 13.106

[2]



13.106 A normal shock stands in a constant-area duct. Air approaches the shock with $T_{0_1} = 550\,\text{K}$, $p_{0_1} = 650\,\text{kPa (abs)}$, and $M_1 = 2.5$. Determine the static pressure downstream from the shock. Compare the downstream pressure with that reached by decelerating isentropically to the same subsonic Mach number.

Given: Normal shock

Find: Pressure after shock; Compare to isentropic deceleration

Solution:

The given or available data is:

$R =$ 286.9 J/kg·K
$k =$ 1.4
$T_{01} =$ 550 K
$p_{01} =$ 650 kPa
$M_1 =$ 2.5

Equations and Computations:

Using built-in function *Isenp* (M,k):

$p_{01}/p_1 =$ 17.09 $p_1 =$ 38 kPa

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.513

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 7.13 $p_2 =$ 271 kPa

Using built-in function *Isenp* (M,k) at M_2:

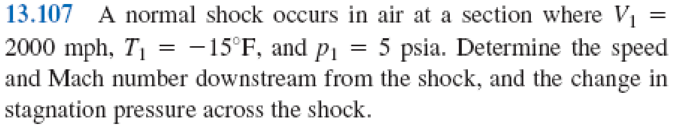
$p_{02}/p_2 =$ 1.20

But for the isentropic case: $p_{02} = p_{01}$

Hence for isentropic deceleration: $p_2 =$ 543 kPa

Problem 13.107

[2]



13.107 A normal shock occurs in air at a section where V_1 = 2000 mph, T_1 = −15°F, and p_1 = 5 psia. Determine the speed and Mach number downstream from the shock, and the change in stagnation pressure across the shock.

Given: Normal shock

Find: Speed and Mach number after shock; Change in stagnation pressure

Solution:

The given or available data is:

R =	53.33	ft·lbf/lbm·R	0.0685	Btu/lbm·R
k =	1.4			
T_1 =	445	$^{\mathrm{o}}$R		
p_1 =	5	psi		
V_1 =	2000	mph	2933	ft/s

Equations and Computations:

From $c_1 = \sqrt{kRT_1}$ c_1 = 1034 ft/s

Then M_1 = 2.84

Using built-in function *NormM2fromM* (M,k):

M_2 = 0.486

Using built-in function *NormdfromM* (M,k):

ρ_2/ρ_1 = 3.70

Using built-in function *Normp0fromM* (M,k):

p_{02}/p_{01} = 0.378

Then $V_2 = \dfrac{\rho_1}{\rho_2}V_1$ V_2 = 541 mph 793 ft/s

Using built-in function *Isenp* (M,k) at M_1:

p_{01}/p_1 = 28.7

From the above ratios and given p_1:

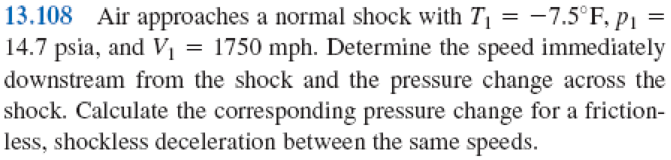
p_{01} = 143 psi

p_{02} = 54.2 psi

$p_{01} - p_{02}$ = 89.2 psi

Problem 13.108

[2]



13.108 Air approaches a normal shock with $T_1 = -7.5°\text{F}$, $p_1 = 14.7$ psia, and $V_1 = 1750$ mph. Determine the speed immediately downstream from the shock and the pressure change across the shock. Calculate the corresponding pressure change for a frictionless, shockless deceleration between the same speeds.

Given: Normal shock

Find: Speed; Change in pressure; Compare to shockless deceleration

Solution:

The given or available data is:

$R =$ 53.33 ft·lbf/lbm·R 0.0685 Btu/lbm·R

$k =$ 1.4

$T_1 =$ 452.5 ^{o}R

$p_1 =$ 14.7 psi

$V_1 =$ 1750 mph 2567 ft/s

Equations and Computations:

From $c_1 = \sqrt{kRT_1}$ $c_1 =$ 1043 ft/s

Then $M_1 =$ 2.46

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.517

Using built-in function *NormdfromM* (M,k):

$\rho_2/\rho_1 =$ 3.29

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 6.90 $p_2 =$ 101 psi

$p_2 - p_1 =$ 86.7 psi

Then $V_2 = \dfrac{\rho_1}{\rho_2} V_1$ $V_2 =$ 532 mph 781 ft/s

Using built-in function *Isenp* (M,k) at M_1:

$p_{01}/p_1 =$ 16.1

Using built-in function *Isenp* (M,k) at M_2:

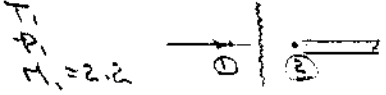
$p_{02}/p_2 =$ 1.20

From above ratios and p_1, for isentropic flow (p_0 = const): $p_2 =$ 197 psi

$p_2 - p_1 =$ 182 psi

Problem 13.109

Given: Supersonic aircraft cruises at $M = 2.2$ at 12 km altitude. Normal shock stands in front of a pitot tube which senses a stagnation pressure, p_{0_2}.



Find: (a) T_{0_1}, p_{0_1} (b) p_{0_2}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock
(4) ideal gas

Use table A.3 to determine properties at state ① At 12 km altitude,

$T_1 = 216.7\ \text{K}$ $\quad p_1 = 19.4\ \text{kPa}$

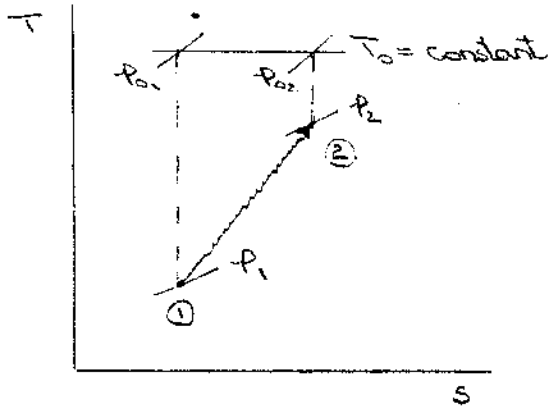
From App. E.1, for $M_1 = 2.2$,

$T_1/T_{0_1} = 0.5081$ $\quad \therefore T_{0_1} = 426\ \text{K}$ $\quad T_{0_1}$

$p_1/p_{0_1} = 0.09352$ $\quad \therefore p_{0_1} = 207\ \text{kPa (abs)}$ $\quad p_{0_1}$

From App. E.4, for $M_1 = 2.2$, $M_2 = 0.5471$

$p_{0_2}/p_{0_1} = 0.6281$ $\quad \therefore p_{0_2} = 130\ \text{kPa (abs)}$ $\quad p_{0_2}$



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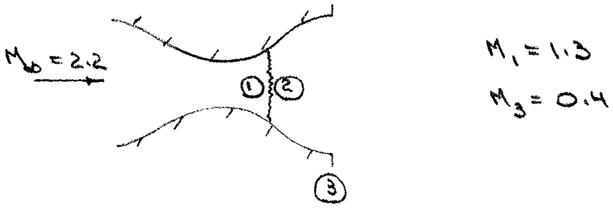
Problem 13.110

Given: Concorde flew at $M = 2.2$ at an altitude of 20 km. In the engine inlet system the air is decelerated isentropically to a local Mach number of 1.3; air undergoes a normal shock and then decelerates isentropically to $M = 0.4$.



$M_1 = 1.3$

$M_3 = 0.4$

Find: T_3, p_3, p_{0_3}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow
(2) ideal gas
(3) flow is isentropic except across the shock.

At 20 km altitude $T_\infty = 217\,K$, $p_\infty = 5.53$ kPa. (Table A.3)

$M_\infty = 2.2$ From App. E.1, $T_\infty / T_0 = 0.5081$ $p_\infty / p_0 = 0.09352$

$\therefore T_0 = 427\,K$ $p_0 = 59.1$ kPa.

$M_1 = 1.3$ From App. E.1, $T_1/T_0 = 0.7474$ $p_1/p_0 = 0.3609$

$\therefore T_1 = 319\,K$ $p_1 = 21.3$ kPa.

From App. E.4, $M_2 = 0.786$ $p_{0_2}/p_{0_1} = 0.9794$ $\frac{T_2}{T_1} = 1.191$ $\frac{p_2}{p_1} = 1.805$

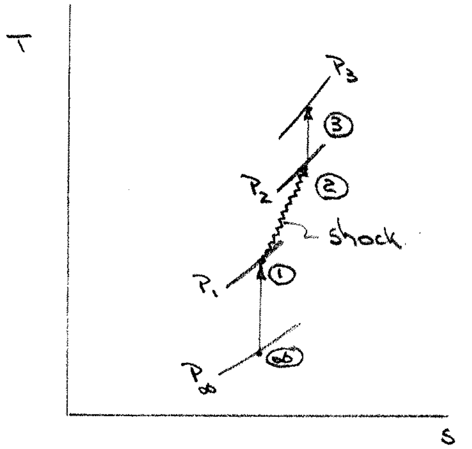
$\therefore p_{0_2} = 57.9$ kPa, $T_2 = 380\,K$, $p_2 = 38.4$ kPa.

$p_{0_3} = p_{0_2} = 57.9$ kPa (abs) ← p_{0_3}

$M_3 = 0.4$ From App. E.1, $T_3/T_0 = 0.9690$ $p_3/p_{0_3} = 0.8956$.

$\therefore T_3 = 414\,K.$ ← T_3

$p_3 = 0.8956\, p_{0_3} = 0.8956\, p_{0_2} = 51.9$ kPa (abs) p_3



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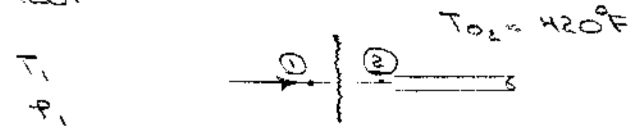
Problem 13.111

[2]

Given: Supersonic aircraft flies at 35000 ft. Normal shock stands in front of stagnation temperature probe. Probe reads $T_0 = 420°F$



Find: (a) M_1, V_1 (b) p_2, p_{0_2}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock, $R_x = 0$
(4) ideal gas

Use table A.3 to determine properties at state ①

$$\text{Altitude} = 35000\,\text{ft} \times 0.3048\,\frac{m}{ft} = 10{,}670\,m$$

From Table A.3 $T_1 = 219\,K = -54°C = -65°F$

$$p_1 = 23.9 \times 10^3\,\frac{N}{m^2} \times \frac{lbf}{4.448\,N} \times \left(\frac{0.3048\,m}{ft}\right)^2 = 499\ \text{psfa}$$

At state ①, $T_1/T_{0_1} = 395/880 = 0.4489$

From App E.1, for $T_1/T_{0_1} = 0.4489$, $M_1 = 2.48$ ← M_1

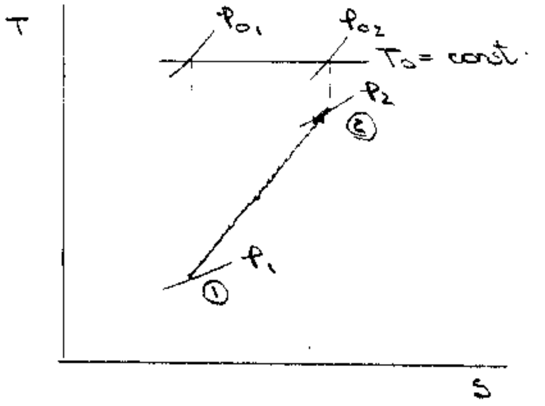
$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.48\left[1.4 \times 53.3\,\frac{ft\cdot lbf}{lbm\,°R} \times 395°R \times 32.2\,\frac{lbm}{slug} \times \frac{slug\cdot ft}{lbf\,s^2}\right]^{1/2} = 2420\,ft/s$$ ← V_1

From App. E.1, for $M_1 = 2.48$, $p_1/p_{0_1} = 0.06038$ $\therefore p_{0_1} = 57.4$ psia

From App. E.4, for $M_1 = 2.48$, $M_2 = 0.5149$

$p_{0_2}/p_{0_1} = 0.5071$ $\therefore p_{0_2} = 29.1$ psia ← p_{0_2}

$p_2/p_1 = 7.009$ $\therefore p_2 = 24.3$ psia ← p_2



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Problem 13.112

[4]

13.112 Equations 13.41 are a useful set of equations for analyzing flow through a normal shock. Derive another useful equation, the Rankine-Hugoniot relation,

$$\frac{p_2}{p_1} = \frac{(k+1)\dfrac{\rho_2}{\rho_1} - (k-1)}{(k+1) - (k-1)\dfrac{\rho_2}{\rho_1}}$$

and use it to find the density ratio for air as $p_2/p_1 \to \infty$.

Given: Normal shock

Find: Rankine-Hugoniot relation

Solution:

Basic equations: Momentum: $p_1 + \rho_1 \cdot V_1^2 = p_2 + \rho_2 \cdot V_2^2$ Mass: $\rho_1 \cdot V_1 = \rho_2 \cdot V_2$

Energy: $h_1 + \frac{1}{2} \cdot V_1^2 = h_2 + \frac{1}{2} \cdot V_2^2$ Ideal Gas: $p = \rho \cdot R \cdot T$

From the energy equation $2 \cdot (h_2 - h_1) = 2 \cdot c_p \cdot (T_2 - T_1) = V_1^2 - V_2^2 = (V_1 - V_1) \cdot (V_1 + V_2)$ (1)

From the momentum equation $p_2 - p_1 = \rho_1 \cdot V_1^2 - \rho_2 \cdot V_2^2 = \rho_1 \cdot V_1 \cdot (V_1 - V_2)$ where we have used the mass equation

Hence $V_1 - V_2 = \dfrac{p_2 - p_1}{\rho_1 \cdot V_1}$

Using this in Eq 1 $2 \cdot c_p \cdot (T_2 - T_1) = \dfrac{p_2 - p_1}{\rho_1 \cdot V_1} \cdot (V_1 + V_2) = \dfrac{p_2 - p_1}{\rho_1} \cdot \left(1 + \dfrac{V_2}{V_1}\right) = \dfrac{p_2 - p_1}{\rho_1} \cdot \left(1 + \dfrac{\rho_1}{\rho_2}\right) = (p_2 - p_1) \cdot \left(\dfrac{1}{\rho_1} + \dfrac{1}{\rho_2}\right)$

where we again used the mass equation

Using the idea gas equation $2 \cdot c_p \cdot \left(\dfrac{p_2}{\rho_2 \cdot R} - \dfrac{p_1}{\rho_1 \cdot R}\right) = (p_2 - p_1) \cdot \left(\dfrac{1}{\rho_1} + \dfrac{1}{\rho_2}\right)$

Dividing by p_1 and multiplying by ρ_2, and using $R = c_p - c_v$, $k = c_p/c_v$

$$2 \cdot \frac{c_p}{R} \cdot \left(\frac{p_2}{p_1} - \frac{\rho_2}{\rho_1}\right) = 2 \cdot \frac{k}{k-1} \cdot \left(\frac{p_2}{p_1} - \frac{\rho_2}{\rho_1}\right) = \left(\frac{p_2}{p_1} - 1\right) \cdot \left(\frac{\rho_2}{\rho_1} + 1\right)$$

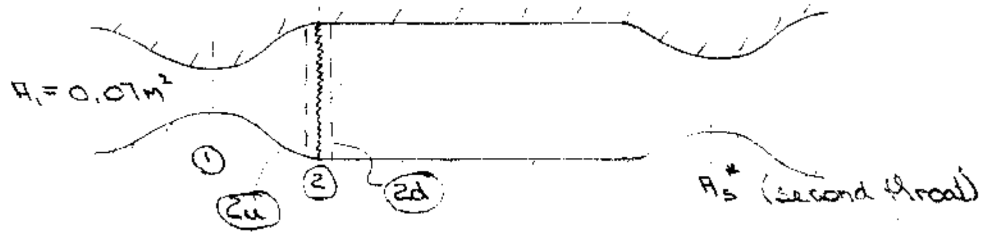
Collecting terms $\dfrac{p_2}{p_1} \cdot \left(\dfrac{2 \cdot k}{k-1} - 1 - \dfrac{\rho_2}{\rho_1}\right) = \dfrac{2 \cdot k}{k-1} \cdot \dfrac{\rho_2}{\rho_1} - \dfrac{\rho_2}{\rho_1} - 1$

$$\frac{p_2}{p_1} = \frac{\dfrac{2 \cdot k}{k-1} \cdot \dfrac{\rho_2}{\rho_1} - \dfrac{\rho_2}{\rho_1} - 1}{\left(\dfrac{2 \cdot k}{k-1} - 1 - \dfrac{\rho_2}{\rho_1}\right)} = \frac{\dfrac{(k+1)}{(k-1)} \cdot \dfrac{\rho_2}{\rho_1} - 1}{\dfrac{(k+1)}{(k-1)} - \dfrac{\rho_2}{\rho_1}} \quad \text{or} \quad \frac{p_2}{p_1} = \frac{(k+1) \cdot \dfrac{\rho_2}{\rho_1} - (k-1)}{(k+1) - (k-1) \cdot \dfrac{\rho_2}{\rho_1}}$$

For an infinite pressure ratio $(k+1) - (k-1) \cdot \dfrac{\rho_2}{\rho_1} = 0$ or $\dfrac{\rho_2}{\rho_1} = \dfrac{k+1}{k-1}$ (= 6 for air)

Problem 13.113

Given: Supersonic wind tunnel, supplied with air at $T_0 = 500\text{K}$ and $p_0 = 1.0\text{ MPa (abs)}$, is to operate at a test section mach number, $M = 2.2$. Normal shock stands in exit plane of inlet nozzle.



Find: a) M_{2d} (b) p_{2d} (c) $p_{0_{2d}}$ (d) A_s

Solution:

Compressible flow functions (Appendix E) to be used in solution.

Assumptions: (1) steady flow (2) uniform flow at a section
(3) isentropic flow in nozzles, adiabatic flow across shock
(4) ideal gas.

At $M_{2u} = 2.2$, from App E.1, $p_{2u}/p_0 = 0.09352$ $\therefore p_{2u} = 93.5\text{ kPa}$

At $M_{2u} = 2.2$, from App E.4, $M_{2d} = 0.547$ (12.34b) $\longleftarrow M_{2d}$

$p_2/p_1 = 5.480$ $\therefore p_{2d} = 512\text{ kPa}$ $\longleftarrow p_{2d}$

$p_{0_2}/p_{0_1} = 0.6281$ $\therefore p_{0_{2d}} = 628\text{ kPa}$ $\longleftarrow p_{0_{2d}}$

At $M_{2u} = 2.2$, from App E.1, $A_2/A^* = A_2/A_1 = 2.005$ $\therefore A_2 = 0.1404\text{ m}^2$

At $M_{2d} = 0.547$, from App E.1, $A_2/A^* = A_2/A_s^* = 1.259$ $\therefore A_s^* = 0.111\text{ m}^2$ $\longleftarrow A_s^*$

Problem 13.114

Given: Supersonic wind tunnel starting as shown.

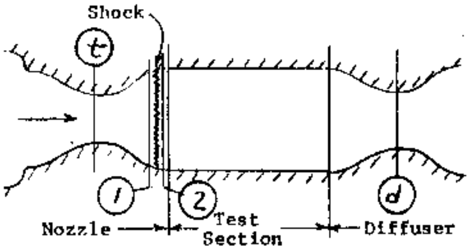
$A_t = 1.25\ ft^2$

$A_1 = A_2 = 3.05\ ft^2$

$M_{design} = 2.50$

$T_0 = 1080\ R$

$p_0 = 115\ psia$



Find: (a) Minimum possible A_d at this condition.
(b) Entropy increase across the shock.

Solution: Use functions for steady, one-dimensional compressible flow.

Computing equations: A/A^* vs. M from isentropic flow functions (App. E.1)

p_{02}/p_{01} vs. M from shock flow functions (App. E.4)

Assumptions:
- (1) Steady flow
- (2) Uniform flow at each section
- (3) Ideal gas
- (4) Isentropic except across shock
- (5) Adiabatic flow
- (6) $F_{Bx} = 0$
- (7) $\Delta z = 0$

Then from App. E.1, $M_1 = 2.416$ at $\frac{A_1}{A^*} = \frac{A_1}{A_t} = \frac{3.05\ ft^2}{1.25\ ft^2} = 2.44$.

From App. E.4, at $M_1 = 2.416$, $\frac{p_{02}}{p_{01}} = 0.5395$. Thus $p_{0d} = 0.5395\ p_{01} = 62.0$ psia.

For adiabatic flow, T_0 = constant and $T^* = \frac{T_0}{1.2} = \frac{1080\ R}{1.2} = 900\ R$ = constant.

From continuity, $\dot{m} = \rho_t A_t V_t = \rho_d V_d A_d$. Substituting $\rho = \frac{p}{RT}$ and $V = M\sqrt{kRT}$,

$$\frac{p_t}{RT_t}\sqrt{kRT_t}\,A_t = \frac{p_d}{RT_d}\sqrt{kRT_d}\,A_d\ ;\quad A_d = A_t\frac{p_t}{p_d} = A_t\frac{p_{0t}}{p_{0d}} = \frac{A_t}{0.5395}$$

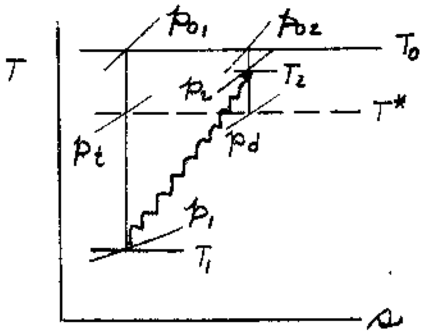
$$A_d = \frac{1.25\ ft^2}{0.5395} = 2.32\ ft^2$$

A_d

From the Gibbs equation,

$$T\,ds = dh - v\,dp\ ;\quad ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$\Delta s = c_p \ln\frac{T_{02}}{T_{01}} - R\ln\frac{p_{02}}{p_{01}}$$



Since T_0 = constant, $\ln(T_{02}/T_{01}) = 0$, and

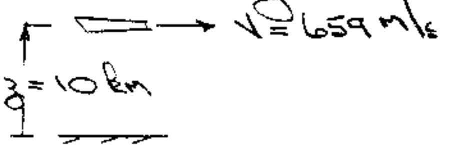
$$\Delta s = -53.3\frac{ft\cdot lbf}{lbm\cdot {}^\circ R}\times\ln(0.5395)\times\frac{Btu}{778\ ft\cdot lbf} = 0.0423\ Btu/lbm\cdot{}^\circ R$$

Δs

{ Note a slightly stronger shock could occur at M = 2.50; a larger diffuser throat ($A_d \simeq 2.51\ ft^2$) would be needed to start this tunnel. }

Problem 13.115

Given: Aircraft in supersonic flight on standard day
Total-head tube senses stagnation pressure.
Mach number computed assuming $s = c$, i.e. ignoring shock in front of tube.



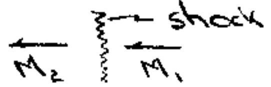
Find:
(a) flight Mach number
(b) pressure sensed by total-head tube
(c) air speed computed assuming $s = \text{constant}$

Solution: (using compressible flow functions - Appendix E)

From Table A.3 at $z = 10\text{ km}$, $T = 223.3\text{ K}$, $p/p_{SL} = 0.2615$

Thus $T = 223\text{ K}$, $p = 0.2615\, p_{SL} = 0.2615 \times 101.3\text{ kPa} = 26.5\text{ kPa}$

$$c_1 = (kRT_1)^{1/2} = \left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 223\text{ K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 299\text{ m/s}$$



$$M_1 = \frac{V_1}{c_1} = \frac{659}{299} = 2.20 \quad M_1$$

From App. E.1 at $M_1 = 2.20$, $p/p_0 = 0.09352$ $\therefore p_{0_1} = 283\text{ kPa}$

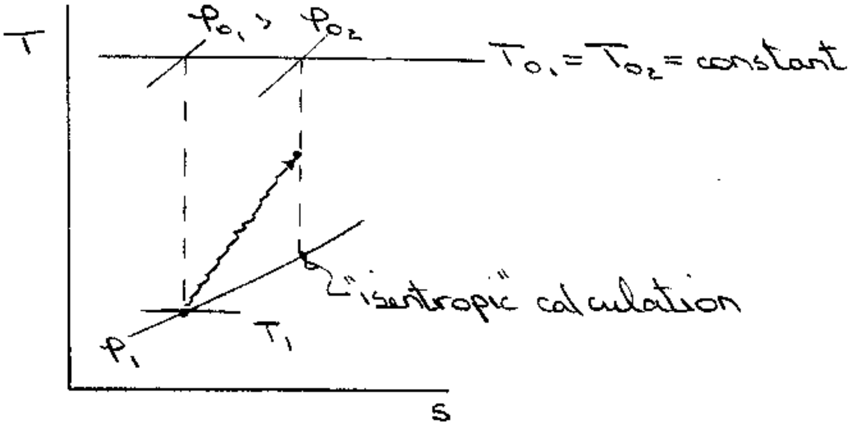
From App E.4 at $M_1 = 2.20$, $p_{0_2}/p_{0_1} = 0.6281$, $p_2/p_1 = 5.480$

$$\therefore p_{0_2} = 0.6281\, p_{0_1} = 0.6281 \times 283\text{ kPa} = 178\text{ kPa (abs)} \quad p_{0_2}$$

If Mach number is calculated neglecting the shock then M is calculated with $p = 26.5\text{ kPa}$ and $p_0 = 178\text{ kPa}$.

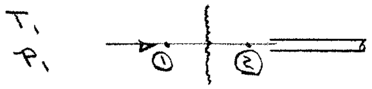
For $p/p_0 = 26.5/178 = 0.149$, from App. E.1, $M_s = 1.90$

$$V_{1_s} = M_s c_1 = 1.90 \times 299\text{ m/s} = 568\text{ m/s} \quad V_{1_s}$$



Problem 13.116

Given: Supersonic aircraft cruises at $M = 2.7$ at 60,000 ft altitude. Normal shock stands in front of a pitot tube which senses a stagnation pressure of 10.4 psia.



$p_{o_2} = 10.4$ psia

Find: (a) T_2, p_2 (b) $p_{o_2} - p_{o_1}$ (c) $s_2 - s_1$

Solution.

Compressible flow functions (Appendix E) to be used in solution

Basic equation: $Tds = dh - v\,dp$

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock, $R_x = 0$
(4) ideal gas

Use table A.3 to determine properties at state ①

Altitude $= 60{,}000\text{ ft} \times 0.3048 \frac{m}{ft} = 18{,}290\text{ m}$

From table A.3 $T_1 = 216.7\text{ K} = -56.3°C = -69°F$

$$p_1 = 7.25 \times 10^3 \frac{N}{m^2} \times \frac{lbf}{4.448\,N} \times \left(\frac{0.3048\,m}{ft}\right)^2 = 151\ lbf/ft^2$$

From App. E.1 for $M_1 = 2.7$ $p_1/p_{o_1} = 0.04295$ $\therefore p_{o_1} = 24.4\ lbf/in^2$

From App. E.4 for $M_1 = 2.7$, $M_2 = 0.4956$

$p_{o_2}/p_{o_1} = 0.4236$ $\therefore p_{o_2} = 10.3$ psia

$T_2/T_1 = 2.343$ $\therefore T_2 = 916\,R$ ← T_2

$p_2/p_1 = 8.338$ $\therefore p_2 = 8.74$ psia ← p_2

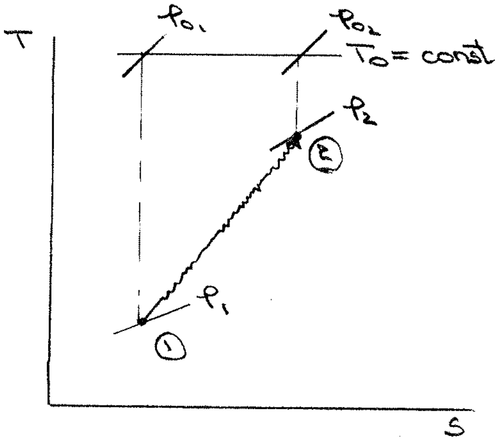
$p_{o_2} - p_{o_1} = 10.3\text{ psi} - 24.4\text{ psi} = -14.1\text{ psi}$ ← $p_{o_2} - p_{o_1}$

From the Tds equation, $Tds = dh - v\,dp = c_p\,dT - RT\frac{dp}{p}$

$$\therefore ds = c_p \frac{dT}{T} - R\frac{dp}{p}$$

Then $s_2 - s_1 = s_{o_2} - s_{o_1} = c_p \ln \cancel{\frac{T_{o_2}}{T_{o_1}}} - R \ln \frac{p_{o_2}}{p_{o_1}}$

$$s_2 - s_1 = -53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \ln \frac{10.3}{24.4} \times \frac{Btu}{778\,ft\,lbf} = 0.0591 \frac{Btu}{lbm \cdot °R}$$ ← $s_2 - s_1$



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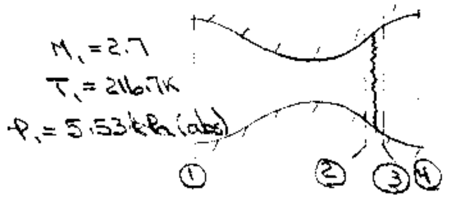
Problem 13.117

Given: Supersonic aircraft flies at $M = 2.7$ at 20 km altitude. Air is slowed isentropically in inlet system to $M = 1.3$. A normal shock occurs at that location. Following the shock, the flow is decelerated adiabatically, but not isentropically, to $p = 104$ kPa and $M = 0.4$



$M_2 = 1.3$
$M_4 = 0.4$
$p_4 = 104$ kPa (abs)

Find: (a) T_0 (b) $p_3 - p_2$ (c) $s_4 - s_1$ d) p_{04}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock, $A_2 = A_3$
(4) ideal gas

For $M_1 = 2.7$, from App. E.1, $T_{01}/T_1 = 2.458$ $\therefore T_{01} = 533$ K $\quad T_0$
$p_{01}/p_1 = 23.283$ $\therefore p_{01} = 128.8$ kPa

Note: $T_0 =$ constant, $p_{02} = p_{01}$

For $M_2 = 1.3$, from App. E.1, $T_{02}/T_2 = 1.338$ $\therefore T_2 = 398$ K
$p_{02}/p_2 = 2.771$ $\quad p_2 = 46.48$ kPa

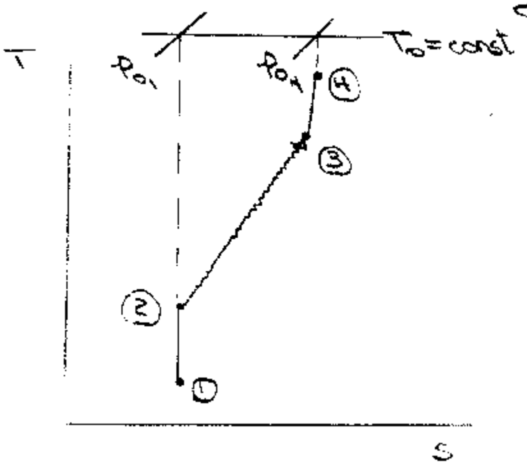
For $M_2 = 1.3$, from App E.4, $M_3 = 0.786$
$p_3/p_2 = 1.805$ $\therefore p_3 = 83.9$ kPa
$p_{03}/p_{02} = 0.9794$ $\quad p_{03} = 126$ kPa
$T_3/T_2 = 1.191$ $\quad T_3 = 474$ K

$p_3 - p_2 = (83.9 - 46.5)$ kPa $= 37.4$ kPa $\quad p_3 - p_2$

For $M_4 = 0.4$, from App. E.1, $p_{04}/p_4 = 1.117$ $\therefore p_{04} = 116$ kPa

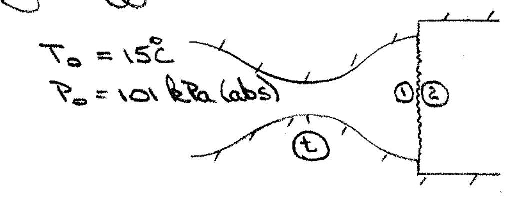
$$s_4 - s_1 = s_{04} - s_{01} = c_p \ln \frac{T_{04}}{T_{01}} - R \ln \frac{p_{04}}{p_{01}} = -287 \frac{\text{J}}{\text{kg}\cdot\text{K}} \ln \frac{116}{129}$$

$s_4 - s_1 = -30.5$ J/kg·K $\quad s_4 - s_1$



Problem 13.118

Given: Steady, adiabatic flow of air through a converging-diverging nozzle with a normal shock in the exit plane



$A_1/A_t = 3.5$

Find: P_2, V_2

Solution:

- Compressible flow tables to be used in the solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) ideal gas (4) isentropic flow except across the shock

$A_1/A^* = 3.5$

From Fig. E.1 and Eq. 12.6, $M_1 = 2.80$; then $T_1/T_0 = 0.3894$ $\quad P_1/P_{0_1} = 0.03665$

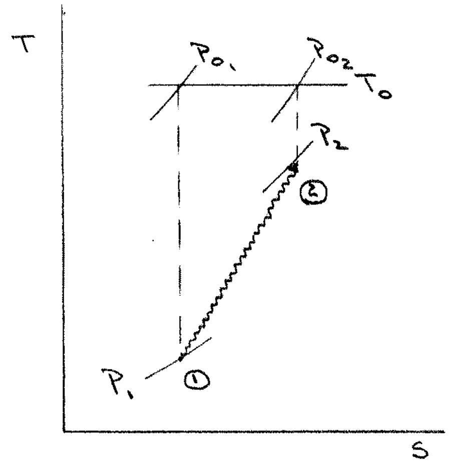
$\therefore T_1 = 112\text{ K} \qquad P_1 = 3.72$ kPa (abs)

For $M_1 = 2.80$, from App. E.4, $M_2 = 0.488$ $\quad T_2/T_1 = 2.451$ $\quad P_2/P_1 = 8.980$

$$\therefore P_2 = 8.98\,P_1 = 8.98 \times 3.72\text{ kPa} = 33.4\text{ kPa (abs)} \longleftarrow P_2$$

$$T_2 = 2.451\,T_1 = 275\text{ K}$$

$$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 0.488\left(1.4 \times 287\,\frac{N\cdot m}{kg\cdot K} \times 275\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 162\text{ m/s} \longleftarrow V_2$$



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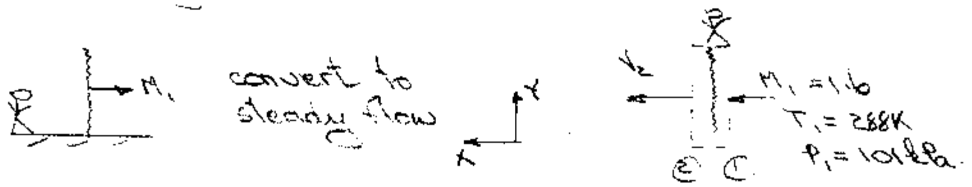
Problem 13.119

Given: Blast wave propagates outward from an explosion. Model the wave as strong normal shock. Wave front travels at $M = 1.6$ through undisturbed air



Find: (a) V_2 relative to wave (b) V_2 relative to ground.

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions:
(1) steady flow as seen by an observer on the wave
(2) uniform flow at a section
(3) thin shock, $R_x = 0$
(4) ideal gas

At state ①

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 1.6\left[1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 288K \times \frac{kg\cdot m}{N\,s^2}\right]^{1/2} = 544\ m/s$$

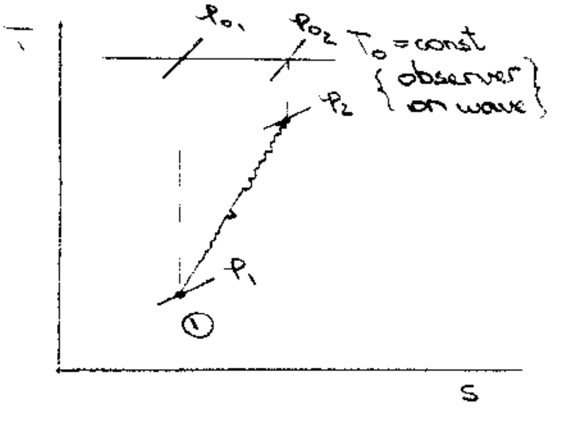
For $M_1 = 1.6$, from Appendix E.4, $V_1/V_2 = \rho_2/\rho_1 = 2.032$

$$\therefore \vec{V}_2 = 268\hat{i}\ m/s \longleftarrow \vec{V}_{2\,\text{rel. wave}}$$

The wave moves to the right at $V_1 = 544$ m/s. Air moves to the left with respect to the wave at $V_{2\,rel} = 268$ m/s.

$$\vec{V}_{2\,abs} = \vec{V}_{2\,wave} + \vec{V}_{2\,rel} = -544\hat{i} + 268\hat{i} = -276\hat{i}\ m/s. \longleftarrow \vec{V}_{2\,abs}$$

{relative to ground, air behind wave moves to right}.



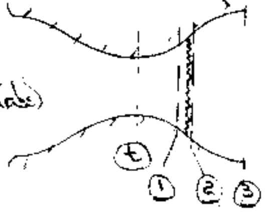
Problem 13.120

[2]

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle with a shock present.

$T_0 = 300\text{K}$

$p_0 = 250 \text{ kPa(abs)}$



$p_t = 132 \text{ kPa (abs)}$

$p_1 = 68.1 \text{ kPa}$

$p_3 = 180 \text{ kPa}$

Find: (a) M_2 (b) p_2 (c) $s_2 - s_1$ (d) Sketch Ts diagram

Solution: Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (3) uniform flow at each section
(2) ideal gas (4) isentropic flow, except across shock

At section ① $p_0 = 250$ kPa (abs), $p_1 = 68.1$ kPa (abs) ∴ $p_1/p_0 = 0.2724$

From App. E.1 $M_1 = 1.50$

From App. E.4 with $M_1 = 1.50$, $M_2 = 0.701$ ⟵ M_2

$p_2/p_1 = 2.458$ ∴ $p_2 = 167$ kPa (abs) ⟵ p_2

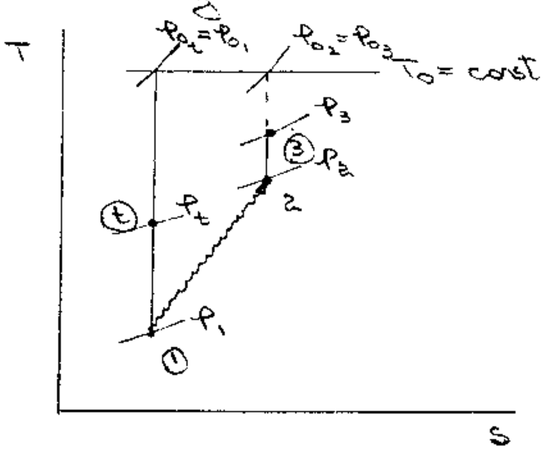
$p_{02}/p_{01} = 0.9298$

From the Tds equation. $T\,ds = dh - v\,dp = c_p\,dT - RT\frac{dp}{p}$

$$\therefore ds = c_p\,dT - R\frac{dp}{p}$$

$$s_2 - s_1 = s_{02} - s_{01} = \overset{0}{\cancel{c_p \ln \frac{T_{02}}{T_{01}}}} - R \ln \frac{p_{02}}{p_{01}}$$

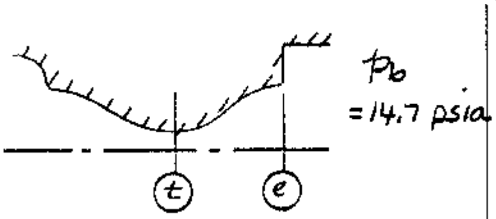
$s_2 - s_1 = -287 \frac{\text{J}}{\text{kg}\cdot\text{K}} \ln 0.9298 = 20.9 \text{ J/kg}\cdot\text{K}$ ⟵ $s_2 - s_1$



Problem 13.121

[2]

Given: C-D nozzle expanding air as shown.



$T_0 = 250°F$

$p_0 = 50.5$ psia

$A_t = 0.801 \text{ in}^2 \quad A_e = 0.917 \text{ in}^2$

Find: (a) Exit Mach number.
(b) Mass flow rate.

Solution: Use functions for steady, one-dimensional compressible flow.

Computing equations: A/A^*, p/p_0, and T/T_0 from isentropic (Appendix E.1)

Assumptions: (1) Steady flow
(2) Uniform flow at each section
(3) Isentropic if no shock
(4) Ideal gas
(5) $F_{Bx} = 0$
(6) $\Delta z = 0$

Check the exit condition: $\frac{A_e}{A^*} = \frac{A_e}{A_t} = \frac{0.917}{0.801} = 1.145 \rightarrow M_e = 1.452$ (App. E.1, Eq. 12.6)

Also from App. E.1, at $M = 1.452$, $\frac{p}{p_0} = 0.2919$; $p_e = 0.2919\, p_0 = 14.74$ psia.

Thus p_e is just slightly above $p_{atm} = 14.7$ psia; flow at exit is supersonic, so $M_t = 1.0$. From continuity

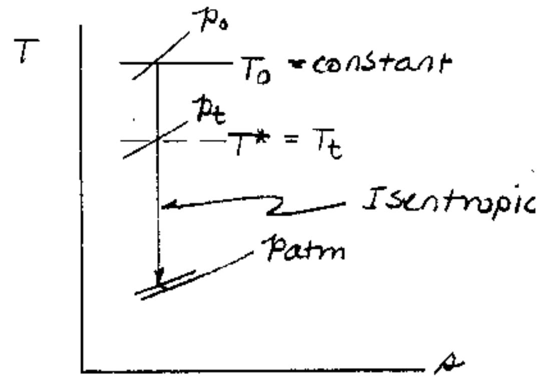
$$\dot m = \rho_t V_t A_t = \frac{p_t}{RT_t} M_t \sqrt{kRT_t}\, A_t = p_t \sqrt{\frac{k}{RT_t}}\, A_t$$

From App. E.1, at $M = 1$, $T/T_0 = 0.8333$ and $p/p_0 = 0.5283$, so

$$T_t = 0.8333 \times (460 + 250)°R = 592°R \quad \text{and} \quad p_t = 0.5283 \times 50.5 \text{ psia} = 26.7 \text{ psia}$$

$$\dot m = 26.7 \frac{\text{lbf}}{\text{in.}^2}\left[1.4 \times \frac{\text{lbm}\cdot°R}{53.3\ \text{ft}\cdot\text{lbf}} \times \frac{1}{592°R} \times \frac{\text{slug}}{32.2\ \text{lbm}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right]^{\frac{1}{2}} 0.801 \text{ in.}^2 \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \times 32.2 \frac{\text{lbm}}{\text{slug}}$$

$\dot m = 0.808$ lbm/s $\qquad \dot m$



{ Flow in the nozzle is just slightly underexpanded, since $p_{exit} > p_{back}$. }

Problem 13.122

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle.

$A_t = 1.0\ in^2$

$A_e = 1.58\ in^2$

$T_0 = 600°R$

$P_0 = 100$ psia

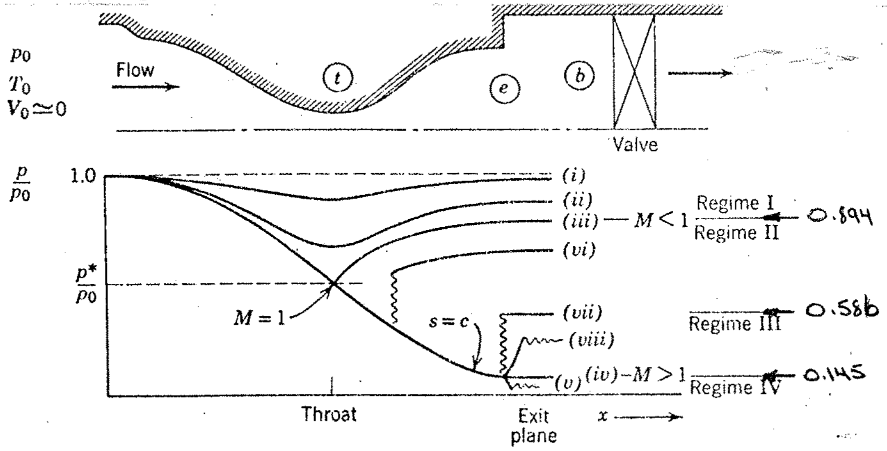


Fig. 13.20 Pressure distributions for flow in a converging-diverging nozzle as a function of back pressure.

Find: M_e at design conditions; P_b corresponding to regime boundaries of Fig 12.20, sketch P(x)

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) isentropic flow except across a shock

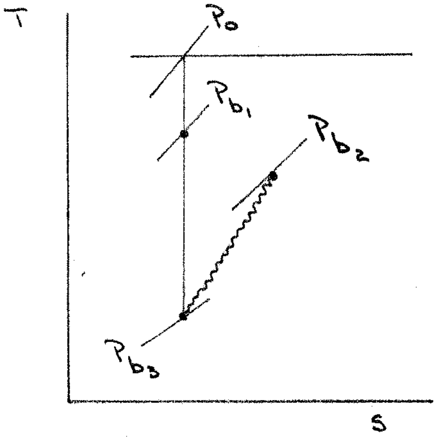
For $A_e/A^* = 1.58$,
from Fig. E.1 and Eq. 12.6, $M_e = 0.403$ $\quad P_e/P_0 = 0.8940$
or $M_e = 1.92$ $\quad P_e/P_0 = 0.1447$ ← M_e

Then $P_{b_1} = \frac{P_{e_1}}{P_0} \cdot P_0 = 0.8940 \times 100$ psia $= 89.4$ psia ← P_{b_1}

$P_{b_3} = \frac{P_{e_3}}{P_0} \cdot P_0 = 0.1447 \times 100$ psia $= 14.5$ psia ← P_{b_3}

For $M_e = 1.92$, from App. E.4, $\frac{P_2}{P_1} = 4.045$

$P_{b_2} = \frac{P_2}{P_1} \times P_{e_3} = 4.045 \times 14.5$ psia $= 58.6$ psia ← P_{b_2}



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Problem 13.123

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle; nozzle is designed to discharge to atmosphere

$A_e/A_t = 4.0$

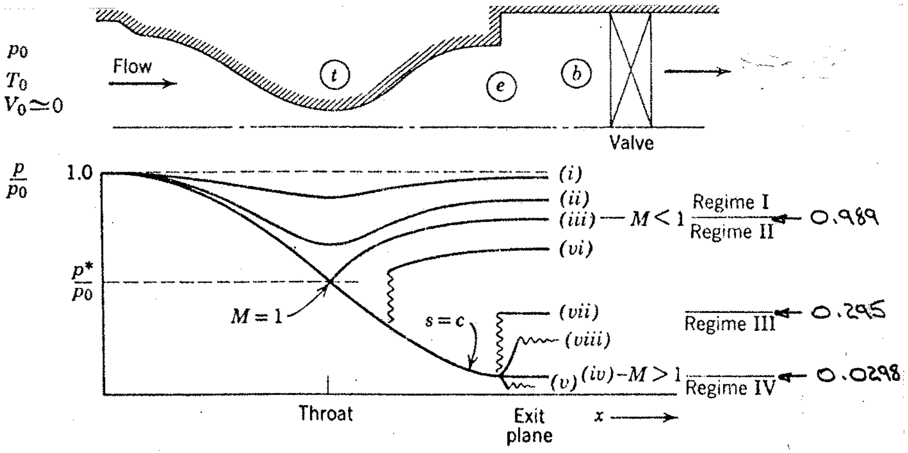


Fig. 13.20 *Pressure distributions for flow in a converging-diverging nozzle as a function of back pressure.*

Find: M_e at design conditions, p_0; p_b corresponding to regime boundaries of Fig 12.20, sketch $p(x)$

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) isentropic flow except across a shock

For $A_e/A^* = 4.00$,
from Fig. E.1 and Eq. 12.6b, $M_e = 2.94$ $p/p_0 = 0.0298$
or $M_e = 0.147$ $p/p_0 = 0.9887$

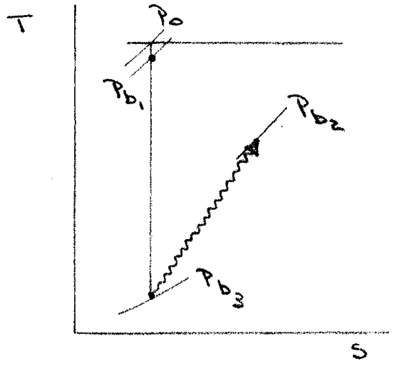
Then
$M_{ed} = 2.94$ $p_{od} = \frac{p_e}{0.0298} = \frac{101 \text{ kPa}}{0.0298} = 3.39 \text{ MPa (abs)}$ M_e, p_0

$p_{b_1} = \frac{p_{e_1}}{p_0} \times p_0 = 0.9887 \times 3.39 \text{ MPa} = 3.35 \text{ MPa (abs)}$ p_{b_1}

$p_{b_3} = 101 \text{ kPa (abs)}$ p_{b_3}

For $M_e = 2.94$ from App. E.4, $p_2/p_1 = 9.918$

$p_{b_2} = \frac{p_2}{p_1} \times p_{e_3} = 9.918 \times 101 \text{ kPa} = 1.00 \text{ MPa (abs)}$ p_{b_2}



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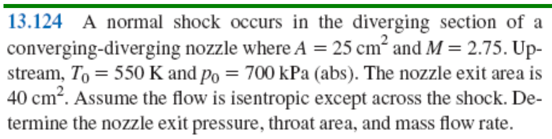
Problem 13.124

[3]



13.124 A normal shock occurs in the diverging section of a converging-diverging nozzle where $A = 25\ \text{cm}^2$ and $M = 2.75$. Upstream, $T_0 = 550$ K and $p_0 = 700$ kPa (abs). The nozzle exit area is $40\ \text{cm}^2$. Assume the flow is isentropic except across the shock. Determine the nozzle exit pressure, throat area, and mass flow rate.

Given: Normal shock in CD nozzle

Find: Exit pressure; Throat area; Mass flow rate

Solution:

The given or available data is:

$R =$ 286.9 J/kg·K
$k =$ 1.4
$T_{01} =$ 550 K
$p_{01} =$ 700 kPa
$M_1 =$ 2.75
$A_1 =$ 25 cm^2
$A_e =$ 40 cm^2

Equations and Computations (assuming State 1 and 2 before and after the shock):

Using built-in function *Isenp* (M,k):
$p_{01}/p_1 =$ 25.14 $\quad p_1 =$ 28 kPa

Using built-in function *IsenT* (M,k):
$T_{01}/T_1 =$ 2.51 $\quad T_1 =$ 219 K

Using built-in function *IsenA* (M,k):
$A_1/A_1^* =$ 3.34 $\quad A_1^* = A_t =$ 7.49 cm^2

Then from the Ideal Gas equation:
$\rho_1 =$ 0.4433 kg/m^3

Also: $c_1 =$ 297 m/s
So: $V_1 =$ 815 m/s

Then the mass flow rate is: $m_{\text{rate}} = \rho_1 V_1 A_1$
$m_{\text{rate}} =$ 0.904 kg/s

For the normal shock:

Using built-in function *NormM2fromM* (M,k):
$M_2 =$ 0.492

Using built-in function *Normp0fromM* (M,k) at M_1:
$p_{02}/p_{01} =$ 0.41 $\quad p_{02} =$ 284 kPa

For isentropic flow after the shock:

Using built-in function *IsenA* (M,k):
$A_2/A_2^* =$ 1.356
But: $A_2 = A_1$
Hence: $A_2^* =$ 18.44 cm^2

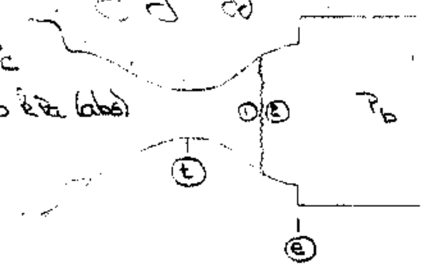
Using built-in function *IsenAMsubfromA* (Aratio,k):
For: $A_e/A_2^* =$ 2.17 $\quad M_e =$ 0.279

Using built-in function *Isenp* (M,k):
$p_{02}/p_e =$ 1.06 $\quad p_e =$ 269 kPa

Problem 13.125

[2]

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle with a shock present



$T_0 = 60°C$

$p_0 = 600$ kPa (abs)

$M_1 = 2.42$

$A_e/A_t = 4.0$

Find: p_b, sketch the pressure distribution.

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions (1) steady flow

(2) uniform flow at a section

(3) ideal gas

(4) isentropic flow except across the shock

$M_1 = 2.42$ From App. E.1, $p/p_0 = 0.06630$ $A_1/A_1^* = A_1/A_t = 2.448$

$\therefore p_1 = 39.8$ kPa(abs)

From App. E.4, $M_2 = 0.521$ $p_2/p_1 = 6.666$ $p_{02}/p_{01} = 0.5318$

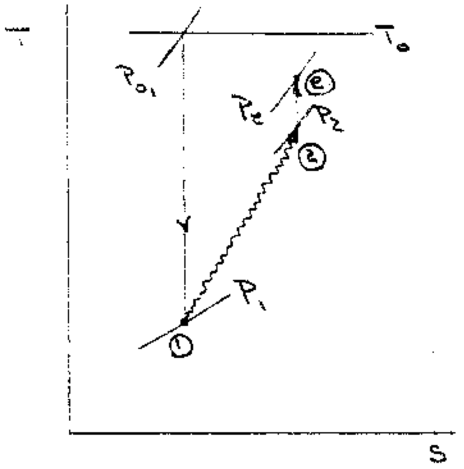
$\therefore p_2 = 265$ kPa(abs) $p_{02} = 319$ kPa (abs)

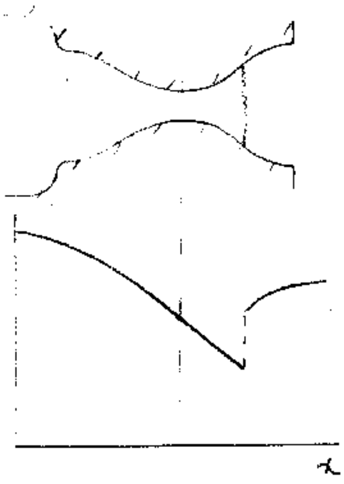
$M_2 = 0.521$ From App. E.1, $A_2/A_2^* = 1.301 = A_1/A_2^*$

$$\text{Then} \quad \frac{A_e}{A_2^*} = \frac{A_e}{A_1^*} \times \frac{A_1^*}{A_1} \times \frac{A_1}{A_2^*} = 4.0 \times \frac{1}{2.448} \times 1.301 = 2.126$$

For $A_e/A^* = 2.126$, from Fig. E.1 and Eq. 12.6, $M_e = 0.286$, $p_e/p_0 = 0.9447$

Then $p_b = p_e = 0.9447\, p_0 = 0.9447 \times 319$ kPa $= 301$ kPa ← p_e



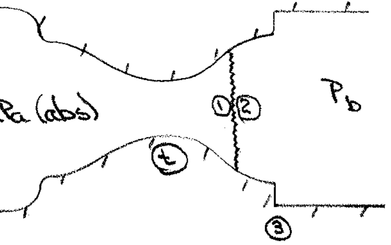


Problem 13.126

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle; nozzle is designed to discharge to atmospheric pressure.

$T_0 = 150°C$

$p_0 = 790$ kPa (abs)



$p_1 = 160$ kPa (abs)

$A_1 = A_2 = 600\ mm^2$

Find: p_b, A_3, A_t

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) ideal gas (4) isentropic flow except across the shock.

At design conditions,

$p_3/p_0 = 101/790 = 0.1278$. From App. E.1, $M_{3\,design} = 2.00$ $\frac{A_3}{A_t} = 1.688$

At section ①, $\frac{p_1}{p_0} = \frac{160}{790} = 0.2025$. From App. E.1, $M_1 = 1.70$ $A_1/A_1^* = 1.338$

$$A_t = A_1^* = \frac{A_1}{1.338} = \frac{600\ mm^2}{1.338} = 448\ mm^2 \longleftarrow A_t$$

Then $A_3 = 1.688\ A_t = 1.688\,(448\ mm^2) = 756\ mm^2 \longleftarrow A_3$

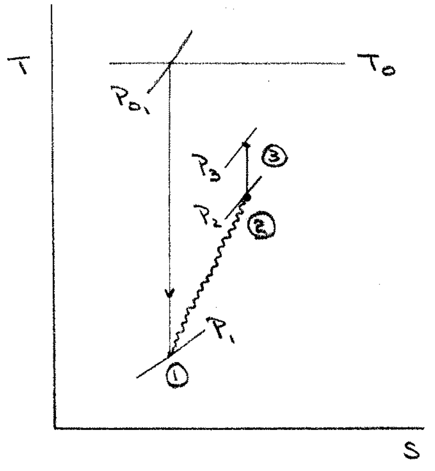
$M_1 = 1.70$. From App. E.4, $M_2 = 0.641$ $p_{0_2}/p_{0_1} = 0.8557$ $p_2/p_1 = 3.205$

$\therefore p_{0_2} = 676$ kPa (abs) $p_2 = 513$ kPa (abs)

$M_2 = 0.641$. From App. E.1, $A_2/A_2^* = 1.145$ $\therefore A_2^* = \frac{A_2}{1.145} = 524\ mm^2$

$A_3/A_2^* = \frac{756}{524} = 1.443$. From Fig. E.1 and Eq. 12.6, $M_3 = 0.453$; $\frac{p_3}{p_{0_3}} = 0.8688$

$$\therefore p_b = p_3 = 0.8688\,p_{0_3} = 0.8688\,p_{0_2} = 587 \text{ kPa (abs)} \longleftarrow p_b$$



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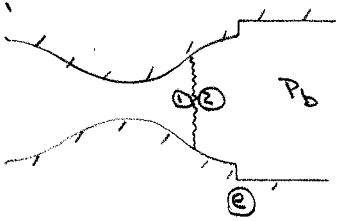
Problem 13.127

Given: Steady, adiabatic flow of air through a converging-diverging nozzle with a shock present; $p_b/p_{0_1} = 0.830$. At design conditions $p_e/p_{0_1} = 0.1278$



$p_b/p_{0_1} = p_e/p_{0_1} = 0.8300$

Find: M_1

Solution:

Compressible flow functions (Appendix E) to be used in solution
Assumptions: (1) steady flow (2) uniform flow at a section
(3) ideal gas (4) isentropic flow except across the shock

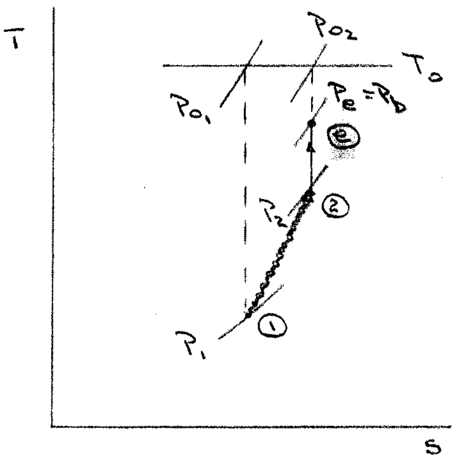
At design conditions, $p_e/p_{0_1} = 0.1278$
From App. E.1, $M_d = 2.0$ and $A_e/A_1^* = 1.688$.

Now $\frac{p_e}{p_{0_1}} = \underbrace{\frac{p_e}{p_{0_2}}}_{f(M_e)} \times \underbrace{\frac{p_{0_2}}{p_{0_1}}}_{f(M_1)}$ and $\frac{A_e}{A_1^*} = \underbrace{\frac{A_e}{A_2^*}}_{f(M_e)} \times \underbrace{\frac{A_2^*}{A_1^*}}_{f(M_1)}$ $\qquad \frac{A_2^*}{A_1^*} = \frac{A_2^*}{A_2} \times \frac{A_1}{A_1^*}$

We thus have a trial and error solution to determine M_1

M_1	$\frac{A_1}{A_1^*}$ (a)	M_2 (b)	$\frac{p_{0_2}}{p_{0_1}}$ (c)	$\frac{A_2}{A_2^*}$ (d)	$\frac{A_2^*}{A_1^*}$	$\frac{A_e}{A_2^*}$	M_e (e)	$\frac{p_e}{p_{0_2}}$ (f)	$\frac{p_e}{p_{0_1}} \overset{?}{=} 0.8300$
1.60	1.25	0.67	0.8952	1.119	1.117	1.511	0.42	0.8857	0.7929
1.40	1.150	0.74	0.9582	1.068	1.077	1.567	0.41	0.8907	0.8534
1.50	1.176	0.70	0.9298	1.094	1.075	1.570	0.403	0.8923	0.830 ✓

$\therefore M_1 = 1.50$ ←———— M_1



a. App. E.1 (12.6)
b. App E.4 (12.34b).
c. App E.4 (12.37)
d. App. E.1 (12.6).
e. App. E.1 Fig. E.1 & Eq. 12.6
f. App. E.1 (11.17a)

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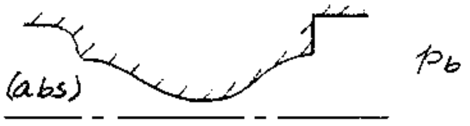
Problem 13.128

[3]

Given: Air flow through a converging-diverging nozzle. $A_e/A_t = 3.5$.

$T_0 = 15°C$

$p_0 = 101$ kPa (abs)



$A_t = 500\ mm^2$

Find: Range of back pressure for which a normal shock will occur in the nozzle, and the corresponding mass flowrate.

Solution: Use compressible flow functions in solution.

Computing equation: $\dot{m} = \rho V A$

Assumptions: (1) Steady flow (3) Uniform flow at a section
(2) Ideal gas (4) Isentropic, except across shock

A normal shock will occur within the nozzle for back pressure conditions in Regime II of Fig. 12.20. For isentropic flow, with $A_e/A_t = 3.5$, from Fig. E.1 and Eq. 12.6,

M	p/p_0	p_b
0.169	0.9858	99.6 kPa
2.80	0.03685	

From Appendix E.4,

M_1	M_2	p_2/p_1
2.80	0.4882	8.980

Thus $p_b = p_2 = p_0 \frac{p_1}{p_0} \frac{p_2}{p_1} = 101\ kPa\ (0.03685)(8.980) = 33.4\ kPa$

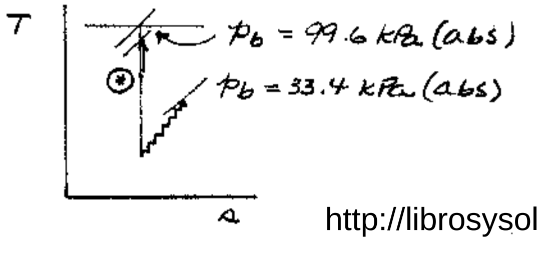
$33.4\ kPa < p_b < 99.6\ kPa\ (abs)$ (for normal shock in nozzle) $\longleftarrow$ p_b

Flow is choked throughout this regime. Thus

$$\dot{m} = \rho_t V_t A_t \qquad \rho_t = \frac{p_t}{RT_t} = (0.5283)\,1.01\times10^5 \frac{N}{m^2} \times \frac{kg\cdot K}{287\ N\cdot m} \times \frac{1}{(0.8333)\,288\ K} = 0.775 \frac{kg}{m^3}$$

$$V_t = c_t = \sqrt{kRT_t} = \left[1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times (0.8333)\,288\ K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{\frac{1}{2}} = 311\ m/s$$

$$\dot{m} = 0.775 \frac{kg}{m^3} \times 311 \frac{m}{s} \times 500\ mm^2 \times \frac{m^2}{10^6\ mm^2} = 0.121 \frac{kg}{s}$$ $\longleftarrow$ $\dot{m}$



Problem 13.129

[3]

Given: Converging-diverging nozzle with $A_e/A_t = 1.633$, designed to operate at atmospheric pressure at the exit plane.

Find: Ranges of stagnation pressure for which nozzle will be free from shocks.

Solution: Use compressible flow tables in solution.

Assume flow in nozzle is isentropic when shock-free. From Appendix E.1 and Eq. 12.6,

M	p/p_0	A/A^*
1.96	0.1360	1.633
0.38	0.9052	1.659
0.40	0.8956	1.590

At design conditions, $M_e = 1.96$, and

$$p_0 \geq \frac{p_e}{(p/p_0)_e} = \frac{101\ \text{kPa}}{0.1360} = 743\ \text{kPa (abs)}$$

By iteration, then the given area ratio corresponds to isentropic choked flow with $M_e = 0.388$ and $(p/p_0)_e = 0.9014$. The corresponding stagnation pressure is

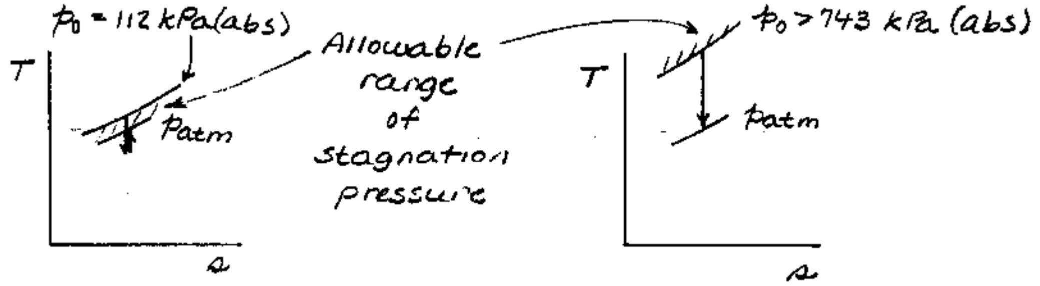
$$p_0 = \frac{p_e}{(p/p_0)_e} = \frac{101\ \text{kPa}}{0.9014} = 112\ \text{kPa (abs)}$$

Flow will be isentropic and shock-free for

(a) $p_{atm} < p_0 < 112$ kPa (abs) $(0 < M_e < 0.388)$

(b) $p_0 > 743$ kPa (abs) $(M_e = 1.96)$

The corresponding Ts diagrams are:



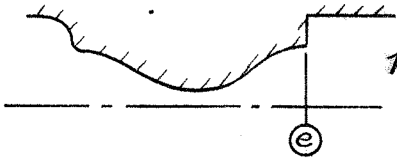
42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE

Problem 13.130

Given: Air flow through a converging-diverging nozzle. $A_e/A_t = 1.87$.

$T_0 = 240°F$

$p_0 = 100$ psia



$p_b = 40$ psia

Find: Mach number and flow velocity in exit plane.

Solution: Use compressible flow functions in solution. Assume ideal gas.

For isentropic flow through the nozzle to $A_e/A_t = 1.87$, from Fig. E.1 and Eq. 12.6,

M	T/T_0	p/p_0	p_d (psia)
2.12	0.5266	0.1060	10.6
~0.33	~0.98	~0.92	~92.0

Neither of these conditions matches the back pressure. Check the case of a shock (at $M = 2.12$) in the exit plane. From Appendix E.4,

M_1	M_2	p_2/p_1
2.12	0.5583	5.077

Then $p_e = 5.077\, p_d = 5.077\,(10.6 \text{ psia}) = 53.8$ psia.

The back pressure of 40 psia is therefore between the design pressure and the pressure that would exist downstream from a normal shock in the exit plane. The flow is in regime III of Fig. 12.20: supersonic in the exit plane with external compression. Thus

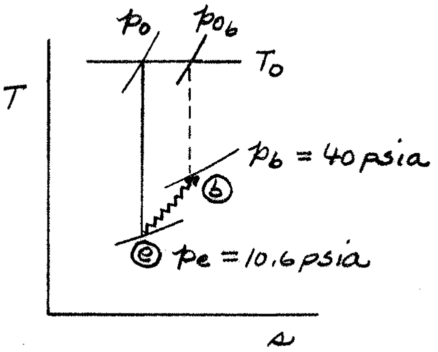
$$M_e = M_d = 2.12 \longleftarrow \quad M_e$$

$$V_e = M_e c_e = M_e \sqrt{kRT_e}$$

$$T_e = \frac{T}{T_0} T_0 = 0.5266\,(460 + 240)°R = 369°R$$

$$V_e = 2.12 \left[1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot °R} \times 369°R \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \right]^{\frac{1}{2}} = 2000 \frac{\text{ft}}{\text{s}} \longleftarrow \quad V_e$$

The Ts diagram is



42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE
NATIONAL

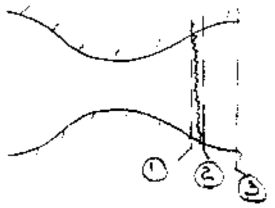
Problem 13.131

[3]

Given: Steady, adiabatic flow of air through a converging-diverging nozzle with shock in diverging section under conditions shown.

$T_0 = 1000°R$

$p_0 = 100$ psia



$A_1 = A_2 = 4.0\ in^2$

$M_1 = 2.0$

$A_3 = 6.0\ in^2$

Find: p_3

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (3) uniform flow at each section
(2) ideal gas (4) isentropic flow, except across shock.

For $M_1 = 2.0$, from App. E.1, $p_1/p_{0_1} = 0.1278$ $\therefore$ $p_1 = 12.78$ psia.

For $M_1 = 2.0$, from App. E.4, $M_2 = 0.5774$

$p_2/p_1 = 4.50$ $\therefore$ $p_2 = 57.5$ psia

$p_{0_2}/p_{0_1} = 0.7209$ $\therefore$ $p_{0_2} = 72.1$ psia

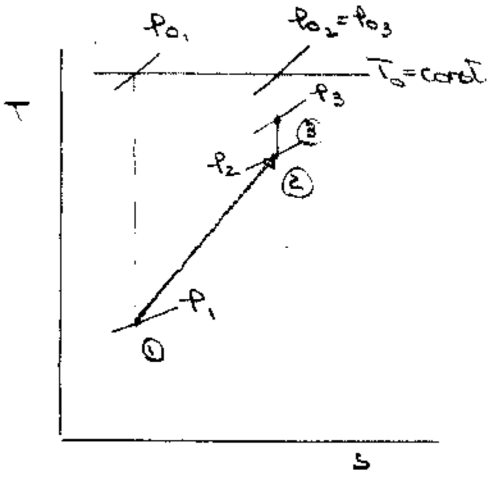
For $M_2 = 0.5774$, from App. E.1, $A_2/A_2^* = 1.216$ $\therefore$ $A_2^* = 3.288\ in^2$

Then $A_3/A_2^* = 6.0/3.288 = 1.825$

With $A_3/A^* = 1.825$, from App. E.1 (Fig. E.1 and Eq. 12.6) $M_3 = 0.340$

With $M_3 = 0.340$, from App. E.1, $p_{0_3}/p_3 = 1.083$

$$p_3 = \frac{p_{0_3}}{1.083} = \frac{p_{0_2}}{1.083} = \frac{72.1 \text{ psia}}{1.083} = 66.6 \text{ psia} \underline{\qquad\qquad p_3}$$



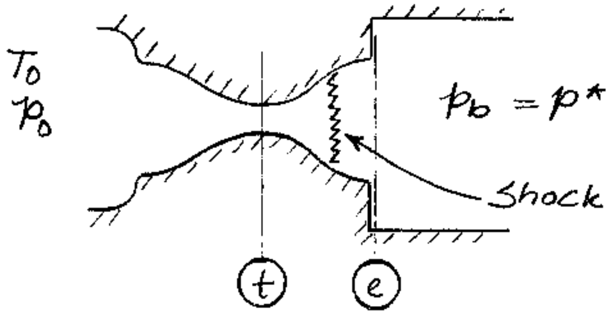
Problem 13.132

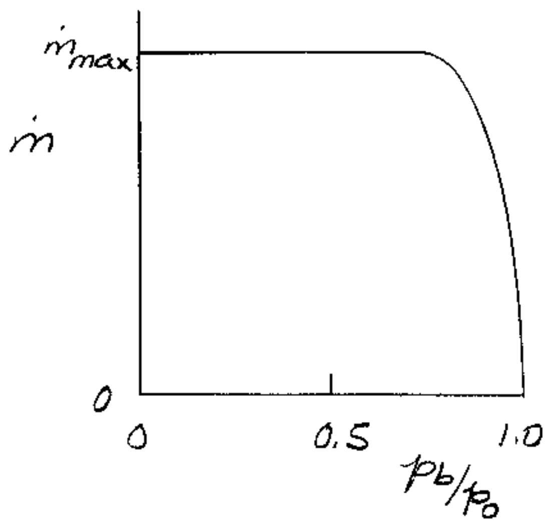
[4]

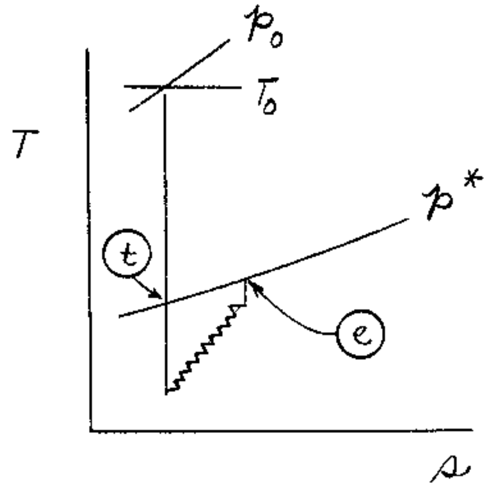
Given: Flow through a converging-diverging nozzle, $p_b = p^*$

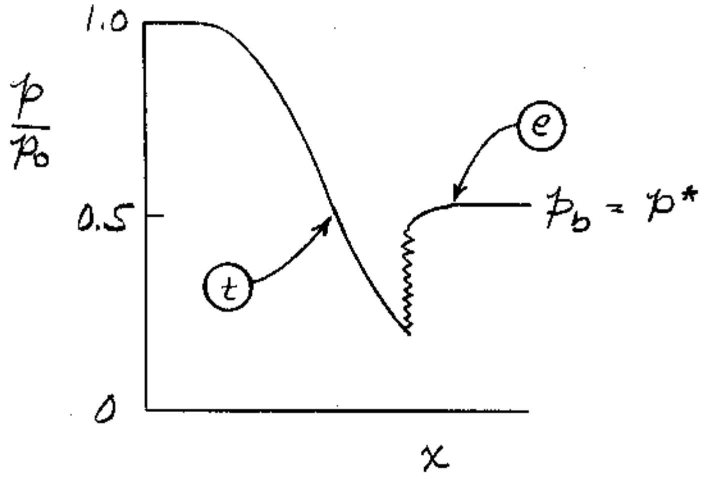
Find: Sketch (a) mass flow rate vs. pressure ratio
(b) pressure vs. distance along nozzle
(c) Ts diagram

Solution: When $p_b = p^*$, flow in the nozzle will be choked and a shock will stand in the diverging section.







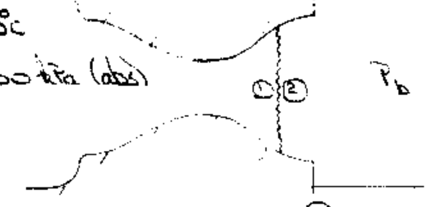


42 381 50 SHEETS 5 SQUARE
42 382 100 SHEETS 5 SQUARE
42 389 200 SHEETS 5 SQUARE
NATIONAL

Problem 13.133

[3]

Given: Steady, adiabatic air flow from a reservoir through a converging - diverging nozzle with a shock present



$T_0 = 60°C$

$p_0 = 600$ kPa (abs)

M_e design $= 2.94$

$M_1 = 2.42$

Find: p_b, sketch the pressure distribution

Solution:

Compressible flow tables (Appendix E) to be used in solution.

Assumptions: (1) steady flow (3) uniform flow at each section
(2) ideal gas (4) isentropic flow, except across shock.

At design, $M_e = 2.94$, from Appendix E.1, $A_e/A_1^* = 3.999$

For $M_1 = 2.42$, from Appendix E.1, $p_1/p_{0_1} = 0.06630$
$A_1/A_1^* = 2.448$

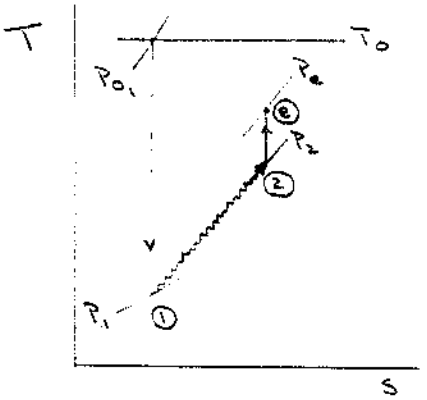
For $M_1 = 2.42$, from Appendix E.4, $M_2 = 0.521$
$p_{0_2}/p_{0_1} = 0.5318$
$p_2/p_1 = 6.666$

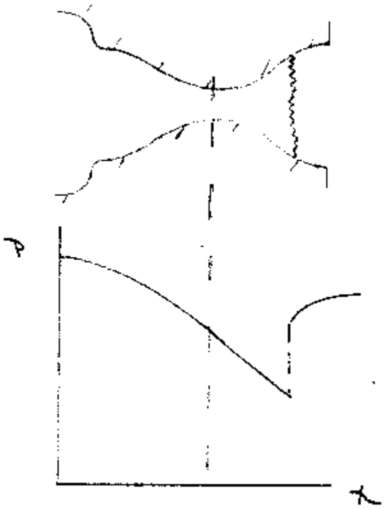
For $M_2 = 0.521$, from Appendix E.1, $A_2/A_2^* = A_1/A_2^* = 1.302$

then, $\dfrac{A_e}{A_2^*} = \dfrac{A_e}{A_1^*} \times \dfrac{A_1^*}{A_1} \times \dfrac{A_1}{A_2^*} = 3.999 \times \dfrac{1}{2.448} \times 1.302 = 2.127$

For $A_e/A_2^* = 2.127$, from App. E.1 (Fig E.1 c Eq 12.6), $M_e = 0.286$, a $\dfrac{p_e}{p_{0_2}} = 0.9447$

$\therefore p_b = p_e = \dfrac{p_e}{p_{0_2}} \times \dfrac{p_{0_2}}{p_{0_1}} \times p_{0_1} = 0.9447 \times 0.5318 \times 600 \text{ kPa} = 301 \text{ kPa} \longleftarrow p_b$



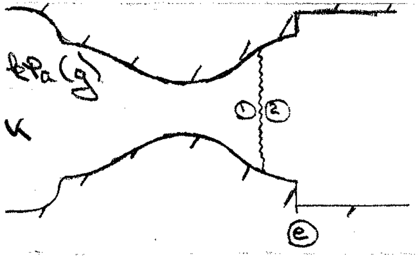


Problem 13.134

Given: Steady, adiabatic flow of air through a converging-diverging nozzle with shock in diverging section under conditions shown.

$p_0 = 1000$ kPa (g)

$T_0 = 400$ K



$M_1 = 3.0$

$A_1 = A_t = 500$ mm^2

$A_e = 600$ mm^2

Find: M_2, p_{02}, p_2, A_t, $s_2 - s_1$, M_e

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (3) uniform flow at each section
(2) ideal gas (4) isentropic flow, except across shock.

For $M_1 = 3.0$, from Appendix E.1, $p_1/p_0 = 0.02722 \quad \therefore p_1 = 29.97$ kPa

$, A_1/A^* = 4.235 \quad \therefore A^* = A_t = 118$ mm^2 $\longleftarrow A_t$

For $M_1 = 3.0$, from Appendix E.4, $M_2 = 0.475 \longleftarrow M_2$

$p_{02}/p_{01} = 0.3283 \quad \therefore p_{02} = 361$ kPa (abs) $\longleftarrow p_{02}$

$p_2/p_1 = 10.33 \quad \therefore p_2 = 310$ kPa (abs) $\longleftarrow p_2$

Since $T_{02} = T_{01}$, then

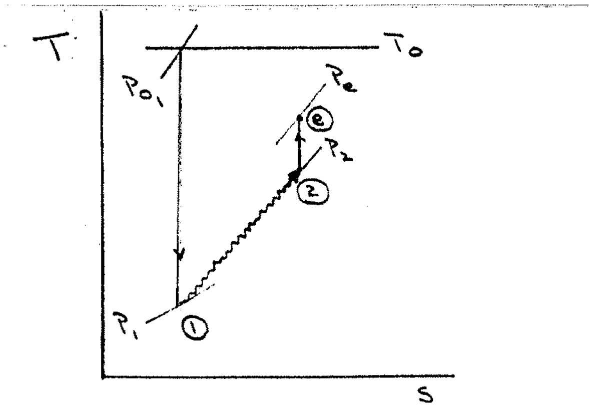
$$s_2 - s_1 = s_{02} - s_{01} = \cancel{c_p \ln \frac{T_{02}}{T_{01}}} - R \ln \frac{p_{02}}{p_{01}} = -287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \ln 0.3283$$

$$s_2 - s_1 = -0.320 \text{ kJ/kg}\cdot\text{K} \longleftarrow s_2 - s_1$$

At $M_2 = 0.475$, from App. E.1 $\quad A_2/A_2^* = 1.391 \quad \therefore A_2^* = 359.5$ mm^2

At exit $A_e/A_2^* = \frac{600}{359.5} = 1.669$. From App E.1 (Fig E.1) & Eq 12.6

$M_e = 0.377 \longleftarrow M_e$



The actual exit Mach number would be higher than the estimate based on isentropic flow downstream from the shock.

Flow downstream from the shock is subsonic. Flow slows in the diverging passage, which acts as a subsonic diffuser, causing pressure to increase in the direction of flow.

The result will be rapid growth of boundary layers on the channel walls. The boundary layers reduce the effective flow area of the passage. Because the boundary layers thicken rapidly, the area ratio for slowing the flow will be less than for isentropic flow. Therefore the actual flow will not slow as much as the isentropic model predicts.

The actual exit Mach number will be higher than the estimate based on isentropic flow.

Problem *13.135

Open-Ended Problem Statement: A supersonic wind tunnel must have two throats, with the second throat larger than the first. Explain why this must be so.

Discussion: The first throat is located in the supersonic nozzle from which flow enters the test section. The second throat is located in the supersonic diffuser that slows flow leaving the test section to subsonic speed for re-compression and re-circulation.

The second throat must be larger than the first for two reasons. First, it is impossible to slow a real flow to a Mach number of exactly one in a supersonic diffuser. The minimum Mach number that can be achieved with stable flow is about $M = 1.3$. Therefore even if the flow were isentropic everywhere the second throat would have to be larger than the first by the area ratio A/A^* corresponding to $M = 1.3$ at the throat.

The second reason is that flow is not isentropic through the tunnel. Some friction exists, which must reduce the stagnation pressure of the flow stream. This also reduces the stagnation density. Therefore a larger area is needed to carry the mass flow at any given flow speed.

For these reasons the second throat area must be larger than the first throat area.

Problem *13.136

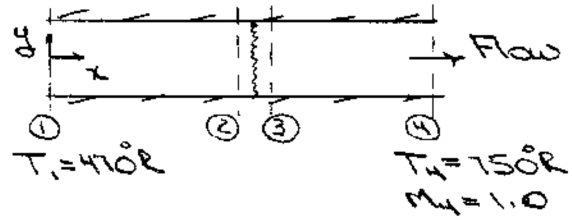
[2]

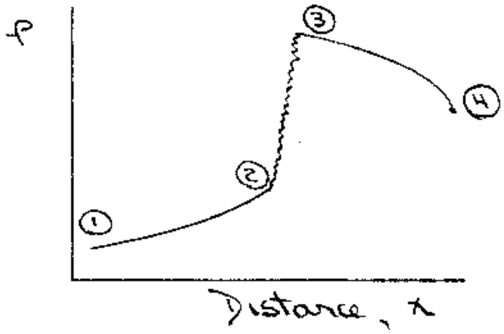
42-381 50 SHEETS 5 SQUARE
42-382 100 SHEETS 5 SQUARE
42-389 200 SHEETS 5 SQUARE

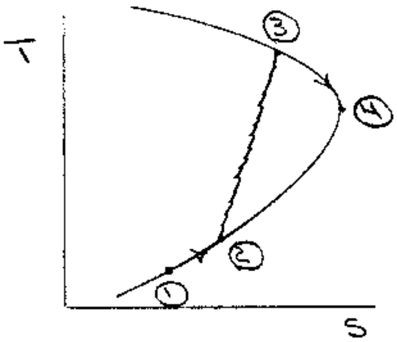
Given: A normal shock stands in a section of insulated constant-area duct; conditions immediately upstream and downstream of the shock are denoted by subscripts 2 and 3, respectively.
Flow in the duct is frictional.
Conditions at ① (some distance upstream) are $T_1 = 470°R$ and at ④ (some distance downstream) are $T_4 = 750°R$ and $M_4 = 1.0$

Find: (a) Sketch pressure distribution all the duct
(b) Sketch a Ts diagram.
(c) Determine M_1

Solution:







For adiabatic flow, T_0 = constant (from energy eq.)

Thus, $T_{0_1} = T_{0_4} = T_4\left[1+\frac{k-1}{2}M_4^2\right] = 750°R\,[1.2] = 900°R$

Since $T_{0_1} = T_1\left[1+\frac{k-1}{2}M_1^2\right]$, then

$$M_1^2 = \frac{2}{k-1}\left[\frac{T_{0_1}}{T_1}-1\right] = \frac{2}{0.4}\left[\frac{900R}{470R}-1\right] = 4.57$$

$$M_1 = \sqrt{4.57} = 2.14 \longleftarrow M_1$$

Problem *13.137

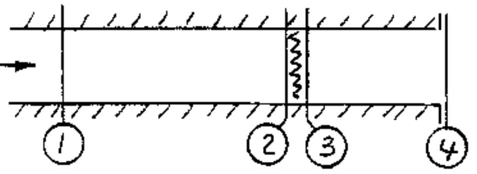
[4]

Given: Flow with shock in insulated constant-area duct, as shown.

$T_1 = 668°R$

$p_{01} = 78.2$ psia

$M_1 = 2.05$



$T_2 = 388\ F$ $\quad M_4 = 1.0$

Find: (a) Speed before shock, V_2.

(b) Entropy change, $s_4 - s_1$.

Solution: Use functions for steady, one-dimensional compressible flow.

Computing equations: $\frac{T}{T_0}$ from isentropic functions (Appendix E.1)

$\frac{p}{p^*}$ from Fanno-line functions (Appendix E.2)

Assumptions: (1) Steady flow
(2) Uniform flow at each cross-section
(3) Ideal gas
(4) Fanno line flow
(5) $F_{B_x} = 0$
(6) $\Delta z = 0$

For adiabatic flow on Fanno line and across shock, T_0 = constant. At $M_1 = 2.05$, $T/T_0 = 0.5433$ (Eq. 11.17b). Thus

$$T_0 = T_{01} = \frac{T_1}{(T/T_0)_1} = \frac{668°R}{0.5433} = 1230°R$$

Using $T_2 = 388\ F\ (848°R)$, $\frac{T}{T_0} = \frac{848°R}{1230°R} = 0.6894$ and $M_2 = 1.50$ (Table E.1).

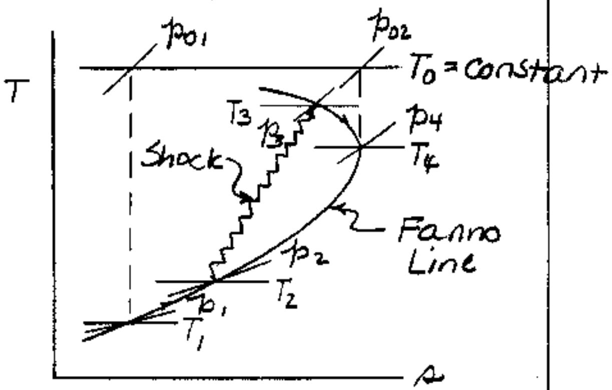
$$V_2 = M_2 c_2 = M_2\sqrt{kRT_2} = 1.50\left[1.4 \times 53.3\frac{ft \cdot lbf}{lbm \cdot °R} \times 848°R \times 32.2\frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}}$$

$$V_2 = 2140\ ft/s$$

V_2

Flow must stay on the same Fanno line. Thus $(p_0/p_0^*)_1 = 1.760$ at $M_1 = 2.05$ (Eq. 12.18e). Thus

$$p_0^* = \frac{p_{01}}{(p_0/p_0^*)_1} = \frac{78.2\ psia}{1.760} = 44.4\ psia$$



From the Gibbs equation

$$T ds = dh - v dp\ ;\ ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$\Delta s = \cancel{c_p \ln\frac{T_{04}}{T_{01}}}^{\,0} - R \ln\frac{p_{04}}{p_{01}}$$

$$\Delta s = -53.3\frac{ft \cdot lbf}{lbm \cdot °R} \times \ln\left(\frac{44.4}{78.2}\right) \times \frac{Btu}{778\ ft \cdot lbf} = 0.0388\ Btu/lbm \cdot °R$$

Δs

{This problem could be solved without using functions, but the solution would require more calculations.}

Problem 13.138

13.138 Show that as the upstream Mach number approaches infinity, the Mach number after an oblique shock becomes

$$M_2 \approx \sqrt{\frac{k-1}{2k\sin^2(\beta-\theta)}}$$

Given: Normal shock

Find: Approximation for downstream Mach number as upstream one approaches infinity

Solution:

Basic equations:

$$M_{2n}^{\;2} = \frac{M_{1n}^{\;2} + \frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_{1n}^{\;2} - 1} \qquad (13.48a) \qquad M_{2n} = M_2\cdot\sin(\beta-\theta) \qquad (13.47b)$$

Combining the two equations

$$M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = \frac{\sqrt{\dfrac{M_{1n}^{\;2} + \frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_{1n}^{\;2} - 1}}}{\sin(\beta-\theta)} = \sqrt{\frac{M_{1n}^{\;2} + \frac{2}{k-1}}{\left[\left(\frac{2\cdot k}{k-1}\right)\cdot M_{1n}^{\;2} - 1\right]\cdot\sin(\beta-\theta)^2}}$$

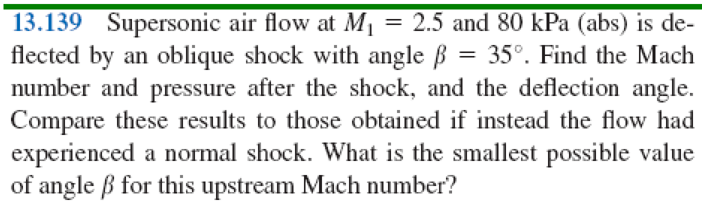
$$M_2 = \sqrt{\frac{1 + \frac{2}{(k-1)\cdot M_{1n}^{\;2}}}{\left[\left(\frac{2\cdot k}{k-1}\right) - \frac{1}{M_{1n}^{\;2}}\right]\cdot\sin(\beta-\theta)^2}}$$

As M_1 goes to infinity, so does M_{1n}, so

$$M_2 = \sqrt{\frac{1}{\left(\frac{2\cdot k}{k-1}\right)\cdot\sin(\beta-\theta)^2}} \qquad M_2 = \sqrt{\frac{k-1}{2\cdot k\cdot\sin(\beta-\theta)^2}}$$

Problem 13.139

[3]



13.139 Supersonic air flow at $M_1 = 2.5$ and 80 kPa (abs) is deflected by an oblique shock with angle $\beta = 35°$. Find the Mach number and pressure after the shock, and the deflection angle. Compare these results to those obtained if instead the flow had experienced a normal shock. What is the smallest possible value of angle β for this upstream Mach number?

Given: Data on an oblique shock

Find: Mach number and pressure downstream; compare to normal shock

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K

$k =$ 1.4

$p_1 =$ 80 kPa

$M_1 =$ 2.5

$\beta =$ 35 °

Equations and Computations:

From M_1 and β

$M_{1n} =$ 1.43

$M_{1t} =$ 2.05

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M* ,*k*))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 178.6 kPa

The tangential velocity is unchanged

$V_{t1} = V_{t2}$

Hence

$c_{t1} M_{t1} = c_{t2} M_{t2}$

$(T_1)^{1/2} M_{t1} = (T_2)^{1/2} M_{t2}$

$M_{2t} = (T_1/T_2)^{1/2} M_{t1}$

From M_{1n}, and Eq. 13.48c
(using built-in function *NormTfromM* (*M* ,*k*))

$T_2/T_1 =$ 1.28

Hence

$M_{2t} =$ 1.81

Also, from M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (*M* ,*k*))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.726

The downstream Mach number is then

$$M_2 = (M_{2t}^2 + M_{2n}^2)^{1/2}$$

$M_2 =$ 1.95

Finally, from geometry

$$V_{2n} = V_2 \sin(\beta - \theta)$$

Hence $\theta = \beta - \sin^{-1}(V_{2n}/V_2)$

or $\theta = \beta - \sin^{-1}(M_{2n}/M_2)$

$\theta =$ 13.2 °

For the normal shock:
From M_1 and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M* ,*k*))

$p_2 =$ 570 kPa

Also, from M_1, and Eq. 13.48a
(using built-in function *NormM2fromM* (*M* ,*k*))

$M_2 =$ 0.513

For the minimum β:
The smallest value of β is when the shock is a Mach wave (no deflection)

$$\beta = \sin^{-1}(1/M_1)$$

$\beta =$ 23.6 °

Problem 13.140

[3]

13.140 Consider supersonic flow of air at $M_1 = 3.0$. What is the range of possible values of the oblique shock angle β? For this range of β, plot the pressure ratio across the shock.

Given: Oblique shock in flow at $M = 3$

Find: Minimum and maximum β, plot of pressure rise across shock

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4
$M_1 =$ 3

Equations and Computations:

The smallest value of β is when the shock is a Mach wave (no deflection)

$$\beta = \sin^{-1}(1/M_1)$$

$\beta =$ 19.5 °

The largest value is $\beta =$ 90.0 °

The normal component of Mach number is

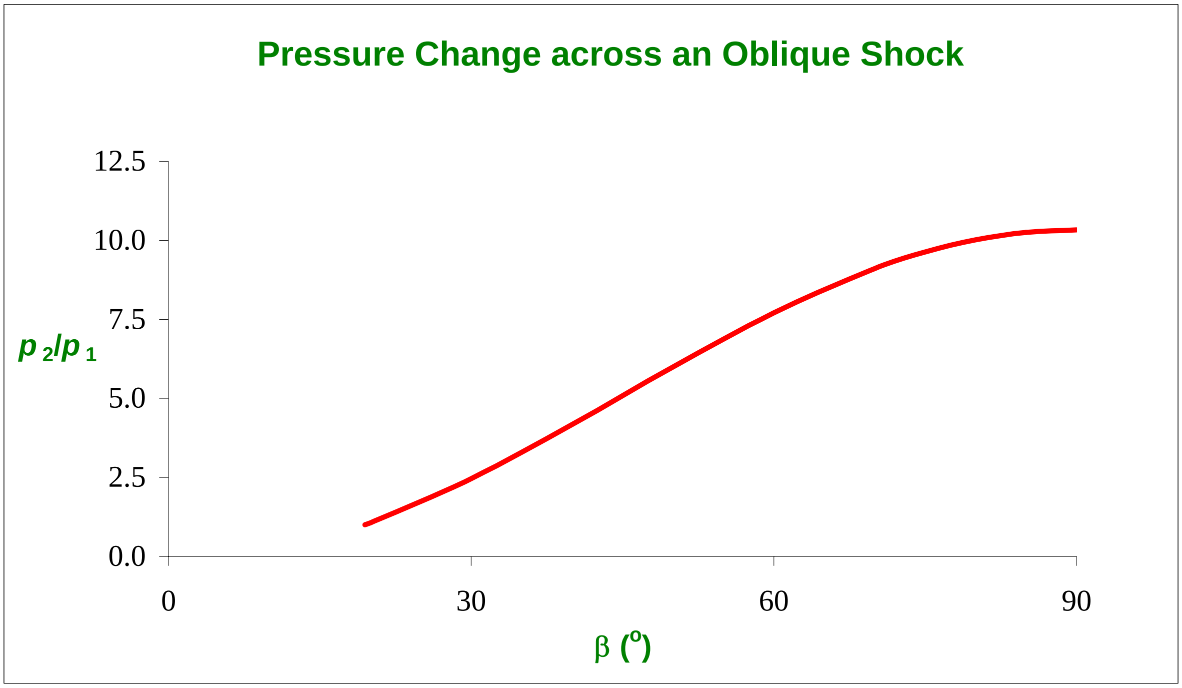
$$M_{1n} = M_1\sin(\beta) \qquad (13.47a)$$

For each β, p_2/p_1 is obtained from M1n, and Eq. 13.48d
(using built-in function *NormpfromM* (M ,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

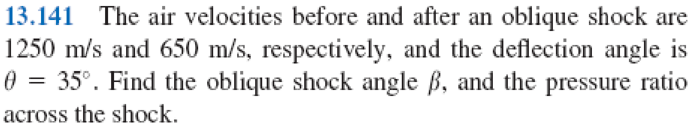
Computed results:

β (^{o})	M_{1n}	p_2/p_1
19.5	1.00	1.00
20	1.03	1.06
30	1.50	2.46
40	1.93	4.17
50	2.30	5.99
60	2.60	7.71
70	2.82	9.11
75	2.90	9.63
80	2.95	10.0
85	2.99	10.3
90	3.00	10.3



Problem 13.141

[3]



13.141 The air velocities before and after an oblique shock are 1250 m/s and 650 m/s, respectively, and the deflection angle is $\theta = 35°$. Find the oblique shock angle β, and the pressure ratio across the shock.

Given: Velocities and deflection angle of an oblique shock

Find: Shock angle β; pressure ratio across shock

Solution:

The given or available data is:

$R = 286.9$ J/kg.K

$k = 1.4$

$V_1 = 1250$ m/s

$V_2 = 650$ m/s

$\theta = 35$ °

Equations and Computations:

From geometry we can write two equations for tangential velocity:

For V_{1t} $$V_{1t} = V_1\cos(\beta) \quad (1)$$

For V_{2t} $$V_{2t} = V_2\cos(\beta - \theta) \quad (2)$$

For an oblique shock $V_{2t} = V_{1t}$, so Eqs. 1 and 2 give

$$V_1\cos(\beta) = V_2\cos(\beta - \theta) \quad (3)$$

Solving for β $$\beta = \tan^{-1}((V_1 - V_2\cos(\theta))/(V_2\sin(\theta)))$$

$$\beta = 62.5\ °$$

(Alternatively, solve Eq. 3 using *Goal Seek*!)

For p_2/p_1, we need M_{1n} for use in Eq. 13.48d

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

We can compute M_1 from θ and β, and Eq. 13.49
(using built-in function *Theta* (M,β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For

$\theta =$ 35.0 °
$\beta =$ 62.5 °

$M_1 =$ 3.19

This value of M_1 was obtained by using *Goal Seek* :
Vary M_1 so that θ becomes the required value.
(Alternatively, find M_1 from Eq. 13.49 by explicitly solving for it!)

We can now find M_{1n} from M_1. From M_1 and Eq. 13.47a

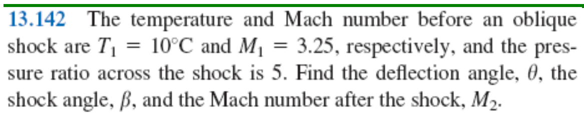
$$M_{1n} = M_1\sin(\beta) \qquad (13.47a)$$

Hence $M_{1n} =$ 2.83

Finally, for p_2/p_1, we use M_{1n} in Eq. 13.48d
(using built-in function *NormpfromM* (M,k)

$p_2/p_1 =$ 9.15

Problem 13.142



13.142 The temperature and Mach number before an oblique shock are $T_1 = 10°\text{C}$ and $M_1 = 3.25$, respectively, and the pressure ratio across the shock is 5. Find the deflection angle, θ, the shock angle, β, and the Mach number after the shock, M_2.

Given: Data on an oblique shock

Find: Deflection angle θ; shock angle β; Mach number after shock

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K

$k =$ 1.4

$M_1 =$ 3.25

$T_1 =$ 283 K

$p_2/p_1 =$ 5

Equations and Computations:

From p_2/p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M,k)
and *Goal Seek* or *Solver*)

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48\text{d})$$

For $p_2/p_1 =$ 5.00

$M_{1n} =$ 2.10

From M_1 and M_{1n}, and Eq 13.47a

$$M_{1n} = M_1 \sin(\beta) \qquad (13.47\text{a})$$

$\beta =$ 40.4 °

From M_1 and β, and Eq. 13.49
(using built-in function *Theta* (M,β, k)

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta) + 2} \qquad (13.49)$$

$\theta =$ 23.6 °

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (M,k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48\text{a})$$

$M_{2n} =$ 0.561

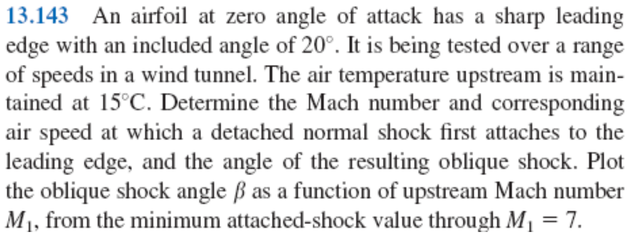
The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2 \sin(\beta - \theta) \qquad (13.47\text{b})$$

Hence $M_2 =$ 1.94

Problem 13.143

[4]



13.143 An airfoil at zero angle of attack has a sharp leading edge with an included angle of 20°. It is being tested over a range of speeds in a wind tunnel. The air temperature upstream is maintained at 15°C. Determine the Mach number and corresponding air speed at which a detached normal shock first attaches to the leading edge, and the angle of the resulting oblique shock. Plot the oblique shock angle β as a function of upstream Mach number M_1, from the minimum attached-shock value through $M_1 = 7$.

Given: Airfoil with included angle of 20^o

Find: Mach number and speed at which oblique shock forms

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4
$T_1 =$ 288 K
$\theta =$ 10 ^{o}

Equations and Computations:

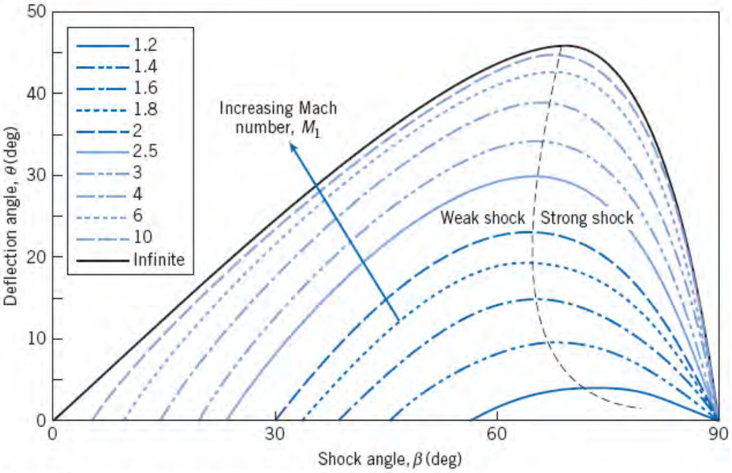


Fig. 13.29 Oblique shock deflection angle.

From Fig. 13.29 the smallest Mach number for which an oblique shock exists at a deflection $\theta = 10^o$ is approximately $M_1 = 1.4$.

By trial and error, a more precise answer is
(using built-in function *Theta* (M ,β, k)

$M_1 =$ 1.42
$\beta =$ 67.4 ^{o}
$\theta =$ 10.00 ^{o}

$c_1 =$ 340 m/s
$V_1 =$ 483 m/s

A suggested procedure is:

1) Type in a guess value for M_1
2) Type in a guess value for β

3) Compute θ from Eq. 13.49
(using built-in function *Theta* (*M* ,β, *k*))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

4) Use *Solver* to maximize θ by varying β
5) If θ is not 10°, make a new guess for M_1
6) Repeat steps 1 - 5 until θ = 10°

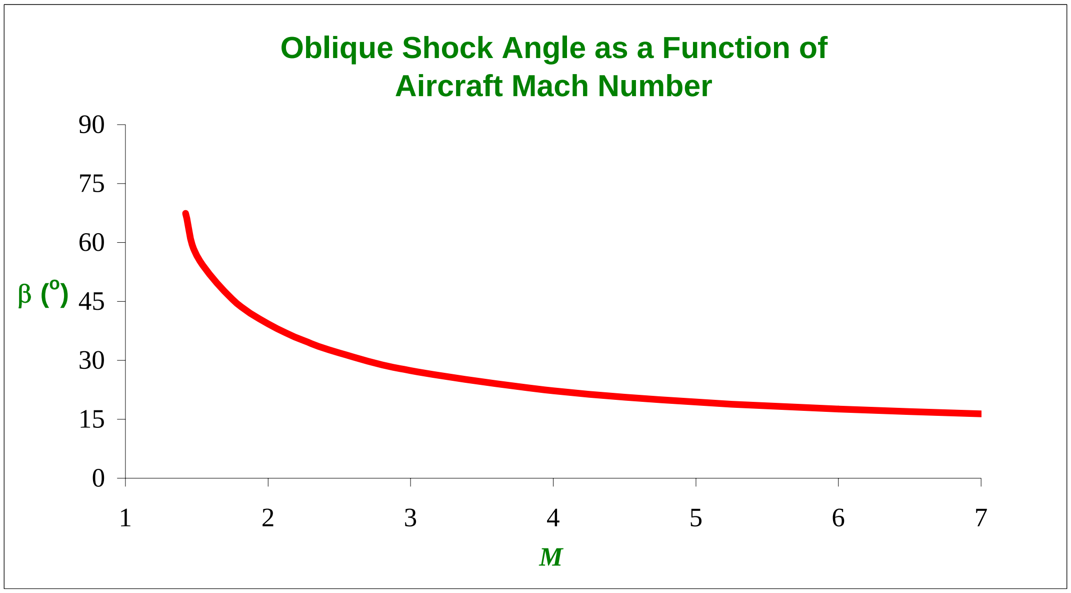
Computed results:

M_1	β (°)	θ (°)	Error
1.42	67.4	10.0	0.0%
1.50	56.7	10.0	0.0%
1.75	45.5	10.0	0.0%
2.00	39.3	10.0	0.0%
2.25	35.0	10.0	0.0%
2.50	31.9	10.0	0.0%
3.00	27.4	10.0	0.0%
4.00	22.2	10.0	0.0%
5.00	19.4	10.0	0.0%
6.00	17.6	10.0	0.0%
7.00	16.4	10.0	0.0%

Sum: 0.0%

To compute this table:

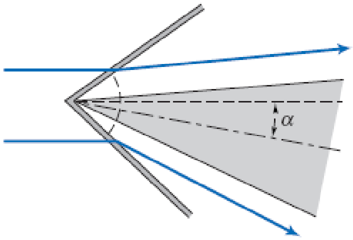
1) Type the range of M_1
2) Type in guess values for β
3) Compute θ from Eq. 13.49
(using built-in function *Theta* (*M* ,β, *k*)
4) Compute the absolute error between each θ and θ = 10°
5) Compute the sum of the errors
6) Use *Solver* to minimize the sum by varying the β values
(Note: You may need to interactively type in new β values
if *Solver* generates β values that lead to no θ, or to
β values that correspond to a strong rather than weak shock)



Problem 13.144

[4]

13.144 An airfoil has a sharp leading edge with an included angle of $\delta = 60°$. It is being tested in a wind tunnel running at 1200 m/s (the air pressure and temperature upstream are 75 kPa and 3.5°C). Plot the pressure and temperature in the region adjacent to the upper surface as functions of angle of attack, α, ranging from $\alpha = 0°$ to 30°. What are the maximum pressure and temperature? (Ignore the possibility of a detached shock developing if α is too large; see Problem 13.145.)



Given: Airfoil with included angle of 60^o

Find: Plot of temperature and pressure as functions of angle of attack

Solution:

The given or available data is:

R = 286.9 J/kg.K

k = 1.4

T_1 = 276.5 K

p_1 = 75 kPa

V_1 = 1200 m/s

δ = 60 ^{o}

Equations and Computations:

From T_1 c_1 = 333 m/s

Then M_1 = 3.60

Computed results:

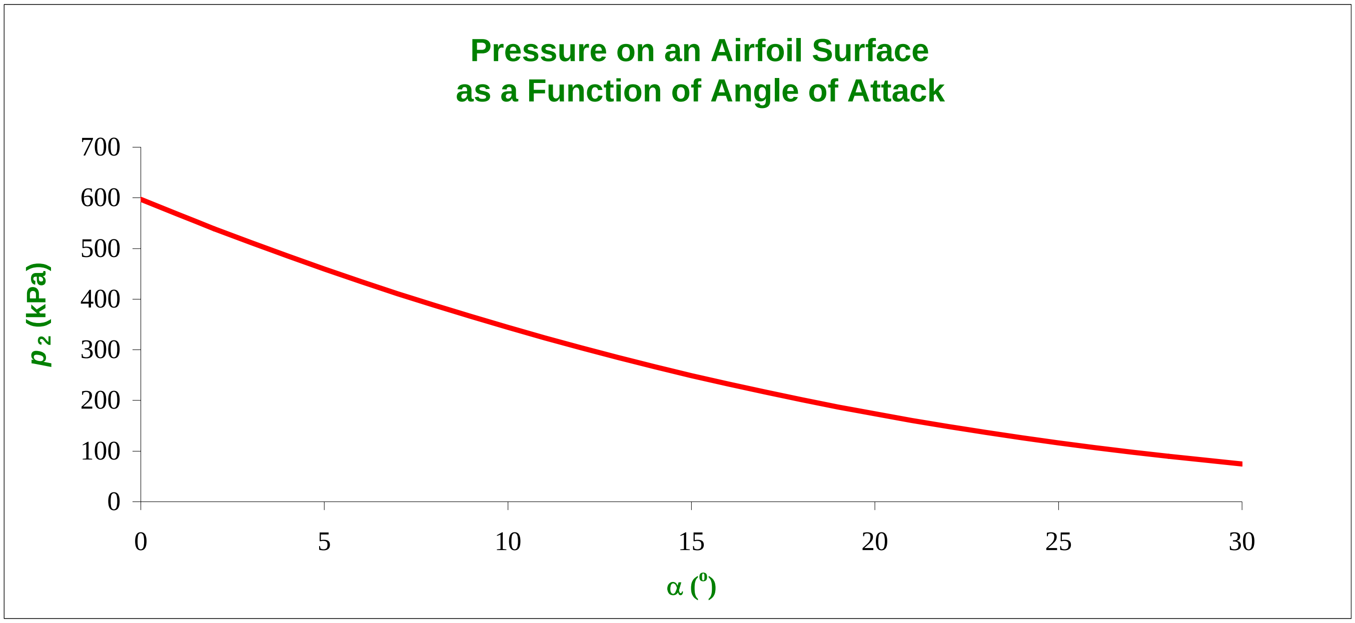
α (^{o})	β (^{o})	θ (^{o}) Needed	θ (^{o})	Error	M_{1n}	p_2 (kPa)	T_2 (^{o}C)
0.00	47.1	30.0	30.0	0.0%	2.64	597	357
2.00	44.2	28.0	28.0	0.0%	2.51	539	321
4.00	41.5	26.0	26.0	0.0%	2.38	485	287
6.00	38.9	24.0	24.0	0.0%	2.26	435	255
8.00	36.4	22.0	22.0	0.0%	2.14	388	226
10.00	34.1	20.0	20.0	0.0%	2.02	344	198
12.00	31.9	18.0	18.0	0.0%	1.90	304	172
14.00	29.7	16.0	16.0	0.0%	1.79	267	148
16.00	27.7	14.0	14.0	0.0%	1.67	233	125
18.00	25.7	12.0	12.0	0.0%	1.56	202	104
20.00	23.9	10.0	10.0	0.0%	1.46	174	84
22.00	22.1	8.0	8.0	0.0%	1.36	149	66
24.00	20.5	6.0	6.0	0.0%	1.26	126	49
26.00	18.9	4.0	4.0	0.0%	1.17	107	33
28.00	17.5	2.0	2.0	0.0%	1.08	90	18
30.00	16.1	0.0	0.0	0.0%	1.00	75	3

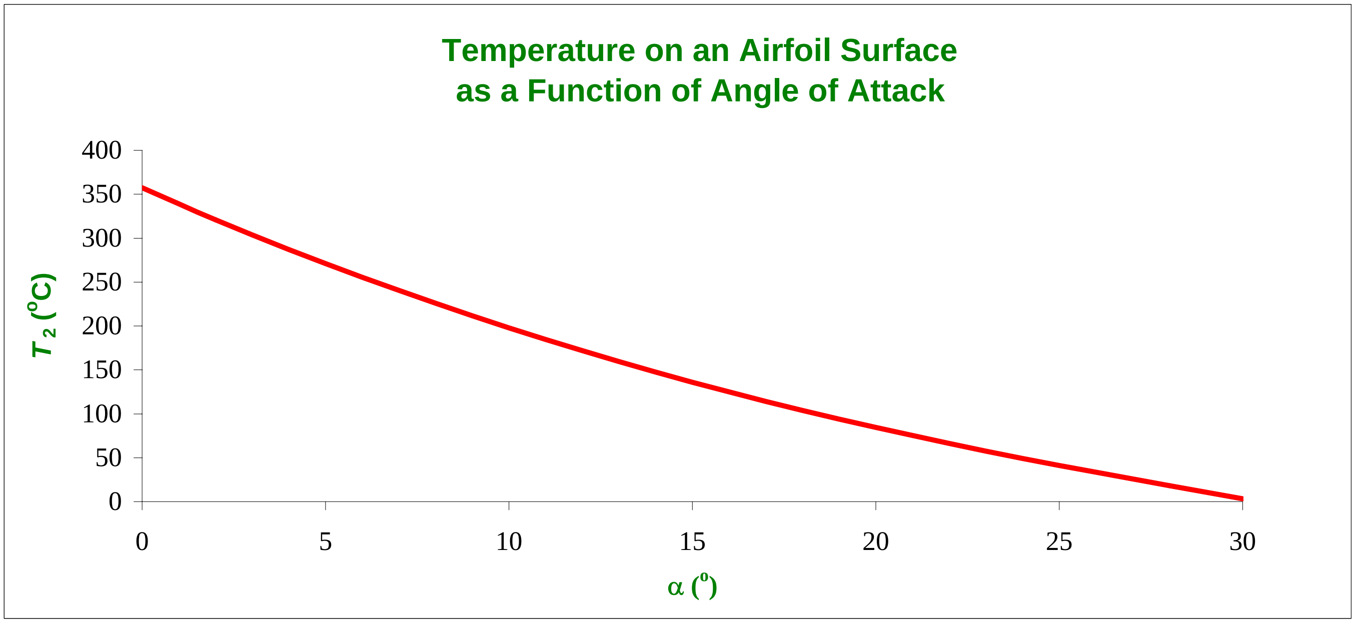
Sum: 0.0%

Max: 597 357

To compute this table:

1) Type the range of α
2) Type in guess values for β
3) Compute θ_{Needed} from $\theta = \delta/2 - \alpha$
4) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M, β, k)
5) Compute the absolute error between each θ and θ_{Needed}
6) Compute the sum of the errors
7) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ)
8) For each α, M_{1n} is obtained from M_1, and Eq. 13.47a
9) For each α, p_2 is obtained from p_1, M_{1n}, and Eq. 13.48d
   (using built-in function *NormpfromM* (M, k))
10) For each α, T_2 is obtained from T_1, M_{1n}, and Eq. 13.48c
   (using built-in function *NormTfromM* (M, k))

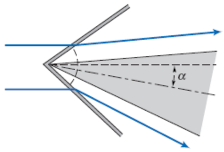




Problem 13.145

[4]

13.145 The airfoil of Problem 13.144 will develop a detached shock on the lower surface if the angle of attack, α, exceeds a certain value. What is this angle of attack? Plot the pressure and temperature in the region adjacent to the lower surface as functions of angle of attack, α, ranging from $\alpha = 0°$ to the angle at which the shock becomes detached. What are the maximum pressure and temperature?



Given: Airfoil with included angle of 60°

Find: Angle of attack at which oblique shock becomes detached

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
T_1 = 276.5 K
p_1 = 75 kPa
V_1 = 1200 m/s
δ = 60 °

Equations and Computations:

From T_1 c_1 = 333 m/s

Then M_1 = 3.60

From Fig. 13.29, at this Mach number the smallest deflection angle for which an oblique shock exists is approximately $\theta = 35°$.

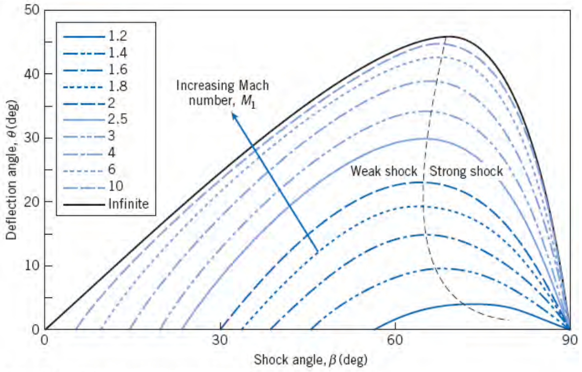


Fig. 13.29 Oblique shock deflection angle.

By using *Solver*, a more precise answer is
(using built-in function *Theta* (M, β, k)

M_1 = 3.60
β = 65.8 °
θ = 37.3 °

A suggested procedure is:

1) Type in a guess value for β
2) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

3) Use *Solver* to maximize θ by varying β

For a deflection angle θ the angle of attack α is

$\alpha = \theta - \delta/2$
α = 7.31 °

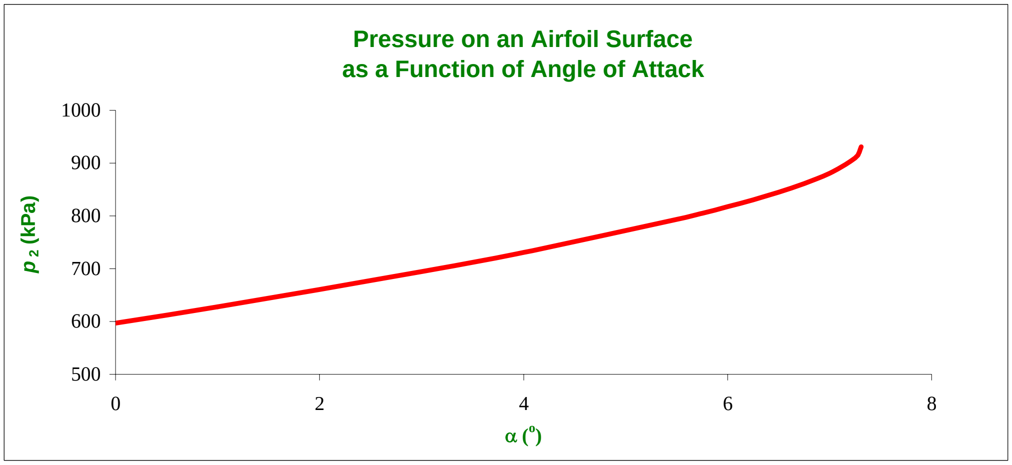
Computed results:

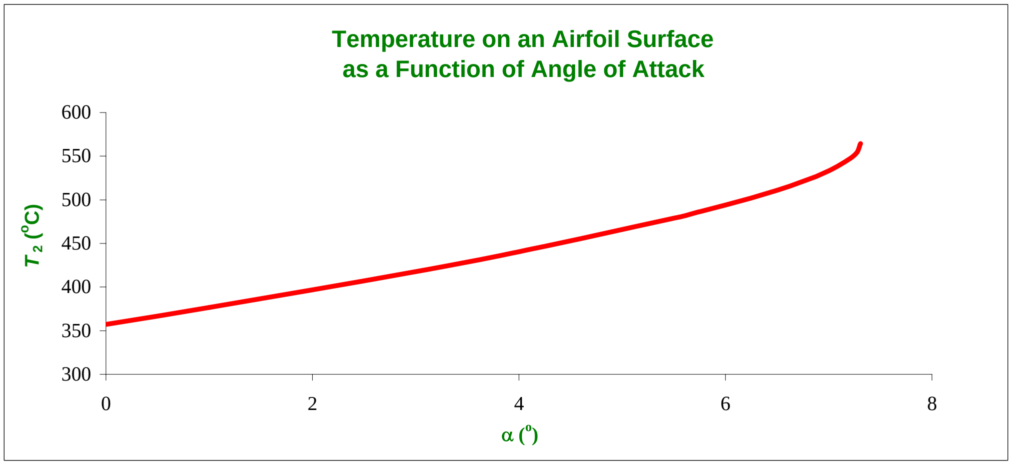
α (^{o})	β (^{o})	θ (^{o}) Needed	θ (^{o})	Error	M_{1n}	p_2 (kPa)	T_2 (^{o}C)
0.00	47.1	30.0	30.0	0.0%	2.64	597	357
1.00	48.7	31.0	31.0	0.0%	2.71	628	377
2.00	50.4	32.0	32.0	0.0%	2.77	660	397
3.00	52.1	33.0	33.0	0.0%	2.84	695	418
4.00	54.1	34.0	34.0	0.0%	2.92	731	441
5.50	57.4	35.5	35.5	0.0%	3.03	793	479
5.75	58.1	35.8	35.7	0.0%	3.06	805	486
6.00	58.8	36.0	36.0	0.0%	3.08	817	494
6.25	59.5	36.3	36.2	0.0%	3.10	831	502
6.50	60.4	36.5	36.5	0.0%	3.13	845	511
6.75	61.3	36.8	36.7	0.0%	3.16	861	521
7.00	62.5	37.0	37.0	0.0%	3.19	881	533
7.25	64.4	37.3	37.2	0.0%	3.25	910	551
7.31	65.8	37.3	37.3	0.0%	3.28	931	564

			Sum:	0.0%	Max:	931	564

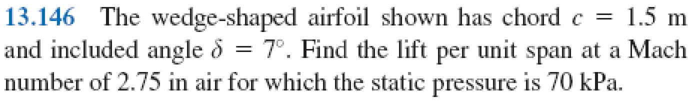
To compute this table:

1) Type the range of α
2) Type in guess values for β
3) Compute θ_{Needed} from $\theta = \alpha + \delta/2$
4) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M, β, k))
5) Compute the absolute error between each θ and θ_{Needed}
6) Compute the sum of the errors
7) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ)
8) For each α, M_{1n} is obtained from M_1, and Eq. 13.47a
9) For each α, p_2 is obtained from p_1, M_{1n}, and Eq. 13.48d
   (using built-in function *NormpfromM* (M, k))
10) For each α, T_2 is obtained from T_1, M_{1n}, and Eq. 13.48c
   (using built-in function *NormTfromM* (M, k))





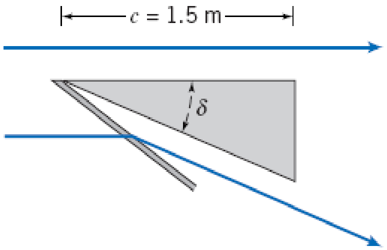
Problem 13.146



13.146 The wedge-shaped airfoil shown has chord $c = 1.5$ m and included angle $\delta = 7°$. Find the lift per unit span at a Mach number of 2.75 in air for which the static pressure is 70 kPa.

Given: Data on airfoil flight

Find: Lift per unit span



Solution:

The given or available data is:

$R =$	286.9	J/kg.K
$k =$	1.4	
$p_1 =$	70	kPa
$M_1 =$	2.75	
$\delta =$	7	°
$c =$	1.5	m

Equations and Computations:

The lift per unit span is

$$L = (p_L - p_U)c \qquad (1)$$

(Note that p_L acts on area $c/\cos(\delta)$, but its normal component is multiplied by $\cos(\delta)$)

For the upper surface:

$$p_U = p_1$$

$$p_U = 70.0 \text{ kPa}$$

For the lower surface:

We need to find M_{1n}

The deflection angle is $\theta = \delta$

$\theta = 7$ °

From M_1 and θ, and Eq. 13.49

(using built-in function *Theta* (M, β,k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

For $\theta =$ 7.0 °

$\beta =$ 26.7 °

(Use *Goal Seek* to vary β so that $\theta = \delta$)

From M_1 and β $M_{1n} =$ 1.24

From M_{1n} and p_1, and Eq. 13.48d

(using built-in function *NormpfromM* (M,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 113 kPa

$p_L = p_2$

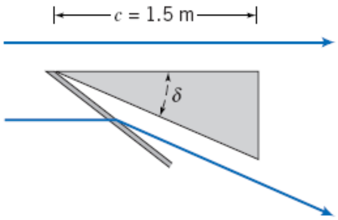
$p_L =$ 113 kPa

From Eq 1 $L =$ 64.7 kN/m

Problem 13.147

[3]

13.147 The wedge-shaped airfoil shown has chord $c = 2$ m and angles $\delta_{\text{lower}} = 15°$ and $\delta_{\text{upper}} = 5°$. Find the lift per unit span at a Mach number of 2.75 in air at a static pressure of 75 kPa.



Given: Data on airfoil flight

Find: Lift per unit span

Solution:

The given or available data is:

R = 286.9 J/kg.K

k = 1.4

p_1 = 75 kPa

M_1 = 2.75

δ_U = 5 °

δ_L = 15 °

c = 2 m

Equations and Computations:

The lift per unit span is

$$L = (p_L - p_U)c \qquad (1)$$

(Note that each p acts on area $c/\cos(\delta)$, but its normal component is multiplied by $\cos(\delta)$)

For the upper surface:

We need to find $M_{1n(U)}$

The deflection angle is $\theta_U = \delta_U$

θ_U = 5 °

From M_1 and θ_U, and Eq. 13.49

(using built-in function *Theta* (M, β,k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For θ_U = 5.00 °

β_U = 25.1 °

(Use *Goal Seek* to vary β_U so that $\theta_U = \delta_U$)

From M_1 and β_U $M_{1n(U)} =$ 1.16

From $M_{1n(U)}$ and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M* ,*k*))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 106 kPa

$p_U = p_2$

$p_U =$ 106 kPa

For the lower surface:

We need to find $M_{1n(L)}$

The deflection angle is $\theta_L = \delta_L$

$\theta_L =$ 15 °

From M_1 and θ_L, and Eq. 13.49
(using built-in function *Theta* (*M* , β,*k*))

For $\theta_L =$ 15.00 °
$\beta_L =$ 34.3 °

(Use *Goal Seek* to vary β_L so that $\theta_L = \delta_L$)

From M_1 and β_L $M_{1n(L)} =$ 1.55

From $M_{1n(L)}$ and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M* ,*k*))

$p_2 =$ 198 kPa

$p_L = p_2$

$p_L =$ 198 kPa

From Eq 1 $L =$ 183 kN/m

Problem 13.148

[3]

13.148 An oblique shock causes a flow that was at $M = 4$ and a static pressure of 75 kPa to slow down to $M = 2.5$. Find the deflection angle and the static pressure after the shock.

Given: Oblique shock Mach numbers

Find: Deflection angle; Pressure after shock

Solution:

The given or available data is:

$k =$ 1.4

$p_1 =$ 75 kPa

$M_1 =$ 4

$M_2 =$ 2.5

Equations and Computations:

We make a guess for β: $\beta =$ 33.6 o

From M_1 and β, and Eq. 13.49 (using built-in function *Theta*(*M*, β,*k*))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

$\theta =$ 21.0 o

From M_1 and β: $M_{1n} =$ 2.211

From M_2, θ, and β: $M_{2n} =$ 0.546 (1)

We can also obtain M_{2n} from Eq. 13.48a (using built-in function *normM2fromM*(*M*,*k*))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.546 (2)

We need to manually change β so that Eqs. 1 and 2 give the same answer.

Alternatively, we can compute the difference between 1 and 2, and use *Solver* to vary β to make the difference zero

Error in $M_{2n} =$ 0.00%

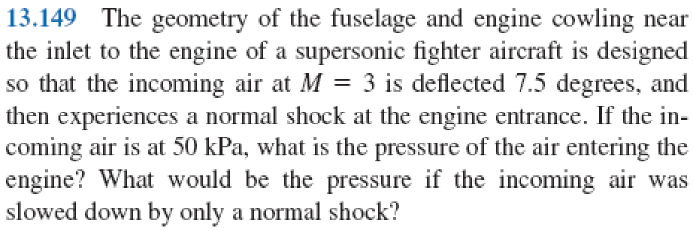
Then p_2 is obtained from Eq. 13.48d (using built-in function *normpfromm*(*M*,*k*))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 415 kPa

Problem 13.149

[4]



13.149 The geometry of the fuselage and engine cowling near the inlet to the engine of a supersonic fighter aircraft is designed so that the incoming air at $M = 3$ is deflected 7.5 degrees, and then experiences a normal shock at the engine entrance. If the incoming air is at 50 kPa, what is the pressure of the air entering the engine? What would be the pressure if the incoming air was slowed down by only a normal shock?

Given: Air flow into engine

Find: Pressure of air in engine; Compare to normal shock

Solution:

The given or available data is:

$k =$ 1.4

$p_1 =$ 50 kPa

$M_1 =$ 3

$\theta =$ 7.5 °

Equations and Computations:

Assuming isentropic flow deflection

$p_0 =$ constant

$p_{02} = p_{01}$

For p_{01} we use Eq. 13.7a (using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{01} =$ 1837 kPa

$p_{02} =$ 1837 kPa

For the deflection $\theta =$ 7.5 °

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

$\omega_1 =$ 49.8 °

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (1)

Applying Eq. 1 $\omega_2 = \omega_1 - \theta$ (Compression!)

$\omega_2 =$ 42.3 °

From ω_2, and Eq. 13.55 (using built-in function *Omega* (M, k))

For ω_2 = 42.3 °
M_2 = 2.64

(Use *Goal Seek* to vary M_2 so that ω_2 is correct)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$p_2 = p_{02}/(p_{02}/p_2)$$

p_2 = 86.8 kPa

For the normal shock (2 to 3) M_2 = 2.64

From M_2 and p_2, and Eq. 13.41d (using built-in function *NormpfromM* (M,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1} \qquad (13.41d)$$

p_3 = 690 kPa

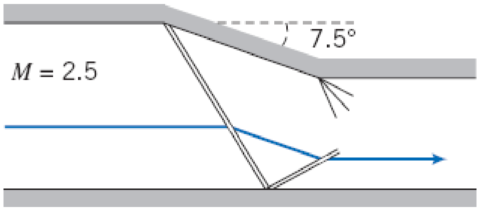
For slowing the flow down from M_1 with only a normal shock, using Eq. 13.41d

p = 517 kPa

Problem 13.150

[3]

13.150 Air flows isentropically at $M = 2.5$ in a duct. There is a 7.5° contraction that triggers an oblique shock, which in turn reflects off a wall generating a second oblique shock. This second shock is necessary so the flow ends up flowing parallel to the channel walls after the two shocks. Find the Mach number and pressure in the contraction and downstream of the contraction. (Note that the convex corner will have expansion waves to redirect the flow along the upper wall.)



Given: Air flow in a duct

Find: Mach number and pressure at contraction and downstream;

Solution:

The given or available data is:

k = 1.4
M_1 = 2.5
θ = 7.5 °
p_1 = 50 kPa

Equations and Computations:

For the first oblique shock (1 to 2) we need to find β from Eq. 13.49

$$\tan\theta = \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

We choose β by iterating or by using *Goal Seek* to target θ (below) to equal the given θ
Using built-in function *theta* (*M*, β,*k*)

θ = 7.50 °
β = 29.6 °

Then M_{1n} can be found from geometry (Eq. 13.47a)

M_{1n} = 1.233

Then M_{2n} can be found from Eq. 13.48a)
Using built-in function *NormM2fromM* (*M*,*k*)

$$M_{2_n} = f(M_{1_n}) \qquad (13.48a)$$

M_{2n} = 0.822

Then, from M_{2n} and geometry (Eq. 13.47b)

M_2 = 2.19

From M_{1n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

$$\frac{p_2}{p_1} = f(M_{1_n}) \qquad (13.48d)$$

p_2/p_1 = 1.61 Pressure ratio

p_2 = 80.40

We repeat the analysis of states 1 to 2 for states 2 to 3, to analyze the second oblique shock

We choose β for M_2 by iterating or by using *Goal Seek* to target θ (below) to equal the given θ
Using built-in function *theta* (M, β,k)

θ = 7.50 °

β = 33.5 °

Then M_{2n} (normal to second shock!) can be found from geometry (Eq. 13.47a)

M_{2n} = 1.209

Then M_{3n} can be found from Eq. 13.48a)
Using built-in function *NormM2fromM* (M,k)

M_{3n} = 0.837

Then, from M_{3n} and geometry (Eq. 13.47b)

M_3 = 1.91

From M_{2n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

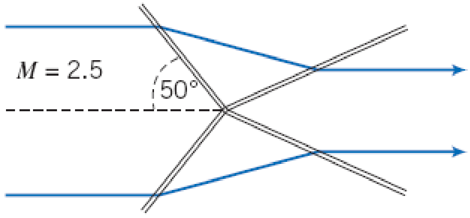
p_3/p_2 = 1.54 Pressure ratio

p_3 = 124

Problem 13.151

[3]

13.151 A flow at $M = 2.5$ is deflected by a combination of interacting oblique shocks as shown. The first shock pair is aligned at 50° to the flow. A second oblique shock pair deflects the flow again so it ends up parallel to the original flow. If the pressure before any deflections is 50 kPa, find the pressure after two deflections.



NOTE: Angle is 30^o not 50^o!

Given:
Air flow in a duct

Find:
Mach number and pressure at contraction and downstream;

Solution:

The given or available data is:

k = 1.4

M_1 = 2.5

β = 30 ^{o}

p_1 = 50 kPa

Equations and Computations:

For the first oblique shock (1 to 2) we find θ from Eq. 13.49

$$\tan\theta = \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

Using built-in function *theta* (*M*, β,*k*)

θ = 7.99 ^{o}

Also, M_{1n} can be found from geometry (Eq. 13.47a)

M_{1n} = 1.250

Then M_{2n} can be found from Eq. 13.48a)

Using built-in function *NormM2fromM* (*M*,*k*)

$$M_{2_n} = f(M_{1_n}) \qquad (13.48a)$$

M_{2n} = 0.813

Then, from M_{2n} and geometry (Eq. 13.47b)

M_2 = 2.17

From M_{1n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

$$\frac{p_2}{p_1} = f(M_{1_n}) \qquad (13.48d)$$

p_2/p_1 = 1.66 Pressure ratio
p_2 = 82.8

We repeat the analysis for states 1 to 2 for 2 to 3, for the second oblique shock

We choose β for M_2 by iterating or by using *Goal Seek* to target θ (below) to equal the previous θ
Using built-in function *theta* (M, β,k)

θ = 7.99 °
β = 34.3 °

Then M_{2n} (normal to second shock!) can be found from geometry (Eq. 13.47a)

M_{2n} = 1.22

Then M_{3n} can be found from Eq. 13.48a)
Using built-in function *NormM2fromM* (M,k)

M_{3n} = 0.829

Then, from M_{3n} and geometry (Eq. 13.47b)

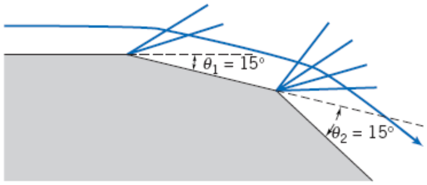
M_3 = 1.87

From M_{2n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

p_3/p_2 = 1.58 Pressure ratio
p_3 = 130

Problem 13.152 [3]

13.152 Air flows at Mach number of 1.5, static pressure 95 kPa, and is expanded by angles $\theta_1 = 15°$ and $\theta_2 = 15°$, as shown. Find the pressure changes.



Given: Deflection of air flow

Find: Pressure changes

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
p = 95 kPa
M = 1.5
θ_1 = 15 °
θ_2 = 15 °

Equations and Computations:

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_a - \omega_b = \omega(M_a) - \omega(M_b) \qquad (1)$$

From M and Eq. 13.55 (using built-in function *Omega* (M, k))

ω = 11.9 °

For the first deflection:

Applying Eq. 1

$\theta_1 = \omega_1 - \omega$

$\omega_1 = \theta_1 + \omega$

ω_1 = 26.9 °

From ω_1, and Eq. 13.55
(using built-in function *Omega* (M, k))

For $\omega_1 =$ 26.9 °

$M_1 =$ 2.02

(Use *Goal Seek* to vary M_1 so that ω_1 is correct)

Hence for p_1 we use Eq. 13.7a

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

The approach is to apply Eq. 13.7a twice, so that
(using built-in function *Isenp* (M, k))

$$p_1 = p\,(p_0/p)/(p_0/p_1)$$

$p_1 =$ 43.3 kPa

For the second deflection:

We repeat the analysis of the first deflection

Applying Eq. 1

$$\theta_2 + \theta_1 = \omega_2 - \omega$$

$$\omega_2 = \theta_2 + \theta_1 + \omega$$

$\omega_2 =$ 41.9 °

(Note that instead of working from the initial state to state 2 we could have worked from state 1 to state 2 because the entire flow is isentropic)

From ω_2, and Eq. 13.55
(using built-in function *Omega* (M, k))

For $\omega_2 =$ 41.9 °

$M_2 =$ 2.62

(Use *Goal Seek* to vary M_2 so that ω_2 is correct)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

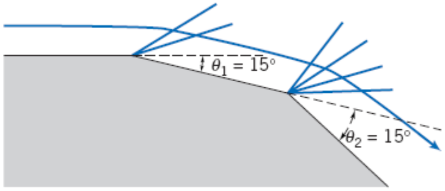
$$p_2 = p\,(p_0/p)/(p_0/p_2)$$

$p_2 =$ 16.9 kPa

Problem 13.153

[3]

13.153 Find the incoming and intermediate Mach numbers and static pressures if, after two expansions of $\theta_1 = 15°$ and $\theta_2 = 15°$, the Mach number is 4 and static pressure is 10 kPa.



Given: Deflection of air flow

Find: Mach numbers and pressures

Solution

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4
$p_2 =$ 10 kPa
$M_2 =$ 4
$\theta_1 =$ 15 ^{o}
$\theta_2 =$ 15 ^{o}

Equations and Computations:

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_a - \omega_b = \omega(M_a) - \omega(M_b) \qquad (1)$$

From M and Eq. 13.55 (using built-in function *Omega* (M , k))

$\omega_2 =$ 65.8 ^{o}

For the second deflection:

Applying Eq. 1

$\omega_1 = \omega_2 - \theta_2$

$\omega_1 =$ 50.8 ^{o}

From ω_1, and Eq. 13.55
(using built-in function *Omega* (M , k))

For $\omega_1 =$ 50.8 ^{o}
$M_1 =$ 3.05

(Use *Goal Seek* to vary M_1 so that ω_1 is correct)

Hence for p_1 we use Eq. 13.7a

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

The approach is to apply Eq. 13.7a twice, so that
(using built-in function *Isenp* (*M* , *k*))

$$p_1 = p_2(p_0/p_2)/(p_0/p_1)$$

$p_1 =$ 38.1 kPa

For the first deflection:

We repeat the analysis of the second deflection

Applying Eq. 1

$$\theta_2 + \theta_1 = \omega_2 - \omega$$

$$\omega = \omega_2 - (\theta_2 + \theta_1)$$

$\omega =$ 35.8 ^{o}

(Note that instead of working from state 2 to the initial state we could have worked from state 1 to the initial state because the entire flow is isentropic)

From ω, and Eq. 13.55
(using built-in function *Omega* (*M* , *k*))

For $\omega =$ 35.8 ^{o}

$M =$ 2.36

(Use *Goal Seek* to vary *M* so that ω is correct)

Hence for *p* we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$$p = p_2(p_0/p_2)/(p_0/p)$$

$p =$ 110 kPa

Problem 13.154

[4]

13.154 Compare the static and stagnation pressures produced by (a) an oblique shock and (b) isentropic *compression* waves as they each deflect a flow at a Mach number of 3.5 through a deflection angle of 35° in air for which the static pressure is 50 kPa.

Given: Mach number and deflection angle

Find: Static and stagnation pressures due to: oblique shock; compression wave

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4
$p_1 =$ 50 kPa
$M_1 =$ 3.5
$\theta =$ 35 $^\circ$

Equations and Computations:

For the oblique shock:

We need to find M_{1n}

The deflection angle is $\theta =$ 35 $^\circ$

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M , β,k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

For $\theta =$ 35.0 $^\circ$
$\beta =$ 57.2 $^\circ$

(Use *Goal Seek* to vary β so that $\theta = 35^\circ$)

From M_1 and β $M_{1n} =$ 2.94

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M ,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48\text{d})$$

$p_2 =$ 496 kPa

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (M ,k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.479

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2 \sin(\beta - \theta) \qquad (13.47b)$$

Hence $M_2 =$ 1.27

For p_{02} we use Eq. 12.7a
(using built-in function *Isenp* (M , k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{02} = p_2/(p_{02}/p_2)$

$p_{02} =$ 1316 kPa

For the isentropic compression wave:

For isentropic flow $p_0 =$ constant

$p_{02} =$ p_{01}

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (M , k))

$p_{01} =$ 3814 kPa

$p_{02} =$ 3814 kPa

(Note that for the oblique shock, as required by Eq. 13.48b

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[\frac{\frac{k+1}{2} M_{1_n}^2}{1 + \frac{k-1}{2} M_{1_n}^2}\right]^{k/(k-1)}}{\left[\frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1}\right]^{1/(k-1)}} \qquad (13.48b)$$

$p_{02}/p_{01} =$ 0.345
(using built-in function *Normp0fromM* (M ,k)

p_{02}/p_{01} = 0.345

(using p_{02} from the shock and p_{01})

For the deflection θ = $-\theta$ (*Compression*)

θ = -35.0 °

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (13.55)$$

and

$$\text{Deflection} = \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1) \quad (1)$$

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

ω_1 = 58.5 °

Applying Eq. 1 ω_2 = $\omega_1 + \theta$

ω_2 = 23.5 °

From ω_2, and Eq. 13.55

(using built-in function *Omega* (M, k))

For ω_2 = 23.5 °

M_2 = 1.90

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 23.5^\circ$)

Hence for p_2 we use Eq. 13.7a

(using built-in function *Isenp* (M, k))

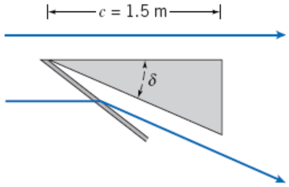
$p_2 = p_{02}/(p_{02}/p_2)$

p_2 = 572 kPa

Problem 13.155

[3]

13.155 Consider the wedge-shaped airfoil of Problem 13.146. Suppose the oblique shock could be replaced by isentropic *compression* waves. Find the lift per unit span at the Mach number of 2.75 in air for which the static pressure is 70 kPa.



Given: Wedge-shaped airfoil

Find: Lift per unit span assuming isentropic flow

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
p = 70 kPa
M = 2.75
δ = 7 °
c = 1.5 m

Equations and Computations:

The lift per unit span is

$$L = (p_L - p_U)c \qquad (1)$$

(Note that p_L acts on area $c/\cos(\delta)$, but its normal component is multiplied by $\cos(\delta)$)

For the upper surface:

$$p_U = p$$

$$p_U = 70 \text{ kPa}$$

For the lower surface:

$$\theta = -\delta$$

$$\theta = -7.0 \ ^{\circ}$$

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_L - \omega = \omega(M_L) - \omega(M) \qquad (2)$$

From M and Eq. 13.55 (using built-in function *Omega* (M, k))

$\omega =$ 44.7 $^{\circ}$

Applying Eq. 2

$\theta =$ $\omega_L - \omega$

$\omega_L =$ $\theta + \omega$

$\omega_L =$ 37.7 $^{\circ}$

From ω_L, and Eq. 13.55

(using built-in function *Omega* (M, k))

For $\omega_L =$ 37.7 $^{\circ}$

$M_L =$ 2.44

(Use *Goal Seek* to vary M_L so that ω_L is correct)

Hence for p_L we use Eq. 13.7a

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

The approach is to apply Eq. 13.7a twice, so that
(using built-in function *Isenp* (M, k))

$p_L = p(p_0/p)/(p_0/p_L)$

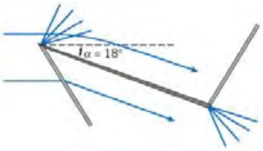
$p_L =$ 113 kPa

From Eq 1 $L =$ 64.7 kN/m

Problem 13.156

[4]

13.156 Find the lift and drag per unit span on the airfoil shown for flight at a Mach number of 1.75 in air for which the static pressure is 50 kPa. The chord length is 1 m.



Given: Mach number and airfoil geometry

Find: Lift and drag per unit span

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
p_1 = 50 kPa
M_1 = 1.75
α = 18 °
c = 1 m

Equations and Computations:

The net force per unit span is $F = (p_L - p_U)c$

Hence, the lift force per unit span is

$$L = (p_L - p_U)c\cos(\alpha) \qquad (1)$$

The drag force per unit span is

$$D = (p_L - p_U)c\sin(\alpha) \qquad (2)$$

For the lower surface (oblique shock):

We need to find M_{1n}

The deflection angle is $\theta = \alpha$

θ = 18 °

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M, β,k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

For θ = 18.0 °
β = 62.9 °

(Use *Goal Seek* to vary β so that θ is correct)

From M_1 and β M_{1n} = 1.56

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

p_2 = 133.2 kPa

$p_L = p_2$

p_L = 133.2 kPa

For the upper surface (isentropic expansion wave):

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{01} =$ 266 kPa

$p_{02} =$ 266 kPa

For the deflection $\theta = \alpha$ (*Compression*)

$\theta =$ 18.0 °

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1) \qquad (3)$$

From M_1 and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

$\omega_1 =$ 19.3 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 37.3 °

From ω_2, and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

For $\omega_2 =$ 37.3 °
$M_2 =$ 2.42

(Use *Goal Seek* to vary M_2 so that ω_2 is correct)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 17.6 kPa

$p_U = p_2$

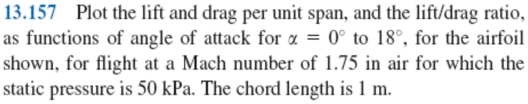
$p_U =$ 17.6 kPa

From Eq. 1 $L =$ 110.0 kN/m

From Eq. 2 $D =$ 35.7 kN/m

Problem 13.157

[4]



13.157 Plot the lift and drag per unit span, and the lift/drag ratio, as functions of angle of attack for $\alpha = 0°$ to $18°$, for the airfoil shown, for flight at a Mach number of 1.75 in air for which the static pressure is 50 kPa. The chord length is 1 m.

Given: Mach number and airfoil geometry

Find: Plot of lift and drag and lift/drag versus angle of attack

Solution:

The given or available data is:

k = 1.4
p_1 = 50 kPa
M_1 = 1.75
α = 12 °
c = 1 m

Equations and Computations:

The net force per unit span is

$$F = (p_L - p_U)c$$

Hence, the lift force per unit span is

$$L = (p_L - p_U)c\cos(\alpha) \qquad (1)$$

The drag force per unit span is

$$D = (p_L - p_U)c\sin(\alpha) \qquad (2)$$

For each angle of attack the following needs to be computed:

For the lower surface (oblique shock):

We need to find M_{1n}

Deflection $\theta = \alpha$

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

find β

(Use *Goal Seek* to vary β so that θ is the correct value)

From M_1 and β find M_{1n}

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

find p_2

and $p_L = p_2$

For the upper surface (isentropic expansion wave):

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp*(M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

find $p_{02} =$ 266 kPa

Deflection $\theta = \alpha$

we use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

Deflection $= \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega*(M, k))

find $\omega_1 =$ 19.3 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$ (4)

From ω_2, and Eq. 12.55 (using built-in function *Omega*(M, k))

From ω_2 find M_2

(Use *Goal Seek* to vary M_2 so that ω_2 is the correct value)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp*(M, k))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_U = p_2$

Finally, from Eqs. 1 and 2, compute L and D

Computed results:

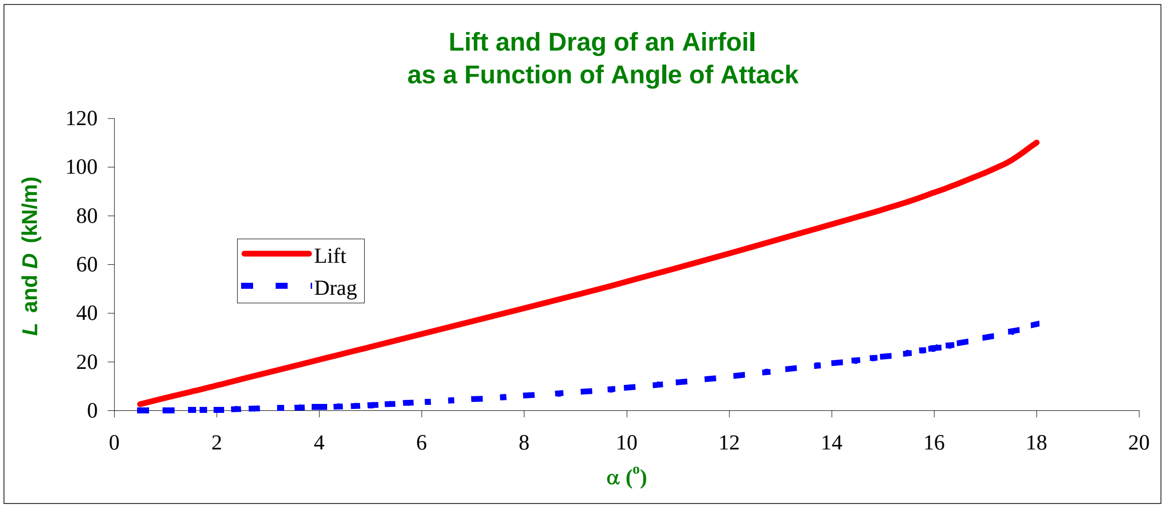
α (°)	β (°)	θ (°)	**Error**	M_{1n}	p_L **(kPa)**	ω_2 (°)	ω_2 **from** M_2 (°)	**Error**	M_2	p_U **(kPa)**	L **(kN/m)**	D **(kN/m)**	***L/D***
0.50	35.3	0.50	0.0%	1.01	51.3	19.8	19.8	0.0%	1.77	48.7	2.61	0.0227	115
1.00	35.8	1.00	0.0%	1.02	52.7	20.3	20.3	0.0%	1.78	47.4	5.21	0.091	57.3
1.50	36.2	1.50	0.0%	1.03	54.0	20.8	20.8	0.0%	1.80	46.2	7.82	0.205	38.2
2.00	36.7	2.00	0.0%	1.05	55.4	21.3	21.3	0.0%	1.82	45.0	10.4	0.364	28.6
4.00	38.7	4.00	0.0%	1.09	61.4	23.3	23.3	0.0%	1.89	40.4	20.9	1.46	14.3
5.00	39.7	5.00	0.0%	1.12	64.5	24.3	24.3	0.0%	1.92	38.3	26.1	2.29	11.4
10.00	45.5	10.0	0.0%	1.25	82.6	29.3	29.3	0.0%	2.11	28.8	53.0	9.35	5.67
15.00	53.4	15.0	0.0%	1.41	106.9	34.3	34.3	0.0%	2.30	21.3	82.7	22.1	3.73
16.00	55.6	16.0	0.0%	1.44	113.3	35.3	35.3	0.0%	2.34	20.0	89.6	25.7	3.49
16.50	56.8	16.5	0.0%	1.47	116.9	35.8	35.8	0.0%	2.36	19.4	93.5	27.7	3.38
17.00	58.3	17.0	0.0%	1.49	121.0	36.3	36.3	0.0%	2.38	18.8	97.7	29.9	3.27
17.50	60.1	17.5	0.0%	1.52	125.9	36.8	36.8	0.0%	2.40	18.2	102.7	32.4	3.17
18.00	62.9	18.0	0.0%	1.56	133.4	37.3	37.3	0.0%	2.42	17.6	110	35.8	3.08

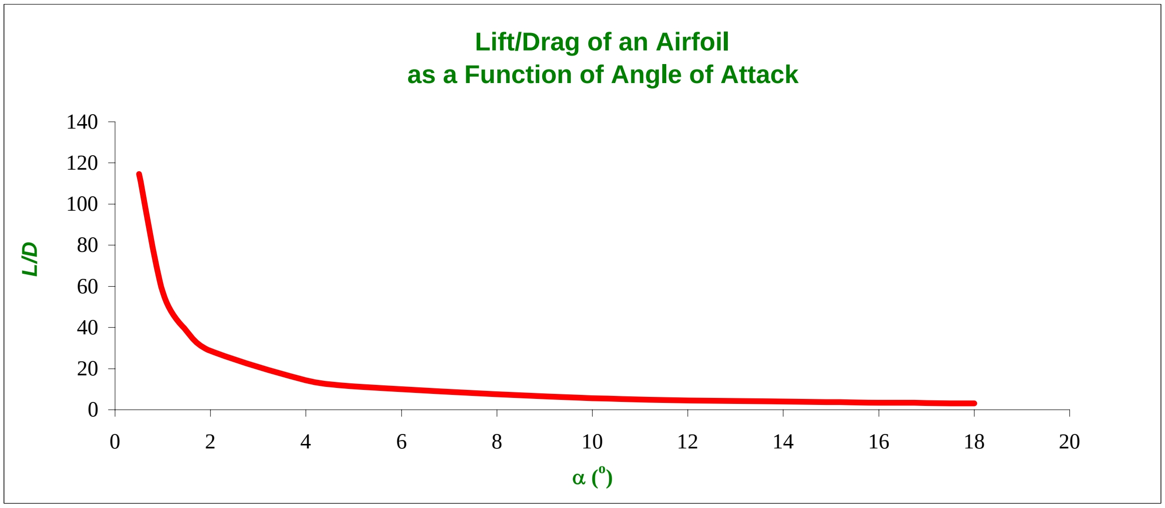
Sum: 0.0%

Sum: 0.0%

To compute this table:

1) Type the range of α
2) Type in guess values for β
3) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M,β, k)
4) Compute the absolute error between each θ and α
5) Compute the sum of the errors
6) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ)
7) For each α, M_{1n} is obtained from M_1, and Eq. 13.47a
8) For each α, p_L is obtained from p_1, M_{1n}, and Eq. 13.48d
   (using built-in function *NormpfromM* (M,k))
9) For each α, compute ω_2 from Eq. 4
10) For each α, compute ω_2 from M_2, and Eq. 13.55
    (using built-in function *Omega* (M,k))
11) Compute the absolute error between the two values of ω_2
12) Compute the sum of the errors
13) Use *Solver* to minimize the sum by varying the M_2 values
    (Note: You may need to interactively type in new M_2 values)
    if *Solver* generates β values that lead to no θ)
14) For each α, p_U is obtained from p_{02}, M_2, and Eq. 13.47a
    (using built-in function *Isenp* (M, k))
15) Compute L and D from Eqs. 1 and 2

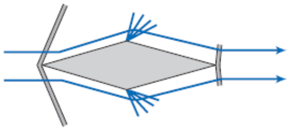




Problem 13.158

[4]

13.158 Find the drag coefficient of the symmetric, zero angle of attack airfoil shown for a Mach number of 2.0 in air for which the static pressure is 95 kPa and temperature is 0°C. The included angles at the nose and tail are each 10°.



Given: Mach number and airfoil geometry

Find: Drag coefficient

Solution:

The given or available data is:

$R = 286.9$ J/kg.K

$k = 1.4$

$p_1 = 95$ kPa

$M_1 = 2$

$\alpha = 0$ °

$\delta = 10$ °

Equations and Computations:

The drag force is

$$D = (p_F - p_R)cs\tan(\delta/2) \qquad (1)$$

(s and c are the span and chord)

This is obtained from the following analysis

Airfoil thickness (frontal area) = $2s(c/2\tan(\delta/2))$

Pressure difference acting on frontal area = $(p_F - p_R)$

(p_F and p_R are the pressures on the front and rear surfaces)

The drag coefficient is $C_D = D/(1/2\rho V^2 A)$ (2)

But it can easily be shown that

$$\rho V^2 = pkM^2$$

Hence, from Eqs. 1 and 2

$$C_D = (p_F - p_R)\tan(\delta/2)/(1/2pkM^2) \qquad (3)$$

For the frontal surfaces (oblique shocks):

We need to find M_{1n}

The deflection angle is $\theta = \delta/2$

$\theta = 5$ °

From M_1 and θ, and Eq. 13.49

(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

For $\theta =$ 5.0 °

$\beta =$ 34.3 °

(Use *Goal Seek* to vary β so that θ = 5°)

From M_1 and β $M_{1n} =$ 1.13

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M*,*k*))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 125.0 kPa

$p_F = p_2$

$p_F =$ 125.0 kPa

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (*M*,*k*))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.891

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2\sin(\beta - \theta) \qquad (13.47b)$$

Hence $M_2 =$ 1.82

For p_{02} we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{02} =$ 742 kPa

For the rear surfaces (isentropic expansion waves):

Treating as a new problem

Here: M_1 is the Mach number after the shock
and M_2 is the Mach number after the expansion wave
p_{01} is the stagnation pressure after the shock
and p_{02} is the stagnation pressure after the expansion wave

$M_1 = M_2$ (shock)

$M_1 =$ 1.82

$p_{01} = p_{02}$ (shock)

$p_{01} =$ 742 kPa

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

$p_{02} =$ 742 kPa

For the deflection $\theta = \delta$

$\theta =$ 10.0 ^{o}

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1) \qquad (3)$$

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

$\omega_1 =$ 21.3 ^{o}

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 31.3 ^{o}

From ω_2, and Eq. 13.55 (using built-in function Omega(M, k))

For $\omega_2 =$ 31.3 ^{o}

$M_2 =$ 2.18

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 31.3^{o}$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 71.2 kPa

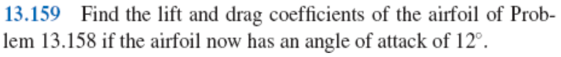
$p_R = p_2$

$p_R =$ 71.2 kPa

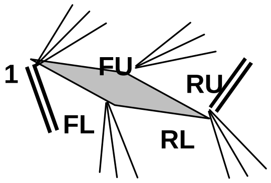
Finally, from Eq. 1 $C_D =$ 0.0177

Problem 13.159

[4]



13.159 Find the lift and drag coefficients of the airfoil of Problem 13.158 if the airfoil now has an angle of attack of 12°.



Given: Mach number and airfoil geometry

Find: Lift and Drag coefficients

Solution:

The given or available data is:

$R =$ 286.9 J/kg.K

$k =$ 1.4

$p_1 =$ 95 kPa

$M_1 =$ 2

$\alpha =$ 12 °

$\delta =$ 10 °

Equations and Computations:

Following the analysis of Example 13.14
the force component perpendicular to the major axis, per area, is

$$F_V/sc = 1/2\{(p_{FL} + p_{RL}) - (p_{FU} + p_{RU})\} \quad (1)$$

and the force component parallel to the major axis, per area, is

$$F_H/sc = 1/2\tan(\delta/2)\{(p_{FU} + p_{FL}) - (p_{RU} + p_{RL})\} \quad (2)$$

using the notation of the figure above.
(s and c are the span and chord)

The lift force per area is

$$F_L/sc = (F_V\cos(\alpha) - F_H\sin(\alpha))/sc \quad (3)$$

The drag force per area is

$$F_D/sc = (F_V\sin(\alpha) + F_H\cos(\alpha))/sc \quad (4)$$

The lift coefficient is $\quad C_L = F_L/(1/2\rho V^2 A) \quad (5)$

But it can be shown that

$$\rho V^2 = pkM^2 \quad (6)$$

Hence, combining Eqs. 3, 4, 5 and 6

$$C_L = (F_V/sc\cos(\alpha) - F_H/sc\sin(\alpha))/(1/2pkM^2) \quad (7)$$

Similarly, for the drag coefficient

$$C_D = (F_V/sc\sin(\alpha) + F_H/sc\cos(\alpha))/(1/2pkM^2) \quad (8)$$

For surface FL (oblique shock):

We need to find M_{1n}

The deflection angle is $\theta = \alpha + \delta/2$

$\theta = 17$ °

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For $\theta = 17.0$ °
$\beta = 48.2$ °

(Use *Goal Seek* to vary β so that $\theta = 17^{\circ}$)

From M_1 and β $\quad M_{1n} = 1.49$

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 = 230.6$ kPa

$p_{FL} = p_2$

$p_{FL} = 230.6$ kPa

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (M, k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} = 0.704$

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2\sin(\beta - \theta) \qquad (13.47b)$$

Hence $\quad M_2 = 1.36$

For p_{02} we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{02} = 693$ kPa

For surface RL (isentropic expansion wave):

Treating as a new problem

Here: M_1 is the Mach number after the shock
and M_2 is the Mach number after the expansion wave
p_{01} is the stagnation pressure after the shock
and p_{02} is the stagnation pressure after the expansion wave

$M_1 = M_2$ (shock)

$M_1 =$ 1.36

$p_{01} = p_{02}$ (shock)

$p_{01} =$ 693 kPa

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

$p_{02} =$ 693 kPa

For the deflection $\theta = \delta$

$\theta =$ 10.0 °

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (13.55)$$

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (M , k))

$\omega_1 =$ 7.8 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 17.8 °

From ω_2, and Eq. 13.55 (using built-in function *Omega* (M , k))

For $\omega_2 =$ 17.8 °

$M_2 =$ 1.70

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 17.8^{\circ}$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M , k))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 141 kPa

$p_{RL} = p_2$

$p_{RL} =$ 141 kPa

For surface FU (isentropic expansion wave):

$M_1 =$ 2.0

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$p_{01} =$ 743
$p_{02} =$ 743 kPa

For the deflection $\theta = \alpha - \delta/2$

$\theta =$ 7.0 ^{o}

We use Eq. 13.55

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

$\omega_1 =$ 26.4 ^{o}

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 33.4 ^{o}

From ω_2, and Eq. 13.55 (using built-in function Omega(M, k))

For $\omega_2 =$ 33.4 ^{o}
$M_2 =$ 2.27

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 33.4^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 62.8 kPa

$p_{FU} = p_2$

$p_{FU} =$ 62.8 kPa

For surface RU (isentropic expansion wave):

Treat as a new problem.

Flow is isentropic so we could analyse from region FU to RU
but instead analyse from region 1 to region RU.

$M_1 =$ 2.0

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

$p_{01} =$ 743 kPa
$p_{02} =$ 743 kPa

TOTAL deflection $\theta = \alpha + \delta/2$

$\theta =$ 17.0 °

We use Eq. 13.55

and

$$\text{Deflection} = \omega_2 - \omega_1 = \omega(M_2) - \omega(M_1) \qquad (3)$$

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

$\omega_1 =$ 26.4 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 43.4 °

From ω_2, and Eq. 13.55 (using built-in function Omega(M, k))

For $\omega_2 =$ 43.4 °
$M_2 =$ 2.69

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 43.4^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$p_2 = p_{02}/(p_{02}/p_2)$$

$p_2 =$ 32.4 kPa

$p_{RU} = p_2$

$p_{RU} =$ 32.4 kPa

The four pressures are:

$p_{FL} =$ 230.6 kPa
$p_{RL} =$ 140.5 kPa
$p_{FU} =$ 62.8 kPa
$p_{RU} =$ 32.4 kPa

From Eq 1 $F_V/sc =$ 138 kPa

From Eq 2 $F_H/sc =$ 5.3 kPa

From Eq 7 $C_L =$ 0.503

From Eq 8 $C_D =$ 0.127

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Answers to Selected Problems, Chapter 1

1.5 $M = 5913$ kg
1.7 $L = 27.25$ in. $D = 13.75$ in.
1.9 $y = 2.05\dfrac{W^2}{g\,t^2}$
1.11 $d = 0.109$ mm
1.15 $y = 0.922$ mm
1.17 a) N·m/s, lbf·ft/s b) N/m^2, lbf/ft^2 c) N/m^2, lbf/ft^2, d) 1/s, 1/s e) N·m, lbf·ft
f) N·s, lbf·s g) N/m^2, lbf/ft^2 h) m^2/s^2·K, ft^2/s^2·R i) 1/K, 1/R j) N·m·s, lbf·ft·s
1.19 a) 6.89 kPa b) 0.264 gal 47.9 N·s/m^2
1.21 a) 0.0472 m^3/s b) 0.0189 m^3 c) 29.1 m/s d) 2.19×10^4 m^2
1.23 101 gpm
1.25 SG = 13.6 $v = 7.37 \times 10^{-5}$ m^3/kg $\gamma_E = 847$ lbf/ft^3, $\gamma_M = 144$ lbf/ft^3
1.27 2.25 kgf/cm^2
1.29 $c = 0.04$ K$^{1/2}$·s/m $\dot{m}_{\max} = 2.36\dfrac{A_t \cdot p_0}{\sqrt{T_0}}$ (ft^2, psi, R)
1.31 C_D is dimensionless
1.33 c: N·s/m, lbf·s/ft k: N/m, lbf/ft f: N, lbf
1.35 $H(\text{m}) = 0.457 - 3450 \cdot (Q(\text{m}^3/\text{s}))^2$
1.37 $\rho = 0.0765 \pm 2.66 \times 10^{-4}$ lbm/ft^3 (± 0.348%)
1.39 $\rho = 1130 \pm 21.4$ kg/m^3 SG = 1.13 ± 0.0214
1.41 $\rho = 930 \pm 27.2$ kg/m^3
1.43 $t = 1, 5, 5$ s Flow rate uncertainty = ± 5.0, 1.0, 1.0%
1.45 $\mu = 1.01 \times 10^{-3}$ N·s/m^2 ± 0.609%
1.47 $\delta x = \pm 0.158$ mm
1.49 $H = 57.7 \pm 0.548$ ft $\theta_{\min} = 31.4^\circ$

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Answers to Selected Problems, Chapter 2

2.1 1) 1D, Unsteady 2) 1D, Steady 3) 2D, Unsteady 4) 2D, Unsteady
5) 1D, Unsteady 6) 3D, Steady 7) 2D, Unsteady 8) 3D, Steady

2.3 Streamlines: $y = \dfrac{c}{\sqrt{x}}$

2.5 Streamlines: $y = c\,x^{-\frac{b}{a}t}$

2.7 Streamlines: $y = \dfrac{1}{\sqrt{2\left(\dfrac{b}{a\,x} + c\right)}}$

2.9 Streamlines: $y = 3x$ $\Delta t = 0.75$ s

2.11 Streamlines: $x^2 + y^2 = c$

2.13 Pathlines: $y = 2/x$ Streamlines: $y = 2/x$

2.15 $\omega = \dfrac{K}{2\pi a}$

2.17 Pathlines: $y = y_0 e^{Ct}$, $x = x_0 e^{B\left(t + \frac{1}{2}At^2\right)}$ Streamlines: $y = x^{\frac{1}{1+0.5t}}$

2.19 Pathlines: $y = y_0 e^{-bt}$, $x = x_0 e^{\frac{1}{2}at^2}$ Streamlines: $y = Cx^{-\frac{b}{at}}$

2.21 Pathlines: $y = 4t + 1$, $x = 3e^{0.05t^2}$ Streamlines: $y = 1 + \dfrac{40}{t}\ln\left(\dfrac{x}{3}\right)$

2.23 Streamlines: $y(t_0) = v_0 \sin(\omega t)(t - t_0)$, $x(t_0) = u_0(t - t_0)$

2.25 Streaklines: $y = e^{(t-\tau)}$, $x = e^{(t-\tau)+0.1\left(t^2-\tau^2\right)}$ Streamlines: $y = x^{\frac{1}{(1+0.2t)}}$

2.29 Streamlines: $y = \dfrac{x^2}{4} + 4$ (4 m, 8 m) (5 m, 10.25 m)

2.31 (2.8 m, 5 m) (3 m, 3 m)

2.33 $\nu = \dfrac{b' T^{\frac{3}{2}}}{1 + {S'}/{T}}$ $b' = 4.13 \times 10^{-9}\ \mathrm{m^2/s{\cdot}K^{3/2}}$ $S' = 110.4$ K

2.35 $\tau_{yx} = -4.56\ \mathrm{N/m^2}$

2.39 $a = -0.491\ \mathrm{ft/s^2}$

2.41 $L = 2.5$ ft

2.43 $t = 1.93$ s

2.45 $V = \dfrac{Mgd\sin\theta}{\mu A}\left(1 - e^{-\frac{\mu A}{Md}t}\right)$ $V = 0.404$ m/s $\mu = 1.08\ \mathrm{N{\cdot}s/m^2}$

2.47 $F = 2.83$ N

2.49 $F = 0.0254$ N

2.51 $\mu = 8.07 \times 10^{-4}\ \mathrm{N{\cdot}s/m^2}$

2.53 $\mu = \dfrac{Mgr^2a}{2\pi V_m R^3 H}$ $\mu = 0.0651\ \mathrm{N{\cdot}s/m^2}$

2.55 $t = 4.00$ s

2.57 $\omega = \frac{A}{B}\left(1 - e^{-\frac{B}{C}t}\right)$ $\omega_{\max} = 25.1$ rpm $t = 0.671$ s

2.59 $\tau_{z\theta} = \frac{\mu\omega r}{h}$ $T = \frac{\pi\mu\omega R^4}{2h}$

2.61 Dilatant $k = 0.0499$ $n = 1.21$ $\mu = 0.191\ \text{N·s/m}^2$, $0.195\ \text{N·s/m}^2$

2.63 Bingham plastic $\mu_{\text{p}} = 0.652\ \text{N·s/m}^2$

2.65 $T = \frac{2\pi\mu\omega R^3 H}{a}$ $T = \frac{\pi\mu\omega R^4}{2b}$

2.69 $\tau = \frac{\mu\omega R\sin\theta}{a + R(1-\cos\theta)}$ $\tau_{\max} = 79.2\ \text{N/m}^2$

2.75 $a = 0$ $b = 2U$ $c = -U$

2.77 $V = 229$ mph

2.79 $M = 2.5$ $x_{\text{trans}} = 0.327$ m

2.81 SG = 0.9 $\gamma = 8830$ N/m3 Laminar flow

2.83 $V = 667$ km/hr

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Answers to Selected Problems, Chapter 3

3.1 $M = 62.4$ kg $t = 22.3$ mm
3.3 $z = 9303$ ft $\Delta z = 5337$ ft
3.5 $F = 45.6$ N
3.9 $\Delta p = 972$ Pa $\rho = 991$ kg/m^3
3.11 $D = 0.477$ in
3.13 $\Delta\rho/\rho_0 = 4.34\%$ $\Delta p/p_0 = 2.15\%$
3.15 $p = -0.217$ psig
3.17 $p = 6.39$ kPa (gage) $h = 39.3$ mm
3.19 $p = 128$ kPa (gage)
3.21 $\Delta p = 59.5$ Pa
3.23 $H = 30$ mm
3.25 SG = 0.900
3.27 $\Delta p = 1.64$ psi
3.29 $L = \frac{\Delta p}{\rho_{\text{oil}}\left[1+\left(\frac{d}{D}\right)^2\right]}$ $L = 27.2$ mm
3.31 $h = 1.11$ in
3.33 $\theta = 11.1^{\text{o}}$ $S = 5/\text{SG}$
3.35 $p_{\text{atm}} = 14.4$ psi Shorter column at higher temperature
3.37 $\Delta h = 38.1$ mm $\Delta h = 67.8$ mm
3.39 $\Delta h = 0.389$ cm
3.43 $\Delta z = 587$ ft $\Delta z = 3062$ ft
3.45 $p = 57.5$ kPa $p = 60.2$ kPa
3.49 $p_{\text{A}} = 1.96$ kPa $p_{\text{B}} = 8.64$ kPa $p_{\text{C}} = 21.9$ kPa $p_{\text{air}} = -11.3$ kPa $p_{\text{air}} = 1.99$ kPa
3.51 $F_{\text{A}} = 79{,}600$ lbf
3.53 $W = 68$ kN
3.55 $F_{\text{R}} = 0.407$ lbf
3.57 $F_{\text{R}} = 8.63$ MN $R = (8.34\text{ MN}, 14.4\text{ MN})$
3.59 $F = 600$ lbf
3.63 $D = 8.66$ ft
3.65 $d = 2.66$ m
3.67 SG = 0.542
3.69 $F = -137$ kN
3.71 $F_{\text{V}} = 7.62$ kN $x'F_{\text{V}} = 3.76$ kN·m $F_{\text{A}} = -5.71$ kN
3.73 $F_{\text{V}} = -\rho g w R^2\pi/4$ $x' = 4R/3\pi$
3.75 $F_{\text{V}} = 1.05 \times 10^6$ N $x' = 1.61$ m
3.77 $F_{\text{V}} = 1.83 \times 10^7$ N $\alpha = 19.9^{\text{o}}$
3.79 $F_{\text{V}} = 416$ kN $F_{\text{H}} = 370$ kN $\alpha = 48.3^{\text{o}}$ $F = 557$ kN
3.81 $F_{\text{V}} = 2.47$ kN $x' = 0.645$ m $F_{\text{H}} = 7.35$ kN $y' = 0.217$ m
3.83 $M = \frac{4\rho L(H-d)^{\frac{3}{2}}}{3\sqrt{a}}$ $M = 583$ kg
3.85 $M = 2\rho LR\left[(d-R)\sin(\theta_{\max}) + R\left(\frac{\theta_{\max}}{2}+\frac{\sin(2\theta_{\max})}{4}\right)\right]$ $M = 631$ kg

3.87 $\gamma = 8829\ \text{N/m}^3$ $h = 0.292\ \text{m}$

3.89 $V\!\!\!\!-_{\text{Not submerged}}/V\!\!\!\!-_{\text{Submerged}} = 10.5\%$

3.91 $\text{SG} = W_\text{a}/(W_\text{a} - W_\text{w})$

3.93 $F_\text{B} = 1.89 \times 10^{-11}\ \text{lbf}$ $V = 1.15 \times 10^{-3}\ \text{ft/s}$ (0.825 in/min)

3.95 $L/V\!\!\!\!-_{\text{He}} = 0.0659\ \text{lbf/ft}^3$ $L/V\!\!\!\!-_{\text{H2}} = 0.0712\ \text{lbf/ft}^3$ $L/V\!\!\!\!-_{\text{air}} = 0.0172\ \text{lbf/ft}^3/0.0249\ \text{lbf/ft}^3$

3.97 $D = 116\ \text{m}$ $M = 703\ \text{kg}$

3.99 $\theta = 9.1^\text{o}$ (with $A = 25\ \text{cm}^2$ not $A = 20\ \text{cm}^2$)

3.101 $x = 0.257\ \text{m}$ $f = 6.1\ \text{N}$

3.103 $D = 2.57\ \text{ft}$

3.105 $f = 0.288$ cycle/s ($\omega = 1.81$ rad/s)

3.107 $F = 34.2\ \text{lbf}$

3.113 $a = g(h/L)$

3.115 $\omega = 185$ rad/s (1764 rpm)

3.117 $\Delta p = \rho\omega^2 R^2/2$ $\omega = 7.16$ rad/s

3.119 $dy/dx = -\,0.25$ $p = 105 - 1.96x$ (p: kPa, x: m)

3.121 $\alpha = 30^\text{o}$ $dy/dx = 0.346$

3.123 $T = 47.6\ \text{lbf}$ $p = 55.3\ \text{lbf/ft}^2$ (gage)

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Answers to Selected Problems, Chapter 4

4.1 $x = 0.934$ m
4.3 $x = 747$ m $t = 23.9$ mm
4.5 $V_0 = 87.5$ km/hr
4.7 $\tau = 1.50$ hr
4.9 $y_c = \dfrac{3.47h^2 + 2400}{6.94h + 40}$ (y_c, h: mm) $h = 21.2$ mm $\mu_s \geq 0.604$
4.11 $Q = -90$ ft^3/s $\rho\int \vec{V}\left(\vec{V}\cdot d\vec{A}\right) = -450\rho\hat{i} + 360\rho\hat{j}$ (slug·ft/s/s; ρ: slug/ft^3)
4.13 $\int \vec{V}\cdot d\vec{A} = -24\,\mathrm{m^3/s}$ $\int \vec{V}\left(\vec{V}\cdot d\vec{A}\right) = \left(64\hat{i} - 96\hat{j} - 60\hat{k}\right)$ $\mathrm{m^4/s^2}$
4.15 $Q = \frac{1}{2}u_{\max}\pi R^2$ $\text{m.f.} = \frac{1}{3}u_{\max}^2\pi R^2\hat{i}$
4.17 $V = 1.56$ ft/s $Q = 3.82$ gpm
4.19 $Q_{\text{cool}} = 489$ gpm $\dot{m}_{\text{cool}} = 2.45\times10^5\,\mathrm{lb/hr}$ $\dot{m}_{\text{moist}} = 19404\,\mathrm{lb/hr}$ $\dot{m}_{\text{air}} = 14331\,\mathrm{lb/hr}$
4.21 $V_3 = (4.33$ ft/s, -2.50 ft/s)
4.23 Eight pipes
4.25 1400 units/hr 4220 units/hr Outflow = 9 units/s
4.27 $\rho = 10.7$ lb/ft^3
4.29 $\dfrac{\dot{m}}{w} = \dfrac{\rho^2 g\sin(\theta)h^3}{3\mu}$
4.31 $U = 1.5$ m/s
4.33 $Q = 1.05 \times 10^{-5}$ m^3/s (10.45 mL/s) $V_{\text{ave}} = 0.139$ m/s $u_{\max} = 0.213$ m/s
4.35 $v_{\min} = 5.0$ m/s
4.37 $\partial\cancel{V}_{\text{oil}}/\partial t = -2.43 \times 10^{-2}$ ft^3/s (0.18 gal/s)
4.39 $dh/dt = -8.61$ mm/s
4.41 $dh/dt = -0.289$ mm/s
4.43 $Q = 1.5 \times 10^4$ gal/s $A = 4.92 \times 10^7$ ft^2
4.45 $t = 22.2$ s
4.47 $dy/dt = -9.01$ mm/s
4.49 $Q_{\text{cd}} = 4.50 \times 10^{-3}$ m^3/s $Q_{\text{ad}} = 6.0 \times 10^{-4}$ m^3/s $Q_{\text{bc}} = 1.65 \times 10^{-3}$ m^3/s
4.51 $t = \dfrac{2}{5}\dfrac{\pi\tan^2(\theta)y_0^{5/2}}{\sqrt{2g}A}$ $t = \dfrac{6}{5}\dfrac{\cancel{V}_0}{Q_0}$
4.53 mf = (-2406, 2113) lbf
4.55 $\text{mf}_2/\text{mf}_1 = 1.33$
4.57 mf = (-320, 332) N
4.61 $T = 3.12$ N
4.63 $F = 35.7$ lbf
4.65 $\dot{m}_1 = 31.2\,\mathrm{lbm/s}$ $\dot{m}_2 = 32.0\,\mathrm{lbm/s}$ (because of weight plus momentum loss)
4.67 $V = 51$ m/s $V = 18.0$ m/s $V = 67.1$ m/s
4.69 $F = 1.81$ kN (tension)
4.71 $R_x = -\rho V^2\dfrac{\pi D^2}{4}\left(1+\sin\theta\right)\left[1-\left(\dfrac{d}{D}\right)^2\right]$ $R_x = -314$ N

4.73 $F = 11.6$ kN

4.75 $F = (-714, 498)$ N

4.77 $F = 1.70$ lbf

4.79 $F = 22.7$ kN

4.81 $d/D = 0.707$ No-dimensional pressure = 0.5

4.83 $t = 1.19$ mm $F = 3.63$ kN

4.85 $R_x = -4.68$ kN $R_y = 1.66$ kN

4.87 $R_x = -1040$ N $R_y = -667$ kN

4.89 $F = 2456$ N

4.91 $Q = 0.141\ \mathrm{m^3/s}$ $R_x = -1.65$ kN $R_y = -1.34$ kN

4.93 $F = 37.9$ N

4.95 $f = \rho U^2\left(\frac{5\pi}{8} - \frac{2}{\pi}\right)$

4.97 $u_{\max} = 60$ ft/s $\Delta p = 0.699\ \mathrm{lbf/ft^2}$

4.99 $D = 0.446$ N

4.101 $D/w = 0.163$ N/m

4.103 $h_2/h = (1 + \sin\theta)/2$

4.105 $h = H/2$

4.107 $Q = 257$ L/min

4.109 $V = \sqrt{V_0^2 - 2gh}$ $h = 4.28$ m

4.111 $V = 175$ ft/s $F = 2.97$ lbf

4.113 $p_1 = 68.4$ kPa (gage0 $F = 209$ N

4.115 $V = \sqrt{V_0^2 - 2gz}$ $A = \dfrac{A_0}{\sqrt{1 - \dfrac{2gz}{V_0^2}}}$

4.117 $p = p_0 - \rho\left(\frac{Q}{wh}\right)^2\left(\frac{x}{L}\right)^2$

4.119 $V = \dfrac{V_0 r}{2(h_0 - V_0 t)}$

4.123 $R_x = -2400$ N $R_y = 1386$ N

4.125 $V = -\dfrac{\rho Q}{2k} + \sqrt{\left(\dfrac{\rho Q}{2k}\right)^2 + \dfrac{\rho Q}{k}V_j}$ $V_j = 80$ m/s

4.127 $F = 3840$ lbf

4.129 $F = 4.24$ kN $t = 4.17$ s

4.131 $\alpha = 30^{\circ}$ $F = 10.3$ kN

4.133 $a = 13.5\ \mathrm{m/s^2}$

4.135 $t = 0.680$ s

4.137 $\dfrac{U}{V} = \ln\left(\dfrac{M_0}{M_0 - \rho VAt}\right)$ V = 0.61 m/s

4.139 $a_{\max}$ at $t = 0$ $\theta = 90^{\circ}$ $U \rightarrow V$

4.141 $A = \dfrac{2aM}{3\rho(V - at)^2}$ $A = 111\ \mathrm{mm^2}$

4.143 $h = 17.9$ mm

4.145 $a = \dfrac{2\rho(V - U)^2 A - kU^2}{M}$ $a = 5.99\ \mathrm{m/s^2}$ $U/U_t = 0.667$

4.147 $a = 14.2\ \mathrm{m/s^2}$ $t = 15.2$ m/s

4.149 $a = -\frac{\rho (V+U)^2 A}{M}$ $t = \frac{M}{\rho V A\left(1+\frac{V}{U_0}\right)}$

4.151 $V = 5$ m/s $x_{\max} = 1.93$ m $t = 2.51$ s

4.153 $a = 2.28$ m/s^2

4.155 $U = U_0 + V_e \ln\left(1-\frac{\dot{m}t}{M_0}\right)$ $U = 227$ m/s

4.157 $V_{\max} = 834$ m/s $a_{\max} = 96.7$ m/s^2

4.159 $m_f = 82.7$ kg

4.161 $a = 83.3$ m/s^2 $U = 719$ m/s

4.163 $V = 3860$ ft/s $Y = 33{,}500$ ft

4.165 $V = 641$ m/s

4.167 $\theta = 19^{o}$

4.169 $U = \frac{U_0}{\sqrt{1+\frac{2\rho U_0 A}{M_0}t}}$

4.171 $V_{\max} = 138$ m/s $y_{\max} = 1085$ m

4.175 $h = 10.7$ m

4.181 $M = -192$ N·m

4.183 $T = 0.193$ N·m $\dot{\omega} = 2610\,\text{rad/s}^2$

4.185 $\dot{\omega} = \frac{3}{2\rho A R^3}\left(T - \rho Q R V - 2\omega\rho V A R^2\right)$ $\omega_{\max} = -20.2$ rad/s (-193 rpm)

4.187 $T = 30$ N·m $\omega = 1434$ rpm

4.189 $\omega = 78.3\sin(\theta)$ rad/s $\omega = 39.1$ rad/s

4.191 $\dot{\omega} = 0.161\,\text{rad/s}^2$

4.195 $T = \rho\omega Q L^2 \sin^2\alpha$ $\Delta T = \frac{2}{3}\rho\dot{\omega} A L^3 \sin^2\alpha$

4.197 $\alpha = 0^{o}$ $\alpha \approx 42^{o}$

4.199 $dT/dt = -0.064$ K/s (^{o}C/s)

4.201 $\Delta p = 75.4$ kPa

4.203 $\dot{W}_s = -96.0\,\text{kW}$

4.205 $\dot{W}_{s,actual} = -3.41\,\text{kW}$

4.207 $\frac{\Delta mef}{\dot{m}} = -1.88\,\text{J/kg}$ $\Delta t = 4.49 \times 10^{-4}$ K (^{o}C)

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Answers to Selected Problems, Chapter 5

5.1 a) $\rho \neq$ const b) Possible c) Possible d) Possible

5.3 $A + E + J = 0$ (Others arbitrary)

5.5 $v = -3x^2y + y^3 + f(x)$

5.7 $u = x^4/2 - 3x^2y^3$

5.11 $\left.\frac{v}{U}\right)_{\max} = 0.00167 \quad (0.167\%)$

5.13 $\left.\frac{v}{U}\right)_{\max} = \frac{3}{8}\frac{\delta}{x}\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{2}\left(\frac{y}{\delta}\right)^4\right]$ $\frac{y}{\delta} = 1$ $\left.\frac{v}{U}\right)_{\max} = \frac{3}{8}\frac{\delta}{x}\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{2}\left(\frac{y}{\delta}\right)^4\right]$

5.15 $v = -2Axy^3/3 + f(x)$ $\psi = xy^{3/2}$

5.19 $V_\theta = -\frac{\Lambda \sin\theta}{r^2} + f(r)$

5.23 $\psi = \frac{Uy^2}{2h}$ $y = \frac{h}{\sqrt{2}}$

5.25 $\vec{V} = \left(-U\cos\theta + \frac{q}{2\pi r}\right)\hat{i}_r + U\sin\theta\hat{i}_\theta$ $(r,\theta) = \left(\frac{q}{2\pi U}, 0\right)$

5.27 2D Incompressible $\psi = zy^3 - z^3y$

5.29 $\psi = \frac{Uy^2}{2h}$ $h_{\text{half}} = 1.06$ m

5.31 $\psi = U\delta\left[\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3\right]$ $\frac{y}{\delta} = 0.442$ (1/4 flow) $\frac{y}{\delta} = 0.652$ (1/2 flow)

5.33 $\psi = U\delta\left[\frac{3}{4}\left(\frac{y}{\delta}\right)^2 - \frac{1}{8}\left(\frac{y}{\delta}\right)^4\right]$ $\frac{y}{\delta} = 0.465$ (1/4 flow) $\frac{y}{\delta} = 0.671$ (1/2 flow)

5.35 $\psi = -C\ln(r) + C_1$ $Q/b = 0.0912$ $\text{m}^3/\text{s/m}$

5.37 2D Incompressible $\vec{a} = \frac{16}{3}\hat{i} + \frac{32}{3}\hat{j} + \frac{16}{3}\hat{k} \quad \text{m/s}^2$

5.39 $\vec{a} = -2.86\left(10^{-2}\,\hat{i} + 10^{-4}\,\hat{j}\right) \quad \text{m/s}^2$ $dy/dx = 0.0025$

5.41 Incompressible $a_x = -\frac{\Lambda^2 x}{\left(x^2 + y^2\right)^2}$ $a_y = -\frac{\Lambda^2 y}{\left(x^2 + y^2\right)^2}$ $a = -\frac{100}{r^3}$

5.43 $a_x = -\frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)$

5.45 $a = -\left(\frac{Q}{2\pi h}\right)^2 \frac{1}{r^3}$ (Radial)

5.47 $\frac{Dc}{Dt} = \frac{UA}{a}\left(\frac{1}{2}e^{-\frac{Ut}{2a}} - e^{-\frac{Ut}{a}}\right)$ $x = 2.77$ m $\left.\frac{Dc}{Dt}\right|_{\max} = 1.25 \times 10^{-5}\ \frac{\text{ppm}}{\text{s}}$

5.49 $\partial T/\partial x = -0.0873$ $^{\text{o}}$F/mile

5.53 $\vec{a} = A^2\left(x\hat{i} + y\hat{j}\right)$ $\quad \vec{a} = \frac{1}{2}\hat{i} + 2\hat{j}$ $\quad \vec{a} = \hat{i} + \hat{j}$ $\quad \vec{a} = 2\hat{i} + \frac{1}{2}\hat{j}$

5.55 $xy = 8$ $\quad \vec{V} = 12\hat{i} - 24\hat{j}$

$\vec{V} = 6\pi\hat{i} - 12\pi\hat{j}$ (Local) $\quad \vec{V} = 72\hat{i} + 144\hat{j}$ (Convective) $\quad \vec{V} = 9.8\hat{i} + 106\hat{j}$ (Total)

5.57 $a_x = \frac{U^2}{x}\left[-\left(\frac{y}{\delta}\right)^2 + \frac{4}{3}\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^2\right]$ $\quad a_{x(\mathrm{max})} = -5.22\ \mathrm{m/s^2}$

5.59 $v = v_0\left(1 - \frac{y}{h}\right)$ $\quad \vec{a} = \frac{v_0^2 x}{h^2}\hat{i} + \frac{v_0^2}{h}\left(\frac{y}{h} - 1\right)\hat{j}$

5.61 $a_r = \frac{2U^2}{R}\left[1 - \left(\frac{R}{r}\right)^2\right]\left(\frac{R}{r}\right)^3$ $\quad a_\theta = 0$ $\quad r = 1.29R$ (max a)

$a_r = -\frac{4U^2}{R^2}\sin^2\theta$ $\quad a_\theta = \frac{4U^2}{R^2}\sin\theta\cos\theta$ $\quad \theta = \pm\pi/2$ (max a)

5.63 $a_x = 32\left[20 + 2\sin(\omega t)\right]^2 + 2.4\cos(\omega t) \quad \left(\mathrm{m/s^2}\right)$ at $x = L$

5.67 Not incompressible $\quad$ Not irrotational

5.69 $\Gamma = 0$

5.71 Incompressible $\quad$ Irrotational

5.73 Incompressible $\quad$ Not irrotational

5.75 $\vec{V} = -2y\hat{i} - 2x\hat{j}$

5.77 $\vec{\omega} = -\hat{k}$ (rad/s)

5.79 $\omega = -\frac{u}{2h}$ $\quad \omega = -0.5\ \mathrm{s^{-1}}$

5.81 $\Gamma = -UL\frac{h}{b}\left(1 - \frac{h}{b}\right)$ $\quad \Gamma = -UL/4$ ($h = b/2$) $\quad \Gamma = 0$ ($h = b$)

5.83 $\gamma = -\frac{2yU_{\mathrm{max}}}{b^2}$ $\quad \vec{\zeta} = \frac{2yU_{\mathrm{max}}}{b^2}\hat{k}$

5.85 $\frac{dF_{\mathrm{max}}}{dV\!\!\!-} = -\mu U\left(\frac{\pi}{2\delta}\right)^2$ $\quad \frac{dF_{\mathrm{max}}}{dV\!\!\!-} = -1.85\,\frac{\mathrm{kN}}{\mathrm{m^3}}$

Introduction to Fluid Mechanics 7th Edition

Fox, Pritchard, & McDonald

Answers to Selected Problems, Chapter 6

6.1 $\vec{a} = 9\hat{i} + 7\hat{j}$ ft/s^2 $\nabla p = -0.125\hat{i} - 0.544\hat{j}$ psi/ft

6.3 $\vec{a}_{\text{local}} = 10\left(\hat{i} + \hat{j}\right)$ ft/s^2 $\vec{a}_{\text{conv}} = A\left(Ax + Bt\right)\hat{i} - A\left(-Ay + Bt\right)\hat{j}$

$\vec{a}_{\text{conv}} = 300\hat{i} - 200\hat{j}$ ft/s^2 $\nabla p = -4.17\hat{i} + 2.56\hat{j} - 0.43\hat{k}$ psi/ft

6.5 $\vec{a} = 1\hat{i} + 7\hat{j}$ ft/s^2 $\nabla p = -0.0139\hat{i} - 0.544\hat{j}$ psi/ft

6.7 $v = -Ay$ $\vec{a} = 8\hat{i} + 4\hat{j}$ m/s^2

$\nabla p = -12\hat{i} - 6\hat{j} - 14.7\hat{k}$ Pa/m $p(x) = 190 - 3x^2/1000$ (p in Pa, x in m)

6.9 Incompressible Stagnation point: (2.5, 1.5)

$\nabla p = -\rho\left[\left(4x - 10\right)\hat{i} + \left(4y - 6\right)\hat{j} + g\hat{k}\right]$ $\Delta p = 9.6$ Pa

6.11 $a_x = -\dfrac{U^2}{2L}\left(1 - \dfrac{x}{2L}\right)$ $\dfrac{dp}{dx} = \rho\dfrac{U^2}{2L}\left(1 - \dfrac{x}{2L}\right)$ $p_{\text{out}} = 43.3$ kPa (gage)

6.13 $\vec{a} = -0.101\hat{e}_r + 0.0507\hat{e}_\theta$ m/s^2 $\vec{a} = -0.101\hat{e}_r + 0.0507\hat{e}_\theta$ m/s^2

$\vec{a} = -0.0127\hat{e}_r + 0.00633\hat{e}_\theta$ m/s^2 $\nabla p = 101\hat{e}_r - 50.5\hat{e}_\theta$ Pa/m

$\nabla p = 101\hat{e}_r - 50.5\hat{e}_\theta$ Pa/m $\nabla p = 12.7\hat{e}_r - 6.33\hat{e}_\theta$ Pa/m

6.15 $\dfrac{\partial p}{\partial x} = -\dfrac{\rho A_i^2 L^2 u_i^2\left(A_e - A_i\right)}{\left(A_i L + A_e x - A_i x\right)^3}$ $p = p_i + \dfrac{\rho u_i^2}{2}\left\{1 - \left[\dfrac{A_i}{A_i - \dfrac{\left(A_i - A_e\right)}{L}x}\right]^2\right\}$

6.17 $a_x = -\dfrac{2V_i^2\left(D_o - D_i\right)}{D_i L\left[1 + \dfrac{\left(D_o - D_i\right)}{D_i L}x\right]^5}$ $\left.\dfrac{\partial p}{\partial x}\right|_{\max} = -10$ MPa/m $L \geq 1$ m

6.19 $F_z = -1.56$ N (Acts downwards)

6.21 $a_x = \dfrac{2V^2 x}{b^2}$ $\dfrac{\partial p}{\partial x} = -\dfrac{2\rho V^2 x}{b^2}$

$p = p_{\text{atm}} + \dfrac{\rho V^2 L^2}{b^2}\left[1 - \left(\dfrac{x}{L}\right)^2\right]$ $F_y = \dfrac{4\rho V^2 L^3 W}{3b^2}$

6.23 $a_x = \dfrac{q^2 x}{h^2}$ $\dfrac{\partial p}{\partial x} = -\dfrac{\rho q^2 x}{h^2}$

6.25 $\nabla p = \dfrac{2\Lambda^2\rho}{r^5}\hat{e}_r + 0\hat{e}_\theta$

6.27 $\Delta p = -30.6$ Pa

6.31 $B = -0.6$ m$^{-2}\cdot$s^{-1} $\vec{a} = 6\hat{i} + 3\hat{j}$ m/s^2 $a_{\text{n}} = 6.45$ m/s^2

6.33 $\vec{a} = 4\hat{i} + 2\hat{j}$ ft/s^2 $R = 5.84$ ft

6.35 $\vec{a} = 0.5\hat{i} + \hat{j}$ m/s^2 $R = 5.84$ ft

6.37 $\Delta h = 1.37$ in

6.39 $F = 0.379$ lbf $F = 1.52$ lbf

6.41 $h = 628$ mm

6.47 $p_2 = 291$ kPa (gage)

6.49 $p = 9.53$ psig

6.51 $h = 17.0$ ft

6.53 $A = A_1\sqrt{\dfrac{1}{1+\dfrac{2g(z_1 - z)}{V_1^2}}}$

6.55 $V = 262$ m/s

6.57 $Q = 304$ gpm (0.676 ft^3/s)

6.59 $p = p_\infty + \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta\right)$ $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

6.61 $F = -278$ N/m

6.63 $Q = 2.55 \times 10^{-3}$ m^3/s

6.65 $p_1 = 7.11$ psig $K_x = 12.9$ lbf

6.67 $p_2 = 13.2$ kPa (gage) (98.9 mm Hg) $p_3 = 706$ Pa (gage) (5.29 mm Hg)
$R_x = 0.375$ N $R_y = 0.533$ N

6.69 $p_1 = 1.35$ psig $p_0 = 1.79$ psig

6.73 $\Delta h = 6.60$ in $F = 0.105$ lbf $F = 18.5$ lbf

6.75 $F = 83.3$ kN

6.77 $p_1 = 11.4$ psig $F = 14.1$ lbf

6.79 $\dot{m} = A\sqrt{2p\rho}$ $\dfrac{dM}{dt} = -\rho_w \dfrac{d\mathcal{V}_{air}}{dt}$

$$M_w = \rho_w \mathcal{V}_0\left\{\frac{V_t}{\mathcal{V}_0} - \left[1 + 1.70\sqrt{\frac{2p_0}{\rho_w}}\,\frac{At}{\mathcal{V}_0}\right]^{0.588}\right\}$$

6.83 $C_c = ½$

6.87 $a_x = 10.5$ ft/s^2

6.89 $dQ/dt = 0.516$ m^3/s/s

6.91 $D_j/D_1 = 0.32$

6.93 Bernoulli can be applied

6.95 Incompressible Unsteady Irrotational $\phi = \left[\frac{A}{2}\left(y^2 - x^2\right) + Bxy\right]t$

6.97 $\psi = \dfrac{q}{2\pi}\left[\tan^{-1}\left(\dfrac{y-h}{x}\right) + \tan^{-1}\left(\dfrac{y+h}{x}\right)\right]$ $\phi = -\dfrac{q}{2\pi}\ln\left\{\left[x^2 + (y-h)^2\right]\left[x^2 + (y+h)^2\right]\right\}$

6.99 NOTE: Error – function is $\psi = Ax^2y - By^3$ $\phi = 3Bxy^2 - \dfrac{A}{3}x^3$

6.101 $\psi = \dfrac{B}{2}\left(x^2 - y^2\right) - 2Axy$

6.105 $\vec{V} = -(A + 2Bx)\hat{i} + 2By\hat{j}$ $\psi = -(Ay + 2Bxy)$ $\Delta p = 12$ kPa

6.107 $V = x^2 + y^2$ $\psi = 3Ax^2y - \dfrac{B}{3}x^3$

6.109 Incompressible Irrotational Stagnation point: $(-2, 4/3)$
$\phi = \dfrac{A}{2}\left(y^2 - x^2\right) - Bx - Cy$ $\Delta p = 55.8$ kPa

6.113 $\psi = \dfrac{q}{2\pi}(\theta_1 - \theta_2) + Ur\sin\theta$ $\phi = \dfrac{q}{2\pi}\ln\left(\dfrac{r_2}{r_1}\right) - Ur\cos\theta$

$$\vec{V} = \left[\frac{q}{2\pi} \left(\frac{\cos\theta_1}{r_1} - \frac{\cos\theta_2}{r_2} \right) + U \right] \hat{i} + \frac{q}{2\pi} \left(\frac{\sin\theta_1}{r_1} - \frac{\sin\theta_2}{r_2} \right) \hat{j}$$

Stagnations points: $\theta = 0, \pi$ $r = 0.367$ m $\psi_{\text{stag}} = 0$

6.115 $\psi = Ur\sin\theta - \frac{K}{2\pi}\ln r$ $\phi = -Ur\cos\theta - \frac{K}{2\pi}\theta$

$$\vec{V} = U\cos\theta\hat{e}_r + \left(\frac{K}{2\pi r} - U\sin\theta \right) \hat{e}_\theta$$

Stagnations point: $\theta = \pi/2$ $r = \frac{K}{2\pi U}$

6.117 Stagnations points: $\theta = 63^\circ, 297^\circ$ $r = 1.82$ m $\Delta p = 317$ Pa

Introduction to Fluid Mechanics 7th Edition

Fox, Pritchard, & McDonald

Answers to Selected Problems, Chapter 7

7.1 $\dfrac{V_0^2}{gL}$

7.3 $\dfrac{E}{\rho L^2 \omega^2}$

7.5 $\dfrac{\nu}{V_0 L} \quad \left(= \dfrac{1}{\mathrm{Re}}\right)$

7.7 $\dfrac{\Delta p}{\rho \bar{V}^2}, \dfrac{\nu}{D\bar{V}}, \dfrac{L}{D}$

7.9 $F \propto \mu V D$

7.11 $\dfrac{\Delta p}{\rho V^2} = f\left(\dfrac{\mu}{\rho V D}, \dfrac{d}{D}\right)$

7.13 $\dfrac{\tau_w}{\rho U^2} = f\left(\dfrac{\mu}{\rho U L}\right)$

7.15 $\dfrac{W}{g\rho p^3}, \dfrac{\sigma}{g\rho p^3}$

7.17 $V = \sqrt{gD}\, f\left(\dfrac{\lambda}{D}\right)$

7.19 $Q = h^2 \sqrt{gh}\, f\left(\dfrac{b}{h}\right)$

7.21 $\dfrac{W}{D^2 \omega \mu} = f\left(\dfrac{L}{D}, \dfrac{c}{D}\right)$

7.23 $\dfrac{\mathscr{P}}{\rho D^5 \omega^3}, \dfrac{\Delta p}{\rho D^2 \omega^2}, \dfrac{d}{D}, \dfrac{d_i}{D}, \dfrac{d_o}{D}$

7.25 Four parameters $\quad \Pi_1 = \dfrac{\mu}{\rho d^{3/2} g^{1/2}}$

7.27 $\dfrac{Q}{Vh^2} = f\left(\dfrac{\rho V h}{\mu}, \dfrac{V^2}{gh}\right)$

7.29 $\dfrac{d}{D}, \dfrac{\mu}{\rho V D}, \dfrac{\sigma}{\rho D V^2}$

7.31 $\dfrac{W}{\rho V^2 d^2}, \dfrac{\mu}{\rho V d}, \dfrac{h}{d}, \dfrac{D}{d}$

7.33 $\dfrac{d}{D}, \dfrac{\mu^2}{\rho \Delta p D^2}, \dfrac{\sigma}{D \Delta p}$

7.35 $\dfrac{\delta}{D}, \dfrac{L}{D}, \dfrac{\mu \omega D^3}{T}, \dfrac{I\omega^2}{T}$

7.37 $\dfrac{\Delta p}{\rho D^2\omega^2}, \dfrac{\mu}{\rho D^2\omega}, cD^3, N, \dfrac{\rho_p}{\rho}, \dfrac{g}{D\omega^2}$

7.39 $\dfrac{\mathcal{P}}{p\omega D^3} = f\left(\dfrac{\rho\omega}{p}, \dfrac{c}{D}, \dfrac{l}{D}\right)$

7.41 Four primary dimensions $\dot{Q} = \rho V^3 L^2 f\left(\dfrac{c_p\Theta}{V^2}, \dfrac{\mu}{\rho VL}\right)$

7.43 $\dfrac{dT}{dt}\dfrac{Lc_p}{V^3} = f\left(\dfrac{c}{c_p}, \dfrac{k}{\rho L^2 c_p}, \dfrac{\mu}{\rho LV}\right)$

7.45 $\Pi_1 = \dfrac{u}{U}$ $\Pi_2 = \dfrac{y}{\delta}$ $\Pi_3 = \dfrac{(dU/dy)\delta}{U}$ $\Pi_4 = \dfrac{\nu}{\delta U}$

7.47 V_{w} = 6.90 m/s F_{air} = 522 N
7.49 $V_{\mathrm{air}} > V_{\mathrm{water}}$ $V_{\mathrm{air}} = 15.1\cdot V_{\mathrm{water}}$
7.51 ω_{m} = 395 rpm ω_{m} = 12500 rpm Froude number modeling is most likely
7.53 V_{m} = 40.3 m/s V_{p} = 40.3 m/s
7.55 V_{m} = 5.07 m/s $F_{\mathrm{m}}/F_{\mathrm{p}}$ = 3.77
7.57 Q_{m} = 0.125 m^3/s $\mathcal{P}_{\mathrm{p}}$ = 127 kW
7.59 p_{m} = 2.96 psia

7.61 $\dfrac{fd}{V} = F\left(\dfrac{\rho Vd}{\mu}\right)$ $\dfrac{V_1}{V_2} = \dfrac{1}{2}$ $\dfrac{f_1}{f_2} = \dfrac{1}{4}$

7.63 V_{m} = 0.618 m/s – 1.03 m/s

7.65 $\dfrac{F_D}{\rho V^2 A^{1/2}} = f\left(\dfrac{\mu}{\rho VA^{1/2}}\right)$ $F_{D_p} = 2.46$ kN $\mathcal{P}$ =55.1 kW (73.9 hp)

7.67 τ_{p} = 1070 hr (~ 45 days)
7.69 V_{m} = 1.88 m/s V_{p} = 7.36 m/s $F_{\mathrm{p}}/F_{\mathrm{m}}$ = 1.13 (submerged), = 2.77 x 10^4 (surface)
7.71 CD = 1.028 F_{D_p} = 3.89 kN V_{m} = 250 m/s (model is impractical, compressible flow)

7.73 Model = $\dfrac{1}{50}$ x Prototype Adequate Reynolds number not achievable

7.77 D_{Total} = 1305 N D_{Total} = 2316 N (Wave drag negligible)
7.79 h_{m} = 13.8 J/kg Q_{m} = 0.166 m^3/s D_{m} = 0.120 m

7.81 $\dfrac{F_t}{\rho\omega^2 D^4} = f_1\left(\dfrac{V}{\omega D}, \dfrac{g}{\omega^2 D}\right)$ $\dfrac{T}{\rho\omega^2 D^5} = f_2\left(\dfrac{V}{\omega D}, \dfrac{g}{\omega^2 D}\right)$ $\dfrac{\mathcal{P}}{\rho\omega^3 D^5} = f_3\left(\dfrac{V}{\omega D}, \dfrac{g}{\omega^2 D}\right)$

7.83 K.E. ratio = 7.22
7.85 $F_{\mathrm{B}} \approx 0.273$ N (to right) $C_{\mathrm{D_m}} = 0.443$ $F_{\mathrm{D_p}} = 1.64$ kN

Introduction to Fluid Mechanics 7th Edition

Fox, Pritchard, & McDonald

Answers to Selected Problems, Chapter 8

8.1 $Q = 0.146\ \text{ft}^3/\text{s}$ $L = 12.5 - 20$ ft (turbulent) $L = 69.0$ ft (laminar)

8.3 Smallest turbulent first $Q_{\text{large}} = 0.0244\ \text{ft}^3/\text{s}$ Non are fully developed
$Q_{\text{mid}} = 0.0122\ \text{ft}^3/\text{s}$ Smallest fully developed
$Q_{\text{small}} = 0.00610\ \text{ft}^3/\text{s}$ Smallest fully developed

8.7 $\bar{V}/u_{\max} = 2/3$

8.9 $\tau_{yx} = -1.88$ Pa $Q/b = -5.63 \times 10^{-6}\ \text{m}^2/\text{s}$

8.11 $Q = 1.25 \times 10^{-5}\ \text{ft}^3/\text{s}$ ($0.0216\ \text{in}^3/\text{s}$)

8.13 $Q = 3.97 \times 10^{-9}\ \text{m}^3/\text{s}$ (3.97×10^{-6} L/s)

8.15 $M = 4.32$ kg $Q = \dfrac{\pi}{12}\dfrac{\Delta p D}{\mu L}a^3$ $a = 1.28 \times 10^{-5}$ m ($12.8\ \mu$m)

8.17 $\bar{V} = \dfrac{Q}{2\pi r h}$ $\dfrac{dp}{dr} = -\dfrac{6\mu Q}{\pi r h^3}$ $p = p_{\text{atm}} - \dfrac{6\mu Q}{\pi r h^3}\ln\left(\dfrac{r}{R}\right)$ ($p = p_0$, $r < R_0$)

8.19 $n = 1.48$ (dilatant)

8.21 $\partial p/\partial x = -92.6$ Pa/m

8.23 $u_{\text{interface}} = 15$ ft/s

8.25 $\partial p/\partial x = -2U\mu/a^2$ $\partial p/\partial x = 2U\mu/a^2$

8.27 $\nu = 1.00 \times 10^{-4}\ \text{m}^2/\text{s}$

8.29 $\tau = \rho g \sin(\theta)(h - y)$ $Q/w = 217\ \text{mm}^3/\text{s/mm}$ $Re = 0.163$

8.31 $Q/w = 0.0208\ \text{ft}^3/\text{s/ft}$ $\tau = 1.58 \times 10^{-6}$ psi $\partial p/\partial x = 7.58 \times 10^{-4}$ psi/ft

8.33 $\partial p/\partial x = 34.4$ Pa/m

8.35 B.C.: $y = 0$, $u = U_0$; $y = h$, $\tau = 0$ $u = \dfrac{\rho g}{\mu}\left(\dfrac{y^2}{2} - hy\right) + U_0$

8.37 $\dfrac{dV}{dt} = -\dfrac{\pi w L}{mh}V$ $t = 1.06$ s

8.39 $\Delta p = \dfrac{6\mu L R\omega}{a^2}\left(1 - \dfrac{2Q}{abR\omega}\right)$ $\mathscr{P} = \dfrac{\mu L b (R\omega)^2}{a}\left(4 - \dfrac{6Q}{abR\omega}\right)$ $\eta = \dfrac{6Q}{abR\omega}\dfrac{\left(1 - \dfrac{2Q}{abR\omega}\right)}{\left(4 - \dfrac{6Q}{abR\omega}\right)}$

8.41 $\mathscr{P}_{\text{v}} = \dfrac{\pi\mu\omega^2 D^3 L}{4a}$ $\mathscr{P}_{\text{p}} = \dfrac{\pi D a^3 \Delta p^2}{12\mu L}$ $\mathscr{P}_{\text{v}} = 3\mathscr{P}_{\text{p}}$

8.45 $r = 0.707R$

8.47 $Q = 1.43 \times 10^{-3}\ \text{in}^3/\text{s}$ ($0.0857\ \text{in}^3/\text{min}$)

8.49 $\tau = c_1/r$ $u = \dfrac{c_1}{\mu}\ln r + c_2$ $c_1 = \dfrac{\mu V_0}{\ln(r_i/r_o)}$ $c_2 = -\dfrac{V_0 \ln r_o}{\ln(r_i/r_o)}$

8.51 $r = R\left[\dfrac{1}{2}\dfrac{\left(1 - k^2\right)}{\ln(1/k)}\right]^{1/2}$

8.53 % change $= -100/(1 + \ln k)$

8.55 $\tau_{\text{w}} = -131$ Pa

8.57 $\tau_{\text{w}} = -0.195\ \text{lbf/ft}^2$ $\tau_{\text{w}} = -1.35 \times 10^{-3}$ psi

8.59 $Q = 4.52 \times 10^{-7}$ m^3/s $\Delta p = 235$ kPa $\tau_w = 294$ Pa
8.61 $n = 6.21$ $n = 8.55$
8.63 $\beta_{\text{lam}} = 4/3$ $\beta_{\text{turb}} = 1.02$
8.65 $\alpha = 2$
8.67 $H_{lT} = 1.33$ m $h_{lT} = 13.0$ J/kg
8.69 $V_1 = 3.70$ m/s
8.71 $Q = 411$ gpm
8.73 $\bar{V}_1 = 2$ m/s
8.75 $h_{lT} = 913$ ft^2/s^2 ($H_{lT} = 28.4$ ft)
8.77 $\frac{d\bar{u}}{dy} = 963\,\text{s}^{-1}$ $\tau_w = 3.58 \times 10^{-4}$ lbf/ft^2 $\tau_w = 4.13 \times 10^{-4}$ lbf/ft^2
8.79 $f = 0.0390$ $Re = 3183$ Turbulent
8.81 Maximum = 2.12% at $Re = 10000$ and $e/D = 0.01$
8.85 $p_2 = 177$ kPa $p_2 = 175$ kPa
8.87 $Q = 0.0406$ ft^3/s (2.44 ft^3/min, 18.2 gpm)
8.91 $K = 9.38 \times 10^{-4}$
8.93 $Q = 12.7$ gpm $Q = 11.6$ gpm ($\Delta Q = -1.1$ gpm) $Q = 13.7$ gpm ($\Delta Q = 1.0$ gpm)
8.95 $\Delta p = 23.7$ psi $K = 0.293$
8.97 $h_{l_m} = (1 - AR)^2 \frac{V_1^2}{2}$
8.99 $\bar{V}_1 = \sqrt{\frac{2\Delta p}{\rho(1 - AR^2 - K)}}$ Inviscid assumption: Lower indicated flow/larger Δp
8.101 $Q = 0.345$ L/min $d = 3.65$ m
8.103 $d = 6.13$ m (or 6.16 m if $\alpha = 2$, laminar)
8.105 Analogy fails at $Q = 7.34 \times 10^{-7}$ m^3/s
8.107 118800%! (A huge increase because $V \sim 1/d^2$, and $\Delta p \sim V^2$)
8.111 (a) $\Delta p = 25.2$ kPa (b) $\Delta p = 32.8$ kPa (c) $\Delta p = 43.3$ kPa ((a) is best)
8.113 $V_B = 4.04$ m/s $L_A = 12.8$ m (Not feasible!) $\Delta p = 29.9$ kPa
8.115 $\Delta p/L = 7.51 \times 10^{-3}$ lbf/ft^2/ft (round) $\Delta p/L = 8.68 \times 10^{-3}$ lbf/ft^2/ft (1:1) (+15.6%)
$\Delta p/L = 9.32 \times 10^{-3}$ lbf/ft^2/ft (2:1) (+24.1%) $\Delta p/L = 0.010$ lbf/ft^2/ft (3:1) (+33.2%)
8.117 $p_1 = 179$ psig
8.121 $L = 26.5$ m
8.123 Friction ≈ 77%, Gravity ≈ 23% (Turbulent)
8.125 $Q = 0.0395$ m^3/s
8.127 $V_1 = 0.0423$ m/s (down) (Falls at 42.3 mm/s)
8.129 Rate of downpour = 0.418 cm/min
8.135 $Q = 6.68 \times 10^{-3}$ m^3/s $p_{\text{min}} = -35.5$ kPa (gage)
8.137 $Q = 5.30 \times 10^{-4}$ m^3/s $Q = 5.35 \times 10^{-4}$ m^3/s (diffuser)
8.139 $L = 0.296$ m
8.141 Your boss was wrong (which is s-w-e-e-e-e-e-t!)
8.143 $D = 5.0 - 5.1$ cm (corresponds to standard 2 in. pipe)
8.145 $D = 6$ in. (nominal)
8.149 $\bar{V} = 6.46$ m/s $p_F = 705$ kPa (gage) $\mathscr{P} = 832$ kW $\tau_w = 88.6$ Pa
8.151 $dQ/dt = -0.524$ m^3/s/min
8.153 $\mathscr{P} = 8.13$ hp
8.155 $\Delta p = 53.2$ psi
8.157 $D = 48$ mm $\Delta p = 3840$ kPa $\mathscr{P}_{\text{pump}} = 24.3$ kW (32.6 hp)
8.159 $Q = 5.58 \times 10^{-3}$ m^3/s (0.335 m^3/min) $V = 37.9$ m/s $\mathscr{P} = 8.77$ kW
8.161 Cost = \$4980/year

8.163 $Q = 0.0419\ \text{m}^3/\text{s}$ $\Delta p = 487\ \text{kPa}$ $\mathscr{P} = 29.1\ \text{kW}$

8.165 $Q = 2.51\ \text{m}^3/\text{s}$

8.167 $Q_0 = 0.00928\ \text{m}^3/\text{s}$ $Q_1 = 0.00306\ \text{m}^3/\text{s}$ $Q_2 = Q_3 = 0.00311\ \text{m}^3/\text{s}$ $Q_4 = 0.00623\ \text{m}^3/\text{s}$

$Q_0 = 0.00862\ \text{m}^3/\text{s}$ $Q_1 = 0.0\ \text{m}^3/\text{s}$ $Q_2 = Q_3 = 0.00431\ \text{m}^3/\text{s}$ $Q_4 = 0.00862\ \text{m}^3/\text{s}$

8.169 $\Delta p = 22.2\ \text{psi}$ $Q_{22} \approx 5.2\ \text{gpm}$ $Q_{24} \approx 24.8\ \text{gpm}$

8.171 $\Delta p = 25.8\ \text{kPa}$

8.173 $Q = 1.49\ \text{ft}^3/\text{s}$

8.175 $Q = 0.00611\ \text{m}^3/\text{s}$

8.177 $\Delta t = 40.8\ \text{mm}$ $\dot{m}_{\text{min}} = 0.0220\ \text{kg/s}$

8.179 $Q = 1.37\ \text{ft}^3/\text{s}$

8.183 $Re_{\text{d}} = 1800$ $f = 0.0356$ $p_2 = -290\ \text{Pa (gage)}\ (29.6\ \text{mm Hg})$

8.185 $K_{\text{c}} = \$9890/\text{in}^2$ $K_{\text{p}} = 1.81 \times 10^{13}\ \$\cdot\text{in}^5$

Introduction to Fluid Mechanics 7th Edition

Fox, Pritchard, & McDonald

Answers to Selected Problems, Chapter 9

9.1 $x_p = 18.6$ cm $x_m = 10.3$ mm
9.3 $x_p = 10.4$ cm $x_p = 7.47$ mm
9.5 $Re_D = 1$ (reasonable) $Re_D = 2.5 \times 10^5$ (not reasonable) Use water and $D = 10$ cm
9.7 L increases with elevation
9.9 $A = U$ $B = \dfrac{\pi}{2\delta}$ $C = 0$
9.11 $\dfrac{\delta^*}{\delta} = 0.375$ $\dfrac{\theta}{\delta} = 0.139$
9.13 $\dfrac{\delta^*}{\delta} = 0.396$ $\dfrac{\theta}{\delta} = 0.152$
9.15 Linear: $\dfrac{\theta}{\delta} = 0.167$ Sinusoidal: $\dfrac{\theta}{\delta} = 0.137$ Parabolic: $\dfrac{\theta}{\delta} = 0.133$
9.17 Power: $\dfrac{\delta^*}{\delta} = 0.125$, $\dfrac{\theta}{\delta} = 0.0972$ Parabolic: $\dfrac{\delta^*}{\delta} = 0.333$, $\dfrac{\theta}{\delta} = 0.133$
9.19 $\dot{m}_{ab} = 50.4\ \text{kg/s}$ $F_D = 50.4$ N
9.21 $d_{\text{exit}} = 3.13$ mm $\Delta U = 3.91\%$
9.23 $U_2 = 13.8$ m/s $\Delta p = 20.6$ Pa
9.25 $\Delta p = -1.16\ \text{lbf/ft}^2$
9.27 $U_2 = 24.6$ m/s $p_2 = -44.5$ mm H_2O
9.29 $\delta_2^* = 2.54$ mm $\Delta p = -107$ Pa $F_D = 2.00$ N
9.35 $y = 0.121$ in $dy/dx = 0.00326$ $\tau_w = 0.3321\dfrac{\rho U^2}{\sqrt{\text{Re}_x}}$ $F_D = 0.6642\dfrac{\rho U^2 bL}{\sqrt{\text{Re}_L}}$

$\theta_L = 0.0454$ in
9.39 $\theta_L = 0.0454$ in $F_D = 0.850$ N
9.41 $F_D = 26.3$ N $F_D = 45.5$ N
9.43 $F_D = 8.40 \times 10^{-4}$ N (or $F_D = 1.68 \times 10^{-3}$ N for two sides) (Higher than Problem 9.42)
9.45 $F_D = 3.45 \times 10^{-3}$ N (or $F_D = 6.90 \times 10^{-3}$ N for two sides) (Higher than Problem 9.44)
9.51 $F_D = 0.557$ N
9.53 $U = 1.81$, 2.42, 3.63, and 7.25 m/s
9.55 $\tau_w = 0.0297\dfrac{\rho U^2}{\text{Re}_x^{1/5}}$ $F_D = 0.0360\dfrac{\rho U^2 bL}{\text{Re}_L^{1/5}}$ $F_D = 2.34$ N
9.57 $F_D = 4.57 \times 10^{-3}$ N (or $F_D = 9.14 \times 10^{-3}$ N for two sides)
9.59 $F_D = 55.8$ N (or $F_D = 112$ N for two sides)
9.61 $\dfrac{\delta}{x} = \dfrac{0.353}{\text{Re}_x^{1/5}}$ $c_f = \dfrac{0.0612}{\text{Re}_x^{1/5}}$ $F_D = 2.41$ N
9.63 $\delta_L = 31.3$ mm $\tau_{w_L} = 0.798$ Pa $F_D = 0.700$ N
9.65 $w_2 = 80.3$ mm
9.67 $\Delta p = 6.16$ Pa $L = 0.233$ m
9.69 Petroleum used ≈ 0.089% (about 15% of pipeline use)

9.71 $\dot{m}_{\text{lam}} = 0.525\rho U^2\delta$ $\dot{m}_{\text{turb}} = 0.777\rho U^2\delta$

9.73 $a = 0$ $b = 0$ $c = 3$ $d = -2$ $H = 3.89$

9.75 $U_2 = 2.50$ m/s $\Delta p = 0.00370$ in H_2O

9.77 $\frac{U}{U_1} = \sqrt{\left[1+\frac{c_f}{\theta(H+2)}x\right]}$ (constant τ_w) $\frac{U}{U_1} = \left[1+0.00583\left(\frac{\nu}{U_1\delta}\right)^{\frac{1}{4}}\frac{x}{\theta(H+2)}\right]^4$ ($\tau_w \neq$ constant)

9.79 $F_D = 5.58$ N (11.2 N for both sides) One system: $F_D = 4.23$ N (8.46 N for both sides)

9.81 $F_D = 1500$ lbf $\mathscr{P} = 2000$ hp

9.85 $x_{\text{lam}}/L = 0.0352\%$ $F_D = 5.49 \times 10^5$ N $\mathscr{P} = 7.63$ MW

9.87 $F_D = 3.02 \times 10^4$ N Savings = $F_D = 7.94 \times 10^4$ kg/yr

9.91 $F_D = 3610$ lbf $\mathscr{P} = 77.6$ hp

9.93 $d_{\text{i}} = 96.5$ mm

9.95 $t = 9.29$ s, $x = 477$ m ($t = 7.93$ s, $x = 407$ m for three parachutes) "g" = – 3.66

9.97 B is 20.8% better than A ($H > D$)

9.99 $\overline{C}_D = 0.299$

9.101 $V = 24.7/35.8$ km/hr New tires: $V = 26.8/32.6/39.1$ km/hr Plus fairing: $V = 29.8/35.7/42.1$ km/hr

9.105 $M = 0.0451$ kg

9.107 $V = \sqrt{\left[\frac{2mg\sin\theta}{C_D A\rho\cos^2\theta}\right]}$ $t = 1.30$ mm

9.109 $t = 2.95$ s $x = 624$ ft

9.111 $V = 47.3$ mph (1970's car) $V = 59.0$ (current car)

9.113 $FE = 6.72$ mpg $\Delta Q = 1720$ gal/yr (8.78%)

9.115 $C_D = 1.17$

9.117 $V = \sqrt{\left[\frac{2mg}{\rho A}\frac{1}{\left(\frac{A_1}{A_2}\right)^2 - 2\left(\frac{A_1}{A_2}\right) + 1}\right]}$

9.119 $F_D = C_D A\frac{1}{2}\rho(V-U)^2$ $T = C_D A\frac{1}{2}\rho(V-U)^2 R$ $P = C_D A\frac{1}{2}\rho(V-U)^2 U$ $\omega_{\text{opt}} = \frac{V}{3R}$

9.121 $M = 11.0$ N·m

9.123 $\mathscr{P} = 3.00$ kW

9.125 $V = 23.3$ m/s $Re = 48{,}200$ $F_D = 0.111$ N

9.127 $x = 13.9$ m

9.129 $C_D = 61.9$ $\rho_s = 3720$ kg/m^3 $V = 0.731$ m/s

9.131 $M = 0.0471$ kg

9.133 $C_L = 1.01$ $C_D = 0.0654$

9.135 $F_D = \frac{7}{9}C_D\frac{1}{2}\rho U^2 DH$ $M = \frac{7}{16}C_D\frac{1}{2}\rho U^2 DH^2$ $\frac{F_D}{F_{D_{\text{uniform}}}} = \frac{7}{9}$ $\frac{M}{M_{\text{uniform}}} = \frac{7}{8}$

9.137 $D = 7.99$ mm $y = 121$ mm

9.139 $t = 4.69$ s $x = 70.9$ m

9.141 $x_{\max} = 48.7$ m (both methods)

9.143 $C_D = 0.606$ $V = 37.4$ mph

9.145 $V_b = V_w - \sqrt{\frac{2F_R}{\rho(C_{Du}A_u + C_{Db}A_b)}}$ $V_b = 4.56$ m/s (16.4 km/hr)

9.147 $x \approx 203$ m

9.151 $\Delta\mathcal{P} = 16.3$ kW (94%)

9.157 $V_{\text{min}} = 5.62$ m/s (10.9 kt) $\mathcal{P}_{\text{min}} = 31.0$ kW $V_{\text{max}} = 19.9$ m/s (38.7 kt)

9.159 $V_{\text{min}} = 144$ m/s $R = 431$ m

9.161 $M = 37.9$ kg $\mathcal{P} = 1.53$ kW (or 3.02 kW if treated as two wings)

9.163 $T = 17{,}300$ lbf

9.165 $F_D = 524$ lbf $\mathcal{P} = 209$ hp

9.167 $\theta = 3.42^{\text{o}}$ $L = 168$ km

9.169 For a race car, effective; for a passenger car, not effective

9.175 $F_L = 0.00291$ lbf

9.177 $F_L = 50.9$ kN $F_D = 18.7$ kN $F = 54.2$ kN $\mathcal{P} = 5.94$ kW

9.179 $\omega = 14{,}000 – 17{,}000$ rpm $x = 3.90$ ft

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Fox, Pritchard, & McDonald

Answers to Selected Problems, Chapter 11

11.1 $Q = 3.18\ \text{m}^3/\text{s}$
11.3 $y = 2.61$ ft
11.5 $y_2 = 0.507$ m $Fr_2 = 2.51$
11.7 $S_0 = 0.00186$
11.9 $S_0 = 0.00160$
11.11 $Q = 0.194\ \text{m}^3/\text{s}$
11.13 $y = 2.47$ ft
11.15 $y = 0.775$ m
11.19 $y = 4.83$ ft $V = 3.69$ ft/s
11.23 $y = 7.38$ ft
11.25 $y_c = 0.365$ ft, $E_c = 0.547$ ft $y_c = 0.759$ ft, $E_c = 1.14$ ft
$y_c = 1.067$ ft, $E_c = 1.60$ ft $y_c = 1.46$ ft, $E_c = 2.19$ ft
11.27 $y_c = 0.637$ m
11.31 $\Delta x = 197$ ft
11.33 $y = 0.645$ ft $y = 4.30$ ft
11.35 $y_c = \left(\dfrac{2Q^2}{z^2 g}\right)^{\frac{1}{5}}$
11.37 $Q = 3.24\ \text{ft}^3/\text{s}$
11.39 $y_2 = 0.610$ ft (– 32.2%)
11.41 $y_2 = 1.31$ ft
11.43 $Q = 49.5\ \text{ft}^3/\text{s}$
11.45 $Q = 10.6\ \text{m}^3/\text{s}$ $y_c = 0.894$ m $H_l = 0.808$ m
11.47 $y_2 = 5.94$ ft
11.49 $Q = 54.0\ \text{ft}^3/\text{s}$ $H_l = 1.62$ ft
11.51 $y_2 = 4.45$ m $H_l = 9.31$ m
11.53 $Q = 26.6\ \text{ft}^3/\text{s}$
11.55 $H = 0.514$ m
11.57 $C_w = 1.45$

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Fox, Pritchard, & McDonald

Answers to Selected Problems, Chapter 12

12.1 T = const. p decreases ρ decreases (Irreversible adiabatic process)

12.3 $\Delta s > 0$ so it is feasible for a real (irreversible) adiabatic process

12.5 $T_2 = 20^{\circ}\text{F}$ $p_2 = 100$ kPa

12.7 $\Delta s = -346$ kJ/kg·K ($\Delta S = -1729$ J/K) $\Delta u = -143$ kJ/kg ($\Delta U = -717$ kJ)
$\Delta h = -201$ kJ/k ($\Delta H = -1004$ kJ)

12.9 $h = 57.5\%$

12.11 $W = 176$ MJ $W_s = 228$ MJ T_s (max) = 858 K $Q_s = -317$ MJ

12.13 $\dot{m} = 36.7$ kg/s $T_2 = 572$ K $V_2 = 4.75$ m/s $\dot{W} = 23$ MW

12.15 $\Delta t = 828$s (≈ 14 min)

12.17 $\Delta\rho = 1.70 \times 10^{-4}$kg/m^3 $\Delta T = 0.017$ K $\Delta V = 0.049$ m/s

12.19 $\Delta t = 198$ μs $E_v = 12.7$ GN/m^2

12.21 $x = 19.2$ km

12.23 $c = 299$ m/s $V = 987$ m/s $V/V_{\text{bullet}} = 1.41$

12.29 $c = 340$ m/s (sea level)

12.31 $V = 1471$ mph $\alpha = 31.8^{\circ}$

12.33 $V = 642$ m/s (2110 ft/s)

12.35 $V = 493$ m/s $\Delta t = 0.398$ s

12.37 $V = 515$ m/s $t = 6.92$ s

12.39 $\Delta x \approx 1043 - 1064$ m

12.41 Density change < 1.21%, so incompressible

12.43 $M = 0.142$ (1%) $M = 0.322$ (5%) $M = 0.464$ (10%)

12.45 $\Delta\rho/\rho = 48.5\%$ (Not incompressible)

12.47 $p_{\text{dyn}} = 54.3$ kPa $p_0 = 152$ kPa

12.49 $p_0 = 546$ kPa $h_0 - h = 178$ kJ/kg $T_0 = 466$ K

12.51 $p_0 - p = 8.67$ kPa $V = 195$ m/s $V = 205$ m/s Error using Bernoulli = 5.13%

12.55 T_0 = const (isoenergetic) p_0 decreases (irreversible adiabatic)

12.57 $V = 890$ m/s $T_0 = 677$ K $p_0 = 212$ kPa

12.59 T_0 = const = 294 K (20.6o) (isoenergetic) $p_{0_1} = 1.01$ MPa, $p_{0_2} = 189$ kPa (irreversible adiabatic)
$\Delta s = 480$ J/kg·K Flow accelerates even with friction due to large pressure drop

12.61 $T_{0_1} = T_{0_2} = 344$ K $p_{0_1} = 223$ kPa $p_{0_2} = 145$ kPa $\Delta s = 0.124$ kJ/kg·K

12.63 $T_{0_1} = T_{0_2} = 445$ K $p_{0_1} = 57.5$ kPa $p_{0_2} = 46.7$ kPa $\Delta s = 59.6$ J/kg·K

12.65 $\Delta p = 48.2$ kPa (inside higher)

12.67 $T^* = 260$ K $p^* = 24.7$ MPa $V^* = 252$ m/s

12.69 $T_t = 2730$ K $p_t = 25.5$ MPa $V_t = 1030$ m/s